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Algebra I

Polynomials

2015-11-02

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Definitions of Monomials, Polynomials and Degrees

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Monomial

A monomial is a one-term expression formed by a number, a variable, or the product of numbers and variables. Examples of monomials....

$4x$ 28 $81y^4z$ $\frac{rt}{6}$ $32,457$ $17x^2$ mn^3

We usually write the variables in exponential form - exponents must be whole numbers.

Monomials

Drag the following terms into the correct sorting box. If you sort correctly, the term will be visible. If you sort incorrectly, the term will disappear.

$x^2(5 + 7y)$ $7x^3y^5 - 4$ $\frac{t}{16}$ -12 $a + b - 5$ $6+5rs$ $\frac{xy^4}{7}$ $5x + 7$ 15 $48x^2yz^3$ $4(5a^2bc^2)$

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Polynomials

A polynomial is an expression that contains one or more monomials. Examples of polynomials....

$5a^2$

$7+b+c^2+4d^3$

c^2+d

$8a^3-2b^2$

$8x^3+x^2$

$\frac{rt}{6}+\frac{a^4b}{15}$

$4c-mn^3$

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Polynomials

What polynomials DO have:

One or more terms made up of...

- Numbers
- Variables raised to whole-number exponents
- Products of numbers and variables

What polynomials DON'T have:

- Square roots of variables
- Negative exponents
- Fractional exponents
- Variables in the denominators of any fractions

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Polynomials

What is the exponent of the variable in the expression 5x?

What is the exponent of the variable in the expression 5?

Degrees of Monomials

The degree of a monomial is the sum of the exponents of its variables. The degree of a nonzero constant such as 5 or 12 is 0. The constant 0 has no degree.

Examples:

1) The degree of 3x is?

2) The degree of -6x3y is?

3) The degree of 9 is?

1 What is the degree of x^2?

- A 0
- B 1
- C 2
- D 3

2 What is the degree of *mn*?

- A 0
- B 1
- C 2
- D 3

3 What is the degree of *3* ?

- A 0
- B 1
- C 2
- D 3

4 What is the degree of $7t^8$?

Degrees of Polynomials

The degree of a polynomial is the same as that of the term with the greatest degree.

Example:
Find degree of the polynomial $4x^3y^2 - 6xy^2 + xy$.

$4x^3y^2$ has a degree of 5,
$-6xy^2$ has a degree of 3,
xy has a degree of 2.

The highest degree is 5, so the degree of the polynomial is 5.

Find the degree of each polynomial

1) 3
2) $12c^3$
3) ab
4) $8s^4t$
5) 2 - 7n
6) h^4 - 8t
7) $s^3 + 2v^2y^2 - 1$

5 What is the degree of the following polynomial:

$$a^2b^2 + c^4d - x^2y$$

- A 3
- B 4
- C 5
- D 6

6 What is the degree of the following polynomial:

$$a^3b^3 + c^4d - x^3y^2$$

- A 3
- B 4
- C 5
- D 6

Adding and Subtracting Polynomials

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Standard Form

A polynomial is in standard form when all of the terms are in order from highest degree to the lowest degree.
Standard form is commonly accepted way to write polynomials.

Example: $9x^7 - 8x^5 + 1.4x^4 - 3x^2 + 2x - 1$ is in standard form.

Drag each term to put the following equation into standard form:

67 $-11x^4$ $-21x^9$ $-9x^4$ $-x^8$ $+2x^3$ $-x$

Vocabulary

Monomials with the same variables and the same power are like terms.

The number in front of each term is called the coefficient of the term. If there is no variable in the term, the term is called the constant term.

Like Terms	Unlike Terms
$4x$ and $-12x$	$-3b$ and $3a$
x^3y and $4x^3y$	$6a^2b$ and $-2ab^2$

Like Terms

Like terms can be combined by adding the coefficients, but keeping the variables the same. WHY?

3x + 5x means 3 times a number x added to 5 times the same number x.

So altogether, we have 8 times the number x.

What we are really doing is the distributive property of multiplication over addition in reverse:

$$3x + 5x = (3+5)x = 8x$$

One big mistake students often make is to multiply the variables:

~~$3x + 5x = 8x^2$~~

Like Terms

Combine these like terms using the indicated operation.

$4x + 3x$

$5a^2 - 2a^2$

$7xy + 8xy - 5xy$

$2x^2y + 3xy^2$

7 Simplify $7y + 5y$

- A $12y^2$
- B $12y$
- C $2y^2$
- D $2y$

8 Simplify $5y - 7y$

- A $-2y^2$
- B $-2y$
- C $2y^2$
- D $2y$

9 Simplify $5x^2y + 4xy^2 - 3x^2y$

- A $2x^2y + 4xy^2$
- B $5x^2y + 4xy^2 - 3x^2y$
- C $5x^2y + xy^2$
- D $6x^2y$

Add Polynomials

To add polynomials, combine the like terms from each polynomial.

To add vertically, first line up the like terms and then add.

Examples:

$(3x^2 + 5x - 12) + (5x^2 - 7x + 3)$

line up the like terms

$3x^2 + 5x - 12$
(+) $5x^2 - 7x + 3$

click

$(3x^4 - 5x) + (7x^4 + 5x^2 - 14x)$

line up the like terms

$3x^4 \quad - 5x$
(+) $7x^4 + 5x^2 - 14x$

10

click

Add Polynomials

We can also add polynomials horizontally.
$(3x^2 + 12x - 5) + (5x^2 - 7x - 9)$

Use the communitive and associative properties to group like terms.

$(3x^2 + 5x^2) + (12x + -7x) + (-5 + -9)$

$8x^2 + 5x - 14$

10 Add $(4x+1)+(5x+8)$

- A $9x+9$
- B $9x^2+9$
- C $9x+8$
- D $9x+7$

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Subtract Polynomials

To subtract polynomials, subtract the coefficients of like terms.

Example:

$-3x - 4x = -7x$

$13y - (-9y) = 22y$

$6xy - 13xy = -7xy$

Subtract Polynomials

We can subtract polynomials vertically .

To subtract a polynomial, change the subtraction to adding -1. Distribute the -1 and then follow the rules for adding polynomials

$(3x^2 +4x -5) - (5x^2 -6x +3)$

$(3x^2+4x-5) +(-1) (5x^2-6x+3)$

$(3x^2+4x-5) + (-5x^2+6x-3)$

$3x^2 + 4x - 5$

$\underline{(+) -5x^2 - 6x + 3}$

click

Subtract Polynomials

We can subtract polynomials vertically .

Example:

$(4x^3 -3x -5) - (2x^3 +4x^2 -7)$

$(4x^3 -3x -5) +(-1)(2x^3 +4x^2 -7)$

$(4x^3 -3x -5) + (-2x^3 -4x^2 +7)$

$4x^3 \quad - 3x - 5$

$\underline{(+) -2x^3 - 4x^2 \quad + 7}$

click

Subtract Polynomials

We can also subtract polynomials horizontally.

$(3x^2 + 12x - 5) - (5x^2 - 7x - 9)$

Change the subtraction to adding a negative one and distribute the negative one.

$(3x^2 + 12x - 5) +(-1)(5x^2 - 7x - 9)$

$(3x^2 + 12x - 5) + (-5x^2 + 7x + 9)$

Use the communitive and associative properties to group like terms.

$(3x^2 +-5x^2) + (12x +7x) + (-5 +9)$

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Summary

Is the sum or difference of two polynomials always a polynomial?

When we add polynomials, we are adding the terms of the first to the terms of the second, and each of these sums is a new term of the same degree. Each new term consists of a constant times variables raised to whole number powers, so the sum is in fact a polynomial.

Therefore, we say that the set of polynomials is "closed under addition".

Since subtraction is just adding the opposite, the set of polynomials is also closed under subtraction.

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Multiplying a Polynomial by a Monomial

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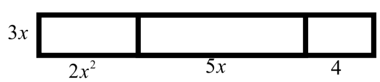
Multiplying Polynomials

Find the total area of the rectangles.



$3x(2x^2+5x+4)=3x(2x^2)+3x(5x)+3x(4)=6x^3+15x^2+12x$ square units

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Multiplying Polynomials

To multiply a polynomial by a monomial, you use the distributive property together with the laws of exponents for multiplication.

Example:

$-2x(5x^2 - 6x + 8)$

$(-2x)(5x^2) + (-2x)(-6x) + (-2x)(8)$

$-10x^3 + 12x^2 - 16x$

Multiplying Polynomials

Let's Try It! Multiply to simplify.

1. $-x(2x^3 - 4x^2 + 7x)$

2. $4x^2(5x^2 - 6x - 3)$

3. $3xy(4x^3y^2 - 5x^2y^3 + 8xy^4)$

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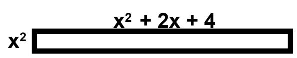
21 What is the area of the rectangle shown?



- A $x^4 + 3x^2 + 4x^2$
- B $2x^2 + 2x^3 + 4x^2$
- C $x^4 + 2x^3 + 4x^2$
- D $2x^4 + 2x^3 + 4x^2$

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22 Multiply $2x(3x^2 + 4x - 6)$

- A $6x^2 + 8x - 12$
- B $6x^2 + 8x^2 - 12$
- C $6x^2 + 8x^2 - 12x$
- D $6x^3 + 8x^2 - 12x$

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23 Multiply $-3x^4\left(5xy-2xy^3\right)$

- A $-15x^4y+6x^4y^3$
- B $-15x^5y+6x^5y^3$
- C $-15x^5y-6x^5y^3$
- D $-15x^4y-6x^4y^3$

24 Find the area of a triangle ($A=\frac{1}{2}bh$) with a base of 4x and a height of 2x - 8. (All answers are in square units.)

- A $8x^2-32$
- B $6x^2-32x$
- C $3x^2-16x$
- D $4x^2-16x$

25 Rewrite the expression

-3a(a + b - 5) + 4(-2a + 2b) + b(a + 3b - 7)

to find the coefficients of each term. Enter the coefficients into the appropriate boxes.

Students type their answers here

☐ a^2 + ☐ b^2 + ☐ ab + ☐ a + ☐ b

From PARCC EOY sample test calculator #9

Multiplying a Polynomial by a Polynomial

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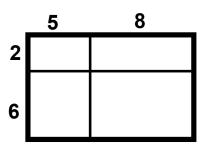
26 Find the area of the rectangle in two different ways.



Multiply Polynomials

To multiply a polynomial by a polynomial, you multiply each term of the first polynomial by each term of the second. Then, add like terms.

Example 1:

(2x + 4y)(3x + 2y)

Example 2:

(x + 3)(x2 + 2x + 4)

FOIL Method

The FOIL Method is a shortcut that can be used to remember how multiply two binomials. To multiply two binomials, find the sum of the products of the....

$(a + b)(c + d) =$

First terms of each binomial	ac +
Outer terms - the terms on the outsides	ad +
Inner Terms- the terms on the inside	bc +
Last Terms of each binomial	bd

Remember - FOIL is just a mnemonic to help you remember the steps for binomials. What you are really doing is multiplying each term in the first binomial by each term in the second.

Multiply Polynomials

Try it!Find each product.

1) $(x - 4)(x - 3)$

2) $(x + 2)(3x - 8)$

Multiply Polynomials

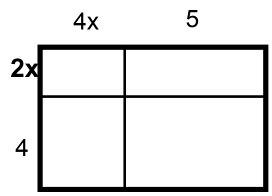
Try it!Find each product.

3) $(2x - 3y)(4x + 5y)$

4) $(3x - 6)(x^2 - 2x)$

27 What is the total area of the rectangles shown?

- A $8x^2 + 20$
- B $8x^2 + 10x + 20$
- C $8x^2 + 16x + 20$
- D $8x^2 + 26x + 20$



28 Multiply: $(x+3)(7x+2)$

- A $7x^2 + 27x + 6$
- B $7x^2 + 23x + 6$
- C $7x^2 + 21x + 6$
- D $7x^2 + 13x + 6$

29 Multiply: $(2x+3)(-3x-4)$

- A $-6x^2 - 17x - 12$
- B $-6x^2 + 17x - 12$
- C $-6x^2 - 17x + 12$
- D $-6x^2 + 17x + 12$

30 Multiply: $(x-2)(5x^2+3x-1)$

- A $5x^3-13x^2-7x+2$
- B $5x^3-10x^2+7x+2$
- C $5x^3-7x^2-7x+2$
- D $5x^3+7x^2+7x+2$

31 Multiply: $(x^2-5)(x^2+3)$

- A x^2-2x^2-15
- B x^4-2x^2-15
- C $x^2-2x-15$
- D $x^4-2x-15$

32 Find the area of a square with a side of x^2-2x

- A x^4-4x^2
- B x^4+4x^2
- C $x^4-2x^3+4x^2$
- D $x^4-4x^3+4x^2$

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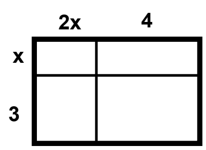
34 Find the total area of the rectangles.

Students type their answers here



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Special Binomial Products

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Square of a Sum

$(a + b)^2$
$= (a + b)(a + b)$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$

	a	b
a	a^2	ab
b	ab	b^2

Notice that there are two of the term ab!

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Square of a Difference

$(a - b)^2$
$= (a - b)(a - b)$
$= a^2 - ab - ab + b^2$
$= a^2 - 2ab + b^2$

	a	- b
a	a^2	- ab
- b	- ab	$+ b^2$

Notice that there are two of the term -ab!

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Product of a Sum and a Difference

$(a + b)(a - b)$

$= a^2 - ab + ab + -b^2$

$= a^2 - b^2$

	a	- b
a	a^2	- ab
+ b	+ ab	$- b^2$

This time, the **+ ab** and the **- ab** add up to 0, and so the middle term drops out.

Special Products

Try It! Find each product.

1) $(3p + 9)^2$

2) $(6 - p)^2$

3) $(2x - 3)(2x + 3)$

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Fill in the missing pieces

$(3x - 5y)^2 = \square x^2 + \square xy + \square y^2$

$(\square x + \square y)^2 = 9x^2 + \square xy + 36y^2$

$(\square x + \square y)^2 = 121x^2 - 66xy + \square y^2$

$(12x - \square y)(\square x + 9y) = \square x^2 - \square y^2$

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37 $(x - 5)^2$

- A $x^2 + 25$
- B $x^2 + 10x + 25$
- C $x^2 - 10x + 25$
- D $x^2 - 25$

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38 $(x-6)(x+6)$

- A $x^2-12x-36$
- B x^2+36
- C $x^2+12x-36$
- D x^2-36

39 What is the area of a square with sides 2x + 4?

- A $4x^2+16$
- B $4x^2-16$
- C $4x^2+8x+16$
- D $4x^2+16x+16$

40 $(3x+y^2)^2$

- A $9x^2+6xy^2+y^4$
- B $6x^2+3xy^2+y^4$
- C $9x^2+y^4$
- D $6x^2+y^4$

Solving Equations

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Zero Product Property

Given the following equation, what conclusion(s) can be drawn?

$ab = 0$

Since the product is 0, one of the factors, a or b, must be 0.

Zero Product Property

If $ab = 0$, then either $a = 0$ or $b = 0$.

Think about it: if $3x = 0$, then what is x?

Zero Product Property

What about this? $(x - 4)(x + 3) = 0$

Since $(x - 4)$ is being multiplied by $(x + 3)$, then each binomial is a FACTOR of the left side of the equation.

Since the product is 0, one of the factors must be 0.
Therefore, either $x - 4 = 0$ or $x + 3 = 0$.

$$\begin{array}{rcl} x - 4 = 0 & \text{or} & x + 3 = 0 \\ +4 \quad +4 & & -3 \quad -3 \\ \hline x = 4 & \text{or} & x = -3 \end{array}$$

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Zero Product Property

Therefore, our solution set is {-3, 4}. To verify the results, substitute each solution back into the original equation.

To check $x = -3$:

$$\begin{aligned} (x - 4)(x + 3) &= 0 \\ (-3 - 4)(-3 + 3) &= 0 \\ (-7)(0) &= 0 \\ 0 &= 0 \end{aligned}$$

To check $x = 4$:

$$\begin{aligned} (x - 4)(x + 3) &= 0 \\ (4 - 4)(4 + 3) &= 0 \\ (0)(7) &= 0 \\ 0 &= 0 \end{aligned}$$

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Solve

What if you were given the following equation?

$(x - 6)(x + 4) = 0$

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41 Solve (a + 3)(a - 6) = 0.

- A {3 , 6}
- B {-3 , -6}
- C {-3 , 6}
- D {3 , -6}

42 Solve (a - 2)(a - 4) = 0.

- A {2 , 4}
- B {-2 , -4}
- C {-2 , 4}
- D {2 , -4}

43 Solve (2a - 8)(a + 1) = 0.

- A {-1 , -16}
- B {-1 , 16}
- C {-1 , 4}
- D {-1 , -4}

Factors
and
Greatest Common Factors

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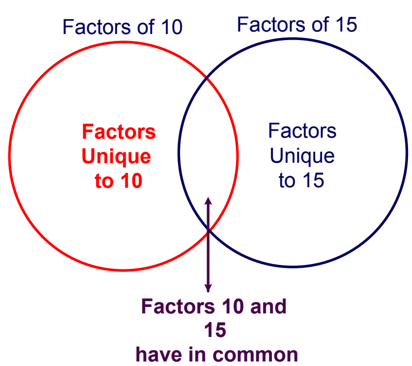
GCF

What is the greatest common factor (GCF) of 10 and 15?

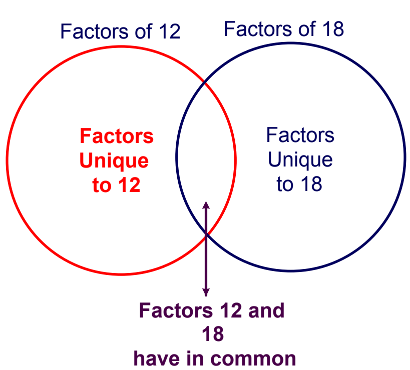


Number Bank

1	11
2	12
3	13
4	14
5	15
6	16
7	17
8	18
9	19
10	20

GCF

What is the greatest common factor (GCF) of 12 and 18?



Number Bank

1	11
2	12
3	13
4	14
5	15
6	16
7	17
8	18
9	19
10	20

44 What is the GCF of 12 and 15?

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45 What is the GCF of 24 and 48?

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46 What is the GCF of 72 and 54?

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47 What is the GCF of 28, 56 and 42?

GCF

Variables also have a GCF.

The GCF of variables is the variable(s) that is in each term raised to the least exponent given.

Example: Find the GCF

x^2 and x^3

r^4, r^5 and r^8

x^3y^2 and x^2y^3

$20x^2y^2z^5$ and $15x^4y^4z^4$

48 What is the GCF of x^8 and x^9?

- A x^8
- B x^9
- C x
- D 1

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Factoring out GCFs

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Factoring

Factoring a number means to find other numbers you can multiply to get the number.

48 = 6 × 8, so 6 and 8 are both factors of 48.

Factoring a polynomial means to find other polynomials that can be multiplied to get the original polynomial.

$(y + 1)(y - 4) = y^2 - 3y - 4$, so $y + 1$, and $y - 4$ are factors of $y^2 - 3y - 4$.

Factoring

Example:

Factor $10x^2 - 30x$

We might notice quickly that both terms have 10 as a factor, so we could have $10(x^2 - 3x)$.

But both terms also have x as a factor. So the greatest common factor of both terms is 10x.

$$10x^2 - 30x = 10x\,(x - 3)$$

The left side of the equation is in expanded form, and the right side is in factored form.

Factoring

The first step in factoring is to look for the greatest monomial factor. If there is a greatest monomial factor other than 1, use the distributive property in reverse to rewrite the given polynomial as the product of this greatest monomial factor and a polynomial.

Example Factor

$6x^4 - 15x^3 + 3x^2$

Factoring

Factor:

$4m^3n - 7m^2n^2$

$100x^5 - 20x^3 + 30x - 50$

$\frac{1}{2}x^2 - \frac{1}{2}x$

Factoring

Sometimes we can factor a polynomial that is not in simplest form but has a common binomial factor.

Consider this problem:

$y(y - 3) + 7(y - 3)$

In this case, y - 3 is the common factor.

If we divide out the y - 3's we get:

$(y - 3)(\ y\cancel{(y - 3)} + 7\cancel{(y - 3)}\ ) = (y - 3)(y + 7)$

Factoring

Factor each polynomial:

$a(z^2 + 5) - (z^2 + 5)$

$3x(x + y) + 4y(x + y)$

$7mn(x - y) - 2(x + y)$

Factoring

In working with common binomial factors, look for factors that are opposites of each other.

For example: $(x - y) = - (y - x)$ because

$x - y = x + (-y) = -y + x = -1(y - x)$

so $x - y$ and $y - x$ are opposites or additive inverses of each other.

You can check this by adding them together: $x - y + y - x = 0$!

Additive Inverse

Name the additive inverse of each binomial:

$3x - 1$

$5a + 3b$

$x + y$

$4x - 6y$

Prove that each pair are additive inverses by adding them together - what do you get?

52 True or False: $y - 7 = -7 - y$

- True
- False

53 True or False: $8 - d = -1(d + 8)$

- True
- False

54 True or False: The additive inverse of $8c - h$ is $-8c + h$.

- True
- False

55 True or False: -a - b and a + b are opposites.

- True
- False

Opposites

In working with common binomial factors, look for factors that are opposites of each other.

Example 3 Factor the polynomial.

n(n - 3) - 7(3 - n)

Rewrite 3 - n as -1(n - 3)	n(n - 3) - 7(-1)(n - 3)
Simplify	n(n - 3) + 7(n - 3)
Factor	(n - 3)(n + 7)

Factor the polynomial.

p(h - 1) + 4(1 - h)

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56 If possible, Factor $7r + 14s$

- A $7r(1+2s)$
- B $7s(r+2)$
- C $7(r+2s)$
- D Already Simplified

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57 If possible, Factor $10a^3 - 35a^2 + 12$

- A $2a(5a^2 - 7a + 6)$
- B $5a(2a^2 - 7a + 2)$
- C $2(5a^3 - 7a^2 + 6)$
- D Already Simplified

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58 If possible, Factor $z(z-1) + 2(z-1)$

- A $(z-1)(z+2)$
- B $(z-1)(z-2)$
- C $(z+1)(z-2)$
- D Already Simplified

59 If possible, Factor $9(1-x)-x(x-1)$

- A $(x-1)(x-9)$
- B $(1-x)(9+x)$
- C $(x-9)(x-1)$
- D Already Simplified

Factoring Using Special Patterns

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Special Patterns in Multiplying

When we were multiplying polynomials we had special patterns.

Square of Sums $(a+b)^2 = a^2 + 2ab + b^2$

Difference of Sums $(a-b)^2 = a^2 - 2ab + b^2$

Product of a Sum and a Difference $(a+b)(a-b) = a^2 - b^2$

If we learn to recognize these squares and products we can use them to help us factor.

Perfect Square Trinomials

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The Square of a Sum and the Square of a difference have products that are called Perfect Square Trinomials.

How to Recognize a Perfect Square Trinomial:

$$(\square + \square)^2 = \square^2 + 2\square\square + \square^2$$

$$(\square - \square)^2 = \square^2 - 2\square\square + \square^2$$

Fill in the blanks with any monomial (or any expression!!) Try it!!

Perfect Square Trinomials

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Examples:

$x^2 + 10x + 25$	$t^2 + 2t + 1$
$b^2 - 8b + 16$	$x^2 - 18xy + 81y^2$
$h^2 + 12h + 36$	$c^4 - 6c^2 + 9$

What do these trinomials have in common?
What patterns do you see?

Perfect Square Trinomials

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Complete these perfect square equations:

$(x + \_\_\_)^2 = x^2 + \_\_\_\_ + 25$

$(x - \_\_\_)^2 = x^2 - \_\_\_\_ + 49$

$(x - 10)^2 = x^2 + \_\_\_\_ + \_\_\_\_$

$(2x + \_\_\_)^2 = \_\_x^2 + \_\_\_\_ + 81$

Perfect Square Trinomials

Is the trinomial a perfect square?

$16-24j+3j^2$

$9-6y+y^2$

$x^2+10x+25$

$4c^2+6c+9$

$d^2-8d-16$

b^2-2b+1

$4h^2+20h+5$

$4m^2+24mn+36n^2$

Drag the Perfect Square Trinomials into the Box.

Only Perfect Square Trinomials will remain visible.

60 Factor x^2+4x+4

- A $(x+2)^2$
- B $(x-2)^2$
- C $(x+4)^2$
- D Not a perfect Square Trinomial

61 Factor $x^2 - 10x + 100$

- A $(x+10)^2$
- B $(x-10)^2$
- C $(x-5)^2$
- D Not a perfect Square Trinomial

62 Factor $16x^2 - 40x + 25$

- A $(4x+5)^2$
- B $(4x-5)^2$
- C $(8x-5)^2$
- D Not a perfect Square Trinomial

Difference of Squares Binomials

The product of a sum and difference of two monomials has a product called a Difference of Squares.

How to Recognize a Difference of Squares Binomial:

$$(\square + \square)(\square - \square) = \square^2 - \square^2$$

What happens to the middle term?

Fill in the blanks with any monomial (or any expression!!) Try it!!

Difference of Squares

Examples:

$x^2 - 16$ $16b^2 - 16$

$d^2 - 100$ $4c^2 - 1$

$j^2 - 49$ $j^4 - 16$

Factoring a Difference of Squares

Once a binomial is determined to be a Difference of Squares, it factors following the pattern:

(sq rt of 1st term - sq rt of 2nd term)(sq rt of 1st term + sq rt of 2nd term)

Factor each of the following:

$x^2 - 25$

$9 - y^2$

$4m^2 - 36n^2$

$y^4 - 1$

63 Factor $x^2 - 9$

- A $(x-3)(x-3)$
- B $(x-3)(x+3)$
- C $(x+3)(x+3)$
- D Not a Difference of Squares

64 Factor $100 - 4h^2$

- A $(10-2h)(10+2h)$
- B $(50-2h)(50+2h)$
- C $(10-2h)(10-2h)$
- D Not a Difference of Squares

65 Factor $x^2 + 9$

- A $(x-3)(x-3)$
- B $(x-3)(x+3)$
- C $(x+3)(x+3)$
- D Not a Difference of Squares

66 Factor using Difference of Squares: $4y^2 - 6$

- A $(2y-3)(2y-3)$
- B $(2y-3)(2y+3)$
- C $(2y+3)(2y+3)$
- D Not a Difference of Squares

Identifying & Factoring: $x^2 + bx + c$

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Classifying Polynomials

Polynomials can be classified by the number of terms. The table below summarizes these classifications.

Number of terms	Name	Examples
1	Monomial	10 $-5x$ $-5x^3$
2	Binomial	$10+x$ $8x^3y^2-4$
3	Trinomial	$7x^2+5x-2$ $a+b+c$
> 3	No special name	$11x^3+9x^2-\frac{1}{2}x+\frac{2}{3}$

Classifying Polynomials

Polynomials can be desribed based on something called their "degree".

For a polynomial with one variable, the degree is the largest exponent of the variable.

the degree of this polynomial is 7

$3x^7 - 5x^4 + 8x - 1$

Classifying Polynomials

Polynomials can also be classified by degree. The table below summarizes these classifications.

Degree	Type	Examples
0	Constant	10 $\frac{1}{3}$
1	Linear	$-5x$ $-5x+4$
2	Quadratic	$8x^2-5x+3$
3	Cubic	$7x^3+5x-2$
4	Quartic	$11x^4+9x^2-\frac{1}{2}x+\frac{2}{3}$

Classifying Polynomials

Classify each polynomial based on the number of terms and its degree.

$x^2 - 3x + 7$ ____________

$3x + 8$ ____________

$4x^2$ ____________

$4x^2 - 8$ ____________

3 ____________

Cubic
Quadratic
Linear
Constant

Trinomial
Binomial
Monomial

68 Choose all of the descriptions that apply to:

$$-4x^2 + 9$$

- ☐ A Quadratic
- ☐ B Linear
- ☐ C Constant
- ☐ D Trinomial
- ☐ E Binomial
- ☐ F Monomial

70 Choose all of the descriptions that apply to:

$$5x^2 + x + 2x$$

- ☐A Quadratic
- ☐B Linear
- ☐C Constant
- ☐D Trinomial
- ☐E Binomial
- ☐F Monomial

71 Choose all of the descriptions that apply to:

$$2$$

- ☐A Quadratic
- ☐B Linear
- ☐C Constant
- ☐D Trinomial
- ☐E Binomial
- ☐F Monomial

Simplify

1) (x + 2)(x + 3) = ____________________

2) (x - 4)(x - 1) = ____________________

3) (x + 1)(x - 5) = ____________________

4) (x + 6)(x - 2) = ____________________

Answer Bank

$x^2 - 5x + 4$

$x^2 - 4x - 5$

$x^2 + 5x + 6$

$x^2 + 4x - 12$

Slide each polynomial from the circle to the correct expression.

RECALL … What did we do?? Look for a pattern!!

Multiply:

(x + 3)(x +4)

(x +3)(x - 4)

(x - 3)(x + 4)

(x - 3)(x - 4)

What is the same and what is different about each product? What patterns do you see? What generalizations can be made about multiplication of binomials?

Work in your groups to make a list and then share with the class. Make up your own example like the one above. Do your generalizations hold up?

Factor

Examples:

$x^2 - 4x + 3$

$x^2 + 7x + 10$

$x^2 - 12x + 20$

Factor

Examples:

$x^2 - 9x + 8$

$x^2 + 8x + 12$

$x^2 + 7x + 12$

72 What kind of signs will the factors of 12 have, given the following equation? $x^2 - 8x + 12$

- A Both positive
- B Both Negative
- C Bigger factor positive, the other negative
- D The bigger factor negative, the other positive

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73 The factors of 12 will have what kind of signs given the following equation? $x^2 + 13x + 12$

- A Both positive
- B Both negative
- C Bigger factor positive, the other negative
- D The bigger factor negative, the other positive

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74 Factor $x^2 - 7x + 12$

- A (x + 12)(x + 1)
- B (x + 6)(x + 2)
- C (x + 4)(x + 3)
- D (x - 12)(x - 1)
- E (x - 6)(x - 1)
- F (x - 4)(x - 3)

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75 Factor $x^2 + 8x + 12$

- A (x + 12)(x + 1)
- B (x + 6)(x + 2)
- C (x + 4)(x + 3)
- D (x - 12)(x - 1)
- E (x - 6)(x - 1)
- F (x - 4)(x - 3)

76 Factor $x^2 + 13x + 12$

- A (x + 12)(x + 1)
- B (x + 6)(x + 2)
- C (x + 4)(x + 3)
- D (x - 12)(x - 1)
- E (x - 6)(x - 1)
- F (x - 4)(x - 3)

77 Factor $x^2 - 8x + 12$

- A (x + 12)(x + 1)
- B (x + 6)(x + 2)
- C (x + 4)(x + 3)
- D (x - 12)(x - 1)
- E (x - 6)(x - 2)
- F (x - 4)(x - 3)

Factor

Examples

$x^2 - x - 20$

$x^2 + 6x - 16$

$x^2 + 4x - 32$

Factor

Examples

$x^2 + 9x - 36$

$x^2 - 3x - 18$

$x^2 - 3x - 10$

78 The factors of -12 will have what kind of signs given the following equation? $x^2 - 1x - 12$

- A Both positive
- B Both negative
- C Bigger factor positive, the other negative
- D The bigger factor negative, the other positive

79 The factors of -12 will have what kind of signs given the following equation? $x^2 + 4x - 12$

- A Both positive
- B Both negative
- C Bigger factor positive, the other negative
- D The bigger factor negative, the other positive

80 Factor $x^2 + x - 12$

- A (x + 12)(x - 1)
- B (x + 6)(x - 2)
- C (x + 4)(x - 3)
- D (x - 12)(x + 1)
- E (x - 6)(x + 1)
- F (x - 4)(x + 3)

81 Factor $x^2 - 5x - 12$

- A (x + 12)(x - 1)
- B (x + 6)(x - 2)
- C (x + 4)(x - 3)
- D (x - 12)(x + 1)
- E (x - 6)(x + 1)
- F unable to factor using this method

Mixed Practice

83 Factor the following $x^2 + 2x - 8$

- A (x - 2)(x - 4)
- B (x + 2)(x + 4)
- C (x - 2)(x +4)
- D (x + 2)(x - 4)

84 Factor the following $x^2 - 8x + 15$

- A (x - 3)(x - 5)
- B (x + 3)(x + 5)
- C (x - 3)(x +5)
- D (x + 3)(x - 5)

85 Factor the following $x^2 + 7x + 12$

- A (x - 3)(x - 4)
- B (x + 3)(x + 4)
- C (x +2)(x +6)
- D (x + 1)(x+12)

86 Factor the following $x^2 - 3x - 10$

- A (x - 2)(x - 5)
- B (x + 2)(x + 5)
- C (x - 2)(x +5)
- D (x + 2)(x - 5)

Factoring Trinomials: $ax^2 + bx + c$

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a does not = 1

How to factor a trinomial of the form $ax^2 + bx + c$.

Example: Factor $2d^2 + 15d + 18$

First, find ac: $2 \cdot 18 = 36$

Now find two integers whose product is ac and whose sum is equal to b or 15.

Factors of 36	Sum = 15?
1, 36	1 + 36 = 37
2, 18	2 + 18 = 20
3, 12	**3 + 12 = 15**

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a does not = 1

$2d^2 + 15d + 18$

ac = 36, b = 15

Our numbers: 3 and 12

Split the middle term, 15d, into 3d + 12d: $2d^2 + 3d + 12d + 18$

first 2 terms — last 2 terms

Factor the first two terms and the last two terms:

d(2d + 3) + 6(2d + 3)

Factor out the common binomial (2d + 3)(d + 6)

Remember to check by multiplying!

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a does not = 1

Factor. $15x^2 - 13x + 2$

ac = 30, but b = -13
Since ac is positive, and b is negative we need
to find two negative factors of 30 that add up to -13

Factors of 30	Sum = -13?
-1, -30	-1 + -30 = -31
-2, -15	-2 + -15 = -17
-3, -10	**-3 + -10 = -13**
-5, -6	-5 + -6 = -11

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a does not = 1

$15x^2 - 13x + 2$

ac = 30, b = -13

Our numbers: -3 and -10

a does not = 1

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Factor. $2b^2 - b - 10$

$a = 2$, $c = -10$, and $b = -1$
Since ac is negative, and b is negative we need to find two factors with opposite signs whose product is -20 and that add up to -1. Since b is negative, larger factor of -20 must be negative.

Factors of -20	Sum = -1?

a does not = 1

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Factor

$6y^2 - 13y - 5$

Berry Method to Factor

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Step 1: Calculate ac.

Step 2: Find a pair of numbers m and n, whose product is ac, and whose sum is b.

Step 3: Create the product $(ax + m)(ax + n)$.

Step 4: From each binomial in step 3, factor out and discard any common factor. The result is your factored form.

Example: $4x^2 - 19x + 12$ ac = 48, b = -19

m = -3, n = -16

$(4x - 3)$~~$(4x - 16)$~~ Factor 4 out of $4x - 16$ and toss it!

$(4x - 3)(x - 4)$ THE ANSWER!

Prime Polynomial

A polynomial that cannot be factored as a product of two polynomials is called a prime polynomial .

How can you tell if a polynomial is prime? *Discuss with your table.*

click to reveal

um is b.

87 Factor $3a^2 + 13a + 4$

- A $(3a+2)(a+2)$
- B $(3a+4)(a+1)$
- C $(3a+1)(a+4)$
- D Prime Polynomial

88 Factor $14a^2 - 43a + 20$

- A $(7a-4)(2a-5)$
- B $(7a-5)(2a-4)$
- C $(7a-10)(2a-2)$
- D Prime Polynomial

89 Factor $8a^2 - 10a - 3$

- A $(8a-6)(a+2)$
- B $(2a-3)(4a+1)$
- C $(4a-3)(2a+1)$
- D Prime Polynomial

Factoring 4 Term Polynomials

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4 Terms

Polynomials with four terms like ab - 4b + 6a - 24, can sometimes be factored by grouping terms of the polynomials.

Example 1:

ab - 4b + 6a - 24

(ab - 4b) + (6a - 24) Group terms into binomials that can be factored using the distributive property

b(a - 4) + 6(a - 4) Factor the GCF

(a - 4) (b + 6)

4 Terms

Example

6xy + 8x - 21y - 28

What are the relationships among the following:

Some are equivalent, some are opposites, some are not related at all. Mix and match by dragging pairs for each category:

Equivalent Opposites Not related

x + 3 -x + 3 -x - 3 x - 3 3 - x 3 + x

Additive Inverses

You must be able to recognize additive inverses!!!
(3 - a and a - 3 are additive inverses because their sum is equal to zero.)
Remember 3 - a = -1(a - 3).

Example
15x - 3xy + 4y - 20

(15x - 3xy) + (4y - 20)	Group
3x(5 - y) + 4(y - 5)	Factor GCF
3x(-1)(y - 5) + 4(y - 5)	Rewrite based on additive inverses
-3x(y - 5) + 4(y - 5)	Simplify
(y - 5) (-3x + 4)	Factor common binomial

Remember to check each problem by using FOIL.

90 Factor 15ab - 3a + 10b - 2

- A (5b - 1)(3a + 2)
- B (5b + 1)(3a + 2)
- C (5b - 1)(3a - 2)
- D (5b + 1)(3a - 1)

91 Factor $10m^2n$ - 25mn + 6m - 15

- A (2m-5)(5mn-3)
- B (2m-5)(5mn+3)
- C (2m+5)(5mn-3)
- D (2m+5)(5mn+3)

92 Factor 20ab - 35b - 63 +36a

- A (4a - 7)(5b - 9)
- B (4a - 7)(5b + 9)
- C (4a + 7)(5b - 9)
- D (4a + 7)(5b + 9)

93 Factor $a^2 - ab + 7b - 7a$

- A (a - b)(a - 7)
- B (a - b)(a + 7)
- C (a + b)(a - 7)
- D (a + b)(a + 7)

Mixed Factoring

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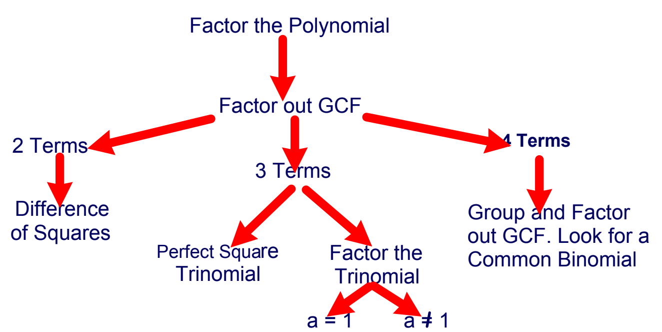
Summary of Factoring



Check each factor to see if it can be factored again.
If a polynomial cannot be factored, then it is called prime.

94 Factor completely: $4cd^2 + 12cd + 8c$

- A $4c(d+3)(d+2)$
- B $4c(d+2)(d+1)$
- C $(d+3)(4d+2)$
- D $4c(d^2+3d+2)$

95 Factor completely $10a^3 - 35a^2 + 12$

- A $2a(5a^2 - 7a + 6)$
- B $5a(2a^2 - 7a + 2)$
- C $2(5a^3 - 7a^2 + 6)$
- D prime polynomial

96 Factor $4y^2 - 15$

- A $(2y-5)(2y-3)$
- B $(2y-5)(2y+3)$
- C $(2y+5)(2y+3)$
- D prime polynomial

97 Factor completely $10w^2x^2 - 100w^2x + 1000w^2$

- A $10w^2(x + 10)^2$
- B $10w^2(x - 10)^2$
- C $10(wx - 10)^2$
- D $10w^2(x^2 - 10x + 100)$

98 Factor $4a^2 - 2a - 30$

- A $2(2a-5)(a+3)$
- B $2(2a+5)(a-3)$
- C $2(2a-3)(a+5)$
- D Prime Polynomial

Solving Equations by Factoring

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Given the following equation, what conclusion(s) can bedrawn?

$$ab = 0$$

Recall ~ Given the following equation, what conclusion(s) can be drawn?

(x - 4)(x + 3) = 0

Since the product is 0, one of the factors must be 0.
Therefore, either x - 4 = 0 or x + 3 = 0.

$$\begin{array}{rcl} x - 4 = 0 & \text{or} & x + 3 = 0 \\ +4 \quad +4 & & -3 \quad -3 \\ \hline x = 4 & \text{or} & x = -3 \end{array}$$

Therefore, our solution set is {-3, 4}. To verify the results, substitute each solution back into the original equation.

To check x = -3:

$$\begin{aligned} (x - 4)(x + 3) &= 0 \\ (-3 - 4)(-3 + 3) &= 0 \\ (-7)(0) &= 0 \\ 0 &= 0 \end{aligned}$$

To check x = 4:

$$\begin{aligned} (x - 4)(x + 3) &= 0 \\ (4 - 4)(4 + 3) &= 0 \\ (0)(7) &= 0 \\ 0 &= 0 \end{aligned}$$

What if you were given the following equation?

$$x^2 - 2x - 24 = 0$$

How would you solve it?

We can use the Zero Product Property to solve it.

How can we turn this polynomial into a multiplication problem? Factor it

Factoring yields: (x - 6)(x + 4) = 0

By the Zero Product Property:

x - 6 = 0 or x + 4 = 0

After solving each equation, we arrive at our solution:

{-4, 6}

Trinomial

Recall the Steps for Factoring a Trinomial

1) See if a monomial can be factored out.
2) Need 2 numbers that multiply to the constant
3) and add to the middle number.
4) Write out the factors.

Solve $2a^3 - 4a^2 - 30 = 0$

$$2a\left(a^2 - 2a - 15\right) = 0$$

$$2a\left(a - 5\right)\left(a + 3\right) = 0$$

Now...

1) Set each binomial equal to zero.
2) Solve each binomial for the variable.

$2a = 0 \quad a - 5 = 0 \quad a + 3 = 0$

click to reveal

99 Choose all of the solutions to: $14a^2 - 43a + 20 = 0$

- ☐A $\frac{4}{7}$
- ☐B $\frac{2}{5}$
- ☐C $\frac{7}{4}$
- ☐D $\frac{5}{2}$
- ☐E $-\frac{4}{7}$
- ☐F $-\frac{5}{2}$

100 Choose all of the solutions to: $g^3 - 16g = 0$

- ☐A -4
- ☐B -2
- ☐C 0
- ☐D 2
- ☐E 4
- ☐F 16

101 Choose all of the solutions to: $m^2 = 4m$

- ❑A -4
- ❑B -2
- ❑C 0
- ❑D 2
- ❑E 4
- ❑F 16

102 A ball is thrown with its height at any time given by

$$h = -16t^2 + 144t + 160$$

When does the ball hit the ground?

- ❍A -1 seconds
- ❍B 0 seconds
- ❍C 9 seconds
- ❍D 10 seconds