Algebra and	Functions			
Part 1	; Equation	Of	3	Line



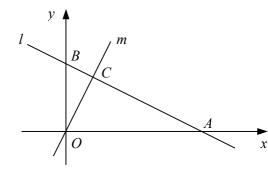
**AS Level** Pt 1: Equation of a Line

Pt. 2: Circles

A-Level Pt 3: Parametric and Cartesian Equations

1. Find in the form $y = mx + c$ , the equation of the straight line passing through the pair of co-ordinates $(-\frac{1}{2}, -2)$ and $(2, 8)$ .	(3)
<ul> <li>2. The straight line <i>l</i> passes through the points A (-6, 8) and B (3, 2).</li> <li>a. Find an equation of the line <i>l</i></li> <li>b. Show that the points C (9, -2) lies on <i>l</i>.</li> </ul>	(3) (2)
3. The straight line $l_1$ passes through the points P(-2, 1) and Q(4, -1). a. Find the equation of $l_1$ in thr form $ax + by + c = 0$ , where <i>a</i> , <i>b</i> , and <i>c</i> are integers. The straight line $l_2$ passes through the points R (2, 4) and through the mid-point PQ. b. Find the equation of $l_2$ in the form $y = mx + c$ .	(3) (3)
4. The straight line <i>p</i> has the equation $3x - 4y + 8 = 0$ . The straight line <i>q</i> is parallel to <i>p</i> and passes through the point with coordinates (8, 5). a. Find the equation of <i>q</i> in the form $y = mx + c$ . The straight line <i>r</i> is perpendicular to <i>p</i> and passes through the point with coordinates (-4, 6). b. Find the equation of <i>r</i> in the form $ax + by + c = 0$ , where <i>a</i> , <i>b</i> and <i>c</i> are integers. c. Find the coordinates of the point where lines <i>q</i> and <i>r</i> intersect.	(2) (3) (4)
<ul> <li>5. The vertices of a triangle are the points A (5, 4), B (-5, 8) and C (1, 11).</li> <li>a. Find the equation of the straight line passing through A and B, giving your answer in the form ax + by + c = 0, where a, b and c are integers.</li> <li>b. Find the coordinates of the point M, the mid-point of AC.</li> <li>c. Show that OM is perpendicular to AB, where O is the origin.</li> </ul>	(2) (1) (2)
6. The diagram shows the straight line <i>l</i> with equation $x + 2y - 20 = 0$ and the straight line <i>m</i> which is normal disulate <i>l</i> and reasons through the arisin $Q$ .	

which is perpendicular to l and passes through the origin O.



a. Find the coordinates of the points A and B where l meets the x-axis and *y*-axis respectively.

(2) Given that *l* and *m* intersect at the point *C*, b. find the ratio of the area of triangle *OAC* to the area of triangle *OBC*. (5)

7. The vertices of a triangle are the points P(3, c), Q(9, 2) and R(3c, 11) where c is a constant.

- Given that  $\angle PQR = 90^{\circ}$ a. Find the value of *c* (5) b. Show that the length of PQ is  $k\sqrt{10}$ , where k is an integer to be found (3) (4)
- c. Find the area of triangle PQR.

#### Mark Scheme

1.	
Gradient = $\frac{8+2}{2+0.5} = 4$	M1
y - 8 = 4(x - 2)	M1
y = 4x	M1

2a	•

24.	
Gradient $=\frac{2-8}{3+6} = -\frac{2}{3}$	<b>M1</b>
$y-8 = -\frac{2}{3}(x+6)$	<b>M1</b>
2x + 3y - 12 = 0	M1

## 2b.

2(9) + 3(-2) - 12 = 0	M1
Therefore, C lies on C.	M1

#### 3a.

Gradient $=\frac{-1-1}{4+2} = -\frac{1}{3}$	M1
$y - 1 = -\frac{1}{3}(x + 2)$ 3y - 3 = -x - 2	M1
x + 3y - 1 = 0	M1

## <u>3b.</u>

Mid-point of PQ = $(\frac{-2+4}{2}, \frac{1-1}{2}) = (1, 0)$	M1
Gradient of $l_2 = \frac{0-4}{1-2} = 4$	M1
y = 4(x - 1) y = 4x - 4	M1

#### 4a.

$p \rightarrow y = \frac{3}{4}x = 2$ gradient = $\frac{3}{4}$	M1
$y - 5 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 1$	M1

4b.

Perpendicular gradient = $-\frac{4}{3}$	M1
$y - 6 = -\frac{4}{3}(x + 4)$ 3y - 18 = -4x - 16	M1
4x + 3y - 2 = 0	M1

4c.	
$q \rightarrow 3x - 4y - 4 = 0 \rightarrow 9x - 12y - 12 = 0$	M1
$\mathbf{r} \to 16x + 12y - 8 = 0$	M1
Adding, $25x - 20 = 0$	
$x = \frac{4}{5}$	M1
$y = \frac{3(0.8) - 4}{4} = -\frac{2}{5}$	
Co-oridnates = $\left(\frac{4}{5}, -\frac{2}{5}\right)$	M1
	149 Maths

5a.	
Gradient = $\frac{8-4}{-5-5} = -\frac{2}{5}$	M1
$y - 4 = -\frac{2}{5}(x - 5)$	
5y - 20 = -2x + 10 2x + 5y - 30 = 0	M1
2x + 5y - 30 = 0	

## 5b.

Midpoint = $\left(\frac{5+1}{2}, \frac{4+11}{2}\right) = (3, 3.5)$	M1
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# 5c.

Gradient of OM = $3.5 \div 3 = \frac{5}{2}$	M1
Gradient OM x Gradient $AB = \frac{5}{2}x - \frac{2}{5} = -1$ Therefore, OM is perpendicular to AB.	M1

#### 6a.

At A, $y = 0, x = 20 \rightarrow A(20, 0)$	M1
At B, $x = 0, y = 10 \rightarrow B(0, 10)$	M1

## <u>6</u>b.

$1 \rightarrow y = 10 - 0.5x$	M1
Gradient of $1 = -0.5$	IVII
Gradient of $m = 2$	M1
Equation of line m: $y = 2x$	IVII
At C, $10 - 0.5x = 2x$	
x = 4	M1
Therefore, $C = (4,8)$	
Area of $\triangle OAC$ : area of $\triangle OBC$	M1
0.5 x 20 x 8 : 0.5 x 10 x 4	IVII
4:1	M1

### 7a.

74.	
Gradient of PQ = $\frac{2-c}{9-3} = \frac{2-c}{6}$	M1
Gradient of QR = $\frac{11-2}{3c-9} = \frac{3}{c-3}$	<b>M1</b>
$\angle PQR = 90^{\circ}$ , therefore PQ is perpendicular to QR	
$\frac{2-c}{2} \times \frac{3}{2} = -1$	
6 c-3	
3(2-c) = -6 (c-3) 3c = 12	
c = 4	

7b.	
$PQ^2 = 6^2 + 2^2 = 40$	M1
$PQ = \sqrt{40} = 2\sqrt{10}$ k = 2	M1

#### 7c.

$QR = \sqrt{3^2 + 9^2} = \sqrt{90} = 3\sqrt{10}$	M1
Area = $\frac{1}{2} \times PQ \times QR = 30$	M1

Bring Maths

Par	rt 2; Circ		String V	
	AS L Pt 1: Equation of a Line	.evel Pt. 2: Circles	<b>A-Level Pt 3:</b> Parametric and Cartesian Equations	
1. Find the c	coordinates of the cente a	nd the radius of the	circles $9x^2 + 9y^2 + 6x - 24y + 8 = 0$	(3
2. Find whe	ther the (7, -3) lies inside	or outside the circle	$e x^2 + y^2 + 10x - 4y = 140$	(3
3. Find the e	equation of the normal to	the circle with equa	attion $x^2 + y^2 + 4x = 13$ at the point (-1, 4).	(3
	with equaton $y = 1 - x$ integration of the chord AB, givi		h equation $x^2 + y^2 + 6x + 2y = 27$ at the points <i>h</i> he form $k\sqrt{2}$	A and B.
				(3
	e C has centre $(3, -2)$ and	radius 5.		
	wn an equation of $C$ in ca	rtesian form.		(1
The line $y =$	wn an equation of C in ca 2x - 3 intersects C at the t $AB = 4\sqrt{5}$ .	rtesian form.		(1 (5
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The line $y =$ 5. Show that 6. The circle a. Find an eac b. Find an eac c. Show that 7. The circle a. Find the c	= $2x - 3$ intersects <i>C</i> at the t $AB = 4\sqrt{5}$ . e <i>C</i> touches the <i>y</i> -axis at t quation of the perpendicu quation for <i>C</i> . at the tangent to <i>C</i> at <i>B</i> ha e <i>C</i> has equation $x^2 + y^2 - c$	rtesian form. e points A and B. the point A (0, 3) and lar bisector of AB. as equation $3x - 4y + 3x $	+22 = 0.	
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### Mark Scheme

1.	
$x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0$	M1
$\left(x+\frac{1}{3}\right)^2 - \frac{1}{9} + \left(y-\frac{4}{3}\right)^2 - \frac{16}{9} + \frac{8}{9} = 0$	M1
$\left(x + \frac{1}{3}\right)^2 + \left(y - \frac{4}{3}\right)^2 = 1$	
Centre $\left(\frac{1}{3}, 0\right)$ Radius 1	M1
Radius 1	

2	
4	•

2.	
$(x+5)^2 - 25 + (y-2)^2 - 4 = 140$	M1
$(x+5)^2 + (y-2)^2 = 169$	
Centre (-5, 2)	M1
Radius 13	
Distance to centre = $\sqrt{144 + 25} = 13$	M1
Therefore point is on circle.	

## 3.

$(x+2)^2 - 4 + y^2 = 13$ Therefore, centre (-2, 0)	M1
Gradient $=\frac{0-4}{-2+1}=4$	M1
Therefore, $y - 4 = 4(x + 1)$ y = 4x + 8	M1

#### 4.

$x^{2} + (1 - x)^{2} + 6x + 2(1 - x) = 27$	M1
$x^2 + x - 12 = 0$	
(x+4)(x-3)=0	M1
x = -4, y = 14 = 5	1411
x = 3, y = 1 - 3 = -2	
Therefore, AB = $\sqrt{49 + 49} = 7\sqrt{2}$	M1

5a.  $(x-3)^2 + (y+2)^2 = 25$ **M1** 

# <u>5b.</u>

$(x-3)^2 + [(2x-3)+2]^2 = 25$	M1
$(x-3)^2 + (2x-1)^2 = 25$	M1
$x^2 - 2x - 3 = 0$	IVII
(x+1)(x-3) = 0	
x = -1, y = 2(-1) - 3 = -5	M1
x = 3, y = 2(3) - 3 = 3	
$AB^2 = 4^2 + 8^2 = 80$	M1
$AB = \sqrt{80} = 4\sqrt{5}$	M1

6a.

04.		
Midpoint AB = $(\frac{0+2}{2}, \frac{3+7}{2}) = (1, 5)$		M1
Gradient AB = $\frac{7-3}{2-0} = 2$		M1
Therefore perpendicular gradient = $-\frac{1}{2}$	6	M1
$y-5 = -\frac{1}{2}(x-1)$	- Star	M1
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11 1	
y = x	

<u>6b.</u>

Circle touches <i>y</i> -axis at (0, 3)	M1
Therefore y-coordordinate of centre $= 3$	IVII
$3 = \frac{11}{2} - \frac{1}{2}x$	M1
x = 5	IVII
Centre (5, 3) radius 5. $(x-5)^2 + (y-3)^2 = 25$	M1
$(x-5)^2 + (y-3)^2 = 25$	IVII

## 6c.

Gradient of radius $=\frac{7-3}{2-5} = -\frac{4}{3}$	M1
Therefore gradient of tangent = $\frac{3}{4}$	M1
$y - 7 = \frac{3}{4}(x - 2)$ 4y - 28 = 3x - 6	M1
3x - 4y + 22 = 0	M1

## 7a.

$(x-4)^2 - 16 + (y+2)^2 - 4 + 12 = 0$	M1
$(x-4)^2 + (y+2)^2 = 8$	IVII
Centre: (4, -2)	<b>M1</b>
Radius: $2\sqrt{2}$	IVII

# 7b.

Distance P to centre = $\sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$	M1
Therefore, max PQ = $5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$	M1
$Minimum PQ = 5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$	M1

7c.

Tangent perpendicular to radius:	M1
$PQ^{2} = (5\sqrt{2})^{2} - (5\sqrt{2})^{2} = 50 - 8 = 42$	IVII
$PQ = \sqrt{42} = 6.48$	M1

8a.

$ \begin{aligned} x^2 + (y-a)^2 - a^2 &= 0 \\ x^2 + (y-a)^2 &= a^2 \end{aligned} $	M1
Centre: (0, a) Radius: a	M1

8b.

00.	
C <sub>2</sub> : $(x - b)^2 - b^2 + y^2 = 0$ $(x - b)^2 + y^2 = b^2$	M1
Centre: (b, 0)	M1
Radius: b	
$C_1$ $a$ $C_2$ $C_2$ $b$ $x$	M1
Jelking ,	Matus

9a.	
$ \begin{aligned} & (x-4)^2 - 16 + (y-8)^2 - 64 + 72 = 0 \\ & (x-4)^2 + (y-8)^2 = 8 \end{aligned} $	M1
Centre: $(4, 8)$ Radius: $2\sqrt{2}$	M1

<u>9b.</u>

$\sqrt{16+64} = \sqrt{80}$	M1
$=4\sqrt{5}$	M1

9c.

Tangent perpendicular to radius:	M1
$OA^2 = (\sqrt{80})^2 - (2\sqrt{2})^2 = 72$	M1
$OA = \sqrt{72} = 6\sqrt{2}$	1711

