Algebra 2/Trig: Chapter 6 – Sequences and Series

In this unit, we will…

- Identify an arithmetic or geometric sequence and find the formula for its nth term
- Determine the common difference in an arithmetic sequence
- Determine the common ratio in a geometric sequence
- Determine a specified term of an arithmetic or geometric sequence
- Specify terms of a sequence, given its recursive definition
- Represent the sum of a series, using sigma notation
- Determine the sum of the first n terms of an arithmetic or geometric series

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- Determine the common difference in an arithmetic sequence
- Determine the common ratio in a geometric sequence
- Determine a specified term of an arithmetic or geometric sequence
- Specify terms of a sequence, given its recursive definition

Pgs. 1 – 6 in Packet

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SWBAT:

- Identify an arithmetic or geometric sequence and find the formula for its nth term
- Determine the common difference in an arithmetic sequence
- Determine the common ratio in a geometric sequence
- Determine a specified term of an arithmetic or geometric sequence
- Specify terms of a sequence, given its recursive definition

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Formulas for Sequences and Series

Formulas You Must Memorize

Generator for an Arithmetic Sequence

$$a_n = a_1 + (n-1) \cdot d$$

Generator for a Geometric Sequence

$$a_n = a_1 \cdot r^{n-1}$$

Formulas Appearing on the Formula Sheet

Sum of a Finite Arithmetic Series

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Sum of a Finite Geometric Series

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Note: To use these formulas, you need to know the meaning of each variable within the formulas.

$n =$ the index which indicates the position in a sequence
$a_n =$ the n^{th} term in a sequence
$a_1 =$ the 1^{st} term in a sequence
$d =$ the common difference in an arithmetic sequence
$r =$ the common ratio in a geometric sequence/series
$S_n =$ the sum of the first n terms in a series

Lesson #1: Arithmetic and Geometric Sequences

Definition of a sequence:

Example 1:
Example 2:

Concept 1: Ways to define a sequence

There are two ways to define a sequence: _____________or ________________.

An **explicitly** defined sequence is like a formula. Plugging into the formula gives the terms of the sequence.

- **Subscripts name terms**. They are not values in the problem.

Example 3: Consider a_n = 3n + 2. Find the first 3 terms (a_1, a_2, a_3) of this sequence.

A **recursively** defined sequence has two parts;

(1) It gives the **first term** and

(2) all of the other terms are **found using operations on the previous term(s)**

Key Points for Recursive Formulas

- **Subscripts name terms**. They are not values in the problem.
- If the next term is a_n the term before it will be a_{n-1} because **n-1 is one smaller than n**.

  For example, if a_n is a_3, a_{n-1} is a_2.
- If the next term is a_{n+1} the term before it will be a_n because **n is one smaller than n+1**.

  For example, if a_{n+1} is a_3, a_n is a_2.

Bottom Line: Build off the last term!

For each problem, find the **next** four terms.

Example 4: $a_1 = 4$ $a_{n+1} = (a_n)^2 - 10$	Example 5: $a_1 = 5$ $a_n = a_{n-1} + n$

Label the following as either an Explicit Formula or a Recursive Formula.

a) $t_1 = 5$
$t_n = (t_{n-1}) + 3$

____

b) $t_n = n + 3$

____

c) $t_n = 10 - 4(n - 1)$

____

d) $t_1 = 6$
$t_n = 7(t_{n-1})^2$

____

e) $t_n = (n - 1)^2$

____

f) $t_n = 5 + 3(n - 1)$

____

Arithmetic Sequences

If a sequence of values follows a pattern of **adding a fixed amount** from one term to the next, it is referred to as an arithmetic sequence. The number added to each term is constant (always the same). The fixed amount is called the **common difference, *d***.

a) The following is an example of an arithmetic sequence: 3,8,13,18,23 . . .
   What is the common difference, d?

b) What is the common difference of the following arithmetic sequence?: $5, \frac{7}{2}, 2, \frac{1}{2}, \ldots$

Geometric Sequences

If a sequence of values follows a pattern of **multiplying a fixed amount** (not zero) times each term to arrive at the following term, it is referred to as a geometric sequence. The number multiplied each time is constant (always the same). The fixed amount is called the **common ratio, *r***.

a) The following is an example of a geometric sequence: $8, 56, 392, 2744$ . . .
   What is the common ratio, r?

b) What is the common ratio of the following geometric sequence?: 27, 9, 3, 1, $\frac{1}{3}, \frac{1}{9}$, . . .

Concept 2: Generating a Sequence

A sequence can be defined by a formula (or generator) which generates each term. *(Note: the variable "n" appears in most generator. It is used to indicate the position of a term in a sequence.)*

- The **formula** to generate any **arithmetic sequence** can be written in the form:

- The **formula** to generate any **geometric sequence** can be written in the form:

Example 6: Find a formula to generate the ***arithmetic*** sequence 3, 5, 7, … and use it to generate the 50th term.

Step 1:

Step 2:

Example 7: Find a formula to generate the ***geometric*** sequence 4, 12, 36, … and use it to determine the 19th term.

Step 1:

Step 2:

<u>You Try it!</u>

Determine if the sequence is arithmetic. If it is, find the common difference, the term named in the problem, and the explicit formula.

$25,\ 33,\ 41,\ 49,\ \ldots$

Find a_{31}

Determine if the sequence is geometric. If it is, find the common ratio, the term named in the problem, and the explicit formula.

$2,\ -6,\ 18,\ -54,\ \ldots$

Find a_{11}

SUMMARY

Solve as specified

Ex 2: Find the 100th term of $9,\ 8\frac{2}{3},\ 8\frac{1}{3},\ 8,\ 7\frac{2}{3},\ldots$

$d = -\frac{1}{3}$ $\quad a_n = 9 + (n-1)(-\frac{1}{3})$

$a_1 = 9$ $\quad a_{100} = 9 + 99(-\frac{1}{3}) = \boxed{-24}$

Ex 3: If $a_{41} = 53$ and $d = -\frac{4}{3}$, find a_7. *first need a_1!

$53 = a_1 + 40(-\frac{4}{3})$ $\quad a_7 = 106\frac{1}{3} + 6(-\frac{4}{3})$

$a_1 = 106\frac{1}{3}$ $\quad \boxed{a_7 = 98\frac{1}{3}}$

Exit Ticket

1. Which arithmetic sequence has a common difference of 4?
   1) $\{0, 4n, 8n, 12n, \ldots\}$
   2) $\{n, 4n, 16n, 64n, \ldots\}$
   3) $\{n+1, n+5, n+9, n+13, \ldots\}$
   4) $\{n+4, n+16, n+64, n+256, \ldots\}$

2. What is the common ratio of the geometric sequence shown below?

   $$-2, 4, -8, 16, \ldots$$

   1) $-\frac{1}{2}$
   2) 2
   3) -2
   4) -6

ID: 1

A2T/Ms. Williams Name___________________________________

Sequences and Series HW - Day 1

Date__________________ Period____

Determine if the sequence is arithmetic. If it is, find the common difference.

1) $-31,\ 169,\ 369,\ 569,\ \ldots$

2) $121,\ 1218,\ 12188,\ 121888,\ \ldots$

3) $6,\ 4,\ 2,\ 0,\ \ldots$

4) $3,\ 9,\ 27,\ 81,\ \ldots$

Determine if the sequence is geometric. If it is, find the common ratio.

5) $-2,\ -4,\ -12,\ -48,\ \ldots$

6) $4,\ 16,\ 64,\ 256,\ \ldots$

7) $3,\ 5,\ 7,\ 9,\ \ldots$

8) $-4,\ -\frac{4}{3},\ -\frac{4}{9},\ -\frac{4}{27},\ \ldots$

Find the first four terms in each sequence.

9) $a_n = \frac{2 + a_{n-1}}{2}$

$a_1 = 10$

10) $a_n = a_{n-1} + n$

$a_1 = -5$

11) $a_n = n^2$

12) $a_n = -\frac{10}{n+3}$

Find the term named in the problem and the explicit formula.

13) 11, 18, 25, 32, ...
Find a_{40}

14) −13, −43, −73, −103, ...
Find a_{40}

15) 36, 28, 20, 12, ...
Find a_{40}

16) 23, 33, 43, 53, ...
Find a_{20}

17) 2, 4, 8, 16, ...
Find a_{12}

18) 1, −4, 16, −64, ...
Find a_{10}

19) 2, 10, 50, 250, ...
Find a_9

20) 1, 2, 4, 8, ...
Find a_{10}

Answers to Sequences and Series HW - Day 1 (ID: 1)

1) $d = 200$

2) Not arithmetic

3) $d = -2$

4) Not arithmetic

5) Not geometric

6) $r = 4$

7) Not geometric

8) $r = \frac{1}{3}$

9) 10, 6, 4, 3

10) −5, −3, 0, 4

11) 1, 4, 9, 16

12) $-\frac{5}{2}, -2, -\frac{5}{3}, -\frac{10}{7}$

13) $a_{40} = 284$
Explicit: $a_n = 4 + 7n$

14) $a_{40} = -1183$
Explicit: $a_n = 17 - 30n$

15) $a_{40} = -276$
Explicit: $a_n = 44 - 8n$

16) $a_{20} = 213$
Explicit: $a_n = 13 + 10n$

17) $a_{12} = 4096$
Explicit: $a_n = 2 \cdot 2^{n-1}$

18) $a_{10} = -262144$
Explicit: $a_n = (-4)^{n-1}$

19) $a_9 = 781250$
Explicit: $a_n = 2 \cdot 5^{n-1}$

20) $a_{10} = 512$
Explicit: $a_n = 2^{n-1}$

Lesson #2: More with Arithmetic & Geometric Sequences

WARM-UP!
Writing Sequences using equations practice

1) $a_n = -3n + 2$ What type of sequence is this? ____________________

Write the first 5 terms of the sequence: ______________________________

2) $a_n = 162\left(\frac{1}{3}\right)^{n-1}$ What type of sequence is this? ____________________

Write the first 4 terms of the sequence: ______________________________

3) $a_n = 3(-2)^n$ What type of sequence is this? ____________________

Write the first 5 terms of the sequence: ______________________________

4) $a_n = 6 + \frac{1}{2}(n-1)$ What type of sequence is this? ____________________

Write the first 5 terms of the sequence: ______________________________

5) $a_1 = 1$ and the common difference of this arithmetic sequence is -3.

Write the first 4 terms of the sequence:______________________________

Write the explicit equation of this sequence:______________________________

6) $a_1 = 2$ and the common ratio of this geometric sequence is 10.

Write the first 4 terms of the sequence:______________________________

Write the explicit equation of this sequence:______________________________

7) Write an explicit equation for the sequence: $\frac{9}{2}, 3, 2, \frac{4}{3}, \frac{4}{9}, ...$

8) Write an explicit equation for the sequence: **-7.8, -5.6, -3.4, -1.2, . . .**

Concept 1: ***Arithmetic Mean***

Arithmetic mean- the mean average between any two numbers of a sequence

-a missing term can be found by finding the arithmetic mean of two terms.

Ex: Given the arithmetic sequence 84,_____, 110 , find the missing term.

Arithmetic mean $= \frac{84+110}{2}$

$= \frac{194}{2}$

$= 97$

Ex: Find the missing term of each arithmetic sequence.

1) 16, ______, 36

2) 23, ______, ______, 17

3) 12, _______,_______,_______, 0

4) If $a_4 = 80$ and $a_{12} = 32$ in a arithmetic sequence, find the 24th term.

Concept 2: ***Geometric Mean***

<u>Geometric mean</u>- the positive square root of the product of two numbers of a sequence

- A missing term can be found by finding the geometric mean of two terms

Ex: Given: 20, _____, 80

Step 1:
Step 2:
Step 3:

Find the missing term of each geometric sequence.

1) 3, ___ , 18.75

2) 28, ____, ____, 9604

3) 19,683, ______, ______ , ______, 243

4) If a_2 = -6 and a_5 = -1296 in a geometric sequence, find the 14th term.

SUMMARY

If $t_2 = 6$ and $t_5 = 162$ in a geometric sequence, find the 15th term.

2nd — — 5th

$162 \div 6 = 27$

$27^{(1/3)} = 3$

$a_n = 6(3)^{n-2}$

$a_{15} = 6(3)^{15-2}$

$a_{15} = 9{,}565{,}938$

Exit Ticket

1) What is the common ratio of the geometric sequence whose first term is 27 and fourth term is 64?

1) $\frac{3}{4}$

2) $\frac{64}{81}$

3) $\frac{4}{3}$

4) $\frac{37}{3}$

2) A sequence has the following terms: $a_1 = 4$, $a_2 = 10$, $a_3 = 25$, $a_4 = 62.5$. Which formula represents the nth term in the sequence?

1) $a_n = 4 + 2.5n$

2) $a_n = 4 + 2.5(n - 1)$

3) $a_n = 4(2.5)^n$

4) $a_n = 4(2.5)^{n-1}$

HW – Day 2

Two important types of sequences are **arithmetic** sequences and **geometric** sequences. Try to figure out which is which, and fill in the missing number in each sequence.

arithmetic or geometric

1) 4, 7, 10, _____, 16, . . . _____ _____

2) 2, 4, 8, 16, _____, . . . _____ _____

3) 1, _____, 9, 27, 81, . . . _____ _____

4) 3.5, 6, 8.5, 11, _____, . . . _____ _____

5) 8, 12, 18, _____, 40.5, . . . _____ _____

6) _____, -5.5, -9.5, -13.5, . . . _____ _____

7) 256, 64, 16, 4, _____, . . . _____ _____

8) -4, 8, -16, 32, -64, _____, . . . _____ _____

Given two terms in a geometric sequence find the term named in the problem.

9) $a_3 = 16$ and $a_6 = -128$
Find a_{10}

10) $a_4 = -54$ and $a_5 = -162$
Find a_{11}

11) $a_6 = 128$ and $a_5 = -64$
Find a_{12}

12) $a_6 = 3072$ and $a_3 = -48$
Find a_{10}

Given two terms in an arithmetic sequence find the term named in the problem.

13) $a_{13} = 121$ and $a_{34} = 331$
Find a_{31}

14) $a_{13} = 6$ and $a_{34} = 69$
Find a_{27}

15) $a_{13} = -2374$ and $a_{35} = -6774$
Find a_{40}

16) $a_{12} = -17$ and $a_{38} = -147$
Find a_{21}

Day 2 - Answers

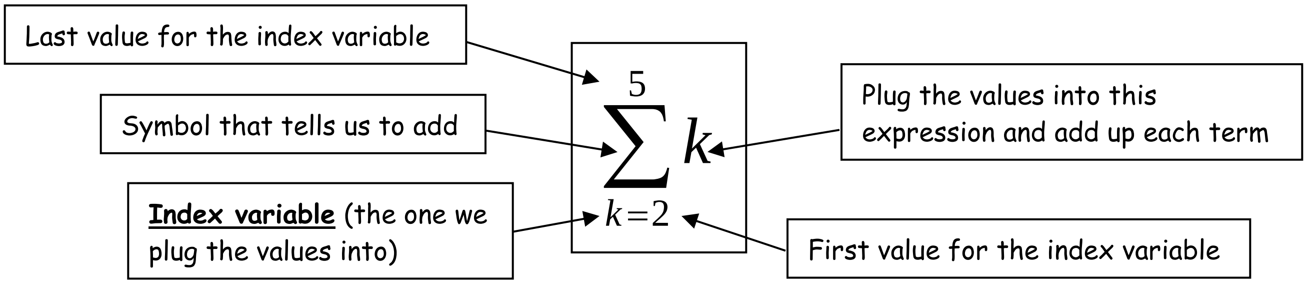
	arithmetic	or geometric
1) 4, 7, 10, __13__, 16, . . .	✓	
2) 2, 4, 8, 16, __32__, . . .		✓
3) 1, __3__, 9, 27, 81, . . .		✓
4) 3.5, 6, 8.5, 11, __13.5__, . . .	✓	
5) 8, 12, 18, __27__, 40.5, . . .		✓
6) __-1.5__, -5.5, -9.5, -13.5, . . .	✓	
7) 256, 64, 16, 4, __1__, . . .		✓

9) $a_{10} = -2048$ 10) $a_{11} = -118098$ 11) $a_{12} = 8192$ 12) $a_{10} = 786432$

13) $a_{31} = 301$ 14) $a_{27} = 48$ 15) $a_{40} = -7774$ 16) $a_{21} = -62$

Day 3 – Series

Review of Summation Notation



- Find each sum.

1. $$\sum_{a=1}^{4} a^2 - 2a$$

2. $$\sum_{m=1}^{4} (-1)^{m+1}(m^2 + 2m)$$

A **series** is the sum of the terms in a sequence. On the first page of this lesson, you reviewed the different ways we have used summation notation so far this year.

1. Which of the following represents the sum 8+12+16+20+24?

(1) $\sum_{a=0}^{4} 8 + a$

(2) $\sum_{a=1}^{4} 8 + 4(a-1)$

(3) $\sum_{a=0}^{5} 8 + 4a$

(4) $\sum_{a=0}^{4} 8 + 4a$

Solution:

You try it!

2. Which of the following represents the sum 3+8+15+24+ . . . 80?

(1) $\sum_{n=1}^{9} 3+5(n-1)$ (2) $\sum_{n=2}^{9} n^2-1$

(3) $\sum_{n=1}^{9} n^2-1$ (4) $\sum_{n=3}^{80} n$

3. *Exercise* #5: Which of the following represents the sum $3+6+12+24+48$?

(1) $\sum_{i=1}^{5} 3^i$ (3) $\sum_{i=0}^{4} 6^{i-1}$

(2) $\sum_{i=0}^{4} 3(2)^i$ (4) $\sum_{i=3}^{48} i$

Imagine having to find the sum of the arithmetic series 3+6+9+ . . . 129+132 by hand.

Imagine having to find the sum of the geometric series $\sum_{k=1}^{25} 4(3)^{k-1}$ by hand.

Luckily there are formulas to find the sum of the first *n* terms of any arithmetic or geometric series. Even more luckily, you do not have to memorize them because they are given to you on the **A2&T reference sheet**.

Sum of a Finite Arithmetic Series

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Sum of a Finite Geometric Series

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

The rest of this lesson will deal with some problems you might encounter where you need to use these formulas. You will also need to use the other formulas from the previous lessons to find the numbers to plug into the formulas.

Finding the nth term in an arithmetic sequence:

Finding the nth term in a geometric sequence:

1. Find the sum of the first 28 terms of the series 3 + 6 + 9 + 12 + ...
(Hint: You need to find a_{28} first).

2. Find the sum of the first 12 terms of the series, -3 + 6 - 12 + 24 - 48 + ...
(Since this one is geometric, we do not need a_{12})

3. Evaluate this series using the series formula: $\sum_{k=1}^{14} (1-2k)$

4. Evaluate using the series formula: $\sum_{k=1}^{5} 3^k$

5. ***Exercise* #4:** Find the sum of each arithmetic series described or shown below.

(a) The sum of the sixteen terms given by:

$$-10+-6+-2+\cdots+46+50.$$

(b) The first term is -8, the common difference, d, is 6 and there are 20 terms

(c) The last term is $a_{12}=-29$ and the common difference, d, is -3.

(d) The sum $5+8+11+\cdots+77$.

6. ***Exercise* #3:** Which of the following represents the sum of a geometric series with 8 terms whose first term is 3 and whose common ratio is 4?

(1) 32,756

(3) 42,560

(2) 28,765

(4) 65,535

____

7. ***Exercise* #4:** Find the value of the geometric series shown below. Show the calculations that lead to your final answer.

$$6+12+24+\cdots+768$$

8. Find the sum of the first 20 terms of the sequence 4, 6, 8, 10, ...

9. Find the sum of the sequence -8, -5, -2, ..., 7

10. Find the sum of the first 8 terms of the sequence -5, 15, -45, 135, ...

SUMMARY

Evaluate each arithmetic series.

a) $a_1 = 4, \quad a_n = 22, \quad n = 10$

$$S_{10} = \frac{10(4+22)}{2}$$

$$= \boxed{130}$$

Evaluate each geometric series.

1, -4, 16, -64, . . . n=9

$$S_9 = \frac{1(1-(-4)^9)}{1-(-4)}$$

$$= \boxed{52{,}429}$$

Exit Ticket

Which summation represents
$5+7+9+11+\ldots+43$?

1) $\sum_{n=5}^{43} n$

2) $\sum_{n=1}^{20} (2n+3)$

3) $\sum_{n=4}^{24} (2n-3)$

4) $\sum_{n=3}^{23} (3n-4)$

Day 3 – Series HW

Sum of Arithmetic Series **Name________________________**

Sum of a Finite Arithmetic Series: $S_n = \dfrac{n(a_1 + a_n)}{2}$

1) Evaluate each arithmetic series.

a) $a_1 = 4, \quad a_n = 22, \quad n = 10$

b) $a_1 = -2, \quad a_n = -156, \quad n = 10$

c) $\sum_{n=1}^{100} (2n - 1)$

d) $\sum_{n=1}^{50} (3n + 2)$

e) 20, 23, 26, 29, . . . n=25

f) 20, 30, 40, 50, . . . n=30

2) Determine the number of terms *n* in the following arithmetic series.

a) $a_1 = 19, \quad a_n = 96, \quad S_n = 690$

b) $a_1 = 15, \quad a_n = 79, \quad S_n = 423$

3) Determine the sum of the all of the even integers from 2 to 2000.

4) A company offers Sue a starting salary of $50,000 plus a guaranteed pay increase of 5,000 each year. What is the total amount of money Sue will have earned after working for 25 years?

Sum of Geometric Series **Name________________________**

Sum of a Finite Geometric Series: $S_n = \dfrac{a_1(1-r^n)}{1-r}$

1) Evaluate each geometric series.

a) $\sum_{k=1}^{7}(4^{k-1})$

b) $\sum_{n=1}^{12} 4\cdot 3^{n-1}$

c) 1, -4, 16, -64, . . . n=9

d) 1, 2, 4, 8, . . . n=30

e) $a_1 = 4, \quad a_n = 8748, \quad r = 3$

f) $a_1 = 4, \quad a_n = 1024, \quad r = 2$

2) Determine the number of terms *n* in the following geometric series.

a) $a_1 = -2, \quad r = 5, \quad S_n = -62$

b) $a_1 = 3, \quad r = -3, \quad S_n = -60$

3) A company offers Jim a starting salary of $50,000 plus a guaranteed pay increase of 5% each year. What is the total amount of money Jim will have earned after working for 25 years?

Answers to Arithmetic Series

1) Evaluate each arithmetic series.

a) $a_1 = 4, \quad a_n = 22, \quad n = 10$

$S_{10} = \frac{10(4+22)}{2} = \boxed{130}$

b) $a_1 = -2, \quad a_n = -156, \quad n = 10$

$S_{10} = \frac{10(-2+-156)}{2} = \boxed{-790}$

c) $\sum_{n=1}^{100}(2n-1)$

1st find 1st term → $2(1)-1 = 1$

+ find last term → $2(100)-1 = 199$

then use eq:

$S_{100} = \frac{100(1+199)}{2} = \boxed{10{,}000}$

d) $\sum_{n=1}^{50}(3n+2)$

1st term → $3(1)+2 = 5$

last term → $3(50)+2 = 152$

$S_{50} = \frac{50(5+152)}{2} = \boxed{3{,}925}$

e) 20, 23, 26, 29, . . . n=25

$a_1 = 20 \quad a_{25} = ?$

find 25th term →

$a_{25} = 20 + 3(25-1) = 92$

$S_{25} = \frac{25(20+92)}{2} = \boxed{1{,}400}$

f) 20, 30, 40, 50, . . . n=30

$a_1 = 20 \quad a_{30} = ?$

$a_{30} = 20 + 10(30-1) = 310$

$S_{30} = \frac{30(20+310)}{2} = \boxed{4{,}950}$

2) Determine the number of terms n in the following arithmetic series.

a) $a_1 = 19, \quad a_n = 96, \quad S_n = 690$

$S_n = \frac{n(19+96)}{2}$

$690 = \frac{n(115)}{2}$

$\boxed{12 = n}$ 12 terms

b) $a_1 = 15, \quad a_n = 79, \quad S_n = 423$

$423 = \frac{n(15+79)}{2}$

$\boxed{9 = n}$ 9 terms

3) Determine the sum of the all of the even integers from 2 to 2000.

$2 + 4 + 6 + \ldots + 1998 + 2000$

1st find # terms

$a_n = 2 + 2(n-1)$

$2000 = 2 + 2n - 2$

$1000 = n$ ← # terms

$S_{1000} = \frac{1000(2+2000)}{2} = \boxed{1{,}001{,}000}$

4) A company offers Sue a starting salary of \$50,000 plus a guaranteed pay increase of 5,000 each year. What is the total amount of money Sue will have earned after working for 25 years?

$n = 25$

$a_1 = 50{,}000$

$a_{25} = 50000 + 5000(25-1)$

$a_{25} = 170{,}000$

$S_{25} = \frac{25(50000 + 170000)}{2}$

$= \boxed{2{,}750{,}000}$

Answers to Geometric Series

1) Evaluate each geometric series.

a) $\sum_{k=1}^{7}(4^{k-1})$

Do by hand $= 4^0 + 4^1 + 4^2 + 4^3 + 4^4 + 4^5 + 4^6 = 5{,}461$

or use formula

$S_7 = \frac{1(1-4^7)}{1-4} = \boxed{5461}$

b) $\sum_{n=1}^{12} 4 \cdot 3^{n-1} = 4 + 12 + 36 + \ldots$ → $a_1 = 4 \cdot 3^0 = 4$

$S_{12} = \frac{4(1-3^{12})}{1-3} = \boxed{1{,}062{,}880}$

c) 1, -4, 16, -64, . . . n=9

by hand: $1 + -4 + 16 + -64$ etc

or use formula

$S_9 = \frac{1(1-(-4)^9)}{1-(-4)} = \boxed{52{,}429}$

d) 1, 2, 4, 8, . . . n=30 (·2 ·2 ·2)

$S_{30} = \frac{1(1-2^{30})}{1-2} = \boxed{1{,}073{,}741{,}823}$

e) $a_1 = 4, \quad a_n = 8748, \quad r = 3$

first find # of terms: $n = 8$

use → $8748 = 4(3)^{n-1}$

or just do it: $4 + 12 + 36 + \ldots$

$S_8 = \frac{4(1-3^8)}{1-3} = 13{,}120$

f) $a_1 = 4, \quad a_n = 1024, \quad r = 2$

$4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1024 = \boxed{2{,}044}$

2) Determine the number of terms n in the following geometric series.

a) $a_1 = -2, \quad r = 5, \quad S_n = -62$

Just do it:

$-2 + -10 + -50 = -62$ (3 terms)

or

$-62 = \frac{-2(1-5^n)}{1-5}$

$2 \cdot -62 = \frac{1-5^n}{2} \cdot 2$

$-124 = 1 - 5^n$

$-125 = -5^n$

$125 = 5^n$

$n = 3$

b) $a_1 = 3, \quad r = -3, \quad S_n = -60$

$3 + -9 + 27 + -81 = -60$ (4 terms)

or use formula

$-60 = \frac{3(1-(-3)^n)}{1-(-3)}$

$-60 = \frac{3(1-(-3)^n)}{4}$

$-240 = 3(1-(-3)^n)$

$-80 = 1 - (-3)^n$

$+81 = (-3)^n$

$n = 4$

3) A company offers Jim a starting salary of \$50,000 plus a guaranteed pay increase of 5% each year. What is the total amount of money Jim will have earned after working for 25 years?

$S_{25} = \frac{50{,}000(1-(1.05)^n)}{1-1.05}$

$\boxed{\$2{,}356{,}354}$

Day 4 - Sequences and Series Mixed Practice 2014

1. What is the difference between an arithmetic and a geometric sequence?

2. Find the next three terms of each sequence

   a. 9, 16, 23, ________, ________, ________

   b. 100, -200, 400, ________, ________, ________

   c. -8, -5, -2, ________, ________, ________

3. Find the first three terms of each sequence where *d* is the common difference, and *r* is the common ratio

   a. $a_1 = 576, r = -\frac{1}{2}$ ________, ________, ________

   b. $a_1 = 2, d = 13$ ________, ________, ________

   c. $a_1 = \frac{5}{8}, d = \frac{3}{8}$ ________, ________, ________

4. Find a_8 if $a_n = 4 + 3n$.

5. Find a_7 if $a_n = 12\left(\frac{1}{2}\right)^{n-1}$.

6. Find a_{12} for -17, -13, -9, …

7. Find a_8 for 4, -12, 36, …

8. Find a_{14} if $a_1 = 3$ and the common difference is $d = 7$

9. Find a_8 if the common ratio $r = 3$ and $a_1 = \frac{1}{3}$

10. Write the equation for the *n*th term for each sequence

   a. 7, 16, 25, 34, …

   b. 36, 12, 4, …

11. Find the 10th term of the arithmetic sequence given the following information: $a_3 = 55, a_7 = 115$

12. Find the 7th term of the geometric sequence given the following information: $a_1 = 9, a_5 = 144$

Write the first 5 terms of each sequence described below with recursive equations:

13. $$a_1 = -3$$ $$a_{n+1} = 3a_n + 10$$

14. $$a_1 = 5$$ $$a_n = a_{n-1} - 4$$

15. $$a_1 = 2$$ $$a_2 = -1$$ $$a_{n+2} = a_{n+1} + 4a_n$$

Series Questions

Find the value of each series below.

16. $\sum_{j=1}^{3}(2j-6)$

17. $\sum_{k=0}^{3} 2(4)^k$

18. $6\sum_{j=1}^{4} j^j$

19. Find $2\sum_{k=3}^{6} x_k$ if $x_3 = 2$, $x_4 = -4$, $x_5 = 8$ and $x_6 = 10$

20. Write this series using sigma notation: $2+9+16$

21. Write this series using sigma notation: $3+12+48+192+768$

Evaluate each arithmetic series.

22. $a_1 = -2, \quad a_8 = 33, \quad S_8 = ?$

23. $\sum_{n=1}^{50} (7n - 1)$

Evaluate each geometric series.

24. $\sum_{n=1}^{11} 5 \cdot 3^{n-1}$

25. 1, 2, 4, 8, . . . n = 25

26. Determine the number of terms *n* in the following geometric series.

$$a_1 = 2, \quad r = 5, \quad S_n = 62$$

27) Expand the following binomials. (Remember to fully simplify each term.)

a) $(x+3)^5$

b) $(a-2)^6$

c) $(3x+2y)^3$

d) Find the 3^{rd} term of $(2a^2+3y)^7$.