HIGLEY UNIFIED SCHOOL DISTRICT
INSTRUCTIONAL ALIGNMENT

Algebra II Semester 1 (Quarter 1)

Unit 1: Polynomial, Rational, and Radical Relationships (45 days)
Topic A: Polynomials – From Base Ten to Base X (11 instructional days)

In Topic A, students draw on their foundation of the analogies between polynomial arithmetic and base ten computation, focusing on properties of operations, particularly the distributive property. In Lesson 1, students write polynomial expressions for sequences by examining successive differences. They are engaged in a lively lesson that emphasizes thinking and reasoning about numbers and patterns and equations. In Lesson 2, they use a variation of the area model referred to as the tabular method to represent polynomial multiplication and connect that method back to application of the distributive property.

In Lesson 3, students continue using the tabular method and analogies to the system of integers to explore division of polynomials as a missing factor problem. In this lesson, students also take time to reflect on and arrive at generalizations for questions such as how to predict the degree of the resulting sum when adding two polynomials. In Lesson 4, students are ready to ask and answer whether long division can work with polynomials too and how it compares with the tabular method of finding the missing factor. Lesson 5 gives students additional practice on all operations with polynomials and offers an opportunity to examine the structure of expressions such as recognizing that $n(n+1)(2n+1)$ /6
is a 3rd degree polynomial expression with leading coefficient 13 without having to expand it out.

In Lesson 6, students extend their facility with dividing polynomials by exploring a more generic case; rather than dividing by a factor such as $(x+3)$, they divide by the factor $(x+a)$ or $(x-a)$. This gives them the opportunity to discover the structure of special products such as $(x-a)(x^2+ax+a^2)$ in Lesson 7 and go on to use those products in Lessons 8–10 to employ the power of algebra over the calculator. In Lesson 8, they find they can use special products to uncover mental math strategies and answer questions such as whether or not 2100 − 1 is prime. In Lesson 9, they consider how these properties apply to expressions that contain square roots. Then, in Lesson 10, they use special products to find Pythagorean triples.

The topic culminates with Lesson 11 and the recognition of the benefits of factoring and the special role of zero as a means for solving polynomial equations.

Big Idea:	• Polynomials form a system analogous to the integers. • Polynomials can generalize the structure of our place value system and of radical expressions.
Essential Questions:	• How is polynomial arithmetic similar to integer arithmetic? • What does the degree of a polynomial tell you about its related polynomial function?
Vocabulary	Numerical symbol, variable symbol, algebraic expression, numerical expression, monomial, binomial, polynomial expression, sequence, arithmetic sequence, equivalent polynomial expressions, polynomial identity, coefficient of a monomial, terms of a polynomial, like terms of a polynomial, standard form of a polynomial in one variable, degree of a polynomial in one variable, radical, conjugate, Pythagorean Theorem, converse to the Pythagorean Theorem, Pythagorean Triple, function, polynomial function, degree of a polynomial function, constant function, linear function, quadratic function, cubic function, zeros or roots of a function
Assessments	Galileo: Geometry Module 1 Foundational Skills Assessment; Galileo: Topic A Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources

A.SSE.A.2	**A. Interpret the structure of expressions** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra II, tasks are limited to polynomial, rational, or exponential expressions. In Algebra I, students focused on rewriting algebraic expressions in different equivalent forms by combining like terms and using the associative, commutative and distributive properties. If students have difficulties with these skills, refer to Algebra I Module 1 L6-9 and Module 4 L2. In Algebra II, students should use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely. **Examples:** • See $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. • In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center (−1, 0). • See $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$. • Factor: $x^3 – 2x^2 - 35x$ • Rewrite $m^{2x} + m^x -6$ into an equivalent form. • Factor: $x^3 – 8$	**Eureka Math:** Module 1 Lesson 2 - 9 Module 1 Lesson 10-11 **This standard is revisited in Topic B (Factoring)**

A.APR.C.4	**C. Use polynomial identities to solve problems** Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2+y^2)^2 = (x^2- y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.	**Explanation:** Students prove polynomial identities algebraically by showing steps and providing reasons or explanation. Polynomial identities should include but are not limited to: • The product of the sum and difference of two terms, • The difference of two squares, • The sum and difference of two cubes, • The square of a binomial Students prove polynomial identities by showing steps and providing reasons and describing relationships. For example, determine $81^2 - 80^2$ by applying differences of squares which leads to $(81 + 80)(81 - 80) = 161$. Illustrate how polynomial identities are used to determine numerical relationships; such as $25^2 = (20 + 5)^2 = 20^2 + 2 \bullet 20 \bullet 5 + 5^2$. **Examples:** • Explain why $x^2 - y^2 = (x- y)(x+ y)$ for any two numbers x and y. • Verify the identity $(x- y)^2 = x^2 - 2x+ y^2$ by replacing ***y*** with –***y*** in the identity $(x + y)^2 = x^2 + 2x+ y^2$. • Show that the pattern shown below represents an identity. Explain. $2^2 - 1^2 = 3$ $3^2 - 2^2 = 5$ $4^2 - 3^2 = 7$ $5^2 - 4^2 = 9$ ***Solution:*** $(n+ 1)^2 - n^2 = 2n+ 1$ for any whole number n.	**Eureka Math:** Module 1 Lesson 2 – 7 Module 1 Lesson 10

MP.1	**Make sense of problems and persevere in solving them.**	Students discover the value of equating factored terms of a polynomial to zero as a means of solving equations involving polynomials.	**Eureka Math:** Module 1 Lesson 1 Module 1 Lesson 2 Module 1 Lesson 11
MP.2	**Reason abstractly and quantitatively.**	Students apply polynomial identities to detect prime numbers and discover Pythagorean triples.	**Eureka Math:** Module 1 Lesson 4 Module 1 Lesson 8
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.	**Eureka Math:** Module 1 Lesson 5 Module 1 Lesson 7 Module 1 Lesson 8, 9
MP.6	**Attend to precision.**	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	**Eureka Math:** Module 1 Lesson 10
MP.7	**Look for and make use of structure.**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities.	**Eureka Math:** Module 1 Lesson 1 – 6 Module 1 Lesson 8, 9, 10
MP.8	**Look for and express regularity in repeated reasoning.**	Students understand that polynomials form a system analogous to the integers. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students recognize factors of expressions and develop factoring techniques.	**Eureka Math:** Module 1 Lesson 1 - 4 Module 1 Lesson 6 Module 1 Lesson 8 Module 1 Lesson Module 1 Lesson

Algebra II Semester 1 (Quarter 1)

Unit 1: Polynomial, Rational, and Radical Relationships (45 days)
Topic B: Factoring – Its Use and Its Obstacles (10 instructional days)

Armed with a newfound knowledge of the value of factoring, students develop their facility with factoring and then apply the benefits to graphing polynomial equations in Topic B. In Lessons 12–13, students are presented with the first obstacle to solving equations successfully. While dividing a polynomial by a given factor to find a missing factor is easily accessible, factoring without knowing one of the factors is challenging. Students recall the work with factoring done in Algebra I and expand on it to master factoring polynomials with degree greater than two, emphasizing the technique of factoring by grouping.

In Lessons 14–15, students find that another advantage to rewriting polynomial expressions in factored form is how easily a polynomial function written in this form can be graphed. Students read word problems to answer polynomial questions by examining key features of their graphs. They notice the relationship between the number of times a factor is repeated and the behavior of the graph at that zero (i.e., when a factor is repeated an even number of times, the graph of the polynomial will touch the x-axis and "bounce" back off, whereas when a factor occurs only once or an odd number of times, the graph of the polynomial at that zero will "cut through" the x-axis). In these lessons, students will compare hand plots to graphing- calculator plots and zoom in on the graph to examine its features more closely.

In Lessons 16–17, students encounter a series of more serious modeling questions associated with polynomials, developing their fluency in translating between verbal, numeric, algebraic, and graphical thinking. One example of the modeling questions posed in this lesson is how to find the maximum possible volume of a box created from a flat piece of cardboard with fixed dimensions.

In Lessons 18–19, students are presented with their second obstacle: "What if there is a remainder?" They learn the Remainder Theorem and apply it to further understand the connection between the factors and zeros of a polynomial and how this relates to the graph of a polynomial function. Students explore how to determine the smallest possible degree for a depicted polynomial and how information such as the value of the y-intercept will be reflected in the equation of the polynomial.

The topic culminates with two modeling lessons (Lessons 20–21) involving approximating the area of the cross-section of a riverbed to model the volume of flow. The problem description includes a graph of a polynomial equation that could be used to model the situation, and students are challenged to find the polynomial equation itself.

Big Idea:	• Through deeper understanding of multiplication and division, students will develop higher-level and abstract thinking skills.
Essential Questions:	• How is the Zero Property helpful in writing equations in factored form? • Why is factoring polynomials beneficial? • What impact does an even- or odd-degree polynomial function have on its graph? • How do polynomials help solve real-world problems? • How does factoring relate to multiplication?
Vocabulary	Difference of squares identity, multiplicities, zeros or roots, relative maximum (maxima), relative minimum (minima), end behavior, even function, odd function, remainder theorem, factor theorem
Assessments	Galileo: Topic B Assessment

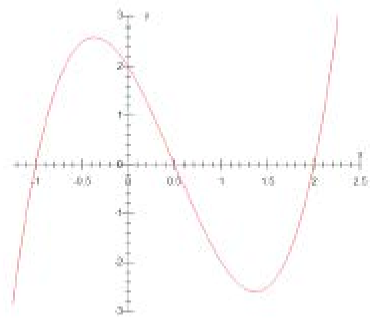
Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
N.Q.A.2	**A. Reason qualitatively and units to solve problems** Define appropriate quantities for the purpose of descriptive modeling. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** Students define appropriate quantities for the purpose of describing a mathematical model in context. This standard is taught in Algebra I and Algebra II. In Algebra II, the standard will be assessed by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades/courses) require the student to <u>create</u> a quantity of interest in the situation being described (i.e. this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude. **Example:** • Explain how the unit cm, cm^2, and cm^3 are relate. Describe situations where each would be an appropriate unit of measure.	**Eureka Math:** Module 1 Lesson 15, 16, 17, 20, 21 **This standard is revisited in Unit 2.**
A.SSE.A.2	**A. Interpret the structure of expressions** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra II, tasks are limited to polynomial, rational, or exponential expressions. In Algebra I, students focused on rewriting algebraic expressions in different equivalent forms by combining like terms and using the associative, commutative and distributive properties. If students have difficulties with these skills, refer to Algebra I Module 1 L6-9 and Module 4 L2. In Algebra II, students should use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely. **Examples:** • See $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.	**Eureka Math:** Module 1 Lesson 12 – 14 Module 1 Lesson 17 - 21

		• In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center (−1, 0). • See $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$. • Factor: $x^3 – 2x^2 - 35x$ • Rewrite $m^{2x} + m^x -6$ into an equivalent form. • Factor: $x^3 – 8$	
A.APR.B.2 ●	**B. Understand the relationship between zeros and factors of polynomials** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	**Explanation:** The Remainder theorem says that if a polynomial $p(x)$ is divided by any factor, $(x – c)$, *which does not need to be a factor of the polynomial*, the remainder is the same as if you evaluate the polynomial for c (meaning $p(c)$). If the remainder $p(c) = 0$ then $(x - c)$ is a factor of $p(x)$. Include problems that involve interpreting the Remainder Theorem from graphs and in problems that require long division. **Examples:** • Let $(x) = x^3 - x^4 + 8x^2 - 9x + 30$. Evaluate $p(-2)$. What does the solution tell you about the factors of $p(x)$? • Consider the polynomial function: $P(x) = x^4 - 3x^3 + ax^2 - 6x + 14$, where a is an unknown real number. If $(x – 2)$ is a factor of this polynomial, what is the value of a? • Use the Factor Theorem to determine whether $x – 1$ is a factor of $f(x) = 2x^4 + 3x^2 – 5x + 7$ 1 | 2 0 3 −5 7 2 2 5 0 2 2 5 0 7 $x – 1$ is not a factor of $f(x)$	**Eureka Math:** Module 1 Lesson 19 - 21

		Using the Factor Theorem, verify that $x + 4$ is a factor of $f(x) = 5x^4 + 16x^3 - 15x^2 + 8x + 16$ $\begin{array}{r\|rrrrr} -4 & 5 & 16 & -15 & 8 & 16 \\ & & -20 & 16 & -4 & -16 \\ \hline & 5 & -4 & 1 & 4 & 0 \end{array}$ $x + 4$ is a factor of $5x^4 + 16x^3 - 15x^2 + 8x + 16$	
A.APR.B.3	**B. Understand the relationship between zeros and factors of polynomials** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks were limited to quadratic and cubic polynomials, in which linear and quadratic factors are available. For example, find the zeros of *$(x - 2)(x^2 - 9)$*. In Algebra II, tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$. Students identify the multiplicity of the zeroes of a factored polynomial and explain how the multiplicity of the zeroes provides a clue as to how the graph will behave when it approaches and leaves the x-intercept. Students sketch a rough graph using the zeroes of a polynomial and other easily identifiable points such as the y-intercept. **Examples:** • Factor the expression $x^3 + 4x^2 - 64x - 256$ and explain how your answer can be used to solve the equation $x^3 + 4x^2 - 64x - 256 = 0$. Explain why the solutions to this equation are the same as the x-intercepts of the graph of the function $(x) = x^3 + 4x^2 - 64x - 256$. • For a certain polynomial function, $x = 3$ is a zero with multiplicity two, $x = 1$ is a zero with multiplicity three, and $x = -3$ is a zero with multiplicity one. Write a possible equation for this function and sketch its graph.	**Eureka Math:** Module 1 Lesson 14, 15, 17, 19, 20, 21

A.APR.D.6	**D. Rewrite rational expressions** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	**Explanation:** Students define rational expressions and determine the best method of simplifying a given rational expression. Students rewrite rational expressions in the form of $\frac{a(x)}{b(x)}$, in the form $q(x)+\frac{r(x)}{b(x)}$ by using inspection (factoring) or long division. The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. **Examples:** • Express $\frac{-x^2+4x+87}{x+1}$ in the form $(x)+\frac{r(x)}{b(x)}$. • Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. Students determine the best method of simplifying a given rational expression. • Simplify $\frac{x^2+9x+14}{x+7}$ $x+2$ $x+7\overline{)x^2+9x+14}$ $-x^2-7x$ $2x+14$ $-2x-14$ 0 • Simplify (using inspection): o $\frac{6x^3+15x^2+12x}{3x}$	**Eureka Math:** Module 1 Lesson 12, 13, 18, 19, 20, 21

		o $\frac{x^2+9x+14}{x+7}$ • Simplify (using long division): o $\frac{x^2+3x}{x^2-4}$ o $\frac{x^3+7x^2+13x+6}{x+4}$ ***Note:*** The use of synthetic division may be introduced as a method but students should recognize its limitations (division by a linear term). When students use methods that have not been developed conceptually, they often create misconceptions and make procedural mistakes due to a lack of understanding as to why the method is valid. They also lack the understanding to modify or adapt the method when faced with new and unfamiliar situations.	
F.IF.B.4	**B. Interpret functions that arise in applications in terms of context** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.* ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and they are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. In Algebra II, tasks have a real-world context and they may involve **polynomial**, exponential, logarithmic, and trigonometric functions. (Trigonometric functions will be explored 2nd semester in Unit 4) **Examples:** • For the function below, label and describe the key features. Include intercepts, relative max/min, intervals of increase/decrease, and end behavior.	**Eureka Math** Module 1 Lesson 15 **This standard will be revisited in Unit 2 Topic C (exponential and logarithmic) and again 2nd semester in Unit 4 (trigonometric).**



- The number of customers at a coffee shop vary throughout the day. The coffee shop opens at 5:00am and the number of customers increase slowly at first and increase more and more until reaching a maximum number of customers for the morning at 8:00 am. The number of customers slowly decrease until 9:30 when they drop significantly and then remain steady until 11:00 am when the lunch crowd begins to show. Similar to the morning, the number of customers increase slowly and then begin to increase more and more. The maximum customers is less at lunch than breakfast and is largest at 12:20pm. The smallest number of customers since opening occurs at 2:00 pm. There is a third spike in customers around 5:00 pm and then a late night crowd around 9:00 pm before closing at 10:00 pm. Sketch a graph that would model the number of customers at the coffee shop during the day.

Examples from Algebra I:

- A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet.
  - What is a reasonable domain restriction for t in this context?
  - Determine the height of the rocket two seconds after it was launched.
  - Determine the maximum height obtained by the rocket.
  - Determine the time when the rocket is 100 feet above the

		ground. o Determine the time at which the rocket hits the ground. o How would you refine your answer to the first question based on your response to the second and fifth questions? • Marla was at the zoo with her mom. When they stopped to view the lions, Marla ran away from the lion exhibit, stopped, and walked slowly towards the lion exhibit until she was halfway, stood still for a minute then walked away with her mom. Sketch a graph of Marla's distance from the lions' exhibit over the period of time when she arrived until she left. • A *relative minimum* for the function f occurs at the x-coordinate of $\left(\frac{2}{3}\sqrt{3}, -\frac{16}{9}\sqrt{3}\right)$. A similar calculation as you did above shows that this point is also a solution to $y = f(x)$. Plot this point on your graph. *Answer: Students should plot the point* $\left(\frac{2}{3}\sqrt{3}, -\frac{16}{9}\sqrt{3}\right)$ *on their graphs approximately at* $(1.15, -3.08)$. Look at your graph. On what interval(s) is the function f decreasing? *Answer:* $-\frac{2}{3}\sqrt{3} \le x \le \frac{2}{3}\sqrt{3}$ *or* $[-\frac{2}{3}\sqrt{3}, \frac{2}{3}\sqrt{3}]$. Look at your graph. On what interval(s) is the function f increasing? *Answer:* $x \le -\frac{2}{3}\sqrt{3}$ *or* $\frac{2}{3}\sqrt{3} \le x$ *or* $\left(-\infty, -\frac{2}{3}\sqrt{3}\right]$ *or* $\left[\frac{2}{3}\sqrt{3}, \infty\right)$.	
F.IF.C.7c	**C. Analyze functions using different representation** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and	**Explanation:** In Algebra II, students graph polynomial functions, exponential functions (Unit 2), and logarithmic functions (Unit 2), in addition to other functions types learned in previous courses. **Examples:** • Graph $g(x) = x^3 + 5x^2 + 2x - 8$ o Identify the zeroes o Discuss the end behavior	**Eureka Math:** Module 1 Lesson 14, 15, 17, 19, 20, 21

	showing end behavior. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	o In what intervals is the function increasing? Decreasing? Students explore the end behavior of a polynomial and develop ideas about the impact of the leading coefficient on the output values as the input values increase. **Examples:** • Many computer applications use very complex mathematical algorithms. The faster the algorithm, the more smoothly the programs run. The running time of an algorithm depends on the total number of steps needed to complete the algorithm. For image processing, the running time of an algorithm increases as the size of the image increases. For an n-by-n image, algorithm 1 has running time given by $p(n) = n^3 + 3n + 1$ and algorithm 2 has running time given by $q(n) = 15n^2 + 5n + 4$ (measured in nanoseconds, or 10^{-9} seconds. a. Compute the running time for both algorithms for images of size 10-by-10 pixels and 100-by-100 pixels. b. Graph both running time polynomials in an appropriate window (or several windows if necessary). c. Which algorithm is more efficient? Explain your reasoning. (*illustrative mathematics*)	
MP.1	**Make sense of problems and persevere in solving them.**	Students discover the value of equating factored terms of a polynomial to zero as a means of solving equations involving polynomials.	**Eureka Math:** Module 1 Lesson 12 Module 1 Lesson 20
MP.2	**Reason abstractly and quantitatively.**	Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students also learn to make sense of remainders in polynomial long division problems.	**Eureka Math:** Module 1 Lesson 17
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.	**Eureka Math:** Module 1 Lesson 14, 15, 16, 17

		They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.	
MP.5	**Use appropriate tools strategically.**	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to use technological tools to explore and deepen their understanding of concepts.	**Eureka Math:** Module 1 Lesson 14 Module 1 Lesson 21
MP.7	**Look for and make use of structure.**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. Students recognize structure in the graphs of polynomials in factored form and develop refined techniques for graphing.	**Eureka Math:** Module 1 Lesson 12 Module 1 Lesson 13 Module 1 Lesson 14 Module 1 Lesson 18 Module 1 Lesson 20
MP.8	**Look for and express regularity in repeated reasoning.**	Students understand that polynomials form a system analogous to the integers. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students recognize factors of expressions and develop factoring techniques.	**Eureka Math:** Module 1 Lesson 15 Module 1 Lesson 19

Algebra II Semester 1 (Quarter 1)

Unit 1: Polynomial, Rational, and Radical Relationships (45 days)

Topic C: Solving and Applying Equations – Polynomial, Rational and Radical (14 instructional days)

In Topic C, students continue to build upon the reasoning used to solve equations and their fluency in factoring polynomial expressions. In Lesson 22, students expand their understanding of the division of polynomial expressions to rewriting simple rational expressions (**A-APR.D.6**) in equivalent forms. In Lesson 23, students learn techniques for comparing rational expressions numerically, graphically, and algebraically. The practice of rewriting rational expressions in equivalent forms in Lessons 22–25 is carried over to solving rational equations in Lessons 26 and 27 (**In regular Algebra II, the rational expressions are limited to simple expressions in the form** $\frac{a(x)}{b(x)} = \frac{c(x)}{d(x)}$ or $\frac{a(x)}{b(x)} = c(x)$).Lesson 27 also includes working with word problems that require the use of rational equations. In Lessons 28–29, we turn to radical equations. Students learn to look for extraneous solutions to these equations as they did for rational equations.

In Lessons 30–32, students solve and graph systems of equations including systems of one linear equation and one quadratic equation and systems of two quadratic equations. Next, in Lessons 33–35, students study the definition of a parabola as they first learn to derive the equation of a parabola given a focus and a directrix and later to create the equation of the parabola in vertex form from the coordinates of the vertex and the location of either the focus or directrix. Students build upon their understanding of rotations and translations from Geometry as they learn that any given parabola is congruent to the one given by the equation $y = ax^2$ for some value of a and that all parabolas are similar.

Big Idea:	• Systems of non-linear functions create solutions more complex than those of systems of linear functions. • Mathematicians use the focus and directrix of a parabola to derive an equation.
Essential Questions:	• How do you reduce a rational expression to lowest terms? • How do you compare the values of rational expressions? • Why is it important to check the solutions of a rational or radical equation? • Why are solving systems of nonlinear functions different than systems of linear functions? • What does the focus and directrix define a parabola? • What conditions will two parabolas be congruent?
Vocabulary	Rational expression, complex fraction, equating numerators method, equating fractions method, extraneous solution, linear systems, parabola, axis of symmetry of a parabola, vertex of a parabola, paraboloid, focus, directrix, conic sections, eccentricity, vertical scaling, horizontal scaling, dilation
Assessments	Galileo: Topic C Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
A.APR.D.6	**D. Rewrite rational expressions** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long	**Explanation:** Students define rational expressions and determine the best method of simplifying a given rational expression. Students rewrite rational expressions in the form of $\frac{a(x)}{b(x)}$, in the form $q(x) + \frac{r(x)}{b(x)}$ by using inspection (factoring) or long division. The polynomial $q(x)$ is called	**Eureka Math:** Module 1 Lesson 22 – 23, 26,27

	division, or, for the more complicated examples, a computer algebra system.	the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. **Examples:** • Express $\frac{-x^2+4x+87}{x+1}$ in the form $(x)+\frac{r(x)}{b(x)}$. • Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. Students determine the best method of simplifying a given rational expression. • Simplify $\frac{x^2+9x+14}{x+7}$ $$\begin{array}{r} x+2 \\ x+7\overline{)x^2+9x+14} \\ -x^2-7x \\ \hline 2x+14 \\ -2x-14 \\ \hline 0 \end{array}$$ • Simplify (using inspection): o $\frac{6x^3+15x^2+12x}{3x}$ o $\frac{x^2+9x+14}{x+7}$ • Simplify (using long division): o $\frac{x^2+3x}{x^2-4}$	

		o $\frac{x^3+7x^2+13x+6}{x+4}$ ***Note:*** The use of synthetic division may be introduced as a method but students should recognize its limitations (division by a linear term). When students use methods that have not been developed conceptually, they often create misconceptions and make procedural mistakes due to a lack of understanding as to why the method is valid. They also lack the understanding to modify or adapt the method when faced with new and unfamiliar situations.	
A.REI.A.1	**A. Understand solving equations as a process of reasoning and explain the reasoning** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks were limited to linear and quadratic equations. In Algebra II, tasks are limited to simple rational or radical equations. Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions. When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method. **Examples:** • Explain why the equation $x/2 + 7/3 = 5$ has the same solutions as the equation $3x + 14 = 30$. Does this mean that $x/2 + 7/3$ is equal to $3x + 14$? • Show that $x = 2$ and $x = -3$ are solutions to the equation $x^2 + x = 6$. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning. • Prove $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$. Justify each step. • Explain each step in solving the quadratic equation	**Eureka Math:** Module 1 Lesson 26 -29 **Note: Only include simple rational equations in Lessons 26 and 27.**

		$x^2 + 10x = -7$. $$x^2 + 10x = -7$$ $$x^2 - 10x + 25 = 18$$ $$(x-5)^2 = 18$$ $$x - 5 = \pm 3\sqrt{2}$$ $$x = 5 \pm 3\sqrt{2}$$ • Explain the steps involved in solving the following: a. $5(2^t) + 20 = 60$ $5(2^t) = 40$ $(2^t) = 8$ $2^t = 2^3$ $t = 3$ • The rational equation has been solved using two different methods. Solve the following equation: $\frac{x+3}{12} = \frac{5}{6}$. *Equating Numerators Method: Obtain expressions on both sides with the same denominator and equate numerators.* $$\frac{x+3}{12} = \frac{5}{6} \cdot \frac{2}{2}$$ $$\frac{x+3}{12} = \frac{10}{12}$$ *Thus, $x + 3 = 10$, and $x = 7$; therefore, 7 is the solution to our original equation.* *Clearing Fractions Method: Multiply both sides by a common multiple of the denominators to clear the fractions, and then solve the resulting equation.* $$12 \cdot \left(\frac{x+3}{12}\right) = 12 \cdot \left(\frac{5}{6}\right)$$ $$x + 3 = 10$$ *We can see, once again, that the solution is 7.*	
A.REI.A.2 ●	**A. Understand solving equations as a process of reasoning and explain the reasoning** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**Explanation:** Students should be proficient with solving simple rational and radical equations that do not have extraneous solutions before moving on to equations that result in quadratics and possible solutions that need to be eliminated. It is very important that students are able to reason how and why extraneous solutions arise. The square root symbol (like all even roots) is defined to be the positive square root, so a positive root can never be equal to a negative	**Eureka Math:** Module 1 Lesson 26-29 **Note: Only include simple rational equations in Lessons 26 and 27.**

number. Squaring both sides of the equation will make that discrepancy disappear; the square of a positive number is positive but so is the square of a negative number, so we'll end up with a solution to the new equation even though there was no solution to the original equation. This is not the case with odd roots – a cube root of a positive number is positive, and a cube root of a negative number is negative. When we cube both sides of the last equation, the negative remains, and we end up with a true solution to the equation.

Examples:

- Solve $5 - \sqrt{-(x-4)} = 2$ for *x*.
- Mary solved $x = \sqrt{2-x}$ for *x* and got *x* = -2 and *x* = 1. Evaluate her solutions and determine if she is correct. Explain your reasoning.
- When raising both sides of an equation to a power we sometimes obtain an equation which has more solutions than the original one. (Sometimes the extra solutions are called extraneous solutions.) Which of the following equations result in extraneous solutions when you raise both sides to the indicated power? Explain.
    a. $\sqrt{x} = 5$, Square both sides
    b. $\sqrt{x} = -5$, Square both sides
    c. $\sqrt[3]{x} = 5$, Cube both sides
    d. $\sqrt[3]{x} = -5$, Cube both sides
- Create a square root equation that when solved algebraically introduces an extraneous solution. Show the algebraic steps you would follow to look for a solution and indicate where the extraneous solution arises.

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Sample solution: $\sqrt{2x+1}+7=2$
$$\begin{aligned}\sqrt{2x+1}+7&=2\\ \left(\sqrt{2x+1}\right)^2&=(-5)^2\\ 2x+1&=25\\ 2x&=24\\ x&=12\end{aligned}$$
This is where the extraneous solution comes in. The square root can't be negative, but by squaring both sides, we're losing that information.
Solve $\frac{2x-8}{x-4}=4$
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<td>A.REI.B.4b
▲</td>
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B. Solve equations and inequalities in one variable
Solve quadratic equations in one variable.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.
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Explanation:
Part b of this standard is taught in Algebra I and Algebra II. In Algebra I, tasks did not require students to write solutions for quadratic equations that had roots with nonzero imaginary parts. However, tasks did require that students recognize cases in which a quadratic equation had no real solutions. In Algebra II, tasks include equations having roots with nonzero imaginary parts. Students write the solutions as $a \pm bi$ where a and b are real numbers. In Topic C, students will only be using the content required in Algebra I. The complex solutions will be explored in Topic D of this unit.
Examples from Algebra I:
Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.
<table><tr><th>Value of</th><th>Nature of</th><th>Nature of Graph</th></tr></table>
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Eureka Math:
Module 1 Lesson 31
This standard is revisited in Topic D
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<tr><th>Discriminant</th><th>Roots</th><th></th></tr>
<tr><td>$b^2 - 4ac = 0$</td><td>1 real roots</td><td>intersects x-axis once</td></tr>
<tr><td>$b^2 - 4ac > 0$</td><td>2 real roots</td><td>intersects x-axis twice</td></tr>
<tr><td>$b^2 - 4ac < 0$</td><td>2 complex roots</td><td>does not intersect x-axis</td></tr>
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Examples:

Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have?
What is the nature of the roots of $x^2 + 6x - 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?
Elegant ways to solve quadratic equations by factoring for those involving expressions of the form: ax^2 and $a(x-b)^2$
$3x^2 - 9 = 0$
$3x^2 = 9 \rightarrow x^2 = 3 \rightarrow x = \pm\sqrt{3}$

$(x-3)^2 = 1$
$(x-3) = \pm 1 \rightarrow x = 3 \pm 1 \rightarrow x = 2 \text{ or } 4$

Example: Ryan used the quadratic formula to solve an equation and his result was $x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-2)}}{2(1)}$.
a. Write the quadratic equation Ryan started with.
b. Simplify the expression to find the solutions.
c. What are the x-intercepts of the graph of the corresponding quadratic function?
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<td>A.REI.C.6</td>
<td>C. Solve systems of equations

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</td>
<td>Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I tasks were limited to <u>pairs</u> of linear equations in two variables. In Algebra II, systems of three linear equations in three variables are introduced.

Examples:
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<td>Eureka Math:
Module 1 Lesson 30-31</td>
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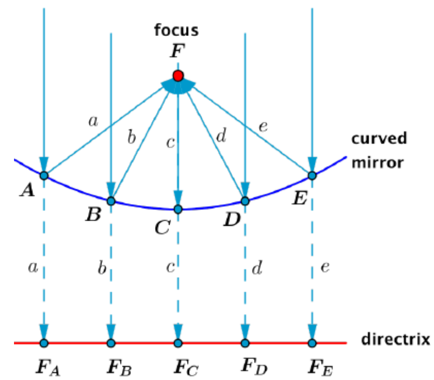
		Determine the values for x, y, and z in the following system: $2x + 3y - z = 5$ (1) $4x - y - z = -1$ (2) $x + 4y + z = 12$ (3) • Given the system below, determine the values of r, s, and u that satisfy all three equations. $r + 2s - u = 8$ $s + u = 4$ $r - s - u = 2$ *Adding the second and third equation together produces the equation $r = 6$. Substituting this into the first equation and adding it to the second gives $6 + 3s = 12$, so that $s = 2$. Replacing s with 2 in the second equation gives $u = 2$. The solution to this system of equations is $(6, 2, 2)$.* • Find the equation of the form $y = ax^2 + bx + c$ that satisfies the points $(1, 6)$, $(3, 20)$, and $(-2, 15)$. *$a = 2$, $b = -1$, $c = 5$; therefore, the quadratic equation is $y = 2x^2 - x + 5$.*	
A.REI.C.7 ●	**C. Solve systems of equations** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.	**Explanation:** Students solve a system containing a linear equation and a quadratic equation in two-variables. Students solve graphically and algebraically. *Note: Quadratics may include conic sections such as a circle.* **Examples:** • Solve $\begin{cases} y = x^2 - x - 6 \\ 2x - y = 2 \end{cases}$ both graphically and algebraically • Describe the possible number of solutions of a linear and quadratic system. Illustrate the possible number of solutions with graphs.	**Eureka Math:** Module 1 Lesson 31-32
G.GPE.A.2 ▲	**A. Translate between the geometric description and the equation for a conic section** Derive the equation of a parabola given a focus and directrix.	**Explanations:** Students have used parabolas to represent *y* as a funciton of *x*. This standard intorduces the parabola as a geometric figure that is the set of all points an equal distrance from a fixed point (focus) and a fixed	**Eureka Math:** Module 1 Lesson 33-34

line (directrix). Students derived the equation of a parabola given the focus and directrix.

Students may derive the equation by starting with a horizontal directrix and a focus on the y-axis, and use the distance formula to obtain an equation of the resulting parabola in terms of y and x^2. Next, they use a vertical directrix and a focus on the x-axis to obtain an equation of a parabola in terms of x and y^2. Make generalizations in which the focus may be any point, but the directrix is still either horizontal or vertical. Students may use the generalization sin their future work. Allow sufficient tiem for stuents to become familiar with new vocabulary and notation.



Examples:

- Given a focus and a directrix, create an equation for a parabola.
  - Focus: $\boldsymbol{F}=(\mathbf{0},\mathbf{2})$
  - Directrix: $\boldsymbol{x}$-axis

Parabola: $\boldsymbol{P}=\{(\boldsymbol{x},\boldsymbol{y})\mid (\boldsymbol{x},\boldsymbol{y})$ is equidistant to *F* and to the $\boldsymbol{x}$-axis.}

Let $\boldsymbol{A}$ be any point $(\boldsymbol{x},\ \boldsymbol{y})$ on the parabola $\boldsymbol{P}$. Let $\boldsymbol{F'}$ be a point on the directrix with the same $\boldsymbol{x}$-coordinate as point $\boldsymbol{A}$.

What is the length of $\boldsymbol{A}$F'? $\boldsymbol{AF'}=\boldsymbol{y}$

Use the distance formula to create an expression that represents the length of AF. $AF = \sqrt{(x-0)2 + (y-2)2}$

Create an equation that relates the two lengths and solve it for y.

Therefore,	$P = \left\{(x,y) \mid \sqrt{(x-0)^2 + (y-2)^2} = y\right\}$.
The two segments have equal lengths.	$AF' = AF$
The length of each segment.	$y = \sqrt{(x-0)^2 + (y-2)^2}$
Square both sides of the equation.	$y^2 = x^2 + (y-2)^2$
Expand the binomial.	$y^2 = x^2 + y^2 - 4y + 4$
Solve for y.	$4y = x^2 + 4$
	$y = \frac{1}{4}x^2 + 1$

Replacing this equation in the definition of $P = \{(x,y) \mid (x,y)$ is equidistant to F and to the x-axis$\}$ gives the statement $P = \left\{(x,y) \mid y = \frac{1}{4}x^2 + 1\right\}$.

Thus, the parabola P is the graph of the equation $y = \frac{1}{4}x^2 + 1$.

Verify that this equation appears to match the graph shown.

- Write and graph an equation for a parabola with focus (2,3) and directrix y=1.
- A parabola has focus (-2,1) and directrix y=-3. Determine whether or not the point (2,1) is part of the parabola. Justify your answer.
- Given the equation $20(y-5) = (x+3)^2$, find the focus, vertex and directrix.
- Identify the focus and directrix of the parabola given by $y^2=-4x$
- Identify the focus and directrix of the parabola given by $x^2 = 12y$
- Write the standard form of the equation of the parabola

		with its vertex at (0,0) and focus at (0,-4) • Write the standard form of the equation of the parabola with its vertex at (0,0) and directrix y=5. • Write the standard form of the equation of the parabola with its vertex at (0,0) and directrix x=2.	
MP.1	**Make sense of problems and persevere in solving them.**	Students solve systems of linear equations and linear and quadratic pairs in two variables. Further, students come to understand that the complex number system provides solutions to the equation $x^2 + 1 = 0$ and higher-degree equations.	**Eureka Math:** Module 1 Lesson 26-27 Module 1 Lesson 29-31 Module 1 Lesson 33
MP.2	**Reason abstractly and quantitatively.**	Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students also learn to make sense of remainders in polynomial long division problems.	**Eureka Math:** Module 1 Lesson 23 Module 1 Lesson 27 Module 1 Lesson 34
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.	**Eureka Math:** Module 1 Lesson 23 Module 1 Lesson 28-29 Module 1 Lesson 34-35
MP.4	**Model with mathematics.**	Students use primes to model encryption. Students transition between verbal, numerical, algebraic, and graphical thinking in analyzing applied polynomial problems. Students model a cross-section of a riverbed with a polynomial, estimate fluid flow with their algebraic model, and fit polynomials to data. Students model the locus of points at equal distance between a point (focus) and a line (directrix) discovering the parabola.	**Eureka Math:** Module 1 Lesson 27 Module 1 Lesson 33
MP.5	**Use appropriate tools strategically.**	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of	**Eureka Math:** Module 1 Lesson 31

		functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge.	
MP.7	**Look for and make use of structure.**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. Students recognize structure in the graphs of polynomials in factored form and develop refined techniques for graphing. Students discern the structure of rational expressions by comparing to analogous arithmetic problems. Students perform geometric operations on parabolas to discover congruence and similarity.	**Eureka Math:** Module 1 Lesson 22 Module 1 Lesson 24-26 Module 1 Lesson 28-30 Module 1 Lesson 34
MP.8	**Look for and express regularity in repeated reasoning.**	Students understand that polynomials form a system analogous to the integers. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students recognize factors of expressions and develop factoring techniques. Further, students understand that all quadratics can be written as a product of linear factors in the complex realm.	**Eureka Math:** Module 1 Lesson 22 Module 1 Lesson 31

Algebra II Semester 1 (Quarter 2)

Unit 1: Polynomial, Rational, and Radical Relationships (45 days)

Topic D: A Surprise from Geometry-Complex Numbers Overcome All Obstacles (5 instructional days)

In Topic D, students extend their facility with finding zeros of polynomials to include complex zeros. Lesson 36 presents a third obstacle to using factors of polynomials to solve polynomial equations. Students begin by solving systems of linear and non-linear equations to which no real solutions exist, and then relate this to the possibility of quadratic equations with no real solutions. Lesson 37 introduces complex numbers through their relationship to geometric transformations. That is, students observe that scaling all numbers on a number line by a factor of −1 turns the number line out of its one-dimensionality and rotates it 180° through the plane. They then answer the question, “What scale factor could be used to create a rotation of 90°?” In Lesson 38, students discover that complex numbers have real uses; in fact, they can be used in finding real solutions of polynomial equations. In Lesson 39, students develop facility with properties and operations of complex numbers.

Big Idea:	• Every polynomial can be rewritten as the product of linear factors. • The properties of the real number system extend to the complex number system.
Essential Questions:	• Are real numbers complex numbers? Explain. • What are the subsets of the set of complex numbers? • What do imaginary numbers represent?
Vocabulary	Complex numbers, imaginary, discriminant, conjugate pairs, [Fundamental Theorem of Algebra (Honors only)]
Assessments	Galileo: Topic D Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Comments
N.CN.A.1	**A. Perform arithmetic operations with complex numbers** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real.	**Explanation:** Students will review the structure of the complex number system realizing that every number is a complex number that can be written in the form *a* + *bi* where *a* and *b* are real numbers. If *a* = 0, then the number is a pure imaginary number however when *b* = 0 the number is a real number. Real numbers are complex numbers; the real number *a* can be written as the complex number *a* + 0*i*. The square root of a negative number is a complex number. Multiplying by *i* rotates every complex number in the complex plane by 90˚ about the origin.	**Eureka Math:** Module 1 Lesson 37

	Problem	Solution	*bi* Form
1.	$\sqrt{-36}$	$\sqrt{-36} = \sqrt{-1} \cdot \sqrt{36} = 6i$	$6i$
2.	$2\sqrt{-49}$	$2\sqrt{-49} = 2\sqrt{-1} \cdot \sqrt{49} = 2 \cdot 7i = 14i$	$14i$
3.	$-3\sqrt{-10}$	$-3\sqrt{-10} = -3\sqrt{-1} \cdot \sqrt{10} = -3 \cdot i \cdot \sqrt{10} = -3i\sqrt{10}$	$-3i\sqrt{10}$
4.	$5\sqrt{-8}$	$5\sqrt{-8} = 5\sqrt{-1} \cdot \sqrt{8} = 5 \cdot i \cdot 2\sqrt{2} = 10i\sqrt{2}$	$10i\sqrt{2}$

		Examples: • Explore the powers of i and apply a pattern to simplify i^{126}. • **Express each of the following in $a + bi$ form.** a. i^5 — $0 + i$ b. i^6 — $-1 + 0i$ c. i^7 — $0 - i$ d. i^8 — $1 + 0i$ e. i^{102} — $-1 + 0i$	
N.CN.A.2	**A. Perform arithmetic operations with complex numbers** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	**Explanation:** Students recognize the relationships between different number sets and their properties. The complex number system possesses the same basic properties as the real number system: that addition and multiplication are commutative and associative; the existence of an additive identity and a multiplicative identity; the existence of an additive inverse of every complex number and the existence of a multiplicative inverse or reciprocal for every non-zero complex number; and the distributive property of multiplication over the addition. An awareness of the properties minimizes students' rote memorization and links the rules for manipulations with the complex number system to the rules for manipulations with binomials with real coefficients of the form $a + bx$. The commutative, associative and distributive properties hold true when adding, subtracting, and multiplying complex numbers. Addition and subtraction with complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$ Multiplication with complex numbers $(a + bi) \cdot (c + di) = ac + bci + adi + bd\,i^2$ $= (ac - bd) + (bc + ad)i$ **Examples:**	**Eureka Math:** Module 1 Lesson 37

		• **Example 1: Addition with Complex Numbers** Compute $(3 + 4i) + (7 - 20i)$. $(3 + 4i) + (7 - 20i) = 3 + 4i + 7 - 20i = (3 + 7) + (4 - 20)i = 10 - 16i$ **Example 2: Subtraction with Complex Numbers** Compute $(3 + 4i) - (7 - 20i)$. $(3 + 4i) - (7 - 20i) = 3 + 4i - 7 + 20i = (3 - 7) + (4 + 20)i = -4 + 20i$ • Verify that $-1 + 2i$ and $-1 - 2i$ are solutions to $x^2 + 2x + 5 = 0$. $-1 + 2i$: $(-1 + 2i)^2 + 2(-1 + 2i) + 5 = 1 - 4i + 4i^2 - 2 + 4i + 5$ $= 4i^2 - 4i + 4i + 1 - 2 + 5$ $= -4 + 0 + 4$ $= 0$ $-1 - 2i$: $(-1 - 2i)^2 + 2(-1 - 2i) + 5 = 1 + 4i + 4i^2 - 2 - 4i + 5$ $= 4i^2 + 4i - 4i + 1 - 2 + 5$ $= -4 + 0 + 4$ $= 0$ • Simplify the following expression. Justify each step using the commutative, associative and distributive properties. $(3 - 2i)(-7 + 4i)$	

		Solutions may vary; one solution follows: $(3-2i)(-7+4i)$ $3(-7+4i)-2i(-7+4i)$ Distributive Property $-21+12i+14i-8i^2$ Distributive Property $-21+(12i+14i)-8i^2$ Associative Property $-21+i(12+14)-8i^2$ Distributive Property $-21+26i-8i^2$ Computation $-21+26i-8(-1)$ $i^2=-1$ $-21+26i+8$ Computation $-21+8+26i$ Commutative Property $-13+26i$ Computation	

N.CN.C.7	**C. Use complex numbers in polynomial identities and equations**
	
	Solve quadratic equations with real coefficients that have complex solutions.
		Explanation:
		Students solve quadratic equations with real coefficients that have solutions of the form *a* + *bi* and *a* – *bi*. They determine when a quadratic equation in standard form, $ax^2+bx+c=0$, has complex roots by looking at a graph of $f(x)=ax^2+bx+c$ or by calculating the discriminant.
		
		Examples:
		• Use the quadratic formula to write quadratic equations with the following solutions:
		    o One real number solution
		    o Solutions that are complex numbers in the form *a* + *bi*, *a* ≠ 0 and *b* ≠ 0.
		    o Solutions that are imaginary numbers *bi*.
		• Within which number system can $x^2 = -2$ be solved? Explain how you know.
		• Solve $x^2 + 2x + 2 = 0$ over the complex numbers.
		• Find all solutions of $2x^2 + 5 = 2x$ and express them in the form *a* + *bi*.
		• Given the quadratic equation $ax^2+bx+c=0$ that has a solution of 2 + 3*i*, determine possible values for *a*, *b* and *c*.
			Eureka Math:
			Module 1 Lesson 37-39
			
			Note: In lesson 39, do not include exercises 1-3. That standard addresses standard N.CN.C.8 which is an extended standard in Honors Algebra II.

		Are there other combinations possible? Explain. • Write a polynomial P with the lowest possible degree that has the given solutions. Explain how you generated each answer. a. $-2, 3, -4i, 4i$ *The polynomial P has two real zeroes and two complex zeros. Since the two complex zeros are members of a conjugate pair, P may have as few as four total factors. Therefore, P has degree at least 4.* $P(x) = (x+2)(x-3)(x+4i)(x-4i)$ $= (x^2 - x - 6)(x^2 - 16i^2)$ $= (x^2 - x - 6)(x^2 + 16)$ $= x^4 - x^3 - 6x^2 + 16x^2 - 16x - 96$ $= x^4 - x^3 + 10x^2 - 16x - 96$	
A.REI.B.4b	**B. Solve equations and inequalities in one variable** Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and **write them as a ± bi for real numbers a and b.**	**Explanation:** Part ***b*** of this standard is taught in Algebra I and Algebra II. In Algebra I, tasks did not require students to write solutions for quadratic equations that had roots with nonzero imaginary parts. However, tasks did require that students recognize cases in which a quadratic equation had no real solutions. In Algebra II, tasks include equations having roots with nonzero imaginary parts. Students write the solutions as $a \pm bi$ where a and b are real numbers. **Examples:** • Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation. • What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related? • How does the value of the discriminant relate the number of solutions to a quadratic equation? ***If the discriminant is negative, we get complex solutions. If the discriminant is zero, we get one real solution. If the***	**Eureka Math:** Module 1 Lesson 38

		discriminant is positive, we get two real solutions. • Consider the equation $3x + x^2 = -7$. ▪ What does the value of the discriminant tell us about number of solutions to this equation? ▫ The equation in standard form is $x^2 + 3x + 7 = 0$. ▫ $a = 1, b = 3, c = 7$ ▫ The discriminant is $3^2 - 4(1)(7) = -19$. The negative discriminant indicates that no real solutions exist. There are two complex solutions. ▪ Solve the equation. Does the number of solutions match the information provided by the discriminant? Explain. ▫ *Using the quadratic formula,* $x = \frac{-3+\sqrt{-19}}{2}$ *or* $x = \frac{-3-\sqrt{-19}}{2}$. ▫ *The solutions, in $a + bi$ form, are* $-\frac{3}{2} + \frac{\sqrt{19}}{2}i$ *and* $-\frac{3}{2} - \frac{\sqrt{19}}{2}i$. ▫ *The two complex solutions are consistent with the rule for a negative discriminant.*	
A.REI.C.7 ●	**C. Solve systems of equations** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.	**Explanation:** Students solve a system containing a linear equation and a quadratic equation in two-variables. Students solve graphically and algebraically. *Note: Quadratics may include conic sections such as a circle.* In Geometry, students used completing the square to put an equation in standard form in order to find the center and radius of a circle. (**G-GPE.A.1**) **Examples:** • Solve $\begin{cases} y = x^2 - x - 6 \\ 2x - y = 2 \end{cases}$ both graphically and algebraically • Describe the possible number of solutions of a linear and quadratic system. Illustrate the possible number of solutions with graphs. • Does the line y = -x intersect the circle $x^2 + y^2 = 1$? If so, how	**Eureka Math:** Module 1 Lesson 36-38

		many times and where? Draw graphs on the same set of axis. • Solve the following system of equations algebraically. Confirm your answer graphically. $3x^2 + 3y^2 = 6$ $x - y = 3$	
MP.2	**Reason abstractly and quantitatively.**	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	**Eureka Math:** Module 1 Lesson 37-38
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.	**Eureka Math:** Module 1 Lesson 36 Module 1 Lesson 38
MP.7	**Look for and make use of structure.**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. Students recognize structure in the graphs of polynomials in factored form and develop refined techniques for graphing. Students discern the structure of rational expressions by comparing to analogous arithmetic problems. Students perform geometric operations on parabolas to discover	**Eureka Math:** Module 1 Lesson 37 Module 1 Lesson 39

		congruence and similarity.	

Algebra II Semester 1 (Quarter 2)

Unit 2: Exponential and Logarithmic Functions (45 days)

Topic A: Real Numbers (6 Instructional Days)

In Topic A, students prepare to generalize what they know about various function families by examining the behavior of exponential functions. One goal of the module is to show that the domain of the exponential function, $(x) = bx$, where b is a positive number not equal to 1, is all real numbers.

Big Idea:	• Real world situations can be modeled with exponential functions.
Essential Questions:	• How can the properties of exponents help us to rewrite expressions? • Why are the properties of exponents useful when working with large or small numbers? • What are some characteristics of the graph of an exponential function? • How can you determine if an exponential function represents exponential growth or exponential decay? • When are exponential models appropriate for two-variable data?
Vocabulary	Properties of exponents, leading digit, scientific notation, order of magnitude, n^{th} root of a number, principal n^{th} root of a number, Euler's number, e, average rate of change
Assessment	Galileo: Module 3 Pre-assessment of Foundation Skills; Topic A Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Comments
N.RN.A.1	**A. Extend the properties of exponents to rational exponents** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	**Explanation:** Students were first introduced to exponents in 6^{th} grade by writing and evaluating expressions containing *integer* exponents (6.EE.A.1). In 8^{th} grade, students applied the properties of *integer* exponents to generate equivalent expressions (8.EE.A.1). In Algebra I, students continued their work with *integer* exponents as they applied the properties when multiplying polynomials (A-APR.A.1) and used the properties to transform expressions for exponential functions (A-SSE.B.3c). In Algebra II, students build on their previous understanding of integer exponents to understanding ***rational*** exponents. Students may explain orally or in written format how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	**Eureka Math** Module 3 Lesson 3,4,5

The meaning of an exponent relates the frequency with which a number is used as a factor. So 5^3 indicates that product where 5 is a factor 3 times. Extend this meaning to a rational exponent, then $125^{\frac{1}{3}}$ indicates one of three equal factors whose product is 125.

Students recognize that a fractional exponent can be expressed as a radical or root. For example, $\frac{1}{3}$ is equivalent to a cube root; and exponent of $\frac{1}{4}$ is equivalent to a fourth root.

Students extend the use of the power rule, $(b^n)^m = b^{nm}$ from whole number exponents to rational exponents. They compare examples such as $\left(7^{\frac{1}{2}}\right)^2 = 7^{\frac{1}{2}*2} = 7^1 = 7$ to $\left(\sqrt{7}\right)^2 = 7$ to establish a connection between radicals and rational exponents: $7^{\frac{1}{2}} = \sqrt{7}$ and , in general, $b^{\frac{1}{2}} = \sqrt{b}$.

Examples:

- Determine the value of x
  - $64^{\frac{1}{2}} = 8^x$
  - $(12^5)^x = 12$
- A biology student was studying bacterial growth. The population of bacterial doubled every hour as indicated in the following table:

# of hours of observation	0	1	2	3	4
Number of bacteria cells (thousands)	4	8	16	32	64

How could the student predict the number of bacteria every half hour? Every 20 minutes?

If every hour the number of bacteria cells is being multiplied by a factor of 2, then on the half hour the number of cells is increasing by a factor of $2^{\frac{1}{2}}$. For every 20 minutes, the number of cells is increasing by a factor of $2^{\frac{1}{3}}$.

		• Provide a written explanation for each question below. a. Is it true that $\left(4^{\frac{1}{2}}\right)^3 = (4^3)^{\frac{1}{2}}$? Explain how you know. $\left(4^{\frac{1}{2}}\right)^3 = \left(\sqrt{4}\right)^3 = 2^3 = 8$ $(4^3)^{\frac{1}{2}} = 64^{\frac{1}{2}} = \sqrt{64} = 8$ *So the first statement is true.* b. Is it true that $\left(1000^{\frac{1}{3}}\right)^3 = (1000^3)^{\frac{1}{3}}$? Explain how you know. *Similarly the left and right sides of the second statement are equal to one another.* $\left(1000^{\frac{1}{3}}\right)^3 = \left(\sqrt[3]{1000}\right)^3 = 10^3 = 1000$ $(1000^3)^{\frac{1}{3}} = (1000000000)^{\frac{1}{3}} = 1000$	
N.RN.A.2 ●	**A. Extend the properties of exponents to rational exponents** Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**Explanation:** The foundation for this standard was set in 8th grade where students used the properties of integer exponents to rewrite equivalent expessions (**8.EE.A.1**). In Algebra II, students rewrite expressions involving radicals and rational exponents using the properties of exponents. Students rewrite expressions involving rational exponents as expressions involving radicals and simplify those expressions. **Examples:** • Rewrite the expression $8^{\frac{2}{3}}$ in exponential form. Explain how they are equivalent. $$8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = \left(8^{\frac{1}{3}}\right)^2$$ In the first expression, the base number is 8 and the	**Eureka Math** Module 3 Lesson 3-4 **Review from 8th Grade:** Module 3 Lesson 1-2

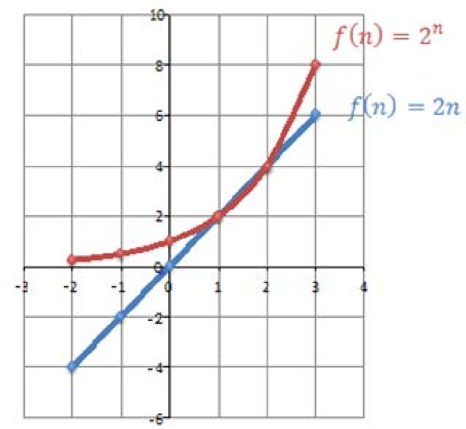
		exponent is *2/3*. This means that the expression represents *2* of the *3* equal factors whose product is *8*, thus the value is 4, since *2×2×2 = 8*; there are three factors of *2*; and two of these factors multiply to be 4. In the second expression, there are 2 equal factors of 8 or 64. The exponent *1/3* represents *1* of the *3* equal factors of *64*. Since *4×4×4 = 64* then one of the three factors is *4*. The last expression there is *1* of *3* equal factors of 8 which is 2 since *2×2×2 = 8*. Then there are *2* of the equal factors of *2*, which is 4. • Using the properties of exponents, simplify a. $\left(\sqrt[4]{32^3}\right)^2$ b. $\frac{\sqrt[5]{b^3}}{b^{\frac{4}{3}}}$ Students rewrite expression involving radicals as expressions using rational exponents and use the properties of exponents to simplify the expressions. **Examples:** • Given $81^{\frac{3}{4}} = \sqrt[4]{81^3} = \left(\sqrt[4]{81}\right)^3$, which form would be easiest to calculate without using a calculator. Why? • Determine whether each equation is true or false. Justify using the properties of exponents. a. $\sqrt{32} = 2^{\frac{5}{2}}$ d. $2^8 = \left(\sqrt[3]{16}\right)^6$ b. $16^{\frac{3}{2}} = 8^2$ e. $\left(\sqrt{64}\right)^{\frac{1}{3}} = 8^{\frac{1}{6}}$ c. $4^{\frac{1}{2}} = \sqrt[4]{64}$	
N.Q.A.2 ▲	**A. Reason qualitatively and units to solve problems** Define appropriate quantities for the purpose of	**Explanation:** Students define appropriate quantities for the purpose of describing a mathematical model in context. This standard is taught in Algebra I and Algebra II. In Algebra II, the	**Eureka Math** Module 3 Lesson 2

	descriptive modeling. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	standard will be assessed by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades/courses) require the student to <u>create</u> a quantity of interest in the situation being described (i.e. this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude. **Example:** Explain how the unit cm, cm^2, and cm^3 are relate. Describe situations where each would be an appropriate unit of measure.	
F.IF.B.6 ●	**B. Interpret functions that arise in applications in terms of context** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** Students were first introduced to the concept of rate of change in grade 6 and continued exploration of the concept throughout grades 7 and 8. This standard is taught in Algebra I and Algebra II. In Algebra I, students extended their knowledge from previous grades to non-linear functions (quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers). In Algebra II, tasks have a real-world context and involve polynomial, exponential, logarithmic and trigonometric functions. In this topic the focus will be on **exponential functions**. The average rate of change of a function $y = f(x)$ over an interval [a,b] is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation. **Examples:** • Let us understand the difference between $f(n) = 2n$ and $f(n) = 2^n$. Complete the tables below, and then graph the points	**Eureka Math** Module 3 Lesson 6 **This standard is revisited in Unit 2 Topic D.**

(*n,f(n)*) on a coordinate plane for each of the formulas.

n	$f(n) = 2n$
-2	-4
-1	-2
0	0
1	2
2	4
3	6

n	$f(n) = 2^n$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



Describe the change in each sequence when n increases by 1 unit for each sequence.

For the sequence $f(n) = 2n$, for every increase in n by 1 unit, the $f(n)$ value increases by 2 units. For the sequence $f(n) = 2^n$, for every increase in n by 1 unit, the $f(n)$ value increases by a factor of 2.

- In the table below, assume the function f is deifined for all real numbers. Calculate $\Delta f = f(x+1) - f(x)$ in the last column. What do you notice about Δf? Could the function be linear or

		exponential? Write a linear or exponential function formula that generates the same input-output pairs as given in the table. | x	$f(x)$	$\Delta f = f(x+1) - f(x)$ | |---|---|---| | 0 | 2 | $6-2=4$ | | 1 | 6 | $18-6=12$ | | 2 | 18 | $54-18=36$ | | 3 | 54 | $162-54=108$ | | 4 | 162 | | *If the entries in the table were considered as a geometric sequence, then the common quotient would be $r = 3$. Since $f(0) = 2$, $a = 2$. Since $f(1) = 6$, we must have $6 = 2 \cdot b$ or $b = 3$. Hence, $f(x) = 2(3)^x$.* In this table, students should see that Δf is not constant for any two inputs that have a difference of 1 unit, which implies that the function cannot be a linear function. However, there is a common quotient between inputs that have a difference of 1 unit: $\frac{6}{2} = \frac{18}{6} = \frac{54}{18} = \frac{162}{54}$. Hence the function f could be exponential. • How do the average rates of change help to support an argument of whether a linear or exponential model is better suited for a set of data? *If the model Δf was growing linearly, then the average rate of change would be constant. However, if it appears to be growing multiplicatively, then it indicates an exponential model.*	
F.BF.A.1a 🔴	**A. Build a function that models a relationship between two quantities** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra II, tasks have a real-world context and are limited to linear functions, quadratic functions, and exponential functions. Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically.	**Eureka Math** Module 3 Lesson 5,6 **This standard is revisited in Unit 2 Topics B, C and D.**

	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. **Examples:** • You buy a \$10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of \$250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • The radius of a circular oil slick after t hours is given in feet by $r=10t2-0.5t$, for $0 \le t \le 10$. Find the area of the oil slick as a function of time. • Suppose you deposit \$100 in a savings account that pays 4% interest, compounded annually. At the end of each year you deposit an additional \$50. Write a recursive function that models the amount of money in the account for any year.	

F.LE.A.2	**A. Construct and compare linear, quadratic, and exponential models and solve problems**
	
	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
	
	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.
		Explanation:
		This standard is taught in Algebra I and Algebra II. In Algebra II, tasks are limited to solving multi-step problems by constructing linear and exponential functions.
		
		Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.
		
		Examples:
		• After a record setting winter storm, there are 10 inches of snow on the ground! Now that the sun is finally out, the snow is melting. At 7 am there were 10 inches and at 12 pm there were 6 inches of snow.
		    o Construct a linear function rule to model the amount of snow.
		    o Construct an exponential function rule to model
			Eureka Math
			Module 3 Lesson 1
			
			This standard is revisited in Unit 2 Topic C.

the amount of snow.

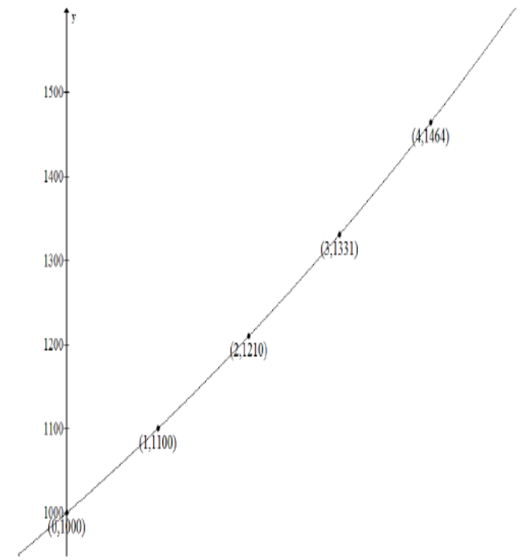
- Which model best describes the amount of snow? Provide reasoning for your choice.

Note: In order to write the exponential function as the amount of snow for every hour, connect to F.IF.8b. Students could start with $10(.6)^x$ *where x is the number of 5 hour periods then rewrite it to be* $10(.6)^{\left(\frac{1}{5}x\right)} = 10\left(.6^{\frac{1}{5}}\right)^x \approx 10(.9)^x$ *where x is the number of hours since 7am.*

▪

Lewis' dad put **1,000** dollars in a money market fund when he was **16**. Lewis can't touch the money until he is **26**, but he gets updates on the balance of his account.

Years After Lewis Turns 16	Account Balance in Dollars
0	1000
1	1100
2	1210
3	1331
4	1464

		a. Develop a model for this situation. *We might try graphing this data. However, in the viewing window that shows our data points (see graph below), it appears that the function might be linear. Let's try zooming out to see more of the key features of this graph (See graph below.)* x: [0, 5] y: [975, 1500]  *This viewing window gives us a close-up of the data points and their relation to each other. However, we cannot really see the features of the graph that represents the data.*	
MP.1	**Make sense of problems and persevere in solving them.**	Students make sense of rational and real number exponents and in doing so are able to apply exponential functions to solve problems involving exponential growth and decay for continuous domains such as time. They explore logarithms numerically and graphically to understand their meaning and how they can be used to solve exponential equations. Students have multiple opportunities to make connections between information presented graphically, numerically, and algebraically and search for similarities between these representations to further understand the underlying mathematical properties of exponents and logarithms. When presented with a wide variety of information related to financial planning, students make sense of the given information and use appropriate formulas to	**Eureka Math** Module 3 Lesson 1

		effectively plan for a long-term budget and savings plan.	
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.	**Eureka Math** Module 3 Lesson 1-4
MP.4	**Model with mathematics.**	Students use exponential functions to model situations involving exponential growth and decay. They model the number of digits needed to assign identifiers using logarithms. They model exponential growth using a simulation with collected data. The application of exponential functions and logarithms as a means to solve an exponential equation is a focus of several lessons that deal with financial literacy and planning a budget. Here, students must make sense of several different quantities and their relationships as they plan and prioritize for their future financial solvency.	**Eureka Math** Module 3 Lesson 1,6
MP.6	**Attend to precision.**	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	**Eureka Math** Module 3 Lesson 3
MP.7	**Look for and make use of structure.**	Students extend the laws of exponents for integer exponents to rational and real number exponents. They connect how these laws are related to the properties of logarithms and understand how to rearrange an exponential equation into logarithmic form. Students analyze the structure of exponential and logarithmic functions to	**Eureka Math** Module 3 Lesson 2,3,4

		understand how to sketch graphs and see how the properties relate to transformations of these types of functions. They analyze the structure of expressions to reveal properties such as recognizing when a function models exponential growth versus decay. Students use the structure of equations to understand how to identify an appropriate solution method.	
MP.8	**Look for and express regularity in repeated reasoning.**	Students discover the properties of logarithms and the meaning of a logarithm by investigating numeric examples. They develop formulas that involve exponentials and logarithms by extending patterns and examining tables and graphs. Students generalize transformations of graphs of logarithmic functions by examining several different cases.	**Eureka Math** Module 3 Lesson 3,4,5,6

Algebra II Semester 1 (Quarter 2)

Unit 2: Exponential and Logarithmic Functions (45 days)

Topic B: Logarithms (9 Instructional Days)

The lessons covered in Topic A familiarize students with the laws and properties of real-valued exponents. In Topic B, students extend their work with exponential functions to include solving exponential equations numerically and to develop an understanding of the relationship between logarithms and exponentials.

Big Idea:	• Logarithms can be used to solve the exponential equations modeling many real-life situations. • Logarithmic equations can be solved graphically through the use of technology. • Logarithmic functions (and logarithmic scales) can be useful to represent numbers that are very large or that vary greatly and are used to describe real-world situations (Richter scale, Decibels, pH scale, etc.). • The logarithm of a number is the exponent that another value (the base) must be raised to produce the given number. • $\text{Log}_b\ y = x$ is another way of expressing $b^x = y$ and that this logarithmic expression can be used to determine the solution of an equation where the unknown is in the exponent.
Essential Questions:	• What can be modeled using logarithmic functions? • What type of function is best to model a given situation? • How can logarithmic equations be solved? • What is a logarithm? • How are logarithms and exponentials related? • What are the key features of the graph of a logarithmic function? • How can a logarithmic function be represented numerically or in a table?
Vocabulary	Logarithm, common logarithm, change of base formula
Assessment	Galileo: Topic B Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
N.Q.A.2 ▲	**A. Reason qualitatively and units to solve problems** Define appropriate quantities for the purpose of descriptive modeling.	**Explanation:** Students define appropriate quantities for the purpose of describing a mathematical model in context. This standard is taught in Algebra I and Algebra II. In Algebra II, the standard will be assessed by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades/courses) require the student to <u>create</u> a quantity of interest in the situation being described (i.e. this is not provided in the task). For	**Eureka Math** Module 3 Lesson 9-10

		example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude. **Example:** Explain how the unit cm, cm^2, and cm^3 are relate. Describe situations where each would be an appropriate unit of measure.	
A.CED.A.1 ●	**A. Create equations that describe numbers or relationships** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra II, tasks are limited to exponential equations with rational or real exponents and rational functions. Students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem. Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. **Examples:** ▪ Two cups of coffee are poured from the same pot. The initial temperature of the coffee is 180°F and k is 0.2337 (for time in minutes). 1. Suppose both cups are poured at the same time. Cup 1 is left sitting in the room that is 75°F, and cup 2 is taken outside where it is 42°F. a. Use Newton's Law of Cooling to write equations for the temperature of each cup of coffee after t minutes has elapsed. *Cup 1:* $T_1(t) = 75 + (180 - 75) \cdot 2.718^{-0.2337t}$ *Cup 2:* $T_2(t) = 42 + (180 - 42) \cdot 2.718^{-0.2337t}$ ▪ Phil purchases a used truck for $11,500. The value of the truck is expected to decrease by 20% each year. When will the truck first be worth less than $1,000? ▪ A scientist has 100 grams of a radioactive substance. Half of	**Eureka Math** Module 3 Lesson 7 **This standard is revisited in Unit 2 Topic D.** **Supplement needed for the "rational functions" portion of the standard. It should be incorporated in Unit 1.**

		it decays every hour. How long until 25 grams remain? Be prepared to share any equations, inequalities, and/or representations used to solve the problem. **Simple rational function example (inverse variation)** ▪ In kickboxing, it is found that the force, *f*, needed to break a board, varies inversely with the length, *l*, of the board. If it takes 5 lbs. of pressure to break a board 2 feet long, how many pounds of pressure will it take to break a board that is 6 feet long? **Examples from Algebra I:** ▪ Lava coming from the eruption of a volcano follows a parabolic path. The height h in feet of a piece of lava t seconds after it is ejected from the volcano is given by $h\ \ t= -16t^2 + 64t + 936$. After how many seconds does the lava reach it's a height of 1000 feet? ▪ The function $h\ \ x= 0.04x^2 - 3.5x + 100$ defines the height (in feet) of a major support cable on a suspension bridge where x is the horizontal distance (in feet) from the left end of the bridge. o Where is the cable less than 40 feet above the bridge surface? o Where is the cable at least 60 feet above the bridge surface? ▪ To be considered a 'fuel efficient' vehicle, a car must get more than 30 miles per gallon. Consider a test run of 200 miles. What is the possible amount of gallons of fuel a car can use and be considered 'fuel-efficient'?	
F.BF.A.1a	**A. Build a function that models a relationship between two quantities** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra II, tasks have a real-world context and are limited to linear functions, quadratic functions, and exponential functions. Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible,	Eureka Math Module 3 Lesson 7 **This standard is revisited in Unit 2 Topics C and D.**

	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.	relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. **Examples:** • You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • The radius of a circular oil slick after *t* hours is given in feet by $r=10t2-0.5t$, for $0 \le t \le 10$. Find the area of the oil slick as a function of time. • Suppose you deposit $100 in a savings account that pays 4% interest, compounded annually. At the end of each year you deposit an additional $50. Write a recursive function that models the amount of money in the account for any year.	
F.LE.A.4	**A. Construct and compare linear, quadratic, and exponential models and solve problems** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** Students recognize how to rewrite values using bases 2, 10, or *e*. Students use calculators to approximate answers. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms. **Examples:** • Solve $200e^{0.04t} = 450$ for t. • Rewrite each of the following in the form $\log_b(x) = L$. a. $16^{\frac{1}{4}} = 2$ $\log_{16}(2) = \frac{1}{4}$ b. $10^3 = 1,000$ $\log_{10}(1,000) = 3$	**Eureka Math** Module 3 Lesson 8,10-15 **This standard is revisited in Unit 2 Topics C and D.**

		• Rewrite each of the following in the form $b^L = x$. a. $\log_5(625) = 4$ b. $\log_{10}(0.1) = -1$ $5^4 = 625$ $10^{-1} = 0.1$	
MP.1	**Make sense of problems and persevere in solving them.**	Students make sense of rational and real number exponents and in doing so are able to apply exponential functions to solve problems involving exponential growth and decay for continuous domains such as time. They explore logarithms numerically and graphically to understand their meaning and how they can be used to solve exponential equations. Students have multiple opportunities to make connections between information presented graphically, numerically, and algebraically and search for similarities between these representations to further understand the underlying mathematical properties of exponents and logarithms. When presented with a wide variety of information related to financial planning, students make sense of the given information and use appropriate formulas to effectively plan for a long-term budget and savings plan.	**Eureka Math** Module 3 Lesson 9
MP.2	**Reason abstractly and quantitatively.**	Students consider appropriate units when exploring the properties of exponents for very large and very small numbers. They reason about quantities when solving a wide variety of problems that can be modeled using logarithms or exponential functions. Students relate the parameters in exponential expressions to the situations they model. They write and solve equations and then interpret their solutions within the context of a problem.	**Eureka Math** Module 3 Lesson 9
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or	**Eureka Math** Module 3 Lesson 7,9,12,13,14,15

		reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.	
MP.4	**Model with mathematics.**	Students use exponential functions to model situations involving exponential growth and decay. They model the number of digits needed to assign identifiers using logarithms. They model exponential growth using a simulation with collected data. The application of exponential functions and logarithms as a means to solve an exponential equation is a focus of several lessons that deal with financial literacy and planning a budget. Here, students must make sense of several different quantities and their relationships as they plan and prioritize for their future financial solvency.	**Eureka Math** Module 3 Lesson 9
MP.5	**Use appropriate tools strategically.**	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to use technological tools to explore and deepen their understanding of concepts.	**Eureka Math** Module 3 Lesson 15
MP.6	**Attend to precision.**	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	**Eureka Math** Module 3 Lesson 15
MP.7	**Look for and make use of structure.**	Students extend the laws of exponents for integer exponents to rational and real number exponents. They connect how these laws are	**Eureka Math** Module 3 Lesson 7,10

		related to the properties of logarithms and understand how to rearrange an exponential equation into logarithmic form. Students analyze the structure of exponential and logarithmic functions to understand how to sketch graphs and see how the properties relate to transformations of these types of functions. They analyze the structure of expressions to reveal properties such as recognizing when a function models exponential growth versus decay. Students use the structure of equations to understand how to identify an appropriate solution method.	
MP.8	**Look for and express regularity in repeated reasoning.**	Students discover the properties of logarithms and the meaning of a logarithm by investigating numeric examples. They develop formulas that involve exponentials and logarithms by extending patterns and examining tables and graphs. Students generalize transformations of graphs of logarithmic functions by examining several different cases.	**Eureka Math** Module 3 Lesson 7,8,10,11

Algebra II Semester 1 (Quarter 2)

Unit 2: Exponential and Logarithmic Functions (45 days)

Topic C: Exponential and Logarithmic Functions and their Graphs (7 Instructional Days)

The lessons covered in Topic A and Topic B build upon students' prior knowledge of the properties of exponents, exponential expression, and solving equations by extending the properties of exponents to all real number exponents and positive real number bases before introducing logarithms. This topic reintroduces exponential functions, introduces logarithmic functions, explains their inverse relationship, and explores the features of their graphs and how they can be used to model data.

Big Idea:	• Functions and relations can accurately model real-world relationships between variables. • Functions and relations can be represented in many ways. • Switching form one representation to another can reveal new information about a relationship. • Functions may be combined or decomposed using composition to obtain new functions and inverses.
Essential Questions:	• How can you determine if a relationship is growing or decaying? • What is the relationship between an exponential function and a logarithmic function? • Why is it beneficial to convert between exponential equations and logarithmic equations?
Vocabulary	Invertible function, general form of a logarithmic function, general form of an exponential function
Assessment	Galileo: Topic C Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
F.IF.B.4	**B. Interpret functions that arise in applications in terms of context** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.* ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model***	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and they are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. In Algebra II, tasks have a real-world context and they may involve polynomial, **exponential**, **logarithmic**, and trigonometric functions. (Trigonometric functions will be explored 2nd semester in Unit 4) **Examples:** • Jack planted a mysterious bean just outside his kitchen window. Jack kept a table (shown below) of the plant's growth. He measured the height at 8:00 am each day.	**Eureka Math** Module 3 Lesson 17-18,21 **This standard will be revisited 2nd semester in Unit 4 (trigonometric).**

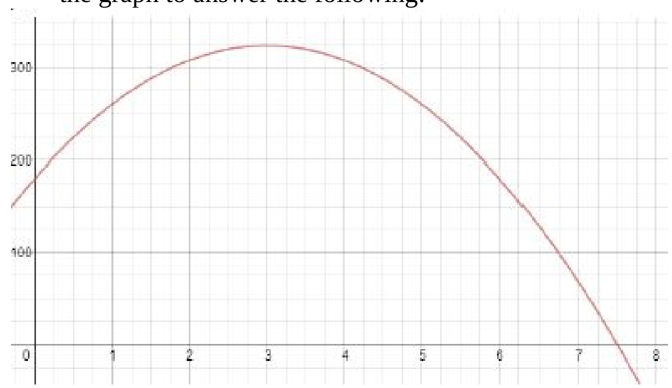
and use the model to solve problems are essential.

Day	0	1	2	3	4
Height (cm)	2.56	6.4	16	40	100

- What was the initial height of Jack's plant?
- How is the height changing each day?
- If this pattern continues, how tall should Jack's plant be after 8 days?

Examples from Algebra I:

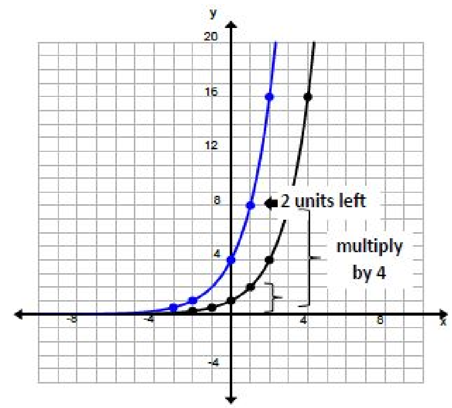
- The graph represents the height (in feet) of a rocket as a function of the time (in seconds) since it was launched. Use the graph to answer the following:

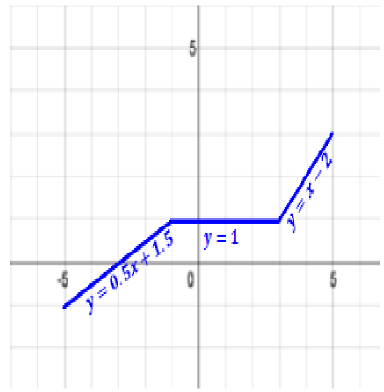
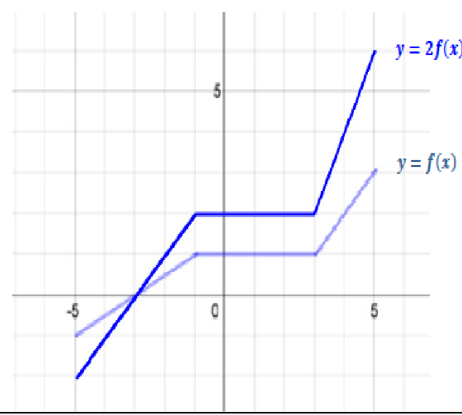


a. What is the practical domain for t in this context? Why?
b. What is the height of the rocket two seconds after it was launched?
c. What is the maximum value of the function and what does it mean in context?
d. When is the rocket 100 feet above the ground?
e. When is the rocket 250 feet above the ground?
f. Why are there two answers to part *e* but only one practical answer for part *d*?
g. What are the intercepts of this function? What do they mean in the context of this problem?
h. What are the intervals of increase and decrease on the practical domain? What do they mean in the context of

		the problem? • Marla was at the zoo with her mom. When they stopped to view the lions, Marla ran away from the lion exhibit, stopped, and walked slowly towards the lion exhibit until she was halfway, stood still for a minute then walked away with her mom. Sketch a graph of Marla's distance from the lions' exhibit over the period of time when she arrived until she left.	
F.IF.C.7e	**C. Analyze functions using different representation** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude~~. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. **Examples:** • Graph $f(x) = 10^x$ and $g(x) = \log x$. Compare the key features of intercepts and end behavior. Discuss how they are related. • Graph the function $f(x) = \log_3 x$ without using a calculator and identify its key features. • Sketch the graphs of $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \left(\frac{3}{4}\right)^x$ on the same sheet of graph paper. o Identify the key features of each graph. o Where do the graphs intersect?	**Eureka Math** Module 3 Lesson 16-18,20-21 **This standard will be revisited 2nd semester in Unit 4.**
F.BF.A.1a	**A. Build a function that models a relationship between two quantities** **Write a function that describes a relationship between two quantities.** a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ***This is a modeling standard which means students***	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra II, tasks have a real-world context and are limited to linear functions, quadratic functions, and exponential functions. Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.	Eureka Math Module 3 Lesson 22 **This standard is revisited 2nd semester in Unit 2 Topic D.**

	choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.	**Examples:** • You buy a \$10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of \$250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • The radius of a circular oil slick after *t* hours is given in feet by $r=10t2-0.5t$, for $0 \le t \le 10$. Find the area of the oil slick as a function of time. • Suppose you deposit \$100 in a savings account that pays 4% interest, compounded annually. At the end of each year you deposit an additional \$50. Write a recursive function that models the amount of money in the account for any year.	
F.BF.B.3	**A. Build a function that models a relationship between two quantities** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, the focus was on linear and quadratic functions and did not involve recognizing even and odd functions. In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. Students will apply transformations to functions and recognize even and odd functions from their graphs and algebraic expression for them. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. **Examples:** • The general form of a logarithmic function is given by $f(x) = k + a\log_b(x-h)$, where *a*, *b*, *k*, and *h* are real numbers such that *b* is a positive number not equal to 1, and $x - h > 0$. Given $g(x) = 3 + 2\log(x-2)$, describe the graph of *g* as a transformation of the common logarithm function. The graph of *g* is a horizontal translation 2 units to the right, a	Eureka Math Module 3 Lesson 20-21 **This standard will be revisited 2nd semester in Unit 4.**

		vertical scaling by a factor of 2, and a vertical translation up 3 units of the graph of the common logarithm function. • **Use properties of exponents to explain why graphs of $f(x) = 4^x$ and $g(x) = 2^{2x}$ are identical.** *Using the power property of exponents, $2^{2x} = (2^2)^x = 4^x$. Since the expressions are equal, the graphs of the functions would be the same.* **Use the properties of exponents to predict what the graphs of $f(x) = 4 \cdot 2^x$ and $g(x) = 2^{x+2}$ will look like compared to one another. Describe the graphs of f and g as transformations of the graph of $f = 2^x$. Confirm your prediction by graphing f and g on the same coordinate axes.** *The graphs of these two functions will be the same since $2^{x+2} = 2^x \cdot 2^2 = 4 \cdot 2^x$ by the multiplication property of exponents and the commutative property. The graph of f is the graph of $y = 2^x$ scaled vertically by a factor of 4. The graph of g is the graph of $y = 2^x$ translated horizontally 2 units to the left.*  	

		The graph $y = f(x)$ of a piecewise function f is shown. The domain of f is $-5 \le x \le 5$, and the range is $-1 \le y \le 3$. a. Mark and identify four strategic points helpful in sketching the graph of $y = f(x)$. $(-5,-1), (-1,1), (3,1)$, *and* $(5,3)$  • Sketch the graph of $y = 2f(x)$ and state the domain and range of the transformed function. How can you use part (a) to help sketch the graph of $y = 2f(x)$? *Domain: $-5 \le x \le 5$, range: $-2 \le y \le 6$. For every point (x, y) in the graph of $f(x)$, there is a point $(x, 2y)$ on the graph of $y = 2f(x)$. The four strategic points can be used to determine the line segments in the graph of $y = 2f(x)$ by graphing points with the same original x-coordinate and 2 times the original y-coordinate ($(-5,-2), (-1,2), (3,2)$, and $(5,6)$).* 	

F.BF.B.4a	**B. Build new functions from existing functions** Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{(x+1)}{(x-1)}$ for $x \neq 1$.	**Explanation:** Students solve a function for the dependent variable and write the inverse of a function by interchanging the values of the dependent and independent variable. They connect the concept of the inverse to the effect on the graph and the input-output pairs. Students find inverse functions for linear and exponential functions. Also, include simple situations where the domain of the functions must be restricted in order for the inverse to be a function, such as $(x) = {}^2, x \le 0$. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. **Examples:** • Graph the inverse of $(x) = -\frac{3}{2}x - 3$. How does $f^{-1}(x)$ relate to $f(x)$? • Find the inverse of the function $g(x) = 2^x$ and demonstrate it is the inverse using input-output pairs. • Let h ( )= x^3. Find the inverse function.	**Eureka Math** Module 3 Lesson 19
F.LE.A.2	**A. Construct and compare linear, quadratic, and exponential models and solve problems** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra II, tasks are limited to solving multi-step problems by constructing linear and exponential functions. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions. **Examples:** • After a record setting winter storm, there are 10 inches of snow on the ground! Now that the sun is finally out, the snow is melting. At 7 am there were 10 inches and at 12 pm there were 6 inches of snow. o Construct a linear function rule to model the amount of snow. o Construct an exponential function rule to model	**Eureka Math** Module 3 Lesson 22 **This standard is revisited 2nd semester in Unit 2 Topic D.** **Note: The focus in this topic is on exponential functions. It is explored further via geometric sequences in Topic D, Lesson 25.**

the amount of snow.

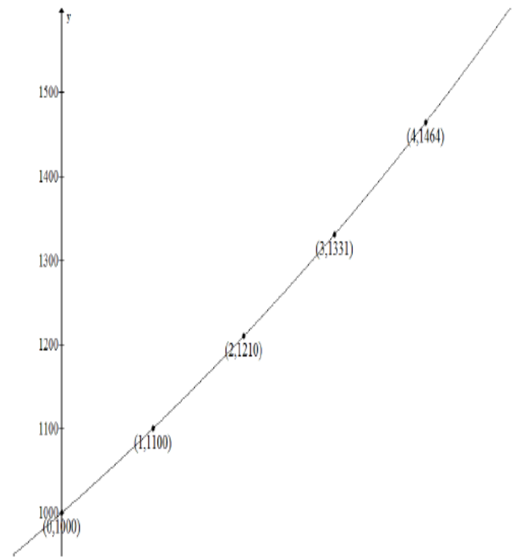
- Which model best describes the amount of snow? Provide reasoning for your choice.

Note: In order to write the exponential function as the amount of snow for every hour, connect to F.IF.8b. Students could start with $10(.6)^{x}$ *where x is the number of 5 hour periods then rewrite it to be* $10(.6)^{\left(\frac{1}{5}x\right)} = 10\left(.6^{\frac{1}{5}}\right)^{x} \approx 10(.9)^{x}$ *where x is the number of hours since 7am.*

▪

Lewis' dad put **1,000** dollars in a money market fund when he was **16**. Lewis can't touch the money until he is **26**, but he gets updates on the balance of his account.

Years After Lewis Turns 16	Account Balance in Dollars
0	1000
1	1100
2	1210
3	1331
4	1464

		a. Develop a model for this situation. *We might try graphing this data. However, in the viewing window that shows our data points (see graph below), it appears that the function might be linear. Let's try zooming out to see more of the key features of this graph (See graph below.)* x: [0,5] y: [975,1500]  *This viewing window gives us a close-up of the data points and their relation to each other. However, we cannot really see the features of the graph that represents the data.*	
F.LE.A.4	**A. Construct and compare linear, quadratic, and exponential models and solve problems** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** Students recognize how to rewrite values using bases 2, 10, or *e*. Students use calculators to approximate answers. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms. **Examples:** • Solve $200e^{0.04t} = 450$ for t. • Rewrite each of the following in the form $\log_b(x) = L$. a. $16^{\frac{1}{4}} = 2$ b. $10^3 = 1{,}000$ $\log_{16}(2) = \frac{1}{4}$ $\log_{10}(1{,}000) = 3$	**Eureka Math** Module 3 Lesson 19 **This standard is revisited 2nd semester in Unit 2 Topic D.**

		• Rewrite each of the following in the form $b^L = x$. a. $\log_5(625) = 4$ $5^4 = 625$ b. $\log_{10}(0.1) = -1$ $10^{-1} = 0.1$	
MP.1	**Make sense of problems and persevere in solving them.**	Students make sense of rational and real number exponents and in doing so are able to apply exponential functions to solve problems involving exponential growth and decay for continuous domains such as time. They explore logarithms numerically and graphically to understand their meaning and how they can be used to solve exponential equations. Students have multiple opportunities to make connections between information presented graphically, numerically, and algebraically and search for similarities between these representations to further understand the underlying mathematical properties of exponents and logarithms. When presented with a wide variety of information related to financial planning, students make sense of the given information and use appropriate formulas to effectively plan for a long-term budget and savings plan.	**Eureka Math** Module 3 Lesson 20,21
MP.2	**Reason abstractly and quantitatively.**	Students consider appropriate units when exploring the properties of exponents for very large and very small numbers. They reason about quantities when solving a wide variety of problems that can be modeled using logarithms or exponential functions. Students relate the parameters in exponential expressions to the situations they model. They write and solve equations and then interpret their solutions within the context of a problem.	**Eureka Math** Module 3 Lesson 20
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an	**Eureka Math** Module 3 Lesson 16,18,20,21

		argument—explain what it is.	
MP.7	**Look for and make use of structure.**	Students extend the laws of exponents for integer exponents to rational and real number exponents. They connect how these laws are related to the properties of logarithms and understand how to rearrange an exponential equation into logarithmic form. Students analyze the structure of exponential and logarithmic functions to understand how to sketch graphs and see how the properties relate to transformations of these types of functions. They analyze the structure of expressions to reveal properties such as recognizing when a function models exponential growth versus decay. Students use the structure of equations to understand how to identify an appropriate solution method.	**Eureka Math** Module 3 Lesson 17-21
MP.8	**Look for and express regularity in repeated reasoning.**	Students discover the properties of logarithms and the meaning of a logarithm by investigating numeric examples. They develop formulas that involve exponentials and logarithms by extending patterns and examining tables and graphs. Students generalize transformations of graphs of logarithmic functions by examining several different cases.	**Eureka Math** Module 3 Lesson 18,20,21