An Alternative Approach to Cognitive and Achievement Relations Research: An Introduction to Quantile Regression



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Abstract

A large body of prior work shows that cognitive abilities and basic academic skills explain individual differences in performance on reading, writing, and math achievement measures. However, this research focuses exclusively on the average relations between cognitive and achievement scores without consideration of whether effects vary at different thresholds of academic performance. To address this limitation, we employed unconditional quantile regression to explore the effects of cognitive abilities and basic and intermediate academic skills on advanced achievement outcomes as a function of reading, writing, or math skill level in a large nationally representative sample of youth and adolescent children (N = 3891). Quantile regression is a methodological technique that allows for a more nuanced examination of whether differential effects along the distribution of an outcome skill exist, which often goes undetected when employing more conventional regression methods that focus on mean effects. Findings from this exploratory study generally showed that cognitive abilities and basic and intermediate academic skills had a pattern of differential effects depending on achievement level, with stronger effects often observed when performance on the academic outcome measure was lower. This exploratory study provides an illustrative example of unconditional quantile regression and how it can be interpreted and applied within an area of relevance to pediatric neuropsychology.

Keywords Quantile regression · Cognitive abilities · Academic achievement · Woodcock-Johnson Tests · Ordinary least squares · Pediatric neuropsychology

Introduction

When pediatric neuropsychology researchers explore how neuropsychological constructs relate to academic achievement, they typically use (OLS) regression or structural equation modeling (SEM) analytic frameworks. In the traditional use of these techniques, a single estimated slope represents each predictor's "effect" on an outcome variable. An

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important limitation of these methods is that they inform us about the *conditional mean* of an outcome variable and have little to say about how the predictors relate to the rest of the outcome variable's distribution.

Techniques such as polynomial regression test whether a predictor's effect changes at different levels of the predictor. Cross-product regression tests interactions such that a predictor's effect changes dynamically as a function of other predictors (Keith 2019). However, techniques have been developed to explore how predictors' effect on an outcome can change at different levels of the outcome variable. That is, a predictor may have a strong relationship with the outcome variable when the outcome variable is low but a weak relationship at higher levels of the outcome variable. Quantile regression may be a more suitable technique when the level of analysis is concerned with how variables differentially predict the whole outcome distribution or at the margins of performance.

The use of quantile regression has grown steadily since it was first introduced in the literature (Koenker and Bassett 1978). Historically, conditional quantile regression (CQR; Koenker and Hallock 2001) has been more frequently used; however, more recently unconditional quantile regression



(UQR; Firpo et al. 2009) has gained traction in the literature (Porter 2015). The current paper will focus on the use of quantile regression in an area of pediatric neuropsychology; the appropriateness of CQR versus UQR; and specifically, an example of UQR examining cognitive abilities and basic and intermediate academic skills as predictors of advanced academic achievement for children and adolescents in kindergarten through twelfth grade using the standardization samples for the Woodcock-Johnson IV (Schrank et al. 2014a). Last, the paper will cover basic issues in the implementation and interpretation of quantile regression using the aforementioned data as an illustration.

Why Use Quantile Regression in Pediatric Neuropsychology Research?

Quantile regression is an analytic technique that addresses practical and theoretical questions of interest when more conventional regression methods may fall short. Quantile regression allows for the examination of changes in distributions and provides more specific information than would OLS regression techniques that focus on the effects of the mean of the dependent variable. Specifically, unconditional quantile regression estimates effects at different points of the distribution of y (e.g., 0.1 quantile, 0.5 quantile, 0.9 quantile) or across the whole distribution, allowing researchers within pediatric neuropsychology to better understand how pertinent variables affect the entire distribution of an outcome variable, rather than just the mean of the distribution. This is particularly beneficial when trying to understand the unique learning needs of struggling students and higher performing students (Schneider and Kaufman 2017), as neuropsychologists are more likely to interact and intervene with youth functioning outside the average range. In other words, UQR extends the focus of effects on typical levels of the dependent variable to how effects differ at different thresholds of the outcome variable.

Researchers and educators are frequently interested in understanding which cognitive variables contribute to different levels of academic performance. For example, does working memory affect the reading comprehension performance of all readers equally, or more so for struggling readers? If vocabulary and general knowledge show positive effects on writing, does it become more important in more competent products of writing quality? The same logic may also apply to intervention research, where it can only be concluded that an intervention has (or does not have) an effect on the average of the measured dependent variable of interest. However, it may be the case that an intervention has a significant impact on those individuals at greatest risk (as measured by the lowest scores on a curriculum-based measure) but minimal to no impact for those at relatively less risk (as measured by the highest scores on a curriculum-based measure) despite showing an overall positive effect for the whole group of individuals. In this manner, an intervention may show larger effects when scores on the outcome measure are lowest, but this trend may be masked when examining the effects on the overall group. With this in mind, an optimal use of quantile regression within pediatric neuropsychology is when a more specific understanding of how relevant explanatory variables play a role in outcomes for those with differing characteristics on the outcome measure.

Another advantage of quantile regression is that it better aligns with current conceptualizations of developmental models of achievement. The acquisition of advanced academic competencies follows a hierarchical developmental sequence where proficiency in basic academic skills and underlying cognitive processes capacitate as well as facilitate more advanced academic skills. Some academic skills are less important as they become fluent and automatized, whereas others become increasingly important with age. For example, research has shown that reading decoding skills become less important for reading comprehension as students mature (Hajovsky et al. 2014), whereas other academic skills such as math computation become increasingly important for math problem-solving (Villeneuve et al. 2019). As such, it would be expected that at different levels of academic performance (or maturational development), different cognitive and academic skill variables would account for performance differences. Stated differently, the effect of certain variables may not be constant across achievement levels but instead may differ at the lower and higher margins of the unconditional distribution of the dependent variable. Quantile regression allows us to observe these differences rather than simply observing the effect on mean levels of y.

While quantile regression allows greater specificity when examining the distribution of the outcome measure compared with what can be gleaned from OLS regression, there has been some discussion and clarification about the appropriate use of quantile regression models in educational and developmental science research (cf. Petscher and Logan 2014; Wenz 2019). In quantile regression, there are two primary modeling choices, CQR and UQR. The current paper primarily focuses on UQR as it is more applicable for this scope of work relevant to pediatric neuropsychologists. Specifically, CQR "estimates the effect of an independent variable on the conditional distribution of y, such that the coefficient must be interpreted as a within-group effect, where the groups are defined by the independent variables used in the model" (Porter 2015, p. 374). Thus, in CQR "interpretation of coefficients is in relation to the quantiles of the distributions defined by the covariates (the conditional distribution), rather than the unconditional distribution of y" (Porter 2015, p. 342). In this manner, CQR is useful when the focus is on understanding the varying influences of covariates along the distribution of an outcome. However, with each covariate that is added to a CQR model,



the results may not be consistent across all quantiles (percentiles) as the conditional quantiles may vary, making interpretation of the CQR coefficients less clear. The CQR approach is less useful within pediatric neuropsychology when the focus is on understanding how explanatory variables differ in importance as a function of a child's achievement level.¹

A more suitable approach is the use of UQR, which examines the effect of an independent variable across the entire unconditional distribution of the dependent variable (Borah and Basu 2013; Firpo et al. 2009; Porter 2015). This latter approach is typically used as it focuses on how an effect varies depending on the location in the unconditional distribution of y, which is similar to OLS regression in its interpretation of effects even when multiple covariates are added in the UQR model. Stated differently, UQR estimates coefficients without reference to values of other variables in the model by defining quantiles before fitting regressions (Fuchs et al. 2020; Porter 2015). For the purposes of this paper, when we indicate "quantile regression," it is in reference to UQR.

Figure 1 provides a hypothetical illustration of a UQR analysis in which the regression coefficients differ across the quantiles of the outcome variable. In this example, the *y*-axis represents the strength of the relation between the predictor variable and the outcome variable, whereas the *x*-axis represents the selected quantiles for the unconditional distribution of the outcome variable. The black line connects points located at each quantile, which represents the coefficient at that quantile. The shading around the black line represents the 95% confidence interval for the regression coefficient at each quantile. As shown in Fig. 1, the strength of the effect of the predictor variable is strongest at the lower end of the outcome variable distribution, with the effect decreasing or leveling off at the middle and upper end of the outcome variable distribution, respectively.

Interpretation in Quantile Regression

Quantile regression offers researchers a method for analyzing distributions instead of means and provides insight into how independent variables affect the entire distribution of the dependent variable. For example, a regression coefficient in the parlance of OLS tells us the effect of x on the mean of y, while all other x's are controlled (Keith 2019; Wenz 2019). Conversely, quantile regression estimates the change in a specified quantile of y by a one unit change in each x, which permits a comparison of how quantiles on y are affected by each x (Koenker 2015; Fuchs et al. 2020). Said differently, quantile regression can address the

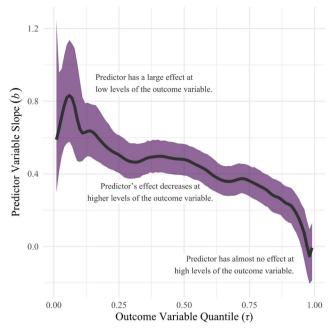


Fig. 1 Unconditional quantile regression analysis of hypothetical variables in which the regression coefficients differ across the quantiles of the outcome variable

following question: If x increases (or decreases) by one unit, how much does the distribution of y change (Porter 2015)?

When interpreting coefficients from UQR models, it is helpful to compare how it differs from OLS regression. Take for example an OLS regression model with a continuous variable (e.g., vocabulary knowledge) and a dichotomous variable (e.g., gender) included as simultaneous predictors of writing achievement. These variables represent main effects where the coefficient of vocabulary knowledge is interpreted as the effect on the mean of writing achievement, after controlling for differences in writing achievement due to gender. Similarly, the gender coefficient is interpreted as the mean difference in writing achievement between males and females, after controlling for differences in writing due to vocabulary knowledge. Quantile regression (specifically UQR) differs by focusing on the effect of an independent variable on a particular quantile of the outcome variable (Hajovsky et al. 2018a; Porter 2015). Using the example above, let us assume that vocabulary knowledge has a standardized regression coefficient of 0.45 at the 0.1 quantile (percentile) on writing achievement, when controlling for gender differences in writing. The interpretation of this coefficient can be understood as follows: for children at the 10th percentile in writing, a one standard deviation increase in vocabulary knowledge is associated with a 0.45 standard deviation increase in writing achievement, controlling for gender differences (Fuchs et al. 2020). The coefficient and its interpretation may differ for children at the 90th percentile in writing.



¹ However, CQR may be appropriate when analyzing the growth between past and current scores (see Betebenner 2009 or Porter 2015).

Cognitive-Achievement Relations Research Using Quantile Regression

The cognitive abilities used in this study and within some of the prior research are interpreted through the framework of the Cattell-Horn-Carroll (CHC) theory. CHC theory is a three-stratum hierarchical structure of cognitive abilities with the most general cognitive ability, g or overall intelligence, at the apex of the model. Overall intelligence subsumes several broad abilities, which in turn subsume many narrow abilities (Schneider and McGrew 2018). Eight primary CHC broad abilities are examined in our study. Fluid reasoning (Gf) is the ability to solve novel problems; comprehension-knowledge (Gc) involves the depth and breadth of acquired cultural knowledge including language²; learning efficiency (Gl) is the ability to learn and store information over a period of time; retrieval fluency (Gr) involves the rate and fluency in retrieving information stored in long-term memory; visual processing (Gv) is the ability to perceive visual images and use mental imagery; auditory processing (Ga) is the ability to perceive and process sounds; short-term working memory (Gwm) is the ability to hold information in immediate awareness and manipulate the information; and processing speed (Gs) is the ability to quickly and accurately perform simple tasks (Schneider and McGrew 2018).

Cognitive-achievement relations research is primarily based on results from OLS regression or SEM (Caemmerer et al. 2018; Cormier et al. 2016Hajovsky et al. 2019; Lewno-Dumdie and Hajovsky 2019; Niileksela, Reynolds, Keith, & McGrew, 2016). Quantile regression results, however, illustrate important differences between high and low academic performers. For example, in a sample of 245 third-grade children, basic word reading skills predicted lower reading comprehension skills for lower skilled readers (0.4 quantile and lower), whereas language abilities (i.e., Gc) were more uniquely associated with reading comprehension for higher skilled readers (Language and Reading Research Consortium and Logan 2017). Thus, the relation between explanatory variables and reading comprehension varied as a function of reading comprehension skill (i.e., poor, average, or good readers).

In another study using quantile regression and including CHC cognitive explanatory variables, Hajovsky et al. (2018a) examined how CHC-based cognitive abilities and basic writing skills (i.e., spelling) predicted written expression across writing ability levels using the Kaufman Assessment Battery for Children, Second Edition (KABC-II: Kaufman and Kaufman 2004a) and the Kaufman Tests of

 $^{^2}$ Scholars disagree on the interpretation of the CHC factor Gc as either an underlying cause of individual differences (biological capacity) or a statistical entity (Kan et al. 2011).



Educational Achievement, Second Edition (KTEA-II; Kaufman and Kaufman 2004b), co-normed standardization sample data. While spelling predicted written expression across all writing ability levels, more language-based variables (i.e., learning efficiency (Gl), crystallized ability (Gc)) influenced higher writing ability level. This study showed that CHC-based cognitive predictors differ in strength across the continuum of writing achievement skill (Hajovsky et al. 2018b). However, a limitation of this study is that it did not include other relevant measures of cognitive constructs for writing (because the KABC-II does not measure processing speed and auditory processing), lending additional strength to the current study.

Purpose of Current Study

The purpose of this exploratory study was to provide an illustrative example regarding the use of UQR in the study of cognitive abilities and academic achievement relations and to compare findings drawn from a more conventional regression method. Research shows CHC-based cognitive abilities influence academic achievement with the majority of this correlational evidence based on techniques that focus on the mean relations between cognitive and achievement scores (cf. Caemmerer et al. 2018; Hajovsky et al. 2019; Villeneuve et al. 2019). A consequence of these analytic choices is that the relations between cognitive ability and academic skills for various thresholds of academic performance are relatively unknown. UQR overcomes this limitation as influences on academic achievement can be estimated for those with differing levels of achievement (i.e., low [0.10], average [0.50], or high [0.90] performance). This exploratory study addressed the following research questions within a large, nationally representative pediatric sample:

- 1. Which CHC-based cognitive abilities (i.e., Gc, Gf, Gs, Ga, Gwm, Gl, Gr, and Gv) have differential effects on basic academic skills (i.e., letter-word identification, spelling, and math facts fluency) in the domains of reading, writing, and mathematics achievement across grade groups (i.e., K–2, 3–6, and 7–12)?
- 2. Do CHC-based cognitive abilities and basic academic skills (e.g., letter-word identification) have differential effects on intermediate reading, writing, and mathematics skill (e.g., sentence reading fluency) levels across grade groups?
- 3. How consistent are the differential effects of CHC-based cognitive abilities and basic and intermediate academic skills on advanced reading, writing, and mathematics skill levels across grade groups?

Methods

Participants

The dataset utilized in this introductory UOR study is from the Woodcock-Johnson IV Tests of Cognitive Abilities (WJ IV COG; Schrank et al. 2014b), the Woodcock-Johnson IV Tests of Academic Achievement (WJ IV ACH: Schrank et al. 2014c), and the Woodcock-Johnson IV Tests of Oral Language (WJ IV OL; Schrank et al. 2014d) co-normed standardization sample data. The WJ IV co-normed sample includes a diverse array of participants from 100 geographic areas across the USA, where data were gathered according to a stratified random sampling design (McGrew et al. 2014). The demographic features of the study sample matched those of the general US population according to 2010 Census data (McGrew et al. 2014). For the current study, we only used children and youth in kindergarten through twelfth grade (N =3891). Specifically, the effects were examined across three disaggregated grade groups: kindergarten through second grade (n = 973), third through sixth grade (n = 1293), and seventh through twelfth grade (n = 1625).

Measures

The WJ IV COG, WJ IV OL, and WJ IV ACH include multiple measures to represent cognitive abilities and academic skills according to the rich history of CHC theory research and expert consensus (Carroll 1993; McGrew et al. 2014; Schneider and McGrew 2018). The WJ IV test batteries are individually administered, where after administration, raw scores are converted to normative standard scores with a mean of 100 and a standard deviation of 15. Utilizing standardized scores in the current analysis mirrors clinical practice, thus providing valuable insights when linking cognitive and basic and intermediate academic skills scores across various thresholds of advanced academic skills.

Cognitive Abilities

Age-referenced standardized scores from both WJ IV COG and WJ IV OL measures were used to index eight CHC-based cognitive abilities, where each cognitive ability index served as an independent variable in all predictive models. The Gc composite is comprised of subtests oral vocabulary, general information, and picture vocabulary; Gf, concept formation and analysis-synthesis; Gwm, verbal attention, numbers reversed, and object-number sequencing; Gs, letter-pattern matching and pair cancelation; Ga, segmentation and sound blending; Gl, Story recall and visual-auditory learning; Gr, rapid picture naming and retrieval fluency; and last, Gv, visualization and picture recognition. The majority of internal consistency reliability coefficients for tests range from 0.77 to

0.99 for individuals ages 5–19 years with the exception of picture recognition which range from 0.61 to 0.81 (McGrew et al. 2014). The median test-retest reliability coefficients for timed tests are 0.79–0.91 for individuals ages 7–19 years (McGrew et al. 2014). Concurrent, criterion, developmental, and structural validity evidence is included in the WJ IV Technical (McGrew et al. 2014).

Academic Skills

Age-referenced standardized scores from the WJ IV ACH were utilized to represent the hierarchy of 10 academic skills-basic, intermediate, and advanced-in reading, writing, and mathematics. Three subtests represent basic academic skills: letter-word identification (reading), spelling (writing), and math facts fluency (mathematics). Four subtests represent intermediate academic skills: word reading fluency and sentence reading fluency (reading), sentence writing fluency (writing), and calculation (mathematics). Three subtests represent advanced academic skills: passage comprehension (reading comprehension), writing samples (written expression), and applied problems (math problem-solving). The internal consistency reliability coefficients for subtests range from 0.81 to 0.98 for individuals ages 5-19 years, and the median test-retest reliability coefficients for timed tests are from 0.79 to 0.91 for individuals ages 7-19 years (McGrew et al. 2014).

Analysis Plan

To answer the three research questions of the current study, UQR models were employed. UQR allows for the evaluation of the cognitive ability (and academic subskill) variables across levels of performance in basic (or more advanced) academic skills. The UQR model results are the primary focus of the current study, whereas the OLS results are only illustrated for comparison purposes.

The ten academic skills were separately modeled as outcome variables. In the three basic academic skill models, the eight CHC-based cognitive abilities served as independent variables (see Table 1 for a detailed list of the independent and dependent variables in each model). In the four intermediate academic skill models, the eight CHC cognitive abilities and the associated basic skill for each academic domain served as independent variables. In the three advanced academic skill models, the eight CHC cognitive abilities, basic skill, and intermediate skill for each academic domain were tested as predictor variables. Letter-word identification (basic reading) was included as a predictor in all writing skill outcome models as letter-word identification is an important predictor of writing skill (Decker et al. 2016). Before running each model, all independent variables were mean-centered to aid in interpretation of results (Keith 2019). Ten UQR and



Table 1 Predictors tested in each of the 10 UQR and 10 OLS models

Outcome variables	Independent/predictor variables
Letter-word identification (basic reading)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv
2. Spelling (basic writing)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv, letter-word identification
3. Math facts fluency (basic math)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv
4. Word reading fluency (intermediate reading)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv, letter-word identification
5. Sentence reading fluency (intermediate reading)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv, letter-word identification, word reading fluency
6. Sentence writing fluency (intermediate writing)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv, letter-word identification, spelling
7. Calculation (intermediate math)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv, math facts fluency
8. Passage comprehension (advanced reading)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv, letter-word identification, word reading fluency, sentence reading fluency
9. Writing samples (advanced writing)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv, letter-word identification, spelling, sentence writing fluency
10. Applied problems (advanced math)	Gc, Gf, Gwm, Gs, Ga, Gl, Gr, Gv, math facts fluency, calculation

Gc comprehension-knowledge, Gf fluid reasoning, Gwm short-term working memory, Gs processing speed, Ga auditory processing, Gl learning efficiency, Gr retrieval fluency, Gv visual processing

OLS models were tested across each grade-level group (i.e., K-2, 3-6, 7-12) for a total of 30 models analyzed. To answer our research questions, observed data for the measures at each grade-level group were used to estimate the OLS and UQR regression models in R (R Core Team 2019, ver. 3.5.3).

Results

Means and standard deviations as well as univariate skewness and kurtosis values for each variable across each grade-level group (i.e., K-2, 3-6, 7-12) are located in Supplemental Tables 1–3. In addressing our research questions, we briefly report findings for the first two research questions that focus on which CHC-based cognitive abilities affect basic academic skills and whether CHC-based cognitive abilities and basic academic skills have differential effects on intermediate academic skills across grade groups (K-2, 3-6, and 7-12; see Table 1 for a list of predictor and outcome variables). As the main focus of our study is on the application of UOR in the study of CHC-based cognitive abilities and basic and intermediate academic skill predictors of advanced academic achievement, we provide a more in-depth interpretation of quantile coefficients and discuss effects across grade groups (see Tables 2, 3, and 4, respectively). Next, we then briefly compare results from the UQR analysis with findings from OLS regression to demonstrate how findings differ across methodological approaches. All other UQR and OLS parameters for grades K-2, 3-6, and 7-12 are presented in Supplemental Tables 4–30. Coefficients are reported for three quantiles only—0.1, 0.5, 0.9—for the sake of simplicity, but all nine quantiles were tested (0.1–0.9).

Findings for one basic academic skill—letter-word identification—are discussed, but the OLS regression and UQR results for spelling and math facts fluency across grade groups are reported in Supplemental Tables 13–15 and 22–24, respectively. The most consistent and largest cognitive ability effect on letter-word identification was Gc, with effects generally decreasing across letter-word identification performance (see

Table 2 Ordinary least squares (OLS) and unconditional quantile multiple regression coefficients for predictors of passage comprehension in grades K through 2

		Quantile coefficients		
Predictor	OLS	0.1	0.5	0.9
Intercept	100.81	82.21	101.03	119.79
Letter-word identification	0.50	0.65	0.44	0.43
Word reading fluency	0.03	-0.02	0.11	- 0.12
Sentence reading fluency	0.16	0.11	0.18	0.28
Fluid reasoning (Gf)	0.03	0.04	-0.01	0.08
Comprehension-knowledge (Gc)	0.16	0.21	0.14	0.15
Short-term working memory (Gwm)	-0.03	- 0.05	- 0.02	- 0.09
Visual processing (Gv)	0.04	0.00	0.05	0.00
Processing speed (Gs)	-0.03	- 0.07	- 0.09	0.09
Learning efficiency (Gl)	-0.02	0.09	-0.01	- 0.06
Retrieval fluency (Gr)	-0.02	- 0.05	0.00	- 0.07
Auditory processing (Ga)	0.20	0.16	0.18	0.24

Statistically significant estimates (p < 0.05) are in italics



Table 3 Ordinary least squares (OLS) and unconditional quantile multiple regression coefficients for predictors of passage comprehension in grades 3 through 6

		Quantile coefficients		
Predictor	OLS	0.1	0.5	0.9
Intercept	100.75	81.48	100.45	119.99
Letter-word identification	0.45	0.65	0.39	0.44
Word reading fluency	0.05	0.06	0.05	0.01
Sentence reading fluency	0.17	0.24	0.07	0.14
Fluid reasoning (Gf)	-0.02	-0.01	0.02	- 0.20
Comprehension-knowledge (Gc)	0.24	0.15	0.29	0.32
Short-term working memory (Gwm)	-0.02	- 0.17	0.02	-0.05
Visual processing (Gv)	0.05	0.09	0.11	0.03
Processing speed (Gs)	-0.05	- 0.09	-0.06	0.06
Learning efficiency (Gl)	-0.01	0.08	-0.06	0.00
Retrieval fluency (Gr)	-0.02	0.04	- 0.03	- 0.07
Auditory processing (Ga)	0.18	0.19	0.16	0.26

Statistically significant estimates (p < 0.05) are in italics

Supplemental Tables 4–6). Ga also had consistent effects with stronger effects observed at the lower end of letter-word identification performance in grades K–2. Gs demonstrated consistently stronger effects on the lower end of the distribution of letter-word identification across all grades. The effects of Gwm were mostly observed in grades 3 or higher, whereas Gv and Gr had less consistent effects across letter-word identification performance.

Table 4 Ordinary least squares (OLS) and unconditional quantile multiple regression coefficients for predictors of passage comprehension in grades 7 through 12

		Quantile coefficients		
Predictor	OLS	0.1	0.5	0.9
Intercept	99.47	78.89	99.36	120.40
Letter-word identification	0.34	0.33	0.26	0.38
Word reading fluency	0.05	0.09	- 0.02	0.02
Sentence reading fluency	0.24	0.30	0.27	0.22
Fluid reasoning (Gf)	0.02	0.03	0.02	0.04
Comprehension-knowledge (Gc)	0.26	0.22	0.31	0.23
Short-term working memory (Gwm)	0.00	0.03	0.00	- 0.07
Visual processing (Gv)	0.03	0.02	- 0.06	0.03
Processing speed (Gs)	- 0.06	- 0.04	- 0.06	- 0.05
Learning efficiency (Gl)	0.01	-0.01	- 0.02	0.07
Retrieval fluency (Gr)	- 0.06	- 0.12	- 0.02	- 0.10
Auditory processing (Ga)	0.23	0.16	0.24	0.25

Statistically significant estimates (p < 0.05) are in italics

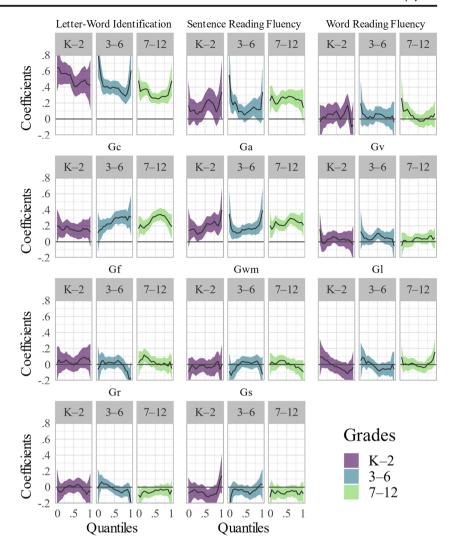
Findings for word reading fluency, an intermediate academic skill, are briefly described, but results for sentence reading fluency, sentence writing fluency, and calculation are also reported in Supplemental Tables 10–12, 16–18, and 25–27, respectively. The most consistent and largest effects on word reading fluency were letter-word identification and Gs. The effect of letter-word identification consistently decreased across grades as performance on word reading fluency increased (see Supplemental Tables 7–9). Additionally, Gs had consistent effects, but in grades K-2, the effect increased across the word reading fluency continuum, whereas the effect was variable in grades 3-6 and slightly decreased in grades 7-12. Gr also had some consistent effects with stronger effects observed as grade level increased; at grades 7–12, Gr had the strongest effect for those at the lower end of the distribution of word reading fluency (see Supplemental Table 9). The effects of Gl, Gf, Gc, or Gv were mostly negligible and inconsistent.

Figure 2 visually presents the UQR coefficients for predicting an advanced academic skill, passage comprehension (reading comprehension), at each grade-level group. In this example, the *y*-axis represents the strength of the effect of the 11 cognitive ability and basic and intermediate academic skill predictors (see Table 1) on different locations of the unconditional distribution of Passage Comprehension. Within the UQR models, the intercept for the 0.1 quantile of passage comprehension at K–2 can be interpreted as the mean value of passage comprehension (standard score = 82.21) at the 10th percentile (see Table 2). Results for the other advanced academic skills—writing samples and applied problems—are presented in Supplemental Tables 19–21 and 28–30, respectively.

Results from the UQR analyses show that letter-word identification had a statistically significant association with passage comprehension across all quantiles and grades when controlling for the effects of all other 10 predictors (see Tables 2, 3, and 4). Specifically, when controlling for the effects of eight cognitive ability predictors (i.e., Gf, Gc, Gwm, Gv, Gs, Gl, Gr, and Ga) and two intermediate academic skill predictors (i.e., word reading fluency, sentence reading fluency) in the 0.1 quantile model, a 1 SD increase in performance on letterword identification is associated with a 0.65 SD increase in performance on passage comprehension in grades K-2 and 3-6 (see Tables 2 and 3) and a 0.33 SD increase in grades 7–12 (see Table 4). Similarly, when controlling for all 10 predictors in the 0.5 quantile model, a 1 SD increase in performance on letter-word identification is associated with SD increases of 0.44, 0.39, and 0.26 in performance on passage comprehension at grades K-2, 3-6, and 7-12, respectively. Last, when controlling for all 10 predictors in the 0.9 quantile model, a 1 SD increase in performance on letter-word identification is associated with SD increases of 0.43, 0.44, and 0.38 in performance on passage comprehension at grades K-2, 3-6, and 7-12, respectively.



Fig. 2 Unconditional quantile regression coefficients for predicting WJ IV Passage Comprehension



As shown in Fig. 2 (top left graph), letter-word identification effects were generally largest at the lower end of the passage comprehension distribution (0.65 at the 0.1 quantile at grades K–6); however, with older children, the effect of letter-word identification was slightly stronger at the upper end of the passage comprehension distribution (0.33 at the 0.1 quantile versus 0.38 at the 0.9 quantile at grades 7–12). Letter-word identification is important for children who perform at both tails of the passage comprehension distribution in grades 3–12. For example, Fig. 3 illustrates that in grades 3–6, the effect of letter-word identification is greatest at both extremes of passage comprehension even though effects are much stronger at the lower end of the passage comprehension distribution.

When compared with the findings from the UQR analyses, OLS results show that when controlling for the other 10 predictors, the effect of letter-word identification is strongest at grades K-2 (β = 0.50; see Table 2) compared with that at grades 3-6 (β = 0.45; see Table 3) or grades 7-12 (β = 0.34; see Table 4). There appears to be a downward trend where the effects decrease as grade level increases, which were similar to

the findings from the UQR analyses, where effects tended to decrease across grades within each quantile. While the OLS coefficients provide a rough approximation of the average effect of Letter-Word Identification on Passage Comprehension, the UQR results provide a more nuanced depiction of effects for those with different levels of competency in passage comprehension; namely, letter-word identification is relatively more important for passage comprehension for those at the lower end of the continuum on passage comprehension, but it is still important at all levels of passage comprehension.

When considering the intermediate academic skills (i.e., word reading fluency, sentence reading fluency) and cognitive ability predictors of passage comprehension in the UQR models across grades K-12, a few consistent findings emerged. In addition to letter-word identification, the most important predictors across grades K-12 were Gc, Ga, and sentence reading fluency (see Fig. 2). Gc at grades K-2 generally has the same effect across the performance distribution of passage comprehension; however, at and around the 0.1



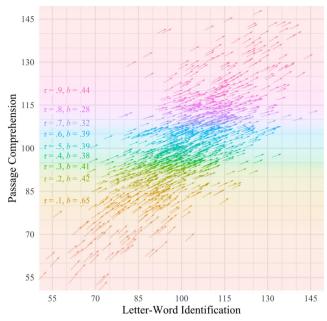


Fig. 3 For grades 3–6, after controlling for the cognitive and academic predictors listed in Table 2 and Fig. 2, the conditional effects of WJ IV letter-word identification on passage comprehension are greatest at both extremes of passage comprehension

quantile, Gc is more important compared with other levels of Passage Comprehension performance. When considering Gc at grades 3–6, Gc steadily increases in importance across the distribution from low to high performance on Passage Comprehension. Similarly, Gc generally increased in importance across the distribution of passage comprehension from low to high performance at grades 9–12. Furthermore, Gc also generally increased in importance from grades K–2 to grades 3–12. As an example, Fig. 4 shows that in grades 3–6, after controlling for the cognitive and intermediate skill predictors, the conditional effects of Gc on passage comprehension increase at higher quantiles of passage comprehension.

When examining the effects of Ga on passage comprehension across grades K–12 and across the passage comprehension distribution (i.e., 0.1–0.9 quantiles), Ga generally increased in importance as performance on passage comprehension increased. Last, sentence reading fluency varied in importance for passage comprehension performance across grades K–12. Specifically, at grades K–2, sentence reading fluency generally increased in importance across the passage comprehension distribution from low to high performance; however, sentence reading fluency appears to be the most important at the low end of the passage comprehension distribution in grades 3–12 (see Tables 2, 3, and 4).

Discussion

The purpose of the current exploratory study was to apply the unconditional quantile regression modeling technique to study

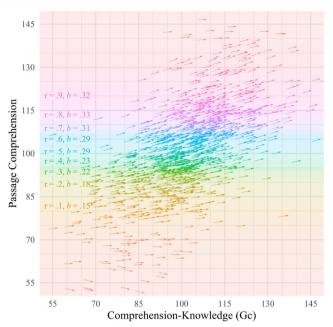


Fig. 4 For grades 3–6, after controlling for the cognitive and academic predictors listed in Table 2 and Fig. 2, the conditional effects of WJ IV comprehension-knowledge (Gc) on passage comprehension increase at higher quantiles of passage comprehension

the effects of cognitive abilities and basic and intermediate academic skills on advanced reading, writing, and math achievement in a nationally representative pediatric sample. Quantile regression allows for an examination of whether the influence of predictors differs across the spectrum of reading, writing, and math performance. We used the results from ordinary least squares regression analyses, which allows estimation of average relations between predictors and outcomes, as a comparison to highlight important differences between approaches to examining cognitive and achievement relations. As such, this study offers researchers and practitioners an alternative and potentially more useful method for understanding how cognitive abilities and basic and intermediate academic skills affect advanced achievement outcomes, especially for those who are performing at the lower and higher margins.

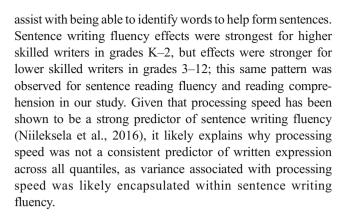
Cognitive-Achievement Relations Findings

The current findings are based on an exploratory analysis using UQR. When discussing our findings within the context of prior research, we focus primarily on the advanced academic achievement skills of reading comprehension, written expression, and math problem-solving. Basic and intermediate academic skills were influenced by a number of unique cognitive and academic variables, and the strength of these effects tended to vary according to performance on the outcome measure. Results for these analyses are located in Supplemental Tables.



Reading Comprehension Verbal comprehension-knowledge, auditory processing, decoding and word recognition skill, and sentence reading fluency consistently influenced reading comprehension performance, across all quantiles, for kindergarteners through twelfth graders. The cognitive-achievement relations of vocabulary, language, and acquired knowledge and auditory processing are well-supported in the literature based on OLS regression and SEM (Caemmerer et al. 2018; Hajovsky et al. 2014; McGrew and Wendling 2010; Niileksela et al., 2016). An examination of the reading comprehension distribution, however, reveals that the influence of comprehension-knowledge was stronger for young children with lower reading comprehension, stronger for third through sixth graders with higher reading comprehension, and variable for seventh through twelfth graders. The influence of children's auditory processing increased with higher reading comprehension performance. Youth's basic reading skill, decoding and word recognition (letter-word identification), had consistently large effects across all grades. Consistent with previous findings, the influence of decoding was stronger for younger children (Vellutino et al. 2007). The quantile approach revealed that the influence of reading decoding was strongest for kindergarteners through sixth graders with lower reading comprehension performance and was variable for older students. Another quantile regression study also found that reading decoding was relatively more influential for third graders with lower reading comprehension skills and verbal comprehension-knowledge was more influential for third graders with higher reading comprehension skills (Language and Reading Research Consortium & Logan, 2017). At the intermediate level, sentence reading fluency was more influential for third through twelfth graders with low reading comprehension scores, but the opposite trend was found for the youngest children. This fluency, or automaticity of syntactic processing, may allow more cognitive resources to be directed to reading comprehension (Klauda and Guthrie 2008).

Written Expression Learning efficiency, letter-word identification (basic reading), and sentence writing fluency (intermediate writing) consistently influenced written expression performance across all quantiles for kindergarten through twelfth graders. Learning efficiency showed stronger effects on lower written expression performance for younger children, whereas stronger effects were observed for higher written expression performance for older children. Hajovsky et al. 2018b found that learning efficiency influenced written expression across first through twelfth graders using a different sample and different measures, but these effects diminished over time. Letter-word identification influences were much stronger for less skilled writers in kindergarten through second grade but shifted to being more important for higher written expression performance in older children. Letter-word identification has been shown to influence writing (Decker et al. 2016) and may



Math Problem-Solving Verbal comprehension-knowledge, fluid reasoning, visual processing, and calculation consistently influenced math problem-solving performance across all quantiles for kindergarten through twelfth graders. Fluid reasoning and verbal comprehension-knowledge effects tended to be stronger for lower to average performers in math problem-solving across grades. Visual processing effects were strongest for lower performers in math problem-solving at grades K-2 but had variable effects across quantiles for older children. Verbal comprehension-knowledge, fluid reasoning (or g), and visual processing have been shown to influence math problem-solving for first through twelfth graders (McGrew and Wendling 2010; Villeneuve et al. 2019). Verbal comprehension-knowledge may assist with languagebased reasoning required of mathematical problem-solving (Decker and Roberts 2015). Calculation influences were slightly stronger for lower performing math problem-solvers across grades, but these effects were much more consistent across quantiles in grades 3–12. Villeneuve et al. (2019) found consistent calculation effects on math problem-solving that increased over time. Math facts fluency has variable effects across quantiles across grades. Short-term working memory had inconsistent effects across quantiles of math problemsolving in grades K-2, but was more consistent and variable in grades 3-12.

Implications

While these findings highlight how explanatory cognitive and achievement variables influence advanced reading, writing, and math across the continuum of performance, many findings showed that influences were stronger for lower academic performers. If these findings are generalizable to the population, then it may support the premise that children and youth who demonstrate low academic performance (i.e., at and around the 0.10 quantile or percentile of the outcome skill) may get a disproportionately stronger boost in achievement when there are increases in particular cognitive and basic academic skill variables. In other words, unlike ordinary least squares regression, where coefficients are interpreted as



effects on the mean of *y* and hence effects are assumed to be linear, our preliminary results show that effects are nonlinear, with stronger effects in many cases observed at the lower end of achievement. The ordinary least squares results show that cognitive abilities and basic and intermediate academic skills influence achievement across the entire spectrum of academic performance, whereas the quantile regression results suggest there are qualitative differences in the predictive weight of explanatory variables between low and average to high performers on the outcome measure.

Evidence of cognitive and achievement relations informs our understanding of learning difficulties and diagnostic decision-making, but this large body of work has mostly utilized mean-based analyses which is not where our critical focus most often exists. Instead, the practice of pediatric neuropsychology typically involves working with individuals demonstrating academic functioning near or at the margins of performance. Therefore, quantile regression is an ideal quantitative framework for pediatric neuropsychology research, as quantile regression aims to examine the correlates of learning across thresholds of achievement.

A potential implication of our findings for pediatric neuropsychologists working with children demonstrating academic difficulties is to assess the most important cognitive and basic academic skill variables related to the focal academic deficiency. This practice may encourage a narrower and more selective assessment approach of only the most critical variables related to the person's underlying achievement characteristics. While outside the scope of the current study, it is important to note that findings may inform clinical practices related to specific learning disability (SLD) identification. For example, several alternative, research-based methods commonly operationalize SLD as consisting of academic difficulties resulting from weaknesses in corresponding basic psychological processes, which typically require academic and cognitive weaknesses be theoretically aligned and empirically related (Alfonso and Flanagan 2018). Currently, some research suggests that these common identification methods may be better at identifying when SLD is not present; that is, academic weaknesses often exist in the presence of no specific cognitive deficits (cf. Kranzler et al. 2016a; Kranzler et al. 2016b; Stuebing et al. 2012). Although our findings do not directly address this limitation, results from our study may help better identify predictors of low achievement (versus typical, average achievement) and provide a more natural mapping of how specific cognitive abilities correspond to the dimension of low achievement. Stated differently, findings from this study suggest which cognitive abilities provide the strongest associated improvement when an individual's target skill is at the low end of the achievement distribution. From an intervention perspective, Fuchs et al. (2020) suggest that quantile regression may lend empirical strength to developing instructional differentiation strategies that correspond to different degrees of performance on a desired domain. Studies employing quantile regression may help facilitate the conversation regarding appropriate assessment and intervention practices for children struggling academically.

Limitations

This illustrative exploratory study should be interpreted in light of several important measurement and design limitations. While the WJ IV includes tests that are important measures of neuropsychological processes relevant to pediatric neuropsychologists (e.g., attentional focus, cognitive flexibility), the WJ IV battery does not include "pure" measures of these processes. However, as all behavioral measures contain some degree of construct irrelevant variance, neuropsychological tests are likely not uncontaminated measures of neuropsychological processes either. In support, previous research suggests there is measurement overlap between CHC cognitive abilities and executive functions and that they are not easily distinguished (Floyd et al. 2010; Jewsbury et al. 2016; Salthouse 2005). In addition, some scholars argue that the observed CHC broad ability composite scores used to represent the cognitive constructs employed in this study largely contain variance attributable to psychometric g (e.g., Dombrowski et al. 2018), in which case less weight should be given to interpreting the broad ability factors. Nonetheless, neuropsychologists and other clinicians frequently use composite scores for diagnostic purposes to inform their decisionmaking.

A statistical design limitation is the use of cross-sectional data, which does not account for within-person variability and as a result increases between-group sampling error. Longitudinal designs allow for control of previous levels of academic achievement over time (Hajovsky et al. 2017), a possible internal validity threat in nonexperimental research when not conducted (Keith 2019). Similarly, there was no experimental manipulation of CHC cognitive abilities and basic academic skills to determine their subsequent effects on advanced academic skills because nonexperimental data were used. Therefore, statements regarding "effects," "influences," or "predictions" are limited to interpretations of the coefficients within the regression models.

Conclusion

The current study provides an illustrative example of the application of unconditional quantile regression to the study of cognitive and achievement relations within a school-aged sample. Unconditional quantile regression examines whether differential effects along the distribution of an outcome skill exist, which differs from conventional regression methods that focus on means. Importantly, this exploratory study showed



that the pattern of effects was often relatively stronger when performance on the outcome measure was lowest, indicating influences differ across the continuum of achievement. These findings help shed light on the contribution of cognitive abilities and basic and intermediate academic skills to different levels of advanced academic performance and provide a better understanding of the unique learning needs of students with differing achievement characteristics. Although quantile regression offers pediatric neuropsychologists an alternative method to conduct prediction-based research, more work is needed to clarify how findings can be integrated into assessment practices.

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Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

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