

Amortization - Depreciation and Depletion

Amortization describes the equivalence of a capital sum over a period of time. In process industries, it may be considered as a program or policy whereby the owners (stock-holders) of the company have their investment on depreciable capital that is partly protected against loss.

An analysis of costs and profits for any business operation requires recognition of the fact that physical assets decrease in value with age. This decrease in value may be due to physical deterioration, technological advances, economic changes, or other factors, which ultimately will cause retirement of the property. The reduction in value due to any of these causes is a measure of the *depreciation*. The economic function of depreciation, therefore, can be employed as a means of distributing the original expense for a physical asset over the period during which the asset is in use. Because the engineer thinks of depreciation as a measure of the decrease in value of property with time, depreciation can immediately be considered from a cost viewpoint.

For example, suppose a piece of equipment had been put into use 10 years ago at a total cost of \$31,000. The equipment is now worn out and is worth only \$1000 as scrap material. The decrease in value during the 10-year period is \$30,000; however, the engineer recognizes that this \$30,000 is in reality a cost incurred for the use of the equipment. This depreciation cost was spread over a period of 10 years, and sound economic procedure would require part of this cost to be charged during each of the years. The application of depreciation in engineering design, accounting, and tax studies is almost always based on costs prorated throughout the life of the property.

According to the United States, Internal Revenue Service, depreciation is defined as “A reasonable allowance for the exhaustion, wear, and tear of property used in the trade or business including a reasonable allowance for obsolescence.” The terms *amortization* and *depreciation* are often used interchangeably. Amortization is usually associated with a definite period of cost distribution, while depreciation usually deals with an unknown or estimated period over which the asset costs are distributed. Depreciation and amortization are of particular significance as an accounting concept that serves to reduce taxes.

Depreciation

Depreciation has many meanings, but only two are discussed in our syllabus

1. loss of value of capital with the time when equipment wears out or becomes obsolete.
2. the systematic allocation of costs of an asset that produces an income from operations.

In short, depreciation may be considered as a cost for protection of depreciating capital without interest over a period, which the capital (asset or equipment) is used.

Before beginning the problems on depreciation let us see some of the terminologies involved in the depreciation calculations.

Terminology	Definition
Book value	It is the difference between original cost of a property and all depreciation charged up to a time.
Salvage value	It is the net amount of money obtained from the sale of used property. The term <i>salvage value</i> implies that the property can be of further use or in service. If the property is not useful, it can be sold for material recovery then the income obtainable from this type of disposal is known as <i>scrap value</i> .
Current value	The current value of an asset is value of the asset in its condition at the time of evaluation.
Replacement Value	The cost necessary to replace an existing property at any given time with one at least equally capable of rendering the same service is known as the <i>replacement value</i> .
Market value	The price that could be obtained for an asset if it were sold on the open market is designated as the <i>market value</i> .
Service life	The period during which the use of a property is economically feasible is known as the <i>service life</i> of the property.
Recovery period	The period over which the use of property is economically feasible is known as the <i>service life</i> of the property. The period over which the depreciation is charged is the <i>recovery period</i> .
Capital recovery	It is defined as the repayment of original capital plus interest. Capital recovery = Original cost – Salvage value

Table 1: Terminologies involved in depreciation calculations

The following tabulation (table 2) for estimating the life of equipment in years is an abridgement of information from "Depreciation-Guidelines and Rules" (Rev. Proc. 62-21) issued by the Internal Revenue Service of the U.S. Treasury Department as Publication No. 456 (7-62) in July 1962.

	Life, years
Group I: General business assets	
1. Office furniture, fixtures, machines, equipment	10
2. Transportation	
a. Aircraft	6
b. Automobile	3
c. Buses	9
d. General-purpose trucks	4-6
e. Railroad cars (except for railroad companies)	15
f. Tractor units	4
g. Trailers	6
h. Water transportation equipment	18
3. Land and site improvements (not otherwise covered)	20
4. Buildings (apartments, banks, factories, hotels, stores, warehouses)	40-60
Group II: Nonmanufacturing activities (excluding transportation, communications, and public utilities)	
1. Agriculture	
a. Machinery and equipment	10
b. Animals	3-10
c. Trees and vines	variable
d. Farm buildings	25
2. Contract construction	
a. General	5
b. Marine	12
3. Fishing	variable
4. Logging and sawmilling	6-10
5. Mining (excluding petroleum refining and smelting and refining of minerals)	10
6. Recreation and amusement	10
7. Services to general public	10
8. Wholesale and retail trade	10
Group III: Manufacturing	
1. Aerospace industry	8
2. Apparel and textile products	9
3. Cement (excluding concrete products)	20
4. Chemicals and allied products	11
5. Electrical equipment	
a. Electrical equipment in general	12
b. Electronic equipment	8
6. Fabricated metal products	12
7. Food products, except grains, sugar and vegetable oil products	12
8. Glass products	14
9. Grain and grain-mill products	17

(Continued)

Table 2 Estimated life of equipment

Group III: Manufacturing (continued)

10. Knitwear and knit products	9
11. Leather products	11
12. Lumber, wood products, and furniture	10
13. Machinery unless otherwise listed	12
14. Metalworking machinery	12
15. Motor vehicles and parts	12
16. Paper and allied products	
a. Pulp and paper	16
b. Paper conversion	12
17. Petroleum and natural gas	
a. Contract drilling and field service	6
b. Company exploration, drilling, and production	14
c. Petroleum refining	16
d. Marketing	16
18. Plastic products	11
19. Primary metals	
a. Ferrous metals	18
b. Nonferrous metals	14
20. Printing and publishing	11
21. Scientific instruments, optical and clock manufacturing	12
22. Railroad transportation equipment	12
23. Rubber products	14
24. Ship and boat building	12
25. Stone and clay products	15
26. Sugar products	18
27. Textile mill products	12-14
28. Tobacco products	15
29. Vegetable oil products	18
30. Other manufacturing in general	12

Group IV: Transportation, communications, and public utilities

1. Air transport	6
2. Central steam production and distribution	28
3. Electric utilities	
a. Hydraulic	50
b. Nuclear	20
c. Steam	28
d. Transmission and distribution	30
4. Gas utilities	
a. Distribution	35
b. Manufacture	30
c. Natural-gas production	14
d. Trunk pipelines and storage	22
5. Motor transport (freight)	8
6. Motor transport (passengers)	8
7. Pipeline transportation	22
8. Radio and television broadcasting	6

(Continued)

Estimated life of equipment (Continued)

	Life, years
Group IV: Transportation, communications, and public utilities (continued)	
9. Railroads	
a. Machinery and equipment	14
b. Structures and similar improvements	30
c. Grading and other right of way improvements	variable
d. Wharves and docks	20
10. Telephone and telegraph communications	variable
11. Water transportation	20
12. Water utilities	50

Some of the methods used to calculate the depreciation are as follows:

1. Straight-line method
2. Fixed percentage (or declining-balance method)
3. Sinking-fund method
4. Sum-of-the-years-digits method

Equations used in the above methods are given below:

1. Straight-line method

$$A_D = \frac{P - L}{n}$$

$$D_n = \frac{n'(P - L)}{n}$$

$$B_v = P - n'(D_n)$$

where, A_D = annual depreciation

D_n = depreciation up to any age n in life service of the asset or
accumulated/cumulative depreciation at any age n in life service.

P = principal, or investment or fixed capital cost

L = salvage value; n = total number of life service; n' = number of years of service
upto n age

B_v = book value at the end of year

2. Fixed percentage method or Declining–Balance method

$$f = 1 - \sqrt[n]{\frac{L}{P}}$$

A_D = Depreciation factor \times Book value at the beginning of the year

$$B_v = P(1 - f)^n$$

Where, f = depreciation rate (or) depreciation factor expressed in percentage; L = salvage value; P = principal/ original sum or fixed capital investment; B_v = book value;

3. Sinking–fund method

$$A_D = (P - L) \left(\frac{i'}{(1 + i')^n - 1} \right)$$

$$D_n = (P - L) \left(\frac{\frac{i'}{(1 + i')^n - 1}}{\frac{i'}{(1 + i')^{n'} - 1}} \right)$$

$$B_v = P - (P - L) \left(\frac{\frac{i'}{(1 + i')^n - 1}}{\frac{i'}{(1 + i')^{n'} - 1}} \right)$$

where, A_D = annual depreciation

D_n = depreciation up to any age n in life service of the asset (or)
accumulated/cumulative depreciation at any age n in life service.

P = principal (or) investment or fixed capital cost

L = salvage value; n = total number of life service; n' = number of years of service upto n age

B_v = book value; i = interest rate or interest; i' = sinking-fund interest

4. Sum-of-the-years-digits method

$$D_n = (P - L) \times (\text{Depreciation factor})$$

$$B_v = (P - D_n)$$

Problem No. 1

If a heat exchanger costs \$1,100 with 10 years of service life had a salvage value of \$100. Estimate the annual depreciation of heat exchanger using the following methods

- (1) Straight-line method
- (2) Fixed percentage (or) Declining-Balance method
- (3) Sinking-fund method
- (4) Sum-of-the-years-digits method.

Solution:

Given:

Principal (or) Original sum (or) Initial Investment (or) Fixed capital cost = \$1,100

Service life of the heat exchanger = 10 years

Salvage value of the heat exchanger at the end of 10th year is = \$100

Required:

Annual depreciation by

- (1) Straight-line method
- (2) Fixed percentage (or) Declining-Balance Method
- (3) Sinking-fund method
- (4) Sum-of-the-years-digits method and show the behavior of book value and depreciation in graph for each of the above mentioned methods.

Calculation:

- (1) Straight-line method

The annual depreciation (A_D), depreciation up to any age n in life service of the asset (D_n), and book value (B_v) at the end of each year from 0 to 10 is calculated and tabulated in table 1 as follows,

$$B_v = P - n'(D_n)$$

where, B_v = book value at the end of year

D_n = depreciation up to any age n in life service of the asset or

accumulated/cumulative depreciation at any age n in life service.

P = principal, or investment or fixed capital cost

n' = number of years of service upto n age

therefore,

$$B_{V_0} = 1100 - 0(0)$$

$$B_{V_0} = 1100$$

$$B_{V_1} = 1100 - 1(100)$$

$$\underline{\underline{B_{V_1} = 1000}} \quad \text{Similarly for other years as follows,}$$

$$B_{V_2} = 900$$

$$B_{V_3} = 800$$

$$B_{V_4} = 700$$

$$B_{V_5} = 600$$

$$B_{V_6} = 500$$

$$B_{V_7} = 400$$

$$B_{V_8} = 300$$

$$B_{V_9} = 200$$

$$B_{V_{10}} = 100$$

Accumulated depreciation/cumulative depreciation:

$$A_D = \frac{P - L}{n}$$

$$D_n = \frac{n'(P - L)}{n}$$

where, A_D = annual depreciation

D_n = depreciation up to any age n in life service of the asset or
accumulated/cumulative depreciation at any age n in life service.

P = principal, or investment or fixed capital cost

L = salvage value; n = total number of life service; n' = number of years of service
upto n age.

$$A_D = \left(\frac{P - L}{n} \right)$$

$$A_D = \left(\frac{1100 - 100}{10} \right)$$

$$\underline{\underline{A_D = 100}}$$

$$D_n = \frac{n'(P - L)}{n}$$

$$D_0 = \left(\frac{0(1100 - 100)}{10} \right) = 0$$

$$D_1 = \left(\frac{1(1100 - 100)}{10} \right) = 100$$

$$D_2 = \left(\frac{2(1100 - 100)}{10} \right) = 200$$

$$D_3 = \left(\frac{3(1100 - 100)}{10} \right) = 300$$

$$D_4 = \left(\frac{4(1100 - 100)}{10} \right) = 400$$

$$D_5 = \left(\frac{5(1100 - 100)}{10} \right) = 500$$

$$D_6 = \left(\frac{6(1100 - 100)}{10} \right) = 600$$

$$D_7 = \left(\frac{7(1100 - 100)}{10} \right) = 700$$

$$D_8 = \left(\frac{8(1100 - 100)}{10} \right) = 800$$

$$D_9 = \left(\frac{9(1100 - 100)}{10} \right) = 900$$

$$D_{10} = \left(\frac{0(1100 - 100)}{10} \right) = 1000$$

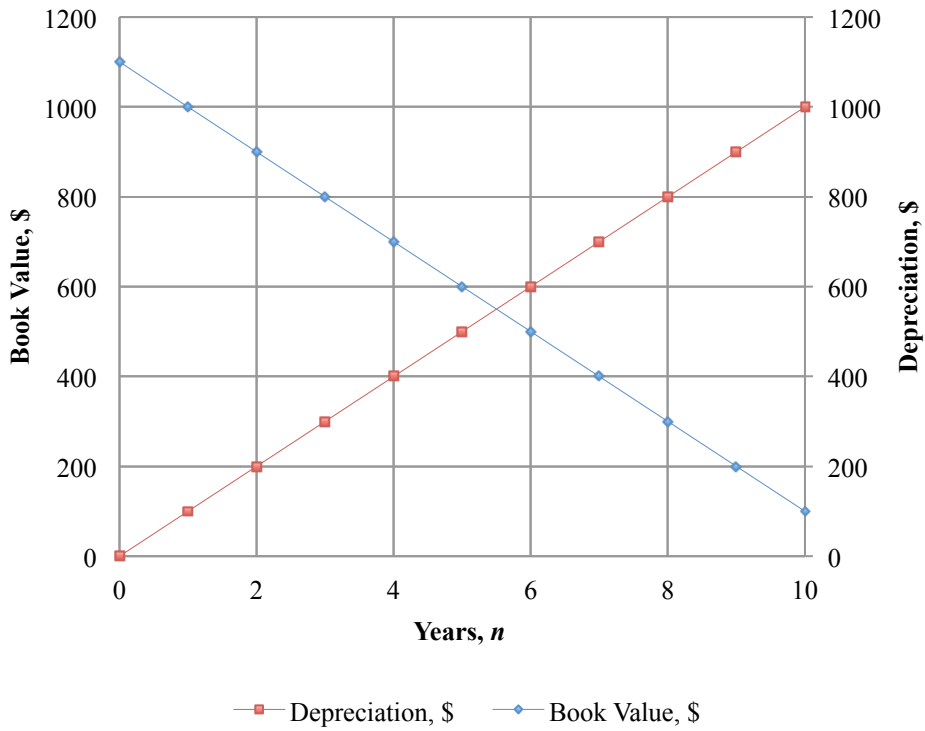
Thus, the calculations were carried out and the results are tabulated in table 3. To understand the behavior of book value and depreciation (depreciation up to any age n in life service of the asset or accumulated/cumulative depreciation at any age n in life service) a graph is plotted and shown in figure 1.

Years (<i>n</i>)	Book value at the beginning of the year (<i>B_v</i>), \$	Depreciation expense (or) Annual Depreciation (<i>A_D</i>), \$	Accumulated depreciation (or) Cumulative depreciation (<i>C_D</i>), \$	Book value at the end of the year (<i>B_v</i>), \$
0	1100	0	0	1100
1	1100	100	100	1000
2	1000	100	200	900
3	900	100	300	800
4	800	100	400	700
5	700	100	500	600
6	600	100	600	500
7	500	100	700	400
8	400	100	800	300
9	300	100	900	200
10	200	100	1000	100

Table 3: Straight–line method

From the above table 3, it is noted that the sum of the cumulative depreciation/ or accumulated depreciation and the book value at the end of 10th year (i.e. \$1,000 + \$100 = \$1,100) will be the original sum invested. The graph given below will show the behavior of book value and the depreciation (Annual depreciation /or Cumulative depreciation) from original time of investment till the expected service life of the equipment. Now, as mentioned above we will look into the behavior of the book value and the depreciation of heat exchanger in the graph. The graph says that the book value of heat exchanger decreases, when depreciation of the heat exchanger increases as time passes. The values used for plotting the graph are given in the table 4.

Depreciation by straight line method



Years (n)	Accumulated depreciation (or) Cumulative depreciation (C_D), \$	Book value at the end of the year (B_v), \$
0	0	1100
1	100	1000
2	200	900
3	300	800
4	400	700
5	500	600
6	600	500
7	700	400
8	800	300
9	900	200
10	1000	100

Table 4. Book value and Depreciation by Straight-line method

(2) Fixed percentage (or) Declining–Balance Method

Book value at the end of the year =

Book value at the beginning of the year – Annual depreciation

Therefore,

$$B_{V_1} = 1100 - 234 = 866$$

$$B_{V_2} = 866 - 185 = 681$$

$$B_{V_3} = 681 - 145 = 536$$

$$B_{V_4} = 536 - 114 = 422$$

$$B_{V_5} = 422 - 90 = 332$$

$$B_{V_6} = 332 - 71 = 261$$

$$B_{V_7} = 261 - 56 = 205$$

$$B_{V_8} = 205 - 44 = 161$$

$$B_{V_9} = 161 - 34 = 127$$

$$B_{V_{10}} = 127 - 27 = 100$$

The annual depreciation (A_D), depreciation up to any age n in life service of the asset (D_n), and book value (B_v) at the end of each year from 0 to 10 is calculated and tabulated in table 5 as follows,

$$f = 1 - \sqrt[n]{\frac{L}{P}}$$

$$f = 1 - \sqrt[10]{\frac{100}{1100}} = 0.213$$

A_D or D_n = Depreciation rate (f) \times Book value at the beginning of the year

Where, A_D = Annual depreciation

D_n = Depreciation up to any age n in life service of the asset or accumulated/cumulative depreciation at any age n in life service.

Therefore,

D_n = Depreciation rate (f) \times Book value at the beginning of the year

$$D_1 = 0.213 \times 1100 = 234$$

$$D_2 = 0.213 \times 866 = 185$$

$$D_3 = 0.213 \times 681 = 145$$

$$D_4 = 0.213 \times 536 = 114$$

$$D_5 = 0.213 \times 422 = 90$$

$$D_6 = 0.213 \times 332 = 71$$

$$D_7 = 0.213 \times 261 = 56$$

$$D_8 = 0.213 \times 205 = 44$$

$$D_9 = 0.213 \times 161 = 34$$

$$D_{10} = 0.213 \times 127 = 27$$

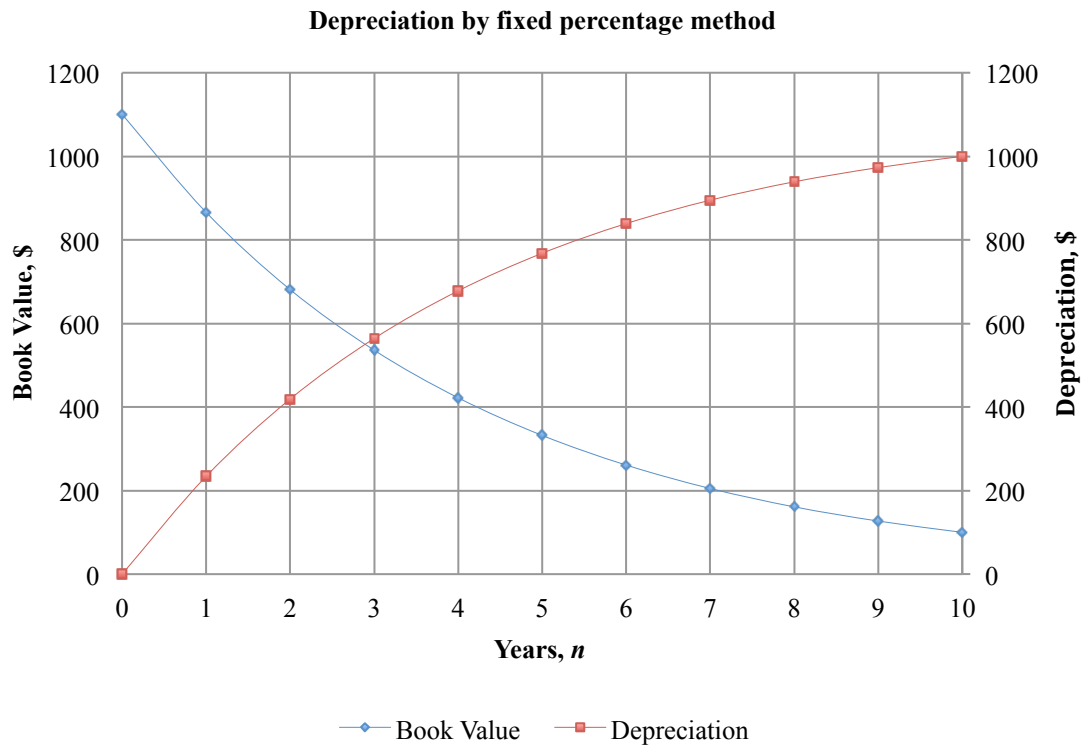
Where, f = depreciation rate (or) depreciation factor expressed in percentage; L = salvage value; P = principal/ original sum or fixed capital investment; B_v = book value;

Years 'n'	Book value at the beginning of the year ' B_v '	Depreciation rate ' f '	Depreciation expense (or) Annual Depreciation (A_D), \$	Accumulated depreciation (or) Cumulative depreciation (C_D), \$	Book value at the end of the year (B_v), \$
0	1100	0	0	0	1100
1	1100	0.213	234	234	866
2	866	0.213	185	419	681
3	681	0.213	145	564	536
4	536	0.213	114	678	422
5	422	0.213	90	768	332
6	332	0.213	71	839	261
7	261	0.213	56	895	205
8	205	0.213	44	939	161
9	161	0.213	34	973	127
10	127	0.213	27	1000	100

Table: 5 Fixed percentage or Declining-Balance Method

From the above table 5 it is once again observed that the sum of the cumulative depreciation/ or accumulated depreciation and the book value at the end of 10th year (i.e. \$1,001 + \$99 = \$1,100) will

be the original sum invested. The graph given below shows the behavior of book value and the depreciation (Annual depreciation /or Cumulative depreciation) from original time of investment till the end of expected service life of the equipment.



Years ' n '	Accumulated depreciation (or) Cumulative depreciation (C_D), \$	Book value at the end of the year (B_v), \$
0	0	1100
1	234	866
2	419	681
3	564	536
4	678	422
5	768	332
6	839	261
7	895	205
8	939	161
9	973	127
10	1000	100

Table 6. Book value and Depreciation by Fixed Percentage method

(3) Sinking-fund Method

Calculation:

The annual depreciation (A_D), depreciation up to any age n in life service of the asset (D_n), and book value (B_v) at the end of each year from 0 to 10 is calculated and tabulated in table 7 using the formulas given below,

$$A_D = (P - L) \left(\frac{i'}{(1 + i')^n - 1} \right)$$

$$D_n = (P - L) \left(\frac{\frac{i'}{(1 + i')^n - 1}}{\frac{i'}{(1 + i')^{n'} - 1}} \right)$$

$$B_v = P - (P - L) \left(\frac{\frac{i'}{(1 + i')^n - 1}}{\frac{i'}{(1 + i')^{n'} - 1}} \right)$$

where, A_D = annual depreciation

D_n = depreciation up to any age n in life service of the asset (or)
accumulated/cumulative depreciation at any age n in life service.

P = principal (or) investment or fixed capital cost

L = salvage value; n = total number of life service; n' = number of years of service
upto n age

B_v = book value; i = interest rate or interest; i' = sinking-fund interest

Therefore,

$$B_v = P - (P - L) \left(\frac{\frac{i'}{(1 + i')^n - 1}}{\frac{i'}{(1 + i')^{n'} - 1}} \right)$$

$$B_1 = 1100 - (1100 - 100) \left(\frac{\frac{0.06}{(1+0.06)^{10} - 1}}{\frac{0.06}{(1+0.06)^2 - 1}} \right) = 1100 - 1000 \left(\frac{0.0758}{1} \right) = 1100 - 156 = 1024$$

$$B_2 = 1100 - (1100 - 100) \left(\frac{\frac{0.06}{(1+0.06)^{10} - 1}}{\frac{0.06}{(1+0.06)^2 - 1}} \right) = 1100 - 1000 \left(\frac{0.075867}{0.48543} \right) = 1100 - 156 = 943$$

$$B_3 = 1100 - (1100 - 100) \left(\frac{\frac{0.06}{(1+0.06)^{10} - 1}}{\frac{0.06}{(1+0.06)^3 - 1}} \right) = 1100 - 1000 \left(\frac{0.075867}{0.314109} \right) = 1100 - 242 = 859$$

$$B_4 = 1100 - (1100 - 100) \left(\frac{\frac{0.06}{(1+0.06)^{10} - 1}}{\frac{0.06}{(1+0.06)^4 - 1}} \right) = 1100 - 1000 \left(\frac{0.075867}{0.228591} \right) = 1100 - 332 = 768$$

In the same manner up to the end of tenth year book value is calculated and tabulated in table 4.

Similarly, the annual depreciation (A_D) and the depreciation up to any age n in life service of the asset (D_n) is calculated and tabulated in table 4.

$$A_D = (P - L) \left(\frac{i'}{(1+i')^n - 1} \right)$$

$$A_D = (1100 - 100) \left(\frac{0.06}{(1+0.06)^{10} - 1} \right) = 76$$

$$D_n = (P - L) \left(\frac{\frac{i'}{(1+i')^n - 1}}{\frac{i'}{(1+i')^{n'} - 1}} \right)$$

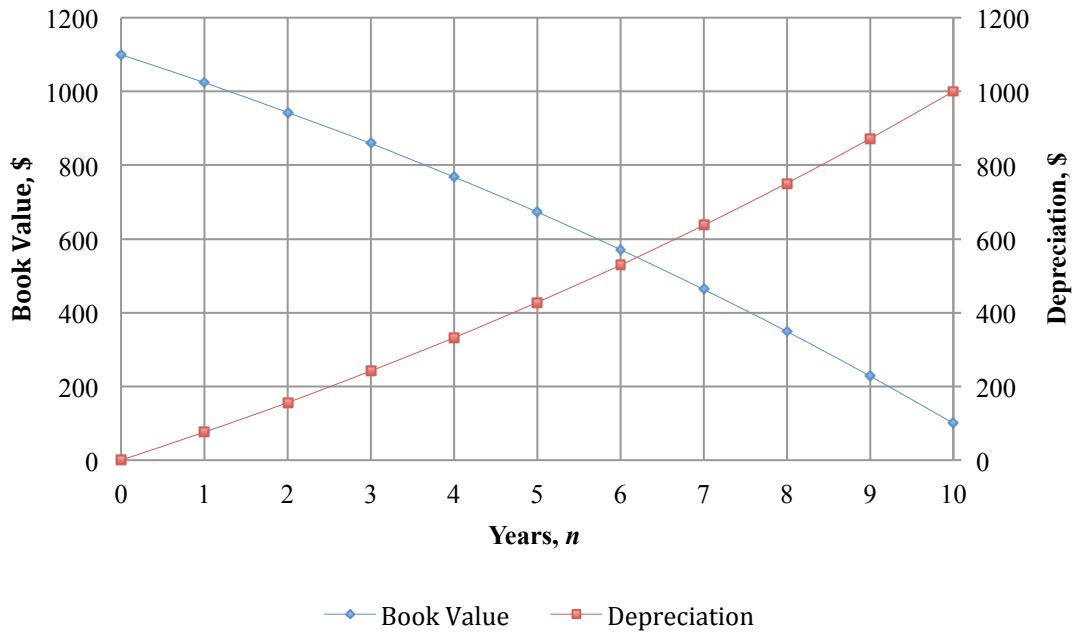
$$D_1 = (1100 - 100) \left(\frac{\frac{0.06}{(1+0.06)^{10} - 1}}{\frac{0.06}{(1+0.06)^1 - 1}} \right) = 76$$

Years 'n'	Book value at the beginning of the year 'B _v '	Interest rate 'i'	Depreciation expense (or) Annual Depreciation (A _D), \$	Accumulated depreciation (or) Cumulative depreciation (C _D), \$	Book value at the end of the year (B _v), \$
0	1100	0.06	0	0	1100
1	1100	0.06	76	76	1024
2	1024	0.06	76	156	943
3	943	0.06	76	242	859
4	859	0.06	76	332	768
5	768	0.06	76	428	673
6	673	0.06	76	529	571
7	571	0.06	76	637	463
8	463	0.06	76	751	349
9	349	0.06	76	872	228
10	228	0.06	76	1000	100

Table 7: Book value and depreciation by sinking-fund method

From the above table 7 it is noted that the sum of the cumulative depreciation/ or accumulated depreciation and the book value at the end of 10th year (i.e. \$1,000 + \$100 = \$1,100) will be the original sum invested. The graph given below will show the behavior of book value and the depreciation (Annual depreciation /or Cumulative depreciation) from original time of investment till the expected service life of the equipment. Now as mentioned above we will look into the behavior of the book value and the depreciation of heat exchanger in the graph. The graph given below says that the book value decreases when the depreciation increases as the time passes. The values used for plotting the graph are given in the table 8

Depreciation by sinking-fund method



Years ' n '	Book value at the end of the year (B_v), \$	Accumulated depreciation (or) Cumulative depreciation (C_D), \$
0	1100	0
1	1024	76
2	943	156
3	859	242
4	768	332
5	673	428
6	571	529
7	463	637
8	349	751
9	228	872
10	100	1000

Table 8. Book value and Depreciation by Sinking-fund method

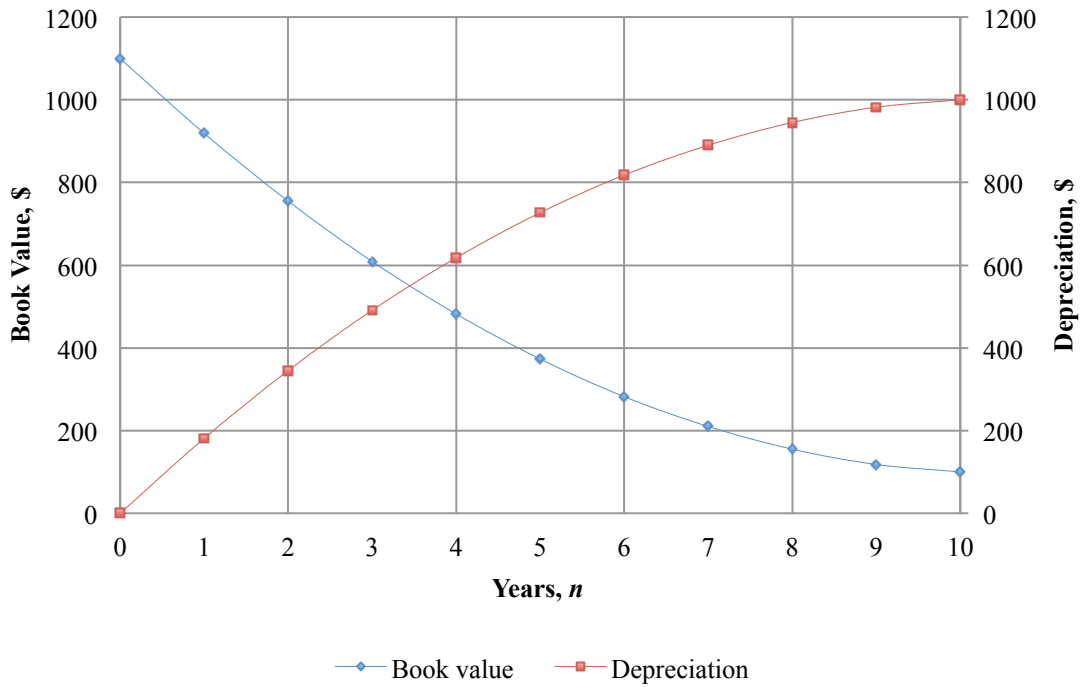
(4) Sum-of-the-years-digits method

$$\text{Sum-of-the-years-digits} = 1+2+3+4+5+6+7+8+9+10 = 50 \text{ (or) } \frac{n+n^2}{2} = \frac{10+10^2}{2} = 55$$

Years 'n'	Book value at the beginning of the year 'B _v ', \$	Total depreciable cost (P-L), \$	Depreciation factor	Accumulated depreciation (A _D), \$	Accumulated depreciation (or) Cumulative depreciation (C _D),	Book value at the end of the year (B _v), \$
0	1100	0	0	0	0	1100
1	1100	1000	$\frac{10}{55}$	$1000 \times \frac{10}{55} = 181$	181	919
2	919	1000	$\frac{09}{55}$	$1000 \times \frac{09}{55} = 164$	345	755
3	755	1000	$\frac{08}{55}$	$1000 \times \frac{08}{55} = 146$	491	609
4	609	1000	$\frac{07}{55}$	$1000 \times \frac{07}{55} = 127$	618	482
5	482	1000	$\frac{06}{55}$	$1000 \times \frac{06}{55} = 109$	727	373
6	373	1000	$\frac{05}{55}$	$1000 \times \frac{05}{55} = 91$	818	282
7	282	1000	$\frac{04}{55}$	$1000 \times \frac{04}{55} = 72$	890	210
8	210	1000	$\frac{03}{55}$	$1000 \times \frac{03}{55} = 55$	945	155
9	155	1000	$\frac{02}{55}$	$1000 \times \frac{02}{55} = 37$	982	118
10	118	1000	$\frac{01}{55}$	$1000 \times \frac{01}{55} = 18$	1000	100

Table 9. Depreciation by sum-of-the-years-digits method

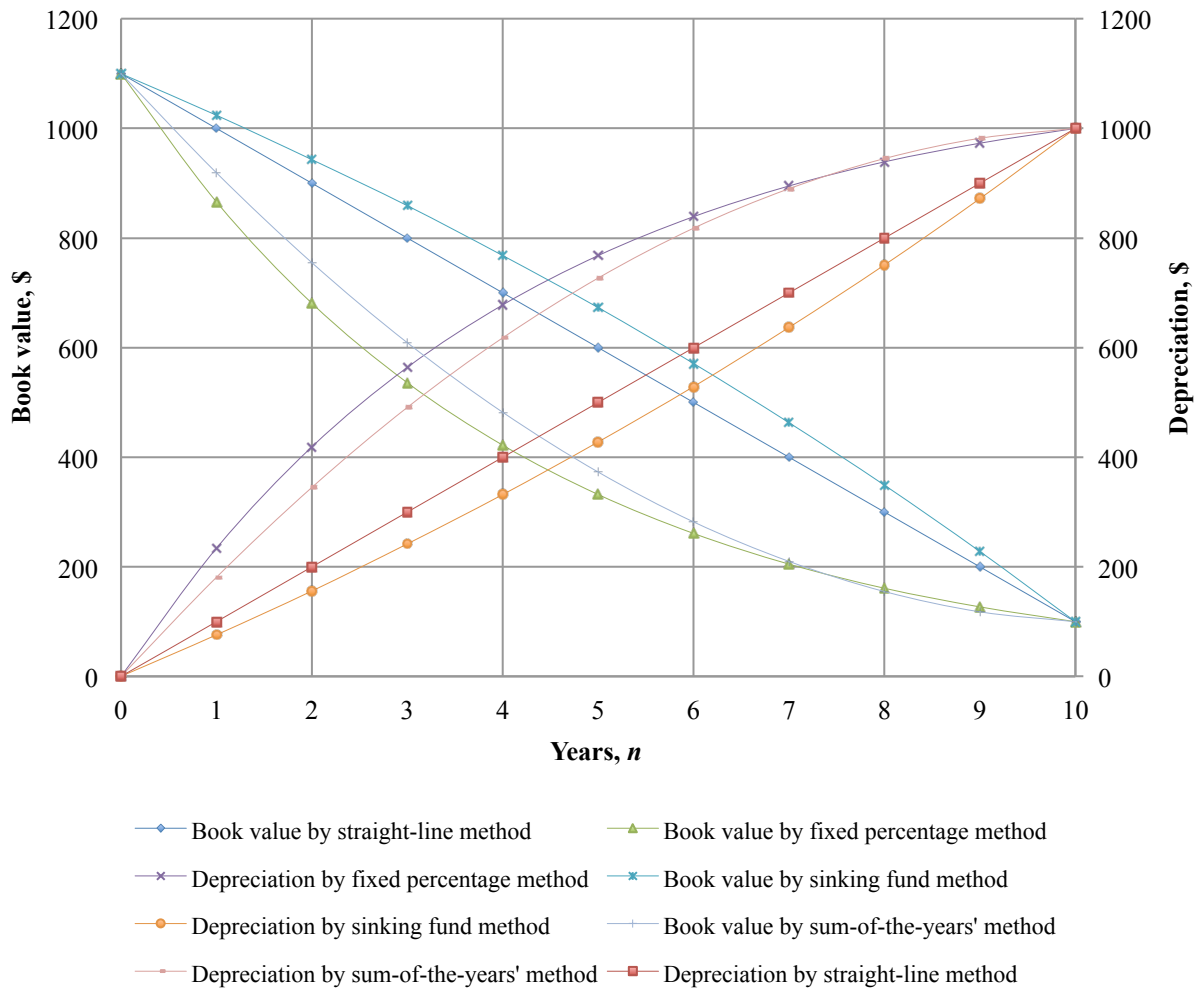
Depreciation by sum-of-the -years-digits method



Years ' n '	Book value at the end of the year (B_v), \$	Accumulated depreciation (or) Cumulative depreciation (C_D),
0	1100	0
1	919	181
2	755	345
3	609	491
4	482	618
5	373	727
6	282	818
7	210	890
8	155	945
9	118	982
10	100	1000

Table 10. Book value and Depreciation by sum-of-the-years-digits method

When all the data's are combined together the plot becomes



Years	Straight-line		Fixed percentage		Sinking fund		Sum-of-the-years	
	B_V	A_D^*	B_V	A_D^*	B_V	A_D^*	B_V	A_D^*
0	1100	0	1100	0	1100	0	1100	0
1	1000	100	866	234	1024	76	919	181
2	900	200	681	419	943	156	755	345
3	800	300	536	564	859	242	609	491
4	700	400	422	678	768	332	482	618
5	600	500	332	768	673	428	373	727
6	500	600	261	839	571	529	282	818
7	400	700	205	895	463	637	210	890
8	300	800	161	939	349	751	155	945
9	200	900	127	973	228	872	118	982
10	100	1000	100	1000	100	1000	100	1000

A_D^* represents annual/cumulative depreciation
 B_V represents book value at the end of the year

The above graph shows that from all the four methods, book value of the asset decreases, when

depreciation of the asset increases. Similarly, the sum of annual/cumulative depreciation and book value at the end of the year in all the four methods will be the original investment of the asset

Problem No. 2 The original value of a piece of equipment is \$22,000, completely installed and ready for use. Its salvage value is estimated to be \$2000 at the end of a service life of 10 years. Determine the asset (or book) value of the equipment at the end of 5 years using:

- (1) Straight–line method.
- (2) Fixed percentage or declining–balance method.

Solution

- (1) Straight–line method.

$$A_D = \frac{P - L}{n}$$

$$B_v = P - n'(D_n)$$

where, A_D = annual depreciation

D_n = depreciation up to any age n in life service of the asset or
accumulated/cumulative depreciation at any age n in life service.

P = principal, or investment or fixed capital cost

L = salvage value; n = total number of life service; n' = number of years of service
upto n age

B_v = book value at the end of year

$$A_D = \frac{P - L}{n}$$

$$A_D = \frac{22000 - 2000}{10} = \$2000$$

$$B_v = P - n'(A_D)$$

$$B_v = 22,000 - 5(2000) = \$12,000$$

(or)

the other way of solving the same problem by straight–line method is as follows,

$$D_n = \frac{n'(P-L)}{n}$$

$$B_v = P - n'(D_n)$$

where, A_D = annual depreciation

D_n = depreciation up to any age n in life service of the asset or accumulated/cumulative depreciation at any age n in life service.

P = principal, or investment or fixed capital cost

L = salvage value; n = total number of life service; n' = number of years of service upto n age

B_v = book value at the end of year

$$D_n = \frac{n'(P-L)}{n}$$

$$D_1 = \frac{1(22,000 - 2000)}{10} = \$2000$$

$$D_2 = \frac{2(22,000 - 2000)}{10} = \$4,000$$

$$D_3 = \frac{3(22,000 - 2000)}{10} = \$6,000$$

$$D_4 = \frac{4(22,000 - 2000)}{10} = \$8,000$$

$$D_5 = \frac{5(22,000 - 2000)}{10} = \$10,000$$

$$B_v = P - n'(D_n)$$

$$B_1 = 22,000 - 1(2000) = \$20,000 \quad B_1 = 22,000 - 1(2000) = \$20,000$$

$$B_2 = 22,000 - 2(2000) = \$18,000 \quad B_2 = 22,000 - 2(2000) = \$18,000$$

$$B_3 = 22,000 - 3(2000) = \$16,000 \quad B_3 = 22,000 - 3(2000) = \$16,000$$

$$B_4 = 22,000 - 4(2000) = \$14,000 \quad B_4 = 22,000 - 4(2000) = \$14,000$$

$$B_5 = 22,000 - 5(2000) = \$12,000 \quad B_5 = 22,000 - 5(2000) = \$12,000$$

(2) Fixed percentage or declining-balance method.

$$f = 1 - \sqrt[n]{\frac{L}{P}}$$

$$B_v = P(1 - f)^n$$

Where, f = depreciation rate (or) depreciation factor expressed in percentage; L = salvage value; P = principal/ original sum or fixed capital investment; B_v = book value;
 n' = number of years of service upto n age.

$$B_v = P(1 - f)^n$$

$$B_v = 22,000(1 - 0.213)^5$$

$$B_v = 22,000(0.30191)$$

$$B_v = 6,641.96$$

Result

The asset (or book) value of the equipment at the end of 5 years using:

(1) Straight-line method is \$12,000

(2) Fixed percentage method is \$6,642

Depletion

When exhaustible resources are sold, part of the sales realization is a return of capital, and the income tax should adjust for that. Also, it is desirable to have an incentive to encourage exploration for new resources as existing resources are used up. Depletion is the term used to describe the write-off certain exhaustible natural resources such as minerals, oils and gas, timber. Depletion implies to production units withdrawn from the property, whereas depreciation limited to original cost less the estimated salvage value.

In other words, Capacity loss due to materials actually consumed is measured as *depletion*. Depletion cost equals the initial cost times the ratio of amount of material used to original amount of material purchased. This type of depreciation is particularly applicable to natural resources, such as stands of timber or mineral and oil deposits.

There are two methods for computing depletion:

1. Cost depletion
2. Percentage depletion

Cost depletion:

The value of depletion, unit, say, a ton of ore is arrived at by calculating the total value depleted (or reduced) divided by the tons of ore to be depleted.

$$\text{Value of depletion unit(a ton of ore)} = \frac{\text{total value depleted}}{\text{tons of ore to be depleted}}$$

Deduction for a tax year = depletion unit \times the number of units sold within the year

Percentage depletion:

In the percentage method, the depletion allowance for the year is a specified percentage of the “gross income from the property” but must not exceed 50% of the taxable income figured without depletion allowance. The cost depletion method can always be used but the percentage depletion method has certain shortcomings. The deduction should be computed in both the ways, if applicable, and the large deduction taken.

Percentage depletion varies from 5% to a maximum of 22% Oil and gas wells have recently lost the percentage depletion allowance, except for certain small producers subjected to limitations. It should be noted that under cost depletion, when total cost and accumulated depletion are equal, no further cost depletion is allowed. However, percentage depletion is not limited to original cost less salvage, as is ordinarily true with depreciation of assets.

Problem: 1 A mining property with an estimated 1 megaton ($Mt=1\times 10^6$ t) of ore originally cost \$50,00,000 (50 lakhs). In one year 100 kilotons (kt) of ore is sold for \$16/t with expenses of \$10,00,000 (10 lakhs). The percentage depletion allowance is 50%, and the tax rate is 46%. Calculate the annual cash flow. Which is more advantageous, cost depletion or percentage depletion?

Solution:

Given :

Available or estimated resources	=	1×10^6 t	
Original cost of the resource	=	\$50,00,000	
Quantity of ore sold in a year	=	100 kt	= 100×10^3 t
Price of ore sold in a particular year	=	\$16/t	
Percentage depletion allowance	=	50%	
The tax rate for the year	=	46%	

Required:

1. Annual cash flow for the year
2. Suggestion for the advantageous method (Cost depletion or Percentage depletion) or Money saved or Income retained after depletion (Cost and Percentage) allowance

Calculation:

$$\text{Cost depletion} = \text{Value of depletion unit(a ton of ore)} = \frac{\text{total value depleted}}{\text{tons of ore to be depleted}}$$

$$= \frac{50,00,000}{10,00,000} = \frac{\$5}{t}$$

$$\text{Price of ore sold in a year} = \frac{\$16}{t}$$

$$1 \text{ ton} = \$16$$

$$100 \times 10^3 \text{ ton} = \frac{100 \times 10^3 t \times \$16}{1t}$$

$$= 100 \times 10^3 \times 16$$

$$= \$16,00,000$$

$$\left. \begin{array}{l} \text{Gross income on the sale} \\ \text{of 100 kt of ore} \end{array} \right\} = \$16,00,000$$

Particulars	Cost depletion	Percentage depletion
1. Gross income	\$16,00,000.00	\$16,00,000.00
2. Expenses for the year, excluding depletion	\$10,00,000.00	\$10,00,000.00
3. Gross income after expenses, taxable income [1 – 2]	\$6,00,000.00	\$6,00,000.00
4. (a) Cost of depletion at \$5/t for 100 kt i.e 1t = \$5 for 100 kt it is = \$5,00,000.00	\$5,00,000.00	NA*
4. (b) Percentage depletion, at 50% i.e. 50% of [3]	NA*	\$3,00,000.00
5. Actual income after the above deductions [cost and percentage depletion] or actual taxable income [3 – 4a]	\$1,00,000.00	\$3,00,000.00 [3 – 4b]
6. Tax at 46% on actual income	\$46,000.00	\$1,38,000.00
7. Net cash flow or Available cash after all tax deduction [3 – 6]	\$5,54,000.00	\$4,62,000.00

*NA – Not Applicable

Therefore, the cost depletion is advantageous that the percentage depletion.

since the larger deduction is usually taken into account as mentioned above.

Result :

1. The annual cash flow by

a) Cost depletion = $\boxed{\$5,54,000.00}$

b) Percentage depletion = $\boxed{\$4,62,000.00}$

2. The cost depletion is advantageous than the percentage depletion.