Algebra II
Unit 3: Rational Equations and Inequalities

Time Frame: Approximately three weeks

Unit Description

The study of rational equations reinforces the students' abilities to multiply polynomials and factor algebraic expressions. This unit develops the process for simplifying rational expressions, adding, multiplying, and dividing rational expressions, and solving rational equations and inequalities.

Student Understandings

Students symbolically manipulate rational expressions in order to solve rational equations. They determine the domain restrictions that drive the solutions of rational functions. They relate the domain restrictions to vertical asymptotes on a graph of the rational function but realize that the calculator does not give an easily readable graph of rational functions. Therefore, they solve rational inequalities by the sign chart method instead of the graph. Students also solve application problems involving rational functions.

Guiding Questions

1. Can students simplify rational expressions in order to solve rational equations?
2. Can students add, subtract, multiply, and divide rational expressions?
3. Can students simplify a complex rational expression?
4. Can students solve rational equations?
5. Can students identify the domain and vertical asymptotes of rational functions?
6. Can students solve rational inequalities?
7. Can students solve real world problems involving rational functions?

Unit 3 Grade-Level Expectations (GLEs)

Teacher Note: The individual Algebra II GLEs are sometimes very broad, encompassing a variety of functions. To help determine the portion of the GLE that is being addressed in each unit and in each activity in the unit, the key words have been underlined in the GLE list, and the number of the predominant GLE has been underlined in the activity.

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Number and Number Relations	
2.	Evaluate and perform basic operations on expressions containing rational exponents (N-2-H)
Algebra	
4.	Translate and show the relationships among non-linear graphs, related tables of values, and algebraic symbolic representations (A-1-H)
5.	Factor simple quadratic expressions including general trinomials, perfect squares, difference of two squares, and polynomials with common factors(A-2-H)
6.	Analyze functions based on zeros, asymptotes, and local and global characteristics of the function (A-3-H)
7.	Explain, using technology, how the graph of a function is affected by change of degree, coefficient, and constants in polynomial, <u>rational</u>, radical, exponential, and logarithmic functions (A-3-H)
9.	Solve quadratic equations by factoring, completing the square, using the quadratic formula, and graphing (A-4-H)
10.	Model and solve problems involving quadratic, polynomial, exponential, logarithmic, step function, <u>rational</u>, and absolute value equations using technology (A-4-H)
Patterns, Relations, and Functions	
24.	Model a given set of real-life data with a non-linear function (P-1-H) (P-5-H)
25.	Apply the concept of a function and function notation to represent and evaluate functions (P-1-H) (P-5-H)
27.	Compare and contrast the properties of families of polynomial, <u>rational</u>, exponential, and logarithmic functions, with and without technology (P-3-H)
29.	Determine the family or families of functions that can be used to represent a given set of real-life data, with and without technology (P-5-H)
CCSS for Mathematical Content	
CCSS #	**CCSS Text**
Reasoning with Equations & Inequalities	
A.REI.2	Solve simple <u>rational</u> and radical equations in one variable, and give examples showing how extraneous solutions may arise.
ELA CCSS	
CCSS #	**CCSS Text**
Reading Standards for Literacy in Science and Technical Subjects 6-12	
RST.11-12.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.
Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12	
WHST.11-12.2d	Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.

Sample Activities

Ongoing Activity: Little Black Book of Algebra II Properties

Materials List: Black marble composition book, Little Black Book of Algebra II Properties BLM

Activity:

- Have students continue to add to the Little Black Books they created in previous units which are modified forms of *vocabulary cards* (view literacy strategy descriptions). When students create *vocabulary cards*, they see connections between words, examples of the word, and the critical attributes associated with the word, such as a mathematical formula or theorem. *Vocabulary cards* require students to pay attention to words over time, thus improving their memory of the words. In addition, *vocabulary cards* can become an easily accessible reference for students as they prepare for tests, quizzes, and other activities with the words. These self-made reference books are modified versions of *vocabulary cards* because, instead of creating cards, the students will keep the vocabulary in black marble composition books (thus the name "Little Black Book" or LBB). Like *vocabulary cards*, the LBBs emphasize the important concepts in the unit and reinforce the definitions, formulas, graphs, real-world applications, and symbolic representations.
- At the beginning of the unit, distribute copies of the Little Black Book of Algebra II Properties BLM for Unit 3. These are lists of properties in the order in which they will be learned in the unit. The BLM has been formatted to the size of a composition book so students can cut the list from the BLM and paste or tape it into their composition books to use as a table of contents.
- The student's description of each property should occupy approximately one-half page in the LBB and include all the information on the list for that property. The student may also add examples for future reference.
- Periodically check the Little Black Books and require that the properties applicable to a general assessment be finished by the day before the test, so pairs of students can use the LBBs to quiz each other on the concepts as a review.

Rational Equations and Inequalities

3.1 Rational Terminology – define rational number, rational expression, and rational function, least common denominator (LCD), complex rational expression.
3.2 Rational Expressions – explain the process for simplifying, adding, subtracting, multiplying, and dividing rational expressions; define reciprocal, and explain how to find denominator restrictions.
3.3 Complex Rational Expressions – define and explain how to simplify.
3.4 Vertical Asymptotes of Rational Functions – explain how to find domain restrictions and what the domain restrictions look like on a graph; explain how to determine end-behavior of a rational function around a vertical asymptote.
3.5 Solving Rational Equations – explain the difference between a rational expression and a

rational equation; list two ways to solve rational equations and define extraneous roots.

3.6 Solving Rational Inequalities – list the steps for solving an inequality by using the sign chart method.

Activity 1: Simplifying Rational Expressions (GLEs: 2, 5, 7)

Materials List: paper, pencil, graphing calculator, Math Log Bellringer BLM, Simplifying Rational Expressions BLM

In this activity, the students will review non-positive exponents and use their factoring skills from the previous unit to simplify rational expressions.

Math Log Bellringer: Simplify:

(1) $(x^2)(x^5)$, (2) $\left(x^2y^5\right)^4$, (3) $\frac{\left(x^5\right)^3}{x^8}$, (4) $\frac{x^7}{x^7}$, (5) $\frac{x^3}{x^5}$

(6) Choose one problem above and write in a sentence the Law of Exponents used to determine the solution.

Solutions:

(1) x^7, *Law of Exponents: When you multiply variables with exponents, you add the exponents.*

(2) x^8y^{20}, *Law of Exponents: When you raise a variable with an exponent to a power, you multiply exponents.*

(3) x^7, $x \neq 0$, *Law of Exponents: Same as #2 plus when you divide variables with exponents, you subtract the exponents.*

(4) *1*, $x \neq 0$, *Law of Exponents: Same as #3 plus any variable to the 0 power equals 1.*

(5) $x^{-2} = \frac{1}{x^2}$, $x \neq 0$, *Law of Exponents: Same as #3 plus a variable to a negative exponent moves to the denominator.*

(6) *See Laws of Exponents above.*

Activity:

- Overview of the Math Log Bellringers:
  - As in previous units, each in-class activity in Unit 3 is started with an activity called a Math Log Bellringer that either reviews past concepts to check for understanding (reflective thinking about what was learned in previous classes or previous courses) or sets the stage for an upcoming concept (predictive thinking for that day's lesson).
  - A math log is a form of a *learning log* (view literacy strategy descriptions) that students keep in order to record ideas, questions, reactions, and new understandings. Documenting ideas in a log about how content's being studied forces students to "put into words" what they know or do not know. This process offers a reflection of understanding that can lead to further study and alternative learning paths. It combines writing and reading with

content learning. The Math Log Bellringers will include mathematics done symbolically, graphically, and verbally.
- Since Bellringers are relatively short, Blackline Masters have not been created for each of them. Write them on the board before students enter class, paste them into an enlarged *Word*® document or *PowerPoint*® slide, and project using a TV or digital projector, or print and display using a document or overhead projector. A sample enlarged Math Log Bellringer *Word*® document has been included in the Blackline Masters. This sample is the Math Log Bellringer for this activity.
- Have the students write the Math Log Bellringers in their notebooks preceding the upcoming lesson during beginning–of–class record keeping and then circulate to give individual attention to students who are weak in that area.

- It is important for future mathematics courses that students find denominator restrictions throughout this unit. They should never write $\frac{x}{x}=1$ unless they also write if $x \neq 0$ because the two graphs of these functions are not equivalent.

- Write the verbal rules the students created in Bellringer #6 on the board or overhead projector. Use these rules and the Bellringer problems to review Laws of Exponents and develop the meaning of zero and negative exponents. The rules in their words should include the following:
  (1) *"When you multiply like variables with exponents, you add the exponents."*
  (2) *"When you raise a variable with an exponent to a power, you multiply exponents."*
  (3) *"When you divide like variables with exponents, you subtract the exponents."*
  (4) *"Any variable raised to the zero power equals 1."*
  (5) *"A variable raised to a negative exponent moves the variable to the denominator and means reciprocal."*

- Simplifying Rational Expressions:
  - Distribute the Simplifying Rational Expressions BLM. This is a guided discovery/review in which students work only one section at a time and draw conclusions.
  - Connect negative exponents to what they have already learned about scientific notation in Algebra I and science. To reinforce the equivalencies, have students enter the problems in Section I of the Simplifying Rational Expressions BLM in their calculators. This can be done by getting decimal representations or using the TEST feature of the calculator: Enter $2^{-3}=\frac{1}{2^3}$ (The "=" sign is found under $\boxed{2^{nd}}$, TEST (above the $\boxed{\text{MATH}}$)). If the calculator returns a "1" then the statement is true; if it returns a "0" then the statement is false.
  - Use guided practice with problems in Section II of the Simplifying Rational Expressions BLM which students simplify and write answers with only positive exponents.
  - Have students define rational number to review the definition as the quotient of two integers $\frac{p}{q}$ in which $q \neq 0$, and then define rational algebraic expression as the quotient

of two polynomials *P(x)* and *Q(x)* in which *Q(x)* $\neq$ 0. Discuss the restrictions on the denominator and have students find the denominator restrictions Section III of the Simplifying Rational Expressions BLM.

- Have students simplify $\frac{24}{40}$ in Section IV and let one student explain the steps he/she used. Make sure there is a discussion of dividing out and cancelling of a common factor. Then have students apply this concept to simplify the expressions in Section V of the Simplifying Rational Expressions BLM.
- Remind students that the domain restrictions on any simplified rational expression are obtained from the original expression and apply to all equivalent forms; therefore, they should find the domain restrictions (due to a denominator = 0) prior to simplifying they expression. To stress this point, have students work Section VI of the Simplifying Rational Expressions BLM.
- Conclude the worksheet by having students work the application problem.

Activity 2: Multiplying and Dividing Rational Expressions (GLEs: 2, 5; CCSS: WHST.11-12.2d)

Materials List: paper, pencil

This activity has not changed because it already incorporates this CCSS. In this activity, the students will multiply and divide rational expressions and use their factoring skills to simplify the answer. They will also express domain restrictions.

Math Log Bellringer:

Simplify the following:

(1) $\frac{3}{4}\cdot\frac{10}{11}$

(2) $\frac{7}{8}\cdot 4$

(3) $\frac{4x^2}{5y}\cdot\frac{y^3}{12x^5}$

(4) $\frac{x+2}{x-3}\cdot\frac{4}{5}$

(5) $\frac{2x+3}{x-5}\cdot(x-2)$

(6) $\frac{x-2}{x+4}\cdot\frac{x+3}{x-5}$

(7) Write in a sentence the rule for multiplying and simplifying fractions.

(8) What mathematical rule allows you to cancel constants?

(9) What restrictions should you state when you cancel variables?

Solutions:

(1) $\frac{15}{22}$, *(2)* $\frac{7}{2}$, *(3)* $\frac{y^2}{15x^3}$, *$y \neq 0$,* *(4)* $\frac{4x+8}{5x-15}$,
(5) $\frac{2x^2-x-6}{x-5}$, *(6)* $\frac{x^2+x-6}{x^2-x-20}$
(7) When you multiply fractions, you multiply the numerators and multiply the denominators. Then you find any common factors in the numerator and denominator and cancel them to simplify the fractions.
(8) If "a" is a constant, $\frac{a}{a}=1$*, the identity element of multiplication; therefore, you can cancel common factors without changing the value of the expression.*
(9) If you cancel variables, you must state the denominator restrictions of the cancelled factor or the expressions are not equivalent. (Teacher Note: If the original problem already has a factor with a variable in the denominator, then the domain is assumed to already be restricted; these domain restrictions do not have to be repeated in the solution even though the factor is still in the denominator. It is not incorrect to restate the original domain restrictions, such as $x \neq 0$ in #3 or $x \neq 3$ in #4, but it is redundant.)

Activity:

- Use the Bellringer to review the process of multiplying numerical fractions and have students extend the process to multiplying rational expressions. Students should simplify and state domain restrictions.
- Have students multiply and simplify $\frac{x^2-4}{x+3}\cdot\frac{2x+6}{x^2+7x+10}$ and let students that have different processes show their work on the board. Examining all the processes, have students choose the most efficient (factoring, canceling, and then multiplying). Make sure to include additional domain restrictions.

  Solution: $\frac{2x-4}{x+5}, x \neq -2, x \neq -3$

- Have the students work the following $\frac{3}{4}\div\frac{10}{11}=$ and $\frac{7}{8}\div 4$. Define <u>reciprocal</u> and have students rework the Bellringers with a division sign instead of multiplication.

  (1) $\frac{3}{4}\div\frac{10}{11}$ (2) $\frac{7}{8}\div 4$ (3) $\frac{4x^2}{5y}\div\frac{y^3}{12x^5}$ (4) $\frac{x+2}{x-3}\div\frac{4}{5}$ (5) $\frac{2x+3}{x-5}\div(x-2)$ (6) $\frac{x-2}{x+4}\div\frac{x+3}{x-5}$

  Solutions: (1) $\frac{33}{40}$ *(2)* $\frac{7}{32}$ *(3)* $\frac{48x^7}{5y^4}$ *(4)* $\frac{5x+10}{4x-12}$ *(5)* $\frac{2x+3}{x^2-7x+10}, x \neq 2$ *(6)* $\frac{x^2-7x+10}{x^2+7x+12}, x \neq -3$

- Application:

  Density is mass divided by volume. The density of solid brass is $\frac{x+5}{2} g/cm^3$. If a sample of an unknown metal in a laboratory experiment has a mass of $\frac{x^2+2x-15}{2x-8}$ g and a volume of $\frac{x^2+x-12}{x^2-16}$ cm^3, determine if the sample is solid brass.

  Solution: yes

- Critical Thinking Writing Assessment: *(See Activity-Specific Assessments at end of unit.)*

Activity 3: Adding and Subtracting Rational Expressions (GLEs: <u>2</u>, 5, <u>10</u>, 24, 25; CCSS: WHST.11-12.2d)

Materials List: paper, pencil, Adding & Subtracting Rational Expressions BLM

This activity has not changed because it already incorporates this CCSS. In this activity, the students will find common denominators to add and subtract rational expressions.

Math Log Bellringer:

Simplify and express answer as an improper fraction:

(1) $\frac{2}{5}+\frac{7}{11}$ (2) $\frac{2}{15}+\frac{7}{25}$ (3) $\frac{2}{5}+6$

(4) Write the mathematical process used to add fractions.

Solutions: (1) $\frac{57}{55}$*, (2)* $\frac{31}{75}$*, (3)* $\frac{32}{5}$*, (4) When you add fractions, you have to find a common denominator. To find the least common denominator, use the highest degree of each factor in the denominator.*

Activity:

- Use the Bellringer to review the rules for adding and subtracting fractions and relate them to rational expressions.

- Adding/Subtracting Rational Functions BLM:
  - Distribute the Adding & Subtracting Rational Expressions BLM and have students work in pairs to complete. On this worksheet, the students will apply the rules they know about adding and subtracting fractions to adding and subtracting rational expressions with variables.
  - In Section I, have the students write the rule developed from the Bellringers, then apply the rule to solve the problems in Section II. Have two of the groups write the problems on the board and explain the process they used.

- Have the groups work Section III and IV and again have two of the groups write the problems on the board and explain the process they used.
- Have students work the application problem and one of the groups explain it on the board.
- Finish by giving the students additional problems adding and subtracting rational expressions from the math textbook.

- Critical Thinking Writing Assessment: *(See Activity-Specific Assessments at end of unit.)*

Activity 4: Complex Rational Expressions (GLEs: 5; CCSS: WHST.11-12.2d)

Materials List: paper, pencil

This activity has not changed because it already incorporates this CCSS. In this activity, the students will simplify complex rational fractions.

Math Log Bellringer: Multiply and simplify the following:

(1) $6x^2y^2\left(\dfrac{x}{6y^2}+\dfrac{3y}{2x^2}\right)$

(2) $(x+2)(x-5)\left(\dfrac{3}{x+2}+\dfrac{7}{x-5}\right)$

(3) What mathematical properties are used to solve the above problems?

Solutions: (1) $x^3 + 9y^3$, $x \neq 0$, $y \neq 0$, (2) $10x - 1$, $x \neq -2$, $x \neq 5$ (3) First, you use the Distributive Property of Multiplication over Addition. Second, you cancel like factors which uses the identity element of multiplication. Then you combine like terms.

Activity:

- Use the Bellringer to review the Distributive Property.
- Define complex fraction and ask students how to simplify $\dfrac{\frac{1}{6}}{\frac{5}{9}}$. Most students will invert and multiply. Discuss an alternate process of multiplying by 18/18 or the LCD ratio equivalent to 1. *Solution: 3/10*
- Define complex rational expression and have students determine the best way to simplify $\dfrac{\frac{1}{x}+4}{5+\frac{3}{y}}$. Discuss why it would be wrong to work this problem this way: $\left(\dfrac{1}{x}+4\right)\left(\dfrac{1}{5}+\dfrac{y}{3}\right)$.

  Solution: $\dfrac{y+4xy}{5xy+3x}$

- Have students determine the process to simplify $\dfrac{\dfrac{2}{x+3}+\dfrac{5x}{x^2-9}}{\dfrac{4}{x+3}+\dfrac{2}{x-3}}$

  Solution: $\dfrac{7x-6}{6x-6}$

- Use the math textbook for additional problems.

- Critical Thinking Writing Assessment: *(See Activity-Specific Assessments at end of unit.)*

Activity 5: Solving Rational Equations (GLEs: 5, 6, 10; CCSS: A.REI.2)

Materials List: paper, pencil

This activity has not changed because it already incorporates this CCSS. In this activity, the students will solve rational equations.

Math Log Bellringer:

$\dfrac{x}{2}+\dfrac{3x}{4}=5$ Solve for x showing all the steps. Is this a rational equation or linear equation? Why?

Solution: x = 4. This is a linear equation because x is raised to the first power and there are no variables in the denominators.

Activity:

- Use a *SPAWN* writing prompt (view literacy strategy descriptions) to set the stage for solving rational functions with a variable in the denominator. *SPAWN* is an acronym that stands for five categories of writing prompts (*Special Powers*, *Problem Solving*, *Alternative Viewpoints*, *What If*?, and *Next*), which can be crafted in numerous ways to stimulate students' predictive, reflective, and critical thinking about content-area topics.
  - Write this "*Problem Solving*" writing prompt on the board and give students a few minutes to complete the *SPAWN* writing prompt individually. "In the Bellringer, there are constants in the denominator. Discuss what you would do differently if there were variables in the denominator."
  - Ask several students with alternate methods to put their comments on the board to share their answers to the writing prompt.
- Have students solve and check the following:

  1. $\dfrac{1}{4x}-\dfrac{3}{4}=\dfrac{7}{x}$
  2. $\dfrac{x}{x-2}=\dfrac{1}{2}+\dfrac{2}{x-2}$

Solutions: (1) x = – 9, (2) no solution, 2 is an extraneous root

- When students have finished the two problems, revisit the *SPAWN* prompt and refine the procedure for solving rational equations. Discuss alternate ways to solve this rational equation:
  - ➢ finding the LCD and adding fractions.
  - ➢ multiplying both sides of the equation by the LCD to remove fractions, then solve for *x*. Always check the solution because the answer may be an extraneous root, meaning it is a solution to the transformed equation but not the original equation because of the denominator restrictions.
- Use the following problems to develop the concept of zeros of the function. Find the denominator restrictions and the solutions for the following:

  (1) $\frac{x-2}{x+3}=0$

  (2) $\frac{x^2-x-12}{3x^2}=0$

  (3) $\frac{x^2-6x+5}{x^2-3x-10}=0$

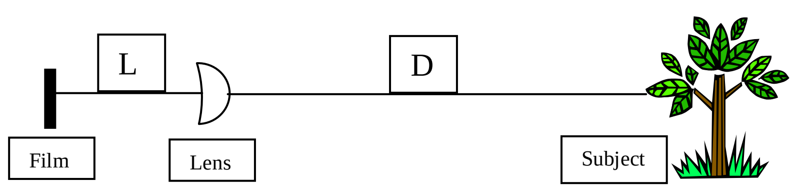
  (4) Write the process you used to find the zeros.

  Solutions:

  (1) $x=2, x\neq-3$, *(2)* $x=-3, x=4, x\neq 0$, *(3) x = 1, x ≠ 5, x ≠ –2*

  (4) To find zeros of a rational function, cancel common factors in the numerator and denominator, set the numerator equal to zero, and solve for x.
- Application:
  Every camera lens has a characteristic measurement called focal length, *F*. When the object is in focus, its distance, *D*, from the lens to the subject and the distance, *L*, from the lens to the film, satisfies the following equation. $\frac{1}{L}+\frac{1}{D}=\frac{1}{F}$. If the distance from the lens to an object is 60 cm and the distance from the lens to the film is 3 cm greater than the focal length, what is the focal length of the lens? Draw a picture of the subject, the film, and the lens and write the variables on the picture. Set up the equation and solve. Discuss the properties used.

Solution:



$\frac{1}{3+F}+\frac{1}{60}=\frac{1}{F}$, *F = 12 cm,*

Properties used: Answers may vary but could include (1) the multiplication property of equality multiplying both sides of the equation by (3+F)(60)(F), (2) distributive property and combined like terms, (3) found a common denominator *using the identity element* $1 = \frac{60}{60}$ *and* $1 = \frac{3+F}{3+F}$.

Activity 6: Applications Involving Rational Expressions (GLEs: 5, 9, 10, 29; CCSS: A.REI.2)

Materials List: paper, pencil, graphing calculator, Rational Expressions Applications BLM

This activity has not changed because it already incorporates this CCSS. In this activity, students will solve rate problems that are expressed as rational equations.

Math Log Bellringer:

In an Algebra II class, 2 out of 5 of the students are wearing blue. If 14 of the students are wearing blue, how many are there in the class? Set up a rational equation and solve. Describe the process used.

Solution: $\frac{2}{5} = \frac{14}{x}$, *x = 35*

Activity:

- Use the Bellringer to review the meaning of ratio (part to part) and proportion (part to whole). Ask the students what is the rate of blue wearers to any color wearers, and have them define rate as a comparison of two quantities with different units. Define *proportion* as an equation setting two rates equal to each other (with the units expressed in the same order).

- Rational Expressions Applications BLM:
  - On this worksheet, the students will set up rational equations, using the concepts of rate and proportion, and solve.
  - Distribute the Rational Expressions Applications BLM and have students work with a partner to set up and solve the application problems. Stop after each problem to check for understanding and to discuss the process used.

- Give additional problems in the math textbook for practice.

Activity 7: Vertical Asymptotes on Graphs of Rational Functions (GLEs: 4, 5, 6, 7, 9, 10, 25, 27; CCSS: RST.11-12.4)

Materials List: paper, pencil, graphing calculator, Vertical Asymptotes Discovery Worksheet BLM

This activity has not changed because it already incorporates this CCSS. In this activity, students will use technology to look at the graphs of rational functions in order to locate vertical asymptotes and to relate them to the domain restrictions.

Math Log Bellringer:

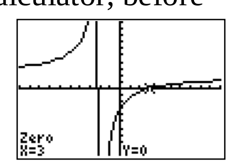
Find the solution and the domain restrictions for the following rational equation and describe the process used. $\dfrac{2x-6}{x+2}=0$

Solution: x = 3, $x \neq -2$ *, Process: Set the numerator = 0 to find the solution for x and set the denominator = 0 to find the domain restrictions.*

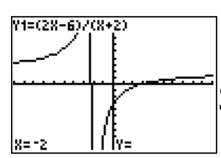
Activity:

- Use *SQPL (Student Questions for Purposeful Learning)* (view literacy strategy descriptions) to set the stage for graphing rational functions with a horizontal asymptote at $y = 0$ and one or more vertical asymptotes. *(Teacher note: Finding complex horizontal asymptotes and graphing complicated rational functions is a skill left to Precalculus because of its relationship to limits.)*
  - In this literacy strategy, create an *SQPL* lesson by first looking over the material to be covered in the day's lesson. Then, generate a statement related to the material that would cause students to wonder, challenge, and question. The statement does not have to be factually true as long as it provokes interest and curiosity.
  - Before graphing rational functions on their graphing calculators, the students will generate questions they have about the graphs based on an *SQPL* prompt.
  - Tell students they are going to be told something about the graphs before they graph them. State the following: ***"The graphs of rational functions follow the same rules learned in Unit 2 about the graphs of polynomials."*** Write it on the board or a piece of chart paper. Repeat it as necessary.
  - Next, ask students to turn to a partner and think of one good question they have about the graphs based on the statement: ***The graphs of rational functions follow the same rules learned in Unit 2 about the graphs of polynomials.*** As students respond, write their questions on the chart paper or board. A question that is asked more than once should be marked with a smiley face to signify that it is an important question. When students finish asking questions, contribute additional questions to the list as needed. Make sure the following questions are on the list:
    1. Is the end-behavior the same for odd and even degree factors?
    2. How do you locate the zeros?
    3. How do the domain restrictions affect the graph?
    4. Is there a hole in the graph at the domain restrictions?
    5. How do you find the y-intercept?
  - Proceed with the following calculator practice before addressing the questions.

- Since graphs of rational functions are difficult to see on the graphing calculator, before distributing the discovery worksheet, have the students graph $f(x)=\dfrac{2x-6}{x+2}$ from the Bellringer on their calculators.

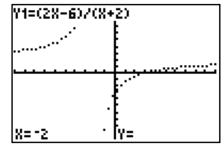


  - Ask them to find the zero of the graph.
  - Ask the question, "What do you see at $x = -2$?" *(Some students*

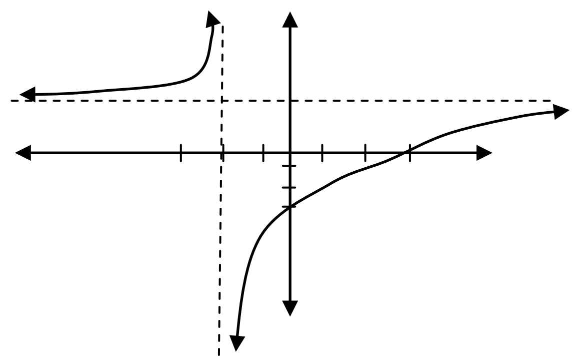


may see a line and some may not depending on the tolerance and number of pixels in their calculators. This line is simply connecting the pixels because the calculator is in connected mode.)

- Have students find *f(–2)* by tracing to $x = -2$ which has no *y* value.
- Change the calculator from connected mode to dot mode to show that there really is no graph at $x = -2$. *(The connected mode gives an easier graph to see as long as the students realize that the line is not part of the graph.)*



- Define *asymptote* as a line a graph approaches near infinity (i.e., as *x* or *y* gets extremely large ($x \to +\infty$ or $y \to +\infty$) or small ($x \to -\infty$ or $y \to -\infty$)) and demonstrate how to draw a dotted line at the vertical asymptote on a graph.
- Have students trace to a large *x*-value and define the *y*–value it approaches as the horizontal asymptote. Have students draw a dotted line at $y = 2$. Tell students that all the graphs today will have a horizontal asymptote at $y = 0$. Other horizontal asymptotes will be explored in Precalculus next year.



- After this practice, have students add questions to the previously generated *SQPL,* such as:
  1. Can a graph cross the vertical asymptote?
  2. How do you know end–behavior on either side of the vertical asymptote?
  3. Can a graph cross the horizontal asymptote?
  4. When is the graph above and/or below the horizontal asymptote?
  5. What effect do ± signs in the numerator have?
- Vertical Asymptotes Discovery Worksheet BLM:
  - This discovery worksheet will explore how the factors in the denominator and the exponents on the factors in the denominator affect the graphs of equations in the form $y = \frac{1}{(x-a)^n}$.
  - Distribute the Vertical Asymptotes Discovery Worksheet BLM and have students graph the example on their calculators with the specified window setting. Remind students to dot the asymptotes and review how to find the *y*-intercepts.
  - Tell students to look carefully for the answers to the questions generated from the *SQPL* prompts as they graph the functions. Have students work with their partners to graph #1 – 3. Stop after graph #3 and ask students if they have found answers to any of their questions. Allow students to confer with their partners before responding. Mark questions that are answered.
  - Continue this process until all the graphs are completed. Go back to the list of questions to check which ones may still need to be answered. Remind students they should ask questions before they learn something new, then listen and look for answers to their questions.
  - Now have students answer the questions on the back of the worksheet and complete the

worksheet.

- Assign the Activity-Specific Assessment to check for individual understanding.

Activity 8: Rational Equation Lab "Light at a Distance" (GLEs: 4, 6, 7, 10, 25, 29)

Materials List: one set of the following for each lab group of students: graphing calculator with EasyData application or BULB program, CBL data collection interface, light sensor probe, meter stick or tape measure, masking tape, dc–powered point light source, Rational Equations Lab BLM, Rational Equations Lab Data Collection & Analysis BLM

It is important that students get to experience the use of rational functions in applications. In the lab in this activity, the students use a light sensor along with a CBL unit to record light intensity as the sensor moves away from the light bulb.

- Rational Equations Lab:
  - This lab is "Light at a Distance: Distance and Intensity," Activity 16 in *Real World Math Made Easy*, Texas Instruments Incorporated (2005). In this activity, the students will explore the relationship between distance and intensity for a light bulb which results in a rational equation. The Rational Equation Lab Teacher Information BLM explains the best way to conduct the lab.
  - Distribute the Rational Equations Lab BLM, the Rational Equations Lab Data Collection and Analysis Sheet BLM, and the equipment listed in the lab and materials list above.
  - Divide the students in groups and allow them to proceed on their own. When the lab is complete, they should turn in the Rational Equations Lab Data Collection & Analysis BLM to be graded using the rubric in the Activity–Specific Assessments.
  - Have students write a paragraph outlining what they learned from the lab and what they liked and disliked about the lab.

- Alternate Projects if CBL equipment is unavailable:
  - *Whelk-Come to Mathematics: Using Rational Functions to Investigate the Behavior of Northwestern Crows*, http://illuminations.nctm.org/index_o.aspx?id=143 – Students make conjectures, conduct an experiment, analyze the data, and work to a conclusion using rational functions to investigate the behavior of Northwestern Crows.
  - *Alcohol and Your Body* by Rosalie Dance and Hames Sandifer (1998), http://www.georgetown.edu/projects/handsonmath/downloads/alcohol.htm - Students use rational functions to model elimination of alcohol from the body and learn to interpret horizontal and vertical asymptotes in context.

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Activity 9: Rational Inequalities (GLEs: 4, 5, 6, 9, 10, 24, 27, 29)

Materials List: paper, pencil, graphing calculator, Rational Inequalities BLM

In this activity, the students will solve rational inequalities using a sign chart.

Math Log Bellringer:

Solve for x using a sign chart: $(x – 3)(x + 4)(x – 5) \geq 0$. Explain why $x = –2$ is a solution to the inequality even though $–2 < 0$ when the problem says "≥ 0."

Solution: $[-4, 3] \cup [5, \infty)$

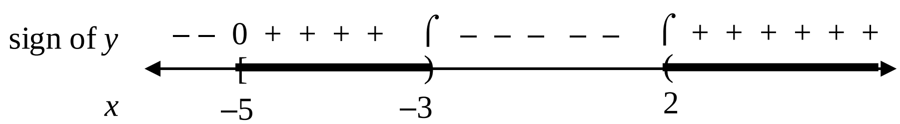
Activity:

- Use the Bellringer to review the concept that the zeros create endpoints to the intervals of possible solutions to polynomial inequalities. Have students locate the zeros on a number line and check numbers in each interval. Determine that the students' explanations include a review of the properties of inequalities.
  (1) $ab > 0$ if and only if $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$. (Review compound sentence use of *and* and *or*.)
  (2) $ab < 0$ if and only if $a < 0$ and $b > 0$ or $a > 0$ and $b < 0$.

- Use an *anticipation guide* (view literacy strategy descriptions) to set the stage for solving rational inequalities. The *anticipation guide* involves giving students a list of statements about the topic to be studied and asking them to respond to them before reading and learning, and then again after reading and learning. This strategy is especially helpful to struggling and reluctant learners as it heightens motivation and helps students focus on important content. Write the following statements on the board and tell students to respond individually to the statements as "true" or "false" and be prepared to explain their responses:

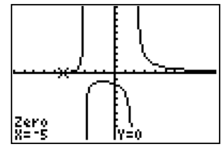
(1) The only solutions for $\frac{2x+10}{(x-2)(x+3)} \geq 0$ would occur when $2x+10 \geq 0$, or $x-2 \geq 0$, or $x+3 \geq 0$.
(2) The answers will always be closed intervals because of the $\geq$ sign.

  - Put students in pairs to solve the inequality under a heading on their paper entitled "My Original Solution." They will usually answer $x > –5$. Ask if $x = 1$ is a solution.
  - Remind students of the sign chart used previously to solve polynomial inequalities and discuss how it could be used with rational inequalities with intervals created by the zeros and the denominator restrictions.
  - Have students work the problem again using a sign chart under a heading on their paper entitled "My Final Solution."
    (Ans: $[-5, -3) \cup (2, \infty)$. *The symbol* ʃ *on the sign chart means "does not exist.")*



- Stop periodically as answers are generated to consider the statements from the *anticipation guide* and have students reconsider their pre-lesson responses. Students should revise their original responses to reflect their new learning.
- Discuss inclusion or non-inclusion of endpoints, thus open or closed intervals, based on denominator domain restrictions.

- Have students graph $y=\dfrac{2x+10}{(x-2)(x+3)}$ on their graphing calculator and discuss that since graphs of rational functions are not easily graphed by hand, finding the solution intervals for inequalities is easier using a sign chart rather than a graph.



- Have students solve $\dfrac{-1}{x-3}\le 1$. Discuss why they cannot multiply both sides of the equation by $x-3$ to solve (*because the inequality sign would change if the denominator is negative.*). Have students understand that they must isolate zero and find the LCD to form one rational expression, then find the zeros and the domain restrictions to mark the number line intervals for the sign chart.

  Solution: $\dfrac{-1}{x-3}\le 1 \Rightarrow \dfrac{2-x}{x-3}\le 0 \Rightarrow x\le 2$ or $x>3$

- Have students refer to their "Final Solutions" in their *anticipation guides* to develop the steps for solving rational inequalities with a sign chart:
  (1) Isolate zero and find the LCD to form one rational expression.
  (2) Set the numerator and denominator equal to 0 and solve the equations.
  (3) Use the solutions to divide the number line into regions.
  (4) Find the intervals that satisfy the inequality.
  (5) Consider the endpoints and exclude any values that make the denominator zero.

- Guided Practice in Solving Rational Inequalities:
  - Distribute the Rational Inequalities BLM. Allow students to work in pairs to practice solving rational inequalities using a sign chart.
  - When they have completed the worksheet, have each pair of students put the sign charts and answers on the board for others to agree or disagree. Clarify.

Sample Assessments

General Assessments

- Use Bellringers as ongoing informal assessments.
- Collect the Little Black Books of Algebra II Properties and grade for completeness at the end of the unit.
- Monitor student progress using small quizzes to check for understanding during the unit

on such topics as the following:
  (1) multiplying and dividing rational expressions
  (2) adding and subtracting rational expressions
  (3) solving rational equations
  (4) finding domain restrictions and vertical asymptotes

- Administer two comprehensive assessments:
  (1) adding, subtracting, multiplying, dividing, and simplifying rational expressions, specifying denominator restrictions
  (2) solving rational equations and inequalities, finding vertical asymptotes and matching graphs of rational functions

Activity-Specific Assessments

Teacher Note: Critical Thinking Writings are used as activity-specific assessments in many of the activities in every unit. Post the following grading rubric on the wall for students to refer to throughout the year.

2 pts.	- answers in paragraph form in complete sentences with proper grammar and punctuation
2 pts.	- correct use of mathematical language
2 pts.	- correct use of mathematical symbols
3 pts./graph	- correct graphs (if applicable)
3 pts./solution	- correct equations, showing work, correct answer
3 pts./discussion	- correct conclusion

- Activity 2: Critical Thinking Writing

  (1) Describe what is similar about simplifying both expressions:
  $\frac{42}{72}=\frac{7}{12}$ and $\frac{x^2-2x}{12x+12}\cdot\frac{7x^2+21x+14}{x^3-4x}=\frac{7}{12}$
  (2) What error did the student make when writing the statement below?
  The reciprocal of $\left(\frac{x}{5}+\frac{7}{x}\right)$ *is* $\left(\frac{5}{x}+\frac{x}{7}\right)$.
  (3) Describe how to find the correct reciprocal. Find the correct reciprocal and simplify.
  Solutions:
  (1) Both expressions have common factors that should be cancelled.
  (2) The reciprocal of a sum is not the sum of the reciprocals.
  (3) Find a common denominator first, then reciprocate. $\frac{5x}{x^2-35}$

- Activity 3: Critical Thinking Writing

  (1) Describe the common process used to find both sums:

$$\frac{5}{12}+\frac{4}{15}=\frac{41}{60} \text{ and } \frac{5}{2x^2y}+\frac{4}{3xy^3}=\frac{15y^2+8x}{6x^2y^3}$$

(2) What error did the student make when subtracting the rational expressions below?

$$\frac{a}{c}-\frac{b-d}{c}=\frac{a-b-d}{c}$$

(3) Describe the process to simplify $\frac{a}{c}-\frac{b-d}{c}$ and simplify it correctly.

Solutions:

(1) I found a least common denominator (LCD) then multiplied each term on the left side of the expression by the identity element that would equal the LCD. In the first equation, the LCD is 60 so I multiplied $\left(\frac{5}{12}\right)\left(\frac{5}{5}\right)$ *and* $\left(\frac{4}{15}\right)\left(\frac{4}{4}\right)$ *and then added numerators. In the second equation, the LCD is* $6x^2y^3$*, so I multiplied* $\left(\frac{5}{2x^2y}\right)\left(\frac{3y^2}{3y^2}\right)$ *and* $\left(\frac{4}{3xy^3}\right)\left(\frac{2x}{2x}\right)$ *and then added numerators.*

(2) The student did not distribute the negative sign.

(3) Distribute the negative sign to change the expression to $\frac{a}{c}+\frac{-b+d}{c}$ *then add numerators and put over the common denominator.* $\frac{a-b+d}{c}$

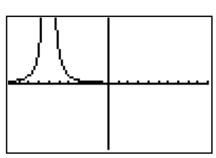
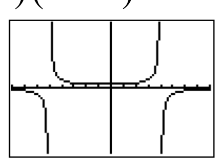
- <u>Activity 4</u>: Critical Thinking Writing

Put students in groups of three to simplify three complex fractions from the text. Have each student take one of the problems and write a verbal explanation of the step-by-step process used to simplify the problem, including all the properties used and why. They should critique each other's explanations before handing in the assessment.

- <u>Activity 7</u>:

Graph the following rational functions without a graphing calculator. Label and dot the vertical and horizontal asymptotes and locate and label the *y*–intercepts.

(1) $f(x)=\frac{1}{(x+6)^2}$ (2) $g(x)=\frac{-1}{(x+5)(x-4)^3}$

Solutions: (1)  *(2)* 

- Activity 8:

Evaluate the Lab Report for "Light at a Distance" (see activity) using the rubric below:
Grading Rubric for Labs –

10 pts./ question	- correct graphs and equations showing all the work
2 pts.	- answers in paragraph form in complete sentences with proper grammar and punctuation
2 pts.	- correct use of mathematical language
2 pts.	- correct use of mathematical symbols

- Activity 9:

Solve the following rational inequalities using a sign chart.

(1) $\dfrac{x+3}{x+6} > 0$ (2) $\dfrac{5-x}{x^2-5x+4} \le 0$

Solutions: (1) $(-\infty, -6) \cup (-3, \infty)$, (2) $(1, 4) \cup [5, \infty)$