Name: _____ Block: _____ Teacher: _____

Algebra 1

Unit 2A Notes: Reasoning with Linear Equations and Inequalities

DISCLAIMER: We will be using this note packet for Unit 2A. You will be responsible for bringing this packet to class EVERYDAY. If you lose it, you will have to print another one yourself. An electronic copy of this packet can be found on my class blog.

Standard	Lesson
MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions, ~~quadratic, simple rational, and exponential functions (integer inputs only).~~	
MGSE9-12.A.CED.2 Create linear, ~~quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.	
MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations.	
MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.	
MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. For example, given $ax + 3 = 7$, solve for x.	
MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.	
MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.	
MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation $f(x) = g(x)$ is the x-value where the y-values of $f(x)$ and $g(x)$ are the same.	
MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables	

Unit 2A:Equations & Inequalities

After completion of this unit, you will be able to…

Learning Target #1: Creating and Solving Linear Equations
- Solve one, two, and multi-step equations (variables on both sides)
- Justify the steps for solving a linear equation
- Create and solve an equation from a context

Learning Target #2: Creating and Solving Linear Inequalities
- Solve and graph a linear inequality
- Create and solve an inequality from a context

Learning Target #3: Isolating a Variable
- Solve a literal equation (multiple variables) for a specified variable
- Use a Formula to Solve Problems

Learning Target #4: Creating and Solving Systems of Equations
- Identify the solution to a system from a graph or table
- Graph systems of equations
- Determine solutions to a system of equations
- Use a graphing calculator to solve a system of equations
- Use substitution & elimination to solve a system of equations
- Determine the best method for solving a system of equations
- Apply systems to real world contexts

Learning Target #5: Creating and Solving Systems of Inequalities
- Graph linear inequalities & systems of linear inequalities
- Create a linear inequality or system of inequalities from a graph
- Determine the solution to a linear inequality or system of inequalities
- Determine if a given solution is a solution to an inequality or system of inequalities
- Apply inequalities to real world contexts

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August 19th	**20th**	**21st**	**22nd** Day 1 – Solve 1, 2 & Multi-Step Equations	**23rd** Day 2 – Solving Equations with Fractions & Justifying Solving
26th Day 3 – Graphing & Solving Inequalities	**27th** Day 4 – Creating Equations & Inequalities from a Context	**28th** Early Release Day Mixed Practice – Solving Equations & Inequalities	**29th** Day 5 – Solving For a Variable <mark>Quiz on Creating & Solving Equations and Inequalities</mark>	**30th** Day 6 – Graphing Systems of Equations
2nd <mark>No School – Labor Day</mark>	**3rd** Day 7 – Solving Systems of Equations by Substitution	**4th** Day 8 – Solving Systems of Equations by Elimination	**5th** Day 9 – Real World Applications Systems of Equations	**6th** Day 10 – More Real World Applications <mark>Systems of Equations Quiz</mark>
9th Day 11 – Graphing Linear Inequalities	**10th** Day 12 – Graphing Systems of Inequalities	**11th** Unit 2A Test Review	**12th** <mark>Unit 2A Test</mark>	**13th**

Day 1 – Solving One & Two Step Equations

Standard(s):

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. For example, given ax + 3 = 7, solve for x.

Expression:
- A mathematical "phrase" composed of terms, coefficients, and variables that stands for a single number, such as $3x + 1$ or $x^2 – 1$.
- We use Properties of Operations to _simplify_ algebraic expressions. Expressions do NOT contain equal signs.

An **Algebra Expression** does NOT have an = sign.

$$4n^2 + 7$$

Equation:
- A mathematical "sentence" that says two expressions are equal to each other such as $3x + 1 = 5$.
- We use Properties of Equality (inverse operations) to _solve_ algebraic equations.
- Equations contain equal signs.

An "Equation" does have an Equals sign.

$$4n^2 + 7 = 11$$

When solving equations, you must perform **inverse operations**, which means you have to perform the operation opposite of what you see. You must also remember the operation you perform on one side of the equation must be performed to the other side.

Informal		Formal		
Operation	**Inverse**	**Property**	**General Example**	**Example 1**
Addition		Addition Property of Equality	If $a = b$, then $a + c = b + c$	If $x – 4 = 8$, then $x = 12$
Subtraction		Subtraction Property of Equality	If $a = b$, then $a – c = b - c$	If $x + 5 = 7$, then $x = 2$
Multiplication		Multiplication Property of Equality	If $a = b$, then $ac = bc$	If $\frac{x}{2} = 9$, then $x = 18$
Division		Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$	If $2x = 10$, then $x = 5$

Additive Inverse	A number plus its inverse equals 0.	$a + -a = 0$	$7 + -7 = 0$
Multiplicative Inverse (Reciprocal)	A number times its reciprocal equals 1.	$a \cdot \frac{1}{a} = 1$	$3 \cdot \frac{1}{3} = 1$

Solving One Step Equations Practice

Practice: Solve each equation.

1. $x - 4 = 3$ Operation You See: _____   Inverse Operation: _____

2. $y + 4 = 3$ Operation You See: _____   Inverse Operation: _____

3. $\dfrac{s}{3} = 9$ Operation You See: _____   Inverse Operation: _____

4. $6p = 12$ Operation You See: _____   Inverse Operation: _____

Practice: Solve each equation on your own.

a. $x - 6 = 10$ b. $-5d = 25$ c. $8 + m = -4$

d. $\dfrac{x}{7} = 1$ e. $y - (-9) = 2$ f. $\dfrac{1}{3}x = 6$

Solving Two Step Equations

When solving equations with more than one step, you still want to think about how you can "undo" the operations you see.

Practice: Solve each equation, showing all steps, for each variable.

1. $3x - 4 = 14$ 2. $2x + 4 = 10$ 3. $7 - 3y = 22$

4. $0.5m - 1 = 8$ 5. $-6 + \dfrac{x}{4} = -5$ 6. $\dfrac{x-8}{4} = -5$

Solving Multi-Step Equations

Multi-step equations mean you might have to add, subtract, multiply, or divide all in one problem to isolate the variable. When solving multi-step equations, you are using inverse operations, which is like doing PEMDAS in reverse order.

Multi - Step Equations with Combining Like Terms

Practice: Solve each equation, showing all steps, for each variable.
a. $-5n + 6n + 15 - 3n = -3$ b. $3x + 12x - 20 = 25$ c. $-2x + 4x - 12 = 40$

Multi - Step Equations with the Distributive Property

Practice: Solve each equation, showing all steps, for each variable.
a. $2(n + 5) = -2$ b. $4(2x - 7) + 5 = -39$ c. $6x - (3x + 8) = 16$

Multi – Step Equations with Variables on Both Sides

Practice: Solve each equation, showing all steps, for each variable
a. $5p - 14 = 8p + 4$ b. $8x - 1 = 23 - 4x$ c. $5x + 34 = -2(1 - 7x)$

Day 2 - Equations with Fractions and Decimals

Standard(s):

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. For example, given ax + 3 = 7, solve for x.

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

When you solve equations with decimals, you solve them as if you would an equation without decimals.

1. 3.5x – 37.9 = .2x

2. 14.7 + 2.3x = 4.06

3. -1.6 – 0.9w = 11.6 + 2.4w

Equations with Fractions

When solving equations with fractions, you want to find a way to eliminate the fraction.

To eliminate the fraction, multiply by a Common Denominator

1. $\dfrac{-2}{3}m = 10$

2. $\dfrac{3x}{4} = 6$

3. $-\dfrac{3}{2}x - 1 = 8$

4. $\dfrac{2m}{3} + 5 = 12$

1. $w + \dfrac{1}{7} = \dfrac{6w}{7} - 1$

2. $\dfrac{x}{6} + \dfrac{2x}{3} = 5$

3. $\dfrac{x+3}{8} - \dfrac{x}{2} = 5$

Special Types of Solutions

Solve the following equations. What do you notice about the solutions?

a. $2x – 7 + 3x = 4x + 2$ 	 	 b. $3(x – 5) + 11 = x + 2(x + 5)$ 	 	 c. $3x + 7 = 5x + 2(3 – x) + 1$

Justifying the Solving of Equations

Properties of Addition Operations			
Property	**What It Means**	**General Example**	**Example 1**
Commutative Property of Addition	Rearrange the order and the sum will stay the same.	$a + b = b + a$	$2 + 4 = 4 + 2$
Associative Property of Addition	Change the order of the grouping and the sum will stay the same.	$(a + b) + c = a + (b + c)$	$(4 + 6) + 1 = 4 + (6 + 1)$
Additive Identity	Zero added to any number will equal that number.	$a + 0 = a$	$4 + 0 = 4$
Additive Inverse	A number plus its inverse equals 0.	$a + -a = 0$	$7 + -7 = 0$
Properties of Multiplication Operations			
Commutative Property of Multiplication	Rearrange the order and the product will stay the same.	$a \cdot b = b \cdot a$	$5 \cdot 2 = 2 \cdot 5$
Associative Property of Multiplication	Change the order of the grouping and the product will stay the same.	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(3 \cdot 4) \cdot 2 = 3 \cdot (4 \cdot 2)$
Multiplicative Identity	One times any number equals that number.	$a \cdot 1 = a$	$8 \cdot 1 = 8$
Multiplicative Inverse (Reciprocal)	A number times its reciprocal equals 1.	$a \cdot \frac{1}{a} = 1$	$3 \cdot \frac{1}{3} = 1$
Zero Property of Multiplication	Any number times 0 will always equal 0.	$a \cdot 0 = 0$	$7 \cdot 0 = 0$
Distributive Property	Multiply a number to every term within a quantity (parenthesis).	$a(b + c) = ab + ac$	$4(x + 5) = 4x + 4(5)$ $= 4x + 20$

Properties of Equality		
Property	**General Example**	**Example 1**
Addition Property	If $a = b$, then $a + c = b + c$	If $x - 4 = 8$, then $x = 12$
Subtraction Property	If $a = b$, then $a - c = b - c$	If $x + 5 = 7$, then $x = 2$
Multiplication Property	If $a = b$, then $ac = bc$	If $\frac{x}{2} = 9$, then $x = 18$
Division Property	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$	If $2x = 10$, then $x = 5$
Reflexive Property	$a = a$	$5 = 5$
Symmetric Property	If $a = b$, then $b = a$	If $2 = x$, then $x = 2$
Transitive Property	If $a = b$ and $b = c$, then $a = c$	If $x + 2 = y$ and $y = 4x + 3$, then $x + 2 = 4x + 3$
Substitution Property	If $x = y$, then y can be substituted for x in any expression	If $x = 3$ and the expression is $2x - 7$, then $2(3) - 7$

Justifying the Solutions to Two & Multi-Step Equations

Practice: Identify the property or simplification that is used in each step to solve the equation.

Example 1

3x + 5 = -13	**Given**
3x = -18	
x = -6	

Example 2

12 = 2(x – 4)	**Given**
12 = 2x – 8	
20 = 2x	
10 = x	
x = 10	

Example 3

5n – 3 = 2(n + 3) + 9	**Given**
5n – 3 = 2n + 6 + 9	
5n – 3 = 2n + 15	
3n – 3 = 15	
3n = 18	
n = 6	

Day 3 – Solving Inequalities

Standard(s):

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. For example, given ax + 3 = 7, solve for x.

An **inequality** is a statement that that compares two quantities. The quantities being compared use one of the following signs:

$A < B$	$A > B$	$A \le B$	$A \ge B$	$A \ne B$
A is less than B.	A is greater than B.	A is less than or equal to B.	A is greater than or equal to B.	A is not equal to B.
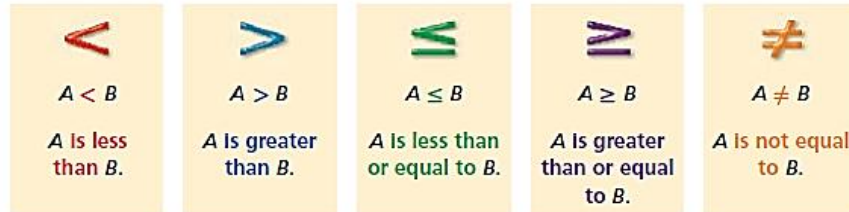
When reading an inequality, you always to want to read from the variable. Translate the inequalities into words:

A. x > 2 _____

B. -3 > p _____

C. y ≤ 0 _____

D. -2 ≤ z _____

E. x ≠ 1 _____

When graphing an inequality on a number line, you must pay attention to the sign of the inequality.

Words	Example	Circle	Number Line
Greater Than	x > 2	Open	-5 -4 -3 -2 -1 0 1 2 3 4 5
Less Than	p < -3	Open	-5 -4 -3 -2 -1 0 1 2 3 4 5
Greater Than or Equal To	z ≥ -2	Closed	-5 -4 -3 -2 -1 0 1 2 3 4 5
Less Than or Equal To	y ≤ 0	Closed	-5 -4 -3 -2 -1 0 1 2 3 4 5
Not Equal To	x ≠ 1	Open	-5 -4 -3 -2 -1 0 1 2 3 4 5

Solutions to Inequalities

A **solution** to an inequality is any number that makes the inequality true.

Value of x	$2x - 4 \geq -12$	Is the inequality true?
-2		
-4		
-6		

Solving and Graphing Linear Inequalities

Solving linear inequalities is very similar to solving equations, but there is one minor difference. See if you can figure it out below:

Experiment
Take the inequality 6 > 2. Is this true?

1. Add 3 to both sides. What is your new inequality?

2. Subtract 3 from both sides. What is your new inequality?

3. Multiply both sides by 3. What is your new inequality?

4. Divide both sides by 3. What is your new inequality?

5. Multiply both sides by -3. What is your new inequality?

6. Divide both sides by -3. What is your new inequality?

Conclusion:
"Golden Rule of Inequalities"

When you _____ or _____ an inequality by a _____ number, you MUST _____ the inequality.

Practice: Solve each inequality and graph on a number line.

1. $x - 4 < -2$

2. $-3x > 12$
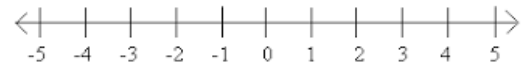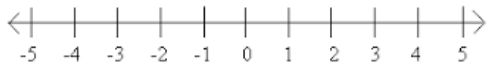
3. $7 \le \frac{1}{2}x$

4. $\frac{x}{4} - 1 > 9$
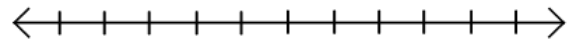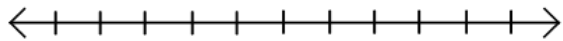
5. $-2(x + 1) \ge 6$

6. $6x - 5 \le 7 + 2x$
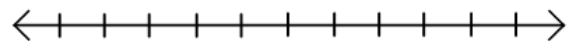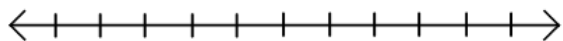
Day 4 - Creating Equations from a Context

Standard(s):

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

Include equations arising from linear functions, ~~quadratic, simple rational, and exponential functions.~~

Earlier in our unit, you learned to write expressions involving mathematical operations. You used the following table to help you decode those written expressions. We are going to use those same key words along with words that indicate an expression will become part of an equation or inequality.

Addition	Subtraction	Multiplication	Division	Equals
Sum	Difference	Of	Quotient	Is
Increased by	Decreased by	Product	Ratio of	Equals
More than	Minus	Times	Percent	Will be
Combined	Less	Multiplied by	Fraction of	Gives
Together	Less than	Double	Out of	Yields
Total of	Fewer than	Twice	Per	Costs
Added to	Withdraws	Triple	Divided by	
Gained				
Raised				
Plus				

When taking a word problem and translating it to an equation or inequality, it is important to "**talk to the text**" or underline/highlight key phrases or words. By doing this it helps you see what is occurring in the problem.

Practice Examples: *In the examples below, "talk to the text" as you translate your word problems into equations. Define a variable to represent an unknown quantity, create your equation, and then solve your equation.*

1. Six less than four times a number is 18. What is the number?

Variables: _____

Equation: _____

2. You and three friends divide the proceeds of a garage sale equally. The garage sale earned $412. How much money did each friend receive?

Variables: _____

Equation: _____

3. On her iPod, Mia has rock songs and dance songs. She currently has 14 rock songs. She has 48 songs in all. How many dance songs does she have?

Variables: _____

Equation: _____

4. Brianna has saved $600 to buy a new TV. If the TV she wants costs $1800 and she saves $20 a week, how many months will it take her to buy the TV (4 weeks = 1 month)?

Variables: _____

Equation: _____

5. It costs Raquel $5 in tolls to drive to work and back each day, plus she uses 3 gallons of gas. It costs her a total of $15.50 to drive to work and back each day. How much per gallon is Raquel paying for her gas?

Variables: _____

Equation: _____

6. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width (Hint: p = 2w + 2l).

Variables: _____

Equation: _____

Day 4 - Creating Inequalities from a Context

Standard(s):

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions~~, quadratic, simple rational, and exponential functions (integer inputs only).~~

When creating problems that involve inequalities, you will use the same methods as creating equations, except you have new keywords that will replace the equal sign with an inequality sign.

<	≤	>	≥
Less than	Less than or equal to	Greater than	Greater than or equal to
Fewer than	At most	More than	At least
	Maximum		Minimum
	No more than		No less than

Examples: Define a variable for the unknown quantity, create an inequality, and then solve.

1. One half of a number decreased by 3 is no more than 33.

Variables: _____

Inequality: _____

2. Alexis is saving to buy a laptop that costs $1,100. So far she has saved $400. She makes $12 an hour babysitting. What's the least number of hours she needs to work in order to reach her goal?

Variables: _____

Inequality: _____

3. Keith has $500 in a savings account at the bank at the beginning of the summer. He wants to have at least $200 in the account by the end of the summer. He withdraws $25 each week for food, clothes, and movie tickets. How many weeks can Keith withdraw money from his account?

Variables: _____

Inequality: _____

Day 5 – Isolating a Variable

Standard(s): MGSE9-12.A.CED.4

Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations.

Isolating a variable simply means to solve for that variable or get the variable "by itself" on one side of the equal sign (usually on the left).   Sometimes we may have more than one variable in our equations; these types of equations are called **literal equations**. We solve literal equations the same way we solve "regular" equations.

Steps for Isolating Variables

1. Locate the variable you are trying to isolate.
2. Follow the rules for solving equations to get that variable by itself.

Solving an Equation You're Familiar with	Solving a Literal Equation
$2x = 10$	$gh = m$ solve for h
$2x + 5 = 11$	$ax + b = c$ solve for x

Practice:

1. Solve the equation for b: $a = bh$

2. Solve the equation for b: $y = mx + b$

3. Solve the equation for x: $2x + 4y = 10$

4. Solve the equation for m: $y = mx + b$

5. Solve the equation for w: $p = 2l + 2w$

6. Solve the equation for a: $\frac{a}{2} - 1 = b$

Your Turn:

7. Solve the equation for y: $6x - 3y = 15$

8. Solve the equation for h: $V = \frac{1}{3}Bh$

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1. You are visiting a foreign county over the weekend. The forecast is predicted to be 30 degrees Celsius. Are you going to pack warm or cold clothes? Use Celsius = $\frac{5}{9}(F - 32)$.

2. The area of a triangle is given by the formula A = ½bh or $A = \frac{bh}{2}$, where b is the base and h is the height.

      a. Use the formula given to find the height of the triangle that has a base of 5 cm and an area of 50 cm.

      b. Solve the formula for the height.

Day 6 – Graphing Systems of Equations

Standard(s):

MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.
MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Graphing a Line in Slope-Intercept Form

When we write an equation of a line, we use **slope intercept form** which is $y = mx + b$, where **m** represents the **slope** and **b** represents the **y-intercept**.

> ### Slope Intercept Form
> $$y = mx + b$$
> m: slope   b: y=intercept

Slope can be described in several ways:	**Y-intercept**
- Steepness of a line - Rate of change – rate of increase or decrease $\dfrac{\text{Rise}}{\text{Run}}$ - Change (difference) in y over change (difference) in x	- The point where the graph crosses the _____ - Its coordinate will always be the point (0, b) *y-intercept*
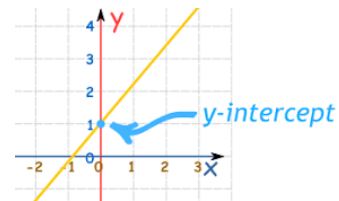
Graphing Linear Functions

When you graph equations, you must be able to identify the slope and y-intercept from the equation.
Step 1: Solve for y (if necessary)
Step 2. Plot the y-intercept
Step 3: From the y-intercept, use the slope to calculate another point on the graph.
Step 4: Connect the points with a ruler or straightedge.

$$\textbf{Slope} = \frac{change\ in\ y}{change\ in\ x} = \frac{+\uparrow \quad -\downarrow}{+\rightarrow \quad -\leftarrow}$$

Ex. Graph the following lines:

A. $y = -\frac{2}{3}x + 4$ m = _____   b = _____

-3x + y = 2   m = _____   b = _____

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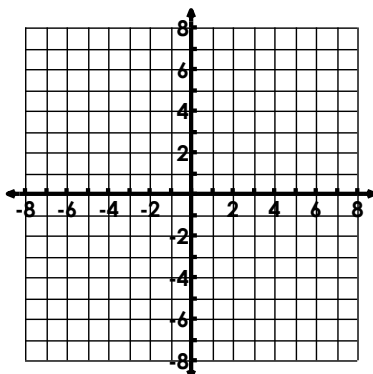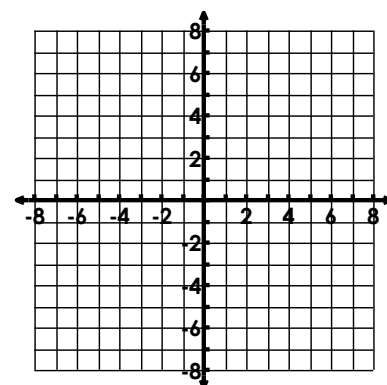
Graphing Horizontal and Vertical Lines

Ex. $y = -4$

H
O
Y

Ex. $x = -4$

V
U
X
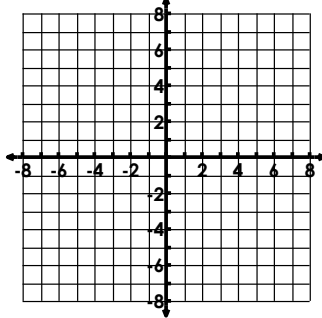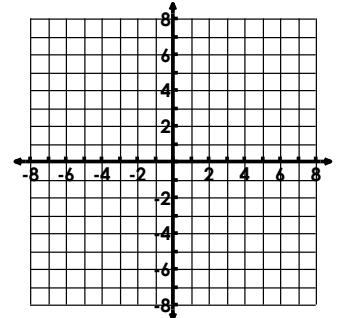
Solving Systems of Equations by Graphing

Two or more linear equations in the same variable form a **system of equations**.
Example:

Solution to a System of Equations
- An ordered pair (x, y) that makes each equation in the system a _____ statement
- The point where the two equations _____ each other on a graph.
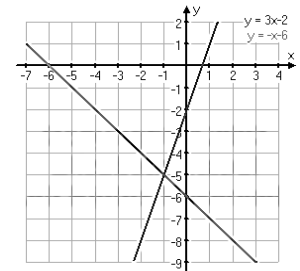
$y = 3x-2$
$y = -x-6$

Examples: Check whether the ordered pair is a solution of the system of linear equations.

Ex. (1, 1)

$2x + y = 3$
$x - 2y = -1$

Ex. (-2, 4)

$4x + y = -4$
$-x - y = 1$

Identify Solutions to a System from a Table

The solution to a system of equations is where the two lines intersect each other.
The solution is where the x-value (input) produces the same y-value (output) for both equations.

Using the tables below, identify the solution.

a.

x	$y = -x$	$y = x - 6$
0	0	−6
3	−3	−3
6	−6	0
9	−9	3

b.

x	$y = 2x + 4$	$y = 4x + 2$
−2	0	−6
−1	2	−2
0	4	2
1	6	6

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Practice: Tell how many solutions the systems of equations has. If it has one solution, name the solution.
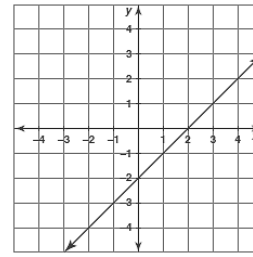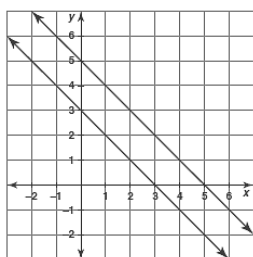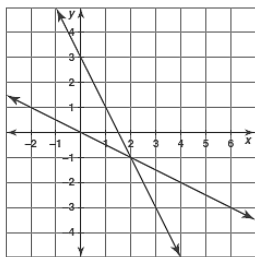
Solving a Linear System by Graphing

Step 1: Write each equation in slope intercept form (y = mx + b).

Step 2: Graph both equations in the same coordinate plane.

Step 3: Estimate the coordinates of the point of intersection.

Step 4: Check whether the coordinates give a true solution (Substitute into each equation)

Example: Use the graph and check method to solve the linear equations.

A. $y = x - 2$ $y = -x + 4$
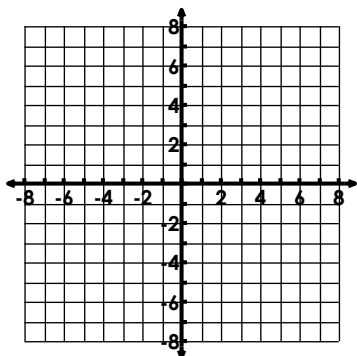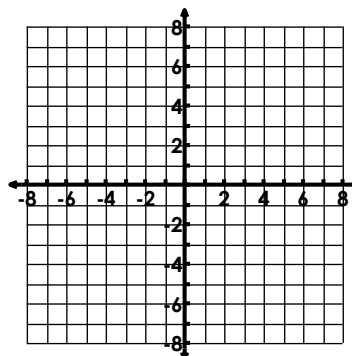
      m= _____   m= _____

      b = _____   b = _____

Solution: _____

B. $y = -\frac{1}{2}x - 1$ $y = \frac{1}{4}x - 4$

      m= _____   m= _____

      b = _____   b = _____

Solution: _____

C.   3x + y = 6   -x + y = -2

      m= _____   m= _____

      b = _____   b = _____

Solution: _____

D.   y = - 2   4x – 3y = 18

      m= _____   m= _____
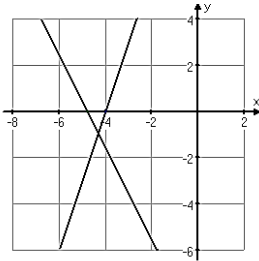
      b = _____   b = _____

Solution: _____

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Day 7 – Solving Systems Using Substitution

Standard(s):

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Name the solution of the systems of equations below:

Were you able to figure out an exact solution???
- Unless a solution to a system of equations are integer coordinate points, it can be very hard to determine the solution.
- Therefore, we need algebraic methods that allow us to find exact solutions to Systems of Equations.
- We will learn two methods: **Substitution** and **Elimination**

Think About It

How would you find the x and y values for the following systems (i.e a point or solution to the systems)?

a. $-4x + 2y = 24$
$\quad\quad y = 8$

b. $x = 1$
$\quad\quad -2x + 8y = 14$

Steps for Solving a System by Substitution

Example:

$y = x + 1$

$2x + y = -2$

Step 1: Select the equation that already has a variable isolated.	**Step 2**: Substitute the expression from Step 1 into the other equation for the variable you isolated in step 1 and solve for the other variable.	**Step 3**: Substitute the value from Step 2 into the revised equation from Step 1 & solve for the other variable. Create an ordered pair (x, y).	**Step 4**: Check the solution in each of the original equations.

Example 1: Solve the system below:

$2x + 2y = 3$

$x = 4y - 1$

Solution:

Example 2: Solve the system below:

$y = x + 1$

$y = -2x + 4$

Solution:

Example 3: Solve the system below:

$x = 3 - y$
$x + y = 7$

Solution:

Example 4: Solve the system below:

$y = -2x + 4$

$4x + 2y = 8$

Solution:

When the variables drop out and the resulting equation is **FALSE**, the answer is **NO SOLUTIONS.**

When the variables drop out and the resulting equation is **TRUE**, the answer is **INFINITE SOLUTIONS.**

Day 8 – Solving Systems Using Elimination

Standard(s):

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

- Another method for solving systems of equations when one of the variables is not isolated by a variable is to use **elimination.**

- Elimination involves adding or multiplying one or both equations until one of the variables can be eliminated by adding the two equations together.

Steps for Solving Systems by Elimination

Step 1: Arrange the equations with like terms in columns.

Step 2: Analyze the coefficients of x or y. Multiply one or both equations by an appropriate number to obtain new coefficients that are opposites

Step 3: Add the equations and solve for the remaining variable.

Step 4: Substitute the value into either equation and solve.

Elimination by Adding the Systems Together

Ex 1. $-2x + y = -7$

$2x - 2y = 8$

Ex 2. $4x - 2y = 2$

$3x + 2y = 12$

Solution: **Solution:**

Elimination by Rearranging and Adding the Systems Together

Ex 3. $8x = -16 - y$

       $3x - y = 5$

Ex 4. $2x + y = 8$

       $-y = 3 + 2x$

Solution:

Solution:

Elimination by Multiplying the Equations and Then Adding the Equations Together

Ex 5. $x + 12y = -15$

       $-2x - 6y = -6$

Ex 6. $6x + 8y = 12$

       $2x - 5y = -19$
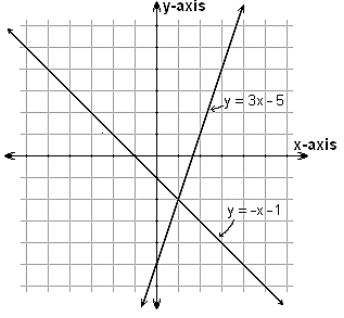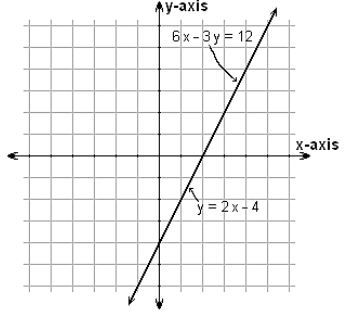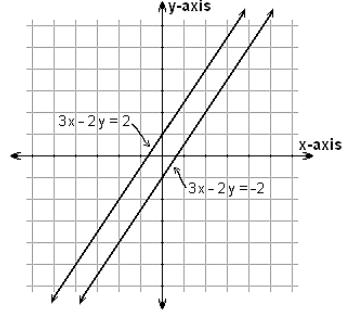
Solution:

Solution:

Elimination by Multiplying Both Equations by a Constant and then Adding

a. $5x - 4y = -1$
     $8x + 7y = -15$

b. $-6x + 12y = -6$
     $-5x + 10y = -5$

Solution: Solution:

		Number of Solutions		
		One Solution	**Infinitely Many Solutions**	**No Solution**
Solving Methods	**Graphing**	When graphed, the 2 lines **intersect once**.	When graphed, the 2 lines **lie on top of one another**.	When graphed, the 2 lines are strictly **parallel**.
	Substitution	When using either substitution or elimination, you should get a value for either x or y. You should be able to find the other value by substituting either x or y back into the original equation.	When using either substitution or elimination, you will get an equation that has **no variable** and is **always true**. For example: 2=2 or -5=-5	When using either substitution or elimination, you will get an equation that has **no variable** and is **never true**. For example: 0=6 or -2=4
	Elimination			

Day 9 – Real World Applications of Systems

Standard(s):

MGSE9-12.A.CED.2 Create linear, ~~quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

Problem Solving with Substitution

Example 1: Loren's marble jar contains plain marbles and colored marbles. If there are 32 more plain marbles than colored marbles, and there are 180 marbles total, how many of each kind of marble does she have?

a. Define your variables (what two things are you comparing?)

b. Create two equations to describe the scenario.

    Equation 1: _____ (relationship between plain and colored marbles)

    Equation 2: _____ (number of marbles total)

c. Solve the system:

Example 2: A bride to be had already finished assembling 16 wedding favors when the maid of honor came into the room for help. The bride assembles at a rate of 2 favors per minute. In contrast, the maid of honor works at a speed of 3 favors per minute. Eventually, they will both have assembled the same number of favors. How many favors will each have made? How long did it take?

a. Define your variables (what two things are you comparing?)

b. Create two equations to describe the scenario.

    Equation 1: _____ (bride's rate)

    Equation 2: _____ (maid of honor's rate)

c. Solve the system:

Problem Solving with Elimination

Example 3: Love Street is having a sale on jewelry and hair accessories. You can buy 5 pieces of jewelry and 8 hair accessories for 34.50 or 2 pieces of jewelry and 16 hair accessories for $33.00. This can be modeled by the

equations: $\begin{cases} 5x + 8y = 34.50 \\ 2x + 16y = 33.00 \end{cases}$. How much is each piece of jewelry and hair accessories?
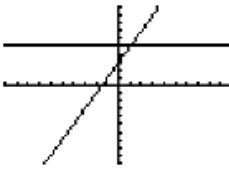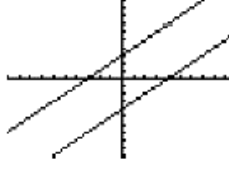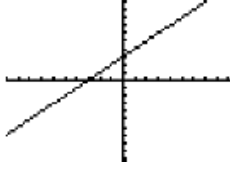
a. What does x and y represent?   d. Solve the system of equations:

b. Explain what the first equation represents:

c. Explain what the second equation represents:

Example 4: A test has twenty questions worth 100 points. The test consists of True/False questions worth 3 points

each and multiple choice questions worth 11 points each. This can be modeled by $\begin{cases} x + y = 20 \\ 3x + 11y = 100 \end{cases}$. How

many multiple choice and True/False questions are on the test?

a. What does x and y represent?   d. Solve the system of equations:

b. Explain what the first equation represents:

c. Explain what the second equation represents:

How Many Solutions to the System?

	Method	One Solution	No Solutions	Infinite Solutions
Graphing	**Best to use when:** Both equations are in slope intercept form. $(y = mx + b)$ EX: $y = 3x - 1$ $y = -x + 4$ Solutions are integer coordinate points (no decimals or fractions)	Solution is the point of intersection. Different Slope Different y-intercept	Lines are parallel and do not intersect. (Slopes are equal) Same Slope Different y-intercept	Lines are identical and intersect at every point. Same Slope Same y-intercept (Same Equations)
Substitution	**Best to use when:** One equation has been solved for a variable or both equations are solved for the same variable. EX: $y = 2x + 1$ or $y = 3x - 1$ $3x - 2y = 10$ $y = -x + 4$	After substituting and simplifying, you will be left with: $x = \#$ $y = \#$ Solution will take the form of (x, y)	After substituting, variables will form **zero pairs** and you will be left with a **FALSE** equation. $3 = 6$	After substituting, variables will form **zero pairs** and will leave you with a **TRUE** equation. $4 = 4$
Elimination	**Best to use when:** Both equations are in standard form. $(Ax + By = C)$ Coefficients of variables are opposites. $3x + 6y = 5$ $-3x - 8y = 2$ Equations can be easily made into opposites using multiplication. $-2(4x + 2y = 5)$ $8x - 6y = -5$	After eliminating and simplifying, you will be left with: $x = \#$ $y = \#$ Solution will take the form of (x, y)	After eliminating, variables will form **zero pairs** and you will be left with a **FALSE** equation. $0 = 5$	After eliminating, variables will form **zero pairs** and will leave you with a **TRUE** equation. $0 = 0$

Day 10 – Real World Applications of Systems (More Practice)

Standard(s):
MGSE9-12.A.CED.2 Create linear, ~~quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

Scenario 1: The admission fee for the county fair includes parking, amusement rides, and admission to all commercial, agricultural, and judging exhibits. The cost for general admission is $7 and the price for children is $4. There were 449 people who attended the fair on Thursday. The admission fees collected amounted to $2768. How many children and adults attended the fair?

Scenario 2: Ms. Ross told her class that tomorrow's math test will have 20 questions and be worth 100 points. The multiple-choice questions will be 3 points each and the open-ended response questions will be 8 points each. Determine how many multiple-choice and open-ended response questions are on the test.

Scenario 3: The Strauss family is deciding between two lawn care services. Green Lawn charges a $49 startup fee, plus $29 per month. Grass Team charges a $25 startup fee, plus $37 per month.

   a. In how many months will both lawn care services costs the same? What will that cost be?

   b. If the family will use the service for only 6 months, which is the better option? Explain.

Scenario 4: The following graph shows the cost for going to two different skating rinks.
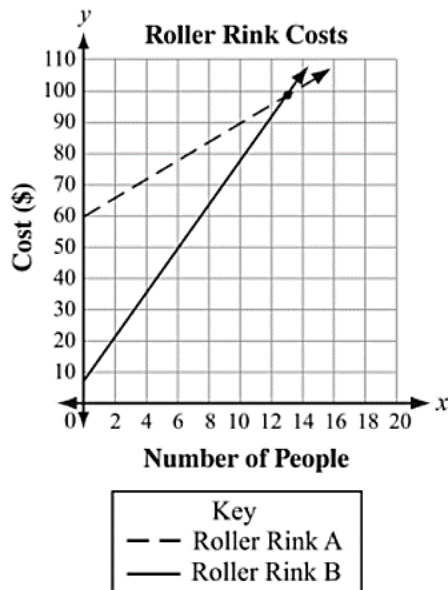
a. When is it cheaper to go to Roller Rink A?

b. When it is cheaper to go to Roller Rink B?

c. When does it cost the same to go to either roller rink?

Profits, Costs, and Break-Even Points

Production Cost
- The cost incurred when manufacturing a good or providing a service

Income
- Money earned from the sale of goods or services

Profit
- Production Cost – Income
- When income is higher than production costs, you make a _____
- When income is lower than production costs, you make a _____

Break Even-Point
- The point where **Production Costs = Income**
- Can be found by finding the intersection of the two lines (x, y)
- x-coordinate represents how many of an item you need to make and sell to break-even
- y-coordinate represents how much the company spent making the item and then selling the item

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Practice 1: Find the break-even point for the following graphs. How many of each item will the company need to sell to make a profit?

Point of Intersection: _____

Break Even Point:

They need to sell at least _____
basketballs to make a profit.
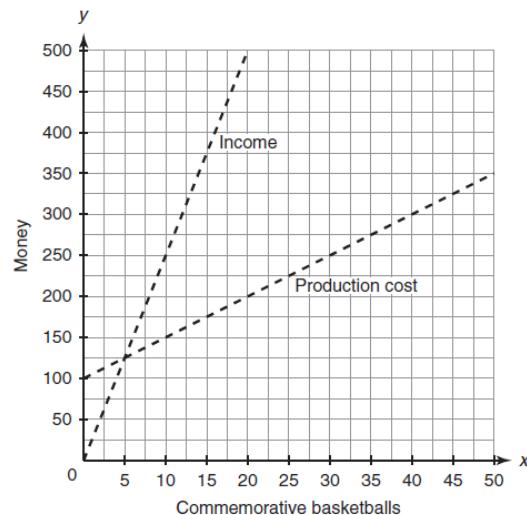
Practice 2: The cost to take pictures at a school dance is $200 for the photographer and $3 per print. The dance committee decides to charge $5 per print. How many pictures need to be taken for the dance committee to break-even? How many pictures need to be taken to make a profit?

Day 11 – Graphing Linear Inequalities

Standard(s):

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Linear Inequalities

- A **linear inequality** is like an equation, but the equal sign is replaced with an _____ symbol.

- A **solution** to an inequality is any ordered pair that makes the inequality _____.

Examples: Tell whether the ordered pair is a solution to the inequality.

(7, 3); $y < 2x - 3$ (4, 5); $y < x + 1$ (4, 5); $y \le x + 1$

A linear inequality describes a region of a coordinate plane called a **half-plane**. All the points in the shaded region are solutions of the linear inequality. The **boundary line** is the line of the equation you graph.

half plane
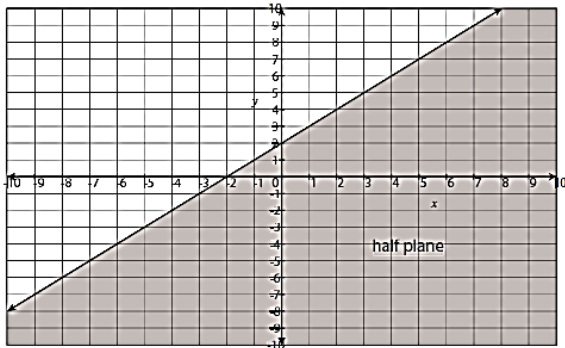
Symbol	Type of Line	Shading
<	Dashed	Below boundary line
>	Dashed	Above boundary line
≤	Solid	Below boundary line
≥	Solid	Above boundary line

Graphing Linear Inequalities

Step 1: Solve the inequality for y (if necessary).

Step 2: Graph the boundary line using a solid line for ≤ or ≥ OR a dashed line for < or >.

Step 3:
If the inequality is > or ≥, shade **above** the boundary line
If the inequality is < or ≤, shade **below** the boundary line

OR

Select a test point and substitute it into linear inequality.
- If the test point gives you a **true** inequality, you shade the region where the test point is located.
- If the test point gives you a **false** inequality, you shade the region where the test point is NOT located.
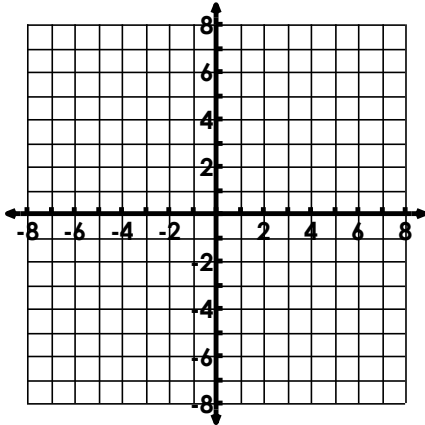
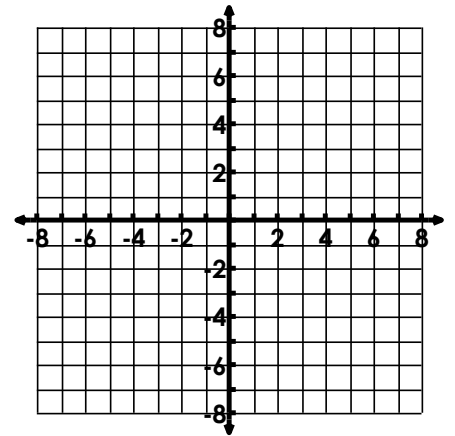## Practice Graphing Linear Inequalities

a. $y < 3x + 4$

Type of Line: Solid OR Dashed

Shade: Above OR Below

Slope: _____　　　　**Y-Int:** _____

Two Solutions: _____ _____

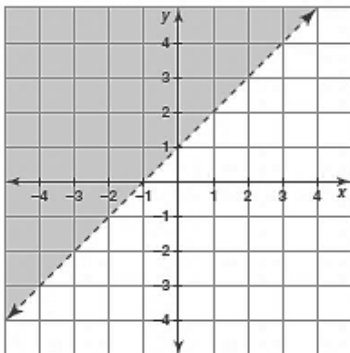**Two Non-Solutions:** _____ _____

Test Point:

b. $4x - 3y \leq 12$

Type of Line: Solid OR Dashed

Shade: Above OR Below

Slope: _____　　　　**Y-Int:** _____

Two Solutions: _____ _____

Two Non-Solutions: _____ _____

Test Point:

Naming Linear Inequalities

What information do you need to look at to name a linear inequality from a graph?

- _____
- _____

- _____
- _____
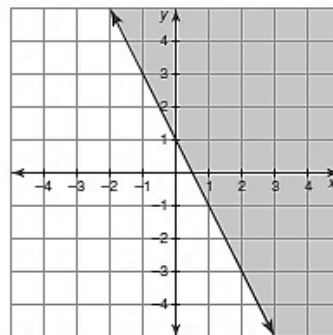
**Practice:** Name each linear inequality from the graph:

a.

Inequality: _____

b.

Inequality: _____

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Problem Solving with Linear Inequalities

Noah plays football. His team's goal is to score at least 24 points per game. A touchdown is worth 6 points and a field goal is worth 3 points. Noah's league does not allow the teams to try for the extra point after a touchdown. The inequality $6x + 3y \geq 24$ represents the possible ways Noah's team could score points to reach their goal.

a. Graph the inequality on the graph.
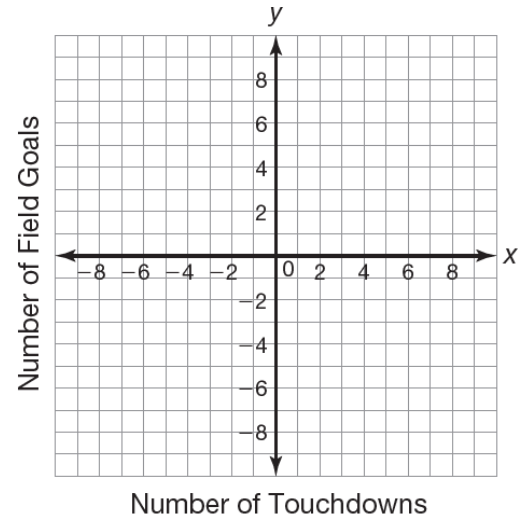
b. Are the following combinations solutions to the problem situation? Use your graph AND algebra to answer the following:

   i. 2 touchdowns and 1 field goal

   ii. 1 touchdown and 5 field goals

   iii. 3 touchdowns and 3 field goals

Day 12 - Graphing Systems of Inequalities

Standard(s):
MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

- The **solution of a system of linear inequalities** is the intersection of the solution to each inequality.
- Every point in the intersection regions satisfies the solution.

Determine if the following points are a solution to the inequality:

$$x + 5y < -1$$
$$2y \geq -3x - 2$$

(0, -1)   (2, 3)

Graphing Systems of Inequalities in Slope Intercept Form

Steps for Graphing Systems of Inequalities

Step 1: Graph the boundary lines of each inequality. Use dashed lines if the inequality is < or >. Use a solid line if the inequality is ≤ or ≥.

Step 2: Shade the appropriate half plane for each inequality.

Step 3: Identify the solution of the system of inequalities as the intersection of the half planes from Step 2.
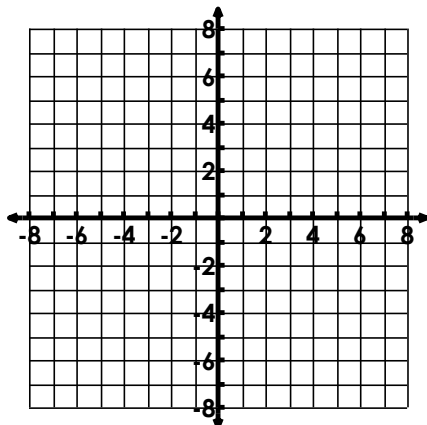
A.   y < -2x - 3
         y ≤ ½x + 2

B.   y < 3
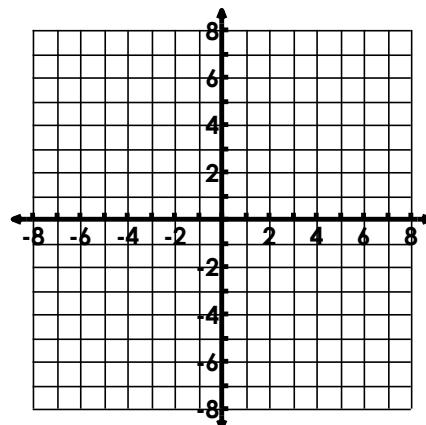   x > 1

Type of Line:

Shade:

Slope:

Y-Int:

Type of Line:

Shade:

Slope:

Y-Int:

Type of Line:

Shade:

Slope:

Y-Int:

Type of Line:

Shade:

Slope:

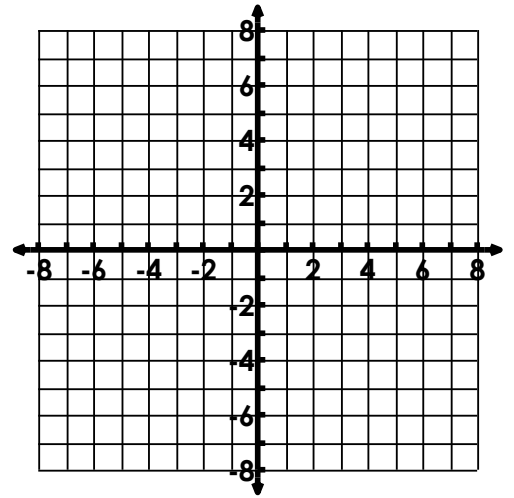**Y-Int:**

Two Solutions:

Two Non-Solutions:

Two Solutions:

Two Non-Solutions:

Graphing a System of Inequalities in Standard Form

Think Back…What is the "Golden Rule" of inequalities?

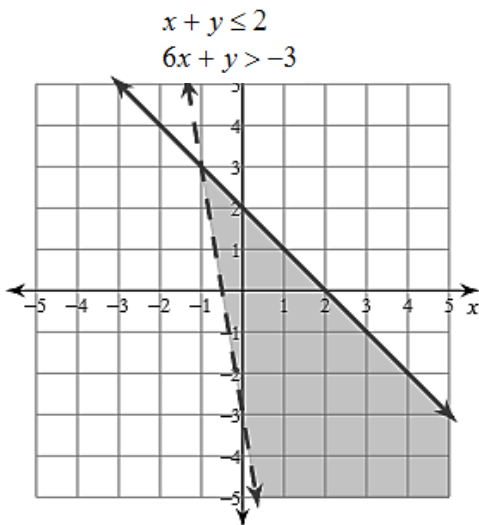C. $x + 3y \leq -9$ $5x - 3y \geq -9$

| Type of Line: |
| Shade: |
| Slope: |
| Y-Int: |

| Type of Line: |
| Shade: |
| Slope: |
| Y-Int: |

| Two Solutions: |
| Two Non-Solutions: |

Warning…Potential Misconception!!!

$x + y \leq 2$
$6x + y > -3$

Do you think the point (-1, 3) is a solution to the inequality?

Determining Solutions Located on a Boundary Line

If a point lies on a **solid** line, it is _____.

If a point lies on a **dashed** line, it is _____.

It must be true or a solution for both inequalities/boundary lines to be a solution!

Create a System of Inequalities from a Graph

What information do you need to look at to name a system of inequalities from a graph?

- _____
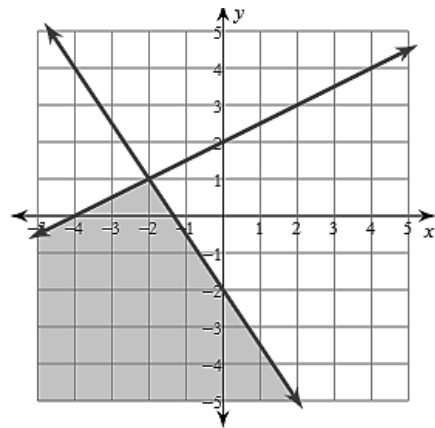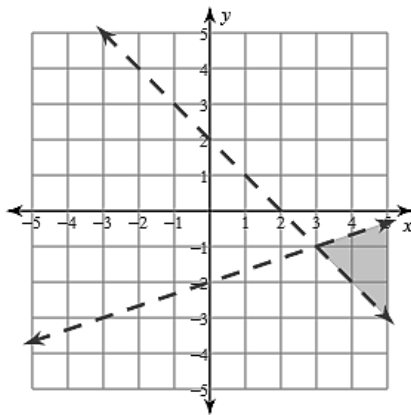
- _____

- _____

- _____

Practice: Name each system of inequalities from the graph:

Line 1: _____   Line 1: _____

Line 2: _____   Line 2: _____

Creating Systems of Inequalities

Write a system of inequalities to describe each scenario.

a. Jamal runs the bouncy house a festival. The bouncy house can hold a maximum of 1200 pounds at one time. He estimates that adults weight approximately 200 pounds and children under 16 weight approximately 100 pounds. For 1 four minute session of bounce time, Jamal charges adults $3 each and children $2 each. Jamal hopes to make at least $18 for each session.

- Define your variables:

- Write a system of inequalities

Inequality 1: _____ describes _____

Inequality 2: _____ describes _____

- Solve the system of inequalities

 a. If 4 adults and 5 children are in 1 session, will that be a solution to the inequalities?

 b. If 2 adults and 7 children are in 1 session, will that be a solution to the inequalities?

---

b. Charles works at a movie theater selling tickets. The theater has 300 seats and charges $7.50 for adults and $5.50 for children. The theater expects to make at least $1500 for each showing.

- Define your variables:

- Write a system of inequalities

Inequality 1: _____ describes _____

Inequality 2: _____ describes _____

- Solve the system of inequalities:

 a. If 150 adults and 180 children attend, will that be a solution to the inequalities?

 b. If 175 adults and 105 children attend, will that be a solution to the inequalities?