AN EXPLORATION OF THE CONCEPTUAL UNDERSTANDING OF GEOMETRIC CONCEPTS: A CASE OF GRADE 8 LEARNERS IN MT AYLIFF DISTRICT

Zingiswa Mybert Monica Jojo
University of South Africa
South Africa
jojozmm@unisa.ac.za

ABSTRACT-The aim of teaching geometry is to provide learners with abilities on critical thinking, problem- solving and high levels of geometric thinking skills. This paper reports on an exploratory study conducted with 82 Grade 8 learners from two of fifteen purposely selected secondary schools in the Mt Ayliff district of the Eastern Cape in South Africa. A qualitative approach was used to explore the learners' conceptual understanding of Grade 8 pre-requisite geometric concepts on a preliminary test administered through questionnaires. Follow-up semi-structured interviews were further conducted with eight learners, four from each school, on the basis of their responses to get clarity on how they constructed geometric meaning. The paper discusses the actual questions on lines, triangles and quadrilaterals given, together with the learners' interpretations on the analysis of the interview transcripts using van Hieles' levels of geometrical thinking to discern relevant characteristics among learners' geometrical tasks. The researcher investigated the meaning with respect to mental constructs made by the learners in understanding the concepts, visual impact sensitivity, conceptions and misconceptions made. Findings from this study indicated the absence of discernment of critical features defining the different geometric figures, properties, linguistic and hierarchical characteristics of van Hieles' theory.

Keywords: Geometry, triangles, vertically opposite angles, quadrilaterals, van Hieles' theory.

INTRODUCTION

Ali, Bhagawati and Sarmah (2014) assert that geometry is a unifying theme to the entire mathematics curriculum and a rich source of visualization for arithmetical algebraic and statistical concepts. In addition Jones (2002) refers to geometry as an integral part of people's cultural experience and serves as a vital component of numerous aspects of life from architecture to design in all manifestations. The author then emphasizes the importance of studying geometry since it contributes to students' development of visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and conducting proof. Taking into consideration all these attributes, geometry teaching therefore aims at making the learners to have a habit of critical thinking, precise understanding bout geometric concepts and how these concepts relate to each other. Learners should be equipped to consider the criteria for making thoughtful decisions such that they assess the relevance of any rule that they use without simply guessing or applying it. For example in problems involving parallel lines, rules for alternating, corresponding and co-interior angles should be applied relevantly. In this way learners can make meaning of learning geometry and apply this knowledge to complete shapes where it is relevant.

Making meaning in geometry is a process where learners learn to think and think to learn (The Critical Thinking Consortium, 2013). According to this consortium, learners who are critically thoughtful develop, (i) deeper engagement and understanding when teachers create conditions that encourage students to "turn on" (p. 2) their brains and actively engage in learning mathematics through critical inquiry; (ii) greater independence and self-regulation when teachers help learners to develop a repertoire of thinking tools that they can use independently to support a growing confidence and monitoring of their own learning; and, (iii) stronger competence with mathematical processes like problem-solving, reasoning, representations and communication. Teaching geometry involves knowing how to recognize interesting geometrical problems and theorems, appreciating the history and cultural context of geometry, and understanding the many and varied uses to which geometry is put. It means appreciating what a full and rich geometry education can offer to learners when the mathematics curriculum is often dominated by other considerations like the demands of numeracy and algebra. In a mathematics classroom, learners can develop critical thinking skills through; (i) their own decision making on how to approach non-procedural geometric problems; (ii) their own choice of the most appropriate ways of geometric representations; (iii) monitoring progress in problem solving and adjustments; (iv) analysis of their own responses to ensure sense making; and, (v) communication of their mathematical ideas effectively such that they connect geometry with their own lives and the wider world. These are some of the ways in which learners are taught to apply critical thinking to their learning.

Learners experience geometry in grade 8 through identification, distinguishing and writing clear definitions of all types of 2-D and 3-D shapes in terms of their sides and angles, and properties of congruency and similarity (Department of Basic Education [DBE], 2011). The shape and space strand in this grade also requires the learners to solve geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties of geometric figures and definitions. Learners in Grade 8 are expected to be able to sort, recognize and describe 2-D shapes while also classifying 3-D shapes according to shape, number of faces, vertices and edges (DBE, 2011). New Jersey Mathematics Curriculum Framework (1997) suggests that for learners to learn properties of geometric figures, they need to deal explicitly with the identification and classification of standard geometric objects by the number of edges and vertices, the number and shapes of the faces, the acuteness of the angles, and so on. This framework suggests exercises that allow learners to (i) cut-and-paste constructions of paper models, (ii) combine shapes to form new shapes and (iii) decompose complex shapes into simpler ones are suitable to promote understanding of geometric properties. This concurs with the perspective of the van Hiele model of the development of geometric thought, where the learner moves in levels from observing and identifying a figure to recognition of its properties, after which the learner understands the interrelationships of the properties of the figures and the axiomatic system within which they are placed (Usiskin, 2003). This paper explores the learners' conceptual understanding of geometric concepts in Grade 8 of the senior phase in Mt Ayliff district of South Africa.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Euclidean Geometry is in many ways the primary geometry in terms of everyday experiences (Rips, Bloemfield, & Asmuth, 2006). It informs the understanding of other geometries like perspective geometry, hyperbolic geometry and other non-Euclidean geometries. Adolphus (2011) noted that geometry is a difficult area where, (i) students' performance has always been low; and, (ii) the problems of teaching and learning occur most in mathematics. This concurs with what Kutama (2009) found wen investigating the process-based instruction in the teaching of Euclidean geometry in Grades 8 and 9. Kutama (2009) found that learners in those grades could not communicate thought using either talking, writing or drawing in the activities given. He noticed that learners only recognized few shapes, made meaningless sentences and mathematical statements that lacked cohesion, and could also not explain in words what they had observed during those activities which would lead to the construction of concepts. Kutama (2009) therefore recommended that in situations where Euclidean geometry is offered in second language, learners should be encouraged to communicate their ideas in their mother tongue and the language of instruction interchangeably. The idea is that through talking and sharing their experiences with geometric concepts, might help them to operate in higher van Hieles' levels of geometric thinking.

This study was conducted in secondary schools located in predominantly rural and socioeconomic disadvantaged settlements characterized by both second language speaking teachers and learners. Interchanging the mother tongue and the language of instruction would work to the advantage of both the teacher and learners for better understanding of geometric concepts. The understanding of concepts in this study was tested through questionnaires written only in the language of instruction with no explanations. It was only during interviews that probing was done in mother tongue to get clarity on some of the responses given in the questionnaire.

Pournara, Mpofu and Saunders (2015,p41) in their content analysis of the 2012, 2013 and 2014 Grade 9 ANA tests, suggest the use of questions 'with a twist' as a useful idea for questions that can be used for productive learning which can bring out the typical errors that learners make. Bankov (2013) asserts that what usually is taught in geometry classes concerns the knowledge and skills needed to solve standard geometrical problems, most of which require only simple computational procedures. This (Bankov, 2013) associates with the fact that teachers are less prepared to teach geometry and therefore only present geometric facts and show solution of geometric problems. On the contrary geometry needs imagination, methodological skills of and deep understanding of geometric ideas coupled with constructions and appreciation of its beauty in the world. Jones, Fujita, & Miyazaki (2013) argue that the use of web-based learning experience can enhance students' acquisition skills in geometry and influence students to proceed to more complex and formal learning in geometry. This does not apply to learners in the rural setting where the study was conducted. The study intends to introduce the teachers to the use of GeoGebra in the teaching of geometry during intervention, but the challenge is that they are not exposed to ICT communication tools.

Elchuck (1992) asserts that learning geometry may not be easy, and a large number of the students fail to develop an adequate understanding of geometry concepts, geometry reasoning and geometry problem solving skills. Fulton (2013) attributes the learners' lack of understanding geometry to the tendency of some people to have a natural proclivity to either an arithmetical approach to mathematics or a visual and geometric one. The latter group of learners therefore enters high school without the pre-requisite knowledge except knowing the names of the shapes and some formulas without remembering their properties. This is contradictory to the discovery made by the van Hieles, that geometry is learnt in five sequential levels of geometric thinking. This paper reports on a study underpinned by the van Hieles' Theory of Geometric thinking, a theory that offers a model for explaining and describing how learners think as they engage with geometry problems. The five levels of acquisition of geometric thinking are, (i) level 0-Visualization; (ii) level 1-Analysis; (iii) level 2-Abstraction; (iv) level 3-Deduction; and, (v) level 4-Rigor. The geometric thinking that is developed is illustrated in Table 1.

Table 1: Levels of geometric thinking (adapted from Hoffer (1981)

(a) **Visual skills**	recognition, observation of properties, interpreting maps, imaging, recognition from different angles,
(b) **Verbal skills**	correct use of terminology and accurate communication in describing spatial concepts and relationships
(c) **Drawing skills-**	communicating through drawing, ability to represent geometric shapes in 2-d and 3-d, to make scale diagrams, sketch isometric figures,
(d) **Logical skills**	classification, recognition of essential properties as criteria, discerning patterns, formulating and testing hypothesis, making inferences, using counter-examples carried out in different ways.
(e) **Applied skills**	The learner engages in real-life applications using geometric results learnt and real uses of geometry, as for designing packages

Both van Hieles' and Hoffer's interpretations have been used to analyze data in this paper since the former theory does not outline detailed descriptions of children's thinking and representations of geometric concepts.

3. METHODOLOGY

This paper reports on an exploratory study conducted with 82 grade 8 learners from two of fifteen purposely selected secondary schools in the Mt Ayliff district of the Eastern Cape in South Africa. A qualitative approach was used to explore the learners' conceptual understanding of grade 8 pre-requisite geometric concepts on a preliminary test administered through questionnaires. The questionnaires were administered to grade 8 learners at the beginning of the second term. They responded to five different tasks on applications of understanding intersecting lines, triangles and quadrilaterals in the form of a pen and pencil test conducted for 45 minutes. The responses were marked and scrutinized not for write or wrong responses, but for clarity of meaning made by the learners.

Follow-up semi-structured interviews were further conducted with eight learners, four from each school, on the basis of their responses to get clarity on how they constructed geometric meaning. This was done after the results were analyzed and validated with the two schools. It is during this time that learners who displayed some interesting responses were interviewed to gain access to their geometrical thinking. The paper discusses the actual questions on lines, triangles and quadrilaterals given, together with the learners' interpretations on the analysis of the interview transcripts using van Hieles' levels of geometrical thinking to discern the van Hieles' characteristics among learners' geometric tasks. This paper specifically responds to the following research questions: (i) How do learners construct their meaning for the conceptual understanding of Grade 8 geometric concepts?; and, (ii) What were the learners' difficulties in the interpretation of Grade 8 geometric concepts?

Findings and discussions

The researcher investigated the meaning with respect to mental constructs made by the learners in understanding the concepts, visual impact sensitivity, conceptions and misconceptions made. Results indicated a pass % of 38,2 and 34,1 from school 1 and 2 respectively (see fig 1 and fig 2 below). On average this reflected a low and poor performance on the entire test indicating that although learners had an idea about geometry concepts, they hardly understood the geometry content.

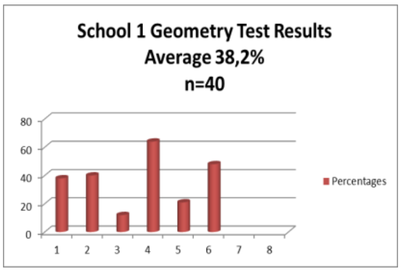


Figure 1: School 1 geometry results

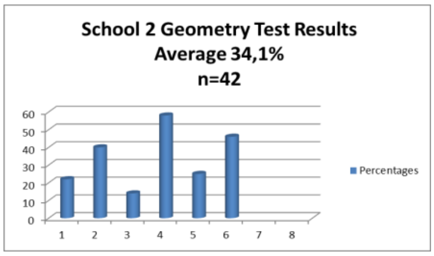
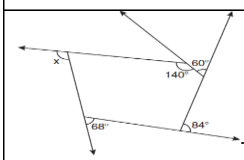


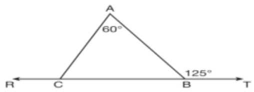
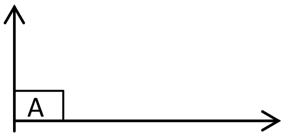
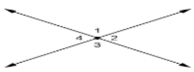
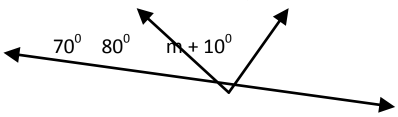
Figure 2: School 2 geometry results

Question 1

In this question the overall performance of learners in schools 1 & 2 indicated an average of 38% and 20% respectively. Most learners were able to find the adjacent angles for 84^0 and 68^0 in the diagram. Only 16% of the learners could find the value of the angle adjacent to 60^0. One of the popular responses was, 'x=68^0 - vertically opposite angles. Probing further in an interview, the learner indicated that he imagined an elongated form of the vertical line. The questions asked and the skills displayed by learners in each question are represented in Table 2.

Table 2: Test Questions and the skills displayed

Questions	Skills displayed
 The pentagon in the diagram below is formed by	Figure interpreted as a regular pentagon Lines assumed to be // Adjacent angle to 60^0 not calculated Sum of interior angles taken as 360^0

five rays. What is the degree measure of angle *x*?	
Find the measure of $A\hat{C}R$ 	No relationship between interior and adjacent angles Learners indicated the sum of angles of a triangle Learners treated both exterior angles as equal
In the diagram below: The angle A is called…………………………… 	64% and 58% of correct responses were displayed from school 1 &2 respectively. Some popular responses include rectangle, square and others
In the diagram  Angle 1 and angle 3 are known as……………… Angle 2 and angle 3 are known as……………… The sum of Angle 1 and angle 2 is ……………… Angle 1 and angle 2 are therefore said to be.. .	Identification of vertical opposite correctly done by most learners. Understanding of basic concepts like adjacent angles, supplementary angles and their application
Find the value of m in the following diagram if the first angle is 	Assumptions made by other learners implied that **$m + 10^0 = 90^0$.**

About 82% of the learners did not understand the concept of the sum of interior angles of a pentagon. They also could not discern the different features of both regular and irregular polygons. When the learner who wrote $x=68^0$ was interviewed, he said:

*'Ndidibanise (140+68+84) kwaphuma 292, then nda subtracta from 360' (*I subtracted the sum of (140+68+84) from 360).

The researcher probed furthered:

Researcher: Why did you subtract from 360^0?
Learner: *Because sinikwe I quad* (because we have been given a quadrilateral)
Researcher: Are you sure this is a quad? How many sides does a quad have?
Learner: *4 sides, let me count.* (He then count the sides) *Mh,…they are five*
Researcher: Why did you add the angles that are outside the figure?
Learner: *Because x is also outside the figure.*
Researcher: What about 60^0?
Learner: *Yen' uphezulu*. (it's up there)

This is a learner operating below level 0 of van Hieles' hierarchy sequence of geometrical thinking. This was also observed in the schools that learners operated in this level. The learner cannot recognize the pentagon and did not know its properties. Visual impact sensitivity was poor such that interior and external angles in the figure were treated the same. The learner also thought he could leave out angles like 60^0 at the top of the figure because of their positioning. This also implies that usually geometry problems given often required learners to find the sum of interior angles of a quadrilateral.

In *question 2*, the learner wrote $A\hat{C}R = 125^0$. His reasoning was that he recognizes them as occupying the same position on either side as the one given in the paper. Such a learner cannot make abstractions based on mental constructions related to geometric thinking. This concurs with (Abu et al., 2012; van Hieles', 1986 & 1999) who suggest that students pass through numerous levels of geometry thinking merely from recognizing geometrical shapes to construction of a formal geometry proof. These learners still need to be taught basic geometric concepts. Visualization, recognition of geometric properties in a figure and interpretation of what they see in geometric terms could not relate. This associated with lack of conception of meaning of basic geometric terms.

The expected response to question three was given by three learners in school 1 while 5 learners gave correct responses in school two. Those who did not write the name of the angle as 'right angle' indicated 90^0. The 'right angle' is one of the popular frequently used terms in geometry. It is then built into the classifications of quadrilaterals like squares, rectangles, and later chords in circles. Thus learners lacking such geometric knowledge often experience difficulties in understanding and developing geometric thinking. It is therefore crucial that learners make much sense of the meaning and implications incurred by the contribution of perpendicular stance of lines to whole figures. This can then help learners to recognize essential properties as criteria for classification of other polygons. Then through developed logic learners can develop discerning patterns, which could help them to formulate and test hypothesis regarding the proofs in geometry. These learners therefore need exposure to calculations involving a mixture of given and unknown angles in polygons where they can make inferences. In this way they can further apply the learnt geometric knowledge and use it in counter-examples presented to them in different ways.

Question 4 enjoyed the highest correct responses in both schools. Sub-questions (i)-(iii) received the maximum number of correct responses, nonetheless only 28% learners gave a correct response for item (iv). Learners' verbal skills with respect to the correct use of terminology and accurate communication in describing spatial concepts and relationships in this question seemed well connected. However learners' responses also indicated that although they could label supplementary angles, they don't relate them to adjacent angles. Consequently some learners who indicated and labelled angles 2 and 3 as adjacent angles could not realize that 1 and 2 were supplementary. Thus the abstraction and mental constructs lacked coherence and deprived learners to develop geometric thinking and build a basis of constructive arguments regarding these geometric terms.

Responses to *question 5* required learners to display their knowledge of geometric concept related to the sum of angles on straight lines. Learners in both schools performed on average below 25%. The concepts and identification of the different angles like adjacent, supplementary, vertically opposite angles were not easily recalled by the four learners interviewed for this question. For example, when one learner was asked on why her response was 30^0, she said:

Learner 3: *I added $70^0 + 80^0$, ndafumana u 150^0. Ngoku ke kwabe kushota u 30^0 kuze ndifumane u 180^0 we straight line* (I added $70^0 + 80^0$ get 150^0 and now I was short of 30^0 to make 180^0 for the requirement of angles on a straight line.

Researcher: What about the $m+10^0$, because you were required to find m?
Learner 3: *I did not know what to do, I am not used to something like that.*
Researcher: Like what?

Learner 3: *like variables and angles zibekwe zombini* (placed together)

The spatial concept relationships were correctly described by this learner. However he lacked the logical skills to deal with equations in order to find the value of m. This learner understood the concept of adjacent angles on a line. Through probing, use of guiding questions and re-drawing the figure, some of these concepts were recalled but not appropriately assigned through lack of conceptual understanding. According to the van Hieles' levels this learner operated on the second level since he could clearly display his reasoning ability with respect to the concept of sum of angles on a straight line. On the contrary when looking at his response to question 1, it appears that he lacked understanding of the relationship of the sum of the interior angles of a pentagon. He found all the values of adjacent angles but did not know what to do with them. He just added and subtracted from 360^0. This indicates that his geometric meaning abstraction could not go beyond calculations on quadrilaterals. It also shows that such problems were never tackled in class.

In this question, although many learners knew that the sum of angles on a line is 180^0, about 31% of them still felt that the value of the angle marked, $m+10^0$, should be 90^0, hence they found the value of m to be 80^0. The other challenge for those who made the statement, $70^0+80^0+m+10^0=180^0$ was to find the correct value of m. During interviews, the researcher rephrased the question in words as: What is the value of m that would make the sum of all given angles to be 180^0? The value of m = 20^0 was easily found. This is an indication that these learners cannot connect the diagrammatic representations of geometric concepts, not to mention the absence of the application of procedures for doing geometric calculations. Seemingly these learners cannot even solve what Bankov (2013) refers to as simple computational standard problems. They can therefore be classified to operate at a level lower that the visualization level of geometric thinking.

4. CONCLUSION

Although the geometric content covered in the test were concepts that learners in grade 8 are familiar with, results of this study indicated lack of understanding with regard to basic geometric concepts. Moreover when these learners were interviewed, their responses revealed that they were never exposed to shapes beyond quadrilaterals. This concurs with Bankov (2013) who accuses teachers of not presenting geometric facts and solutions without allowing the learners to interact geometrical and develop their reasoning skills. Consequently, Pournara, Mpofu and Saunders (2015) suggest that teachers must use problems with a twist challenging enough for the learners to build their geometric thinking.

Learners' performance in the preliminary test revealed the absence of grounding in visual impact sensitivity. This resulted in learners showing incompetence in forming mental construct relevant to making meaning and understanding of the geometric concepts. The concept of adjacent angles is not explored to the fourth level required in van Hieles' levels of thinking where learners classify, and discern patterns beyond location of such angles in straight lines. Learners' conceptions of geometric concepts are absent, they can therefore not be classified as misconceptions made. Findings from this study indicated the absence of discernment of critical features defining the different geometric figures, properties, linguistic and hierarchical characteristics of van Hieles' theory. Learners can construct meaning in geometry if they are exposed to hands on experiences with examples and non-examples of all types of polygons and calculations involving interior and exterior angles of all polygons.

5. RECOMMENDATION

The study recommends the development of geometric manuals to help train the grade 8 mathematics teachers in the Mt Ayliff district with clear distinct examples on basic geometric concepts. This will help the learners to develop their geometric thinking and be empowered with basic knowledge of geometric concepts before they are introduced to problems with challenging problem solving skills that will help them with abstraction of relevant geometric knowledge.

REFERENCES

Abu, M. S., Ali, M. B., & Tan, T. H. (2012). Assisting primary school children to progress through their van Hiele's levels of geometry thinking using Google SketchUp. *Procedia Social and Behavioral Sciences*, *64*, 75–84. doi:10.1016/j.sbspro.2012.11.010.

Adolphus,T.(June 2011). Problems of Teaching and Learning of Geometry in Secondary Schools in Rivers State, Nigeria. Int. J. Emerg. Sci., 1(2), ISSN: 2222-4254 , 143-151.

Ali, I., Bhagawati, S. & Sarmah, J. (2014). Performance of Geometry among the Secondary School Students of Bhurbandha CD Block of Morigaon District, Assam, India. International Journal of Innovative Research & Development, Vol. 3, No.11.

Bankov, K. (2013). Teaching of geometry in Bulgaria. *European Journal of Science and Mathematics Education* Vol. 1, No. 3.

Department of Basic Education (2011). Integrated Strategic Planning Framework for Teacher Education and Development in South Africa. Pretoria: Department of Basic Education.

Elchuck, L.M. (1992). The effects of software type, mathematics Achievement, spatial visualization, locus of control, independent time of investigation, and van Heile level on geometric conjecturing ability. A thesis at the Graduate School, College of Education, The Pennsylvania University.

Hoffer, A. (1981). Geometry is more than proof. Mathematics Teacher, 74, 11-18.

Jones, K. (2002), Issues in the Teaching and Learning of Geometry. In: Linda Haggarty (Ed), Aspects of Teaching Secondary Mathematics: perspectives on practice. London: Routledge Falmer. Chapter 8, pp 121-139. ISBN: 0-415-26641-6.

Jones, K., Fujita, T. & Miyazaki, M. (2013). Learning congruency-based proofs in geometry via a web-based learning system, *Proceedings of the British Society for Research into Learning Mathematics*, 33(1), 31-36. ISSN: 14636840.

Kutama, M. E. (2009). An investigation into process-based instruction in the teaching of grade 8 and 9 Euclidean geometry, University of South Africa, Pretoria, New Jersey Mathematics Curriculum Framework (1997) Standard 7 — Geometry and Spatial Sense.

Parzysz, B. (1991). Espace, géométrie et dessin. Une ingénierie didactique pourl' apprentissage, l'enseignement et l'utilisation de la perspective parallèle aulycée. Recherche en didactique des mathématiques, 11, 23, p. 211-240.

Piaget, J. (1972). The Principles of Genetic Epistemology (W. Mays trans.). Routledge & Kegan Paul: London.

Pournara, C., Mpofu, S. and Saunders, Y. (2013). The Grade 9 Maths ANA –What Can We See After Three Years? *Learning and Teaching Mathematics*, No. 18, 2015, pp. 34

Rips, L. J., Bloomfield, A., & Asmuth, J. (2006). Number sense and number nonsense. Manuscript submitted for publication The critical Thinking consortium (2013).

Usiskin, Z. (2003). Current trends in school mathematics, and the challenges they create. Paper presented at the International Conference on Science and Mathematics Education: Which Way Now? University Malaya, Perak.

Van Hiele, P. M. (1986). Structure and insight: A theory of mathematics education. Orlando, FL: Academic Press.

van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. In Developing Geometric Thinking Through Activities That Begin With Play (Vol. 6, pp. 310–316).