Algebra and Functions

Part 1: Equation of a Line

AS Level		A-Level
Pt 1: Equation of a Line	**Pt. 2**: Circles	**Pt 3:** Parametric and Cartesian Equations

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1. Find in the form $y = mx + c$, the equation of the straight line passing through the pair of co-ordinates $(-\frac{1}{2}, -2)$ and $(2, 8)$. **(3)**

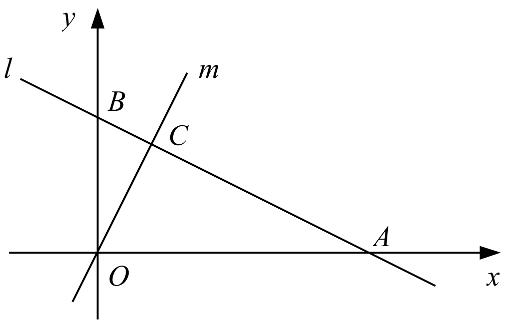
2. The straight line l passes through the points A (-6, 8) and B (3, 2).
a. Find an equation of the line l **(3)**
b. Show that the points C (9, -2) lies on l. **(2)**

3. The straight line l_1 passes through the points P(-2, 1) and Q(4, -1).
a. Find the equation of l_1 in thr form $ax + by + c = 0$, where a, b, and c are integers. **(3)**
The straight line l_2 passes through the points R (2, 4) and through the mid-point PQ.
b. Find the equation of l_2 in the form $y = mx + c$. **(3)**

4. The straight line p has the equation $3x - 4y + 8 = 0$.
The straight line q is parallel to p and passes through the point with coordinates (8, 5).
a. Find the equation of q in the form $y = mx + c$. **(2)**
The straight line r is perpendicular to p and passes through the point with coordinates $(-4, 6)$.
b. Find the equation of r in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**
c. Find the coordinates of the point where lines q and r intersect. **(4)**

5. The vertices of a triangle are the points A (5, 4), B (−5, 8) and C (1, 11).
a. Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(2)**
b. Find the coordinates of the point M, the mid-point of AC. **(1)**
c. Show that OM is perpendicular to AB, where O is the origin. **(2)**

6. The diagram shows the straight line l with equation $x + 2y - 20 = 0$ and the straight line m which is perpendicular to l and passes through the origin O.



a. Find the coordinates of the points A and B where l meets the x-axis and y-axis respectively. **(2)**

Given that l and m intersect at the point C,
b. find the ratio of the area of triangle OAC to the area of triangle OBC. **(5)**

7. The vertices of a triangle are the points P (3, c), Q (9, 2) and R (3c, 11) where c is a constant.
Given that $\angle PQR = 90°$
a. Find the value of c **(5)**
b. Show that the length of PQ is $k\sqrt{10}$, where k is an integer to be found **(3)**
c. Find the area of triangle PQR. **(4)**

Mark Scheme

1.

Gradient = $\frac{8+2}{2+0.5} = 4$	**M1**
$y - 8 = 4(x - 2)$	**M1**
$y = 4x$	**M1**

2a.

Gradient = $\frac{2-8}{3+6} = -\frac{2}{3}$	**M1**
$y - 8 = -\frac{2}{3}(x + 6)$	**M1**
$2x + 3y - 12 = 0$	**M1**

2b.

$2(9) + 3(-2) - 12 = 0$	**M1**
Therefore, C lies on C.	**M1**

3a.

Gradient = $\frac{-1-1}{4+2} = -\frac{1}{3}$	**M1**
$y - 1 = -\frac{1}{3}(x + 2)$ $3y - 3 = -x - 2$	**M1**
$x + 3y - 1 = 0$	**M1**

3b.

Mid-point of PQ = $(\frac{-2+4}{2}, \frac{1-1}{2}) = (1, 0)$	**M1**
Gradient of l_2 = $\frac{0-4}{1-2} = 4$	**M1**
$y = 4(x - 1)$ $y = 4x - 4$	**M1**

4a.

p $\rightarrow y = \frac{3}{4}x = 2$ gradient = $\frac{3}{4}$	**M1**
$y - 5 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 1$	**M1**

4b.

Perpendicular gradient = $-\frac{4}{3}$	**M1**
$y - 6 = -\frac{4}{3}(x + 4)$ $3y - 18 = -4x - 16$	**M1**
$4x + 3y - 2 = 0$	**M1**

4c.

q $\rightarrow 3x - 4y - 4 = 0 \rightarrow 9x - 12y - 12 = 0$	**M1**
r $\rightarrow 16x + 12y - 8 = 0$	**M1**
Adding, $25x - 20 = 0$ $x = \frac{4}{5}$	**M1**
$y = \frac{3(0.8)-4}{4} = -\frac{2}{5}$ Co-oridnates = $(\frac{4}{5}, -\frac{2}{5})$	**M1**

5a.

Gradient = $\frac{8-4}{-5-5} = -\frac{2}{5}$	**M1**
$y - 4 = -\frac{2}{5}(x - 5)$ $5y - 20 = -2x + 10$ $2x + 5y - 30 = 0$	**M1**

5b.

Midpoint = $(\frac{5+1}{2}, \frac{4+11}{2})$ = (3, 3.5)	**M1**

5c.

Gradient of OM = 3.5 ÷ 3 = $\frac{5}{2}$	**M1**
Gradient OM x Gradient AB = $\frac{5}{2}$ x $-\frac{2}{5}$ = -1 Therefore, OM is perpendicular to AB.	**M1**

6a.

At A, $y = 0$, $x = 20 \rightarrow$ A(20, 0)	**M1**
At B, $x = 0$, $y = 10 \rightarrow$ B(0, 10)	**M1**

6b.

l $\rightarrow y = 10 - 0.5x$ Gradient of l = -0.5	**M1**
Gradient of m = 2 Equation of line m: $y = 2x$	**M1**
At C, $10 - 0.5x = 2x$ $x = 4$ Therefore, C = (4,8)	**M1**
Area of ΔOAC: area of ΔOBC 0.5 x 20 x 8 : 0.5 x 10 x 4	**M1**
4 : 1	**M1**

7a.

Gradient of PQ = $\frac{2-c}{9-3} = \frac{2-c}{6}$	**M1**
Gradient of QR = $\frac{11-2}{3c-9} = \frac{3}{c-3}$	**M1**
$\angle PQR = 90^0$, therefore PQ is perpendicular to QR $\frac{2-c}{6} \times \frac{3}{c-3}$ = -1 3(2 – c) = -6 (c – 3) 3c = 12 c = 4	

7b.

$PQ^2 = 6^2 + 2^2 = 40$	**M1**
PQ = $\sqrt{40} = 2\sqrt{10}$ k = 2	**M1**

7c.

QR = $\sqrt{3^2 + 9^2} = \sqrt{90} = 3\sqrt{10}$	**M1**
Area = ½ x PQ x QR = 30	**M1**

Algebra and Functions

Part 2: Circles

Jethwa Maths

AS Level		A-Level
Pt 1: Equation of a Line	**Pt. 2**: Circles	**Pt 3:** Parametric and Cartesian Equations

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1. Find the coordinates of the cente and the radius of the circles $9x^2 + 9y^2 + 6x - 24y + 8 = 0$ **(3)**

2. Find whether the (7, -3) lies inside or outside the circle $x^2 + y^2 + 10x - 4y = 140$ **(3)**

3. Find the equation of the normal to the circle with equation $x^2 + y^2 + 4x = 13$ at the point (-1, 4). **(3)**

4. The line with equaton $y = 1 - x$ intersects the circle with equation $x^2 + y^2 + 6x + 2y = 27$ at the points A and B. Find the length of the chord AB, giving your answer in the form $k\sqrt{2}$
(3)

5. The circle C has centre $(3, -2)$ and radius 5.
a. Write down an equation of C in cartesian form. **(1)**
The line $y = 2x - 3$ intersects C at the points A and B.
b. Show that $AB = 4\sqrt{5}$. **(5)**

6. The circle C touches the y-axis at the point A (0, 3) and passes through the point B (2, 7).
a. Find an equation of the perpendicular bisector of AB. **(4)**
b. Find an equation for C. **(3)**
c. Show that the tangent to C at B has equation $3x - 4y + 22 = 0$. **(4)**

7. The circle C has equation $x^2 + y^2 - 8x + 4y + 12 = 0$.
a. Find the coordinates of the centre of C and the radius of C. **(2)**
The point P has coordinates (3, 5) and the point Q lies on C.
b. Find the largest and smallest values of the length PQ, giving your answers in the form $k\sqrt{2}$. **(3)**
c. Find the length of PQ correct to 3 significant figures when the line PQ is a tangent to C. **(2)**

8. Circle C_1 has the equation $x^2 + y^2 - 2ay = 0$, where a is a positive constant.
a. Find the coordinates of the centre and the radius of C_1. **(2)**
Circle C_2 has the equation $x^2 + y^2 - 2bx = 0$, where b is a constant and $b > a$.
b. Sketch C_1 and C_2 on the same diagram. **(3)**

9. The circle C has equation $x^2 + y^2 - 8x - 16y + 72 = 0$.
a. Find the coordinates of the centre and the radius of C. **(2)**
b. Find the distance of the centre of C from the origin in the form $k\sqrt{5}$. **(2)**
The point A lies on C and the tangent to C at A passes through the origin O.
c. Show that $OA = 6\sqrt{2}$. **(2)**

Mark Scheme

1.

$x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0$	**M1**
$(x + \frac{1}{3})^2 - \frac{1}{9} + \left(y - \frac{4}{3}\right)^2 - \frac{16}{9} + \frac{8}{9} = 0$ $(x + \frac{1}{3})^2 + \left(y - \frac{4}{3}\right)^2 = 1$	**M1**
Centre $(-\frac{1}{3}, 0)$ Radius 1	**M1**

2.

$(x + 5)^2 - 25 + (y - 2)^2 - 4 = 140$ $(x + 5)^2 + (y - 2)^2 = 169$	**M1**
Centre (-5, 2) Radius 13	**M1**
Distance to centre = $\sqrt{144 + 25} = 13$ Therefore point is on circle.	**M1**

3.

$(x + 2)^2 - 4 + y^2 = 13$ Therefore, centre (-2, 0)	**M1**
Gradient $= \frac{0-4}{-2+1} = 4$	**M1**
Therefore, $y - 4 = 4(x + 1)$ $y = 4x + 8$	**M1**

4.

$x^2 + (1 - x)^2 + 6x + 2(1 - x) = 27$	**M1**
$x^2 + x - 12 = 0$ $(x + 4)(x - 3) = 0$ $x = -4, y = 1 - -4 = 5$ $x = 3, y = 1 - 3 = -2$	**M1**
Therefore, AB = $\sqrt{49 + 49} = 7\sqrt{2}$	**M1**

5a.

$(x - 3)^2 + (y + 2)^2 = 25$	**M1**

5b.

$(x - 3)^2 + [(2x - 3) + 2]^2 = 25$	**M1**
$(x - 3)^2 + (2x - 1)^2 = 25$ $x^2 - 2x - 3 = 0$	**M1**
$(x + 1)(x - 3) = 0$ $x = -1, y = 2(-1) - 3 = -5$ $x = 3, y = 2(3) - 3 = 3$	**M1**
$AB^2 = 4^2 + 8^2 = 80$	**M1**
AB = $\sqrt{80} = 4\sqrt{5}$	**M1**

6a.

Midpoint AB = $(\frac{0+2}{2}, \frac{3+7}{2}) = (1, 5)$	**M1**
Gradient AB = $\frac{7-3}{2-0} = 2$	**M1**
Therefore perpendicular gradient = -$-\frac{1}{2}$	**M1**
$y - 5 = -\frac{1}{2}(x - 1)$	**M1**

$y = \frac{11}{2} - \frac{1}{2}x$	

6b.

Circle touches y-axis at (0, 3) Therefore y-coordordinate of centre = 3	**M1**
$3 = \frac{11}{2} - \frac{1}{2}x$ $x = 5$	**M1**
Centre (5, 3) radius 5. $(x - 5)^2 + (y - 3)^2 = 25$	**M1**

6c.

Gradient of radius = $\frac{7-3}{2-5} = -\frac{4}{3}$	**M1**
Therefore gradient of tangent = $\frac{3}{4}$	**M1**
$y - 7 = \frac{3}{4}(x - 2)$ $4y - 28 = 3x - 6$	**M1**
$3x - 4y + 22 = 0$	**M1**

7a.

$(x - 4)^2 - 16 + (y + 2)^2 - 4 + 12 = 0$ $(x - 4)^2 + (y + 2)^2 = 8$	**M1**
Centre: (4, −2) Radius: $2\sqrt{2}$	**M1**

7b.

Distance P to centre = $\sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$	**M1**
Therefore, max PQ = $5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$	**M1**
Minimum PQ = $5\sqrt{2}$ - $2\sqrt{2} = 3\sqrt{2}$	**M1**

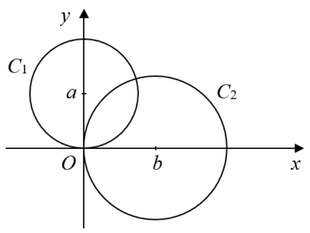
7c.

Tangent perpendicular to radius: $PQ^2 = (5\sqrt{2})^2 - (5\sqrt{2})^2 = 50 - 8 = 42$	**M1**
$PQ = \sqrt{42} = 6.48$	**M1**

8a.

$x^2 + (y - a)^2 - a^2 = 0$ $x^2 + (y - a)^2 = a^2$	**M1**
Centre: (0, a) Radius: a	**M1**

8b.

C_2: $(x - b)^2 - b^2 + y^2 = 0$ $(x - b)^2 + y^2 = b^2$	**M1**
Centre: (b, 0) Radius: b	**M1**
	**M1**

9a.

$(x-4)^2 - 16 + (y-8)^2 - 64 + 72 = 0$ $(x-4)^2 + (y-8)^2 = 8$	**M1**
Centre: (4, 8) Radius: $2\sqrt{2}$	**M1**

9b.

$\sqrt{16+64} = \sqrt{80}$	**M1**
$= 4\sqrt{5}$	**M1**

9c.

Tangent perpendicular to radius:	**M1**
$OA^2 = (\sqrt{80})^2 - (2\sqrt{2})^2 = 72$ $OA = \sqrt{72} = 6\sqrt{2}$	**M1**