ALGEBRA 2 CHAPTER 8 RATIONAL FUNCTIONS
Section 8-1 Direct and Inverse Variation
<u>Objective:</u>

Solve problems involving direct, inverse, and combined variation.

CC.9-12.A.CED.2; CC.9-12.A.CED.3

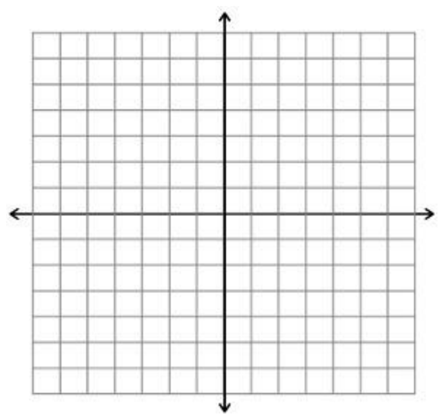
One special type of linear function is called ______________________________

A ____________________________________is a relationship between two variables *x* and *y* that can be written in the form **y =**_____, where $k \neq 0$.

In this relationship, *k* is the____.

For the equation $y = kx$,

Given: *y* varies directly as *x*, and *y* = 27 when *x* = 6. Write and graph the direct variation function. First find k:



When you want to find specific values in a direct variation problem, you can solve for *k* and then use substitution **<u>or</u>** you can use the proportion derived below.

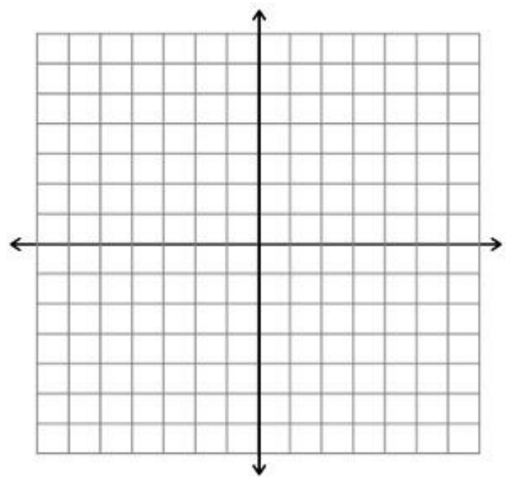
The perimeter *P* of a regular dodecagon varies directly as the side length *s*, and *P* = 18 in. when *s* = 1.5 in. Find *s* when *P* = 75 in.

Another type of variation describes a situation in which one quantity increases and the other decreases.

This type of variation is an inverse variation. An ______________ is a relationship between two variables *x* and *y* that can be written in the form $y =$, where $k \neq 0$.

For the equation $y =$, *y* varies ______________________ as *x*.

Given: *y* varies inversely as *x*, and $y = 4$ when $x = 5$. Write and graph the inverse variation function.



When you want to find specific values in an inverse variation problem, you can solve for *k* and then use substitution **or** you can use the equation derived below.

The time *t* that it takes for a group of volunteers to construct a house varies inversely as the number of volunteers *v*. If 20 volunteers can build a house in 62.5 working hours, how many working hours would it take 15 volunteers to build a house?

You can use algebra to rewrite variation functions in terms of *k*.

Direct Variation

$$y = kx \rightarrow k = \underbrace{\frac{y}{x}}_{\text{Constant ratio}}$$

Inverse Variation

$$y = \frac{k}{x} \rightarrow k = \underbrace{xy}_{\text{Constant product}}$$

Determine whether each data set represents a direct variation, an inverse variation, or neither

x	6.5	13	104
y	8	4	0.5

x	5	8	12
y	30	48	72

ALGEBRA 2 CHAPTER 8 RATIONAL FUNCTIONS

Section 8-2 Rational Expressions

Objective:

Simplify rational expressions.

Multiply and divide rational expressions. CC.9-12.A.APR.7

A ______________________________ is a quotient of two polynomials. Other examples of rational expressions include the following:

Simplify. Identify any *x*-values for which the expression is undefined.

$\frac{10x^8}{6x^4}$ $\frac{x^2+x-2}{x^2+2x-3}$ $\frac{3x+4}{3x^2+x-4}$ $\frac{6x^2+7x+2}{6x^2-5x-5}$

Simplify the following. Identify any *x* values for which the expression is undefined.

$\frac{4x-x^2}{x^2-2x-8}$ $\frac{-x^2+3x}{2x^2-7x-3}$

Multiply. Assume that all expressions are defined.

$\frac{3x^5y^3}{2x^3y^7} \cdot \frac{10x^3y^4}{9x^2y^5}$ $\frac{x-3}{4x+20} \cdot \frac{x+5}{x^2-9}$

Divide. Assume that all expressions are defined.

$$\frac{5x^4}{8x^2y^2} \div \frac{15}{8y^5}$$

$$\frac{x^4-9x^2}{x^2-4x+3} \div \frac{x^4+2x^3-8x^2}{x^2-16}$$

Solve and check.

$$\frac{x^2-25}{x-5} = 14$$

$$\frac{4x^2-9}{2x+3} = 5$$

ALGEBRA 2 CHAPTER 8 RATIONAL FUNCTIONS
Section 8-3 Adding and Subtracting
<u>Objective:</u>
Add and subtract rational expressions.
Simplify complex fractions.
CC.9-12.A.APR.7

Add or subtract. Identify any *x*-values for which the expression is undefined.

$$\frac{x-3}{x+4} + \frac{x-2}{x+4}$$

$$\frac{3x-4}{x^2+1} - \frac{6x+1}{x^2+1}$$

Find the least common multiple for each pair.

A. $4x^2y^3$ and $6x^4y^5$

B. $x^2 - 2x - 3$ and $x^2 - x - 6$

C. $x^2 - 4$ and $x^2 + 5x + 6$

Add. Identify any *x*-values for which the expression is undefined.

$$\frac{x-3}{x^2+3x-4} + \frac{2}{x+4}$$

$$\frac{2x^2-30}{x^2-9} - \frac{x+5}{x+3}$$

$$\frac{x}{x+2} + \frac{-8}{x^2-4}$$

Some rational expressions are *complex fractions.* A <u>complex fraction</u> contains one or more fractions in its numerator, its denominator, or both.

$$\frac{\frac{x-2}{x+1}}{\frac{x-3}{x+5}}$$

$$\frac{\frac{x-3}{x^2-1}}{\frac{x}{x-1}}$$

ALGEBRA 2 CHAPTER 8 RATIONAL FUNCTIONS

Section 8-4 Rational Functions

<u>Objective:</u>

Graph rational functions. CC.9-12.A.CED.2

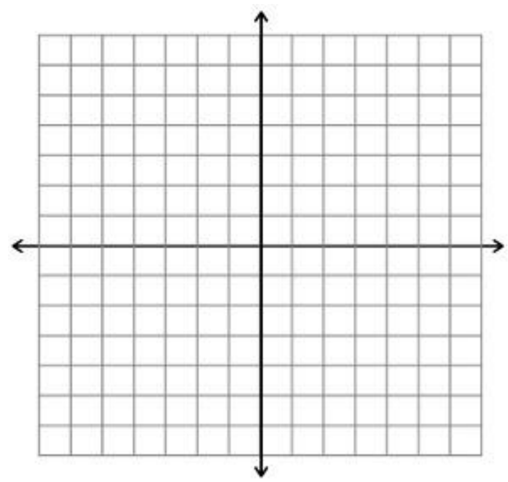
Transform rational functions by changing parameters. CC.9-12.A.CED.3

A ________________________________is a function whose rule can be written as a ratio of two polynomials.

The parent rational function is $f(x) =$.

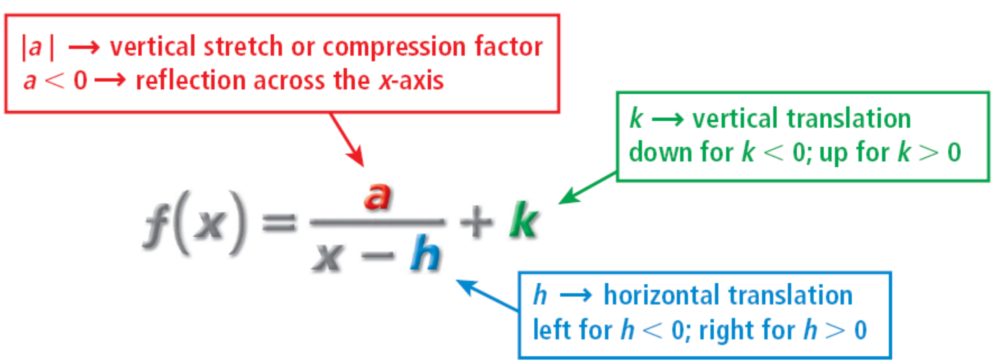
Its graph is a ______________________, which has two separate branches.

Like logarithmic and exponential functions, rational functions may have asymptotes.



Vertical Asymptote:

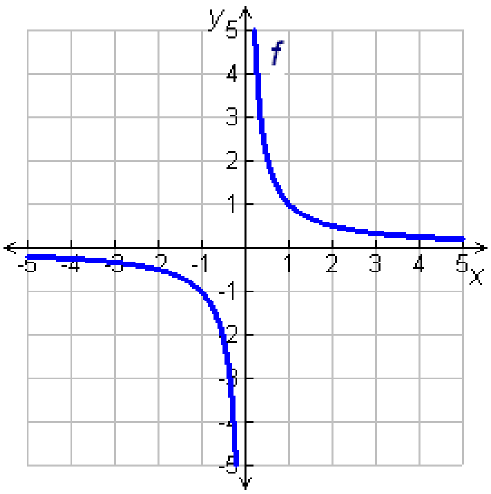
Horizontal Asymptote:

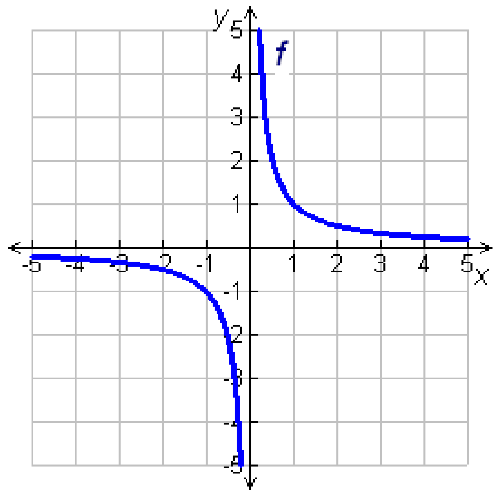


Transform the following:

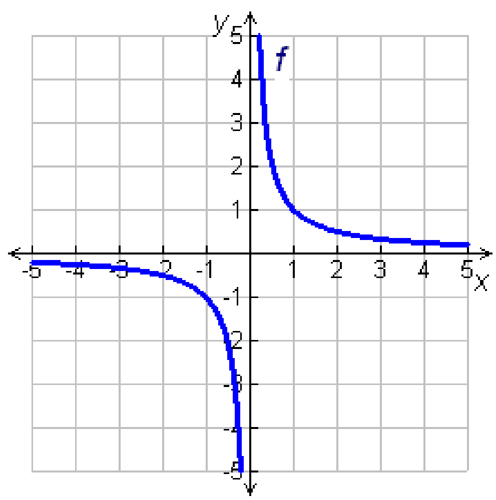
$g(x) = \frac{1}{x+2}$

$h(x) = \frac{1}{x} - 3$





$k(x) = \frac{-1}{x+4}$



Identify the asymptotes, domain, and range of the functions:

$g(x) = \frac{1}{x+3} + 2$

$g(x) = \frac{1}{x-3} - 5$

A ______________________________________ is a function whose graph has one or more gaps or breaks. A _________________________ **function** is a function whose graph has no gaps or breaks. The functions you have studied before this, including linear, quadratic, polynomial, exponential, and logarithmic functions, are continuous functions.

ALGEBRA 2 CHAPTER 8 RATIONAL FUNCTIONS

Section 8-4 Rational Functions (continued)

Objective:

Horizontal Asymptotes and Holes in Rational Functions

Identify the zeros and the vertical asymptotes in the following:

$$f(x) = \frac{x^2+3x-4}{x+3}$$

$$g(x) = \frac{x^2+7x+6}{x+3}$$

Finding Horizontal Asymptotes:

Identify the vertical and horizontal asymptotes and the intercepts:

$$f(x) = \frac{x-2}{x^2-1}$$

$$f(x) = \frac{4x-12}{x-1}$$

$$f(x) = \frac{x^2-3x-4}{x}$$

Finding Holes:

A ________________ is an omitted point in a graph.

To find holes:

Find the holes in the following functions:

$$f(x) = \frac{x^2-9}{x-3}$$

$$f(x) = \frac{x^2+x-6}{x-2}$$

ALGEBRA 2 CHAPTER 8 RATIONAL FUNCTIONS
Section 8-5 Solving Rational Equations

Objectives:

Solve rational equations and inequalities. CC.9-12.F.IF.5

Solve the following:

$$x - \frac{18}{x} = 3 \qquad\qquad \frac{10}{3} = \frac{4}{x} + 2$$

An ____is a solution of an equation derived from an original equation that is not a solution of the original equation.

When you solve a rational equation, it is ______________ to get extraneous solutions.

Solve:

$$\frac{5x}{x-2} = \frac{3x+4}{x-2}$$

Solve:

$$\frac{1}{x-1} = \frac{x}{x-1} + \frac{x}{6}$$

A ______________________________________ is an inequality that contains one or more rational expressions. One way to solve rational inequalities is by using graphs and tables

$$\frac{4}{x-3} \geq 2$$

ALGEBRA 2 CHAPTER 8 RATIONAL FUNCTIONS

Section 8-6 Solving Rational Exponents

Objectives:

Rewrite radical expressions by using rational exponents.

Simplify and evaluate radical expressions and expressions containing rational exponents. CC.9-12.A.REI.12

You are probably familiar with finding the square root of a number. These two operations are inverses of each other. Similarly, there are roots that correspond to larger powers.

The *n*th root of a real number *a* can be written as the radical expression , where *n* is the ________________(plural: *indices*) of the radical and *a* is the ____________________.

When a number has more than one root, the radical sign indicates only the principal, or________________________________, root.

Find all real roots.

a. fourth roots of –256

b. sixth roots of 1

c. cube roots of 125

Properties of *n*th Roots

For $a > 0$ and $b > 0$,

WORDS	NUMBERS	ALGEBRA
Product Property of Roots The *n*th root of a product is equal to the product of the *n*th roots.	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Roots The *n*th root of a quotient is equal to the quotient of the *n*th roots.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplify each expression. Assume that all variables are positive

$\sqrt[4]{81x^{12}}$

$\sqrt[4]{\frac{16x^8}{5}}$

$\sqrt[4]{\frac{x^8}{3}}$

$\sqrt[3]{x^7}\sqrt[3]{x^2}$

A ____ is an exponent that can be expressed as ,
where m and n are integers and $n \neq 0$.

Radical expressions can be written by using ____________________________ exponents.

Write each expression by using rational exponents.

$\sqrt[8]{13^4}$ $\sqrt[15]{13^5}$ $\left(\sqrt[4]{81}\right)^3$

Properties of Rational Exponents

For all nonzero real numbers a and b and rational numbers m and n,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property To multiply powers with the same base, add the exponents.	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property To raise one power to another, multiply the exponents.	$\left(8^{\frac{2}{3}}\right)^3 = 8^{\frac{2}{3}\cdot 3} = 8^2 = 64$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property To find the power of a product, distribute the exponent.	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^m = a^m b^m$
Power of a Quotient Property To find the power of a quotient, distribute the exponent.	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$\frac{16^{\frac{3}{4}}}{16^{\frac{5}{4}}}$ $36^{\frac{3}{8}} \bullet 36^{\frac{1}{8}}$ $(-8)^{-\frac{1}{3}}$

ALGEBRA 2 CHAPTER 8 RATIONAL FUNCTIONS

Section 8-7 Graphing Radical Functions

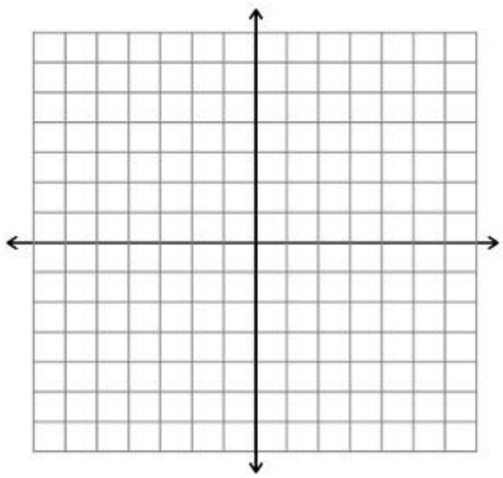
Objectives:

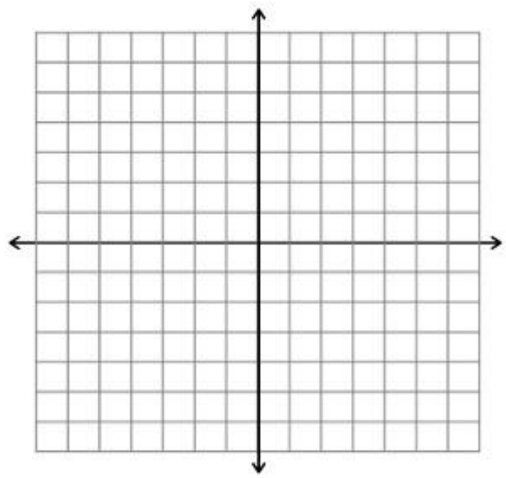
Graph radical functions and inequalities. CC.9-12.F.IF.7b

Transform radical functions by changing parameters CC.9-12.F.BF.3

Recall that exponential and logarithmic functions are inverse functions.

Quadratic and cubic functions have inverses as well. The graphs below show the inverses of the quadratic parent function and cubic parent function.

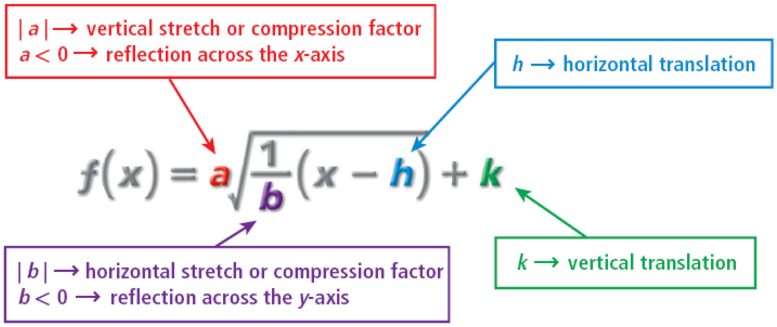




A ______________________________ is a function whose rule is a radical expression.

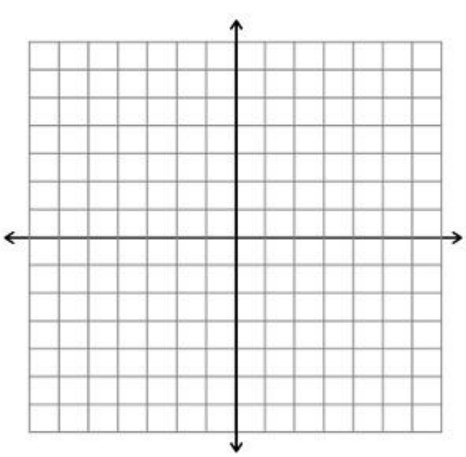
A ____ is a radical function involving _______.

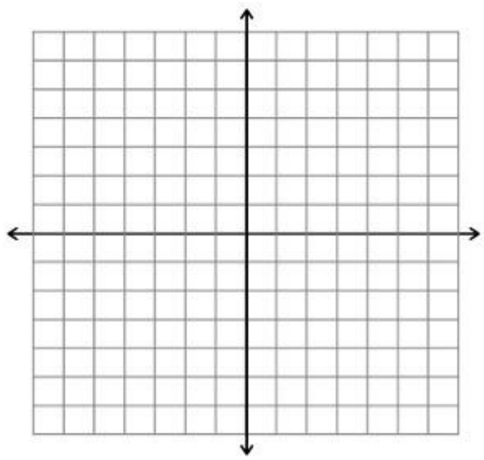
Transformations of the Square-Root Parent Function $f(x) = \sqrt{x}$		
Transformation	$f(x)$ Notation	Examples
Vertical translation	$f(x) + k$	$y = \sqrt{x} + 3$ 3 units up $y = \sqrt{x} - 4$ 4 units down
Horizontal translation	$f(x - h)$	$y = \sqrt{x - 2}$ 2 units right $y = \sqrt{x + 1}$ 1 unit left
Vertical stretch/ compression	$af(x)$	$y = 6\sqrt{x}$ vertical stretch by 6 $y = \frac{1}{2}\sqrt{x}$ vertical compression by $\frac{1}{2}$
Horizontal stretch/ compression	$f\left(\frac{1}{b}x\right)$	$y = \sqrt{\frac{1}{5}x}$ horizontal stretch by 5 $y = \sqrt{3x}$ horizontal compression by $\frac{1}{3}$
Reflection	$-f(x)$ $f(-x)$	$y = -\sqrt{x}$ across x-axis $y = \sqrt{-x}$ across y-axis

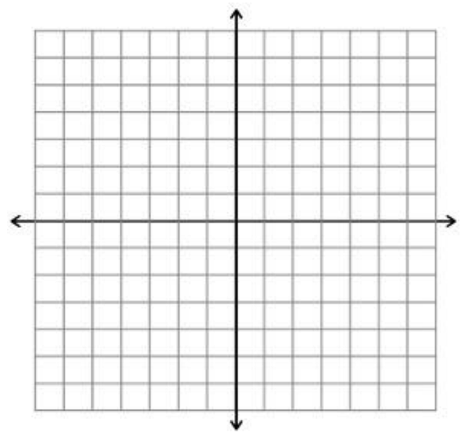


Graph each function and identify its domain and range.

$f(x) = \sqrt{x - 3}$ $\quad g(x) = -\sqrt{x - 4}$ $\quad g(x) = \sqrt{-x} + 3$





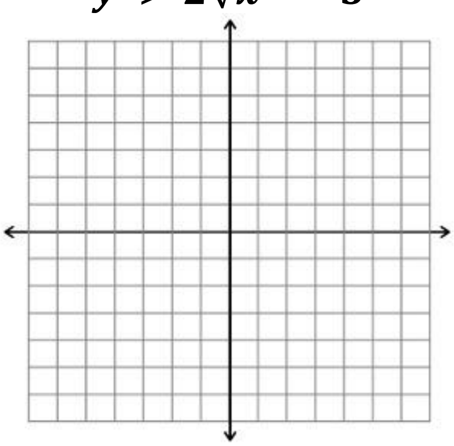


Use the description to write the square-root function *g*. The parent function is reflected across the *x*-axis, compressed vertically by a factor of 1/5, and translated down 5 units

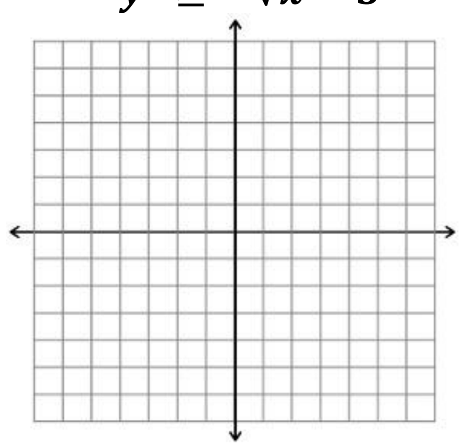
Reflected across the *x*-axis, stretched vertically by a factor of 2, and translated 1 unit up.

Graph the inequality:

$y > 2\sqrt{x} - 3$



$y \geq \sqrt[3]{x - 3}$



ALGEBRA 2 CHAPTER 8 RATIONAL FUNCTIONS

Section 8-8 Solving Radical Functions

Objectives:

Solve radical equations and inequalities. CC.9-12.A.CED.1

A ______________________________________ contains a variable within a radical.

Solving Radical Equations	
Steps	**Example**
1. Isolate the radical.	$\sqrt[3]{x} - 2 = 0$ $\sqrt[3]{x} = 2$
2. Raise both sides of the equation to the power equal to the index of the radical.	$\left(\sqrt[3]{x}\right)^3 = (2)^3$
3. Simplify and solve.	$x = 8$

$\sqrt[3]{3x - 4} = 2$

$5 + \sqrt{x + 1} = 16$

$6\sqrt{x + 10} = 42$

$\sqrt{7x + 2} = 3\sqrt{3x - 2}$

$\sqrt[3]{x + 6} = 2\sqrt[3]{x - 1}$

$(5x + 7)^{\frac{1}{3}} = 3$

A ______________________________________ is an inequality that contains a variable within a radical. You can solve radical inequalities by graphing or using algebra.

$\sqrt{2x - 6} + 3 \leq 9$

$\sqrt{x - 3} + 2 \leq 5$