Algebra 2/Pre-Calculus

Name__________________

Arithmetic and Beginning Factoring (Polynomials, Day 1) (DDMath.com)

In our first problem set of the year, "Pattern Recognition: An Introduction to this Course" we explored a variety of patterns. Many of these patterns can be described by polynomial functions. In this handout, we will begin by introducing polynomials. Then we will practice adding, subtracting, multiplying, and factoring polynomials. (Some of these skills may be review.) Factoring will be particularly important for us, as we will use factoring both for solving polynomial equations and graphing polynomials by hand. (We will learn about this in future problem sets.)

Definition We say $p(x)$ is a *polynomial* if $p(x)$ can be written in the following way:

$p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_nx^n$, where the coefficients, a_0, a_1, $a_2, \ldots, a_n$ are real numbers and $a_n \neq 0$.

Polynomials can be written in different forms. Here's an example showing four ways of writing the same polynomial.

Standard form: $2x^3 + 4x^2 - 8x - 16$

Similar to standard form: $-16 - 8x + 4x^2 + 2x^3$

Factored form: $2(x-2)(x+2)^2$

Some other form: $2(x+2)^3 - 8(x+2)^2$

1. To add or subtract polynomials, we simply combine like terms. Do this in the problems below. ***Note:*** In this context, "simply" means put into standard form.

a. $4x^3 + 7x^2 - 9x + 8 + 10x^4 - 8x^2 - 11$

b. $x^4 + 7 - (x^5 + 3x^4 + 7x^2 - 10)$

c. $5(x^2 - 4x + 5) - 6(x^2 - 4x + 8)$

d. $x(x+4) - (x+8)$

Answers a. $10x^4 + 4x^3 - x^2 - 9x - 3$ b. $-x^5 - 2x^4 - 7x^2 + 17$ c. $-x^2 + 4x - 23$ d. $x^2 + 3x - 8$

2. In this problem, we will multiply polynomials. To do this, we first distribute (multiply every term in the first polynomial by every term in the second polynomial), and then combine like terms. An example is provided below:

$$\begin{aligned}&(x+3)(x^2+5x+9)\\&=x^3+5x^2+9x+3x^2+15x+27\\&=x^3+8x^2+24x+27\end{aligned}$$

Multiply each of the following polynomials. ***Note:*** Answers are provided at the end of this problem.

a. $(x+7)(x-9)$

b. $(x^2+6)(x^2-5)$

c. $(2x+3)(x-4)$

d. $(3x-2)(2x-5)$

e. $2(x^2-8)(x-7)$

f. $5x^3(x^5-7x^2+10)$

g. $(x-4)(x^2-6x-4)$

h. $(x+3)(x^2-3x+9)$

i. $(x+3)(x+4)(x+5)$

j. $(x-2)^3$

Answers a. $x^2-2x-63$ b. x^4+x^2-30 c. $2x^2-5x-12$ d. $6x^2-19x+10$
e. $2x^3-14x^2-16x+112$ f. $5x^8-35x^5+50x^3$ g. $x^3-10x^2+20x+16$ h. x^3+27
i. $x^3+12x^2+47x+60$ j. $x^3-6x^2+12x-8$

3. In the last problem, we practiced multiplying polynomials. Often times, we want to do this process in reverse. We call this *factoring*.

Factor each of the following polynomials.

a. $x^2 + 16x + 63$

b. $x^2 - 13x + 36$

c. $x^2 + 5x - 24$

d. $x^2 + 20x + 100$

Answers a. $(x+7)(x+9)$ b. $(x-4)(x-9)$ c. $(x-3)(x+8)$ d. $(x+10)(x+10)$

4. Here are some harder ones. Check your answers as you do them.

a. $2x^2 + 5x + 3$

b. $3x^2 - 8x + 4$

c. $2x^2 + 9x - 5$

d. $6x^2 + 11x + 3$

Answers a. $(2x+3)(x+1)$ b. $(3x-2)(x-2)$ c. $(2x-1)(x+5)$ d. $(2x+3)(3x+1)$

5. Factor each of the following. Look for a pattern.

a. $x^2 - 16$ **b.** $x^2 - 49$ **c.** $x^2 - 1$

d. $16x^2 - 81$ **e.** $49x^2 - 100$ **f.** $4x^2 - 1$

Answers a. $(x-4)(x+4)$ b. $(x-7)(x+7)$ c. $(x-1)(x+1)$ d. $(4x-9)(4x+9)$
e. $(7x-10)(7x+10)$ f. $(2x-1)(2x+1)$

6. Emily and Maddie were both trying to factor the polynomial $x^2 - 25$. Emily said the answer was $(x-5)(x+5)$ whereas Maddie said the answer was $(x+5)(x-5)$. Who was right?

Answer They are both right.

7. Factor each of the following.

a. $a^2 - b^2$ **b.** $x^2 - y^2$ **c.** $9x^2 - 4y^2$

Answers a. $(a+b)(a-b)$ b. $(x+y)(x-y)$ c. $(3x-2y)(3x+2y)$

8. In the last few problems, we have explored factoring a difference of squares. Now we will attempt to factor a sum of squares. Can you factor x^2+25? What about x^2+49? Or x^2+100?

Answer None of these can be factored in the real number system.

9. Sometimes factoring is as simple as "pulling" out a common term. For example, $x^2+10x = x(x+10)$. Factor each of the following.

a. x^2+9x **b.** $10x^3+15x^2$ **c.** $14x+35$

Answers a. $x(x+9)$ b. $5x^2(2x+3)$ c. $7(2x+5)$

10. Often times factoring is a multistep process. Here are two examples:

$$x^3-7x^2+12x = x(x^2-7x+12) = x(x-3)(x-4)$$

$$14-5x-x^2 = -x^2-5x+14 = -(x^2+5x-14) = -(x+7)(x-2)$$

Unfortunately, sometimes you can only do one step. Here's an example $x^3+x^2+x = x(x^2+x+1)$. Since x^2+x+1 doesn't factor, we can't go any further. Factor each of the following as much as possible. ***Note:*** Answers are provided at the end of this problem. Check your answer after factoring each of these!

a. x^3+18x^2+77x **b.** $3x^4-27x^2$ **c.** $2x^3+2x^2+14x$

d. $2x^2 + 22x - 24$

e. $2x^2 + 9x - 5$

f. $3x^2 + 14x + 8$

g. $x^2 - 4$

h. $x^2 - 4x$

i. $100x^2 - 1$

j. $49x^5 - 4x^3$

k. $x^6 + 13x^5 + 42x^4$

l. $-x^2 + 2x + 15$

m. $-x^2+11x-24$ **n.** $5x^5-21x^4+4x^3$ **o.** $-8x^3-22x^2+6x$

Answers a. $x(x+7)(x+11)$ b. $3x^2(x-3)(x+3)$ c. $2x(x^2+x+7)$ d. $2(x-1)(x+12)$
e. $(2x-1)(x+5)$ f. $(3x+2)(x+4)$ g. $(x+2)(x-2)$ h. $x(x-4)$ i. $(10x+1)(10x-1)$
j. $x^3(7x-2)(7x+2)$ k. $x^4(x+6)(x+7)$ l. $-(x+3)(x-5)$ m. $-(x-3)(x-8)$
n. $x^3(5x-1)(x-4)$ o. $-2x(4x-1)(x+3)$

11. **Optional Exploration: Hidden Difference of Squares** Perform each of the following factoring problems. Look for patterns. Be clever.

a. Factor $(x+5)^2-9$

b. Factor $x^4-(x-7)^2$

c. Factor $x^4 - (x^2 - 4x + 4)$. ***Hint:*** Start by factoring $x^2 - 4x + 4$.

d. Factor $x^4 - x^2 - 12x - 36$.

e. Factor $x^4 - 14x^2 + 49 - x^2 - 6x - 9$

f. Factor as much as you can: $x^6 - 64$