

An Animated Guide©: Proc UCM (Unobserved Components Model)

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ABSTRACT

This paper explores the underlying model and several of the features of Proc UCM, new in the Econometrics and Time Series (ETS) module of SAS ®. This procedure can be used by programmers in many fields, not just Econometrics. Time series data is generated by marketers as they monitor “sales by month” and by medical researchers who collect vital sign information over time. This technique is well suited to modeling the effect of interventions (drug administration or a change in a marketing plan). This new procedure combines the flexibility of Proc ARIMA with the ease of use and interpretability of Smoothing models. UCM does not have the capability to easily model transfer functions, a useful ARIMA function that is planned for Proc UCM.

INTRODUCTION

This paper explains the underlying model several of the features of Proc UCM, new in the Econometrics and Time Series (ETS) module.

This procedure can be used by programmers in many fields, not just Econometrics. Time series data is generated by marketers as they monitor “sales by month” and by medical researchers who collect vital sign information over time. This technique is well suited to modeling the effect of interventions (drug administration or a change in a marketing plan). This new procedure combines the flexibility of Proc ARIMA with the ease of use and interpretability of Smoothing models.

THE MODEL NEW DEFINITIONS

One thing that makes UCM useful is its similarity to regression. A useful conceptual framework for UCM is that of a regression model ($Y = B_0 + B_1X_1 + B_2X_2 + \varepsilon$) where the betas are allowed to be time varying. A major difference between data properly modeled with regression and data typically modeled by time series techniques is the presence of autocorrelation, or serial correlation. In “time series data” observations close together tend to behave similarly. If observation number n is above a fitted regression line, it is likely that observations $N-1$ and $N+1$ will also be above the regression line. This pattern of correlation between observations (and errors) breaks down as observations get farther apart in time. These characteristics suggest that a model for the data should place more “weight” or “importance” on “recent” observations and not give all observations in the data set equal importance. Proc ARIMA, and Proc UCM, both create models that are “local”, that is they attribute more importance to “close” observations.

The model for UCM is:

$$\begin{aligned}
 Y_t &= \mu_t + \gamma_t + \psi_t + r_t + \sum \phi_i Y_{t-1} \\
 Y_t &= \text{trend} + \text{Season} + \text{Cycle} + \text{Autoregressive term} + \text{A regressive terms involving lagged dep. Variables} \\
 &+ \sum \beta_j X_{jt} + \varepsilon_t \\
 &+ \text{A regressive term on indep. vars.} + \text{error term}
 \end{aligned}$$

The model components μ_t , γ_t , ψ_t and r_t are assumed to be independent of each other and model underlying “drivers” of the time series.

Y_t	Dependent Variable	
μ_t	<p>Trend is implemented through the <i>combination</i> of level and slope statements, and their options.</p> <p>A UCM with just a level statement, models a time series with 0 slope. A UCM with just a slope statement gives an error.</p>	<p>Trend is the natural tendency of a series in the absence of seasonality, cycles or the effect of any independent variables. In UCM, this is a mean and a slope, so it corresponds to B_0 and B_1 in regression. Trend is modeled in two ways and it's relationship to B_0 and B_1 can be seen below.</p> <p>One method is a random walk $\mu_t = \mu_{t-1} + \eta_t$ (where η_t is an IID error term).</p> <p>The second method is a locally linear trend with a slope that varies, only, with time.</p> <p>$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t$ (where η_t is \simi.i.d $N(0, \sigma^2_\eta)$ IID error term). As beta goes forward, it can vary with time as $\beta_{t-1} = \beta_{t-1} + \zeta_t$ (where ζ_t is \simi.i.d $N(0, \sigma^2_\zeta)$ IID error term).</p>
γ_t	<p>Season is implemented through the season statement and it's options.</p>	<p>Season is the effect of seasonal effects and does not imply a yearly period to the season. The main characteristic of seasonally is that it's period (the time it takes to get through one full cycle) is known. The effects of seasonality sum to zero over the cycle. Seasonality is modeled in two ways.</p> <p>One method is a dummy variable method $\Sigma \gamma = \omega_t$ (where ω_t is \simi.i.d $N(0, \sigma^2_\omega)$ IID error term).</p> <p>The second method is a Uses a trigonometric form and seasonality is the sum of different cycles.</p> <p>Proc UCM allows blocking of cycles, or specifying cycles within cycles. The need for this can occur in many instances. One example is admissions at an Emergency Room. There is a weekly cycle, where Monday admissions are low and Saturday admissions are high. There is also a daily cycle that starts slow in early AM and has early PM and evening peaks. These cycles of admission nest and produce very high admissions on Saturday evening.</p>
ψ_t	<p>Cycle is important</p>	<p>Cycles are like seasons, but with an unknown period. They are not often used in their "pure form", but are employed as building blocks. Cycle effects are similar to seasonal effects but the period is not known and determined from the data. A periodic pattern, no matter how complex, can be expressed as a sum of cycles. UCM has implemented cycles as having fixed periods but time varying amplitude and phase.</p>
r_t	<p>Autoregressive term</p>	<p>UCM considers an autoregressive term as a cycle where frequency is either 0 or π.</p> <p>The expression for UCM autoregression is: $r_t = \rho r_{t-1} + v_t$ (where v_t is \simi.i.d $N(0, \sigma^2_v)$ IID error term).</p>
$\Sigma \phi_i Y_{t-1}$	<p>A regressive terms involving lagged dep. Variables</p>	<p>These two terms allow the programmer/statistician to great flexibility in describing the process under study.</p>
$\Sigma \beta_j X_{jt}$	<p>A regressive term on indep. vars.</p>	<p>$\Sigma \beta_j X_{jt}$ allows the determination of effects of outside intervention and support dummy variable and continuous variable coding. They can be used to model the effect of investigator interventions like drug administration or a change in a marketing plan.</p>
ϵ_t	<p>Irregular term or error term</p>	<p>ϵ_t is \simi.i.d $N(0, \sigma^2_\epsilon)$ IID error term).</p>

The programmer/statistician can create a great many types of time series by adding and deleting components from the model as well as changing options associated with statements in the model. Some knowledge of this is required because the determination of the best model will involve a process that is similar to the stepwise removal process in regression. While a parsimonious model is the goal of any modeling project, there is no general agreement in the literature on how this is best to be done. This paper, not in conflict with the literature but perhaps foolishly, makes an attempt to simplify a model.

UCM output parameters are different from regression and this has impact on how UCM is used. Proc UCM can

interpolate missing/new values of Y within the time span of the estimating data set. It can also forecast future values of Y. UCM produces two tables that show components of the model and their associated P values. Below, please find some rules on interpreting these P values and how the interpretation can be used to purify the model.

- 1) variances of the disturbance terms of the unobserved components
 -if not significant, the term is not *time varying and should be made deterministic*
- 2) Dampening coefficients and Frequency of cycles-
 -if not significant, the term is not *contributing to the model and should be removed*
- 3) Dampening coefficient of autoregression terms
 - If not significant, the term is not *contributing to the model and should be removed*
- 4) Regression coefficient of Regression terms
 -if not significant, the term is not *contributing to the model and should be removed*

UCM allows the programmer/statistician to set the above parameters to a specific value. This is important because the stepwise process of improving the model involves: 1) removing statements from the model to remove an underlying process from the model and/or 2) setting variance parameters to zero to change the associated underlying process from time varying to fixed.

As an example of changing the form of the model by setting parameters to be fixed at zero, examine the sub-models below that are associated with trend. Trend, we should remember, is only one component of the model. The common conditions of the UCM, that of a locally linear trend is implied in the two equations below.

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \quad (\text{where } \eta \text{ is } \sim \text{i.i.d } N(0, \sigma^2_{\eta} \text{ IID error term})$$

Interpret this as the mean of the current period =last period's mean, + effect of 1 period of time + a random term

$$\beta_{t-1} = \beta_{t-1} + \zeta_t \quad (\text{where } \zeta \text{ is } \sim \text{i.i.d } N(0, \sigma^2_{\zeta} \text{ IID error term}).$$

Interpret this as the slope changes randomly. The change is the effect of "time" and not an independent variable.

If $\sigma^2_{\zeta} = 0$	$B_t = B_{t-1}$ or B is a constant. This transforms the above equations to just one. $\mu_t = \mu_{t-1} + \beta + \eta_t$. This is called a linear trend with fixed slope model
If $\sigma^2_{\eta} = 0$	This transforms the above equations to: $\mu_t = \mu_{t-1} + \beta_{t-1} + 0$ <-- like the model above but with one less error term. $\beta_{t-1} = \beta_{t-1} + \zeta_t$ (where ζ is $\sim \text{i.i.d } N(0, \sigma^2_{\zeta}$ IID error term). Which often produces a smoother trend than the original two equation UCM model.
If both $\sigma^2_{\zeta} = 0$ and $\sigma^2_{\eta} = 0$	B is a constant and there is no error term in the trend component. The trend is no longer random and is modeled as: $\mu_t = \mu_0 + \beta t$

PROJECT1

To demonstrate Proc UCM, a dataset was created by data step programming. Components of the UCM model

were calculated individually in a dataset and summed to get the total sales, shown below. The program is attached to the article. The task was to use Proc UCM to model this dataset.

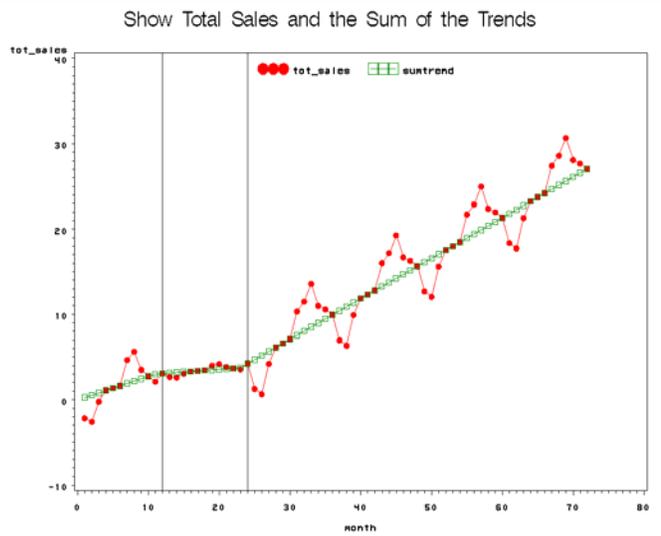
To the right is a plot of total sales for the hypothetical company. The company makes a very high tech kind of eyeglasses. These glasses are appropriate for High altitude hiking, where UV is strong and where there is likelihood of reflection off the ground. The imaginary glasses are sold to high altitude hikers and to construction workers, and others, who work in areas of high ground/water reflectivity.

There was a yearlong recession from month 12 to 24 (see vertical lines) and there was a different effect on the retail vs commercial sales. Both were affected, but retail sales were affected more due to a large reduction in travel vacations (to high altitude spots). Commercial was affected, but not as much.

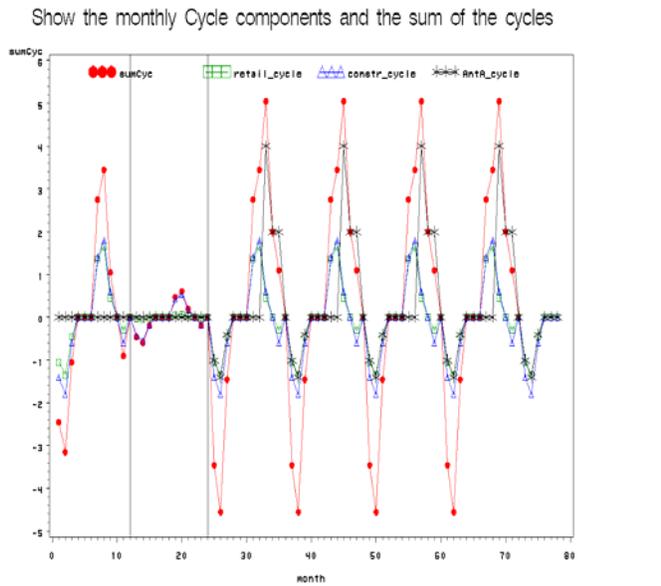
During the recession, the company decided to expand internationally and started shipping to the southern hemisphere (the company calls this volume "Sales to Antarctica" though shipments go to New Zealand a, Peru and other destinations). Since the winter/summer seasons are reversed, the seasonality of total sales was changed. International sales started the same month that the recession ended.

The figure to the right shows how the three cycles (retail construction/commercial and Antarctic) are out of phase and how they add to the total cycle component for the data set.

As American sales started to tail off the international sales (New Zealand, Australia, Peru) started up and not only drove the total sales up, but extended the selling "season".



OUT HYPOTHETICAL MANAGEMENT ISSUE IS TO DETERMINE THE EFFECT OF THE RECESSION AND OF THE STRATEGIC DECISION TO "GO INTERNATIONAL".

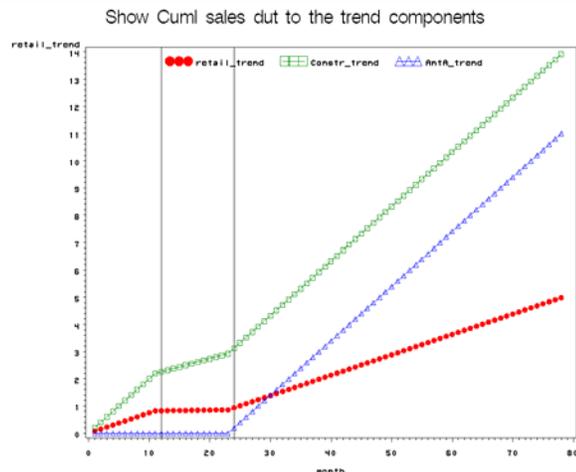


This chart shows the trends of the components of the model.

The blue line shows the calculated trend in the international sales. There were no international sales until the recession forced the company to a strategic change.

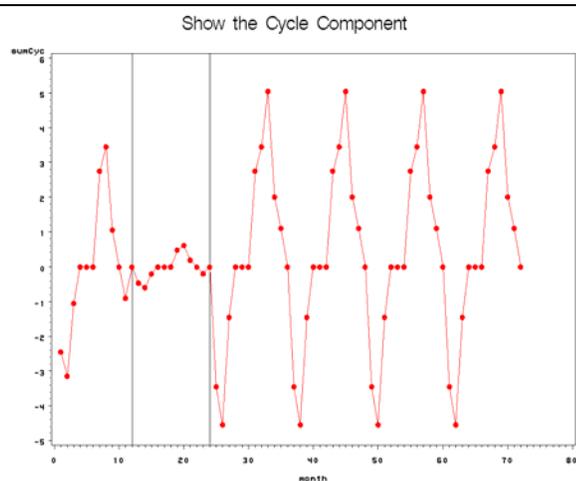
The red line shows that the recession made retail sales flat during the recession.

The green line, the commercial trend, shows that the people who used the glasses at work, continued to buy through the recession, though at a lower rate.



To the right is a plot of the summed cycle components that were calculated by data step programming.

The business challenge was to remove this noise and to recover the effect of the recession and of going international.



The code submitted, matched by some explanation of the commands, is shown below:

```
Proc UCM data=for_ucm
  PRINTALL ;
  id idmonth interval=month;

  model tot_sales =Rcsn_dv Int_DV;

  irregular ;
  level ;
  slope;
  cycle;

  SEASON LENGTH=12
  TYPE=TRIG ;

  deplag lags=1;

  estimate
    OUTEST=UCM_ESTIMATES;

  forecast lead=6
    print=decomp
    OUTFOR=UCM_FORECASTS ; run;
```

Printall turns on all printing options for the procedure. ID specifies a variable to be used as an identifier. The model statement says Y is total sales and is to be explained by a time series and two independent variables. Irregular instructs SAS to include the error term (irregular term) ϵ_t in the model. Level and slope, with no options, combine to tell Proc UCM to model with time varying slope and mean. Cycle says include a cyclical component. This behavior is complex. The season statement and options instruct SAS to look for a 12-month cycle and not to use the dummy variable coding. The observed cyclical behavior seems to be too irregular for dummy variable coding. Deplags 1 instructs SAS to include $\phi_i Y_{t-1}$ in the model. ϕ_i will be estimated from the model. The estimate command tells SAS to estimate parameters and put them in a file called UCM_ESTIMATES. The forecast command tells SAS to forecast values for 6 periods, to calculate the components of the model and put them in a file called UCM_FORECASTS

The procedure outputs several sub-tables that will be described and not included in this paper

First, UCM prints some summary statistics on the data that was used for creation of the model. This includes min, max, mean, date of first obs. and date of last obs. This information is useful in data checking and is a valuable QC feature.

Second, UCM prints some summary statistics on the data that was used for estimation. This includes min, max, mean, date of first obs. and date of last obs. One method for forecasting future observations is to put them in the input data set with no Y values. Additionally, programmers/statistician can check to see how well the model is performing by using options to tell SAS to not use the last n observations in the data set (which do have Y values) in creating the model. This allows the model to forecast these time periods and the programmer/statistician to check forecasted vs actual values. This information is useful in model checking.

Included tables are:

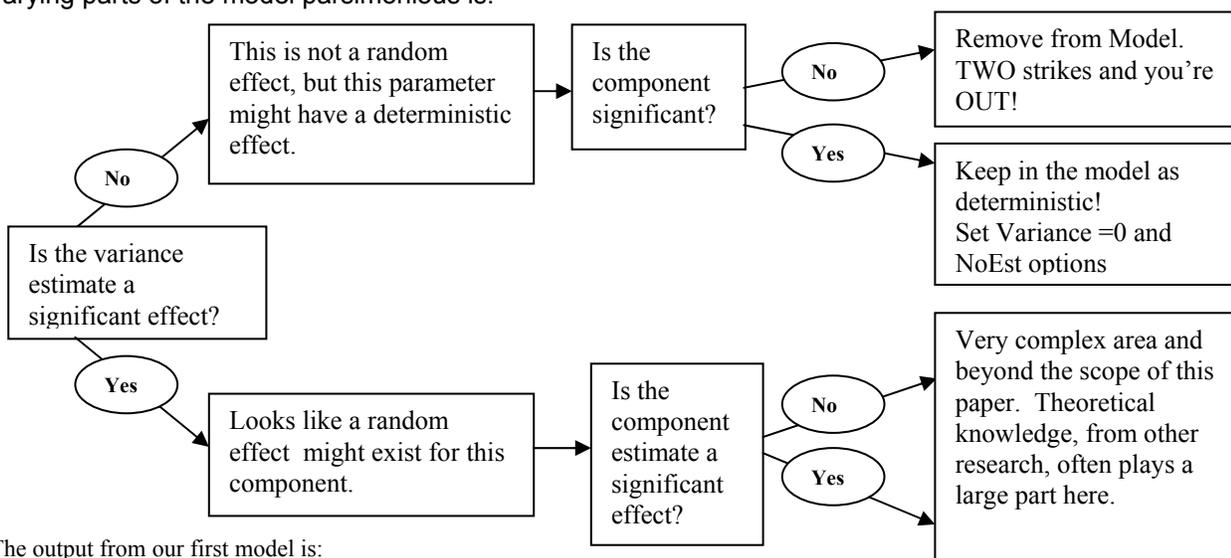
The two tables that must be examined as part of the model selection (stepwise process) are shown below. They are the **“Final Estimates of the Free Parameters”** table and the **“Significance Analysis of Components (Based on the Final State)”** tables.

The parameters of the models, as reported above, are a mixture of variance components and regression-like parameter estimates (Rcsn_dv, Int_DV and DepLag). Variance parameters show up in both tables. Regression-like parameters only show up in one. These different types of parameters have different uses. While the literature has not shown agreement on a procedure for creating a parsimonious model, stepwise logic has not been judged incorrect and can produce a model that predicts and is interesting.

For Regression-like parameters: If there are insignificant p-values, the variables should be eliminated, one variable at a time, in a stepwise fashion. The worst performing variable should be eliminated first.

For variance type parameters: Creating a parsimonious model involves two steps. The first step is to decide if the component of the model is time varying. The second step is to determine if is contributing. The starting assumption is that the components (Irregular, slope, cycle and season) are both time varying and significant. This model shows indications that these assumptions are not true (see bold below).

A component can be significant but not time varying. This means that the non-stochastic part of the component could be left in the model, but as deterministic contributor to Y (like parameters in a regression). A component of the model can not be time varying and “not significant”. A rough outline of the process for making the time varying parts of the model parsimonious is:



The output from our first model is:

Final Estimates of the Free Parameters					
Component	Parameter	Estimate	Std Error	Approx t Value	Approx Pr > t
Irregular	Error Variance	0.00001323	8.60115E-6	1.54	0.1241
Level	Error Variance	6.50277E-12	7.94696E-9	0.00	0.9993
Slope	Error Variance	0.00000483	1.41347E-6	3.42	0.0006
Season	Error Variance	0.00001169	2.76826E-6	4.22	<.0001
Cycle	Damping Factor	0.93881	0.04069	23.07	<.0001
Cycle	Period	16.60643	2.22289	7.47	<.0001
Cycle	Error Variance	7.006927E-7	6.11491E-7	1.15	0.2518
Rcsn_dv	Coefficient	3.55172	0.04018	88.40	<.0001
Int_DV	Coefficient	5.67481	0.06373	89.04	<.0001
DEPLAG	PHI_1	0.34053	0.02848	11.96	<.0001
Significance Analysis of Components (Based on the Final State)					
Component	DF	Chi-Square	Pr > ChiSq	Full Model	
Irregular	1	0.00	0.9868		
Level	1	37311.7	<.0001		
Slope	1	4073.99	<.0001		
Cycle	2	0.94	0.6241		
Season	11	113948	<.0001		

Step two would be to remove level (modify the model for the highest P value in the Free parameter table) **as a time varying component** of the model by adding the options Variance=0 and NoEst to the level statement. This option tells Proc UCM to start the model with a variance estimate equal to zero, and not to attempt to estimate a better value (fix the value at zero). The code for step two is below.

<pre>Proc UCM data=for_ucm PRINTALL ; id idmonth interval=month; model tot_sales=Rcsn_dv Int_DV; irregular ; level variance=0 Noest ; slope; cycle; SEASON LENGTH=12 TYPE=TRIG ; Deplag lags=1; estimate OUTEST=UCM_ESTIMATES; forecast lead=6 print=decomp OUTFOR=UCM_FORECASTS ;run;</pre>	<p>Printall turns on all printing options for the procedure. ID specifies a variable to be used as an identifier.</p> <p>The model statement says Y is total sales and is to be explained by a time series and two independent variables Irregular instructs SAS to include the error term (irregular term) ϵ_t in the model.</p> <p>The variance of this time varying component, level, has been assigned a starting estimate of at 0 with the NOEST option - to make level NOT time variant. NoEst tells SAS not to try to estimate the variance from the data.</p> <p>Slope can still be time varying.</p> <p>Cycle says include a cyclical component.</p> <p>The season statement and options instruct SAS to look for a 12-month cycle and not to use the dummy variable coding. The observed cyclical behavior seems to be too irregular for dummy variable coding.</p> <p>Deplags 1 instructs SAS to include $\phi_1 Y_{t-1}$ in the model. ϕ_1 will be estimated from the model.</p> <p>The estimate command tells SAS to estimate parameters and put them in a file called UCM_ESTIMATES.</p> <p>The forecast command tells SAS to forecast values for 6 periods, to calculate the components of the model and put them in a file called UCM_FORECASTS.</p>
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The SAS output is below:

Final Estimates of the Free Parameters					
Component	Parameter	Estimate	Std Error	Approx t Value	Approx Pr > t
Irregular	Error Variance	0.00001323	8.60123E-6	1.54	0.1241
Slope	Error Variance	0.00000483	1.4139E-6	3.42	0.0006
Season	Error Variance	0.00001169	2.76829E-6	4.22	<.0001
Cycle	Damping Factor	0.93881	0.04070	23.07	<.0001
Cycle	Period	16.60643	2.22861	7.45	<.0001
Cycle	Error Variance	7.006937E-7	6.11598E-7	1.15	0.2519
Rcsn_dv	Coefficient	3.55172	0.04018	88.40	<.0001
Int_DV	Coefficient	5.67481	0.06373	89.04	<.0001
DEPLAG	PHI_1	0.34053	0.02848	11.96	<.0001

Significance Analysis of Components (Based on the Final State)			
Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	0.00	0.9868
Level	1	37311.7	<.0001
Slope	1	4073.99	<.0001
Cycle	2	0.94	0.6241
Season	11	113948	<.0001

Model With Level variance=0

Step three would be to remove irregular (modify the model for the highest P value in the Free parameter table) as a time varying component of the model by adding the options Variance=0 and NoEst =(variance) to the cycle statement. It does not seem to be significant but, alternatively, it might be mis-specified.

The logic is that a mis-specified variable can look insignificant. Try to properly specify it (usually only two ways to specify it) before removing it from the model. The code for step three is below.

<pre>Proc UCM data=for_ucm PRINTALL ; id idmonth interval=month; model tot_sales=Rcsn_dv Int_DV ; irregular variance=0 noest; level variance=0 noest; slope; cycle; SEASON LENGTH=12 TYPE=TRIG ; deplag lags=1; estimate OUTEST=UCM_ESTIMATES; forecast lead=6 print=decomp OUTFOR=UCM_FORECASTS ; run;</pre>	<p>Printall turns on all printing options for the procedure</p> <p>ID specifies a variable to be used as an identifier</p> <p>Y is total sales and is to be explained by a time series and two independent variables</p> <p>Irregular instructs SAS to include the error term (irregular term) ϵ_t in the model</p> <p>The variance of this time varying component has been assigned a starting estimate of at 0 to make it NOT time variant. NoEst tells SAS not to try to estimate the variance from the data.</p> <p>Slope can still be time varying.</p> <p>Cycle has been commented out of the model.</p> <p>The season statement and options instruct SAS to look for a 12-month cycle and not to use the dummy variable coding. The observed cyclical behavior seems to be too irregular for dummy variable coding.</p> <p>Deplags 1 instructs SAS to include $\phi_1 Y_{t-1}$ in the model. ϕ_1 will be estimated from the model.</p> <p>The estimate command tells SAS to estimate parameters and put them in a file called UCM_ESTIMATES.</p> <p>The forecast command tells SAS to forecast values for 6 periods, to calculate the components of the model and put them in a file called UCM_FORECASTS.</p>
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Final Estimates of the Free Parameters					
Component	Parameter	Estimate	Std Error	Approx t Value	Approx Pr > t
Slope	Error Variance	0.00000369	9.5664E-7	3.86	0.0001
Season	Error Variance	0.00000961	2.2805E-6	4.21	<.0001
Cycle	Damping Factor	0.97764	0.01071	91.25	<.0001
Cycle	Period	6.61696	0.05084	130.15	<.0001
Cycle	Error Variance	0.00000222	1.50905E-6	1.47	0.1411
Rcsn_dv	Coefficient	3.46256	0.03653	94.78	<.0001
Int_DV	Coefficient	5.68550	0.05788	98.23	<.0001
DepLag	Phi_1	0.36535	0.02035	17.96	<.0001

Significance Analysis of Components (Based on the Final State)			
Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	.	.
Level	1	40450.6	<.0001
Slope	1	4842.97	<.0001
Cycle	2	0.08	0.9599
Season	11	127708	<.0001

Model With:
 Level Variance=0
 Irregular Variance=0

Step four would be to remove cycle (modify the model for the highest P value in the Free parameter table) as a time varying component of the model by adding the options Variance=0 and NoEst =(variance) to the cycle statement. It does not seem to be significant but, alternatively, it might be mis-specified.

The logic is that a mis-specified variable can look insignificant. Try to properly specify it (usually only two ways to specify it) before removing it from the model. The code for step three is below.

<pre>Proc UCM data=for_ucm PRINTALL ; id idmonth interval=month; model tot_sales=Rcsn_dv Int_DV ; irregular variance=0 noest; level variance=0 noest; slope; *cycle; SEASON LENGTH=12 TYPE=TRIG ; deplag lags=1; estimate OUTEST=UCM_ESTIMATES; forecast lead=6 print=decomp OUTFOR=UCM_FORECASTS ; run;</pre>	<p>Printall turns on all printing options for the procedure ID specifies a variable to be used as an identifier Y is total sales and is to be explained by a time series and two independent variables Irregular instructs SAS to include the error term (irregular term) ϵ_t in the model The variance of this time varying component has been assigned a starting estimate of at 0 to make it NOT time variant. NoEst tells SAS not to try to estimate the variance from the data. Slope can still be time varying. Cycle has been commented out of the model. The season statement and options instruct SAS to look for a 12-month cycle and not to use the dummy variable coding. The observed cyclical behavior seems to be too irregular for dummy variable coding. Deplags 1 instructs SAS to include $\phi_1 Y_{t-1}$ in the model. ϕ_1 will be estimated from the model. The estimate command tells SAS to estimate parameters and put them in a file called UCM_ESTIMATES. The forecast command tells SAS to forecast values for 6 periods, to calculate the components of the model and put them in a file called UCM_FORECASTS.</p>
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The SAS output is below:

Final Estimates of the Free Parameters

Final Estimates of the Free Parameters

Component	Parameter	Estimate	Std Error	Approx t Value	Approx Pr > t
Slope	Error Variance	0.00000676	1.79115E-6	3.78	0.0002
Season	Error Variance	0.00001586	3.74634E-6	4.23	<.0001
Rcsn_dv	Coefficient	3.63449	0.04622	78.63	<.0001
Int_DV	Coefficient	5.81361	0.07322	79.40	<.0001
DEPLAG	PHI_1	0.34204	0.02439	14.02	<.0001

Significance Analysis of Components

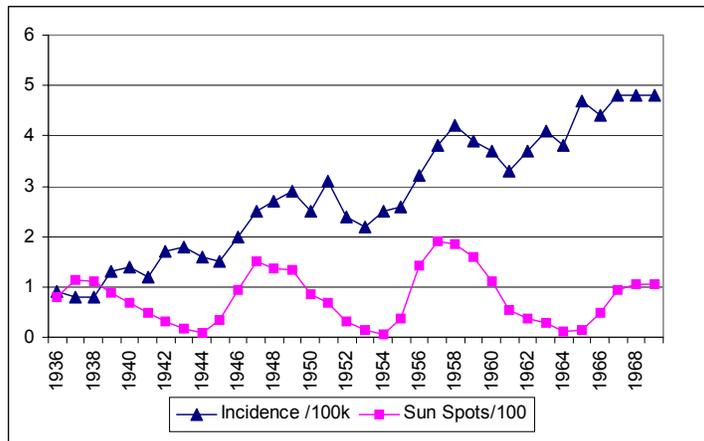
(Based on the Final State)

Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	.	.
Level	1	27514.0	<.0001
Slope	1	2944.99	<.0001
Season	11	89956.5	<.0001

All of these variables are significant, in both tables, though there might well be better models. Cycle was originally put in the model in hopes that it would automatically model the effects of the three different cycles. It did not end up as a strong predictor.

PROJECT 2 TAKEN FROM SAS DOCUMENTATION

The goal of this project is to model some cancer data from the Connecticut Tumor registry as reported by Houghton, Flannery and Viola in the International Journal of Cancer. They calculated the age-adjusted incidence of Melanoma per 100,000 subjects. This data, and sunspot data, is shown in the graph below. The original article did not make any link to sunspot activity (that idea is from a helpful SAS, Inc. employee).



The code for step 1 is

The plot suggests a cycle or seasonality. We will let the data tell us about the cycle.

```
proc ucm data = both;
  id year interval = year ;
  model Incidences ;
  irregular ;
  level variance=0 noest ;
  slope variance=0 noest ;
  cycle plot=smooth ;
  estimate plot=(residual normal acf);
  forecast lead=10 plot=(decomp forecasts); run ;
```

Output is shown below

Final Estimates of the Free Parameters

Component	Parameter	Estimate	Std Error	Approx t Value	Approx Pr > t
Irregular	Error Variance	0.05706	0.01750	3.26	0.0011
Cycle	Damping Factor	0.96476	0.04857	19.86	<.0001
Cycle	Period	9.68327	0.62860	15.40	<.0001
Cycle	Error Variance	0.00302	0.0022975	1.31	0.1893

Significance Analysis of Components

(Based on the Final State)

Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	0.03	0.8698
Level	1	3097.46	<.0001
Slope	1	694.83	<.0001
Cycle	2	2.54	0.2810

Name	Type	Period	Frequency	Damping Factor	Final Amplitude	%Relative Cycle to Level Variance
Cycle	Stationary	9.68327	0.64887	0.96476	0.20439	4.29794 0.04356

Cycle is a poor performer, but the plot really *seems* to show cycles. The old model vs. theory issue arises if we posit a sun connection. While cycle performs poorly, a 9.68327 -year cycle suggests sunspots (to the smart folks at SAS- not to the original authors).

Step 2 is shown below

Sunspot data is available on the web. Get it and create a SAS data set. Use Sunspots and an independent variable in the model.

```
proc ucm data = both;
  id year interval = year ;
  model Incidences = sunspots ;
  irregular ;
  level variance=0 noest ;
  slope variance=0 noest ;
  cycle plot=smooth ;
  estimate plot=(residual normal acf);
  forecast lead=10 plot=(decomp forecasts); run ;
```

Output is shown below.

Final Estimates of the Free Parameters

Component	Parameter	Estimate	Std Error	Approx t Value	Approx Pr > t
Irregular	Error Variance	0.11227	0.02807	4.00	<.0001
Cycle	Damping Factor	0.99999	0.0015394	649.60	<.0001
Cycle	Period	2.60497	0.13294	19.59	<.0001
Cycle	Error Variance	4.91638E-8	1.94826E-7	0.25	0.8008
sunspots	Coefficient	0.00084084	0.0010898	0.77	0.4404

Significance Analysis of Components
(Based on the Final State)

Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	0.15	0.6942
Level	1	1170.86	<.0001
Slope	1	456.73	<.0001
Cycle	2	0.69	0.7075

Name	Type	Period	Dampening Frequency	Final Factor	Amplitude	%Relative to Level Variance	Cycle
Cycle	Stationary	2.60497	2.41200	0.99999	0.03292	0.69612	0.00211

With sunspots in the model, cycle is a very poor performer, but the variable sunspots performs poorly as well. Looking at the picture at the start, we can see multicollinearity. Note that the cycle period (2,60497) does not seem to “agree” with the observed cycle period in the plot.

The code for step 3 is shown below

Follow theory. Keep sunspots in the model and lag them

Lag them by two years (or one...or three). Create the lagged variable in a data step (not shown).
Folks at SAS found lag two to work well and I will follow their advice.

Comment cycle out of the model.

```
ods html ;
ods graphics on ;
  proc ucm data = both;
  id year interval = year ;
  model Incidences =lag2SP;
  irregular ;
  level variance=0 noest ;
  slope variance=0 noest ;
  ****cycle plot=smooth ;
  estimate plot=(residual normal acf);
  forecast lead=10 plot=(decomp forecasts);
run ;
ods graphics off ;
ods html close ;
```

Output is shown below

Final Estimates of the Free Parameters

Component	Parameter	Estimate	Std Error	Approx t Value	Approx Pr > t
Irregular	Error Variance	0.06283	0.01524	4.12	<.0001
lag2SP	Coefficient	0.00429	0.0007966	5.39	<.0001

Significance Analysis of Components

(Based on the Final State)

Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	0.73	0.3937
Level	1	1731.59	<.0001
Slope	1	764.32	<.0001

Components of model are strong. All P values are very small. There is a strong sunspot link.

With the code above, SAS also gives lots of very interesting and useful output. The output from the final model is shown below.

Input Data Set	
Name	WORK.BOTH
Time ID Variable	year

Estimation Span Summary									
Variable	Type	First Obs	Last Obs	NObs	NMiss	Min	Max	Mean	Standard Deviation
Incidences	Dependent	1936	1972	37	0	0.80000	4.80000	2.80811	1.23904
lag2SP	Predictor	1936	1972	37	0	4.40000	190.20000	75.35135	52.88085

Forecast Span Summary									
Variable	Type	First Obs	Last Obs	NObs	NMiss	Min	Max	Mean	Standard Deviation
Incidences	Dependent	1936	1972	37	0	0.80000	4.80000	2.80811	1.23904
lag2SP	Predictor	1936	1972	37	0	4.40000	190.20000	75.35135	52.88085

Fixed Parameters in the Model		
Component	Parameter	Value
Level	Error Variance	0
Slope	Error Variance	0

Preliminary Estimates of the Free Parameters		
Component	Parameter	Estimate
Irregular	Error Variance	3.41052

Likelihood Based Fit Statistics	
Full Log-Likelihood	-15.04479
Diffuse Part of Log-Likelihood	-2.14211
Normalized Residual Sum of Squares	34.00000
Akaike Information Criterion	38.08957
Bayesian Information Criterion	44.53324
Number of non-missing observations used for computing the log-likelihood = 37	

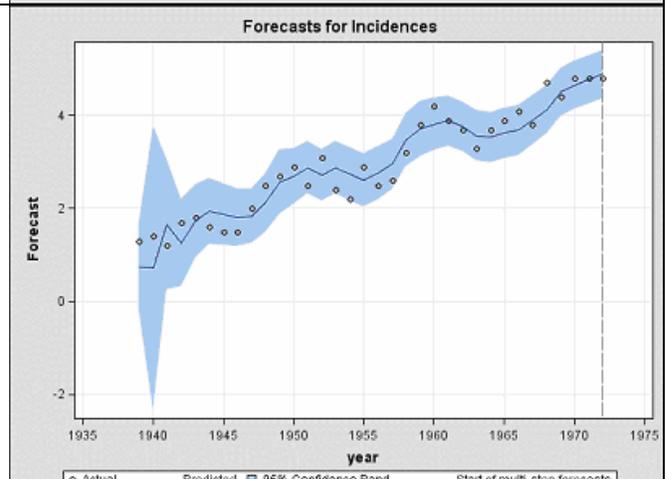
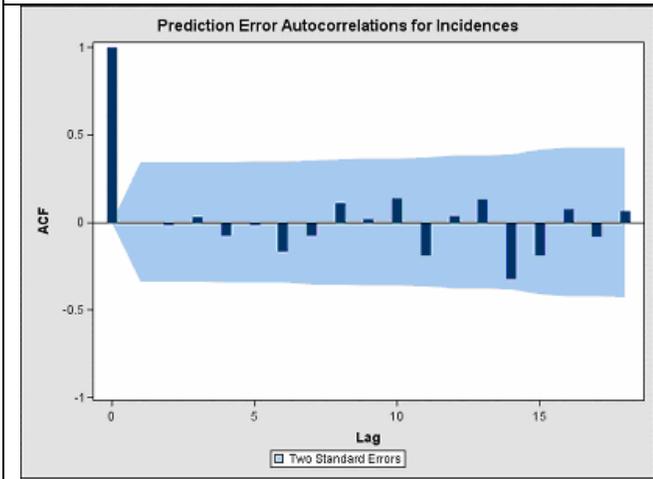
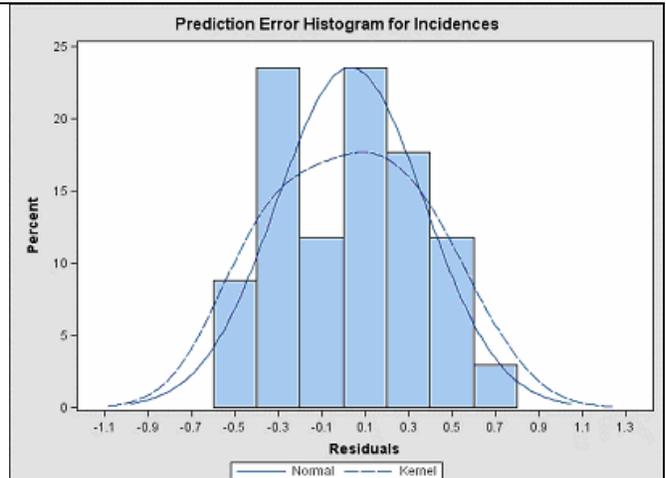
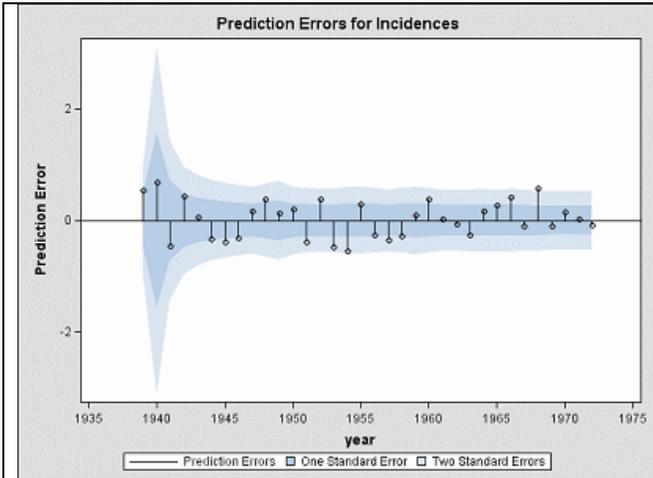
Likelihood Optimization Algorithm Converged in 7 Iterations.

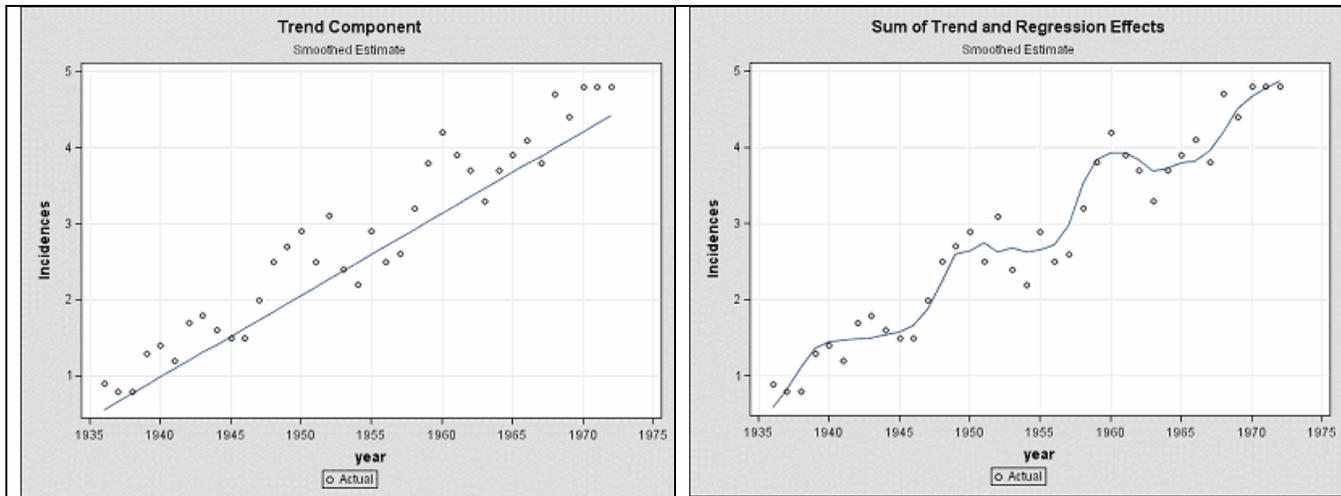
Final Estimates of the Free Parameters					
Component	Parameter	Estimate	Approx Std Error	t Value	Approx Pr > t
Irregular	Error Variance	0.06283	0.01524	4.12	<.0001
lag2SP	Coefficient	0.00429	0.0007966	5.39	<.0001

Fit Statistics Based on Residuals

Mean Squared Error	0.11260
Root Mean Squared Error	0.33556
Mean Absolute Percentage Error	13.00476
Maximum Percent Error	48.79725
R-Square	0.90998
Adjusted R-Square	0.90998
Random Walk R-Square	0.38472
Amemiya's Adjusted R-Square	0.90452
Number of non-missing residuals used for computing the fit statistics = 34	

Significance Analysis of Components (Based on the Final State)			
Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	0.73	0.3937
Level	1	1731.59	<.0001
Slope	1	764.32	<.0001





CONCLUSIONS

Proc UCM, with some study, can be a strong competitor for ARIMA modeling. It is easy to code and has lots of very attractive and useful output.

While the procedure for creating a parsimonious model is not well defined, there is hope that logical playing with the model will result in models with few components and predictive power.

REFERENCES

Houghton, A., Flannery, N, and Viola.V.M. (1980). "Malignant Melanoma in Connecticut", International Journal of Cancer, 25, 95-114

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CONTACT INFORMATION

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```

**the Sun_glasses project*****
data for_ucm;
retain  retail_trend
        Constr_trend
        AntA_trend  ;

infile datalines firstobs=6 missover;
input @1 idmonth monyy7. @9 month 2.0 Rcsn_dv Int_DV ret_rec_fct constr_rec_fct AntA_rec_fct;

if _n_=1 then
    do; /*initialize*/
        retail_trend=0;
        Constr_trend=0;
        AntA_trend =0;
    end;

*Explanation of the recession factors below: 1=good times 0=no sales at all;
*RECESSION FACTORS COULDE BE NAMED BETTER.  THEY MODIFY THE CYCLE AND THE TRAND;

*** TREND SECTION ****;
*calculate trend components;
*there is a constant trend and a modifier to produce recessions and no sales;
*add a bit to last quarters data- we retained it;
retail_trend =retail_trend + (.15 *ret_rec_fct );/*if factor is close to 0, little change*/
Constr_trend =constr_trend + (.20 *constr_rec_fct);
AntA_trend  =AntA_trend  + (.10 *AntA_rec_fct );

** CYCLE SHAPE SECTION ****;
*calculate cyclical components - this section sets the shape of the cycle;
*domestic retail cycle;
if      mod(month,12) = (7)  then retail_cycle= .9 *ret_rec_fct; /*vacation sales*/
else if mod(month,12) = (8)  then retail_cycle= 1.1*ret_rec_fct;
else if mod(month,12) = (9)  then retail_cycle= .3 *ret_rec_fct; /*poser/school sales*/
else if mod(month,12) = (11) then retail_cycle=-.2 *ret_rec_fct; /*winter sales downturn*/
else if mod(month,12) = (12) then retail_cycle= .7 *ret_rec_fct; /*holiday sales*/
else if mod(month,12) = (1)  then retail_cycle=-.7 *ret_rec_fct;
else if mod(month,12) = (2)  then retail_cycle=-.9 *ret_rec_fct;
else if mod(month,12) = (3)  then retail_cycle=-.3 *ret_rec_fct;
else retail_cycle =0;

*domestic construction cycle ;
if      mod(month,12) = (7)  then constr_cycle=.7 *constr_rec_fct; /*START OF WORK PERIOD*/
else if mod(month,12) = (8)  then constr_cycle=.9 *constr_rec_fct;
else if mod(month,12) = (9)  then constr_cycle=.3 *constr_rec_fct; /*replacing lost items*/
else if mod(month,12) = (11) then constr_cycle=-.3*constr_rec_fct;
/*holidays*/
else if mod(month,12) = (1)  then constr_cycle=-.7*constr_rec_fct; /* less work in winter*/
else if mod(month,12) = (2)  then constr_cycle=-.9*constr_rec_fct;
else if mod(month,12) = (3)  then constr_cycle=-.3*constr_rec_fct;
else constr_cycle =0;

*Antarctic sales - Sun from Sept to March - people buy early protection is important;
if      mod(month,12) = (9)  then AntA_cycle= 2 *AntA_rec_fct; /*START OF WORK PERIOD*/
else if mod(month,12) = (10) then AntA_cycle= 1 *AntA_rec_fct;
else if mod(month,12) = (11) then AntA_cycle= 1 *AntA_rec_fct;
else if mod(month,12) = (12) then AntA_cycle=-.1 *AntA_rec_fct;
else if mod(month,12) = (1)  then AntA_cycle=-.5 *AntA_rec_fct; /*replacing lost items*/
else if mod(month,12) = (2)  then AntA_cycle=-.7 *AntA_rec_fct;
else if mod(month,12) = (3)  then AntA_cycle=-.2 *AntA_rec_fct;
else AntA_cycle =0;

** EXPAND OR CONTRACT THE CYCLE SECTION **;
*MODIFY cyclical components - this section sets the magnitude of the cycle;
* this section is just to make for easy changes in the curves;
retail_cycle =retail_cycle *3 ; *set to three to make it show on plot;
constr_cycle =constr_cycle *2 ;
AntA_cycle   =AntA_cycle   *1 ;

*ADD UP COMPONENTS SECTION;

```

```

tot_sales =sum(retail_trend,retail_cycle,constr_trend,constr_cycle,AntA_trend,AntA_cycle) ;
sumtrend  =sum(retail_trend,constr_trend,AntA_trend) ;
sumCyc    =sum(retail_cycle,constr_cycle,AntA_cycle) ;

** blank Y values for last 6 months so that they can be c=forcasted by this proc;
if month GE 73 then do;
    tot_sales=.;
    sumtrend =.;
    sumCyc=.;
    end;

datalines;
this is monthly data
--
Explanation of the recession factors below: 1=good times 0=no sales at all
--
Cmnth mnth Rcsn_dv Int_DV ret_rec_fct constr_rec_fct AntA_rec_fct
Jan2000 01 0 0 .5 1 0 /*good times :-)*/*
Feb2000 02 0 0 .5 1 0 /*selling to Us sports and construction*/
Mar2000 03 0 0 .5 1 0
Apr2000 04 0 0 .5 1 0
May2000 05 0 0 .5 1 0
Jun2000 06 0 0 .5 1 0
Jul2000 07 0 0 .5 1 0
Aug2000 08 0 0 .5 1 0
Sep2000 09 0 0 .5 1 0
Oct2000 10 0 0 .5 1 0
Nov2000 11 0 0 .5 1 0
Dec2000 12 1 0 .02 .3 0 /*start of 12 month recession :-( */
Jan2001 13 1 0 .02 .3 0 /* 1 IN Rcsn_dv INDICATES A RECESSION */
Feb2001 14 1 0 .02 .3 0
Mar2001 15 1 0 .02 .3 0
Apr2001 16 1 0 .02 .3 0
May2001 17 1 0 .02 .3 0
Jun2001 18 1 0 .02 .3 0
Jul2001 19 1 0 .02 .3 0
Aug2001 20 1 0 .02 .3 0
Sep2001 21 1 0 .02 .3 0
Oct2001 22 1 0 .02 .3 0
Nov2001 23 1 0 .02 .3 0 /*end of 12 month recession :-) */
Dec2001 24 0 1 .5 1 2 /*Enter international sports market*/
Jan2002 25 0 1 .5 1 2 /*1 IN Int_DV SHOWS INTERNAIONAL SALES*/
Feb2002 26 0 1 .5 1 2
Mar2002 27 0 1 .5 1 2
Apr2002 28 0 1 .5 1 2
May2002 29 0 1 .5 1 2
Jun2002 30 0 1 .5 1 2
Jul2002 31 0 1 .5 1 2
Aug2002 32 0 1 .5 1 2
Sep2002 33 0 1 .5 1 2
Oct2002 34 0 1 .5 1 2
Nov2002 35 0 1 .5 1 2
Dec2002 36 0 1 .5 1 2
Jan2003 37 0 1 .5 1 2
Feb2003 38 0 1 .5 1 2
Mar2003 39 0 1 .5 1 2
Apr2003 40 0 1 .5 1 2
May2003 41 0 1 .5 1 2
Jun2003 42 0 1 .5 1 2
Jul2003 43 0 1 .5 1 2
Aug2003 44 0 1 .5 1 2
Sep2003 45 0 1 .5 1 2
Oct2003 46 0 1 .5 1 2
Nov2003 47 0 1 .5 1 2
Dec2003 48 0 1 .5 1 2
Jan2004 49 0 1 .5 1 2
Feb2004 50 0 1 .5 1 2
Mar2004 51 0 1 .5 1 2
Apr2004 52 0 1 .5 1 2
May2004 53 0 1 .5 1 2
Jun2004 54 0 1 .5 1 2
Jul2004 55 0 1 .5 1 2
Aug2004 56 0 1 .5 1 2

```

```

Sep2004 57 0 1 .5 1 2
Oct2004 58 0 1 .5 1 2
Nov2004 59 0 1 .5 1 2
Dec2004 60 0 1 .5 1 2
Jan2005 61 0 1 .5 1 2
Feb2005 62 0 1 .5 1 2
Mar2005 63 0 1 .5 1 2
Apr2005 64 0 1 .5 1 2
May2005 65 0 1 .5 1 2
Jun2005 66 0 1 .5 1 2
Jul2005 67 0 1 .5 1 2
Aug2005 68 0 1 .5 1 2
Sep2005 69 0 1 .5 1 2
Oct2005 70 0 1 .5 1 2
Nov2005 71 0 1 .5 1 2
Dec2005 72 0 1 .5 1 2
Jan2006 73 0 1 .5 1 2 /*blank calculations and forecast w/ UCM*/
Feb2006 74 0 1 .5 1 2 /*blank calculations and forecast w/ UCM*/
Mar2006 75 0 1 .5 1 2 /*blank calculations and forecast w/ UCM*/
Apr2006 76 0 1 .5 1 2 /*blank calculations and forecast w/ UCM*/
May2006 77 0 1 .5 1 2 /*blank calculations and forecast w/ UCM*/
Jun2006 78 0 1 .5 1 2 /*blank calculations and forecast w/ UCM*/
;
run;

**set some display options;
symbol1 color=red
        interpol=join
        value=dot
        height=1;
run;

symbol2 color=green
        interpol=join
        value=square
        height=1;
run;

symbol3 color=blue
        interpol=join
        value=triangle
        height=1;

symbol4 color=black
        interpol=join
        value=star
        height=2;
run;

legend1 label=none
        shape=symbol(4,2)
        position=(top center inside)
        mode=share;

**plotting interesting stuff;
proc gplot data=for_ucm;
title "Show Total Sales and the Sum of the Trends";
plot tot_sales *month
      sumtrend *month /overlay legend=legend1 href=12 24;
run;
quit;

proc gplot data=for_ucm; *trends;
title "Show Cuml sales dut to the trend components";
plot retail trend *month
      Constr_trend*month
      AntA_trend *month /overlay legend=legend1 href=12 24 ;

```

```

run;
quit;

proc gplot data=for_ucm;
title "Show the monthly Cycle components and the sum of the cycles";
plot sumCyc *month
      retail cycle *month
      constr cycle *month
      AntA_cycle *month /overlay legend=legend1 href=12 24;
run;
quit;

proc gplot data=for_ucm;
title "Show the Cycle Component";
plot sumCyc*month / legend=legend1 href=12 24;
run;
quit;

title "";

***from paper step 1 initial model****;

id idmonth interval=month;
model tot_sales = Rcsn_dv Int_DV ;
irregular ;

level ;
slope;

cycle;

SEASON LENGTH=12
      TYPE=TRIG ;
deplag lags=1;

estimate
      OUTEST=UCM_ESTIMATES;
forecast lead=6
      print=decomp
      OUTFOR=UCM_FORECASTS ;
run;

***from paper step 2 variance of level is set to 0****;
Proc UCM data=for_ucm PRINTALL ;
id idmonth interval=month;
model tot_sales = Rcsn_dv Int_DV ;
irregular ;

level variance=0 noest;
slope;

cycle;

SEASON LENGTH=12
      TYPE=TRIG ;
deplag lags=1;

estimate
      OUTEST=UCM_ESTIMATES;
forecast lead=6
      print=decomp
      OUTFOR=UCM_FORECASTS ;
run;

***from paper step 3 set irregular to zero****;

id idmonth interval=month;
model tot_sales = Rcsn_dv Int_DV ;
irregular variance=0 noest;

level variance=0 noest;

```

```

slope;

cycle;

SEASON LENGTH=12
      TYPE=TRIG ;
deplag lags=1;

estimate
      OUTEST=UCM_ESTIMATES;
forecast lead=6
      print=decomp
      OUTFOR=UCM_FORECASTS ;

***from paper step 4 get rid of cycle****;

id idmonth interval=month;
model tot_sales = Rcsn_dv Int_DV ;
irregular variance=0 noest;

level variance=0 noest;
slope;

*cycle;

SEASON LENGTH=12
      TYPE=TRIG ;
deplag lags=1;

estimate
      OUTEST=UCM_ESTIMATES;
forecast lead=6
      print=decomp
      OUTFOR=UCM_FORECASTS ;

*****sunspot project *****;

data melanoma ;
input Incidences @@ ;
year = intnx('year', '1jan1936'd, _n_-1) ;
*year=1936+_n_-1;
label Incidences = 'Adg. Inc/100k';
datalines ;
    0.9 0.8 0.8 1.3 1.4 1.2 1.7 1.8 1.6 1.5
    1.5 2.0 2.5 2.7 2.9 2.5 3.1 2.4 2.2 2.9
    2.5 2.6 3.2 3.8 4.2 3.9 3.7 3.3 3.7 3.9
    4.1 3.8 4.7 4.4 4.8 4.8 4.8
;

Proc sort data=melanoma;
by year;

data sunspots;
infile datalines missover;
input @1year @10 sunspots ;
      year = intnx('year', '1jan1931'd, _n_-1) ;
      format year year4. ;

/*
http://ww1.physik.tu-muenchen.de/~gammel/matpack/html/Astronomy/Sunspots.html#yearly\_data
1.2.2 Yearly Data
In the table below the yearly sunspot counts from 1700 to 1992 can be found.

```

Each M marks a sunspot cycle maximum and each m a minimum.
 Through 1944 yearly means were calculated as the average of the 12 monthly means.
 Since 1945 yearly means have been calculated as the average of the daily means.

```
Year Number      Year Number      Year Number      Year Number      Year Number
*/
```

```
datalines;
1931      21.2
1932      11.1
1933       5.7
1934       8.7
1935      36.1
1936      79.7
1937     114.4
1938     109.6
1939      88.8
1940      67.8
1941      47.5
1942      30.6
1943      16.3
1944       9.6
1945      33.2
1946      92.6
1947     151.6
1948     136.3
1949     134.7
1950      83.9
1951      69.4
1952      31.5
1953      13.9
1954       4.4
1955      38.0
1956     141.7
1957     190.2
1958     184.8
1959     159.0
1960     112.3
1961      53.9
1962      37.6
1963      27.9
1964      10.2
1965      15.1
1966      47.0
1967      93.8
1968     105.9
1969     105.5
1970     104.5
1971      66.6
1972      68.9
1973      38.0
1974      34.5
1975      15.5
1976      12.6
1977      27.5
1978      92.5
1979     155.4
1980     154.6
1981     140.4
1982     115.9
1983      66.6
1984      45.9
1985      17.9
1986      13.4
1987      29.4
1988     100.2
1989     157.6
1990     142.6
1991     145.7
1992      94.3
;
run;
proc sort data=sunspots;
by year;

data both;
```

```
merge sunspots melanoma(in=M) ;
by year;
smallsun=sunspots/100; * for plotting;
lag2SP=lag2(sunspots);
if M;
run;

ods html ;
ods graphics on ;
proc ucm data = both;
  id year interval = year ;
  model Incidences ;
  irregular ;
  level variance=0 noest ;
  slope variance=0 noest ;
  cycle plot=smooth ;
  estimate plot=(residual normal acf);
  forecast lead=10 plot=(decomp forecasts);
run ;

proc ucm data = both;
  id year interval = year ;
  model Incidences = sunspots ;
  irregular ;
  level variance=0 noest ;
  slope variance=0 noest ;
  cycle plot=smooth ;
  estimate plot=(residual normal acf);
  forecast lead=10 plot=(decomp forecasts);
run ;

ods html ;
ods graphics on ;

proc ucm data = both;
  id year interval = year ;
  model Incidences =lag2SP;
  irregular ;
  level variance=0 noest ;
  slope variance=0 noest ;
  *cycle plot=smooth ;
  estimate plot=(residual normal acf);
  forecast lead=10 plot=(decomp forecasts);
run ;

ods graphics off ;
ods html close ;
```