Algebra I

Pacing Guide and Curriculum Reference

Based on the 2009 Virginia Standards of Learning

2013-2014

Henrico County Public Schools

Introduction

The Mathematics Curriculum Guide serves as a guide for teachers when planning instruction and assessments. It defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessment. It provides additional guidance to teachers as they develop an instructional program appropriate for their students. It also assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This Guide delineates in greater specificity the content that all teachers should teach and all students should learn.

The format of the Curriculum Guide facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each objective. The Curriculum Guide is divided by unit and ordered to match the established HCPS pacing. Each unit is divided into two parts: a one page unit overview and a Teacher Notes and Resource section. The unit overview contains the suggested lessons for the unit and all the DOE curriculum framework information including the related SOL(s), strands, Essential Knowledge and Skills, and Essential Understandings. The Teacher Notes and Resource section is divided by Resources, Key Vocabulary, Essential Questions, Teacher Notes and Elaborations, Honors/AP Extensions, and Sample Instructional Strategies and Activities. The purpose of each section is explained below.

Vertical Articulation: This section includes the foundational objectives and the future objectives correlated to each SOL.

Unit Overview:

- *Curriculum Information*: This section includes the SOL and SOL Reporting Category, focus or topic, and pacing guidelines.
- *Essential Knowledge and Skills*: Each objective is expanded in this section. What each student should know and be able to do in each objective is outlined. This is not meant to be an exhaustive list nor is a list that limits what taught in the classroom. This section is helpful to teachers when planning classroom assessments as it is a guide to the knowledge and skills that define the objective. (Taken from the Curriculum Framework)
- *Essential Understandings*: This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the objectives. (Taken from the Curriculum Framework)

Teacher Notes and Resources:

- *Resources*: This section gives textbook resources, links to related Algebra 1 Online! modules, and links to VDOE's Enhanced Scope and Sequence lessons.
- *Key Vocabulary*: This section includes vocabulary that is key to the objective and many times the first introduction for the student to new concepts and skills.
- *Essential Questions*: This section explains what is meant to be the key knowledge and skills that define the standard.
- *Teacher Notes and Elaborations*: This section includes background information for the teacher. It contains content that is necessary for teaching this objective and may extend the teachers' knowledge of the objective beyond the current grade level.
- *Extensions*: This section provides content and suggestions to differentiate for honors/Pre-AP level classes.
- *Sample Instructional Strategies and Activities*: This section provides suggestions for varying instructional techniques within the classroom.

Special thanks to Prince William County Public Schools for allowing information from their curriculum documents to be included in this document.

Algebra 1 Pacing and Curriculum Guide

Course Outline

First Marking Period at a Glance	Second Marking Period at a Glance	Third Marking Period at a Glance	Fourth Marking Period at a Glance
A.1 - Expressions	A.7 - Relations and Functions	A.4e/A.5d - Systems	A.2c - Factoring (cont.)
A.4 - Solving Equations	A.6 - Linear Equations	A.2a - Rules of Exponents	A.4c - Quadratics
A.5 - Solving Inequalities	A.8 - Variation	A.2b - Polynomials	A.3 - Radicals
A.9 – Standard Deviation		A.2c - Factoring	A.10/A.11 - Data Analysis

Big Ideas

1. Expressions	2. Equations & Inequalities	3. Relations and Functions	4. Linear Equations	5. Systems
6. Polynomials	7. Factoring	8. Quadratics	9. Data Analysis	10. Radicals

ALGEBRA I SOL TEST BLUEPRINT (50 QUESTIONS TOTAL)

Expressions and Operations	12 Questions	24 % of the Test
Equations and Inequalities	18 Questions	36 % of the Test
Functions and Statistics	20 Questions	40 % of the Test

Resources

Text: Prentice Hall Mathematics, Algebra I, 2006, Prentice Hall	HCPS Mathematics Website http://blogs.henrico.k12.va.us/math/	HCPS Algebra 1 Online Module Homepage http://teachers.henrico.k12.va.us/math/HCPSAlgebra1/modules.html
Virginia Department of Education Mathematics SOL Resources http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml	DOE Enhanced Scope and Sequence Lesson Plans http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/	

SOL Vertical Articulation

	Previous Standards	Algebra 1 Standards	Future Standards
Expressions	**7.1**b) determine scientific notation for numbers > 0; c) compare/order fractions/decimals/percents, and scientific notation **7.13** a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; b) evaluate algebraic expressions **8.1** a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, properties; b) compare/order fractions/decimals/percents, and scientific notation **8.4** evaluate algebraic expressions using order of operations	**A.1** represent verbal quantitative situations algebraically/evaluate expressions for given replacement values of variables	**AII.1** given rational/radical/poly expressions - a) add/subtract/multiply/divide/simplify rational algebraic expressions; b) add/subtract/multiply/ divide/simplify radical expressions containing rational numbers/variables, and expressions containing rational exponents; c) write radical expressions as expressions containing rational exponents; d) factor polynomials completely
Properties and Operations	**7.16** apply properties w/ real numbers: a) commutative and associative properties for add/multi; b) distributive property; c) add/multi identity properties; d) add/multi inverse properties; e) multi property of zero **8.15** c) ID properties of operations used to solve equations	**A.2** perform operations on polynomials - a) apply laws of exponents to perform ops on expressions; b) add/subtract/multiply/divide polynomials; c) factor first and second degree binomials/trinomials (1 or 2 variables)	**AII.3** perform operations on complex numbers/express results in simplest form using patterns of the powers of i/ID field properties for complex numbers
Square/ Cube Roots	**7.1** d) determine square roots **8.5** a) determine if a number is a perfect square; b) find two consecutive whole numbers between which a square root lies	**A.3** express square roots/cube roots of whole numbers/the square root of monomial algebraic expression (simplest radical form)	
Solving and Graphing Equations and Inequalities	**7.14** a) solve one- and two-step linear equations; b) solve practical problems in one variable **8.15** a) solve multistep linear equations in one variable (variable on one and two sides of equations); c) ID properties of operations used to solve	**A.4** solve multistep linear/quad equation (in 2 variables) - a) solve literal equation; b) justify steps used in simplifying expressions and solving equations; c) solve quad equations (algebraically/graphically); d) solve multistep linear equations (algebraically/ graphically); e) solve systems of two linear equation (2 variable- algebraically/graphically); f) solve real-world problems involving equations and systems of equations	**AII.4** solve (algebraically/graphically) a) absolute value equation/inequalities; b) quadratic equations over complex; c) equations containing rational algebraic expression; d) equations containing radical expressions
	7.15 a) solve one-step inequalities; b) graph solutions on number line **8.15** b) solve two-step linear inequalities and graph results on number line; c) ID properties of operations used to solve	**A.5** solve multistep linear inequalities (2 variables) - a) solve multistep linear inequalities (algebraically/ graphically); b) justify steps used in solving inequalities; c) solve real-world problems involving inequalities; d) solve systems of inequalities	**AFDA.5** determine opt values in problem situations by identifying constraints/using linear programming techniques **AII.5** solve nonlinear systems (algebraically/ graphically)

Solv/Graph Equ/Inequal.	**7.15** a) solve one-step inequalities; b) graph solutions on number line **8.15** b) solve two-step linear inequalities and graph results on number line; c) ID properties of operations used to solve **8.16** graph linear equations in two variables	**A.6** graph linear equations/linear inequalities (in 2 variables) **-** a) determine slope of line given equation of line/ graph of line or two points on line **-** slope as rate of change; b) write equation of line given graph of line, two points on line or slope & point on line	**AFDA.2** use transformations to write equations, given graph of function (linear/quad/exponential/log) **AII.6** recognize general shape of function (absolute value/square root/cube root/rational/poly/exponential/ log) families/ convert between graphic and symbolic forms of functions **-** transformational approach to graphing
Function Analysis	**7.12** represent relationships with tables, graphs, rules, and words **8.17** ID domain, range, independent/dependent variable **8.14** make connections between any two representations (tables, graphs, words, rules)	**A.7** investigate/analyze function (linear/ quadratic) families and characteristics (algebraically/graphically) **-** a) determine relation is function; b) domain/range; c) zeros; d) *x*- and *y*-intercepts; e) find values of function for elements in domain; f) make connect between/among multiple representation of functions (concrete/verbal/numeric/graphic/algebraic)	**AFDA.1** investigate/analyze function (linear/quadratic exponential/log) families/ characteristics: a) continuity; b) local/abs max/min; c) domain/range; d) zeros; e) intercepts; f) intervals of increasing/ decreasing; g) end behaviors; h) asymptotes **AFDA.4** transfer between/analyze multiple representations of functions (algebraic formulas/ graphs/ tables/words) **AII.7** investigate/analyze functions (algebraically/ graphically) a) domain/range; b) zeros; c) *x*- and *y*-intercepts; d) intervals of increasing/decreasing; e) asymptotes; f) end behavior; g) inverse of a function; h) composition of multiple functions
Vari-ation		**A.8** given real-world context, analyze relation to determine direct/inverse variation; represent direct variation (algebraically/graphically) and inverse variation (algebraically)	**AII.10** ID/create/solve real-world problems involving inverse/joint variation/ combo of direct/inverse variations
Measures of Data Spread	**5.16** a) describe mean/median/mode; b) describe mean as fair share; c) find the mean/median/mode/ range; d) describe range as measure of variation. **6.15** a) describe mean as balance point; b) decide which measure of center is appropriate	**A.9** given a set of data, interpret variation in real-world contexts/calculate/interpret mean absolute deviation/standard deviation /*z*-scores	**AFDA.7** analyze norm distribution **-** a) characteristics of normally distribution of data; b) percentiles; c) normalizing data, using *z*-scores; d) area under standard norm curve/ probability **AII.11** ID properties of norm distribution/apply properties to determine probabilities associated with areas under the standard normal curve
Data Analysis	**7.11** a) construct/analyze histograms; b) compare/ contrast histograms	**A.10** compare/contrast multiple univariate data sets with box-and-whisker plots	
	8.13 a) make comparisons/predictions/ inferences, using information displayed in graphs; b) construct/analyze scatterplots	**A.11** collect/analyze data/determine equation of curve best fit to make predictions/solve real-world problems, using models (linear/quadratic)	**AFDA.3** collect data/generate equation for the curve (linear/quadratic/exponential/log) of best fit/use best fit equations to interpolate function values/make decisions/justify conclusions (algebraic/graph models) **AII.9** collect/analyze data/determine equations of the curve of best fit/make predictions/solve real world problems, using models (polynomial/exponential /log)

Expressions

First Marking Period

Lessons	Essential Knowledge and Skills	Essential Understandings
• Variables and Expressions • Order of Operations • Open Sentences • Identity and Equality Properties • Distributive Property • Commutative and Associative Properties **Strand**: Expressions and Operations; Equations and Inequalities **SOL A.1** The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. **SOL A.4b** The student will solve multi-step linear and quadratic equations in two variables, including justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets. Return to Course Outline	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Translate verbal quantitative situations into algebraic expressions and vice versa. • Model real-world situations with algebraic expressions in a variety of representations (concrete, pictorial, symbolic, verbal). • Evaluate algebraic expressions for a given replacement set to include rational numbers. • Evaluate expressions that contain absolute value, square roots, and cube roots. • Simplify expressions and solve equations using the field properties of the real numbers and properties of equality to justify simplification and solution.	• Algebra is a tool for reasoning about quantitative situations so that relationships become apparent. • Algebra is a tool for describing and representing patterns and relationships. • Mathematical modeling involves creating algebraic representations of quantitative real-world situations. • The numerical values of an expression are dependent upon the values of the replacement set for the variables. • There is a variety of ways to compute the value of a numerical expression and evaluate an algebraic expression. • The operations and the magnitude of the numbers in an expression impact the choice of an appropriate computational technique. • An appropriate computational technique could be mental mathematics, calculator, or paper and pencil. • Properties of real numbers and properties of equality can be used to justify equation solutions and expression simplification. *(continued)*

Teacher Notes and Resources

Expressions

Resources

Textbook:

1-1 Using Variables
1-2 Exponents and Order of Operations
1-7 The Distributive Property
1-8 Properties of Real Numbers

HCPS Web Site:

Expressions

DOE Lesson Plans:

- Evaluating and Simplifying Expressions (PDF) (Word)
- Translate and Evaluate (PDF) (Word) - Representing quantitative situations algebraically and evaluating and simplifying algebraic expressions
- A Mystery to Solve (PDF) (Word) - Investigating properties with an undefined operation

Key Vocabulary

absolute value
algebraic expression
cube root
field properties:
- closure
- commutative
- associative
- inverse
- identity
- distributive

negative root
positive root
properties of equality:
- reflexive
- symmetric
- transitive
- substitution
- addition
- subtraction
- multiplication
- division

square root
variable

Return to Course Outline

Essential Questions

- What is Algebra?
- How is a variable used in an algebraic expression?
- Why is it necessary to have an agreed upon order of operations?
- How are algebraic expressions modeled?
- How is order of operations applied when simplifying and evaluating expressions?
- How are the field properties and properties of equality of real numbers used to solve equations?

Teacher Notes and Elaborations

A *variable* is a symbol, usually a letter, used to represent a quantity. This quantity represents an element of any subset of the real numbers.

An *algebraic expression* may contain numbers, variables, operations, and grouping symbols. An algebraic expression may be evaluated by substituting values for the variables in the expression.

The numerical values of an expression are dependent upon the values of the replacement set for the variables.

The *absolute value* of a number is the distance from 0 on the number line regardless of direction $\left(\text{e.g., } \left|-\frac{1}{2}\right| = \frac{1}{2}, \left|\frac{-1}{2}\right| = \frac{1}{2}, \left|\frac{1}{-2}\right| = \frac{1}{2}, \text{ and } \left|\frac{1}{2}\right| = \frac{1}{2}\right)$.

In evaluating algebraic expressions, the laws of the order of operations must be followed to find the value of an expression.

The *square root* of a number is any number which when multiplied by itself equals the number. Whole numbers have both positive and negative roots. For example, the square root of 25 is 5 and –5, where 5 is the *positive root* and –5 is the *negative root* (written as ± 5).

(continued)

Teacher Notes and Resources

Expressions *(continued)*

Teacher Notes and Elaborations *(continued)*

The *cube root* of a number, *n*, is a number whose cube is that number. For example, the cube root of 125 is 5 ($\sqrt[3]{125} = 5$) because $5^3 = 125$. In general, $\sqrt[3]{n} = a$ if $a^3 = n$.

Algebra tiles/algeblocks may be used as concrete or pictorial models of real world situations.

Word phrases, which describe characteristics of given conditions, can be translated into algebraic expressions for the purpose of evaluation. There is a variety of methods to compute the value of an algebraic or numerical expression, such as mental math, calculator or paper and pencil methods.

Real world situations are problems expressed in words from day to day life. These problems can be understood and represented using manipulatives, pictures, equations/expressions, and in written and spoken language.

Each step in the solution of the equation will be justified using the field properties of real numbers and the properties of equations. These properties may be modeled using manipulatives and pictorial representations.

Properties of Equality: reflexive, symmetric, transitive, substitution, addition, subtraction, multiplication, and division.

Field Properties of Real Numbers: closure, commutative, associative, inverse, identity, and distributive.

Return to Course Outline

Sample Instructional Strategies and Activities

- Using groups of three, have one student write a mathematical expression. Have another student write the expression in words. Next, have a third student translate the words back to the expression. Compare the initial and final expressions. If they differ, verbalize each step to determine what was done incorrectly.
- Ask students to evaluate a list of algebraic expressions or give values using a calculator, pencil, or mental mathematics. They will then make up four to five expressions of their own to share with other groups.
- Have students evaluate expressions using the graphing calculator. Show students how to enter values into the calculator.
- Use algeblocks or algebra tiles to physically model substituting values into a variable expression by replacing the variable blocks with the appropriate number of ones.
- Write an algebraic expression for students to see. Roll a die to determine the replacement values for each variable in the expression. Students determine the value of the expression.

Solving Equations

First Marking Period

Lessons
- Solving One-Step Equations
- Solving Multistep Equations
- Solving Equations with Variables on Both Sides
- Solving Literal Equations and Formulas

Strand: Equations and Inequalities

SOL A.4abdf
The student will solve multistep linear equations in two variables, including
a) solving literal equations (formulas) for a given variable;
b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets.
d) solving multistep linear equations algebraically and graphically; and
f) solving real-world problems involving equations.
Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

Return to Course Outline

Essential Knowledge and Skills
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Solve a literal equation (formula) for a specified variable.
- Solve multistep linear equations in one variable.
- Determine if a linear equation in one variable has one, an infinite number, or no solutions.
- Confirm algebraic solutions to linear equations, using a graphing calculator.

Essential Understandings
- A solution to an equation is the value or set of values that can be substituted to make the equation true.
- The solution of an equation in one variable can be found by graphing the expression on each side of the equation separately and finding the *x*-coordinate of the point of intersection.
- Real-world problems can be interpreted, represented, and solved using linear equations.
- The process of solving linear equations can be modeled in a variety of ways, using concrete, pictorial, and symbolic representations.

(continued)

Teacher Notes and Resources

Solving Equations *(continued)*

Resources

Textbook:

2-1 Solving One-Step Equations
2-2 Solving Two-Step Equations
2-3 Solving Multi-Step Equations
2-4 Equations with Variables on Both Sides
2-6 Formula

HCPS Algebra 1 Online!:

Solving Equations

DOE Lesson Plans:

- Cover-Up Problems (PDF) (Word) - Deriving a method of solving simple equations in one variable
- Algebra Tiles and Equation Solving (PDF) (Word)
- Solve for the Unknown (PDF) (Word) - Solving literal equations
- Solving Linear Equations (PDF) (Word) - Exploring the functions of the graphing calculator; solving multistep linear equations

Key Vocabulary

coefficient
constant
field properties:
  closure
  commutative
  associative
  inverse
  identity
  distributive
formula
linear equation
literal equation
properties of equality:
  reflexive
  symmetric
  transitive
  substitution
  addition
  subtraction
  multiplication
  division

Essential Questions

- How are the field properties and properties of equality of real numbers used to solve equations?
- What is a literal equation?
- How are equations modeled?

Return to Course Outline

Teacher Notes and Elaborations

Real numbers are composed of rational and irrational numbers. Experiences with solving equations should include simple roots (square and cube) and 2nd and 3rd degree exponents (e.g. $(3x+8)^3 - 5 = 22$).

Linear and Literal Equations

In a *linear equation*, the exponent of the variable(s) is one. For example: $x + 5 = 9$ or $y = 3x - 8$.

A *literal equation* is an equation that shows the relationship between two or more variables. A *formula* is a special type of literal equation.

Each step in the solution of the equation will be justified using the field properties of real numbers and the properties of equations. These properties may be modeled using manipulatives and pictorial representations.

- *Properties of Equality*: reflexive, symmetric, transitive, substitution, addition, subtraction, multiplication, and division.
- *Field Properties* of Real Numbers: closure, commutative, associative, inverse, identity, and distributive.

A solution to an equation is the value or set of values (*solution set*) that can be substituted to make the equation true.

Set builder notation is used to represent solutions. If the solution is $y = 10$ then in set notation the answer is written $\{y : y = 10\}$ or $\{y | y = 10\}$.

(continued)

Teacher Notes and Resources

Solving Equations *(continued)*

Teacher Notes and Elaborations *(continued)*

The *coefficient* is the numerical part of a term. A *constant* is a symbol representing a value that does not change. Coefficients and constants as rational numbers will be emphasized. Real-life situations involving literal equations (formulas) will be investigated and solved.

Extension for PreAP Algebra I

Solve absolute value equations in one variable graphically and algebraically.

To solve an equation in the form $|A| = b$, where A represents a variable expression and $b > 0$, solve $A = b$ and $A = -b$.

Algebraic Example:

Solve $|x-5| = 7$

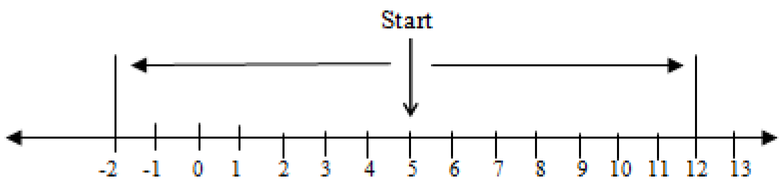
Write $|x-5| = 7$ as $x-5=7$ or $x-5=-7$ and solve both equations.

$x=12$ or $x=-2$

The value of x is 12 or -2.

Graphing Example:

$|x-5| = 7$ means that the distance between x and 5 is 7 units. To find x on the number line, start at 5 and move 7 units in either direction.



The value of x is 12 or -2.

Return to Course Outline

Sample Instructional Strategies and Activities

- Give students a list of equations to solve graphically. Equations should include multi-step equations, equations with variables on both sides, and equations with distributive property and variables on both sides. Using a graphing calculator, let y_1 equal the left member of the equation and let y_2 equal the right member. Use calculator functions to determine the point of intersections. Substitute the solution in the equation to check the problem.
- Students will use a graphing calculator to find the distance, rate, or time, given two of the unknowns. They are asked to tell how each answer was determined and write the literal equations.

  Example: $d = rt$; $t = \dfrac{d}{r}$; $r = \dfrac{d}{t}$
- Students, working in groups of two or three, will be given a set of cards. The names of the properties will be on one set of colored cards. Several examples of each property will be on cards of a different color. The cards should be shuffled. Students will try to match the examples with the property name.
- Groups will be given a list of solved equations and asked to name the property that justifies each step.

Solving Inequalities

First Marking Period

Lessons	Essential Knowledge and Skills	Essential Understandings
• Solving One-Step Inequalities (include graphing) • Solving Multistep Inequalities • Solving Compound Inequalities (optional) **Strand**: Equations and Inequalities **SOL A.5abc** The student will solve multistep linear inequalities in two variables, including a) solving multistep linear inequalities algebraically and graphically; b) justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets; and c) solving real-world problems involving inequalities. Return to Course Outline	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Solve multistep linear inequalities in one variable. • Justify steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers. • Solve real-world problems involving inequalities.	• A solution to an inequality is the value or set of values that can be substituted to make the inequality true. • Properties of inequality and order can be used to solve inequalities. • Set builder notation may be used to represent solution sets of inequalities. • Real-world problems can be modeled and solved using linear inequalities. *(continued)*

Teacher Notes and Resources

Solving Inequalities *(continued)*

Resources

Textbook:

3-1 Inequalities and Their Graphs
3-2 Solving Inequalities Using Addition and Subtraction
3-3 Solving Inequalities Using Multiplication and Division
3-4 Solving Multi-Step Inequalities
3-5 Compound Inequalities

HCPS Algebra 1 Online!:

Solving Inequalities

DOE Lesson Plans:

- Greetings (PDF) (Word) - Writing a function to describe an inequality and solving the inequality
- Inequalities (PDF) (Word) - Relating the graphing of a line to solving an inequality in one variable

Key Vocabulary

inequality	set builder notation

Essential Questions

- How are the properties of real numbers used to solve inequalities?
- What is the same about solving equations and solving inequalities and what is different?
- How are the solutions of systems of linear inequalities the same or different from the solutions of systems of equations?
- How are solutions written in set builder notation?
- What do the endpoints of the graph of an inequality represent?

Teacher Notes and Elaborations

An *inequality* is a statement that one quantity is less than (or greater than) another. The nature of the inequality is not changed when adding or subtracting any real number or when multiplying or dividing by a positive real number. However, when multiplying or dividing by negative numbers you must reverse the inequality symbol to maintain a true statement. A solution to an inequality is the value or set of values, which can be substituted to make the inequality true. A multi-step inequality will involve the combination of two or more operations. Each step in the solution of the inequality will be justified using the axioms of inequality.

An axiom is a statement universally recognized as true without proof.

Graphing can be used to demonstrate that both $x < 5$ and $5 > x$, represent the same solution set. Examples should include solutions with variables on either side.

Extension for Pre AP Algebra I

- Solve compound inequalities.

Extension for Pre AP Algebra I

- A compound inequality is two inequalities connected by "and" or "or". Compound inequalities can model many real-world situations, such as being above the timberline on a mountain but below the snow line. The solution of compound inequalities joined by "and" is the intersection of the solution sets of both inequalities. The solution of compound inequalities joined by "or" is the union of the solutions sets of both inequalities. Inequalities should be solved both algebraically and graphically.

Sample Instructional Strategies and Activities

- Divide the class into four or five small groups. Have each member create an inequality and then graph its solution set. Have groups switch graphs and identify the inequality that each solution set solves. Have groups return papers and verify each other's answers with the use of a graphing calculator. Equations can also be used in this type of activity. Their solution can be checked with the graphing calculator.

Return to Course Outline

Data Analysis – Standard Deviation

First Marking Period

Lessons
- Mean Absolute Deviation, Standard Deviation, z-scores
- Box and Whisker Plots
- Best Fit Lines and Curves

Strand: Statistics

SOL A.9
The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.

Return to Course Outline

Essential Knowledge and Skills
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Analyze descriptive statistics to determine the implications for the real-world situations from which the data derive.
- Given data, including data in a real-world context, calculate and interpret the mean absolute deviation of a data set.
- Given data, including data in a real-world context, calculate variance and standard deviation of a data set and interpret the standard deviation.
- Given data, including data in a real-world context, calculate and interpret z-scores for a data set.
- Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.
- Compare and contrast mean absolute deviation and standard deviation in a real-world context.

Essential Understandings
- Descriptive statistics may include measures of center and dispersion.
- Variance, standard deviation, and mean absolute deviation measure the dispersion of the data.
- The sum of the deviations of data points from the mean of a data set is 0.
- Standard deviation is expressed in the original units of measurement of the data.
- Standard deviation addresses the dispersion of data about the mean.
- Standard deviation is calculated by taking the square root of the variance.
- The greater the value of the standard deviation, the further the data tend to be dispersed from the mean.
- For a data distribution with outliers, the mean absolute deviation may be a better measure of dispersion than the standard deviation or variance.
- Statistical techniques can be used to organize, display, and compare sets of data.
- A *z*-scores (standard score) is a measure of position derived from the mean and standard deviation of data
- A *z*-score derived from a particular data value tells how many standard deviations that data value is above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean.

(continued)

Teacher Notes and Resources

Data Analysis – Standard Deviation and Z-Scores

Resources

HCPS Algebra 1 Online!:

Data Analysis – Lesson 5

DOE Lesson Plans:

- Calculating Measures of Dispersion (PDF) (Word)- Calculating mean absolute deviation, variance, and standard deviation
- Exploring Statistics (PDF) (Word) - Calculating mean absolute deviation, variance, standard deviation, and z-scores
- z-Scores (PDF) (Word) - Calculating and interpreting z-scores
- Analyzing and Interpreting Statistics (PDF) (Word)

Key Vocabulary

dispersion	summation notation
mean	variance
mean absolute deviation	z-score
standard deviation	

Essential Questions

- What is the importance of statistics?
- What is the mean absolute deviation for a set of data?
- What is the variance and standard deviation for a set of data?
- What is the z-score?

Teacher Notes and Elaborations

The following information is taken from the VDOE **Technical Assistance Document – A.9**.

This objective is intended to extend the study of descriptive statistics beyond the measures of center studied during the middle grades.

Return to Course Outline

Teacher Notes and Elaborations

Although calculation is included in this objective, instruction and assessment emphasis should be on understanding and interpreting statistical values associated with a data set including standard deviation, mean absolute deviation, and z-score. While not explicitly included in this objective, the arithmetic mean will be integral to the study of descriptive statistics.

The study of statistics includes gathering, displaying, analyzing, interpreting, and making predictions about a larger group of data (population) from a sample of those data. Data can be gathered through scientific experimentation, surveys, and/or observation of groups or phenomena. Numerical data gathered can be displayed numerically or graphically (examples would include line plots, histograms, and stem-and-leaf plots). Methods for organizing and summarizing data make up the branch of statistics called descriptive statistics.

Sample vs. Population Data

Sample data can be collected from a defined statistical population. Examples of a statistical population might include *SOL scores of all Algebra I students in Virginia, the heights of every U.S. president*, or *the ages of every mathematics teacher in Virginia*. Sample data can be analyzed to make inferences about the population. A data set, whether a sample or population, is comprised of individual data points referred to as elements of the data set.

An element of a data set will be represented as x_i, where i represents the i^{th} term of the data set.

When beginning to teach this standard, start with small, defined population data sets of approximately 30 items or less to assist in

(continued)

Teacher Notes and Resources

Data Analysis – Standard Deviation and Z-Scores

Teacher Notes and Elaborations *(continued)*
focusing on development of understanding and interpretation of statistical values and how they are related to and affected by the elements of the data set.

Related to the discussion of samples versus populations of data are discussions about notation and variable use. In formal statistics, the arithmetic *mean* (average) of a population is represented by the Greek letter μ (mu), while the calculated arithmetic mean of a sample is represented by $\bar{x}$, read "x bar." In general, a bar over any symbol or variable name in statistics denotes finding its mean.

The arithmetic mean of a data set will be represented by μ.

On both brands of approved graphing calculators in Virginia, the calculated arithmetic mean of a data set is represented by $\bar{x}$.

Mean Absolute Deviation vs. Variance and Standard Deviation
Statisticians like to measure and analyze the *dispersion* (spread) of the data set about the mean in order to assist in making inferences about the population. One measure of spread would be to find the sum of the deviations between each element and the mean; however, this sum is always zero. There are two methods to overcome this mathematical dilemma: 1) take the absolute value of the deviations before finding the average or 2) square the deviations before finding the average. The mean absolute deviation uses the first method and the variance and standard deviation uses the second. If either of these measures is to be computed by hand, do not require students to use data sets of more than about 10 elements.

NOTE: Students have not been introduced to *summation notation* prior to Algebra I. An introductory lesson on how to interpret the notation will be necessary.

Return to Course Outline

Examples of summation notation:

$$\sum_{i=1}^{5} i = 1+2+3+4+5 \qquad \sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4$$

Mean Absolute Deviation
Mean absolute deviation is one measure of spread about the mean of a data set, as it is a way to address the dilemma of the sum of the deviations of elements from the mean being equal to zero. The *mean absolute deviation* is the arithmetic mean of the absolute values of the deviations of elements from the mean of a data set.

Mean absolute deviation $= \dfrac{\sum_{i=1}^{n} |x_i - \mu|}{n}$, where μ represents the mean of the data set, n represents the number of elements in the data set, and x_i represents the i^{th} element of the data set.

The mean absolute deviation is less affected by outlier data than the variance and standard deviation. Outliers are elements that fall at least 1.5 times the interquartile range (IQR) below the first quartile (Q_1) or above the third quartile (Q_3). Graphing calculators identify Q_1 and Q_3 in the list of computed 1-varible statistics. Mean absolute deviation cannot be directly computed on the graphing calculator as can the standard deviation. The mean absolute deviation must be computed by hand or by a series of keystrokes using computation with lists of data. More information (keystrokes and screenshots) on using graphing calculators to compute this can be found in the Sample Instructional Strategies and Activities.

Variance
The second way to address the dilemma of the sum of the deviations of elements from the mean being equal to zero is to square the

(continued)

Teacher Notes and Resources

Data Analysis – Standard Deviation and Z-Scores

Teacher Notes and Elaborations *(continued)*

deviations prior to finding the arithmetic mean. The average of the squared deviations from the mean is known as the *variance*, and is another measure of the spread of the elements in a data set.

Variance $(\sigma^2) = \dfrac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$, where μ represents the mean of the data set, n represents the number of elements in the data set, and x_i represents the i^{th} element of the data set.

The differences between the elements and the arithmetic mean are squared so that the differences do not cancel each other out when finding the sum. When squaring the differences, the units of measure are squared and larger differences are "weighted" more heavily than smaller differences. In order to provide a measure of variation in terms of the original units of the data, the square root of the variance is taken, yielding the standard deviation.

The *standard deviation* is the positive square root of the variance of the data set. The greater the value of the standard deviation, the more spread out the data are about the mean. The lesser (closer to 0) the value of the standard deviation, the closer the data are clustered about the mean.

Standard deviation $(\sigma) = \sqrt{\dfrac{\sum_{i=1}(x_i - \mu)^2}{n}}$, where μ represents the mean of the data set, n represents the number of elements in the data set, and x_i represents the i^{th} element of the data set.

Often, textbooks will use two distinct formulas for standard deviation.

Return to Course Outline

In these formulas, the Greek letter " σ ", written and read "sigma", represents the standard deviation of a population, and "*s*" represents the sample standard deviation. The population standard deviation can be estimated by calculating the sample standard deviation. The formulas for sample and population standard deviation look very similar except that in the sample standard deviation formula, $n – 1$ is used instead of n in the denominator. The reason for this is to account for the possibility of greater variability of data in the population than what is seen in the sample. When $n – 1$ is used in the denominator, the result is a larger number. So, the calculated value of the sample standard deviation will be larger than the population standard deviation. As sample sizes get larger (n gets larger), the difference between the sample standard deviation and the population standard deviation gets smaller. The use of $n – 1$ to calculate the sample standard deviation is known as Bessel's correction. Use the formula for standard deviation with n in the denominator as noted in the shaded box above.

When using Casio or Texas Instruments (TI) graphing calculators to compute the standard deviation for a data set, two computations for the standard deviation are given, one for a population (using n in the denominator) and one for a sample (using $n - 1$ in the denominator). Students should be asked to use the computation of standard deviation for population data in instruction and assessments. On a Casio calculator, it is indicated with "*x* σ *n*" and on a TI graphing calculator as " σ *x*".

z-Scores

A *z-score*, also called a standard score, is a measure of position derived from the mean and standard deviation of the data set. In can also be used to determine the value of the element, given the z-score of an unknown element and the mean and standard deviation of a data set. The z-score has a positive value if the element lies

(continued)

Teacher Notes and Resources

Data Analysis – Standard Deviation and Z-Scores

Teacher Notes and Elaborations *(continued)*

above the mean and a negative value if the element lies below the mean. A z-score associated with an element of a data set is calculated by subtracting the mean of the data set from the element and dividing the result by the standard deviation of the data set.

z-score $(z) = \frac{x-\mu}{\sigma}$, where x represents an element of the data set, μ represents the mean of the data set, and σ represents the standard deviation of the data set.

A z-score can be computed for any element of a data set; however, they are most useful in the analysis of data sets that are normally distributed. In Algebra II, z-scores will be used to determine the relative position of elements within a normally distributed data set, to compare two or more distinct data sets that are distributed normally, and to determine percentiles and probabilities associated with occurrence of data values within a normally distributed data set.

Sample Instructional Strategies and Activities

There are seven navigable rivers that feed into the Ohio River. The lengths of these rivers are shown in the table.

Return to Course Outline

Length of Rivers Feeding into the Ohio River	
Monongahela	129 miles
Allegheny	325 miles
Kanawha	97 miles
Kentucky	259 miles
Green	360 miles
Cumberland	694 miles
Tennessee	166 miles

Compare standard deviation and absolute mean deviation of the lengths of the rivers. What conclusions can you reach based on these two measures? The information needed is best organized in a table.

$$\mu = \frac{129+325+97+259+360+694+166}{7}$$

$$\mu = \frac{2030}{7}$$

$$\mu = 290$$

Value (x)	Mean μ	$\lvert x_1 - \mu \rvert$
129	290	161
325	290	35
97	290	193
259	290	31
360	290	70
694	290	404
166	290	124
		Σ = 1018

The absolute mean deviation = $\frac{\sum_{i=1}^{n} \lvert x_i - \mu \rvert}{n}$

$$= \frac{1018}{7}$$

$$= 145$$

(continued)

Teacher Notes and Resources

Data Analysis – Standard Deviation and Z-Scores

Sample Instructional Strategies and Activities *(continued)*

The standard deviation =

$$\sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}$$

$$=\sqrt{\frac{248,848}{7}}$$

$$=\sqrt{35,550}$$

$$\approx 188.5$$

Value (x)	Mean μ	$(x_i-\mu)^2$
129	290	25,921
325	290	1,225
97	290	37,249
259	290	961
360	290	4,900
694	290	163,216
166	290	15,376
		$\Sigma = 248,848$

The z-score for the Monongahela River $=\dfrac{\text{value} - \text{mean}}{\text{standard deviation}}$

$$=\frac{129-290}{188.5}\approx -0.85$$

Example 1:
(Computation of descriptive statistics using graphing calculators)
Maya's company produces a special product on 14 days only each year. Her job requires that she report on production at the end of the 14 days. She recorded the number of products produced each day (below) and decided to use descriptive statistics to report on product production.

Number of Products produced daily:
40, 30, 50, 30, 50, 60, 50, 50, 30, 40, 50, 40, 60, 50

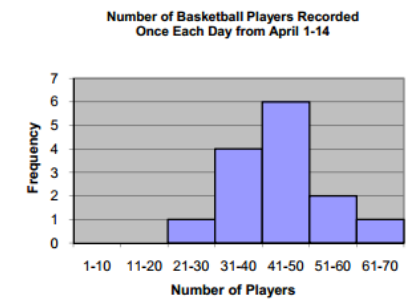
The Mean Absolute Deviation is 8.57142857. View the Technical Assistance Document for calculator keystrokes (TI-83/84 and CASIO) and solution.

Return to Course Outline

Example 2 - Interpretation of descriptive statistics

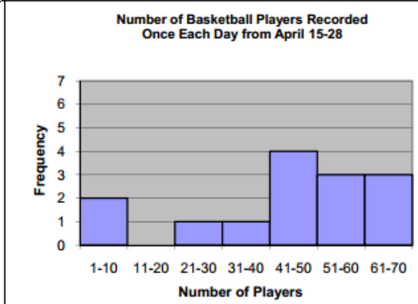
What types of inferences can be made about the data set with the given information?

Data set 1



Mean = 45.0
Variance = 106.1
Standard Deviation = 10.3
Mean Absolute Deviation = 9.1

Data set 2



Mean = 45.0
Variance = 420.3
Standard Deviation = 20.5
Mean Absolute Deviation = 16

- The standard deviation of data set 1 is less than the standard deviation of data set 2. That tells us that there was less variation (more consistency) in the number of people playing basketball during April 1 – 14 (data set 1).
- When comparing the variance of the two data sets, the difference between the two indicates that data set 2 has much more dispersion of data than data set 1.
- The standard deviation of data set 2 is almost twice the standard deviation of data set 1, indicating that the elements of data set 2 are more spread out with respect to the mean.
- Given a standard deviation and graphical representations of different data sets, the standard deviation could be matched to the appropriate graph by comparing the spread of data in each graph.
- When a data set contains clear outliers (the elements with values of 1 and 2 in data set 2), the outlying elements have a lesser affect on the calculation of the mean absolute deviation than on the standard deviation.

(continued)

Teacher Notes and Resources

Data Analysis – Standard Deviation and Z-Scores

Sample Instructional Strategies and Activities *(continued)*

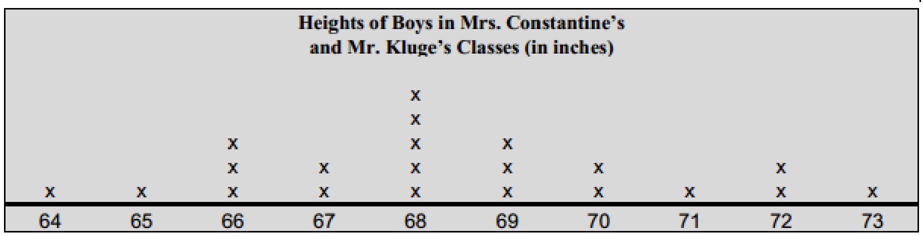
How can z-scores be used to make inferences about data sets?

A z-score can be calculated for a specific element's value within the set of data. The z-score for an element with value of 30 can be computed for data set 2. $z = \frac{30-45}{20.5} = -0.73$

The value of -0.73 indicates that the element falls just under one standard deviation below (negative) the mean of the data set. If the mean, standard deviation, and z-score are known, the value of the element associated with the z-score can be determined. For instance, given a standard deviation of 2.0 and a mean of 8.0, what would be the value of the element associated with a z-score of 1.5? Since the z-score is positive, the associated element lies above the mean. A z-score of 1.5 means that the element falls 1.5 standard deviations above the mean. So, the element falls 1.5(2.0) = 3.0 points above the mean of 8. Therefore, the z-score of 1.5 is associated with the element with a value of 11.0.

Example 3 – Interpretation of descriptive statistics

Maya represented the heights of boys in Mrs. Constantine's and Mr. Kluge's classes on a line plot and calculated the mean and standard deviation.



Mean = 68.4 Standard Deviation = 2.3

Note: In this problem, a small, defined population of the boys in Mrs. Constantine's and Mr. Kluge's classes is assumed.

Return to Course Outline

How many elements are above the mean?

There are 9 elements above the mean value of 68.4.

How many elements are below the mean?

There are 12 elements below the mean value of 68.4.

How many elements fall within one standard deviation of the mean?

There are 12 elements that fall within one standard deviation of the mean. The values of the mean plus one standard deviation and the mean minus one standard deviation ($\bar{x} - \sigma = 66.1$ and $\bar{x} + \sigma = 70.7$) determine how many elements fall within one standard deviation of the mean. In other words, all 12 elements between $\bar{x} - \sigma$ and $\bar{x} + \sigma$ (boys that measure 67", 68", 69", or 70") are within one standard deviation of the mean.

Application Scenarios

1. Dianne oversees production of ball bearings with a diameter of 0.5 inches at three locations in the United States. She collects the standard deviation of a sample of ball bearings each month from each location to compare and monitor production.

Standard deviation of 0.5 inch diameter ball bearing production (in inches)			
	July	August	September
Plant location #1	0.01	0.01	0.02
Plant location #2	0.02	0.04	0.05
Plant location #3	0.02	0.01	0.01

Compare and contrast the standard deviations from each plant location, looking for trends or potential issues with production. What conclusions or questions might be raised from the statistical data provided? What other statistical information and/or other data might need to be gathered in order for Dianne to determine next steps?

(continued)

Teacher Notes and Resources

Data Analysis – Standard Deviation and Z-Scores

Sample Instructional Strategies and Activities *(continued)*

Sample response: Plant #1 and Plant #3 had standard deviations that seemed steady, but one would be wise to keep an eye on Plant #1 in the coming months, because it had increases in August and September. A potential concern with the standard deviation of Plant #2 exists. The growing standard deviation indicates that there might be an issue with growing variability in the size of the ball bearings. Plant #2 should be asked to take more frequent samples and continue to monitor, to check the calibration of the equipment, and/or to check for an equipment problem.

2. Jim needs to purchase a large number of 20-watt florescent light bulbs for his company. He has narrowed his search to two companies offering the 20-watt bulbs for the same price. The Bulb Emporium and Lights-R-Us claim that their 20-watt bulbs last for 10,000 hours. Which descriptive statistic might assist Jim in making the best purchase? Explain why it would assist him.

   Sample response: The standard deviation of the lifespan of each company's 20-watt bulbs should be compared. The bulbs with the lowest lifespan standard deviation will have the slightest variation in number of hours that the bulbs last. The bulbs with the slightest variation in the number of hours that they last means that they are more likely to last close to 10,000 hours.

3. In a school district, Mr. Mills is in charge of SAT testing. In a meeting, the superintendent asks him how many students scored less than one standard deviation below the mean on the mathematics portion of the SAT in 2009. He looks through his papers and finds that the mean of the scores is 525 and 1653 students took the SAT in 2009. He also found a chart with percentages of z-scores on the SAT in 2009 as follows:

z-score (mathematics)	Percent of students
$z < -3$	0.1
$-3 \le z < -2$	2.1
$-2 \le z < -1$	13.6
$-1 \le z < 0$	34.0
$0 \le z < 1$	34.0
$1 \le z < 2$	13.6
$2 \le z < 3$	2.1
$z > 3$	0.1

How can Mr. Mills determine the number of students that scored less than one standard deviation below the mean on the mathematics portion of the SAT?

Sample response: The z-score tells you how many standard deviations an element (in this case a score) is from the mean. If the z-score of a score is -1 , then that score is 1 standard deviation below the mean. There are 15.8% (13.6% + 2.1% + 0.1%) of the scores that have a z-score < -1, so there are about 261 (0.158 · 1653) students that scored less than one standard deviation below the mean.

Questions to explore with students

1. Given a frequency graph, a standard deviation of _______, and a mean of _______, how many elements fall within _______ standard deviation(s) from the mean? Why?
2. Given the standard deviation and mean or mean absolute deviation and mean, which frequency graph would most likely represent the situation and why?

Return to Course Outline

(continued)

Teacher Notes and Resources

Data Analysis – Standard Deviation and Z-Scores

Sample Instructional Strategies and Activities *(continued)*

3. Given two data sets with the same mean and different spreads, which one would best match a data set with a standard deviation or mean absolute deviation of _______? How do you know?
4. Given two frequency graphs, explain why one might have a larger standard deviation.
5. Given a data set with a mean of _______, a standard deviation of _______, and a z-score of _______, what is the value of the element associated with the z-score?
6. What do z-scores tell you about position of elements with respect to the mean? How do z-scores relate to their associated element's value?
7. Given the standard deviation, the mean, and the value of an element of the data set, explain how you would find the associated z-score.

Return to Course Outline

Relations and Functions

Second Marking Period

Lessons
- The Coordinate Plane
- Relations
- Equations as Relations
- Graphing Linear Equations
- Functions
- Writing Equations from Patterns (optional)

Strand: Equations and Inequalities, Functions

SOL A.7abcef
The student will investigate and analyze linear function families and their characteristics both algebraically and graphically, including
a) determining whether a relation is a function;
b) domain and range;
c) zeros of a function;
e) finding the values of a function for elements in its domain; and
f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

Return to Course Outline

Essential Knowledge and Skills
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Determine whether a relation, represented by a set of ordered pairs, a table, or a graph is a function.
- Identify the domain, range, and zeros of a function presented algebraically or graphically.
- For each x in the domain of f, find $f(x)$.
- Represent relations and functions using concrete, verbal, numeric, graphic, and algebraic forms. Given one representation, students will be able to represent the relation in another form.
- Detect patterns in data and represent arithmetic and geometric patterns algebraically.

Essential Understandings
- A set of data may be characterized by patterns, and those patterns can be represented in multiple ways.
- Graphs can be used as visual representations to investigate relationships between quantitative data.
- Inductive reasoning may be used to make conjectures about characteristics of function families.
- Each element in the domain of a relation is the abscissa of a point of the graph of the relation.
- Each element in the range of a relation is the ordinate of a point of the graph of the relation.
- A relation is a function if and only if each element in the domain is paired with a unique element of the range.
- The values of $f(x)$ are the ordinates of the points of the graph of f.
- The object $f(x)$ is the unique object in the range of the function f that is associated with the object x in the domain of f.
- For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $[x, f(x)]$ is a member of f.
- An object x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$.
- Set builder notation may be used to represent domain and range of a relation.

Teacher Notes and Resources

Relations and Functions *(continued)*

Resources

Textbook:

1-9 Graphing Data on the Coordinate Plane
5-2 Relations and Functions
5-3 Function Rules, Tables, and Graphs
5-4 Writing a Function Rule

HCPS Algebra 1 Online!:

Relations and Functions

DOE Lesson Plans:

- Functions 1 (PDF) (Word) - Investigating relations and functions
- Functions 2 (PDF) (Word) - Investigating domain, range, intercepts, and zeros
- Square Patios (PDF) (Word)- Connecting different representations of functions

Key Vocabulary

abscissa	ordinate
function	relation
family of functions	vertical line test
function notation	zeros of a function

Essential Questions

- What is a relation and when does it become a function?
- How are domain/range, abscissa/ordinate, and independent/ dependent variables related in a set of ordered pairs, table, or a graph?
- How can the ordered pair (x, y) be represented using function notation?
- What is the zero of a function?
- How are domain and range represented in set builder notation?

Return to Course Outline

Teacher Notes and Elaborations

A set of data may be characterized by patterns and those patterns can be represented in multiple ways. Algebra is a tool for describing patterns, generalizing, and representing a relationship in which output is related to input. Mathematical relationships are readily seen in the translation of quantitative patterns and relations to equations or graphs. Collected data can be organized in a table or visualized in a graph and analyzed for patterns.

Pattern recognition and analysis might include:

- patterns involving a given sequence of numbers;
- a set of ordered pairs from a given pattern;
- a pattern using the variable(s) in an algebraic format so that a specific term can be determined;
- patterns demonstrated geometrically on the coordinate plane when appropriate; and
- patterns that include the use of the ellipsis such as 1, 2, 3 . . . 99, 100.

Patterns may be represented as relations and/or functions. A *relation* can be represented by a set of ordered pairs of numbers or paired data values. In an ordered pair, the first number is termed the *abscissa* (x-coordinate) and the second number is the *ordinate* (y-coordinate).

A *function* is a special relation in which each different input value is paired with exactly one output value (a unique output for each input). The set of input values forms the domain and the set of output values forms the range of the function. Sets of ordered pairs that do not represent a function should also be identified. Graphs of functions with similar features are called a *family of functions*.

Graphs can be used as visual representations to investigate relationships between quantitative data. Students should have multiple

(continued)

Teacher Notes and Resources

Relations and Functions *(continued)*

Teacher Notes and Elaborations *(continued)*

experiences constructing linear and quadratic graphs utilizing **both** paper and pencil and the graphing calculator. Graphically, a function may be determined by applying the *vertical line test* (A graph is a function if there exists no vertical line that intersects the graph in more than one point.).

If a sequence does not have a last term, it is called an infinite sequence. Three dots called an ellipsis are used to indicate an omission. If a sequence stops at a particular term, it is a finite sequence.

Set builder notation is a method for identifying a set of values. For example, the domain for $y = x^2 - 5$ would be written as $\{x : x \in \Re\}$. This is read, "The set of all x such that x is an element of the real numbers." The range for this equation would be written as $\{y : y \geq -5\}$.

In a function, the relationship between the domain and range may be represented by a rule. This rule may be expressed using *function notation*, $f(x)$, which means the value of the function at x and is read "f of x".

Zeros of a function (roots or solutions) are the x-intercepts of the function and are found algebraically by substituting 0 for y and solving the subsequent equation. An object x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$.

Sample Instructional Strategies and Activities

- Give students a list of ordered pairs such as (-2, -3), (-1, -1), (0, 1), (1, _), (_, 5), (_, _). Have students identify the rule and complete the pattern.
- Students develop patterns and determine if they are linear, quadratic or neither.
- Draw a coordinate plane on a flat surface outside or on the floor. Students pick a number on the x-axis to be used as a domain element. Give them a rule for which they will find the range value for their number. Students will move to this point on the plane. "Connect" the students using yarn or string. Analyze the different types of graphs obtained.
- Divide students into groups. Give each group a folder. Have them write a relation on the outside as well as a list of five numbers to be used as domain elements. They should write the domain and range elements in set notation on a piece of paper and place it in the folder. Folders are passed to each group. When the folder is returned to the group it began with, the results will be analyzed and verified.
- Using a graphing calculator, a series of relations can be graphed. Use a piece of paper to represent a vertical line, and use the vertical line test to test if it is a function.

  Examples:

  $y = x^2$ $\quad$ $y = |x|$

  $y = x$ $\quad$ $y = \frac{1}{x}$

  $y = x + 4 - x^2$ $\quad$ $y = 4 - x^2$

Return to Course Outline

Linear Equations

Second Marking Period

Lessons	Essential Knowledge and Skills	Essential Understandings
Lessons • Slope • Point-Slope and Standard Form • *x*- and *y*-intercepts • Slope-Intercept Form • Graphing Linear Equations Review (all methods) • Graphing Linear Inequalities **Strand**: Equations and Inequalities, Functions **SOL A.6** The student will graph linear equations and linear inequalities in two variables. a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line. **SOL A.7d** The student will investigate and analyze linear function families and their characteristics both algebraically and graphically including *x*- and *y*-intercepts. Return to Course Outline	**Essential Knowledge and Skills** The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Graph linear equations and inequalities in two variables, including those that arise from a variety of real-world situations. • Use the parent function $y = x$ and describe transformations defined by changes in the slope or *y*-intercept. • Find the slope of the line, given the equation of a linear function. • Find the slope of a line, given the coordinates of two points on the line. • Find the slope of a line, given the graph of a line. • Recognize and describe a line with a slope that is positive, negative, zero, or undefined. • Use transformational graphing to investigate effects of changes in equation parameters on the graph of the equation. • Write an equation of a line when given the graph of the line. • Write an equation of a line when given two points on the line whose coordinates are integers. • Write an equation of a line when given the slope and a point on the line whose coordinates are integers. • Write an equation of a vertical line as $x = a$. • Write the equation of a horizontal line as $y = c$. • Identify the x- and y-intercepts of a function presented algebraically or graphically.	**Essential Understandings** • Changes in slope may be described by dilations or reflections or both. • Changes in the *y*-intercept may be described by translations. • Linear equations can be graphed using slope, *x*- and *y*-intercepts, and/or transformations of the parent function. • The slope of a line represents a constant rate of change in the dependent variable when the independent variable changes by a constant amount. • The equation of a line defines the relationship between two variables. • The graph of a line represents the set of points that satisfies the equation of a line. • A line can be represented by its graph or by an equation. • The graph of the solutions of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the boundary are included unless it is a strict inequality. • Parallel lines have equal slopes. • The product of the slopes of perpendicular lines is -1 unless one of the lines has an undefined slope. *(continued)*

Teacher Notes and Resources

Linear Equations *(continued)*

Resources

Textbook:

6-1 Rate of Change and Slope
6-2 Slope-Intercept Form
6-3 Standard Form
6-4 Point-Slope Form and Writing Linear Equations
6-5 Parallel and Perpendicular Lines
7-5 Linear Inequalities

HCPS Algebra 1 Online!:

Linear Equations

DOE Lesson Plans:

- The Submarine (PDF) (Word) - Investigating slope as rate of change
- Slope-2-Slope (PDF) (Word)- Investigating slope of horizontal and vertical lines and graphing a line
- Transformationally Speaking (PDF) (Word) - Identifying patterns in families of graphs and their equations
- Transformation Investigation (PDF) (Word)- Investigating the components of the equation of a line
- Slippery Slope (PDF) (Word) - Writing equations of lines
- Equations of Lines (PDF) (Word) - Writing equations of lines

Key Vocabulary

horizontal line form	slope-intercept form
parent function	standard form
point-slope form	vertical line form
rate of change	*x*-intercept
slope	*y*-intercept

Essential Questions

- What are the appropriate techniques to graph a linear equation in two variables?
- How can a graph of a linear equation be used to represent a real-world situation?
- What are the appropriate techniques to graph a linear inequality in two variables?
- How can a graph of a linear inequality be used to represent a real-world situation?
- How does the slope of a line relate to real-world situations?
- What is meant by the term rate of change?
- How does slope relate to graphs, equations, and points on a line?
- What are positive, negative, zero, and undefined slopes and why are they important?
- How does changing the coefficient of the independent variable of an equation affect the slope of a line?
- What is the standard form of a linear equation?
- What does the slope intercept form of a linear equation mean?
- What is the parent function of a linear equation?
- What is the role of transformations in graphing linear equations?

Teacher Notes and Elaborations

A graph is a picture of an equation or inequality. The graphs of linear equations are straight lines. The graphs of linear inequalities are regions.

The *slope* of a line is the ratio of the change in the *y*-coordinates to the corresponding change in the *x*-coordinates.

The slope of a line can be described as $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$ (Given two ordered pairs, the slope may be found by dividing the change in the y-coordinates by the change in the x-coordinates). The slope *m* of a line that passes through the points (x_1, y_1) and (x_2, y_2) *(continued)*

Return to Course Outline

Linear Equations *(continued)*

Teacher Notes and Resources

Teacher Notes and Elaborations *(continued)*

is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Students should understand that the subscripts are not interchangeable with exponents.

The slope of a linear equation represents a constant rate of change in the dependent variable when the independent variable changes by a fixed amount.

The slope of a line determines its relative steepness. Changing the relationship between the rise of the graph (change in the y-values) and the run (change in the x-values) affects the rate of change or "steepness" of a slope.

The slope of a line can be determined in a variety of ways. Changes in slope affect the graph of a line. The slope-intercept form of a linear equation is $y = mx + b$ where m is the slope and b is the y-intercept. The slope of a line, m, is described as a "*rate of change*," which may be positive, negative, zero, or undefined. A vertical line has an undefined slope and a horizontal line has a slope of zero.

Emphasis should be placed on the difference between zero slope and undefined slope. The use of "no slope" instead of "zero slope" should be avoided because it is confusing to students.

The graphing calculator is an effective tool to illustrate the effect of changes in the slope on the graph of the line.

Slopes and y-intercepts are found in everyday life where a relationship exists (e.g., motion, temperature, light variations, finance, etc.).

A line can be represented by its graph or by an equation. The equation of a line defines the relationship between two variables. The graph of a line represents the set of points that satisfies the equation of a line.

Linear equations can be written in a variety of forms (Linear inequalities can also be written in a variety of forms.):

- *Slope-intercept*: $y = mx + b$, where m is the slope and b is the y-intercept.
- *Standard*: $Ax + By = C$**,** where A, B, and C are integers and A is positive.
- *Point-slope*: $y - y_1 = m(x - x_1)$
- *Vertical line*: $x = a$
- *Horizontal line* (constant function): $y = b$

The *parent function* for a linear equation is $y = x$.

The *x-intercept* of the line is the value of x when $y = 0$. The *y-intercept* of the line is the value of y when $x = 0$.

If the x- and y-intercepts are given, the equation of the line may be determined using these two points just as it is with any two points on the line.

Equations of the line may be written using two ordered pairs (two points on the line), the x- and y-intercepts, or the slope and a point on the line.

Using the slope and one of the coordinates, the equation may be written using the point-slope form of the equation, $y - y_1 = m(x - x_1)$.

Justification of an appropriate technique for graphing linear equations and inequalities is dependent upon the application of slope, x- and y-intercepts, and graphing by transformations

(continued)

Return to Course Outline

Linear Equations *(continued)*

Teacher Notes and Resources

Teacher Notes and Elaborations *(continued)*
Appropriate techniques for graphing linear equations and inequalities are determined by the given information and/or the tools available.

The solution set for an inequality in two variables contains many ordered pairs when the domain and range are the set of real numbers. The graphs of all of these ordered pairs fill a region on the coordinate plane called a half-plane. An equation defines the boundary or edge for each half-plane.

An appropriate technique for graphing a linear inequality is to graph the associated equation, determine whether the line is solid or broken, and then determine the shading by testing points in the region.

Graphs can be used as visual representations to investigate relationships between quantitative data. Students should have multiple experiences constructing linear and quadratic graphs utilizing **both** paper and pencil and the graphing calculator.

A.7
The x-intercept of a line is the point at which the line crosses the x-axis (i.e. where the y value equals 0).

$$x\text{-intercept} = (x, 0)$$

The y-intercept of a line is the point at which the line crosses the y-axis (i.e. where the x value equals 0)

$$y\text{-intercept} = (0, y)$$

The x-intercept (root or solution) can be found algebraically by substituting 0 for y and solving the subsequent equation. *Zeros of a function* (roots or solutions) are the x-intercepts of the function. An object x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$.

Return to Course Outline

The y-intercept can be found algebraically by substituting 0 for x and solving the subsequent equation.

Sample Instructional Strategies and Activities

- In small groups, students take an equation of a line and describe all the different information that can be determined about it. The students will discuss the most efficient techniques of graphing the equation.
- Divide students into groups. Give all groups the same equation, but have students use different graphing techniques. Next, they will compare their results with the class to show that they have the same result.
- Use graphing calculators to investigate the changes in the graph caused by changing the value of the constant and coefficient. This allows the student to visually compare several equations at the same time. Describe how changes in the m and b transform the graph from the parent function.
- Have students calculate the slope of several staircases in the school. Have students calculate the slope of several delivery ramps and/or handicap ramps. Have students find the equation of the line that would represent the ramps or stairs. Use the equation to draw the graph of the lines on the graphing calculator. Discuss how the changes in slope affect the "steepness" of the line.
- Divide students into pairs. One student has a card with a graph or line which he/she describes as accurately and precise as possible to their partner. The other student will write the equation of the line. Students should then look at the graphs to check the answers. The partners then switch positions and repeat.
- Students should describe the strategy used to find an equation of the line that passes through two given points. Sketch a flow chart that shows the steps.

Variation

Second Marking Period	Lessons	Essential Knowledge and Skills	Essential Understandings
	Lessons • Direct and Inverse Variation **Strand**: Functions **SOL A.8** The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically. Return to Course Outline	**Essential Knowledge and Skills** The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Given a situation, including a real-world situation, determine whether a direct variation exists. • Given a situation, including a real-world situation, determine whether an inverse variation exists. • Write an equation for a direct variation, given a set of data. • Write an equation for an inverse variation, given a set of data. • Graph an equation representing a direct variation, given a set of data.	**Essential Understandings** • The constant of proportionality in a direct variation is represented by the ratio of the dependent variable to the independent variable. • The constant of proportionality in an inverse variation is represented by the product of the dependent variable and the independent variable. • A direct variation can be represented by a line passing through the origin. • Real-world problems may be modeled using direct and/or inverse variations. *(continued)*

Teacher Notes and Resources

Variation *(continued)*

Resources

Textbook:

5-5 Direct Variation
12-1 Inverse Variation

DOE Lesson Plans:

Direct Variation (PDF) (Word)
Inverse Variation (PDF) (Word)

Key Vocabulary

constant of proportionality (variation)
dependent variable
direct variation
independent variable
inverse variation

Essential Questions

- What is direct variation?
- What is inverse variation?
- How is a relation analyzed to determine direct variation?
- How is a relation analyzed to determine inverse variation?
- What is the constant of proportionality in a direct variation?
- What is the constant of proportionality in an inverse variation?
- How do you represent a direct variation algebraically and graphically?
- How do you represent an inverse variation algebraically?

Teacher Notes and Elaborations

Direct variation involves a relationship between two variables. The patterns for direct variation relationships can be observed using equations, tables, and graphs. *Direct variation* is used to represent a constant rate of change in real-world situations.

Direct variation is defined by $y = kx$, $(k \neq 0)$ where k is the *constant of proportionality (variation)*. The constant, k, in a direct variation is represented by the ratio of y to x, $k = \frac{y}{x}$, where y is the *dependent variable* and x is the *independent variable*. Emphasis should be placed on finding the constant of proportionality (k).

A table and graph provide visual confirmation that as x increases, y increases or as x decreases, y decreases.

The value of k is determined by substituting a pair of known values for x and y into the equation and solving for k. The graph and table illustrate a linear pattern where the slope is the constant of variation and the y-intercept is zero (the graph goes through the origin).

Inverse variation is defined by $xy = k$, where $k \neq 0$. This can also be written as $y = \frac{k}{x}$, $(x \neq 0)$ and k is the constant of proportionality (variation). In an inverse variation as the values of x increase the values of y decrease.

(continued)

Return to Course Outline

Teacher Notes and Resources

Variation *(continued)*

Sample Instructional Strategies and Activities

- A bicycle travels at a certain constant rate. The distance the bicycle travels varies with time. Suppose the bicycle travels at a rate of 10 km/h. The distance it travels will vary depending only on time. A bicycle will travel a distance (*d*) in a time (*t*) while traveling at 10 km/h.

t	D
1	10
2	20
3	30
4	40

We can say that the distance "varies directly" as time passed or $d = 10t$

If a seesaw is balanced, each person's distance from the fulcrum varies inversely as his weight.

w	d
200	3
180	$3\frac{1}{3}$
150	4
120	5
100	6

$d = \frac{k}{w}$ so that as the person's weight decreases the distance from the fulcrum increases. Solving this equation for k gives the constant of proportionality so k = 600. Using this value, if a person weighs 140 pounds, how far will he sit from the fulcrum?

What other examples can you think of where direct and inverse variations occur?

- Students are given unlabeled graphs. Next, students will make up stories of events that could be happening to describe the situation depicted by the graph or vice-versa. Students will determine whether the situation represents a direct or inverse variation or neither and represent the situation algebraically, if possible.

Return to Course Outline

Systems of Equations and Inequalities

Third Marking Period

Lessons

- Graphing Systems of Equations
- Solving by Substitution
- Solving by Elimination (Add/Subtract)
- Solving by Elimination (Multiplication)
- Graphing Systems of Inequalities

Strand: Equations and Inequalities

SOL A.4e
The student will solve multistep linear and quadratic equations in two variables, including solving systems of two linear equations in two variables algebraically and graphically. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

SOL A.5d
The student will solve multistep linear inequalities in two variables, including solving systems of inequalities.

Return to Course Outline

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to find the ordered pair which satisfies both equations.
- Given a system of two linear equations in two variables that has a unique solution, solve the system graphically by identifying the point of intersection.
- Determine whether a system of two linear equations has one solution, no solution, or infinite solutions.
- Write a system of two linear equations that models a real-world situation.
- Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that models a real-world situation.
- Solve systems of linear inequalities algebraically and graphically.

Essential Understandings

- A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations.
- A system of two linear equations with no solution is characterized by the graphs of two lines that are parallel.
- A system of two linear equations having infinite solutions is characterized by two graphs that coincide (the graphs will appear to be the graph of one line), and the coordinates of all points on the line satisfy both equations.
- The product of the slopes of perpendicular lines is -1 unless one of the lines has an undefined slope.
- Systems of two linear equations can be used to model two real-world conditions that must be satisfied simultaneously.
- Equations and systems of equations can be used as mathematical models for real-world situations.
- Set builder notation may be used to represent solution sets of equations.
- Real-world problems can be modeled and solved using linear inequalities

(continued)

Teacher Notes and Resources

Systems of Equations and Inequalities *(continued)*

Resources

Textbook:

7-1 Solving Systems by Graphing
7-2 Solving Systems using Substitution
7-3 Solving Systems using Elimination
7-4 Applications of Linear Systems
7-6 Systems of Linear Inequalities

HCPS Algebra 1 Online!:

Systems of Equations and Inequalities

DOE Lesson Plans:

- The Exercise Fields (PDF) (Word) - Setting up a system of equations to solve a real-world problem
- How Much Is That Tune? (PDF) (Word) - Solving systems of two linear equations in two variables
- Spring Fling Carnival (PDF) (Word) - Writing equations and finding the solutions (intersections) graphically
- Graphing Systems of Inequalities (PDF) (Word)

Key Vocabulary

infinite number of solutions	system of equations
set builder notation	system of inequalities

Essential Questions

- What is a system of equations?
- In what instances will there be one solution, no solutions, or an infinite number of solutions for a system of equations?
- What methods are used to solve a system of linear equations?
- How are solutions written in set builder notation?
- How are the properties of real numbers used to solve inequalities?
- What is the same about solving equations and solving inequalities and what is different?
- How are the solutions of systems of linear inequalities the same or different from the solutions of systems of equations?
- How are solutions written in set builder notation?

Return to Course Outline

Teacher Notes and Elaborations

By transforming given linear equations into simpler forms, the number of solutions can be determined. An example of a linear equation with one solution is $6x-2=x+13$ where $x = 3$. An example of a linear equation with no solution is $2x = 2x + 1$. An example of a linear equation with an *infinite number of solutions* (identity, all real numbers) is $5x+10-2x=3x+10$ where $x = x$, $10 = 10$, or $0 = 0$.

Systems of Equations

A *system of equations* (simultaneous equations) is two or more equations in two or more variables considered together or simultaneously. The equations in the system may or may not have a common solution.

A linear system may be solved algebraically by the substitution or elimination methods, or by graphing. Graphing calculators are used to solve, compare, and confirm solutions.

A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations. A point shared by two intersecting graphs and the ordered pair that satisfies the equations characterizes a system of equations with only one solution. A system of two linear equations with no solution is characterized by the graphs of two lines that do not intersect, they are parallel. A system of two linear equations that has infinite solutions is characterized by two graphs that coincide (the graphs will appear to be the graph of one line), and all the coordinates on this one line satisfy both equations.

(continued)

Systems of Equations and Inequalities *(continued)*

Teacher Notes and Resources

Teacher Notes and Elaborations *(continued)*

Systems of two linear equations can be used to represent two conditions that must be satisfied simultaneously.

Systems of inequalities (simultaneous inequalities) are two or more inequalities in two or more variables that are considered together or simultaneously. The system may or may not have common solutions. Practical problems can be interpreted, represented, and solved using linear inequalities.

Set builder notation is used to represent solutions. For example, if the solution is the set of all real numbers less than 5 then in set notation the answer is written $\{x : x < 5\}$ or $\{x | x < 5\}$.

Extension for PreAP Algebra 1

Students will develop an understanding that representations of math ideas (equations, models, etc.) are an essential part of learning, doing, and communicating mathematics. They will engage in extensive problem solving using real-world problems. Instruction should include numerous opportunities to investigate multiple strategies to solve word problems. Students should learn to apply appropriate strategies to find solutions to these problems.

Sample Instructional Strategies and Activities

- Students will be divided into groups. Have each group solve a system of equations by a prescribed method. Make sure that all methods are assigned. Have students display their solutions to the class and discuss the most appropriate method for solving the system.

Return to Course Outline

- Given an equation such as $3x + 4y = 12$ find two or more equations that satisfy each of these requirements.
  1. The graphs of the given equation and a second equation intersect at a single point.
  2. The graphs of the given equation and a second equation intersect at an infinite number of points.

Rules of Exponents

Third Marking Period

Lessons	Essential Knowledge and Skills	Essential Understandings
• Multiplying Monomials • Dividing Monomials • Scientific Notation **Strand**: Expressions and Operations **SOL A.2a** The student will perform operations on polynomials, including applying the laws of exponents to perform operations on expressions. Return to Course Outline	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Simplify monomial expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents. • Model sums, differences, products, and quotients of polynomials with concrete objects and their related pictorial representations.	• The laws of exponents can be investigated using inductive reasoning. • A relationship exists between the laws of exponents and scientific notation. *(continued)*

Teacher Notes and Resources

Rules of Exponents *(continued)*

Resources

Textbook:

8-1 Zero and Negative Exponents
8-2 Scientific Notation
8-3 Multiplication Properties of Exponents
8-4 More Multiplication Properties of Exponents

HCPS Algebra 1 Online!:

Polynomials

DOE Lesson Plans:

- Exponents (PDF) (Word)
- Scientifically Speaking (PDF) (Word) - Working with scientific notation

Key Vocabulary

Monomial

Essential Questions

- What is inductive reasoning?
- What are the laws of exponents?
- How are numbers written in scientific notation computed?
- When is it beneficial to use scientific notation?
- What is the difference between a monomial and a polynomial?

Teacher Notes and Elaborations

Inductive reasoning is a process of reaching a conclusion based on a number of observations that form a pattern.

Repeated multiplication can be represented with exponents. The laws of exponents can be used to evaluate algebraic exponential expressions. Each exponential expression (b^n) is made up of a base (b) and an exponent (n).

Return to Course Outline

A positive exponent indicates the number of times the base occurs as a factor. a^{-n} is the reciprocal of a^n. Using this definition, the laws of exponents may be developed.

The laws of exponents can be investigated using patterns.

Multiplication Rules

Product of Powers
For any number a and all integers m and n,

$$a^m a^n = a^{m+n}$$

Power of a Power
For any number a and all integers m and n,

$$(a^m)^n = a^{mn}$$

Power of a Product
For all numbers a and b, and any integers m,

$$(ab)^m = a^m b^m$$

Power of a Monomial
For all numbers a and b, and all integers m, p, and n,

$$(a^m b^n)^p = a^{mp} b^{np}$$

Division Rules

Quotient of Powers
For all integers m and n, and any nonzero number a,

$$\frac{a^m}{a^n} = a^{m-n}$$

Power of a Quotient
For all integers m and any nonzero number b,

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Other Rules

Zero Exponent
For any nonzero number a, $a^0 = 1$

Exponential Identity
For any value a, $a^1 = a$

Negative Exponents
For any nonzero number a and any integer n, $a^{-n} = \dfrac{1}{a^n}$

(continued)

Rules of Exponents *(continued)*

Teacher Notes and Resources

Teacher Notes and Elaborations *(continued)*

A relationship exists between the laws of exponents and scientific notation. Writing numbers in scientific notation is a convenient method of expressing very large or very small numbers for solving real-world problems involving space travel, populations, measurements, etc. In scientific notation, powers of ten are used to express decimal numbers. Numbers in scientific notation are written as: $a \times 10^n$ where $1 \le a < 10$ and n is any integer. The following are examples of computations with scientific notation.

Example 1:
$$\begin{aligned}(7\times10^2)(4\times10^5) &= (7\times4)(10^2\times10^5)\\ &= 28\times10^7\\ &= 2.8\times10^1\times10^7\\ &= 2.8\times10^{1+7}\\ &= 2.8\times10^8\end{aligned}$$

Example 2:
$$\begin{aligned}10^{-3}\times(3\times10^8)^2 &= 10^{-3}\times3^2\times(10^8)^2\\ &= 10^{-3}\times3^2\times10^{16}\\ &= 3^2\times10^{-3}\times10^{16}\\ &= 3^2\times10^{-3+16}\\ &= 9\times10^{13}\end{aligned}$$

Return to Course Outline

Example 3:
$$\begin{aligned}\frac{35\text{ million tons}}{270.5\text{ million people}} &= \frac{3.5\times10^7\text{ tons}}{2.705\times10^8\text{ people}}\\ &= \frac{3.5}{2.705}\times10^{7-8}\\ &= \frac{3.5}{2.705}\times10^{-1}\\ &\approx 1.3\times10^{-1}\\ &= 0.13\end{aligned}$$

Therefore: $\frac{35\text{ million tons}}{270.5\text{ million people}} \approx 0.13$ tons per person

The following are examples of applying the laws of exponents:

$$\begin{aligned}\frac{10b^{-4}}{5b^{-6}} &= \frac{10}{5}\cdot\frac{b^{-4}}{b^{-6}}\\ &= 2\cdot b^{-4-(-6)}\\ &= 2b^2\end{aligned}$$

$$\begin{aligned}\frac{6a^{-3}\cdot3a^{10}}{2a^{-4}} &= \frac{6\cdot3}{2}\cdot\frac{a^{-3}\cdot a^{10}}{a^{-4}}\\ &= \frac{18}{2}\cdot\frac{a^{-3+10}}{a^{-4}}\\ &= 9\cdot a^{7-(-4)}\\ &= 9a^{11}\end{aligned}$$

Polynomials can be represented in a variety of forms. Physical representations such as Algeblocks should be used to support understanding of these concepts. A *monomial* is a constant, a variable, or the product of a constant and one or more variables.

Polynomials

Third Marking Period

Lessons	Essential Knowledge and Skills	Essential Understandings
• Degree, Ascending and Descending Order • Adding and Subtracting Polynomials • Multiplying Polynomials by Monomials • Multiplying Polynomials • Special Products • Dividing Polynomials/Long Division **Strand**: Expressions and Operations **SOL A.2b** The student will perform operations on polynomials, including adding, subtracting, multiplying, and dividing polynomials. Return to Course Outline	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Model sums, differences, products, and quotients of polynomials with concrete objects and their related pictorial representations. • Relate concrete and pictorial manipulations that model polynomial operations to their corresponding symbolic representations. • Find sums and differences of polynomials. • Find products of polynomials. The factors will have no more than five total terms (i.e. $(4x+2)(3x+5)$ represents four terms and $(x+1)(2x^2+x+3)$ represents five terms). • Find the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor.	• Operations with polynomials can be represented concretely, pictorially, and symbolically. • Polynomial expressions can be used to model real-world situations. • The distributive property is the unifying concept for polynomial operations. *(continued)*

Teacher Notes and Resources

Polynomials *(continued)*

Resources

Textbook:

9-1 Adding and Subtracting Polynomials
9-2 Multiplying and Factoring
9-3 Multiplying Binomials
9-4 Multiplying Special Cases
12-5 Dividing Polynomials

HCPS Algebra 1 Online!:

Polynomials

DOE Lesson Plans:

- Adding and Subtracting Polynomials Using Algebra Tiles (PDF) (Word)
- Exponents (PDF) (Word)
- Scientifically Speaking (PDF) (Word) - Working with scientific notation
- Dividing Polynomials Using Algebra Tiles (PDF) (Word)

Key Vocabulary

binomial
polynomial
trinomial

Essential Questions

- What is the difference between a monomial and a polynomial?
- How are polynomials added, subtracted, multiplied, and divided?
- How are manipulatives used to model operations of polynomials?

Teacher Notes and Elaborations

Polynomials can be represented in a variety of forms. Physical representations such as Algeblocks should be used to support understanding of these concepts. A *monomial* is a constant, a variable, or the product of a constant and one or more variables. A *polynomial* is an expression of two or more terms. A *binomial* is a polynomial of two terms. A *trinomial* is a polynomial of three terms. Polynomials can be added and subtracted by combining like terms.

The following are examples of **adding and subtracting polynomials**:

$$(y^2 - 7y - 2) + (3y^2 + 8)$$

$$\begin{array}{r} y^2 - 7y - 2 \\ \underline{+\ 3y^2 \qquad + 8} \\ 4y^2 - 7y + 6 \end{array}$$

$$\begin{aligned} &(5x - 3y - 2) - (4x - 5) \\ &= 5x - 3y - 2 - 4x + 5 \\ &= (5x - 4x) - 3y + ((-2) + 5) \\ &= x - 3y + 3 \end{aligned}$$

Polynomial multiplication requires that each term in the first expression will be multiplied by each term in the second expression using the distributive property. The distributive property is the unifying concept for polynomial operations. This property is better understood if students can use a physical model to help them develop understanding. The area model of multiplication should be demonstrated and used by students. Physical models to use include Algeblocks or Algebra Tiles. Students should be able to sketch the physical models, and record the process as they progress.

The following are examples of multiplying polynomials.

$$\begin{array}{r} 5a^2 - 3a - 7 \\ \underline{3a + 2} \\ 10a^2 - 6a - 14 \\ \underline{15a^3 - 9a^2 - 21a \qquad} \\ 15a^3 + a^2 - 27a - 14 \end{array}$$

$$\begin{aligned} &(x - 4)(2x^2 - x + 3) \\ &= x(2x^2 - x + 3) - 4(2x^2 - x + 3) \\ &= 2x^3 - x^2 + 3x - 8x^2 + 4x - 12 \\ &= 2x^3 - 9x^2 + 7x - 12 \end{aligned}$$

(continued)

Return to Course Outline

Teacher Notes and Resources

Polynomials *(continued)*

Teacher Notes and Elaborations *(continued)*

Area Model of $(2y + 7)(-3y^2 + 4y - 8)$

	$-3y^2$	$4y$	-8
$2y$			
$+7$			

→

	$-3y^2$	$4y$	-8
$2y$	$-6y^3$	$8y^2$	$-16y$
$+7$	$-21y^2$	$28y$	-56

→

	$-3y^2$	$4y$	-8
$2y$	$-6y^3$	$8y^2$	$-16y$
$+7$	$-21y^2$	$28y$	-56
$-6y^3$	$-13y^2$	$+12y$	-56

Division of a polynomial by a monomial requires each term of the polynomial be divided by the monomial.

$$\frac{4x^4+8x^3y-12x^2y^2}{4x^2}=\frac{4x^4}{4x^2}+\frac{8x^3y}{4x^2}-\frac{12x^2y^2}{4x^2}$$

$$=x^2+2xy-3y^2$$

Extension for PreAP Algebra I

- Add and subtract fractions with polynomial numerators and monomial factors in the denominator.
- Factor third-degree polynomials with at least one monomial as a factor.
- Factor third-degree polynomials with four terms by grouping.

Extension for PreAP Algebra I

To add or subtract fractions (rational expressions) with monomial factors in denominators, a common denominator must be found.

Return to Course Outline

For example: $$\frac{x+1}{x}+\frac{x-3}{3x}=\frac{3(x+1)}{3x}+\frac{x-3}{3x}$$
$$=\frac{3x+3}{3x}+\frac{x-3}{3x}$$
$$=\frac{3x+3+x-3}{3x}$$
$$=\frac{4x}{3x}$$
$$=\frac{4}{3}$$

Sample Instructional Strategies and Activities

- Students compare and contrast the different methods for operating on polynomials.
- Have each group of students model a polynomial with algeblocks or self-constructed tiles. Allow groups to exchange models and determine the polynomial represented by the other group.
- Physical models such as algeblocks or algebra tiles should be used to model factoring.
- Graphing calculators can be used demonstrate the connection between x-intercepts and factors.

Factoring

Third Marking Period

Lessons	Essential Knowledge and Skills	Essential Understandings
- Factors and Greatest Common Factors - Factoring using Distributive Property - Factoring Trinomials - Difference of Squares - Perfect Squares and Factoring - Solving by Factoring **Strand**: Equations and Inequalities **SOL A.2c** The student will perform operations on polynomials, including factoring completely first- and second degree binomials and trinomials in one or two variables.Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations. **SOL A.4c** The student will solve multistep quadratic equations in two variables, including solving quadratic equations algebraically and graphically. **SOL A.7c** The student will investigate and analyze linear function families and their characteristics both algebraically and graphically, including zeros of a function. Return to Course Outline	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to - Find the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor. - Factor completely first- and second-degree polynomials with integral coefficients. - Identify prime polynomials. - Use the *x*-intercepts from the graphical representation of the polynomial to determine and confirm its factors. - Solve quadratic equations. - Identify the roots or zeros of a quadratic function over the real number system as the solution(s) to the quadratic equation that is formed by setting the given quadratic expression equal to zero. - Identify the zeros of a function presented algebraically or graphically.	- Factoring reverses polynomial multiplication. - Some polynomials are prime polynomials and cannot be factored over the set of real numbers. - Polynomial expressions can be used to define functions and these functions can be represented graphically. - There is a relationship between the factors of any polynomial and the *x*-intercepts of the graph of its related function. - An object x in the domain of f is an *x*-intercept or a zero of a function f if and only if $f(x) = 0$. *(continued)*

Teacher Notes and Resources

Factoring *(continued)*

Resources

Textbook:

9-2 Multiplying and Factoring
9-4 Multiplying Special Cases (Difference of Squares)
9-5 Factoring Trinomials of the Type $x^2 + bx + c$
9-6 Factoring Trinomials of the Type $ax^2 + bx + c$
9-7 Factoring Special Cases
9-8 Factoring by Grouping
10-5 Factoring to Solve Quadratic Equations

HCPS Algebra 1 Online!:

Factoring

DOE Lesson Plans:

- Factoring (PDF) (Word)
- Multiplying Polynomials Using Algebra Tiles (PDF) (Word)
- Factoring for Zeros (PDF) (Word) - Relating the roots (zeros) of a quadratic equation and the graph of the equation

Key Vocabulary

binomial	prime polynomial
monomial	trinomial
polynomial	x-intercept

Essential Questions

- What methods are used to factor polynomials?
- What is the relationship between factoring polynomials and multiplying polynomials?
- What is the relationship between the factors of a polynomial and the x-intercepts of the related function?

Return to Course Outline

Teacher Notes and Elaborations

Factoring is the reverse of polynomial multiplication. The same models for multiplication can be used to factor. A factor of an algebraic polynomial is one of two or more polynomials whose product is the given polynomial. Some polynomials cannot be factored over the set of real numbers and these are called *prime polynomials*. There is a relationship between the factors of a polynomial and the x-intercepts of its related graph. The *x-intercept* is the point at which a graph intersects the x-axis. Polynomial expressions in a variable x and their factors can be used to define functions by setting y equal to the polynomial expression or y equal to a factor, and these functions can be represented graphically.

Guidelines for Factoring

1. Factor out the greatest monomial factor first.
2. Look for a difference of squares.
3. Look for a trinomial square.
4. If a trinomial is not a square, look for a pair of binomial factors.
5. If a polynomial has four or more terms, look for a way to group the terms in pairs or in a group of three terms that is a binomial square.
6. Make sure that each factor is prime. Check the work by multiplying the factors.

To divide a polynomial by a binomial, several methods may be used such as factoring, long division, or using an area model. Factoring and simplifying is the preferred method but does not always work.

Factoring Example:

$$\frac{x^2+8x+15}{x+3} = \frac{(x+5)(x+3)}{x+3}$$
$$= \frac{(x+5)\cancel{(x+3)}}{\cancel{x+3}}$$
$$= (x+3)$$

(continued)

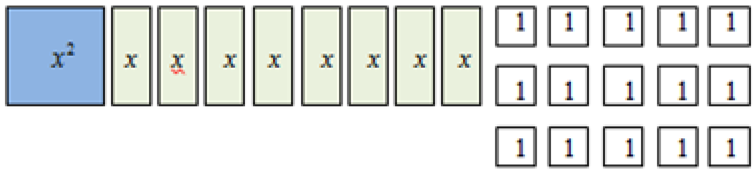
Teacher Notes and Resources

Factoring *(continued)*

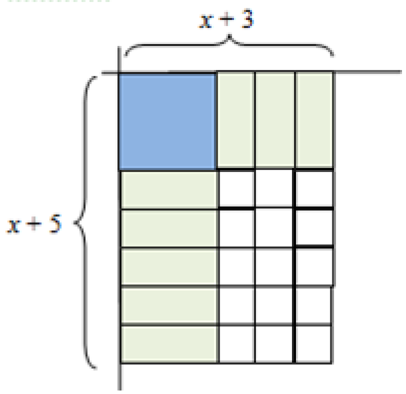
Teacher Notes and Elaborations *(continued)*

Area Model Example:

Step 1: Model the polynomial $x^2+8x+15$ (the dividend)



Step 2: Place the x2 tile at the corner of the product mat. Using all the tiles, make a rectangle with a length of x + 3 (the divisor). The width of the array x + 5, is the quotient.



Long Division Example:

$$\frac{x^2+8x+15}{x+3} \longrightarrow x+3\overline{\smash{)}\,x^2+8x+15\,}^{\;x+5} \longrightarrow x+5$$

$$\begin{array}{r} -(x^2+3x) \\ \hline 5x+15 \\ -(5x+15) \\ \hline 0 \end{array}$$

Return to Course Outline

Extension for PreAP Algebra I

Factor third-degree polynomials with at least one monomial as a factor such as:

$$3x^3-12x^2+9x$$
$$3x(x^2-4x+3)$$
$$3x(x-3)(x-1)$$

Factor four terms by grouping such as:

$$ay+by+3a+3b$$
$$y(a+b)+3(a+b)$$
$$(a+b)(y+3)$$

Quadratics

Fourth Marking Period

Lessons	Essential Knowledge and Skills	Essential Understandings
Lessons • Graphing Quadratic Functions • Solving Quadratic Functions by Graphing • Quadratic Formula **Strand**: Equations and Inequalities, Functions **SOL A.4cf** The student will solve multistep quadratic equations in two variables including c) solving quadratic equations algebraically and graphically; and f) solving real-world problems involving equations. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions. **SOL A.7bcdf** The student will investigate and analyze quadratic function families and their characteristics both algebraically and graphically, including b) domain and range; c) zeros of a function; d) *x*- and *y*-intercepts; f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. Return to Course Outline	**Essential Knowledge and Skills** The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Solve quadratic equations. • Identify the roots or zeros of a quadratic function over the real number system as the solution(s) to the quadratic equation that is formed by setting the given quadratic expression equal to zero. • Confirm algebraic solutions to quadratic equations, using a graphing calculator • Identify the domain, range, zeros and intercepts of a function presented algebraically or graphically.	**Essential Understandings** • Real-world problems can be interpreted, represented, and solved using quadratic equations. • The process of solving quadratic equations can be modeled in a variety of ways, using concrete, pictorial, and symbolic representations. • The zeros or the *x*-intercepts of the quadratic function are the real root(s) or solution(s) of the quadratic equation that is formed by setting the given quadratic expression equal to zero. • Set builder notation may be used to represent solution sets of equations. • An object x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$. *(continued)*

Teacher Notes and Resources

Quadratics *(continued)*

Resources

Textbook:

10-1 Exploring Quadratic Graphs
10-2 Quadratic Functions
10-4 Solving Quadratic Equations
10-7 Using the Quadratic Formula

HCPS Algebra 1 Online!:

Quadratics – Factoring Lesson 6

DOE Lesson Plans:

- Road Trip (PDF) (Word) - Writing equations to describe real-life data; creating statistical plots

Key Vocabulary

family of functions
parabola
quadratic equation
real numbers
root(s)
set builder notation
solution set
standard form of a quadratic equation
x-intercept
zeros of a function

Essential Questions

- What is a quadratic equation?
- What is the standard form of a quadratic equation?
- What methods are used to solve quadratic equations?
- What is the relationship between the solutions of quadratic equations and the roots of a function?
- How are solutions written in set builder notation?
- How can symmetry be helpful in graphing a quadratic function?
- What is the zero of a function?

Return to Course Outline

Teacher Notes and Elaborations

Quadratic Equations

A *quadratic equation* is an equation that can be written the form $ax^2 + bx + c = 0$ where $a \neq 0$. This form is called the *standard form of a quadratic equation*. The graph of a quadratic equation is a *parabola*.

The *zeros of a function* or *the x-intercepts* of the quadratic function are the real *root(s)* or solution(s) of the quadratic equation that is formed by setting the given quadratic expression equal to zero.

To find the *y*-intercept in a quadratic function, let $x = 0$.

Quadratic equations can be solved in a variety of ways:

1. factoring
2. graphing
3. completing the square
4. using the graphing calculator

Given the set of real numbers, quadratic equations may have no solutions, one solution, or two solutions. The solution(s) to a quadratic equation make the equation true when the value is substituted into the equation.

Set builder notation is a method for identifying a set of values. For example, the domain for $y = x^2 - 5$ would be written as $\{x : x \in \Re\}$. This is read, "The set of all *x* such that *x* is an element of the real numbers." The range for this equation would be written as $\{y : y \geq -5\}$.

Extension for PreAP Algebra I

- Explore transformations of quadratic equations in (*h*, *k*) form using the graphing calculator.
- Use the quadratic formula to solve problems.

(continued)

Quadratics *(continued)*

Teacher Notes and Resources

Teacher Notes and Elaborations *(continued)*
Extension for PreAP Algebra 1 *(continued)*

- Solve cubic equations.
- Determine the number of real roots for a quadratic equation using the discriminant.
- Use the equation for the axis of symmetry to graph a quadratic equation.
- Investigate and analyze real-world problems to determine the best method to solve a problem.

Quadratic equations can be solved using the quadratic formula –

If $ax^2+bx+c=0$, and $a \neq 0$, then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Extension for PreAP Algebra 1
Quadratic equations can be written in (h, k) form ($y = a(x – h)^2 + k$). Characteristics about the graph including whether the parabola opens up or down, whether it is wide or narrow, and the location of the vertex can be determined by looking at the values for a, h, and k.

Extension for Pre AP Algebra I
The expectations for solving cubic equations should be limited to equations that result in the elimination of all variable terms except for the cubed term. Experiences in solving equations should include problems with solutions that require finding the cube root of an integer.

Example1:
$$x^3=8$$
$$\sqrt[3]{x^3}=\sqrt[3]{8}$$
$$x=2$$

Example 2:
$$3x^3+2=-22$$
$$3x^3=-24$$
$$x^3=-8$$
$$\sqrt[3]{x^3}=\sqrt[3]{-8}$$
$$x=-2$$

Extension for Pre AP Algebra I

Example 3:
$$4x^3+3(x^2-x)-32=76+3x^2-3x$$
$$4x^3+3x^2-3x-32=76+3x^2-3x$$
$$4x^3-32=76$$
$$4x^3=108$$
$$x^3=27$$
$$\sqrt[3]{x^3}=\sqrt[3]{27}$$
$$x=3$$

Sample Instructional Strategies and Activities

- Strategy for solving quadratic equations:
  - Graphing calculators are given to students, who then enter the equation, investigate its graph, and identify the x intercepts as the roots of the equation.
  - Check the solutions algebraically.

Emphasize that quadratic equations may have no real roots, but this does not preclude the equation from having no solution.

Return to Course Outline

Radicals

Fourth Marking Period

Lessons
- Square Roots and Cube Roots of Whole Numbers
- Square Roots of Monomial Algebraic Expressions
- Operations with Radical Expressions

Strand: Expressions and Operations

SOL A.3
The student will express the square roots and cube roots of whole numbers and the square root of a monomial algebraic expression in simplest radical form.

Return to Course Outline

Essential Knowledge and Skills
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
- Express square roots of a whole number in simplest form.
- Express the cube root of a whole number in simplest form.
- Express the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values.

Essential Understandings
- A square root in simplest form is one in which the radicand (argument) has no perfect square factors other than one.
- A cube root in simplest form is one in which the argument has no perfect cube factors other than one.
- The cube root of a perfect cube is an integer.
- The cube root of a nonperfect cube lies between two consecutive integers.
- The inverse of cubing a number is determining the cube root.
- In the real number system, the argument of a square root must be nonnegative while the argument of a cube root may be any real number.

(continued)

Radicals *(continued)*

Teacher Notes and Resources

Resources

Textbook:

10-3 Finding and Estimating Square Roots
11-1 Simplifying Radicals
11-4 Operations with Radical Expressions

HCPS Algebra 1 Online!:

Radicals

DOE Lesson Plans:

- Simply Radical (PDF) (Word) - Simplifying square roots and cube roots of algebraic expressions
- Simplifying Square Roots (PDF) (Word)

Key Vocabulary

cube root	radical expression
principal square root	radicand
radical	

Essential Questions

- What is a radical?
- How are radical expressions simplified?
- What are the restrictions on the radicands for both square roots and cube roots?

Teacher Notes and Elaborations

The square root of a number is any number which when multiplied by itself equals the number. Whole numbers have both positive and negative roots. For example, the square root of 25 is 5 and –5, where 5 is the positive root and –5 is the negative root (written as ± 5).

Return to Course Outline

The non-negative square root of a number is called the *principal square root*. Most frequently used perfect squares must be committed to memory to allow for reasonable approximations of non-perfect squares.

A root of a number is a *radical* (e.g., $\sqrt{5}$ is called a radical and 5 is the *radicand*).

The square root of a whole number is in simplest form when the radicand has no perfect square factors other than one.

The *cube root* of a number, *n*, is a number whose cube is that number. For example, the cube root of 125 is 5 ($\sqrt[3]{125} = 5$) because $5^3 = 125$. In general, $\sqrt[3]{n} = a$ if $a^3 = n$.

A *radical expression* is in simplest form when all three statements are true.

1. The expression under the radical sign has no perfect square factors other than one.
2. The expression under the radical sign does not contain a fraction.
3. The denominator does not contain a radical expression.

Examples:

$$\sqrt{54}+\sqrt{24}+\sqrt{48}$$
$$\sqrt{9\cdot 6}+\sqrt{4\cdot 6}+\sqrt{16\cdot 3}$$
$$3\sqrt{6}+2\sqrt{6}+4\sqrt{3}$$
$$5\sqrt{6}+4\sqrt{3}$$

$$\left(\sqrt{3x}\right)^4$$
$$\sqrt{3x}\cdot\sqrt{3x}\cdot\sqrt{3x}\cdot\sqrt{3x}$$
$$\left(\sqrt{3x}\cdot\sqrt{3x}\right)\cdot\left(\sqrt{3x}\cdot\sqrt{3x}\right)$$
$$3x\cdot 3x$$
$$9x^2$$

(continued)

Teacher Notes and Resources

Radicals *(continued)*

Teacher Notes and Elaborations *(continued)*

Extension for Pre AP Algebra I

- Simplify multiplication (including the distributive property) of expressions that contain radicals and variables.
- Simplify expressions by rationalizing monomial denominators.
- Express the cube root of an integer in simplest form. Integers are limited to perfect cubes.

Extension for Pre AP Algebra I

Negative numbers can also be cubed. If –5 is cubed the result is –125 ($-5\cdot-5\cdot-5=-125$). The cube root of –125 is –5 ($\sqrt[3]{-125}=-5$).

Extension for Pre AP Algebra I

$$\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{6}\right)$$
$$\left(\sqrt{2}\cdot\sqrt{5}\right)+\left(\sqrt{2}\cdot\sqrt{6}\right)+\left(\sqrt{3}\cdot\sqrt{5}\right)+\left(\sqrt{3}\cdot\sqrt{6}\right)$$
$$\sqrt{10}+\sqrt{12}+\sqrt{15}+\sqrt{18}$$
$$\sqrt{10}+\sqrt{4\cdot3}+\sqrt{15}+\sqrt{9\cdot2}$$
$$\sqrt{10}+2\sqrt{3}+\sqrt{15}+3\sqrt{2}$$

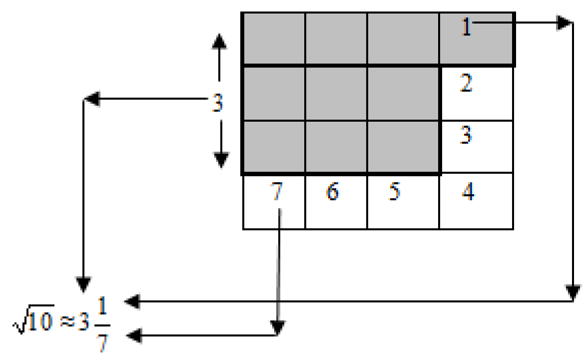
Extension for Pre AP Algebra I

Rationalizing a denominator is the process of expressing a fraction with an irrational denominator as an equal fraction with a rational denominator.

For example: $\frac{-3}{\sqrt{2}}=\frac{-3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{3\sqrt{2}}{2}$

Return to Course Outline

- Make a set of approximately 35 (3×5) index cards with the numbers from 0 to 10 to include $\sqrt{57}$, $3\sqrt{5}$, etc. Stretch a string across the front of the classroom and have students draw a card from the hat and pick a clothespin. (Some students may draw two cards and get two clothespins). Students arrange their numbers on the string from the least to the greatest using a clothespin.
- Students, working in pairs, write numbers on cards and the number's square on other cards. Groups exchange cards and shuffle. Next, students match cards using a format similar to "Concentration".
- Estimate square roots by using 1" tiles
  1. Select a number that is not a perfect square.
  2. Using paper squares or tiles, students make the largest square possible from the total number of tiles. The length of the side of this square is the whole number part of the solution.
  3. The number of paper shapes (tiles) left over when making this square form the numerator of the fraction.
  4. The denominator is formed by counting the total number of paper shapes (tiles) necessary to make the next size square.



Data Analysis

Fourth Marking Period

Lessons	Essential Knowledge and Skills	Essential Understandings
• Box and Whisker Plots • Best Fit Lines and Curves **Strand**: Statistics **SOL A.10** The student will compare and contrast multiple univariate data sets, using box-and-whisker plots. **SOL A.11** The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions. Return to Course Outline	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Compare, contrast, and analyze data, including data from real-world situations displayed in box-and-whisker plots. • Write an equation for a curve of best fit, given a set of no more than twenty data points in a table, a graph, or real-world situation. • Make predictions about unknown outcomes, using the equation of the curve of best fit. • Design experiments and collect data to address specific, real-world questions. • Evaluate the reasonableness of a mathematical model of a real-world situation.	• Box-and-whisker plots can be used to analyze data. • Statistical techniques can be used to organize, display, and compare sets of data. • The graphing calculator can be used to determine the equation of a curve of best fit for a set of data. • The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. • Many problems can be solved by using a mathematical model as an interpretation of a real-world situation. The solution must then refer to the original real-world situation. • Considerations such as sample size, randomness, and bias should affect experimental design. *(continued)*

Teacher Notes and Resources

Data Analysis – Box-and-Whisker Plot

Resources

Textbook:

pg. 740 Box-and-Whisker Plot Skills Handbook

HCPS Algebra 1 Online!:

Data Analysis – Lesson 3

DOE Lesson Plans:

- Box-and-Whisker Plots (PDF) (Word) - Comparing and contrasting data sets, using box-and-whisker plots

Key Vocabulary

box-and-whisker plot	median
extreme values	quartiles
inter-quartile range	range

Essential Questions

- What is a box-and-whisker plot?
- How is a box-and-whisker plot constructed?
- How is a box-and-whisker plot used in a real-world situation?

Teacher Notes and Elaborations *(continued)*

Statistical techniques can be used to organized, display, and compare sets of data.

Descriptive and visual forms of numerical data help interpret and analyze data from real-world situations.

A univariate data set consists of observations on a single variable.

Box-and-whisker plots can be used to summarize and analyze data.

Return to Course Outline

These plots graphically display the median, quartiles, interquartile range, and *extreme values* (minimum and maximum) in a set of data. They can be drawn vertically or horizontally. A box-and-whisker plot consists of a rectangular box with the ends located at the first and third quartiles. The segments extending from the ends of the box to the extreme values are called whiskers.

The *range* of the data is the difference between the greatest and the least values of the set.

The median of an odd collection of numbers, arranged in order, is the middle number. The median of an even collection of numbers, arranged in order, is the average of the two middle numbers.

The *median* of an ordered collection of numbers roughly partitions the collection into two halves, those below the median and those above. The first quartile is the median of the lower half. The second quartile is the median of the entire collection. The third quartile is the median of the upper half.

Box and whisker plots are uniform in their use of the box: the bottom and top of the box are always the 25^{th} and 75^{th} percentile (the lower and upper quartiles, respectively), and the band near the middle of the box is always the 50^{th} percentile (the median). Each quartile represents 25% of the data.

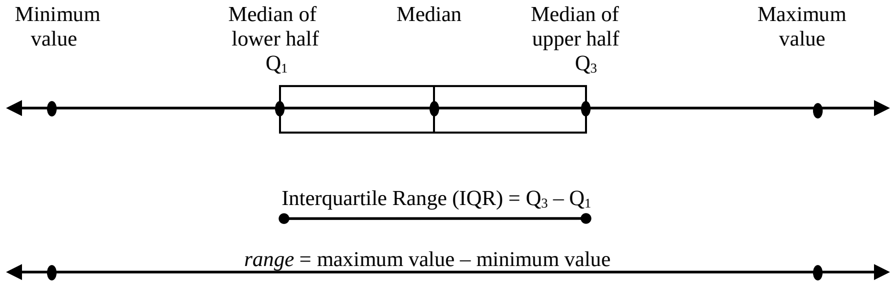
(continued)

Teacher Notes and Resources

Data Analysis – Box-and-Whisker Plot *(continued)*

Teacher Notes and Elaborations *(continued)*

"box" extends from Q_1 to Q_3.



The *interquartile range*, abbreviated IQR, is just the width of the box in the box-and-whisker plot. That is, IQR = $Q_3 - Q_1$. The IQR can be used as a measure of how spread-out the values are. Statistics assumes that values are clustered around some central value. The IQR tells how spread out the "middle" values are; it can also be used to tell when some of the other values are too far from the central value. These "too far away" points are called outliers, because they lie outside the expected range.

A box-and-whisker plot makes it easy to see where the data are spread out and where they are concentrated.

Multiple box-and-whisker plots can be used to compare and contrast sets of data. Graphing calculators may be used to compare multiple box-and-whisker plots.

Return to Course Outline

Sample Instructional Strategies and Activities

- Students are given a list of their previous class test scores. Use the information obtained to make box-and-whiskers graphs. Discuss how summarizing data is helpful to analyzing data. The students will use their individual scores over a period of time.
- Give the students the scores of a high school sport's team for the past four years. From this set of data, ask your students to compare and analyze the team's performance using box and whisker plots. Ask your students to use their plots to check their analysis and make new conclusions.

Teacher Notes and Resources

Data Analysis – Curve of Best Fit

Resources

Textbook:

6-6 Scatter Plots and Equations of Lines
pg. 329 Mixed Review Exercises
10-9 Choosing a Linear, Quadratic or Exponential Model

HCPS Algebra 1 Online!:

Data Analysis – Lesson 4

DOE Lesson Plans:

- Line of Best Fit (PDF) (Word)
- Linear Curve of Best Fit (PDF) (Word)
- Quadratic Curve of Best Fit (PDF) (Word)

Key Vocabulary

Curve of best fit

Essential Questions

- What is a curve of best fit?
- How is a curve of best fit used to make predictions in real-world situations?
- How do sample size, randomness, and bias affect the reasonableness of a mathematical model of a real-world situation?

Teacher Notes and Elaborations

When real-life data is collected, the data graphed usually does not form a perfectly straight line or a perfect quadratic curve. However, the graph may approximate a linear or quadratic relationship. A *curve of best fit* is a line that best represents the given data. The line may pass through some of the points, none of the points, or all of the points. When this is the case, a curve of best fit can be drawn, and a prediction equation that models the data can be determined. A curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. Accuracy of the equation can depend on sample size, randomness, and bias of the collection.

Return to Course Outline

A linear curve of best fit (line of best fit) may be determined by drawing a line and connecting any two data points that seem to best represent the data. An equal number of points should be located above and below the line. This line represents the equation to be used to make predictions.

A quadratic curve of best fit may be determined by drawing a graph and connecting any three points that seem to best represent the data. Putting these data points into a graphing calculator will result in a quadratic function. Since different people may make different judgments for which points should be used, one person's equation may differ from another's.

The graphing calculator can be used to determine the equation of the curve of best fit for both linear and quadratic.

Sample Instructional Strategies and Activities

- The students will measure the height and weight of 10 students in the class. With *x* representing the height and *y* representing the weight, the students organize the data in table form. Then they draw a scatter plot and best-fit line on graph paper. After finding the equation of the best- fit line, the students predict the weight of a mystery student based upon his height. After the predictions have been made, the mystery student stands up.
- Dry spaghetti, string, and thread are great for students to use to informally determine where a line of best fit would be. These show up on the overhead projector so work well for demonstration there as well.

(continued)

Teacher Notes and Resources

Data Analysis – Curve of Best Fit *(continued)*

Teacher Notes and Elaborations *(continued)*

Sample Instructional Strategies and Activities *(continued)*

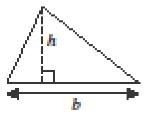
- This table shows data for speed and stopping distances of cars. Discuss with students why this will not be a linear curve of best fit. Using a graphing calculator find a quadratic equation that best represents this data. After finding the equation, students make predictions for speeds not in the table.

Speed	Stopping Distance
10	12.5
20	36
30	69.5
40	114
50	169.5
60	249
70	325.5

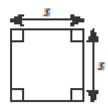
Return to Course Outline

Algebra I Formula Sheet
2009 *Mathematics Standards of Learning*

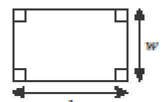
Geometric Formulas:



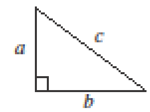
$A = \frac{1}{2}bh$



$p = 4s$
$A = s^2$



$p = 2l + 2w$
$A = lw$



$a^2 + b^2 = c^2$

Quadratic Formula:

$x = \frac{^-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $ax^2 + bx + c = 0$ and $a \neq 0$

Statistics Formulas:

Given:
x represents an element of the data set,
x_i represents the i^{th} element of the data set,
n represents the number of elements in the data set,
μ represents the mean of the data set, and
σ represents the standard deviation of the data set.

variance $(\sigma^2) = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$

standard deviation $(\sigma) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$

mean absolute deviation $= \frac{\sum_{i=1}^{n}\left|x_i - \mu\right|}{n}$

z-score $(z) = \frac{x - \mu}{\sigma}$

http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_algebra1.pdf