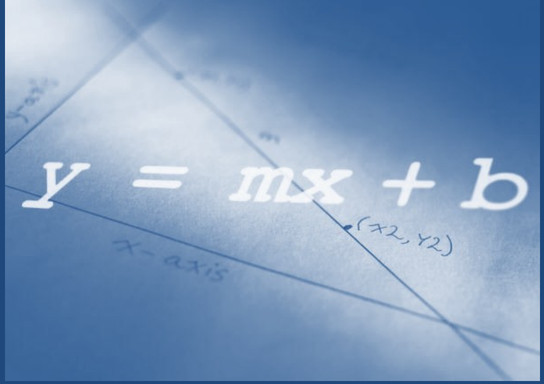
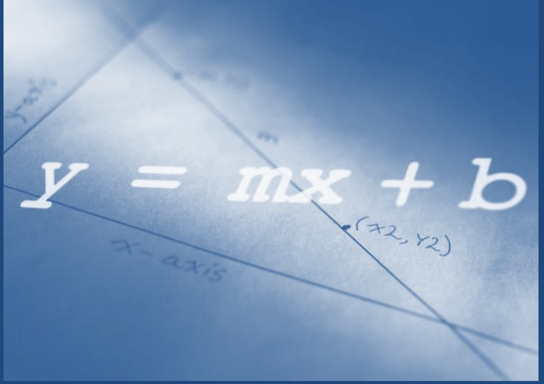


Algebra

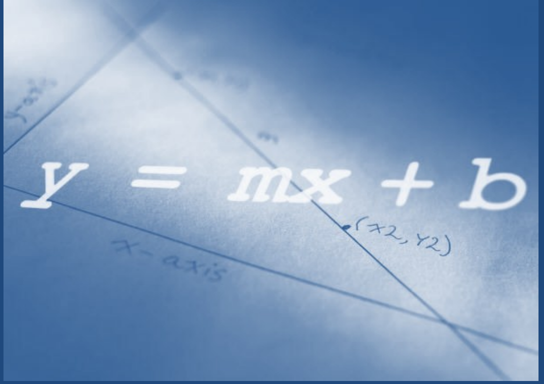
Overall Aim in First Year



The Common Introductory Course

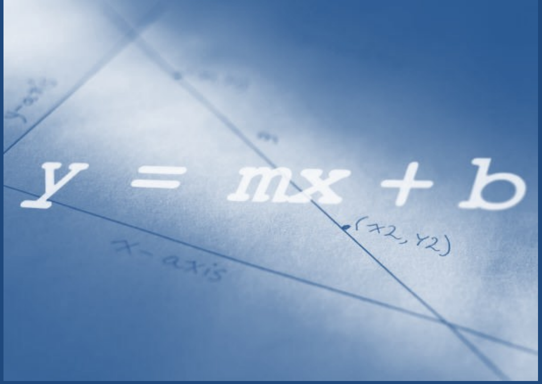
The CIC is only intended as an introduction. For most students they will achieve more than the CIC in First Year. It’s desirable that students will have done their basic algebra skills and equations by the end of First Year.

Algebra Session

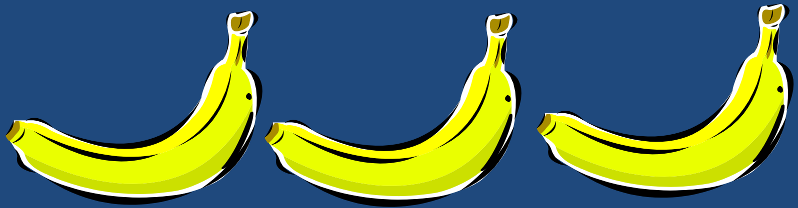


1. Why is it important to do “Patterns” before algebraic skills?
   – It’s all about the variable
2. Unknowns: Solving Equations (Teaching & Learning Plan)
3. Addressing common misconceptions in algebra
4. Factors
5. Graph Matching Activity
6. Syllabus Review

1 Why is it important to do "Patterns" before Algebra?

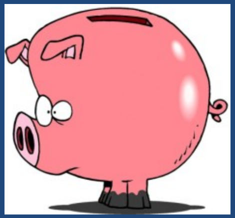


Traditionally, we may have introduced algebra something like this…….



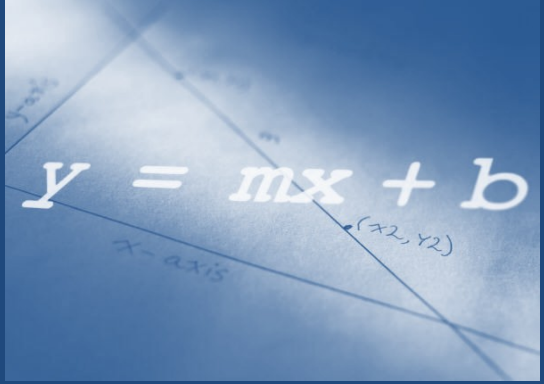
$$4a \quad + \quad 3b$$

Money Box Problem

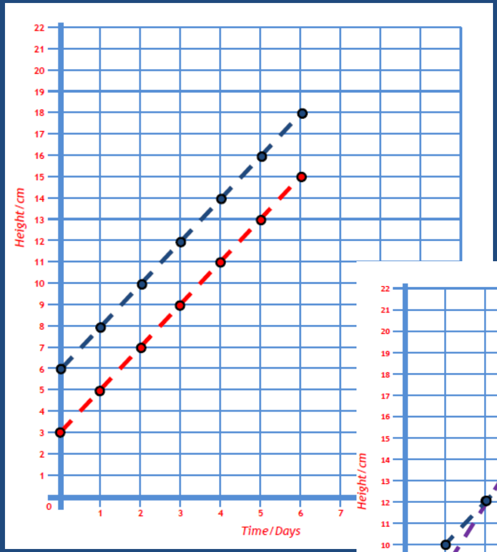


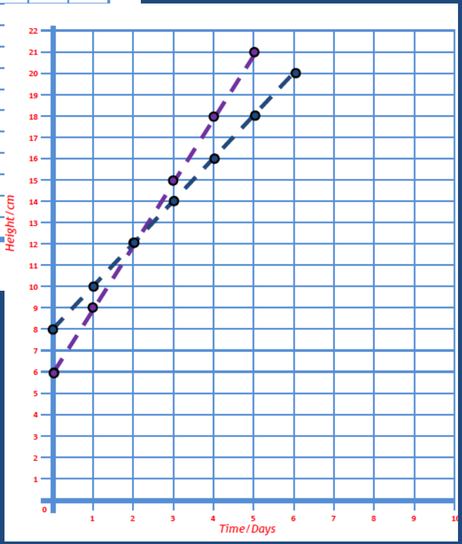
Sunflowers Question

Problem in Words



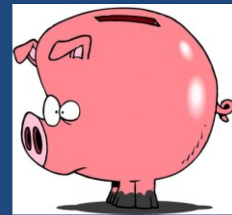
t/d	h/cm	
0	3	
1	5	
2	7	
3	9	
4	11	
5	13	
6	15	





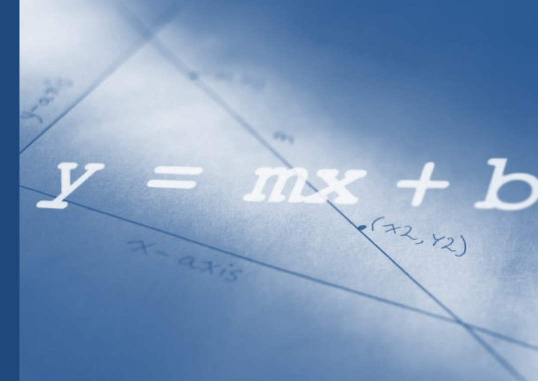
$$h = 3 + 2d$$

Money Box Problem



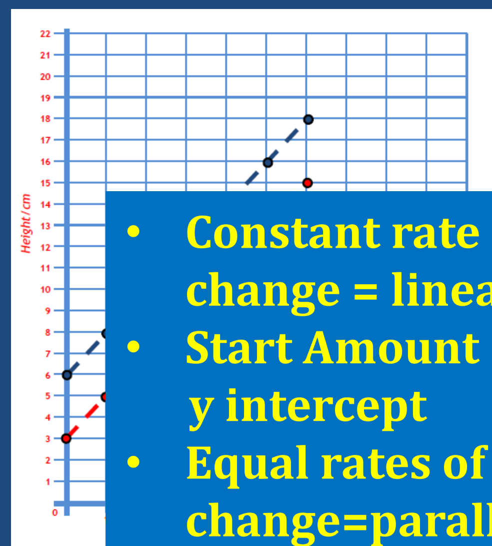
Sunflowers Question

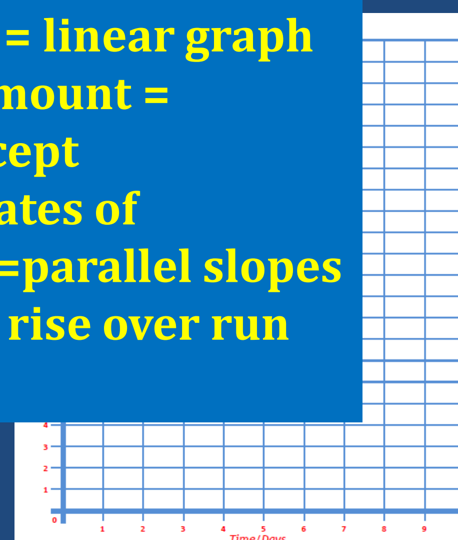
Problem in Words



t/d	h/cm	
0	3	
1	5	
5	13	
6	15	

- Start Amount
- Variables
- Constant rate of change





- Constant rate of change = linear graph
- Start Amount = y intercept
- Equal rates of change=parallel slopes
- Slope = rise over run

$$h = 3 + 2d$$

- Made a formula to describe the pattern

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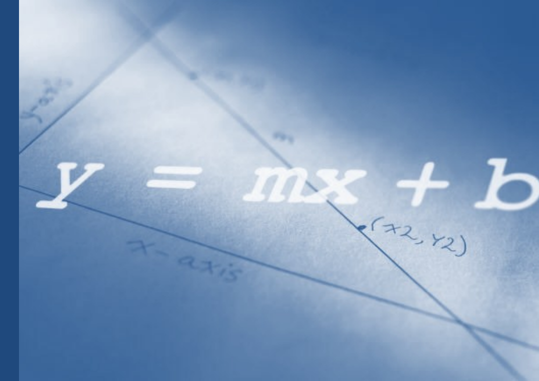
2

Unknowns

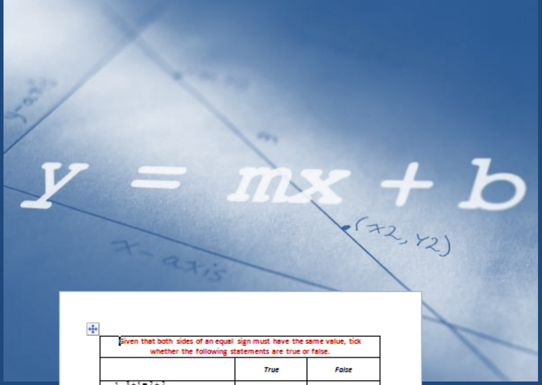


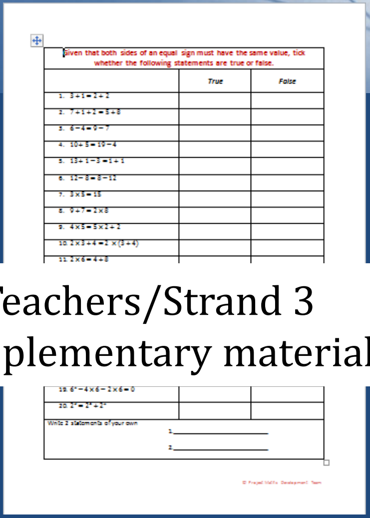
Once a variable is understood as a varying quantity, the Money Box Problem can then be used to introduce the concept of an unknown

The formula/rule for John's savings in his money box was: $\boxed{a = 3 + 2d}$

Question: For how many days did John need to save his pocket money in order to accumulate €42 for a video game?

Equations
Teaching & Learning Plan
from Workshop 4





Given that both sides of an equal sign must have the same value, tick whether the following statements are true or false.		
	True	*False*
1. $3+1=2+2$		
2. $7+1+2=5+8$		
3. $6-4=9-7$		
4. $10+5=19-4$		
5. $13+1-3=1+1$		
6. $12-8=8-12$		
7. $3\times5=15$		
8. $9+7=2\times8$		
9. $4\times5=5\times2+2$		
10. $2\times3+4=2\times(3+4)$		
11. $2\times6=4+8$		
19. $6^2-4\times6-2\times6=0$		
20. [illegible]		
Write 2 statements of your own 1.________ 2.________		

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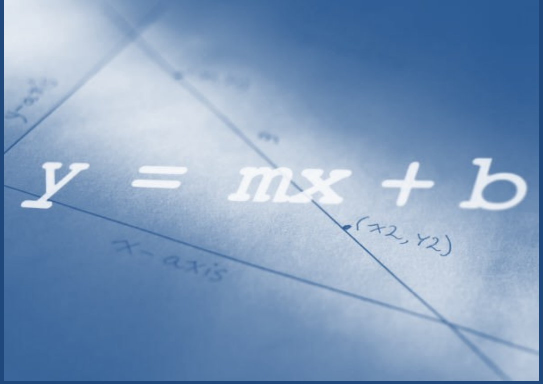
Consolidate the idea of Equality

Projectmaths.ie/Teachers/Strand 3
/Junior Cycle/Supplementary material

Useful methodology of Stabilisers for getting started!

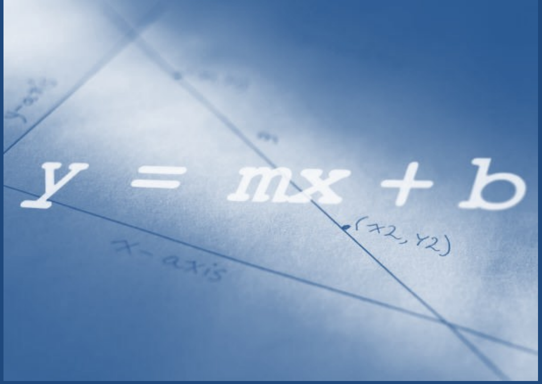
3

Addressing Common Misconceptions



What are some of the misconceptions that students make with their algebra skills?

Look at Common Misconceptions and Methodologies for:



- Numbers and letters
- Displaying expressions
- Understanding that only like terms can be added or subtracted
- Multiplying and dividing in algebra
- Order of operations
- Factorising

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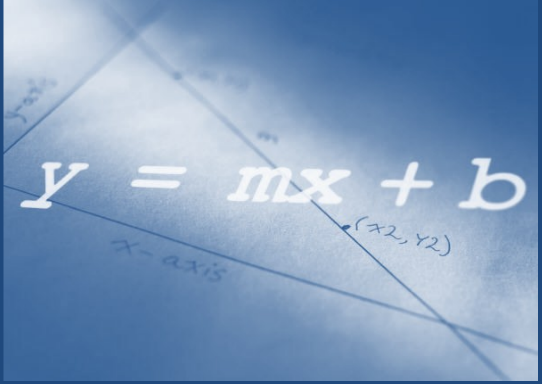
Expressions, Substitution, Adding , Subtracting



a b c 1

a b c 9

a b -4

a b

b

+

$4a \quad + 5b \quad + 2c \quad + \quad 6$

$4(5) \quad + 5(2) \quad + 2(8) \quad + \quad 6$ **= 52**

Misconception:
Symbols don't stand for numbers

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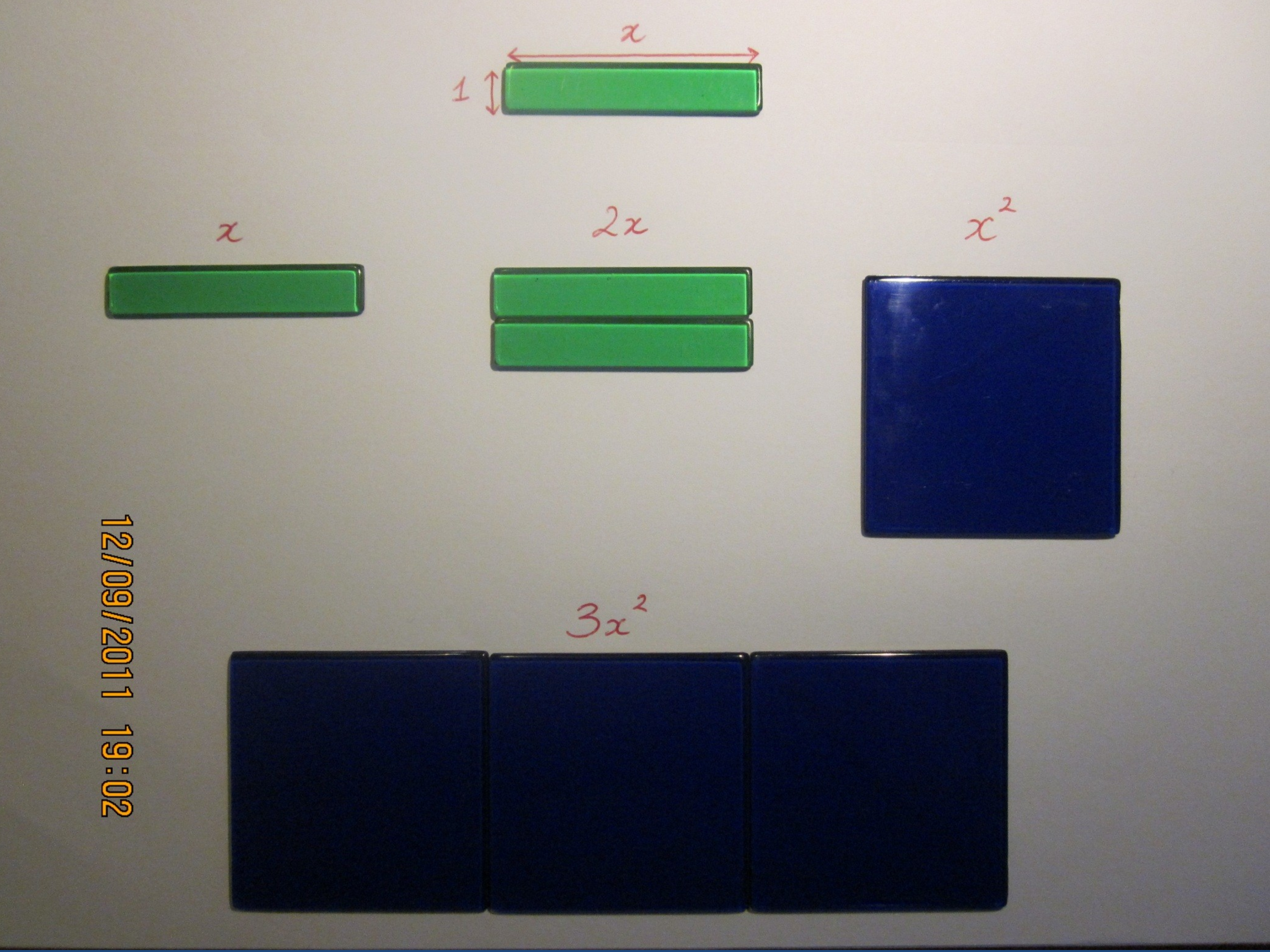
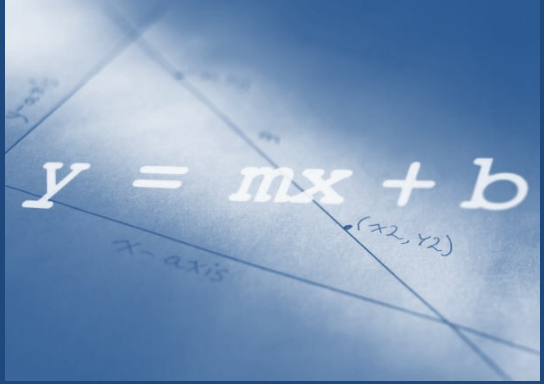
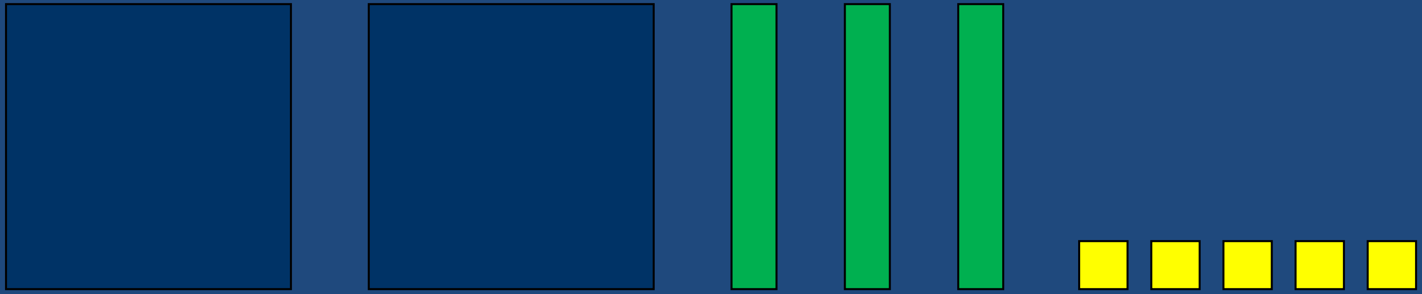


x
1
x
$2x$
x^2
$3x^2$
12/09/2011 19:02

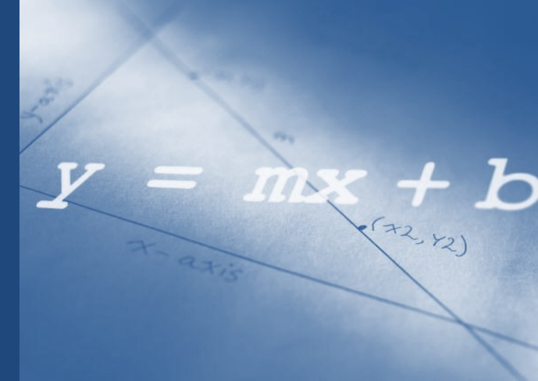
Displaying Expressions: Adding Terms: The Array Model



- $2x^2 + 3x + 5$



Displaying Expressions
Activity: Array Model Activity



Page 22

Draw the following areas:

x, y, $2x$, x^2, $4x^2$, $2(x+y)$, $2x+2y$

where $x \neq y$

Question: Is $2(x+y) = 2x + 2y$? Discuss.

Question: Is $2x \neq x^2$ always, sometimes or never?

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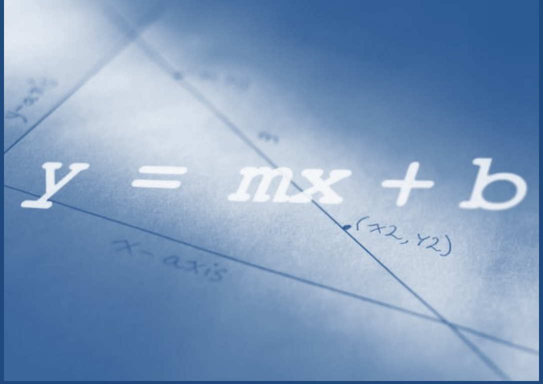
What Have Students Learned?

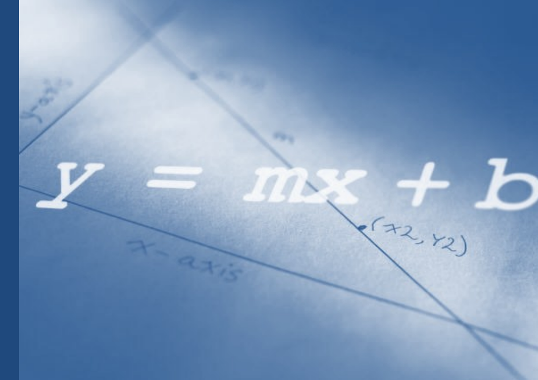


- $2x \neq x^2$, except when $x = 2$

- $2(x+y) = 2x+2y$

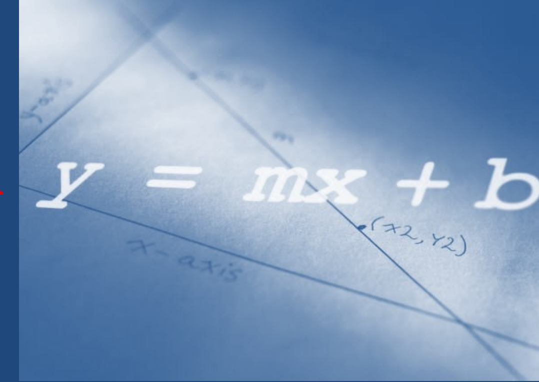
- They are comfortable that $2x+2y$ is an expression that does not need any more work. $2x+2y$ can represent a finished answer.

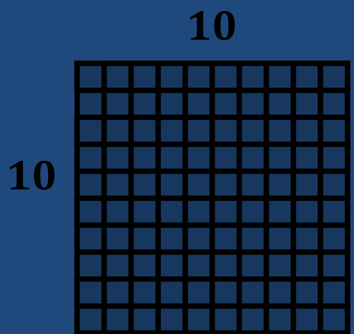
Multiplying and Dividing Expressions

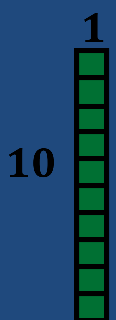


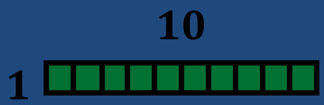
$$(x+3)^2 \neq x^2+3^2$$

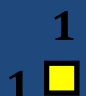
Generate Array Models
Using Numbers as an Introduction

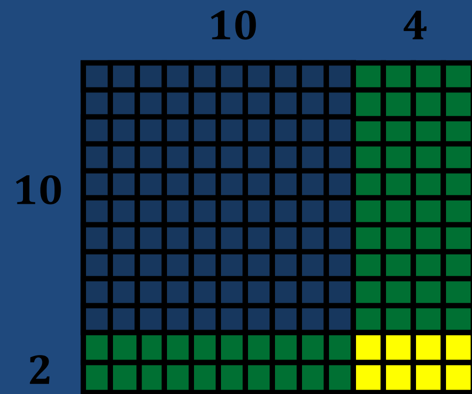




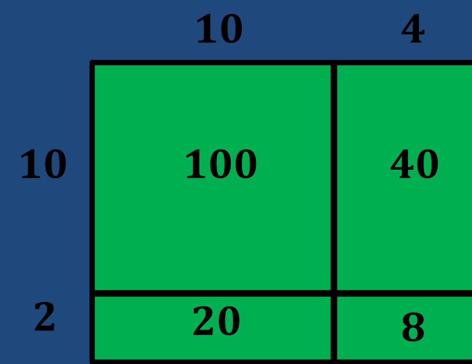








Total Area = $10 \times 10 + 10 \times 4 + 2 \times 10 + 2 \times 4$
= $100 + 40 + 20 + 8$
= 168



Total Area = $100 + 40 + 20 + 8$
= 168

Check

$$\begin{array}{r} 14 \\ \times\ 12 \\ \hline 168 \end{array}$$

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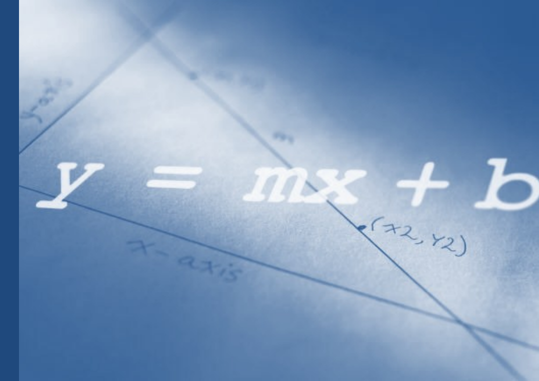
Distributive Law

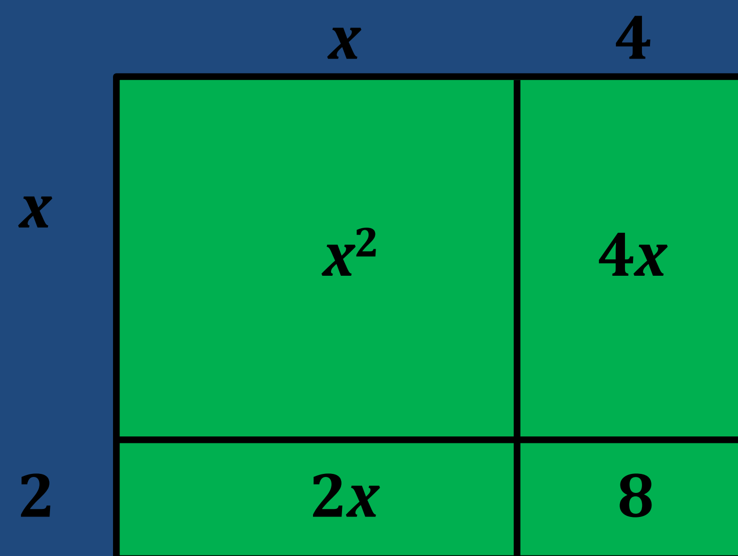


Multiply $(x + 2)$ by $(x + 4)$

$= (x + 2)(x + 4)$

$= x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$



Using the Distributive Law

$= \quad (x + 2)(\underline{x + 4})$

$x(\underline{x + 4}) + 2(\underline{x + 4})$

$= x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$

Activity: Area Model with Numbers

Fill in the answers to the following multiplication sums using array models

1. 13 x 17 = (10 + ___) x (10 + ___)

   = 100 + 30 + ___ + ___

   = 221

	10	___
10	100	30
7	_____	___

2. 27 x 32 = (20 + ___) x (30 + ___)

   = ____ + ____ + ____ + ____

   = _______

	20	___
30	600	___
2	_____	14

3. 35 x 41 = (___ + 5) x (___ + 1)

   = ____ + ____ + ____ + ____

   = _______

	30	___
40	___	200
___	30	___

4. 22 x 15 =

5. 27 x 41 =

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Find the answers to the following multiplication sums using the Distributive Law, then check your answer using an area model.

Name:

Example

$= (x + 2)(x + 4)$

$x(x + 4) + 2(x + 4)$

$= x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$

CHECK WORK USING AN AREA MODEL

	x	2
x	x^2	2x
4	4x	8

→ PLEASE USE THE ARROWS IN YOUR WORK

1. $(x + 5)(x + 3)$

   =

   =

   =

2. $(a + 1)(a + 7)$

   =

   =

   =

4. $(x + y)(x + y)$

   =

   =

   =

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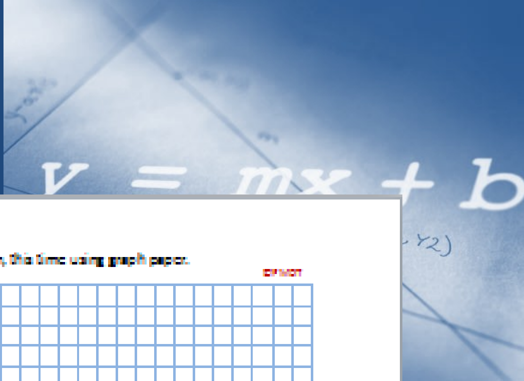
Excerpts from these Worksheets on Page 23

Resource Worksheets



Solution Strategies for Multiplication Name___________________

1. Story: 4 students have 5 balloons each. How many balloons do they have between them in total? Please use 3 different ways to represent your answer – *Diagram, Arithmetic Sentence and Words.*

Diagram	
Arithmetic Sentence (ex. 5 x 8)	Words

2. Story: A chef bought 15 boxes of a dozen eggs to make deserts for a wedding meal. How many eggs did he buy in total? Please use arrays to represent this answer in a diagram.

Diagram using arrays

3. Now, represent the number of eggs the chef bought again, this time using graph paper.

4. Is it still taking too long? Try to find

5. State if the following Arithmetic Sentences are true or false (T/F). Use your diagrams to help you.

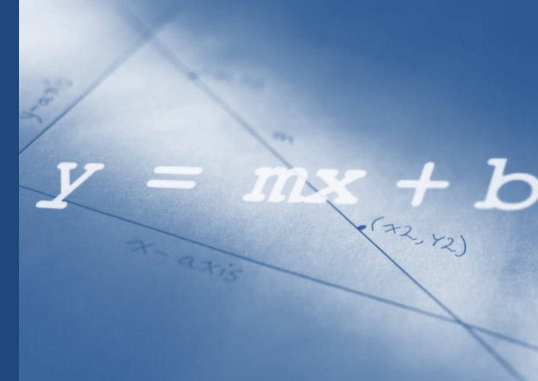
a. 15x 12 *is the same as* (5x 12) + (5 x 12) + (5 x 12) _____
b. 15x 12 *is the same as* (10x 15) + (2 x 15) _____
c. 15 x 12 = (8 x 12) + (7 x 12) _____
d. 15 x 12 = (10x 12) x (5 x 12) _____
e. 15 x 12 = (10 x 12) + (5 x 12) _____
f. 15x 12 = (10 x 10) + (5 x 2) _____
g. 15 x 12 = (15 x 10) + (15 x 2) = 15 x (10 + 2) ____

Worksheets with Teachers Notes

Projectmaths.ie/Teachers/ Strand 3/Junior Cycle/ Supplementary material

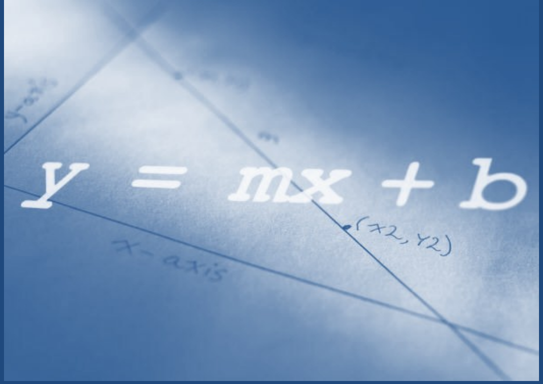
Project Maths

Multiplying and Dividing Expressions



$$(x+3)^2 \neq x^2+3^2$$

More Multiplying



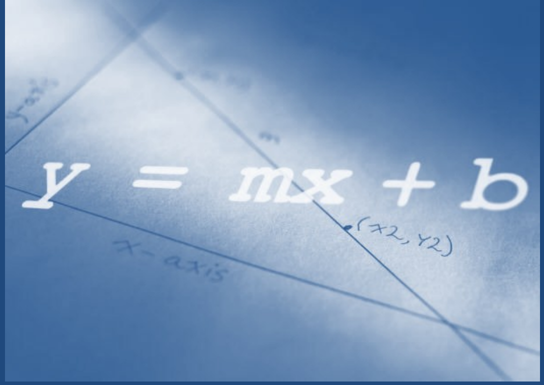
Multiply $(x - 2)(x^2 - 2x + 3)$

	x^2	$-2x$	$+3$
x	x^3	$-2x^2$	$+3x$
-2	$-2x^2$	$+4x$	-6

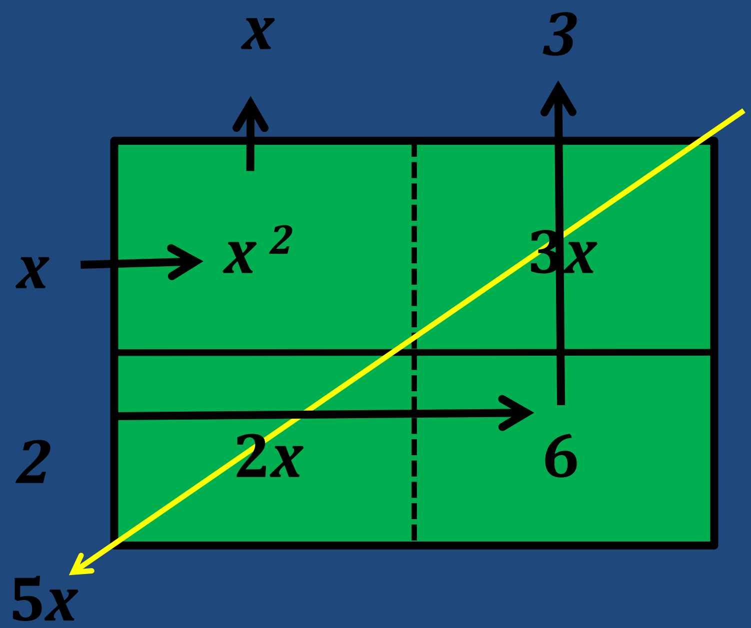
Total Area $= x^3 - 2x^2 - 2x^2 + 3x + 4x - 6$

$= x^3 - 4x^2 + 7x - 6$

Long Dividing



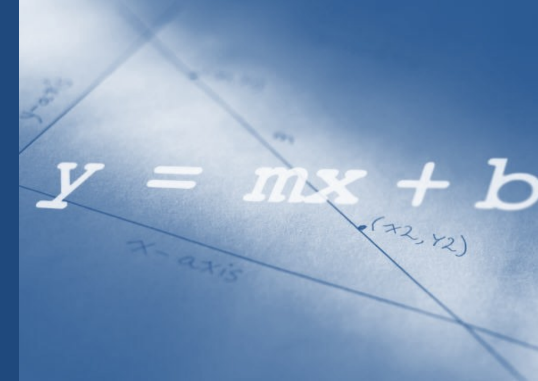
Divide $x^2 + 5x + 6$ by $x + 2$



Check

$3x + 2x = 5x$

Students Work



Leaving Cert HL 2005 1 (c)

2005

$(x-p)^2 = x^2 - 2xp + p^2$

Ans	x	$+2p$
x^2	x^3	$+2px^2$
$-2xp$	$-2px^2$	$-4p^2x$
$+p^2$	$+p^2x$	$+r$

$0x^2$

qx

$x^3 + 0x^2 + qx + r$

$-4p^2x + p^2x = qx$

$-3p^2x = qx$

$2p(p^2) = r$

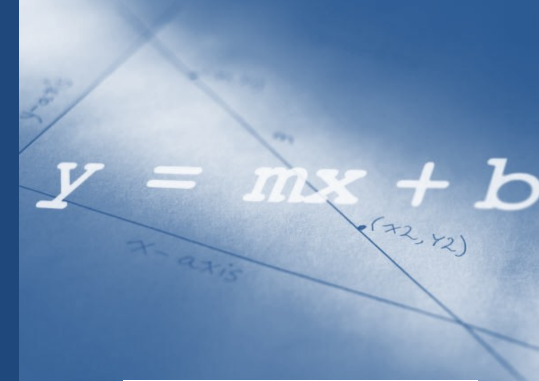
$2p^3 = r$

$27r^2 + 4q^3$

$27(2p^3)^2 + 4(-3p^2)^3$

$27(4p^6) + 4(-27p^6) = 0$

Activity: Order of Operations



Page 24

WS5.19 *Order of Operations*

1. Evaluate $2+3\times4=?$

Answer: $\square$

Class discussion on everyone's answers

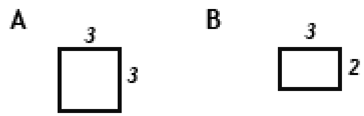
Mathematicians made an agreement that:

4. Another operation to consider is powers. Match the numerical expressions with their corresponding area models by placing A, B, C or D into the box.

(i) 3×2 $\square$

3^2 $\square$

Place A or B in the boxes



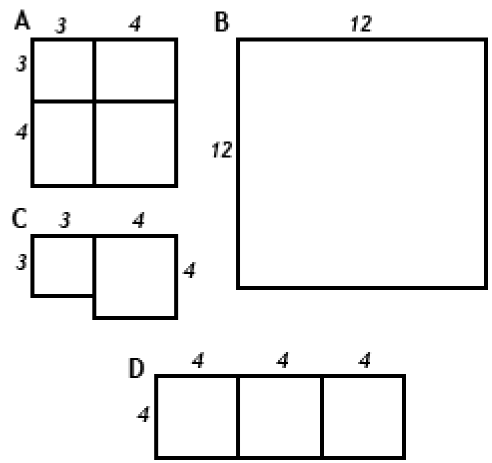
(ii) 3^2+4^2 $\square$

3×4^2 $\square$

$(3\times4)^2$ $\square$

$(3+4)^2$ $\square$

Place one of the letters A, B, C or D in each box



Class discussion on where the powers come in the order of operations and formalise:

B I
M D
A S

D 4 4 4
4

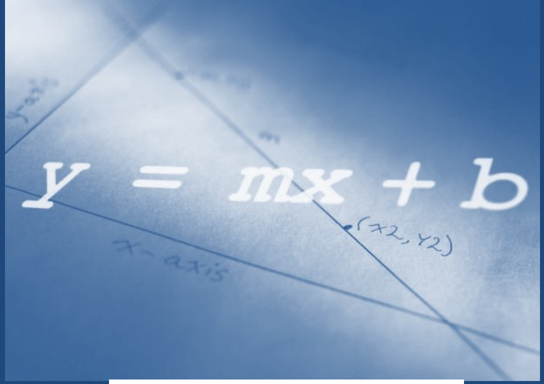
Class discussion on where the powers come in the order of operations and formalise:

B I
M D
A S

Find this worksheet at http://www.projectmaths.ie under Teachers/Strand 3/Junior Cycle/Supplementary Material

<u>Misconception:</u>
Order of Operations

Redo Order of Operations in Algebra



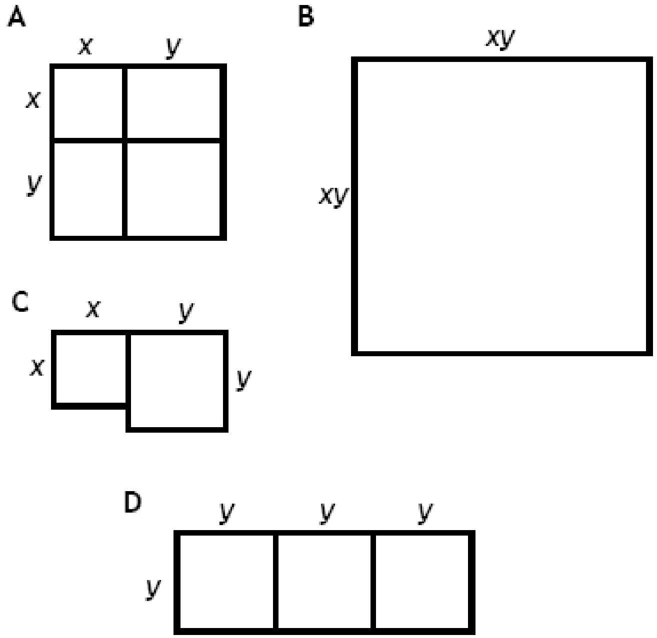
Page 25

(iv) $x^2 + y^2$ ☐

$3 \times y^2$ ☐

$(x \times y)^2$ ☐

$(x + y)^2$ ☐

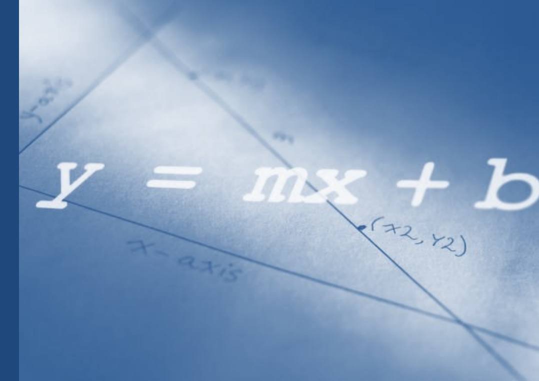


(v) Is $x^2 + y^2 = (x + y)^2$?

Justify your answer:

4

Factorising



JC: Linear & Quadratic Expressions

4 Methods of Factorising

1. Taking Out a Common Factor

2. Grouping

3. Quadratics: $ax^2 + bx + c$

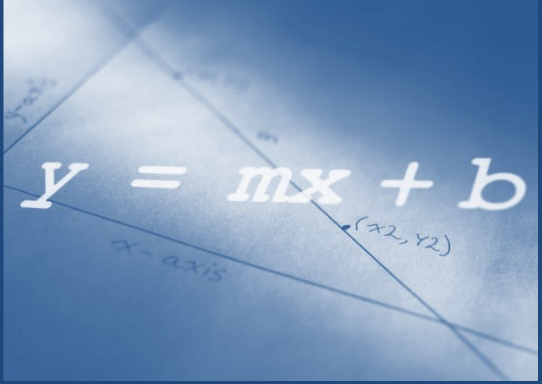
   $ax^2 + bx$

   $ax^2 + c$

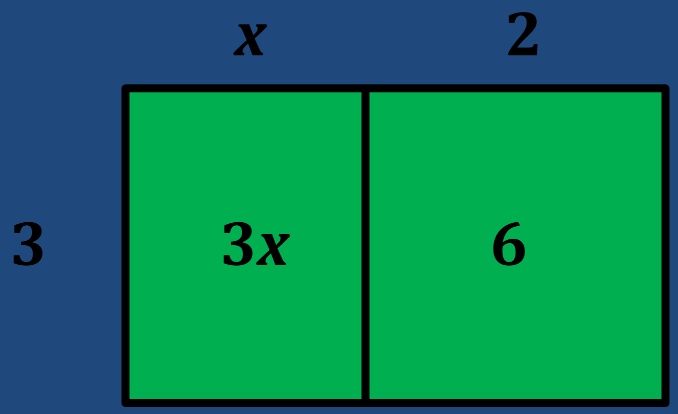
   a, b, c may be equal

4. Difference of Two Squares

1. Taking Out a Common Factor



Factorise $3x + 6$



The factors are $3\,(x + 2)$

Activity

Page 27

Project Maths
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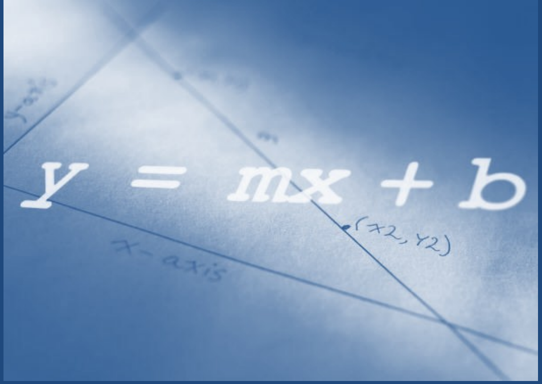
2. Grouping



Factorise $ab - bc + da - dc$

	a	$-c$
b	ab	$-bc$
d	da	$-dc$

The factors are $(b + d)(a - c)$

Activity

Page 27

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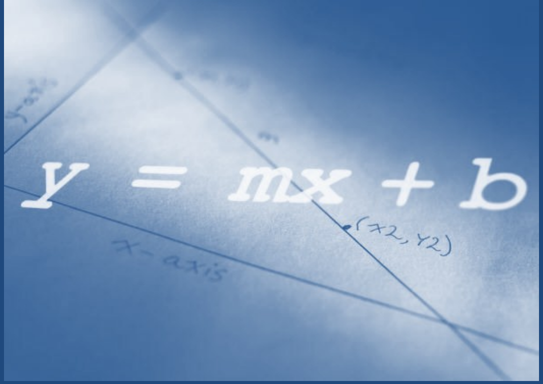
Solving Quadratic Equations
x squared, *x*'s, number equals zero



SOLVE $x^2-5x-14=0$

Can we FACTORISE ?

$(x \quad\quad)(x \quad\quad) = 0$

Guide number
14
1x14
2x7

Subtract to 13
No Good

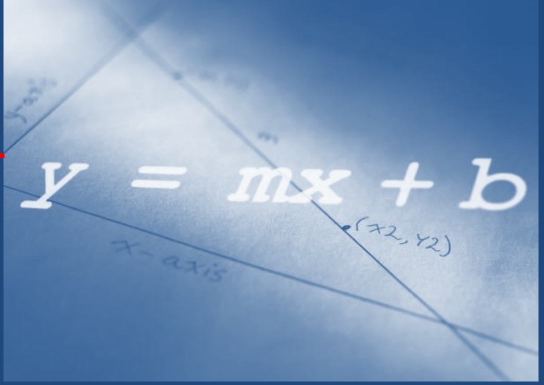
Subtract to 5
Yes! Perfect

Factorised $(x+2)(x-7)=0$

$(x+2)=0 \qquad (x-7)=0$

Solved $x=-2 \qquad x=7$

Reuse the Grouping Method



$\underline{+6}$

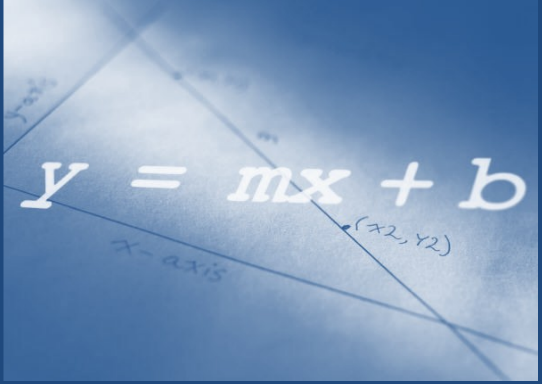
3×2

-3×-2

	x	-3
x	x^2	$-3x$
-2	$-2x$	$+6$

$x^2 - 5x + 6$

x^2 coefficients greater than 1



-42

3×14

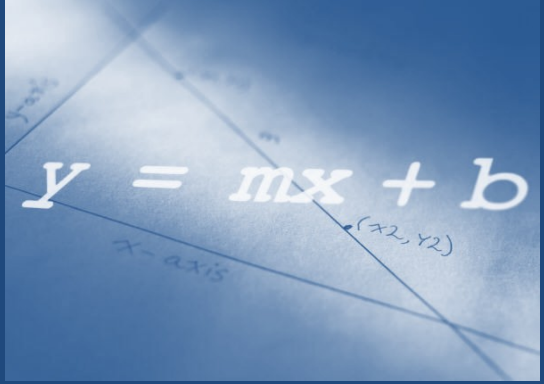
6×7

$2x^2 - 11x - 21$

	x	-7
$2x$	$2x^2$	$-14x$
$+3$	$+3x$	-21

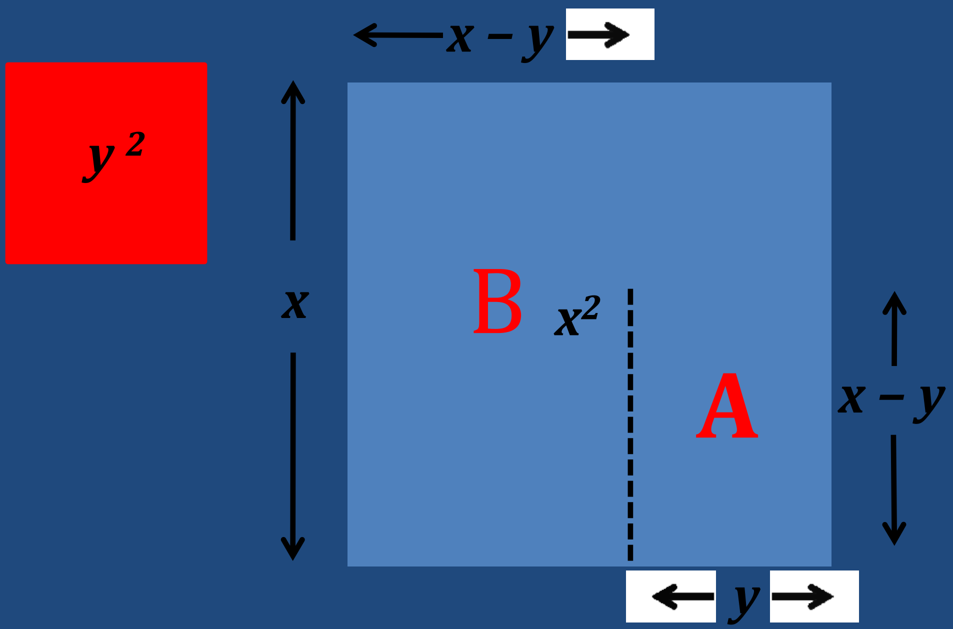
Activity

Page 28



4. Difference of Two Squares

Factorise $x^2 - y^2$

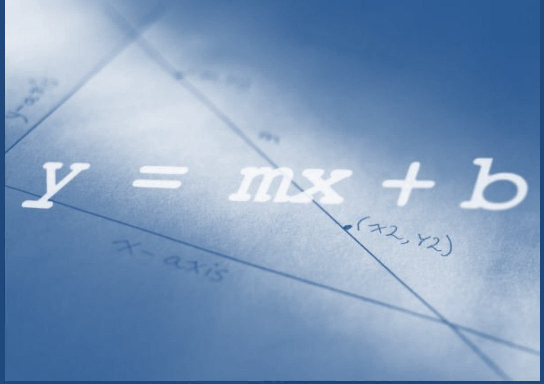


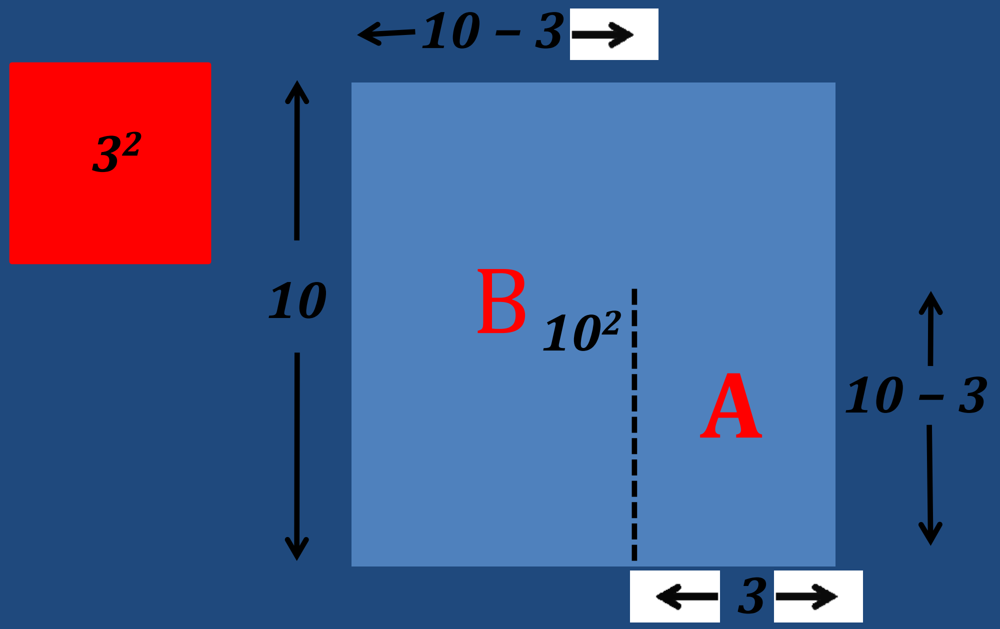
Area of A = $y(x - y)$

Area of B = $x(x - y)$

Area of A + B = $y(x - y) + x(x - y)$
$= (x - y)(x + y)$

$10^2 - 3^2$





Activity

Page 29

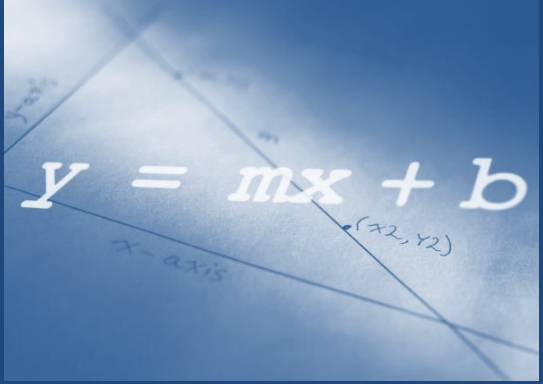
Area of A = $3(10 - 3)$

Area of B = $10(10 - 3)$

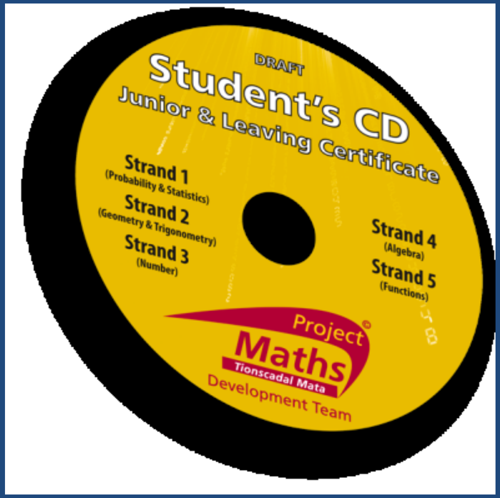
Area of A + B = $3(10 - 3) + 10(10 - 3)$

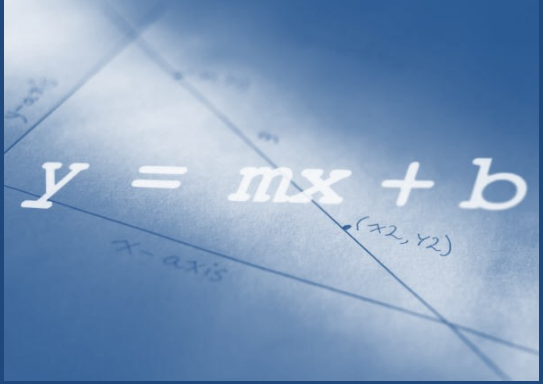
$= (10 - 3)(10 + 3) = 91$

Question to Ponder.....



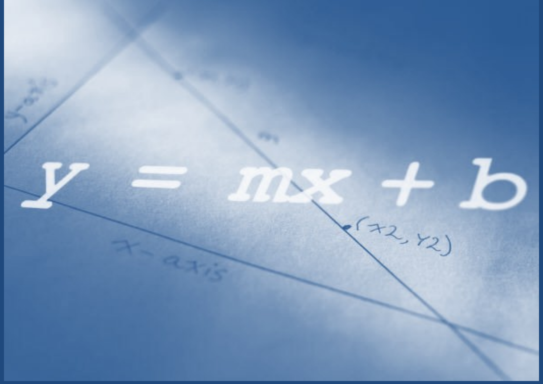
Can you draw a model for the difference of two cubes?





Student’s CD Demo of the Difference of Two Squares Quiz

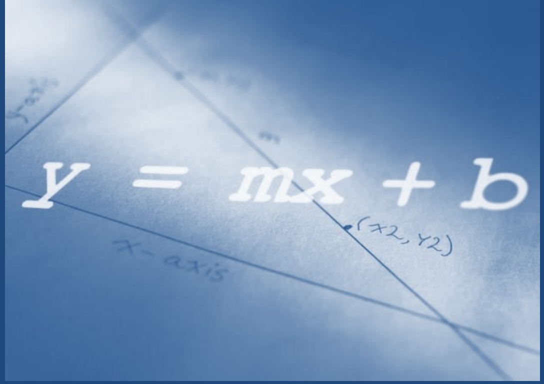
5 Graph Matching Activity



Syllabus:

"The relationships based approach to learning algebra should culminate in students having a deep understanding of algebra which allows easy movement between story, table, graph and equation."

6 Syllabus Review



3.1

Learning outcomes
Students should be able to:

- investigate models such as decomposition, skip counting, arranging items in arrays and accumulating groups of equal size to make sense of the operations of addition, subtraction, multiplication and division, in **N** where the answer is in **N**

- investigate the properties of arithmetic: commutative, associative and distributive laws and the relationships between them including the inverse operation

- appreciate the order of operations, including the use of brackets

- investigate models such as the number line to illustrate the operations of addition, subtraction, multiplication and division in **Z**

- generalise and articulate observations of arithmetic operations

- investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers

- consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value

Learning outcomes
Students should be able to:

- analyse solution strategies to problems

- engage with the idea of mathematical proof

- calculate percentages

- use the equivalence of fractions, decimals and percentages to compare proportions

- consolidate their understanding and their learning of factors, multiples and prime numbers in **N**

- consolidate their understanding of the relationship between ratio and proportion

- check a result by considering whether it is of the right order of magnitude

- check a result by working the problem backwards

- justify approximations and estimates of calculations

- apply the rules for indices (where a ∈ Z, p, q ∈ N):
- $a^p a^q = a^{p+q}$
- $= a^{p-q}, \; p > q$
- $)^q = a^{pq}$

3.2

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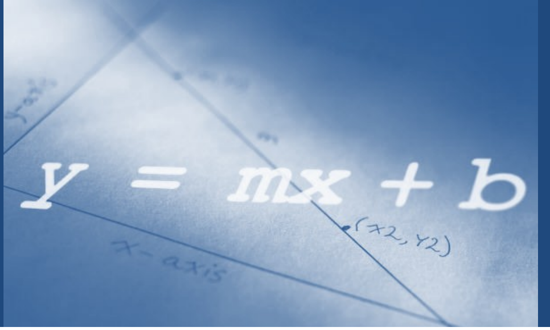
Problem Solving



Syllabus

Students learn about	*Students should be able to*
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar conte – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques o draw relevant conclusions.

- Words
- Real-world Context
- Problem Solving

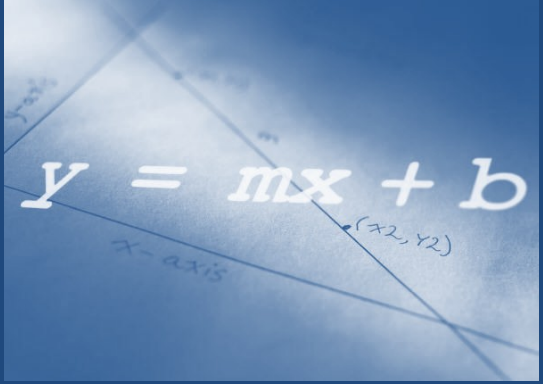
Exam

At all three levels (HL, OL, FL), Section B questions will be of a problem-solving nature.

New Problem Solving Tab on Projectmaths.ie

Page of Problems Page 26

Recap on Algebra



1. Why is it important to do “Patterns” before algebraic skills?
   – It’s all about the variable
2. Unknowns: Solving Equations (Teaching & Learning Plan)
3. Addressing common misconceptions in algebra
4. Factors
5. Graph Matching Activity
6. Syllabus Review

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