Algebra 2 – Midterm Exam Review by Topic

Solving Linear Equations and Inequalities

- ***Solving Equations***
  - Absolute value – get the absolute value part by itself first (no distributing! Can't touch the absolute value!) **(#2)**
    - Set 2 equations – one is regular, the second one has a NEGATIVE answer (don't change the signs of anything else but the answer)
    - If the original absolute problem equals a negative number, it has no solution. Ex. $|5x - 1| = -10$ impossible! **(#6)**
    - If your absolute value equation has x on both sides, like $|4x - 1| = 2x$, then plug your final two answers back in to see if one will turn out to be extraneous! (Extraneous if you plug back in and get a negative number)
  - Fractions – multiply the WHOLE thing by the least common denominator to get rid of fractions **(#1)**
    - ONE fraction (as a coefficient of x) – solve like normal, then multiply by the reciprocal **(#3)**
- ***Solving Inequalities***
  - Graph on a number line! Open or closed circle?
  - Shade the side that is "true"
  - Compound inequalities – TWO inequalities or absolute value inequalities; shade in the middle or shade on the outsides **(#4)**
  - Absolute value inequalities **(#5)**
    - One stays the same
    - For second one, switch the inequality AND make it negative!
    - Solve and graph both

1) $\frac{1}{4}x - \frac{1}{3}x = x - \frac{1}{2}$	2) $2\|3x - 7\| = 30$	3) $\frac{1}{2}a + \frac{3}{5} = \frac{7}{10}$
4) $-1 < 2x - 3 < 11$ *Graph on number line!	5) $\|6x + 2\| > 1$ *Graph on number line!	6) a) $\|x + 7\| = -9$ b) $\|x + 7\| = 9$

<u>Systems of Equations or Inequalities</u>

Solve systems of equations by using:

Graphing: Graph the lines and interpret the solution. **Use if both are in y= already.**

- If the lines cross, there is one solution at the point of intersection.
- If the lines are parallel, there is no intersection and no solution.
- If the lines have the same graph, there are many solutions.

Substitution: Solve one equation for a variable (x or y). Substitute into the other equation and solve. Substitute to find the other variable. Make sure you find the values for both variables! **Use if one is already in x= or y= or it is easy to do so.**

- If you get a number as your answer, there's one solution.
- If you get a TRUE statement (like 3 = 3), there are infinitely many solutions.
- If you get a FALSE statement (like 3 = 5), there is no solution.

Elimination: Make sure that the coefficients of one of the variables differ only by the sign (**<u>multiply by the opposite one to create opposites</u>** if needed). Add the equations together. One variable should cancel. Substitute into one of the original equations to find the other variable. Make sure you find the values for both variables!!! Use if both are in standard form Ax + By = C.

- If you get a number as your answer, there's one solution.
- If you get a TRUE statement (like 3 = 3), there are infinitely many solutions.
- If you get a FALSE statement (like 3 = 5), there is no solution.

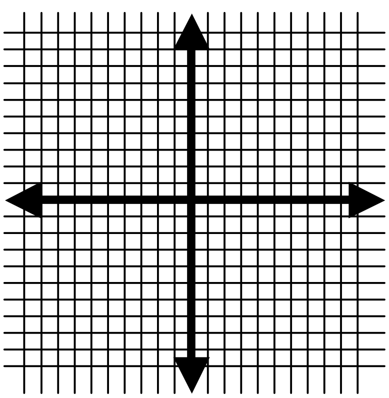
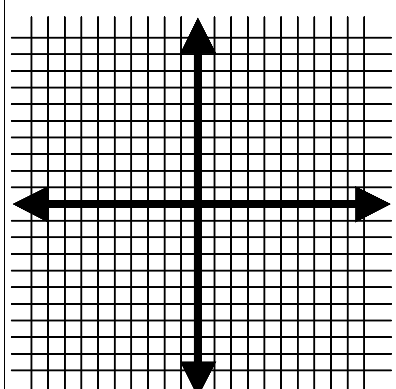
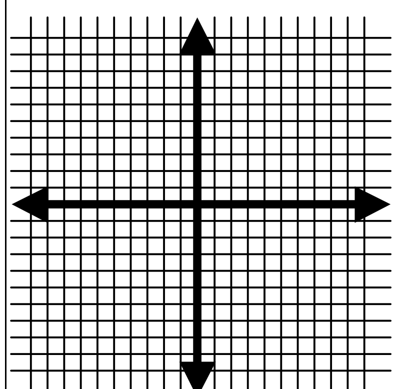
Solve using a different method for each problem. Use each method at least once.

7) $-3x + 4y = 1$ $x = 2y + 1$	8) $2x - 3y = 5$ $-6x + 9y = 12$	9) $9x + 2y = 0$ $3x - 5y = 17$
10) $y = 3x$ $y = -\frac{1}{2}x - 6$	11) $2x + 3y = -7$ $4x + y = 1$	12) $3x - y = 4$ $2x - 3y = 26$

Systems of Inequalities:

1. Get graph to be in y = form
2. Calculator – graph → Push "back space" and the inequality symbols will show up
3. Use a dotted line for $<$ or $>$ and a solid line for $\leq$ or $\geq$.
4. By hand: Graph the equation(s).
   a. To graph: turn to **<u>y = mx + b</u>** form
   b. OR: if in standard form Ax + By = C, do the **<u>cover up method</u>**!
5. Choose a point (choose (0,0) if you can) and check to see if it's a solution.
6. If it's a **solution, shade the side** that the point is on. If it's **not a solution, shade the other side**.
7. If you have TWO inequalities, the overlapped shaded region is your final answer!

Graph and shade on a coordinate plane.

13) $2x - 5y > -10$	14) $x > -3$ $y \leq 7$ *Hint: these are vertical and horizontal lines!	15) $2x + y \leq -4$ $y > x - 1$
		

<u>Simplifying & Rationalizing:</u>

- **Simplifying**
  1. Gets rid of parentheses by distributing **(#16)**
  2. If adding/subtracting – combine like terms (same variables, same powers) **(#16)**
  3. If multiplying – FOIL or box method **(#20)**
  4. Exponent rules! **(#21)**
- **Rationalizing** - Takes **<u>square roots (radicals)</u>** and **<u>imaginary numbers (i)</u>** out of the denominator
  1. One Term in the denominator – Multiply the numerator and denominator by the radical part of the denominator or by the i unit. **(#17)**
  2. Two terms in the denominator – Multiply the numerator and denominator by the denominator's **<u>conjugate!</u>** **(#18, 19)**
  3. **i values** → divide exponent by 4, use the REMAINDER for the value.
     - $i = \sqrt{-1}$ → Remainder of 1
     - $i^2 = -1$ → $i^2 = -1$ is used a lot when simplifying!
     - $i^3 = -\sqrt{-1}$ → Same thing as –i
     - $i^4 = 1$ → If no remainder, use i^4

Simplify.

16) a) $(2x^3 - 4x^2 + 5) - (-x^2 - 3x + 1)$ b) $x^4y(x^2 + 6) - x^3(x^2y - 7xy)$ Hint: distributive property first!	17) a) $\sqrt{\frac{9}{7}}$ b) $\frac{7}{\sqrt{14}}$	18) a) $\frac{5}{3-\sqrt{5}}$ b) $\frac{3-\sqrt{4}}{2+\sqrt{7}}$
19) a) $\frac{3}{2+6i}$ b) $\frac{3+8i}{4-5i}$	20. a) $(2x)^2$ b) $(2x-9)^2$ c) $(2x-9)^3$	21) a) $\frac{(xy)^4}{xy^{-1}}$ b) $yz^{-2}(x^2y)^3z$

Factoring:

- ***See factoring packet notes and the FLOW CHART for more examples***
  - Steps

1. Take out the GCF first.
2. **2 terms**
   a) Difference of squares: $a^2 - b^2 = (a + b)(a - b)$
   b) Difference of Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ SOAP
   c) Sum of Cubes: $a^3 - b^3 = (a + b)(a^2 - ab + b^2)$ SOAP
3. **3 terms** – Find factors of the first and last terms that create two binomials whose product is equal to the trinomial.
   a) If coefficient of x^2 is 1, then do BIG X
   b) If coefficient of x^2 is bigger than 1, then slide and divide (and reduce!)
4. **4 terms** – Grouping. Take out the greatest common factor for pairs of terms.
5. ***Always check if you can keep factoring more!***

Factor.

22) $2x^3 - 3x^2 - 10x + 15$	23) a) $4x^2 - 5x - 9$ b) $4x^4 - 5x^2 - 9$	24) $2x^3 - 128$
25) $w^4 - 16$	26) $9x^4 - 12x^2 + 4$	27) $3x^3 + 7x^2 - 12x - 28$

<u>Quadratics</u> (x^2 means quadratic! Two solutions!) **SEE YELLOW FOLDABLE!**

Graphing

Forms of Quadratic Functions	**Vertex Information**
Standard Form: $ax^2 + bx + c$	**Coordinates of the vertex:** $x = -\dfrac{b}{2a}$ **plug in to find y**
Vertex Form: $y = a(x - h)^2 + k$	**Coordinates of the vertex:** **(h, k)** ***h is opposite, k is the same!**
Intercept Form: $y = a(x - p)(x - q)$	$x = \dfrac{p+q}{2}$ **plug in to find y** ***p and q are opposites!**

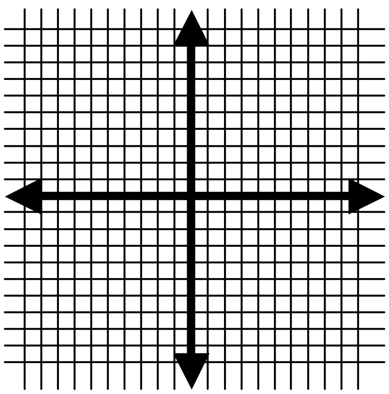
Note: "a" in the equations above tell you if it opens up or down or if it is wide or thin!

Axis of symmetry: x = the x-coordinate of the vertex VERTICAL LINE!

28. $y = -2(x - 3)^2 + 7$

Opening:_____ Vertex:_____

x	y

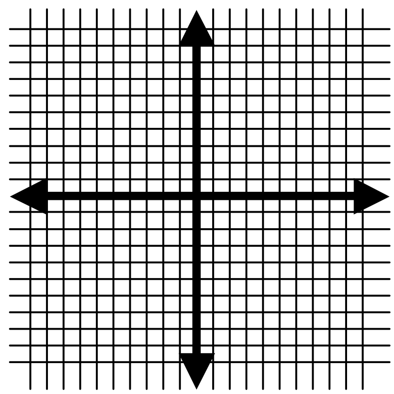


Axis of symmetry:___________

29. $y = 2x^2 - 4x - 6$

Opening:_____ Vertex:_____

x	y

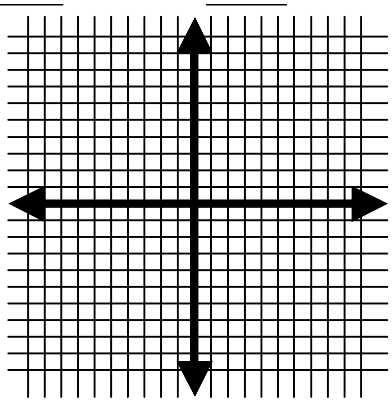


Axis of symmetry:___________

30. $y = \frac{1}{4}(x + 7)(x + 3)$

Opening:_____ Vertex:_____

x	y

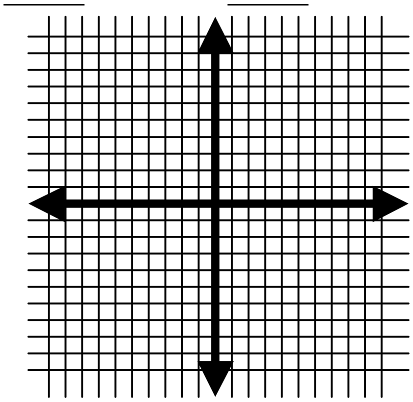


Axis of symmetry:___________

31. $y = x^2 - 2x - 8$

Opening:_____ Vertex:_____

x	y



Axis of symmetry:___________

Solving → SEE PINK GRAPHIC ORGANIZER!

Factoring:	**Inverse operations (square roots):**
• Look for the GCF first. • Factor. • **Set each factored part equal to zero** and solve.	• Use if there is only ONE "x" in the problem. • Solve for the x^2 or for the $(x-1)^2$. • Square root both sides remembering the ± symbol. • Finish solving as necessary.
Completing the square: • Use if you can't factor by BIG X. • Make sure that the <u>leading coefficient is positive one</u> and that the quadratic is in standard form. • Move constants to one side and variables to the other, and draw your blanks! • Find half of b, square it, then add to **<u>both</u>** sides. Factor. Square root both sides and finish solving. • **Use this when converting to vertex form.**	**Quadratic formula:** • Use if you can't factor by slide and divide! • Get the quadratic into standard form, $y = ax^2 + bx + c$. Substitute into the quadratic formula, **using parentheses to keep the correct signs**. • Simplify completely, including radicals and imaginary numbers. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve the following quadratic equations using a different method for each problem.

32) $x^2 + 8x - 5 = 0$ Method: ______________________	33) $x^2 + 7x + 10 = 0$ Method: ______________________
34) $3x^2 - 8x - 4 = 0$ Method: ______________________	35) $5(x-3)^2 + 35 = 0$ Method: ______________________
36) $\frac{1}{2}(x-7)^2 - 17 = 0$ Method: ______________________	37) $x^2 - 12x + 54 = 0$ Method: ______________________

Other Topics to Know (see practice problems packet)

- **Properties**
  - Commutative – change order a + b = b + a ab = ba
  - Associative – group differently a+(b+c) = (a+b)+c a(bc) = (ab)c
  - Identity – answer is the same a + 0 = a a*1 = a
  - Inverse – reverse; answer is "identity" a + (-a) = 0 $a \cdot \frac{1}{a} = 1$
  - Distributive – distribute and multiply a(b+c) = ab + ac

- **Functions**
  - Domain – all the x values ex. $-4 \leq x \leq 2$
  - Range – all the y values ex. $-3 \leq y \leq 5$
  - Is the graph a function? If yes, it must pass the vertical line test
  - Parent functions – linear, quadratic, absolute value, cubic, cube root, square root, exponential
  - Transformations – "h" shifts a graph left/right and is opposite; "k" shifts a graph up or down

- **Piecewise functions (#76 – 78)**
  - Circle the domain
  - Draw a line at the domain(s)
  - Left and right side – which equation is on which side?
  - Plug in the domain point to the equation and plot it
  - Plug in another point to the left or right (depending which side it is on) and plot it
  - Connect the two dots
  - **If f(x) = 6, just a constant number, it is a HORIZONTAL line! No need to plug in points. Just draw the line.

- **Rational Exponents and Radical Equations**
  - Use factor tree to simplify radicals (and grouping system)
  - To rewrite from radical to exponent, *power over index*
    - Ex. $\sqrt[3]{a^5} = a^{\frac{5}{3}}$
  - To solve, take both sides to the RECIPROCAL power
    - Cube root and 3^{rd} power cancel each other out

- **Discriminant: $b^2 - 4ac$**
  - Positive = 2 real roots (zeroes)
  - Negative = 2 imaginary roots (zeroes)
  - Zero = 1 real root (zero)