

Name:
Period:

Algebra 1

Unit 8: Quadratic Expressions (Polynomials)

Note Packet

Date		Topic/Assignment	HW Page	Due Date
	8-A	Naming Polynomials and Combining Like Terms		
	8-B	Adding and Subtracting Polynomials		
	8-C	Multiplying Polynomials		
	8-D	All Operations with Polynomials		
	8-E	Multiplying Special Products of Polynomials		
	8-F	Factoring out the GCF & Factor by Grouping		
	8-G	Factoring Quadratic Trinomials		
	8-H	Factoring Perfect Square Trinomials and Difference of Squares Binomials		
	8-I	Factoring Word Problems		
		Review		
		Test		
		Quizzes will be “pop”...2 or 3 per unit. You may use this packet to complete the quizzes.		

Warm-Up Date:

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8-A: Naming Polynomials and Combining Like Terms

Polynomial	Term	Coefficient
Monomial		
Binomial	Degree of Term	Degree of Polynomial
Trinomial		

POLYNOMIALS

<i>Monomials</i>	<i>Binomials</i>	<i>Trinomials</i>	<i>More than Three Terms</i>
ax^2	$x + y$	$x^2 + 4xy + y^2$	$5x^3 - 6x^2 + 3x - 6$
$-3z$	$3p + 2$	$x^5 + 7x^2 - x$	$-y^5 + y^4 - 3y^3 - y^2 + y$
4	$4x^2 - 7$	$-q^4 + q^3 - 2q$	$x^6 + x^4 - x^3 + 1$

<i>Term</i>	<i>Degree</i>	<i>Polynomial</i>	<i>Degree</i>
$3x^4$	4	$3x^4 - 5x^2 + 6$	4
$5x$, or $5x^1$	1	$5x + 7$	1
-7 , or $-7x^0$	0	$x^2y + xy - 5y^2$	3
$2x^2y$, or $2x^2y^1$	$2 + 1 = 3$	$x^5 + 3x^6$	6

EXAMPLE 1 Find the degree of each term.

- a. $3x^2$ b. -2^3x^5 c. y d. $12x^2yz^3$ e. 5

PRACTICE

1 Find the degree of each term.

- a. $5y^3$ b. $10xy$ c. z d. $-3a^2b^5c$ e. 8

EXAMPLE 2 Find the degree of each polynomial and tell whether the polynomial is a monomial, binomial, trinomial, or none of these.

- a. $-2t^2 + 3t + 6$ b. $15x - 10$ c. $7x + 3x^3 + 2x^2 - 1$

PRACTICE

2 Find the degree of each polynomial and tell whether the polynomial is a monomial, binomial, trinomial, or none of these.

- a. $5b^2 - 3b + 7$ b. $7t + 3$
c. $5x^2 + 3x - 6x^3 + 4$

EXAMPLE 3 Complete the table for the polynomial

$$7x^2y - 6xy + x^2 - 3y + 7$$

Use the table to give the degree of the polynomial.

<i>Term</i>	<i>Numerical Coefficient</i>	<i>Degree of Term</i>

The degree of the polynomial is:_____.

PRACTICE

3

Complete the table for the polynomial $-3x^3y^2 + 4xy^2 - y^2 + 3x - 2$.

<i>Term</i>	<i>Numerical Coefficient</i>	<i>Degree of Term</i>
$-3x^3y^2$		
$4xy^2$		
$-y^2$		
$3x$		
-2		

The degree of the polynomial is:_____.

Simplifying by Combining Like Terms

<i>Like Terms</i>	<i>Unlike Terms</i>

Example 4

Simplify by combining like terms.

(a) $-4x^3 + 6x^3$

(b) $9x^6 - 14x^6 + x^6$

(c) $12m^2 + 5m + 4m^2$

(d) $3x^2y + 4x^2y - x^2y$

Practice

a) $3x^2 - x^2 + 2x$

b) $3ab^2 - 5ab - 7ab^2 + ab^2$

Example 5

Simplify.

a) $3x^2 + (2x^2 - 5x)$

b) $(-3x^2 + x) - 7x^2 - 5x$

c) $(-2x^2 + 3x + 4) + (5x^2 - 6x - 1)$

Practice

a) $(2x^2 - 4x) + 7x^2$

b) $(4x^2 - 5x) - 10x^2 - 8x$

c) $(2x^2 - 5x - 7) + (-3x^2 + 9x - 2)$

Example 6

Simplify by distributing the negative and combining like terms where possible.

a) $-(2x^2 - 7x + 8)$

b) $-8x^2 - (3x^2 - 5x)$

c) $-(5x^2 - 6x + 10) - 5x$

Example 7

a) $-(-3x^2 + 7x - 5)$

b) $4x^2 - (-7x^2 + 3x)$

c) $-(3x^2 + 5x - 12) - 7x^2$

8-B: Adding and Subtracting Polynomials

Example 1

Add vertically.

(a) Add $6x^3 - 4x^2 + 3$ and $-2x^3 + 7x^2 - 5$.

(b) Add $2x^2 - 4x + 3$ and $x^3 + 5x$.

Practice

Add $4y^3 - 2y^2 + y - 1$ and $y^3 - y - 7$ vertically.

Example 2

Add horizontally.

(a) Add $6x^3 - 4x^2 + 3$ and $-2x^3 + 7x^2 - 5$.

(b) Add $2x^2 - 4x + 3$ and $x^3 + 5x$.

Practice

Add $10x^4 - 3x^2 - x$ and $x^4 - 3x^2 + 5x$ horizontally.

Example 3

Subtract horizontally.

(a) Perform the subtraction $(5x - 2) - (3x - 8)$.

(b) Subtract $6x^3 - 4x^2 + 2$ from $11x^3 + 2x^2 - 8$.

Practice

Subtract $5t^4 - 3t^2 + 1$ from $4t^4 - t^2 + 7$.

Example 4

Subtract vertically (by columns).

$$(14y^3 - 6y^2 + 2y - 5) - (2y^3 - 7y^2 - 4y + 6).$$

Practice

Subtract by columns.

$$\begin{array}{r} (12x^2 - 9x + 4) \\ - (-10x^2 - 3x + 7) \end{array}$$

Example 5

Add or subtract as indicated.

(a) $(4a + 2ab - b) + (3a - ab + b)$

(b) $(2x^2y + 3xy + y^2) - (3x^2y - xy - 2y^2)$

Practice

Subtract.

$$\begin{array}{r} (4x^2 - 2xy + y^2) \\ - (6x^2 - 7xy + 2y^2) \end{array}$$

EXAMPLE 1 Multiplying Monomials and Polynomials

Find each product.

(a) $4x^2(3x + 5)$

(b) $-8m^3(4m^3 + 3m^2 + 2m - 1)$

 **NOW TRY**
EXERCISE 1

Find the product.

$-3x^5(2x^3 - 5x^2 + 10)$

EXAMPLE 2 Multiplying Two PolynomialsMultiply $(m^2 + 5)(4m^3 - 2m^2 + 4m)$. **NOW TRY**
EXERCISE 2
Multiply.

$(x^2 - 4)(2x^2 - 5x + 3)$

EXAMPLE 3 Multiplying Polynomials VerticallyMultiply $(x^3 + 2x^2 + 4x + 1)(3x + 5)$ vertically. **NOW TRY**
EXERCISE 3
Multiply.

$$\begin{array}{r} 5t^2 - 7t + 4 \\ \underline{ 2t - 6} \end{array}$$

Now try it using the box method.

(Multiply vertically or using the box method).

EXAMPLE 4 Multiplying Polynomials with Fractional Coefficients Vertically

Find the product of $4m^3 - 2m^2 + 4m$ and $\frac{1}{2}m^2 + \frac{5}{2}$.

 **NOW TRY**
EXERCISE 4

Find the product of $9x^3 - 12x^2 + 3$ and $\frac{1}{3}x^2 - \frac{2}{3}$.

EXAMPLE 5: Multiplying a binomial times a binomial

- a) $(x + 2)(x + 5)$ b) $(x - 2)(x + 5)$ c) $(x + 2)(x - 5)$ d) $(x - 2)(x - 5)$

NOW TRY

- a) $(x + 1)(x + 7)$ b) $(x - 3)(x + 9)$ c) $(x + 2)(x - 8)$ d) $(x - 3)(x - 10)$

FOIL

Used for ONLY:

EXAMPLE 5 Using the FOIL Method

Use the FOIL method to find the product $(x + 8)(x - 6)$. Compare FOIL with box method.

EXAMPLE 6 Using the FOIL Method

Multiply $(9x - 2)(3y + 1)$.

 **NOW TRY
EXERCISE 5**

Use the FOIL method to find the product.

$$(t - 6)(t + 5)$$

 **NOW TRY
EXERCISE 6**

Multiply.

$$(7y - 3)(2x + 5)$$

EXAMPLE 7 Using the FOIL Method

Find each product.

(a) $(2k + 5y)(k + 3y)$ (b) $(7p + 2q)(3p - q)$ (c) $2x^2(x - 3)(3x + 4)$

 **NOW TRY
EXERCISE 7**

Find each product.

(a) $(3p - 5q)(4p - q)$

(b) $5x^2(3x + 1)(x - 5)$

Example 8

Find an expression that represents the area in (a) **FIGURE 1** and (b) **FIGURE 2**.

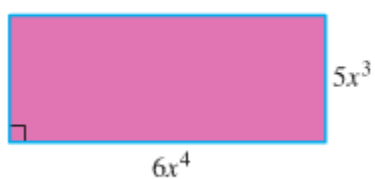


FIGURE 1

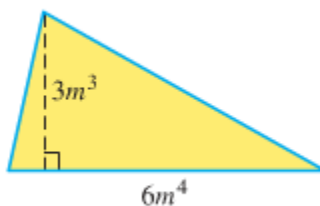
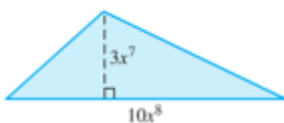


FIGURE 2

Assume $x > 0$, $m > 0$.

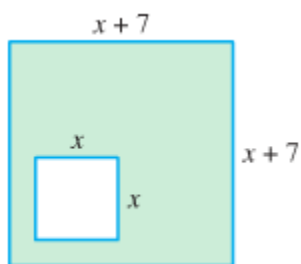
NOW TRY
EXERCISE 8

Write an expression that represents the area of the figure. Assume $x > 0$.

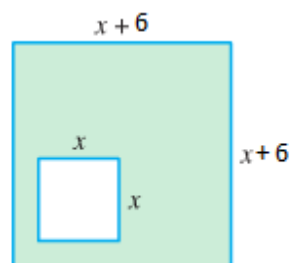


Example 9

Find the area of the shaded region.



Now Try



Example 10

Identify whether the problem is adding, subtracting, or multiplying then simplify.

a) $(3x^2 + 5x + 1) - (x + 7)$ b) $(2x^2 - x + 1) + (x - 7)$ c) $-2x(3x^2 - 4x + 5)$


d) $(x + 2)(x^2 - 4)$ e) $(x - 3)(2x^2 - 4x + 1)$

Now Try

a) $(3x^2 + 5x + 1)(x + 7)$ b) $-4x^2(2x^2 - x + 1)$ c) $(x^2 - 2x + 1) - (3x^2 - 4x + 5)$

d) $(x + 2) + (x^2 - 4)$

EXAMPLE 1 Squaring a BinomialFind $(m + 3)^2$.

 **NOW TRY**
EXERCISE 1
Find $(x + 5)^2$.

Square of a binomial

EXAMPLE 2 Squaring Binomials

Find each binomial square and simplify.

(a) $(5z - 1)^2$

(b) $(3b + 5r)^2$

(c) $(2a - 9x)^2$

(d) $\left(4m + \frac{1}{2}\right)^2$

(e) $x(4x - 3)^2$

 **NOW TRY**
EXERCISE 2

Find each binomial square and simplify.

(a) $(3x - 1)^2$

(b) $(4p - 5q)^2$

(c) $\left(6t - \frac{1}{3}\right)^2$

(d) $m(2m + 3)^2$

EXAMPLE 3 Finding the Product of the Sum and Difference of Two Terms

Find each product.

(a) $(x + 4)(x - 4)$

(b) $(x + 9)(x - 9)$

(c) $(x - 5)(x + 5)$

(d) $\left(\frac{2}{3} - w\right)\left(\frac{2}{3} + w\right)$

NOW TRY
EXERCISE 3

Find the product.

$(t + 10)(t - 10)$

EXAMPLE 4 Finding the Product of the Sum and Difference of Two Terms

Find each product.

(a) $(5m + 3)(5m - 3)$

(b) $(4x + y)(4x - y)$

(c) $\left(z - \frac{1}{4}\right)\left(z + \frac{1}{4}\right)$

(d) $p(2p + 1)(2p - 1)$

NOW TRY
EXERCISE 4

Find each product.

(a) $(4x - 6)(4x + 6)$

(b) $\left(5r - \frac{4}{5}\right)\left(5r + \frac{4}{5}\right)$

(c) $y(3y + 1)(3y - 1)$

EXAMPLE 5 Finding Greater Powers of Binomials

Find each product.

(a) $(x + 5)^3$

(b) $(2y - 3)^4$

(c) $-2r(r + 2)^3$

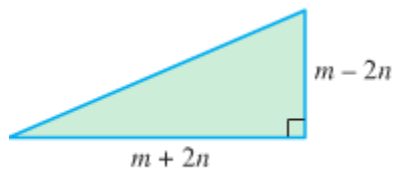
NOW TRY
EXERCISE 5

Find the product.

$$(2m - 1)^3$$

Example 6

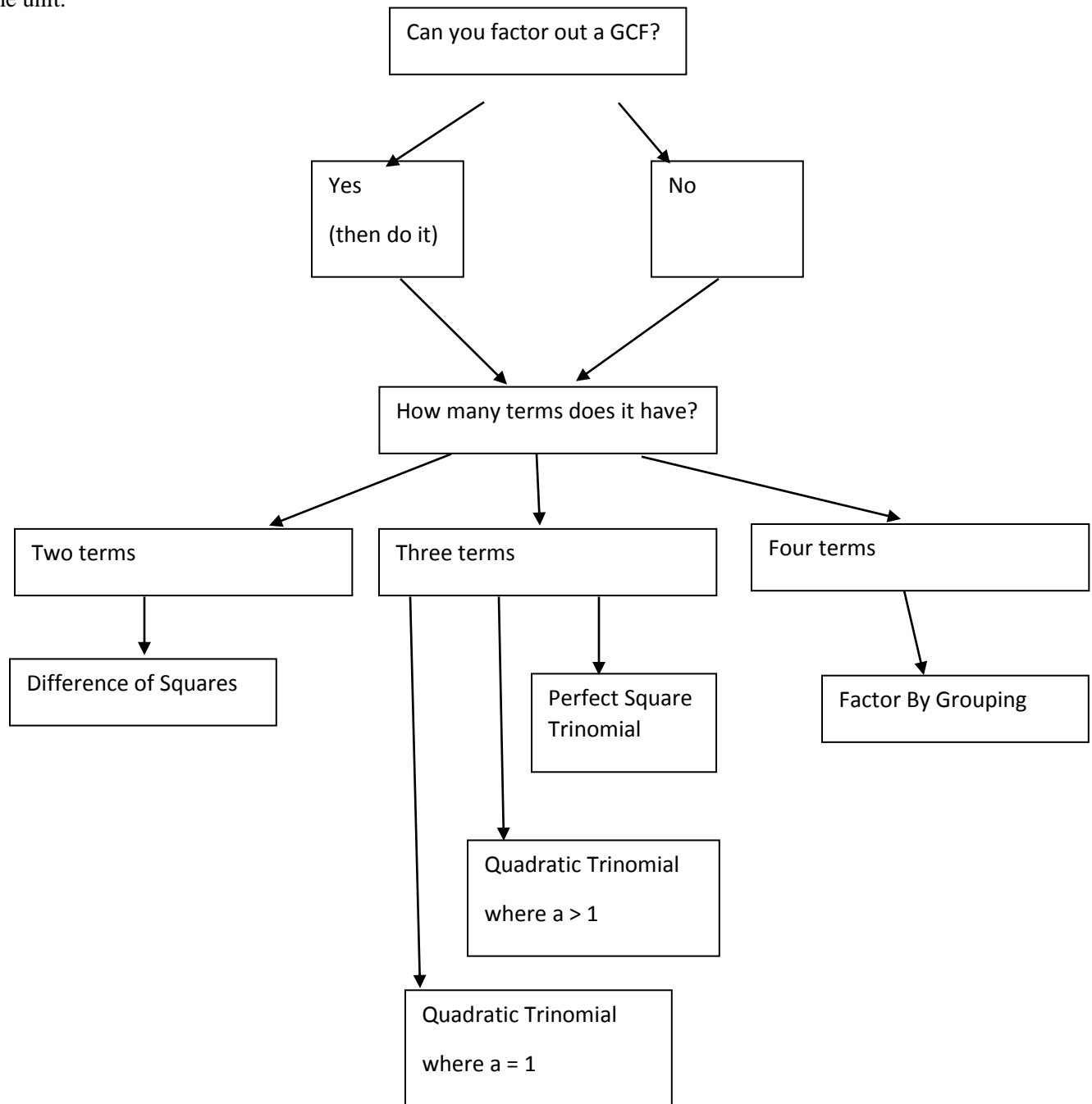
Write a polynomial that represents the area of the figure.



8-F: Factoring out the GCF & Factor by Grouping

Factoring Flow Chart – We will refer to this flow chart throughout the rest of the unit.

to this flow chart throughout the rest of the unit.



Example 1

Write in factored form by factoring out the greatest common factor.

(a) $5y^2 + 10y$

(b) $20m^5 + 10m^4 + 15m^3$

(c) $x^5 + x^3$

(d) $20m^7p^2 - 36m^3p^4$

Practice

(a) $7t^4 - 14t^3$

(b) $8x^6 - 20x^5 + 28x^4$

(c) $30m^4n^3 - 42m^2n^2$

Example 2

Write in factored form by factoring out the greatest common factor.

(a) $a(a + 3) + 4(a + 3)$

(b) $x^2(x + 1) - 5(x + 1)$

Practice

(a) $x(x + 2) + 5(x + 2)$

(b) $a(t + 10) - b(t + 10)$

Example 3

Factor out a negative.

a. $(3x^2 - 2x + 5)$

b. $(-2x^3 + 3x^2 - 5x - 1)$

Practice

a. $(5x^2 + 4x - 7)$

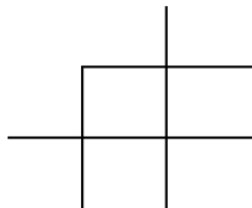
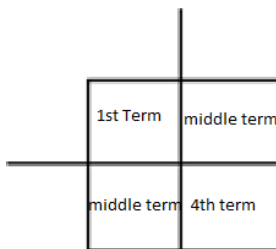
b. $(-5x^3 - 2x^2 + 7x - 4)$

Example 4

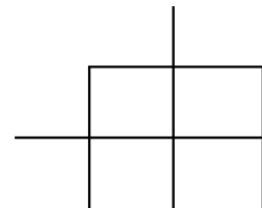
Factor by grouping.

(a) $2x + 6 + ax + 3a$

(b) $6ax + 24x + a + 4$

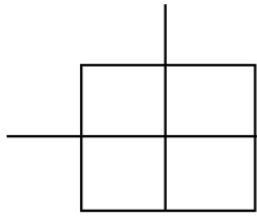


Answer:



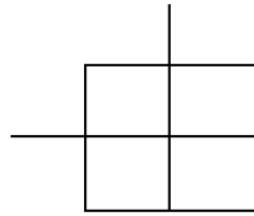
Answer:

(c) $2x^2 - 10x + 3xy - 15y$



Answer:

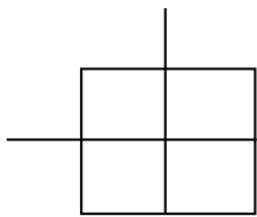
(d) $t^3 + 2t^2 - 3t - 6$



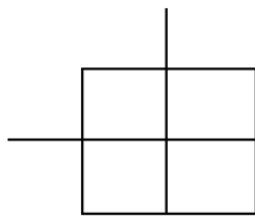
Answer:

Practice

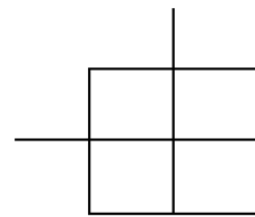
(a) $ab + 3a + 5b + 15$



(b) $12xy + 3x + 4y + 1$



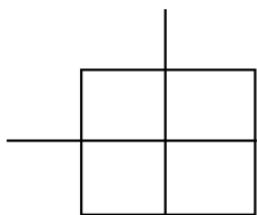
(c) $x^3 + 5x^2 - 8x - 40$



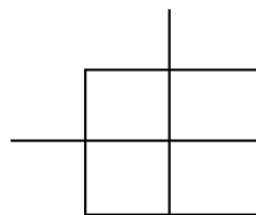
Example 5

Rearranging Terms before Factoring by Grouping

(a) $10x^2 - 12y + 15x - 8xy$

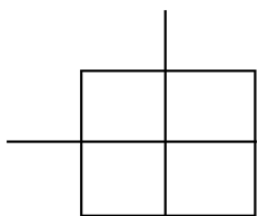


(b) $2xy + 12 - 3y - 8x$

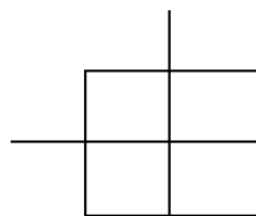


Practice

(a) $12p^2 - 28q - 16pq + 21p$

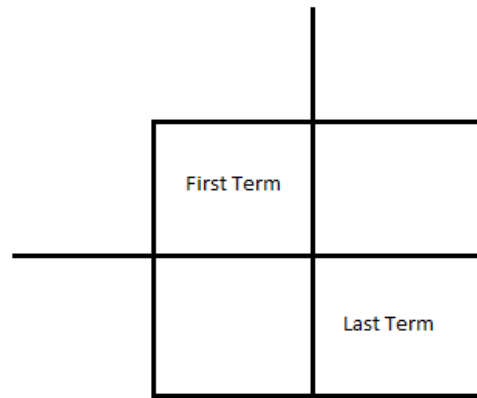
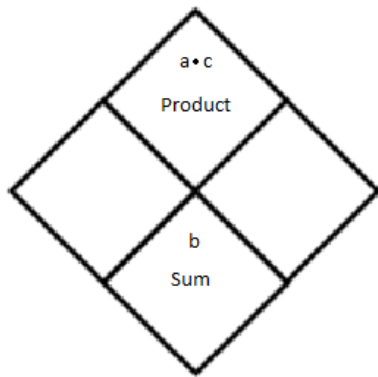


(b) $5xy - 6 - 15x + 2y$



8-G: Factoring Quadratic Trinomials where $a > 1$

Quadratic Trinomial in Standard Form: $ax^2 + bx + c$



Step 1: Use the diamond problem to rewrite the trinomial with FOUR terms.

Step 2: Continue by factor by grouping.

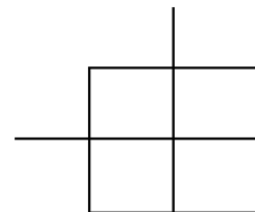
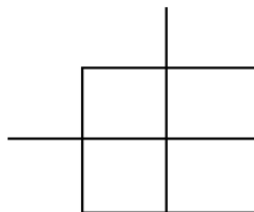
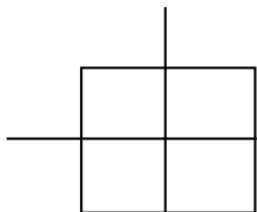
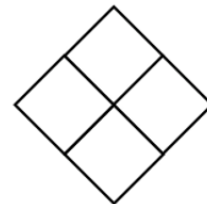
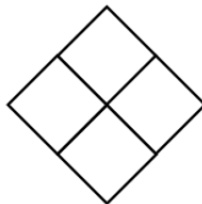
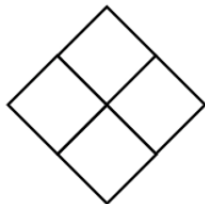
Example 1

Factor each trinomial.

(a) $6r^2 + r - 1$

(b) $12z^2 - 5z - 2$

(c) $10m^2 + mn - 3n^2$



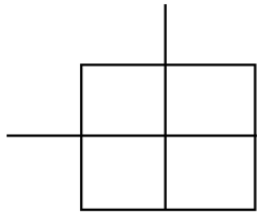
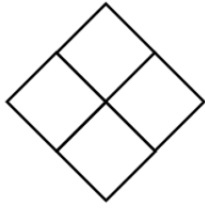
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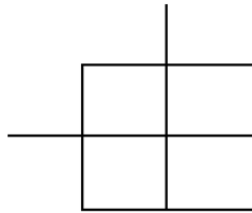
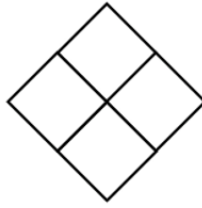
Practice

(a) $2z^2 + 5z + 3$



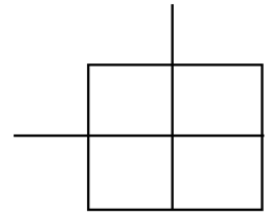
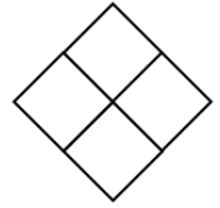
Answer:

(b) $15m^2 + m - 2$



Answer:

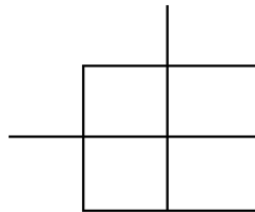
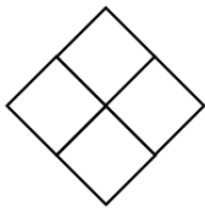
(c) $8x^2 - 2xy - 3y^2$



Answer:

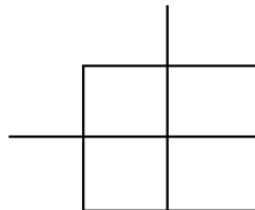
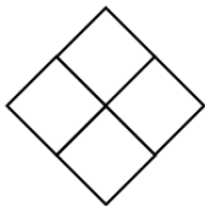
Example 2 **Factoring a Trinomial with a Common Factor by Grouping**

Factor $28x^5 - 58x^4 - 30x^3$.



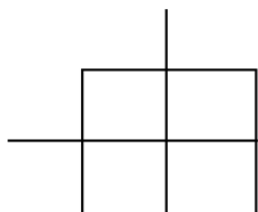
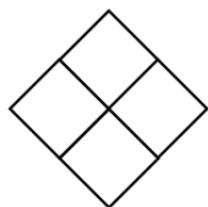
Practice

Factor $15z^6 + 18z^5 - 24z^4$.

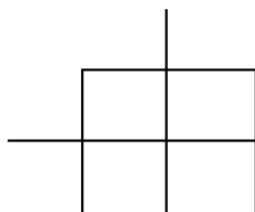
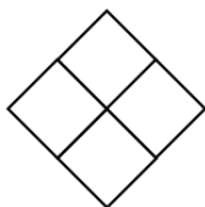


Example 3

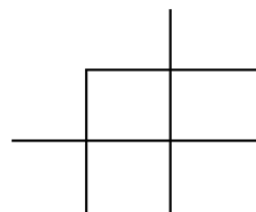
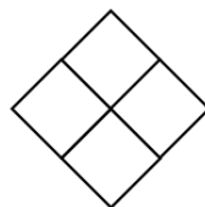
a. $8p^2 + 14p + 5$



b. $6x^2 - 11x + 3$

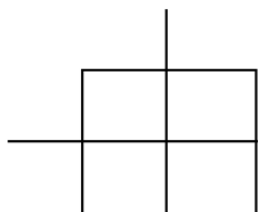
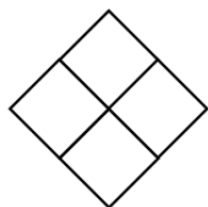


c. $8x^2 + 6x - 9$

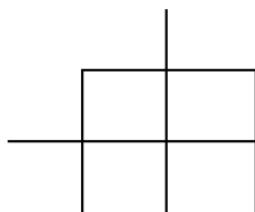
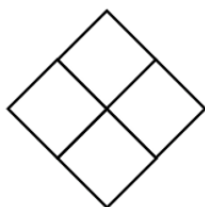


Practice

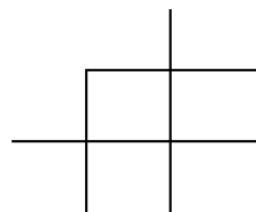
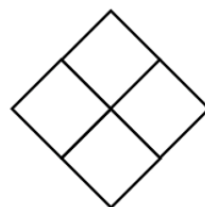
a. $8y^2 + 22y + 5.$



b. $10x^2 - 9x + 2.$



c. $10a^2 + 31a - 14.$



Example 4 **Factoring Trinomials with Common Factors**

(a) $15y^3 + 55y^2 + 30y$

(b) $-24a^3 - 42a^2 + 45a$

Practice

$-10x^3 - 45x^2 + 90x.$

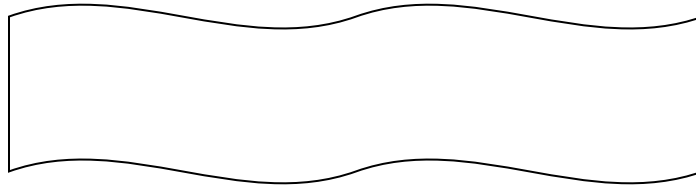
Example 5

Factor $2x^2 + 5x + 1$

Practice

$3x^2 + 10x + 2$

Difference of Squares:

**EXAMPLE 1** Factoring Differences of Squares

Factor each binomial if possible.

(a) $a^2 - 49$

(b) $y^2 - m^2$

(c) $x^2 - 8$

(d) $p^2 + 16$

Practice

(a) $x^2 - 100$

(b) $x^2 + 49$

EXAMPLE 2 Factoring Differences of Squares

Factor each difference of squares.

(a) $25m^2 - 16$

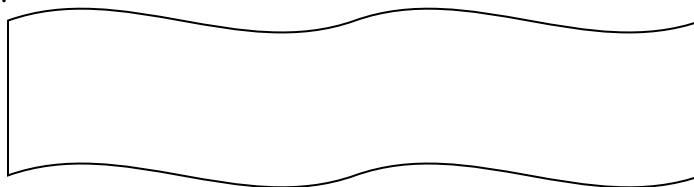
(b) $49z^2 - 64t^2$

Practice

(a) $9t^2 - 100$

(b) $36a^2 - 49b^2$

Perfect Square Trinomials:



EXAMPLE 4 Factoring a Perfect Square Trinomial

Factor $x^2 + 10x + 25$.

NOW TRY
EXERCISE 4

Factor $y^2 + 14y + 49$.

EXAMPLE 5 Factoring Perfect Square Trinomials

Factor each trinomial.

(a) $x^2 - 22x + 121$

(b) $9m^2 - 24m + 16$

(c) $25y^2 + 20y + 16$

(d) $12z^3 + 60z^2 + 75z$

Practice

(a) $t^2 - 18t + 81$

(b) $4p^2 - 28p + 49$

(c) $9x^2 + 6x + 4$

(d) $80x^3 + 120x^2 + 45x$

Warm-up Factor.

1. $x^2 - 6x - 16$

2. $x^2 + x - 12$

3. $6x^2 - 13x - 5$

Guided Practice

1. The square of a number equals nine times that number. Find the number

2. The area of a square is equal to five times its perimeter. Find the length of the side of the square.

Your Turn

3. Suppose that four times the square of a number equals twenty times that number. What is the number?

4. The area of a square is equal to twice its perimeter. Find the length of the side of the square.

Guided Practice

5. The combined area of two squares is 20 square centimeters. Each side of one square is twice as long as the side of the other square. Find the lengths of the sides of each square.

Your Turn

6. The combined area of two squares is 250 square feet. Each side of one square is three times as long as the side of the other square. Find the lengths of the sides of each square.

Guided Practice

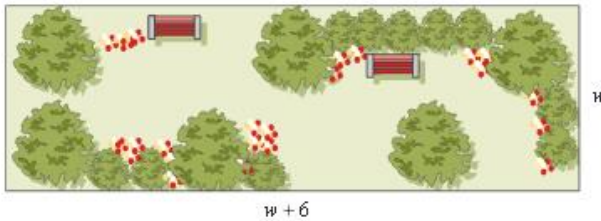
7. Find two consecutive integers whose product is 72.

Your Turn

1. Find two consecutive integers whose product is 110.

Guided Practice

9. A rectangular plot is 6 meters longer than it is wide. The area of the plot is 16 square meters. Find the length and width of the plot.

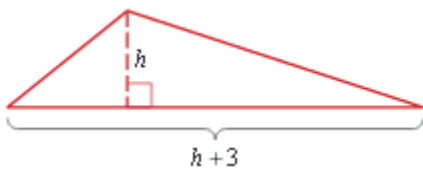


Your Turn

10. A rectangular plot is 2 yards longer than it is wide. The area of the plot is 80 square yards. Find the width and length of the plot.

Guided Practice

11. The area of a triangular sheet of paper is 14 square inches. One side of the triangle is 3 inches longer than the altitude to that side. Find the length of the one side and the length of the altitude to that side.



Your Turn

12. The area of a triangular piece of glass is 30 square inches. One side of the triangle is 4 inches longer than the altitude to that side. Find the length of that side and the length of the altitude to that side.