Heritage High School

EOC Review

Algebra I

Order of Operations

PEMDAS = **P**arentheses, **E**xponents, **M**ultiplication/**D**ivision, **A**dd/**S**ubtract from left to right.

A. Simplify each expression using appropriate Order of Operations.

1. $1 \bullet 5 - 6 \div 2 + 3^2$
2. $125 \div [5(2 + 3)]$
3. $4 + 2(10 - 4 \bullet 6)$
4. $3(2 + 7)^2 \div 5$
5. $12(20 - 17) - 3 \bullet 6$
6. $3^2 \div 3 + 2^2 \bullet 7 - 20 \div 5$

Solving Equations

The five steps to solving an equation are:

- ✓ Get rid of parentheses
- ✓ Simplify the left side and the right side of the equation as much as possible, i.e. combine any and all like terms
- ✓ Get the variable term on just one side
- ✓ Get the variable term by itself
- ✓ Solve for the variable

B. Solve for the variable in each problem.

7. $5(3x - 2) = 35$
8. $\frac{1}{3}(6x + 24) - 20 = -\frac{1}{4}(12x - 72)$
9. $5r - 2(2r + 8) = 16$
10. $13 - (2c + 2) = 2(c + 2) + 3c$
11. $\frac{1}{4}(8y + 4) - 17 = -\frac{1}{2}(4y - 8)$
12. $12 - 3(x - 5) = 21$

Solving Proportions

Remember:

- Use Cross Productions to write and equation
- Solve the equation

Examples

1. $\frac{6}{t+4} = \frac{42}{77}$

$42(t+4) = 6(77)$

$42t + 168 = 462$

$42t = 294$

$t = 7$

2. $\frac{11}{w} = \frac{33}{w+24}$

$33w = 11(w+24)$

$33w = 11w + 264$

$22w = 264$

$w = 12$

C. Solve the following:

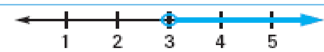
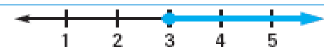
13. $\frac{a}{9a-2} = \frac{1}{8}$
14. $\frac{24}{5z+4} = \frac{4}{z-1}$
15. $\frac{x-8}{-2} = \frac{11-4x}{11}$

Answer the following:

16. A recipe that yields 12 buttermilk biscuits calls for 2 cups of flour. How much flour is needed to make 30 biscuits?

17. It took 7.2 minutes to upload 8 digital pictures from your computer to a website. At this rate, how long will it take to upload 20 pictures?

Solving Inequalities

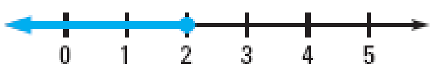
Symbol	Meaning	Equation or Inequality	Graph
=	equals	$x = 3$	
<	is less than	$x < 3$	
≤	is less than or equal to	$x \leq 3$	
>	is greater than	$x > 3$	
≥	is greater than or equal to	$x \geq 3$	

Examples:

$2x + 1 \leq 5$

$2x \leq 4$ Subtract 1 from each side

$x \leq 2$ Divide each side by 2



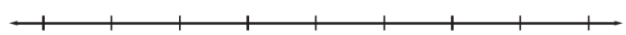
$-4y < 18$

$\frac{-4y}{-4} > \frac{18}{-4}$ Divide by -4 and change < to >

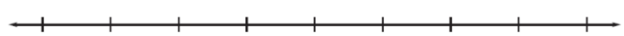
$y > -4.5$ Simplify

D. Solve and graph the following inequalities.

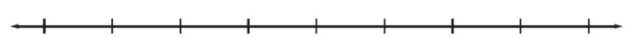
18. $3f - 4 < 2f + 5$



19. $5(1 - x) \geq 4(3 - x)$



20. $12 - \frac{3}{2}c < 0$



Graphs and Equations of Lines

Slope–Intercept Form

y = mx + b, where m = slope and b = y-intercept

<u>Graphing Equations in Slope-Intercept Form</u>

1. Write the equation in slope-intercept form for y
2. Find the y-intercept and use it to plot the point where the line crosses the y-axis.
3. Find the slope and use it to plot at least two more points on the line.
4. Draw a line through the points.

<u>Writing the Equation: Given the Slope and a y - intercept</u>

Example: Write an equation of the line that passes through (0, 4) and has a slope of -5. (These can also be given on a graph)

Step 1: Substitute – 5 for m. y = -5x + b

Step 2: Substitute 4 for b (since it is the y–intercept) y = -5x + 4

Point–Slope Form

$y - y_1 = m(x - x_1)$ where m = slope and (x_1, y_1) is the point.

Graphing Equations in Slope-Intercept Form

1. Plot the point (x_1, y_1).
2. Find the slope and use it to plot a second point on the line.
3. Draw a line through the two points.

Writing the Equation: Given a point and a slope

Example: Write an equation of the line that passes through the point (2, 5) and has a slope of 4. (These can also taken from a graph)

Substitute 2 for x_1, 5 for y_1, and 4 for x   y – 5 = 4(x -2)

Given Two Points

Step 1: Find the slope of the line using the two points and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Step 2: Choose either point and follow the steps above depending on the form you are asked to use.

Standard Form

ax + by = c where a is a *positive*, and a and b are *whole* numbers.

Graphing in Standard Form: Find the x and y – intercepts and graph the line that contains them.

Writing the Equation: Write the equation using slope-intercept or point-slope form, then rearrange to standard form.

Example: Write the equation of the line that passes through the point (4, 5) and has a slope of ½.

Step 1:	Write in Point-Slope Form	y – 5 = ½ (x – 4)
Step 2:	Distribute	y – 5 = ½ x – 3
Step 3:	Subtract ½ x and add 5	- ½ x + y = 2
Step 4:	Multiply by -2 to make *a* a positive, whole number	x – 2y = -4

E. Find the slope of the line containing each pair of points.

21. (5, 0) and (6, 8)
22. (4, -3) and (6, -4)
23. (-2, -4) and (-9, -7)

F. Find the slope of each line

24. y = 7
25. x = -4
26. 2x + y = 15
27. x – 2y = 7

G. Find the equation of the line with the given slope through the given point. Write the answer in *slope-intercept form*.

28. m = 4; (3, 2)
29. m = -2; (4, 7)
30. $m = -\frac{4}{3}$; (3, -1)

H. Write an equation of the line that passes through the given point and is parallel to the given line.

31. (-1, 3); y = 2x +2
32. (1, 7); -6x + y = -1
33. (-10, 0); -y + 3x = 16

I. Write an equation of the line that passes through the given point and is perpendicular to the given line.

34. (3, -3); y = x + 5
35. (8, -1); 4y + 2x = 12
36. (5, 1); y = 5x – 2

J. Write the equation of the line in point-slope form.

37. The line containing (-3, -2) and (5, 2)

38. The horizontal line passing through (2, 5)

K. Write the equation of the line in slope-intercept form.

39. The line containing (3, 1) and (4, 8)

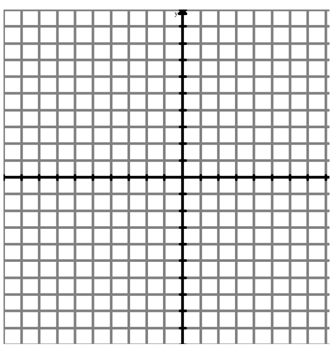
40. The line containing (3, 3) and (-6, 9)

41. The line with slope $\frac{4}{5}$ and containing (-1, 7)

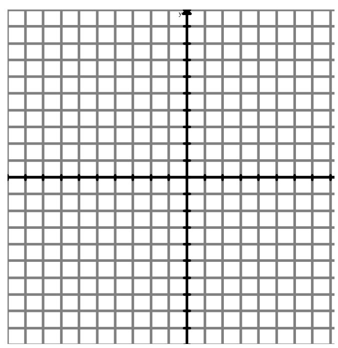
Graph the following equations. Graph three points and label the line with its equation.

L. Point-Slope Form

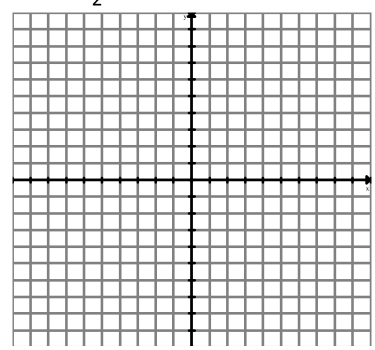
42. $y - 3 = 2(x - 1)$



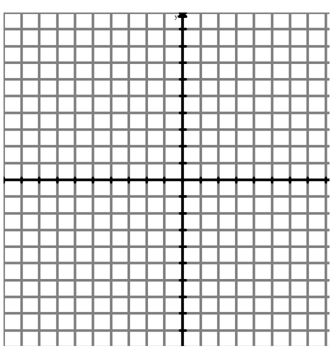
44. $y - 4 = -3(x - 5)$



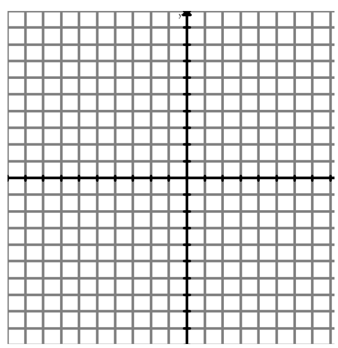
46. $y - 3 = -\frac{1}{2}(x + 2)$



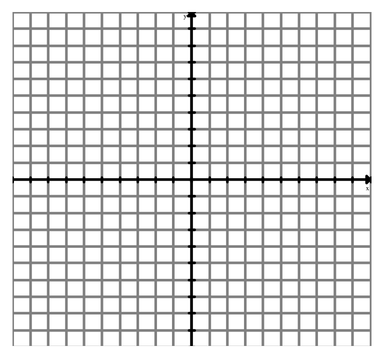
43. $y - 5 = \frac{2}{3}(x - 2)$



45. $y + 2 = -5(x - 3)$

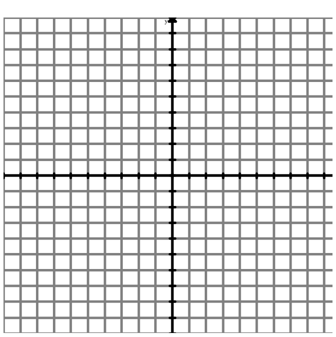


47. $y - 1 = \frac{4}{3}(x + 6)$

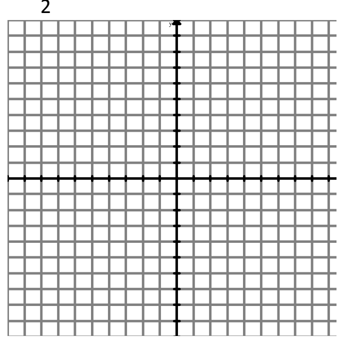


M. Slope-Intercept Form

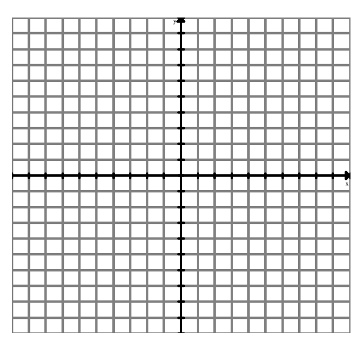
48. $y = 2x - 3$



49. $y = \frac{1}{2}x - 5$

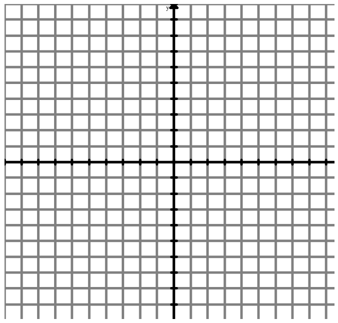


50. $y = -2x + 3$

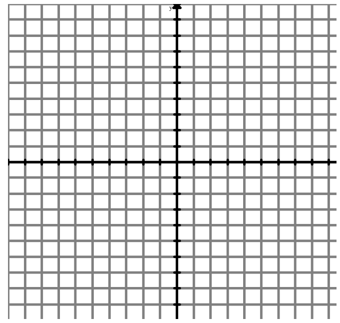


M. Slope-Intercept Form

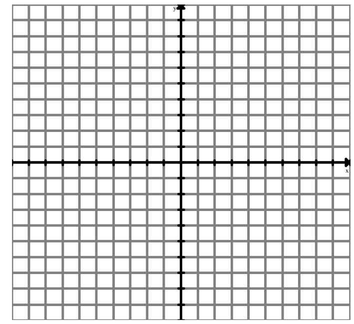
51. $y = -\frac{2}{3}x + 4$



52. $y = -\frac{5}{2}x + 4$

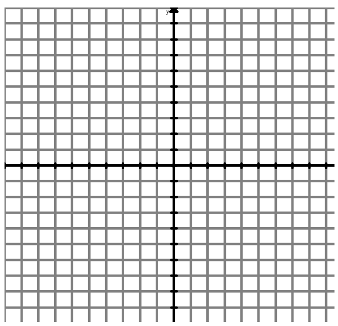


53. $y = -4x - 1$

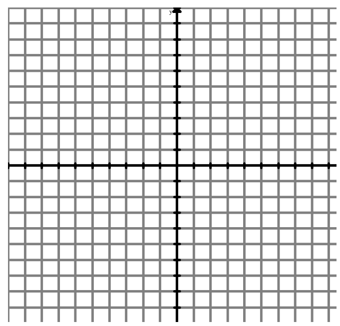


N. Standard Form

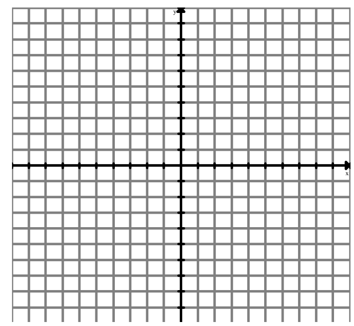
54. $4x + 2y = 8$



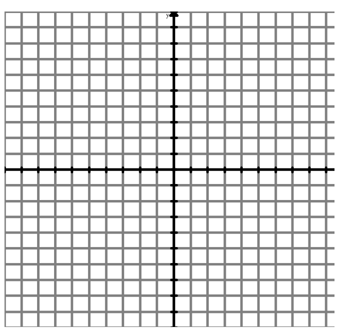
56. $4x + 6y = 12$



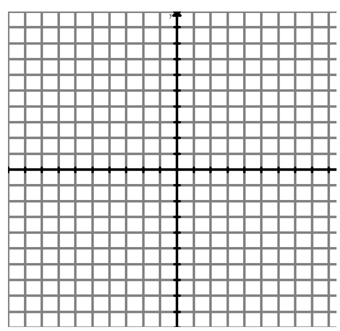
58. $2x - y = 4$



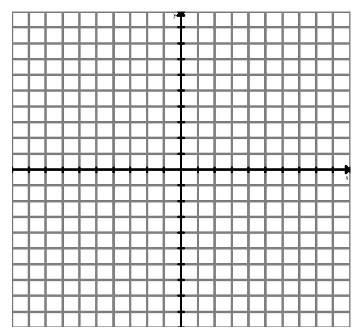
55. $x - 3y = 6$



57. $2x - 3y = 12$



59. $x + y = 5$



Systems of Linear Equations

Substitution Method: Use when an equation is solved for one variables (y = … or x = …)

Solve: $\begin{cases} y = 5 - 2x \\ 5x - 6y = 21 \end{cases}$

Solution: Substitute 5 – 2x for y.

$5x - 6(5 - 2x) = 21$
$5x - 30 + 12x = 21$
$17x - 30 = 21$
$17x = 51$
$x = 3$

Then substitute 3 for x:

$y = 5 - 2(3)$
$y = -1$

Answer: (3, -1)

Solve each system by substitution.

60. $\begin{cases} x = y + 3 \\ 2x - y = 5 \end{cases}$

61. $\begin{cases} 4x - 7y = 10 \\ y = x - 7 \end{cases}$

62. $\begin{cases} x = 16 - 4y \\ 3x + 4y = 8 \end{cases}$

Elimination Method: Use addition when the coefficients of a variable are opposites. Use subtraction when the coefficients are the same.

Example 1 - Solve: $\begin{cases} 3x+4y=9 \\ -3x-2y=-3 \end{cases}$

Solution:
3x + 4y = 9
(+) -3x – 2y = -3
ADD -2y = 6
y = -3

Then substitute -3 for y:
3x + 4(-3) = 9
3x = 21
x = 7

Answer: (7, -3)

Use multiplication when you have neither same or opposite coefficients

Example 2 – Solve: $\begin{cases} 5x-2y=-19 \\ 2x+3y=0 \end{cases}$

Solution: 3(5x – 2y = -19)
2(2x + 3y = x)

→
15x – 6y = -57
(+) 4x + 6y = 0
ADD 19x = -57
x = -3

Then substitute -3 for x:
2(-3) + 3y = 0
3y = 6
x = 2

Answer: (-3, 2)

Solve each system by elimination.

63. $\begin{cases} -5x-6y=8 \\ 5x+2y=4 \end{cases}$

65. $\begin{cases} x+y=1 \\ -2x+y=4 \end{cases}$

67. $\begin{cases} 12x-7y=-2 \\ -8x+11y=14 \end{cases}$

64. $\begin{cases} 6x-4y=14 \\ -3x+4y=1 \end{cases}$

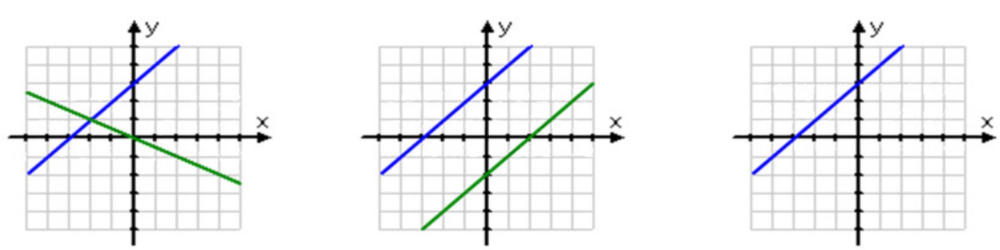
66. $\begin{cases} 7x+3y=-12 \\ 2x+5y=38 \end{cases}$

68. $\begin{cases} 7x-6y=-1 \\ 5x-4y=1 \end{cases}$

Graphing Method: Graph 2 or more equations on the same coordinate plane

- Scenario 1 – Intersecting lines (1 solution – point of intersection)
- Scenario 2 – Parallel Lines (no solution)
- Scenario 3 – Coinciding Lines (Infinitely Many Solutions {IMS})

One Solution | **No Solution** | **Infinitely Many Solutions**



-Consistent & Independent
-Different Slopes
-Lines Intersect

-Inconsistent
-Same Slope
-Lines Parallel

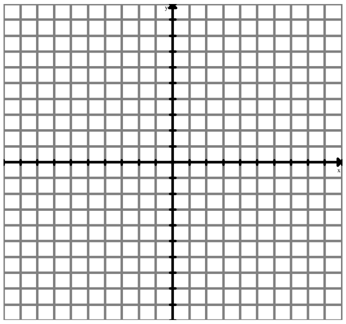
-Consistent & Dependent
-Same Slope
-Same y-intercept
-Lines Coincide (Collinear)

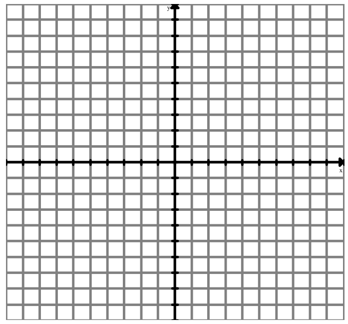
Solve each system by graphing.

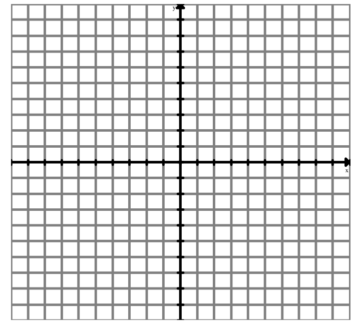
69. $\begin{cases} y = 2x + 3 \\ y = 2x - 2 \end{cases}$

70. $\begin{cases} y = -x + 4 \\ y = 2x - 8 \end{cases}$

71. $\begin{cases} y = 2x - 4 \\ -6x + 3y = -12 \end{cases}$





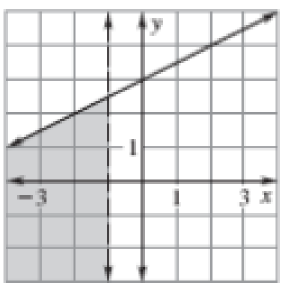


Systems of Inequalities

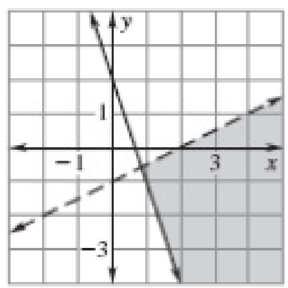
Remember:

- < or > Graph with a dotted line
- ≤ or ≥ Graph with a solid line
- < or ≤ Shade below the line (shade left of a vertical line
- > or ≥ Shade above the line (shade right of a vertical line
- Solutions are where the shaded regions overlap or on a solid boundary line

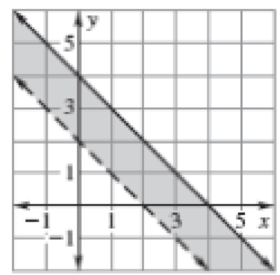
$\begin{cases} y - \frac{1}{2}x \le 3 \\ x < -1 \end{cases}$



$\begin{cases} y < \frac{1}{2}x - 1 \\ y \ge -3x + 2 \end{cases}$



$\begin{cases} y > -x + 2 \\ y \le -x + 5 \end{cases}$

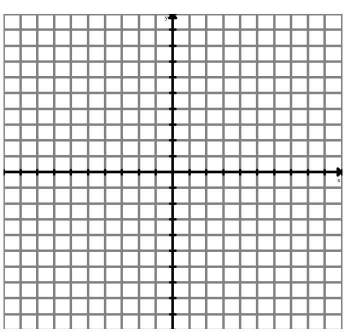


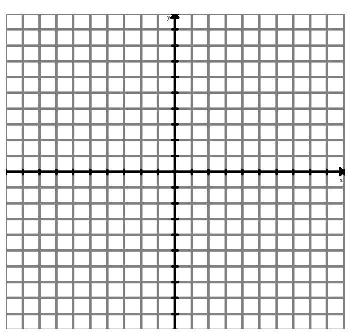
Graph each system of inequalities.

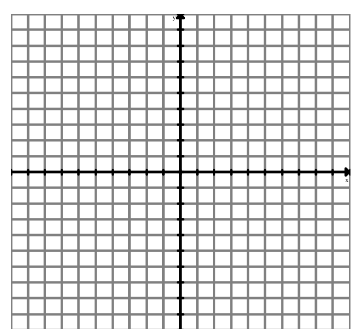
72. $\begin{cases} y < -2x + 3 \\ y \ge 4 \end{cases}$

73. $\begin{cases} y \ge 2x + 1 \\ y < -x + 4 \end{cases}$

74. $\begin{cases} x > 3 \\ y > x \end{cases}$







Domain and Range

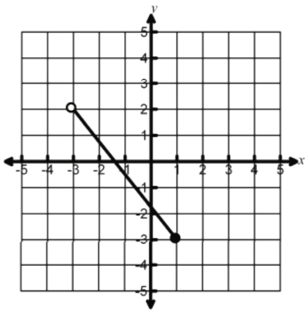
Domain: Set of values of the independent variable (x) for which a function is defined (Can also be seen as input or cause)
Range: Set of y values of a function (dependent variable, output, f(x), or effect)

Examples: Find the domain and range for each of the following.

1. (3, 2), (5, 6), (2, -4), (-3, 5), (7, 2)

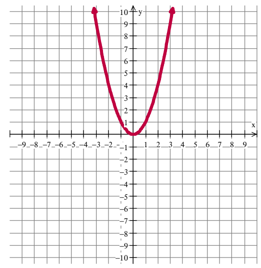
Ans: Domain: {-3, 2, 3, 5, 7}
Range: { -4, 2, 5, 6}
**Written Least to Greatest, No Repeats!

2.



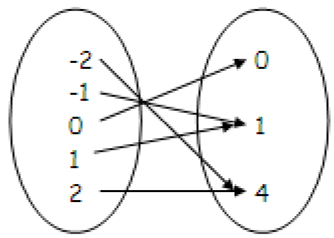
Ans: Domain: $-3 < x \leq 1$
Range: $-3 \leq y < 2$
**Remember (1) Open dots means not included (< or >), (2) Closed dots mean includes ($\leq$ or $\geq$)

3.



Ans: Domain: x = All Real Numbers (ARN)
Range: $y \geq 0$
**Remember that arrows mean continues on infinitely in that direction.

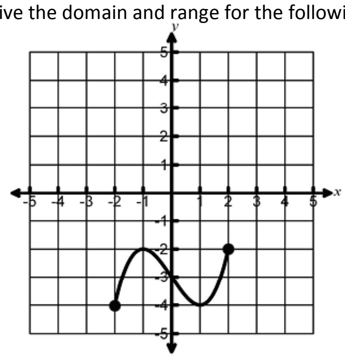
4.



Ans: Domain: {-2, -1, 0, 1, 2}
Range: {0, 1, 4}

P. Give the domain and range for the following:

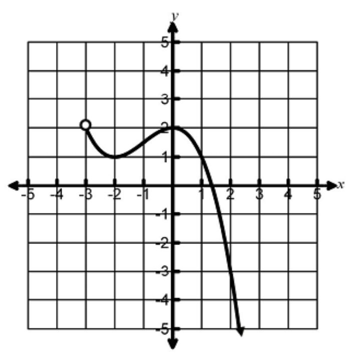
75.



Domain: ____________________

Range: ______________________

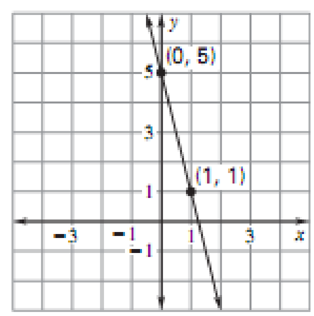
76.



Domain: ____________________

Range: ______________________

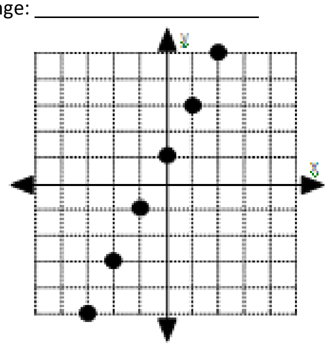
77.



Domain: ____________________

Range: ______________________

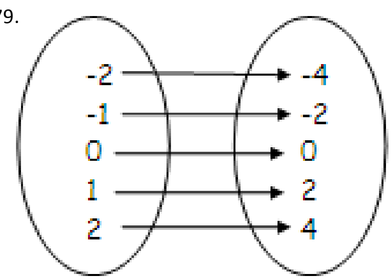
78.



Domain: ____________________

Range: ______________________

79.



Domain: ____________________

Range: ______________________

80.

x	y
-2	-4
-2	4
-1	2
1	2
0	0

Domain: ____________________

Range: ______________________

Exponents

$a^0 = 1$	**Example:** $5^0 = 1$	$a^m \bullet a^n = a^{m+n}$	**Example:** $x^2 \bullet x^4 = x^{2+4} = x^6$
$\dfrac{a^m}{a^n} = a^{m-n}$	**Example:** $\dfrac{b^7}{b^3} = b^{7-3} = b^4$	$(a^m)^n = a^{m(n)}$	**Example:** $(y^3)^4 = y^{3(4)} = y^{12}$
$a^{-m} = \dfrac{1}{a^m}$	**Example:** $6^{-2} = \dfrac{1}{6^2} = \dfrac{1}{36}$		

Q. Simplify each expression.

81. $\left(\dfrac{2}{3}\right)^{-2}$
82. $\left(\dfrac{5}{3}\right)^{-3}$
83. $x^{-1} \bullet x^{-2}$
84. $a \bullet a^{-1}$
85. $\left(x^2\right)^{-2}$
86. $\left(5a^2b^3\right)\left(a^{-2}b\right)$
87. $\left(-2ab^5\right)\left(-4ab^{-3}\right)$
88. $x^3 \bullet x^6$
89. $\left(2a^4\right)\left(5a^3\right)$
90. $\left(a^2\right)^3$
91. $\left(5a\right)^2$
92. $c \bullet c^5 \bullet c^2$
93. $\left(-2xy^2\right)\left(-3x^2y\right)$

Multiplying Polynomials

Monomial x Polynomial

$$\underbrace{3c^3(8c^4 - c^2 - 3c + 5)}_{\textit{Distribute by multiplying } 3c^3 \textit{ by every term inside the ()}} = 24c^7 - 3c^5 - 9c^4 + 15c^3$$

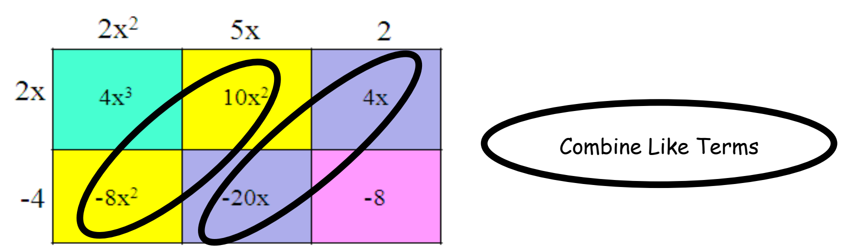
Binomial x Binomial

$$(2x-4)(3x+5) = \underset{\text{First terms}}{6x^2} + \underset{\text{Outer terms}}{10x} - \underset{\text{Inner terms}}{12x} - \underset{\text{last terms}}{20} = \underbrace{6x^2 - 2x - 20}_{\text{combine like terms}}$$

$$(3x-4)^2 = (3x-4)(3x-4) = \underset{\text{First terms}}{9x^2} - \underset{\text{Outer terms}}{12x} - \underset{\text{Inner terms}}{12x} + \underset{\text{last terms}}{16} = \underbrace{9x^2 - 24x + 16}_{\text{combine like terms}}$$

Binomial x Polynomial – Use Punnett Squres

Ex: $(2x-4)(2x^2+5x+2)$



The answer is:

$$4x^3 + 2x^2 - 16x - 8$$

R. Find each product.

94. $(x + 3)(4x^2 - 2x + 9)$

95. $(2x + 1)(x + 4)$

96. $(6x + 5)(2x - 1)$

97. $(x - 4)(x + 4)$

98. $-5b^3(4b^5 - 2b^3 + b - 11)$

99. $(6x + 5y)^2$

Factoring Polynomials

Examples:

1) $a^2 - b^2 = (a+b)(a-b)$ **EX:** $a^2 - 16 = (a+4)(a-4); 25a^2 - 36x^6 = (5a+6x^3)(5a-6x^3)$

2) $a^2 + 2ab + b^2 = (a+b)^2$ **EX:** $k^2 + 10k + 25 = (k+5)(k+5) = (k+5)^2$

k^2 & 25 are perfect squares & 10k = 2(1k*5)

3) $a^2 - 2ab + b^2 = (a-b)^2$ **EX:** $4x^2 - 12x + 9 = (2x-3)(2x-3) = (2x-3)^2$

$4x^2$ & 9 are perfect squares & 12x = 2(2x*3)

4) $ax^2 + bx + c$ **EX:** $x^2 + 6x + 8 = (x+4)(x+2)$ since 4 + 2 = 6 and 4 * 2 = 8

$ax^2 - bx + c$ $x^2 - 8x + 15 = (x-3)(x-5)$ since -3 + -5 = -8 and -3 * -5 = 15

$ax^2 + bx - c$ $a^2 + 12a - 45 = (a+15)(a-3)$ since 15 + -3 = 12 and 15 * -3 = -45

$ax^2 - bx - c$ $y^2 - y - 12 = (y+3)(y-4)$ since 3 + -4 = -1 and 3 * -4 = -12

S. Factor each of the following polynomials.

100. $x^2 + 8x + 15$

101. $a^2 - 14a + 48$

102. $x^2 + x - 42$

103. $x^2 - 7x - 18$

104. $x^2 - 16x + 64$

105. $x^2 - 81$

Solving Quadratic Equations

Solve using Square Roots

Problem:	$5x^2 - 75 = 0$	Problem	$(x+6)^2 = 21$
Get numbers on one side of equation	$\frac{5x^2}{5} = \frac{75}{5}$	Square root both sides	$\sqrt{(x+6)^2} = \pm\sqrt{21}$
Divide by 5	$x^2 = 15$	Square root of $\sqrt{(x+6)^2} = (x+6)$ subtract 6 from both sides	$x+6 = \pm\sqrt{21}$ $-6 \quad -6$
Square root both sides	$x = \pm\sqrt{15}$	Answer:	$x = \pm\sqrt{21} - 6$

T. Solve each quadratic equation using square roots.

106. $x^2 = 121$

107. $3x^2 = 30$

108. $(x-2)^2 = 49$

Solve using Factoring

Problem	$a^2 + 12a - 45$
Factor the problem	$(a+15)(a-3)$
Make each factor equal to zero and solve for "x"	$a+15=0 \quad and \quad a-3=0$
	$-15 \ \ -15 \qquad +3 \ \ +3$
Answer	a = -15 a = 3

Solve each quadratic equation using factoring.

109. $x^2 + 7x = 0$

111. $x^2 + 7x + 6 = 0$

113. $t^2 = 9t - 14$

110. $p^2 - 16p + 48 = 0$

112. $m^2 + 4m = 21$

114. $2x^2 + 12x = -10$

Solve Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Put equation in proper format ($ax^2 + bx + c = 0$)
- Find a, b, and c
- Plug into the formula
- Do the math a little at a time.
- If the discriminate (b^2-4ac) is positive there are 2 real solutions, if 0, there is 1 real solution, if negative, then there is NO real solution.

Examples

1.

$$x^2 - 2x - 15 = 0$$
$$a = 1, b = -2, c = -15$$
$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-15)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{4 + 60}}{2}$$
$$x = \frac{2 \pm \sqrt{64}}{2}$$
$$x = \frac{2 \pm 8}{2}$$
$$x = \frac{2+8}{2}, x = \frac{2-8}{2}$$
$$x = 5, x = -3$$

2

$$2x^2 + 7x - 3 = 0$$
$$a = 2, b = 7, c = -3$$
$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(2)(-3)}}{2(2)}$$
$$x = \frac{-7 \pm \sqrt{49 + 24}}{4}$$
$$x = \frac{-7 \pm \sqrt{73}}{4}$$
$$x = \frac{-7 + \sqrt{73}}{4}, x = \frac{-7 - \sqrt{73}}{4}$$
$$x = .39, x = -3.89$$

Solve using the Quadratic Equation

115. $-2x^2 + 6x + 9 = 0$

116. $3x^2 - 4x + 9 = 0$

117. $2x^2 - 12x - 1 = -7x + 6$

Solve by Graphing

- Rearrange to $y = ax^2 + bx + c$
- Find a, b, and c
- Find axis of symmetry $$x = \frac{-b}{2a}$$
- Plug in the axis of symmetry x-value into the equation and find y which together make the vertex (x, y)
- Make a table using two x-values to the left of the vertex, and two x-values to the right of the vertex.
- Graph all five points and connect with a smooth curved line.
- Solutions to Quadratics are called x-intercepts, zeros, roots, and solutions.
- If the graph does not touch the x-axis, there is no solution.

Example

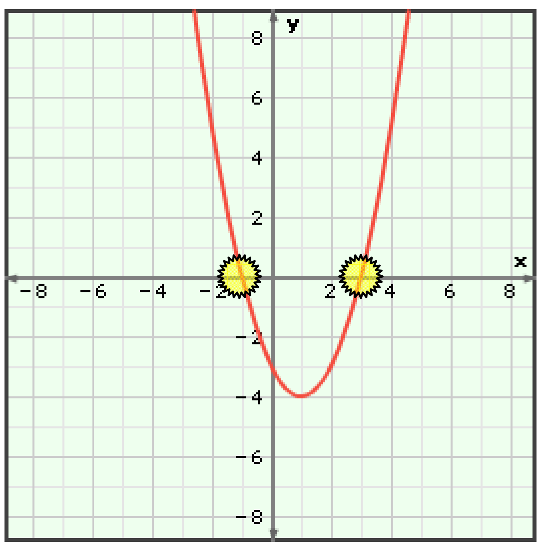
1. $y = x^2 - 2x - 3$

   a = 1, b = -2, c = -3

   $x = \frac{-2(-2)}{2(2)} = 1$

   y = (1)2 – 2(1) – 3 = -4

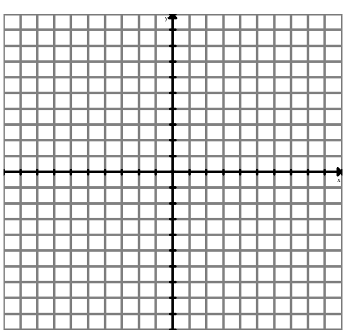
   vertex: (1, -4)



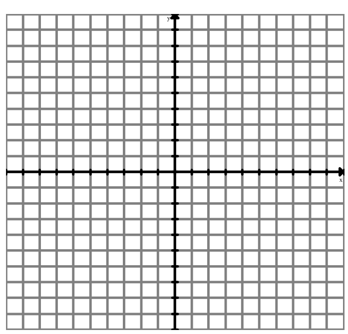
Solutions: x = -1 and x = 3

Solve by Graphing

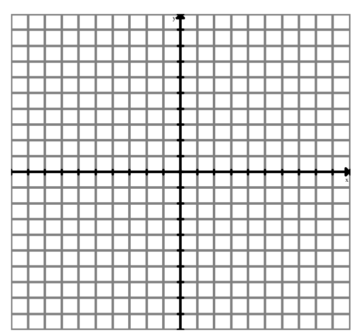
118. $y = x^2 - 4x - 5$



119. $y = x^2 + x + 2$



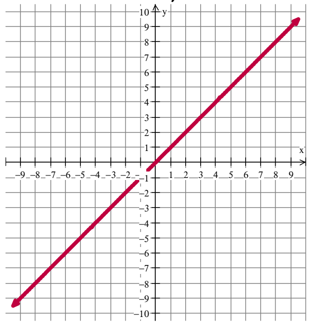
120. $y = x^2 + 16x + 64$



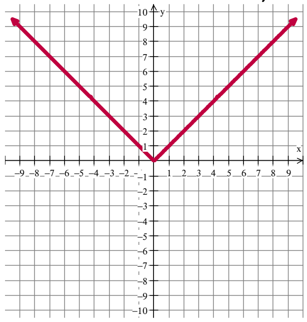
Parent Functions

Parent Function: The simplest version of any function

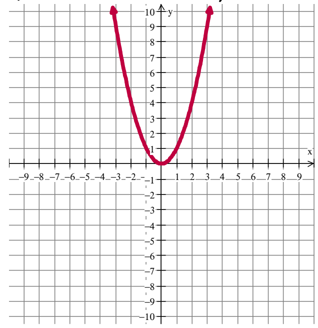
Linear Functions: $y = x$



Absolute Value Functions: $y = |x|$



Quadratic Functions: $y = x^2$



Cubic Functions: $y = x^3$