INTRODUCTION
TO PARTICLE PHYSICS

From atoms to quarks

An elementary historical review of concepts, discoveries and achievements

Recommended reading:

D.H. Perkins, *Introduction to High Energy Physics*
F.E. Close, *The cosmic onion*

Luigi DiLella, Summer Student Program 2004

The “elementary particles” in the 19th century:

The Atoms of the 92 Elements

1. Hydrogen — Mass $M_H \approx 1.7 \times 10^{-24}$ g
2. Helium
3. Lithium

....

....

92. Uranium — Mass $\approx 238\, M_H$

increasing mass

Estimate of a typical atomic radius

Number of atoms /cm^3: $n = \frac{N_A}{A}\rho$

$N_A \approx 6 \times 10^{23}$ mol^{-1} (Avogadro costant)

A: molar mass

ρ: density

Atomic volume: $V = \frac{4}{3}\pi R^3$

Packing fraction: $f \approx 0.52$ — 0.74

$$\Rightarrow \quad R = \left(\frac{3f}{4\pi n}\right)^{1/3}$$

Example: Iron (A = 55.8 g; ρ = 7.87 g cm^{-3})

$$\boldsymbol{R = (1.1 — 1.3) \times 10^{-8}\ \text{cm}}$$

1894 – 1897: Discovery of the electron

J.J. Thomson

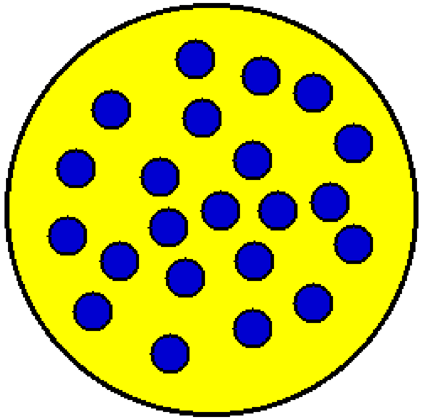
Study of "cathode rays": electric current in tubes at very low gas pressure ("glow discharge")

Measurement of the electron mass: $m_e \approx M_H/1836$

"Could anything at first sight seem more impractical than a body which is so small that its mass is an insignificant fraction of the mass of an atom of hydrogen?" (J.J. Thomson)

➡ ATOMS ARE NOT ELEMENTARY

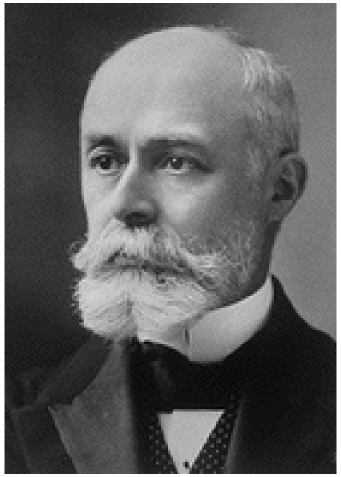
Thomson's atomic model:



- Electrically charged sphere
- Radius ~ 10^{-8} cm
- Positive electric charge
- Electrons with negative electric charge embedded in the sphere

1896: Discovery of natural radioactivity

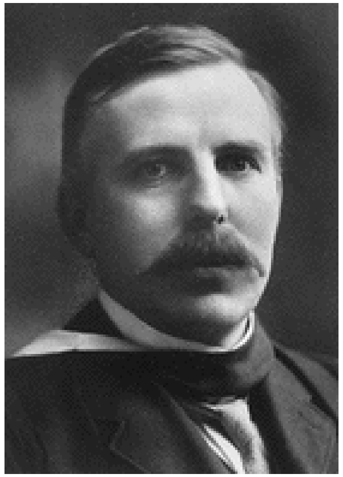
(Henri Becquerel)



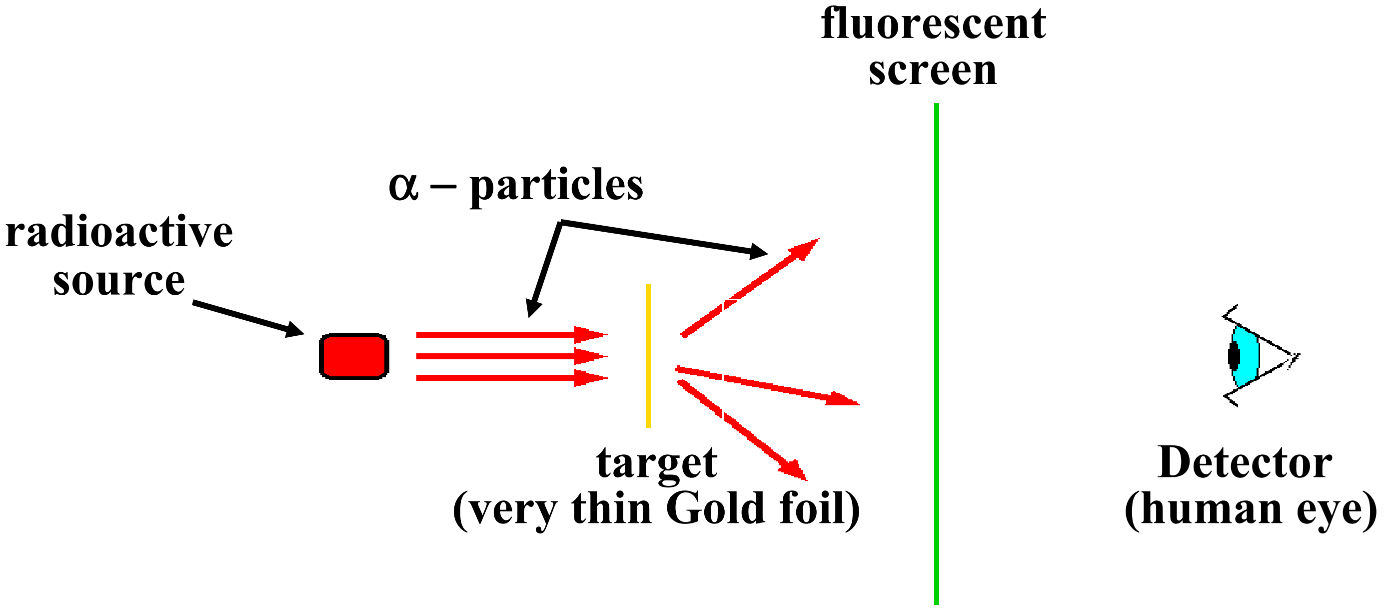
Henri Becquerel

1909 – 13: Rutherford's scattering experiments

Discovery of the atomic nucleus



Ernest Rutherford



fluorescent screen

α – particles

radioactive source

target (very thin Gold foil)

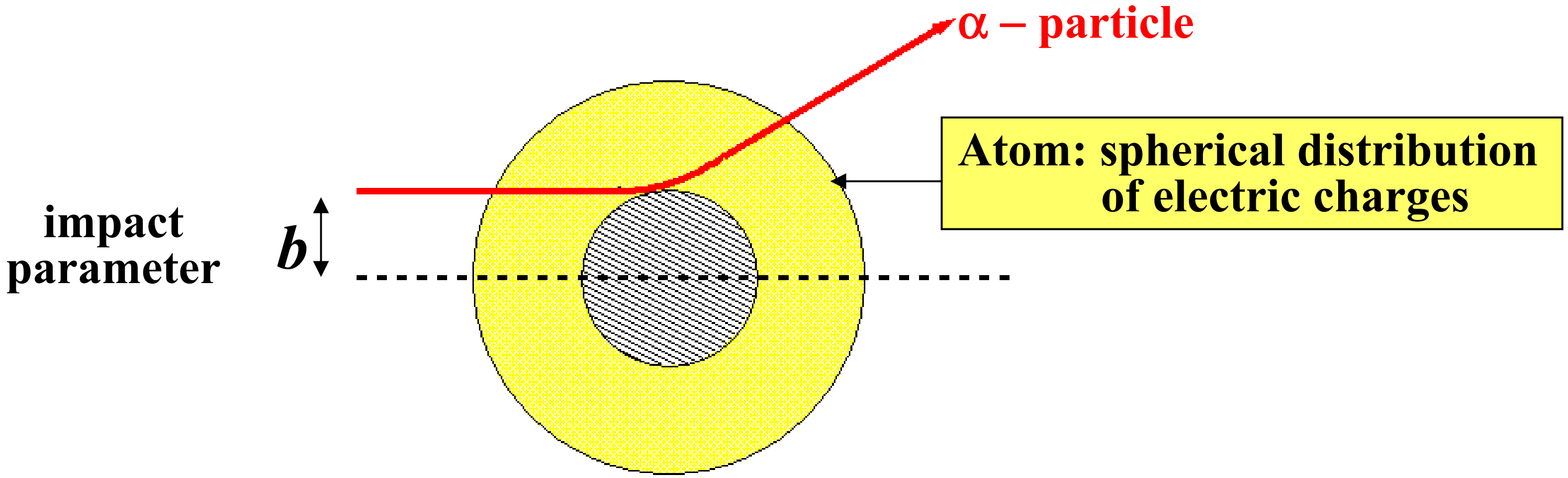
Detector (human eye)

α – particles : nuclei of Helium atoms spontaneously emitted by heavy radioactive isotopes

Typical α – particle velocity $\approx 0.05\,c$ (*c* : speed of light)

Expectations for α – atom scattering

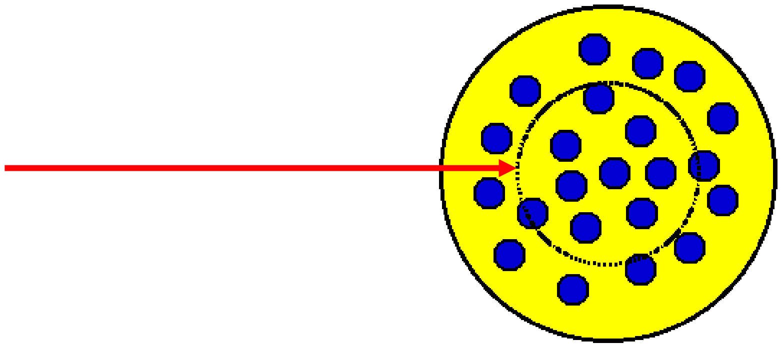
α – atom scattering at low energies is dominated by Coulomb interaction



Atom: spherical distribution of electric charges

α – particles with impact parameter = *b* "see" only electric charge within sphere of radius = *b* (Gauss theorem for forces proportional to r^{-2})

For Thomson's atomic model the electric charge "seen" by the α – particle is zero, independent of impact parameter

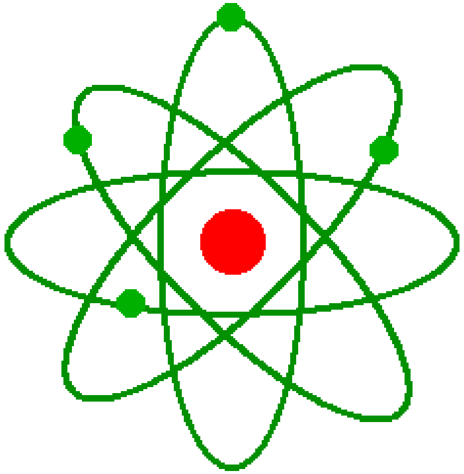


⇒ no significant scattering at large angles is expected

Rutherford's observation:

significant scattering of α – particles at large angles, consistent with scattering expected for a sphere of radius $\approx$ few x 10^{-13} cm and electric charge = Ze, with $Z = 79$ (atomic number of gold) and e = |charge of the electron|

→ **an atom consists of**
a positively charged nucleus
surrounded by a cloud of electrons



Nuclear radius $\approx 10^{-13}$ cm $\approx 10^{-5}$ x atomic radius

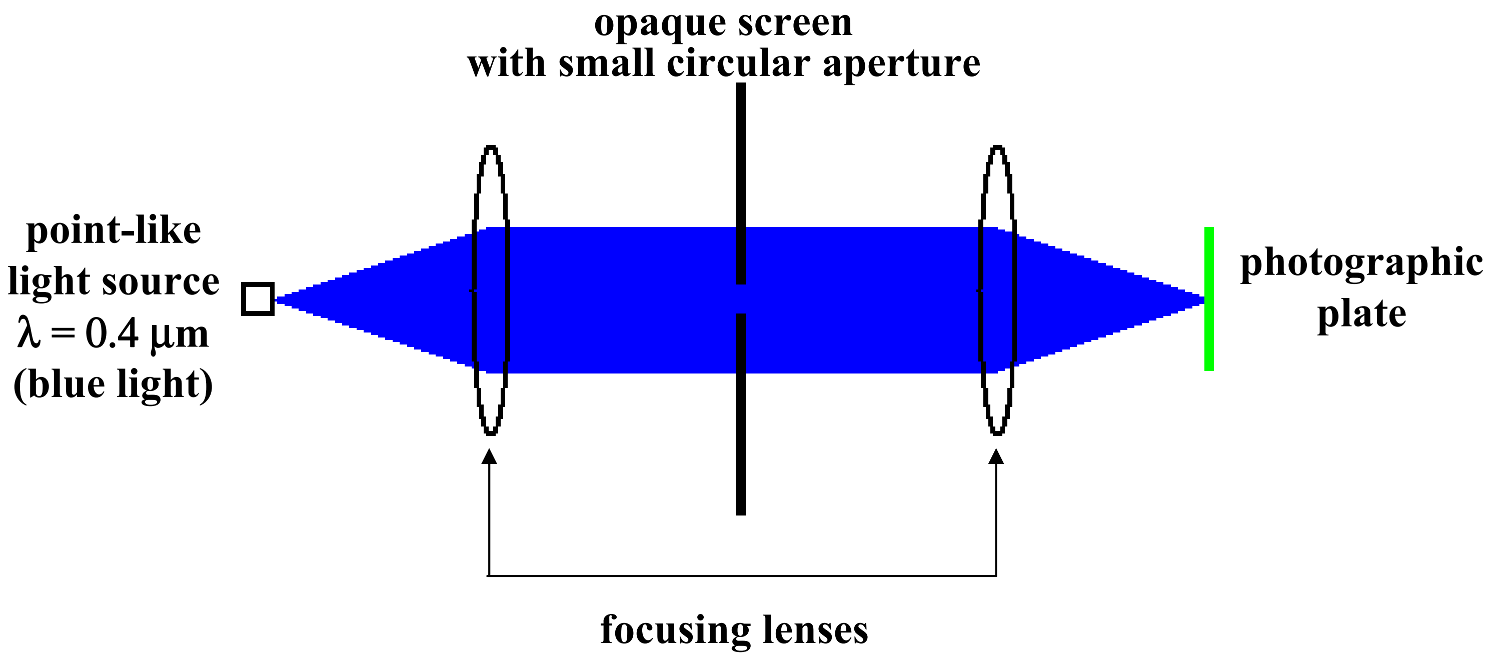
Mass of the nucleus $\approx$ mass of the atom
(to a fraction of 1‰)

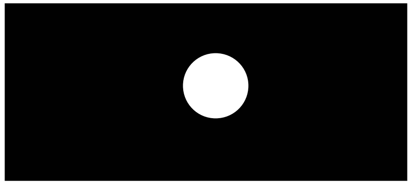
Two questions:

- Why did Rutherford need α – particles to discover the atomic nucleus?
- Why do we need huge accelerators to study particle physics today?

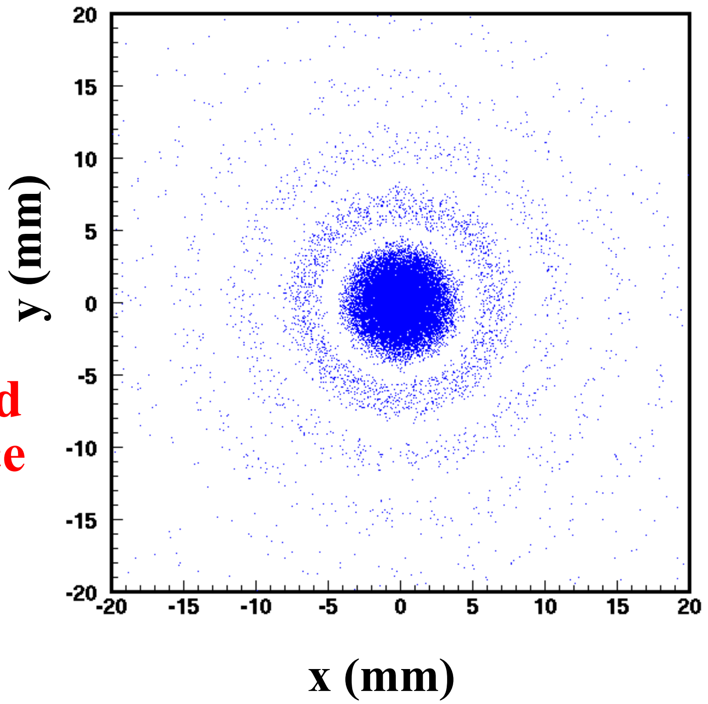
Answer to both questions from basic principles of Quantum Mechanics

Observation of very small objects using visible light





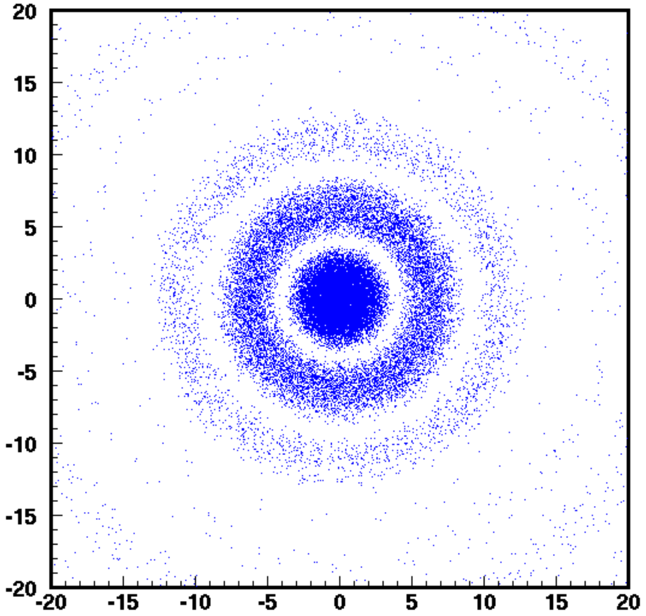
Aperture diameter: $D = 20\ \mu\text{m}$
Focal length: 20 cm

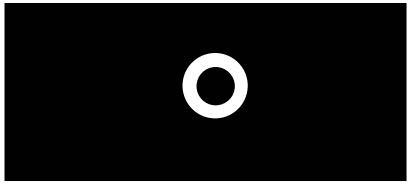


Observation of light diffraction, interpreted as evidence that light consists of waves since the end of the 17th century

Angular aperture of the first circle (before focusing):

$$\alpha = 1.22\, \lambda / D$$



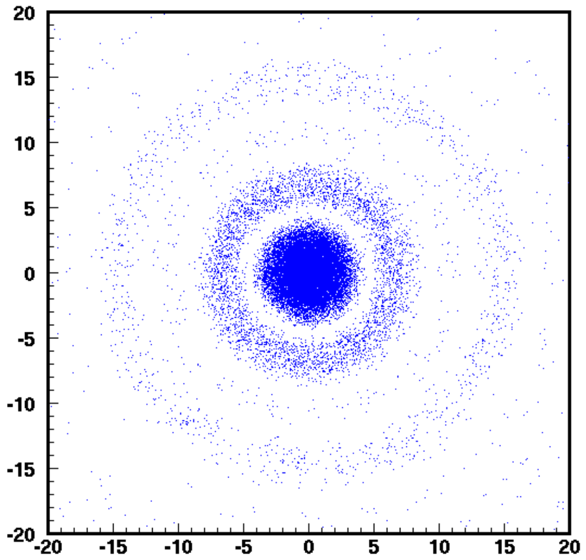


Opaque disk, diam. 10 μm
in the centre

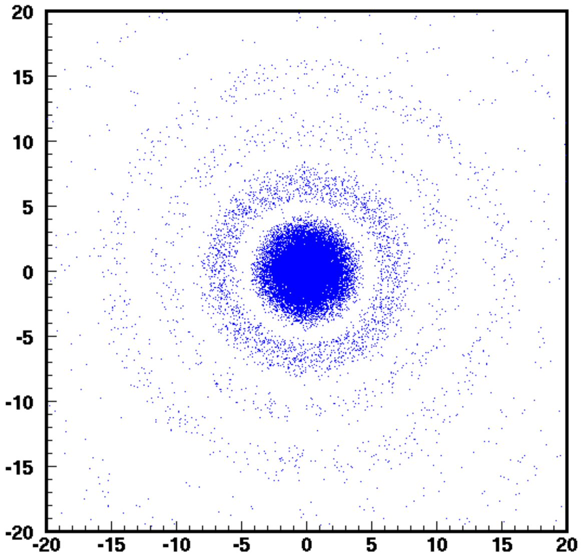
Presence of opaque disk is detectable

Opaque disk of variable diameter

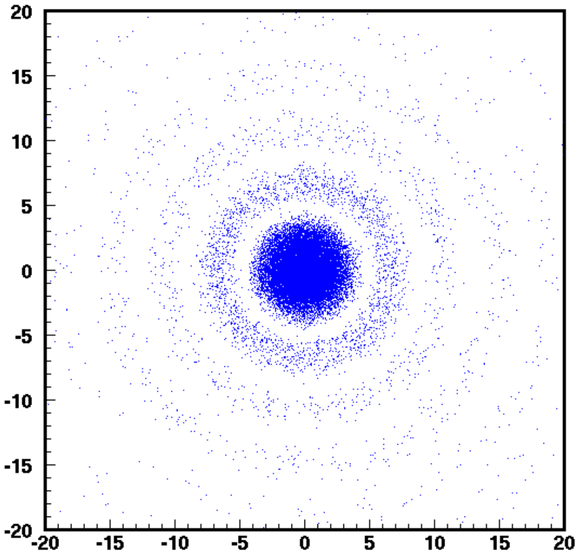
diameter = 4 µm



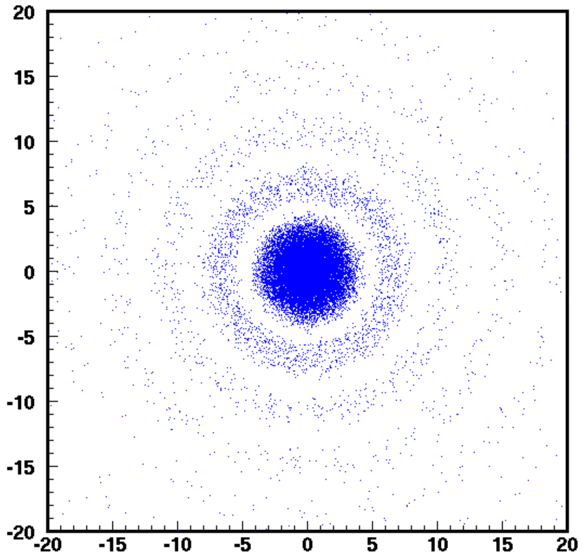
diameter = 2 µm



diameter = 1 µm



no opaque disk

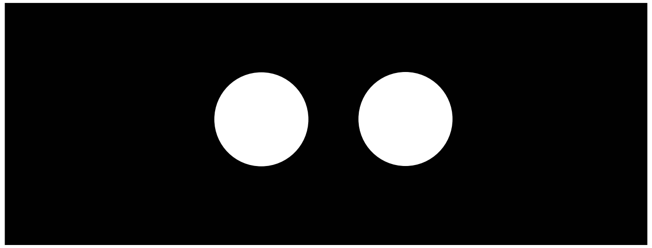


The presence of the opaque disk in the centre is detectable if its diameter is larger than the wavelength λ of the light

The <u>RESOLVING POWER</u> of the observation depends on the wavelength λ

Visible light: not enough resolution to see objects smaller than 0.2 – 0.3 µm

Opaque screen with two circular apertures



aperture diameter: 10 μm
distance between centres: 15 μm

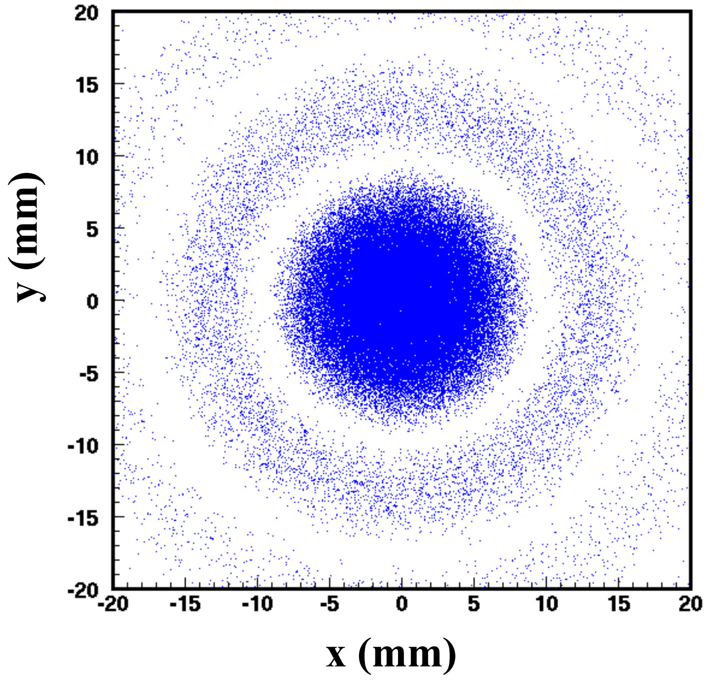
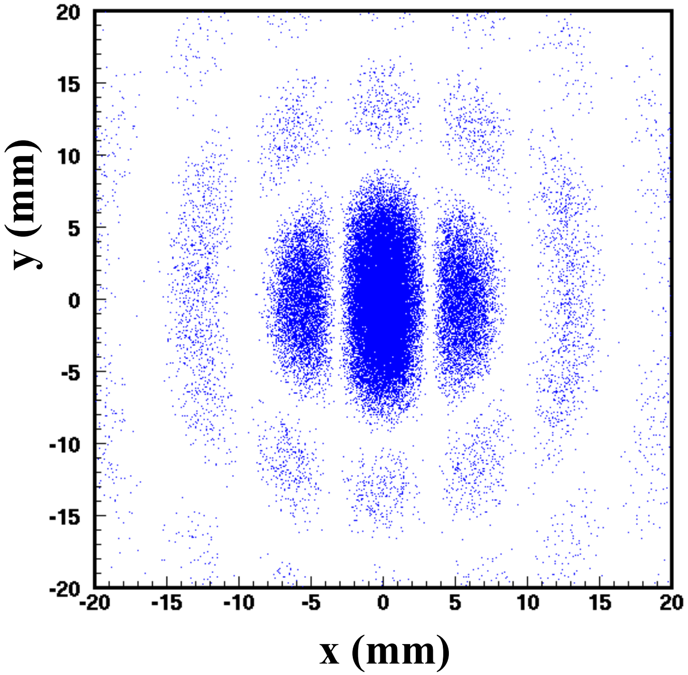
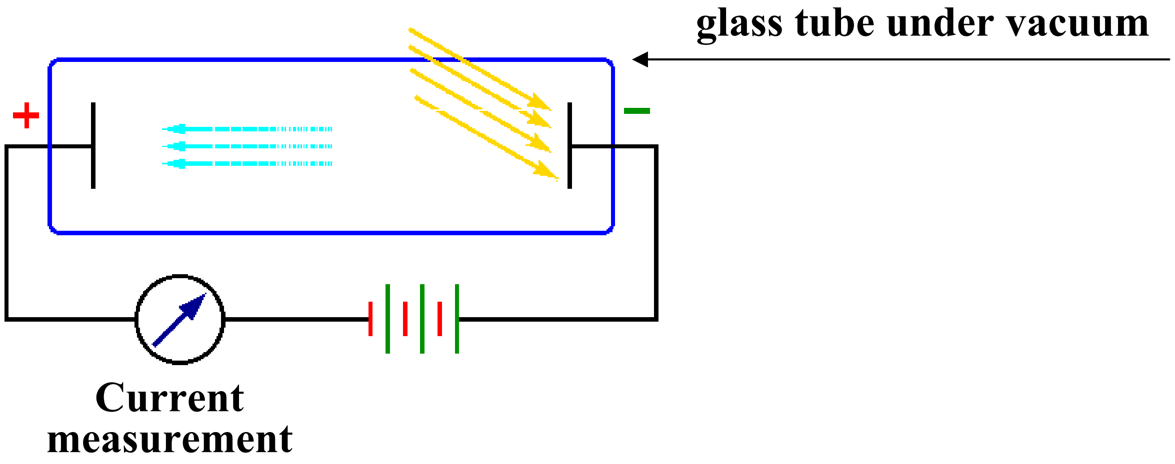


Image obtained by shutting one aperture alternatively for 50% of the exposure time

Image obtained with both apertures open simultaneously



Photoelectric effect: evidence that light consists of particles



Observation of a threshold effect as a function of the frequency of the light impinging onto the electrode at negative voltage (cathode):

Frequency $\nu < \nu_0$: electric current = zero, independent of luminous flux;

Frequency $\nu > \nu_0$: current > 0, proportional to luminous flux

INTERPRETATION (A. Einstein):

- **Light consists of particles ("photons")**
- **Photon energy proportional to frequency:**

$$\boxed{E = h\,\nu}$$

(Planck constant $h = 6.626 \times 10^{-34}$ J s)

- **Threshold energy $E_0 = h\nu_0$: the energy needed to extract an electron from an atom (depends on the cathode material)**

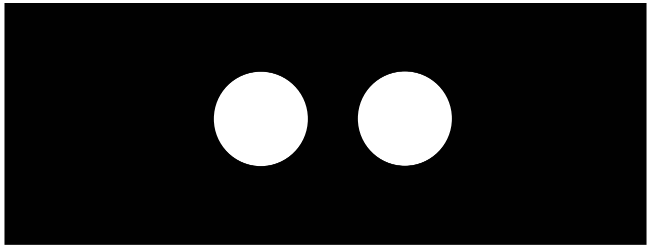
Albert Einstein

Repeat the experiment with two circular apertures using a very weak light source

Luminous flux = 1 photon /second

(detectable using modern, commercially available photomultiplier tubes)

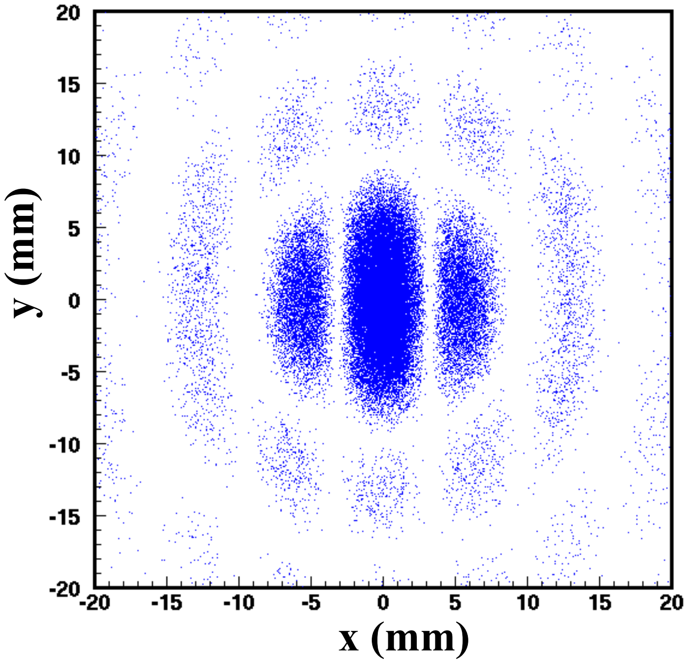
Need very long exposure time



aperture diameter: 10 μm
distance between centres: 15 μm

Question: which aperture will photons choose?

Answer: diffraction pattern corresponds to both apertures simultaneously open, independent of luminous flux



Photons have both particle and wave properties simultaneously

It is impossible to know which aperture the photon traversed

The photon can be described as a coherent superposition of two states

1924: De Broglie's principle

Louis de Broglie

Not only light, but also matter particles possess both the properties of waves and particles

Relation between wavelength and momentum:

$$\boxed{\lambda = \frac{h}{p}}$$

h: Planck constant

$p = m\,\mathrm{v}$: particle momentum

Hypothesis soon confirmed by the observation of diffraction pattern in the scattering of electrons from crystals, confirming the wave behaviour of electrons **(Davisson and Germer, 1927)**

Wavelength of the α – particles used by Rutherford in the discovery of the atomic nucleus:

$$\lambda = \frac{h}{m_\alpha \mathrm{v}} \approx \frac{6.626 \times 10^{-34}\ \mathrm{J\ s}}{(\underset{\uparrow}{6.6 \times 10^{-27}\ \mathrm{kg}}) \times (\underset{\uparrow}{1.5 \times 10^{7}\ \mathrm{m\ s^{-1}}})} \approx 6.7 \times 10^{-15}\ \mathrm{m} = \underset{\uparrow}{6.7 \times 10^{-13}}\ \mathrm{cm}$$

α–particle mass

0.05 c

~ resolving power of Rutherford's experiment

Typical tools to study objects of very small dimensions

		Resolving power
Optical microscopes	Visible light	~ 10^{-4} cm
Electron microscopes	Low energy electrons	~ 10^{-7} cm
Radioactive sources	α–particles	~ 10^{-12} cm
Accelerators	High energy electrons, protons	~ 10^{-16} cm

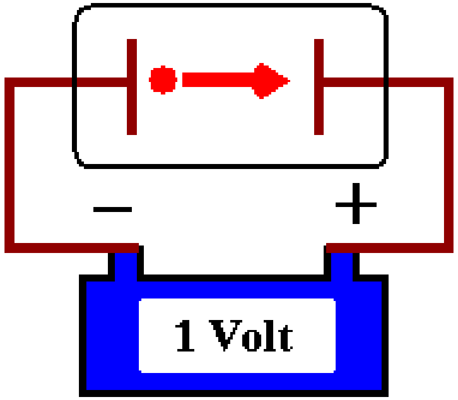
Units in particle physics

Energy

1 electron-Volt (eV):

the energy of a particle with electric charge = |e|, initially at rest, after acceleration by a difference of electrostatic potential = 1 Volt

(e = 1.60 x 10 $^{-19}$ C)



$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

Multiples:

1 keV = 10^3 eV ; 1 MeV = 10^6 eV

1 GeV = 10^9 eV; 1 TeV = 10^{12} eV

Energy of a proton in the LHC (in the year 2007):

7 TeV = 1.12 x 10 $^{-6}$ J

(the same energy of a body of mass = 1 mg moving at speed = 1.5 m /s)

Energy and momentum for relativistic particles

(velocity v comparable to *c*)

Speed of light in vacuum $c = 2.99792 \times 10^8$ m / s

Total energy: $$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-(\mathrm{v}/c)^2}}$$

m: relativistic mass
m_0: rest mass

Expansion in powers of (v/*c*): $$E = \underbrace{m_0 c^2}_{\text{energy associated with rest mass}} + \underbrace{\frac{1}{2} m_0 \mathrm{v}^2}_{\text{“classical” kinetic energy}} + \ldots$$

Momentum: $$p = m\mathrm{v} = \frac{m_0 \mathrm{v}}{\sqrt{1-(\mathrm{v}/c)^2}}$$

$$\frac{pc}{E} = \frac{\mathrm{v}}{c} \equiv \beta$$

$E^2 - p^2c^2 = (m_0c^2)^2$ "relativistic invariant"
(same value in all reference frames)

Special case: the photon (v = c in vacuum)

$$E = h\,\nu \qquad\qquad E / p = \nu\,\lambda = c \;\; \text{(in vacuum)}$$
$$\lambda = h / p \quad \Longrightarrow \quad E^2 - p^2c^2 = 0$$

photon rest mass $m_\gamma = 0$

Momentum units: eV/c (or MeV/c, GeV/c, ...)
Mass units: eV/c^2 (or MeV/c^2, GeV/c^2, ...)

Numerical example: electron with v = 0. 99 c

Rest mass: $m_e = 0.511$ MeV/c^2

$$\gamma \equiv \frac{1}{\sqrt{1-(\mathrm{v}/c)^2}} = 7.089$$ (often called "Lorentz factor")

Total energy: $E = \gamma\, m_e\, c^2 = 7.089 \text{ x } 0.511 = 3.62$ MeV

Momentum: $p = (\mathrm{v} / c) \text{ x } (E / c) = 0.99 \text{ x } 3.62 = 3.58$ MeV/c

First (wrong) ideas about nuclear structure (before 1932)

Observations

- **Mass values of light nuclei ≈ multiples of proton mass (to few %) (proton ≡ nucleus of the hydrogen atom)**
- **β decay: spontaneous emission of electrons by some radioactive nuclei**

Hypothesis: the atomic nucleus is a system of protons and electrons strongly bound together

Nucleus of the atom with atomic number Z and mass number A:

a bound system of A protons and $(A - Z)$ electrons

Total electric charge of the nucleus = $[A - (A - Z)]e = Z\,e$

Problem with this model: the "Nitrogen anomaly"

Spin of the Nitrogen nucleus = 1

Spin: intrinsic angular momentum of a particle (or system of particles)

In Quantum Mechanics only integer or half-integer multiples of $\hbar \equiv (h / 2\pi)$ are possible:

- **integer values for orbital angular momentum** (e.g., for the motion of atomic electrons around the nucleus)
- **both integer and half-integer values for spin**

Electron, proton spin = ½ħ (measured)

Nitrogen nucleus (*A* = 14, *Z* = 7): 14 protons + 7 electrons = 21 spin ½ particles

TOTAL SPIN MUST HAVE HALF-INTEGER VALUE

Measured spin = 1 (from hyperfine splitting of atomic spectral lines)

DISCOVERY OF THE NEUTRON (Chadwick, 1932)

James Chadwick

Neutron: a particle with mass ≈ proton mass
but with zero electric charge

Solution to the nuclear structure problem:

Nucleus with atomic number *Z* and mass number *A*:
a bound system of *Z* protons and (*A* – *Z*) neutrons

Nitrogen anomaly: no problem if neutron spin = ½ħ

Nitrogen nucleus (A = 14, Z = 7): 7 protons, 7 neutrons = 14 spin ½ particles

⇒ total spin has integer value

Neutron source in Chadwick's experiments: a ^{210}Po radioactive source (5 MeV α – particles) mixed with Beryllium powder ⇒ emission of electrically neutral radiation capable of traversing several centimetres of Pb:

$$\underset{\substack{\uparrow \\ \alpha\text{ - particle}}}{{}^{4}\mathrm{He}_2} + {}^{9}\mathrm{Be}_4 \rightarrow {}^{12}\mathrm{C}_6 + \text{neutron}$$

Basic principles of particle detection

Passage of charged particles through matter

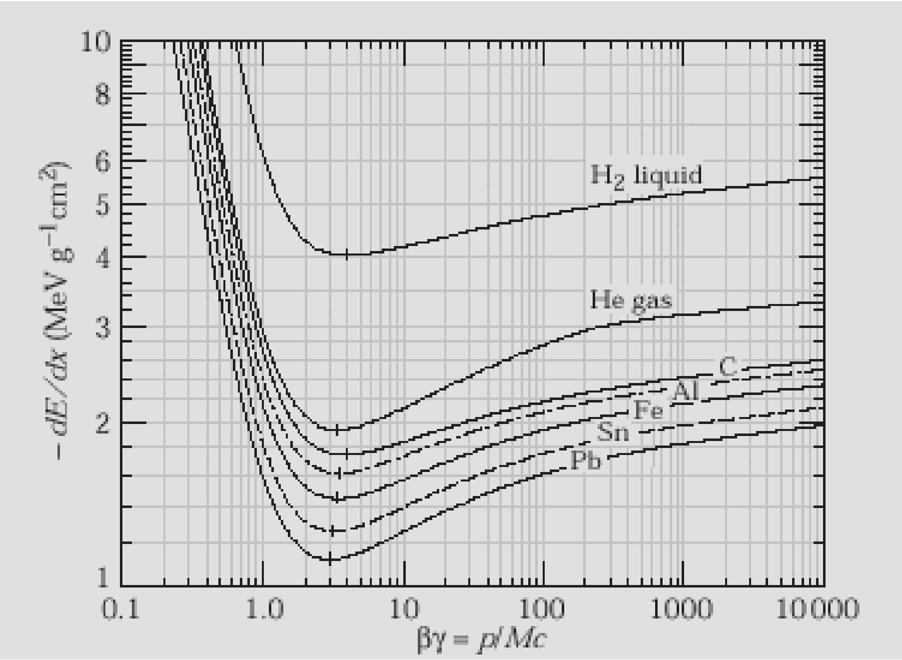
Interaction with atomic electrons → **ionization**
(neutral atom → ion$^+$ + free electron)

→ **excitation of atomic energy levels**
(de-excitation → photon emission)

Ionization + excitation of atomic energy levels → **energy loss**

Mean energy loss rate – *dE /dx*

- **proportional to (electric charge)^2 of incident particle**
- **for a given material, function only of incident particle velocity**
- **typical value at minimum:**
  $-dE/dx = 1 - 2$ MeV /(g cm^{-2})



NOTE: traversed thickness (*dx*) is given in g /cm^2 to be independent of material density (for variable density materials, such as gases) – multiply *dE /dx* by density (g/cm^3) to obtain *dE /dx* in MeV/cm

Residual range

Residual range of a charged particle with initial energy E_0 losing energy only by ionization and atomic excitation:

$$R = \int_{0}^{R} dx = \int_{E_0}^{Mc^2} \frac{1}{dE/dx} dE = MF(\mathrm{v})$$

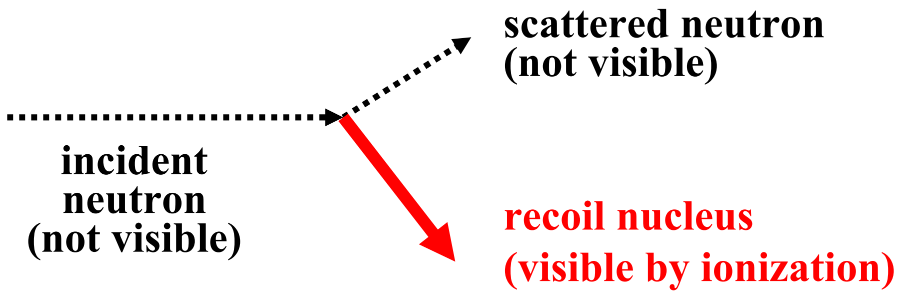
M: particle rest mass
v: initial velocity
$E_0 = Mc^2 / \sqrt{1-(\mathrm{v}/c)^2}$

⇒ the measurement of R for a particle of known rest mass M is a measurement of the initial velocity

<u>Passage of neutral particles through matter</u>: no interaction with atomic electrons
⇒ detection possible only in case of collisions producing charged particles

<u>Neutron discovery</u>:
observation and measurement of nuclear recoils in an "expansion chamber" filled with Nitrogen at atmospheric pressure

An old gaseous detector based on an expanding vapour; ionization acts as seed for the formation of liquid drops. Tracks can be photographed as strings of droplets



scattered neutron (not visible)

incident neutron (not visible)

recoil nucleus (visible by ionization)

Incident neutron direction

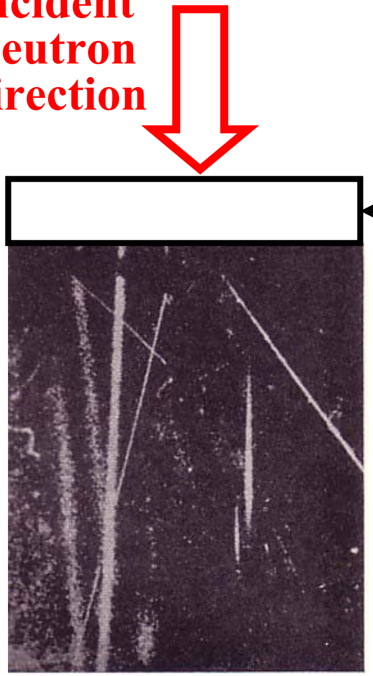
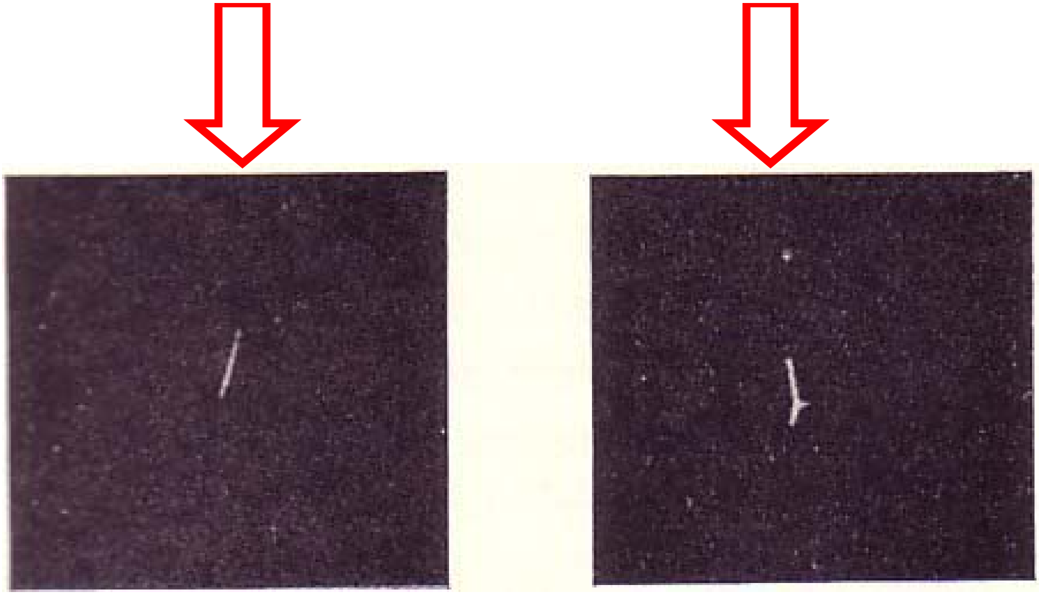


Plate containing free hydrogen (paraffin wax)

proton tracks ejected from paraffin wax



Recoiling Nitrogen nuclei

Assume that incident neutral radiation consists of particles of mass *m* moving with velocities v < V_{max}

Determine max. velocity of recoil protons (U_p) and Nitrogen nuclei (U_N) from max. observed range

$$U_p = \frac{2m}{m + m_p} V_{max} \qquad U_N = \frac{2m}{m + m_N} V_{max}$$

From non-relativistic energy-momentum conservation
m_p: proton mass; m_N: Nitrogen nucleus mass

$$\Rightarrow \quad \frac{U_p}{U_N} = \frac{m + m_N}{m + m_p}$$

From measured ratio U_p / U_N and known values of m_p, m_N determine neutron mass: $m \equiv m_n \approx m_p$

Present mass values : $m_p = 938.272$ MeV/c^2; $m_n = 939.565$ MeV/c^2

Pauli's exclusion principle

In Quantum Mechanics the electron orbits around the nucleus are "quantized":
only some specific orbits (characterized by integer quantum numbers) are possible.

Example: allowed orbit radii and energies for the Hydrogen atom

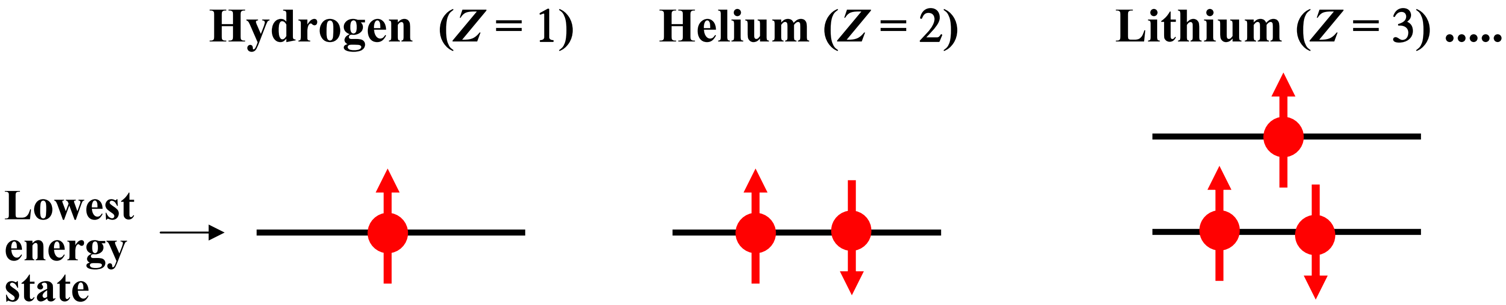
$$R_n = \frac{4\pi\varepsilon_0 \hbar^2 n^2}{me^2} \approx 0.53\times10^{-10} n^2 \text{ [m]}$$

$$E_n = -\frac{me^4}{2(4\pi\varepsilon_0)^2 \hbar^2 n^2} \approx -\frac{13.6}{n^2} \text{[eV]}$$

$$\begin{pmatrix} m = m_e m_p/(m_e + m_p) \\ n = 1, 2, \ldots\ldots \end{pmatrix}$$

In atoms with $Z > 2$ only two electrons are found in the innermost orbit – WHY?

<u>ANSWER</u> (Pauli, 1925): two electrons (spin = ½) can never be in the same physical state



Hydrogen ($Z = 1$) **Helium ($Z = 2$)** **Lithium ($Z = 3$) .....**

Lowest energy state →

Wolfgang Pauli

Pauli's exclusion principle applies to <u>all</u> particles with half-integer spin (collectively named Fermions)

ANTIMATTER

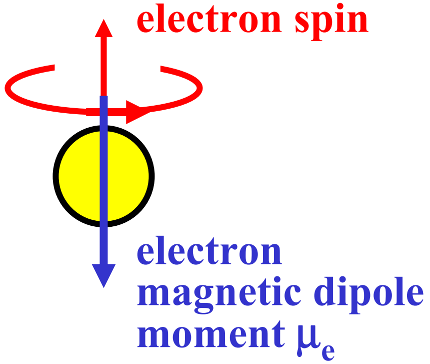
Discovered "theoretically" by P.A.M. Dirac (1928)

P.A.M. Dirac

Dirac's equation: a relativistic wave equation for the electron

Two surprising results:

- **Motion of an electron in an electromagnetic field: presence of a term describing (for slow electrons) the potential energy of a magnetic dipole moment in a magnetic field**
  ⇒ existence of an intrinsic electron magnetic dipole moment opposite to spin



$$\mu_e = \frac{e\hbar}{2m_e} \approx 5.79\times10^{-5}\ [\mathrm{eV/T}]$$

- **For each solution of Dirac's equation with electron energy $E > 0$ there is another solution with $E < 0$**
  What is the physical meaning of these "negative energy" solutions ?

Generic solutions of Dirac's equation: complex wave functions $\Psi(\vec{r}, t)$

In the presence of an electromagnetic field, for each negative-energy solution the complex conjugate wave function Ψ^* is a positive-energy solution of Dirac's equation for an electron with opposite electric charge (+*e*)

Dirac's assumptions:

- **nearly all electron negative-energy states are occupied and are not observable.**
- **electron transitions from a positive-energy to an occupied negative-energy state are forbidden by Pauli's exclusion principle.**
- **electron transitions from a positive-energy state to an empty negative-energy state are allowed ⇒ electron disappearance. To conserve electric charge, a positive electron (positron) must disappear ⇒ e^+e^- annihilation.**
- **electron transitions from a negative-energy state to an empty positive-energy state are also allowed ⇒ electron appearance. To conserve electric charge, a positron must appear ⇒ creation of an e^+e^- pair.**

⇒ empty electron negative–energy states describe positive energy states of the positron

Dirac's perfect vacuum: a region where all positive-energy states are empty and all negative-energy states are full.

Positron magnetic dipole moment = μ_e but oriented parallel to positron spin

Experimental confirmation of antimatter

(C.D. Anderson, 1932)

Carl D. Anderson

Detector: a Wilson cloud – chamber (visual detector based on a gas volume containing vapour close to saturation) in a magnetic field, exposed to cosmic rays

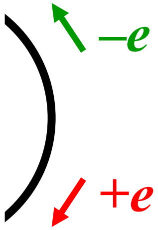
Measure particle momentum and sign of electric charge from magnetic curvature

Lorentz force $\vec{f} = e\vec{\mathrm{v}} \times \vec{B}$ ➡ **projection of the particle trajectory in a plane perpendicular to $\vec{B}$ is a circle**

Circle radius for electric charge |*e*|:

$$R\,[\mathrm{m}] = \frac{10\,p_\perp[\mathrm{GeV/c}]}{3B\,[\mathrm{T}]}$$

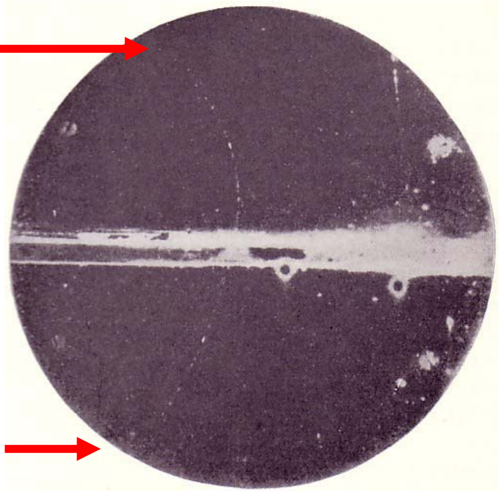
$p_\perp$: momentum component perpendicular to magnetic field direction



NOTE: impossible to distinguish between positively and negatively charged particles going in opposite directio

⇒ need an independent determination of the particle direction of motion

First experimental observation of a positron



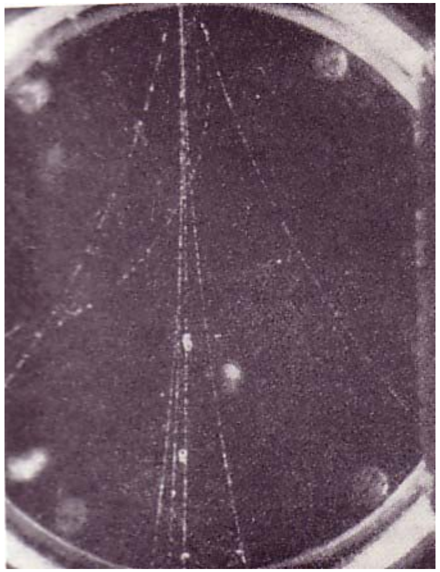
23 MeV positron

6 mm thick Pb plate

63 MeV positron

direction of
high-energy photon

Production of an
electron-positron pair
by a high-energy photon
in a Pb plate

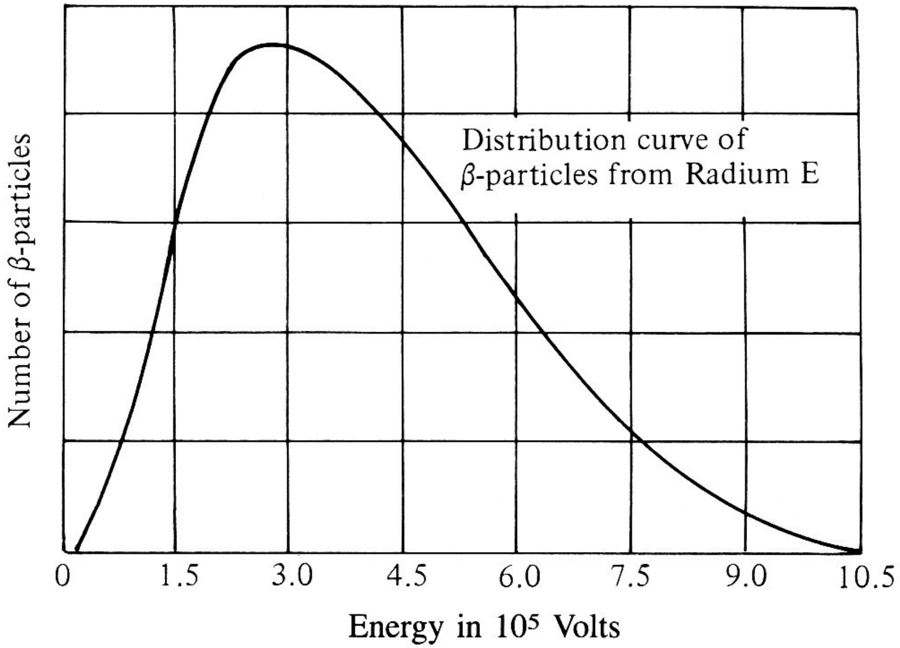


Cosmic-ray “shower”
containing several $e^+ e^-$ pairs

Neutrinos

A puzzle in β – decay: the continuous electron energy spectrum

First measurement by Chadwick (1914)



Radium E: $^{210}Bi_{83}$
(a radioactive isotope produced in the decay chain of ^{238}U)

If β – decay is $(A, Z) \rightarrow (A, Z{+}1) + e^-$, then the emitted electron is mono-energetic:

electron total energy $E = [M(A, Z) - M(A, Z{+}1)]c^2$

(neglecting the kinetic energy of the recoil nucleus $\frac{1}{2}p^2/M(A,Z{+}1) << E$)

Several solutions to the puzzle proposed before the 1930's (all wrong), including violation of energy conservation in β – decay

December 1930: public letter sent by W. Pauli to a physics meeting in Tübingen

Zürich, Dec. 4, 1930

Dear Radioactive Ladies and Gentlemen,

...because of the “wrong” statistics of the N and ^{6}Li nuclei and the continuous β-spectrum, I have hit upon a desperate remedy to save the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin ½ and obey the exclusion principle ..... The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous β-spectrum would then become understandable by the assumption that in β-decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant.

.... For the moment, however, I do not dare to publish anything on this idea ......

So, dear Radioactives, examine and judge it. Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December. ....

W. Pauli

NOTES

- Pauli’s neutron is a light particle ⇒ not the neutron that will be discovered by Chadwick one year later
- As everybody else at that time, Pauli believed that if radioactive nuclei emit particles, these particles must exist in the nuclei before emission

Theory of β-decay (E. Fermi, 1932-33)

Enrico Fermi

β^- decay: $n \rightarrow p + e^- + \bar{\nu}$

β^+ decay: $p \rightarrow n + e^+ + \nu$ (e.g., $^{14}O_8 \rightarrow {}^{14}N_7 + e^+ + \nu$)

ν: the particle proposed by Pauli
(named "neutrino" by Fermi)

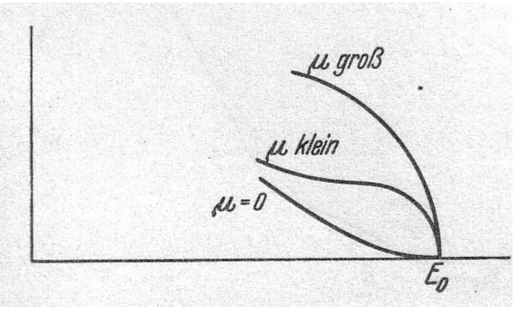
$\bar{\nu}$: its antiparticle (antineutrino)

Fermi's theory: a point interaction among four spin ½ particles, using the mathematical formalism of creation and annihilation operators invented by Jordan
⇒ particles emitted in β – decay need not exist before emission – they are "created" at the instant of decay

Prediction of β – decay rates and electron energy spectra as a function of only one parameter: Fermi coupling constant G_F (determined from experiments)

Energy spectrum dependence on neutrino mass μ
(from Fermi's original article, published in German on Zeitschrift für Physik, following rejection of the English version by Nature)

Measurable distortions for μ > 0 near the end-point (E_0: max. allowed electron energy)

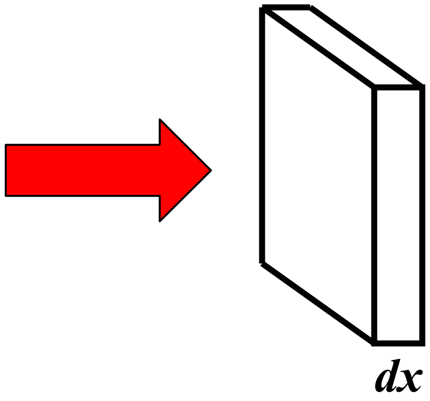


Neutrino detection

Prediction of Fermi's theory: $\bar{\nu} + p \rightarrow e^+ + n$

$\bar{\nu}$ – p interaction probability in thickness *dx* of hydrogen-rich material (e.g., H_2O)

Incident $\bar{\nu}$:
Flux Φ [$\bar{\nu}$ $cm^{-2}\,s^{-1}$]
(uniform over surface S)



Target:
surface *S*, thickness *dx*
containing *n* protons cm^{-3}

$\bar{\nu}$ p interaction rate = $\Phi\, S\, n\, \sigma\, dx$ interactions per second

σ : $\bar{\nu}$ – proton cross-section (effective proton area, as seen by the incident $\bar{\nu}$)

$\bar{\nu}$ p interaction probability = $n\,\sigma\,dx = dx / \lambda$

Interaction mean free path: $\lambda = 1 / n\,\sigma$
Interaction probability for finite target thickness T = $1 - \exp(-T/\lambda)$

$\sigma(\bar{\nu}\,p) \approx 10^{-4\,3}\ cm^2$ for 3 MeV $\bar{\nu}$ $\Rightarrow$ $\lambda \approx$ 150 light-years of water !

Interaction probability $\approx T/\lambda$ very small ($\sim 10^{-18}$ per metre H_2O)
$\Rightarrow$ need very intense sources for antineutrino detection

Nuclear reactors: very intense antineutrino sources

Average fission: $n + {}^{235}U_{92} \rightarrow \underbrace{(A_1, Z) + (A_2, 92 - Z)}_{\substack{\text{nuclei with} \\ \text{large neutron excess}}} + 2.5$ free neutrons + 200 MeV

➡ **a chain of β decays with very short lifetimes:**

$$(A, Z) \xrightarrow[e^- \bar{\nu}]{} (A, Z+1) \xrightarrow[e^- \bar{\nu}]{} (A, Z+2) \xrightarrow[e^- \bar{\nu}]{} \text{....} \quad \text{(until a stable or long lifetime nucleus is reached)}$$

On average, 6 $\bar{\nu}$ per fission

$$\bar{\nu}\ \text{production rate} = \frac{6P_t}{200\ \text{MeV} \times \underbrace{1.6 \times 10^{-13}}_{\substack{\text{conversion factor} \\ \text{MeV} \rightarrow \text{J}}}} = 1.87 \times 10^{11} P_t\ \bar{\nu}/\text{s}$$

P_t: reactor thermal power [W]

For a typical reactor: $P_t = 3 \times 10^9$ W ⇒ $5.6 \times 10^{20}\ \bar{\nu}$ / s (isotropic)

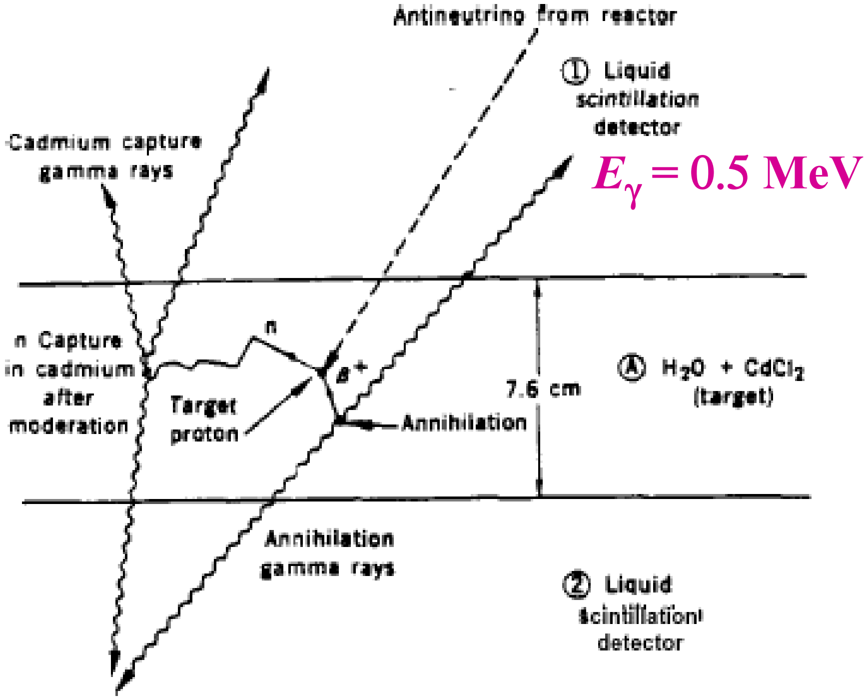
Continuous $\bar{\nu}$ energy spectrum – average energy ~3 MeV

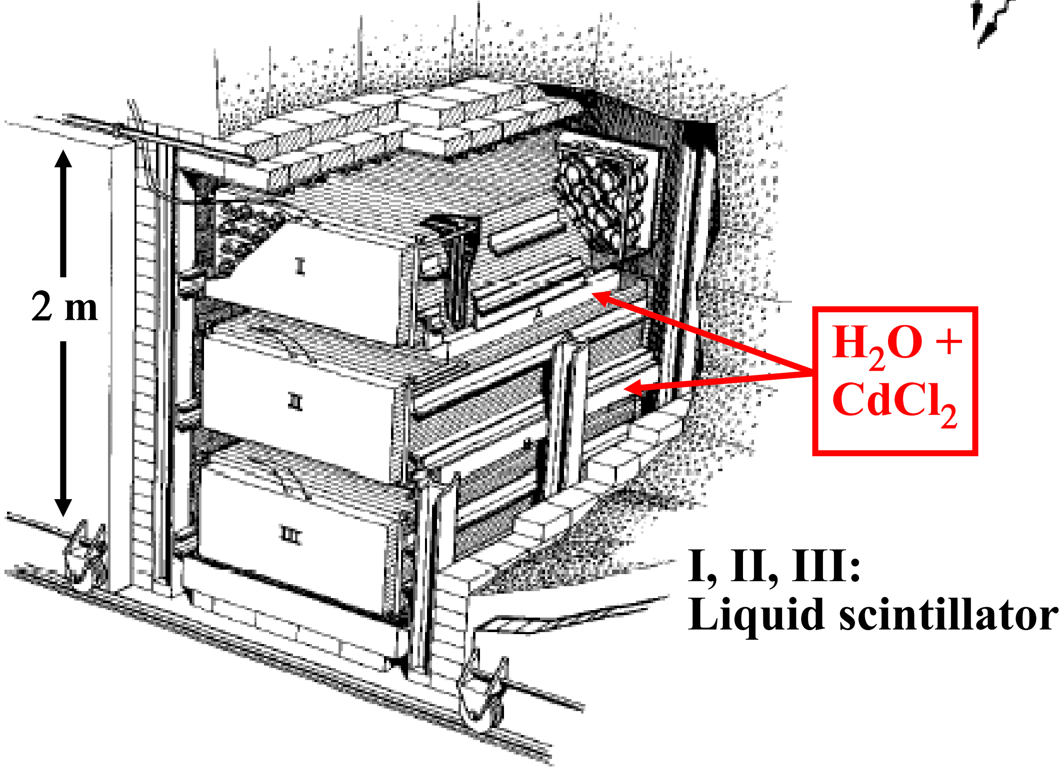
First neutrino detection

(Reines, Cowan 1953)

$$\overline{\nu} + p \rightarrow e^+ + n$$

- detect 0.5 MeV γ-rays from $e^+e^- \rightarrow \gamma\gamma$ ($t = 0$)
- neutron “thermalization” followed by capture in Cd nuclei ⇒ emission of delayed γ-rays (average delay ~30 μs)





Event rate at the Savannah River nuclear power plant:

3.0 ± 0.2 events / hour

(after subracting event rate measured with reactor OFF)

in agreement with expectations

COSMIC RAYS

- **Discovered by V.F. Hess in the 1910's by the observation of the increase of radioactivity with altitude during a balloon flight**
- **Until the late 1940's, the only existing source of high-energy particles**

Composition of cosmic rays at sea level – two main components

- **Electromagnetic "showers", consisting of many $e^{\pm}$ and γ-rays, mainly originating from:**

  γ + nucleus $\rightarrow e^{+}e^{-}$ + nucleus (pair production);

  $e^{\pm}$ + nucleus $\rightarrow e^{\pm} + \gamma$ + nucleus ("bremsstrahlung")

  The typical mean free path for these processes ("radiation length", x_0) depends on Z.

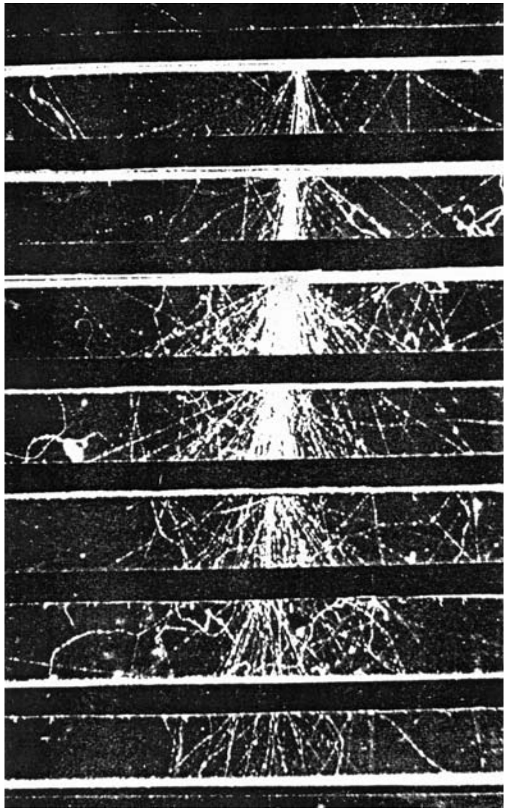
  For Pb ($Z = 82$) $x_0 = 0.56$ cm

  Thickness of the atmosphere $\approx 27\, x_0$

- **Muons ($\mu^{\pm}$) capable of traversing as much as 1 m of Pb without interacting; tracks observed in cloud chambers in the 1930's.**

  Determination of the mass by simultaneous measurement of momentum $p = mv(1 - v^2/c^2)^{-1/2}$ (track curvature in magnetic field) **and velocity v** (ionization)**:**

$$m_{\mu} = 105.66 \text{ MeV}/c^2 \approx 207\, m_e$$

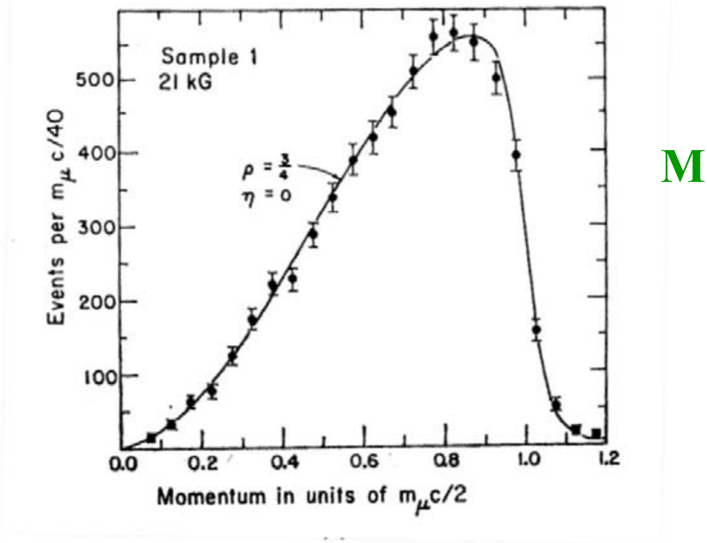


Cloud chamber image of an electromagnetic shower.
Pb plates, each 1. 27 cm thick

Muon decay

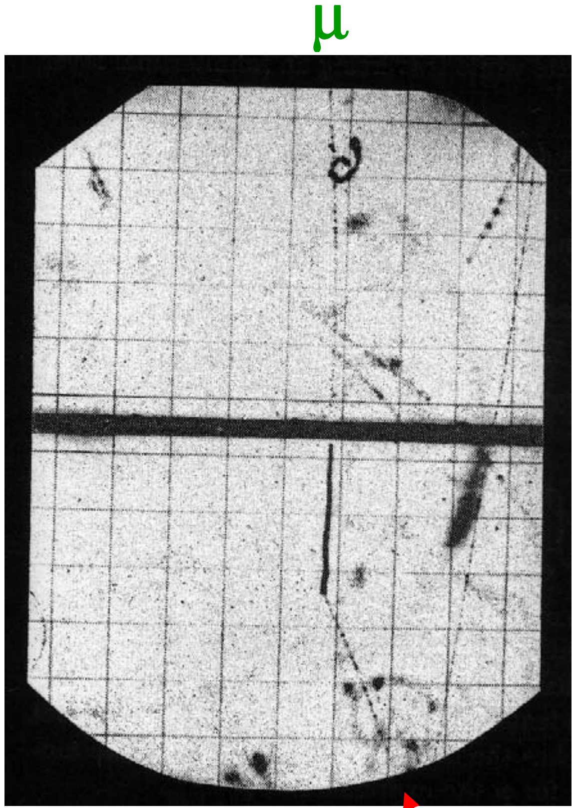
$\mu^{\pm} \rightarrow e^{\pm} + \nu + \bar{\nu}$

Decay electron momentum distribution



Muon spin = ½

Cosmic ray muon stopping in a cloud chamber and decaying to an electron



decay electron track

Muon lifetime at rest: $\tau_{\mu} = 2.197 \times 10^{-6}$ s ≡ 2.197 μs

Muon decay mean free path in flight:

$$\lambda_{decay} = \frac{v\,\tau_{\mu}}{\sqrt{1-(v/c)^2}} = \frac{p\,\tau_{\mu}}{m_{\mu}} = \frac{p}{m_{\mu}c}\tau_{\mu}c$$

$\boldsymbol{p}$: muon momentum
$\tau_{\mu}\,c \approx 0.66$ km

⇒ muons can reach the Earth surface after a path ≥ 10 km because the decay mean free path is stretched by the relativistic time expansion

Particle interactions (as known until the mid 1960's)

In order of increasing strength:

- **Gravitational interaction (all particles)**

  Totally negligible in particle physics

  Example: static force between electron and proton at distance D

  Gravitational: $f_G = G_N \frac{m_e m_p}{D^2}$ **Electrostatic:** $f_E = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{D^2}$

  Ratio $f_G / f_E \approx 4.4 \times 10^{-40}$

- **Weak interaction (all particles except photons)**

  Responsible for β decay and for slow nuclear fusion reactions in the star core

  Example: in the core of the Sun ($T = 15.6 \times 10^6$ °K) $4p \to {}^4He + 2e^+ + 2\nu$

  Solar neutrino emission rate ~ 1.84×10^{38} neutrinos / s

  Flux of solar neutrinos on Earth ~ 6.4×10^{10} neutrinos cm^{-2} s^{-1}

  Very small interaction radius R_{int} (max. distance at which two particles interact)

  ($R_{int} = 0$ in the original formulation of Fermi's theory)

- **Electromagnetic interaction (all charged particles)**

  Responsible for chemical reactions, light emission from atoms, etc.

  Infinite interaction radius

  (example: the interaction between electrons in transmitting and receiving antennas)

- **Strong interaction (neutron, proton, ... NOT THE ELECTRON !)**

  Responsible for keeping protons and neutrons together in the atomic nucleus

  Independent of electric charge

  Interaction radius $R_{\text{int}} \approx 10^{-13}$ cm

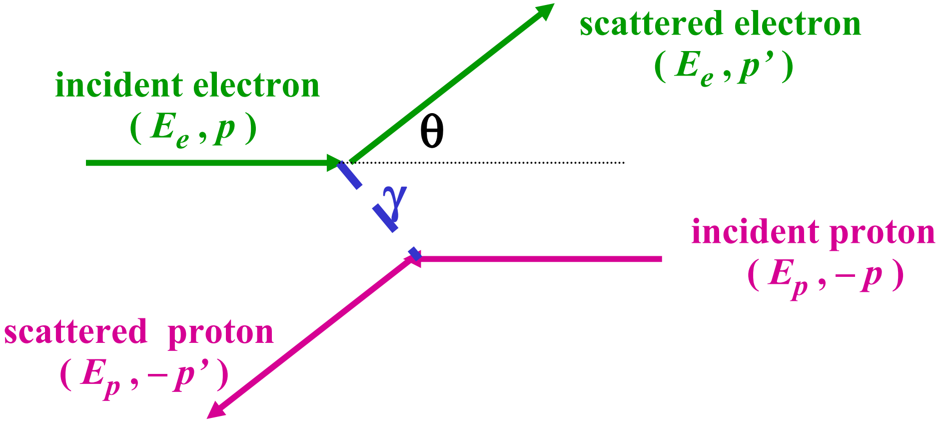
In Relativistic Quantum Mechanics static fields of forces DO NOT EXIST ; the interaction between two particles is "transmitted" by intermediate particles acting as "interaction carriers"

Example: electron – proton scattering (an effect of the electromagnetic interaction) is described as a two-step process : 1) incident electron → scattered electron + photon

2) photon + incident proton → scattered proton

The photon (γ) is the carrier of the electromagnetic interaction

In the electron – proton centre-of-mass system



Energy – momentum conservation:

$$E_\gamma = 0$$

$$\vec{p}_\gamma = \vec{p} - \vec{p}\,' \quad (|\vec{p}| = |\vec{p}\,'|)$$

"Mass" of the intermediate photon: $Q^2 \equiv E_\gamma^{\,2} - p_\gamma^{\,2} c^2 = -2\, p^2 c^2 (1 - \cos\theta)$

The photon is in a VIRTUAL state because for real photons $E_\gamma^{\,2} - p_\gamma^{\,2} c^2 = 0$
(the mass of real photons is ZERO) – **virtual photons can only exist for a very short time interval thanks to the "Uncertainty Principle"**

The Uncertainty Principle

Werner Heisenberg

CLASSICAL MECHANICS

Position and momentum of a particle can be measured independently and simultaneously with arbitrary precision

QUANTUM MECHANICS

Measurement perturbs the particle state ⇒ position and momentum measurements are correlated:

$$\boxed{\Delta x \Delta p_x \approx \hbar}$$

(also for *y* and *z* components)

Similar correlation for energy and time measurements:

$$\boxed{\Delta E \Delta t \approx \hbar}$$

Quantum Mechanics allows a violation of energy conservation by an amount ΔE for a short time $\Delta t \approx \hbar / \Delta E$

Numerical example: $\Delta E = 1\,\mathrm{MeV} \Longrightarrow \Delta t \approx 6.6 \times 10^{-22}\ \mathrm{s}$

1937: Theory of nuclear forces (H. Yukawa)

Hideki Yukawa

Existence of a new light particle ("meson")
as the carrier of nuclear forces

Relation between interaction radius and meson mass *m*:

$$R_{\text{int}} = \frac{\hbar}{mc} \Rightarrow \begin{array}{l} mc^2 \approx 200\ \text{MeV} \\ \text{for } R_{\text{int}} \approx 10^{-13}\ \text{cm} \end{array}$$

Yukawa's meson initially identified with the muon – in this case μ^- stopping in matter should be immediately absorbed by nuclei $\Rightarrow$ nuclear breakup
(not true for stopping μ^+ because of Coulomb repulsion - μ^+ never come close enough to nuclei, while μ^- form "muonic" atoms)

Experiment of Conversi, Pancini, Piccioni (Rome, 1945):

study of μ^- stopping in matter using μ^- magnetic selection in the cosmic rays

In light material ($Z \leq 10$) the μ^- decays mainly to electron (just as μ^+)
In heavier material, the μ^- disappears partly by decaying to electron, and partly by nuclear capture (process later understood as $\mu^- + p \rightarrow n + \nu$).
However, the rate of nuclear captures is consistent with the weak interaction.

$\Rightarrow$ **the muon is not Yukawa's meson**

1947: Discovery of the π - meson (the “real” Yukawa particle)

Observation of the $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay chain in nuclear emulsion exposed to cosmic rays at high altitudes

Nuclear emulsion: a detector sensitive to ionization with ~1 μm space resolution (AgBr microcrystals suspended in gelatin)

In all events the muon has a fixed kinetic energy (4.1 MeV, corresponding to a range of ~ 600 μm in nuclear emulsion) ⇒ two-body decay

$m_\pi = 139.57$ MeV/c^2 ; spin = 0

Dominant decay mode: $\pi^+ \rightarrow \mu^+ + \nu$

(and $\pi^- \rightarrow \mu^- + \overline{\nu}$)

Mean life at rest: $\tau_\pi = 2.6 \times 10^{-8}$ s = 26 ns

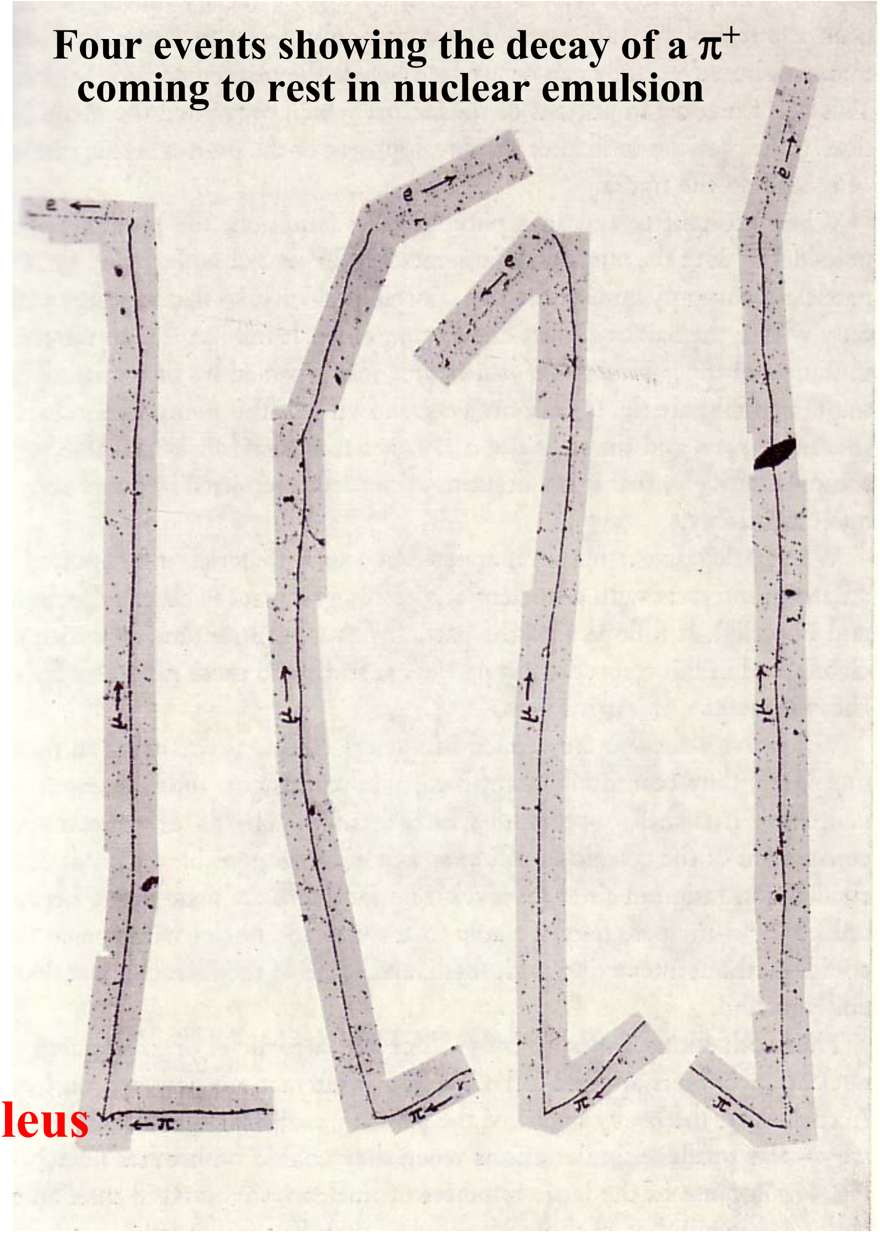
π^- at rest undergoes nuclear capture, as expected for the Yukawa particle

A neutral π – meson (π°) also exists:

m (π°) = 134. 98 MeV /c^2

Decay: $\pi^\circ \rightarrow \gamma + \gamma$, mean life = 8.4×10^{-17} s

π – mesons are the most copiously produced particles in proton – proton and proton – nucleus collisions at high energies



Four events showing the decay of a π^+ coming to rest in nuclear emulsion

CONSERVED QUANTUM NUMBERS

Why is the free proton stable?

Possible proton decay modes (allowed by all known conservation laws: energy – momentum, electric charge, angular momentum):

$$p \rightarrow \pi^\circ + e^+$$
$$p \rightarrow \pi^\circ + \mu^+$$
$$p \rightarrow \pi^+ + \nu$$

. . . . .

No proton decay ever observed – the proton is STABLE

Limit on the proton mean life: $\tau_p > 1.6 \times 10^{25}$ **years**

Invent a new quantum number : "Baryonic Number" B

B = 1 for proton, neutron

B = -1 for antiproton, antineutron

B = 0 for $e^\pm$, $\mu^\pm$, neutrinos, mesons, photons

Require conservation of baryonic number in all particle processes:

$$\sum_i B_i = \sum_f B_f$$

(***i*** **:** initial state particle ; ***f*** **:** final state particle)

Strangeness

Late 1940's: discovery of a variety of heavier mesons (K – mesons) and baryons ("hyperons") – studied in detail in the 1950's at the new high-energy proton synchrotrons (the 3 GeV "cosmotron" at the Brookhaven National Lab and the 6 GeV Bevatron at Berkeley)

Examples of mass values

Mesons (spin = 0): $m(K^\pm) = 493.68$ MeV/c^2 ; $m(K^\circ) = 497.67$ MeV/c^2

Hyperons (spin = ½): $m(\Lambda) = 1115.7$ MeV/c^2 ; $m(\Sigma^\pm) = 1189.4$ MeV/c^2

$m(\Xi^\circ) = 1314.8$ MeV/c^2; $m(\Xi^-) = 1321.3$ MeV/c^2

Properties

- Abundant production in proton – nucleus , π – nucleus collisions
- Production cross-section typical of strong interactions ($\sigma > 10^{-27}$ cm^2)
- Production in pairs (example: $\pi^- + p \rightarrow K^\circ + \Lambda$; $K^- + p \rightarrow \Xi^- + K^+$)
- Decaying to lighter particles with mean life values $10^{-8} - 10^{-10}$ s (as expected for a weak decay)

Examples of decay modes

$K^\pm \rightarrow \pi^\pm \pi^\circ$; $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$; $K^\pm \rightarrow \pi^\pm \pi^\circ \pi^\circ$; $K^\circ \rightarrow \pi^+ \pi^-$; $K^\circ \rightarrow \pi^\circ \pi^\circ$; . . .

$\Lambda \rightarrow p\, \pi^-$; $\Lambda \rightarrow n\, \pi^\circ$; $\Sigma^+ \rightarrow p\, \pi^\circ$; $\Sigma^+ \rightarrow n\, \pi^+$; $\Sigma^+ \rightarrow n\, \pi^-$; . . .

$\Xi^- \rightarrow \Lambda\, \pi^-$; $\Xi^\circ \rightarrow \Lambda\, \pi^\circ$

Invention of a new, additive quantum number “Strangeness” (S)

(Gell-Mann, Nakano, Nishijima, 1953)

- **conserved in strong interaction processes:** $\sum_i S_i = \sum_f S_f$
- **not conserved in weak decays:** $\left| S_i - \sum_f S_f \right| = 1$

$S = +1$: K^+, K° ; $S = -1$: Λ, $\Sigma^\pm$, $\Sigma°$; $S = -2$: $\Xi°$, Ξ^- ; $S = 0$: all other particles
(and opposite strangeness $-S$ for the corresponding antiparticles)

Example of a K^- stopping in liquid hydrogen:

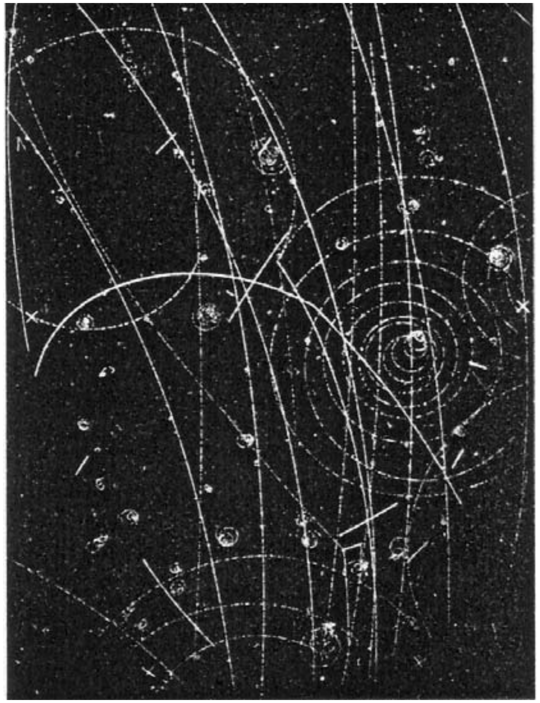
$$K^- + p \rightarrow \Lambda + \pi°$$

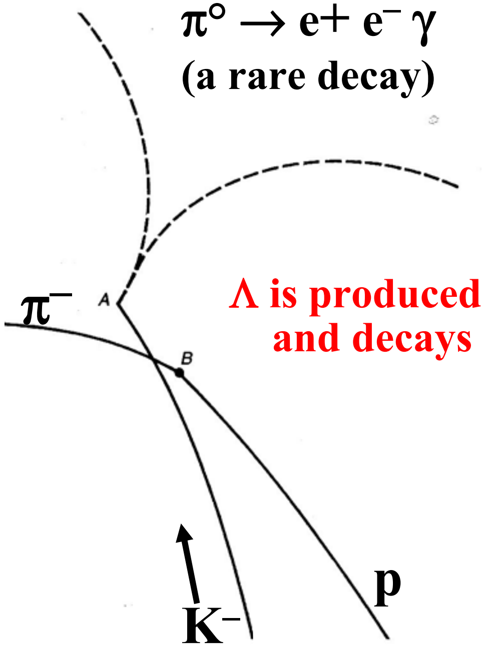
(strangeness conserving)

followed by the decay

$$\Lambda \rightarrow p + \pi^-$$

(strangeness violation)





$\pi° \rightarrow e+ \, e^- \gamma$
(a rare decay)

Λ is produced in A and decays in B

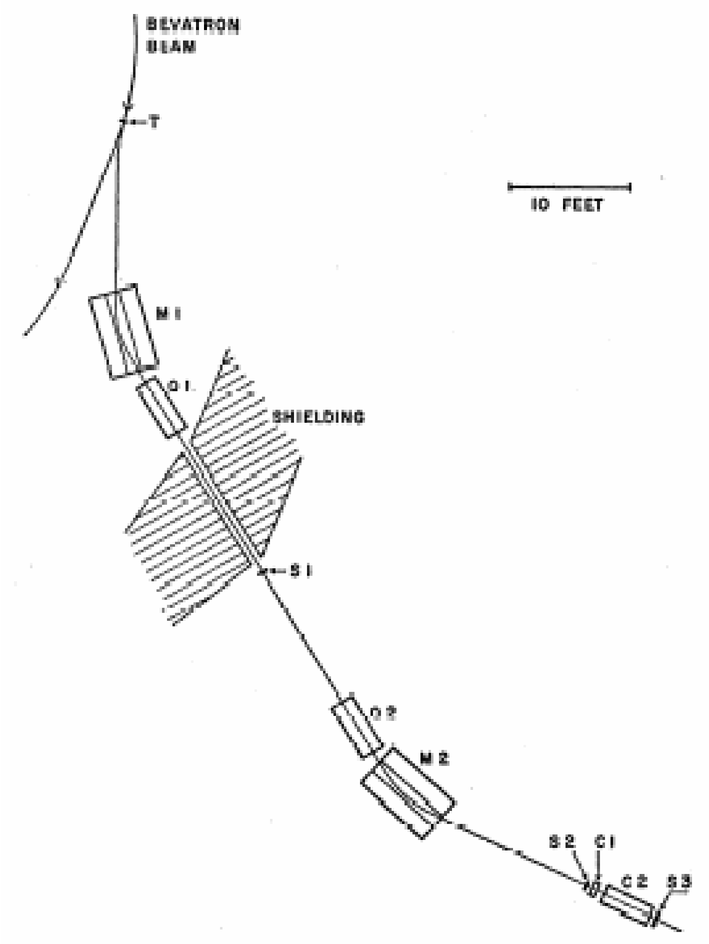
Antiproton discovery (1955)

Threshold energy for antiproton ($\overline{\text{p}}$) production in proton – proton collisions

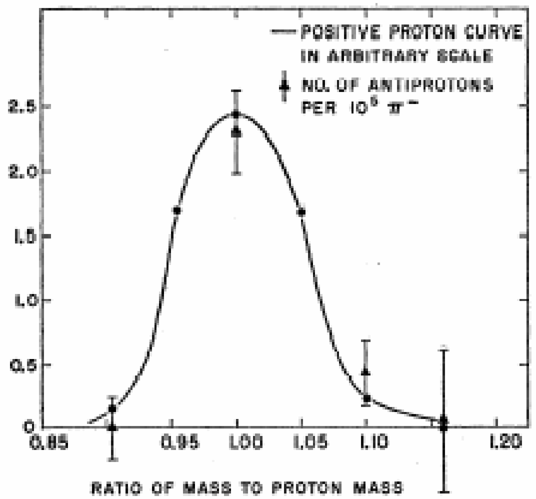
Baryon number conservation ⇒ simultaneous production of $\overline{\text{p}}$ and p (or $\overline{\text{p}}$ and n)

Example: $\text{p} + \text{p} \rightarrow \text{p} + \text{p} + \overline{\text{p}} + \text{p}$ **Threshold energy ~ 6 GeV**

"Bevatron": 6 GeV
proton synchrotron in Berkeley

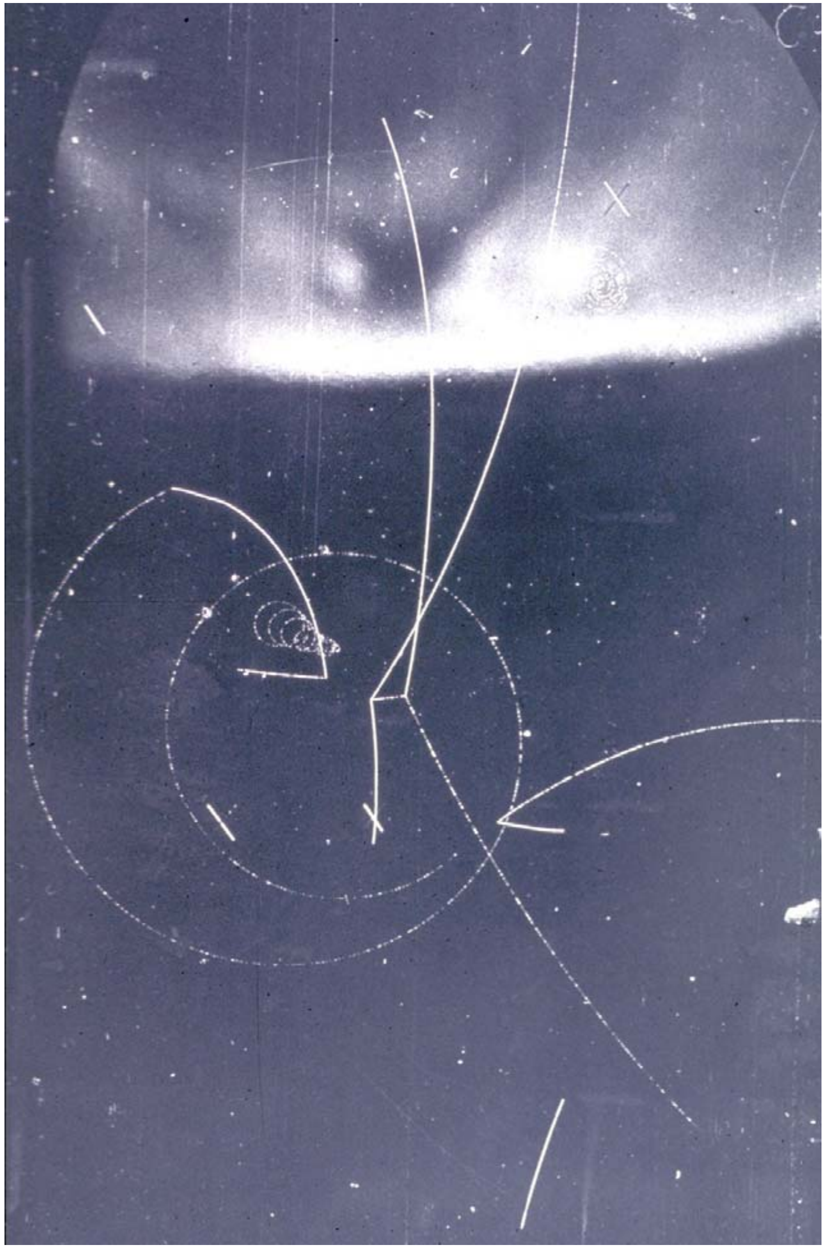


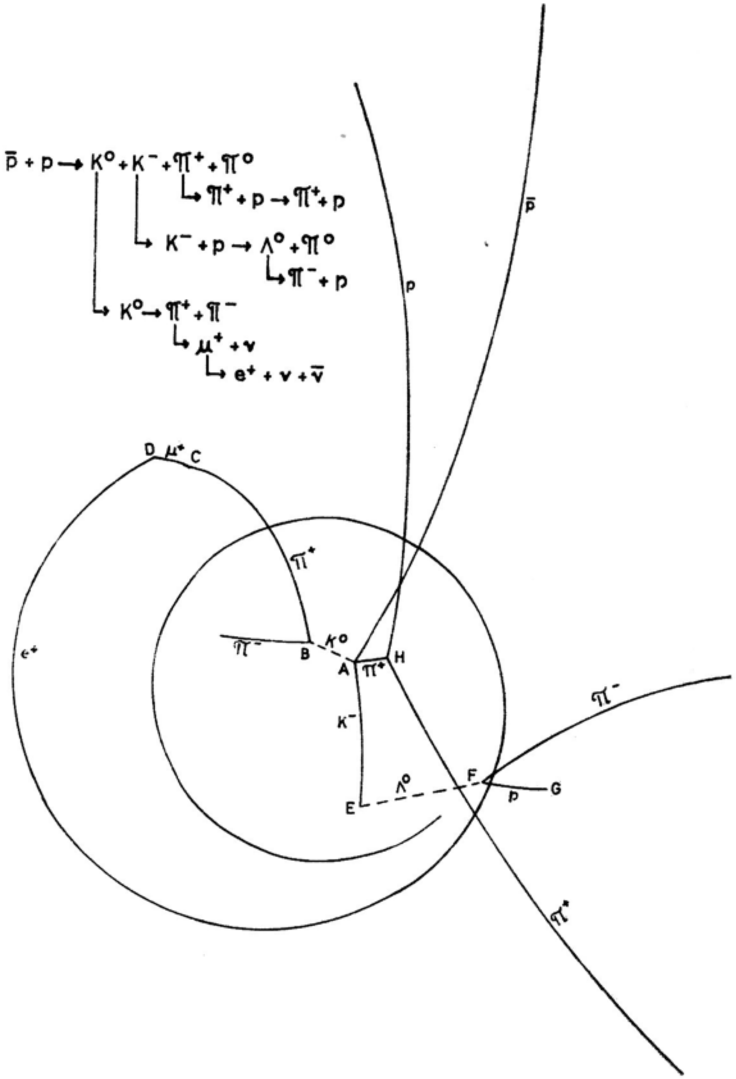
- **build a beam line for 1.19 GeV/c momentum**
- **select negatively charged particles (mostly π^-)**
- **reject fast π^- by Čerenkov effect: light emission in transparent medium if particle velocity v > *c* / *n* (*n*: refraction index) – antiprotons have v < *c* / *n***
  ⇒ no Čerenkov light
- **measure time of flight between counters S_1 and S_2 (12 m path): 40 ns for π^-, 51 ns for antiprotons**



For fixed momentum, time of flight gives particle velocity, hence particle mass

Example of antiproton annihilation at rest in a liquid hydrogen bubble chamber





$\bar{p} + p \rightarrow K^0 + K^- + \pi^+ + \pi^0$

$\hookrightarrow \pi^+ + p \rightarrow \pi^+ + p$

$\hookrightarrow K^- + p \rightarrow \Lambda^0 + \pi^0$

$\hookrightarrow \pi^- + p$

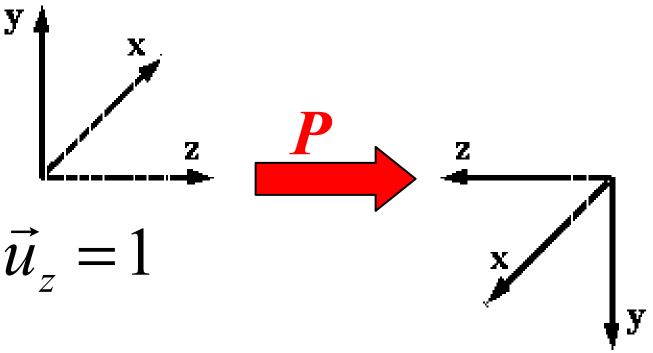
$\hookrightarrow K^0 \rightarrow \pi^+ + \pi^-$

$\hookrightarrow \mu^+ + \nu$

$\hookrightarrow e^+ + \nu + \bar{\nu}$

DISCRETE SYMMETRIES

PARITY: the reversal of all three axes in a reference frame

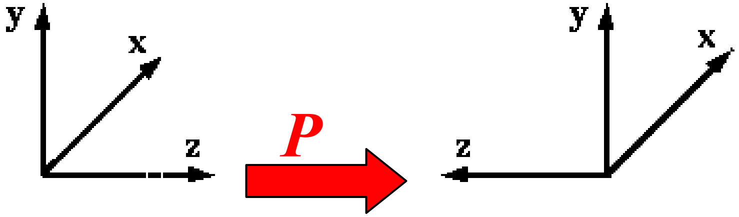


$$\left(\vec{u}_x \times \vec{u}_y\right) \cdot \vec{u}_z = 1$$

$$\left(\vec{u}_x \times \vec{u}_y\right) \cdot \vec{u}_z = -1$$

($\vec{u}$: unit vectors along the three axes)

P transformation equivalent to a mirror reflection



(first, rotate by 180° around the z – axis ; then reverse all three axes)

PARITY INVARIANCE:

All physics laws are invariant with respect to a *P* transformation;

For any given physical system, the mirror-symmetric system is equally probable;

In particle physics Nature does not know the difference between Right and Left.

Vector transformation under *P*

Radial (position) vector $\vec{r} \equiv (x, y, z) \Rightarrow (-x, -y, -z)$

Momentum vector $\vec{p} \equiv (p_x, p_y, p_z) \Rightarrow (-p_x, -p_y, -p_z)$

(all three components change sign)

Angular momentum $\vec{L} \equiv \vec{r} \times \vec{p} \Rightarrow \vec{r} \times \vec{p}$

(the three components do not change)

Spin $\vec{s}$: same behaviour as for angular momentum ($\vec{s} \Rightarrow \vec{s}$)

➡ **a scalar term of type $\vec{s} \cdot \vec{p}$ changes sign under *P***

If the transition probability for a certain process depends on a term of type $\vec{s} \cdot \vec{p}$, the process violates parity invariance

A puzzle in the early 1950's : the decays $K^+ \rightarrow \pi^+ \pi^\circ$ and $K^+ \rightarrow 3\pi$ ($\pi^+ \pi^+ \pi^-$ and $\pi^+ \pi^\circ \pi^\circ$)

A system of two π – mesons and a system of three π – mesons, both in a state of total angular momentum = 0, have OPPOSITE PARITIES

1956: Suggestion (by T.D. Lee and C.N. Yang) Weak interactions are NOT INVARIANT under Parity

$\pi^+ \rightarrow \mu^+ + \nu$ decay

Parity invariance requires that the two states

must be produced with equal probabilities ⇒ the emitted μ^+ is not polarized

Experiments find that the μ^+ has full polarization opposite to the momentum direction ⇒ STATE A DOES NOT EXIST

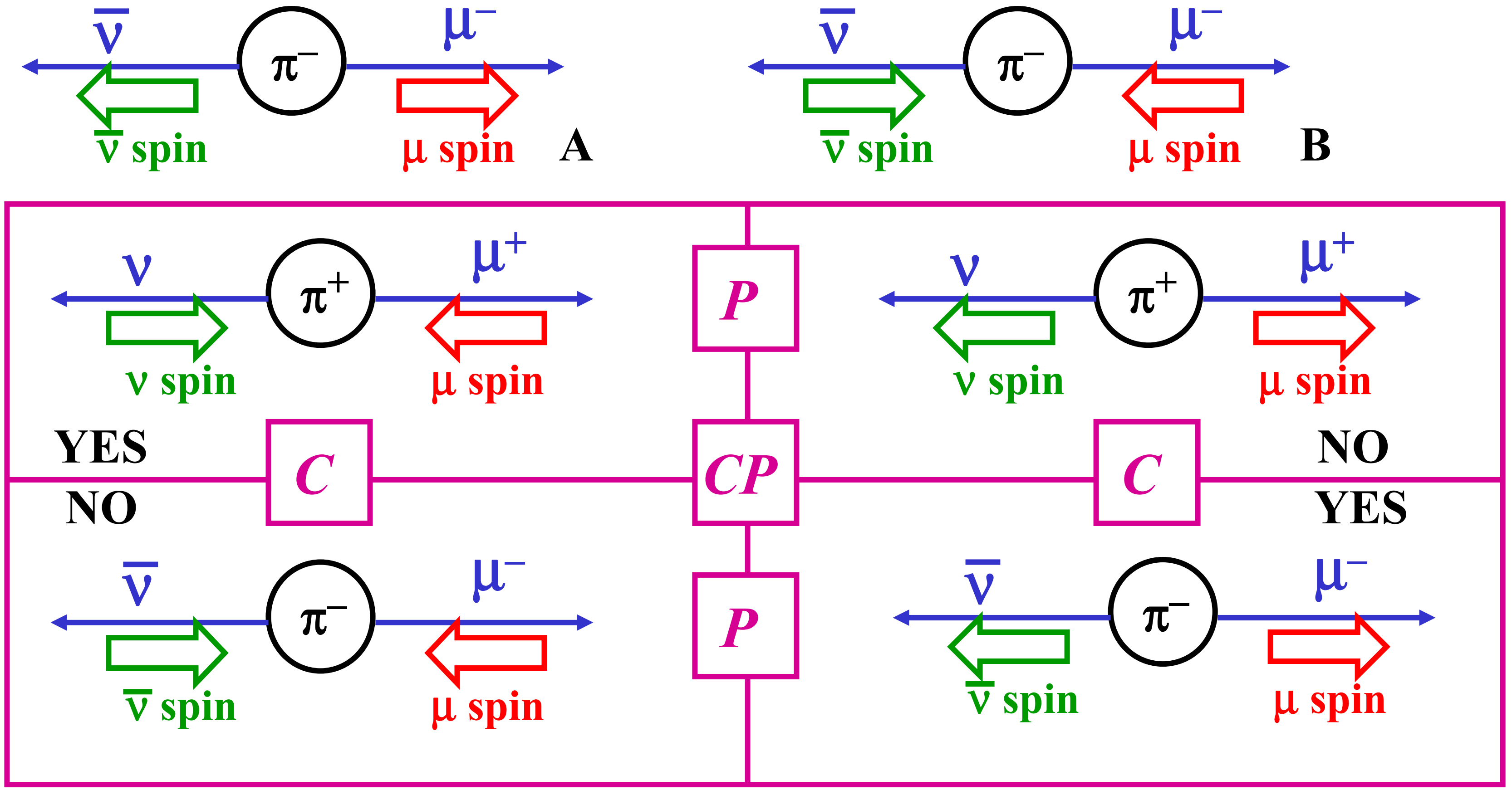
⇒ MAXIMAL VIOLATION OF PARITY INVARIANCE

CHARGE CONJUGATION (*C*)

Particle ⇔ antiparticle transformation

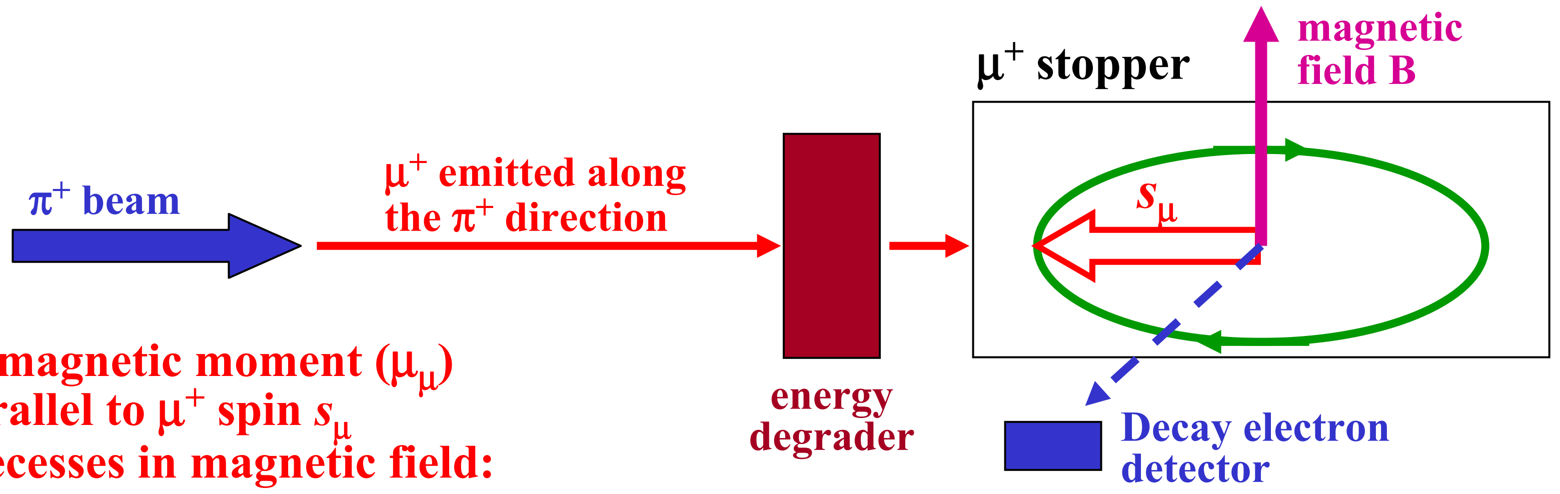
$\pi^- \rightarrow \mu^- + \overline{\nu}$ **decay**

Experiments find that state B does not exist



➡ **π – meson decay violates maximally *C* and *P* invariance, but is invariant under *CP***

Method to measure the μ^+ polarization (R.L. Garwin, 1957)



π^+ beam

μ^+ emitted along the π^+ direction

energy degrader

μ^+ stopper

magnetic field B

s_μ

Decay electron detector

μ^+ magnetic moment (μ_μ) parallel to μ^+ spin s_μ precesses in magnetic field:

precession rate $\omega = 2\,\mu_\mu\,B\,/\,\hbar$

Electron angular distribution from μ^+ decay at rest :

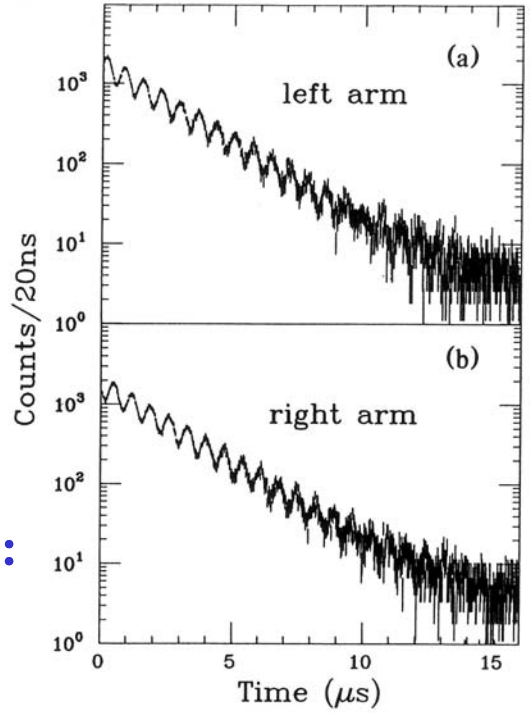
$$dN\,/\,d\,\Omega = 1 + \alpha\cos\theta$$

θ: angle between electron direction and μ^+ spin s_μ

$\cos\theta \propto s_\mu \cdot p_e$ (term violating *P* invariance)

Spin precession: $\cos\theta \Rightarrow \cos(\omega t + \phi)$

$\Rightarrow$ modulation of the decay electron time distribution



Experimental results:

- **$\alpha = -1/3 \Rightarrow$ evidence for *P* violation in μ^+ decay**
- **Simultaneous measurement of the μ^+ magnetic moment:**

$$\mu_\mu = \frac{e\hbar}{2m_\mu} \approx 2.79\times10^{-7}\ [\text{eV/T}]$$

Another neutrino

A puzzle of the late 1950's: the absence of $\mu \rightarrow e\,\gamma$ decays

Experimental limit: < 1 in 10^6 $\mu^+ \rightarrow e^+ \nu\ \bar{\nu}$ decays

A possible solution: existence of a new, conserved "muonic" quantum number distinguishing muons from electrons

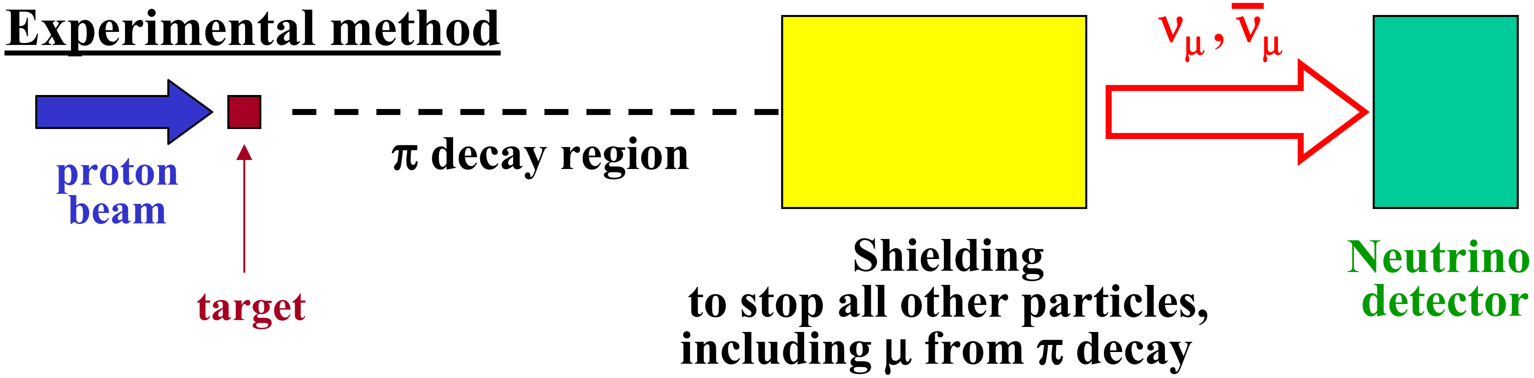
To allow $\mu^+ \rightarrow e^+ \nu\ \bar{\nu}$ decays, $\bar{\nu}$ must have "muonic" quantum number but not ν $\Rightarrow$ in μ^+ decay the $\bar{\nu}$ is not the antiparticle of ν

$\Rightarrow$ two distinct neutrinos (ν_e , ν_μ) in the decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$

Consequence for π – meson decays: $\pi^+ \rightarrow \mu^+ \nu_\mu$; $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ to conserve the "muonic" quantum number

High energy proton accelerators: intense sources of $\pi^\pm$ – mesons $\Rightarrow$ $\nu_\mu , \bar{\nu}_\mu$

Experimental method



$\nu_\mu , \bar{\nu}_\mu$

proton beam

target

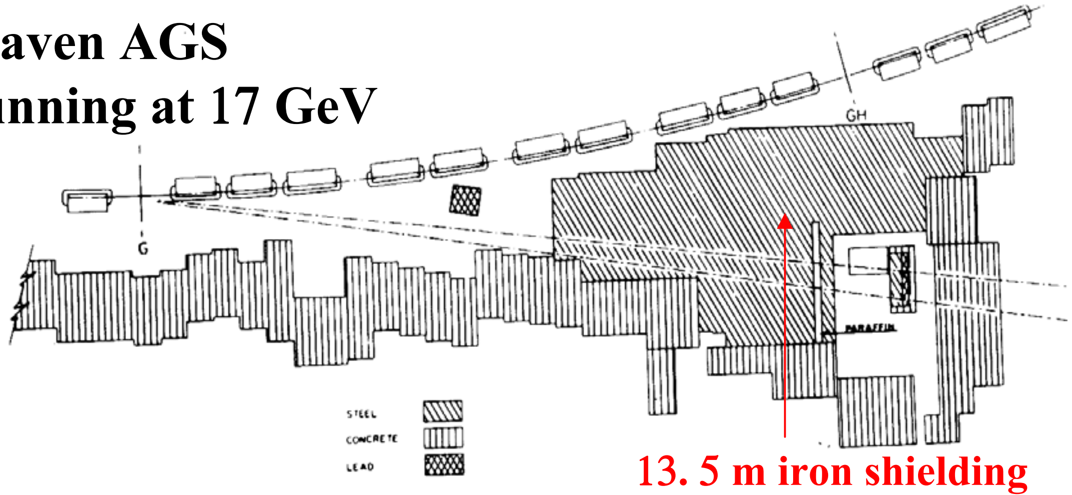
π decay region

Shielding
to stop all other particles,
including μ from π decay

Neutrino detector

If $\nu_\mu \neq \nu_e$, ν_μ interactions produce μ^- and not e^- (example: $\nu_\mu + n \rightarrow \mu^- + p$)

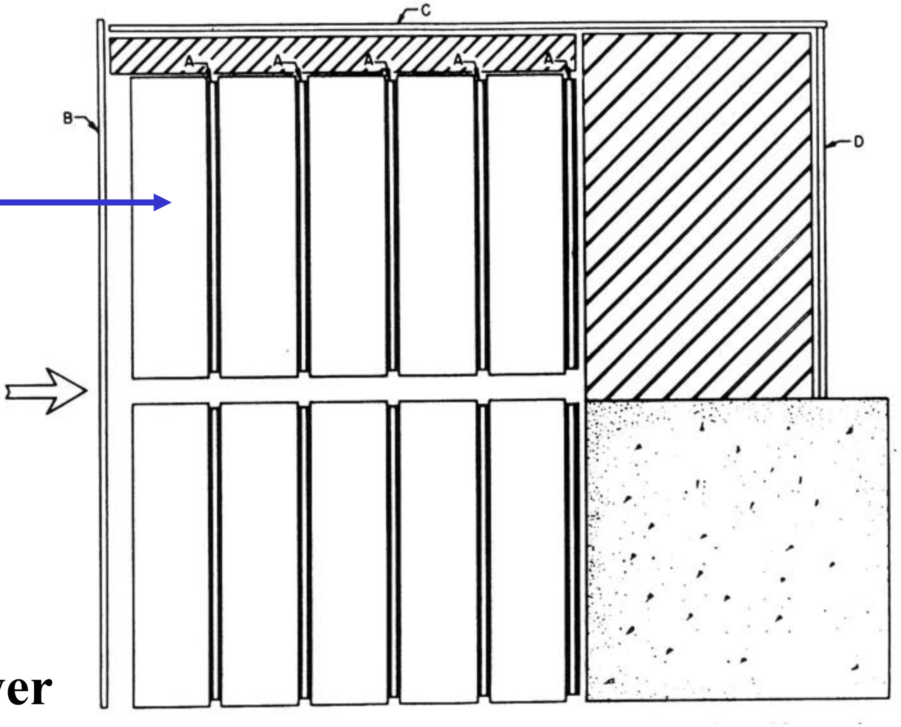
1962: ν_μ discovery at the Brookhaven AGS (a 30 GeV proton synchrotron running at 17 GeV for the neutrino experiment)



13. 5 m iron shielding
(enough to stop 17 GeV muons)

Neutrino energy spectrum
known from π , K production
and $\pi \to \mu$, K $\to \mu$ decay kinematics

Spark chamber
each with 9 Al plates
(112 x 112 x 2.5 cm)
mass 1 Ton



Neutrino detector

Muon – electron separation

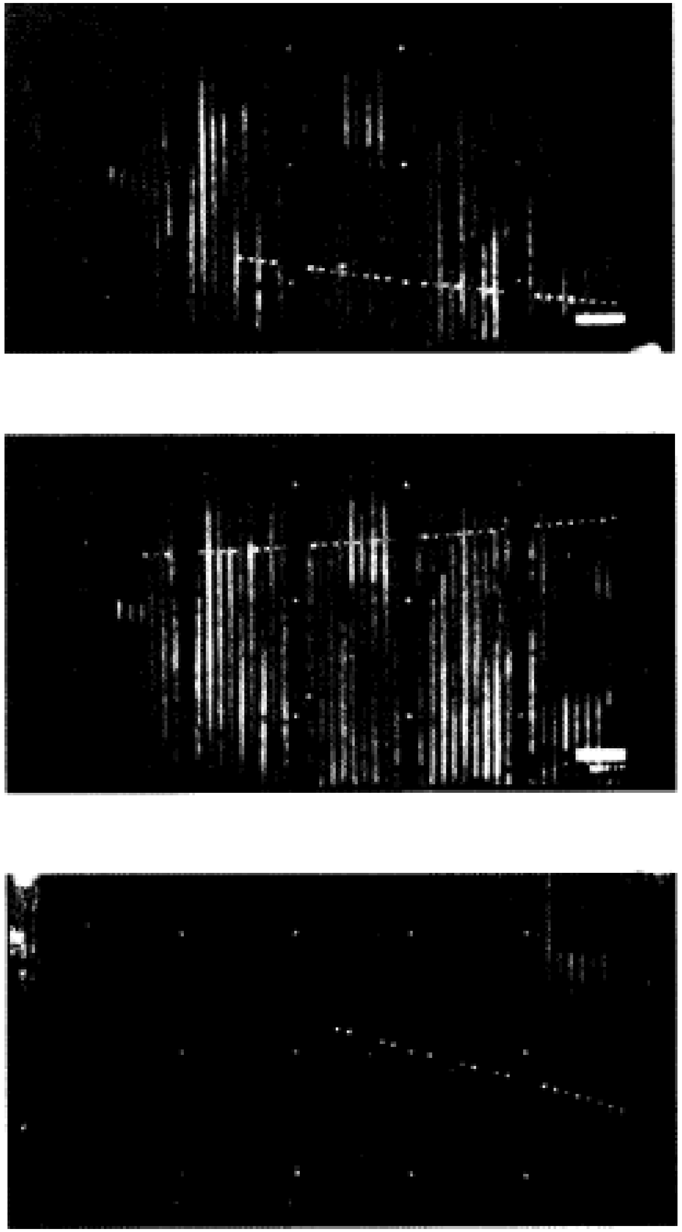
Muon: long track

Electron: short, multi-spark event
from electromagnetic shower

64 “events” from a 300 hour run:

- **34 single track events, consistent with μ track**
- **2 events consistent with electron shower**
  (from small, calculable ν_e contamination in beam)

Clear demonstration that $\nu_\mu \neq \nu_e$



Three typical single-track events in the BNL neutrino experiment

THE "STATIC" QUARK MODEL

Late 1950's – early 1960's: discovery of many strongly interacting particles at the high energy proton accelerators (Berkeley Bevatron, BNL AGS, CERN PS), all with very short mean life times ($10^{-20} - 10^{-23}$ s, typical of strong decays)

⇒ catalog of > 100 strongly interacting particles (collectively named "hadrons")

ARE HADRONS ELEMENTARY PARTICLES?

1964 (Gell-Mann, Zweig): Hadron classification into "families"; observation that all hadrons could be built from three spin ½ "building blocks" (named "quarks" by Gell-Mann):

	u	d	s
Electric charge (units $\|e\|$)	+2/3	–1/3	–1/3
Baryonic number	1/3	1/3	1/3
Strangeness	0	0	–1

and three antiquarks ($\overline{u}$, $\overline{d}$, $\overline{s}$) with opposite electric charge and opposite baryonic number and strangeness

Mesons: quark – antiquark pairs

Examples of non-strange mesons:

$$\pi^+ \equiv u\bar{d} \quad ; \quad \pi^- \equiv \bar{u}d \quad ; \quad \pi^0 \equiv (d\bar{d} - u\bar{u})/\sqrt{2}$$

Examples of strange mesons:

$$K^- \equiv s\bar{u} \quad ; \quad \overline{K}^0 \equiv s\bar{d} \quad ; \quad K^+ \equiv \bar{s}u \quad ; \quad K^0 \equiv \bar{s}d$$

Baryons: three quarks bound together

Antibaryons: three antiquarks bound together

Examples of non-strange baryons:

$$\text{proton} \equiv uud \quad ; \quad \text{neutron} \equiv udd$$

Examples of strangeness –1 baryons:

$$\Sigma^+ \equiv suu \quad ; \quad \Sigma^0 \equiv sud \quad ; \quad \Sigma^- \equiv sdd$$

Examples of strangeness –2 baryons:

$$\Xi^0 \equiv ssu \quad ; \quad \Xi^- \equiv ssd$$

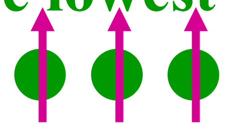
Prediction and discovery of the Ω^- particle

A success of the static quark model

The "decuplet" of spin $\frac{3}{2}$ baryons

Strangeness								**Mass (MeV/c^2)**
0	N^{*++} *uuu*		N^{*+} *uud*		$N^{*\circ}$ *udd*		N^{*-} *ddd*	1232
–1		Σ^{*+} *suu*		$\Sigma^{*\circ}$ *sud*		Σ^{*-} *sdd*		1384
–2			$\Xi^{*\circ}$ *ssu*		Ξ^{*-} *ssd*			1533
–3				Ω^- *sss*				1672 (predicted)

Ω^-: the bound state of three s – quarks with the lowest mass with total angular momentum = 3/ 2 ⇒



Pauli's exclusion principle requires that the three quarks cannot be identical

The first Ω^- event (observed in the 2 m liquid hydrogen bubble chamber at BNL using a 5 GeV/c K^- beam from the 30 GeV AGS)

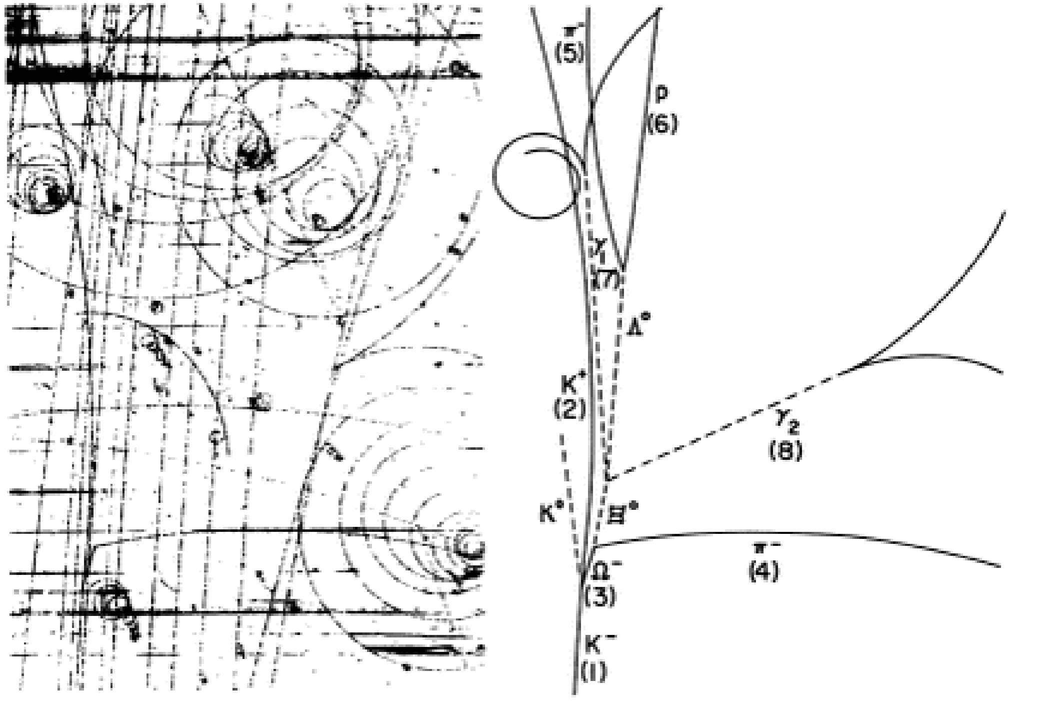
Chain of events in the picture:

$K^- + p \rightarrow \Omega^- + K^+ + K^\circ$
(strangeness conserving)

$\Omega^- \rightarrow \Xi^\circ + \pi^-$
($\Delta S = 1$ weak decay)

$\Xi^\circ \rightarrow \pi^\circ + \Lambda$
($\Delta S = 1$ weak decay)

$\Lambda \rightarrow \pi^- + p$
($\Delta S = 1$ weak decay)



$\pi^\circ \rightarrow \gamma + \gamma$ (electromagnetic decay)
with both γ – rays converting to an e^+e^- in liquid hydrogen
(very lucky event, because the mean free path for $\gamma \rightarrow e^+e^-$ in liquid hydrogen is ~10 m)

Ω^- mass measured from this event = 1686 ± 12 MeV/c^2

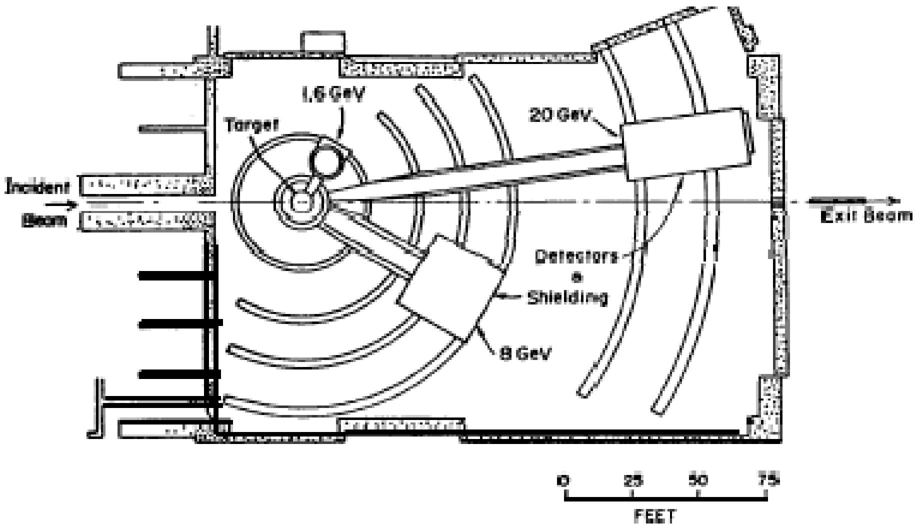
“DYNAMIC” EVIDENCE FOR QUARKS

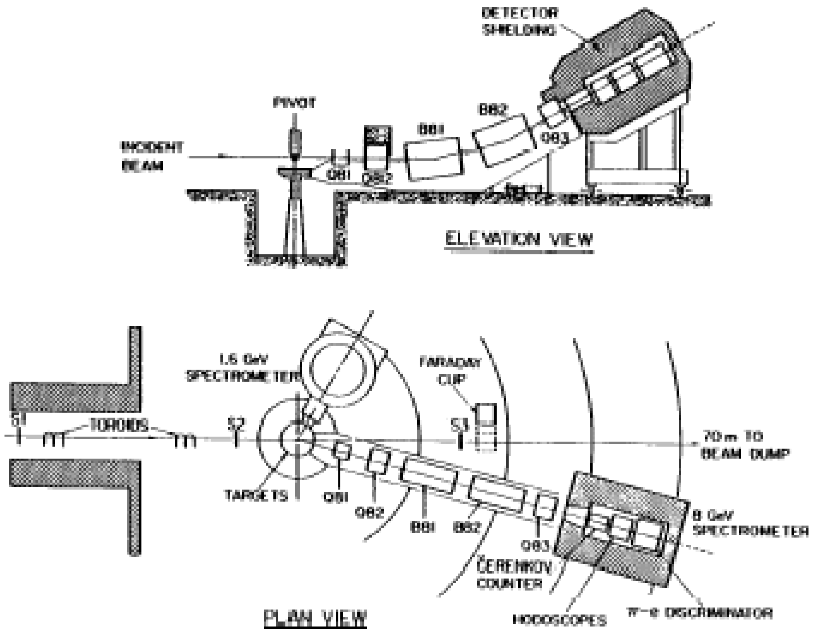
Electron – proton scattering using a 20 GeV electron beam from the Stanford two – mile Linear Accelerator (1968 – 69).

The modern version of Rutherford’s original experiment:
resolving power ≈ wavelength associated with 20 GeV electron ≈ 10^{-15} cm

Three magnetic spectrometers to detect the scattered electron:

- **20 GeV spectrometer (to study elastic scattering $e^- + p \to e^- + p$)**
- **8 GeV spectrometer (to study inelastic scattering $e^- + p \to e^- +$ hadrons)**
- **1.6 GeV spectrometer (to study extremely inelastic collisions)**





The Stanford two-mile electron linear accelerator (SLAC)

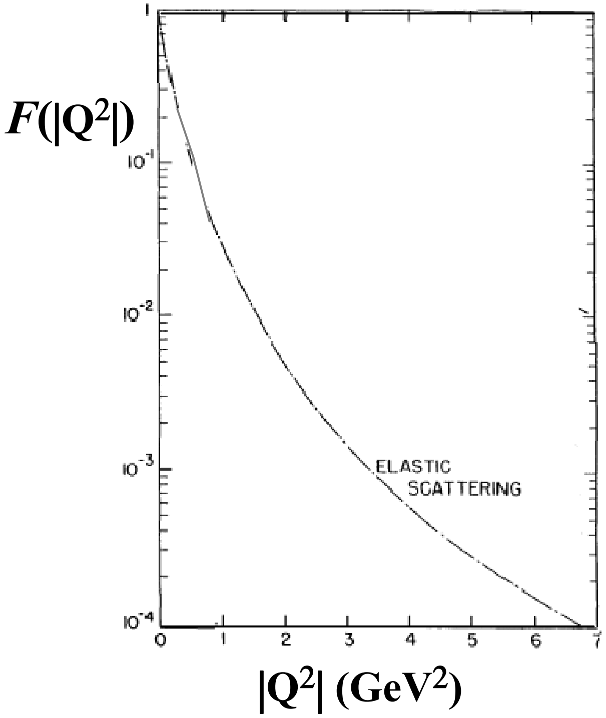
Electron elastic scattering from a point-like charge $|e|$ at high energies: differential cross-section in the collision centre-of-mass (Mott's formula)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 (\hbar c)^2}{8E^2} \frac{\cos^2(\theta/2)}{\sin^4(\theta/2)} \equiv \sigma_M \qquad \left(\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137} \right)$$

Scattering from an extended charge distribution: multiply σ_M by a "form factor":

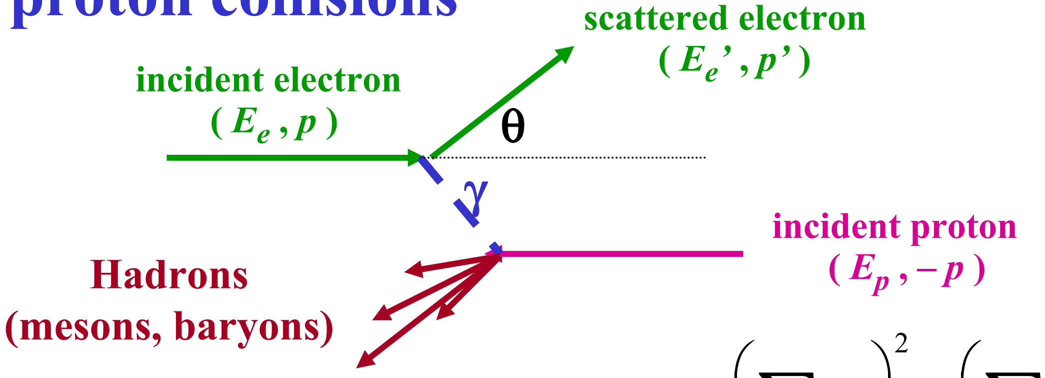
$$\frac{d\sigma}{d\Omega} = F(|Q^2|)\sigma_M$$

$|Q| = \hbar / D$: mass of the exchanged virtual photon
D: linear size of target region contributing to scattering
Increasing $|Q|$ ⇒ decreasing target electric charge



F ($|Q^2|$) = 1 for a point-like particle
⇒ the proton is not a point-like particle

Inelastic electron – proton collisions



scattered electron (E_e' , p')

incident electron (E_e , p)

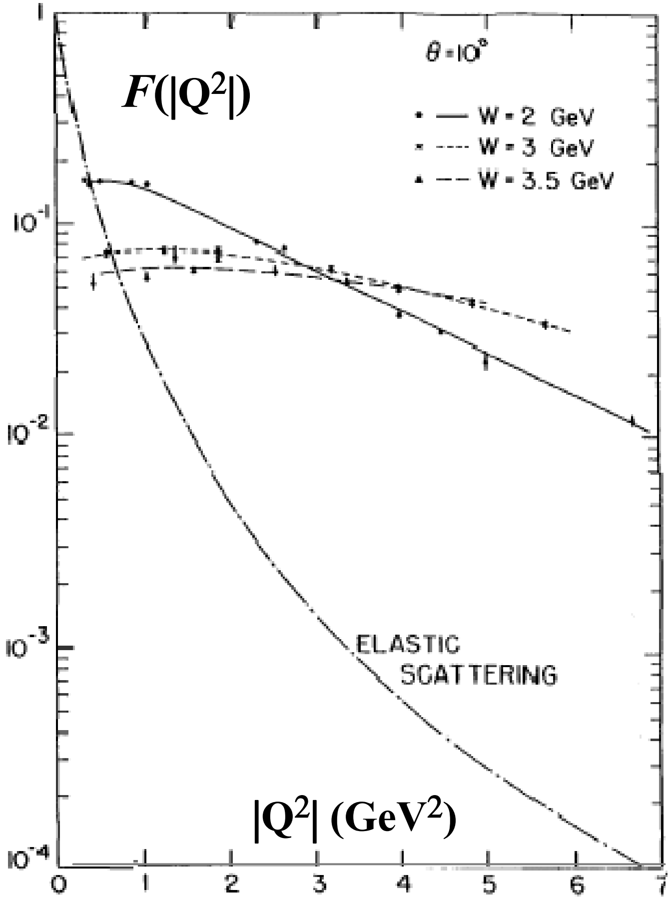
θ

γ

incident proton (E_p , $-p$)

Hadrons (mesons, baryons)

Total hadronic energy : $W^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i \vec{p}_i\right)^2 c^2$



For deeply inelastic collisions, the cross-section depends only weakly on $|Q^2|$, suggesting a collision with a POINT-LIKE object

Interpretation of deep inelastic e - p collisions

Deep inelastic electron – proton collisions are elastic collisions with point-like, electrically charged, spin ½ constituents of the proton carrying a fraction *x* of the incident proton momentum

Each constituent type is described by its electric charge e_i (units of $|e|$) and by its *x* distribution (dN_i/dx) ("structure function")

If these constituents are the *u* and *d* quarks, then deep inelastic e – p collisions provide information on a particular combination of structure functions:

$$\left(\frac{dN}{dx}\right)_{\mathrm{e-p}} = e_u^{\,2}\frac{dN_u}{dx} + e_d^{\,2}\frac{dN_d}{dx}$$

Comparison with ν_μ – p and $\bar{\nu}_\mu$ – p deep inelastic collisions at high energies under the assumption that these collisions are also elastic scatterings on quarks

$\nu_\mu + \mathrm{p} \rightarrow \mu^- + \mathrm{hadrons}$: $\nu_\mu + d \rightarrow \mu^- + u$ **(depends on dN_d / dx)**

$\bar{\nu}_\mu + \mathrm{p} \rightarrow \mu^+ + \mathrm{hadrons}$: $\bar{\nu}_\mu + u \rightarrow \mu^+ + d$ **(depends on dN_u / dx)**

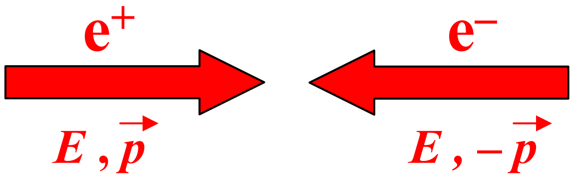
(Neutrino interactions do not depend on electric charge)

All experimental results on deep inelastic e – p , ν_μ – p, $\bar{\nu}_\mu$ – p collisions are consistent with $e_u^2 = 4/9$ and $e_d^2 = 1/9$

➡ the proton constituents are the quarks

PHYSICS WITH e^+e^- COLLIDERS

Two beams circulating in opposite directions in the same magnetic ring and colliding head-on



A two-step process: $e^+ + e^- \rightarrow$ virtual photon $\rightarrow f + \overline{f}$

f : any electrically charged elementary spin ½ particle (μ , quark)

(excluding e^+e^- elastic scattering)

Virtual photon energy – momentum : $E_\gamma = 2E\,,\ p_\gamma = 0 \Rightarrow Q^2 = E_\gamma{}^2 - p_\gamma{}^2 c^2 = 4E^2$

Cross - section for $e^+e^- \rightarrow f\,\overline{f}$:

$$\sigma = \frac{2\pi\alpha^2\hbar^2c^2}{3Q^2} e_f{}^2 \beta(3-\beta)$$

$\alpha = e^2/(\hbar c) \approx 1/137$

e_f: electric charge of particle f (units $|e|$)

$\beta = v/c$ of outgoing particle f

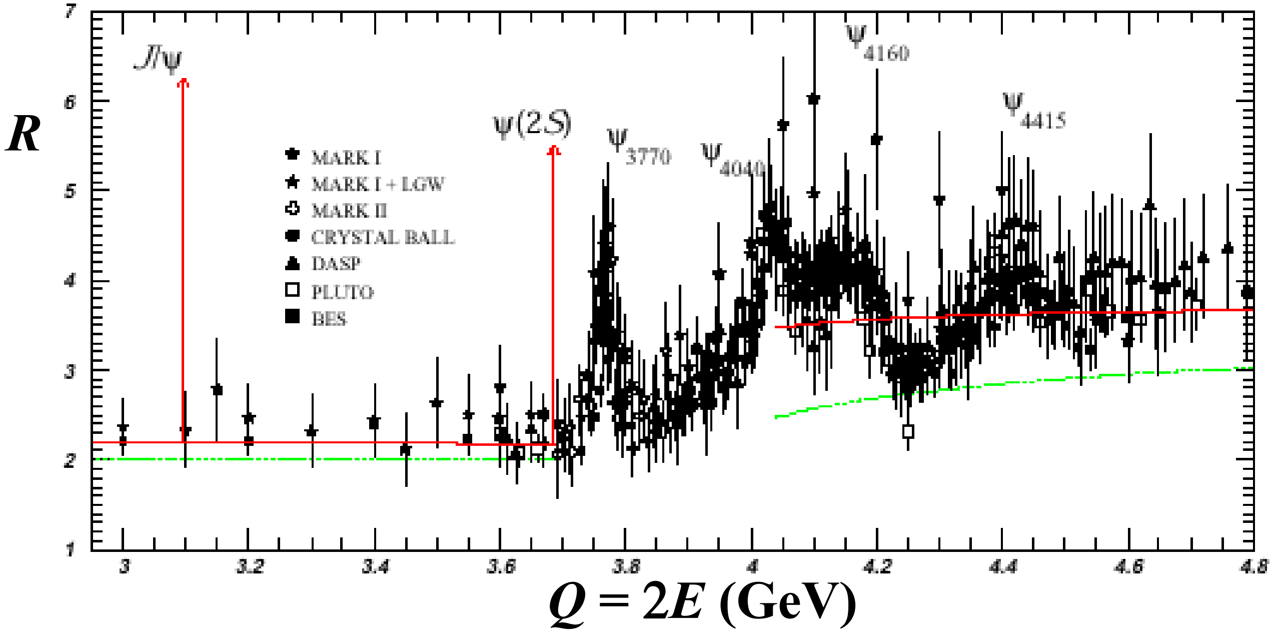
(formula precisely verified for $e^+e^- \rightarrow \mu^+\mu^-$)

Assumption: $e^+e^- \rightarrow$ quark (q) + antiquark ($\overline{q}$) $\rightarrow$ hadrons

$\Rightarrow$ at energies $E >> m_q c^2$ (for $q = u\,,d\,,s$) $\beta \approx 1$:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = e_u{}^2 + e_d{}^2 + e_s{}^2 = \frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{2}{3}$$

Experimental results from the Stanford e^+e^- collider SPEAR (1974 –75):



- For $Q < 3.6$ GeV $R \approx 2$. If each quark exists in three different states, $R \approx 2$ is consistent with 3 x (2 / 3). This would solve the Ω^- problem.

- Between 3 and 4.5 GeV, the peaks and structures are due to the production of quark-antiquark bound states and resonances of a fourth quark ("charm", *c*) of electric charge +2/3

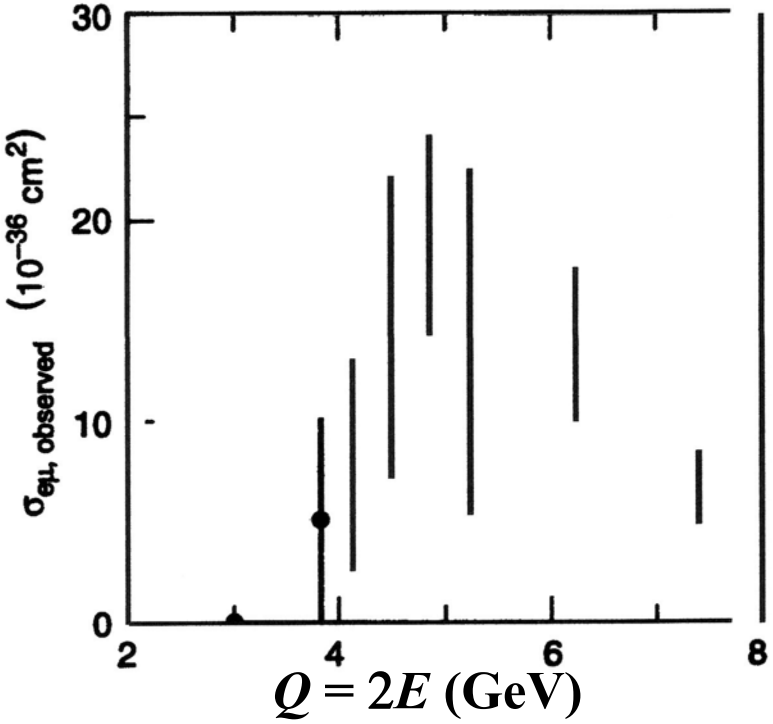
- Above 4.6 GeV $R \approx 4.3$. Expect $R \approx 2$ (from *u*, *d*, *s*) + 3 x (4 / 9) = 3.3 from the addition of the *c* quark alone. So the data suggest pair production of an additional elementary spin ½ particle with electric charge = 1 (later identified as the τ – lepton (no strong interaction) with mass ≈ 1777 MeV/c^2).

$$e^{+} + e^{-} \rightarrow \tau^{\pm} + \tau^{\mp}$$

$$\hookrightarrow \mu^{\mp}\nu\bar{\nu}$$

$$\hookrightarrow e^{\pm}\nu\bar{\nu}$$

Final state : an electron – muon pair + missing energy



Evidence for production of pairs of heavy leptons $\tau^{\pm}$

THE MODERN THEORY OF STRONG INTERACTIONS:

the interactions between quarks based on "Colour Symmetry"
Quantum ChromoDynamics (QCD) formulated in the early 1970's

- **Each quark exists in three states of a new quantum number named "colour"**
- **Particles with colour interact strongly through the exchange of spin 1 particles named "gluons", in analogy with electrically charged particles interacting electromagnetically through the exchange of spin 1 photons**

A MAJOR DIFFERENCE WITH THE ELECTROMAGNETIC INTERACTION

Electric charge: positive or negative

Photons have no electric charge and there is no direct photon-photon interaction

Colour: three varieties

Mathematical consequence of colour symmetry: the existence of eight gluons with eight variety of colours, with direct gluon – gluon interaction

- **The observed hadrons (baryons, mesons) are colourless combinations of coloured quarks and gluons**
- **The strong interactions between baryons, mesons is an "apparent" interaction between colourless objects, in analogy with the apparent electromagnetic interaction between electrically neutral atoms**

Free quarks, gluons have never been observed experimentally; only indirect evidence from the study of hadrons – WHY?

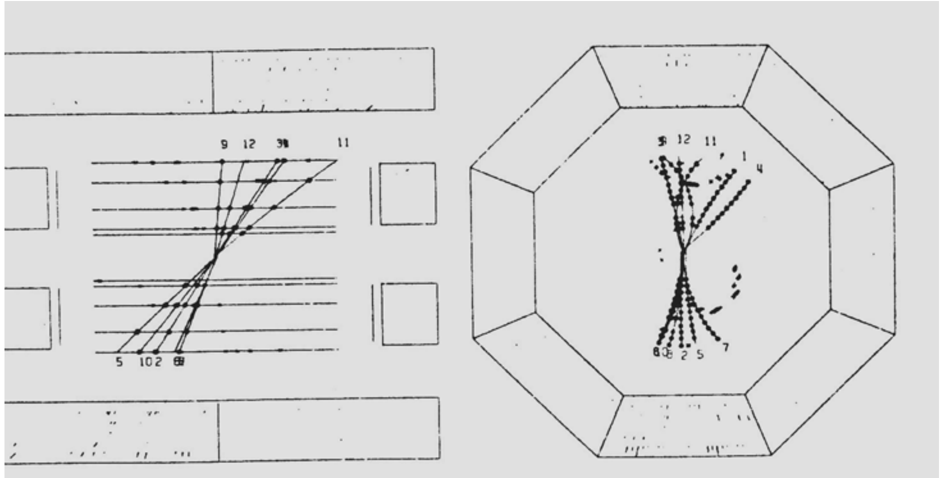
CONFINEMENT: coloured particles are confined within colourless hadrons because of the behaviour of the colour forces at large distances

The attractive force between coloured particles increases with distance ⇒ increase of potential energy ⇒ production of quark – antiquark pairs which neutralize colour ⇒ formation of colourless hadrons (hadronization)

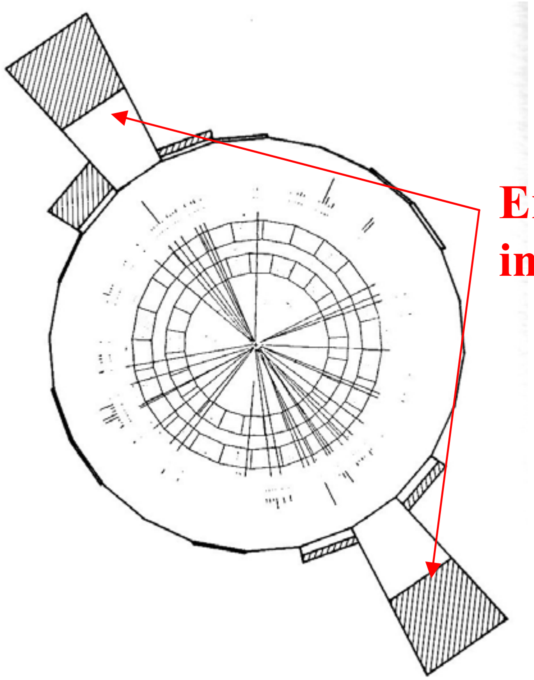
At high energies (e.g., in $e^+e^- \rightarrow q + \overline{q}$) expect the hadrons to be produced along the initial direction of the $q - \overline{q}$ pair ⇒ production of hadronic "jets"

CONFINEMENT, HADRONIZATION: properties deduced from observation. So far, the properties of colour forces at large distance have no precise mathematical formulation in QCD.

$e^+ + e^- \to$ hadrons
A typical event at
$Q = 2E = 35$ GeV:
reconstructed
charged particle tracks



A typical proton-antiproton collision
at the CERN $\overline{\mathrm{p}}$ p collider (630 GeV)
producing high-energy hadrons at
large angles to the beam axis
(UA2 experiment, 1985)



Energy depositions
in calorimeters

1962-66: Formulation of a Unified Electroweak Theory

(Glashow, Salam, Weinberg)

4 intermediate spin 1 interaction carriers (“bosons”):

- the photon (γ)
  responsible for all electromagnetic processes

- three weak, heavy bosons W^{+} W^{-} Z
  $W^{\pm}$ responsible for processes with electric charge transfer = ±1
  (Charged Current processes)

Examples:

$n \rightarrow p\ e^{-}\ \overline{\nu}$: $n \rightarrow p + W^{-}$ followed by $W^{-} \rightarrow e^{-}\overline{\nu}$

$\mu^{+} \rightarrow e^{+}\nu_{e}\overline{\nu}_{\mu}$: $\mu^{+} \rightarrow \overline{\nu}_{\mu} + W^{+}$ followed by $W^{+} \rightarrow e^{+}\nu_{e}$

Z responsible for weak processes with no electric charge transfer
(Neutral Current processes)
PROCESSES NEVER OBSERVED BEFORE
Require neutrino beams to search for these processes, to remove the much larger electromagnetic effects expected with charged particle beams

First observation of Neutral Current processes in the heavy liquid bubble chamber Gargamelle at the CERN PS (1973)

Example of

$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$

(elastic scattering)

Recoil electron energy = 400 MeV

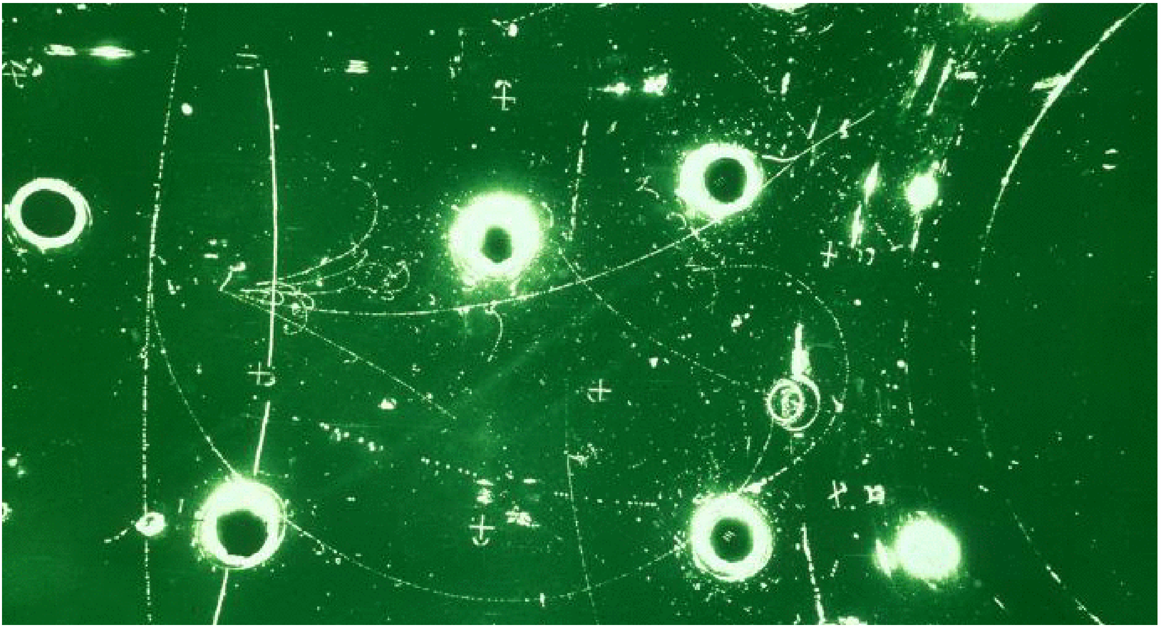
($\bar{\nu}_\mu$ beam from π^- decay in flight)

Example of

$\nu_\mu + p\ (n) \rightarrow \nu_\mu +$ hadrons

(inelastic interaction)

(ν_μ beam from π^+ decay in flight)

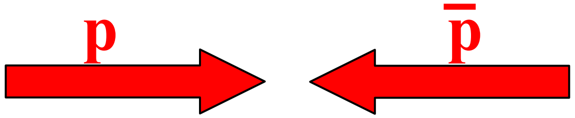


Measured rates of Neutral Current events ⇒ estimate of the W and Z masses (not very accurately, because of the small number of events):

$$M_W \approx 70 - 90\ \mathrm{GeV}/c^2 \quad ; \quad M_Z \approx 80 - 100\ \mathrm{GeV}/c^2$$

too high to be produced at any accelerator in operation in the 1970's

1975: Proposal to transform the new 450 GeV CERN proton synchrotron (SPS) into a proton – antiproton collider (C. Rubbia)



Beam energy = 315 GeV ⇒ total energy in the centre-of-mass = 630 GeV

Beam energy necessary to achieve the same collision energy on a proton at rest :

$$(E + m_p c^2)^2 - p^2c^2 = (630\,\mathrm{GeV})^2 \Longrightarrow \boldsymbol{E} = \mathbf{210\ TeV}$$

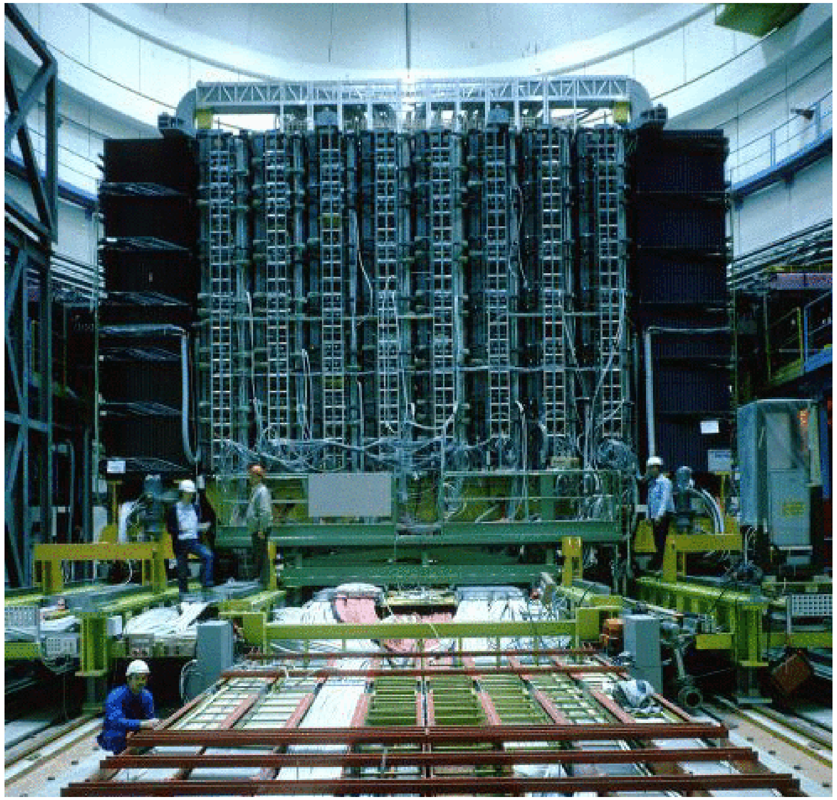
Production of W and Z by quark – antiquark annihilation:

$$u + \bar{d} \rightarrow W^+ \qquad \bar{u} + d \rightarrow W^-$$

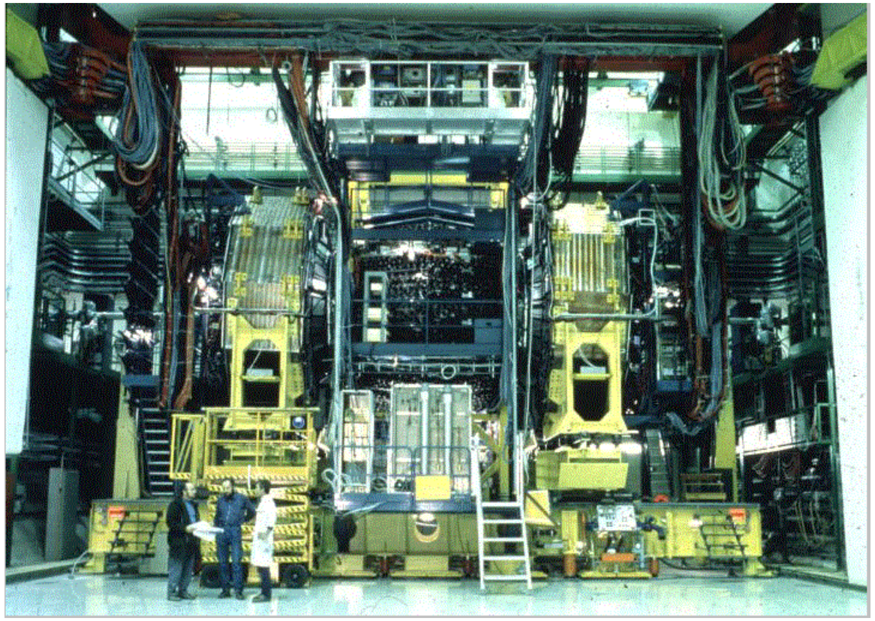
$$u + \bar{u} \rightarrow Z \qquad d + \bar{d} \rightarrow Z$$

UA1 and UA2 experiments (1981 – 1990)

Search for $W^{\pm} \rightarrow e^{\pm} + \nu$ (UA1, UA2) ; $W^{\pm} \rightarrow \mu^{\pm} + \nu$ (UA1)
$Z \rightarrow e^{+}e^{-}$ (UA1, UA2) ; $Z \rightarrow \mu^{+}\mu^{-}$ (UA1)

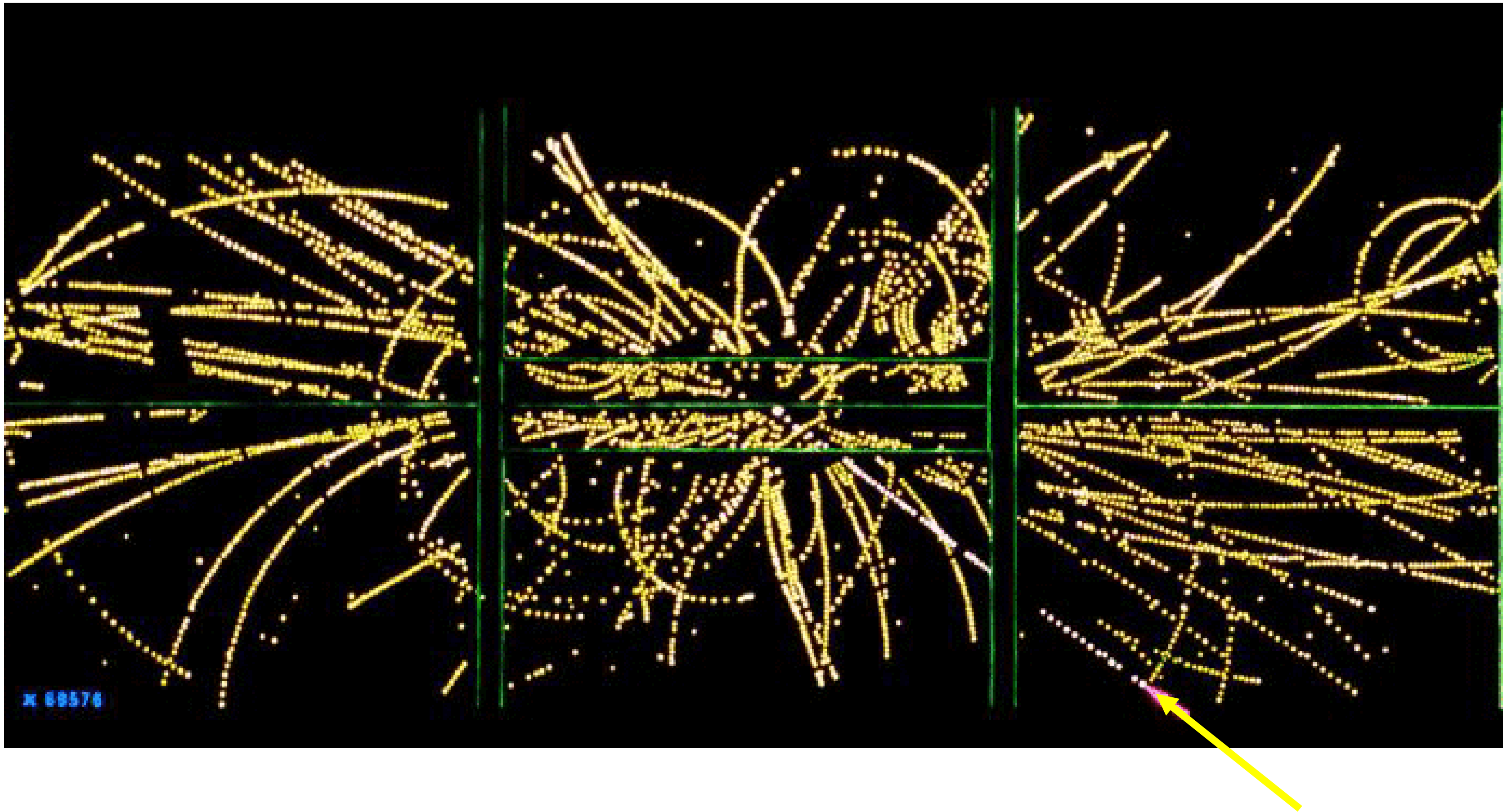


UA1: magnetic volume with trackers, surrounded by "hermetic" calorimeter and muon detectors



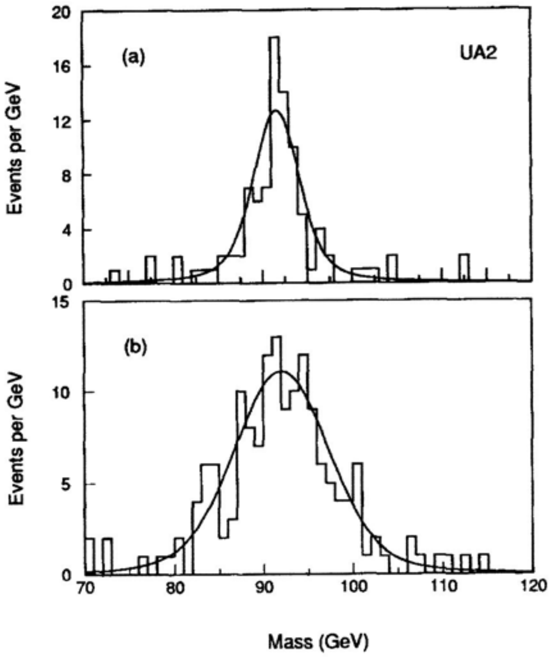
UA2: non-magnetic, calorimetric detector with inner tracker

One of the first $W \rightarrow e + \nu$ events in UA1



48 GeV electron
identified by
surrounding calorimeters

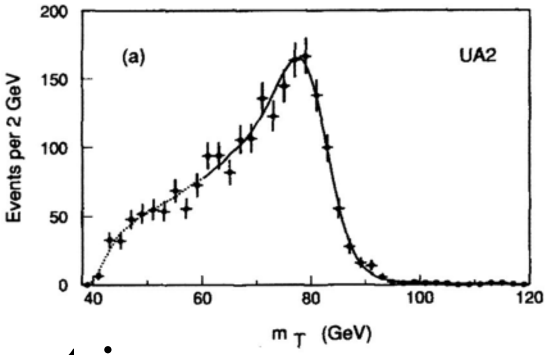
UA2 final results



Events containing two high-energy electrons:
Distributions of the "invariant mass" M_{ee}

$$(M_{ee}c^2)^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 c^2$$

(for $Z \rightarrow e^+e^-$ $M_{ee} = M_Z$)



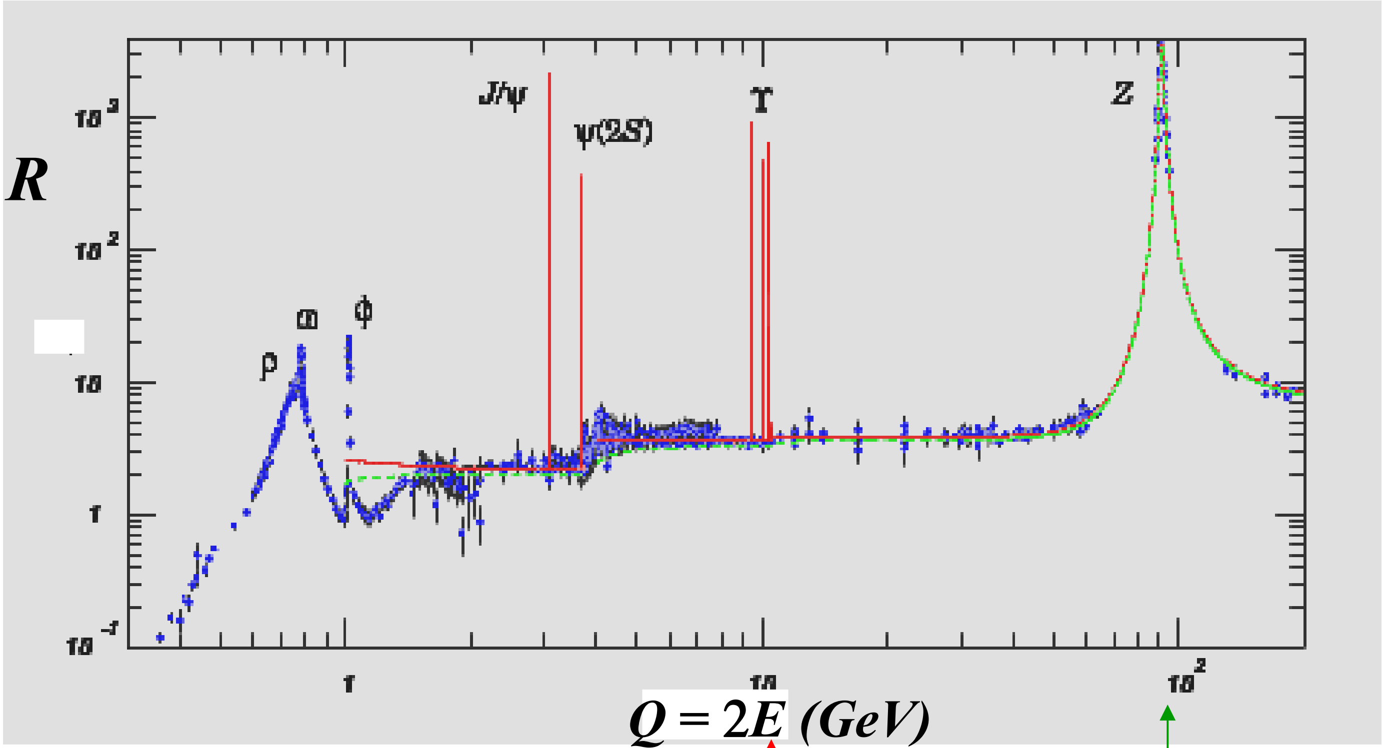
Events containing a single electron with large transverse momentum (momentum component perpendicular to the beam axis) and large missing transverse momentum (apparent violation of momentum conservation due to the escaping neutrino from $W \rightarrow e\nu$ decay)

m_T ("transverse mass"): invariant mass of the electron – neutrino pair calculated from the transverse components only

M_W is determined from a fit to the m_T distribution: $M_W = 80.35 \pm 0.37$ GeV/c^2

e^+e^- colliders at higher energies

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$ between 0.3 and 200 GeV

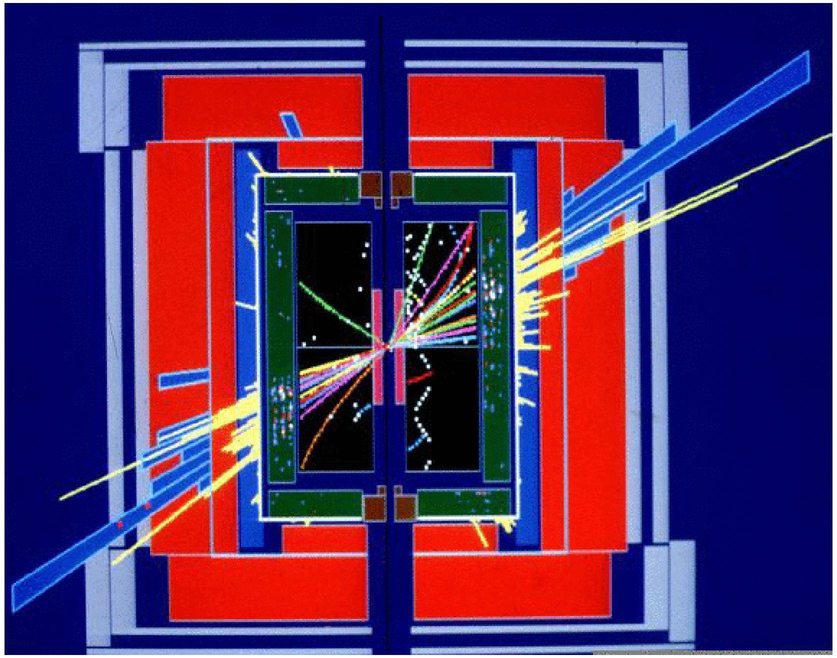


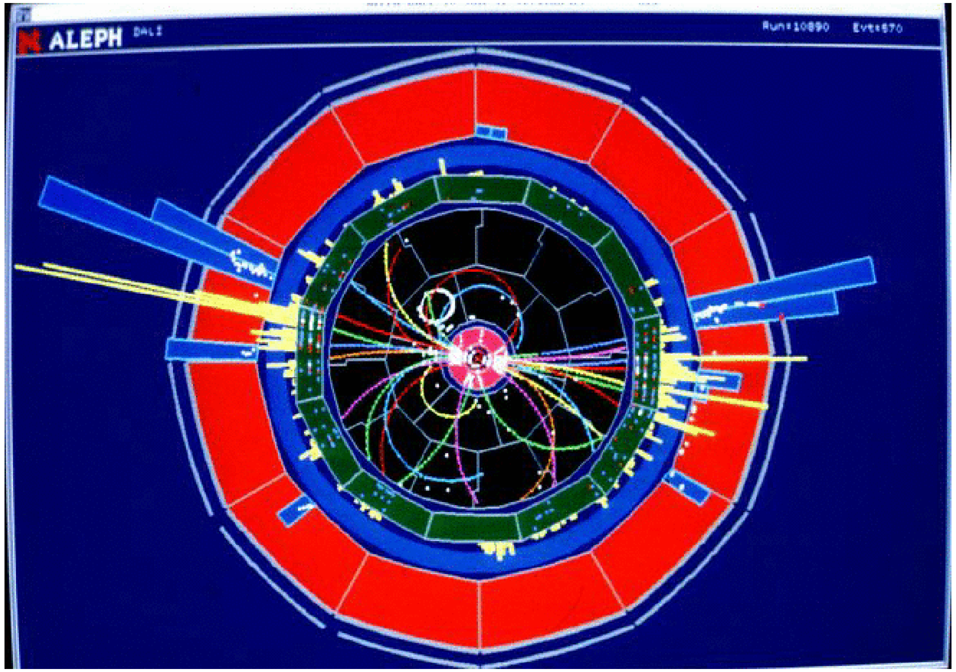
$e^+e^- \rightarrow b\ \bar{b}$

(the 5th quark: $e = -1/3$)

$e^+e^- \rightarrow Z \rightarrow q\ \bar{q}$

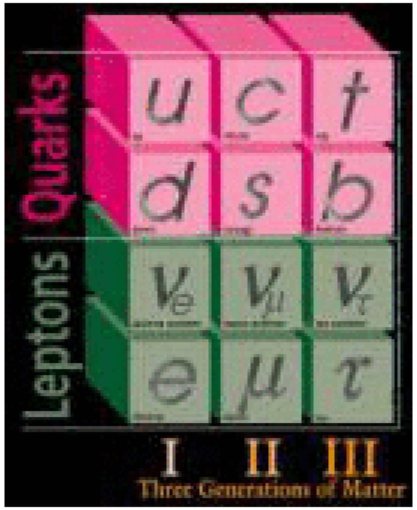
The two orthogonal views of an event $Z \to q\,\overline{q} \to$ hadrons at LEP (ALEPH detector)





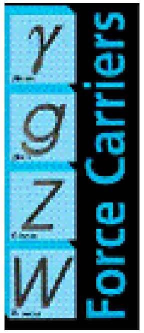
CONCLUSIONS

The elementary particles today:



3 x 6 = 18 quarks
+ 6 leptons
= 24 fermions (constituents of matter)
+ 24 antiparticles
48 elementary particles

consistent with point-like dimensions within the resolving power of present instrumentation (~ 10^{-16} cm)



12 force carriers (γ, $W^{\pm}$, Z, 8 gluons)

+ the Higgs spin 0 particle (NOT YET DISCOVERED)
responsible for generating the masses of all particles