

An Amateur Horizontal Component Broadband Seismometer (Model MkXVII)

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ABSTRACT

A horizontal component broadband seismometer has been designed and fabricated by an amateur instrument builder in a home workshop using commonly available materials. The seismometer's sensitivity and flat response to velocity between 0.022-10 Hz made it capable of detecting teleseismic events greater than 5.6 magnitude at epicentral distances exceeding 160°.

INTRODUCTION

For the detection of teleseismic events, an amateur instrument builder usually chooses a design based on a long period horizontal component seismometer fitted with an electromagnetic transducer, to obtain an electrical output signal proportional to ground velocity. Unfortunately, the transducer's signal becomes increasingly weaker towards the lower frequencies, making it undesirable for the task.

To build a seismometer with higher performance meant attempting a more advanced design. A search of the World Wide Web (WWW) did uncover some information on a variety of products manufactured by Kinematics, Geotech, Guralp and Streckeisen. Their seismometers were designed around a variable capacitance displacement transducer, due to its improved sensitivity to lower frequencies, by providing an output signal proportional to mass displacement..

For simplicity, a long period horizontal pendulum design was chosen over a more complicated vertical component. The new instrument, designated the MkXVII, had its pendulum arranged in the classic "garden gate" configuration. To minimize the restoring force on the seismic mass, the pendulum's mechanical natural period was adjusted for 10 seconds. A small DC electric motor assisted in re-leveling the base plate, compensating for excessive pendulum drift due to ground tilt. Transducer equilibrium was then restored whenever a predetermined offset signal level was exceeded.

The motion sensing transducer and associated electronic circuitry required a low noise floor to achieve the desired sensitivity to weak signals from teleseismic events. A full capacitance bridge displacement transducer was chosen over the mechanically simpler half bridge design, since it was less demanding on the electronics. Having a limited operating range, the acceleration output signal of the displacement transducer was modified and fed back to an electromagnetic transducer (voice coil) mounted on the pendulum, thereby minimizing the relative motion between the seismic mass and support frame. Feedback not only provided the seismometer with a broadband velocity output signal, but improved the system's linearity and increased its bandwidth.

Though the design was capable of providing a very broadband (VBB) response flat to velocity between 0.01-10Hz, the low-end corner frequency was limited to 0.022 Hz (44 seconds) as the instrument was to be installed in a noisy urban site. The following pages of texts and images offer only a brief description of its design, fabrication and performance. It is assumed that the reader has already acquired some basic understanding of mechanics and electronics.

MECHANICAL DETAILS

This section describes the main mechanical features of the seismometer, as well as their contribution to the overall system. See the next major section titled ELECTRONICS, for a description of its circuits. An apology must be made for mixing units of measurements, specifically those relating to physical dimensions.

Being a true "amateur" project, design complexity was limited to the equipment and materials on hand in the home garage. Besides a selection of powered hand tools, more powerful machinery like a mill/drill machine, metal cutting band saw and a 3" metal lathe (hobby type) assisted with cutting and shaping of the thicker materials. Slotting holes with round files combined with shimming compensated for poor machining techniques and limited tooling. Several of the MkXVII's mechanical components were modified salvaged parts from older homemade seismometers, which helped to keep expenses and fabrication time to a minimum.

INSTRUMENT BASE, COLUMN & PENDULUM

From the beginning, an attempt was made to keep the mechanical noise to a minimum. Though aluminum is not the preferred material of choice for a professional instrument designer [1], the availability of an assortment of scrap aluminum pieces stored under the workbench made its use unavoidable. Building the seismometer from mostly one type of material, as opposed to combining different materials with varying rates of thermal expansion, reduced framework warping. Parts deliberately had thick rigid cross sections to add mass and thermal inertia [2]. Figures 1 and 2 will show the general layout of the instrument.

Correctly set up, the pendulum would re-center itself with a 10-second natural period when the hinge axis leaned slightly away from vertical towards the seismic mass. Prior to installing the transducers, pendulum travel was approximately +/- 10 degrees either side of its null position. Circular cross-section and small diameter, the boom was less effected by lateral air currents inside the protective cover. By adjusting the pendulum's mechanical free period to 10 seconds, the instrument's sensitivity to small ground accelerations suitably matched the sensitivity of the displacement transducer [2].

No method for locking the pendulum was provided for. Whenever the seismometer was hand carried, it was tilted at a shallow angle along the sensitive axis, causing two of the transducer's plates to gently rest against each other. A "U" shaped bracket attached to the base and fitted with opposing adjustable screws could have been tried for clamping the pendulum solidly.

The aluminum base plate measured 190mm wide x 420mm long x 20mm thick. Three leveling screws, 1/4-20 UNC thread, with conical points went through the base plate to eventually contact pieces of 1/8" thick plate glass resting on the cement floor of the instrument vault. Attached to the base plate with four SS screws was a vertical column made up of aluminum pieces, providing rigid support for the pendulum with its triangular shaped side plates, see Figures 3A and 3B.

A horizontal aluminum tube 7/8" OD x .050" thick wall x 250mm long formed the main arm of the "T" shaped pendulum. Solid aluminum inserts with tapped central holes, were bonded into each end of the tube. A cylindrical aluminum seismic mass of 0.5 Kg was attached to the free end with a single brass screw and the opposite end of the tube was fastened to a vertical piece of 3/4" square aluminum bar stock with a high tensile stud. A small vent hole was drilled through the side of the tube to equalize internal air pressure. The pendulum was attached to the vertical column by two flexures, or Cardan hinges. The flexures were cut from heat-treated steel feeler gage material 0.002" thick x 0.50" wide x ~25mm long. Both were kept in constant tension, one at the top and bottom of the vertical arm. Their ends were clamped between aluminum plates. Each flexure bridged a gap of ~1mm between the clamps on pendulum and column, establishing a defined hinge axis. Crossed flexures, the configuration of choice for professionally made seismometers, would have been more difficult to custom make and align or expensive to buy.

FEEDBACK TRANSDUCER

An electrical feedback signal flowed through voice coils suspended from the underbelly of the pendulum, see Figure 4. These coils worked in conjunction with permanent magnets attached to the base plate, applying a driving force proportional to ground acceleration that both modified and damped the pendulum's motion. Transducer linearity directly effected the accuracy of the final output signal.

Sean-Thomas Morrissey utilized a large 12" off-the-shelf audio speaker for a feedback transducer, as an alternative to building something from scratch, on his unique STM-8 design [3]. Off the shelf audio speakers can perform reasonably well, as long as they are not expected to control too much mass. A single low resistance voice coil has a low generator constant (calculation of the feedback transducer's generator constant, G_n , will be discussed later in this paper), and given the size of the seismic mass, two feedback transducers were installed in series.

A pair of 8" Woofers (both had damaged cones) had their voice coils carefully extracted from the magnet assemblies using a sharp box knife blade. Scissors trimmed surplus cone material around the periphery of the coil. Existing free hanging conductor wires to the coil were left intact as much as possible, because they would be connected (soldered) to the feedback electronics. Next the sheet metal structure supporting the cone was torn off the magnet. The magnet gaps were thoroughly cleaned out by sliding the end of a piece of pliable vinyl electrical insulation tape into it. Forcing the sticky side of the tape against the inside surfaces of the gaps captured loose magnetic particles, flakes of paint, lint, hair and dead skin. Repeated this process several times to ensure cleanliness. Spent over 20 minutes cleaning each magnet. A piece of tape was temporarily placed over the exposed poles and gap to prevent dirt and metallic particles from re-entering the opening. Tape removed at final assembly with the coils.

Pieces of nylon were turned down, to become plugs that filled the open outer end of each voice coil. The central pole piece of the magnet had a piston like effect on the air confined by the close fitting voice coil, subtly damping pendulum motion. Each plug were generously drilled full of vent holes so air could freely flow in and out of the trapped space. A centrally tapped hole in each plug received a brass screw for mounting the coil assemblies to the pendulum. Note: all fasteners and components installed on the pendulum were made of non-magnetic metals. For ease of assembly and to avoid deforming the magnetic fields too much, the magnets were offset from each other. Each magnet was bonded to a flat

sub-plate and fastened to a piece of aluminum angle. All mounting holes were generously slotted to facilitate alignment of the pendulum mounted voice coils with the gaps in the magnets.

DISPLACEMENT TRANSDUCER

Wanting to ensure decent performance from this crude homemade instrument, a full bridge, or symmetric differential capacitive (SDC) transducer [4] was employed. Compared to a half bridge, an SDC transducer has double the sensitivity and a well defined null position, without the need for a very precise symmetric oscillator. The SDC's main drawback is the need for two variable capacitors (mounted in tandem) and a precise method for differentiating their output signals. A variable capacitance full bridge transducer is basically quieter electronically and more sensitive than a linear voltage displacement transformer (LVDT) transducer, and perhaps easier to make.

SDC transducers consist of two separate transducers rigidly coupled together to move as a single unit, with each transducer forming a branch of a bridge circuit. Refer to Figures 5 & 8 regarding how this concept was applied to the MkXVII. Responding to a displacement of the seismic mass, i.e. detectable motion between the pendulum and base plate, each transducer must have output signal of opposite polarity to the other. Each outer plate comprised of two separate active areas, facing towards the center plate. Back to back active areas on the center plate were joined electrically together, but not to the adjacent active areas on the same side of the center plate. An active area on one of the outer plates was joined electrically to the active area on the opposing outer plate of the other transducer. Both transducers had the active areas of the outer plates cross-linked in this manner. When one of the outer plates, consisting of two active areas, got closer to the center plate, one half of the center plate received a stronger positive signal and the half of the center plate received a stronger negative signal, providing a true differential output. Fairly fine 40 AWG coax wire carried the modulating signal from the oscillator circuit to the outer plates. These coaxes easily flexed where they bridged the gap at the pendulum's hinges.

A three-plate transducer design balances out the electrostatic forces acting between the plates. Viscosity of the air trapped between the plates posed a significant problem by damping the pendulum's motion. A pattern of "vent" holes was uniformly distributed across all plates of the transducer. Size of gaps between the plates and vent hole diameters had to be carefully considered for an acceptable sensitivity. Critical aspects for half bridge capacitance transducer designs were covered in a paper by Jones & Richards [1] and a out of print book written by Baxter [5], yet quite a bit of the information was applicable to the full bridge SDC design.

Flexing a signal cable can generate noise. The center plate (modulated signal receiver) was fixed to the base framework and the two outer plates (modulated signal transmitters) were attached to the moving pendulum. Rigid, heavily shielded coaxes connected the center plate's active areas to the electronics. The outer plate's active areas overlapped those on the center plate, allowing for some very slight, unavoidable off-axis motions without noticeably effecting sensitivity.

Most important for good sensitivity from a capacitive transducer is a sine wave oscillator with stable amplitude. Total harmonic distortion (THD) of 2-3%, with a similar tolerance for frequency stability, is acceptable. One appropriate technique for the MkXVII was a quartz crystal oscillator for creating a square wave signal, followed by a transistor providing a bipolar +/- 5 V output signal. Next, a 3rd order low pass filter with a corner frequency set below the square wave's fundamental frequency, rounded off the corners of the square wave for the desired result. The stability of the power rails established the stability of the sine wave's amplitude. An EXAR XR-8038 function generator with a sine wave output was evaluated for some time, and then rejected because it had too high a noise floor.

Relatively cheap and readily available high performance op amps with moderate slew rates easily handled the sine wave excitation carrier frequency selected for the MkXVII. Precision DC operational amplifiers have increasing noise towards the lower frequencies, so operating the transducer with a reasonably high AC frequency reduced the 1/f noise problem.

The plates' exposed signal carrying surfaces should have been a non-corrosive grade of brass or stainless steel finished with a special surface treatment, but actually were made from 1/8" thick FR-4 (PCB) type material coated on both sides with a layer of 1 oz, .0014" thick, copper. Relied on the FR-4's manufacturer to have applied some type of corrosion inhibitor to the copper. Material was cut to shape with a saw and a mill/drill machine created shallow isolation grooves through the copper layers to define the active areas. Plate details are presented in Figure 8. As an alternative method to milling, a Dremel Tool fitted with an engraving bit could have made isolation grooves 1-2 mm wide through the copper layers.

Copper layers were left intact over much of the outer plates' back surface, acting as a protective ground plane. Mounting screws shorted the ground plane to the seismometer's metal structure. Outer braided jackets of the modulating signal's coaxes were soldered to the copper ground planes, to tie the seismometer and the electronics' metal enclosure to the same voltage potential.

The actual active area for each of the two separate displacement transducers was calculated to be 1266 square millimeters. This value did not include the area of the 5mm diameter air vent holes, in 6 places. Nominal gaps between the center and outer plates measured 0.6 mm. Parallelism between the center and outer plates were within 0.06 mm at the extremes of pendulum travel. Note: another homemade seismometer having 1 mm nominal gaps between plates of similar

proportions and a total vent hole area greater than 15% of the total plate area (i.e. active area + vent hole area), had noticeably reduced the air viscosity problem while still remaining reasonably sensitive.

MOTORIZED LEVELING SCREW

The pendulum's equilibrium was easily disturbed due to unavoidable long term ground tilting. Drifting of the pendulum/displacement transducer required a re-centering adjustment cycle to restore optimum performance approximately once every few weeks, or longer. Re-centering was recognized on the seismogram as a set of large amplitude pulses that slowly diminished back to normal behavior, taking roughly five minutes.

Whenever the transducer's displacement signal exceeded a pre-determined offset voltage level after a long period of integration, a small DC electric motor became activated. Long period integration eliminated possible false triggering during the detection of large microseisms, wind noise and seismic events. The motor rotated the leveling screw that had the greatest influence on the sensitive axis of the pendulum. Slowly raising or lowering the corner of the base plate would swing the pendulum back to null, at a rate slightly slower than the lagging response of the integrator's output signal (observed when the seismometer is correctly leveled). Optimum motor speed, driven by a voltage set by a feedback resistor, combined with suitable gearing met this goal. Complementary transistors were configured to form a dead band within plus or minus 0.6 volts of null. When the amplified signal of the transducer reached the dead band on either side of null, the output voltage of the transistors quickly dropped off to a level insufficient to drive the motor. Now the integrator's lagging output would catch up and drift into the dead band, placing the pendulum very close to null without any dithering. The motor was given 22 seconds to correct any offset signal. Automatic re-centering also meant that the thermal insulation surrounding the instrument did not get disturbed.

Examining Figure 6, you can see that the motorized leveling screw is a piece of threaded rod with a spur gear bonded to the top end. A 12 VDC motor, fitted with a 1000:1 ratio gearbox, had a small stainless steel spur on the output shaft. This drove a larger brass spur gear mounted on another shaft with a single start worm. The worm engaged the spur gear on the leveling screw. All surfaces that rubbed together were lightly greased to reduce friction. Gearing down was done in order to rotate the leveling screw at the very slow rate, something like 1 revolution in 25 minutes. A built-in safety feature is that the worm and its mating spur gear will eventually disengage themselves if the motor is left to run continuously, without destroying itself (but maybe wear the corners off some teeth).

When setting up the seismometer in the vault, the two non-motorized leveling screws were adjusted first to center the bubble in a spirit level (shown bonded to the base plate in Figures 1 and 4) and to roughly center the transducer visually. A momentary switch installed on the electronics' enclosure was then briefly pressed, manually overriding the long period integrator, for precise centering of the transducer with the motorized screw. The re-centering cycle may be activated manually a couple of times before the monitored output voltage of the integrator had settled down to less than +/- 1 VDC. Finally, the seismometer was covered with thermal insulation and left alone to re-center itself automatically.

INSTRUMENT COVER

Over the entire instrument was a cheap and simple polystyrene foam cover, shown installed in Figure 7. This involved cutting out five rectangular shaped pieces of 15mm thick sheet and permanently bonded together with wood glue. Seams were sealed with a second bead of adhesive to properly fill all gaps. For the cover to locate itself correctly and seal reasonably well on the top surface and outer vertical sides of the base plate, a 10mm x 10mm notch was cut into the inside lower edges of the foam sheets, prior to bonding them together. The cable bundle exiting the base plate passed through a small snug fitting notch cut in the lower edge of the cover. A single layer of aluminum foil was wrapped over the external surfaces of the cover to reflect heat.

The cover prevented unwanted debris falling into the displacement transducer and shielded the pendulum from air drafts. Another bonus was the thermal insulation properties of the cover material, it helped to stratify the air trapped inside. The cover provided absolutely no RFI shielding for the transducer, and during evaluation, ambient electrical noise was not monitored.

ELECTRONICS

The electronic circuits were modularized according to their unique function within the overall system, making it possible to conveniently swap out a board for an upgrade or repair. There were four separate hand-wired boards, the Oscillator Circuit Board (OCB), Displacement Transducer Circuit Board (DTCB), Feedback Circuit Board (FCB) and the Transducer Re-centering Circuit Board (TRCB).

The Oscillator Circuit Board and the Displacement Transducer Circuit Board were mounted together in the same aluminum project box. Likewise for Feedback Circuit Board and the Transducer Re-centering Circuit Board, they too shared space in their own aluminum box.

Power came to the circuit boards from an off-the-shelf +/-12 VDC dual linear supply (1 amp per side). Most of the op amps selected for the MkXVII, though reasonably cheap for their rated performance being priced at about a dollar each (a couple were around \$5 each), consumed their share of current. Measured totals were 119 mA from the positive supply and 90 mA from the negative supply. The DC re-centering motor drew an additional 48 mA from either supply, while operating.

Circuits were constructed using metal film resistors with 1% tolerances, for all values up to 1 meg ohm. Larger values were carbon types with 5% tolerances. All power decoupling caps and feedback cap values under .01 microFarad were ceramic, all others >.01 mF were polyester. The circuits were built on perforated boards with sockets for all ICs. To reduce power rail noise, the power pins on all op amps were wired directly, or via 20-Ohm resistors, to a common point on the output pins of the voltage regulators.

To eliminate or significantly reduce ground loops, the following scheme was adopted. A small piece of adhesive backed copper foil 3 mm wide by 20 mm long attached to the underside of the perf board, centered between the rows of legs dangling from each op amp socket. Decoupling caps and other components associated with each op amp requiring grounding were soldered to the piece of foil. Each individual foil was wired directly to a common ground point on the circuit board, no daisy chaining. Also, .01 mF ceramic decoupling caps were inserted where power-carrying wires entered a circuit board. A short length of coax cable connected the output of the DTCB to the FCB. The coax outer shield was connected to the metal project boxes. Grounding the project boxes to the AC mains ground pin added noise. Tying them to a metal stack in the earth is referred, though good results occurred without doing this.

Any flux residues built up on the circuit boards, seen to be bridging between conductors were scraped off as best as possible to reduce potential signal leakage. Please note; the author has had no formal electronics training, but gained the appropriate skills and knowledge after several years of building similar experimental projects.

OSCILLATOR CIRCUIT BOARD (OCB)

Fixed voltage regulators designated U1 and U2 (see Figure 9) filtered and stepped down the voltage from the common supply, providing +/-5 VDC. The input and output capacitors filtered out most of the voltage spikes on the supply rails. The IN4002 diodes protected the regulators from excess voltages during power down.

Figure 9 shows a 1 MHz crystal oscillator U3 followed by two TTL divide-by-ten counters to obtain a frequency of 10 KHz, with a 0 to +5 V square wave output. A NPN transistor with a bipolar output converted it into a +/- 5 volt square wave signal. A voltage divider prevented the buffer U6 from being saturated. A 3rd order low pass filter with a corner frequency of 5 KHz reduced the square wave to its fundamental frequency by removing its higher order harmonics. Op amp U7 added gain to restore some of the signal lost going through the filter, pumping out a ~ 2 V P-P sine wave signal.

DISPLACEMENT TRANSDUCER CIRCUIT BOARD (DTCB)

The DTCB was mounted inside the same metal enclosure that housed the OCB, since both boards had high frequency circuits. Adjustable voltage regulators U8 and U9 (see Figure 10) filtered and stepped down the voltage from the common supply, providing the circuits with +/-10 VDC. With 10 mF Tantalum capacitors connected to the regulators, better than 65dB ripple and spike rejection was possible. The IN4002 diodes protected the regulators from excess voltages during power down.

AC coupled, inverting op amp U10 boosted the incoming sine wave signal to ~ 10 V P-P. An audio transformer T1 (Digi-Key P/N 237-1121-ND) had 600-600 ohm center tapped coils which were referenced to ground for DC common-mode voltage rejection. The secondary coil produced two modulating signals, 180 degrees out of phase with each other and connected to the active areas on the SDC's outer plates, PA and PB, per the diagonally opposing pattern described earlier in the paper and shown in Figure 11.

Op amp U11 is a zero-crossing detector referenced to ground for converting the AC sine wave into a sharp square wave, oscillating from 0 to +10 volts. Because the demodulator's analog switch U14 required a lower triggering voltage, a divider dropped its peak to +3.8 volts max. The active areas on the capacitor's moving center plates, PC and PD detected the modulated voltage signal's amplitude and phase, in proportion to its displacement in either direction from their null position. Op amps U12 and U13, both high impedance follower amplifiers at unity gain, drove guards (the coax shields) to reduce stray input capacitance for additional sensitivity. The input pins to these op amps were bootstrapped for circuit stability. Their outputs were fed into the analog switch U14, a demodulator synchronized with the sine wave signal from U10, monitoring the modulated voltage of each half cycle. Op amp U15 is an instrumentation amplifier (in-amp) with unity gain, differentiating the weak signal from the SDC. Instead of the in-amp, a differential op amp with unity gain could replace U15. Op amp U16, a 34 Hz LPF, attenuated high frequency circuit noise prior to exiting the board.

Coax audio cables, half a meter long, were soldered to the SDC and connected to the circuit board via RCA phono plugs. The outer shields of the two outer plate's coaxes were attached to chassis ground. Due to the high frequency signals in this circuit and that of the oscillator, the metal structure that surrounded the SDC and the boards' metal enclosure (for RFI shielding) were connected to earth ground on the 115 VAC power supply via a low resistance conductor.

FEEDBACK CIRCUIT BOARD (FCB)

Adjustable voltage regulators U8 and U9 (see Figure 10), including their associated components were duplicated on this board to supply filtered +/- 10 VDC to the circuit.

Referring now to Figure 12, op amp U16 is configured as a special integrator with the intended purpose of improving low frequency sensitivity and stability. Such an "integrator" appeared in papers written by Erhard Wielandt [2][6][7], seen adjacent to the displacement transducer in block diagrams representing the electronic circuitry of Streckeisen seismometers. Erhard Wielandt, via personal communications, explained that because the system would oscillate if high gain is maintained at high frequencies, loop gain had to be frequency-dependent, increasing towards the low frequencies while ensuring unity gain at the higher frequencies. One way to realize this is to insert a special integrator or quasi-integrator, within the feedback loop. Based on the word description he provided and after some experimentation, a workable circuit design evolved. A 1st order 34 Hz LPF attenuated high frequency circuit noise from the displacement transducer prior to entering the quasi-integrator. Op amp 16 is a unity gain instrumentation amplifier (IA) with high impedance FET inputs and excellent common mode rejection (CMR) to differentiate between small input signals. A 2nd order 3.5 Hz high pass filter (HPF) comprising of C2, C3, R2 and R3 was inserted into the integrator's feedback loop between pins 2 and 6 of the IA. A buffer op amp U17 installed after the integrator prevented any unwanted loading of downstream circuits. A temporary shunt was installed across the capacitors C2 and C3 when the system was calibrated and removed later for normal operation.

The multi-path feedback circuit followed a schematic published by Sean-Thomas Morrissey for his vertical component VBB seismometer, the STM-8 [3] which was itself based on similar circuits found on Streckeisen STS-1 & STS-2 VBB seismometers. Multi-path feedback is a combination of differential and integral feedback signals flowing through the feedback transducers mounted on the pendulum. The differential signal dominates across the passband of interest, decreasing towards the lower frequencies where the strength of the integral signal increases. Reading the papers by Morrissey and Wielandt is highly recommended for their discussions on this subject. Resistor Rp and capacitor C provided the proportional feedback. The combined currents of Rp, C and RI flowed through the feedback coils, modifying the pendulum motion for a BB velocity response. The capacitors, designated C, were non-polarized electrolytic, but they will be replaced with quieter Mylar caps the next time the electronics are accessed.

Following the quasi-integrator, op amp U18 inverted the acceleration signal before it entered a composite integrator, comprising of U19 and U20. The RC time constant of the 10 Meg resistor and the 10-microFarad capacitor (at 100 seconds) is the mathematical term TI used in later calculations. A composite integrator [8] takes advantage of the high input impedance of a FET op amp combined with the DC output accuracy of a BJT op amp. Op amp U21 buffered the circuit whenever the displacement signal was monitored with a DVM. The Transducer Re-centering Circuit also monitored the displacement signal at pin 6 on op amp U20.

Buffer U22 prevented down stream loading of a .003 Hz (333 seconds) high pass filter to remove unwanted DC signal drift. Finally, op amp U23 with a 2X gain to drive a capacitive load created by a 30-meter long coax cable connecting the seismometer to the data logger, a PC fitted with an internal A/D card.

TRANSDUCER RE-CENTERING CIRCUIT BOARD (TRCB)

The TRCB was mounted inside the same metal enclosure that housed the FCB, sharing the FCB's two voltage regulators that were a duplication of U8 and U9.

This circuit constantly monitored the voltage level and polarity of the displacement output at pin 6 on op amp U20. Referring to Figure 13, op amps U24A and U24B buffered the circuits from the effects of the long period integrator (LPI), formed by the 2 meg resistor and 2X 1500 mF caps. The LPI's time constant of 1500 seconds, or 25 minutes, was enough to smooth out voltage spikes that would have constantly triggered the motor into action.

Op amps U25A and U25B are voltage comparators, to output >8 volts when the LPI exceeded voltage levels of +/- 1.3 volts. Their high state outputs turned on transistor Q2, driving the input of a LM555 timer U26 low. For the next 22 seconds, the output of U26 remained high, causing transistor Q3 to activate a DPDT relay. One pole of the relay, REL 1A, drained accumulated voltage away from the large capacitors to re-set the LPI.

A momentary push button switch SW1 mounted on the metal enclosure could be used at any time to manually override the function of the LPI, by driving the input of U26 low. As a visual indicator, an LED mounted on the outside of the metal enclosure lit up when the re-centering circuit was activated.

Like U24A, op amp U27 constantly monitored the voltage and polarity at pin 6 on U20. Power transistors Q4 and Q5 boosted the current output of op amp U27 to drive the DC motor at over 50 mA. Once activated, the relay pole REL 1B connected the motor MOT indirectly to the feedback circuit. By being able to track the circuit's polarity, the motor rotated in a direction necessary to null the transducer. The 30K-feedback resistor increased the drive voltage to the motor for optimum speed. At the end of a 22-second timed cycle, the motor was once more isolated from the rest of the circuit.

VELOCITY OUTPUT SIGNAL

To help determine the direction of ground motion on a seismogram produced by a horizontal seismometer, the accepted standard is to have the output voltage signal go positive when the ground is heading north or east, on a N-S or E-W aligned instrument, respectively.

A small direction arrow was first scribed on the base plate, drawn perpendicular to the boom and parallel with its sensitive axis. To verify correct polarity of the output, pushing the pendulum in the opposite direction to the arrow (equivalent to the ground moving in the direction of the arrow) should produce a positive voltage signal coming out of pin 6 on both U16 and U23. Initially a negative voltage signal resulted, requiring the wires to pins 1 and 2 on U14 to be reversed. Powering up the circuits again caused the pendulum to violently oscillate, but reversing the wires to the feedback coils restored stability. Installation of the seismometer in the vault had the direction arrow point either north or east.

Not shown in the electronic schematics was a simple 1:2 voltage divider, installed between the output of U23 and the input port of the A/D card, thus preventing the card from receiving a voltage signal exceeding +/- 5 volts.

TRANSFER FUNCTION

SYSTEM RESPONSE

The transfer function is a mathematical statement that describes the relationship between the input, a set of predetermined parameters, and the output of the system. Calculating a system's output can be a very difficult task. Fortunately the Mathcad file compiled by Morrissey for his STM-8 was available for downloading [9], thereby saving an enormous amount of time and effort. Mathcad Standard 2000 software opened the file on a local PC, whereby the STM-8's values were substituted with different ones (Appendix A at the end of this paper contains the work sheet). Numerous values were tried for C, Rp, RI and TI before achieving an acceptable response, see below:

Input parameters:

r =	101,380	V/m (this value was determined per the method explained below)
M =	0.55	Kg
C =	0.000050	Farads
Rp =	100 K	Ohms
RI =	10 K	Ohms
Gn =	8.7	N/A (this value was determined per the method explained below)
Rf =	9	Ohms
TI =	100	seconds
To =	10	seconds

Calculated system response:

Zeta =	0.72	(damping)
Tn =	44	seconds (closed loop natural period)
V/m/s =	1,266	(max sensitivity)

The plotted curve for the velocity response showed the high frequency end rolling off above 10 Hz (see Appendix A), more than enough range for the intended goal of teleseismic event detection.

DISPLACEMENT TRANSDUCER CONSTANT: V/M

The transducer's displacement constant Volts/Meter (V/M), was determined by measuring the voltage at pin 6 of op amp U17, i.e. at TP1, as the pendulum was moved in approximately 0.0005" (0.0127 mm) increments about its null position. The jumper (adjacent to C1) must be temporarily removed to avoid the influences of the capacitor. A micrometer's spindle contacted the pendulum at the seismic mass' Center of Gravity. The collected voltage/displacement data points were then plotted on graph paper. Pushed the pendulum for a distance of 0.004" (0.1016 mm), which was well within the linear portion of the plotted points, roughly centered about the null position. Over this length the SDC transducer produced a total output of 10.3 volts, which equaled 101,380 V/M. Looking at the plotted data points, transducer linearity exceeded 85% of the range of travel, in either direction from its null position. Note: additional amplification of the seismic signal in later circuits saturated the final output well before the linear range of the transducer was exceeded. Resistor values for R1 and R2 were adjusted for an acceptable gain. Replaced the jumper after performing these measurements.

GENERATOR CONSTANT: Gn

This test set-up applied a horizontal force to the pendulum. Per Figure 14, a piece of fine cotton thread ~300 mm long had one end taped to the side of the pendulum at its CG, and the other end taped to a rigid support off to the side of the pendulum. The thread's two ends were of equal height above the bench top and far enough apart so the thread formed a 90°

vee (within 2°). Next, the pendulum was adjusted for an optimum natural period (10+ seconds) with the transducer centered. Temporarily disconnected the feedback coil from the electronic circuit board and connected a DVM to pin 6 of U16. Connected a 1.5 V battery, an on/off momentary switch, a mA meter (another DVM or analog meter) and a 10K potentiometer in series with the feedback coil. Powered up the circuit, then centered the transducer using the leveling screws to obtain ~0 V at pin 6 of U16. Placed a small test weight of known mass at the bottom of the “V” in the thread, which was heavy enough to deflect the pendulum horizontally towards the rigid support. Now the battery circuit was turned on and the 10K potentiometer adjusted until just enough current flowed through the coils to re-center the pendulum, indicated by ~0 volts on the DVM connected to pin 6 of U16. Did not allow more than a few milliamps to flow through the coils, otherwise the coils could have burned out. Recorded the amount of current necessary to keep the transducer centered while counter-acting the load exerted by the test weight.

The test mass was a paper clip measuring 0.00145 Kg, but the actual load applied to the pendulum was one half of that amount: $0.00145/2 = 0.00072$ Kg. Re-centering current measured: 0.00082 Amp.

The generator constant, Gn, is defined in units of Newton/Amperes (N/A).

$N = \text{mass} \times \text{accel.} = 0.00072 \times 9.8 = 0.0071$ N

$G_n = N/A = 0.0071/0.00082 = 8.7$ N/A

SEISMOMETER VAULT

Co-ordinates: Latitude: 47.849 N, Longitude: 122.328 W, Elevation: 110 meters

The seismometer was installed in a small surface vault squeezed between a 1.5-meter tall hedge and a 2-meter high wooden fence, in the backyard of a house located in a noisy urban environment. Vault was separated from bedrock by a 1.8 meter thick layer of unconsolidated backfill having 16+ years of settling, on top of 200+ meters of glacial till according to geologic maps.

Long period noise occurred when northerly or southerly winds struck the E-W aligned hedge and fence. Major levels of short period noise came from the N-S aligned railroad tracks located one kilometer west of the vault, where on an average day, twenty to thirty freight trains passed by. Large bodies of water to the west, the Pacific Ocean at a distance of 160 kilometers and the eastern shoreline of Puget Sound just west of the railroad tracks, generated noise (microseisms) with periods of 4-15 seconds, peaking in the autumn/winter months.

Inside the vault was a small rectangular pier, measuring 450mm wide x 800mm long x 250mm thick, cast from a 50/50 mixture of sand and cement (no aggregate) poured directly onto compacted earth. Styrofoam sheet, 75mm thick, isolated the pier from the vault's, 203mm thick, hollow concrete block walls. A hollow insulated plywood roof was hinged off one of the block walls to permit access to the vault's interior from the top. Styrofoam insulation 25mm thick lined the internal wall surfaces and 50mm thick sheets covered the external surfaces of the concrete block walls.

Under each leveling screw of the seismometer was a piece of smooth glass plate 40mm x 40mm x 5mm thick for reducing thermal noise and torsion forces necessary to operate the screws. Metal enclosures, at chassis ground housing the electronics, were electrically connected to a common metal stake sunk in the ground outside the vault.

Setting up the seismometer in the vault required adjustment of the three leveling feet in conjunction with the spirit level bonded on the base plate. A momentary switch installed on the electronics' enclosure was briefly pressed to manually override the long period integrator for precise centering of the transducer using the motorized leveling screw. The re-centering cycle was activated manually several times, until the monitored output voltage of the integrator, at pin 6 of U21, came within +/- 0.75 VDC.

Output signal of the displacement transducer drifted enough to cause automatic re-centering cycles many times during the first few days after initial installation to restore optimum performance. Subsequent re-centering cycles (after the settling-in period) occurred in response to changing ambient conditions. Re-centering is recognized on the seismograms as a very large amplitude wave (or several waves) that slowly diminished back to normal behavior in approximately 5-7 minutes. People walking on the soft surrounding earth within 8 meters of the vault (clearly detectable), along with earthquakes, apparently did not cause auto re-centering. Sustained long term disturbances such as frost heave, temperature distortion of the vault's structure, accumulation or evaporation of large amounts of snow/rain on the surrounding earth, did effect transducer equilibrium.

DATA PROCESSING

Collected data through an 8 channel, 16 bit, A/D card installed on the motherboard of a Gateway 486 DX2 66 MHz personal computer. Larry Cochrane of Redwood City, California USA, manufactured the card, model PSN-ADC-12/16 Rev 2. The card had a temperature-controlled oven for the clock crystal and a WWV time correction feature accurate to within +/-20 milliseconds when locked onto an appropriate frequency provided by a short wave radio receiver. One channel of the A/D card recorded the raw output signal (0.003-10 Hz passband, defined by the seismometer's internal filters), for detecting high frequency (short period) local and regional events. A second channel recorded data passband

filtered at 0.007-0.05 Hz, for detecting long period teleseismic events. Passband's high end was controlled by a 4th order 0.05 Hz Bessel LPF.

Data was displayed and saved using Seismic Data Recorder (SDR) software, release 2.71, a DOS program written by Larry Cochran, loaded on the Gateway computer. Data collected at a rate of 50 samples per second, time and date stamped by the software. Seismic events files were saved in PSN format for later analysis, at a lesser sampling rate for manageable file sizes.

WinQuake software, release 2.8.7, also written by Larry Cochran, is a Windows based program used for analyzing the data files created with the SDR software. Features include distance calculations, FFT graphic displays, digital filtering, data integration and much more. Adjustable digital filters controlled bandpass limits and roll-offs, for extracting useful seismic data from the clutter of unwanted background noise. After entering the event's origin time and co-ordinates into WinQuake, the embedded JB Tables calculated the theoretical arrival times for P, S and other wave phases, placing markers at the proper locations on the seismograms. Significant "spikes" seen on the signal trace could then be judged as either random instrument/environmental noise, or waves originating from a distant teleseismic event.

SAMPLE SEISMOGRAMS

In this paper are several seismograms saved as GIF images using WinQuake software, starting with Figure 15. When viewing this document as a PDF image, you may need to zoom in on the traces to reveal finer details. Orientation of the sensitive axis was North-South (N-S) for all seismograms.

During the first week of January 2005, the electronic feedback circuit was slightly modified. Prior to this date the MkXVII was installed in the surface vault, producing the seismograms presented in Figures 15 and 16. After the alteration date, the seismometer was operated temporarily on the basement floor of the house for easy access, in case further changes were necessary, see Figure 7. While on the floor two other seismograms, Figures 17 and 20, were created. Figure 17 can be compared to a seismogram, Figure 18, made by a Streckeisen STS-2 seismometer at station PAL in New York State.

RESULTS

Without the appropriate test equipment and skills suitable for determining the instrument's sensitivity and noise levels, visual comparisons were made with near real time seismograms accessible on the WWW.

Closest station was PGC (CGS), located 124 km to the NW in Sidney BC, Canada. Their Guralp CMG-3ESP BB seismometer provided Z-axis LP data passband filtered to 0.01-0.07 Hz. Its seismogram's signal to noise ratios appeared similar to the MkXVII.

Event detection by two widely separated high gain VBB stations was the most helpful. First was station COR (USGS) 360 km to the south in Corvallis Oregon, providing filtered Z-axis LP data at 0.01-0.05 Hz from a Streckeisen STS-1 VBB seismometer. The other was station PAL (Lamont-Doherty) situated across the country in Palisades New York. It actually displayed N-S aligned LP data, from a Streckeisen STS-2 VBB seismometer. Station COR daily detected several small teleseismic events, proving to be the most convenient system for evaluating the MkXVII's overall sensitivity. Background noise levels were much higher on the MkXVII seismograms during an average day, when compared to COR or PAL. During favorable local weather conditions, the MkXVII successfully detected many of the same small events captured by COR.

It's difficult to compare data from a tilt sensitive horizontal instrument supported by unconsolidated earth, to a Z-axis component residing on solid bedrock some distance away, i.e. stations COR and PGC. Comparisons to N-S LP data from station PAL were possible if the differences in station locations could be easily factored in, i.e. both at similar distances to a given event, see Figures 18,19 & 20. Without a doubt, coupling the seismometer to the basement's concrete floor slab with carpeting during temporary testing attenuated a portion of the LP wave energy from teleseismic events, and at the same time added a certain level of both SP and LP noise.

CONCLUSIONS

The mechanical components and assemblies took approximately 70 hours to design, build and calibrate. Manually wiring the electronic components on three perforated circuit boards, attaching hardware to the metal enclosures and system de-bugging took an additional 30 hours.

As expected from an amateur design, the MkXVII's signal to noise ratio was worse than the top quality professional systems. Any differences in performance are acceptable, knowing that the cost of the MkXVII at ~\$250 (mostly for electronic components) equaled just a small fraction of its commercial counterpart's cost.

. Digital filtering can only do so much, really need a quieter site for the seismometer vault. Further design enhancements like a smaller seismic mass combined with a more forceful feedback transducer, could have increased the top end response, but the added performance may be quickly overwhelmed by local environmental noise.

ACKNOWLEDGEMENTS

The author is indebted to the late Sean-Thomas Morrissey who for several years generously shared his professional knowledge and experiences regarding seismometer theory, operation and installation with subscribers to the Public Seismic Network (PSN), particularly the construction details of the STM-8. Erhard Wielandt must be thanked for his mentoring concerning the “quasi-integrator”.

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http://mnw.eas.slu.edu/People/STMorrissey/etc_export

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Usher M. J. & Guralp C., 1978. *The design of miniature wideband seismometers*. Journal of Geophysics, **48**, 281-292.

FIGURES

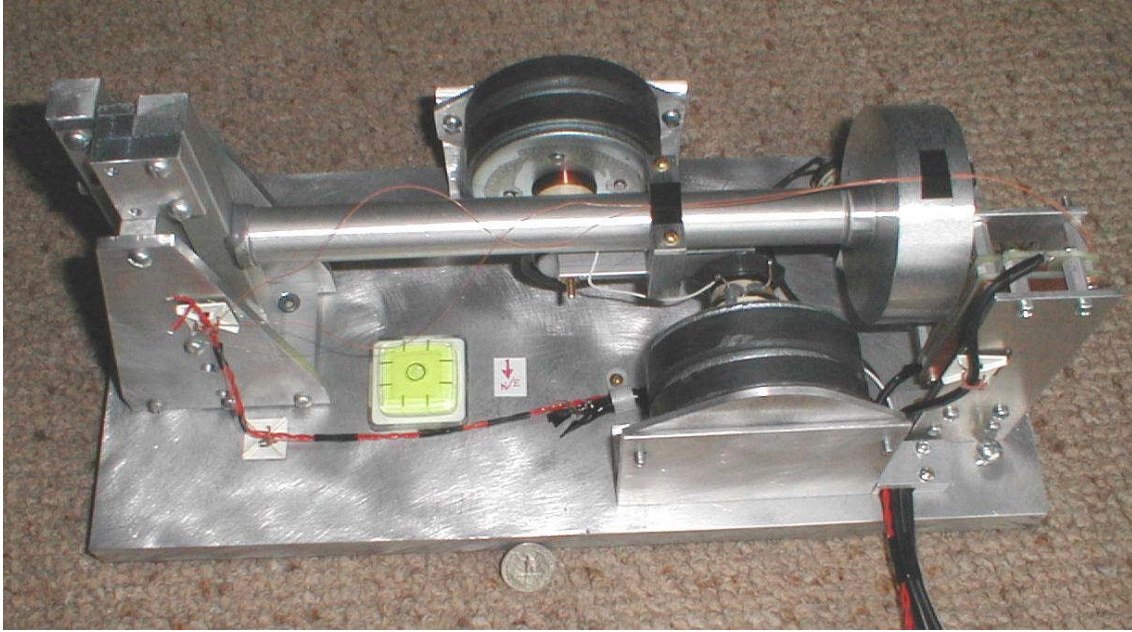


FIGURE 1

OVERVIEW OF THE MkXVII SESIMOMETER
(Coin leaning against base plate is a U.S. quarter dollar, 24 mm in diameter)

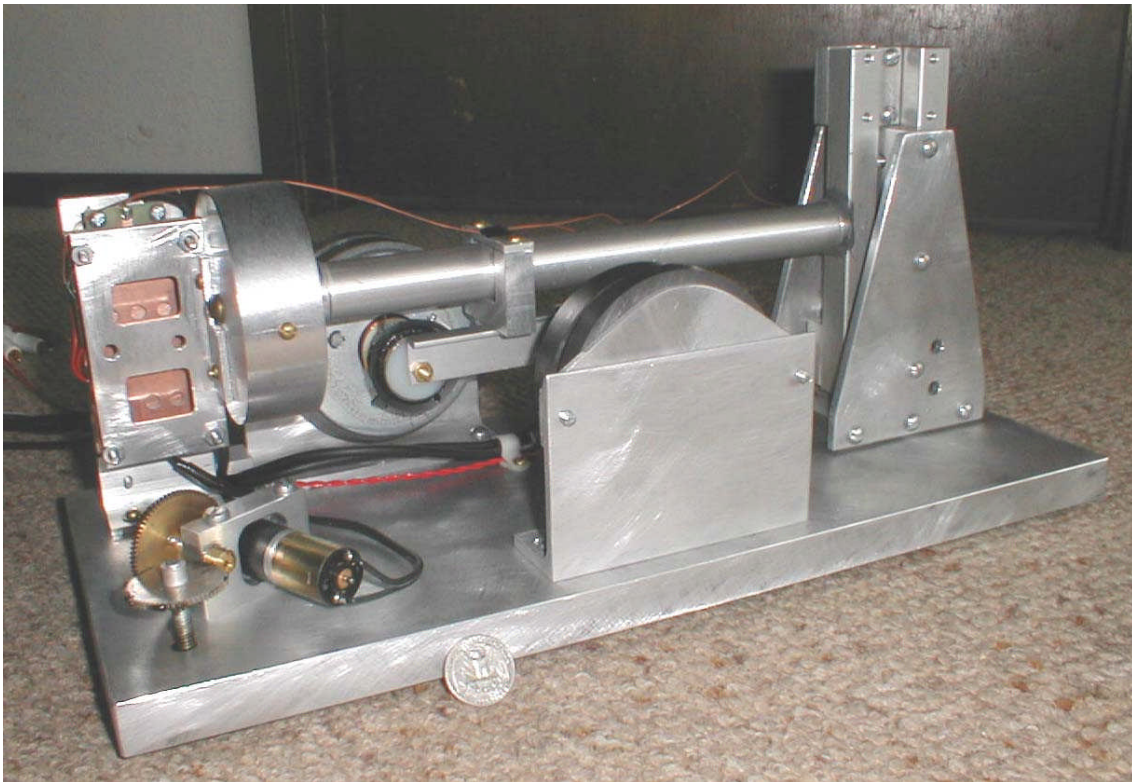
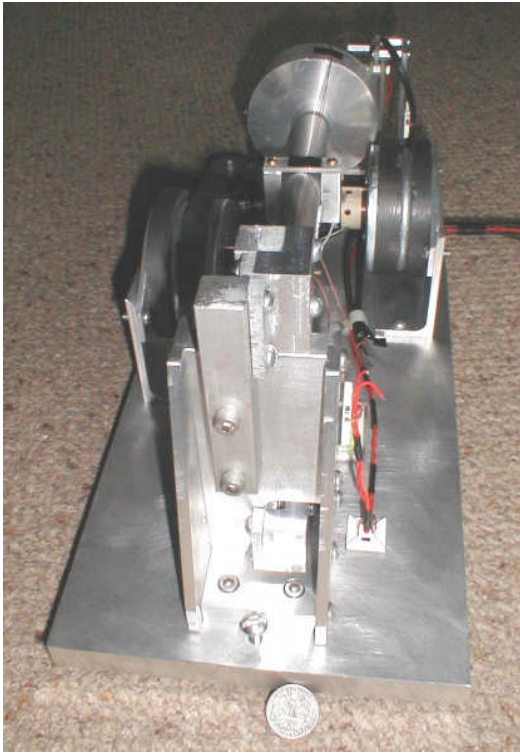
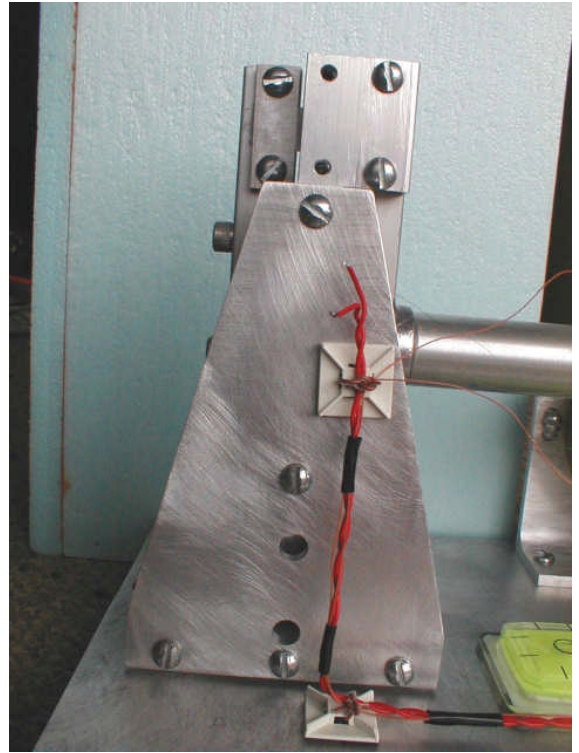


FIGURE 2

SIDE VIEW OF THE SEISMOMETER



(A) Lower hinge, hidden at base of column.



(B) Upper hinge, at top of column.

FIGURE 3

HINGES - PENDULUM TO COLUMN

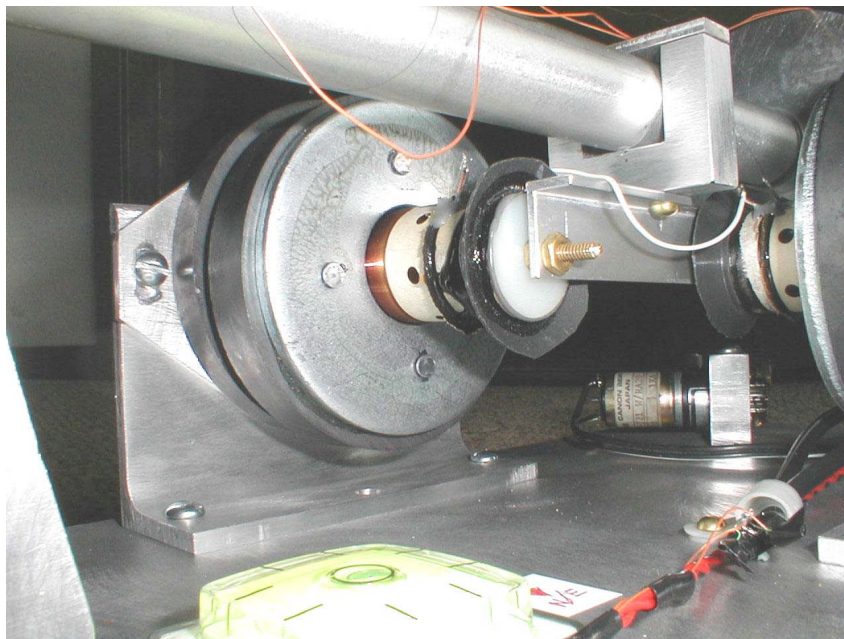


FIGURE 4

FEEDBACK TRANSDUCERS

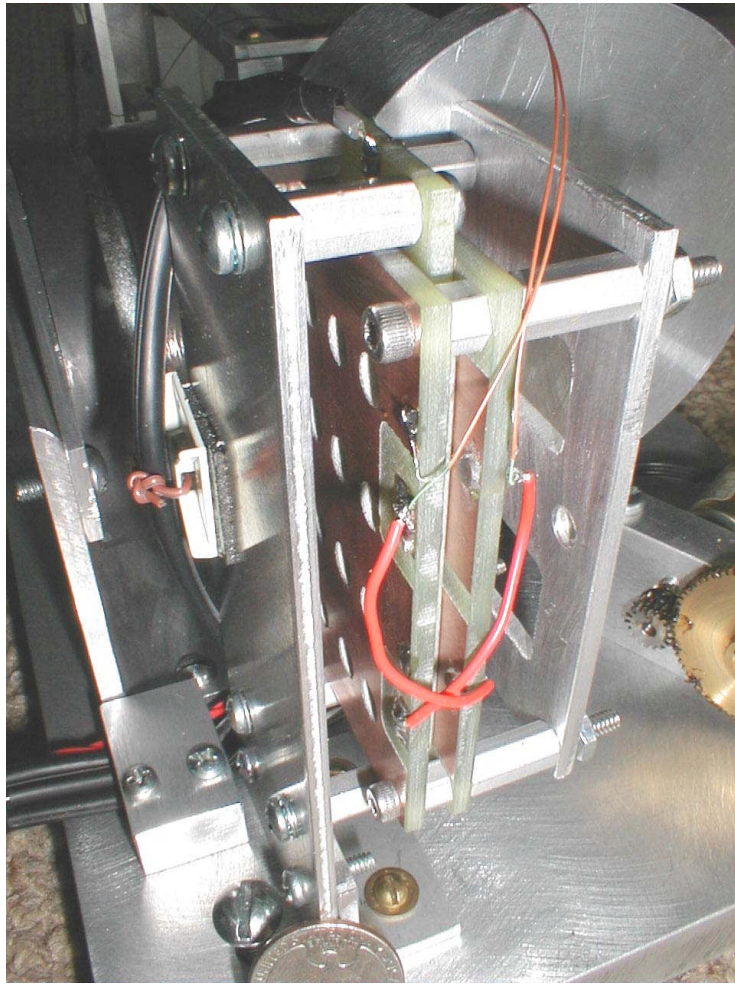


FIGURE 5

CAPACITANCE DISPLACEMENT TRANSDUCER

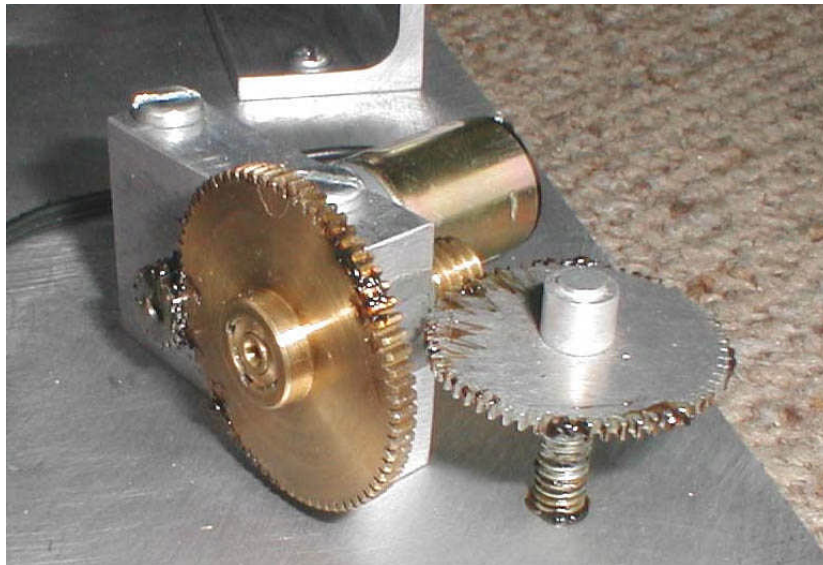


FIGURE 6

MOTORIZED LEVELLING SCREW

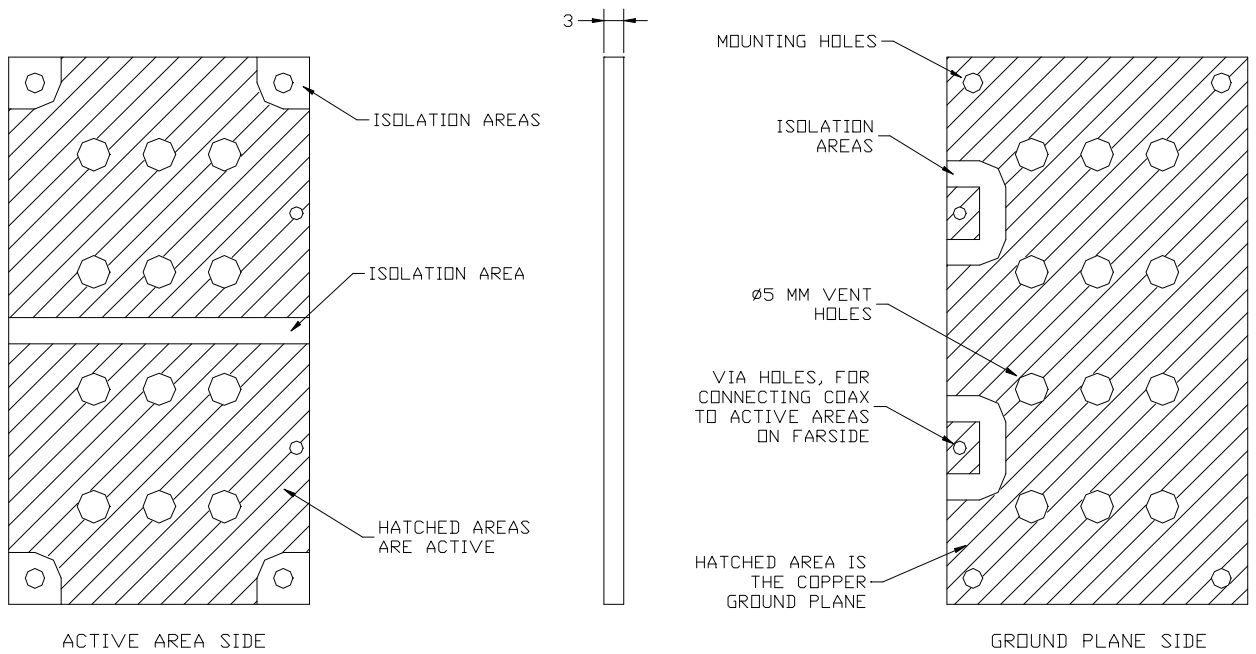


FIGURE 7

PROTECTIVE COVER INSTALLED ON THE INSTRUMENT

Protective cover was fabricated from Styrofoam sheet. External surfaces were wrapped in heat reflecting aluminum foil. The two small aluminum boxes off to the left of the seismometer enclose the electronic circuit boards. An old galvanometer (with the round white dial) continually monitored the “position of the mass”.

Whenever the electronic circuits were modified, system reliability was verified by continuous operation over several weeks, with the instrument temporarily installed on the basement floor of the house (see above photo). The seismometer rested on pieces of $\frac{1}{4}$ " thick glass plate, spaced off the concrete floor slab by a layer of carpet and foam under-pad compressed to a thickness of $\sim 6\text{mm}$ due to the weight of the unit. During verification several seismograms were generated, some eventually became figures in this paper. Ambient basement temperatures were fairly constant $13\pm 2^\circ\text{C}$ (winter months). After exhibiting satisfactory performance, the instrument would be transferred to a small surface vault out side the house.



OUTER PLATE (2 REQUIRED)

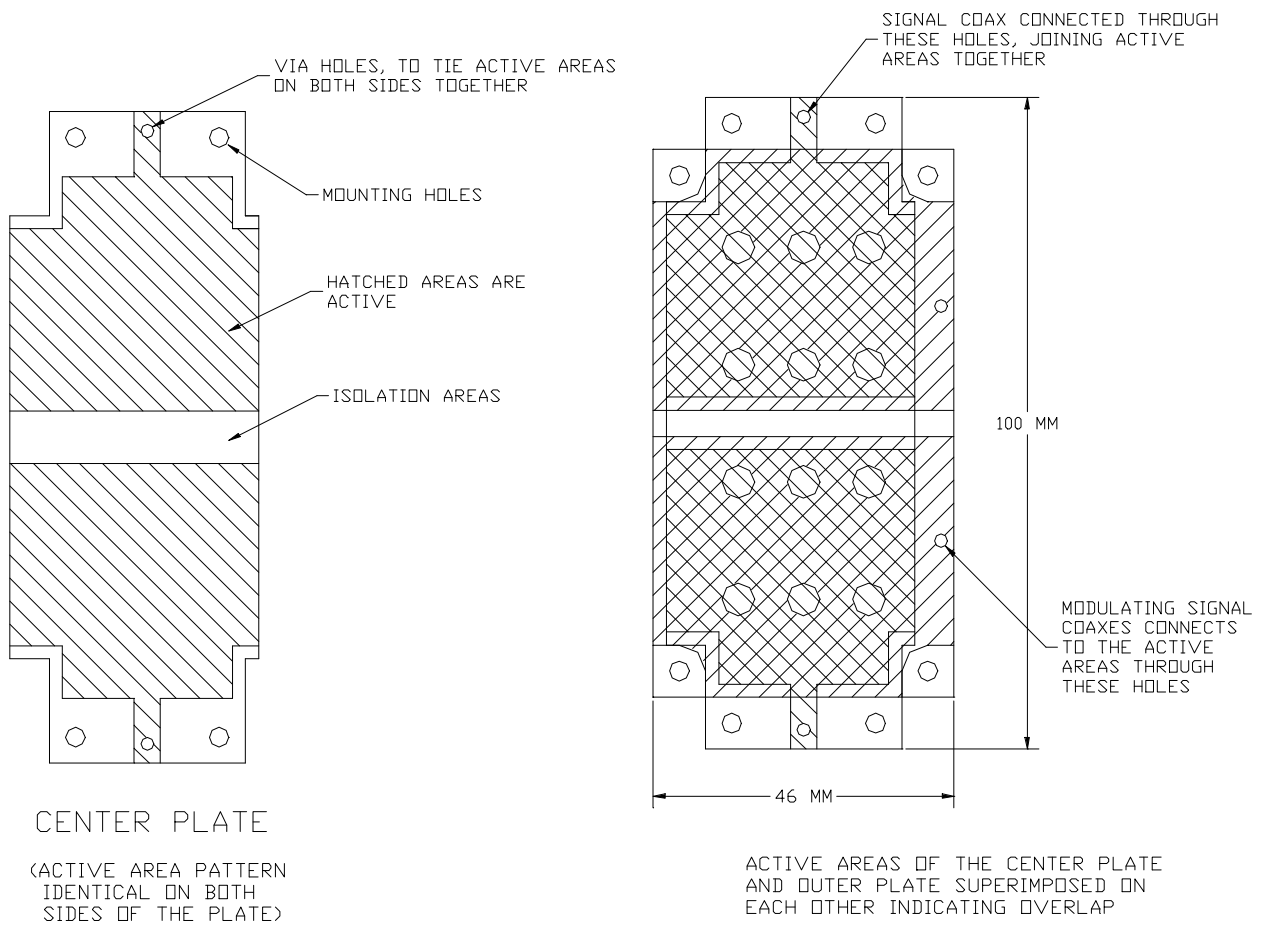


FIGURE 8
CAPACITOR PLATE DETAILS

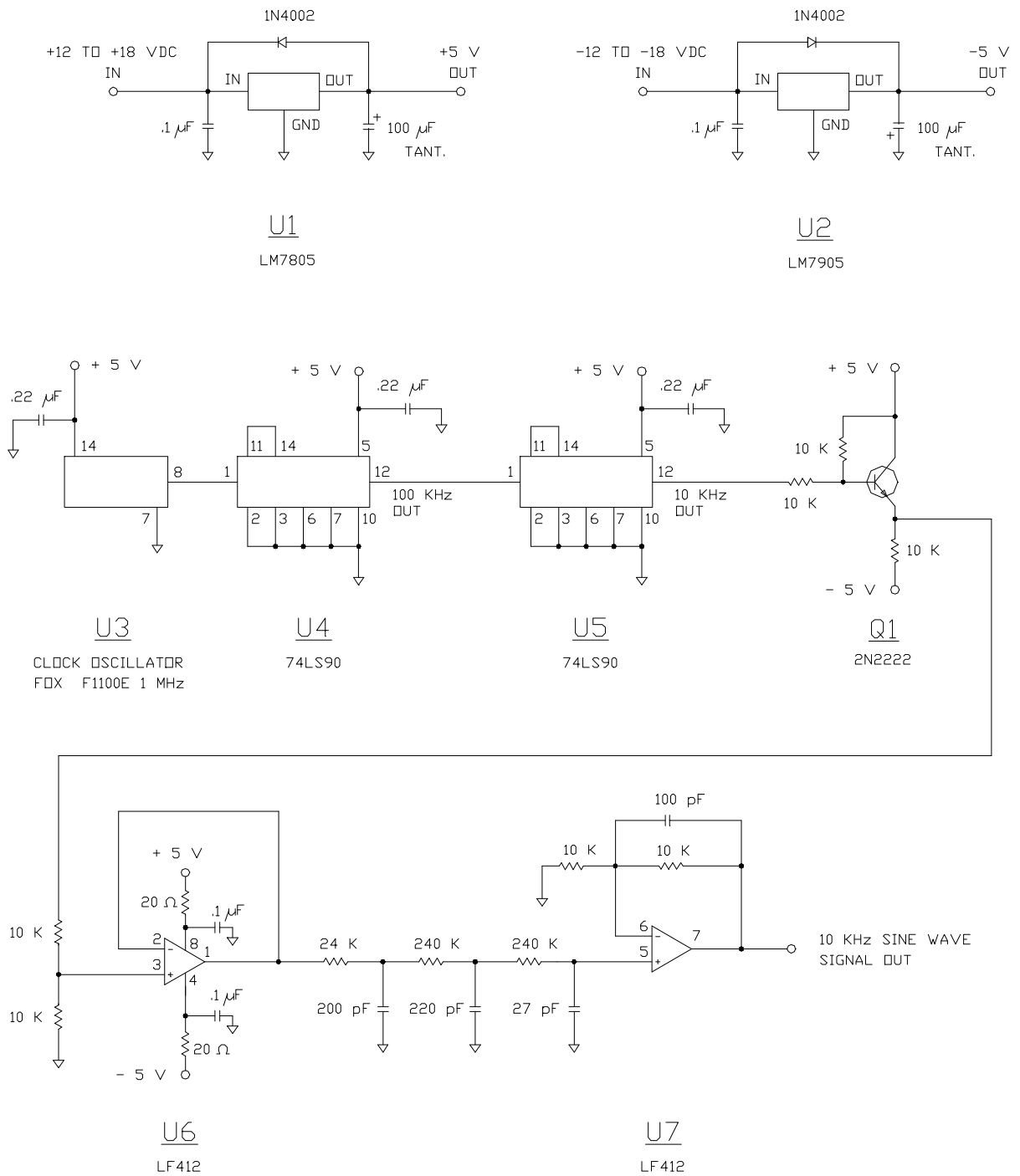
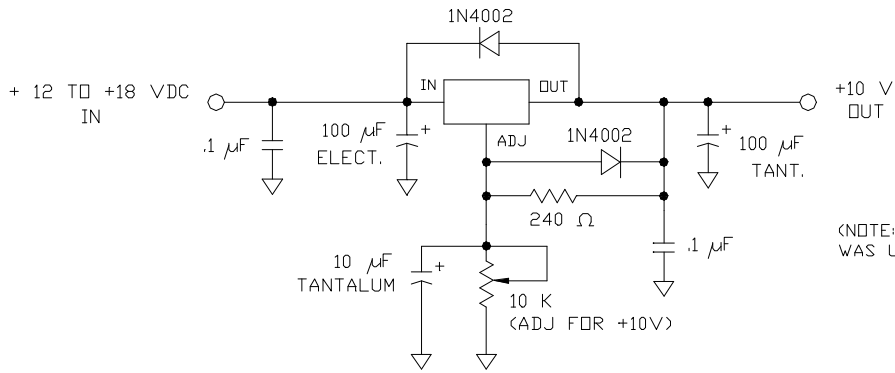


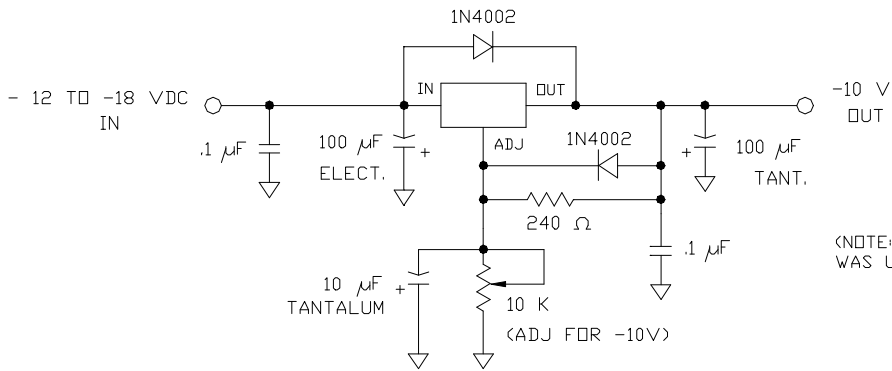
FIGURE 9
 OSCILLATOR CIRCUIT BOARD (OCB)

REVISED 14 DECEMBER 2003



(NOTE: THIS VOLTAGE REGULATOR WAS USED ON THE DTCB AND FCB)

U8
LM317



(NOTE: THIS VOLTAGE REGULATOR WAS USED ON THE DTCB AND FCB)

U9
LM337

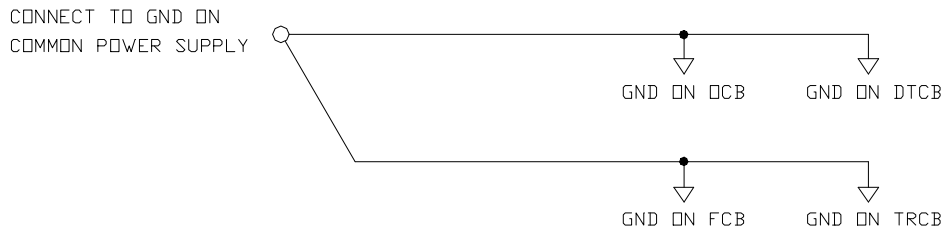


FIGURE 10
VOLTAGE REGULATOR CIRCUITS

REVISED 14 DECEMBER 2003

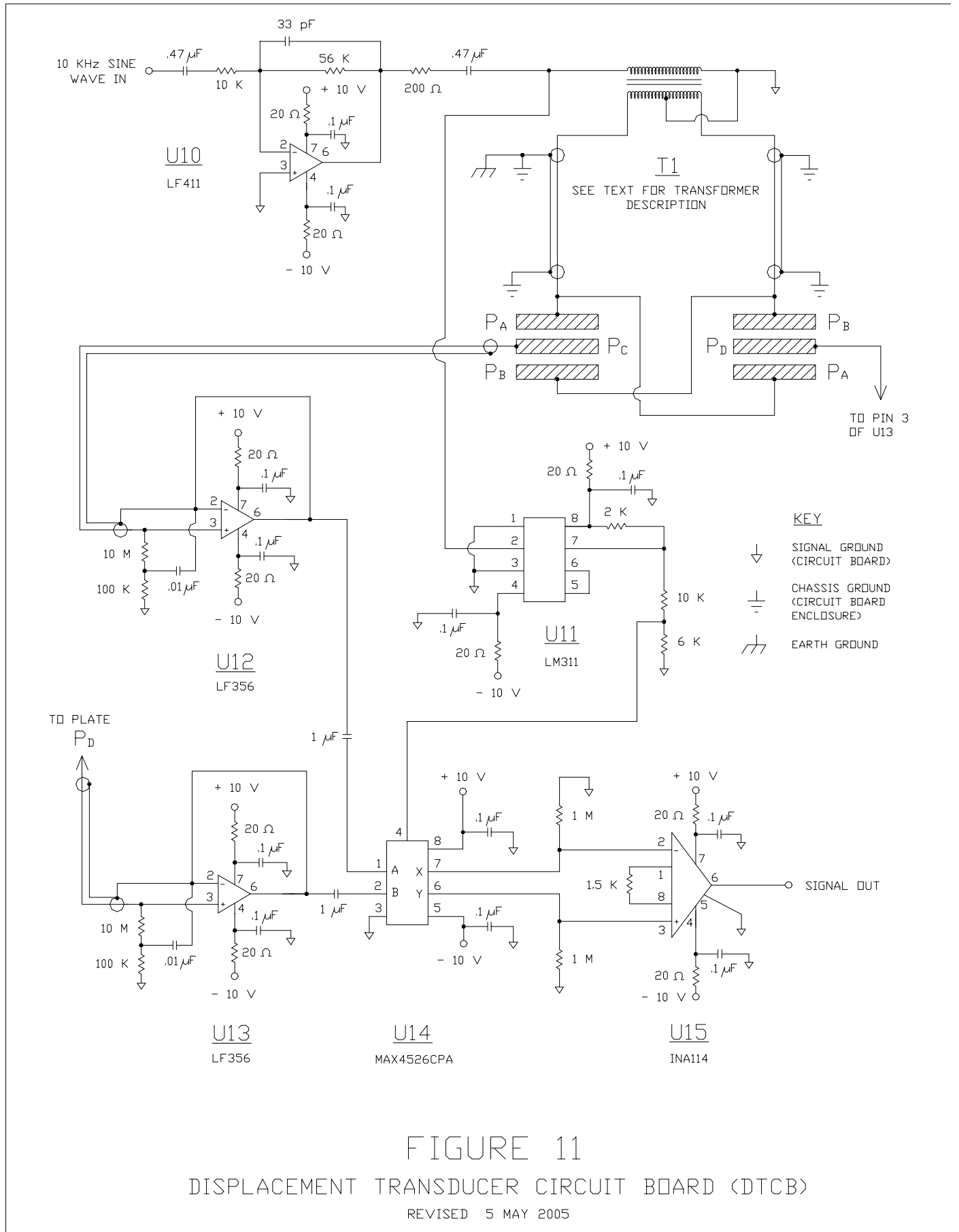


FIGURE 11
 DISPLACEMENT TRANSDUCER CIRCUIT BOARD (DTCB)
 REVISED 5 MAY 2005

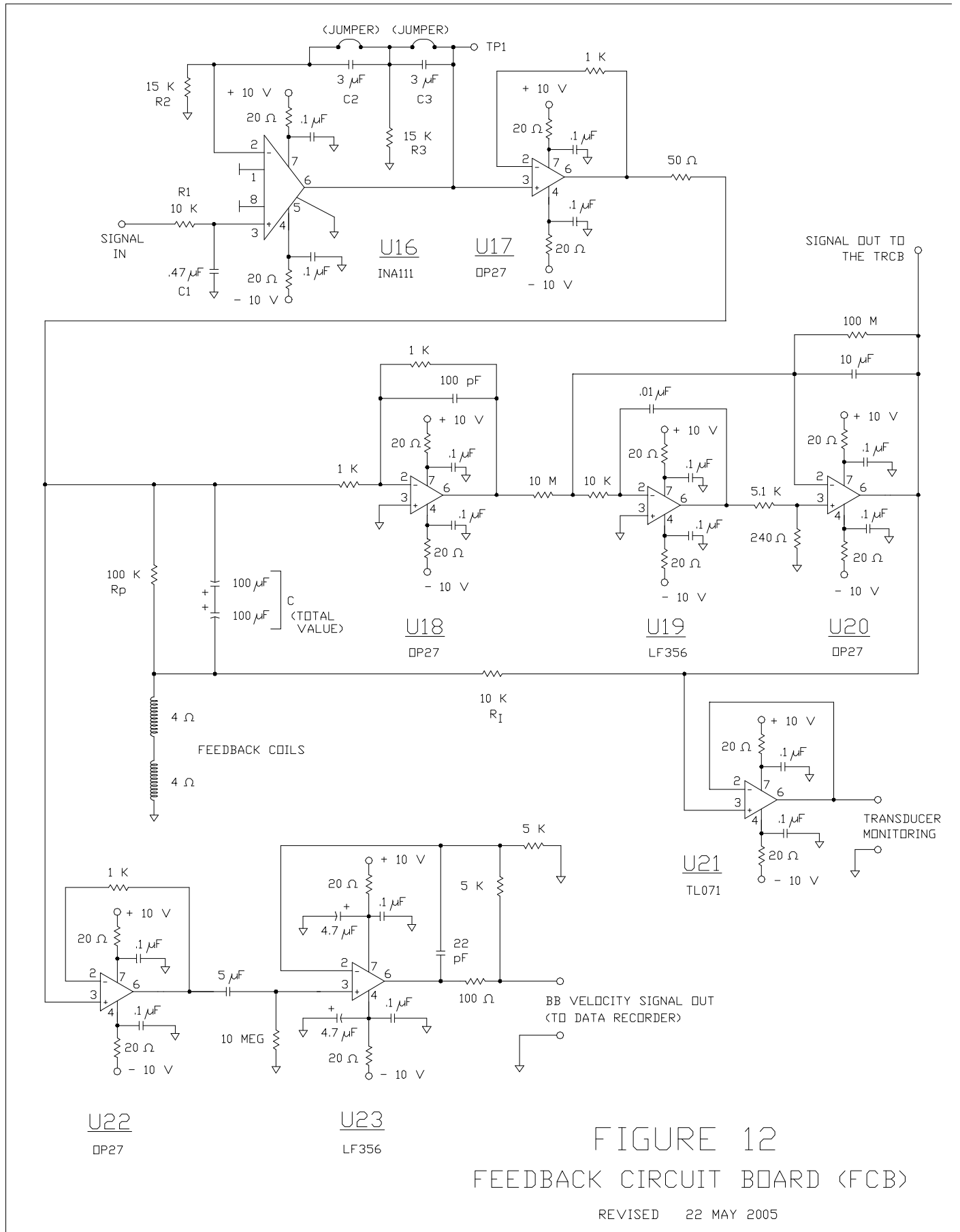


FIGURE 12
 FEEDBACK CIRCUIT BOARD (FCB)

REVISED 22 MAY 2005

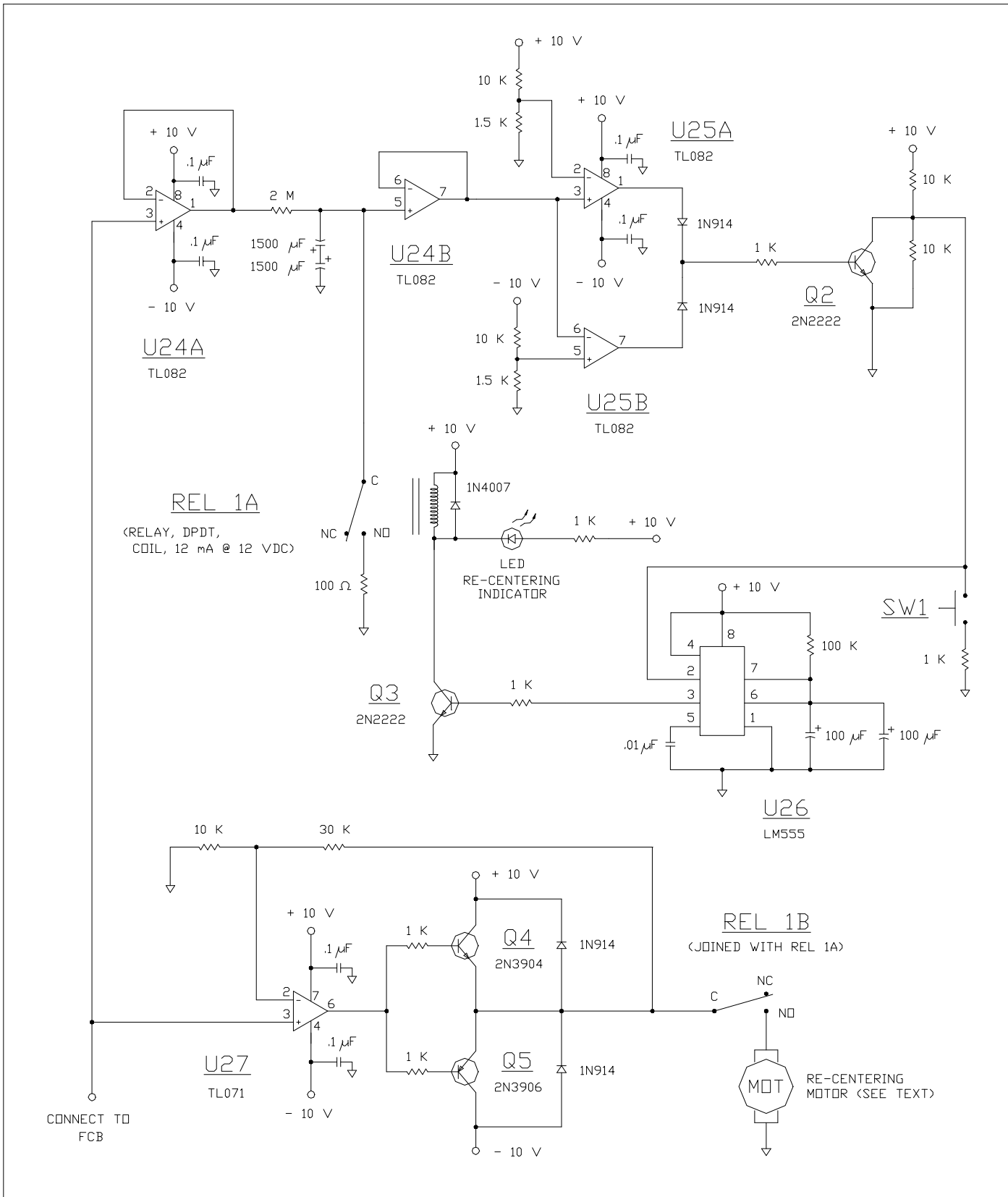


FIGURE 13
 TRANSDUCER RE-CENTERING CIRCUIT BOARD (TRCB)

REVISED 2 JAN 2005

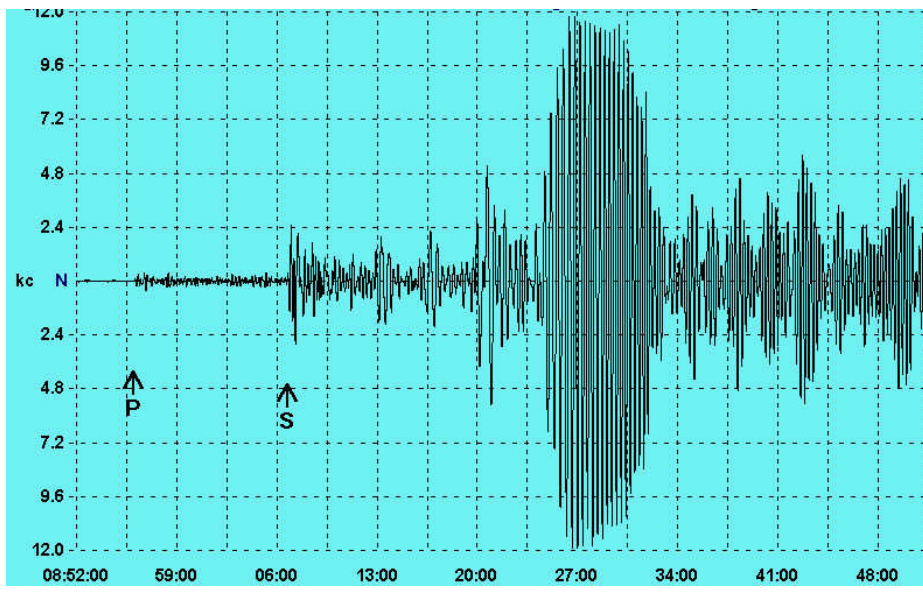
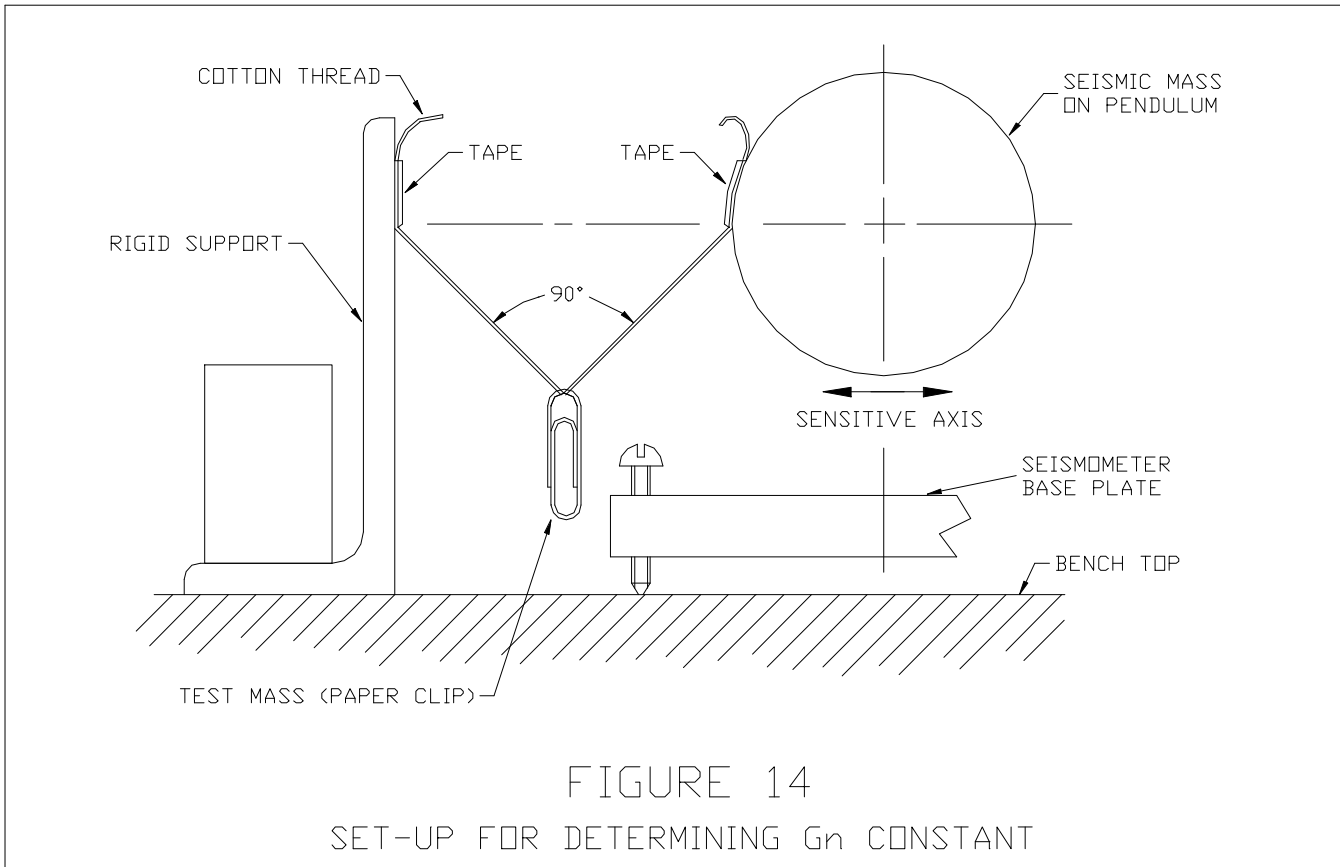


FIGURE 15

SEISMOGRAM: SOLOMON ISLANDS, Mw 7.3

Origin date and time: 20 January 2003, 08:43:06 UTC
 Event coordinates: 10.48° S, 160.75° W, 33 km deep
 Distance: delta = 89°, 9902 km, 6153 miles
 Passband: 0.01-0.07 Hz, 12th order roll-offs for LPF and HPF
 Comments: UTC time indicated across the bottom of the seismogram.

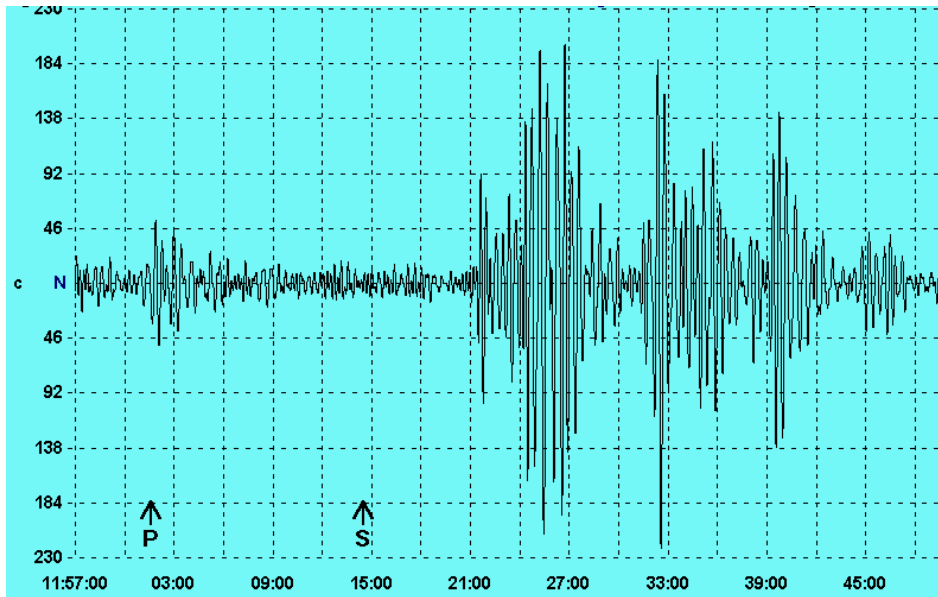


FIGURE 16

SEISMOGRAM: SOUTHERN SUMATRA, INDONESIA, Mb 5.4

Origin date and time: 3 February 2003, 11:46:17 UTC

Event coordinates: 2.70° S, 101.08° E, 33 km deep

Distance: delta = 121°, 13514 km, 8397 miles

Passband: 0.03-0.08 Hz, 12th order roll-offs for LPF and HPF

Comments: UTC time indicated across the bottom of the seismogram.

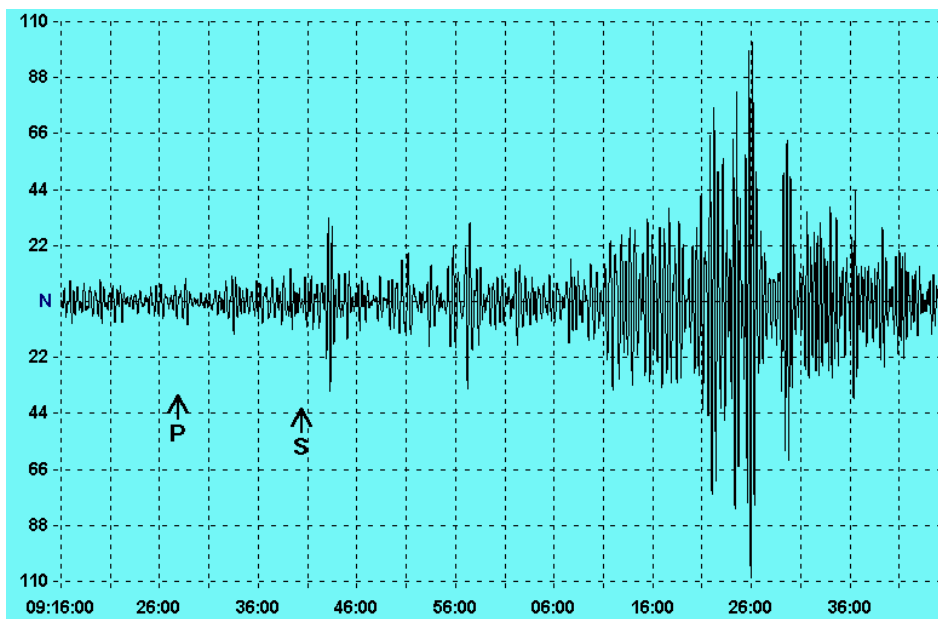


FIGURE 17

SEISMOGRAM: ANDAMAN ISLANDS, INDIA REGION, Mw 6.1

Origin date and time: 4 January, 2005, 09:13:12 UTC

Event coordinates: 10.67° N, 92.40° E, 20 km deep

Distance: delta = 114°, 12682 km, 78801 miles

Passband: 0.03-0.05 Hz, 4th order roll-offs for LPF and HPF

Comments: UTC time indicated across the bottom of the seismogram.

Seismograms in Figures 17 & 18 are used for a comparison study. See caption under Figure 18 for more details.

GMT 2 micrometer/s Station PAL, channel LPN - Jan 4, 2005 gain 0.1 bandpass 0.003- 0.10 Hz

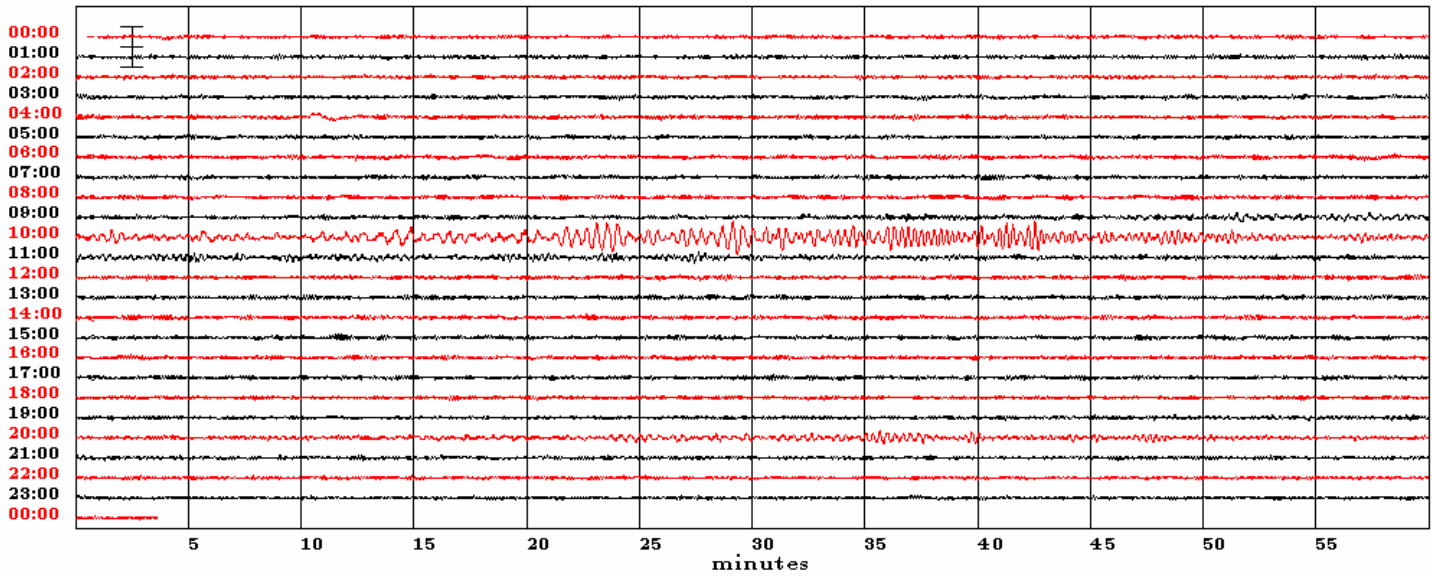


FIGURE 18

SEISMOGRAM: ANDAMAN ISLANDS, INDIA REGION, Mw 6.0

Seismogram of the same event recorded in Figure 17, this one made by station PAL in Palisades New York using an STS-2 seismometer, with a delta of 127° to the event.

UTC time indicated on the left-hand side and across the bottom of the seismogram.

Passband: 0.003-0.10 Hz, LPF and HPF roll-offs are unknown.

Comments: Size of the surface waves, when compared to normal background noise levels, appear similar to the MkXVII seismogram (Figure 17). It should be noted that the MkXVII detected this event while resting on the carpeted basement floor of the house, per Figure 7. Sensitive axis of both the MkXVII and STS-2 were aligned N-S. See Figure 19 for how the seismometers were located to the event.

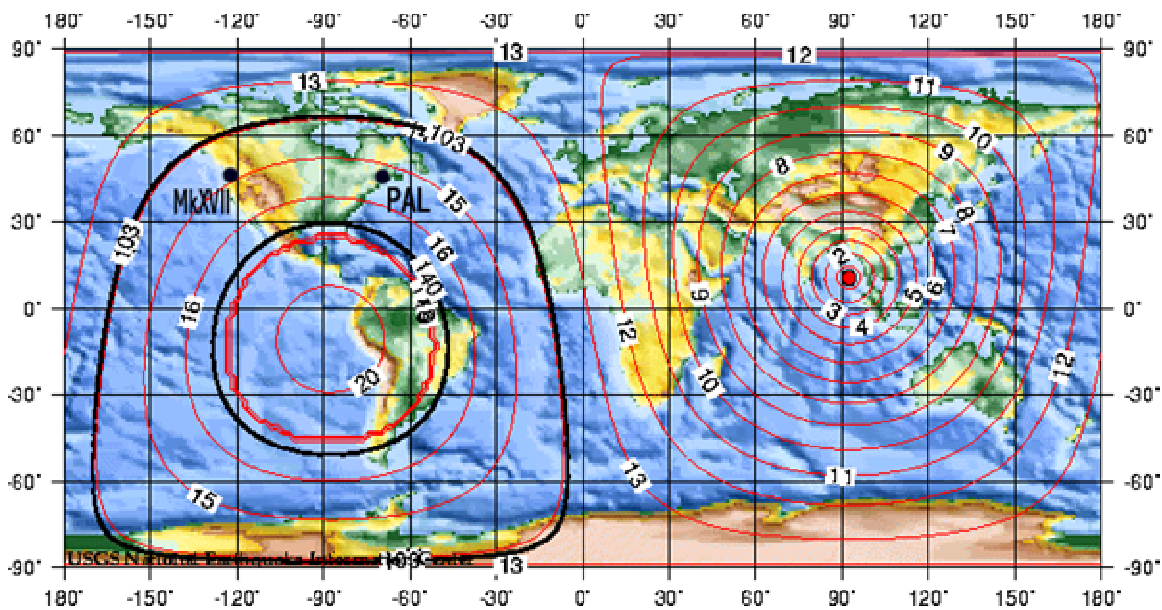


FIGURE 19

Map showing locations of the MkXVII (Fig 17) and PAL (Fig 18), relative to the Mw 6.1 Andaman event of 4 Jan 2005

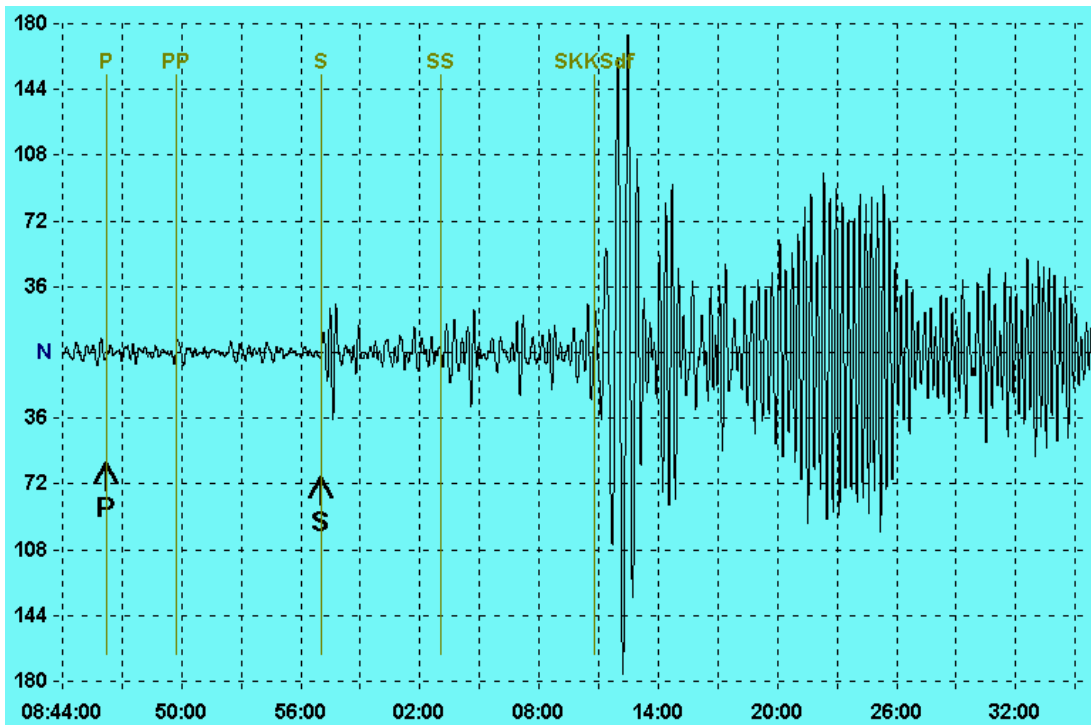


FIGURE 20

SEISMOGRAM: NEW BRITAIN REGION, P.N.G., Mw 6.1

Origin date and time: 14 January 2005, 08:33:13 UTC

Event coordinates: 4.33° S, 152.73° E, 10 km deep

Distance: delta = 89.8°, 9981 km, 6202 miles

Passband: 0.03-0.067 Hz, 4th order roll-offs for LPF and HPF

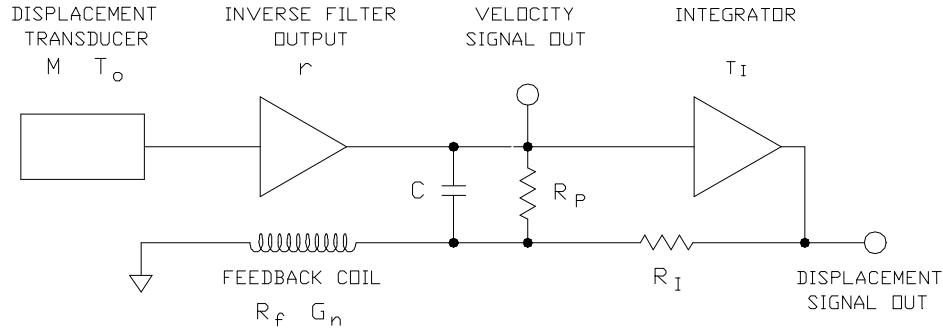
Comments: UTC time indicated across the bottom of the seismogram.

It should be noted that the MkXVII detected this event while resting on the carpeted basement floor of the house, per Figure 7.

APPENDIX A

TRANSFER FUNCTION FOR THE MK XVII BROADBAND SEISMOMETER.

This Mathcad work sheet was originally created by Sean-Thomas Morrissey.
After downloading the document was reorganized for clarity by Allan Coleman.
Last revised: 01/02/05



CIRCUIT BLOCK DIAGRAM

- | | |
|---|--------------------------|
| r = displacement transducer constant, volts/meter
(measured at output of inverse filter) | r := 101378 |
| M = sensor mass, kg | M := .55 |
| C = feedback capacitor, farads | C := .000050 |
| R _p = Proportional (DC) feedback resistor, ohms | R _p := 100000 |
| R _I = Integrator feedback resistor, ohms | R _I := 10000 |
| R _F = force transducer coil resistance | R _F := 9 |
| T _I = Integrator time constant, seconds | T _I := 100 |
| T ₀ = Mechanical free period of sensor, seconds | T ₀ := 10 |
| T _d = Displacement Xducer time constant (~= 0) | |

Generator Constant, Newton/Amps

.....function requires m/s/s/amp := $G \frac{\left| \frac{m}{\text{sec}^2} \right|}{\text{amp}} \quad G_n := 8.7 \quad G := \frac{G_n}{M} \quad G = 15.8182$

Damping := $S\omega_0 := 2 \cdot \frac{\pi}{T_0} \quad S\omega_0 = 0.6283 \quad \zeta := \frac{1}{2} \cdot \left[\frac{1}{R_p} + \frac{(S\omega_0)^2}{r \cdot G} \right] \cdot \left(T_I \cdot \frac{R_I}{C} \right)^{.5} \quad \zeta = 0.7245$

Effective "natural Period" (seconds) := $T_n := 2 \cdot \pi \cdot \sqrt{C \cdot R_I \cdot T_I} \quad T_n = 44.4288$

Equation Generalized:
$$Y_i := \frac{1}{G \cdot C} \cdot \frac{s_i \cdot (1 + s_i \cdot C \cdot R_F)}{(s_i)^4 \cdot \left(\frac{R_F}{r \cdot G} \right) + (s_i)^3 \cdot \left(\frac{1}{r \cdot G \cdot C} \right) + (s_i)^2 + s_i \cdot \left[\frac{(S\omega_0)^2}{r \cdot G \cdot C} + \frac{1}{C \cdot R_p} \right] + \frac{1}{C \cdot R_I \cdot T_I}}$$

Setting Constants: $k := \frac{1}{G \cdot C} \quad n := (C \cdot R_F) \quad a := \frac{R_F}{r \cdot G} \quad b := \frac{1}{r \cdot G \cdot C} \quad c := \frac{(S\omega_0)^2}{r \cdot G \cdot C} + \frac{1}{C \cdot R_p} \quad d := \frac{1}{C \cdot R_I \cdot T_I}$

$$i := 1, 2.. 600 \quad f_i := 10^{(i-300) \cdot 0.01} \quad \omega_i := 2 \cdot \pi \cdot f_i \quad s_i := j \cdot \omega_i \quad s_{200} = 0.6283i$$

Evaluating:
$$A_i := k \cdot \left[\frac{s_i \cdot (1 + s_i \cdot n)}{(s_i)^4 \cdot a + (s_i)^3 \cdot b + (s_i)^2 + s_i \cdot c + d} \right]$$

$$\text{vel}_i := \text{Re}(A_i \cdot s_i)$$

$$\text{maxv} := \max(\text{vel})$$

Ref. line, V/m/s: $\text{maxv} = 1.2662 \times 10^3$

Marking effective periods: $F_1 := \frac{1}{T_1} \quad F_n := \frac{1}{T_n}$

