NEW JERSEY CENTER
FOR TEACHING & LEARNING

Algebra II

Rational Expressions & Equations

2015-08-15

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Working with Rational Expressions

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Goals and Objectives

- Students will simplify rational expressions, as well as be able to add, subtract, multiply, and divide rational expressions.
- Students will solve rational equations and use them in applications.
- Students will graph rational functions and identify their holes, vertical asymptotes, and horizontal asymptotes.

What is a rational expression?

A rational expression is the ratio of two polynomials. It is written as a fraction with polynomial expressions in the numerator and denominator.

Why do we need this?

Rational expressions are often used to simplify expressions with long polynomials in both the numerator and denominator. Since it is more efficient to work with simple problems and situations, knowing how to simplify rational expressions makes looking at graphs and other problems easier.

Rational expressions and equations are often used to model more complex equations in fields such as science and engineering. Rational expressions are applicable in working with forces and fields in physics and aerodynamics.

Inverse and Joint Variation

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Variation

Variation describes the relationship between variables.

There are three types of variation:

direct,

inverse and

joint variation.

Each type describes a different relationship.

Inverse Variation ↑ ↓

With Inverse variation, when one element increases, the other element decreases. Or, vice versa, when one element decreases, the other element increases.

Examples:

As you pull on a rubber band to make it longer, the width of the band gets smaller.	As you increase your amount of spending, you decrease the amount of money available to you.	As you increase your altitude by hiking up a mountain, you will feel a decrease in the temperature.

Joint Variation

Joint variation is the same as direct variation, but is used when two or more elements affect what another element does. If one or both elements increase, the other element increases. Or, vice versa, when one or both elements decrease, the other element also decreases.

Examples:

As you increase the radius and/ or the height of a cone, you increase the volume.

As you either decrease the speed you drive or decrease the time you drive, you will decrease the distance you cover.

As you increase the length or width of your backyard fence, you increase the area of your backyard.

Variation

Using more mathematical vocabulary...

Inverse variation: The temperature of the air varies inversely with the altitude.

written as: $Temp = \frac{k}{Alt}$

Joint variation: The volume of a cone varies jointly with the square of its radius and its height.

written as: $V = \frac{1}{3}\pi r^2 h$

Variation

Notice that in each of these variations there is an additional number whose value does not change:

$Temp = \frac{k}{Alt}$ ← $\quad V = \frac{1}{3}\pi r^2 h$ ↑

This number is called the constant of variation and is denoted by k.

Variation

Steps to solving a variation problem:

1) Determine an equation based on each type of variation.
   - Inverse: $y = {}^{k}/_{x}$
   - Joint: $y = kxz$
2) Find the constant of variation (k)
3) Rewrite the equation substituting a value for k.
4) Use the final equation to find the missing value.

Variation

Example:

If y varies inversely with x, and y = 10 when x = 4, find x when y = 80.

Variation

Example: The volume of a square pyramid varies jointly with the area of the base (s^2) and the height. If the volume is 75 when the base side is 5 and the height is 9, find the volume when the height is 12 and the base side is 4.

1 If y varies inversely with x, and y = 10 when x = -4, find y when x = 8.

2 If y varies inversely with x, and y = 3 when x = 15, find y when x = 5.

3 If y varies jointly with x and z, and y = 6 when x = 3 and z = 9, find y when x = 5 and z = 4.

4 If y varies jointly with x and z, and y = 3 when x = 4 and z = 6, find y when x = 6 and z = 8.

Simplifying Rational Expressions

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Simplifying Rationals

A rational expression is an expression that can be written in the form $\frac{polynomial}{polynomial}$, where a variable is in the denominator.

The domain of a rational expression is all real numbers excluding those that would make the denominator 0. (This is very important when solving rational equations.)

For example, in the expression $\frac{3}{x^2-4}$, 2 and -2 are restricted from the domain.

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Simplifying Rationals

Remember to use properties of exponents and/or factoring to simplify the rational expressions.

$$\frac{3a^4b^7}{6a^8b^5} \quad \Bigg| \quad \frac{(2xy^3)^3}{4x^2y^7} \quad \Bigg| \quad \frac{x^2-16}{x^2-6x+8}$$

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5 Simplify $\frac{12x^4y^{10}}{24x^5y^9}$

- A $\frac{2y}{x}$
- B $\frac{1}{2}$
- C $\frac{y}{2x}$
- D $2xy$

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6 Simplify $\frac{\left(4x^2y^3\right)^2}{24x^3y^3}$

- A $\frac{xy^3}{3}$
- B $\frac{2xy^3}{3}$
- C $\frac{2y^3}{3x}$
- D $\frac{y^3}{3}$

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7 Simplify $\frac{x^2+2x-15}{x^2-10x+21}$

- A $12x-2$
- B $\frac{x+5}{x-7}$
- C $\frac{x+5}{x+7}$
- D $-12x+2$

Multiplying Rational Expressions

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Multiply

$$\frac{3k^2m}{4ab^3}\cdot\frac{8ab}{9km}=$$

$$\frac{4x^2}{9x^2-1}\cdot\frac{3x+1}{2x}=$$

10 Simplify $\frac{8x^2}{12xy}\cdot\frac{36y^2}{6x}=$

- **A** $\frac{4}{x}$
- **C** $4xy$
- **B** $\frac{4x}{y}$
- **D** $4y$

Dividing Rational Expressions

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14 Simplify $\frac{7}{m^2} \div \frac{14}{m} =$

- A $\frac{98}{m^3}$
- B $\frac{m}{2}$
- C $\frac{1}{2m}$
- D $\frac{m^3}{98}$

15 Simplify $\frac{y-4}{9} \div \frac{4-y}{3} =$

- A $\frac{y-4}{3(y-4)}$
- B $-\frac{1}{3}$
- C $\frac{y^2-16}{27}$
- D $\frac{1}{3}$

18 Simplify $\dfrac{\dfrac{e^2-f^2}{ef}}{\dfrac{e-f}{e}} =$

- A $\dfrac{f}{e-f}$
- C $\dfrac{e}{e+f}$
- B $\dfrac{e+f}{e}$
- D $\dfrac{e+f}{f}$

Adding and Subtracting Rational Expressions

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Adding and Subtracting Rational Expressions

Just as in multiplication and division, when adding or subtracting rationals, use the same rules as basic fractions.

Recall: When adding and subtracting fractions, you MUST use common denominators.

Adding and Subtracting Rational Expressions

To add and subtract rational expressions they must have common denominators. Identify the LCD and rewrite the rational expressions with the same denominator.

Example:
No common denominator. → $\frac{3}{x}+\frac{4}{x^2}$

x^2 is the least common denominator.(LCD)

Multiply by an expression equal to 1. → $\frac{3}{x}\cdot\frac{x}{x}+\frac{4}{x^2}=\frac{3x}{x^2}+\frac{4}{x^2}$
(Multiply numerator and denominator by the same quantity.)

Then Add. → $\frac{3x+4}{x^2}$

Simplify if possible.

Adding and Subtracting Rational Expressions

Step 1: Find LCD

Step 2: Multiply each term by an expression equal to 1 to obtain LCD for each term.

Step 3: Add or subtract numerators

Step 4: Simplify

Example

Solve: $\frac{3}{x+2}+\frac{4}{x-2}$

Step 1: LCD = $(x + 2)(x - 2)$

Step 2: $\frac{3}{x+2}\bullet\frac{x-2}{x-2}+\frac{4}{x-2}\bullet\frac{x+2}{x+2}$

Step 3: $\frac{3x-6}{(x+2)(x-2)}+\frac{4x+8}{(x+2)(x-2)}=\frac{3x-6+4x+8}{(x+2)(x-2)}$

Step 4: $\frac{7x+2}{(x+2)(x-2)}$

Example

Solve: $\frac{2}{x-3}+\frac{5}{3-x}$ ← *The denominators are additive inverses.*

Step 1: $LCD: x-3$

Step 2: $\frac{2}{x-3}+\frac{5}{3-x}\cdot\frac{-1}{-1}$

Step 3: $\frac{2}{x-3}+\frac{-5}{x-3}$

Step 4: $\frac{-3}{x-3}$

Example Continued

Click

Remember: you can always check results by substituting values for the variables, being sure to avoid values for which the expression is undefined.

Common Denominator

Find the LCDs for the following. Describe any restrictions on the variables.

$\frac{3}{h-2}+\frac{1}{h+2}$

$\frac{4}{t^2-25}-\frac{3}{t+5}$

$\frac{2}{s^2-36}+\frac{7}{s^2+2s-24}$

23 Simplify $\frac{3}{h-2}+\frac{1}{h+2}$

- **A** $\frac{4h+4}{(h-2)(h+2)}$
- **C** $\frac{4h}{(h-2)(h+2)}$
- **B** $\frac{2}{h-2}$
- **D** $\frac{4h-4}{(h-2)(h+2)}$

24 Simplify $\frac{4}{t^2-25}-\frac{3}{t+5}$

A $\frac{-3t+19}{(t-5)(t+5)}$

B $\frac{-3t-11}{(t-5)(t+5)}$

C $\frac{-3t-1}{(t-5)(t+5)}$

D $\frac{-3t+9}{(t-5)(t+5)}$

25 Simplify $\frac{2}{s^2-36}+\frac{7}{s^2+2s-24}$

A $\frac{9s+34}{(s-6)(s+6)(s-4)}$

B $\frac{9s-50}{(s-6)(s+6)(s-4)}$

C $\frac{9s+10}{(s-6)(s+6)(s-4)}$

D $\frac{9s-10}{(s-6)(s+6)(s-4)}$

Division by Zero

Find an equivalent rational expression in lowest terms, and identify the value(s) of the variables that must be excluded to prevent division by zero.

$$\frac{x^2-7x+12}{6-5x+x^2}$$

(Derived from engage[ny])

Equivalent Expressions

Determine whether or not the rational expressions below are equivalent for $x \neq -1, x \neq -2, x \neq 3$. Explain how you know.

$$\frac{x+4}{(x+2)(x-3)} \qquad \frac{x^2+5x+4}{(x+1)(x+2)(x-3)}$$

(Derived from engage[ny])

Adding and Subtracting Rationals

Problem is from:

Illustrative Mathematics

Illustrations

Click for link for commentary and solution.

a. What is the combined fuel economy for the 2012 Volkswagen Jetta? Give your answer to three significant digits.

b. For most conventional cars, the highway fuel economy is 10 mpg higher than the city fuel economy. If we set the city fuel economy to be x mpg for such a car, what is the combined fuel economy in terms of x? Write your answer as a single rational function, $a(x)/b(x)$.

c. Rewrite your answer from (b) in the form of $q(x) + \frac{r(x)}{b(x)}$ where $q(x), r(x)$ and $b(x)$ are polynomials and the degree of $r(x)$ is less than the degree of $b(x)$.

d. Use your answer in (c) to conclude that if the city fuel economy, x, is large, then the combined fuel economy is approximately $x + 5$.

Solving Rational Equations

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Solving Rational Equations

Step 1: Find LCD

Step 2: Multiply EACH TERM by LCD

Step 3: Simplify

Step 4: Solve

Step 5: Check for Extraneous Solutions

Example Continued

Step 5: $$\frac{1}{2+2}+\frac{1}{2-2}=\frac{4}{2^2-4}$$

$$\frac{1}{4}+\frac{1}{0}\neq\frac{4}{0}$$

Explanation

When the solution of h = 2 is substituted into the original equation, it creates two undefined terms:

$$\frac{1}{0} \qquad \frac{4}{0}$$

This means that h = 2 is an extraneous solution and the rational equation has no solution.

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Solving Rational Equations

Example: Remember to find LCD and check all solutions.

$$\frac{3}{x} - \frac{2}{3x} = \frac{-7}{3x^2 - 6x}$$

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27 Use Steps 1 - 4 to solve for x:

$$\frac{4}{x} + \frac{3}{7} = \frac{1}{7x}$$

A -9

B 9

C 24

D 30

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28 Is the solution to the previous question valid when substituted into the original equation?

○ **A Yes, the solution is valid.**

○ **B No, the solution creates a false mathematical statement and is therefore an extraneous solution.**

○ **C No, the solution creates an undefined term(s) and is therefore an extraneous solution.**

Answer

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29 Use Steps 1 - 4 to solve for m:

$$\frac{5}{2m} + \frac{2m}{m+1} = 2$$

A -12

B -5

C 5

D 12

30 Is the solution to the previous question valid when substituted into the original equation?

- A Yes, the solution is valid.
- B No, the solution creates a false mathematical statement and is therefore an extraneous solution.
- C No, the solution creates an undefined term(s) and is therefore an extraneous solution.

31 Use Steps 1 - 4 to solve for x:

(Choose all that apply)

$$\frac{-3}{x^2-5x+6}-\frac{2}{x^2-9}=-\frac{1}{x-2}$$

- A -3
- B -2
- C 5
- D 7

32 Are the solutions to the previous question valid when substituted into the original equation?

- A Yes, both solutions are valid.
- B No, both of the solutions create a false mathematical statement and are therefore extraneous solutions.
- C No, one of the solutions creates an undefined term(s) and is therefore an extraneous solution.

34 Is the solution to the previous question valid when substituted into the original equation?

- A Yes, the solution is valid.
- B No, the solution creates a false mathematical statement and is therefore an extraneous solution.
- C No, the solution creates an undefined term(s) and is therefore an extraneous solution.

35 What is the solution of the equation

$$\frac{2m^2+3m-5}{m^2+4m-5}=4$$

From PARCC sample test

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Basketball

Problem is from:

Illustrative Mathematics Illustrations

Click for link for commentary and solution.

Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.

a) How many games would Chase have to win in a row in order to have a 75% winning record?

b) How many games would Chase have to win in a row in order to have a 90% winning record?

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Basketball

Problem is from:

Illustrative Mathematics Illustrations

Click for link for commentary and solution.

Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.

c) Is Chase able to reach a 100% winning record? Explain why or why not.

d) Suppose that after reaching a winning record of 90% in part (b), Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again?

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Applications of Rational Equations

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Applications

Rational equations can be used to solve a variety of problems in real-world situations.

We will see how to use rational equations in multi-rate work problems, and distance-speed-time problems.

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Applications

Here's a video showing the use of a rational equation to solve a simple multi-rate work problem:

click here

This is the problem described in the video:

Tom can wash a car in 45 minutes. Jerry can wash the same car in 30 minutes. How long will it take to wash the car if they work together?

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Applications

To solve the problem, the instructor used the fact that the amount of work completed is equal to the rate of work multiplied by the time spent working: $W = rt$

This formula might also be used as $t = \frac{W}{r}$

or $r = \frac{W}{t}$ depending upon which quantity is unknown.

Applications

Example:

Underground pipes can fill a swimming pool in 4 hours. A regular garden hose can fill the pool in 16 hours. If both are used at the same time, how long will it take to fill the pool?

The unknown quantity is time, or *t*. Discuss the table entries for use in this solution.

	rate	time	work
pipes	$\frac{1}{4}$	t	$\frac{1}{4}t$
hose	$\frac{1}{16}$	t	$\frac{1}{16}t$

Example (continued):

	rate	time	work
pipes	$\frac{1}{4}$	t	$\frac{1}{4}t$
hose	$\frac{1}{16}$	t	$\frac{1}{16}t$

The total amount of work by the pipes and the hose should equal 1 job completed.

$$\frac{1}{4}t+\frac{1}{16}t=1$$
$$16\cdot\frac{1}{4}t+16\cdot\frac{1}{16}t=16\cdot 1$$
$$4t+t=16$$
$$5t=16$$
$$t=3.2$$

With the pipes and hose working together, the pool will be filled in 3.2 hours.

Applications

Example:

Working alone, Tony's dad can complete the yard work in 3 hours. If Tony helps his dad, the yard work takes 2 hours. How long would it take Tony working alone to complete the yard work?

The unknown is the number of hours for Tony working alone. Discuss the table entries for use in this solution. Then write an equation and solve.

Answer

	rate	time	work
Dad	$\frac{1}{3}$	2	$\frac{2}{3}$
Tony	$\frac{1}{x}$	2	$\frac{2}{x}$

36 James can paint the office by himself in 7 hours. Manny paints the office in 10 hours. How long will it take them to paint the office working together?

37 Working together, it takes Sam, Jenna, and Francisco 2 hours to clean one house. When Sam is working alone, he can clean the house in 6 hours. When Jenna works alone, she can clean the house in 4 hours. Determine how long it would take Francisco to clean the house on his own.

38 Allison can complete a sales route by herself in 5 hours. Working with an associate, she completes the route in 3 hours. How long would it take her associate to complete the route by himself?

- **A 8 hours**
- **B 6.5 hours**
- **C 7.5 hours**
- **D 5 hours**

Applications

Another application of rational equations is solving distance-speed-time problems. Recall that distance traveled is equal to the speed (rate) multiplied by the time.

$$D = rt$$

This formula may also be used as $r = \frac{D}{t}$ or $t = \frac{D}{r}$ depending upon which quantity is unknown.

39 James can jog twice as fast as he can walk. He was able to jog the first 9 miles to his grandmother's house, but then he tired and walked the remaining 1.5 miles. If the total trip took 2 hours, then what was his average jogging speed?

- **A 3 mph**
- **B 4.5 mph**
- **C 2.5 hours**
- **D 3 hours**

40 A passenger car averages 16 miles per hour faster than a bus. If the bus travels 56 miles in the same time it takes the passenger car to travel 84 miles, then what is the speed of each?

(Hint: use r for the smaller unknown speed)

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Graphing Rational Functions

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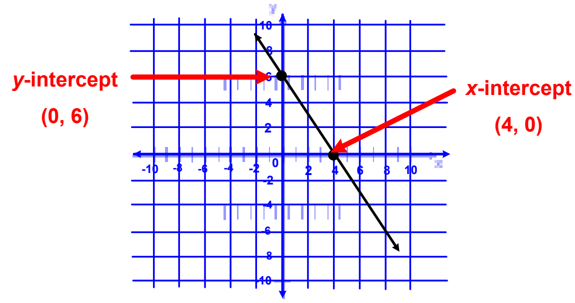
Vocabulary Review

x-intercept: The point where a graph intersects with the *x*-axis and the *y*-value is zero.

y-intercept: The point where a graph intersects with the *y*-axis and the *x*-value is zero.



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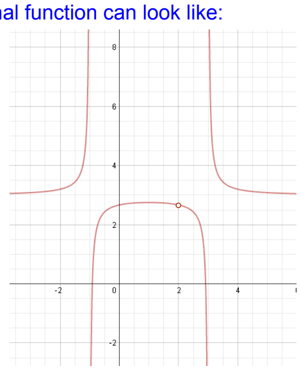
Graphs

Rational Functions have unique graphs that can be explored using properties of the function itself. Here is a general example of what the graph of a rational function can look like:



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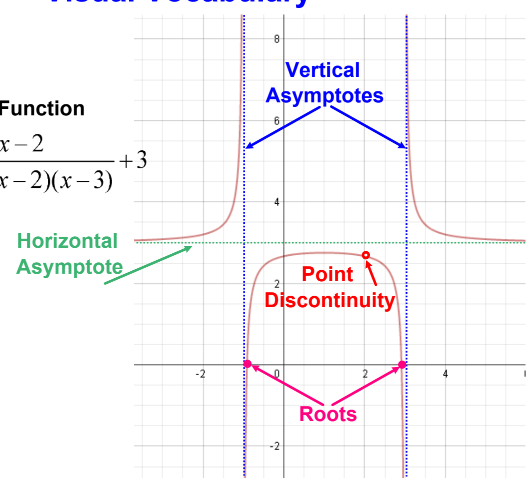
Visual Vocabulary

Rational Function

$$f(x)=\frac{x-2}{(x+1)(x-2)(x-3)}+3$$



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Vocabulary

Rational Function: $f(x)=\frac{polynomial}{polynomial}$

Roots: *x*-intercept(s) of the function; *x* values for which the numerator = 0

Discontinuities: *x*-values for which the function is undefined;

Infinite discontinuity: *x*-values for which only the denominator = 0 (vertical asymptote)

Point discontinuity: *x*-values for which the numerator & denominator = 0 (hole)

Asymptote: A line that the graph continuously approaches but does not intersect

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Graphing a Rational Function

Step 1: Find and graph vertical discontinuities

Step 2: Find and graph horizontal asymptotes

Step 3: Find and graph *x*- and *y*-intercepts

Step 4: Use a table to find values between the *x*- and *y*-intercepts

Step 5: Connect the graph

Step 1 Continued

$$f(x) = \frac{x-2}{(x+1)(x-2)(x-3)} + 3$$

B) Set remaining denominator factors equal to zero and solve - Vertical Asymptotes

$x + 1 = 0$

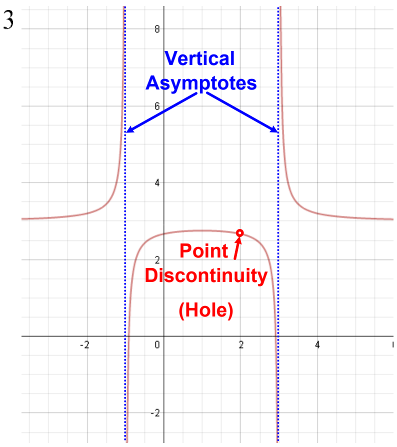
$x = -1$

Vertical Asymptote at $x = -1$

$x - 3 = 0$

$x = 3$

Vertical Asymptote at $x = 3$



41 What are the point discontinuities of the following function:

$$f(x) = \frac{(x-1)(2x+1)}{(2x+1)(x-3)(x-1)}$$

(Choose all that apply.)

- ☐ **A** $x = -3$
- ☐ **B** $x = -2$
- ☐ **C** $x = -1$
- ☐ **D** $x = -\frac{1}{2}$
- ☐ **E** $x = \frac{1}{2}$
- ☐ **F** $x = 1$
- ☐ **G** $x = 2$
- ☐ **H** $x = 3$

42 What are the point discontinuities of the following function:

$$g(x) = \frac{x^2 + 5x}{x^3 - 9x}$$

(Choose all that apply.)

- ☐ **A** $x = -5$
- ☐ **B** $x = -3$
- ☐ **C** $x = -\frac{5}{3}$
- ☐ **D** $x = 0$
- ☐ $x = \frac{5}{3}$
- ☐ $x = 3$
- ☐ $x = 5$
- ☐ $x = 9$

43 What are the point discontinuities of the following function:

$$h(x) = \frac{x^3 - x^2 - 6x}{x^3 - 3x^2 - 10x}$$

(Choose all that apply.)

- ☐ **A** $x = -5$
- ☐ **B** $x = -3$
- ☐ **C** $x = -2$
- ☐ **D** $x = 0$
- ☐ **E** $x = 2$
- ☐ **F** $x = 3$
- ☐ **G** $x = 5$
- ☐ **H** $x = 10$

44 **Find the vertical asymptotes of the following function:**

$$g(x)=\frac{x^2}{x^3-2x}$$

(Choose all that apply.)

- ❑ **A** $x = -3$
- ❑ **B** $x = -2$
- ❑ **C** $x=-\sqrt{2}$
- ❑ **D** $x = 0$
- ❑ **E** $x=\sqrt{2}$
- ❑ **F** $x = 2$
- ❑ **G** $x = 3$
- ❑ **H** no vertical discontinuities

45 **Find the vertical asymptotes of the following function:**

$$f(x)=\frac{x^2+7x+12}{(x-2)(x^2+x-12)}$$

(Choose all that apply.)

- ❑ **A** $x = -6$
- ❑ **B** $x = -4$
- ❑ **C** $x = -3$
- ❑ **D** $x = -2$
- ❑ **E** $x = 2$
- ❑ **F** $x = 3$
- ❑ **G** $x = 4$
- ❑ **H** $x = 6$

46 **Discuss the discontinuities of:**

$$h(x)=\frac{x}{x-1}$$

47 **Discuss the discontinuities of:**

$$g(x)=\frac{x+2}{(x-3)(x+2)}$$

48 **Discuss the discontinuities of:**

$$y=\frac{x-3}{x^2-9}$$

Notation for Holes

The point discontinuities (holes) in the graph of a rational function should be given as an ordered pair.

Once the *x*-value of the hole is found, substitute for *x* in the simplified rational expression to obtain the *y*-value.

Example

Find the holes in the graph of the following rational function:

$$g(x) = \frac{x+2}{(x-3)(x+2)}$$

Common factor of numerator and denominator:

$x + 2 = 0$

Hole at $x = -2$

Simplified expression: $\frac{1}{(x-3)}$

Evaluate for $x = -2$: $\frac{1}{(-2-3)} = -\frac{1}{5}$

The hole of this function is at (-2, -1/5)

49 Identify the hole(s) of the following function:

$$h(x) = \frac{x}{x-1}$$

(Choose all that apply.)

- ☐ **A (1, 1)**
- ☐ **B (-1, 1)**
- ☐ **C (1, 0)**
- ☐ **D no holes exist**

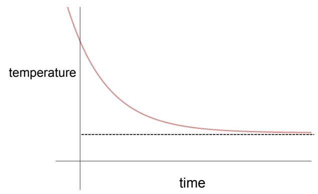
Step 2: Horizontal Asymptotes

The horizontal asymptote of a rational function is determined by comparing the degree of the numerator to the degree of the denominator.

The horizontal asymptote provides guidance for the graph's behavior as *x*-values become very large or very small. In other words, as *x* approaches ∞ or as *x* approaches $-\infty$.

Example

Think of a cup of boiling water left on a table to cool. If you graph the temperature for a period of time, what would be considered the horizontal asymptote and why?



Horizontal Asymptote = Room T emperature

The limiting factor is the room temperature. The water is not able to cool below room temperature, so the graph will have a horizontal asymptote.

Degree

Recall from Algebra I

The degree of a polynomial is the term containing the variable raised to the highest exponent.

Remember: A constant has a degree of 0. A variable with no exponent has a degree of 1.

For Example:

What is the degree of the polynomial $-6x3 + 2x$?

First Term is $-6x3$: x has a power of 3, so the degree is 3

Second Term is $2x$: x has a power of 1, so the degree is 1

The degree of the polynomial is 3.

Horizontal Asymptotes

Try these: Decide if the following functions have horizontal asymptotes. If so, find the equation of the asymptote.

a. $y = \dfrac{x^4}{x^2 - 7}$

b. $y = \dfrac{1}{x^3 + 2x - 7}$

Step 3: Intercepts

x-intercepts

The *x*-intercept(s) occur when *y* = 0, or where the numerator equals zero.

Set the numerator equal to zero and solve to find the *x*-intercepts.

Intercepts should be named as ordered pairs.

****Remember, if this value makes the denominator zero as well, there is a point discontinuity (a hole)****

Intercepts

y-intercepts

The *y*-intercepts occur where *x* is equal to zero.

Substitute zero for all *x*'s and solve to find the *y*-intercepts.

Intercepts should be named as ordered pairs.

56 Identify the y-intercept of

$$f(x)=\frac{x}{x-1}$$

57 Identify the y-intercept of

$$f(x)=\frac{x+1}{x-1}$$

58 Find the y-intercept of

$$f(x)=\frac{x+2}{x^2-9}$$

59 What are the *y*-intercepts for the following function?

$$f(x)=\frac{x+2}{x^2}$$

(Choose all that apply.)

- ☐ **A (0, -6)**
- ☐ **B (0, -3)**
- ☐ **C (0, 0)**
- ☐ **D (0, 3)**
- ☐ **E (0, 6)**
- ☐ **F There are no real intercepts**

60 Find any *x*-intercept(s) of:

$$h(x)=\frac{x}{x-1}$$

- ☐ **A (-3, 0)**
- ☐ **B (-1, 0)**
- ☐ **C (0, 0)**
- ☐ **D (1, 0)**
- ☐ **E (3, 0)**
- ☐ **F There are no real intercepts**

61 Find all *x*-intercept(s) of:

$$g(x)=\frac{x+2}{(x-3)(x+2)}$$

- ☐ **A** (-3, 0)
- ☐ **B** (-2, 0)
- ☐ **C** (0, 0)
- ☐ **D** (2, 0)
- ☐ **E** (3, 0)
- ☐ **F** There are no real intercepts

62 Identify all *x*-intercept(s) of:

$$y=\frac{(x-3)(x^2-4)}{(x^2-9)}$$

- ☐ **A** (-3, 0)
- ☐ **B** (-2, 0)
- ☐ **C** (0, 0)
- ☐ **D** (2, 0)
- ☐ **E** (3, 0)
- ☐ **F** There are no real intercepts

63 Choose all *x*-intercept(s) of :

$$y=\frac{(x^3-9x)}{(x^2-4)}$$

- ☐ **A** (-3, 0)
- ☐ **B** (-2, 0)
- ☐ **C** (0, 0)
- ☐ **D** (2, 0)
- ☐ **E** (3, 0)
- ☐ **F** There are no real intercepts

Step 4: Table

Graphs of rational functions contain curves, and additional points are needed to ensure the shape of the graph.

Once all discontinuities, asymptotes and intercepts are graphed, additional points can be found by creating a table of values.

To create an accurate graph, it is good practice to choose *x*-values near the intercepts and vertical asymptotes.

Example Continued

Step 2: Horizontal Asymptotes $f(x)=\frac{x^2-x-6}{x^2-4}$

Check the degree of numerator and denominator.

Since n = m, the asymptote is $y=\frac{a}{b}$

The asymptote for this graph is $y = 1$

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Example Continued

Step 4: Create a table of additional ordered pairs.

Choose values for *x* on either side of vertical asymptotes and *x*-intercepts.

$$f(x)=\frac{x^2-x-6}{x^2-4}$$

x	y
-4	1.17
-3	1.2
-1	1.3
0	1.5
1	2
3	0
4	0.5
5	0.67

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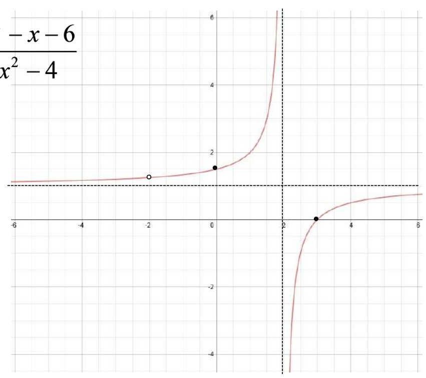
Example Continued

Step 5: Connect the points with a smoothe curve.

$$f(x)=\frac{x^2-x-6}{x^2-4}$$



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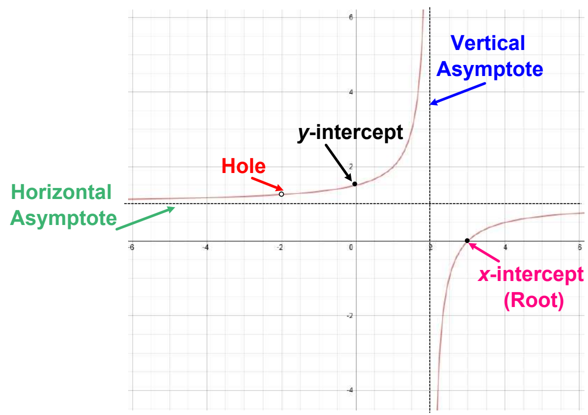
Graph Components



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64 What is the first step to use when graphing rational functions?

- A Finding the intercepts
- B Finding the horizontal asymptote
- C Creating a table of values
- D Creating the graph by connecting all previously found components
- E Finding the discontinuities

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65 The correct notation for a hole in a rational function is:

- A $x = 2$
- B $y = 2$
- C (2, 5)
- D There is no correct notation.

Graph 1

Now, let's put it all together.

Step 1: Find and graph vertical discontinuities

$$f(x) = \frac{x+3}{x^2+4x-12}$$

Graph 1

Step 2: Find and graph horizontal asymptotes

$$f(x) = \frac{x+3}{x^2+4x-12}$$

Graph 1

Step 3: Find and graph *x*- and *y*-intercepts

$$f(x) = \frac{x+3}{x^2+4x-12}$$

Graph 1

Step 4: Use a table to find values between the *x*- and *y*-intercepts

$$f(x) = \frac{x+3}{x^2+4x-12}$$

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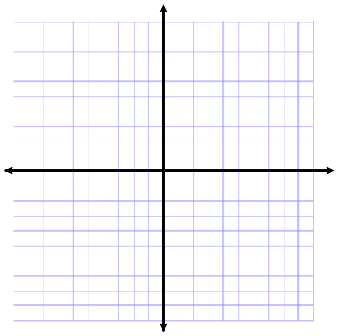
Graph 1

Step 5: Connect the graph

$$f(x)=\frac{x+3}{x^2+4x-12}$$



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Graph 2

Try another example.

Step 1: Find and graph vertical discontinuities

$$f(x)=\frac{3}{x+1}+2$$

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Graph 2

Step 2: Find and graph horizontal asymptotes

$$f(x)=\frac{3}{x+1}+2$$

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Graph 2

Step 3: Find and graph *x*- and *y*-intercepts

$$f(x)=\frac{3}{x+1}+2$$

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Graph 2

Step 4: Use a table to find values between the *x*- and *y*-intercepts

$$f(x)=\frac{3}{x+1}+2$$

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Graph 2

Step 5: Connect the graph

$$f(x)=\frac{3}{x+1}+2$$