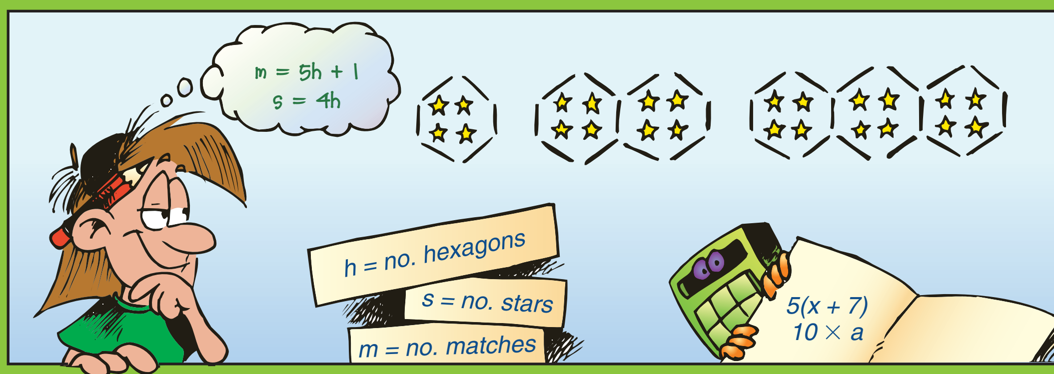
4

Algebraic Expressions



Chapter Contents

Learning Outcomes

Students will be able to:

- Use the algebraic symbols to represent word problems.
- Simplify, expand and factorise simple algebraic expressions.
- Work with expressions involving algebraic fractions.
- Expand binomial products.

Areas of Interaction

Approaches to Learning (Knowledge Acquisition, Problem Solving, Communication, Logical Thinking, Reflection), Human Ingenuity

4:01 | Generalised Arithmetic

Find:

1	the sum of 7 and 5	**2**	the difference between 9 and 2
3	the number 8 less than 25	**4**	the quotient of 48 and 6
5	the product of 7 and 3	**6**	12 more than 8
7	the average of 41 and 47	**8**	the total of 13 and 21
9	the number of times 23 can be taken from 138		
10	the number 8 less than the product of 4 and 5		

prep quiz
4:01

In mathematics, the method of solving a problem is sometimes hard to express in words. In cases like this, pronumerals are often used. The result could be a simple formula.

- Some numbers in a pattern are known. How can we find the others?
  For example: 9, 8, 7, 6, . . .   or   3, 5, 7, 9, . . .
  Patterns like these can be written in a table of values, where n represents the position of the number in the pattern, and T the actual number (or term).

n	1	2	3	4	5
T	9	8	7	6	

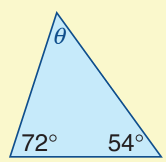
Here we can see that:
$T = 10 - n$
So an algebraic expression that represents this pattern would be:
$10 - n$

n	1	2	3	4	5
T	3	5	7	9	

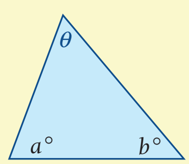
Here we can see that:
$T = 2n + 1$
So an algebraic expression that represents this pattern would be:
$2n + 1$

- Two angles of a triangle are known. How can we find the third?

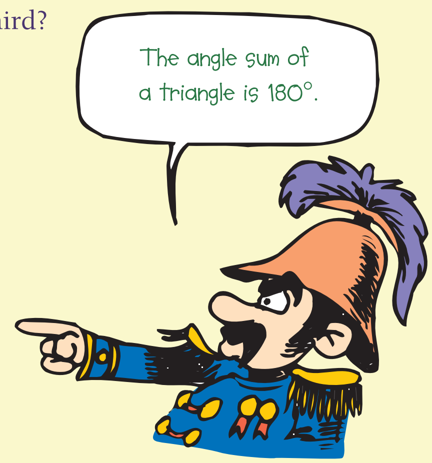
A Consider a numerical example.
$\theta = 180° - (72 + 54)°$
$= 54°$



B Show the general result.
$\theta = 180° - (a + b)°$
or $180° - a° - b°$



$180° - (a + b)°$ is called the *general case*.



worked examples

1 The sum of 8 and 12 $= 8 + 12$
so the sum of x and y $= x + y$

2 The cost of 6 books at 30c each $= 6 \times 30$c
so the cost of x books at 30c each $= x \times 30$
$= 30x$ cents

continued →→→

3 The average of 9 and 13 $= \dfrac{9+13}{2}$

so the average of a and b $= \dfrac{a+b}{2}$

4 The change from \$10 after buying 3 books at \$2 each $= 10 - (2 \times 3)$ dollars

so the change from \$10 after buying x books at \$2 each $= 10 - 2 \times x$

$= 10 - 2x$ dollars

The aim of 'generalised arithmetic' is to write an algebraic expression that shows the steps to be taken, no matter which numbers are involved.

Exercise 4:01

Foundation Worksheet 4:01

Generalised arithmetic

1 Write expressions for:
 a the sum of $3a$ and $2b$
 b the average of m and n

2 a Find the cost of x books at 75c each.
 b Find the age of Bill, who is 25 years old, in another y years.

1 Match each table of values with the correct algebraic expression from the given list to complete the statement $T = \ldots$

a

n	1	2	3	4
T	3	4	5	6

b

n	1	2	3	4
T	−2	−1	0	1

c

n	1	2	3	4
T	3	6	9	16

d

n	1	2	3	4
T	5	8	11	18

e

n	1	2	3	4
T	1	4	9	16

f

n	1	2	3	4
T	1	3	5	7

A $3n$
B n^2
C $n + 2$
D $3n + 2$
E $2n - 1$
F $n - 3$

2 Write down an algebraic expression that represents each pattern of numbers, using n to represent the position of each number in the pattern.

a 2, 4, 6, 8, . . . b 4, 5, 6, 7, . . .
c 7, 6, 5, 4, . . . d 5, 7, 9, 11, . . .
e $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, . . . f −3, −1, 1, 3, . . .

Use a table of values if you need to.

For questions 3 to 8 write expressions for each.

3 a the sum of 5 and 7
 b the sum of 5 and y
 c the sum of x and y

4 a the product of 3 and 7
 b the product of a and 7
 c the product of a and b

5 a the difference between 8 and 3
 b the difference between 8 and p
 c the difference between q and p

6 a the average of 8 and 12
 b the average of 8 and x
 c the average of w and x

7 a the cost of 5 books at 75c each
 b the cost of a books at 75c each
 c the cost of a books at b cents each

8 a dividing 30 cm into 5 equal lengths
 b dividing 30 cm into t equal lengths
 c dividing A cm into t equal lengths

9 **a** If Steve is 15 years old, how old will he be in 6 years?
b How old will Steve be in y years?

10 **a** If a car travels at 60 km/h for 3 hours, how far does it travel?
b If the same car travels for h hours, how far does it travel?

11 **a** If three lengths of rope, each 2 m long, are cut from a piece of rope 10 m long, what length is left?
b If two lengths of rope, each x m long, are cut from a piece of rope X m long, what length is left?

12 **a** A student buys x books and y pens. If each book costs 85c and each pen costs 63c, what is the total cost?
b If the books cost C cents each and the pens D cents each, what is the total cost?

13 Mr Smith is Y years old; his son is 22 years younger. How old is his son? How old will the son be in x years' time?

14 **a** If I travel x km in 2 hours, and then y km in the next 3 hours, how far have I travelled altogether?
b What is my average speed for the whole journey?

15 Bob and Tom have \$1 between them. If Bob has x cents, how much has Tom?

16 **a** What is the next even number after 6?
b What is the next even number after y, if y is an even number?
c What is the largest odd number less than y, if y is even?

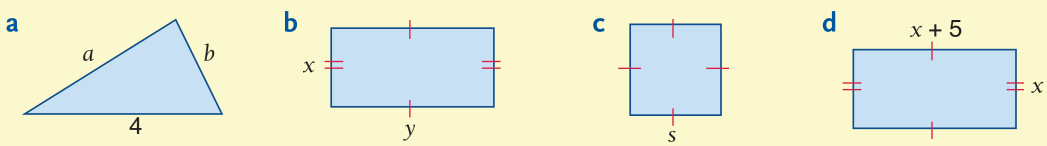
17 The expression $2n$ will give an even number for all values of 'n', where n is an integer. Write down an expression that will always give an odd number.

18 **a** Two angles of a triangle are 25° and 79°. What is the size of the third angle?
b Two angles of a triangle are a° and b°. What is the size of the third angle?

19 **a** How far will a person walk at m km/h in h hours?
b What is the average speed of a car that travels k km in h hours?
c How long will it take to travel k km at m km/h?

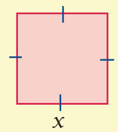
20 **a** A TV set is bought for \$$P$. If it is sold for \$$R$, what is the profit?
b If a gain of \$$G$ is to be made, what should the selling price of the TV be?

21 Write an expression for the perimeter of each figure below.
All measurements are in centimetres.



22 Write an expression for the area of each figure.

a



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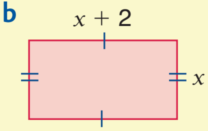

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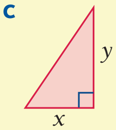

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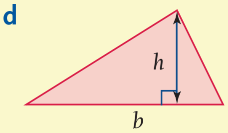


23 In each of the following, use grouping symbols to make the meaning clear.

a the product of 4 and $a + 2$

b subtract $x + y$ from 8

c twice the sum of p and q

d the square of $2a$

e the difference between $3a$ and $b + c$

f the square of $x + 2$

g the product of x and $y - 5$

h the square root of $2m + 3n$

24 **a** What is the cost of 3 books at p cents and 4 pens at q cents each?

b What is the change from \$5 (in cents)?

25 Translate the following algebraic expressions into words.

a xy

b $5 - a$

c $2p + q$

d $3m - 2n$

e $5(x + y)$

f $(a + b)^2$

g $\dfrac{x + y}{2}$

h $\dfrac{m - n}{a}$

i $\sqrt{u + v}$

j $\sqrt[3]{ab}$

26 Translate the following into words to explain the difference between each pair of expressions.

a abc and $a + b + c$

b $a - b$ and $b - a$

c $3(a + b)$ and $3a + b$

d $x^2 + y^2$ and $(x + y)^2$

e $\sqrt{a} + \sqrt{b}$ and $\sqrt{a + b}$

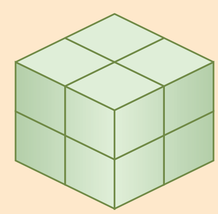
f $\dfrac{a}{b} + \dfrac{2}{3}$ and $\dfrac{a + 2}{b + 3}$

challenge

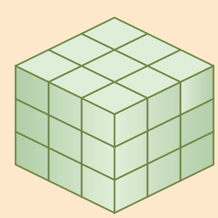
4:01

Challenge 4:01 | Let's play with blocks

Eight blocks have been stacked together here to form a cube. If the outside of the cube were painted, how many sides of each block would be painted?



How many blocks make up the second cube? If this cube were painted, how many blocks would have 3 sides, 2 sides, 1 side or even no sides painted?



What would be the result of painting a cube that had four blocks along each edge?

4:02 | Substitution

prep quiz 4:02

Write the following algebraic expressions in their simplest form.

1 $a \times b$ **2** $x \times y \times y$ **3** $2 \times x + 3 \times y$

Simplify the following:

4 $2 + 4 \times 3$ **5** $3 \times 4 + 2 \times 5$ **6** 4×5^2

7 $3(6 - 10)$ **8** $\frac{1}{2} \times 6 - 5$ **9** $\frac{8-2}{3}$

10 $\frac{5}{3} - \frac{3}{5}$

Algebra involves the use of 'pronumerals' as well as numbers. A pronumeral is usually a letter, such as x, that takes the place of a number in an expression like $3x + 7$.

If a number is substituted for each pronumeral, a value for the expression can then be obtained.

worked examples

Find the value of the following expressions, given that $a = 10$, $b = 4$, $x = 5$ and $y = -3$.

1 $3a + 2b$
$= 3 \times 10 + 2 \times 4$
$= 30 + 8$
$= 38$

2 $x^2 + y^2$
$= 5^2 + (-3)^2$
$= 25 + 9$
$= 34$

3 $\frac{1}{2}ab^2$
$= \frac{1}{2} \times 10 \times 4^2$
$= \frac{1}{2} \times 160$
$= 80$

4 $\frac{1}{x} + \frac{1}{y}$
$= \frac{1}{5} + \frac{1}{(-3)}$
$= \frac{1}{5} - \frac{1}{3}$
$= -\frac{2}{15}$

Exercise 4:02

Foundation Worksheet 4:02

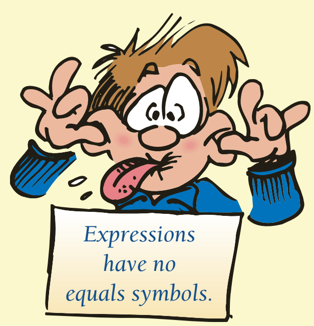
Substitution

1 Find the value of:
a $2x + 3y$ if $x = 3$, $y = -5$

2 If $a = 4$, $b = 5$, $c = -2$, find the value of:
a $a^2 + bc$

1 Evaluate the following expressions if $x = 3$, $y = 4$ and $z = 8$.

a $x + y$ **b** $3x + 2y$ **c** $2x - y$
d x^2y **e** $(x + y)^2$ **f** $x^2 + y^2$
g $z(x + y)$ **h** $xz - 10$ **i** $xy - xz$
j $\frac{x+y}{2}$ **k** $\frac{x}{2} + \frac{y}{4}$ **l** $z^2 - z$
m $xy^2 + x^2y$ **n** $xz - yz$ **o** $4(x - 2y)$
p $x(y^2 - z^2)$ **q** $x - 3y$ **r** $y^2 - z^2$
s $z - xy$ **t** $\frac{x}{y} + \frac{y}{x}$ **u** $\frac{x+y}{xy}$
v $\frac{x+y}{x-y}$ **w** $\frac{1}{2}xyz^2$ **x** $\sqrt{3xy}$



Technology Applications Activities

4:02 Substitution

4:02 Magic squares

2 Find the value of the following expressions if $a = 3$, $b = -4$ and $c = \frac{1}{2}$.

a $a + b$ **b** $a - b$ **c** ab **d** ab^2
e $(ab)^2$ **f** bc **g** abc **h** $10 - ab$
i $\frac{1}{a} + \frac{1}{b}$ **j** $c(2a + b)$ **k** $\frac{1}{a} + c$
l $(a + b)(a - b)$ **m** $b^2 - b^3$ **n** $\frac{1}{c} - \frac{1}{b}$
o $\frac{a}{b} - \frac{b}{c}$ **p** $\frac{a}{b} + \frac{b}{c} + \frac{c}{d}$ **q** $ab + bc + ac$

$\frac{1}{\frac{1}{2}} = 1 \div \frac{1}{2} = 2$

3 **a** Find the value of $mx + c$ if:
i $m = 3, x = 10, c = 1$ **ii** $m = 5, x = 4, c = -3$ **iii** $m = -2, x = 7, c = -4$

b Find the value of $\frac{h}{2}(a + b)$ if:
i $h = 4, a = 7, b = 5$ **ii** $h = 7, a = 4, b = 6$ **iii** $h = 3{\cdot}4, a = 9{\cdot}2, b = 3{\cdot}7$

c Find the value of πr^2 if:
i $\pi = 3{\cdot}1, r = 8$ **ii** $\pi = 3{\cdot}14, r = 2{\cdot}5$ **iii** $\pi = 3{\cdot}142, r = 100$

investigation
4:02

Investigation 4:02 | The history of algebra

Find out as much as you can about the early history of algebra.
You might consider:
- Ahmes Papyrus (Egyptian c. 1700 BC)
- Diophantus (Greek c. AD 250)
- Mohammed ibn Musa al-Khowarizmi (Arab c. AD 825)
- Bhaskara (Hindu c. AD 1150)

4:03 | Simplifying Algebraic Expressions

prep quiz
4:03

Write these expressions in their simplest forms.

1 $7x + 2x$ **2** $9x - 8x$ **3** $3x \times 2y$ **4** $5x \times x$
5 $12x \div 4$ **6** $10ab \div 5a$ **7** $3a + 2b + 5a + 3b$ **8** $6x + 2y - x - y$
9 $3 \times (-2a) \times 4a$ **10** $3a \div (-9b)$

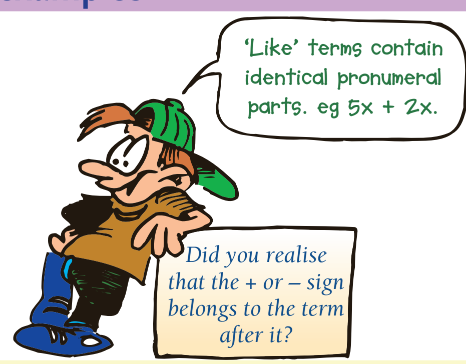
worked examples

Remember that only terms that are alike may be added or subtracted.

1 $5a + 2b - 3a + b = 5a - 3a + 2b + b$
$= 2a + 3b$

2 $5p^2 + 2p - 3p^2 = 5p^2 - 3p^2 + 2p$
$= 2p^2 + 2p$
(*Note:* p^2 and p are *not* like terms.)

3 $6ab - 4ba = 6ab - 4ab$
$= 2ab$



4 $-7x \times -3xy^2 = 21x^2y^2$

5 $3pq \times 4qr = 12pqqr$
$\qquad\qquad\quad = 12pq^2r$

6 $12ac \div 8ab = \dfrac{{}^{3}\cancel{12}\cancel{a}c}{{}_{2}\cancel{8}\cancel{a}b}$
$\qquad\qquad\quad = \dfrac{3c}{2b}$

7 $-6x \div 18xy = \dfrac{-{}^{1}\cancel{6}\cancel{x}}{{}_{3}\cancel{18}\cancel{x}y}$
$\qquad\qquad\quad = -\dfrac{1}{3y}$

8 $10a - 3 \times 2a = 10a - 6a$
$\qquad\qquad\quad = 4a$

9 $(5a + 7a) \times (3b - 2b) = 12a \times b$
$\qquad\qquad\quad = 12ab$

10 $3m \times 2n \div mn = 6mn \div mn$
$\qquad\qquad\quad = \dfrac{6\cancel{mn}}{\cancel{mn}}$
$\qquad\qquad\quad = 6$

11 $\dfrac{7p + 8p - 3p}{2p \times 3q} = \dfrac{{}^{2}\cancel{12}\cancel{p}}{{}_{1}\cancel{6}\cancel{p}q}$
$\qquad\qquad\quad = \dfrac{2}{q}$

Remember the order in which operations should be done.

↓

Grouping symbols

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Exercise 4:03

1 Collect the like terms to simplify these expressions.

a $3x + 2x$ **b** $8a + 5a$ **c** $10p + 21p$ **d** $x + 7x$
e $7a - 4a$ **f** $9b - 3b$ **g** $11q - q$ **h** $12e + 9e$
i $3p + 5p - 6p$ **j** $4x + 2x + x$ **k** $10x - 9x + 3x$ **l** $x + 2x - 3x$
m $2a + p - a + 3p$ **n** $a + m - a + m$ **o** $8 + 2x - 5x - 7$ **p** $8y - 1 - 8y - 1$
q $x^2 + 2x + 2x^2 - x$ **r** $p^2 + 4p + 3p^2 + p$ **s** $3q^2 + 8q - 4q - q^2$ **t** $y^2 + y + y^2 - y$
u $7 - p^2 + p - 5$ **v** $2a + a^2 + 7 + a$ **w** $8x - 7 - 7x - 3x^2$ **x** $5ab - 7 + 3ba - 9$

2 Simplify these products.

a $8y \times 3$ **b** $4 \times 4a$ **c** $3x \times 2y$ **d** $8p \times 4q$ **e** $6a \times b$
f $5x \times x$ **g** $5a \times 3a$ **h** $ab \times ac$ **i** $3pq \times 2p$ **j** $5mn \times mp$
k $4mn \times \frac{1}{2}n$ **l** $9b \times a^2$ **m** $6a^2 \times (-7a)$ **n** $-5x \times -2x$ **o** $x \times 2y \times 3x$
p $14ab \times (-\frac{1}{2}ab)$ **q** $(-ab) \times (-bc)$ **r** $2k \times 3k \times 4k$ **s** $-2 \times 7x \times -5y$ **t** $\frac{1}{4}m \times 4n \times (-p)$

3 Simplify:

a $12x \div 4$	**b** $12x \div 4x$	**c** $9x^2 \div 3$	**d** $8x \div 8x$
e $15m \div 10n$	**f** $32a \div 12b$	**g** $5 \div 20a$	**h** $48ab \div 6b$
i $a \div 3a$	**j** $45ab \div 20ba$	**k** $-20p \div 4p$	**l** $-xy \div xz$
m $14a \div (-a)$	**n** $(-15x) \div (-5xy)$	**o** $-28mnp \div 7mp$	**p** $8a^2b \div 16ab^2$

4 Simplify:

a $mn \times np$	**b** $7 + m + 6 + 3m$	**c** $14 - 2a + 5$	**d** $5x^2 \times 0$
e $3xy \times 2yx$	**f** $8x^2 + 2x + 7x^2 + 3x$	**g** $3 \times 4y \times 5z$	**h** $-4x \times 7x$
i $15ab - 9ba + ab$	**j** $6m - 7m$	**k** $8b + 3b - 11b$	**l** $18ab \div 9bc$
m $x \div 3x$	**n** $2pq \times 9pq$	**o** $3a + b + 2a - c$	**p** $-3y \times (-5z)$
q $\frac{1}{2}y + \frac{1}{2}y$	**r** $m + n - m + n$	**s** $3a \times 2b \times c$	**t** $15at \div 10tx$

5 Write the simplest expression for:

a $(2a + 3a) \times 4$	**b** $(10x - 3x) \div 7$	**c** $(9b - 3b) \times 2$	**d** $(3m + 9m) \div 4$
e $12x \div (2x + x)$	**f** $5a \times (10a + 2a)$	**g** $3m \times (10m - 9m)$	**h** $15y \div (9y - 2y)$
i $5a \times 7 \div a$	**j** $8x \times 4y \div 2xy$	**k** $10a \div 5 \times 3a$	**l** $9xy \div 3x \times 2y$
m $2x + 3x \times 4$	**n** $5x \times 3x + 10x^2$	**o** $20y - 5 \times 2y$	**p** $18m - 12m \div 6$
q $3 \times 2n + 5n \times 4$	**r** $7x + 3 \times 2x - 10x$	**s** $8x \div 4 - x$	**t** $11m + 18m \div 2$
u $\dfrac{6 \times 3x}{2x \times 5}$	**v** $\dfrac{3p + 2p - 1p}{2 \times 2p}$	**w** $\dfrac{11y - y}{6y + 4y}$	**x** $\dfrac{5a \times 4b \times 2c}{10c \times b \times 8c}$

4:04 | Algebraic Fractions

4:04A Addition and subtraction

prep quiz 4:04A

Answer the following:

1 $\frac{3}{5} + \frac{1}{5}$	**2** $\frac{7}{10} - \frac{3}{10}$	**3** $\frac{1}{4} + \frac{1}{3}$
4 $\frac{1}{2} + \frac{3}{8}$	**5** $\frac{2}{5} - \frac{1}{4}$	**6** $\frac{7}{12} - \frac{1}{3}$

Simplify the expressions:

7 $7x + 4x$ **8** $3ab + ab$ **9** $6x - 5x$ **10** $9a - a$

Rewrite each fraction as two equivalent fractions with a common denominator, then add or subtract the numerators.

worked examples

1
$$\frac{3x}{5} + \frac{2x}{5} = \frac{3x + 2x}{5} = \frac{{}^{1}\cancel{5}x}{{}_{1}\cancel{5}} = x$$

2
$$\frac{5}{a} - \frac{3}{a} = \frac{5 - 3}{a} = \frac{2}{a}$$

In these two examples, each fraction already had a common denominator.

3 $\frac{x}{3} + \frac{x}{2} = \frac{x \times 2}{3 \times 2} + \frac{x \times 3}{2 \times 3}$

$= \frac{2x}{6} + \frac{3x}{6}$

$= \frac{5x}{6}$

4 $\frac{4a}{5} - \frac{a}{3} = \frac{4a \times 3}{5 \times 3} - \frac{a \times 5}{3 \times 5}$

$= \frac{12a}{15} - \frac{5a}{15}$

$= \frac{7a}{15}$

5 $\frac{5m}{8} + \frac{m}{2} = \frac{5m}{8} + \frac{m \times 4}{2 \times 4}$

$= \frac{5m}{8} + \frac{4m}{8}$

$= \frac{9m}{8}$

6 $\frac{3x}{4} - \frac{2y}{3} = \frac{9x}{12} - \frac{8y}{12}$

$= \frac{9x - 8y}{12}$

7 $\frac{9}{x} + \frac{2}{3x} = \frac{27}{3x} + \frac{2}{3x}$

$= \frac{29}{3x}$

8 $\frac{5a}{2x} - \frac{2a}{3x} = \frac{15a}{6x} - \frac{4a}{6x}$

$= \frac{11a}{6x}$

Exercise 4:04A

Foundation Worksheet 4:04A

Simplifying algebraic fractions

1 Simplify: **a** $\frac{1}{7} + \frac{3}{7}$

2 Simplify: **a** $\frac{7}{10} - \frac{3}{10}$

3 Simplify: **a** $\frac{x}{5} + \frac{2x}{5}$

1 Simplify the following.

a $\frac{3a}{2} + \frac{a}{2}$ **b** $\frac{3x}{5} - \frac{2x}{5}$ **c** $\frac{a}{3} + \frac{4a}{3}$ **d** $\frac{9m}{10} - \frac{3m}{10}$

e $\frac{x}{4} + \frac{y}{4}$ **f** $\frac{5a}{3} - \frac{2b}{3}$ **g** $\frac{2}{a} + \frac{3}{a}$ **h** $\frac{7}{x} + \frac{1}{x}$

i $\frac{3}{y} - \frac{2}{y}$ **j** $\frac{9}{m} - \frac{1}{m}$ **k** $\frac{5a}{x} + \frac{2a}{x}$ **l** $\frac{2x}{y} - \frac{3x}{y}$

m $\frac{5}{3n} + \frac{7}{3n}$ **n** $\frac{3}{2x} - \frac{1}{2x}$ **o** $\frac{8a}{5b} + \frac{2a}{5b}$ **p** $\frac{7m}{4x} - \frac{3m}{4x}$

2 Reduce each of these expressions to its simplest form.

a $\frac{x}{3} + \frac{x}{5}$ **b** $\frac{a}{2} + \frac{a}{5}$ **c** $\frac{y}{3} - \frac{y}{4}$ **d** $\frac{m}{2} - \frac{m}{4}$

e $\frac{2a}{3} + \frac{a}{2}$ **f** $\frac{5x}{3} + \frac{2x}{4}$ **g** $\frac{3n}{8} - \frac{n}{4}$ **h** $\frac{4p}{5} - \frac{3p}{10}$

i $\frac{x}{4} + \frac{y}{3}$ **j** $\frac{2a}{3} - \frac{3b}{2}$ **k** $\frac{3m}{5} + \frac{n}{2}$ **l** $\frac{k}{6} - \frac{2l}{4}$

m $\frac{2}{x} + \frac{4}{3x}$ **n** $\frac{1}{3a} + \frac{2}{4a}$ **o** $\frac{7}{2m} - \frac{2}{5m}$ **p** $\frac{5}{8x} - \frac{1}{2x}$

q $\frac{2a}{3x} + \frac{3a}{2x}$ **r** $\frac{x}{3m} - \frac{2x}{m}$ **s** $\frac{5m}{2n} + \frac{3m}{4n}$ **t** $\frac{2x}{3a} + \frac{y}{4a}$

4:04B Multiplication and division

prep quiz 4:04B

Answer the following.

1 $\frac{1}{2} \times \frac{3}{4}$ 2 $\frac{2}{5} \times \frac{3}{4}$ 3 $\frac{4}{9} \times \frac{3}{8}$

4 $\frac{1}{2} \div \frac{3}{4}$ 5 $\frac{3}{5} \div \frac{3}{10}$ 6 $\frac{2}{3} \div \frac{5}{4}$

Simplify these expressions.

7 $5 \times 6x$ 8 $3a \times 2a$

9 $15a \div 5$ 10 $12ab \div 6b$

When *multiplying*
- **Cancel any common factors, then**
- **multiply the numerators together and multiply the denominators together.**

When *dividing*
- **Turn the second fraction upside down, then**
- **multiply as above (invert and multiply).**

worked examples

1 $\frac{2}{a} \times \frac{5}{b} = \frac{2 \times 5}{a \times b}$

$= \frac{10}{ab}$

2 $\frac{5}{x} \times \frac{x}{10} = \frac{\cancel{5}^1}{\cancel{x}_1} \times \frac{\cancel{x}^1}{\cancel{10}_2}$

$= \frac{1 \times 1}{1 \times 2}$

$= \frac{1}{2}$

3 $\frac{3b}{2} \times \frac{4}{5b} = \frac{3\cancel{b}}{\cancel{2}_1} \times \frac{\cancel{4}^2}{5\cancel{b}}$

$= \frac{3 \times 2}{1 \times 5}$

$= \frac{6}{5}$ or $1\frac{1}{5}$

4 $\frac{ab}{2} \div \frac{b}{5} = \frac{a\cancel{b}^1}{2} \times \frac{5}{\cancel{b}_1}$

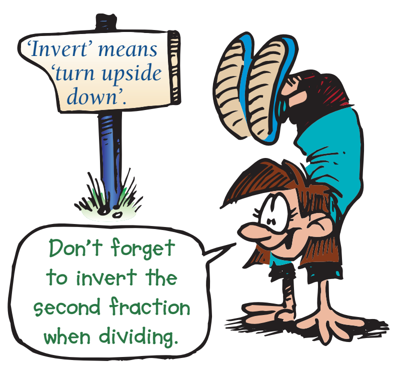
$= \frac{a \times 5}{2 \times 1}$

$= \frac{5a}{2}$

5 $\frac{8a}{3b} \div \frac{2a}{9b} = \frac{{}^4\cancel{8a}}{{}_1\cancel{3b}} \times \frac{{}^3\cancel{9b}}{{}_1\cancel{2a}}$

$= \frac{4 \times 3}{1 \times 1}$

$= 12$



Exercise 4:04B

Foundation Worksheet 4:04B
Simplifying algebraic fractions
1 Simplify: a $\frac{1}{3} \times \frac{4}{5}$
2 Simplify: a $\frac{a}{2} \times \frac{b}{5}$
3 Simplify: a $\frac{a}{2} \div \frac{a}{8}$

1 Simplify these products.

a $\frac{x}{2} \times \frac{y}{3}$ b $\frac{a}{4} \times \frac{b}{3}$ c $\frac{m}{2} \times \frac{m}{5}$ d $\frac{a}{4} \times \frac{a}{10}$

e $\frac{3}{a} \times \frac{4}{m}$ f $\frac{2}{x} \times \frac{1}{y}$ g $\frac{1}{p} \times \frac{4}{p}$ h $\frac{1}{n} \times \frac{1}{3n}$

i $\frac{p}{q} \times \frac{x}{y}$ **j** $\frac{2}{a} \times \frac{a}{4}$ **k** $\frac{m}{5} \times \frac{10}{n}$ **l** $\frac{3x}{5} \times \frac{2}{9x}$

m $\frac{ab}{3} \times \frac{2}{b}$ **n** $\frac{x}{y} \times \frac{y}{x}$ **o** $\frac{6m}{5a} \times \frac{15a}{2m}$ **p** $\frac{8x}{5p} \times \frac{2a}{3x}$

2 Simplify these divisions.

a $\frac{m}{2} \div \frac{m}{4}$ **b** $\frac{n}{3} \div \frac{n}{5}$ **c** $\frac{5a}{3} \div \frac{2a}{9}$ **d** $\frac{x}{5} \div \frac{3x}{10}$

e $\frac{5}{a} \div \frac{2}{a}$ **f** $\frac{3}{2m} \div \frac{1}{3m}$ **g** $\frac{a}{b} \div \frac{2a}{b}$ **h** $\frac{3x}{5y} \div \frac{x}{10y}$

i $\frac{a}{b} \div \frac{x}{y}$ **j** $\frac{2p}{3q} \div \frac{8p}{9q}$ **k** $\frac{10k}{3n} \div \frac{2k}{9n}$ **l** $\frac{a}{2} \div \frac{a}{3}$

m $\frac{xy}{2} \div \frac{y}{4}$ **n** $\frac{b}{2} \div \frac{ab}{6}$ **o** $\frac{xy}{c} \div \frac{y}{cx}$ **p** $\frac{9a}{b} \div \frac{4a}{3b}$

3 Simplify these expressions.

a $\frac{a}{3} \times \frac{12}{5a}$ **b** $\frac{2}{p} \times \frac{p}{3}$ **c** $\frac{15}{x} \div 5$ **d** $3b \div \frac{6}{b}$

e $\frac{xy}{z} \times \frac{2z}{x}$ **f** $\frac{ab}{c} \div \frac{a}{c}$ **g** $\frac{9m}{2} \times \frac{4m}{3}$ **h** $\frac{2x}{y} \div \frac{x}{2y}$

i $\frac{4}{pq} \times \frac{p}{q}$ **j** $\frac{3}{a} \times \frac{2}{b}$ **k** $\frac{4ab}{x} \times \frac{xy}{2ac}$ **l** $\frac{9bc}{2a} \div \frac{6b}{4a}$

m $\frac{2}{x} \times \frac{x}{3} \times \frac{9}{4}$ **n** $\frac{b}{c} \times \frac{c}{a} \times \frac{a}{b}$ **o** $\frac{8bc}{3a} \times \frac{9a}{b} \times \frac{1}{4c}$ **p** $\frac{8}{a} \times \frac{2a}{15} \div \frac{8}{3}$

Fun Spot 4:04 | Try this maths-word puzzle

fun spot 4:04

Hidden in the maze of letters there are many words used in mathematics. Make a list of the words you find and, at the same time, put a line through the letters you use. Words may be written in any direction: up, down, backwards, even diagonally. Also, a letter may be used more than once, but you cannot change direction in order to form a word, ie the letters must be in a straight line.

When you have found all the words there should be four letters that have not been used. These four letters can be arranged to form another 'mystery' maths word.

R	E	T	E	M	A	I	D	C
L	E	L	C	R	I	C	G	U
E	T	C	X	R	Y	O	H	B
L	E	I	T	R	A	N	T	E
L	S	Q	U	A	R	E	G	L
A	L	P	L	A	N	E	N	A
R	O	L	A	I	I	G	E	U
A	P	U	L	C	N	A	L	Q
P	E	S	M	M	E	T	R	E

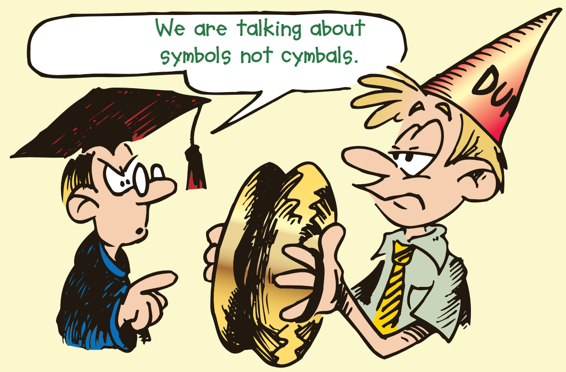
4:05 | Simplifying Expressions with Grouping Symbols

prep quiz 4:05

Simplify: **1** $7x + 3x$ **2** $4a^2 - a^2$ **3** $4x + 3 + 2x + 5$ **4** $2x + 7 - x - 5$
5 $3y^2 + 5y + 2y^2 - y$ **6** $7 - 3a + 6 + 5a$

Expand: **7** $3(x - 7)$ **8** $9(2 - 5y)$ **9** $2a(a + 3)$ **10** $-5(x + 7)$

The two most commonly used grouping symbols are:
parentheses ( )
brackets [ ]



$$\boldsymbol{a(b \pm c) = ab \pm ac}$$

To expand an expression, such as $a(b + c)$, each term inside the grouping symbols is multiplied by the term outside the grouping symbols.

worked examples

1 $p(p + 3) = p \times p + p \times 3$
$= p^2 + 3p$

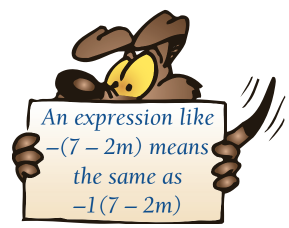
2 $3a(5 - 2a) = 3a \times 5 - 3a \times 2a$
$= 15a - 6a^2$

3 $-5(3x + 4) = (-5) \times 3x + (-5) \times 4$
$= -15x - 20$

4 $-(7 - 2m) = (-1) \times 7 - (-1) \times 2m$
$= -7 + 2m$

5 $x(x - 1) - x^2 + 5 = x^2 - x - x^2 + 5$
$= -x + 5$

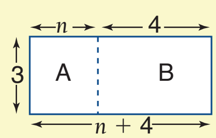
6 $2a(a + b) - a(3a - 4b) = 2a^2 + 2ab - 3a^2 + 4ab$
$= 6ab - a^2$



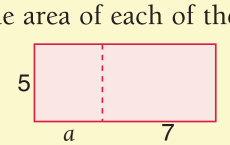
Exercise 4:05

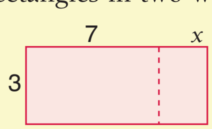
Foundation Worksheet 4:05
Grouping symbols
1 Simplify: **a** $3 \times 4y$
2 Complete the following:
a $3(2m + 5) = 3 \times 2m + 3 \times \ldots$
3 Remove the grouping symbols.
a $3(2a + 5)$

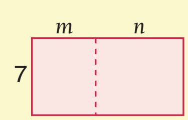
1 The area of rectangle A = $3 \times n = 3n$
The area of rectangle B = $3 \times 4 = 12$
The area of the combined rectangle = $3(n + 4)$
$\therefore\ 3(n + 4) = 3n + 12$

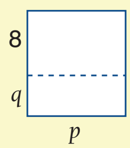


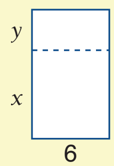
Following the example given above, write down the area of each of the following rectangles in two ways.

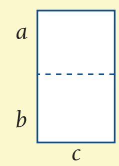
a


b


c


d


e


f


2 Expand:

a $3(x + 4)$
b $4(a + 5)$
c $3(y - 2)$
d $5(x - 3)$
e $3(2a + 3)$
f $4(m + n)$
g $6(3x + 2y)$
h $7(2a - 4b)$
i $x(x + 5)$
j $y(y - 2)$
k $2g(g - 1)$
l $3w(w + 2v)$
m $x(x + t)$
n $2h(3h - 1)$
o $2q(2 - q)$
p $8x(2 - 8x)$
q $y(s + t)$
r $ab(a + b)$
s $6xy(x - 5)$
t $5r(2r + 2s)$

3 Expand by removing the parentheses.

a $-3(x + 4)$
b $-4(a + 5)$
c $-4(a - 2)$
d $-3(p - 3)$
e $-7(2m + 5)$
f $-7(2m - 5)$
g $-7(5 - 2m)$
h $-8(1 - x)$
i $-(a - 3)$
j $-(4m - 3)$
k $-(3 + 6y)$
l $-(2a - 4c)$
m $-a(2a + 1)$
n $-3x(3x + 1)$
o $-3m(3m - 1)$
p $-9h(3h - j)$

4 Simplify:

a $4(x + 1) + x + 3$
b $3(x + 5) + 7x - 8$
c $5(y - 2) + 3y + 7$
d $4(a - 1) + 6a - 5$
e $3(p + 2) - 2p + 4$
f $10(m + 3) - 11m - 15$
g $5a + 6 + 2(a + 7)$
h $2x + 7 + 5(x - 1)$
i $7n - 4 + 3(n - 1)$
j $4h - 1 + 7(h + 2)$
k $6x + 2(x + 1) + 5$
l $4y + 6(y + 2) - 10$
m $3a + 10 - 2(a + 1)$
n $10m + 4 - 5(m + 4)$
o $6 - 2(y - 4) + 4y$
p $20 - 4(x - 2) + 5x$
q $5x + 7 + 2(2x + 7)$
r $4(3a + 1) - 10a + 2$
s $10m + 6 - 3(2m - 1)$
t $8x - 3(1 - 2x) + 10$

5 Simplify each expression by expanding the grouping symbols and then collecting like terms.

a $4(x + 1) + 2(x + 3)$
b $6(m + 3) + 3(m + 2)$
c $a(a + 4) + 3(a + 2)$
d $8(m - 3) + 5(m + 2)$
e $4(3x + 2) + 5(x - 4)$
f $6(x + 7) + 2(2x - 1)$
g $5(x + 7) - 3(x + 4)$
h $6(m + 1) - 3(m + 2)$
i $9(a + 5) - 7(a - 3)$
j $5(n - 5) - 3(n + 7)$
k $x(x + 3) + 3(x + 1)$
l $a(a + 3) + 7(a - 3)$
m $m(m + 3) - 4(m + 3)$
n $t(t - 5) - 4(t - 5)$
o $a(a + 2b) + a(2a + b)$
p $x(x + y) + y(x + y)$

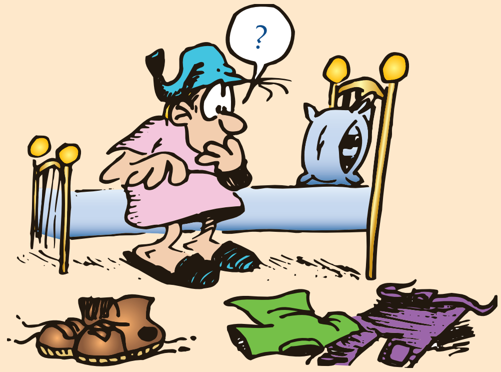
Challenge worksheet 4:05 Fractions and grouping symbols

fun spot

4:05

Fun Spot 4:05 | What is taken off last before you get into bed?

Work out the answer to each part and put the letter for that part in the box that is above the correct answer.



Write down the expression that is:

H 2 more than x **L** twice x

K half of x **N** 2 less than x

R the square of x

Find the value of $u + 10t$ if:

R $u = 12$, $t = 4$ **R** $u = -10$, $t = 2$

Simplify:

T $x \div x$ **T** $11x - x$

T $x + x + x$ **F** $x \times x \times 2$ **F** $x^2 - x - x$ **F** $7x + 3y - x + 3y$

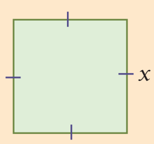
F $3x \times 2y$ **A** $-2x \times -3x$ **A** $12x \div 2$ **Y** $12m \div 4m$

O $14 - 3x + 6$ **E** $\frac{x}{2} + \frac{x}{2}$ **O** $\frac{2x}{5} - \frac{x}{10}$ **O** $\frac{5}{x} \times \frac{x}{10}$

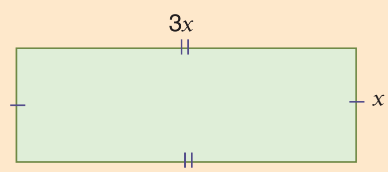
O $x - \frac{x}{10}$ **U** $x \div \frac{1}{8}$ **E** $\frac{x}{3} \div \frac{x}{6}$ **E** $\frac{x}{2} \div 2$

Write an expression for the perimeter of:

E



E



3	$\frac{9x}{10}$	$8x$	x^2	$6xy$	$\frac{x}{4}$	$4x$	1	$6x^2$	52	x	$3x$	$6x$	$\frac{x}{2}$	$8x$	$x - 2$

$\frac{3x}{10}$	$6x + 6y$	$2x^2$	$10x$	$x + 2$	2	$x^2 - 2x$	$2x$	$\frac{1}{2}$	$20 - 3x$	10

4:06 | Binomial Products

Simplify:	**1** $5x + 7x$	**2** $2a - a$	**3** $x^2 + 3x - 5x + 3$
Expand:	**4** $2(x + 5)$	**5** $x(x - 2)$	**6** $-3(a + 1)$
	7 $-y(5 - y)$		
Expand and simplify:	**8** $x(x + 1) + 3(x + 1)$		**9** $5(a + 5) - a(a + 5)$
	10 $2x(3x - 2) - 5(3x + 2)$		

prep quiz
4:06

A binomial expression is one that contains two terms, such as $2x - 7$ or $a + b$. Thus a binomial product is the product of two such expressions. For example, $(2x + 7)(a + 5)$ is a binomial product.

Long multiplication is like a binomial product.

$$\begin{aligned} 26 \times 19 &= (20 + 6) \times (10 + 9) \\ &= 20(10 + 9) + 6(10 + 9) \\ &= [20 \times 10] + [20 \times 9] + [6 \times 10] + [6 \times 9] \\ &= 200 + 180 + 60 + 54 \\ &= 494 \end{aligned}$$

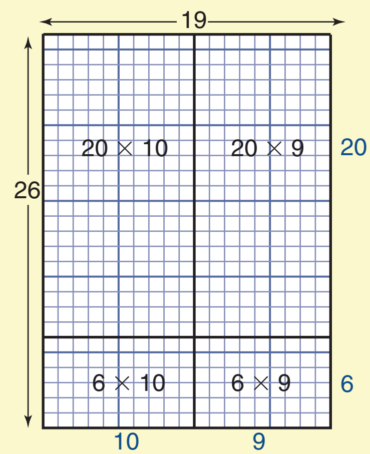
Each part of one number must multiply each part of the other.

$$(20 + 6)\quad(10 + 9)$$

As you can see, the products form a *face*.

$$\begin{array}{r} 19 \\ \times\ 26 \\ \hline 54 \\ 60 \\ 180 \\ 200 \\ \hline 494 \\ \hline \end{array}$$

26 × 19 by areas



Multiplying binomial expressions

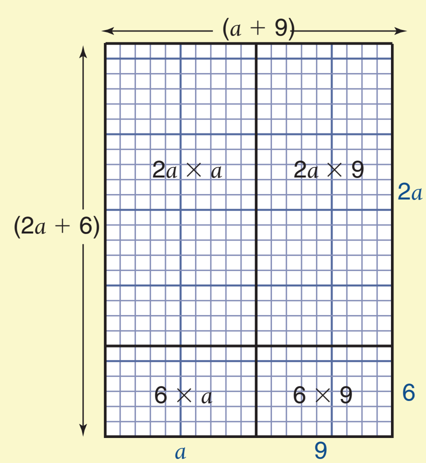
The expansion of binomial products may also be demonstrated by considering the area of a rectangle. This rectangle has dimensions $(2a + 6)$ and $(a + 9)$.

- The area of the whole rectangle must be equal to the sum of the four smaller areas.
- Area = $(2a + 6)(a + 9)$

$$\begin{aligned} &= 2a(a + 9) + 6(a + 9) \\ &= 2a^2 + 18a + 6a + 54 \\ &= 2a^2 + 24a + 54 \end{aligned}$$

- We can see that the product of two binomials yields four terms. Often two of these may be added together to simplify the answer.

(2a + 6)(a + 9) by areas



worked examples

1 $(a + 2)(b + 4) = a(b + 4) + 2(b + 4)$
$= ab + 4a + 2b + 8$

2 $(a - 2)(a + 7) = a(a + 7) - 2(a + 7)$
$= a^2 + 7a - 2a - 14$
$= a^2 + 5a - 14$

3 $(x + 2y)(2x + y) = x(2x + y) + 2y(2x + y)$
$= 2x^2 + xy + 4xy + 2y^2$
$= 2x^2 + 5xy + 2y^2$

4 $(1 - x)(x - 3) = 1(x - 3) - x(x - 3)$
$= x - 3 - x^2 + 3x$
$= 4x - x^2 - 3$

You should notice that each term in the first binomial is multiplied by each term in the second; ie

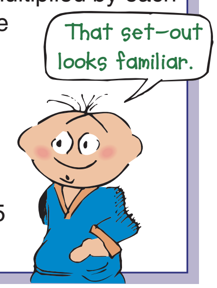
$2x^2$ $\quad -15$

$(x + 5)(2x - 3)$

$10x$

$-3x$

$= 2x^2 + 10x - 3x - 15$
$= 2x^2 + 7x - 15$



$$(a + b)(c + d) = a(c + d) + b(c + d)$$
$$= ac + ad + bc + bd$$

Exercise 4:06

1 Expand the following binomial products.

a $(a + 2)(b + 3)$ **b** $(x + 1)(y + 4)$ **c** $(m + 7)(n + 5)$ **d** $(a + 3)(x + 2)$
e $(p + 5)(q + 4)$ **f** $(2x + 1)(y + 3)$ **g** $(a + 6)(3p + 2)$ **h** $(4x + 1)(2y + 3)$
i $(3a + 1)(2b - 7)$ **j** $(7x + 5)(2p + 1)$ **k** $(5p + 3)(x - 4)$ **l** $(2x + y)(a + 2b)$

2 Expand the following and collect the like terms.

a $(a + 2)(a + 3)$ **b** $(x + 1)(x + 5)$ **c** $(n + 3)(n + 4)$ **d** $(p + 2)(p + 5)$
e $(m + 1)(m - 3)$ **f** $(y + 7)(y - 2)$ **g** $(x + 1)(x - 6)$ **h** $(t + 2)(t - 4)$
i $(x - 2)(x - 4)$ **j** $(n - 7)(n - 1)$ **k** $(a - 6)(a - 3)$ **l** $(x - 10)(x - 9)$
m $(y - 11)(y + 7)$ **n** $(a - 2)(a + 1)$ **o** $(x - 8)(x - 8)$ **p** $(m - 9)(m - 2)$
q $(a - 3)(a + 3)$ **r** $(x - 7)(x + 3)$ **s** $(y + 12)(y + 5)$ **t** $(a - 8)(a + 8)$
u $(q + 5)(q + 5)$ **v** $(x - 1)(x - 9)$ **w** $(t + 3)(t + 10)$ **x** $(k - 8)(k + 11)$

3 Find these products and simplify.

a $(a + 3)(2a + 1)$ **b** $(2x + 1)(x + 2)$ **c** $(3m + 2)(m + 5)$
d $(y + 3)(4y + 1)$ **e** $(2x + 1)(2x + 3)$ **f** $(3n + 2)(2n + 1)$
g $(2x + 3)(4x + 3)$ **h** $(5t + 2)(2t + 3)$ **i** $(2x - 2)(5x - 1)$
j $(8p + 1)(3p - 2)$ **k** $(5m - 2)(2m - 5)$ **l** $(3q + 1)(7q - 2)$
m $(3x + 2)(6x - 2)$ **n** $(2n + 3)(2n - 3)$ **o** $(8y - 1)(8y + 1)$
p $(3k - 2)(5k - 3)$ **q** $(7p - 1)(7p - 1)$ **r** $(3x - 1)(5x - 3)$
s $(5x + 4)(5x + 4)$ **t** $(9y - 4)(3y + 2)$ **u** $(5p + 2)(p - 7)$
v $(10q - 1)(q - 10)$ **w** $(4a + 3)(3a + 4)$ **x** $(7p + 5)(7p - 5)$

4 Expand and simplify:

a $(3 + x)(4 + x)$ **b** $(5 - a)(2 - a)$ **c** $(7 + m)(1 - m)$
d $(3 - n)(3 + n)$ **e** $(4 + y)(y + 5)$ **f** $(x - 7)(5 - x)$
g $(9 + k)(k + 10)$ **h** $(2a + 1)(3 + a)$ **i** $(3n + 1)(7 - 2n)$
j $(x + y)(x + 2y)$ **k** $(2n + m)(n + 2m)$ **l** $(a - b)(2a + 3b)$
m $(2p - q)(2p + q)$ **n** $(3x + y)(2x - 5y)$ **o** $(3a + 2b)(2a + 3b)$
p $(9w - 5x)(9w - 5x)$

4:07 | Special Products

prep quiz 4:07A

Simplify: **1** 4^2 **2** 7^2 **3** $(-2)^2$

4 $(-10)^2$ **5** $(3x)^2$

Complete:

6 $(x + 2)(x + 7) = x^2 + 9x +$ ....

7 $(a + 3)(a + 3) = a^2 + 6a +$ ....

8 $(2m - 1)(2m - 1) =$ .... $m^2 - 4m + 1$

9 $(n + 5)(n + 5) = n^2 +$ .... $n + 25$

10 $(x - 3)(x - 3) = x^2 -$ .... $x + 9$

4:07A Perfect squares

When a binomial is multiplied by itself, we call this product a perfect square. If a perfect square is expanded, we get:

$$\begin{aligned}(x + y)^2 &= (x + y)(x + y)\\ &= x(x + y) + y(x + y)\\ &= x^2 + xy + yx + y^2\\ &= x^2 + 2xy + y^2\end{aligned}$$

Similarly $(x - y)^2 = x^2 - 2xy + y^2$

In words, we could say: 'The square of a binomial is equal to the square of the first term, plus twice the product of the two terms, plus the square of the second term.'

worked examples

1 $(a + 3)^2 = a^2 + 2(3a) + 3^2$
$\quad = a^2 + 6a + 9$

[first term squared] [twice the product of the 2 terms] [second term squared]

2 $(m - 5)^2 = m^2 - 2[5m] + 5^2$
$\quad = m^2 - 10m + 25$

[first term squared] [twice the product of the 2 terms] [second term squared]

3 $(3y - 7)^2 = [3y]^2 - 2[21y] + [-7]^2$
$\quad = 9y^2 - 42y + 49$

$(x + y)^2 = x^2 + 2xy + y^2$
$(x - y)^2 = x^2 - 2xy + y^2$

Investigation 4:07 | The square of a binomial

investigation 4:07

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

The Prep Quiz above suggests that there might be a pattern formed when a binomial is squared. Copy and complete this table.

x	y	x^2	y^2	xy	$(x+y)^2$	$x^2 + 2xy + y^2$	$(x-y)^2$	$x^2 - 2xy + y^2$
5	3							
6	1							
10	4							

What were your findings?

Assessment Grid for Investigation 4:07 | The square of a binomial

The following is a sample assessment grid for this investigation. You should carefully read the criteria *before* beginning the investigation so that you know what is required.

Assessment Criteria (B, C) for this investigation				Achieved ✓
Criterion B Investigating Patterns	a	None of the following descriptors has been achieved.	0	
	b	Some help was needed to be able to expand the brackets and complete the table.	1	
			2	
	c	Mathematical techniques have been selected and applied to complete the table and suggest relationships or general rules.	3	
			4	
	d	The student has completed the table and accurately described the rules for the square of a binomial.	5	
			6	
	e	The above has been completed with justification using the patterns within the columns of the table and further examples.	7	
			8	
Criterion C Communication in Mathematics	a	None of the following descriptors has been achieved.	0	
	b	There is a basic use of mathematical language and representation. Lines of reasoning are insufficient.	1	
			2	
	c	There is satisfactory use of mathematical language and representation. Explanations are clear but not always complete.	3	
			4	
	d	An efficient use of mathematical language and representation has been shown. Explanations of all rules are complete and concise.	5	
			6	

Exercise 4:07A

1 Find the missing term in each example to make the statements true.

a $(x + 2)^2 = x^2 + 4x +$ ....
b $(a + 6)^2 = a^2 + 12a +$ ....
c $(y - 3)^2 = y^2 - 6y +$ ....
d $(m - 10)^2 = m^2 - 20m +$ ....
e $(x + 1)^2 = x^2 +$ .... $+ 1$
f $(y + 7)^2 = y^2 +$ .... $+ 49$
g $(n - 2)^2 = n^2 -$ .... $+ 4$
h $(p - 5)^2 = p^2 -$ .... $+ 25$
i $(q + 8)^2 =$ .... $+ 16q + 64$
j $(x - 4)^2 =$ .... $-8x + 16$
k $(x +$ ....$)^2 = x^2 + 6x + 9$
l $(a +$ ....$)^2 = a^2 + 18a + 81$
m $(y -$ ....$)^2 = y^2 - 14x + 49$
n $(m -$ ....$)^2 = m^2 - 22m + 121$
o $(2x + 3)^2 =$ .... $+ 12x + 9$
p $(5n + 1)^2 =$ .... $+ 10n + 1$
q $(3m + 7)^2 = 9m^2 +$ .... $+ 49$
r $(4x + 5)^2 = 16x^2 +$ .... $+ 25$
s $(2a - 1)^2 = 4a^2 -$ .... $+ 1$
t $(9y - 7)^2 = 81y^2 -$ .... $+ 49$

2 Expand these perfect squares and simplify.

a $(x + 3)^2$ **b** $(x + 5)^2$ **c** $(x + 1)^2$
d $(x - 6)^2$ **e** $(m - 1)^2$ **f** $(n - 5)^2$
g $(x + 2)^2$ **h** $(n - 8)^2$ **i** $(m + 11)^2$
j $(a + 12)^2$ **k** $(x + 10)^2$ **l** $(p - 9)^2$
m $(x + y)^2$ **n** $(a + m)^2$ **o** $(x + t)^2$
p $(a - b)^2$ **q** $(k - m)^2$ **r** $(p - q)^2$

3 Expand and simplify:

a $(2x + 3)^2$ **b** $(2x + 1)^2$ **c** $(3x + 5)^2$ **d** $(4a + 1)^2$
e $(3a + 7)^2$ **f** $(7t + 2)^2$ **g** $(2x - 1)^2$ **h** $(3a - 2)^2$
i $(5m - 4)^2$ **j** $(4t - 7)^2$ **k** $(6q - 1)^2$ **l** $(9n + 4)^2$
m $(2x + y)^2$ **n** $(a + 3b)^2$ **o** $(3t - 2x)^2$

4:07B Difference of two squares

Evaluate:	**1**	$7^2 - 3^2$	**2**	$(7 + 3)(7 - 3)$
	3	$4^2 - 2^2$	**4**	$(4 + 2)(4 - 2)$
	5	$5^2 - 1^2$	**6**	$(5 - 1)(5 + 1)$
	7	$6^2 - 3^2$	**8**	$(6 - 3)(6 + 3)$
	9	$10^2 - 9^2$	**10**	$(10 + 9)(10 - 9)$

prep quiz 4:07B

If the sum of two terms is multiplied by their difference, another special type of product is formed. If $(x + y)$ is multiplied by $(x - y)$ we get:

$$\begin{aligned}(x + y)(x - y) &= x(x - y) + y(x - y)\\ &= x^2 - xy + yx - y^2\\ &= x^2 - y^2\end{aligned}$$

In words, we could say: 'The sum of two terms multiplied by their difference is equal to the square of the first term minus the square of the second term.'

worked examples

1 $(x+3)(x-3) = \underbrace{x^2}_{\text{first term squared}} - \underbrace{3^2}_{\text{second term squared}}$

$= x^2 - 9$

2 $(2a-3b)(2a+3b) = \underbrace{(2a)^2}_{\text{first term squared}} - \underbrace{(3b)^2}_{\text{second term squared}}$

$= 4a^2 - 9b^2$

3 $(p-7)(p+7) = p^2 - 7^2$

$= p^2 - 49$

4 $(5x+y)(5x-y) = (5x)^2 - y^2$

$= 25x^2 - y^2$

■ $(x+y)(x-y) = x^2 - y^2$

Exercise 4:07B

1 Expand these products and simplify.

a $(x+4)(x-4)$	**b** $(a+1)(a-1)$	**c** $(m+2)(m-2)$	**d** $(n+7)(n-7)$
e $(p-5)(p+5)$	**f** $(q-6)(q+6)$	**g** $(x-3)(x+3)$	**h** $(y-9)(y+9)$
i $(10+x)(10-x)$	**j** $(5+a)(5-a)$	**k** $(8-x)(8+x)$	**l** $(11-m)(11+m)$
m $(x+t)(x-t)$	**n** $(a-b)(a+b)$	**o** $(m+m)(m-n)$	**p** $(p-q)(p+q)$

2 Express as the difference of two squares.

a $(2a+1)(2a-1)$	**b** $(3x+2)(3x-2)$	**c** $(5m+3)(5m-3)$
d $(9q+2)(9q-2)$	**e** $(4t-3)(4t+3)$	**f** $(7x-1)(7x+1)$
g $(8n-5)(8n+5)$	**h** $(10x-3)(10x+3)$	**i** $(2x+y)(2x-y)$
j $(4a+3b)(4a-3b)$	**k** $(5p+2q)(5p-2q)$	**l** $(3m-n)(3m+n)$
m $(2m-5n)(2m+5n)$	**n** $(2p-3q)(2p+3q)$	**o** $(x-5y)(x+5y)$
p $(12x-5y)(12x+5y)$		

4:08 | Miscellaneous Examples

- It is important that you are able to expand and simplify algebraic expressions readily and accurately, if you are to use algebra in later problem-solving exercises.
- Work through the miscellaneous questions of Exercise 4:08 after examining the following two examples.

worked examples

1 $(x+3)^2 - (x-1)(x+2) = [x^2 + 6x + 9] - [x^2 + x - 2]$

$= x^2 + 6x + 9 - x^2 - x + 2$

$= 5x + 11$

2 $(3x+5)(x-1) + (x+2)^2 - (2x+1)(2x-1) = [3x^2 + 2x - 5] + [x^2 + 4x + 4] - [4x^2 - 1]$

$= 3x^2 + 2x - 5 + x^2 + 4x + 4 - 4x^2 + 1$

$= 6x$

Exercise 4:08

Expand and simplify, where possible, each of the following expressions.

1
- **a** $5x + 3(x - 7)$
- **b** $(x + 2)(x - 1)$
- **c** $(2x + 1)(x - 1)$
- **d** $5(x + 2) - x(x + 1)$
- **e** $(3x - 1)^2$
- **f** $(x + 5)(x - 5)$
- **g** $(2x - 7)(3x - 1)$
- **h** $(5x - 1)(5x + 1)$
- **i** $4x + 7 + x(x + 2)$
- **j** $9x - (x + 5) + 5$
- **k** $(x + 10)(x - 3)$
- **l** $(9 - y)(9 + y)$
- **m** $3x(x - 5) - 2x^2$
- **n** $3(x + 2)(x + 1)$
- **o** $(x + y)^2$
- **p** $(x + 2y)(2x + y)$
- **q** $5x - 2(x + y) + 2y$
- **r** $(a + 2b)(a - 2b)$
- **s** $a(x + 2) - x(a + 2)$
- **t** $(3a + 7)(5a - 3)$
- **u** $(2m - 5n)^2$
- **v** $(1 - 5y)(1 + 5y)$
- **w** $3x - 7(x - 3)$
- **x** $(9x - 8y)(9x + 8y)$

2
- **a** $(x + 1)^2 + 5(x + 2)$
- **b** $(a - 3)^2 - 3(a + 1)$
- **c** $(x + 2)(x + 3) - 7(x - 2)$
- **d** $8(x + 2) + (x - 7)(x + 1)$
- **e** $(x + 3)^2 + (x + 1)(x + 2)$
- **f** $(a + 5)(a + 3) - (a + 4)^2$
- **g** $(m + 6)^2 - (m - 1)(m + 1)$
- **h** $(y + 7)(y - 7) - (y + 7)^2$
- **i** $(x + 2)^2 + (x + 1)^2$
- **j** $(a + 3)^2 - (a + 2)^2$
- **k** $(x + 1)(x + 2) + (x + 2)(x + 3)$
- **l** $(a + 1)(a - 2) + (a + 2)(a - 1)$
- **m** $(x + 3)(x - 1) - (x + 2)(x - 5)$
- **n** $(y + 7)(y - 2) - (y + 1)(y + 3)$
- **o** $(2x + 1)^2 - 5(x + 3)$
- **p** $2x(x + 5) + (x + 7)^2$
- **q** $(5x + 1)(x - 3) + (2x + 1)^2$
- **r** $(2x + 1)(3x + 1) - (2x - 1)(3x - 1)$
- **s** $(p + 3)(p - 3) - (q + 3)(q - 3)$
- **t** $(x + y)^2 - (x - y)(x + y)$
- **u** $(a + b)(a + 2b) + (a + b)^2$
- **v** $(m - n)^2 + (m + n)^2$
- **w** $3(x + 1)^2 + 5(x + 1)$
- **x** $2(x - 1)(x + 1) + 3(x + 1)^2$
- **y** $(2x + 3y)^2 - (2x - 3y)(2x + 3y)$
- **z** $(3a + 2b)(2a + 3b) - 6(a + b)^2$

3
- **a** $(x + 1)^2 + (x + 2)^2 + (x + 3)^2$
- **b** $(x + 1)(x + 2) + (x + 2)(x + 3) + (x + 3)(x + 4)$
- **c** $(a - 1)(a + 1) + (a + 1)^2 + (a - 1)^2$
- **d** $(x + 2)^2 + (x + 3)^2 - (x + 2)(x + 3)$
- **e** $(3a + 2b)(2a + 3b) + (3a - 2b)(3a + 2b) + (2a + 3b)(2a - 3b)$
- **f** $(4x + 1)(3x - 1) + (x + 2)^2 - (x - 3)(x + 3)$
- **g** $5(m - 5)^2 - 8(m - 4)^2 + 3(m - 3)^2$
- **h** $(3x + 2y)(3x - 2y) - (2x + y)(2x - y) - (x + 1)(x - 1)$
- **i** $(x + 3y)^2 - (2x + 2y)^2 + (3x + y)^2$
- **j** $2(x - y)(x + y) - (x + y)^2 - (x - y)^2$

challenge 4:08

Challenge 4:08 | Patterns in products

The examples below involve the sum of a series of products. Can you see the patterns involved and, hence, find the simplest expression for each sum?

1. $(x + 1)^2 + (x + 2)^2 \ldots + (x + 9)^2 + (x + 10)^2$
2. $(x + 1)(x + 2) + (x + 2)(x + 3) + \ldots + (x + 9)(x + 10)$
3. $(a - 5)^2 + (a - 4)^2 + \ldots + a^2 + \ldots + (a + 4)^2 + (a + 5)^2$
4. $(5m - n)(5m + n) + (4m - 2n)(4m + 2n) + (3m - 3n)(3m + 3n) + (2m - 4n)(2m + 4n) + (m - 5n)(m + 5n)$

Investigation 4:08 | Using special products in arithmetic

A Perfect squares

Example 1

Using $(a \pm b)^2 = a^2 \pm 2ab + b^2$, evaluate $(103)^2$.

Solution 1

Writing 103 as $(100 + 3)$

Then $103^2 = (100 + 3)^2$

$= 100^2 + 2 \times 100 \times 3 + 3^2$

$= 10\,000 + 600 + 9$

$= 10\,609$

Similarly, the square of a number like 98 could be found by writing 98 as $(100 - 2)$.

Exercise A

Following the example above, evaluate:

a 101^2	**b** 205^2	**c** 1004^2	**d** 72^2
e 98^2	**f** 199^2	**g** 995^2	**h** 67^2

B Difference of two squares

Example 2

Using $(a - b)(a + b) = a^2 - b^2$, evaluate $100^2 - 97^2$.

Solution 2

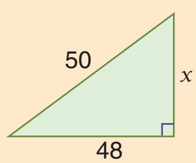
$100^2 - 97^2 = (100 - 97)(100 + 97)$

$= 3 \times 197$

$= 591$

This method can be useful when finding a shorter side of a right-angled triangle.

eg



$x^2 = 50^2 - 48^2$

$= (50 - 48)(50 + 48)$

$= 2 \times 98$

$= 196$

$\therefore x = \sqrt{196}$

$= 14$

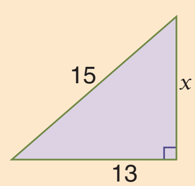
Exercise B

1 Evaluate:

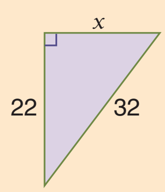
a $100^2 - 98^2$ **b** $73^2 - 67^2$ **c** $145^2 - 140^2$ **d** $651^2 - 641^2$

2 Use the method above to find the value of x for each triangle. (Leave your answer in surd form.)

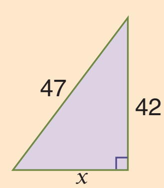
a



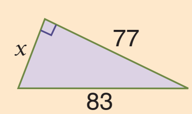
b



c



d



Mathematical Terms 4

algebra
- A branch of mathematics where numbers are represented by symbols, usually letters.

algebraic expression
- A group of terms that are joined by addition or subtraction signs.

binomial
- An algebraic expression consisting of two terms.
  eg $2x + 4$, $3x - 2y$

brackets
- The name given to these grouping symbols: [ ]

cancel
- To simplify a fraction by dividing the numerator and denominator by a common factor.
  eg $\frac{{}^{3}\cancel{21x}y}{{}^{2}\cancel{14x}} \begin{matrix} \div 7x \\ \div 7x \end{matrix} = \frac{3y}{2}$

denominator
- The bottom of a fraction.

difference of two squares
- The result of multiplying two binomials which are the sum and difference of the same terms.
  eg $(a + 3)(a - 3) = a^2 - 3^2$
  $= a^2 - 9$

expand
- To remove grouping symbols by multiplying the terms in each pair of grouping symbols by the term or terms outside.

like terms
- Terms that have identical pronumeral parts.
  eg $7x$ and $10x$
  $5a^2b$ and $-3a^2b$
- Only like terms may be added or subtracted together. This is called 'collecting like terms'.

numerator
- The 'top' of a fraction.

parentheses
- The name given to these grouping symbols: ( )

perfect square
- When a binomial is multiplied by itself.
  eg $(x + 5)^2$ or $(2a - 3b)^2$

pronumeral
- A symbol, usually a letter, that is used to represent a number.

substitution
- The replacing of a pronumeral with a numeral in an expression.
  eg to substitute 3 for a in the expression $4a - 2$ would give:
  $4(3) - 2 = 10$

Mathematical terms 4

- A machine counts coins by weight. What is the value of a pile of $M coins that weighs W grams if each coin weighs w grams?

Diagnostic Test 4: | Algebraic expressions

- These questions reflect the important skills introduced in this chapter.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

Section

1 For each table of values, find the expression in x that completes the rule $y = \ldots$ — 4:01

a

x	0	1	2	3
y	0	4	8	12

b

x	5	6	7	8
y	12	14	16	18

c

x	1	2	3	4
y	1	4	9	16

2 Write an algebraic expression for the following: — 4:01

a the sum of x and y
b the average of 5 and m
c the cost of b books at p dollars each
d \$2 was shared between Sue and Jenny. If Sue received x cents, how many cents did Jenny receive?

3 If $m = 2$, $x = 6$, $c = 1$, $h = 10$, $a = 3$, $b = 6$, $\pi = 3{\cdot}1$ and $r = 10$, evaluate: — 4:02

a $mx + c$ **b** $\frac{h}{2}(a + b)$ **c** πr^2

4 Simplify: — 4:03

a $7x + 6 - 3x - 2$ **b** $6q^2 + 7q - q^2$
c $5xy - 3yx$ **d** $5m + 2n - 3m - 4 - 3n + 7$

5 Simplify: — 4:03

a $-5y \times a$ **b** $2xy \times x$
c $12a \times (-3b)$ **d** $3x^2 \times 4xy$

6 Simplify: — 4:03

a $18m \div 6$ **b** $24ab \div 4a$
c $\frac{16x^2}{8x}$ **d** $-5mn \div 10m^2$

7 Simplify: — 4:04A

a $\frac{3x}{5} + \frac{2x}{5}$ **b** $\frac{x}{3} - \frac{x}{2}$
c $\frac{4a}{5} - \frac{a}{3}$ **d** $\frac{5m}{8} + \frac{m}{2}$

8 Simplify: — 4:04B

a $\frac{3}{4} \times \frac{n}{3}$ **b** $\frac{2}{a} \times \frac{5}{b}$
c $\frac{5}{x} \times \frac{x}{10}$ **d** $\frac{3b}{2} \times \frac{4}{5b}$

9 Simplify: **a** $\frac{3m}{2} \div \frac{1}{4}$ **b** $\frac{x}{3} \div \frac{x}{6}$ **c** $\frac{8a}{3b} \div \frac{2a}{9b}$ **d** $\frac{ab}{2} \div \frac{b}{5}$		4:04B
10 Expand: **a** $4(x - 3)$ **b** $4(2x + 3)$ **c** $4x(2x - 3)$ **d** $3a(5 - a)$		4:05
11 Expand: **a** $-4(x - 3)$ **b** $-4x(x + 3)$ **c** $-5m(3m - 3)$ **d** $-4a(3a + 7)$		4:05
12 Expand and simplify: **a** $x(x - 1) - x^2$ **b** $7n - 4 + 3(n - 1)$ **c** $2a(a + b) - a(3a - 4b)$		4:05
13 Expand and simplify: **a** $(x + 3)(x + 4)$ **b** $(a - 3)(2a - 1)$ **c** $(2 - y)(3 + y)$ **d** $(2x + y)(x - 3y)$		4:06
14 Expand and simplify: **a** $(x + 2)^2$ **b** $(a - 7)^2$ **c** $(2y + 5)^2$ **d** $(m - n)^2$		4:07A
15 Expand and simplify: **a** $(x + 3)(x - 3)$ **b** $(y - 7)(y + 7)$ **c** $(2a + 5)(2a - 5)$ **d** $(x + y)(x - y)$		4:07B

1. Three darts are thrown and all land in the '20' sector. What are the possible total scores for the three darts if all darts can land on either the 20, double 20 or triple 20?
2. Three darts are thrown and all land in the 'x' sector. Write an algebraic expression for the possible total scores.
3. Three darts are thrown and all land in the same sector. The total score is 102. In what sector did the darts land?

assignment

4A

Chapter 4 | Revision Assignment

1 Simplify the following.

- **a** $6a + a$
- **b** $6x \times 3x$
- **c** $a - 5a$
- **d** $x^2 + x^2$
- **e** $18x \div 3x$
- **f** $12y \div 8$
- **g** $2x + 3y$
- **h** $3ab \times 2b$
- **i** $12a^2b \div 6a$
- **j** $5ab + 7ab$
- **k** $6a^2 - a$
- **l** $4x - 3y - 5x$
- **m** $12 + 6x + 7 - x$
- **n** $6x + 2x \times 3$
- **o** $x^2 - 3x + 2x + 3x^2$
- **p** $12x - 6x \div 3$

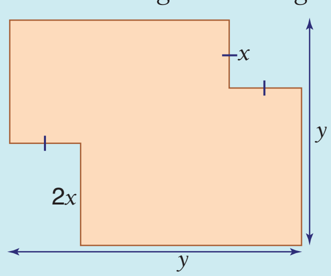
2 Expand and simplify where possible.

- **a** $(x - 1)(x + 2)$
- **b** $5x + 3(x - 1)$
- **c** $2(x + 3) - 2x - 3$
- **d** $(2x + 1)(x - 7)$
- **e** $(x + 5)(x - 5)$
- **f** $(3x + 2)^2$
- **g** $x(x - 3) + 2(x + 1)$
- **h** $(2 - x)(3 - x)$
- **i** $(x + y)(y - x)$
- **j** $(2x - y)^2$
- **k** $5[x + 3(x + 1)]$
- **l** $[3x - (x - 2)]^2$

3 Simplify:

- **a** $\frac{x}{2} + \frac{x}{3}$
- **b** $\frac{2a}{5} - \frac{a}{10}$
- **c** $\frac{3a}{2} \times \frac{5b}{6}$
- **d** $\frac{10y}{3} \div 5y$
- **e** $\frac{7x}{5} - \frac{x}{3}$
- **f** $\frac{3m}{5} + \frac{m}{3} - \frac{m}{2}$
- **g** $\frac{6n}{5} \times \frac{10}{7n} \div \frac{3}{2n}$
- **h** $\frac{x+3}{2} + \frac{x+1}{3}$

4 Find the simplest expression for the perimeter of this figure. All angles are 90°.



5 Find the algebraic rule for these tables of values.

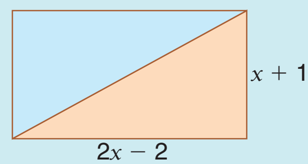
a

x	0	1	2	3
y	12	9	6	3

b

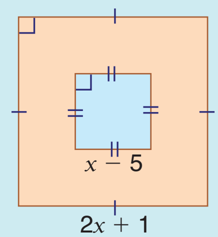
t	−1	1	3	5
s	2	2	10	26

6 Find an expression for the shaded area of this rectangle. Expand and simplify your answer.

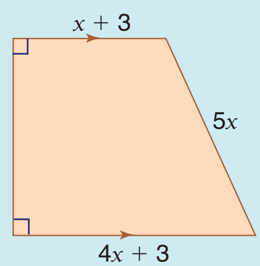


7 Find the simplest expression for the shaded area of each figure.

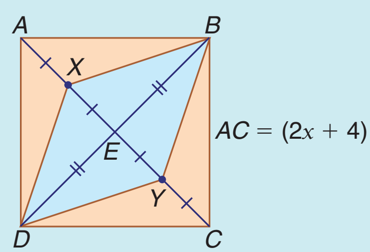
a



b



c



ABCD is a square.
X and *Y* bisect *AE* and *CE* respectively.

Chapter 4 | Working Mathematically

assignment 4B

1 Use ID Card 4 on page xiv to give the mathematical term for:
a 1 b 2 c 3 d 4 e 5
f 6 g 7 h 8 i 9 j 11

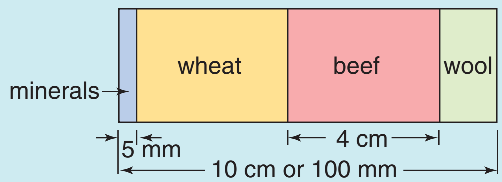
2 a What geometric shape has inspired the design of this coffee cup?

b What would you estimate the capacity of the cup to be?

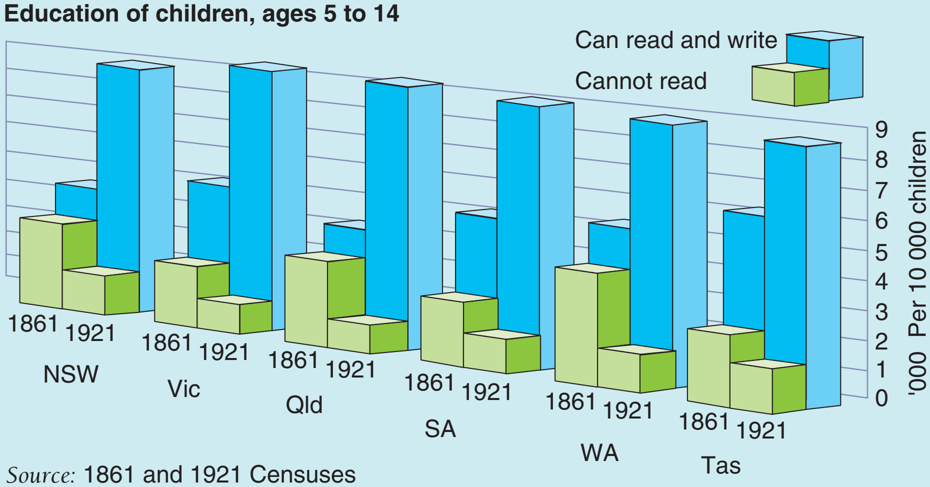
3 Diane and Garry married and had three children. Each child married and had three children. Assuming that no one has died, how many people are now in this extended family altogether?

4 The numerals 1 to 10 are written on ten separate cards, one on each card.
a How many pairs of cards have a sum of 10?
b How many groups of three cards are there that have a sum of 18?

5 A particular country's exports are shown in the bar graph below (reduced in size). Find what percentage of the country's exports are taken up by:
a beef b minerals



6 **Education of children, ages 5 to 14**



Source: 1861 and 1921 Censuses

a In 1861, which state had the greatest number per 10 000 children that could read and write? What percentage was this?
b In 1921, which state had the greatest percentage of children that could read and write? What percentage was this?
c Which state had 4000 per 10 000 children that could read and write in 1861? About how many in that state could not read in 1861?
d Consider Western Australia in 1861. Approximately what percentage could read and write? Approximately what percentage could not read? (To determine this, measure the height of this column and measure this height on the scale.)

Technology Applications — Drag and Drops

1 Addition and subtraction of algebraic fractions
2 Multiplication and division of algebraic fractions
3 Grouping symbols
4 Binomial products
5 Special products

Chapter Review Questions