An Energy Preserving Time Integration Method for Gyric Systems: Development of the Offshore Wind Energy Simulation Toolkit

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Abstract:

This paper presents an overview on the development of a finite element design tool for offshore vertical-axis wind turbines (VAWTs). VAWT configurations possess desirable characteristics for large offshore wind applications, and motivation for considering this configuration is discussed. The modular and flexible finite element framework of the Offshore Wind ENergy Simulation (OWENS) toolkit is presented. This paper also presents an energy preserving time integration method that has been implemented into OWENS. A previously developed time integration method has been extended to rotational systems. The method utilizes system energy to construct an unconditionally stable integration scheme. Consequently, for conservative systems energy is preserved regardless of time step size. The integration method is demonstrated on a representative VAWT configuration with aerodynamic and platform effects, and compared to another popular integration method. Overall, desirable properties of the energy preserving integration scheme are observed.

1. Introduction

Availability of offshore wind resources in coastal regions makes offshore wind energy an attractive opportunity. There are, however, significant challenges in realizing offshore wind energy with an acceptable cost of energy due to increased infrastructure, logistics, and operations and maintenance costs. As this paper will show, the vertical-axis wind turbine (VAWT) [1] has the potential to alleviate many challenges encountered by the application of HAWTs to large offshore wind projects. Although tools exist for offshore [2] and vertical-axis [1,3,4] turbine design, offshore VAWTs require unique considerations better addressed through a new, custom design tool. Furthermore, this software can serve as an open-source, modular foundation for future offshore wind energy research. An overview of the OWENS toolkit is given, and the modular, finite element framework of the tool is highlighted. To aid in transient analysis, an energy preserving time integration method has been extended to rotational systems, and implemented into OWENS. This time integration method may prevent spurious energy trends from being introduced into simulation results. A concept from celestial mechanics known as the Jacobi integral [5] is applied to a general rotational system to arrive at a conserved energy quantity from which this stable, energy preserving integration method is developed.

2. Motivation

The availability of offshore wind resources in coastal regions along with a high concentration of load centers in these areas, makes offshore wind energy an attractive opportunity. Infrastructure costs and operation and maintenance (O&M) costs for offshore wind technology, however, are significant obstacles that need to be overcome to make offshore wind a viable option. It has been estimated that a greater than 20% decrease in cost of energy (COE) will be required to ensure the viability of offshore wind energy. This reduction in COE is likely to come from decreases in installation costs and O&M, while increasing energy production.

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Rotor design has a significant impact on all three of these areas, and therefore is critical in reducing COE. Whereas it is estimated that the entire turbine contributes nearly 28% of the life-cycle cost, the actual rotor is only estimated to contribute about 7% of this cost. Therefore, it is more important to consider design configurations that lower the installation, logistics, and O&M costs while increasing energy capture rather than trying to decrease the cost of the rotor itself.

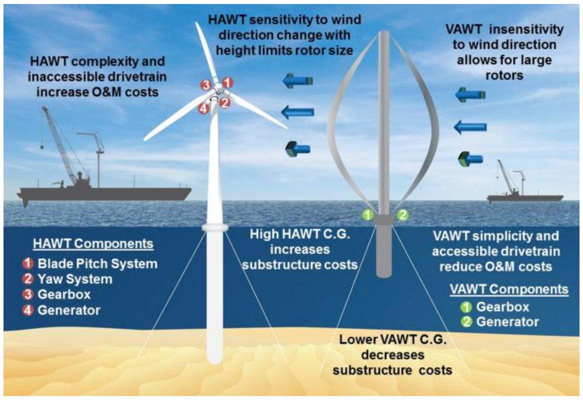

Figure 1. Comparison of VAWT and HAWT for offshore applications

Vertical-axis wind turbines held significant interest in the earlier days of wind energy technology during the 1980s. In the early 1990s, this configuration lost its popularity and the HAWT was adopted as the primary wind turbine configuration. The VAWT configuration, however, can significantly address the need for lower COE in offshore applications. Figure 1 illustrates potential advantages of a VAWT configuration over a HAWT configuration for offshore applications. This is primarily due to the placement of the gearbox and generator at the bottom of the tower. This not only reduces platform cost by lowering the center of gravity of the turbine, but also reduces O&M costs by having components readily accessible near water level. The simplicity of the VAWT configuration compared to the HAWT can also lower rotor costs. The insensitivity of the VAWT to wind direction and the ability to scale the machines to large sizes will increase energy production and further reduce COE. To remain a viable option for offshore wind energy, however, VAWT technology will need to undergo significant development in coming years. Thus, the OWENS toolkit is being developed to assess VAWT designs for offshore environments.

3. Overview of the OWENS Toolkit

To facilitate the development of VAWT technology, robust design tools must be developed to assess innovative design concepts for offshore wind energy technology. Therefore, an aeroelastic design tool is being developed for modeling large offshore VAWT configurations. The OWENS toolkit is able to explore a wide array of offshore VAWT configurations via modal and transient analysis. This tool will interface with aerodynamics, platform dynamics (hydrodynamics), and drive-train/generator modules to predict the response of a VAWT of arbitrary configuration under a variety of conditions. The formulation allows for stability analysis to identify potential resonance and flutter issues. The core of the analysis tool is a robust and flexible finite element framework capable of considering the dynamics of large, flexible, rotating structures.

To facilitate innovative design concepts, a mesh generator has been developed to allow for turbines of arbitrary configuration to be considered. A general beam finite element that accounts for gyroscopic, aeroelastic, and platform effects has been developed. This beam element is the foundation of this modeling effort and has the ability to model couplings within composite materials. This allows an investigation of aeroelastic tailoring in VAWT structures.

Currently, the OWENS toolkit is a robust structural dynamics analysis tool for rotational systems with modal and transient analysis capability. A one-way aerodynamic coupling, and simplified, representative generator /drivetrain, platform and hydrodynamics modules have been implemented. General interfaces for external modules are clearly defined, and external modules may be interfaced with the structural dynamics model via two-way coupling as they become available.

3.1 Analysis framework

The fundamental requirements of the aeroelastic analysis tool for offshore VAWTs necessitate a flexible framework capable of considering arbitrary configuration geometries, arbitrary loading scenarios, and the ability to interface with various modules that account for the

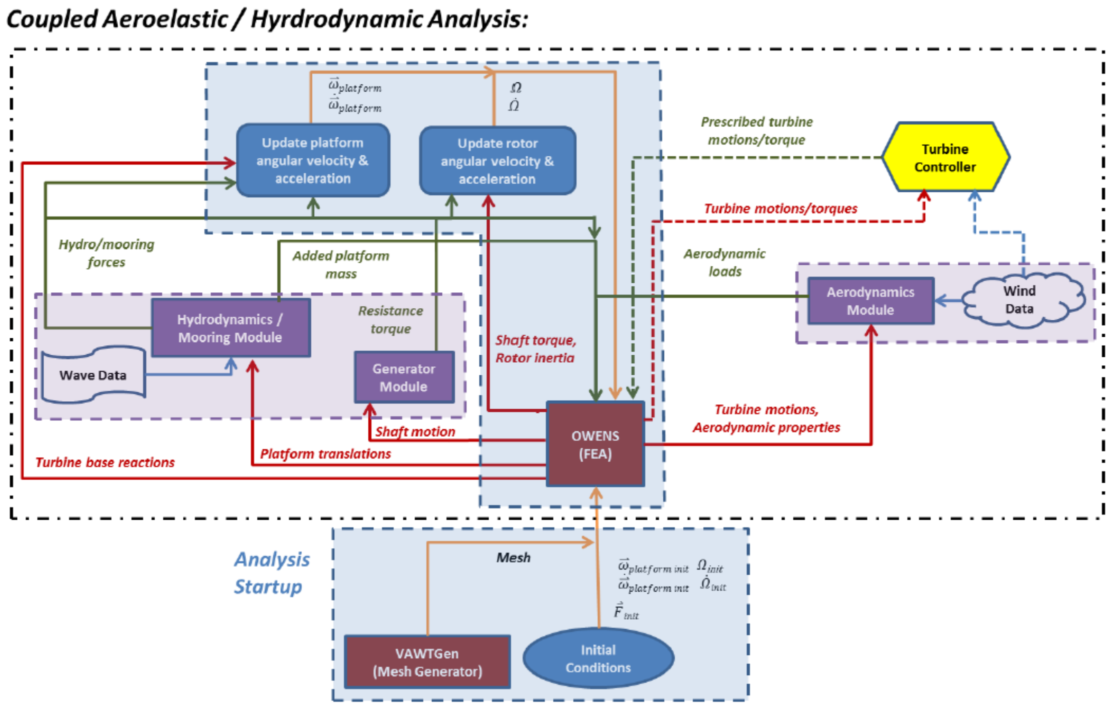

Figure 2. Analysis framework for the OWENS toolkit

interaction of the environment and power generation hardware with motions of the turbine. The finite element method provides a means to satisfy these general requirements.

The finite element method requires boundary conditions to be imposed by specifying loads or displacements at each point in a mesh. These boundary conditions provide a clear interface between aerodynamic and hydrodynamic modules that impart forces on the turbine. With boundary conditions specified, unspecified displacements and loads may be calculated. Next, displacement motions of the turbine may be provided to aerodynamic and hydrodynamic modules to calculate loads on the turbine. The interaction of loadings on the structure and platform will be considered along with generator effects to predict the motions of the turbine. Provisions will be made for a turbine controller as well. Figure 2 shows the analysis framework and the associated flow of information between the core OWENS analysis tool, aerodynamic, hydrodynamic, generator, and controller modules. This implementation is adaptable to modal analysis to assess stability of VAWT configurations and identify potential resonance concerns. Furthermore, the general finite element formulation is easily adaptable to transient analysis for investigation of start-up and shut-down procedures as well as turbulent wind and wave loadings. Such transient analysis requires selection of an appropriate integration method as is discussed in this paper.

Existing commercially available multi-body dynamics software could be adapted to enable the required analysis of VAWTs. Indeed, previous VAWT research [1] required the development of analysis tools that typically utilized a simplified approach [3] or relied heavily on commercial analysis software [4]. There is a need, however, for a VAWT aeroelastic code that can serve the wind research community, one that is modular, open source, and can be run concurrently in a parallel batch processing setting without the need to purchase multiple software licenses. Furthermore, existing offshore turbine design tools such as FAST [2] were designed using an assumed modes approach and are developed for a HAWT configuration. The finite element approach utilized in OWENS allows for arbitrary VAWT configurations to be considered. The modularity of the present approach will also allow re-use of many existing analysis code components, such as existing aerodynamics [6,7] and hydrodynamics [8] modules.

3.2 VAWT mesh generator

A VAWT rotor consists of a tower, blades, and possibly support members (or struts). The blades may be affixed to the tower at their ends as in the Darrieus and V-VAWT configurations or via struts (H-VAWT). Struts may also provide a connection between the tower and blades at any position along the tower and blade spans.

The VAWTGen mesh generator is capable of generating VAWTs of arbitrary geometry, including H-type, V-type, and Darrieus configurations. The blades may be rotated into an arbitrary orientation at arbitrary locations about the tower. Therefore configurations with swept blades may be considered. The VAWT configuration is discretized from continuous structural components into a finite number of beam elements. Figure 3 shows arbitrary configurations VAWTGen is capable of generating. The implementation also allows for concentrated structural components to be considered, and constraints of various joints may be imposed between structural components.

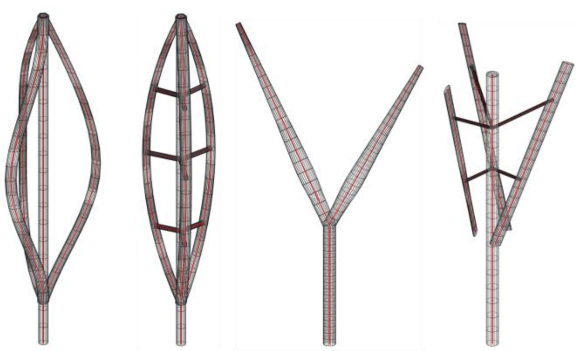

Figure 3. Arbitrary VAWT geometries produced with VAWTGen

4. Gyric Systems

In general, the differential equations of motion (EOM) for a flexible/deformable body may be expressed in the following form

$$M\ddot{q} + C\dot{q} + Kq = Q \qquad (1)$$

Here, M is a symmetric positive definite (SPD) mass matrix, C is a positive semi-definite damping matrix, and K is a SPD stiffness matrix. Q and q are generalized force and displacement vectors respectively.

Considerations for rotating systems are slightly different in that these systems consider linear representations of motion and are subject to a prescribed time-varying angular velocity about fixed axes. Such systems are commonly called *Gyric Systems* [5]. The resulting differential EOM for Gyric systems are

$$\begin{aligned} M\ddot{q} + (G(t) + C)\dot{q} + \big(K - S(t) + H(t)\big)q \\ = Q_c(t) + Q_H(t) + Q_{nc}(t) \end{aligned} \qquad (2)$$

Here, M, K, and C are defined as before. $G(t)$ is a *Gyric* or *Coriolis* matrix and is skew symmetric in nature. $S(t)$ is the *Spin Softening* matrix and is positive definite in nature. $H(t)$ is the *Circulatory* matrix and is skew symmetric in nature. The Gyric matrix is proportional to the angular velocity $\Omega(t)$, the Spin Softening matrix is proportional to $\Omega(t)^2$, and the Circulatory matrix is proportional to $\dot{\Omega}(t)$. $Q_c(t)$ is a conservative force vector, resulting from rotational effects on the reference position coordinates. $Q_H(t)$ is the Circulatory force vector, and $Q_{nc}(t)$ is the non-conservative force vector resulting from external forces. Non-conservative forces are not derivable from potential or kinetic energy of the system. A more detailed discussion of this topic is provided in Reference 5.

Although considerations have been made for time integration of non-conservative Gyric systems, they will not be discussed in this paper due to space limitations. Subsequent discussion will be concerned with conservative Gyric systems. It will be shown that such a system is useful for constructing stable, energy preserving time integration schemes. A conservative Gyric system has no damping, and a constant prescribed angular velocity, Ω. Thus, the EOM for a conservative Gyric system are

$$M\ddot{q} + G\dot{q} + (K - S)q = Q_c \qquad (3)$$

5. Energy in Gyric Systems

The total energy of a system is the sum of the potential *(V)* and kinetic *(T)* energies, *E=T+V*. Furthermore, let the kinetic energy be decomposed into parts that are quadratic (T_2), linear (T_1) and constant (T_0) with respect to velocity ($\dot{q}$) such that

$$T_2(\dot{q}) = \frac{1}{2}\dot{q}^T M \dot{q} \quad (4)$$

$$T_1(q, \dot{q}) = q^T L^T \dot{q} \quad (5)$$

$$T_0(q) = \frac{1}{2} q^T S q + Q_C^T q \quad (6)$$

Note that in the above definitions it has been assumed that T_1 is linear in generalized displacements (q). It can be shown that the relation between L and the Gyric matrix is: $G = L + L^T$. Potential energy is simply due to strain energy.

$$V(q) = \frac{1}{2} q^T K q \quad (7)$$

Examining the time rate of change for the energy of a conservative Gyric system and substituting relations for the EOM reveals that energy is not constant

$$\dot{E} = \dot{q}^T(2[Sq + Q_c] + L^T\dot{q}) + q^T L^T \ddot{q} \quad (8)$$

Nevertheless, considering the Jacobi integral [8] allows an energy function H^* to be introduced.

$$H^* = T_2 - T_0 + V \quad (9)$$

Note for $T = T_2$, that $H^* = E$. Examining the time rate of change of H*

$$\dot{H}^* = -\dot{q}^T(M\ddot{q} + (K - S)q - Q_c) \quad (10)$$

Substitution of the EOM for a conservative Gyric system, and accounting for the skew-symmetry of the Gyric matrix reveals H^* is constant for a conservative Gyric system.

$$\dot{H}^* = -\dot{q}^T G \dot{q} = 0 \quad (11)$$

This property will be utilized to construct a stable, H^* conserving time integration method for conservative Gyric systems.

6. Time Integration of Gyric Systems

Transient structural dynamics analysis requires time integration strategies to integrate second order differential equations of motion. A number of methods exist for time integration and may be explicit or implicit in nature [10,11]. Explicit methods are inexpensive computationally, but require smaller time steps and numerical stability is often a significant concern. Implicit methods require more expense computationally, but allow for larger time steps. Furthermore, implicit methods can allow for unconditional stability when suitable integration parameters are chosen. Although stability of an implicit integration method may be ensured by selection of appropriate integration parameter, accuracy of the solution is not guaranteed. Indeed, without careful tuning of integration parameters, spurious energy trends may be observed in the motions of a system.

The implicit integration method developed by Dean *et al.* is well suited for the transient analysis of flexible structures [9]. This method is unconditionally stable, and has the ability to conserve the energy of a system if non-conservative forces are absent. Such properties are extremely desirable when performing structural dynamics analysis to ensure an accurate representation of motion; integration methods that accumulate numerical error can cause numerical instability or display spurious and artificial energy trends that can cloud any analysis of complex systems.

Using the original ideas of Dean *et al*., the authors have extended the scope of the original method to show that finite difference approximations utilized to construct the integration scheme lead to a constant energy function H^* for conservative Gyric systems regardless of the size of the time step size. This means that large time steps can provide a good representation of motion.

Due to space limitations, the formulation of the Dean method for Gyric systems will not be shown. However, the resulting linear system of effective stiffness and load vectors for time stepping is shown below. This form of the equations is readily implemented in numerical frameworks such as the finite element method in OWENS. For $\alpha = 0.25$, H^* is conserved in conservative Gyric systems. For $0.25 \leq \alpha \leq 0.5$ unconditional stability exists.

$$[\bar{K}]_n q^{n+1} = \bar{F}_n \quad (12)$$

$$[\bar{K}]_n = [K]_n \alpha (\Delta t)^2 + [G]_n \frac{\Delta t}{2} + [M]_n \quad (13)$$

$$\{\bar{F}\} = \{F(n\Delta t)\}(\Delta t)^2 + [M]_n\{A\} + [K]_n\{B\} + [G]_n\{D\} \quad (14)$$

$$\{A\} = 2q^n - q^{n-1} \quad (15)$$

$$\{B\} = -\alpha(\Delta t)^2 q^{n-1} - (1-2\alpha)(\Delta t)^2 q^n \quad (16)$$

$$\{D\} = \frac{\Delta t}{2} q^{n-1} \qquad \frac{1}{4} < \alpha < \frac{1}{2} \quad (17)$$

7. Transient Analysis of a Vertical-axis Wind Turbine

A representative VAWT configuration was considered in a transient analysis using the energy preserving time integration method. Analysis results obtained with this method were compared to results obtained using the Newmark-β method [11], a popular integration scheme used in structural dynamics analysis. First, a representative VAWT configuration, an idealized version of the Sandia National Laboratories 34-meter (500 kW) VAWT [1] is presented. Analysis for a VAWT operating at constant rotor speed under only conservative loads (centrifugal and gravitational forces) and aerodynamic loads is considered, and the performance of the aforementioned time integration methods is assessed. Aerodynamic loads were generated using a one-way coupling of the Sandia National Laboratories CACTUS [6] aerodynamics code. Finally, additional loading on the platform is considered to simulate nominal hydrodynamic loads on a floating offshore turbine configuration.

7.1 Description of VAWT configuration

An idealized version of the Sandia National Laboratories 34-meter VAWT was considered in this study. A simple parabolic blade shape with a uniform cross-section was used to approximate the actual shape of the 34-meter VAWT. A tower height of 42 meters was specified, and the tower and blades were discretized using 20 uniform Timoshenko beam elements each. The ends of each blade were attached to the tower top and base with a fixed connection. The maximum blade radius is 17 meters at mid-height of the tower, resulting in a turbine diameter of 34 meters (height to diameter ratio of 1.235). A fixed boundary condition is specified at the base of the tower, and a pinned boundary condition is specified at the top of the tower. Initially, no platform effects are considered (fixed foundation). A wire-frame visualization of the idealized 34-meter VAWT is shown in Figure 4. Joints, guy cables, and other hardware are not modeled in the idealized configuration.

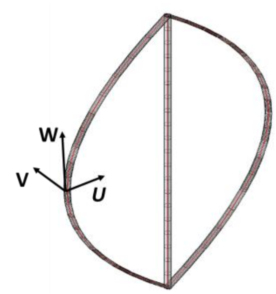

Figure 4. Idealized 34-meter VAWT

7.2 Operation at constant rotor speed with conservative loadings

The idealized VAWT configuration was analyzed at a constant rotor speed of 0.5 Hz. At t=0 displacements, velocities, and accelerations were considered to be zero. For t > 0 the turbine is acted upon by centrifugal and gravitational forces. The resulting vibrations of the rotating VAWT were examined with transient analysis using both the energy preserving and Newmark-β time integration methods. For the energy preserving integration method the tuning parameter α was set to 0.25. For the Newmark-β integration method an unconditionally stable constant average acceleration scheme was utilized.

A reference solution was obtained by performing the analysis with a constant time step size of $\Delta t = 0.001$ seconds with both time integration methods. Results showed nearly identical agreement, and were used as a reference solution. Time step sizes of Δt = 0.01, 0.1, and 1 seconds were considered for both integration methods. The results for the blade mid-span radial (U), edgewise (V), and vertical (W) displacements and were compared along with H*. Directions associated with U, V, and W are shown in Figure 4. Note that results for $\Delta t = 0.01$ seconds using either method were in very good agreement with the reference solution and displacements and velocities are not presented. 8 seconds of simulation time (4 revolutions of the VAWT rotor) were simulated, which resulted in periodic motions.

Figures 5 and 6 show a comparison of blade mid-span displacements and velocities respectively for both integration methods and Δt = 0.1 seconds. With respect to displacements, both integration methods perform reasonably well predicting amplitudes. With regards to velocities, the energy preserving integration method appears to capture amplitudes better than the Newmark-β method.

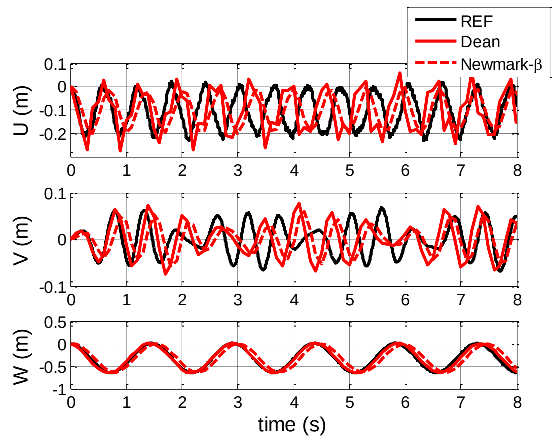

Figure 5. Blade mid-span displacements (Δt = 0.1 sec)

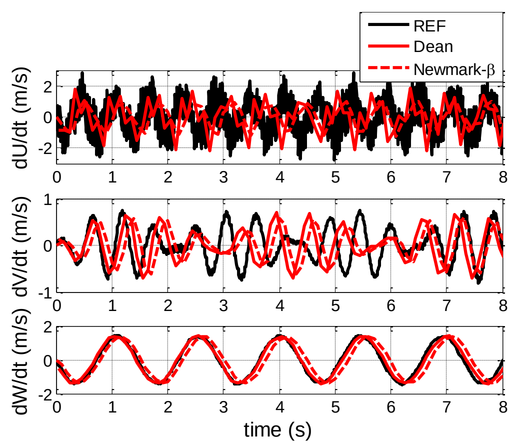

Figure 6. Blade mid-span velocities (Δt = 0.1 sec)

Figure 7 and Figure 8 show displacements and velocities respectively for Δt = 1.0 seconds. This can be considered a very coarse time step given the frequencies observed in the reference solution and corresponds to two time steps per revolution of the VAWT rotor. Inspecting displacements shows that the Newmark-β method predicts very small amplitudes and minimal oscillatory motion. The energy preserving method predicts amplitudes in moderate agreement with the reference solution. Furthermore, it can be noted that the energy preserving method does predicts velocity amplitudes better than the Newmark-β method.

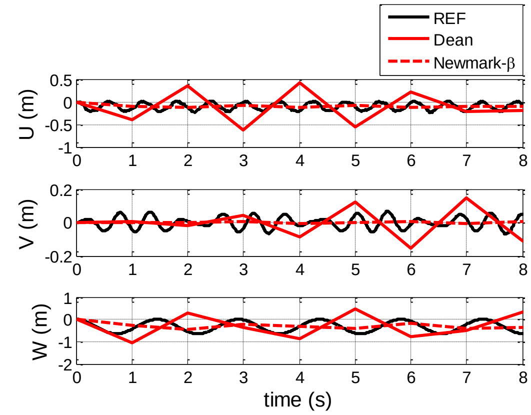

Figure 7. Blade mid-span displacements (Δt = 1.0 sec)

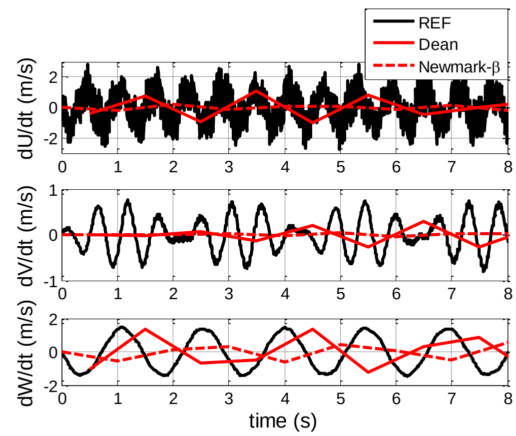

Figure 8. Blade mid-span velocities (Δt = 1.0 sec)

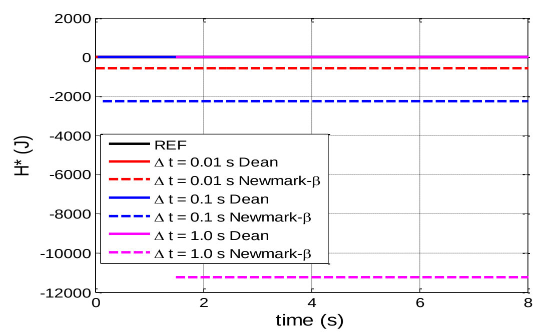

Figure 9. H* vs. time for VAWT with conservative forcing

Figure 9 shows the energy function H^* versus time for both integration methods at various time step sizes. As expected, the energy preserving method with $\alpha = 0.25$ maintains the same constant H^* as the reference solution. Newmark-β method maintains a constant H* value, but the value of H* is sensitive to time step size. By utilizing a constant H^* as a condition for bounded stability, the energy preserving method maintains a constant *H** regardless of time step size, and shows reasonable displacement/ velocity amplitudes at relatively large time step sizes.

7.3 Operation at constant rotor speed with aerodynamic loading

The same configuration was modeled including aerodynamic loads at a constant rotor speed of 0.5 Hz. A steady, uniform wind speed of 8.9 m/s was considered. A one-way coupling is assumed such that rotor speed and position are used to calculate aerodynamic loads. Aeroelastic coupling is not considered in that deformations of the turbine do not influence aerodynamic loads. Future work will implement a two-way coupling between aerodynamics modules and the OWENS analysis tool. 8 seconds of simulation time were observed, as aerodynamic forcing and turbine deformations were observed to be periodic over this time interval.

It is important to note that for external, aerodynamic forcing the energy function *H** will no longer be constant. *H** will equal the work done by aerodynamic forces on the system. Thus, the accuracy of predictions for work is not only related to the temporal discretization of aerodynamic loads, but also to the accuracy of motion predictions. Therefore, even for the energy preserving integration method, the quality of the prediction for H* is related to the time step size. As before, analysis was conducted with $\Delta t = 0.001$ seconds to serve as a reference solution. Examining the frequencies/ periods associated with aerodynamic loads revealed a minimum period of 0.35 seconds. Therefore, only time step sizes of 0.01, 0.05, and 0.1 seconds were considered in this study. Time steps larger than this would result in very poor temporal discretization of aerodynamic loads and inaccurate work predictions.

Figures 10 and 11 show displacement and velocity predictions of a blade mid-span for a simulation with aerodynamic loading. For the most part, time step sizes of 0.01 and 0.05 agree well with the general motions and amplitudes of the reference solution.

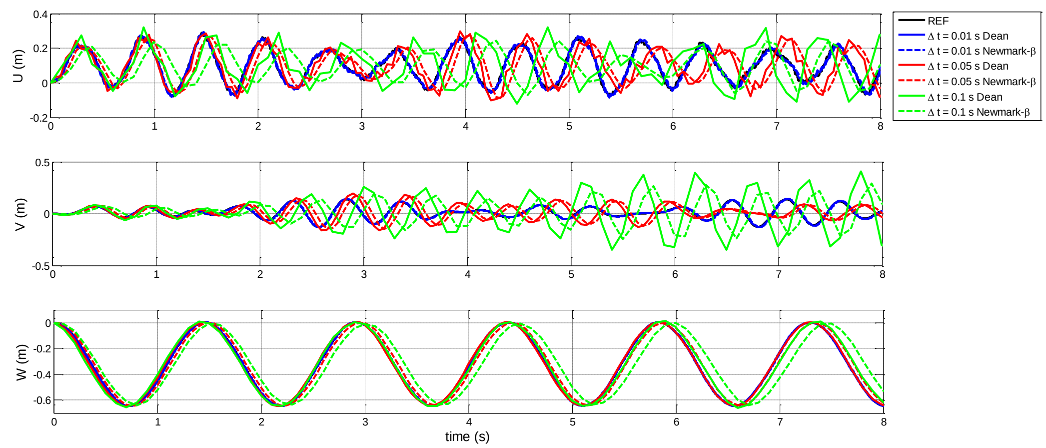

Figure 10. Blade mid-span displacements with aerodynamic forcing for various time step sizes

Similar observations are made for velocity. For the coarser time step of 0.1 seconds, discrepancies are visible with respect to *V* displacement, but *U* and *W* are predicted reasonably well. Similar trends are seen for velocities. Figure 12 shows the predicted *H** for the various time step sizes and integration methods. For $\Delta t = 0.01$ and 0.05 both methods capture the reference *H** well, and the energy preserving method appears to perform slightly better. For time $\Delta t = 0.1$ seconds neither method is in agreement with the energy function, indicating this time step size may be insufficient to accurately predict work in the system. It is notable that the energy preserving integration method with $\Delta t = 0.01$ is in almost identical agreement to the reference solution.

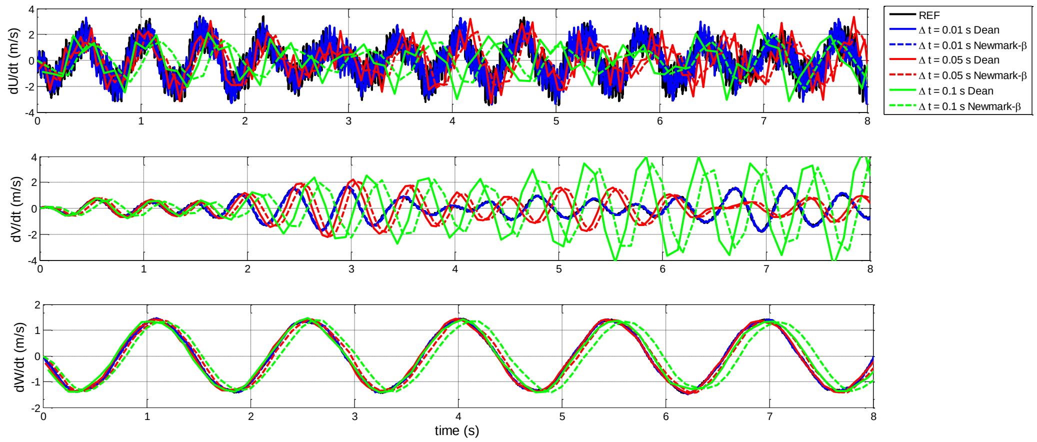

Figure 11. Blade mid-span velocities with aerodynamic forcing for various time step sizes

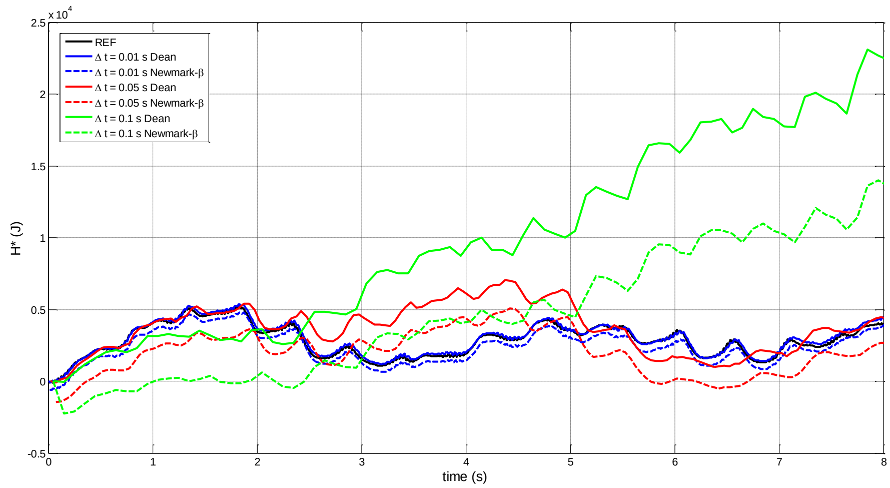

Figure 12. H* vs. time with aerodynamic forcing for various time step sizes

7.4 Operation at constant rotor speed with platform effects

Performance of the time integration methods was assessed for the idealized 34-meter VAWT with platform effects included. A simplified mooring model was considered by attaching three linear springs to the platform to provide restoring forces, and a time varying (sinusoidal) concentrated force was applied to the platform to simulate nominal hydrodynamic loads on the platform. Platform rotations were not included in the current study.

First, a conservative Gyric system was considered. As before, the turbine rotated at a constant rotor speed of 0.5 Hz, but with the addition of platform step relaxation. At t = 0 the platform was released from some initial displacement and allowed to oscillate. The model simulated the two-way coupled motions of turbine deformations and the rigid body motion of the turbine. The time integration methods performed very similar to the conservative Gyric system without platform effects. Therefore, results are not presented. As before, the energy preserving method predicted a consistent, constant H* value regardless of time step size.

Next, the nominal “hydrodynamic" loadings were applied to the rotating turbine with an initially stationary platform. Cases with and without aerodynamic loading were considered. To avoid complications in comparisons, the frequency of the platform loading was lower than that of aerodynamic forcing. Overall, the performance of the methods was similar to the fixed foundation configuration with aerodynamic loadings and the results are not presented.

8. Conclusions

This paper has presented the motivation and overview for the development of the Offshore Wind Energy Simulation toolkit. This modular, finite element framework will allow arbitrary, offshore VAWT configurations to be examined under a variety of conditions by interfacing with various aerodynamic and hydrodynamic modules. Future work will incorporate two-way couplings between external aerodynamics and hydrodynamics modules.

A previously developed energy conserving time integration method has been extended to consider Gyric systems such as wind turbines. While energy in a conservative Gyric system is not constant, a quantity known as the energy function H* is. This physical, energy related quantity was utilized to construct an unconditionally stable time integration method. This energy preserving time integration method may serve as an alternative numerical tool for the time integration of Gyric systems. Coarser time steps will still result in conservation of H^* for a conservative Gyric system, indicating large time steps may give a reasonable characterization of motion amplitudes for preliminary design studies.

A representative VAWT operating at a constant rotor speed under conservative and non-conservative loadings was considered. The energy preserving Gyric integrator performed well when compared to displacements, velocities, and H^* of a reference solution. At large time steps, more pronounced errors were visible in predicted motions. Nevertheless, the energy preserving integration method produced reasonable displacement and velocity amplitudes and predicted the oscillatory motion of the system.

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