An Instructor's Solutions Manual to Accompany

PRINCIPLES OF HEAT TRANSFER, 7TH EDITION, SI

FRANK KREITH
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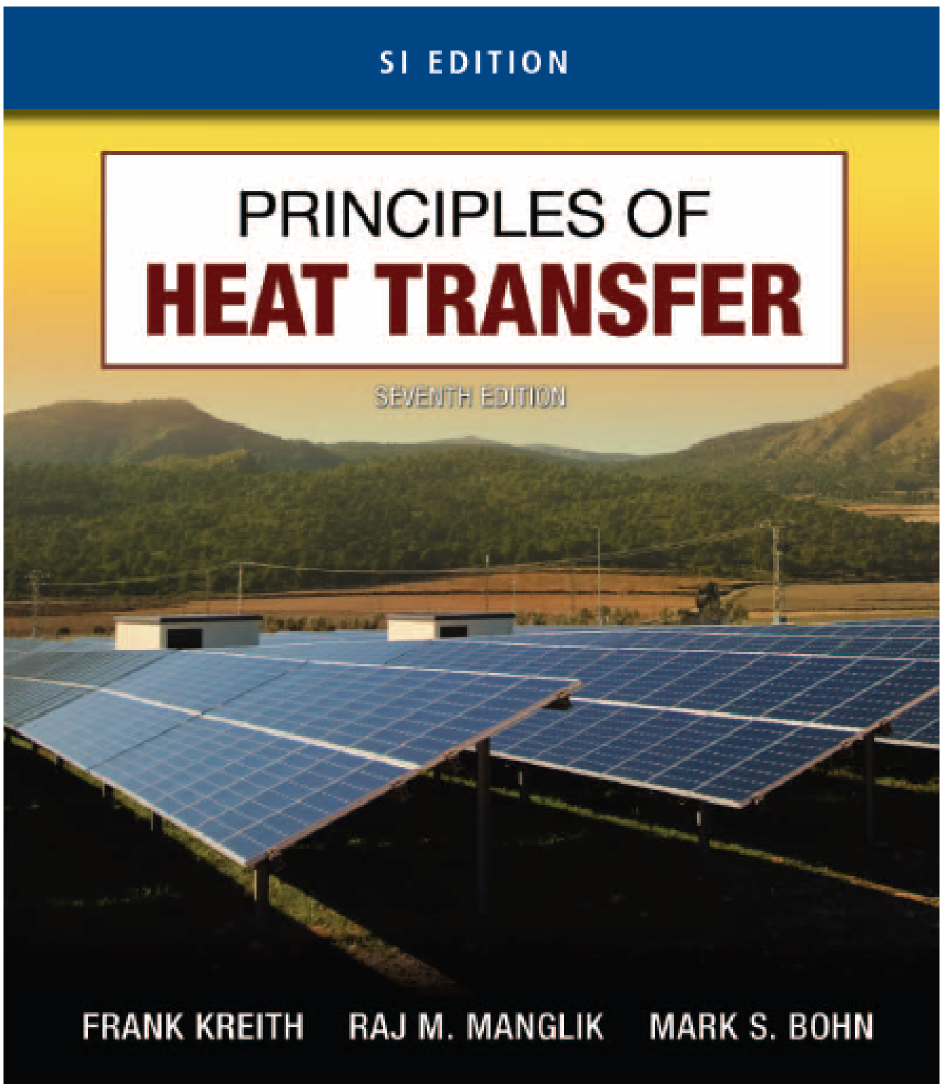
SI EDITION PREPARED BY: **SHALIGRAM TIWARI**



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INSTRUCTOR'S SOLUTIONS MANUAL TO ACCOMPANY

PRINCIPLES OF HEAT TRANSFER

SEVENTH EDITION, SI

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Chapter 1

PROBLEM 1.1

The outer surface of a 0.2m-thick concrete wall is kept at a temperature of –5°C, while the inner surface is kept at 20°C. The thermal conductivity of the concrete is 1.2 W/(m K). Determine the heat loss through a wall 10 m long and 3 m high.

GIVEN

10 m long, 3 m high, and 0.2 m thick concrete wall
Thermal conductivity of the concrete (k) = 1.2 W/(m K)
Temperature of the inner surface (T_i) = 20°C
Temperature of the outer surface (T_o) = –5°C

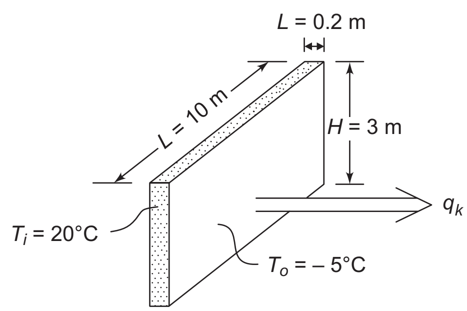
FIND

The heat loss through the wall (q_k)

ASSUMPTIONS

One dimensional heat flow
The system has reached steady state

SKETCH



SOLUTION

The rate of heat loss through the wall is given by Equation (1.2)

$$q_k = \frac{A\,K}{L}(\Delta T)$$

$$q_k = \frac{(10\,\text{m})(3\,\text{m})(1.2\ \text{W/(m K)})}{0.2\,\text{m}}\quad(20°\text{C} - (-5°\text{C}))$$

$$q_k = 4500\ \text{W}$$

COMMENTS

Since the inside surface temperature is higher than the outside temperature heat is transferred from the inside of the wall to the outside of the wall.

PROBLEM 1.2

The weight of the insulation in a spacecraft may be more important than the space required. Show analytically that the lightest insulation for a plane wall with a specified thermal resistance is that insulation which has the smallest product of density times thermal conductivity.

GIVEN

Insulating a plane wall, the weight of insulation is most significant

FIND

Show that lightest insulation for a given thermal resistance is that insulation which has the smallest product of density (ρ) times thermal conductivity (k)

ASSUMPTIONS

One dimensional heat transfer through the wall
Steady state conditions

SOLUTION

The resistance of the wall (R_k), from Equation (1.13) is

$$R_k = \frac{L}{A\,k}$$

where

L = the thickness of the wall

A = the area of the wall

The weight of the wall (w) is

$$w = \rho\,A\,L$$

Solving this for L

$$L = \frac{w}{\rho\,A}$$

Substituting this expression for L into the equation for the resistance

$$R_k = \frac{w}{\rho\,k\,A^2}$$

$$\therefore\ w = \rho\,k\,R_k\,A^2$$

Therefore, when the product of $\rho\,k$ for a given resistance is smallest, the weight is also smallest.

COMMENTS

Since ρ and k are physical properties of the insulation material they cannot be varied individually. Hence in this type of design different materials must be tried to minimize the weight.

PROBLEM 1.3

A furnace wall is to be constructed of brick having standard dimensions 22.5 cm × 11 cm × 7.5 cm. Two kinds of material are available. One has a maximum usable temperature of 1040°C and a thermal conductivity of 1.7 W/(m K), and the other has a maximum temperature limit of 870°C and a thermal conductivity of 0.85 W/(m K). The bricks cost the same and can be laid in any manner, but we wish to design the most economical wall for a furnace with a temperature on the hot side of 1040°C and on the cold side of 200°C. If the maximum amount of heat transfer permissible is 950 W/m^2 for each square foot of area, determine the most economical arrangements for the available bricks.

GIVEN

Furnace wall made of 22.5 cm × 11 cm × 7.5 cm bricks of two types

Type 1 bricks
- Maximum useful temperature (T_1, max) = 1040°C
- Thermal conductivity (k_1) = 1.7 W/(m K)

Type 2 bricks
- Maximum useful temperature (T_2, max) = 870°C
- Thermal conductivity (k_2) = 0.85 W/(m K)

Bricks cost the same

Wall hot side temperature (T_{hot}) = 1040°C and wall cold side temperature (T_{cold}) = 200°C

Maximum permissible heat transfer (q_{max}/A) = 950 W/m^2

FIND

The most economical arrangement for the bricks

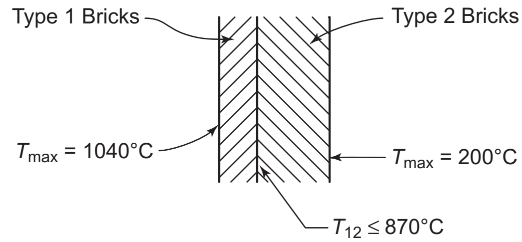
ASSUMPTIONS

One-dimensional, steady state heat transfer conditions

Constant thermal conductivities

The contact resistance between the bricks is negligible

SKETCH



SOLUTION

Since the type 1 bricks have a higher thermal conductivity at the same cost as the type 2 bricks, the most economical wall would use as few type 1 bricks as possible. However, there should be thick enough layer of type 1 bricks to keep the type 2 bricks at 870°C or less.

For one-dimensional conduction through type 1 bricks (from Equation 1.2)

$$q_k = \frac{k\,A}{L}\,\Delta T$$

$$\frac{q_{max}}{A} = \frac{k_1}{L_1}\,(T_{hot} - T_{12})$$

where L_1 is the minimum thickness of the type 1 bricks.

Solving for L_1

$$L_1 = \frac{k_1}{\left(\frac{q_{max}}{A}\right)}\,(T_{hot} - T_{12})$$

$$\Rightarrow \qquad L_1 = \frac{1.7\ \text{W/(m K)}}{950\ \text{W/m}^2}\,(1040 - 870)\text{K} = 0.3042\ \text{m} = 30.42\ \text{cm}$$

This thickness can be achieved by using 4 layers of type 1 bricks using the 7.5 cm dimension.
Similarly, for one-dimensional conduction through type 2 bricks

$$L_2 = \frac{k_2}{\left(\frac{q_{max}}{A}\right)} (T_{12} - T_{cold})$$

$$L_2 = \frac{0.85\ \text{W/(m K)}}{950\ \text{W/m}^2} (870 - 200)\text{K} = 0.6\ \text{m} = 60\ \text{cm}$$

This thickness can be achieved with 8 layers of type 2 bricks using the 7.5 cm dimension.

Therefore, the most economical wall would be built using 4 layers of type 1 bricks and 8 layers of type 2 bricks with the three inch dimension of the bricks used as the thickness.

PROBLEM 1.4

To measure thermal conductivity, two similar 1-cm-thick specimens are placed in an apparatus shown in the accompanying sketch. Electric current is supplied to the 6-cm by 6-cm guarded heater, and a wattmeter shows that the power dissipation is 10 watts (W). Thermocouples attached to the warmer and to the cooler surfaces show temperatures of 322 and 300 K, respectively. Calculate the thermal conductivity of the material at the mean temperature in W/(m K).

GIVEN

Thermal conductivity measurement apparatus with two samples as shown
Sample thickness (L) = 1 cm = 0.01 cm
Area = 6 cm × 6 cm = 36 cm^2 = 0.0036 m^2
Power dissipation rate of the heater (q_h) = 10 W
Surface temperatures
- T_{hot} = 322 K
- T_{cold} = 300 K

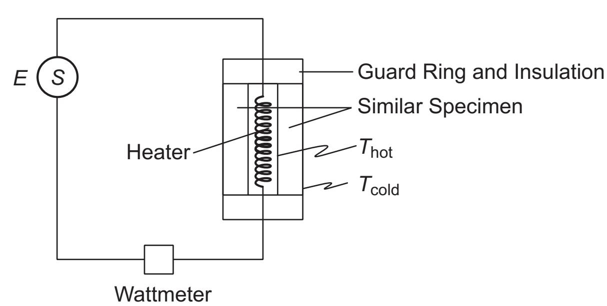
FIND

The thermal conductivity of the sample at the mean temperature in W/(m K)

ASSUMPTIONS

One dimensional, steady state conduction
No heat loss from the edges of the apparatus

SKETCH



SOLUTION

By conservation of energy, the heat loss through the two specimens must equal the power dissipation of the heater. Therefore the heat transfer through one of the specimens is $q_h/2$.

For one dimensional, steady state conduction (from Equation (1.3)

$$q_k = \frac{k\,A}{L}\,\Delta T = \frac{q_h}{2}$$

Solving for the thermal conductivity

$$k = \frac{\frac{q_h}{2}L}{A\Delta T}$$

$$k = \frac{(5\text{ W})(0.01\text{m})}{(0.0036\text{m}^2)(322\text{K} - 300\text{K})}$$

$$k = 0.63\ \text{W/(m K)}$$

COMMENTS

In the construction of the apparatus care must be taken to avoid edge losses so all the heat generated will be conducted through the two specimens.

PROBLEM 1.5

To determine the thermal conductivity of a structural material, a large 15 cm-thick slab of the material was subjected to a uniform heat flux of 2500 W/m^2, while thermocouples embedded in the wall 2.5 cm apart were read over a period of time. After the system had reached equilibrium, an operator recorded the readings of the thermocouples as shown below for two different environmental conditions.

Distance from the surface (cm)	Temperature (°C)
Test 1:	
0	**40**
5	**65**
10	**97**
15	**132**
Test 2:	
0	**95**
5	**130**
10	**168**
15	**208**

From these data, determine an approximate expression for the thermal conductivity as a function of temperature between 40 and 208°C.

GIVEN

Thermal conductivity test on a large, 15 cm slab
Thermocouples are embedded in the wall, 2.5 cm apart
Heat flux (q/A) = 2500 W/m^2
Two equilibrium conditions were recorded (shown in Table above)

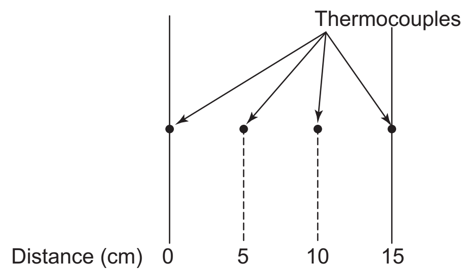
FIND

An approximate expression for thermal conductivity as a function of temperature between 40 and 208°C.

ASSUMPTIONS

One-dimensional conduction

SKETCH



SOLUTION

The thermal conductivity can be calculated for each pair of adjacent thermocouples using the equation for one-dimensional conduction

$$q = k\,A\,\frac{\Delta T}{L}$$

Solving for k

$$k = \frac{q}{A}\,\frac{L}{\Delta T}$$

This will give a thermal conductivity for each pair of adjacent thermocouples which are assigned to the average temperature of the pair of thermocouples. As an example, for the first pair of thermocouples in Test 1, the thermal conductivity (k_o) is

$$k_o = (2500\ \text{W/m}^2)\left(\frac{5\times10^{-2}\ \text{m}}{65\,^{\circ}\text{C} - 40\,^{\circ}\text{C}}\right) = 5\ \text{W/(m K)}$$

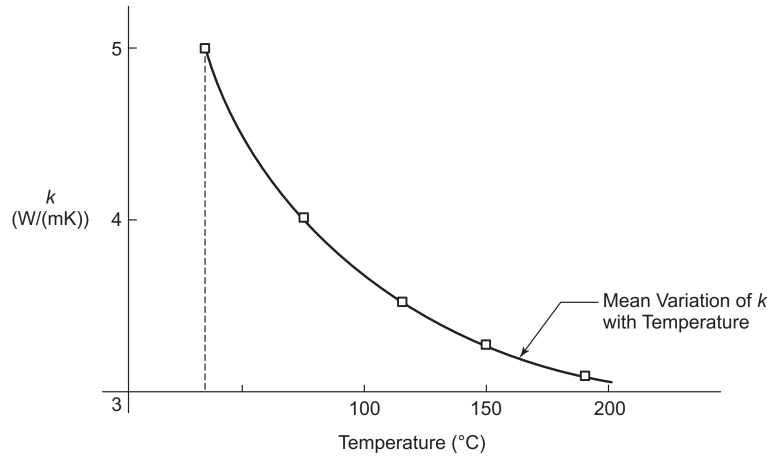
The average temperature for this pair of thermocouples is

$$T_{\text{avg}} = \frac{40+65}{2} = 52.5\ ^{\circ}\text{C}$$

The average temperature and the thermal conductivity for all other pairs of thermocouples are given in the table below.

n	(°C)	Thermal Conductivity W/(m K)
1	52.5	5
2	81	3.9
3	114.5	3.57
4	112.5	3.38
5	149	3.29
6	188	3.125

These points are displayed graphically.



We will use the best fit quadratic function to represent the relationship between thermal conductivity and temperature

$$k(T) = a + b\,T + c\,T^2$$

The constants a, b, and c can be found using a least squares fit.

Let the experimental thermal conductivity at data point n be designated as k_n. A least squares fit of the data can be obtained as follows

The sum of the squares of the errors is

$$S = \sum_N [k_n - k(T_n)]^2$$

$$S = \sum k_n^2 - 2a\sum k_n - N a^2 + 2ab\sum T_n - 2b\sum k_n T_n + 2ac\sum T_n^2 + b^2\sum T_n^2$$
$$- 2c\sum k_n T_n^2 + 2bc\sum T_n^3 + c^2\sum T_n^4$$

By setting the derivatives of S (with respect to a, b, and c) equal to zero, the following equations result

$$N\,a + \Sigma\,T_n b + \Sigma\,T_n^2\,c = \Sigma\,k_n$$
$$\Sigma\,T_n\,a + \Sigma\,T_n^2\,b + \Sigma\,T_n^3\,c = \Sigma\,k_n\,T_n$$
$$\Sigma\,T_n^2\,a + \Sigma\,T_n^3\,b + \Sigma\,T_n^4\,c = \Sigma\,k_n\,T_n^2$$

For this problem

$$\Sigma\,T_n = 697.5$$
$$\Sigma\,T_n^2 = 9.263 \times 10^4$$
$$\Sigma\,T_n^3 = 1.3554 \times 10^7$$
$$\Sigma\,T_n^4 = 2.125 \times 10^9$$
$$\Sigma\,k_n = 22.41$$
$$\Sigma\,k_n\,T_n = 2445.12$$
$$\Sigma\,k_n\,T_n^2 = 3.124 \times 10^4$$

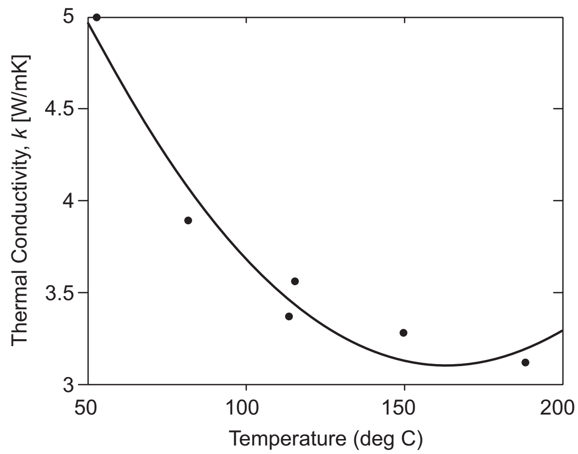
Solving for a, b, and c

$$a = 6.9722$$
$$b = -4.7213 \times 10^{-2}$$
$$c = 1.443 \times 10^{-4}$$

Therefore the expression for thermal conductivity as a function of temperature between 40 and 208°C is

$$k(T) = 6.9722 - 4.7213 \times 10^{-2}\, T + 1.443 \times 10^{-4}\, T^2$$

This is plotted in the following graph



COMMENTS

Note that the derived empirical expression is only valid within the temperature range of the experimental data.

PROBLEM 1.6

> **A square silicone chip 7 mm by 7 mm in size and 0.5 mm thick is mounted on a plastic substrate with its front surface cooled by a synthetic liquid flowing over it. Electronic circuits in the back of the chip generate heat at a rate of 5 watts that have to be transferred through the chip. Estimate the steady state temperature difference between the front and back surfaces of the chip. The thermal conductivity of silicone is 150 W/(m K).**

GIVEN

A 0.007 m by 0.007 m silicone chip
Thickness of the chip (L) = 0.5 mm = 0.0005 m
Heat generated at the back of the chip ($\dot{q}_G$) = 5 W
The thermal conductivity of silicon (k) = 150 W/(m K)

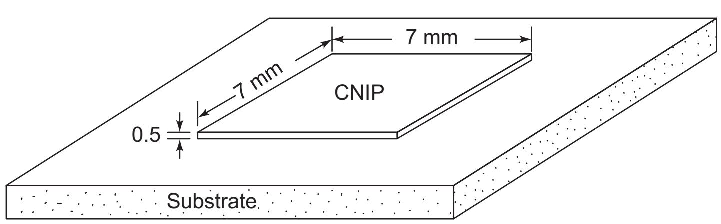
FIND

The steady state temperature difference (ΔT)

ASSUMPTIONS

One dimensional conduction (edge effects are negligible)
The thermal conductivity is constant
The heat lost through the plastic substrate is negligible

SKETCH



SOLUTION

For steady state the rate of heat loss through the chip, given by Equation (1.3), must equal the rate of heat generation

$$q_k = \frac{A\,k}{L}\,(\Delta T) = \dot{q}_G$$

Solving this for the temperature difference

$$\Delta T = \frac{L\,\dot{q}_G}{k\,A}$$

$$\Delta T = \frac{(0.0005)(5\,\mathrm{W})}{(150\ \mathrm{W/(m\,K)})(0.007\,\mathrm{m})(0.007\,\mathrm{m})}$$

$$\Delta T = 0.34°\mathrm{C}$$

PROBLEM 1.7

A warehouse is to be designed for keeping perishable foods cool prior to transportation to grocery stores. The warehouse has an effective surface area of 1860 m^2 exposed to an ambient air temperature of 32°C. The warehouse wall insulation (*k* = 0.17 W/(m K) is 7.5 cm thick. Determine the rate at which heat must be removed from the warehouse to maintain the food at 4°C.

GIVEN

Cooled warehouse
Effective area (A) = 1860 m^2
Temperatures
- Outside air = 32°C
- Food inside = 4°C

Thickness of wall insulation (L) = 7.5 cm
Thermal conductivity of insulation (k) = 0.17 W/(m K)

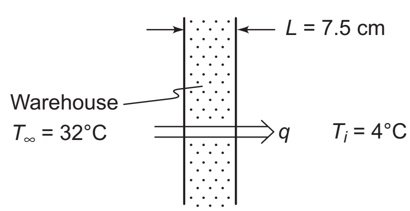
FIND

Rate at which heat must be removed (q)

ASSUMPTIONS

One dimensional, steady state heat flow
The food and the air inside the warehouse are at the same temperature
The thermal resistance of the wall is approximately equal to the thermal resistance of the wall insulation alone

SKETCH



SOLUTION

The rate at which heat must be removed is equal to the rate at which heat flows into the warehouse. There will be convective resistance to heat flow on the inside and outside of the wall. To estimate the upper limit of the rate at which heat must be removed these convective resistances will be neglected. Therefore the inside and outside wall surfaces are assumed to be at the same temperature as the air inside and outside of the wall. Then the heat flow, from Equation (1.2), is

$$q = \frac{k\,A}{L}\,\Delta T$$

$$q = \frac{(0.17\,\text{W/(m K)})\,(1860\,\text{m}^2)}{7.5\times10^{-2}\,\text{m}}\,(32-4)$$

$$q = 118\text{ kW}$$

PROBLEM 1.8

With increasing emphasis on energy conservation, the heat loss from buildings has become a major concern. For a small tract house the typical exterior surface areas and R-factors (area × thermal resistance) are listed below

Element	Area (m^2)	*R*-Factors = Area × Thermal Resistance [(m^2 K/W)]
Walls	**150**	**2.0**
Ceiling	**120**	**2.8**
Floor	**120**	**2.0**
Windows	**20**	**0.1**
Doors	**5**	**0.5**

(a) Calculate the rate of heat loss from the house when the interior temperature is 22°C and the exterior is –5°C.

(b) Suggest ways and means to reduce the heat loss and show quantitatively the effect of doubling the wall insulation and the substitution of double glazed windows (thermal resistance = 0.2 m^2 K/W) for the single glazed type in the table above.

GIVEN

Small house
Areas and thermal resistances shown in the table above
Interior temperature = 22°C
Exterior temperature = –5°C

FIND

(a) Heat loss from the house (q_a)
(b) Heat loss from the house with doubled wall insulation and double glazed windows (q_b). Suggest improvements.

ASSUMPTIONS

All heat transfer can be treated as one dimensional
Steady state has been reached
The temperatures given are wall surface temperatures
Infiltration is negligible
The exterior temperature of the floor is the same as the rest of the house

SOLUTION

(a) The rate of heat transfer through each element of the house is given by Equations (1.33) and (1.34)

$$q = \frac{\Delta T}{R_{th}}$$

The total rate of heat loss from the house is simply the sum of the loss through each element:

$$q = \Delta T \left(\frac{1}{\left(\frac{AR}{A}\right)_{\text{wall}}} + \frac{1}{\left(\frac{AR}{A}\right)_{\text{ceiling}}} + \frac{1}{\left(\frac{AR}{A}\right)_{\text{floor}}} + \frac{1}{\left(\frac{AR}{A}\right)_{\text{windows}}} + \frac{1}{\left(\frac{AR}{A}\right)_{\text{doors}}} \right)$$

$$q = (22°\text{C} - -5°\text{C}) \left(\frac{1}{\left(\frac{2.0\,(\text{m}^2\text{K})/\text{W}}{150\text{ m}^2}\right)} + \frac{1}{\left(\frac{2.8\,(\text{m}^2\text{K})/\text{W}}{120\text{ m}^2}\right)} + \frac{1}{\left(\frac{2.0\,(\text{m}^2\text{K})/\text{W}}{120\text{ m}^2}\right)} + \frac{1}{\left(\frac{0.5\,(\text{m}^2\text{K})/\text{W}}{20\text{ m}^2}\right)} + \frac{1}{\left(\frac{0.5\,(\text{m}^2\text{K})/\text{W}}{5\text{ m}^2}\right)} \right)$$

$$q = (22°\text{C} - -5°\text{C})\ (75 + 42.8 + 60 + 200 + 10)\ \text{W/K}$$

$$q = 10{,}500\ \text{W}$$

(b) Doubling the resistance of the walls and windows and recalculating the total heat loss:

$$q = (22°\text{C} - -5°\text{C}) \left(\frac{1}{\left(\frac{4.0\,(\text{m}^2\text{K})/\text{W}}{150\text{ m}^2}\right)} + \frac{1}{\left(\frac{2.8\,(\text{m}^2\text{K})/\text{W}}{120\text{ m}^2}\right)} + \frac{1}{\left(\frac{2.0\,(\text{m}^2\text{K})/\text{W}}{120\text{ m}^2}\right)} + \frac{1}{\left(\frac{0.2\,(\text{m}^2\text{K})/\text{W}}{20\text{ m}^2}\right)} + \frac{1}{\left(\frac{0.5\,(\text{m}^2\text{K})/\text{W}}{5\text{ m}^2}\right)} \right)$$

$$q = (22°\text{C} - -5°\text{C})\ (37.5 + 42.8 + 60 + 100 + 10)\ \text{W/K}$$

$$q = 6800\ \text{W}$$

Doubling the wall and window insulation led to a 35% reduction in the total rate of heat loss.

COMMENTS

Notice that the single glazed windows account for slightly over half of the total heat lost in case (a) and that the majority of the heat loss reduction in case (b) is due to the double glazed windows. Therefore double glazed windows are strongly suggested.

PROBLEM 1.9

Heat is transferred at a rate of 0.1 kW through glass wool insulation (density = 100 kg/m^3) of 5 cm thickness and 2 m^2 area. If the hot surface is at 70°C, determine the temperature of the cooler surface.

GIVEN

Glass wool insulation with a density (ρ) = 100 kg/m^3
Thickness (L) = 5 cm = 0.05 m
Area (A) = 2 m^2
Temperature of the hot surface (T_h) = 70°C
Rate of heat transfer (q_k) = 0.1 kW = 100 W

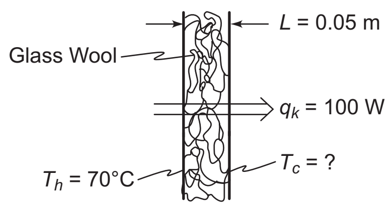
FIND

The temperature of the cooler surface (T_c)

ASSUMPTIONS

One dimensional, steady state conduction
Constant thermal conductivity

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
The thermal conductivity of glass wool at 20°C (k) = 0.036 W/(m K)

SOLUTION

For one dimensional, steady state conduction, the rate of heat transfer, from Equation (1.2), is

$$q_k = \frac{A\,k}{L}\ (T_h - T_c)$$

Solving this for T_c

$$T_c = T_h - \frac{q_k\ L}{A\ k}$$

$$T_c = 70°\text{C} - \frac{(100\,\text{W})(0.05\,\text{m})}{(2\ \text{m}^2)(0.036\,\text{W/m K})}$$

$$T_c = 0.6°\text{C}$$

PROBLEM 1.10

A heat flux meter at the outer (cold) wall of a concrete building indicates that the heat loss through a wall of 10 cm thickness is 20 W/m^2. If a thermocouple at the inner surface of the wall indicates a temperature of 22°C while another at the outer surface shows 6°C, calculate the thermal conductivity of the concrete and compare your result with the value in Appendix 2, Table 11.

GIVEN

Concrete wall
Thickness (L) = 100 cm = 0.1 m
Heat loss (q/A) = 20 W/m^2
Surface temperature
- Inner (T_i) = 22°C
- Outer (T_o) = 6°C

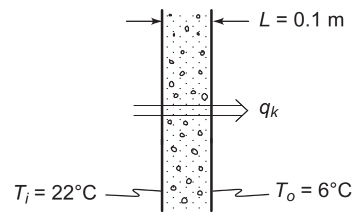
FIND

The thermal conductivity (k) and compare it to the tabulated value

ASSUMPTIONS

One dimensional heat flow through the wall
Steady state conditions exist

SKETCH



SOLUTION

The rate of heat transfer for steady state, one dimensional conduction, from Equation (1.2), is

$$q_k = \frac{k\,A}{L}\,(T_{\text{hot}} - T_{\text{cold}})$$

Solving for the thermal conductivity

$$k = \left(\frac{q_k}{A}\right)\frac{L}{(T_i - T_o)}$$

$$k = (20\ \text{W/m}^2)\left(\frac{0.1\,\text{m}^2}{22^\circ\text{C} - 6^\circ\text{C}}\right) = 0.125\ \text{W/(m K)}$$

This result is very close to the tabulated value in Appendix 2, Table 11 where the thermal conductivity of concrete is given as 0.128 W/(m K).

PROBLEM 1.11

> **Calculate the heat loss through a 1-m by 3-m glass window 7 mm thick if the inner surface temperature is 20°C and the outer surface temperature is 17°C. Comment on the possible effect of radiation on your answer.**

GIVEN

Window: 1 m by 3 m
Thickness (L) = 7 mm = 0.007 m
Surface temperature
- Inner (T_i) = 20°C
- Outer (T_o) = 17°C

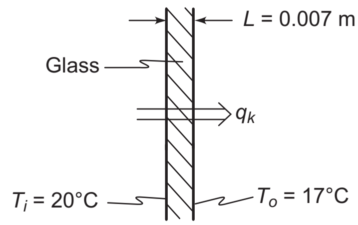
FIND

The rate of heat loss through the window (q)

ASSUMPTIONS

One dimensional, steady state conduction through the glass
Constant thermal conductivity

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
Thermal conductivity of glass (k) = 0.81 W/(m K)

SOLUTION

The heat loss by conduction through the window is given by Equation (1.2)

$$q_k = \frac{k\,A}{L}\,(T_{\text{hot}} - T_{\text{cold}})$$

$$q_k = \frac{(0.81\ \text{W/(m K)})\,(1\,\text{m})\,(3\,\text{m})}{(0.007\,\text{m})}\,(20°\text{C} - 17°\text{C})$$

$$q_k = 1040\ \text{W}$$

COMMENTS

Window glass is transparent to certain wavelengths of radiation, therefore some heat may be lost by radiation through the glass.
During the day sunlight may pass through the glass creating a net heat gain through the window.

PROBLEM 1.12

> **If in Problem 1.11 the outer air temperature is –2°C, calculate the convective heat transfer coefficient between the outer surface of the window and the air assuming radiation is negligible.**
>
> **Problem 1.11: Calculate the heat loss through a 1 m by 3 m glass window 7 mm thick if the inner surface temperature is 20°C and the outer surface temperature is 17°C. Comment on the possible effect of radiation on your answer.**

GIVEN

Window: 1 m by 3 m
Thickness (L) = 7 mm = 0.007 m
Surface temperatures
- Inner (T_i) = 20°C
- Outer (T_o) = 17°C

The rate of heat loss = 1040 W (from the solution to Problem 1.11)
The outside air temperature = –2°C

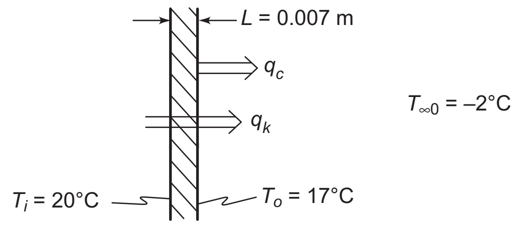
FIND

The convective heat transfer coefficient at the outer surface of the window ($\bar{h}_c$)

ASSUMPTIONS

The system is in steady state and radiative loss through the window is negligible

SKETCH



SOLUTION

For steady state the rate of heat transfer by convection (Equation (1.10)) from the outer surface must be the same as the rate of heat transfer by conduction through the glass

$$q_c = \bar{h}_c\, A\, \Delta T = q_k$$

Solving for $\bar{h}_c$

$$\bar{h}_c = \frac{q_k}{A(T_o - T_\infty)}$$

$$\bar{h}_c = \frac{1040\,\text{W}}{(1\,\text{m})(3\,\text{m})(17\,^{\circ}\text{C} - -2\,^{\circ}\text{C})}$$

$$\bar{h}_c = 18.2\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

This value for the convective heat transfer coefficient falls within the range given for the free convection of air in Table 1.4.

PROBLEM 1.13

> **Using Table 1.4 as a guide, prepare a similar table showing the order of magnitudes of the thermal resistances of a unit area for convection between a surface and various fluids.**

GIVEN

Table 1.4— The order of magnitude of convective heat transfer coefficient ($\bar{h}_c$)

FIND

The order of magnitudes of the thermal resistance of a unit area ($A\ R_c$)

SOLUTION

The thermal resistance for convection is defined by Equation (1.14) as

$$R_c = \frac{1}{\bar{h}_c A}$$

Therefore the thermal resistances of a unit area are simply the reciprocal of the convective heat transfer coefficient

$$A\ R_c = \frac{1}{\bar{h}_c}$$

As an example, the first item in Table 1.4 is 'air, free convection' with a convective heat transfer coefficient of 6–30 W/(m^2 K). Therefore the order of magnitude of the thermal resistances of a unit area for air, free convection is

$$\frac{1}{30\ \text{W/(m}^2\text{K)}} = 0.03\ (\text{m}^2\text{K})/\text{W} \ \text{to} \ \frac{1}{6\ \text{W/(m}^2\text{K)}} = 0.17\ (\text{m}^2\text{K})/\text{W}$$

The rest of the table can be calculated in a similar manner

Order of Magnitude of Thermal Resistance of a Unit Area for Convection

Fluid	W/(m^2 K)
Air, free convection	0.03–0.2
Superheated steam or air, forced convection	0.003–0.03
Oil, forced convection	0.0006–0.02
Water, forced convection	0.0002–0.003
Water, boiling	0.00002–0.0003
Steam, condensing	0.000008–0.0002

COMMENTS

The extremely low thermal resistance in boiling and condensation suggests that these resistances can often be neglected in a series thermal network.

PROBLEM 1.14

A thermocouple (0.8-mm-OD wire) is used to measure the temperature of quiescent gas in a furnace. The thermocouple reading is 165°C. It is known, however, that the rate of radiant heat flow per meter length from the hotter furnace walls to the thermocouple wire is 1.1 W/m and the convective heat transfer coefficient between the wire and the gas is 6.8 W/(m^2 K). With this information, estimate the true gas temperature. State your assumptions and indicate the equations used.

GIVEN

Thermocouple (0.8 mm OD wire) in a furnace
Thermocouple reading (T_p) = 165°C
Radiant heat transfer to the wire (q_r/L) = 1.1 W/m
Heat transfer coefficient ($\bar{h}_c$) = 6.8 W/(m^2 K)

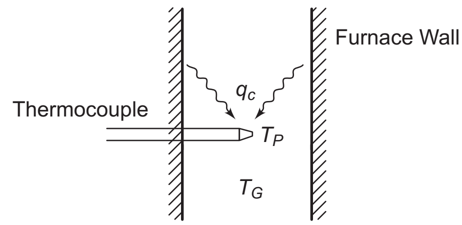
FIND

Estimate the true gas temperature (T_G)

ASSUMPTIONS

The system is in equilibrium
Conduction along the thermocouple is negligible
Conduction between the thermocouple and the furnace wall is negligible

SKETCH



SOLUTION

Equilibrium and the conservation of energy require that the heat gain of the probe by radiation if equal to the heat lost by convection.

The rate of heat transfer by convection is given by Equation (1.10)

$$q_c = \bar{h}_c \, A \, \Delta T = \bar{h}_c \, \pi D \, L \, (T_p - T_G)$$

For steady state to exist the rate of heat transfer by convection must equal the rate of heat transfer by radiation

$$q_c = q_r$$

$$\bar{h}_c \, \pi D \, L \, (T_p - T_G) = \left(\frac{q_r}{L}\right) L$$

$$T_G = T_p - \frac{\left(\frac{q_r}{L}\right) L}{\bar{h}_c \, \pi D L}$$

$$T_G = 165°\text{C} - \frac{(1.1\,\text{W/m})}{\left(6.8\ \text{W/(m}^2\text{K)}\right)\pi(0.0008\,\text{m})}$$

$$T_G = 101°\text{C}$$

COMMENTS

This example illustrates that care must be taken in interpreting experimental measurements. In this case a significant correction must be applied to the thermocouple reading to obtain the true gas temperature. Can you suggest ways to reduce the correction?

PROBLEM 1.15

> **Water at a temperature of 77°C is to be evaporated slowly in a vessel. The water is in a low pressure container which is surrounded by steam. The steam is condensing at 107°C. The overall heat transfer coefficient between the water and the steam is 1100 W/(m^2 K). Calculate the surface area of the container which would be required to evaporate water at a rate of 0.01 kg/s.**

GIVEN

Water evaporated slowly in a low pressure vessel surrounded by steam
Water temperature (T_w) = 77°C
Steam condensing temperature (T_s) = 107°C
Overall transfer coefficient between the water and the steam (U) = 1100 W/(m^2 K)
Evaporation rate $(\dot{m}_w)$ = 0.01 kg/s

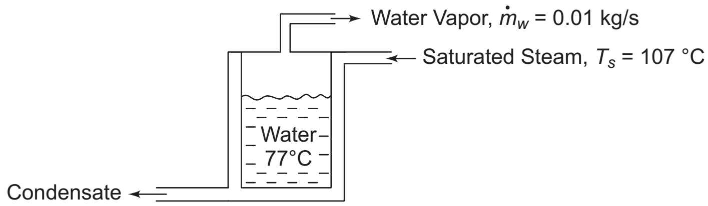
FIND

The surface area (A) of the container required

ASSUMPTIONS

Steady state prevails
Vessel pressure is held constant at the saturation pressure corresponding to 77°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13
The heat of vaporization of water at 77°C (h_{fg}) = 2317 kJ/kg

SOLUTION

The heat transfer required to evaporate water at the given rate is

$$q = \dot{m}_w \, h_{fg}$$

For the heat transfer between the steam and the water

$$q = U A \,\Delta T = \dot{m}_w \, h_{fg}$$

Solving this for the transfer area

$$A = \frac{\dot{m}_w h_{fg}}{U \,\Delta T}$$

$$A = \frac{(0.01\,\text{kg/s})(2317\,\text{kJ/kg})(1000\,\text{J/kJ})}{\left(1100\,\text{W/(m}^2\text{K)}\right)(107\,^\text{o}\text{C} - 77^\text{o}\,\text{C})}$$

$$A = 0.70\ \text{m}^2$$

PROBLEM 1.16

> **The heat transfer rate from hot air at 100°C flowing over one side of a flat plate with dimensions 0.1 m by 0.5 m is determined to be 125 W when the surface of the plate is kept at 30°C. What is the average convective heat transfer coefficient between the plate and the air?**

GIVEN

Flat plate, 0.1 m by 0.5 m, with hot air flowing over it
Temperature of plate surface (T_s) = 30°C
Air temperature (T_∞) = 100°C
Rate of heat transfer (q) = 125 W

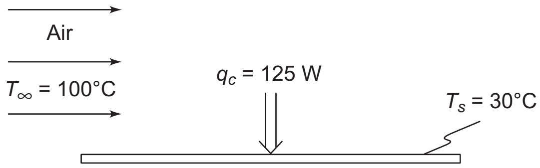
FIND

The average convective heat transfer coefficient, h_c, between the plate and the air

ASSUMPTION

Steady state conditions exist

SKETCH



SOLUTION

For convection the rate of heat transfer is given by Equation (1.10)

$$q_c = \bar{h}_c A \Delta T$$

$$q_c = \bar{h}_c A (T_\infty - T_s)$$

Solving this for the convective heat transfer coefficient yields

$$\bar{h}_c = \frac{q_c}{A(T_\infty - T_s)}$$

$$\bar{h}_c = \frac{125\,\text{W}}{(0.1\,\text{m})(0.5\,\text{m})(100^\circ\text{C} - 30^\circ\text{C})}$$

$$\bar{h}_c = 35.7\ \text{W/(m}^2\,\text{K)}$$

COMMENTS

One can see from Table 1.4 (order of magnitudes of convective heat transfer coefficients) that this result is reasonable for free convection in air.

Note that since $T_\infty > T_s$ heat is transferred from the air to the plate.

PROBLEM 1.17

The heat transfer coefficient for a gas flowing over a thin flat plate 3 m long and 0.3 m wide varies with distance from the leading edge according to

$$\bar{h}_c\ (x) = 10 \times {}^{-\frac{1}{4}}\ \text{W/(m}^2\text{K)}$$

If the plate temperature is 170°C and the gas temperature is 30°C, calculate (a) the average heat transfer coefficient, (b) the rate of heat transfer between the plate and the gas and (c) the local heat flux 2 m from the leading edge.

GIVEN

Gas flowing over a 3 m long by 0.3 m wide flat plate
Heat transfer coefficient (h_c) is given by the equation above
The plate temperature (T_P) = 170°C
The gas temperature (T_G) = 30°C

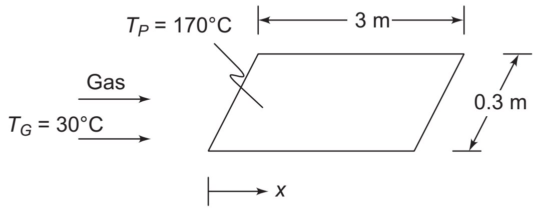
FIND

(a) The average heat transfer coefficient ($\bar{h}_c$)
(b) The rate of heat transfer (q_c)
(c) The local heat flux at x = 2 m (q_c (2)/A)

ASSUMPTIONS

Steady state prevails

SKETCH



SOLUTION

(a) The average heat transfer coefficient can be calculated by

$$\bar{h}_c = \frac{1}{L}\int_0^L h_c(x)\,dx = \frac{1}{L}\int_0^L 10\times^{-\frac{1}{4}} = \frac{10}{L}\frac{4}{3}\times^{\frac{3}{4}}\Big|_0^L = \frac{10}{3}\frac{4}{3}\,3^{\frac{3}{4}}$$

$$\bar{h}_c = 10.13 \text{ W/(m}^2\text{ K)}$$

(b) The total convective heat transfer is given by Equation (1.10)

$$q_c = \bar{h}_c\ A\ (T_P - T_G)$$

$$q_c = \left(10.13 \text{ W/(m}^2\text{K)}\right)\ (3 \text{ m})\ (0.3 \text{ m})\ (170°\text{C} - 30°\text{C})$$

$$q_c = 1273 \text{ W}$$

(c) The heat flux at $x = 2$ m is

$$\frac{q(x)}{A} = h_c(x)\ (T_P - T_G) = 10 \times^{-\frac{1}{4}}\ (T_P - T_G)$$

$$\frac{q(2)}{A} = 10\,(2)^{-\frac{1}{4}}\ (170°\text{C} - 30°\text{C})$$

$$\frac{q(2)}{A} = 1177 \text{ W/m}^2$$

COMMENTS

Note that the equation for h_c does not apply near the leading edge of the plate since h_c approaches infinity as x approaches zero. This behavior is discussed in more detail in Chapter 6.

PROBLEM 1.18

A cryogenic fluid is stored in a 0.3 m diameter spherical container in still air. If the convective heat transfer coefficient between the outer surface of the container and the air is 6.8 W/(m^2 K), the temperature of the air is 27°C and the temperature of the surface of the sphere is –183°C, determine the rate of heat transfer by convection.

GIVEN

A sphere in still air
Sphere diameter (D) = 0.3 m
Convective heat transfer coefficient $\bar{h}_c$ = 6.8 W/(m^2 K)
Sphere surface temperature (T_s) = –183°C
Ambient air temperature (T_∞) = 27°C

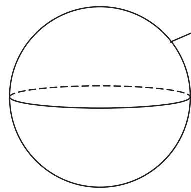
FIND

Rate of heat transfer by convection (q_c)

ASSUMPTIONS

Steady state heat flow

SKETCH



SOLUTION

The rate of heat transfer by convection is given by

$$q_c = \bar{h}_c \; A \, \Delta T$$

$$q_c = \bar{h}_c \; (\pi D^2) \, (T_\infty - T_s)$$

$$q_c = \left(6.8 \,\text{W/(m}^2\text{K)}\right) \pi (0.3 \text{ m})^2 \, [27°\text{C} - (-183°\text{C})]$$

$$q_c = 404 \text{ W}$$

COMMENTS

Condensation would probably occur in this case due to the low surface temperature of the sphere. A calculation of the total rate of heat transfer to the sphere would have to take the rate on condensation and the rate of radiative heat transfer into account.

PROBLEM 1.19

> **A high-speed computer is located in a temperature controlled room of 26°C. When the machine is operating its internal heat generation rate is estimated to be 800 W. The external surface temperature is to be maintained below 85°C. The heat transfer coefficient for the surface of the computer is estimated to be 10 W/(m^2 K). What surface area would be necessary to assure safe operation of this machine? Comment on ways to reduce this area.**

GIVEN

A high-speed computer in a temperature controlled room
Temperature of the room (T_∞) = 26°C
Maximum surface temperature of the computer (T_c) = 85°C
Heat transfer coefficient (U) = 10 W/(m K)
Internal heat generation $(\dot{q}_G)$ = 800 W

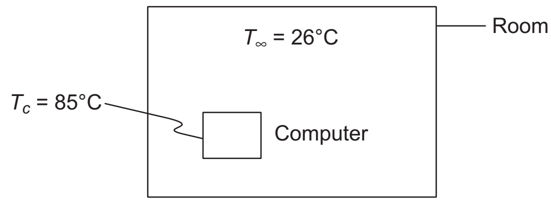
FIND

The surface area (A) required and comment on ways to reduce this area

ASSUMPTIONS

The system is in steady state

SKETCH



SOLUTION

For steady state the rate of heat transfer from the computer (given by Equation (1.33)) must equal the rate of internal heat generation

$$q = U A \Delta T = \dot{q}_G$$

Solving this for the surface area

$$A = \frac{\dot{q}_G}{U \Delta T}$$

$$A = \frac{800\,\text{W}}{\left(10\ \text{W/(m}^2\text{K)}\right)(85^{\circ}\text{C} - 26^{\circ}\text{K})} = 1.4\ \text{m}^2$$

COMMENTS

Possibilities to reduce this surface area include

Increase the convective heat transfer from the computer by blowing air over it

Add fins to the outside of the computer

PROBLEM 1.20

In order to prevent frostbite to skiers on chair lifts, the weather report at most ski areas gives both an air temperature and the wind chill temperature. The air temperature is measured with a thermometer that is not affected by the wind. However, the rate of heat loss from the skier increases with wind velocity, and the wind-chill temperature is the temperature that would result in the same rate of heat loss in still air as occurs at the measured air temperature with the existing wind.

Suppose that the inner temperature of a 3 mm thick layer of skin with a thermal conductivity of 0.35 W/(m K) is 35°C and the ambient air temperature is –20°C. Under calm ambient conditions the heat transfer coefficient at the outer skin surface is about 20 W/(m^2 K) (see Table 1.4), but in a 40 mph wind it increases to 75 W/(m^2 K). (a) If frostbite can occur when the skin temperature drops to about 10°C, would you advise the skier to wear a face mask? (b) What is the skin temperature drop due to wind chill?

GIVEN

Skier's skin exposed to cold air

Skin thickness (L) = 3 mm = 0.003 m

Inner surface temperature of skin (T_{si}) = 35°C

Thermal conductivity of skin (k) = 0.35 W/(m K)

Ambient air temperature (T_∞) = –20°C

Convective heat transfer coefficients
- Still air (h_{c0}) = 20 W/(m^2 K)
- 40 mph air (h_{c40}) = 75 W/(m^2 K)

Frostbite occurs at an outer skin surface temperature (T_{so}) = 10°C

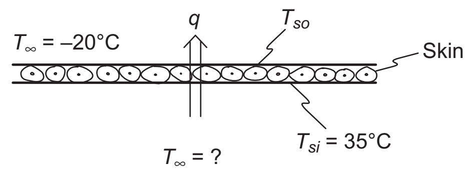
FIND

(a) Will frostbite occur under still or 40 mph wind conditions?
(b) Skin temperature drop due to wind chill.

ASSUMPTIONS

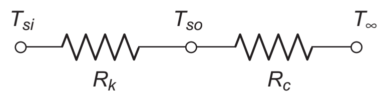
Steady state conditions prevail
One dimensional conduction occurs through the skin
Radiative loss (or gain from sunshine) is negligible

SKETCH



SOLUTION

The thermal circuit for this system is shown below



(a) The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_k + R_c} = \frac{T_{si} - T_\infty}{\left(\dfrac{L}{k\,A}\right) + \left(\dfrac{1}{\bar{h}_c\,A}\right)}$$

$$\therefore \frac{q}{A} = \frac{T_{si} - T_\infty}{\dfrac{L}{k} + \dfrac{1}{\bar{h}_c}}$$

The outer surface temperature of the skin in still air can be calculated by examining the conduction through the skin layer

$$q_k = \frac{k\,A}{L}\,(T_{si} - T_{so})$$

Solving for the outer skin surface temperature

$$T_{so} = T_{si} - \frac{q_k}{A}\frac{L}{k}$$

The rate of heat transfer by conduction through the skin must be equal to the total rate of heat transfer, therefore

$$T_{so} = T_{si} - \left[\frac{T_{si} - T_\infty}{\dfrac{L}{K} + \dfrac{1}{\bar{h}_c}}\right]\frac{L}{k}$$

Solving this for still air

$$(T_{so})_{\text{still air}} = 35°\text{C} - \left[\frac{35^{\text{o}}\,\text{C} - (-20^{\text{o}}\,\text{C})}{\dfrac{0.003\,\text{m}}{0.25\,\text{W/(m K)}} + \dfrac{1}{20\,\text{W/(m}^2\,\text{K)}}}\right]\frac{0.003\,\text{m}}{0.25\,\text{W/(m}^2\text{K)}}$$

$$(T_{so})_{\text{still air}} = 24°\text{C}$$

For a 40 mph wind

$$(T_{so})_{\text{40 mph}} = 35°\text{C} - \left[\frac{35^{\text{o}}\,\text{C} - (-20^{\text{o}}\,\text{C})}{\dfrac{0.003\,\text{m}}{0.25\,\text{W/(m K)}} + \dfrac{1}{75\,\text{W/(m}^2\,\text{K)}}}\right]\frac{0.003\,\text{m}}{0.25\,\text{W/(m}^2\text{K)}}$$

$$(T_{so})_{\text{40 mph}} = 9°\text{C}$$

Therefore, frostbite may occur under the windy conditions.

(b) Comparing the above results we see that the skin temperature drop due to the wind chill was 15°C.

PROBLEM 1.21

Using the information in Problem 1.20, estimate the ambient air temperature that could cause frostbite on a calm day on the ski slopes.

From Problem 1.20

Suppose that the inner temperature of a 3 mm thick layer of skin with a thermal conductivity of 0.35 W/(m K) is a temperature of 35°C. Under calm ambient conditions the heat transfer coefficient at the outer skin surface is about 20 W/(m^2 K). Frostbite can occur when the skin temperature drops to about 10°C.

GIVEN

Skier's skin exposed to cold air
Skin thickness (L) = 3 mm = 0.003 m
Inner surface temperature of skin (T_{si}) = 35°C
Thermal conductivity of skin (k) = 0.35 W/(m K)
Convective heat transfer coefficient in still air ($\bar{h}_c$) = 20 W/(m^2 K)
Frostbite occurs at an outer skin surface temperature (T_{so}) = 10°C

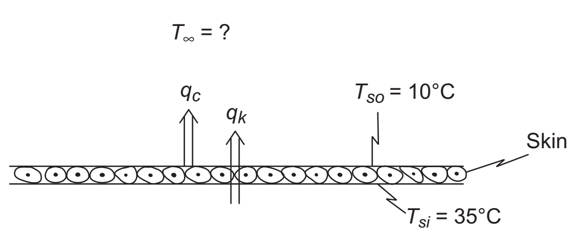
FIND

The ambient air temperature (T_∞) that could cause frostbite

ASSUMPTIONS

Steady state conditions prevail
One dimensional conduction occurs through the skin
Radiative loss (or gain from sunshine) is negligible

SKETCH



SOLUTION

The rate of conductive heat transfer through the skin at frostbite conditions is given by Equation (1.2)

$$q_k = \frac{k\,A}{L}\,(T_{si} - T_{so})$$

The rate of convective heat transfer from the surface of the skin, from equation (1.10), is

$$q_c = \bar{h}_c\,A\,(T_{so} - T_\infty)$$

These heat transfer rates must be equal

$$q_k = q_c$$

$$\frac{k\,A}{L}\,(T_{si} - T_{so}) = \bar{h}_c\,A\,(T_{so} - T_\infty)$$

Solving for the ambient air temperature

$$T_\infty = T_{so}\left(1 + \frac{k}{\bar{h}_c\,L}\right) - T_{si}\left(\frac{k}{\bar{h}_c\,L}\right)$$

$$T_\infty = 10°\text{C}\left[1 + \frac{0.25\,\text{W/(m K)}}{\left[20\,\text{W/(m}^2\text{K)}\right](0.003\,\text{m})}\right] - 35°\text{C}\left[\frac{0.25\,\text{W/(m K)}}{\left[20\,\text{W/(m}^2\text{K)}\right](0.003\,\text{m})}\right]$$

$$T_\infty = -94°\text{C}$$

PROBLEM 1.22

> **Two large parallel plates with surface conditions approximating those of a blackbody are maintained at 816 and 260°C, respectively. Determine the rate of heat transfer by radiation between the plates in W/m^2 and the radiative heat transfer coefficient in W/(m^2 K).**

GIVEN

Two large parallel plates, approximately black bodies

Temperatures
- T_1 = 816°C
- T_2 = 260°C

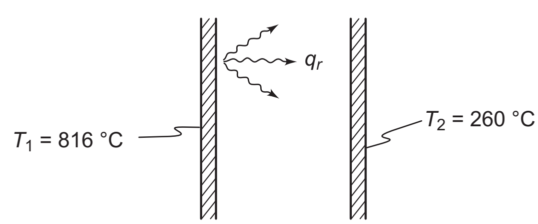
FIND

(a) Rate of radiative heat transfer (q_r/A) in W/m^2
(b) Radiative heat transfer coefficient (h_r) in W/(m^2 K)

ASSUMPTIONS

Steady state prevails
Edge effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: Stefan-Boltzmann constant (σ) = 5.7×10^{-8} W/(m^2 K^4)

SOLUTION

(a) The rate of heat transfer is given by Equation (1.16)

$$\frac{q_r}{A} = \sigma(T_1^4 - T_2^4)$$

$$\frac{q_r}{A} = (5.7 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4))\ [(1089\ \text{K})^4 - (533\ \text{K})^4]$$

$$\Rightarrow \qquad \frac{q_r}{A} = 75.56\ \text{(kW)/m}^2$$

(b) Let h_r represent the radiative heat transfer coefficient

$$q_r = h_r\, A\, \Delta T$$

$$\therefore\ h_r = \frac{q_r}{A}\,\frac{1}{\Delta T} = \frac{7.556\times 10^4\ \text{W/m}^2}{(816-260)^\circ\text{C}}$$

$$h_r = 136\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equation, whereas h_r is based on the assumption that the rate of heat transfer is proportional to the temperature difference. Hence h_r cannot be applied to any other temperatures than those specified.

PROBLEM 1.23

> **A spherical vessel 0.3 m in diameter is located in a large room whose walls are at 27°C (see sketch). If the vessel is used to store liquid oxygen at –183°C and the surface of the storage vessel as well as the walls of the room are black, calculate the rate of heat transfer by radiation to the liquid oxygen in watts.**

GIVEN

A black spherical vessel of liquid oxygen in a large black room
Liquid oxygen temperature (T_o) = –183°C = 90 K
Sphere diameter (D) = 0.3 m
Room wall temperature (T_w) = 27°C = 300 K

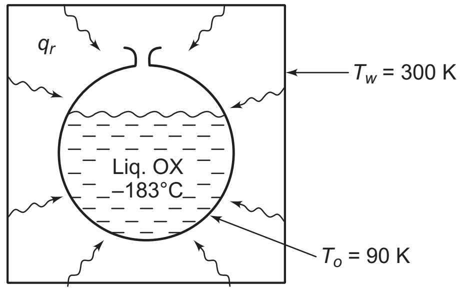
FIND

The rate of radiative heat transfer to the liquid oxygen in W

ASSUMPTIONS

Steady state prevails
The temperature of the vessel wall is the same as the temperature of the oxygen

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The net radiative heat transfer to a black body in a black enclosure is given by Equation (1.16)

$$q_r = A\ \sigma(T_1^4 - T_2^4)$$
$$q_r = \pi D^2\ \sigma(T_w^4 - T_o^4)$$

Converting the net radiative heat transfer into SI units using the conversion factor given on the inside front cover of the text

$$q_r = 133 \text{ W}$$

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equation.

PROBLEM 1.24

> **Repeat Problem 1.23 but assume that the surface of the storage vessel has an absorptance (equal to the emittance) of 0.1. Then determine the rate of evaporation of the liquid oxygen in kilograms per second and pounds per hour, assuming that convection can be neglected. The heat of vaporization of oxygen at –183°C is 213.3 kJ/kg.**
>
> **From Problem 1.23: A spherical vessel of 0.3 m in diameter is located in a large room whose walls are at 27°C (see sketch). If the vessel is used to store liquid oxygen at –183°C and the surface of the storage vessel as well as the walls of the room are black, calculate the rate of heat transfer by radiation to the liquid oxygen in watts.**

GIVEN

A spherical vessel of liquid oxygen in a large black room
Emittance of vessel surface (ε) = 0.1
Liquid oxygen temperature (T_o) = –183°C = 90 K
Sphere diameter (D) = 0.3 m
Room wall temperature (T_w) = 27°C = 300 K
Heat of vaporization of oxygen (h_{fg}) = 213.3 kJ/kg

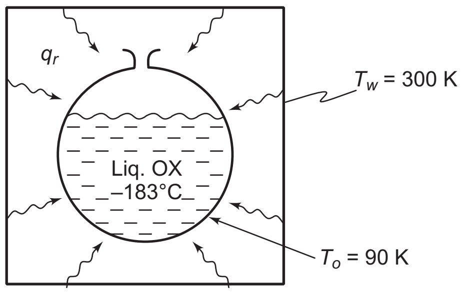
FIND

(a) The rate of radiative heat transfer (q_r) to the liquid oxygen in W
(b) The rate of evaporation of oxygen (m_o) in kg/s and 1b/h

ASSUMPTIONS

Steady state prevails
The temperature of the vessel wall is equal to the temperature of the oxygen
Convective heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) The net radiative heat transfer from a gray body in a black enclosure, from Equation (1.17) is

$$q_r = A_1\, \varepsilon_1\, \sigma (T_1^4 - T_2^4)$$

$$q_r = \pi D^2\, \varepsilon\, \sigma (T_o^4 - T_w^4)$$

$$q_r = \pi (0.3\text{ m})^2\, (0.1)\, (5.67 \times 10^{-8}\,[\text{W/(m}^2\text{ K}^4)]\,[(90\text{ K})^4 - (300\text{ K})^4)]$$

$$q_k = -12.9\text{ W}$$

(b) The rate of evaporation of oxygen is given by

$$\dot{m}_o = \frac{q_r}{h_{fg}}$$

$$\dot{m}_o = \frac{(12.9\text{ W})\,(\text{J/Ws})}{(213.3\text{ kJ/kg})(1000\text{ J/kJ})}$$

$$\dot{m}_o = 6.05 \times 10^{-5}\text{ kg/s}$$

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equation.

The negative sign in the rate of heat transfer indicates that the sphere is gaining heat from the surrounding wall.

Note that the rate of heat transfer by radiation can be substantially reduced (see Problem 1.23) by applying a surface treatment, e.g., applying a metallic coating with low emissivity.

PROBLEM 1.25

> **Determine the rate of radiant heat emission in watts per square meter from a blackbody at (a) 150°C, (b) 600°C, (c) 5700°C.**

GIVEN

A blackbody

FIND

The rate of radiant heat emission (q_r) in W/m^2 for a temperature of

(a) T = 150°C = 423 K

(b) T = 600°C = 873 K

(c) T = 5700°C = 5973 K

PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The rate of radiant heat emission from a blackbody is given by Equation (1.15)

$$q_r = \sigma A_1 T_1^4$$

$$\frac{q_r}{A} = \sigma T^4$$

(a) For T = 423 K

$$\frac{q_r}{A} = [5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4)] (423\text{K})^4$$

$$\frac{q_r}{A} = 1820 \text{ W/m}^2$$

(b) For T = 873 K

$$\frac{q_r}{A} = [5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4)] (873 \text{ K})^4$$

$$\frac{q_r}{A} = 32{,}900 \text{ W/m}^2$$

(c) For T = 5973 K

$$\frac{q_r}{A} = [(5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4)] (5974 \text{ K})^4$$

$$\frac{q_r}{A} = 7.2 \times 10^7 \text{ W/m}^2$$

COMMENTS

Note that absolute temperatures must be used in radiative heat transfer equations.
The rate of heat transfer is proportional to the absolute temperature to the fourth power, this results in a rapid increase in the rate of heat transfer with increasing temperature.

PROBLEM 1.26

> **The sun has a radius of 7 × 10^8 m and approximates a blackbody with a surface temperature of about 5800 K. Calculate the total rate of radiation from the sun and the emitted radiation flux per square meter of surface area.**

GIVEN

The sun approximates a blackbody
Surface temperature (T_s) = 5800 K
Radius (r) = 7×10^8 m

FIND

(a) The total rate of radiation from the sun (q_r)
(b) The radiation flux per square meter of surface area (q_r/A)

PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The rate of radiation from a blackbody, from Equation (1.15), is

$$q_r = \sigma A\, T^4$$

$$q_r = [5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)]\ [4\pi(7 \times 10^8\ \text{m})^2]\ (5800\ \text{K})^4$$

$$q_r = 4.0 \times 10^{26}\ \text{W}$$

The flux per square meter is given by

$$\frac{q_r}{A} = \sigma T^4$$

$$\frac{q_r}{A} = [5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)]\ (5800\ \text{K})^4$$

$$\frac{q_r}{A} = 6.4 \times 10^7\ \text{W/m}^2$$

COMMENTS

The solar radiation flux impinging in the earth's atmosphere is only 1400 W/m^2. Most of the radiation from the sun goes into space.

PROBLEM 1.27

> **A small gray sphere having an emissivity of 0.5 and a surface temperature of 537°C is located in a blackbody enclosure having a temperature of 37°C. Calculate for this system: (a) the net rate of heat transfer by radiation per unit of surface area of the sphere, (b) the radiative thermal conductance in W/K if the surface area of the sphere is 95 cm^2, (c) the thermal resistance for radiation between the sphere and its surroundings, (d) the ratio of thermal resistance for radiation to thermal resistance for convection if the convective heat transfer coefficient between the sphere and its surroundings is 11 W/(m^2 K), (e) the total rate of heat transfer from the sphere to the surroundings, and (f) the combined heat transfer coefficient for the sphere.**

GIVEN

Small gray sphere in a blackbody enclosure
Sphere emissivity (ε_s) = 0.5
Sphere surface temperature (T_1) = 537°C = 810 K
Enclosure temperature (T_2) = 37°C = 310 K
The surface area of the sphere (A) is 95 cm^2
The convective transfer coefficient ($\bar{h}_c$) = 11 W/(m^2 K)

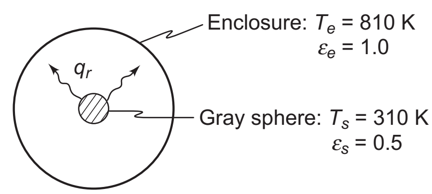
FIND

(a) Rate of heat transfer by radiation per unit surface area
(b) Radiative thermal conductance (K_r) in W/K
(c) Thermal resistance for radiation (R_r)
(d) Ratio of the radiative and conductive resistance
(e) Total rate of heat transfer (q_T) to the surroundings
(f) Combined heat transfer coefficient ($\bar{h}_{cr}$)

ASSUMPTIONS

Steady state prevails
The temperature of the fluid in the enclosure is equal to the enclosure temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) For a gray body radiating to a blackbody enclosure the net heat transfer is given by Equation (1.17)

$$q_r = A_1\, \varepsilon_1\, \sigma (T_1^4 - T_2^4)$$

$$\frac{q_r}{A} = (0.5)\,(5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4))\ [(810\ \text{K})^4 - (310\ \text{K})^4]$$

$$\frac{q_r}{A} = 11.94\ \text{kW/m}^2$$

(b) The radiative thermal conductance must be based on some reference temperature. Let the reference temperature be the enclosure temperature. Then, from Equation (1.21), the radiative thermal conductance is

$$K_r = \frac{A_1 \mathscr{F}_{1-2}\, \sigma (T_1^4 - T_1^4)}{T_1 - T_2^{\prime}} \quad \text{where } \mathscr{F}_{1-2} = \varepsilon_s$$

$$K_r = \frac{(95 \times 10^{-4}\ \text{m}^2)(0.5)(5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4))[(810\,\text{K})^4 - (310\,\text{K})^4]}{810\,\text{K} - 310\,\text{K}}$$

$$\Rightarrow \qquad K_r = 0.227\ \text{W/K}$$

(c) The thermal resistance for radiation is given by

$$R_r = \frac{1}{K_r} = \frac{1}{0.227\,(\text{W/K})} = 4.4\ \text{K/W}$$

(d) The convective thermal resistance is given by Equation (1.14)

$$R_c = \frac{1}{\bar{h}_c\, A} = \frac{1}{(11\,\text{W/(m}^2\text{K}))(95 \times 10^{-4}\ \text{m}^2)} = 9.57\ \text{K/W}$$

Therefore the ratio of the radiative to the convective resistance is

$$\frac{R_r}{R_c} = \frac{4.4\,\text{K/W}}{9.57\,\text{K/W}} = 0.46$$

(e) The radiative and convective resistances are in parallel, therefore the total resistance, from Figure 1.18, is

$$R_{\text{total}} = \frac{R_c\,R_r}{R_c + R_r} = \frac{(9.57)(4.4)}{9.57 + 4.4} = 3.01\ \text{K/W}$$

The total heat transfer is given by

$$q_T = \frac{\Delta T}{R_{\text{total}}} = \frac{810\,\text{K} - 310\,\text{K}}{3.01\,\text{K/W}} = 166.1\ \text{W}$$

(f) The combined heat transfer coefficient can be calculated from

$$q_T = \bar{h}_{cr}\ A\Delta\,T$$

$$\therefore\ \bar{h}_{cr} = \frac{q_T}{A\,\Delta T} = \frac{166.1\,\text{W}}{(95 \times 10^{-4}\,\text{m}^2)\,(810\,\text{K} - 310\,\text{K})}$$

$$\bar{h}_{cr} = 34.97\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equations.
Both heat transfer mechanisms are of the same order of magnitude in this situation.

PROBLEM 1.28

A spherical communications satellite 2 m in diameter is placed in orbit around the earth. The satellite generates 1000 W of internal power from a small nuclear generator. If the surface of the satellite has an emittance of 0.3 and is shaded from solar radiation by the earth, estimate the surface temperature.

GIVEN

Spherical satellite
Diameter (D) = 2 m
Heat generation = 1000 W
Emittance (ε) = 0.3

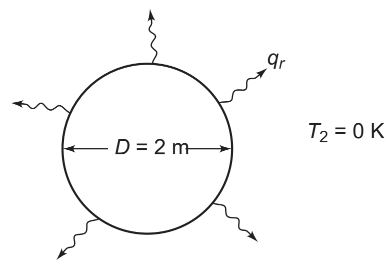
FIND

The surface temperature (T_s)

ASSUMPTIONS

The satellite radiates to space which behaves as a blackbody enclosure at 0 K
The system is in steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

From Equation (1.17), the rate of the heat transfer from a gray body in a blackbody enclosure is

$$q_r = A_1\, \varepsilon_1\, \sigma (T_1^4 - T_2^4)$$

Solving this for the surface temperature

$$T_1 = \left(\frac{q_r}{A_1 \varepsilon_1 \sigma}\right)^{\frac{1}{4}} = \left(\frac{q_r}{\pi D^2 \varepsilon_1 \sigma}\right)^{\frac{1}{4}}$$

For steady state the rate of heat transfer must equal the rate of internal generation, therefore the surface temperature is

$$T_1 = \left(\frac{1000\,\text{W}}{\pi (2\,\text{m})^2 (0.3) 5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)}\right)^{\frac{1}{4}} = 262\text{ K} = -11°\text{C}$$

PROBLEM 1.29

A long wire 0.7 mm in diameter with an emissivity of 0.9 is placed in a large quiescent air space at 270 K. If the wire is at 800 K, calculate the net rate of heat loss. Discuss your assumptions.

GIVEN

Long wire in still air
Wire diameter (D) = 0.7 mm
Wire temperature (T_s) = 800 K
Emissivity (ε) = 0.9
Air temperature (T_∞) = 270 K

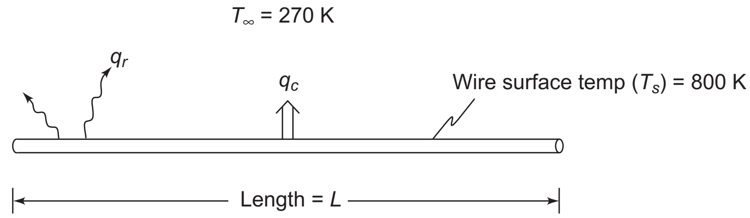
FIND

The net rate of heat loss

ASSUMPTIONS

The enclosure around the wire behaves as a blackbody enclosure at the temperature of the air
The natural convection heat transfer coefficient is 17 W/(m^2 K) (From Table 1.4)
Steady state conditions prevail

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The total rate of heat loss from the wire is the sum of the convective (Equation (1.10)) and radiative (Equation (1.17)) losses

$$q_{\text{total}} = \bar{h}_c\, A\,(T_s - T_\infty) + A\,\varepsilon\,\sigma\,(T_s^4 - T_\infty^4)$$

$$q_{\text{total}} = (17\ \text{W/(m}^2\ \text{K)})\ \pi\,(0.7 \times 10^{-3})\ L\ (800\ \text{K} - 270\ \text{K})$$
$$+\ \pi\,(0.7 \times 10^{-3})\ L\ (0.9)\ (5.67 \times 10^{-8})\ [(800\ \text{K})^4 - (270\ \text{K})^4]$$

$$\frac{q_{\text{total}}}{L} = 65\ \text{W/m} = 65\ \text{W per m of wire length}$$

COMMENTS

The radiative heat transfer is about twice the magnitude of the convective transfer.

The enclosure is more likely a gray body, therefore the actual rate of loss will be smaller than we have calculated.

The convective heat transfer coefficient may differ by a factor of two or three from our assumed value.

PROBLEM 1.30

> **Wearing layers of clothing in cold weather is often recommended because dead-air spaces between the layers keep the body warm. The explanation for this is that the heat loss from the body is less. Compare the rate of heat loss for single 2 cm-thick layer of wool [*k* = 0.04 W/(m K)] with three 0.67 cm layers separated by 1.5 mm air gaps. The thermal conductivity of air is 0.024 W/(m K).**

GIVEN

Wool insulation

Thermal conductivities
- Wool (k_w) = 0.04 W/(m K)
- Air (k_a) = 0.024 W/(m K)

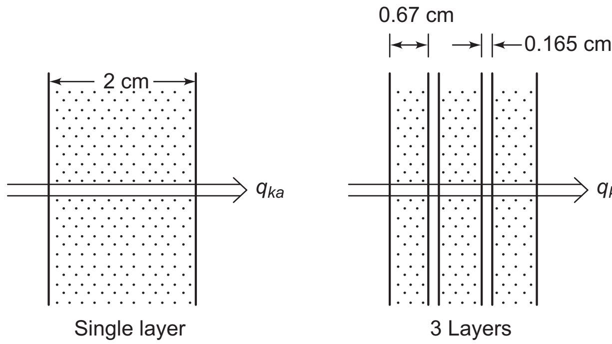
FIND

Compare the rate of heat loss for a single 2 cm layer of wool to that of three 0.67 cm layers separated by 0.165 cm layers of air

ASSUMPTIONS

Heat transfer can be approximated as one dimensional, steady state conduction

SKETCH



SOLUTION

The thermal resistance for the single thick layer, from Equation (1.3), is

$$R_{ka} = \frac{L}{k_w A} = \frac{0.02\,\text{m}}{(0.04\,\text{W}/(\text{m K}))(A\,\text{m}^2)} = \frac{1}{A}\ 0.5\ \text{K/W}$$

(A is the area of the body covered by wool)

The rate of conductive heat transfer is

$$q_{ka} = \frac{\Delta T}{R_{ka}} = \frac{\Delta T\ K}{\frac{1}{A}\ 0.5\,\text{K/W}} = \Delta T\ (\text{K})\ \text{A}\ (\text{m}^2)\ 2\text{W}$$

The thermal resistance for three thin layers equals sum of the resistances of the wool and the air between the layers

$$R_{kb} = \frac{L_w}{k_w A} + \frac{L_a}{k_a A}$$

$$= \frac{(3\,\text{layers})(0.0067\,\text{m/layer})}{A(\text{m}^2)(0.04\,\text{W}/(\text{m K}))} + \frac{(2\,\text{layers})(0.0015\,\text{m/layer})}{A(\text{m}^2)(0.024\,\text{W}/(\text{m K}))}$$

$$= \frac{1}{A}\ [0.5 + 0.125] = \frac{1}{A(\text{m}^2)}\ 0.625\ \text{K/W}$$

The rate of conductive heat transfer for the three layer situation is

$$q_{kb} = \frac{\Delta T}{R_{kb}} = \frac{\Delta T\,(\text{K})}{\frac{1}{A(\text{m}^2)}0.625\,\text{K/W}} = \Delta T\ (\text{K})\ \text{A}(\text{m}^2)\ 1.6\ \text{W}$$

Comparing the rate of heat loss for the two situations

$$\therefore\ \frac{q_{kb}}{q_{ka}} = \frac{1.6\,\text{W}}{2.0\,\text{W}} = 0.8$$

Therefore, for the same temperature difference, the heat loss through the three layers of wool is only 80% of the heat loss through the single layer.

PROBLEM 1.31

A section of a composite wall with the dimensions shown below has uniform temperatures of 200°C and 50°C over the left and right surfaces, respectively. If the thermal conductivities of the wall materials are: k_A = 70 W/(m K), k_B = 60 W/(m K), k_c = 40 W/(m K) and k_D = 20 W/(m K), determine the rate of heat transfer through this section of the wall and the temperatures at the interfaces.

GIVEN

A section of a composite wall

Thermal conductivities	▪ k_A = 70 W/(m K)
	▪ k_B = 60 W/(m K)
	▪ k_C = 40 W/(m K)
	▪ k_D = 20 W/(m K)
Surface temperatures	▪ Left side (T_{As}) = 200°C
	▪ Right side (T_{Ds}) = 50°C

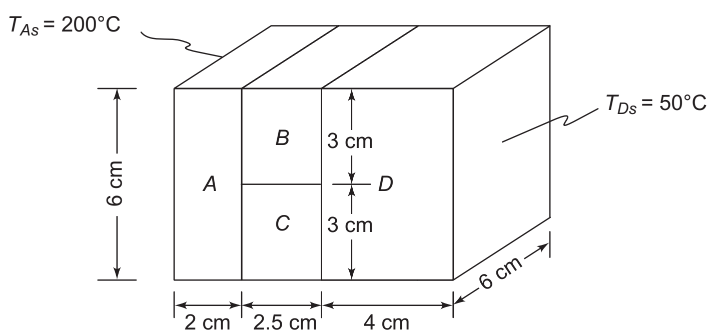
FIND

(a) Rate of heat transfer through the wall (q)
(b) Temperature at the interfaces

ASSUMPTIONS

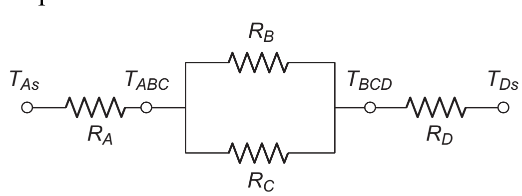
One dimensional conduction
The system is in steady state
The contact resistances between the materials is negligible

SKETCH



SOLUTION

The thermal circuit for the composite wall is shown below



(a) Each of these thermal resistances has a form given by Equation (1.3)

$$R_k = \frac{L}{A\,k}$$

Evaluating the thermal resistance for each component of the wall

$$R_A = \frac{L_A}{A_A k_A} = \frac{0.02\,\text{m}}{(0.06\,\text{m})(0.06\,\text{m})[70\,\text{W/(m K)}]} = 0.0794\ \text{K/W}$$

$$R_B = \frac{L_B}{A_B k_B} = \frac{0.025\,\text{m}}{(0.03\,\text{m})(0.06\,\text{m})[60\,\text{W/(m K)}]} = 0.2315\ \text{K/W}$$

$$R_C = \frac{L_C}{A_C k_C} = \frac{0.025\,\text{m}}{(0.03\,\text{m})(0.06\,\text{m})[40\,\text{W/(m K)}]} = 0.3472\ \text{K/W}$$

$$R_D = \frac{L_D}{A_D k_D} = \frac{0.04\,\text{m}}{(0.06\,\text{m})(0.06\,\text{m})[20\,\text{W/(m K)}]} = 0.5556\ \text{K/W}$$

The total thermal resistance of the wall section, from Section 1.5.1, is

$$R_{\text{total}} = R_A + \frac{R_B R_C}{R_B + R_C} + R_D$$

$$R_{\text{total}} = 0.0794 + \frac{(0.2315)(0.3472)}{0.2315 + 0.3472} + 0.5556\ \text{K/W}$$

$$R_{\text{total}} = 0.7738\ \text{K/W}$$

The total rate of heat transfer through the composite wall is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{200^{\circ}\text{C} - 50^{\circ}\text{C}}{0.7738\,\text{K/W}} = 194\ \text{W}$$

(b) The average temperature at the interface between material A and materials B and C (T_{ABC}) can be determined by examining the conduction through material A alone

$$q_{ka} = \frac{T_{As} - T_{ABC}}{R_A} = q$$

Solving for T_{ABC}

$$T_{ABC} = T_{As} - q\,R_A = 200°\text{C} - (194\ \text{W})\ (0.0794\ \text{K/W}) = 185°\text{C}$$

The average temperature at the interface between material D and materials B and C (T_{BCD}) can be determined by examining the conduction through material D alone

$$q_{kD} = \frac{T_{BCD} - T_{Ds}}{R_D} = q$$

Solving for T_{BCD}

$$T_{BCD} = T_{Ds} + q\,R_D = 50°\text{C} + (194\ \text{W})\ (0.5556\ \text{K/W}) = 158°\text{C}$$

PROBLEM 1.32

Repeat the Problem 1.31 including a contact resistance of 0.1 K/W at each of the interfaces.

Problem 1.31: A section of a composite wall with the dimensions shown in the schematic diagram below has uniform temperatures of 200°C and 50°C over the left and right surfaces, respectively. If the thermal conductivities of the wall materials are: k_A = 70 W/(m K), k_B = 60 W/(m K), k_C = 40 W/(m K), and k_D = 20 W/(m K), determine the rate of heat transfer through this section of the wall and the temperatures at the interfaces.

GIVEN

Composite wall

Thermal conductivities
- k_A = 70 W/(m K)
- k_B = 60 W/(m K)
- k_C = 40 W/(m K)
- k_D = 20 W/(m K)
- Right side (T_{Ds}) = 50°C

Contact resistance at each interface (R_i) = 0.1 K/W

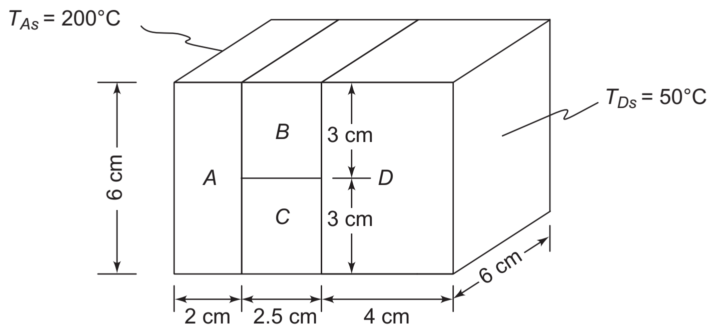
FIND

(a) Rate of heat transfer through the wall (q)
(b) Temperatures at the interfaces

ASSUMPTIONS

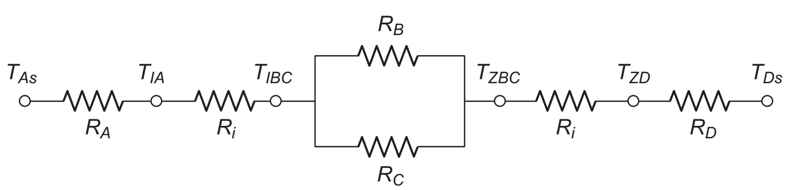
One dimensional conduction
The system is in steady state

SKETCH



SOLUTION

The thermal circuit for the composite wall with contact resistances is shown below



The values of the individual resistances, from Problem 1.31, are

$$R_A = 0.0794 \text{ K/W} \qquad R_B = 0.2315 \text{ K/W} \qquad R_C = 0.3472 \text{ K/W} \qquad R_D = 0.5556 \text{ K/W}$$

(a) The total resistance for this system is

$$R_{\text{total}} = R_A + R_i + \frac{R_B R_C}{R_B + R_C} + R_i + R_D$$

$$R_{\text{total}} = 0.0794 + 0.1 + \frac{(0.2315)(0.3472)}{0.2315 + 0.3472} + 0.1 + 0.5556 \text{ K/W}$$

$$R_{\text{total}} = 0.9738 \text{ K/W}$$

The total rate of heat transfer through the composite wall is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{200°\text{C} - 50°\text{C}}{0.9738 \text{ K/W}} = 154 \text{ W}$$

(b) The average temperature on the *A* side of the interface between material *A* and material *B* and *C* (T_{1A}) can be determined by examining the conduction through material *A* alone

$$q = \frac{T_{As} - T_{1A}}{R_A}$$

Solving for T_{1A}

$$T_{1A} = T_{As} - q\,R_A = 200°\text{C} - (154\text{ W})\,(0.0794\text{ K/W}) = 188°\text{C}$$

The average temperature on the *B* and *C* side of the interface between material *A* and materials *B* and *C* (T_{1BC}) can be determined by examining the heat transfer through the contact resistance

$$q = \frac{T_{1A} - T_{1BC}}{R_i}$$

Solving for T_{1BC}

$$T_{1BC} = T_{1A} - q\,R_i = 188°\text{C} - (154\text{ W})\,(0.1\text{ K/W}) = 172°\text{C}$$

The average temperature on the *D* side of the interface between material *D* and materials *B* and *C* (T_{2D}) can be determined by examining the conduction through material *D* alone

$$q = \frac{T_{2D} - T_{Ds}}{R_D}$$

Solving for T_{2D}

$$T_{2D} = T_{Ds} + q\,R_D = 50°\text{C} + (154\text{ W})\,(0.5556\text{ K/W}) = 136°\text{C}$$

The average temperature on the *B* and *C* side of the interface between material *D* and materials *B* and *C* (T_{2BC}) can be determined by examining the heat transfer through the contact resistance

$$q = \frac{T_{2BC} - T_{2D}}{R_i}$$

Solving for T_{2BC}

$$T_{2BC} = T_{2D} + q\,R_i = 136°\text{C} + (154\text{ W})\,(0.1\text{ K/W}) = 151°\text{C}$$

COMMENTS

Note that the inclusion of the contact resistance lowers the calculated rate of heat transfer through the wall section by about 20%.

PROBLEM 1.33

Repeat the Problem 1.32 but assume that instead of surface temperatures, the given temperatures are those of air on the left and right sides of the wall and that the convective heat transfer coefficients on the left and right surfaces are 6 and 10 W/(m^2 K), respectively.

Problem 1.32: Repeat the Problem 1.31 including a contact resistance of 0.1 K/W at each of the interfaces.

Problem 1.31: A section of a composite wall with the dimensions shown in the schematic diagram below has uniform temperatures of 200°C and 50°C over the left and right surfaces, respectively. If the thermal conductivities of the wall materials are: k_A = 70 W/(m K), k_B = 60 W/(m K), k_C = 40 W/(m K), and k_D = 20 W/(m K), determine the rate of heat transfer through this section of the wall and the temperatures at the interfaces.

GIVEN

Composite wall

Thermal conductivities
- k_A = 70 W/(m K)
- k_B = 60 W/(m K)
- k_C = 40 W/(m K)
- k_D = 20 W/(m K)

Air temperatures
- Left side ($T_{A\infty}$) = 200°C
- Right side ($T_{D\infty}$) = 50°C

Contact resistance at each interface (R_i) = 0.1 K/W

Convective heat transfer coefficients
- Left side ($\overline{h}_{cA}$) = 6 W/(m^2 K)
- Right side ($\overline{h}_{cD}$) = 10 W/(m^2 K)

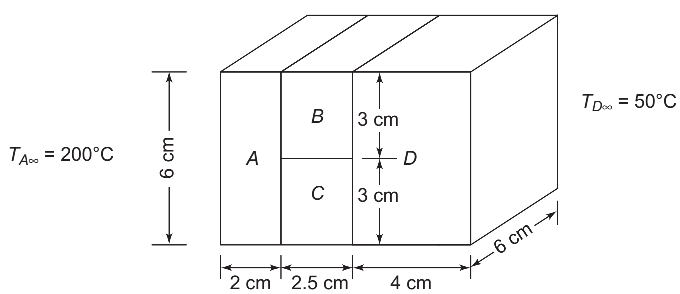
FIND

(a) Rate of heat transfer through the wall (q)
(b) Temperatures at the interfaces

ASSUMPTIONS

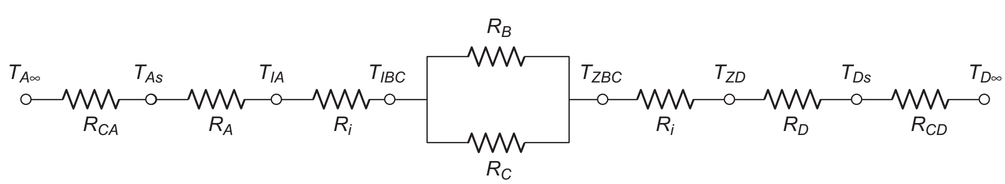
One dimensional, steady state conduction

SKETCH



SOLUTION

The thermal circuit for the composite wall with contact resistances and convection from the outer surfaces is shown below



The values of the individual conductive resistances, from Problem 1.31, are

R_A = 0.0794 K/W R_B = 0.2315 K/W R_C = 0.3472 K/W R_D = 0.5556 K/W

The values of the convective resistances, using Equation (1.14), are

$$R_{cA} = \frac{1}{\overline{h}_{cA} A} = \frac{1}{[6\,\text{W/(m}^2\text{K)}](0.06\,\text{m})(0.06\,\text{m})} = 46.3\ \text{K/W}$$

$$R_{cD} = \frac{1}{\overline{h}_{cD} A} = \frac{1}{[10\,\text{W/(m}^2\text{K)}](0.06\,\text{m})(0.06\,\text{m})} = 27.8\ \text{K/W}$$

(a) The total resistance for this system is

$$R_{\text{total}} = R_{cA} + R_A + R_i + \frac{R_B\,R_C}{R_B + R_C} + R_i + R_D + R_{cD}$$

$$R_{\text{total}} = 46.3 + 0.0794 + 0.1 + \frac{(0.2315)(0.3472)}{0.2315 + 0.34472} + 0.1 + 0.5556 + 27.8 \text{ K/W}$$

$$R_{\text{total}} = 75.1 \text{ K/W}$$

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{200°C - 50°C}{75.1\text{K/W}} = 2.0 \text{ W}$$

(b) The surface temperature on the left side of material A (T_{As}) can be determined by examining the convection from the surface of material A

$$q = \frac{T_{A\infty} - T_{As}}{R_{cA}}$$

Solving for T_{As}

$$T_{As} = T_{A\infty} - q\,R_{cA} = 200°\text{C} - (2 \text{ W})\,(46.3 \text{ K/W}) = 107.4°\text{C}$$

The average temperature on the A side of the interface between material A and material B and C (T_{1A}) can be determined by examining the conduction through material A alone

$$q = \frac{T_{As} - T_{1A}}{R_A}$$

Solving for T_{1A}

$$T_{1A} = T_{As} - q\,R_A = 107.4°\text{C} - (2 \text{ W})\,(0.0794 \text{ K/W}) = 107.2°\text{C}$$

The average temperature on the B and C side of the interface between material A and materials B and C (T_{1BC}) can be determined by examining the heat transfer through the contact resistance

$$q = \frac{T_{1A} - T_{1BC}}{R_i}$$

Solving for T_{1BC}

$$T_{1BC} = T_{1A} - q\,R_i = 107.2°\text{C} - (2 \text{ W})\,(0.1 \text{ K/W}) = 107.0°\text{C}$$

The surface temperature on the D side of the wall (T_{Ds}) can be determined by examining the convection from that side of the wall

$$q = \frac{T_{Ds} - T_{D\infty}}{R_{cD}}$$

Solving for T_{Ds}

$$T_{Ds} = T_{D\infty} + q\,R_{cD} = 50°\text{C} + (2 \text{ W})\,(27.8 \text{ K/W}) = 105.6°\text{C}$$

The average temperature on the D side of the interface between material D and materials B and C (T_{2D}) can be determined by examining the conduction through material D alone

$$q = \frac{T_{2D} - T_{Ds}}{R_D}$$

Solving for T_{2D}

$$T_{2D} = T_{Ds} + q\,R_D = 105.6°\text{C} + (2 \text{ W})\,(0.5556 \text{ K/W}) = 106.7°\text{C}$$

The average temperature on the B and C side of the interface between material D and materials B and C (T_{2BC}) can be determined by examining the heat transfer through the contact resistance

$$q = \frac{T_{2BC} - T_{2D}}{R_i}$$

Solving for T_{2BC} $\quad T_{2BC} = T_{2D} + q\,R_i = 106.7°\text{C} + (2 \text{ W})\,(0.1 \text{ K/W}) = 106.9°\text{C}$

COMMENTS

Note that the addition of the convective resistances reduced the rate of heat transfer through the wall section by a factor of 77.

PROBLEM 1.34

> **Mild steel nails were driven through a solid wood wall consisting of two layers, each 2.5 cm thick, for reinforcement. If the total cross-sectional area of the nails is 0.5% of the wall area, determine the unit thermal conductance of the composite wall and the per cent of the total heat flow that passes through the nails when the temperature difference across the wall is 25°C. Neglect contact resistance between the wood layers.**

GIVEN

Wood wall
Two layers 0.025 m thick each
Nail cross sectional area of nails = 0.5% of wall area
Temperature difference (ΔT) = 25°C

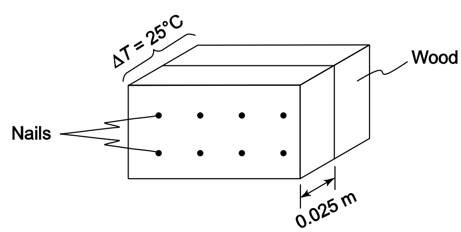
FIND

(a) The unit thermal conductance (k/L) of the wall
(b) Percent of total heat flow that passes through the wall

ASSUMPTIONS

One dimensional heat transfer through the wall
Steady state prevails
Contact resistance between the wall layers is negligible

SKETCH



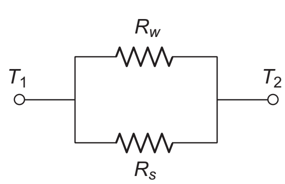
PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10 and 11

Thermal conductivities
- Wood (Pine) (k_w) = 0.15 W/(m K)
- Mild steel (1% C) (k_s) = 43 W/(m K)

SOLUTION

(a) The thermal circuit for the wall is



The individual resistances are

$$R_w = \frac{L_w}{A_w k_w} = \frac{0.05\,\text{m}}{(0.995\,A_{\text{wall}}[0.15\,\text{W/(m K)}]} = \frac{1}{A_{\text{wall}}\,0.335\,(\text{K m}^2)/\text{W}}$$

$$R_s = \frac{L_s}{A_s k_s} = \frac{0.05\,\text{m}}{(0.005\,A_{\text{wall}}[43\,\text{W/(m K)}]} = \frac{1}{A_{\text{wall}}\,0.233\,(\text{K m}^2)/\text{W}}$$

The total resistance of the wood and steel in parallel is

$$R_{\text{total}} = \frac{R_w\,R_s}{R_w + R_s} = \frac{1}{A_{\text{wall}}}\left[\frac{(0.335)(0.233)}{0.335+0.233}\right](\text{K m}^2)/\text{W} = \frac{1}{A_{\text{wall}}}\,0.1374\;(\text{K m}^2)/\text{W}$$

The unit thermal conductance (k/L) is:

$$\frac{k}{L} = \frac{1}{R_{\text{total}}\,A_{\text{wall}}} = \frac{1}{0.1374\,(\text{K m}^2)/\text{W}} = 7.3\ \text{W/(K m}^2)$$

(b) The total heat flow through the wood and nails is given by

$$q_{\text{total}} = \frac{\Delta T}{R_{\text{total}}} = \frac{25°C}{\dfrac{1}{A_{\text{wall}}}0.1374\,(\text{K m}^2)/\text{W}}$$

$$\therefore\ \frac{q_{\text{total}}}{A_{\text{wall}}} = 182\ \text{W/m}^2$$

The heat flow through the nails alone is

$$q_{\text{nails}} = \frac{\Delta T}{R_{\text{nails}}} = \frac{25°C}{\dfrac{1}{A_{\text{wall}}}0.233\,(\text{K m}^2)/\text{W}}$$

$$\therefore\ \frac{q_{\text{nails}}}{A_{\text{wall}}} = 107\ \text{W/m}^2$$

Therefore the percent of the total heat flow that passes through the nails is

Percent of heat flow through nails = $\dfrac{107}{182} \times 100 = 59\%$

PROBLEM 1.35

Calculate the rate of heat transfer through the composite wall in Problem 1.34 if the temperature difference is 25°C and the contact resistance between the sheets of wood is 0.005 m^2 K/W.

Problem 1.34: To reinforce a solid wall consisting of two layers, each 2.5 cm thick, mild steel nails were driven through it. If the total cross sectional area of the nails is 0.5% of the wall area, determine the unit thermal conductance of the composite wall and the percent of the total heat flow that passes through the nails when the temperature difference across the wall is 25°C. Neglect contact resistance between the wood layers.

GIVEN

Wood wall ▪ Two layers 0.025 m thick each, nailed together
Nail cross sectional area of nails = 0.5% of wall area
Temperature difference (ΔT) = 20°C
Contact resistance ($A\,R_i$) = 0.005 (m^2 K)/W

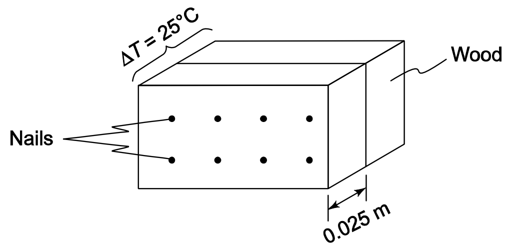
FIND

The rate of heat transfer through the wall

ASSUMPTIONS

One dimensional heat transfer through the wall
Steady state prevails

SKETCH



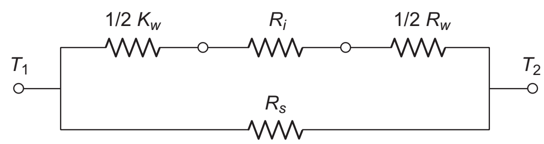
PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10 and 11
Thermal conductivities

- Wood (Pine) (k_w) = 0.15 W/(m K)
- Mild steel (1% C) (k_s) = 43 W/(m K)

SOLUTION

The thermal circuit for the wall with contact resistance is shown below.



From Problem 1.34, the thermal resistance of the wood and the nails are

$$R_w = \frac{1}{A_{\text{wall}}}\ 0.335\ (\text{K m}^2)/\text{W} \qquad R_s = \frac{1}{A_{\text{wall}}}\ 0.233\ (\text{K m}^2)/\text{W}$$

The combined resistance of the wood and the contact resistance in series is

$$R_{wi} = R_w + R_i = R_w + \frac{1}{A}\ (A\ R_i) = \frac{1}{A_{\text{wall}}}\left[0.355\ (\text{K m}^2)/\text{W} + 0.005\ (\text{K m}^2)/\text{W}\right]$$

$$R_{wi} = \frac{1}{A_{\text{wall}}}\ 0.360\ (\text{K m}^2)/\text{W}$$

The total resistance equals the combined resistance of the wood and the contact resistance in parallel with the resistance of the nails

$$R_{\text{total}} = \frac{R_{wi}\ R_s}{R_{wi} + R_s} = \frac{1}{A_{\text{wall}}}\left[\frac{(0.360)(0.233)}{0.360 + 0.233}\right](\text{K m}^2)/\text{W} = \frac{1}{A_{\text{wall}}} = 0.1415\ (\text{K m}^2)/\text{W}$$

Therefore the rate of heat flow through the wall is:

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{25°\text{C}}{\frac{1}{A_{\text{wall}}}\ 0.1415\ (\text{K m}^2)/\text{W}} \qquad \therefore\ \frac{q}{A_{\text{wall}}} = 172\ \text{W/m}^2$$

COMMENTS

In this case the inclusion of the contact resistance lowered the calculated rate of heat transfer by only 3% because most of the heat is transferred through the nails (see Problem 1.34).

PROBLEM 1.36

Heat is transferred through a plane wall from the inside of a room at 22°C to the outside air at –2°C. The convective heat transfer coefficients at the inside and outside surfaces are 12 and 28 W/(m^2 K), respectively. The thermal resistance of a unit area of the wall is 0.5 m^2 K/W. Determining the temperature at the outer surface of the wall and the rate of heat flow through the wall per unit area.

GIVEN

Heat transfer through a plane wall

Air temperature
- Inside wall (T_i) = 22°C
- Outside wall (T_o) = –2°C

Heat transfer coefficient
- Inside wall ($\overline{h_{ci}}$) = 12 W/(m^2 K)
- Outside wall ($\overline{h_{co}}$) = 28 W/(m^2 K)

Thermal resistance of a unit area ($A\,R_w$) = 0.5 (m^2 K)/W

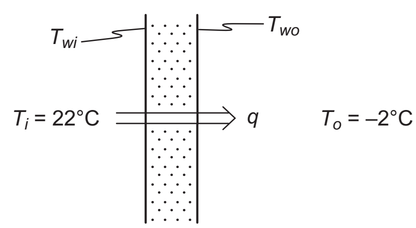
FIND

(a) Temperature of the outer surface of the wall (T_{wo})
(b) Rate of heat flow through the wall per unit area (q/A)

ASSUMPTIONS

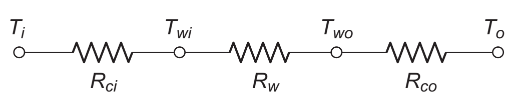
One dimensional heat flow
Steady state has been reached

SKETCH



SOLUTION

The thermal circuit for the wall is shown below



The rate of heat transfer can be used to calculate the temperature of the outer surface of the wall, therefore part (b) will be solved first.

(b) The heat transfer situation can be visualized using the thermal circuit shown above. The total heat transfer through the wall, from Equations (1.33) and (1.34), is

$$q = \frac{\Delta T_{\text{total}}}{R_{\text{total}}}$$

The three thermal resistances are in series, therefore

$$R_{\text{total}} = R_{ci} + R_w + R_{\infty}$$

$$R_{\text{total}} = \frac{1}{A\,\overline{h}_{ci}} + \frac{A\,R_w}{A} + \frac{1}{A\,\overline{h}_{\infty}}$$

The heat flow through the wall is

$$q = \frac{T_i - T_o}{\frac{1}{A}\left(\frac{1}{\overline{h}_{ci}} + A\,R_w + \frac{1}{\overline{h}_{\infty}}\right)}$$

$$\therefore\ \frac{q}{A} = \frac{22\,°\text{C} - (-2\,°\text{C})}{\frac{1}{12\,\text{W/(m}^2\text{K)}} + 0.5\,(\text{m}^2\text{K})/\text{W} + \frac{1}{28\,\text{W/(m}^2\text{K)}}}$$

$$\frac{q}{A} = 38.8\ \text{W/m}^2$$

(a) The temperature of the outer surface of the wall can be calculated by examining the convective heat transfer from the outside of the wall (given by Equation (1.10))

$$\frac{q_c}{A} = \overline{h}_{co}\ (T_{wo} - T_o)$$

Solving for T_{wo}

$$T_{wo} = \frac{q}{A}\frac{1}{\overline{h}_{co}} + T_o = (38.8\ \text{W/m}^2\left(\frac{1}{28\,\text{W/(m}^2\ \text{K)}}\right) + (-2°\text{C}) = -\,0.6°\text{C}$$

COMMENTS

Note that the conductive resistance of the wall is dominant compared to the convective resistance.

PROBLEM 1.37

How much fiberglass insulation [k = 0.035 W/(m K)] is needed to guarantee that the outside temperature of a kitchen oven will not exceed 43°C? The maximum oven temperature to be maintained by the convectional type of thermostatic control is 290°C, the kitchen temperature may vary from 15°C to 33°C and the average heat transfer coefficient between the oven surface and the kitchen is 12 W/(m^2 K).

GIVEN

Kitchen oven wall insulated with fiberglass
Fiberglass thermal conductivity (k) = 0.035 W/(m K)
Convective transfer coefficient on the outside of wall ($\overline{h}_c$) = 12 W/(m^2 K)
Maximum oven temperature (T_i) = 290°C
Kitchen temperature (T_∞) may vary: 15°C < T_∞ < 33°C

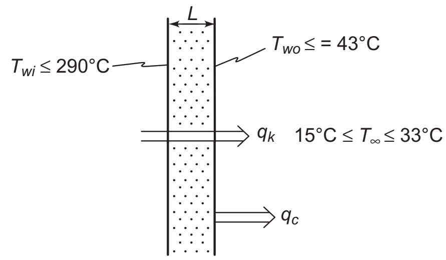
FIND

Thickness of fiberglass (L) to keep the temperature of the outer surface of the oven (T_{wo}) at 43°C or less

ASSUMPTIONS

One dimensional, steady state heat transfer prevails
The temperature of the inside of the wall (T_{wi}) is the same as the oven temperature
The thermal resistance of the metal wall of the oven is negligible

SKETCH



SOLUTION

For steady state conditions, the heat transfer by conduction through the wall, from Equation (1.2), must be equal to the heat transfer by convection from the outer surface of the wall, from Equation (1.10)

$$q_k = \frac{k\,A}{L}\,(T_{wi} - T_{wo}) = q_c = \bar{h}_c\,A\,(T_{wo} - T_\infty)$$

Solving for L

$$L = \frac{k\,(T_{wi} - T_{wo})}{\bar{h}_c\,(T_{wo} - T_\infty)}$$

By examination of the above equation, the greatest thickness required for a given T_{wo} will occur when T_{wi} and T_∞ are at their maximum values

$$L = \frac{0.035\,\text{W/(m K)}\,(290^\circ\text{C} - 43^\circ\text{C})}{12\,\text{W/(m}^2\,\text{K)}\,(43^\circ\text{C} - 33^\circ\text{C})} = 0.072\text{ m} = 7.2\text{ cm}$$

COMMENTS

In a real design a slightly thicker layer of insulation should be chosen to provide a margin of safety in case the convective heat transfer coefficient on the outside of the wall in some circumstances is less than expected due to the location of the oven in the kitchen or other unforseen factors.

PROBLEM 1.38

> **A heat exchanger wall consists of a copper plate 2 cm thick. The heat transfer coefficients on the two sides of the plate are 2700 and 7000 W/(m^{2}K), corresponding to fluid temperatures of 92 and 32°C, respectively. Assuming that the thermal conductivity of the wall is 375 W/(m K), (a) compute the surface temperatures in °C, and (b) calculate the heat flux in W/m^2.**

GIVEN

Heat exchanger wall, thickness (L) = 2 cm = 0.02 m

Heat transfer coefficients
- h_{c1} = 2700 W/(m^{2}K)
- h_{c2} = 7000 W/(m^{2}K)

Fluid temperatures
- T_{f1} = 92°C
- T_{f2} = 32°C

Thermal conductivity of the wall (k) = 375 W/(m K)

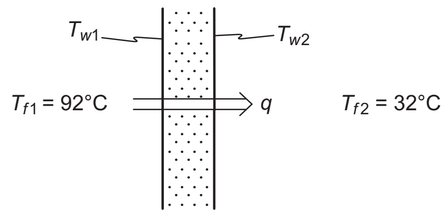
FIND

(a) Surface temperatures (T_{w1}, T_{w1}) in °C
(b) The heat flux (q/A) in W/m^2

ASSUMPTIONS

One dimensional heat transfer prevails
The system has reached steady state
Radiative heat transfer is negligible

SKETCH



SOLUTION

The thermal circuit for the wall is shown below

The surface temperatures can only be calculated after the heat flux has been established, therefore part (b) will be solved before part (a).

(b) The resistances are in series, therefore the total resistance is

$$R_{\text{total}} = \sum_{i=1}^{3} R_i = R_{c1} + R_w + R_{c2}$$

The total rate of heat transfer is given by Equation (1.33) and (1.34)

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{c1} + R_w + R_{c2}} = \frac{T_1 - T_2}{\dfrac{1}{\bar{h}_{c1} A} + \dfrac{L}{kA} + \dfrac{1}{\bar{h}_{c2} A}}$$

Therefore the heat flux (q/A) is

$$\Rightarrow \quad \frac{q}{A} = \frac{92 - 32}{\dfrac{1}{2700\,\text{W/(m}^2\text{K)}} + \dfrac{0.02\,\text{m}}{375\,\text{W/(m K)}} + \dfrac{1}{7000\,\text{W/(m}^2\text{K)}}} = 105.9\ \text{(kW)/m}^2$$

(a) Equation (1.10) can be applied to the convective heat transfer on the fluid 1 side

$$\frac{q_c}{A} = \bar{h}_{c1}\ (T_{f1} - T_{w1})$$

Solving for T_{w1}

$$T_{w1} = T_{f1} - \frac{q}{A}\frac{1}{\bar{h}_{c1}} = 92°\text{C} - (105.9\ \text{W/(m}^2\text{K))} \left(\frac{1}{2700\,\text{W/(m}^2\text{K)}}\right) = 52.8°\text{C}$$

Similarly, on the fluid 2 side

$$\frac{q_c}{A} = \bar{h}_{c2}\ (T_{w2} - T_{f2})$$

$$T_{w2} = T_{f2} - \frac{q}{A}\frac{1}{\bar{h}_{c2}} = 32°\text{C} + (105.9\ \text{W/(m}^2\text{K))} \left(\frac{1}{7000\,\text{W/(m}^2\text{K)}}\right) = 47.13°\text{C}$$

PROBLEM 1.39

A submarine is to be designed to provide a comfortable temperature for the crew of no less than 21°C. The submarine can be idealized by a cylinder 9 m in diameter and 61 m in length. The combined heat transfer coefficient on the interior is about 14 W/(m^{2}K), while on the outside the heat transfer coefficient is estimated to vary from about 57 W/(m^{2}K) (not moving) to 847 W/(m^{2}K) (top speed). For the following wall constructions, determine the minimum size in kilowatts of the heating unit required if the sea water temperatures vary from 1.1 to 12.8°C during operation. The walls of the submarine are (a) 2 cm aluminium (b) 1.8 cm stainless steel with a 2.5 cm thick layer fiberglass insulation on the inside and (c) of sandwich construction with a 1.8 cm thickness of stainless steel, a 2.5 cm thick layer of fiberglass insulation, and a 0.6 cm thickness of aluminium on the inside. What conclusions can you draw?

GIVEN

Submarine
- Inside temperature (T_i) > 21°C

Can be idealized as a cylinder
- Diameter (D) = 9 m
- length (L) = 61 m

Combined heat transfer coefficients
- Inside ($\bar{h}_{ci}$) = 14 W/(m^{2}K)
- Outside ($\bar{h}_{co}$): not moving = 57 W/(m^{2}K)
  : top speed: 847 W/(m^{2}K)

Sea water temperature (T_o) varies: 1.1°C < T_o < 12.8°C

FIND

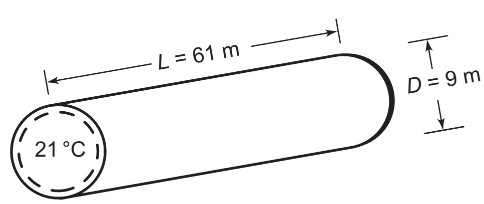
Minimum size of the heating unit (q) in kW for

(a) 1.2 cm thick aluminium walls
(b) 1.8 cm thick stainless steel with 2.5 cm of fiberglass insulation
(c) Sandwich of 1.8 cm stainless steel, 2.5 cm of fiberglass insulation, and 0.6 cm of aluminium

ASSUMPTIONS

Steady state prevails
Heat transfer can be approximated as heat transfer through a flat plate with the surface area of the cylinder
Constant thermal conductivities
Contact resistance between the difference materials is negligible

SKETCH



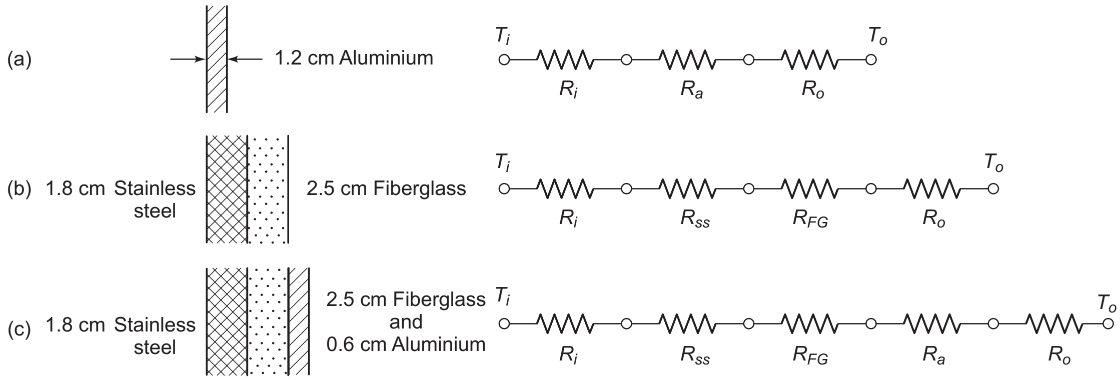
PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10, 11, and 12: The thermal conductivities are

Aluminium (k_a)	= 236 W/(m K) at 0°C
Stainless steel (k_s)	= 14 W/(m K) at 20°C
Fiberglass insulation (k_{fg})	= 0.035 W/(m K) at 20°C

SOLUTION

The thermal circuits for the three cases are shown below



The total surface area of the idealized submarine (A) is

$$A = \pi DL + 2\pi \frac{D^2}{4} = (61 \text{ m})\pi(9 \text{ m}) + \frac{\pi}{2} (9 \text{ m})^2 = 1850 \text{ m}^2$$

(a) For case (a) the total resistance is

$$R_{\text{total}} = \sum_{i=1}^{3} R_i = R_i + R_a + R_o = \frac{1}{\bar{h}_{ci} A} + \frac{L}{k_a A} + \frac{1}{\bar{h}_{co} A}$$

The heat transfer through the wall is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_i - T_o}{\dfrac{1}{\bar{h}_{ci} A} + \dfrac{L_a}{k_a A} + \dfrac{1}{\bar{h}_{co} A}}$$

By examination of the above equation, the heater requirement will be the largest when T_o is at its minimum value and h_{co} is at its maximum value

$$\Rightarrow \quad q = \frac{(1850 \text{ m}^2)(21 - 1.1)\text{K}}{\dfrac{1}{14 \text{ W/(m}^2\text{K)}} + \dfrac{1.2 \times 10^{-2} \text{ m}}{(236 \text{ W/(m K))}} + \dfrac{1}{847 \text{ W/(m}^2\text{K)}}}$$

q = 507 kW

(b) Similarly, for case (b), the total resistance is

$$R_{\text{total}} = \sum_{i=1}^{4} R_i = R_s + R_a + R_{fg} + R_o = \frac{1}{\bar{h}_{ci} A} + \frac{L_s}{k_s A} + \frac{L_{fg}}{k_{fg} A} + \frac{1}{\bar{h}_{co} A}$$

The size of heater needed is

$$q = \frac{(1850 \text{ m}^2)(21 - 1.1)\text{K}}{\dfrac{1}{14 \text{ W/(m}^2\text{K)}} + \dfrac{1.8 \times 10^{-2} \text{ m}}{14 \text{ W/(m K)}} + \dfrac{2.5 \times 10^{-2} \text{ m}}{0.035 \text{ W/(m K)}} + \dfrac{1}{847 \text{ W/(m}^2\text{K)}}}$$

$\Rightarrow \quad q$ = 46.7 kW

(c) The total resistance for case (c) is

$$R_{\text{total}} = \sum_{i=1}^{5} R_i = R_s + R_a + R_{fg} + R_a + R_o = \frac{1}{\bar{h}_{ci} A} + \frac{L_s}{k_s A} + \frac{L_{fg}}{k_{fg} A} + \frac{L_a}{k_a A} + \frac{1}{\bar{h}_{co} A}$$

The size of heater needed is

$$q = \frac{(1850\,\text{m}^2)(21 - 1.1)\text{K}}{\frac{1}{14\ \text{W/(m}^2\text{K)}} + \frac{1.8 \times 10^{-2}\ \text{m}}{14\ \text{W/(m K)}} + \frac{2.5 \times 10^{-2}\ \text{m}}{0.035\ \text{W/(m K)}} + \frac{0.6 \times 10^{-2}}{236\ \text{W/(m K)}} + \frac{1}{847\ \text{W/(m}^2\text{K)}}}$$

$\Rightarrow \qquad q = 46.7$ kW

COMMENTS

Neither the aluminium nor the stainless steel offers any appreciable resistance to heat loss.

Fiberglass or other low conductivity material is necessary to keep the heat loss down to a reasonable level.

PROBLEM 1.40

A simple solar heater consists of a flat plate of glass below which is located a shallow pan filled with water, so that the water is in contact with the glass plate above it. Solar radiation is passing through the glass at the rate of 490 W/m^2. The water is at 92°C and the surrounding air is 27°C. If the heat transfer coefficients between the water and the glass and the glass and the air are 28 W/(m^{2}K), and 7 W/(m^{2}K), respectively, determine the time required to transfer 1.1 MJ/m^2 of surface to the water in the pan. The lower surface of the pan may be assumed to be insulated.

GIVEN

A simple solar heater: shallow pan of water below glass, the water touches the glass
Solar radiation passing through glass (q_r/A) = 490 W/m^2
Water temperature (T_w) = 92°C
Surrounding air temperature (T_∞) = 27°C
Heat transfer coefficients
- Between water and glass ($\bar{h}_{cw}$) = 28 W/(m^{2}K)
- Between glass and air ($\bar{h}_{ca}$) = 7 W/(m^{2}K)

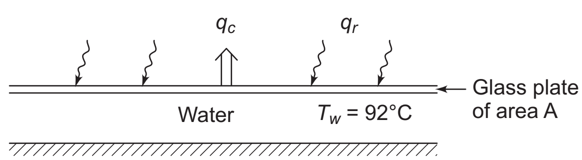
FIND

The time (t) required to transfer 11 (kJ)/m^2 to the water

ASSUMPTIONS

One dimensional, steady state heat transfer prevails
The heat loss from the bottom of the pan is negligible
The radiative loss from the top of the glass is negligible
The thermal resistance of the glass is negligible

SKETCH



SOLUTION

The total thermal resistance between the water and the surrounding air is the sum of the two convective thermal resistances

$$R_{\text{total}} = \sum_{i=1}^{2} R_i = R_{cw} + R_{ca} = \frac{1}{\overline{h}_{cw} A} + \frac{1}{\overline{h}_{ca} A}$$

$$R_{\text{total}} = \frac{1}{A(28\,\text{W/(m}^2\text{K))}} + \frac{1}{A(7\,\text{W/(m}^2\text{K))}} = \frac{1}{A(\text{m}^2)}\ 0.178\ (\text{m}^2\text{K})/\text{W}$$

The net rate of heat transfer to the water is

$$\frac{q_{\text{total}}}{A} = \frac{q_r}{A} = \frac{q_c}{A} = \frac{q_r}{A} = \frac{\Delta T}{A\,R_{\text{total}}}$$

$$\frac{q_{\text{total}}}{A} = 490\ \text{W/m}^2 - \frac{92\,°\text{C} - 27\,°\text{C}}{A\left(\frac{1}{A}\,0.178\,(\text{K m}^2)/\text{W}\right)}$$

$$\frac{q_{\text{Total}}}{A} = 125\ \text{W/m}^2$$

At this rate, the time required to transfer 1.1 M J to the water per m^2 area is

$$t = \frac{\text{total energy incident per unit area}}{\text{heat flux}}$$

$$= \frac{1.1 \times 10^6\ \text{J/m}^2}{125\,\text{W/m}^2} = 2.45\ \text{hours}$$

$$\Rightarrow \qquad t = 2.45\ \text{hours}$$

PROBLEM 1.41

A composite refrigerator wall is composed of 5 cm of corkboard sandwiched between a 1.2 cm thick layer of oak and a 0.8 mm thickness of aluminium lining on the inner surface. The average convective heat transfer coefficients at the interior and exterior wall are 11 and 8.5 W/(m^{2}K), respectively. (a) Draw the thermal circuit. (b) Calculate the individual resistances of the components of this composite wall and the resistances at the surfaces. (c) Calculate the overall heat transfer coefficient through the wall. (d) For an air temperature inside the refrigerator of – 1°C and outside of 32°C, calculate the rate of heat transfer per unit area through the wall.

GIVEN

Refrigerator wall: oak, corkboard, and aluminium

Thicknesses
- Oak (L_o) = 1.2 cm
- Corkboard (L_c) = 5 cm
- Aluminum (L_a) = 0.8 mm

Convective heat transfer coefficients
- Interior ($\overline{h}_{ci}$) = 11 W/(m^{2}K)
- Exterior ($\overline{h}_{co}$) = 8.5 W/(m^{2}K)

Air temperature
- Inside (T_i) = – 1°C;
- Outside (T_o) = 32°C

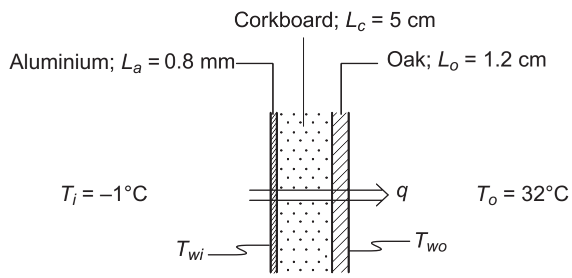
FIND

(a) Draw the thermal circuit
(b) The individual resistances
(c) Overall heat transfer coefficient (U)
(d) Rate of heat transfer per unit area (q/A)

ASSUMPTIONS

One dimensional, steady state heat transfer
Constant thermal conductivities
Contact resistance between the different materials is negligible

SKETCH



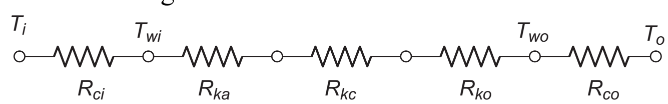
PROPERTIES AND CONSTANTS

From Appendix 2, Tables 11 and 12, the thermal conductivities are

Oak (k_o) = 0.2 W/(m K) at 20°C
Corkboard (k_c) = 0.04 W/(m K) at 20°C
Aluminium (k_a) = 235 W/(m K) at 0°C

SOLUTION

(a) The thermal circuit for the refrigerator wall is shown below



(b) The resistance to convection from the inner and outer surfaces is given by Equation (1.14)

$$R_c = \frac{1}{\bar{h}_c A}$$

This means

$$R_{ci} = \frac{1}{\bar{h}_{ci} A} = \frac{1}{(11\,\text{W/(m}^2\text{K))}A(\text{m}^2)} = \frac{1}{A(\text{m}^2)}\ 0.091\ (\text{K m}^2)/\text{W}$$

$$R_{co} = \frac{1}{\bar{h}_{co} A} = \frac{1}{(8.5\,\text{W/(m}^2\text{K))}A(\text{m}^2)} = \frac{1}{A(\text{m}^2)}\ 0.117\ (\text{K m}^2)/\text{W}$$

The resistance to conduction through the components of the wall is given by Equation (1.3)

$$R_k = \frac{L}{A k}$$

That is

$$R_{ka} = \frac{L_a}{A k_a} = \frac{(0.8 \times 10^{-3}\ \text{m})}{A(\text{m}^2)(235\,\text{W/(m K)})} = \frac{1}{A(\text{m}^2)}\ 3.4 \times 10^{-6}\ (\text{K m}^2)/\text{W}$$

$$R_{kc} = \frac{L_c}{A k_c} = \frac{5 \times 10^{-2}\ \text{m}}{A(\text{m}^2)(0.04\ \text{W/(m K)})} = \frac{1}{A(\text{m}^2)}\ 1.25\ (\text{K m}^2)/\text{W}$$

$$R_{ko} = \frac{L_o}{A k_o} = \frac{1.2 \times 10^{-2}\ \text{m}}{A(\text{m}^2)(0.2\ \text{W/(m K)})} = \frac{1}{A(\text{m}^2)}\ 0.06\ (\text{K m}^2)/\text{W}$$

(c) The overall heat transfer coefficient satisfies Equation (1.34)

$$UA = \frac{1}{R_{\text{total}}}$$

$$\therefore\ U = \frac{1}{A\,R_{\text{total}}} = \frac{1}{A\,(R_{ci} + R_{ka} + R_{kc} + R_{ko} + R_{co})}$$

$$\Rightarrow \qquad U = \frac{1}{(0.091 + 3.4\times10^{-6} + 1.25 + 0.06 + 0.117)(\text{K m}^2)/\text{W}}$$

$$\Rightarrow \qquad U = 0.66\ \text{W/(m}^2\text{K)}$$

(d) The rate of heat transfer through the wall is given by Equation (1.33)

$$\frac{q}{A} = U\,\Delta T = \left(0.66\ \text{W/(m}^2\text{K)}\right)\ (32 + 1)\text{K} = 21.8\ \text{W/m}^2$$

COMMENTS

The thermal resistance of the corkboard is more than three times greater than the sum of the other resistances. The thermal resistance of the aluminum is negligible.

PROBLEM 1.42

An electronic device that internally generates 600 mW of heat has a maximum permissible operating temperature of 70°C. It is to be cooled in 25°C air by attaching aluminum fins with a total surface area of 12 cm^2. The convective heat transfer coefficient between the fins and the air is 20 W/(m^2 K). Estimate the operating temperature when the fins are attached in such a way that: (a) there exists a contact resistance between the surface of the device and the fin array of approximately 50 K/W, and (b) there is no contact resistance but the construction of the device is more expensive. Comment on the design options.

GIVEN

An electronic device with aluminum fin array
Device generates heat at a rate ($\dot{q}_G$) = 600 mW = 0.6 W
Surface area (A) = 12 cm^2
Max temperature of device = 70°C
Air temperature (T_∞) = 25°C
Convective heat transfer coefficient ($\bar{h}_c$) = 20 W/(m^2 K)

FIND

Operating temperature (T_o) for
(a) contact resistance (R_i) = 50 K/W
(b) no contact resistance

ASSUMPTIONS

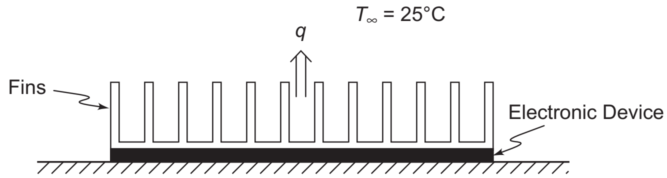
One dimensional heat transfer
Steady state has been reached

The temperature of the device is uniform

The temperature of the aluminum fins is uniform (the thermal resistance of the aluminum is negligible)

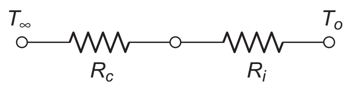
The heat loss from the edges and back of the device is negligible

SKETCH



SOLUTION

(a) The thermal circuit for the case with contact resistance is shown below



The value of the convective resistance, from Equation (1.14), is

$$R_c = \frac{1}{\bar{h}_c A} = \frac{1}{[20\,\mathrm{W/(m^2\,K)}](0.0012\,\mathrm{m^2})} = 41.7 \text{ K/W}$$

For steady state conditions, the heat loss from the device (q) must be equal to the heat generated by the device

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_o - T_\infty}{R_c + R_i} = \dot{q}_G$$

Solving for T_o

$$T_o = T_\infty + \dot{q}_G\ (R_c + R_i) = 25°\text{C} + (0.6 \text{ W})\ (41.7 \text{ K/W} + 50 \text{ K/W}) = 80°\text{C}$$

(b) Similarly, the operating temperature of the device with no contact resistance is

$$T_o = T_\infty + \dot{q}_G\ R_c = 25°\text{C} + (0.6 \text{ W})\ (41.7 \text{ K/W}) = 50°\text{C}$$

COMMENTS

The more expensive device with no contact resistance will have to be used to assure that the operating temperature does not exceed 70°C.

PROBLEM 1.43

To reduce the home heating requirements, modern building codes in many parts of the country require the use of double-glazed or double-pane windows, i.e., windows with two panes of glass. Some of these so called thermopane windows have an evacuated space between the two glass panes while others trap stagnant air in the space.

(a) Consider a double-pane window with the dimensions shown in the following sketch. If this window has stagnant air trapped between the two panes and the convective heat transfer coefficients on the inside and outside surfaces are 4 W/(m^2 K) and 15 W/(m^2 K), respectively, calculate the overall heat transfer coefficient for the system.

(b) If the inside air temperature is 22°C and the outside air temperature is –5°C, compare the heat loss through a 4 m^2 double-pane window with the heat loss through a single-pane window. Comment on the effect of the window frame on this result.

(c) The total window area of a home heated by electric resistance heaters at a cost of \$.10/kWh is 80 m^2. How much more cost can you justify for the double-pane windows if the average temperature difference during the six winter months when heating is required is about 15°C?

GIVEN

Double-pane window with stagnant air in gap

Convective heat transfer coefficients
- Inside ($\bar{h}_{ci}$) = 4 W/(m^2 K)
- Outside ($\bar{h}_{co}$) = 15 W/(m^2 K)

Air temperatures
- Inside (T_i) = 22°C
- Outside (T_o) = –5°C

Single window area (A_w) = 4 m^2
During the winter months, (ΔT) = 15°C
Heating cost = \$.1.0/kWh
Total window area (A_T) = 80 m^2

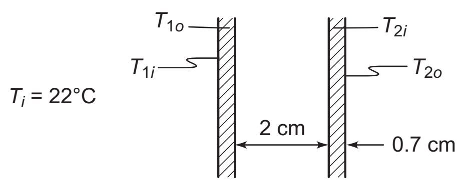
FIND

(a) The overall heat transfer coefficient
(b) Compare heat loss of double- and single-pane window

ASSUMPTIONS

Steady state conditions prevail
Radiative heat transfer is negligible

SKETCH

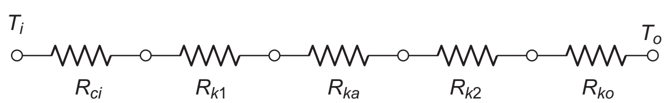


PROPERTIES AND CONSTANTS

From Appendix 2, Tables 11 and 27, the thermal conductivities are
window glass (k_g) = 0.81 W/(m K) at 20°C; dry air (k_a) = 0.0243 W/(m K) at 8.5°C

SOLUTION

The thermal circuit for the system is shown below



The individual resistances are

$$R_{co} = \frac{1}{\bar{h}_{co} A} = \frac{1}{[15\,\text{W/(m}^2\,\text{K)}]\,A} = \frac{1}{A}\ 0.0667\ \text{(K m}^2\text{)/W}$$

$$R_{k1} = R_{k2} = \frac{L_g}{A k_g} = \frac{0.007\,\text{m}}{A[0.81\,\text{W/(m K)}]} = \frac{1}{A}\ 0.00864\ \text{(K m}^2\text{)/W}$$

$$R_{ka} = \frac{L_a}{A k_a} = \frac{0.02\,\text{m}}{A[0.0243\,\text{W/(m K)}]} = \frac{1}{A}\ 0.823\ \text{(K m}^2\text{)/W}$$

$$R_{ci} = \frac{1}{\bar{h}_{ci} A} = \frac{1}{[4\,\text{W/(m}^2\,\text{K)}]A} = \frac{1}{A}\ 0.25\ \text{(K m}^2\text{)/W}$$

The total resistance for the double-pane window is

$$R_{\text{total}} = \sum_{i=1}^{5} R_i = R_{co} + R_{k1} + R_{ka} + R_{k2} + R_{ci}$$

$$R_{\text{total}} = \frac{1}{A}\ (0.0667 + 0.00864 + 0.823 + 0.00864 + 0.25)\ (\text{m}^2\ \text{K})/\text{W} = \frac{1}{A}\ 1.157\ (\text{K m}^2)/\text{W}$$

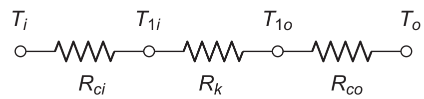
Therefore the overall heat transfer coefficient is

$$U_{\text{double}} = \frac{1}{A\,R_{\text{total}}} = \frac{1}{1.157\,(\text{m}^2\text{K})/\text{W}} = 0.864\ \text{W}/(\text{m}^2\ \text{K})$$

(b) The rate of heat loss through the double-pane window is

$$q_{\text{double}} - U\,A\,\Delta T = [0.864\ \text{W}/(\text{m}^2\ \text{K})]\ (4\ \text{m}^2)\ [22°\text{C} - (-5°\text{C})] = 93\text{W}$$

The thermal circuit for the single-pane window is



The total thermal resistance for the single-pane window is

$$R_{\text{total}} = \sum_{i=1}^{3} R_i = R_{co} + R_{k1} + R_{ci} = \frac{1}{A}\ (0.0667 + 0.00864 + 0.25)\ (\text{m}^2\ \text{K})/\text{W}$$

$$R_{\text{total}} = 0.325\ (\text{m}^2\ \text{K})/\text{W}$$

The overall heat transfer coefficient for the single-pane window is

$$U_{\text{single}} = \frac{1}{A\,R_{\text{total}}} = \frac{1}{0.325\,(\text{m}^2\ \text{K})/\text{W}} = 3.08\ \text{W}/(\text{m}^2\ \text{K})$$

Therefore, the rate of heat loss through the single-pane window is

$$q_{\text{single}} = U\,A\,\Delta T = [3.07\ \text{W}/(\text{m}^2\ \text{K})]\ (4\ \text{m}^2)\ [22°\text{C} - (-5°\text{C})] = 332\ \text{W}$$

The heat loss through the double-pane window is only 28% of that through the single-pane window.

(c) The average heat loss through double-pane windows during the winter months is

$$q_{\text{double}} = U\,A_T\,\Delta T = [0.864\ \text{W}/(\text{m}^2\ \text{K})]\ (80\ \text{m}^2)\ 15°\text{C} = 1040\ \text{W}$$

Therefore, the cost of the heat loss from the double-pane windows is

$\text{Cost}_{\text{double}}$ = q_{double} (heating cost)

$\text{Cost}_{\text{double}}$ = (1040 W) ($0.10/kWh) (24 h/day) (182 heating days/year) (1 kW/1000 W)

$\text{Cost}_{\text{double}}$ = $454/yr

The average heat loss through the single-pane windows during the winter months is

$$q_{\text{single}} = U\,A_T\,\Delta T = [3.07\ \text{W}/(\text{m}^2\ \text{K})]\ (80\ \text{m}^2)\ (15°\text{C}) = 3688\ \text{W}$$

The cost of this heat loss is

$\text{Cost}_{\text{single}}$ = q_{single} (heat cost)

$\text{Cost}_{\text{single}}$ = (3688 W) ($0.10/kWh) (24 h/day) (182 heating days/year) (1 kW/1000 W)

$\text{Cost}_{\text{single}}$ = $1611/yr

The yearly savings of the double-pane windows is $1157. Therefore if we would like to have a payback period of two years, we would be willing to invest $2314 in double panes.

PROBLEM 1.44

A flat roof can be modeled as a flat plate insulated on the bottom and placed in the sunlight. If the radiant heat that the roof receives from the sun is 600 W/m^2, the convection heat transfer coefficient between the roof and the air is 12 W/(m^2 K), and the air temperature is 27°C, determine the roof temperature for the following two cases: (a) Radiative heat loss to space is negligible. (b) The roof is black (ε = 1.0) and radiates to space, which is assumed to be a black-body at 0 K.

GIVEN

A flat plate in the sunlight
Radiant heat received from the sun (q_r/A) = 600 W/m^2
Air temperature (T_∞) = 27°C
Convective heat transfer coefficient ($\bar{h}_c$) = 12 W/(m^2 K)

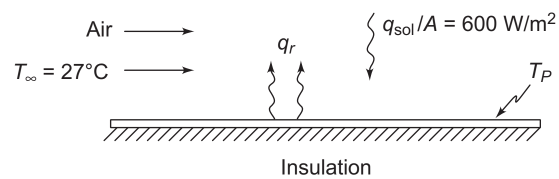
FIND

The plate temperature (T_p)

ASSUMPTIONS

Steady state prevails
No heat is lost from the bottom of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5: Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

(a) For this case steady state and the conservation of energy require the heat lost by conduction, from Equation (1.10), to be equal to the heat gained from the sun

$$q_c = \bar{h}_c\ A\ (T_s - T_\infty) = q_r$$

Solving for T_s

$$T_s = \frac{q_r}{A}\frac{1}{\bar{h}_c} + T_\infty = (600\ \text{W/m}^2)\left(\frac{1}{12\ \text{W/(m}^2\ \text{K)}}\right) + (27°\text{C}) = 77°\text{C}$$

(b) In this case, the solar gain must be equal to the sum of the convective loss, from Equation (1.10), and radiative loss, from Equation (1.16)

$$\frac{q_r}{A} = \bar{h}_c\ (T_p - T_\infty) + \sigma(T_p^{\,4} - T_{sp}^{\,4})$$

$$600\ \text{W/m}^2 = 12\ \text{W/(m}^2\ \text{K)}\ (T_p - 300\text{K}) + 5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)\ (T_p^{\,4} - 0)$$

By trial and error

$$T_p = 308\ \text{K} = 35°\text{C}$$

COMMENTS

The addition of a second means of heat transfer from the plate in part (b) allows the plate to operate at a significantly lower temperature.

PROBLEM 1.45

A horizontal 3-mm-thick flat copper plate, 1 m long and 0.5 m wide, is exposed in air at 27°C to radiation from the sun. If the total rate of solar radiation absorbed is 300 W and the combined radiative and convective heat transfer coefficients on the upper and lower surfaces are 20 and 15 W/(m^2 K), respectively, determine the equilibrium temperature of the plate.

GIVEN

Horizontal, 1 m long, 0.5 m wide, and 3 mm thick copper plate is exposed to air and solar radiation
Air temperature (T_∞) = 27°C
Solar radiation absorbed (q_{sol}) = 300 W
Combined transfer coefficients are

- Upper surface ($\bar{h}_u$) = 20 W/(m^2 K)
- Lower surface ($\bar{h}_1$) = 15 W/(m^2 K)

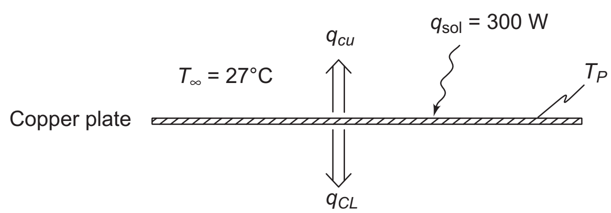
FIND

The equilibrium temperature of the plate (T_p)

ASSUMPTIONS

Steady state prevails
The temperature of the plate is uniform

SKETCH



SOLUTION

For equilibrium the heat gain from the solar radiation must equal the heat lost from the upper and lower surfaces

$$q_{sol} = \bar{h}_u\ A\ (T_p - T_\infty) + \bar{h}_1\ A\ (T_p - T_\infty)$$

Solving for T_p

$$T_p = \frac{q_{sol}}{A}\frac{1}{\bar{h}_u + \bar{h}_1} + T_\infty$$

$$T_p = \left(\frac{300\,\text{W}}{(1\,\text{m})(0.5\,\text{m})}\right)\left(\frac{1}{20\,\text{W/(m}^2\text{K)} + 15\,\text{W/(m}^2\text{K)}}\right) + (27°\text{C})$$

$$T_p = 44°\text{C}$$

PROBLEM 1.46

A small oven with a surface area of 0.28 m^2 is located in a room in which the walls and the air are at a temperature of 27°C. The exterior surface of the oven is at 150°C and the net heat transfer by radiation between the oven's surface and the surroundings is 586 W. If the average convective heat transfer coefficient between the oven and the surrounding air is 11 W/(m^{2}K), calculate: (a) the net heat transfer between the oven and the surroundings in W, (b) the thermal resistance at the surface for radiation and convection, respectively, in K/W, and (c) the combined heat transfer coefficient in W/(m^{2}K).

GIVEN

Small oven in a room
Oven surface area (A) = 0.28 m^2
Room wall and air temperature (T_∞) = 27°C
Surface temperature of the exterior of the oven (T_o) = 150°C
Net radiative heat transfer (q_r) = 586 W
Convective heat transfer coefficient ($\bar{h}_c$) = 11 W/(m^{2}K)

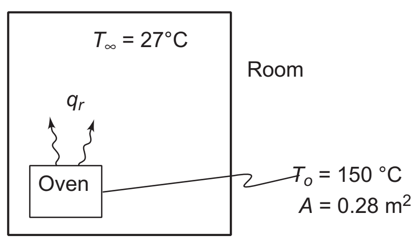
FIND

(a) Net heat transfer (q_T) in W
(b) Thermal resistance for radiation and convection (R_T) in K/W
(c) The combined heat transfer coefficient ($\bar{h}_{cr}$) in W/(m^{2}K)

ASSUMPTIONS

Steady state prevails

SKETCH



SOLUTION

(a) The net heat transfer is the sum of the convective heat transfer, from Equation (1.10), and the net radiative heat transfer

$$q_T = q_c + q_r + \bar{h}_c A\,(T_o - T_\infty) + q_r$$

This gives

$$q_T = 11\ \text{W/(m}^2\text{K)}\ (0.28\ \text{m}^2)\ (150 - 27)\text{K} + 586\ \text{W} = 965\ \text{W}$$

$$= 379\ \text{W} + 586\ \text{W}$$

(b) The radiative resistance is

$$R_r = \frac{T_o - T_\infty}{q_r} = \frac{(150 - 27)\text{K}}{586\,\text{W}} = 0.21\ \text{K/W}$$

The convective resistance is

$$R_c = \frac{T_o - T_\infty}{q_c} = \frac{(150-27)\text{K}}{379\,\text{W}} = 0.325 \text{ K/W}$$

These two resistances are in parallel, therefore the total resistance is given by

$$R_T = \frac{R_c\,R_r}{R_c + R_r} = \left(\frac{(0.21\,\text{K/W})(0.325\,\text{K/W})}{(0.21+0.325)\text{ K/W}}\right) = 0.13 \text{ K/W}$$

(c) The combined heat transfer coefficient can be calculated from

$$q_T = \bar{h}_{cr}\, A\, \Delta T$$

$$\therefore\ \overline{h_{cr}} = \frac{q_r}{A\,\Delta T} = \frac{965\text{ W}}{(0.28\,\text{m}^2)(150-27)\text{K}} = 28 \text{ W/(m}^2\text{K)}$$

COMMENTS

The thermal resistances for the convection and radiation modes are of the same order of magnitude. Hence, neglecting either one would lead to a considerable error in the rate of heat transfer.

PROBLEM 1.47

A steam pipe 200 mm in diameter passes through a large basement room. The temperature of the pipe wall is 500°C, while that of the ambient air in the room is 20°C. Determine the heat transfer rate by convection and radiation per unit length of steam pipe if the emissivity of the pipe surface is 0.8 and the natural convection heat transfer coefficient has been determined to be 10 W/(m^2 K).

GIVEN

A steam pipe passing through a large basement room
Pipe diameter (Δ) = 200 mm = 0.2 m
The temperature of the pipe wall (T_p) = 500°C = 773 K
Temperature of ambient air in the room (T_∞) = 20°C = 293 K
Emissivity of the pipe surface (ε) = 0.8
Natural convection heat transfer coefficient (h_c) = 10 W/(m^2 K)

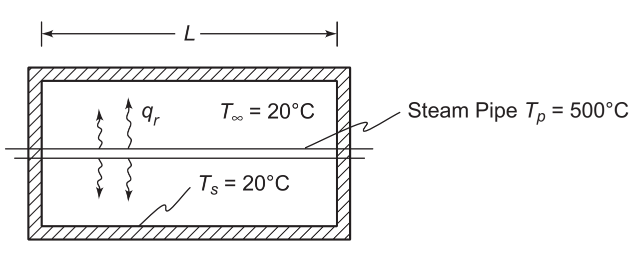
FIND

Heat transfer rate by convection and radiation per unit length of the steam pipe (q/L)

ASSUMPTIONS

Steady state prevails
The walls of the room are at the same temperature as the air in the room
The walls of the room are black (ε = 1.0)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The net radiative heat transfer rate for a gray object in a blackbody enclosure is given by Equation (1.17)

$$q_r = A_1\, \varepsilon_1\, \sigma (T_1^4 - T_2^4) = \pi D\, L\, \varepsilon\, \sigma (T_p^4 - T_s^4)$$

$$\therefore\ \frac{q_r}{L} = \pi (0.2 \text{ m})\, (0.8)\, [5.67 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4)]\, [(773 \text{ K})^4 - (293 \text{ K})^4]$$

$$\frac{q_r}{L} = 9970 \text{ W/m}$$

The convective heat transfer rate is given by

$$q_c = \overline{h_c}\ A\, (T_p - T_\infty) = \overline{h_c}\ (\pi D\, L)\, (T_p - T_\infty)$$

$$\therefore\ \frac{q_c}{L} = [10 \text{ W/(m}^2 \text{ K)}]\ \pi (0.2 \text{ m})\, (500°\text{C} - 20°\text{C})$$

$$\frac{q_c}{L} = 3020 \text{ W/m}$$

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equation.

The radiation heat transfer dominates because of the high emissivity of the surface and the high surface temperature which enters to the fourth power in the rate of radiative heat loss.

PROBLEM 1.48

> **The inner wall of a rocket motor combustion chamber receives 160(kW)/m^2 by radiation from a gas at 2760°C. The convective heat transfer coefficient between the gas and the wall is 110 W/(m^{2}K). If the inner wall of the combustion chamber is at a temperature of 540°C, determine the total thermal resistance of a unit area of the wall in (m^{2}K)/W and the heat flux. Also draw the thermal circuit.**

GIVEN

Wall of a rocket motor combustion chamber
Radiation to inner surface (q_r/A) = 160 (kW)/m^2
Temperature of gas in chamber (T_g) = 2760°C
Convective heat transfer coefficient on inner wall (h_c) = 110 W/(m^{2}K)
Temperature of inner wall (T_w) = 540°C

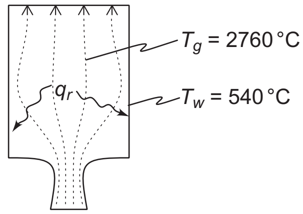
FIND

(a) Draw the thermal circuit
(b) The total thermal resistance of a unit area ($A\ R_{\text{total}}$) in (m^{2}K)/W

ASSUMPTIONS

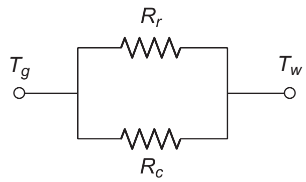
One dimensional heat transfer through the walls of the combustion chamber
Steady state heat flow

SKETCH



SOLUTION

(a) The thermal circuit for the chamber wall is shown below



(b) The total thermal resistance can be calculated from the total rate of heat transfer from the pipe

$$q_{\text{total}} = \frac{\Delta T}{R_{\text{total}}}$$

$$\therefore A\, R_{\text{total}} = \frac{\Delta T}{\left(\dfrac{q_{\text{total}}}{A}\right)}$$

The total rate of heat transfer is the sum of the radiative and convective heat transfer

$$q_{\text{total}} = q_r + q_c = q_r + \overline{h_c}\; A\, \Delta T$$

$$\therefore \frac{q_{\text{total}}}{A} = \frac{q_r}{A} + \overline{h_c}\; \Delta T$$

$$\frac{q_{\text{total}}}{A} = 160000\ \text{W/m}^2 + 110\ \text{W/(m}^2\text{K)}\ (2760 - 540)\text{K} = 404{,}200\ \text{W/m}^2$$

Therefore the thermal resistance of a unit area is

$$A\, R_{\text{total}} = \frac{(2760 - 540)\text{K}}{404{,}200\ \text{W/m}^2} = 0.0055\ (\text{m}^2\text{K})/\text{W}$$

An alternate method of solving part (b) is to calculate the radiative and convective resistances separately and then combine them in parallel as illustrated below.

The convective resistance is

$$R_c = \frac{1}{\overline{h_c}\, A} = \frac{1}{A}\left(\frac{1}{110\ \text{W/(m}^2\text{K)}}\right) = \frac{1}{A}\ 0.0091\ (\text{m}^2\text{K})/\text{W}$$

The radiative resistance is

$$R_r = \frac{\Delta T}{q_r} = \frac{\Delta T}{A\left(\dfrac{q_r}{A}\right)} = \frac{1}{A}\ \frac{(2760 - 540)\text{K}}{160000\ \text{W/m}^2} = \frac{1}{A}\ 0.014\ (\text{m}^2\text{K})/\text{W}$$

Combining these two resistances in parallel yields the total resistance

$$R_{\text{total}} = \frac{R_r R_c}{R_r + R_c}$$

$$\therefore A\, R_{\text{total}} = \frac{(0.0091)(0.014)}{(0.0091 + 0.014)} \text{ (m}^2\text{K)/W} = 0.0055 \text{ (m}^2\text{K)/W}$$

PROBLEM 1.49

A flat roof of a house absorbs a solar radiation flux of 600 W/m^2. The backside of the roof is well insulated, while the outside loses heat by radiation and convection to ambient air at 20°C. If the emittance of the roof is 0.80 and the convective heat transfer coefficient between the roof and the air is 12 W/(m^2 K), calculate: (a) the equilibrium surface temperature of the roof, and (b) the ratio of convective to radiative heat loss. Can one or the other of these be neglected? Explain your answer.

GIVEN

Flat roof of a house
Solar flux absorbed (q_{sol}/A) = 600 W/m^2
Back of roof is well insulated
Ambient air temperature (T_∞) = 20°C = 293 K
Emittance of the roof (ε) = 0.80
Convective heat transfer coefficient ($\bar{h}_c$) = 12 W/(m^2 K)

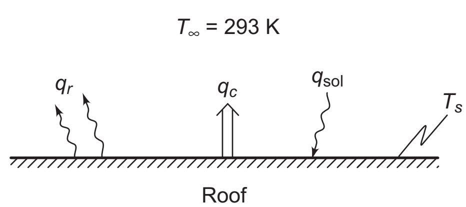
FIND

(a) The equilibrium surface temperature (T_s)
(b) The ratio of the convective to radiative heat loss

ASSUMPTIONS

The heat transfer from the back surface of the roof is negligible
Steady state heat flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) For steady state the sum of the convective heat loss, from Equation (1.10), and the radiative heat loss, from Equation (1.15), must equal the solar gain

$$\frac{q_{\text{sol}}}{A} = \frac{q_c}{A} + \frac{q_r}{A} = \bar{h}_c\ (T_s - T_\infty) + \varepsilon\, \sigma\, T_s^4$$

$$600 \text{ W/m}^2 = 12 \text{ W/(m}^2\text{ K)}\ (T_s - 293\text{K}) + (0.8)\left(5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4)\right)\ T_s^4$$

$$4.535 \times 10^{-8}\, T_s^4 + 12\, T_s - 4116 = 0$$

By trial and error

$$T_s = 309 \text{ K} = 36°\text{C}$$

(b) The ratio of the convective to radiative loss is

$$\frac{q_c}{q_r} = \frac{\overline{h_c}\,(T_s - T_\infty)}{\varepsilon \sigma T_s^4} = \frac{\left(12 \text{ W/(m}^2\text{ K)}\right)(309\text{K} - 293\text{K})}{(0.8)\left(5.67\times10^{-8} \text{ W/(m}^2\text{ K}^4)\right)(309\text{K})^4} = 0.46$$

COMMENTS

Since the radiative and convective terms are of the same order of magnitude, neither one may be neglected without introducing significant error.

PROBLEM 1.50

Determine the power requirement of a soldering iron in which the tip is maintained at 400°C. The tip is a cylinder 3 mm in diameter and 10 mm long. Surrounding air temperature is 20°C and the average convective heat transfer coefficient over the tip is 20 W/(m^2 K). Initially, the tip is highly polished giving it a very low emittance.

GIVEN

Soldering iron tip
- Diameter (D) = 3 mm = 0.003 m
- Length (D) = 10 mm = 0.01 m

Temperature of the tip (T_t) = 400°C
Temperature of the surrounding air (T_∞) = 20°C
Average convective heat transfer coefficient ($\overline{h}_c$) = 20 W/(m^2 K)
Emittance is very low ($\varepsilon = 0$)

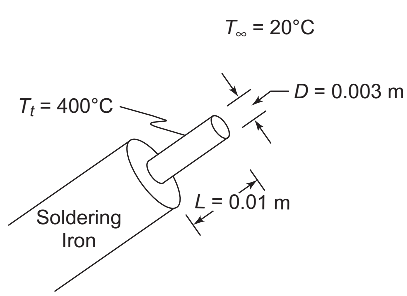
FIND

The power requirement of the soldering iron ($\dot{q}$)

ASSUMPTIONS

Steady state conditions exist
All power used by the soldering iron is used to heat the tip
Radiative heat transfer from the tip is negligible due to the low emittance
The end of the tip is flat
The tip is at a uniform temperature

SKETCH



SOLUTION

The power requirement of the soldering iron, $\dot{q}$, is equal to the heat lost from the tip by convection

$$q_c = \overline{h_{co}}\ A\ \Delta T = \overline{h_c}\ (\pi D^2/4 + \pi D\ L)\ (T_t - T_\infty) = \dot{q}$$

$$\dot{q} = 20\ \text{W/(m}^2\ \text{K)} \left[\frac{\pi (0.003\,\text{m})^2}{4} + \pi (0.003\,\text{m})(0.01\,\text{m}) \right] (400°\text{C} - 20°\text{C})$$

$$\dot{q} = 0.77\ \text{W}$$

PROBLEM 1.51

The soldering iron tip in Problem 1.50 becomes oxidized with age and its gray-body emittance increases to 0.8. Assuming that the surroundings are at 20°C determine the power requirement for the soldering iron.

Problem 1.50:

Determine the power requirement of a soldering iron in which the tip is maintained at 400°C. The tip is a cylinder 3 mm in diameter and 10 mm long. Surrounding air temperature is 20°C and the average convective heat transfer coefficient over the tip is 20 W/(m^2 K). Initially, the tip is highly polished giving it a very low emittance.

GIVEN

Soldering iron tip
- Diameter (D) = 3 mm = 0.003 m
- Length (D) = 10 mm = 0.01 m

Temperature of the tip (T_t) = 400°C
Temperature of the surrounding air (T_∞) = 20°C
Average convective heat transfer coefficient ($\overline{h_c}$) = 20 W/(m^2 K)
Emittance of the tip (ε) = 0.8

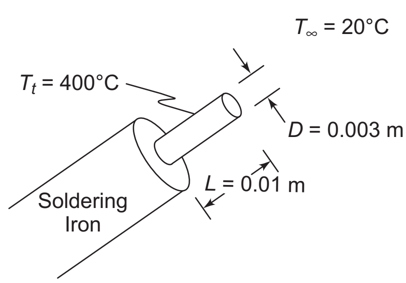
FIND

The power requirement of the soldering iron ($\dot{q}$)

ASSUMPTIONS

Steady state conditions exist
All power used by soldering iron is used to heat the tip
The surroundings of the soldering iron behave as a blackbody enclosure
The end of the tip is flat

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The rate of heat loss by convection, from Problem 1.50, is 0.77 W.

The rate of heat loss by radiation is given by Equation (1.17)

$$q_r = A_1\,\varepsilon_1\,\sigma(T_1^4 - T_2^4) = \left(\frac{\pi D^2}{4} + \pi DL\right)\ \varepsilon\,\sigma(T_t^4 - T_w^4)$$

$$q_r = \left[\frac{\pi(0.003\,\text{m})^2}{4} + \pi(0.003\,\text{m})(0.01\,\text{m})\right]\ (0.8)\ [5.67\times10^{-8}\ \text{W/(m}^2\ \text{K}^4)]\ [(673\ \text{K})^4 - (293\ \text{K})^4]$$

$$q_r = 0.91\ \text{W}$$

The power requirement of the soldering iron, $\dot{q}$, is equal to the total rate of heat loss from the tip. The total heat loss is equal to the sum of the convective and radiative losses

$$\dot{q} = q_c + q_r = 0.77\ \text{W} + 0.91\ \text{W} = 1.68\ \text{W}$$

COMMENTS

Note that the inclusion of the radiative term more than doubled the power requirement for the soldering iron.

The power required to maintain the desired temperature could be provided by electric resistance heating.

PROBLEM 1.52

Some automobile manufacturers are currently working on a ceramic engine block that could operate without a cooling system. Idealize such an engine as a rectangular solid, 45 cm by 30 cm by 30 cm. Suppose that under maximum power output the engine consumes 5.7 liters of fuel per hour, the heat released by the fuel is 9.29 kWh per liter and the net engine efficiency (useful work output divided by the total heat input) is 0.33. If the engine block is alumina with a gray-body emissivity of 0.9, the engine compartment operates at 150°C, and the convective heat transfer coefficient is 30 W/(m^2 K), determine the average surface temperature of the engine block. Comment on the practicality of the concept.

GIVEN

Ceramic engine block, 0.45m by 0.3m by 0.3m
Engine gas consumption is 5.7 1/h
Heat released is 9.29 (kWh)/1
Net engine efficiency (η) = 0.33
Emissivity (ε) = 0.9
Convective heat transfer coefficient (hc) = 30 W/(m^2 K)
Engine compartment temperature (T_c) = 150°C = 423 K

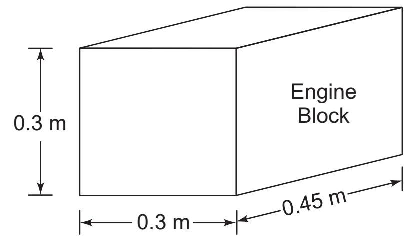
FIND

The surface temperature of the engine block (T_s)
Comment on the practicality

ASSUMPTIONS

Heat transfer has reached steady state
The engine compartment behaves as a blackbody enclosure

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The surface area of the idealized engine block is

$$A = 4\,(0.45\text{m})\,(0.3\text{m}) + 2(0.3\text{m})^2 = 0.72\ \text{m}^2$$

The rate of heat generation within the engine block is equal to the energy from the gasoline that is not transformed into useful work

$$\dot{q}_G = (1 - \eta)\, m_g\, h_g = (1 - 0.33)\ (5.71/\text{h})\ (9.29\ (\text{kWh})/1) = 35.5\ \text{kW}$$

For steady state conditions, the net radiative and convective heat transfer from the engine block must be equal to the heat generation within the engine block

$$q_{\text{total}} = q_r + q_c = \dot{q}_G$$

$$\dot{q}_G = A\,\varepsilon\,\sigma(T_s^4 - T_c^4) + \overline{h_c}\ A\,(T_s - T_c)$$

$$35.5\ \text{kW} = (0.72\ \text{m}^2)\ (0.9)\ \left(5.67\times10^{-8}\,\text{W/(m}^2\,\text{K}^4)\right)\ [T_s^4 - (423\ \text{K})^4] + (0.72\ \text{m}^2)$$

$$\left(30\ \text{W/(m}^2\ \text{K)}\right)\ (T_s - 3.674 \times 10^{-8}\ T_s^4 + 21.6\ T_s - 45656 = 0$$

By trial and error

$$T_s = 916\ \text{K} = 643°\text{C}$$

COMMENTS

The engine operates at a temperature high enough to burn a careless motorist.

Note that absolute temperature must be used in radiation equations.

Hot spots due to the complex geometry of the actual engine may produce local temperatures much higher than 916 K.

PROBLEM 1.53

> **A pipe carrying superheated steam in a basement at 10°C has a surface temperature of 150°C. Heat loss from the pipe occurs by radiation (ε = 0.6) and natural convection [$\overline{h_c}$ = 25 W/(m^2 K)]. Determine the percentage of the total heat loss by these two mechanisms.**

GIVEN

Pipe in a basement
Pipe surface temperature (T_s) = 150°C = 423 K
Basement temperature (T_∞) = 10°C = 283 K

Pipe surface emissivity (ε) = 0.6
Convective heat transfer coefficient ($\overline{h}_c$) = 25 W/(m^2 K)

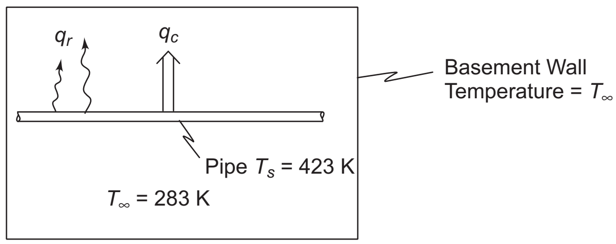
FIND

The percentage of the total heat loss due to radiation and convection

ASSUMPTIONS

The system is in steady state
The basement behaves as a blackbody enclosure at 10°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5: the Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The rate of heat transfer from a gray-body to a blackbody enclosure, from Equation (1.17), is

$$q_r = A_1\, \varepsilon_1\, \sigma(T_1^4 - T_2^4) = A\, \varepsilon\, \sigma(T_s^4 - T_\infty^4)$$

$$\therefore\ \frac{q_r}{A} = (0.6)\,[5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)]\,[(423\ \text{K})^4 - (283\ \text{K})^4]$$

$$\frac{q_r}{L} = 870\ \text{W/m}$$

The rate of heat transfer by convection, from Equation (1.10), is

$$q_c = \overline{h}_c\ A\,(T_s - T_\infty)$$

$$\therefore\ \frac{q_c}{A} = 25\ \text{W/(m}^2\ \text{K)}\ (423\ \text{K} - 283\ \text{K}) = 3500\ \text{W/m}^2$$

The total rate of heat transfer is the sum of the radiative and convective rates

$$\frac{q_{\text{total}}}{A} = \frac{q_r}{A} + \frac{q_c}{A} = 870\ \text{W/m}^2 + 3500\ \text{W/m}^2 = 4370\ \text{W/m}^2$$

The percentage of the total heat transfer due to radiation is

$$\frac{q_r / A}{q_{\text{total}} / A} \times 100 = \frac{870}{4370} \times 100 = 20\%$$

The percentage of the total heat transfer due to convection is

$$\frac{q_c / A}{q_{\text{total}} / A} \times 100 = \frac{3500}{4370} \times 100 = 80\%$$

COMMENTS

This pipe surface temperature and rate of heat loss are much too high to be acceptable. In practice, a layer of mineral wool insulation would be wrapped around the pipe. This would reduce the surface temperature as well as the rate of heat loss.

PROBLEM 1.54

> **For a furnace wall, draw the thermal circuit, determine the rate of heat flow per unit area, and estimate the exterior surface temperature under the following conditions: the convective heat transfer coefficient at the interior surface is 15 W/(m^2 K); rate of heat flow by radiation from hot gases and soot particles at 2000°C to the interior wall surface is 45,000 W/m^2; the unit thermal conductance of the wall (interior surface temperature is about 850°C) is 250 W/(m^2 K); there is convection from the outer surface.**

GIVEN

A furnace wall
Convective heat transfer coefficient ($\bar{h}_c$) = 15 W/(m^2 K)
Temperature of hot gases inside furnace (T_g) = 2000°C
Rate of radiative heat flow to the interior of the wall (q_r/A) = 45,000 W/m^2
Unit thermal conductance of the wall (k/L) = 250 W/(m^2 K)
Interior surface temperature (T_{wi}) is about 850°C
Convection occurs from outer surface of the wall

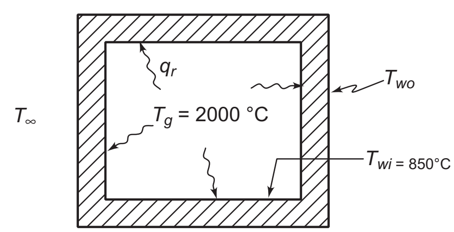
FIND

(a) Draw the thermal circuit
(b) Rate of heat flow per unit area (q/A)
(c) The exterior surface temperature (T_{wo})

ASSUMPTIONS

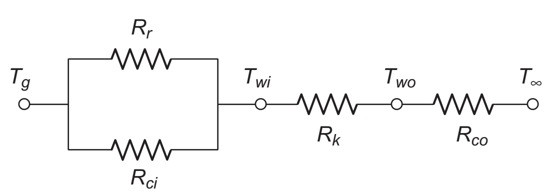
Heat flow through the wall is one dimensional
Steady state prevails

SKETCH



SOLUTION

The thermal circuit for the furnace wall is shown below



The rate of heat flow per unit area through the wall is equal to the rate of convective and radiative heat flow to the interior wall

$$\frac{q}{A} = \frac{q_r}{A} + \frac{q_c}{A} = \frac{q_r}{A} + \overline{h_c}\ (T_g - T_{wi})$$

$$\frac{q}{A} = 45{,}000\ \text{W/m}^2 + 15\ \text{W/(m}^2\ \text{K)}\ (2000°\text{C} - 850°\text{C}) = 62{,}250\ \text{W/m}^2$$

We can calculate the outer surface temperature of the wall by examining the conductive heat transfer through the wall given by Equation (1.2)

$$q_k = \frac{K\,A}{L}\ (T_{wi} - T_{wo})$$

$$\therefore\ T_{wo} = T_{wi} - \frac{q_k}{A}\frac{1}{k/L} = 850°\text{C} - (62{,}250\ \text{W/m}^2)\left(\frac{1}{250\,\text{W/(m}^2\text{K)}}\right) = 601°\text{C}$$

COMMENTS

The corner sections should be analyzed separately since the heat flow there is not one dimensional.

PROBLEM 1.55

> **Draw the thermal circuit for heat transfer through a double-glazed window. Include solar energy gain to the window and the interior space. Identify each of the circuit elements. Include solar radiation to the window and interior space.**

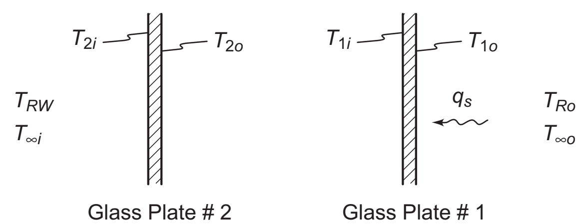
GIVEN

Double-glazed window

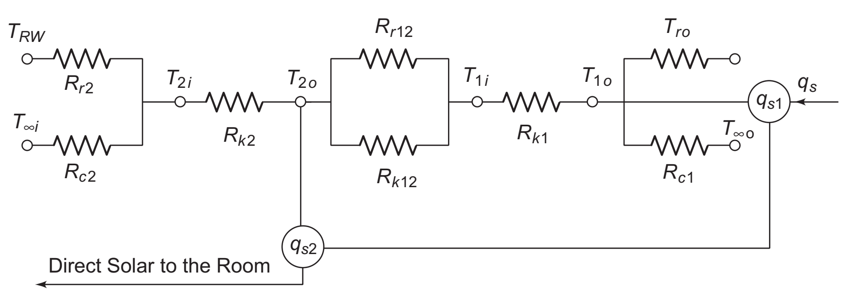
FIND

The thermal circuit

SKETCH



SOLUTION



where R_{r1}, R_{r12}, R_{r2} = Radiative thermal resistances
R_{k1}, R_{k2}, R_{k12} = Conductive thermal resistances
R_{c1}, R_{c2} = Convective thermal resistances
T_{rw}, T_{ro} = Effective temperatures for radiative heat transfer
T_∞ = Air temperatures
T_{1i}, T_{1o}, T_{2i} = Surface temperatures of the glass
q_{s1}, q_{s2} = Solar energy incident on the window panes

PROBLEM 1.56

The ceiling of a tract house is constructed of wooden studs with fiberglass insulation between them. On the interior of the ceiling is plaster and on the exterior is a thin layer of sheet metal. A cross section of the ceiling with dimensions is shown below.

(a) The *R*-factor describes the thermal resistance of insulation and is defined by:

$$R\text{-factor} = L/k_{\text{eff}} = \Delta T/(q/A)$$

Calculate the *R*-factor for this type of ceiling and compare the value of this *R*-factor with that for a similar thickness of fiberglass. Why are the two different?

(b) Estimate the rate of heat transfer per square meter through the ceiling if the interior temperature is 22°C and the exterior temperature is –5°C.

GIVEN

Ceiling of a tract house, construction shown below
Inside temperature (T_i) = 22°C
Outside temperature (T_o) = –5°C

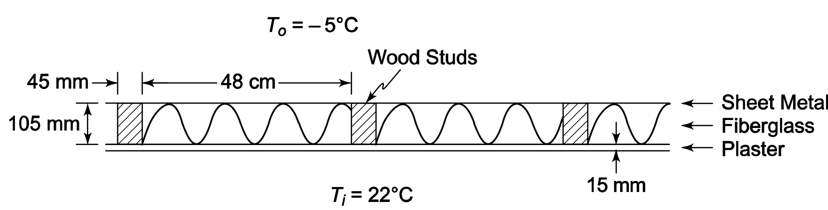
FIND

(a) R-factor for the ceiling (RF_c). Compare this to the R-factor for the same thickness of fiberglass (RF_{fg}). Why do they differ?
(b) Rate of heat transfer (q/A)

ASSUMPTIONS

Steady state heat transfer
One dimensional conduction through the ceiling
Thermal resistance of the sheet metal is negligible

SKETCH



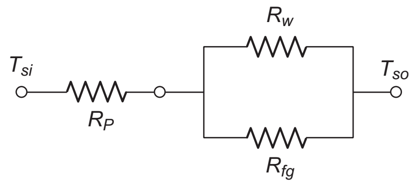
PROPERTIES AND CONSTANTS

From Appendix 2, Table 11, the thermal conductivities of the ceiling materials are

Pine or fir wood studs (k_w) = 0.15 W/(m K) at 20°C
Fiberglass (k_{fg}) = 0.035 W/(m K) at 20°C
Plaster (k_p) = 0.814 W/(m K) at 20°C

SOLUTION

The thermal circuit for the ceiling with studs is shown below



where R_p = thermal resistance of the plaster
R_w = thermal resistance of the wood
R_{fg} = thermal resistance of the fiberglass

Each of these resistances can be evaluated using Equation (1.4)

$$R_p = \frac{L_P}{A_{\text{wall}}\, k_P} = \frac{15\times10^{-3}\ \text{m}}{(A_{\text{wall}})[0.814\ \text{W/(m K)}]} = \frac{1}{A_{\text{wall}}}\ 0.0184\ \text{K m}^2/\text{W}$$

$$R_w = \frac{L_w}{A_w\, k_w} = \frac{105\times10^{-3}\ \text{m}}{(A_w)[0.15\ \text{W/(m K)}]} = \frac{1}{A_{\text{wall}}}\ 0.7\ \text{K m}^2/\text{W}$$

$$R_{fg} = \frac{L_{fg}}{A_{fg}\, k_{fg}} = \frac{105\times10^{-3}\ \text{m}}{(A_{\text{fg}})[0.035\ \text{W/(m K)}]} = \frac{1}{A_{\text{wall}}}\ 3\ \text{K m}^2/\text{W}$$

To convert these all to a wall area basis the fraction of the total wall area taken by the wood studs and the fiberglass must be calculated

$$\text{wood studs} = \frac{A_w}{A_{\text{wall}}} = \frac{45\,\text{mm}}{48\,\text{cm}} = 0.094$$

$$\text{fiberglass} = \frac{A_{fg}}{A_{\text{wall}}} = \frac{43.5\,\text{cm}}{48\,\text{m}} = 0.906$$

Therefore the resistances of the studs and the fiberglass based on the wall area are

$$R_w = \frac{1}{0.094\, A_{\text{wall}}}\ 0.7\ \text{K m}^2/\text{W} = \frac{1}{A_{\text{wall}}}\ 7.45\ \text{K m}^2/\text{W}$$

$$R_{fg} = \frac{1}{0.906\, A_{\text{wall}}}\ 3\ \text{K m}^2/\text{W} = \frac{1}{A_{\text{wall}}}\ 3.31\ \text{K m}^2/\text{W}$$

The R-Factor of the wall is related to the total thermal resistance of the wall by

$$RF_c = A_{\text{wall}}\, R_{\text{total}} = A_{\text{wall}} \left[R_p + \frac{R_w\, R_{fg}}{R_w + R_{fg}} \right] =$$

$$0.0184 + \frac{(7.45)\ (3.31)}{7.45 + 3.31}\ \text{K m/W} = 2.31\ \text{K m}^2/\text{W}$$

For 12 cm of fiberglass alone, the R-factor is

$$RF_{fg} = \frac{L}{k_{fg}} = \frac{12\times10^{-2}\ \text{m}}{0.035\ \text{W/(m K)}} = 3.43\ \text{K m}^2/\text{W}$$

The R-factor of the ceiling is only 67% that of the same thickness of fiberglass. This is mainly due to the fact that the wood studs act as a ‘thermal short’ conducting heat through the ceiling more quickly than the surrounding fiberglass.

(b) The rate of heat transfer through the ceiling is

$$\frac{q}{A} = \frac{\Delta T}{RF_c} = \frac{22\,°\text{C} - (-5\,°\text{C})}{2.31\text{K m}^2/\text{W}} = 11.69\ \text{W/m}^2$$

COMMENTS

R-factors are given in handbooks. For example, *Mark's Standard Handbook for Mechanical Engineers* lists the *R*-factor of a multi-layer masonry wall as 6.36 Btu/(h ft^2) = 20 W/m^2.

PROBLEM 1.57

A homeowner wants to replace an electric hot-water heater. There are two models in the store. The inexpensive model costs $280 and has no insulation between the inner and outer walls. Due to natural convection, the space between the inner and outer walls has an effective conductivity of 3 times that of air. The more expensive model costs $310 and has fiberglass insulation in the gap between the walls. Both models are 3.0 m tall and have a cylindrical shape with an inner wall diameter of 0.60 m and a 5 cm gap. The surrounding air is at 25°C, and the convective heat transfer coefficient on the outside is 15 W/(m^2 K). The hot water inside the tank results in an inside wall temperature of 60°C.

If energy costs 6 cents per kilowatt-hour, estimate how long it will take to pay back the extra investment in the more expensive hot-water heater. State your assumptions.

GIVEN

Two hot-water heaters

- Height (H) = 3.0 m
- Inner wall diameter (D_i) = 0.60 m
- Gap between walls (L) = 0.05 m

Water heater #1

- Cost = $280.00
- Insulation: none
- Effective Conductivity between wall (k_{eff}) = 3(k_a)

Water heater #2

- Cost = $310.00
- Insulation: Fiberglass

Surrounding air temperature (T_∞) = 25°C
Convective heat transfer coefficient (h_c) = 15 W/(m^2 K)
Inside wall temperature (T_{wi}) = 60°C
Energy cost = $0.06/kWh

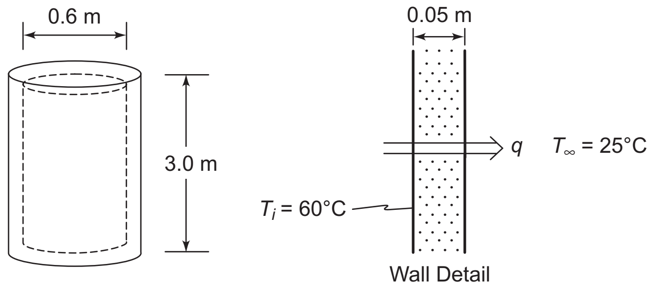
FIND

The time it will take to pay back the extra investment in the more expensive hot-water heater

ASSUMPTIONS

Since the diameter is large compared to the wall thickness, one-dimensional heat transfer is assumed
To simplify the analysis, we will assume there is no water drawn from the heater, therefore the inside wall is always at 60°C
Steady state conditions prevail

SKETCH



PROPERTIES AND CONSTANTS

From Appendix, Table 11 and 27: The thermal conductivities are

fiberglass (k_i) = 0.035 W/(m K) at 20°C

dry air (k_a) = 0.0279 W/(m K) at 60°C

SOLUTION

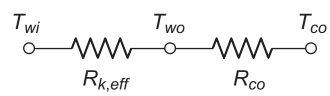
The areas of the inner and outer walls are

$$A_i = 2\,\frac{\pi D_i^2}{4} + \pi D_i\,H = 2\,\frac{\pi(0.6\,\text{m})^2}{4} + \pi(0.6\text{ m})\,(3\text{ m}) = 6.22\text{ m}^2$$

$$A_o = 2\,\frac{\pi D_o^2}{4} + \pi D_o\,H = 2\,\frac{\pi(0.7\,\text{m})^2}{4} + \pi(0.7\text{ m})\,(3\text{ m}) = 7.37\text{ m}^2$$

The average area for the air or insulation between the walls (A_a) = 6.8 m^2.

The thermal circuit for water heater #1 is



The rate of heat loss for water heater #1 is

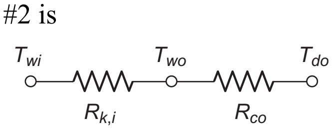
$$q_1 = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{k,\text{eff}} + R_{co}} = \frac{T_{wi} - T_\infty}{\dfrac{L}{k_{\text{eff}}\,A_a} + \dfrac{1}{\bar{h}_\infty A_{co}}}$$

$$q_1 = \frac{60\,°\text{C} - 25\,°\text{C}}{\dfrac{0.05\,\text{m}}{3[0.0279\,\text{W/(m K)}](6.8\,\text{m}^2)} + \dfrac{1}{[15\,\text{W/(m}^2\text{K)}](7.37\,\text{m}^2)}} = 361\text{ W} = 0.361\text{ kW}$$

Therefore the cost to operate water heater #1 is

$\text{Cost}_1 = q_1$ (energy cost) = 0.361 kW ($0.06/kWh) (24 h/day) = $0.52/day

The thermal circuit for water heater #2 is



The rate of heat loss from water heater #2 is

$$q_2 = \frac{60\,°\text{C} - 25\,°\text{C}}{\dfrac{0.05\,\text{m}}{[0.035\,\text{W/(m K)}](6.8\,\text{m}^2)} + \dfrac{1}{[15\,\text{W/(m}^2\text{ K)}](7.37\,\text{m}^2)}} = 160\text{ W} = 0.16\text{ kW}$$

Therefore the cost of operating water heater #2 is

$$\text{Cost}_2 = q_2 \text{ (energy cost)} = 0.16 \text{ kW (\$0.06/kWh) (24 h/day)} = \$0.23\text{/day}$$

The time to pay back the additional investment is the additional investment divided by the difference in operating costs

$$\text{Payback time} = \frac{\$310 - \$280}{\$0.52/\text{day} - \$0.23/\text{day}}$$

$$\text{Payback time} = 103 \text{ days}$$

COMMENTS

When water is periodically drawn from the water heater, energy must be supplied to heat the cold water entering the water heater. This would be the same for both water heaters. However, drawing water from the heater also temporarily lowers the temperature of the water in the heater thereby lowering the heat loss and lowering the cost savings of water heater #2. Therefore, the payback time calculated here is somewhat shorter than the actual payback time.

A more accurate, but much more complex estimate could be made by assuming a typical daily hot water usage pattern and power output of heaters. But since the payback time is so short, the increased complexity is not justified since it will not change the bottom line—buy the more expensive model and save money as well as energy!

PROBLEM 1.58

Liquid oxygen (LOX) for the Space Shuttle can be stored at 90 K prior to launch in a spherical container 4 m in diameter. To reduce the loss of oxygen, the sphere is insulated with superinsulation developed at the US Institute of Standards and Technology's Cryogenic Division that has an effective thermal conductivity of 0.00012 W/(m K). If the outside temperature is 20°C on the average and the LOX has a heat of vaporization of 213 J/g, calculate the thickness of insulation required to keep the LOX evaporation rate below 200 g/h.

GIVEN

Spherical LOX tank with superinsulation
Tank diameter (D) = 4 m
LOX temperature (T_{LOX}) = 90 K
Ambient temperature (T_∞) = 20°C = 293 K
Thermal conductivity of insulation (k) = 0.00012 W/(m K)
Heat of vaporization of LOX (h_{fg}) = 213 kJ/kg
Maximum evaporation rate ($\dot{m}_{Lox}$) = 0.2 kg/h

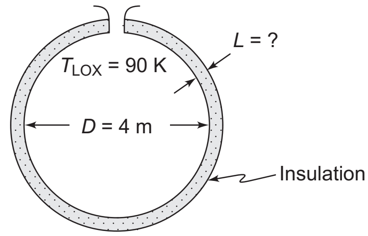
FIND

The minimum thickness of the insulation (L) to keep evaporation rate below 0.2 kg/h

ASSUMPTIONS

The thickness is small compared to the sphere diameter so the problem can be considered one dimensional
Steady state conditions prevail
Radiative heat loss is negligible

SKETCH



SOLUTION

The maximum permissible rate of heat transfer is the rate that will evaporate 0.2 kg/h of LOX

$$q = \dot{m}_{Lox}\, h_{fg} = (0.2\text{ kg/h})\ (213\text{ kJ/kg}) \left(\frac{\text{h}}{3600\text{ s}}\right) \left(\frac{1000\text{ J}}{\text{kJ}}\right) (\text{Ws/J}) = 11.8\text{ W}$$

An upper limit can be put on the rate of heat transfer by assuming that the convective resistance on the outside of the insulation is negligible and therefore the outer surface temperature is the same as the ambient air temperature. With this assumption, heat transfer can be calculated using Equation (1.2), one dimensional steady state conduction

$$q_k = \frac{k\,A}{L}\ (T_{\text{hot}} - T_{\text{cold}}) = \frac{k\,\pi D^2}{L}\ (T_\infty - T_{\text{LOX}})$$

Solving for the thickness of the insulation (L)

$$L = \frac{k\,\pi D^2}{q_k}\ (T_\infty - T_{\text{LOX}}) = \frac{[0.00012\text{ W/(m K)}]\,\pi\,(4\text{ m})^2}{11.8\text{ W}}\ (293\text{ K} - 90\text{ K}) = 0.10\text{ m} = 10\text{ cm}$$

COMMENTS

The insulation thickness is small compared to the diameter of the tank. Therefore, the assumption of one dimensional conduction is reasonable.

PROBLEM 1.59

The heat transfer coefficient between a surface and a liquid is 60 W/(m^{2}K). How many watts per square meter will be transferred in this system if the temperature difference is 10°C?

GIVEN

The heat transfer coefficient between a surface and a liquid (h_c) = 60 W/(m^{2}K)
Temperature difference (ΔT) = 10°C

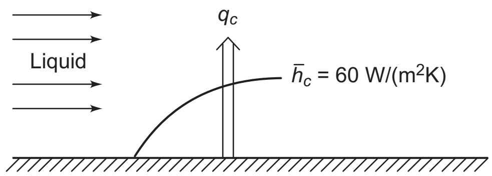
FIND

The rate of heat transfer in watts per square meter

ASSUMPTIONS

Steady state conditions
Surface temperature is higher than the liquid temperature

SKETCH



SOLUTION

The rate of convective heat transfer per unit area (q_c/A) is

$$\frac{q_c}{A} = \overline{h_c}\ \Delta T = 60\ \text{W/(m}^2\text{K)} \times 10°\text{C} = 600\ \text{W}.$$

PROBLEM 1.60

An ice chest (see sketch) is to be constructed from Styrofoam [k = 0.033 W/(m K)]. If the wall of the chest is 5 cm thick, calculate its R-value in (m^{2}K)/(W-cm).

GIVEN

Ice chest constructed of Styrofoam, k = 0.0333 W/(m K)
Wall thickness 5 cm

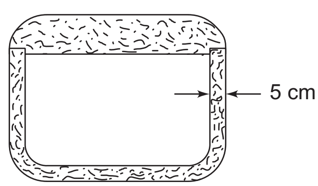
FIND

(a) R-value of the ice chest wall

ASSUMPTIONS

(a) One-dimensional, steady conduction

SKETCH



SOLUTION

From Section 1.6 the R-value is defined as

$$R\text{-value} = \frac{\text{thickness}}{\text{thermal conductivity}}$$

$$R\text{-value} = \frac{5 \times 10^{-2}\ \text{m}}{0.033\ \text{W/(m K)}} = 1.51\ (\text{m}^2\text{K})/\text{W}$$

From the problem statement, it is clear that we are asked to determine the R-value on a 'per-cm' basis. Dividing the above R-value by the thickness in cm, we get

$$R\text{-value} = \frac{1.51}{5} = 0.302\ (\text{m}^2\text{K})/(\text{W-cm})$$

PROBLEM 1.61

Estimate the *R*-values for a 5 cm-thick fiberglass board and a 2.5 cm-thick polyurethane foam layer. Then compare their respective conductivity-times-density products if the density for fiberglass is 50 kg/m^3 and the density of polyurethane is 30 kg/m^3. Use the units given in Figure 1.27.

GIVEN

5 cm-thick fiberglass board, density = 50 kg/m^3
2.5 cm-thick polyurethane, density = 30 kg/m^3

FIND

(a) *R*-values for both
(b) Conductivity-times-density products for both

ASSUMPTIONS

(a) One-dimensional, steady conduction

SOLUTION

Ranges of conductivity for both of these materials are given in Figure 1.28. Using mean values we find:

fiberglass board k = 0.04 W/(m K)
polyurethane foam k = 0.025 W/(m K)

For the 5 cm fiberglass we have

t = 0.05 m
k = 0.04 W/(m K)

From Section 1.6 the *R*-value is given by

$$R\text{-value} = \frac{\text{thickness}}{\text{thermal conductivity}} = \frac{0.050\,\text{m}}{0.04\ \text{W/(m K)}} = 1.25\ (\text{m}^2\ \text{K})/\text{W}$$

and

$$\text{conductivity} \times \text{density} = (0.04\ \text{W/(m K)})\,(50\ \text{kg/m}^3) = 2\ \ (\text{Wkg})/(\text{Km}^4)$$

For the 2.5 cm polyurethane we have

t = 0.025 m
k = 0.025 W/(m K)

$$R\text{-value} = \frac{t}{k} = 1\ (\text{m}^2\ \text{K})/\text{W}$$

$$\text{conductivity} \times \text{density} = (0.025\ \text{W/(m K)})\left(30\ \text{kg/m}^3\right) = 0.75\ \ (\text{Wkg})/(\text{Km}^4)$$

Summarizing, we have

	R-value (m^2 K)/W	conductivity × density (W kg)/(K m^4)
2″ fiberglass board	1.25	2
1″ polyurethane foam	1	0.75

PROBLEM 1.63

How many kilograms of ice can a 3-ton refrigeration unit produce in a 24-hour period? The heat of fusion of water is 330 kJ/kg.

A manufacturer in the US wants to sell a refrigeration system to a customer in Germany. The standard measure of refrigeration capacity used in the United States is the 'ton'; a one-ton capacity means that the unit is capable of making about one ton of ice per day or has a heat removal rate of 3.52 kW. The capacity of the American system is to be guaranteed at three tons. What would this guarantee be in SI units?

GIVEN

A three-ton refrigeration unit
Heat of fusion of ice is 330 kJ/kg

FIND

(a) Kilograms of ice produced by the unit per 24 hour period
(b) The refrigeration unit capacity is the net value, i.e., it includes heat losses

ASSUMPTIONS

(a) Water is cooled to just above the freezing point before entering the unit

SOLUTION

The mass of ice produced in a given period of time Δt is given by

$$m_{\text{ice}} = \frac{q\Delta T}{h_f}$$

where h_f is the heat of fusion and q is the rate of heat removal by the refrigeration unit. From Problem 1.65 we have q = 10,548 W. Inserting the given values we have

$$m_{\text{ice}} = \frac{(10{,}548\,\text{W})(24\,\text{hr})}{\left(3.30\times10^5\ \text{J/kg}\right)(\text{Ws})/\text{J}\left(\dfrac{\text{hr}}{3600\text{s}}\right)} = 2762\ \text{kg}$$

PROBLEM 1.64

Explain a fundamental characteristic that differentiates conduction from convection and radiation.

SOLUTION

Conduction is the only heat transfer mechanism that dominates in solid materials. Convection and radiation play important roles in fluids or, for radiation, in a vacuum. Under certain conditions, e.g., a transparent solid, radiation could be important in a solid.

PROBLEM 1.65

Explain in your own words: (a) what is the mode of heat transfer through a large steel plate that has its surfaces at specified temperatures? (b) what are the modes when the temperature on one surface of the steel plate is not specified, but the surface is exposed to a fluid at a specified temperature.

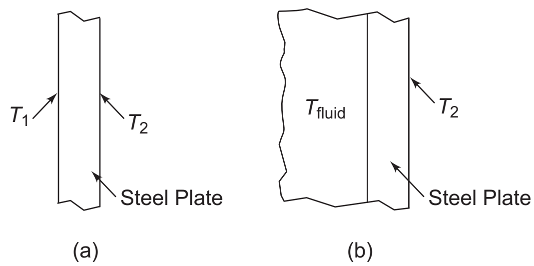
GIVEN

Steel plate with specified surface temperatures
Steel plate with one specified temperature and another surface exposed to a fluid

FIND

(a) Modes of heat transfer

SKETCH



SOLUTION

(a) Since the surface temperatures are specified, the only mode of heat transfer of importance is conduction through the steel plate
(b) In addition to conduction to the steel plate, convection at the surface exposed to the fluid must be considered

PROBLEM 1.66

> **What are the important modes of heat transfer for a person sitting quietly in a room? What if the person is sitting near a roaring fireplace?**

GIVEN

Person sitting quietly in a room
Person sitting in a room with a fireplace

FIND

(a) Modes of heat transfer for each situation

ASSUMPTIONS

The person is clothed

SOLUTION

(a) Since the person is clothed, we would need to consider conduction through the clothing, and convection and radiation from the exposed surface of the clothing.
(b) In addition to the modes identified in (a), we would need to consider that surfaces of the person oriented towards the fire would be absorbing radiation from the flames.

PROBLEM 1.67

> **Explain a fundamental characteristic that differentiates condition from convection and radiation.**

SOLUTION

Conduction is the only heat transfer mechanism that dominates in solid materials. Convection and radiation play important roles in fluids or, for radiation, in a vacuum. Under certain conditions, e.g. a transparent solid, radiation could be important in a solid.

PROBLEM 1.68

Describe and compare the modes of heat loss through the single-pane and double-pane window assemblies shown in the sketch below.

GIVEN

A single-pane and a double-pane window assembly

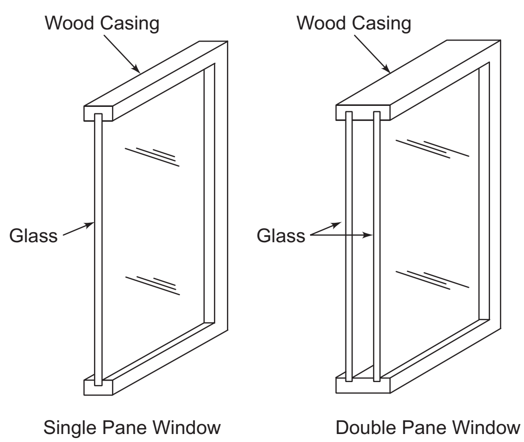
FIND

(a) The modes of heat transfer for each
(b) Compare the modes of heat transfer for each

ASSUMPTIONS

The window assembly wood casing is a good insulator

SKETCH



SOLUTION

The thermal network for both cases is shown above and summarizes the situation. For the single-pane window, we have convection on both exterior surfaces of the glass, radiation from both exterior surfaces of the glass, and conduction through the glass. For the double-pane window, we would have these modes in addition to radiation and convection exchange between the facing surfaces of the glass panes. Since the overall thermal network for the double-pane assembly replaces the pane-conduction with two-pane conductions plus the convection/radiation between the two panes, the overall thermal resistance of the double-pane assembly should be larger. Therefore, we would expect lower heat loss through the double-pane window.

PROBLEM 1.69

A person wearing a heavy parka is standing in a cold wind. Describe the modes of heat transfer determining heat loss from the person's body.

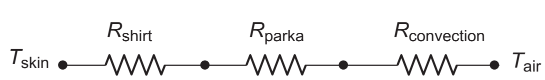
GIVEN

Person standing in a cold wing wearing a heavy parka

FIND

(a) The modes of heat transfer

SKETCH



SOLUTION

The thermal circuit for the situation is shown above. Assume that the person is wearing one other garment, i.e. a shirt, under the parka. The modes of heat transfer include conduction through the shirt and the parka and convection from the outer surface of the parka to the cold wind. We expect that the largest thermal resistance will be the parka insulation. We have neglected radiation from the parka outer surface because its influence on the overall heat transfer will be small compared to the other terms.

PROBLEM 1.70

> **Discuss the modes of heat transfer that determine the equilibrium temperature of the space shuttle Endeavor when it is in orbit. What happens when it reenters the earth's atmosphere?**

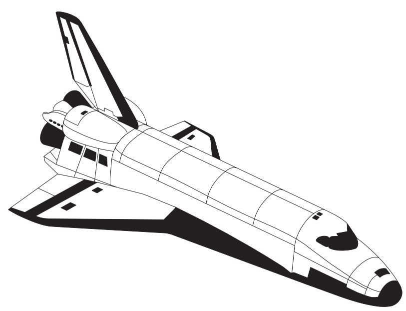
GIVEN

Space shuttle Endeavor in orbit
Space shuttle Endeavor during reentry

FIND

(a) Modes of heat transfer

SKETCH



SOLUTION

Heat generated internally will have to be rejected to the skin of the shuttle or to some type of radiator heat exchanger exposed to space. The internal loads that are not rejected actively, i.e., by a heat exchanger, will be transferred to the internal surface of the shuttle by radiation and convection, transferred by conduction through the skin, then radiated to space. These two paths of heat transfer must be sufficient to ensure that the interior is maintained at a comfortable working temperature.

During reentry, the exterior surface of the shuttle will be exposed to a heat flux that results from frictional heating by the atmosphere. In this case, it is likely that the net heat flow will be into the space shuttle. The thermal design must be such that during reentry the interior temperature does not exceed some safe value.

Chapter 2

PROBLEM 2.1

The heat conduction equation in cylindrical coordinates is

$$\rho c \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

(a) Simplify this equation by eliminating terms equal to zero for the case of steady-state heat flow without sources or sinks around a right-angle corner such as the one in the accompanying sketch. It may be assumed that the corner extends to infinity in the direction perpendicular to the page. (b) Solve the resulting equation for the temperature distribution by substituting the boundary condition. (c) Determine the rate of heat flow from T_1 to T_2. Assume k = 1 W/(m K) and unit depth perpendicular to the page.

GIVEN

- Steady state conditions
- Right-angle corner as shown below
- No sources or sinks
- Thermal conductivity (k) = 1 W/(m K)

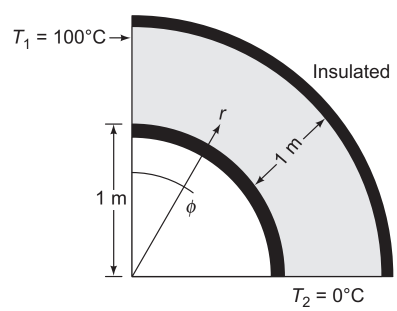
FIND

(a) Simplified heat conduction equation
(b) Solution for the temperature distribution
(c) Rate of heat flow from T_1 to T_2

ASSUMPTIONS

- Corner extends to infinity perpendicular to the paper
- No heat transfer in the z direction
- Heat transfer through the insulation is negligible

SKETCH



SOLUTION

The boundaries of the region are given by

$$1 \text{ m} \le r \ge 2 \text{ m}$$

$$0 \le \phi \ge \frac{\pi}{2}$$

Assuming there is no heat transfer through the insulation, the boundary condition is

$$\frac{\partial T}{\partial r} = 0 \text{ at } r = 1 \text{ m}$$

$$\frac{\partial T}{\partial r} = 0 \text{ at } r = 2 \text{ m}$$

$$T_1 = 100°\text{C at } \phi = 0$$

$$T_2 = 0°\text{C at } \phi = \frac{\pi}{2}$$

(a) The conduction equation is simplified by the following

Steady state

$$\frac{\partial T}{\partial t} = 0$$

No sources or sinks

$$q_k = 0$$

No heat transfer in the z direction

$$\frac{\partial^2 T}{\partial z^2} = 0$$

Since $\frac{\partial T}{\partial r} = 0$ over both boundaries, $\frac{\partial T}{\partial r} = 0$ throughout the region

(Maximum principle); therefore, $\frac{\partial^2 T}{\partial r^2} = 0$ throughout the region also.

Substituting these simplifications into the conduction equation

$$0 = k\left(0 + 0 + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + 0\right)$$

$$\frac{\partial^2 T}{\partial \phi^2} = 0$$

(b) Integrating twice

$$T = c_1\,\phi + c_2$$

The boundary condition can be used to evaluate the constants

At $\phi = 0$, $T = 100°\text{C} : 100°\text{C} = c_2$

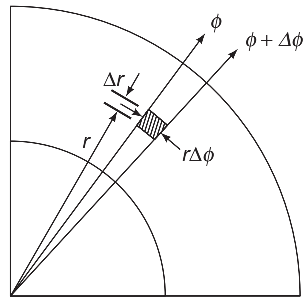
At $\phi = \frac{\pi}{2}$, $T = 0°\text{C} : 0°\text{C} = c_1\,(\pi/2) + 100°\text{C}$

$$\text{s}c_1 = -\frac{200\,^{\circ}\text{C}}{\pi}$$

Therefore, the temperature distribution is

$$T(\phi) = 100 - \frac{200\,°\text{C}}{\pi}\,\phi\,°\text{C}$$

(c) Consider a slice of the corner as follow



The heat transfer flux through the shaded element in the ϕ direction is

$$q'' = \frac{-k\,\Delta T}{\text{thickness}} = \frac{-k(T_\phi - T_{\phi+\Delta\phi})}{r\,\Delta\phi}$$

In the limit as $\Delta\phi \to 0$, $q'' = -k\,\dfrac{dT}{r\,d\phi}$

Multiplying by the surface area *drdz* and integrating along the radius

$$q = \int_{r_1}^{r_o} q''\,drdz = \frac{200°\text{C k}}{\pi}\int_{r_1}^{r_o}\frac{dr}{r} = \frac{200°\text{C k}}{\pi}\ln\frac{r_o}{r_1}$$

$$q = \frac{200°\text{C k}}{\pi}\quad [1\ \text{W/(m K)}]\ \ln(2\ \text{m}/1\ \text{m}) = 44.1\ \text{W/m}$$ 44.1W per meter in the z direction

COMMENTS

Due to the boundary conditions, the heat flux direction is normal to radial lines.

PROBLEM 2.2

Write Equation (2.20) in a dimensionless form similar to Equation (2.17).

GIVEN

- Equation (2.20)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}_G}{k} = \frac{1}{\propto}\frac{\partial T}{\partial t}$$

FIND

- Dimensionless form of the equation

SOLUTION

Let

$$\tau = \frac{t}{t_r} \Rightarrow t = \tau\, t_r$$

$$\theta = \frac{T}{T_r} \Rightarrow T = \theta\, T_r$$

$$\zeta = \frac{r}{R_r} \Rightarrow r = \zeta\, R_r$$

Where T_r, R_r, and t_r are reference temperature, reference radius, and reference time, respectively. Substituting these into Equation (2.20)

$$\frac{1}{\zeta R_r}\frac{\partial}{\partial(\zeta R_r)}\left(\zeta R_r \frac{\partial(\theta T_r)}{\partial(\zeta R_r)}\right) + \frac{\dot{q}_G}{k} = \frac{1}{\alpha}\;\frac{\partial(T_r\theta)}{\partial''(\tau t_r)}$$

$$\frac{1}{\zeta R_r^2}\frac{\partial}{\partial\zeta}\left(\zeta T_r \frac{\partial\theta}{\partial\zeta}\right) + \frac{\dot{q}_G}{k} = \frac{1}{\alpha}\;\frac{T_r}{t_r}\frac{\partial\theta}{\partial\tau}$$

$$\frac{1}{\zeta}\frac{\partial}{\partial\zeta}\left(\zeta\frac{\partial\theta}{\partial\zeta}\right) + \frac{R_r^2\,\dot{q}_G}{T_r\,k} = \frac{R_r^2}{t_r\,\alpha}\frac{\partial\theta}{\partial\tau}$$

$$\text{let } \dot{Q}_G = \frac{R_r^2\,\dot{q}_G}{T_r\,k} \text{ and } F_o = \frac{t_r\,\alpha}{R_r^2}$$

$$\frac{1}{\zeta}\frac{\partial}{\partial\zeta}\left(\zeta\frac{\partial\theta}{\partial\zeta}\right) + \dot{Q}_G = \frac{1}{F_o}\frac{\partial\theta}{\partial\tau}$$

PROBLEM 2.3

Calculate the rate of heat loss per foot and the thermal resistance for a 15 cm schedule 40 steel pipe covered with a 7.5 cm thick layer of 85% magnesia. Superheated steam at 150°C flows inside the pipe ($\bar{h}_c$ = 170 W/(m^2 K)) and still air at 16°C is on the outside ($\bar{h}_c$ = 30 W/(m^2 K)).

GIVEN

- A 15 cm standard steel pipe covered with 85% magnesia
- Magnesia thickness = 7.5 cm
- Superheated steam at 150°C flows inside the pipe
- Surrounding air temperature (T_∞) = 16°C
- Heat transfer coefficients
  - Inside ($\bar{h}_{ci}$) = 170 W/(m^2 K)
  - Outside ($\bar{h}_{co}$) = 30 W/(m^2 K)

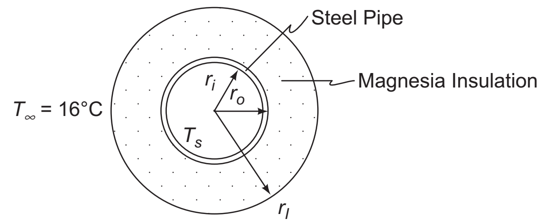
FIND

(a) The thermal resistance (R)
(b) The rate of heat loss per foot (q/L)

ASSUMPTIONS

- Constant thermal conductivity
- The pipe is made of 1% carbon steel

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10, 11, and 41

For a 15 cm schedule 40 pipe

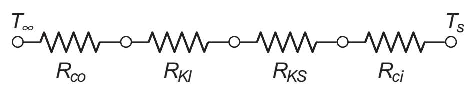
- Inside diameter (D_i) = 15.16 cm
- Outside diameter (D_o) = 16.56 cm

Thermal Conductivities

- 85% Magnesia (k_I) = 0.06 W/(m K) at 20°C
- 1% Carbon steel (k_s) = 43 W/(m K) at 20°C

SOLUTION

The thermal circuit for the insulated pipe is shown below



(a) The values of the individual resistances can be calculated using Equations (1.14) and (2.39)

$$R_{co} = \frac{1}{\overline{h_{co}}\,A_o} = \frac{1}{\overline{h_{co}}\,\pi D_i L} = \frac{1}{L(m)}\frac{1}{\left(30\,\text{W/(m}^2\,\text{K)}\right)\pi(0.1656\,\text{m})}$$

$$\Rightarrow \qquad R_{co} = \frac{1}{L(m)}\,0.064\ \text{(m K)/W}$$

$$R_{kI} = \frac{\ln\left(\frac{r_I}{r_o}\right)}{2\pi L k_I} = \frac{\ln\left(\frac{(0.1656+0.075)}{0.1656}\right)}{L(m)\,2\pi(0.06\ \text{W/(m K)})} = \frac{1}{L(m)}\,1\ \text{(m K)/W}$$

$$R_{ks} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi L k_s} = \frac{1}{L(m)}\frac{\ln\left(\frac{16.56}{15.16}\right)}{2\pi(43\,\text{W/(m K)})} = \frac{1}{L(m)}\ 3.26\times10^{-4}\ \text{(m K)/W}$$

$$R_{ci} = \frac{1}{\overline{h_{ci}}\,A_i} = \frac{1}{\overline{h_{ci}}\,\pi D_i L} = \frac{1}{L(m)}\frac{1}{\pi(0.1516\,\text{m})(170\,\text{W/(m}^2\text{K))}} = \frac{1}{L(m)}\ 0.0124\ \text{(m K)/W}$$

The total resistance is

$$R_{\text{total}} = R_{co} + R_{kI} + R_{ks} + R_{ci}$$

$$= \frac{1}{L}\ (0.064 + 1 + 3.26\times10^{-4} + 0.0124)\ \text{(m K)/W}$$

$$= \frac{1}{L}\ 1.0764\ \text{(m K)/W}$$

(b) The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{(150-16)\text{K}}{\frac{1}{L}1.0764\ \text{(m K)/W}} \qquad \therefore \frac{q}{L} = 124.5\ \text{W/m}$$

COMMENTS

Note that almost all of the thermal resistance is due to the insulation and that the thermal resistance of the steel pipe is negligible.

PROBLEM 2.4

> **Suppose that a pipe carrying a hot fluid with an external temperature of T_i and outer radius r_i is to be insulated with an insulation material of thermal conductivity k and outer radius r_o. Show that if the convective heat transfer coefficient on the outside of the insulation is h and the environmental temperature is T_∞, the addition of insulation can actually increase the rate of heat loss if $r_o < k/\bar{h}$ and that maximum heat loss occurs when $r_o = k/\bar{h}$. This radius, r_c, is often called the critical radius.**

GIVEN

- An insulated pipe
- External temperature of the pipe = T_i
- Outer radius of the pipe = r_i
- Outer radius of insulation = r_o
- Thermal conductivity = k
- Ambient temperature = T_∞
- Convective heat transfer coefficient = $\bar{h}$

FIND

Show that

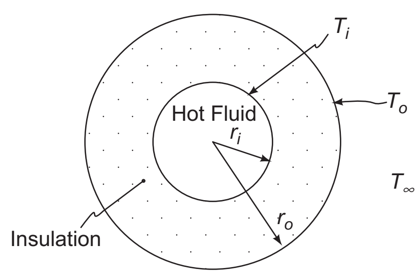
(a) The insulation can increase the heat loss if $r_o < k/\bar{h}$

(b) Maximum heat loss occurs when $r_o = k/\bar{h}$

ASSUMPTIONS

- The system has reached steady state
- The thermal conductivity does not vary appreciably with temperature
- Conduction occurs in the radial direction only

SKETCH



SOLUTION

Radial conduction for a cylinder of length L is given by Equation (2.37)

$$q_k = 2\,\pi L\,k\,\frac{T_i - T_o}{\ln\frac{r_o}{r_i}}$$

Convection from the outer surface of the cylinder is given by Equation (1.10)

$$q_c = \bar{h}_c\,A\,\Delta T = \bar{h}\;2\,\pi r_o L\,(T_o - T_\infty)$$

For steady state

$$q_k = q_c$$

$$2\,\pi L\,k\,\frac{T_i - T_o}{\ln\frac{r_o}{r_i}} = \bar{h}\;2\,\pi r_o\,L\,(T_o - T_\infty)$$

The outer wall temperature, T_o, is an unknown and must be eliminated from the equation
Solving for $T_i - T_o$

$$T_i - T_o = \frac{\bar{h}\,r_o}{k}\;\ln\frac{r_o}{r_i}\;(T_o - T_\infty)$$

$$T_i - T_\infty = (T_i - T_o) + (T_o - T_\infty) = \frac{\bar{h}\,r_o}{k}\;\ln\frac{r_o}{r_i}\;(T_o - T_\infty) + (T_o - T_\infty)$$

$$T_i - T_\infty = \left(\frac{\bar{h}\,r_o}{k}\ln\frac{r_o}{r_i} + 1\right)\;(T_o - T_\infty)$$

or

$$T_o - T_\infty = \frac{T_i - T_\infty}{1 + \frac{\bar{h}\,r_o}{k}\ln\frac{r_o}{r_i}}$$

Substituting this into the convection equation

$$q = q_c = \bar{h}\;2\,\pi r_o\,L\left[\frac{T_i - T_\infty}{1 + \frac{\bar{h}\,r_o}{k}\ln\frac{r_o}{r_i}}\right]$$

$$q = \frac{T_i - T_\infty}{\left(\frac{1}{2\pi r_o\,L\bar{h}} + \frac{\ln\frac{r_o}{r_i}}{2\pi\,Lk}\right)}$$

Examining the above equation, the heat transfer rate is a maximum when the term

$\left(\frac{1}{2\pi r_o\,L\bar{h}} + \frac{\ln\frac{r_o}{r_i}}{2\pi\,Lk}\right)$ is a minimum, which occurs when its differential with respect to r_o is zero

$$\frac{1}{2\pi k\,L}\frac{d}{dr_o}\left(\frac{k}{r_o\bar{h}} + \ln\frac{r_o}{r_i}\right) = 0$$

$$\frac{k}{\bar{h}}\frac{d}{dr_o}\left(\frac{1}{r_o}\right) + \frac{d}{dr_o}\left(\ln\frac{r_o}{r_i}\right) = 0$$

$$\frac{k}{\bar{h}}\left(-\frac{1}{r_o^2}\right) + \frac{1}{r_o} = 0$$

$$r_o = \frac{k}{\bar{h}}$$

PROBLEM 2.5

A solution with a boiling point of 82°C boils on the outside of a 2.5 cm tube with a No. 14 BWG gauge wall. On the inside of the tube flows saturated steam at 4.2 bar (abs). The convective heat transfer coefficients are 8500 W/(m^2 K) on the steam side and 6200 W/(m^2 K) on the exterior surface. Calculate the increase in the rate of heat transfer for a copper over a steel tube.

GIVEN

- Tube with saturated steam on the inside and solution boiling at 82°C outside
- Tube specification: 2.5 cm No. 14 BWG gauge wall
- Saturated steam in the pipe is at 4.2 bar
- Convective heat transfer coefficients
  - Steam side ($\bar{h}_{ci}$) : 8500 W/(m^2 K)
  - Exterior surface ($\bar{h}_{co}$) : 6200 W/(m^2 K)

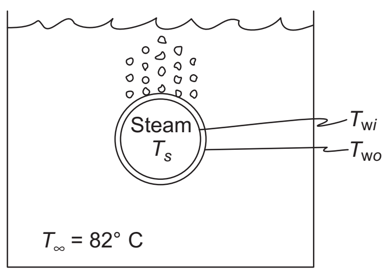
FIND

- The increase in the rate of heat transfer for a copper over a steel tube

ASSUMPTIONS

- The system is in steady state
- Constant thermal conductivities

SKETCH



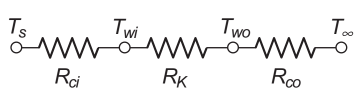
PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10, 12, 13 and 42

- Temperature of saturated steam at 4.2 bar (T_s) = 144°C
- Thermal conductivities
  - Copper (k_c) = 390 W/(m K) at 127°C
  - 1% Carbon steel (k_s) = 43 W/(m K) at 20°C
- Tube inside diameter (D_i) = 0.834 in.

SOLUTION

The thermal circuit for the tube is shown below



The individual resistances are

$$R_{ci} = \frac{1}{\bar{h}_{ci} A_i} = \frac{1}{\bar{h}_{ci}\,\pi D_i L} = \frac{1}{L}\frac{1}{(8500\,\text{W/(m}^2\text{K))}\,\pi\,(2.08\times 10^{-2}\text{ m})} = \frac{0.0018}{L}\ \text{K/W}$$

$$R_{co} = \frac{1}{\bar{h}_{co} A_o} = \frac{1}{\bar{h}_{co}\,\pi D_i L} = \frac{1}{L}\frac{1}{(6200\,\text{W/(m}^2\text{K))}\,\pi\,(2.5\times 10^{-2}\text{ m})} = \frac{0.00205}{L}\ \text{K/W}$$

$$R_{kc} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi L k_c} = \frac{\ln\left(\frac{2.5\,\text{cm}}{2.08\,\text{cm}}\right)}{L\,2\pi(390\,\text{W/(m K)})} = \frac{7.5\times10^{-5}}{L}\ \text{K/W}$$

$$R_{ks} = \frac{\ln\left(\frac{r_o}{r_s}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{2.5\,\text{cm}}{2.08\ \text{cm}}\right)}{L\,2\pi(43\,\text{W/(m K)})} = \frac{6.81\times10^{-4}}{L}\ \text{K/W}$$

For the copper tube

$$\frac{q_c}{L} = \frac{(144-82)\text{K}}{(0.0018+0.000075+0.00205)\,(\text{K m})/\text{W}} = 15800\ \text{W/m}$$

For the steel tube

$$\frac{q_s}{L} = \frac{(144-82)\text{K}}{(0.0018+0.00068+0.00205)(\text{K m})/\text{W}} = 13690\ \text{W/m}$$

The increase in the rate of heat transfer per unit length with the copper tube is

$$= \frac{q_c}{L} - \frac{q_s}{L} = 2110\ \text{W/m}$$

$$\therefore \qquad \%\ \text{increase} = \frac{2110}{13690} \times 100 = 15.4\%$$

COMMENTS

The choice of tubing material is significant in this case because the convective heat transfer resistances are small making the conductive resistant a significant portion of the overall thermal resistance.

PROBLEM 2.6

Steam having a quality of 98% at a pressure of 1.37×10^5 N/m^2 is flowing at a velocity of 1 m/s through a steel pipe of 2.7 cm OD and 2.1 cm ID. The heat transfer coefficient at the inner surface, where condensation occurs, is 567 W/(m^2 K). A dirt film at the inner surface adds a unit thermal resistance of 0.18 (m^2 K)/W. Estimate the rate of heat loss per meter length of pipe if; (a) the pipe is bare, (b) the pipe is covered with a 5 cm layer of 85% magnesia insulation. For both cases assume that the convective heat transfer coefficient at the outer surface is 11 W/(m^2 K) and that the environmental temperature is 21°C. Also estimate the quality of the steam after a 3-m length of pipe in both cases.

GIVEN

- A steel pipe with steam condensing on the inside
- Diameters
  - Outside (D_o) = 2.7 cm = 0.027 m
  - Inside (D_i) = 2.1 cm = 0.021 m
- Velocity of the steam (V) = 1 m/s
- Initial steam quality (X_i) = 98%
- Steam pressure = 1.37×10^5 N/m^2
- Heat transfer coefficients
  - Inside (h_{ci}) = 567 W/(m^2 K)
  - Outside (h_{co}) = 11 W/(m^2 K)
- Thermal resistance of dirt film on inside surface (R_f) = 0.18 (m^2 K)/W
- Ambient temperature (T_∞) = 21°C

FIND

The heat loss per meter (q/L) and the change in the quality of the steam per 3 m length for

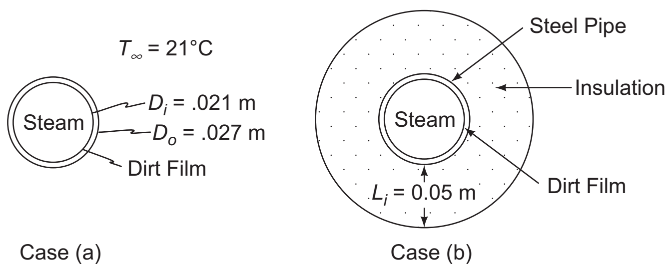
(a) A bare pipe

(b) A pipe insulated with 85% Magnesia: thickness (L_i) = 0.05 m

ASSUMPTIONS

- Steady state conditions exist
- Constant thermal conductivity
- Steel is 1% carbon steel
- Radiative heat transfer from the pipe is negligible
- Neglect the pressure drop of the steam

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10, 11, and 13

The thermal conductivities are: 1% carbon steel (k_s) = 43 W/(m K) at 20°C

85% Magnesia (k_i) = 0.059 W/(m K) at 20°C

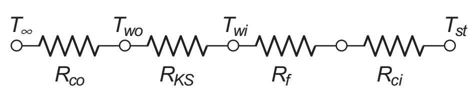
For saturated steam at 1.37×10^5 N/m^2: Temperature (T_{st}) = 107°C

Heat of vaporization (h_{fg}) = 2237 kJ/kg

Specific volume (υ_s) = 1.39 m^3/kg

SOLUTION

(a) The thermal circuit for the uninsulated pipe is shown below



Evaluating the individual resistances

$$R_{co} = \frac{1}{\overline{h_{co}}A_o} = \frac{1}{\overline{h_{co}}\,\pi D_o L} = \frac{1}{[11\,\text{W/(m}^2\text{K)}]\,\pi\,(0.027\,\text{m})\,L} = \frac{1}{L}\ 1.072\ \text{(m K)/W}$$

$$R_{ks} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi L k_i} = \frac{\ln\left(\frac{0.027}{0.021}\right)}{2\pi[43\,\text{W/(mK)}]} = \frac{1}{L}\ 0.00093\ \text{(m K)/W}$$

$$R_f = \frac{r_f}{A} = \frac{r_f}{2\pi D_i L} = \frac{1}{L}\,\frac{0.18\,\text{m}^2\text{K/W}}{\pi\,(0.021\,\text{m})} = \frac{1}{L}\ 2.728\ \text{(m K)/W}$$

$$R_{ci} = \frac{1}{\overline{h_{ci}}A_i} = \frac{1}{\overline{h_{ci}}\,\pi D_i L} = \frac{1}{[567\,\text{W/(m}^2\text{K)}]\,\pi\,(0.021\,\text{m})L} = \frac{1}{L}\ 0.0267\ \text{(m K)/W}$$

The rate of heat transfer through the pipe is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{st} - T_\infty}{R_\infty + R_{ks} + R_i + R_{ci}}$$

$$\frac{q}{L} = \frac{107\,°\text{C} - 21°\text{C}}{(1.072 + 0.00093 + 2.728 + 0.267)\,(\text{mK})/\text{W}} = 22.5 \text{ W/m}$$

The total rate of transfer of a three meter section of the pipe is

$$q = 22.5 \text{ W/m (3 m)} = 67.4 \text{ W}$$

The mass flow rate of the steam in the pipe is

$$\dot{m}_s = \frac{A_i V}{\upsilon_s} = \frac{\pi D_i^2 V}{4\upsilon_s} = \frac{\pi (0.021\,\text{m})^2\,(1\,\text{m/s})}{4\,(1.39\,\text{m}^3/\text{kg})\,(1\,\text{kg}/1000\text{g})} = 0.249 \text{ g/s}$$

The mass rate of steam condensed in a 3 meter section of the pipe is equal to the rate of heat transfer divided by the heat of vaporization of the steam

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{67.4\,\text{W}}{2237\,\text{J/g(Ws/J)}} = 0.030 \text{ g/s}$$

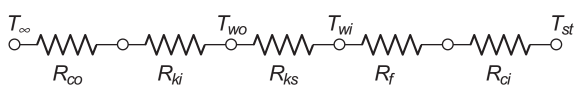
The quality of the saturated steam is the fraction of the steam which is vapor. The quality of the steam after a 3 meter section, therefore, is

$$X_i = \frac{(\text{original vapor mass}) - (\text{mass of vapor condensed})}{\text{total mass of steam}} = \frac{X_i\,\dot{m}_s - \dot{m}_c}{\dot{m}_s}$$

$$X_i = \frac{0.98\,(0.249\,\text{g/s}) - 0.030\,\text{g/s}}{0.249\,\text{g/s}} = 0.86 = 86\%$$

The quality of the steam changed by 12%.

The thermal circuit for the pipe with insulation is shown below



The convective resistance on the outside of the pipe is different than that in part (a) because it is based on the outer area of the insulation

$$R_{co} = \frac{1}{\overline{h_{co}} A_o} = \frac{1}{\overline{h_{co}}\,\pi(D_o + 2L_i)L} = \frac{1}{[11\,\text{W}/(\text{m}^2\text{K})]\,\pi\,(0.027\,\text{m}{+}0.1\,\text{m})L} = \frac{1}{L}\ 0.228 \text{ (m K)/W}$$

The thermal resistance of the insulation is

$$R_{ki} = \frac{\ln\left(\dfrac{D_o + 2L_i}{r_i}\right)}{2\,\pi\,L k_i} = \frac{\ln\left(\dfrac{0.027 + 0.1}{0.027}\right)}{2\,\pi[0.059\,\text{W}/(\text{mK})]} = \frac{1}{L}\ 4.18 \text{ (m K)/W}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{si} - T_\infty}{R_\infty + R_{ki} + R_{ks} + R_f + R_{ci}}$$

$$\therefore\ \frac{q}{L} = \frac{107\,°\text{C} - 21°\text{C}}{(0.228 + 4.18 + 0.00093 + 2.728 + 0.0267)(\text{mK})/\text{W}} = 12.0 \text{ W/m}$$

Therefore, the rate of steam condensed in 3 meters is

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{12.0\,\text{W}}{2237\,\text{J/g}\,(\text{Ws/J})} = 0.016\ \text{g/s}$$

The quality of the steam after 3 meters of pipe is

$$X_f = \frac{0.98(0.249\,\text{g/s}) - 0.016\,\text{g/s}}{0.249\,\text{g/s}} = 0.92 = 92\%$$

The change in the quality of the steam is 6%.

COMMENTS

Notice that the resistance of the steel pipe and the convective resistance on the inside of the pipe are negligible compared to the other resistances.

The resistance of the dirt film is the dominant resistance for the uninsulated pipe.

PROBLEM 2.7

> **Estimate the rate of heat loss per unit length from a 5 cm ID, 6 cm OD steel pipe covered with high temperature insulation having a thermal conductivity of 0.11 W/(m K) and a thickness of 1.2 cm. Steam flows in the pipe. It has a quality of 99% and is at 150°C. The unit thermal resistance at the inner wall is 0.0026 (m^2 K)/W, the heat transfer coefficient at the outer surface is 17 W/(m^2 K), and the ambient temperature is 16°C.**

GIVEN

- Insulated, steam filled steel pipe
- Diameters
  - *ID* of pipe (D_i) = 5 cm
  - *OD* of pipe (D_o) = 6 cm
- Thickness of insulation (L_i) = 1.2 cm
- Steam quality = 99%
- Steam temperature (T_s) = 150°C
- Unit thermal resistance at inner wall ($A\ R_i$) = 0.0026 (m^2 K)/W
- Heat transfer coefficient at outer wall (h_o) = 17 W /(m^2 K)
- Ambient temperature (T_∞) = 16°C
- Thermal conductivity of the insulation (k_I) = 0.11 W /(m K)

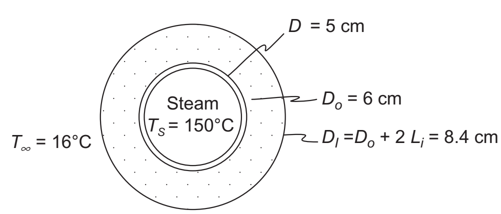
FIND

- Rate of heat loss per unit length (q/L)

ASSUMPTIONS

- 1% carbon steel
- Constant thermal conductivities
- Steady state conditions

SKETCH



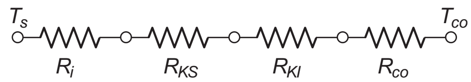
PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

The thermal conductivity of 1% carbon steel (k_s) = 43 W/(m^2 K) at 20 °C

SOLUTION

The outer diameter of the insulation (D_I) = 6 cm + 2 × 1.2 cm = 8.4 cm



The values of the individual resistances are

$$R_i = \frac{AR_i}{A_i} = \frac{AR_i}{\pi D_i L} = \frac{1}{L}\frac{0.0026\,(\text{m}^2\,\text{K})/\text{W}}{\pi(0.05\,\text{m})} = \frac{0.0165}{L}\ \text{K/(W m)}$$

$$R_{ks} = \frac{\ln\left(\dfrac{D_o}{D_i}\right)}{2\pi L k_s} = \frac{\ln\left(\dfrac{6\,\text{cm}}{5\,\text{cm}}\right)}{L\,2\pi(43\,\text{W}/(\text{m K}))} = \frac{6.75\times10^{-4}}{L}\ \text{K/(W m)}$$

$$R_{kI} = \frac{\ln\left(\dfrac{D_o}{D_I}\right)}{2\pi L k_i} = \frac{\ln\left(\dfrac{8.4\,\text{cm}}{6\,\text{cm}}\right)}{L\,2\pi(0.11\,\text{W}/(\text{m K}))} = \frac{0.487}{L}\ \text{K/(W m)}$$

$$R_{co} = \frac{1}{\bar{h}_{co} A_o} = \frac{1}{\bar{h}_{co}\,\pi D_I L} = \frac{1}{L}\frac{1}{(17\,\text{W}/(\text{m}^2\text{K}))\,\pi(8.4\times10^{-2}\,\text{m})} = \frac{0.223}{L}\ \text{K/(W m)}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_\text{i} + R_{ks} + R_{kI} + R_{co}}$$

$$\Rightarrow \qquad \frac{q}{L} = \frac{150-16}{0.0165 + 6.75\times10^{-4} + 0.487 + 0.223} = \text{W/m}$$

$$\therefore \qquad \frac{q}{L} = 184\ \text{W/m}$$

PROBLEM 2.8

The rate of heat flow per unit length q/L through a hollow cylinder of inside radius r_i and outside radius r_o is

$$q/L = (\bar{A}\,k\,\Delta T)/(r_o - r_i)$$

where $A = 2\pi(r_o - r_i)/\ln(r_o/r_i)$. Determine the percent error in the rate of heat flow if the arithmetic mean area $\pi(r_o + r_i)$ is used instead of the logarithmic mean area A for ratios of outside to inside diameters (D_o/D_i) of 1.5, 2.0, and 3.0. Plot the results.

GIVEN

- A hollow cylinder
- Inside radius = r_i
- Outside radius = r_o
- Heat flow per unit length as given above

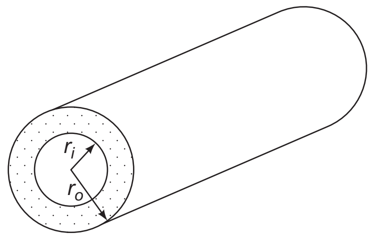
FIND

(a) Percent error in the rate of heat flow if the arithmetic rather than the logarithmic mean area is used for ratios of outside to inside diameters of 1.5, 2.0, and 3.0.
(b) Plot the results

ASSUMPTIONS

- Radial conduction only
- Constant thermal conductivity
- Steady state prevails

SKETCH



SOLUTION

The rate of heat transfer per unit length using the logarithmic mean area is

$$\left(\frac{q}{L}\right)_{\log} = \frac{2\pi(r_o - r_i)}{\ln\left(\frac{r_o}{r_i}\right)} \frac{k\Delta T}{r_o - r_i} = \frac{2\pi\, k\,\Delta T}{\ln\left(\frac{r_o}{r_i}\right)}$$

The rate of heat transfer per unit length using the arithmetic mean area is

$$\left(\frac{q}{L}\right)_{\text{arith}} = \pi(r_o + r_i)\,\frac{k\Delta T}{r_o - r_i} = \pi k\,\Delta T\,\frac{r_o + r_i}{r_o - r_i}$$

The percent error is

$$\%\text{ error} = \frac{\left(\frac{q}{L}\right)_{\log} - \left(\frac{q}{L}\right)_{\text{arith}}}{\left(\frac{q}{L}\right)_{\log}} \times 100 = \frac{\frac{2\pi\, k\,\Delta T}{\ln\left(\frac{r_o}{r_i}\right)} - \pi k\,\Delta T\,\frac{r_o + r_i}{r_o - r_i}}{\frac{2\pi\, k\,\Delta T}{\ln\left(\frac{r_o}{r_i}\right)}} \times 100$$

$$\%\text{ error} = \left[1 - \frac{1}{2}\ln\left(\frac{r_o}{r_i}\right)\frac{\left(\frac{r_o}{r_i}+1\right)}{\left(\frac{r_o}{r_i}-1\right)}\right] \times 100$$

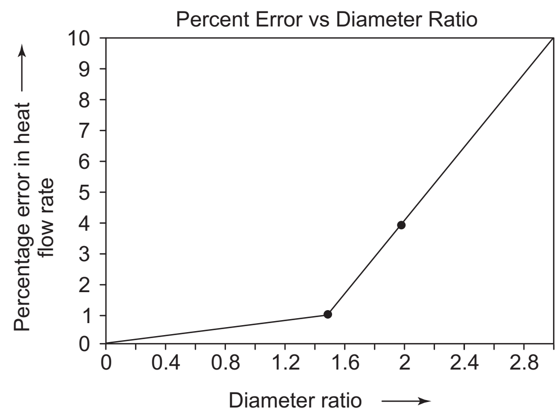
For a ratio of outside to inside diameters of 1.5

$$\%\text{ error} = \left[1 - \frac{1}{2}\ln(1.5)\left(\frac{1.5+1}{1.5-1}\right)\right] \times 100 = -\,1.37\%$$

The percent errors for the other diameter ratios can be calculated in a similar manner with the following results

Diameter ratio	% Error
1.5	–1.37
2.0	–3.97
3.0	–9.86

(b)



COMMENTS

For diameter ratios less than 2, use of the arithmetic mean area will not introduce more than a 4% error.

PROBLEM 2.9

> **A 2.5-cm-*OD*, 2-cm-*ID* copper pipe carriers liquid oxygen to the storage site of a space shuttle at –183°C and 0.04 m^3/min. The ambient air is at 21°C and has a dew point of 10°C. How much insulation with a thermal conductivity of 0.02 W/(m K) is needed to prevent condensation on the exterior of the insulation if $h_c + h_r$ = 17 W/(m^2 K) on the outside?**

GIVEN

- Insulated copper pipe carrying liquid oxygen
- Inside diameter (D_i) = 2 cm = 0.02 m
- Outside diameter (D_o) = 2.5 cm = 0.025 m
- LOX temperature (T_{ox}) = – 183°C
- LOX flow rate (m_{ox}) = 0.04 m^3/min
- Thermal conductivity of insulation (k_i) = 0.02 W/(m K)
- Exterior heat transfer coefficients ($h_o = h_c + h_r$) = 17 W/(m^2 K)
- Ambient air temperature (T_∞) = 21°C
- Ambient air dew point (T_{dp}) = 10°C

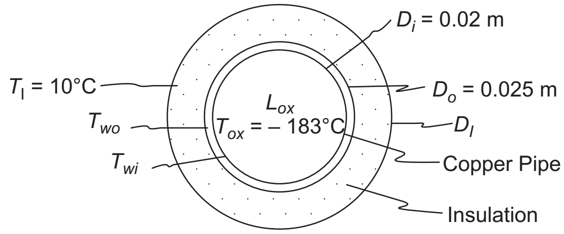
FIND

- Thickness of insulation (L) needed to prevent condensation

ASSUMPTIONS

- Steady-state conditions have been reached
- The thermal conductivity of the insulation does not vary appreciably with temperature
- Radial conduction only
- The thermal resistance between the inner surface of the pipe and the liquid oxygen is negligible, therefore $T_{wi} = T_{ox}$

SKETCH

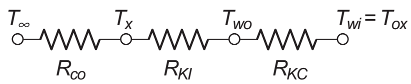


PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, thermal conductivity of copper (k_c) = 401 W/(m K) at 0°C

SOLUTION

The thermal circuit for the pipe is shown below



The rate of heat transfer from the pipe is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_\infty - T_{ox}}{\frac{1}{\bar{h}_o A_I} \frac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L k_I} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_c}}$$

The rate of heat transfer by convection and radiation from the outer surface of the pipe is

$$q = \frac{\Delta T}{R_o} = \frac{T_\infty - T_I}{\frac{1}{h_o A_i}}$$

Equating these two expressions

$$\frac{T_\infty - T_{ox}}{\frac{1}{\bar{h}_o A_I} + \frac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L k_I} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_c}} = \frac{T_\infty - T_I}{\frac{1}{h_o A_I}}$$

$$\frac{T_\infty - T_{ox}}{T_\infty - T_I} = \frac{\frac{1}{\bar{h}_o \pi D_I L} + \frac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L k_I} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_c}}{\frac{1}{h_o \pi D_I L}}$$

$$\frac{T_\infty - T_{ox}}{T_\infty - T_I} = 1 + \frac{h_o}{2} D_I \left(\frac{\ln\left(\frac{D_I}{D_o}\right)}{k_I} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{k_c} \right)$$

$$D_I \left(\frac{\ln D_I}{k_I} + \frac{\ln D_o}{k_I} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{k_c} \right) = \frac{2}{h_o} \left(\frac{T_\infty - T_{ox}}{T_\infty - T_I} - 1 \right)$$

$$D_I \left(\frac{\ln D_I}{0.02\,\text{W/(m K)}} + \frac{\ln(0.025)}{0.02\,\text{W/(m K)}} + \frac{\ln\frac{0.025}{0.02}}{401\,\text{W/(m K)}} \right) = \frac{2}{17\,\text{W/(m}^2\,\text{K)}}$$

$$\left(\frac{21^\circ\text{C} - (183^\circ\text{C})}{21^\circ\text{C} - 10^\circ\text{C}} - D_I \right) \left(\frac{\ln D_I}{0.02} + 184.4 + 0.00056 \right) = 2.064\ (\text{m}^2\ \text{K})/\text{W}$$

Solving this by trial and error

$$D_I = 0.054\ \text{m} = 5.4\ \text{cm}$$

Therefore, the thickness of the insulation is

$$L = \frac{D_I - D_o}{2} = \frac{5.4\,\text{cm} - 2.5\,\text{cm}}{2} = 1.5\ \text{cm}$$

COMMENTS

Note that the thermal resistance of the copper pipe is negligible compared to that of the insulation.

PROBLEM 2.10

A salesman for insulation material claims that insulating exposed steam pipes in the basement of a large hotel will be cost effective. Suppose saturated steam at 5.7 bars flows through a 30 cm *OD* steel pipe with a 3 cm wall thickness. The pipe is surrounded by air at 20°C. The convective heat transfer coefficient on the outer surface of the pipe is estimated to be 25 W/(m^2 K). The cost of generating steam is estimated to be \$5 per 10^9 J and the salesman offers to install a 5 cm thick layer of 85% magnesia insulation on the pipes for \$200/m or a 10 cm thick layer for \$300/m. Estimate the payback time for these two alternatives assuming that the steam line operates all year long and make a recommendation to the hotel owner. Assume that the surface of the pipe as well as the insulation have a low emissivity and radiative heat transfer is negligible.

GIVEN

- Steam pipe in a hotel basement
- Pipe outside diameter (D_o) = 30 cm = 0.3 m
- Pipe wall thickness (L_s) = 3 cm = 0.03 m
- Surrounding air temperature (T_∞) = 20°C
- Convective heat transfer coefficient (h_c) = 25 W/(m^2 K)
- Cost of steam = \$5/10^9 J
- Insulation is 85% magnesia

FIND

Payback time for

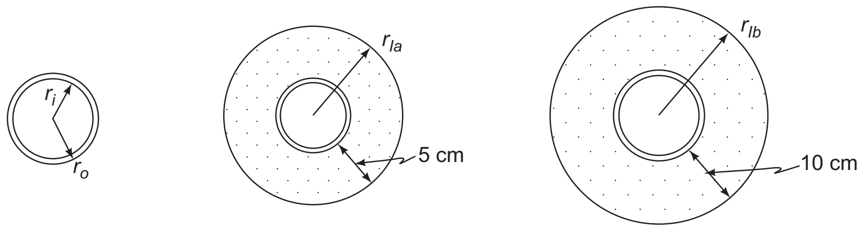
(a) Insulation thickness (L_{Ia}) = 5 cm = 0.05 m; Cost = \$200/m
(b) Insulation thickness (L_{Ib}) = 10 cm = 0.10 m; Cost = \$300/m

Make a recommendation to the hotel owner.

ASSUMPTIONS

- The pipe and insulation are black (ε = 1.0)
- The convective resistance on the inside of the pipe is negligible, therefore the inside pipe surface temperature is equal to the steam temperature
- The pipe is made of 1% carbon steel
- Constant thermal conductivities

SKETCH



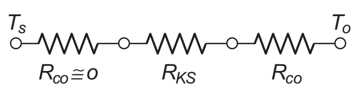
Uninsulated Pipe | (a) 5 cm Thick Insulation | (a) 10 cm Thick Insulation

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Appendix 2, Table 10 and 11
Thermal conductivities: 1% Carbon Steel (k_s) = 43 W/(m K) at 20°C
85% Magnesia (k_I) = 0.059 W/(m K) at 20°C
From Appendix 2, Table 13
The temperature of saturated steam at 5.7 bars (T_s) = 156°C

SOLUTION

The rate of heat loss and cost of the uninsulated pipe will be calculated first.
The thermal circuit for the uninsulated pipe is shown below



Evaluating the individual resistances

$$R_{ks} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{0.15}{0.12}\right)}{2\pi[43\,\text{W/(m K)}]} = \frac{1}{L}\ 0.000826\ \text{(m K)/W}$$

$$R_{co} = \frac{1}{\bar{h}_c A_o} = \frac{1}{\bar{h}_c 2\pi r_o L} = \frac{1}{[25\,\text{W/(m}^2\ \text{K)}]2\pi(0.15\,\text{m})L} = \frac{1}{L}\ 0.0424\ \text{(m K)/W}$$

The rate of heat transfer for the uninsulated pipe is

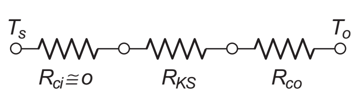
$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{ks}} + R_{\text{co}}}$$

$$\therefore \frac{q}{L} = \frac{156^\circ\text{C} - 20^\circ\text{C}}{(0.000826 + 0.0424)\,(\text{K m})/\text{W}} = 3148\ \text{W/m}$$

The cost to supply this heat loss is

cost = (3148 w/m) (J/W s) (3600 s/h) (24 h/day) (365 days/yr) ($5/109J) = $496/(yr m)

For the insulated pipe the thermal circuit is



The resistance of the insulation is given by

$$R_{kIa} = \frac{\ln\left(\frac{r_{Ia}}{r_o}\right)}{2\pi L k_I} = \frac{\ln\left(\frac{0.2}{0.15}\right)}{2\pi[0.059\,\text{W/(mK)}]} = \frac{1}{L}\ 0.776\ \text{(m K)/W}$$

$$R_{kIb} = \frac{\ln\left(\frac{r_{Io}}{r_o}\right)}{2\pi L k_I} = \frac{\ln\left(\frac{0.25}{0.15}\right)}{2\pi[0.059\,\text{W/(mK)}]} = \frac{1}{L}\ 1.378\ \text{(m K)/W}$$

(a) The rate of heat transfer for the pipe with 5 cm of insulation is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{ks} + R_{kIa} + R_{co}}$$

$$\therefore \frac{q}{L} = \frac{156^{\text{o}}\text{C} - 20^{\text{o}}\text{C}}{(0.000826 + 0.776 + 0.0424)\,\text{(Km)/W}} = 166\ \text{W/m}$$

The cost of this heat loss is

cost = (166 w/m) (J/W s) (3600 s/h) (24 h/day) (365 days/yr) ($\$5/10^9$J) = \$26/yr m

Comparing this cost to that of the uninsulated pipe we can calculate the payback period

$$\text{Payback period} = \frac{\text{Cost of installation}}{\text{uninsulated cost} - \text{insulated cost}} = \frac{\$200/\text{m}}{\$496/(\text{yr m}) - \$26/(\text{yr m})}$$

Payback period = 0.43 yr = 5 months

(b) The rate of heat loss for the pipe with 10 cm of insulation is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{ks} + R_{kIb} + R_{co}}$$

$$\therefore \frac{q}{L} = \frac{156^{\text{o}}\text{C} - 20^{\text{o}}\text{C}}{(0.000826 + 1.378 + 0.0424)\,\text{(Km)/W}} = 95.7\ \text{W/m}$$

The cost of this heat loss

cost= (95.7 w/m) (J/W s) (3600 s/h) (24 h/day) (365 days/yr) ($\$5/10^9$ J) = \$15/yr m

Comparing this cost to that of the uninsulated pipe we can calculate the payback period

$$\text{Payback period} = \frac{\$300/\text{m}}{\$496/\text{yr m} - \$15/\text{yr m}} = 0.62\ \text{yr} = 7.5\ \text{months}$$

COMMENTS

The 5 cm insulation is a better economic investment. The 10 cm insulation still has a short payback period and is the superior environmental investment since it is a more energy efficient design. Moreover, energy costs are likely to increase in the future and justify the investment in thicker insulation.

PROBLEM 2.11

A hollow sphere with inner and outer radii of R_1 and R_2, respectively, is covered with a layer of insulation having an outer radius of R_3. Derive an expression for the rate of heat transfer through the insulated sphere in terms of the radii, the thermal conductivities, the heat transfer coefficients, and the temperatures of the interior and the surrounding medium of the sphere.

GIVEN

- An insulated hollow sphere
- Radii
  - Inner surface of the sphere = R_1
  - Outer surface of the sphere = R_2
  - Outer surface of the insulation = R_3

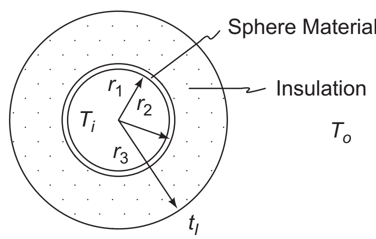
FIND

- Expression for the rate of heat transfer

ASSUMPTIONS

- Steady state heat transfer
- Conduction in the radial direction only
- Constant thermal conductivities

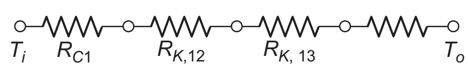
SKETCH



SOLUTION

Let k_{12} = the thermal conductivity of the sphere
k_{23} = the thermal conductivity of the insulation
h_1 = the interior heat transfer coefficient
h_3 = the exterior heat transfer coefficient
T_i = the temperature of the interior medium
T_o = the temperature of the exterior medium

The thermal circuit for the sphere is shown below



The individual resistances are

$$R_{c1} = \frac{-1}{h_1 A_1} = \frac{1}{\bar{h}_1 4\pi R_1^2 L}$$

From Equation (2.48)

$$R_{k12} = \frac{R_2 - R_1}{4\pi k_{12} R_2 R_1}$$

$$R_{k23} = \frac{R_3 - R_2}{4\pi k_{23} R_3 R_2}$$

$$R_{c3} = \frac{1}{\bar{h}_3 A_3} = \frac{1}{\bar{h}_3 4\pi R_3^2 L}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{c1} + R_{k12} + R_{k23} + R_{c3}}$$

$$q = \frac{\Delta T}{\frac{1}{4\pi}\left(\frac{1}{R_1^2 h_1} + \frac{R_2 - R_1}{k_{12} R_2 R_1} + \frac{R_3 - R_2}{k_{23} R_3 R_2} + \frac{1}{R_3^2 h_3}\right)}$$

$$q = \frac{4\pi \Delta T}{\frac{1}{R_1^2 h_1} + \frac{R_2 - R_1}{k_{12} R_2 R_1} + \frac{R_3 - R_2}{k_{23} R_3 R_2} + \frac{1}{R_3^2 h_3}}$$

PROBLEM 2.12

The thermal conductivity of a material may be determined in the following manner. Saturated steam 2.41×10^5 N/m^2 is condensed at the rate of 0.68 kg/h inside a hollow iron sphere that is 1.3 cm thick and has an internal diameter of 51 cm. The sphere is coated with the material whose thermal conductivity is to be evaluated. The thickness of the material to be tested is 10 cm and there are two thermocouples embedded in it, one 1.3 cm from the surface of the iron sphere and one 1.3 cm from the exterior surface of the system. If the inner thermocouple indicates a temperature of 110°C and the outer themocouple a temperature of 57°C, calculate (a) the thermal conductivity of the material surrounding the metal sphere, (b) the temperatures at the interior and exterior surfaces of the test material, and (c) the overall heat transfer coefficient based on the interior surface of the iron sphere, assuming the thermal resistances at the surfaces, as well as the interface between the two spherical shells, are negligible.

GIVEN

- Hollow iron sphere with saturated steam inside and coated with material outside
- Steam pressure = 2.41×10^5 N/m^2
- Steam condensation rate ($\dot{m}_s$) = 0.68 kg/h
- Inside diameter (D_i) = 51 cm = 0.51 m
- Thickness of the iron sphere (Ls) = 1.3 cm = 0.013 m
- Thickness of material layer (L_m) = 10 cm = 0.1 m
- Two thermocouples are located 1.3 cm from the inner and outer surface of the material layer
- Inner thermocouple temperature (T_1) = 110°C
- Outer thermocouple temperature (T_2) = 57°C

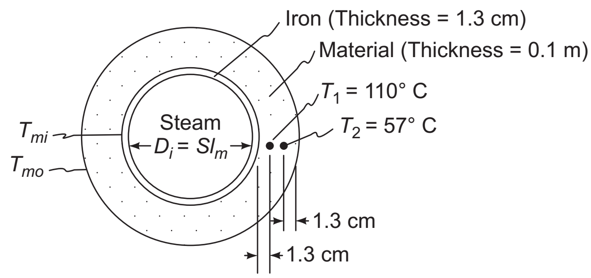
FIND

(a) Thermal conductivity of the material (k_m)
(b) Temperatures at the interior and exterior surfaces of the test material (T_{mi}, T_{mo})
(c) Overall heat transfer coefficient based on the inside area of the iron sphere (U)

ASSUMPTIONS

- Thermal resistance at the surface is negligible
- Thermal resistance at the interface is negligible
- The system has reached steady-state
- The thermal conductivities are constant
- One dimensional conduction radially

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13: For saturated steam at 2.41×10^5 N/m^2,
Saturation temperature (T_s) = 125°C
Heat of vaporization (h_{fg}) = 2187 kJ/kg

SOLUTION

(a) The rate of heat transfer through the sphere must equal the energy released by the condensing steam:

$$q = \dot{m}_s h_{fg} = 0.68 \text{ kg/h}(2187 \text{ kJ/kg})(1000 \text{ J/kJ})\left(\frac{\text{h}}{3600\text{s}}\right)((\text{Ws})/\text{J}) = 413.1 \text{ W}$$

The thermal conductivity of the material can be calculated by examining the heat transfer between the thermocouple radii

$$q = \frac{\Delta T}{R_{k12}} = \frac{T_2 - T_1}{\left(\dfrac{r_2 - r_1}{4\pi k_m r_2 r_1}\right)}$$

Solving for the thermal conductivity

$$k_m = \frac{q(r_2 - r_1)}{4\pi r_2 r_1 (T_2 - T_1)}$$

$$r_1 = \frac{D_i}{2} + L_s + 0.013 \text{ m} = \frac{0.51\text{m}}{2} + 0.013 \text{ m} + 0.013 \text{ m} = 0.281 \text{ m}$$

$$r_2 = \frac{D_i}{2} + L_s + L_m - 0.013 \text{ m} = \frac{0.51\text{m}}{2} + 0.013 \text{ m} + 0.1 \text{ m} - 0.013 \text{ m} = 0.355$$

$$k_m = \frac{413.1\text{W}(0.355 \text{ m} - 0.281 \text{ m})}{4\pi(0.355\text{m})(0.281\text{m})(110^{\circ}\text{C} - 57^{\circ}\text{C})} = 0.46 \text{ W/(m K)}$$

(b) The temperature at the inside of the material can be calculated from the equation for conduction through the material from the inner radius, the radius of the inside thermocouple

$$q = \frac{\Delta T}{R_{ki1}} = \frac{T_{mi} - T_i}{\left(\dfrac{r_1 - r_i}{4\pi k_m r_1 r_i}\right)}$$

Solving for the temperature of the inside of the material

$$T_{mi} = T_1 + \frac{q(r_1 = r_i)}{4\pi k_m r_1 r_i}$$

$$r_i = \frac{D_i}{2} + L_m = \frac{0.51\text{ m}}{2} + 0.013\text{ m} = 0.268\text{ m}$$

$$T_{mi} = 110°\text{C} + \frac{413.1\,\text{W}\,(0.013\,\text{m})}{4\pi[0.46\,\text{W/(mK)}](0.281\text{ m})(0.268\,\text{m})} = 122°\text{C}$$

The temperature at the outside radius of the material can be calculated from the equation for conduction through the material from the radius of the outer thermocouple to the outer radius

$$q = \frac{\Delta T}{R_{k2o}} = \frac{T_2 - T_{mo}}{\left(\dfrac{r_o - r_2}{4\pi k_m r_o r_2}\right)}$$

Solving for the temperature of the outer surface of the material

$$T_{mo} = T_2 - \frac{q(r_o - r_2)}{4\pi k_m r_o r_2}$$

$$r_o = \frac{D_i}{2} + L_s + L_m = \frac{0.51\text{ m}}{2} + 0.013\text{ m} + 0.01\text{ m} = 0.368\text{ m}$$

$$T_{mo} = 57°\text{C} - \frac{413.1\,\text{W}\,(0.013\,\text{m})}{4\pi[0.46\,\text{W/(mK)}](0.368\text{ m})(0.355\,\text{m})} = 50°\text{C}$$

(c) The heat transfer through the sphere can be expressed as

$$q = U A_i \Delta T = U \pi D_1^2 (T_s - T_{mo})$$

$$\therefore U = \frac{q}{\pi D_i^2 (T_s - T_{mo})} = \frac{413.1\text{ W}}{\pi (0.51\text{m})^2 (125°\text{C} - 50°\text{C})} = 6.74\text{ W/(m}^2\text{ K)}$$

PROBLEM 2.13

A cylindrical liquid oxygen (LOX) tank has a diameter of 1.22 m, a length of 6.1 m, and hemispherical ends. The boiling point of LOX is – 179.4°C. An insulation is sought which will reduce the boil-off rate in the steady state to no more than 11.3 kg/hr. The heat of vaporization of LOX is 214 kJ/kg. If the thickness of this insulation is to be no more than 7.5 cm, what would the value of it's thermal conductivity have to be?

GIVEN

- Insulated cylindrical tank with hemispherical ends filled with LOX
- Diameter of tank (D_t) = 1.22 m
- Length of tank (L_t) = 6.1 m
- Boiling point of LOX (T_{bp}) = – 179.4°C

- Heat of vaporization of LOX (h_{fg}) = 214 kJ/kg
- Steady state boil-off rate ($\dot{m}$) = 11.3 kg/hr = 3.14 × 10^{-3} kg/s
- Maximum thickness of insulation (L) = 7.5 cm = 0.075 m
- Outside temperature = 21°C

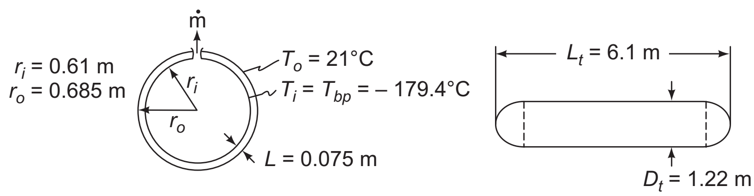
FIND

- The thermal conductivity (k) of the insulation necessary to maintain the boil-off rate below 25 lb/h.

ASSUMPTIONS

- The length given includes the hemispherical ends
- The thermal resistance of the tank is negligible compared to the insulation
- The thermal resistance at the interior surface of the tank is negligible

SKETCH



SOLUTION

The tank can be thought of as a sphere (the ends) separated by a cylindrical section, therefore the total heat transfer is the sum of that through the spherical and cylindrical sections. The steady state conduction through a spherical shell with constant thermal conductivity, from Equation (2.47), is

$$q_s = \frac{4\pi K\, r_o r_i\,(T_o - T_i)}{r_o - r_i}$$

The rate of steady state conduction through a cylindrical shell, from Equation (2.37), is

$$q_c = 2\,\pi L_c\, k\, \frac{T_o - T_i}{\ln\left(\frac{r_o}{r_i}\right)} \qquad (L_c = 6.1 - 1.22 = 4.88 \text{ m})$$

The total heat transfer through the tank is the sum of these

$$q = q_s + q_c = \frac{4\pi k\, r_o r_i\,(T_o - T_i)}{r_o - r_i} + 2\,\pi L_c\, k\, \frac{(T_o - T_i)}{\ln\left(\frac{r_o}{r_i}\right)} = 2\,\pi k\,(T_o - T_i)\left[\frac{2 r_o r_i}{r_o - r_i} + \frac{L_c}{\ln\left(\frac{r_o}{r_i}\right)}\right]$$

The rate of heat transfer required to evaporate the liquid oxygen at m is $m\, h_{\text{fg}}$, therefore

$$\dot{m}_s h_{fg} = 2\,\pi k\,(T_o - T_i)\left[\frac{2 r_o r_i}{r_o - r_i} + \frac{L_c}{\ln\frac{r_o}{r_i}}\right]$$

$$\therefore\ k = \frac{\dot{m} h_{fg}}{2\pi k(T_o - T_i)\left[\frac{2 r_o r_i}{r_o - r_i} + \frac{L_c}{\ln\frac{r_o}{r_i}}\right]}$$

$$k = \frac{(3.14 \times 10^{-3}\ \text{kg/s})(214\,\text{kJ/kg}) \times \dfrac{1000\,\text{J}}{\text{kJ}}}{2\pi[21-(-179.4)](\text{K})\left[\dfrac{2(0.685\,\text{m})(0.61\,\text{m})}{0.075\,\text{m}} + \dfrac{4.88\,\text{m}}{\ln\left(\dfrac{6.685}{0.61}\right)}\right]}$$

$$\Rightarrow \qquad k = \frac{672\,\text{W}}{2\pi(200.4\,\text{K})[11.14 + 42.06]\,\text{m}}$$

$$\Rightarrow \qquad k = 0.01\ \text{W/(m K)}$$

COMMENTS

Based on data given in Appendix 2, Table 11, no common insulation has such low value of thermal conductivity. However, *Marks Standard Handbook for Mechanical Engineers* lists the thermal conductivity of expanded rubber board, 'Rubatex', at – 179.4°C to be 0.007 W/(m K).

PROBLEM 2.14

The addition of insulation to a cylindrical surface, such as a wire, may increase the rate of heat dissipation to the surroundings (see Problem 2.4). (a) For a No. 10 wire (0.26 cm in diameter), what is the thickness of rubber insulation [*k* = 0.16 W/(m K)] that will maximize the rate of heat loss if the heat transfer coefficient is 10 W/(m^2 K)? (b) If the current-carrying capacity of this wire is considered to be limited by the insulation temperature, what percent increase in capacity is realized by addition of the insulation? State your assumptions.

GIVEN

- An insulated cylindrical wire
- Diameter of wire (D_w) = 0.26 cm = 0.0026 m
- Thermal conductivity of rubber (k) = 0.16 W/(m K)
- Heat transfer coefficient ($\bar{h}_c$) = 10 W/(m^2 K)

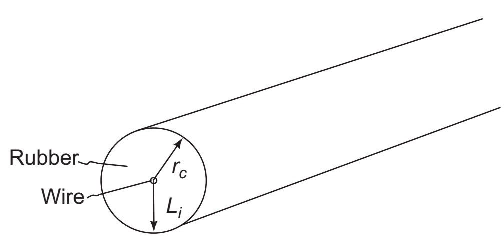
FIND

(a) Thickness of insulation (L_i) to maximize heat loss
(b) Percent increase in current carrying capacity

ASSUMPTIONS

- The system is in steady state
- The thermal conductivity of the rubber does not vary with temperature

SKETCH



SOLUTION

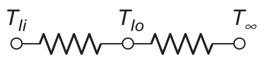
(a) From Problem 2.4, the radius that will maximize the rate of heat transfer (r_c) is:

$$r_c = \frac{k}{h} = \frac{0.16\,\text{W/(mK)}}{10\,\text{W/(m}^2\,\text{K)}} = 0.016\ \text{m}$$

The thickness of insulation needed to make this radius is

$$L_i = r_c - r_w = 0.016\ \text{m} - \frac{0.0026\,\text{m}}{2} = 0.015\ \text{m} = 1.5\ \text{cm}$$

(b) The thermal circuit for the insulated wire is shown below



where $$R_{kI} = \frac{\ln\left(\dfrac{r_o}{r_i}\right)}{2\pi Lk} \quad \text{and } R_c = \frac{1}{\bar{h}_c A} = \frac{1}{\bar{h}_c 2\pi r_o L}$$

The rate of heat transfer from the wire is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{Ii} - T_\infty}{R_{kI} + R_c} = \frac{2\pi L (T_{Ii} - T_\infty)}{\dfrac{\ln\left(\dfrac{r_o}{r_i}\right)}{k} + \dfrac{1}{\bar{h}_c r_o}}$$

If only a very thin coat of insulation is put on the wire to insulate it electrically then $r_o = r_i = D_w/2$ = 0.0013 m. The rate of heat transfer from the wire is

$$\frac{q}{L} = \frac{2\pi(T_{Ii} - T_\infty)}{0 + \dfrac{1}{10\,\text{W/(m}^2\text{K)}(0.0013\,\text{m})}} = 0.082\ (T_{Ii} - T_\infty)$$

For the wire with the critical insulation thickness

$$\frac{q}{L} = \frac{2\pi(T_{Ii} - T_\infty)}{\dfrac{\ln\left(\frac{0.016}{0.0013}\right)}{10\,\text{W/(m K)}} + \dfrac{1}{10\,\text{W/(m}^2\text{K)}(0.016\,\text{m})}} = 0.286\ (T_{Ii} - T_\infty)$$

The current carrying capacity of the wire is directly related to the rate of heat transfer from the wire. For a given maximum allowable insulation temperature, the increase in current carrying capacity of the wire with the critical thickness of insulation over that of the wire with a very thin coating of insulation is

$$\%\ \text{increase} = \frac{\left(\dfrac{q}{L}\right)_{r_a} - \left(\dfrac{q}{L}\right)_{\text{thin coat}}}{\left(\dfrac{q}{L}\right)_{\text{thin coat}}} \times 100 = \frac{0.286 - 0.082}{0.082} \times 100 = 250\%$$

COMMENTS

This would be an enormous amount of insulation to add to the wire changing a thin wire into a rubber cable over an inch in diameter and would not be economically justifiable. Thinner coatings of rubber will achieve smaller increases in current carrying capacity.

PROBLEM 2.15

For the system outlined in Problem 2.11, determine an expression for the critical radius of the insulation in terms of the thermal conductivity of the insulation and the surface coefficient between the exterior surface of the insulation and the surrounding fluid. Assume that the temperature difference, R_1, R_2, the heat transfer coefficient on the interior, and the thermal conductivity of the material of the sphere between R_1 and R_2 are constant.

GIVEN

- An insulated hollow sphere
- Radii
  - Inner surface of the sphere = R_1
  - Outer surface of the sphere = R_2
  - Outer surface of the insulation = R_3

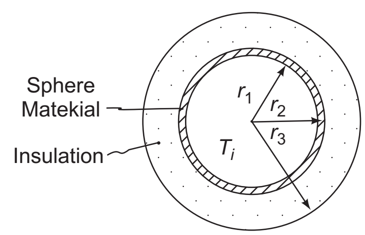
FIND

- An expression for the critical radius of the insulation

ASSUMPTIONS

- Constant temperature difference, radii, heat transfer coefficients, and thermal conductivities
- Steady state prevails

SKETCH



SOLUTION

Let
k_{12} = the thermal conductivity of the sphere
k_{23} = the thermal conductivity of the insulation
h_1 = the interior heat transfer coefficient
h_3 = the exterior heat transfer coefficient
T_i = the temperature of the interior medium
T_o = the temperature of the exterior medium

From Problem 2.11, the rate of heat transfer through the sphere is

$$q = \frac{4\pi \Delta T}{\dfrac{1}{R_1^2 h_1} + \dfrac{R_2 - R_1}{k_{12} R_2 R_1} + \dfrac{R_3 - R_2}{k_{23} R_3 R_2} + \dfrac{1}{R_3^2 h_3}}$$

The rate of heat transfer is a maximum when the denominator of the above equation is a minimum. This occurs when the derivative of the denominator with respect to R_3 is zero

$$\frac{d}{dR_3} = \left(\frac{1}{R_1^2 h_1} + \frac{R_2 - R_1}{k_{12} R_2 R_1} + \frac{R_3 - R_2}{k_{23} R_3 R_2} + \frac{1}{R_3^2 h_3} \right) = 0$$

$$-\frac{2}{h_3 R_3}+\frac{1}{k_{23}}=0$$

$$R_3=\frac{2k_{23}}{h_3}$$

The maximum heat transfer will occur when the outer insulation radius is equal to 2 k_{23}/h_3.

COMMENTS

A more realistic analysis should take the dependence of h_c on temperature into account. Such an analysis was made for a pipe by Sparrow and Kang, Int. J. Heat Mass Transf., 28: 2049-2060, 1985.

PROBLEM 2.16

A standard 10 cm steel pipe (*ID* = 10.066 cm, *OD* = 11.25 cm) carries superheated steam at 650°C in an enclosed space where a fire hazard exists, limiting the outer surface temperature to 38°C. In order to minimize the insulation cost, two materials are to be used; first a high temperature insulation (relatively expensive) applied to the pipe and then magnesia (a less expensive material) on the outside. The maximum temperature of the magnesia is to be 315°C. The following constants are known.

Steam-side coefficient	**h = 500 W/(m^2 K)**
High-temperature insulation conductivity	**k = 0.1 W/(m K)**
Magnesia conductivity	**k = 0.076 W/(m K)**
Outside heat transfer coefficient	**h = 11 W/(m^2 K)**
Steel conductivity	**k = 43 W/(m K)**
Ambient temperature	**T_a = 21°C**

(a) Specify the thickness for each insulating material.
(b) Calculate the overall heat transfer coefficient based on the pipe *OD*.
(c) What fraction of the total resistance is due to (1) steam-side resistance, (2) steel pipe resistance, (3) insulation (combination of the two), and (4) outside resistance?
(d) How much heat is transferred per hour, per foot length of pipe?

GIVEN

- Steam filled steel pipe with two layers of insulation
- Pipe inside diameter (D_i) = 10.066 cm
- Pipe outside diameter (D_o) = 11.25 cm
- Superheated steam temperature (T_s) = 650°C
- Maximum outer surface temperature (T_{so}) = 38°C
- Maximum temperature of the Magnesia (T_m) = 315°C
- Thermal conductivities
  - High-temperature insulation (k_h) = 0.1 W/(m K)
  - Magnesia (k_m) = 0.076 W/(m K)
  - Steel (k_s) = 43 W/(m K)
- Heat transfer coefficients
  - Steam side ($\bar{h}_{ci}$) = 500 W/(m^2 K)
  - Outside ($\bar{h}_{co}$) = 11 W/(m^2 K)
- Ambient temperature (T_a) = 21°C

FIND

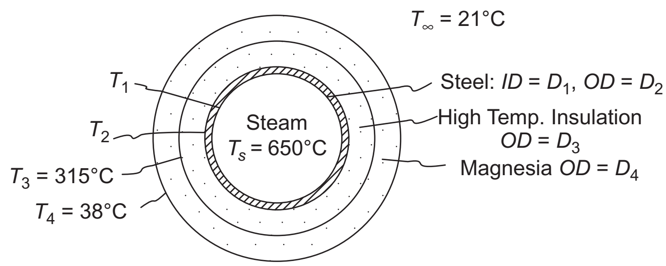
(a) Thickness for each insulation material
(b) Overall heat transfer coefficient based on the pipe *OD*
(c) Fraction of the total resistance due to
- Steam-side resistance
- Steel pipe resistance
- Insulation
- Outside resistance

(d) The rate of heat transfer per unit length of pipe (q/L)

ASSUMPTIONS

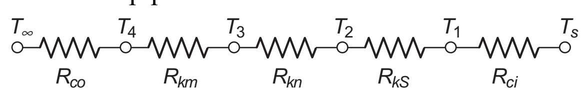
- The system is in steady state
- Constant thermal conductivities
- Contact resistance is negligible

SKETCH



SOLUTION

The thermal circuit for the insulated pipe is shown below



The values of the individual resistances can be evaluated with Equations (1.14) and (2.39)

$$R_{co} = \frac{1}{\bar{h}_{co} A_o} = \frac{1}{\bar{h}_{co}\, 2\pi r_4 L}$$

$$R_{km} = \frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi L k_m}$$

$$R_{kh} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi L k_h}$$

$$R_{ks} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k_s}$$

$$R_{ci} = \frac{1}{\bar{h}_{ci} A_i} = \frac{1}{\bar{h}_{ci}\, 2\pi r_1 L}$$

The variables in the above equations are

r_1 = 5.035 cm

r_2 = 5.625 cm

r_3 = ?

r_4 = ?

k_m = 0.076 W/(m K)

k_s = 43 W/(m K)

k_h = 0.1 W/(m K)

$\bar{h}_{co}$ = 11 W/(m^2 K)

$\bar{h}_{ci}$ = 500 W/(m^2 K)

The temperatures for this problem are

T_s = 650°C

T_1 = ?

T_2 = ?

T_3 = 315°C

T_4 = 38°C

T_a = 21°C

There are five unknowns in this problem: q/L, T_1, T_2, r_3, and r_4. These can be solved for by writing the equation for the heat transfer through each of the five resistances and solving them simultaneously.

1. Steam side convective heat transfer

$$q = \frac{\Delta T}{R_{ci}} = 2\,\pi\,\overline{h_{ci}}\;r_1\,L\,(T_s - T_1) = 2\,\pi L\,(500\text{ W/(m}^2\text{K))}\,(5.035\times 10^{-2}\text{ m})\,(650 - T_1)°\text{C}$$

$$\Rightarrow \quad \frac{q}{L} = 102760 - 158.1 T_1 \text{ W/m} \qquad (1)$$

2. Conduction through the pipe wall

$$q = \frac{\Delta T}{R_{ks}} = \frac{2\pi k_s L}{\ln\left(\frac{r_2}{r_1}\right)}\,(T_1 - T_2) = \frac{2\pi L\,(43\text{ W/(mK)})}{\ln\left(\frac{5.625\text{ cm}}{5.035\text{ cm}}\right)}\,(T_1 - T_2)$$

$$\frac{q}{L} = 2436\,(T_1 - T_2)\text{ W/m} \qquad (2)$$

3. Conduction through the high temperature insulation

$$q = \frac{\Delta T}{R_{kh}} = \frac{2\pi k_h L}{\ln\left(\frac{r_1}{r_2}\right)}\,(T_2 - T_3) = \frac{2\pi L\,(0.1\text{ W/(m K)})}{\ln\left(\frac{r_3\text{ cm}}{5.625\text{ cm}}\right)}\,(T_2 - 315)$$

$$\Rightarrow \quad \frac{q}{L} = \frac{0.628(T_2 - 315)}{\ln\left(\frac{r_3\text{ cm}}{5.625\text{ cm}}\right)}\text{ W/m} \qquad (3)$$

4. Conduction through the magnesia insulation

$$q = \frac{\Delta T}{R_{km}} = \frac{2\pi k_m L}{\ln\left(\frac{r_4}{r_3}\right)} (T_3 - T_4) = \frac{2\pi L(0.076\text{ W/(m K)})}{\ln\left(\frac{r_4}{r_3}\right)} (315 - 38)$$

$$\frac{q}{L} = \frac{132.2}{\ln\left(\frac{r_4}{r_3}\right)} \text{ W/m} \qquad (4)$$

5. Air side convective heat transfer

$$q = \frac{\Delta T}{R_{co}} = 2\pi \overline{h_{co}}\ r_4 L (T_4 - T_a) = 2\ \pi L\ r_4 (11\text{ W/(m}^2\text{ K)) } (38 - 21)\text{K}$$

$$\Rightarrow \qquad \frac{q}{L} = 377\ r_4 \text{ Btu/(h ft)} \qquad (5)$$

To maintain steady state, the heat transfer rate through each resistance must be equal. Equations [1] through [5] are a set of five equations with five unknowns, they may be solved through numerical iterations using a simple program or may be combined algebraically as follows

Substituting Equation (1) into Equation (2) yields

$$T_2 = (1.065\ T_1 - 42.18)°\text{C}$$

Substituting Equation (1) into Equation (2) gives

$$102760 - 158.1\ T_1 = 2436\ (T_1 - T_2)$$

$$\Rightarrow \qquad 42.18 - 0.065\ T_1 = T_1 - T_2$$

$$\Rightarrow \qquad T_2 = 1.065\ T_1 - 42.18$$

Substituting this into Equation (3) and combining the result with Equation (1)

$$\ln\left(\frac{r_3\text{ cm}}{5.625\text{cm}}\right) = 0.628\ (1.065\ T_1 - 357.18)$$

LHS can be arranged as $\ln\left(\frac{r_3}{5.625}\right) = -\ln\left(\frac{r_4}{r_3}\right) + \ln\left(\frac{r_4}{5.625}\right)$

This gives

$$0.628\ (1.065\ T_1 - 357.18) = \frac{132.2}{158.1 T_1 - 102760} + \ln\frac{r_4}{5.625}$$

Thus

$$\ln\frac{r_4}{5.625} = 0.628\ (1.065\ T_1 - 357.18) - \frac{132.2}{102760 - 158.1 T_1} \qquad (6)$$

Again using r_4 from Equation (1) and (5), we get

$$r_4 = \frac{102760 - 158.1 T_1}{1175} \qquad (7)$$

Substituting Equation (7) into Equation (6) and solving by trial and error gives

$$T_1 \simeq 648°\text{C}$$

Then the unknown radii become (are solved)

$$r_3 = 11.43 \text{ cm}; r_4 = 17.96 \text{ cm}$$

The thickness of the high temperature insulation = $r_3 - r_2 = 5.8$ cm
The thickness of the magnesia insulation = $r_4 - r_3 = 6.5$ cm

(b) Substituting $T_1 = 648°C$, Equation (1) gives $\frac{q}{L} = 311$ W/m

Hence $$q = U A_2 (T_s - T_a) = U \pi D_2 L (T_s - T_a)$$

$$\therefore U = \frac{q}{L}\frac{1}{\pi D_2 (T_s - T_a)} = 311 \frac{1}{\pi (11.25 \times 10^{-2} \text{ cm})(650 - 21)}$$

$$\Rightarrow \quad U = 1.4 \text{ W/(m}^2\text{ K)}$$

(c) The overall resistance for the insulated pipe is

$$R_{\text{total}} = \frac{1}{UA_2} = \frac{1}{(1.4\,\text{W/(m}^2\text{K))}\,\pi\,(0.1125\,\text{m})\,L} = \frac{2.02}{L} \text{ K/W}$$

(4) The convective thermal resistance on the air side is

$$R_{co} = \frac{1}{\bar{h}_{co} A_o} = \frac{1}{\bar{h}_{co} 2\pi r_4 L} = \frac{1}{(11\,\text{W/(m}^2\text{K))}2\pi(0.1796)\,L} = \frac{0.08}{L} \text{ K/W}$$

The fraction of the resistance due to air side convection = $\frac{0.08}{2.02} = 0.04$.

(3) The thermal resistance of the magnesia insulation is

$$R_{km} = \frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi L k_m} = \frac{\ln\left(\frac{17.96}{11.43}\right)}{2\pi L\,(0.076\,\text{W/(m K)})} = \frac{0.95}{L} \text{ K/W}$$

The thermal resistance of the high temperature insulation is

$$R_{kh} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi L k_h} = \frac{\ln\left(\frac{11.43}{5.625}\right)}{2\pi L\,(0.1\,\text{W/(m K)})} = \frac{1.13}{L} \text{ K/W}$$

The fraction of the resistance due to the insulation = $\frac{0.95}{1.13} = 0.85$.

(2) The thermal resistance of the steel pipe is

$$R_{ks} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{5.625}{5.035}\right)}{2\pi L\,(43\,\text{W/(m K)})} = \frac{0.0004}{L} \text{ K/W}$$

The fraction of the resistance due to the steel pipe = $\frac{0.004}{2.02} \simeq 0.00$.

(1) The thermal resistance of the steam side convection is

$$R_{ci} = \frac{1}{\bar{h}_{ci} A_i} = \frac{1}{\bar{h}_{ci} 2\pi r_1 L} = \frac{1}{(500 \text{ W/(m}^2\text{K))}\,2\pi(5.035 \times 10^{-2})L} = \frac{0.0063}{L} \text{ K/W}$$

The fraction of the resistance due to steam side convection = $\frac{0.0063}{2.02} \simeq 0.00$.

(d) The rate of heat transfer is

$$q = U A_2 (T_s - T_a) = U \pi D_2 L (T_s - T_a)$$

$$\frac{q}{L} = 1.4 \text{ W/(m}^2\text{ K) } 2\pi (5.625 \times 10^{-2}) (650 - 21) = 311 \text{ W/m}$$

COMMENTS

Notice that the insulation accounts for 97% of the total thermal resistance and that the thermal resistance of the steel pipe and the steam side convection are negligible.

PROBLEM 2.17

Show that the rate of heat conduction per unit length through a long hollow cylinder of inner radius r_i and outer radius r_o, made of a material whose thermal conductivity varies linearly with temperature, is given by

$$\frac{q_k}{L} = \frac{T_i - T_o}{(r_o - r_i)/k_m \bar{A}}$$

where T_i **= temperature at the inner surface**

T_o **= temperature at the outer surface**

$\bar{A} = 2\pi (r_o - r_i)/\ln\left(\frac{r_o}{r_i}\right)$

$k_m = k_o [1 + \beta_k (T_i + T_o)/2]$

L **= length of cylinder**

GIVEN

- A long hollow cylinder
- The thermal conductivity varies linearly with temperature
- Inner radius = r_i
- Outer radius = r_o

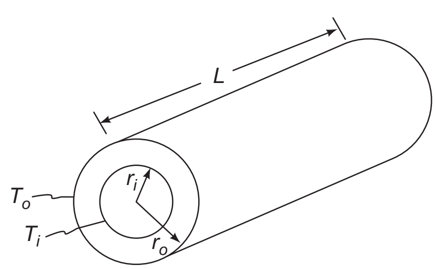
FIND

- Show that the rate of heat conduction per unit length is given by the above equation

ASSUMPTIONS

- Conduction occurs in the radial direction only
- Steady state prevails

SKETCH



SOLUTION

The rate of radial heat transfer through a cylindrical element of radius r is

$$\frac{q}{L} = k A \frac{dT}{dr} = k\, 2\pi r \frac{dT}{dr} = a \text{ constant}$$

But the thermal conductivity varies linearly with the temperature

$$k = k_o\,(1 + \beta\, T)$$

$$\therefore\ \frac{q}{L} = 2\pi r\, k_o\,(1 + \beta\, T)\,\frac{dT}{dr}$$

$$\frac{q}{L}\,\frac{1}{r}\,dr = 2\pi k_o\,(1 + \beta\, T)\,dT$$

Integrating between the inner and outer radii:

$$\frac{q}{L}\int_{r_i}^{r_o}\frac{1}{r}\,dr = 2\pi k_o\int_{T_i}^{T_o}(1+\beta T)dt$$

$$\frac{q}{L}\,(\ln r_o - \ln r_i) = 2\pi k_o\left[T_o + \frac{\beta}{2}T_o^2 - T_i - \frac{\beta}{2}T_i^2\right]$$

$$\frac{q}{L}\left(\ln\frac{r_o}{r_i}\right) = 2\pi k_o\left[(T_o - T_i) + \frac{\beta}{2}(T_o^2 - T_i^2)\right]$$

$$\frac{q}{L} = \left[\frac{2\pi(r_o - r_i)}{\ln\frac{r_o}{r_i}\,(r_o - r_i)}\right] k_o\,(T_o - T_i)\left[1 + \frac{\beta}{2}(T_o - T_i)\right]$$

$$\frac{q}{L} = \frac{\overline{A}}{(r_o - r_i)}\,k_m\,(T_o - T_i)$$

$$\frac{q}{L} = \frac{T_o - T_i}{\left(\frac{r_o - r_i}{k_m\overline{A}}\right)}$$

PROBLEM 2.18

A long, hollow cylinder is constructed from a material whose thermal conductivity is a function of temperature according to *k* = 0.15 + 0.0018 *T*, where *T* is in °C and *k* is in W/(m K). The inner and outer radii of the cylinder are 12.5 cm and 25 cm, respectively. Under steady-state conditions, the temperature at the interior surface of the cylinder is 427°C and the temperature at the exterior surface is 93°C.

(a) Calculate the rate of heat transfer per foot length, taking into account the variation in thermal conductivity with temperature. (b) If the heat transfer coefficient on the exterior surface of the cylinder is 17 W/(m^2 K), calculate the temperature of the air on the outside of the cylinder.

GIVEN

- A long hollow cylinder
- Thermal conductivity (k) = 0.15 + 0.0018 T [T in °C, k in W/(m K)]
- Inner radius (r_i) = 12.5 cm = 0.125 m
- Outer radius (r_o) = 25 cm = 0.25 m
- Interior surface temperature (T_{wi}) = 427°C
- Exterior surface temperature (T_{wo}) = 93°C
- Exterior heat transfer coefficient ($\overline{h}_o$) = 17 W/(m^2 K)
- Steady-state conditions

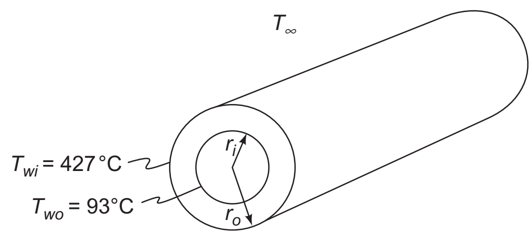
FIND

(a) The rate of heat transfer per foot length (q/L)
(b) The temperature of the air on the outside (T_∞)

ASSUMPTIONS

- Steady state heat transfer
- Conduction occurs in the radial direction only

SKETCH



SOLUTION

(a) The rate of radial conduction is given by Equation (2.37)

$$q = -k\,A\,\frac{dT}{dr}$$

$$q = -(0.15 + 0.0018\,T)\,2\pi r\,L\,\frac{dT}{dr}$$

$$\frac{1}{r}\,dr = \frac{2\pi L}{q}\,(0.15 + 0.0018\,T)\,dT$$

Integrating this from the inside radius to the outside radius

$$\int_{r_i}^{r_o}\frac{1}{r}\,dr = -\frac{2\pi L}{q}\int_{T_{wi}}^{T_{wo}}(0.15 + 0.0018T)dt$$

$$\Rightarrow \quad \ln\left(\frac{r_o}{r_i}\right) = -\frac{2\pi L}{q}\left[0.15(T_{wo} - T_{wi}) + \frac{0.0018}{2}(T_{wo}^2 - T_{wi}^2)\right]$$

$$\therefore \quad \frac{q}{L} = \frac{2\pi}{\ln\left(\frac{r_o}{r_i}\right)}\,[0.15\,(T_{wi} - T_{wo}) + 0.0009\,(T_{wi}^2 - T_{wo}^2)]$$

$$= \frac{2\pi}{\ln\left(\frac{25}{12.5}\right)}\,[0.15\,(427 - 93) + 0.0009\,(427^2 - 93^2)]$$

$$\Rightarrow \quad \frac{q}{L} = 9.06\,[50.1 + 156.3] = 1870\text{ W/m}$$

(b) The conduction through the hollow cylinder must equal the convection from the outer surface in steady state

$$\frac{q}{L} = \overline{h_o}\,A_o\,\Delta T = \overline{h_o}\,2\pi r_o\,(T_{wo} - T_\infty)$$

Solving for the air temperature

$$T_\infty = T_{wo} - \frac{q}{L}\frac{1}{\bar{h}_o 2\pi r_o}$$

$$= 93\text{ °C} - 1870\text{ W/m}\ \frac{1}{(17\text{ W/(m}^2\text{K))}\,2\pi(0.25\text{ m})}$$

$$= 93\text{°C} - 70\text{°C}$$

$$\Rightarrow \quad T_\infty = 23\text{°C}$$

PROBLEM 2.19

A plane wall 15 cm thick has a thermal conductivity given by the relation

$$\boldsymbol{k = 2.0 + 0.0005\ T\ \text{W/(m K)}}$$

where T is in degrees Kelvin. If one surface of this wall is maintained at 150 °C and the other at 50 °C, determine the rate of heat transfer per square meter. Sketch the temperature distribution through the wall.

GIVEN

- A plane wall
- Thickness (L) = 15 cm = 0.15 m
- Thermal conductivity (k) = 2.0 + 0.0005 T W/(m K) (with T in Kelvin)
- Surface temperatures: T_h = 150 °C T_c = 50 °C

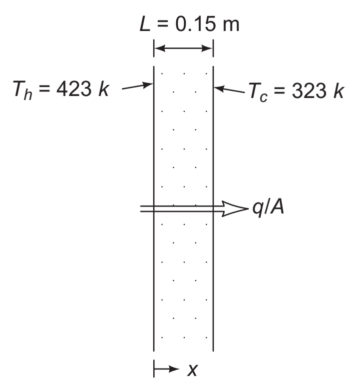
FIND

(a) The rate of heat transfer per square meter (q/A)
(b) The temperature distribution through the wall

ASSUMPTIONS

- The wall has reached steady state
- Conduction occurs in one dimension

SKETCH



SOLUTION

Simplifying Equation (2.2) for steady state conduction with no internal heat generation but allowing for the variation of thermal conductivity with temperature yields

$$\frac{d}{dx}k\frac{dT}{dx} = 0$$

with boundary conditions: $T = 423$ K at $x = 0$

$$T = 323 \text{ K at } x = 0.15 \text{ m}$$

The rate of heat transfer does not vary with x

$$-k\,\frac{dT}{dx} = \frac{q}{A} = \text{constant}$$

$$-(2.0 + 0.0005T)\,dT = \frac{q}{A}\,dx$$

Integrating

$$2.0T + 0.00025\,T^2 = -\frac{q}{A}\,x + C$$

The constant can be evaluated using the first boundary condition

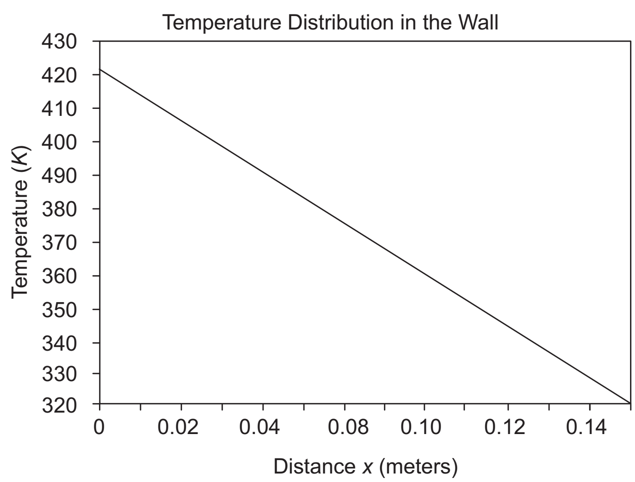
$$2.0\,(423) + 0.00025\,(423)^2 = C - \frac{q}{A}\,(0) \Rightarrow C = 890.7$$

(a) The rate of heat transfer can be evaluated using the second boundary condition:

$$2.0\,(323) + 0.00025\,(323)^2 = 890.7 - \frac{q}{A}\,(0.15 \text{ m}) \Rightarrow q_k = 1457 \text{ W/m}^2$$

(b) Therefore, the temperature distribution is

$$0.00025\,T^2 + 2.0\,T = 890.7 - 1458\,x$$



COMMENTS

Notice that although the temperature distribution is not linear due to the variation of the thermal conductivity with temperature, it is nearly linear because this variation is small compared to the value of the thermal conductivity.

If the variation of thermal conductivity with temperature had been neglected, the rate of heat transfer would have been 1333 W/m^2, an error of 8.5%.

PROBLEM 2.20

A plane wall 7.5 cm thick, generates heat internally at the rate of 10^5 W/m^3. One side of the wall is insulated, and the other side is exposed to an environment at 90°C. The convective heat transfer coefficient between the wall and the environment is 500 W/(m^2 K). If the thermal conductivity of the wall is 12 W/(m K), calculate the maximum temperature in the wall.

GIVEN

- Plane wall with internal heat generation
- Thickness (L) = 7.5 cm = 0.075 m
- Internal heat generation rate ($\dot{q}_G$) = 10^5 W/m^3
- One side is insulated
- Ambient temperature on the other side (T_∞) = 90 °C
- Convective heat transfer coefficient ($\overline{h}_c$) = 500 W/(m^2 K)
- Thermal conductivity (k) = 12 W/(m K)

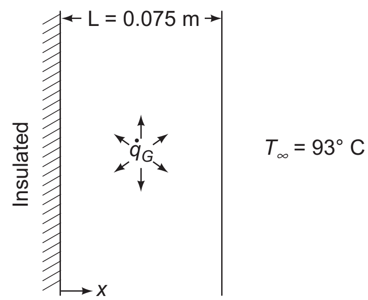
FIND

- The maximum temperature in the wall ($T_{\max}$)

ASSUMPTIONS

- The heat loss through the insulation is negligible
- The system has reached steady state
- One dimensional conduction through the wall

SKETCH



SOLUTION

The one dimensional conduction equation, given in Equation (2.5), is

$$k\,\frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho\, c\,\frac{\partial T}{\partial t}$$

For steady state, $\frac{\partial T}{\partial t} = 0$ therefore

$$k\,\frac{d^2 T}{d x^2} + \dot{q}_G = 0$$

$$\frac{d^2 T}{d x^2} = -\frac{\dot{q}_G}{k}$$

This is subject to the following boundary conditions

No heat loss through the insulation

$$\frac{dT}{dx} = 0 \text{ at } x = 0$$

Convection at the other surface

$$-k\,\frac{dT}{dx} = \overline{h}_c\,(T - T_\infty) \qquad \text{at} \quad x = L$$

Integrating the conduction equation once

$$\frac{dT}{dx} = \frac{\dot{q}_G}{k} + C_1$$

C_1 can be evaluated using the first boundary condition

$$0 = -\frac{\dot{q}_G}{k}(0) + C_1 \Rightarrow \quad C_1 = 0$$

Integrating again

$$T = -\frac{\dot{q}_G}{2k}x^2 + C_2$$

The expression for T and its first derivative can be substituted into the second boundary condition to evaluate the constant C_2

$$-k\left(\frac{\dot{q}_G L}{k}\right) = \overline{h}_c\left(-\frac{\dot{q}_G L^2}{2k} + C_2 - T_\infty\right) \Rightarrow C_2 = \dot{q}_G L\left(\frac{1}{\overline{h}_c} + \frac{L}{2k}\right) + T_\infty$$

Substituting this into the expression for T yields the temperature distribution in the wall

$$T(x) = \frac{\dot{q}_G}{2k}x^2 + \dot{q}_G L\left(\frac{1}{\overline{h}_c} + \frac{L}{2k}\right) + T_\infty$$

$$T(x) = T_\infty + \frac{\dot{q}_G}{2k}\left(L^2 + \frac{2kL}{\overline{h}_c} - x^2\right)$$

Examination of this expression reveals that the maximum temperature occurs at $x = 0$

$$T_{\max} = T_\infty + \frac{\dot{q}_G}{2k}\left(L^2 + \frac{2kL}{\overline{h}_c}\right)$$

$$T_{\max} = 90°\text{C} + \frac{10^5\ \text{W/m}^3}{2[12\text{W/(mK)}]}\left((0.075\,\text{m})^2 + \frac{2[12\ \text{W/(mK)}](0.075\,\text{m})}{500\ \text{W/(m}^2\text{K)}}\right) = 128°\text{C}$$

PROBLEM 2.21

A small dam, which may be idealized by a large slab 1.2 m thick, is to be completely poured in a short period of time. The hydration of the concrete results in the equivalent of a distributed source of constant strength of 100 W/m^3. If both dam surfaces are at 16°C, determine the maximum temperature to which the concrete will be subjected, assuming steady-state condition. The thermal conductivity of the wet concrete may be taken as 0.84 W/(m K).

GIVEN

- Large slab with internal heat generation
- Internal heat generation rate ($\dot{q}_G$) = 100 W/m^3
- Both surface temperatures (T_s) = 16°C
- Thermal conductivity (k) = 0.84 W/(m K)

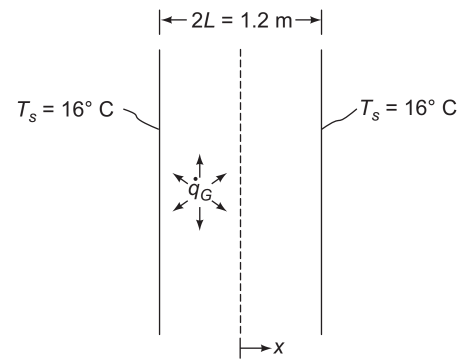
FIND

- The maximum temperature ($T_{\max}$)

ASSUMPTIONS

- Steady state conditions prevail

SKETCH



SOLUTION

The dam is symmetric; therefore x will be measured from the centerline of the dam. The equation for one dimensional conduction is given by Equation (2.5)

$$k \frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho c \frac{\partial T}{\partial t}$$

For steady state, $\frac{\partial T}{\partial t}$ = 0 therefore

$$k \frac{d^2 T}{d x^2} + \dot{q}_G = 0$$

This is subject to the following boundary conditions

1. By symmetry, $dT/dx = 0$ at $x = 0$
2. $T = T_s$ at $x = L$

Also note that for this problem $\dot{q}_G$ is a constant.

Integrating the conduction equation

$$\frac{dT}{dx} = -\frac{\dot{q}_G}{k} x + C_1$$

The constant C_1 can be evaluated using the first boundary condition

$$0 = -\frac{\dot{q}_G}{k} (0) + C_1 \Rightarrow C_1 = 0$$

Integrating once again

$$T = \frac{\dot{q}_G}{2k} x^2 + C_2$$

The constant C_2 can be evaluated using the second boundary condition

$$T_s = \frac{\dot{q}_G}{2k} L^2 + C_2 \Rightarrow C_2 = T_s + \frac{\dot{q}_G}{2k} L^2$$

Therefore, the temperature distribution in the dam is

$$T = T_s + \frac{\dot{q}_G}{2k} (L^2 - x^2)$$

The maximum temperature occurs at $x = 0$

$$T_{\max} = T_s + \frac{\dot{q}_G}{2k}(L^2 - (0)^2) = 16°\text{C} + \frac{100\ \text{W/m}^3}{2[0.84\ \text{W/(m K)}]}(0.6\ \text{m})^2 = 37°\text{C}$$

COMMENTS

This problem is simplified significantly by choosing $x = 0$ at the centerline and taking advantage of the problem's symmetry.

For a more complete analysis, the change in thermal conductivity with temperature and moisture content should be measured. The system could then be analyzed by numerical methods discussed in chapter 3.

PROBLEM 2.22

Two large steel plates at temperatures of 90° and 70°C are separated by a steel rod 0.3 m long and 2.5 cm in diameter. The rod is welded to each plate. The space between the plates is filled with insulation, which also insulates the circumference of the rod. Because of a voltage difference between the two plates, current flows through the rod, dissipating electrical energy at a rate of 12 W. Determine the maximum temperature in the rod and the heat flow rate at each end. Check your results by comparing the net heat flow rate at the two ends with the total rate of heat generation.

GIVEN

- Insulated steel rod with internal heat generation
- Length (L) = 0.3 m
- Diameter (D) = 2.5 cm = 0.025 m
- Internal heat generation rate ($\dot{q}_G V$) = 12 W
- End temperature of the rod: T_1 = 90°C T_2 = 70°C

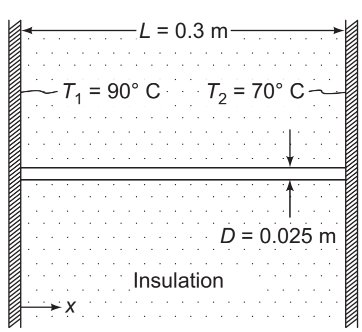
FIND

(a) Maximum temperature in the rod ($T_{\max}$)
(b) Heat flow rate at each end (q_0 and q_L)
(c) Check the results by comparing with the heat generation

ASSUMPTIONS

- The system has reached steady state
- The heat loss through the insulation is negligible
- The steel is 1% carbon steel
- Constant thermal conductivity
- The plate temperatures are constant
- Heat is generated uniformly throughout the rod

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

Thermal conductivity of 1% carbon steel (k) = 43 W/(m K) at 20°C

SOLUTION

The heat generation per unit volume of the rod is

$$\dot{q}_G = \frac{\dot{q}_G V}{V} = \frac{\dot{q}_G V}{\frac{\pi}{4} D^2 L} = \frac{12\,\mathrm{W}}{\frac{\pi}{4}(0.025\,\mathrm{m})^2\,(0.3\text{ m})} = 81{,}487\text{ W/m}^3$$

(a) The temperature distribution in the rod will be evaluated from the conduction equation, Equation (2.5), and the boundary conditions. The one dimensional conduction equation is

$$k\,\frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho\, c\,\frac{\partial T}{\partial t}$$

For steady state, $\frac{\partial T}{\partial t} = 0$ therefore

$$\frac{d^2T}{d x^2} = \frac{\dot{q}_G}{k} = 0$$

This is subject to the following boundary conditions

$$T = T_1 \text{ at } x = 0 \text{ and } T = T_2 \text{ at } x = L$$

Integrating the conduction equation yields

$$\frac{d\,T}{d\,x} = \frac{\dot{q}_G}{k}\,x + C_1$$

Integrating a second time

$$T = -\frac{\dot{q}_G}{2k}\,x^2 + C_1\,x + C_2$$

The constant C_2 can be evaluated using the first boundary condition

$$T_1 = \frac{\dot{q}_G}{2k}\,(0)^2 + C_1\,(0) + C_2 \Rightarrow C_2 = T_1$$

Therefore, the temperature distribution becomes

$$T = -\frac{\dot{q}_G}{2k}\,x^2 + C_1\,x + T_1$$

The second boundary condition can be used to evaluate the constant C_1

$$T_2 = -\frac{\dot{q}_G}{2k}\,L^2 + C_1\,L + T_1 \quad y \Rightarrow C_1 = \frac{1}{L}\,(T_2 - T_1) + \frac{\dot{q}_G L}{2k}$$

The temperature distribution in the rod is

$$T = -\frac{\dot{q}_G}{2k}\,x^2 + \left[\frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right] x + T_1$$

The maximum temperature in the rod occurs where the first derivative of the temperature distribution is zero

$$\frac{dT}{dx} = -\frac{\dot{q}_G}{k} x_m + \frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k} = 0$$

$$x_m = \frac{k}{L q_G}(T_2 - T_1) + \frac{L}{2} = \frac{43\,\text{W/(m K)}}{0.3\,\text{m}\,(81{,}487\,\text{W/m}^3)}(70°\text{C} - 90°\text{C}) + \frac{0.3\,\text{m}}{2} = 0.1148\text{ m}$$

Evaluating the temperature at this value of x

$$T_{\max} = \frac{\dot{q}_G}{2k} x_m^{\,2} + \left[\frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right] x_m + T_1$$

$$T_{\max} = \frac{81{,}847\text{ W/m}^3}{2(43\text{ W/(mK)})}(0.1148\text{ m})^2 + \left[\frac{90°\text{C} - 70°\text{C}}{0.3\text{ m}} + \frac{81{,}847\text{ W/m}^3\,(0.3\,\text{m})}{2(43\text{ W/(mK)})}\right](0.1148\text{m}) + 90°$$

$$T_{\max} = 102°\text{C}$$

(b) The heat flow from the rod at $x = 0$ can be calculated from Equation (1.1)

$$q_0 = -kA\frac{dT}{dx}\Big|_{x=0} = -kA\left[\frac{\dot{q}_G}{k}x + \frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right]_{x=0}$$

$$q_0 = -k\left(\frac{\pi}{4}D^2\right)\left[\frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right]$$

$$q_0 = -43\text{ W/(m K)}\left(\frac{\pi}{4}(0.025\,\text{m})^2\right)\left[\frac{1}{0.3\,\text{m}}(70°\text{C} - 90°\text{C}) + \frac{81{,}847\text{ W/m}^3\,(0.3\,\text{m})}{2(43\text{ W/(m K)})}\right] = -4.6\text{ W}$$

(The negative sign indicates that heat is flowing to the left, out of the rod)

The heat flow from the rod at $x = L$ is

$$q_L = -kA\frac{dT}{dx}\Big|_{x=L} = -kA\left[-\frac{\dot{q}_G}{k}L + \frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right]$$

$$q_L = \frac{\pi}{4}D^2\left[\frac{\dot{q}_G}{2}L - \frac{k(T_2 - T_1)}{L}\right]$$

$$q_L = -\frac{\pi}{4}(0.025\text{m})^2\left[\frac{\left(81{,}847\text{ W/m}^3\right)(0.3\,\text{m})}{2} - \frac{(43\text{ W/(m K)})(70°\text{C} - 90°\text{C})}{0.3\text{ m}}\right] = 7.4\text{ W}$$

(The positive value indicates that heat is flowing to the right, out of the rod)

(c) The total heat loss is the sum of the loss from each end

$$q_{\text{total}} = |\,q_0\,| + |\,q_L\,| = 4.6\text{W} + 7.4\text{W} = 12.0\text{W}$$

The total rate of heat loss is equal to the rate of heat generation within the rod.

PROBLEM 2.23

The shield of a nuclear reactor can be idealized by a large 25 cm thick flat plate having a thermal conductivity of 3.5 W/(m K). Radiation from the interior of the reactor penetrates the shield and produces heat generation in the shield which decreases exponentially from a value of 187.6 (k W)/m^3. at the inner surface to a value of 18.76 (kW)/m^3 at a distance of 12.5 cm from the interior surface. If the exterior surface is kept at 38°C by forced convection, determine the temperature at the inner surface of the shield. Hint: First set up the differential equation for a system in which the heat generation rate varies according to $\dot{q}(x) = \dot{q}(0)e^{-Cx}$.

GIVEN

- Large flat plate with non-uniform internal heat generation
- Thickness (L) = 25 cm = 0.25 m
- Thermal conductivity (k) = 3.5 W/(m K)
- Exterior surface temperature (T_o) = 38°C
- Heat generation is exponential with values of
  - 187.6 kW/m^3 at the inner surface
  - 18.76 kW/m^3 at 12.5 cm from the inner surface

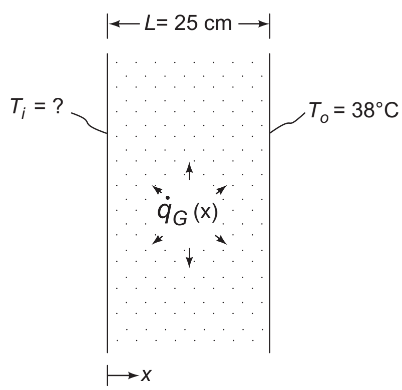
FIND

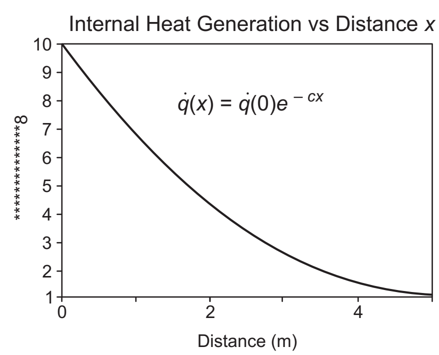
- The inner surface temperature (T_i)

ASSUMPTIONS

- One dimensional, steady state conduction
- The thermal conductivity is constant
- No heat transfer at the inner surface of the shield

SKETCH





SOLUTION

From the hint, the internal heat generation is

$$\dot{q}(x) = \dot{q}(0)\, e^{-cx} \text{ where } \dot{q}(0) = 187.6 \text{ kW/m}^3$$

Solving for the constant c using the fact that $q(x)$ = 1 Btu/h in^3 at x = 5 in = 0.417 ft

$$c = -\frac{1}{x}\ln\left(\frac{\dot{q}(x)}{\dot{q}(0)}\right) = -\frac{1}{0.25}\ln\left[\frac{187.6\,\text{kW/m}^3}{18.76\,\text{kW/m}^3}\right]$$

$$\Rightarrow \qquad c = 9.21 \text{ 1/m}$$

The one dimensional conduction equation is given by Equation (2.5)

$$k\,\frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho\, c\,\frac{\partial T}{\partial t} = 0 \text{ (steady state)}$$

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}_G(x)}{k} = \frac{\dot{q}(0)}{k}\,e^{-cx}$$

The boundary conditions are

$$\frac{dT}{dx} = 0 \text{ at } x = 0$$

$$T(L) = T_o = 38°\text{C at } x = L$$

Integrating the conduction equation

$$\frac{dT}{dx} = -\frac{\dot{q}(0)}{ck}\,e^{-cx} + C_1$$

The constant C_1 can be evaluated by applying the first boundary condition

$$0 = -\frac{\dot{q}(0)}{ck}\,e^{-c(0)} + C_1 \quad \Rightarrow C_1 = \frac{-\dot{q}(0)}{ck}$$

Integrating again

$$T(x) = \frac{-\dot{q}(0)}{c^2k}\,e^{-cx} - \frac{\dot{q}(0)}{c^2k}\,x + C_2$$

The constant C_2 can be evaluated by applying the second boundary condition

$$T(L) = T_o = \frac{-\dot{q}(0)}{c^2k}\,e^{-cL} - \frac{\dot{q}(0)}{ck}\,L + C_2 \quad \Rightarrow C_2 = T_o + \frac{\dot{q}(0)}{ck}\left(L + \frac{1}{c}e^{-cL}\right)$$

Therefore, the temperature distribution is

$$T(x) = T_o + \frac{-\dot{q}(0)}{c^2k}\,[e^{-cL} - e^{-cx} + c(L - x)]$$

Solving for the temperature at the inside surface ($x = 0$)

$$T_i = T(0) = T_o + \frac{\dot{q}(0)}{c^2k}\,[e^{-cL} - 1 + cL]$$

$$T_i = 38°\text{C} + \frac{187.6\times 10^3 \text{ W/m}^3}{\frac{(9.21)^2}{\text{m}^2}(3.5 \text{ W/(m K)})}\,[e^{-(9.21)(0.25)} - 1 + 9.21\times 0.25]°\text{C}$$

$$T_i = 38 + 631.2\,[0.1 - 1 + 2.303] = 922°\text{C}$$

PROBLEM 2.24

Derive an expression for the temperature distribution in an infinitely long rod of uniform cross section within which there is uniform heat generation at the rate of 1 W/m. Assume that the rod is attached to a surface at T_s and is exposed through a convective heat transfer coefficient h to a fluid at T_f.

GIVEN

- An infinitely long rod with internal heat generation
- Temperature at one end = T_s
- Heat generation rate ($\dot{q}_G A$) = 1 W/m
- Convective heat transfer coefficient = h_c
- Ambient fluid temperature = T_f

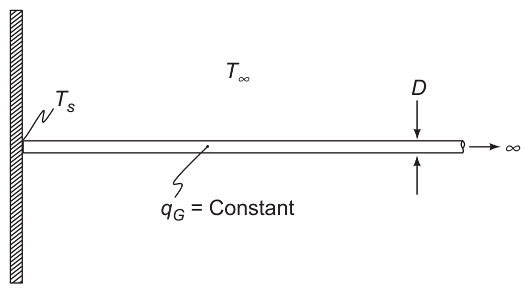
FIND

- Expression for the temperature distribution

ASSUMPTIONS

- The rod is in steady state
- The thermal conductivity (k) is constant

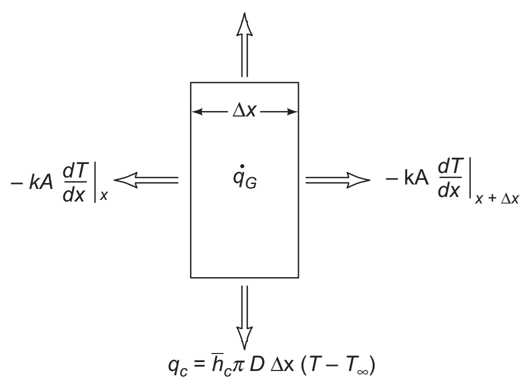
SKETCH



SOLUTION

Let A = the cross sectional area of the rod = $\pi D^2/4$

An element of the rod with heat flows is shown at the right



Conservation of energy requires that

Energy entering the element + Heat generation = Energy leaving the element

$$-k A \left.\frac{dT}{dx}\right|_x + \dot{q}_G A \Delta x = -k A \left.\frac{dT}{dx}\right|_{x+\Delta x} + \overline{h_c}\, \pi D\, \Delta x\, [T(x) - T_f]$$

$$kA \left(\left.\frac{dT}{dx}\right|_{x+\Delta x} - \left.\frac{dT}{dx}\right|_x \right) = \overline{h_c}\, \pi D\, \Delta x\, (T - T_f) - \dot{q}_G\, A\, \Delta x$$

Dividing by Δx and letting $\Delta x \to 0$ yields

$$kA \frac{d^2T}{dx^2} = \overline{h_c}\, \pi D\,(T - T_f) - \dot{q}_G\, A$$

$$\frac{d^2T}{dx^2} = \frac{4\overline{h_c}}{Dk}\,(T - T_f) - \frac{\dot{q}_G}{k}$$

Let $\quad \theta = T - T_f$ and $m^2 = \dfrac{4\,h_c}{(D\,k)}$

$$\frac{d^2\theta}{dx^2} - m^2\,\theta = \frac{-\dot{q}_G}{k}$$

This is a second order, linear, nonhomogeneous differential equation with constant coefficients. Its solution is the addition of the homogeneous solution and a particular solution. The solution to the homogeneous equation

$$\frac{d^2\theta}{dx^2} - m^2\,\theta = 0$$

is determined by its characteristic equation. Substituting $\theta = e^{\lambda x}$ and its derivatives into the homogeneous equation yields the characteristic equation

$$\lambda^2\, e^{\lambda x} - m^2\, e^{\lambda x} = 0 \Rightarrow \lambda = \pm\, m$$

Therefore, the homogeneous solution has the form

$$\theta_h = C_1\, c^{mx} + C_2\, e^{-mx}$$

A particular solution for this problem is simply a constant

$$\theta = a_o$$

Substituting this into the differential equation

$$0 - m^2\, a_o = \frac{-\dot{q}_G}{k} \Rightarrow a_o = \frac{\dot{q}_G}{m^2 k}$$

Therefore, the general solution is

$$q = C_1\, e^{mx} + C_2\, e^{-mx} + \frac{\dot{q}_G}{m^2 k}$$

With the boundary conditions

$$\theta = a \text{ finite number as } x \to \infty$$

$$\theta = T_s - T_f \text{ at } x = 0$$

From the first boundary condition, as $x \to \infty$ $e^{mx} \to \infty$, therefore $C_1 = 0$

From the second boundary condition

$$T_s - T_f = C_2 + \frac{\dot{q}_G}{m^2 k} \Rightarrow C_2 = T_s - T_f - \frac{\dot{q}_G}{m^2 k}$$

The temperature distribution in the rod is

$$q = T(x) - T_f = \left(T_s - T_f - \frac{\dot{q}_G}{m^2 k}\right) e^{-mx} + \frac{\dot{q}_G}{m^2 k}$$

$$T(x) = T_f + \left(T_s - T_f - \frac{\dot{q}_G}{m^2 k}\right) e^{-mx} + \frac{\dot{q}_G}{m^2 k}$$

PROBLEM 2.25

Derive an expression for the temperature distribution in a plane wall in which there are uniformly distributed heat sources which vary according to the linear relation

$$\dot{q}_G = \dot{q}_w \; [1 - \beta(T - T_w)]$$

where q_w is a constant equal to the heat generation per unit volume at the wall temperature T_w. Both sides of the plate are maintained at T_w and the plate thickness is 2L.

GIVEN

- A plane wall with uniformly distributed heat sources as in the above equation
- Both surface temperatures = T_w
- Thickness = 2L

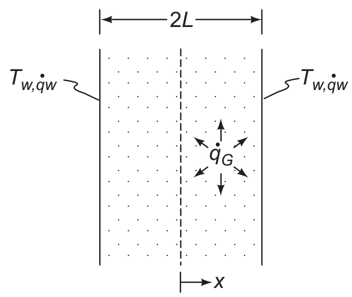
FIND

- An expression for the temperature distribution

ASSUMPTIONS

- Constant thermal conductivity (k)

SKETCH



SOLUTION

The equation for one dimensional, steady state ($dT/dt = 0$) conduction from Equation (2.5) is

$$\frac{d^2T}{dx^2} = \frac{-\dot{q}_G}{k} = \frac{-\dot{q}_w}{k}\,[1 - \beta\,(T - T_w)] = \frac{\dot{q}_w\beta}{k}\,(T - T_w) - \frac{\dot{q}_w}{k}$$

With the boundary conditions

$$\frac{dT}{dx} = 0 \text{ at } x = 0$$

$$T = T_w \text{ at } x = L$$

Let $\theta = T - T_w$ and $m^2 = (\dot{q}_w\,\beta)/k$ then

$$\frac{d^2\theta}{dx^2} - m^2\,\theta = \frac{-\dot{q}_w}{k}$$

This is a second order, linear, nonhomogeneous differential equation with constant coefficients. Its solution is the addition of the homogeneous solution and a particular solution. The solution to the homogeneous equation

$$\frac{d^2\theta}{dx^2} - m^2\,\theta = 0$$

is determined by its characteristics equation. Substituting $\theta = e^{\lambda x}$ and its derivatives into the homogeneous equation yields the characteristics equation

$$\lambda^2 e^{\lambda x} - m^2 e^{\lambda x} = 0 \Rightarrow \lambda = m$$

Therefore, the homogeneous solution has the form

$$\theta_h = C_1\, c^{mx} + C_2\, e^{-mx}$$

A particular solution for this problem is simply a constant: $\theta = a_o$

Substituting this into the differential equation

$$0 - m^2\, a_o = \frac{-\dot{q}_w}{k} \Rightarrow a_o = \frac{\dot{q}_w}{m^2 k}$$

Therefore, the general solution is

$$\theta = C_1\, e^{mx} + C_2\, e^{-mx} + \frac{\dot{q}_w}{m^2 k}$$

With the boundary condition

$$\frac{d\theta}{dx} = 0 \text{ at } x = 0$$

$$\theta = 0 \text{ at } x = L$$

Applying the first boundary condition:

$$\frac{d\theta}{dx} = C_1\, me^{(0)} - C_2\, me^{(0)} = 0 \Rightarrow C_1 = C_2 = C$$

From the second boundary condition

$$0 = C\,(e^{mL} + e^{-mL}) + \frac{\dot{q}_w}{m^2 k} \Rightarrow C = \frac{-\dot{q}_w}{m^2 k\,(e^{mL} + e^{-mL})}$$

The temperature distribution in the wall is

$$\theta = T(x) - T_w = \frac{-\dot{q}_w}{m^2 k\,(e^{mL} + e^{-mL})}\,(e^{mx} + e^{-mx}) + \frac{\dot{q}_w}{m^2 k}$$

$$T(x) = T_w + \frac{\dot{q}_w}{m^2 k}\left(1 - \frac{e^{mx} + e^{-mx}}{e^{mL} + e^{-mL}}\right)$$

$$T(x) = T_w + \frac{\dot{q}_w}{m^2 k}\left(1 - \frac{\cosh(mx)}{\cosh(mL)}\right)$$

PROBLEM 2.26

A plane wall of thickness 2L has internal heat sources whose strength varies according to

$$\dot{q}_G = \dot{q}_0 \cos(ax)$$

where $\dot{q}_0$ is the heat generated per unit volume at the center of the wall (x = 0) and a is a constant. If both sides of the wall are maintained at a constant temperature of T_w, derive an expression for the total heat loss from the wall per unit surface area.

GIVEN

- A plane wall with internal heat sources
- Heat source strength: $\dot{q}_G = \dot{q}_0 \cos(ax)$
- Wall surface temperatures = T_w
- Wall thickness = 2L

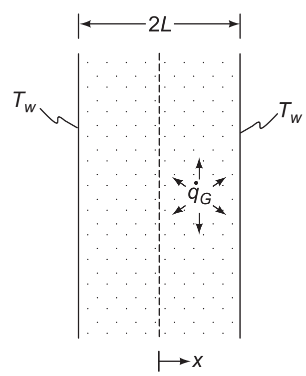
FIND

An expression for the total heat loss per unit area (q/A)

ASSUMPTIONS

- Steady state conditions prevail
- The thermal conductivity of the wall (k) is constant
- One dimensional conduction within the wall

SKETCH



SOLUTION

Equation (2.5) gives the equation for one dimensional conduction. For steady state, dT/dt = 0, therefore

$$k \frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho c \frac{\partial T}{\partial t} = 0$$

$$\frac{d^2 T}{d x^2} = \frac{-\dot{q}_G}{k} = \frac{-\dot{q}_0 \cos(ax)}{k}$$

With boundary conditions:

$$\frac{dT}{dx} = 0 \text{ at } x = 0 \text{ (by symmetry)}$$

$$T = T_w \text{ at } x = L \text{ (given)}$$

Integrating the conduction equation once

$$\frac{dT}{dx} = \frac{\dot{q}_o}{a\,k}\,\sin(ax) + C_1$$

Applying the first boundary condition yields: $C_1 = 0$

The rate of heat transfer from one side of the wall is

$$q_k = -k\,A\,\frac{dT}{dx}\Big|_{x=L} = -k\,A\left[-\frac{\dot{q}_G}{a\,k}\sin(aL)\right] = \frac{\dot{q}_o A}{a}\sin(aL)$$

The total rate of heat transfer is twice the rate of heat transfer from one side of the wall

$$\left(\frac{q_k}{A}\right)_{\text{total}} = \frac{2\dot{q}_o}{a}\,\sin(aL)$$

An alternative method of solution for this problem involves recognizing that at steady state the rate of heat generation within the entire wall must equal the rate of heat transfer from the wall surfaces

$$A\int_{-L}^{L}\dot{q}_G(x)dx = q$$

$$\dot{q}_o\int_{-L}^{L}\cos(ax)\,dx = \frac{q}{A}$$

$$\frac{\dot{q}_o}{a}[\sin(aL)-\sin(-aL)] = \frac{q}{A}$$

$$\frac{q}{A} = \frac{2\dot{q}_o}{a}\,\sin(aL)$$

COMMENTS

The heat loss can be determined by solving for the temperature distribution and then the rate of heat transfer or via the conservation of energy which allows us to equate the heat generation rate with the rate of heat loss.

PROBLEM 2.27

Heat is generated uniformly in the fuel rod of a nuclear reactor. The rod has a long, hollow cylindrical shape with its inner and outer surfaces at temperatures of T_i and T_o, respectively. Derive an expression for the temperature distribution.

GIVEN

- A long, hollow cylinder with uniform internal generation
- Inner surface temperature = T_i
- Outer surface temperature = T_o

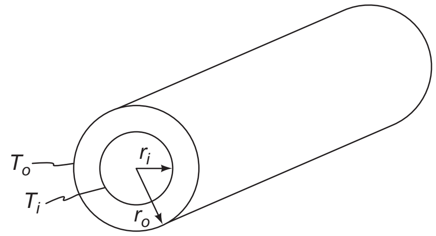
FIND

- The temperature distribution

ASSUMPTIONS

- Conduction occurs only in the radial direction
- Steady state prevails

SKETCH



SOLUTION

Let r_i = the inner radius

r_o = the outer radius

$\dot{q}_G$ = the rate of internal heat generation per unit volume

k = the thermal conductivity of the fuel rod

The one dimensional, steady state conduction equation in cylindrical coordinates is given in Equation (2.21)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}_G}{k} = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-r\,\dot{q}_G}{k}$$

With boundary conditions

$T = T_i$ at $r = r_i$

$T = T_o$ at $r = r_o$

Integrating the conduction equation once

$$r\,\frac{dT}{dr} = \frac{-r^2\,\dot{q}_G}{2k} + C_1$$

$$dT = \left(\frac{-r^2\,\dot{q}_G}{2k} + \frac{C_1}{r}\right) dr$$

Integrating again

$$T = \frac{-r^2\,\dot{q}_G}{4k} + C_1 \ln(r) + C_2$$

Applying the first boundary condition

$$T_i = \frac{-r_i^2\,\dot{q}_G}{4k} + C_1 \ln(r_i) + C_2$$

$$C_2 = T_i + \frac{r_i^2\,\dot{q}_G}{4k} - C_i \ln(r_i)$$

Applying the second boundary condition

$$T_o = \frac{-r_o^2\,\dot{q}_G}{4k} + C_1 \ln(r_o) + C_2$$

$$T_o = \frac{-r_o^2\,\dot{q}_G}{4k} + C_1 \ln(r_o) + T_i + \frac{r_i^2\,\dot{q}_G}{4k} - C_1 \ln(r_i)$$

$$C_1 = \frac{T_o - T_i + \frac{\dot{q}_G}{4k}(r_o^2 - r_i^2)}{\ln\left(\frac{r_o}{r_i}\right)}$$

Substituting the constants into the temperature distribution

$$T = \frac{-r^2\,\dot{q}_G}{4k} + \left(\frac{T_o - T_i + \frac{\dot{q}_G}{4k}(r_o^2 - r_i^2)}{\ln\left(\frac{r_o}{r_i}\right)}\right)\ln(r) + T_i + \frac{r_i^2\,\dot{q}_G}{4k} - \left(\frac{T_o - T_i + \frac{\dot{q}_G}{4k}(r_o^2 - r_i^2)}{\ln\left(\frac{r_o}{r_i}\right)}\right)$$

$$T = \frac{\dot{q}_G}{4k}\left(\frac{(r_o^2 - r_i^2)\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)} + (r_i^2 - r^2)\right) + \frac{(T_o - T_i)\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)} + T_i$$

PROBLEM 2.28

Show that the temperature distribution in a sphere of radius r_o, made of a homogeneous material in which energy is released at a uniform rate per unit volume $\dot{q}_G$, is

$$T(r) = T_o + \frac{\dot{q}_G\, r_o^2}{6\,k}\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

GIVEN

- A homogeneous sphere with energy generation
- Radius = r_o

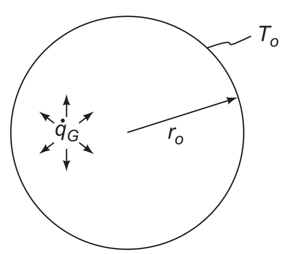
FIND

- Show that the temperature distribution is as shown above.

ASSUMPTIONS

- Steady state conditions persist
- The thermal conductivity of the sphere material is constant
- Conduction occurs in the radial direction only

SKETCH



SOLUTION

Let k = the thermal conductivity of the material

T_o= the surface temperature of the sphere

Equation (2.23) can be simplified to the following equation by the assumptions of steady state and radial conduction only

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{q}_G}{k} = 0$$

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = \frac{-r^2\,\dot{q}_G}{k}$$

With the following boundary conditions

$$\frac{dT}{dr} = 0 \text{ at } r = 0$$

$$T = T_o \text{ at } r = r_o$$

Integrating the differential equation once

$$r^2\,\frac{dT}{dr} = \frac{-r^3\,\dot{q}_G}{3k} + C_1$$

From the first boundary condition

$$C_1 = 0$$

Integrating once again

$$T = \frac{-r^2\,\dot{q}_G}{6k} + C_2$$

Applying the second boundary condition

$$T_o = \frac{-r_o^2\,\dot{q}_G}{6k} + C_2 \Rightarrow C_2 = T_o + \frac{-r_o^2\,\dot{q}_G}{6k}$$

Therefore, the temperature distribution in the sphere is

$$T = \frac{-r^2\,\dot{q}_G}{6k} + T_o + \frac{-r_o^2\,\dot{q}_G}{6k}$$

$$T(r) = T_o + \frac{\dot{q}_G\,r_o^2}{6k}\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

PROBLEM 2.29

In a cylindrical fuel rod of a nuclear reactor, heat is generated internally according to the equation

$$\dot{q}_G = \dot{q}_1\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

where $\dot{q}_g$ **= local rate of heat generation per unit volume at** r

r_o **= outside radius**

$\dot{q}_1$ **= rate of heat generation per unit volume at the centerline**

Calculate the temperature drop from the center line to the surface for a 2.5 cm OD rod having a thermal conductivity of 26 W/(m K) if the rate of heat removal from its surface is 1.6 MW/m^2.

GIVEN

- A cylindrical rod with internal generation and heat removal from its surface
- Outside diameter (D_o) = 2.5 cm = 0.025 m
- Rate of heat generation is as given above
- Thermal conductivity (k) = 26 W/(m K)
- Heat removal rate (q/A) = 1.6 MW/m^2

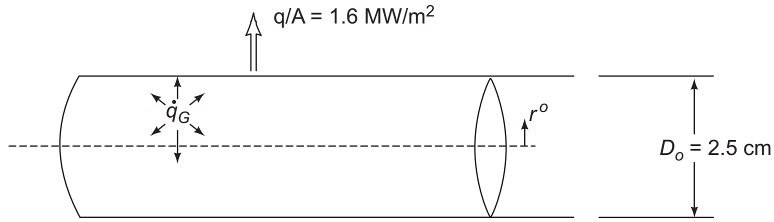
FIND

- The temperature drop from the center line to the surface (ΔT)

ASSUMPTIONS

- The heat flow has reached steady state
- The thermal conductivity of the fuel rod is constant
- One dimensional conduction in the radial direction

SKETCH



SOLUTION

The equation for one dimensional conduction in cylindrical coordinates is given in Equation (2.21)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)\frac{\dot{q}_G}{k} = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-r}{k}\,\dot{q}_1\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

With the boundary conditions

$$\frac{dT}{dr} = 0 \text{ at } r = 0$$

$$T = T_s \text{ at } r = r_o$$

Integrating once

$$r\,\frac{dT}{dr} = \frac{-r^2 q_1}{2k} + \frac{r^4 q_1}{4k\,r_o^2} + C_1$$

From the first boundary condition: $C_1 = 0$, therefore

$$\frac{dT}{dr} = \frac{q_1}{2k}\left(\frac{r^3}{2r_o^2} - r\right)$$

Integrating again

$$T = \frac{q_1}{2k}\left(\frac{r^4}{8r_o^2} - \frac{r^2}{2}\right) + C_2$$

Evaluate this expression at the surface of the cylinder and at the centerline of the Cylinder and subtracting the results gives us the temperature drop in the cylinder

$$\Delta T = T_0 - T_{r_o} = \frac{q_1}{2k}\left(\frac{(0)^4}{8r_o^2} - \frac{(0)^2}{2} - \frac{r_o^4}{8r_o^2} + \frac{r_o^2}{2}\right) = \frac{3q_1 r_o^2}{16k}$$

The rate of heat generation at the centerline (q_1) can be evaluated using the conservation of energy. The total rate of heat transfer from the cylinder must equal the total rate of heat generation within the cylinder

$$\left(\frac{q}{A}\right) A = L \int_{r=0}^{r=r_o} q_1 \left[1 - \frac{r^4}{r_o^2}\right] 2\pi r\, dr$$

$$\left(\frac{q}{A}\right) 2\pi r_o L = 2\pi L\, q_1 \left[\frac{r^2}{2} - \frac{r^4}{4r_o^2}\right]_0^{r_o}$$

$$\left(\frac{q}{A}\right) r_o = q_1 \left[\frac{r_o^2}{2} - \frac{r_o^2}{4}\right] = q_1 \frac{r_o^2}{4}$$

$$\therefore q_1 = \frac{4}{r_o}\left(\frac{q}{A}\right) = 5.12 \times 10^8 \text{ W/m}^3 \; \frac{4}{1.25 \times 10^{-2} \text{ M}} \; (1.6 \times 10^6 \text{ W/m}^2)$$

Therefore, the temperature drop within the cylinder is

$$\Delta T = \frac{3(5.12 \times 10^8 \text{ W/m}^3)(1.25 \times 10^{-2})^2}{16(26 \text{ W/(m K)})} = 577°\text{C}$$

PROBLEM 2.30

An electrical heater capable of generating 10,000 W is to be designed. The heating element is to be a stainless steel wire, having an electrical resistivity of 80 × 10^{-6} ohm-centimeter. The operating temperature of the stainless steel is to be no more than 1260°C. The heat transfer coefficient at the outer surface is expected to be no less than 1720 W/(m^2 K) in a medium whose maximum temperature is 93°C. A transformer capable of delivering current at 9 and 12 V is available. Determine a suitable size for the wire, the current required, and discuss what effect a reduction in the heat transfer coefficient would have. Hint: Demonstrate first that the temperature drop between the center and the surface of the wire is independent of the wire diameter, and determine its value.

GIVEN

- A stainless steel wire with electrical heat generation
- Heat generation rate ($\dot{Q}_G$) = 10,000 W
- Electrical resistivity (ρ) = 80 × 10^{-6} ohms-cm
- Maximum temperature of stainless steel (T_{max}) = 1260°C
- Heat transfer coefficient ($\bar{h}_c$) = 1700 W/(m^2 K)
- Maximum temperature of medium (T_∞) = 93°C

- Voltage (V) = 9 or 12 V

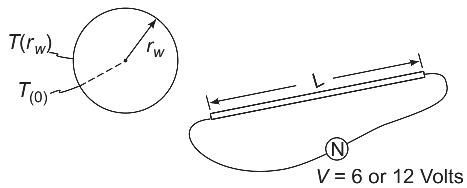
FIND

(a) A suitable wire size: diameter (d_w) and length (L)
(b) The current required (I)
(c) Discuss the effect of reduction in the heat transfer coefficient

ASSUMPTIONS

- Variation in the thermal conductivity of stainless steel is negligible
- The system is in steady-state
- Conduction occurs in the radial direction only

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, Thermal conductivity of stainless steel (k) = 14.4 W/(m^2 K)

SOLUTION

The rate of heat generation per unit volume is

$$\dot{q}_G = \frac{\dot{Q}_G}{\text{volume}} = \frac{\dot{Q}_G}{\pi r_w^2 L}$$

The temperature distribution in a long cylinder with internal heat generation is given in Section 2.3.3

$$T(r) = C_2 - \frac{\dot{q}_G r^2}{4k}$$

where C_2 is a constant determined by boundary conditions. Therefore

$$T(0) - T(r_\text{w}) - = [C_2 - 0] - \left[C_2 - \frac{\dot{q}_G r_w^2}{4k}\right] = \frac{\dot{q}_G r_w^2}{4k} = \frac{\dot{Q}_G}{4\pi kL}$$

The convective heat transfer from the outer surface must equal the internal heat generation

$$q_\text{c} = \overline{h_c}\, A\, [T(r_w) - T_\infty] = \dot{Q}_G$$

$$\therefore\ T(r_w) - T_\infty = \frac{\dot{Q}_G}{2\pi r_w L \overline{h_c}}$$

Adding the two temperature differences calculated above yields

$$[T(0) - T(r_w)] + [T(r_w) - T_\infty] = \frac{\dot{Q}_G}{4\pi kL} + \frac{\dot{Q}_G}{2\pi r_w L \overline{h_c}}$$

$$T(0) - T_\infty = \frac{\dot{Q}_G}{2\pi}\left(\frac{1}{2kL} + \frac{1}{r_w L \overline{h_c}}\right)$$

The wire length and its radius are related through an expression for the electric power dissipation

$$\dot{Q}_G = P_e = \frac{V^2}{R_e} = \frac{V^2}{\frac{\rho L}{A}} = \frac{V^2 \pi r_w^2}{\rho L} \Rightarrow L = \frac{\pi V^2 \pi r_w^2}{\rho \dot{Q}_G}$$

$$\therefore T(0) - T_\infty = \frac{\dot{Q}_G^2 \rho}{2\pi^2 V^2}\left(\frac{1}{2k r_w^2} + \frac{1}{r_w^3 \overline{h_c}}\right)$$

$$r_w^2\,[T(0) - T_\infty] - \frac{\dot{Q}_G^2 \rho}{2\pi^2 V^2}\left(\frac{r_w}{2k} + \frac{1}{\overline{h_c}}\right) = 0$$

For the 12 volt case

$$r_w^3\,(1260°\text{C} - 90°\text{C}) - \frac{(10{,}000\,\text{W})^2 (80\times 10^{-6}\ \text{ohm-cm})}{2\pi^2 (12V^2)(100\,\text{cm/m})}\left(\frac{r_w}{2(14.4\ \text{W/(mK)})} + \frac{1}{1700\,(\text{W/(m}^2\text{K))}}\right) = 0$$

After checking the units, they are dropped for clarity

$$1167\, r_w^3 - 0.0281(0.0347\, r^2 + 0.000581) = 0$$

Solving by trial and error

$$r_w = 0.0025\ \text{m} = 2.5\ \text{mm}$$

For the 12 volt case, the suitable wire diameter is

$$d_w = 2(r_w) = 5\ \text{mm}$$

The length of the wire required is

$$L = \frac{\pi (12\,\text{V})^2 (0.0025\,\text{m})^2 - (100\ \text{cm/m})}{80\times 10^{-6}\ \text{ohm-cm}\,(10{,}000\,\text{W})} = 0.353\ \text{m}$$

The electrical resistance of this wire is

$$R_e = \frac{\rho L}{\pi r_w^2} = \frac{80\times 10^{-6}\ \text{ohm-cm}\,(0.353\,\text{m})}{\pi (0.0025\,\text{m})^2 - (100\ \text{cm/m})} = 0.0144\ \text{ohm}$$

Therefore, the current required for the 12 volt case is

$$I = \frac{V}{R_e} = \frac{12\,\text{V}}{0.0144\,\text{ohm}} = 833\ \text{amps}$$

This same procedure can be used for the 9 volt case yielding

$$d_w = 6.3\ \text{mm}$$
$$L = 0.306\ \text{m}$$
$$R_e = 0.0081\ \text{ohm}$$
$$I = 1111\ \text{amps}$$

COMMENTS

The 5 mm diameter wire would be a better choice since the amperage is less. However 833 amps is still extremely high.

The effect of a lower heat transfer coefficient would be an increase in the diameter and length of the wire as well as an increase in the surface temperature of the wire.

PROBLEM 2.31

> **The addition of aluminum fins has been suggested to increase the rate of heat dissipation from one side of an electronic device 1 m wide and 1 m tall. The fins are to be rectangular in cross section, 2.5 cm long and 0.25 cm thick. There are to be 100 fins per meter. The convective heat transfer coefficient, both for the wall and the fins, is estimated at 35 W/(m^2 K). With this information, determine the percent increase in the rate of heat transfer of the finned wall compared to the bare wall.**

GIVEN

- Aluminum fins with a rectangular cross section
- Dimensions: 2.5 cm long and 0.25 mm thick
- Number of fins per meter = 100
- The convective heat transfer coefficient ($\overline{h_c}$) = 35 W/(m^2 K)

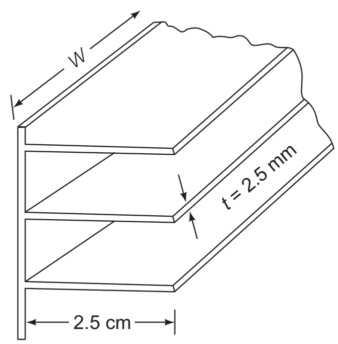
FIND

- The percent increase in the rate of heat transfer of the finned wall compared to the bare wall

ASSUMPTIONS

- Steady state heat transfer

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
The thermal conductivity of aluminum (k) = 240 W/(m K) at 127°C

SOLUTION

Since the fins are of uniform cross section, Table 2.1 can be used to calculate the heat transfer rate from a single fin with convection at the tip

$$q_f = M \frac{\sinh(mL) + \dfrac{\overline{h}_c}{(mk)}\cosh(mL)}{\cosh(mL) + \left(\dfrac{\overline{h}_c}{mk}\right)\sinh(mL)}$$

where $\quad M = \sqrt{\overline{h}_c\, P\, kA}\;\theta_s = \sqrt{\overline{h}_c\; 2(t+w)\,k\,(tw)}\;\theta_s$

$$\theta_s = T_s - T_\infty$$

For a 1 m width (w = 1 m)

$$M = \sqrt{\left(35\,\text{W/(m}^2\text{K)}\right)2\,(1.0025\,\text{m})\,(240\text{W/(m K)})\,(0.025\text{ m}^2)}\ \ \theta_s = 6.49\ \theta_s \text{ W/K}$$

$$m\,L = \sqrt{\frac{\bar{h}_c P}{kA}} = L\sqrt{\frac{\bar{h}_c 2(t+w)}{k\,(tw)}} = 0.025\text{ m}\sqrt{\frac{\left(35\text{ W/(m}^2\text{ K)}\right)2\,(1.0025\text{m})}{(240\text{W/(m K)})\,(0.0025\text{ m}^2)}}$$

$$L\,m = 0.025\text{ m}\left(10.81\frac{1}{\text{m}}\right) = 0.270$$

$$\frac{\bar{h}_c}{\text{m K}} = \frac{35\,\text{W/(m}^2\text{K)}}{\left(10.81\,\frac{1}{\text{m}}\right)(240\text{ W/(m K)})} = 0.0135$$

Therefore, the rate of heat transfer from one fin, 1 meter wide is:

$$q_f = 6.49\ \theta_s\text{ W/K}\frac{\sin\text{h}\,(0.27) + 0.0135\,\cos\text{h}\,(0.27)}{\cos\text{h}\,(0.27) + +0.0135\sin\text{h}\,(0.27)}$$

$$q_f = 1.792\ \theta_s\text{ W/K}$$

In 1 m^2 of wall area there are 100 fins covering 100 tw = 100 (0.0025 m) (1 m) = 0.25 m^2 of wall area leaving 0.75 m^2 of bare wall. The total rate of heat transfer from the wall with fins is the sum of the heat transfer from the bare wall and the heat transfer from 100 fins.

$$q_{\text{tot}} = q_{\text{bare}} + 100\ q_{\text{fin}} = \bar{h}\,A_{\text{bare}}\ \theta_s + 100\ q_{\text{fin}}$$

$$q_{\text{tot}} = \left(35\text{ W/(m}^2\text{K)}\right)\ (0.75\text{ m}^2)\ \theta_s + 100\ (1.792)\ \theta_s\text{ W/K} = 205.3\ \theta_s\text{ W/K}$$

The rate of heat transfer from the wall without fins is

$$q_{\text{bare}} = \bar{h}_c\,A\ \theta_s = \left(35\text{ W/(m}^2\text{K)}\right)\ (1\text{ m}^2)\ \theta_s = 35.0\text{ W/K}$$

The percent increase due to the addition of fins is

$$\%\text{ increase} = \frac{205.3 - 35}{35}\times 100 = 486\%$$

COMMENTS

This problem illustrates the dramatic increase in the rate of heat transfer that can be achieved with properly designed fins.

The assumption that the convective heat transfer coefficient is the same for the fins and the wall is an oversimplification of the real situation, but does not affect the final results appreciably. In later chapters, we will learn how to evaluate the heat transfer coefficient from physical parameters and the geometry of the system.

PROBLEM 2.32

> **The tip of a soldering iron consists of a 0.6-cm-*OD* copper rod, 7.6 cm long. If the tip must be 204°C, what is the required minimum temperature of the base and the heat flow, in watts, into the base? Assume that $\bar{h}$ = 22.7 W/(m^2 K) and T_{air} = 21°C.**

GIVEN

- Tip of soldering iron consists of copper rod
- Outside diameter (D) = 0.6 cm = 0.006 m

- Length (L) = 7.6 cm = 0.076 m
- Temperature of the tip (T_L) = 204°C
- Heat transfer coefficient ($\bar{h}$) = 22.7 W/(m^2 K)
- Ambient temperature (T_∞) = 21°C

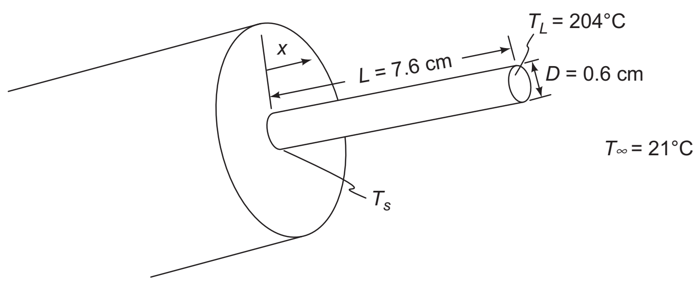
FIND

(a) Minimum temperature of the base (T_s)
(b) Heat flow into the base (q) in W

ASSUMPTIONS

- The tip is in steady state
- The thermal conductivity of copper is uniform and constant, i.e., not a function of temperature
- The copper tip can be treated as a fin

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
The thermal conductivity of copper (K) = 388 W/(m K) at 227°C

SOLUTION

(a) From Table 2.1, the temperature distribution for a fin with a uniform cross section and convection from the tip is

$$\frac{\theta}{\theta_s} = \frac{\cosh[m(L-x)] + \left(\frac{\bar{h}}{mk}\right)\sinh[m(L-x)]}{\cosh(mL) + \left(\frac{\bar{h}}{mk}\right)\sinh(mL)}$$

where $\theta = T - T_\infty$ and $\theta_s = \theta(0) = T_s - T_\infty$

$$L\,m = L\sqrt{\frac{\bar{h}\,P}{kA}} = L\sqrt{\frac{\bar{h}\,\pi D}{k\,\frac{\pi}{4}D^2}} = \sqrt{\frac{4\bar{h}}{kD}} = 0.076\text{ m}\sqrt{\frac{4\left(22.7\text{W/(m}^2\text{K)}\right)}{\left(388\text{W/(m K)}\right)(0.006\text{m})}}$$

$$L\,m = 0.076\text{ m}\left(6.25\frac{1}{\text{m}}\right) = 0.475$$

$$\frac{\bar{h}}{mK} = \frac{22.7\text{ W/(m}^2\text{K)}}{\left(6.25\frac{1}{\text{m}}\right)\left(388\text{W/(mK)}\right)} = 0.00936$$

Evaluating the temperature at $x = L$

$$\frac{\theta_L}{\theta_s} = \frac{T_L - T_\infty}{T_s - T_\infty} = \frac{\cosh(0) + 0.00936\sinh(0)}{\cosh(0.475) + 0.00936\sinh(0.475)} = 0.8932$$

Solving for the base temperature

$$T_s = T_\infty + \frac{T_L - T_\infty}{0.8932} = 21°\text{C} + \frac{204^\circ\text{C} - 21^\circ\text{C}}{0.8932} = 226°\text{C}$$

(b) To maintain steady state conditions, the rate of heat transfer into the base must be equal to the rate of heat loss from the rod. From Table 2.1, the rate of heat loss is

$$q_f = M\,\frac{\sinh(mL) + \left(\frac{\bar{h}}{mk}\right)\cosh(mL)}{\cosh(mL) + \left(\frac{\bar{h}}{mk}\right)\sinh(mL)} \quad \text{where } M = \sqrt{\bar{h}PkA\theta_s} = \sqrt{\bar{h}\,k\,\frac{\pi^2}{4}D^3}\;(T_s - T_\infty)$$

$$M = \sqrt{\left(22.7\text{W/(m}^2\text{K)}\right)\left(388\text{W/(m K)}\right)\frac{\pi^2}{4}(0.006\,\text{m})^3}\;(226°\text{C} - 21°\text{C}) = 14.045\text{ W}$$

$$q_f = 14.045\text{ W}\;\frac{\sinh(0.475) + .00936\cosh(0.475)}{\cosh(0.475) + .00936\sinh(0.475)} = 6.3\text{ W}$$

COMMENTS

A small soldering iron such as this will typically be rated at 30 W to allow for radiation heat losses and more rapid heat-up.

PROBLEM 2.33

One end of a 0.3 m long steel rod is connected to a wall at 204°C. The other end is connected to a wall which is maintained at 93°C. Air is blown across the rod so that a heat transfer coefficient of 17 W/(m^2 K) is maintained over the entire surface. If the diameter of the rod is 5 cm and the temperature of the air is 38°C, what is the net rate of heat loss to the air?

GIVEN

- A steel rod connected to walls at both ends
- Length of rod (L) = 0.3 m
- Diameter of the rod (D) = 5 cm = 0.05 m
- Wall temperatures: T_s = 204°C T_L = 93°C
- Heat transfer coefficient ($\bar{h}_c$) = 17 W/(m^2 K)
- Air temperature (T_∞) = 38°C

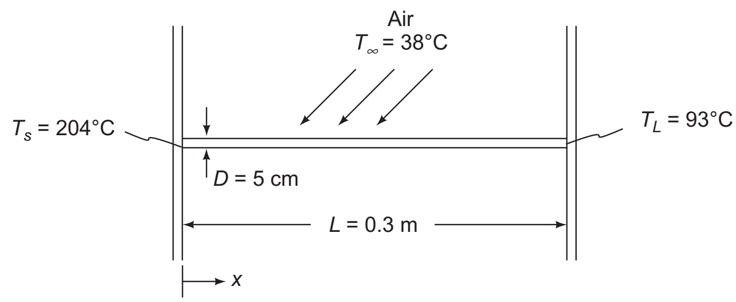
FIND

The net rate of heat loss to the air (q_f)

ASSUMPTIONS

- The wall temperatures are constant
- The system is in steady state
- The rod is 1% carbon steel
- The thermal conductivity of the rod is uniform and not dependent on temperature
- One dimensional conduction along the rod

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

The thermal conductivity of 1% carbon steel (k) = 43 W/(m K) (at 20°C)

SOLUTION

The rod can be idealized as a fin of uniform cross section with fixed temperatures at both ends. From Table 2.1 the rate of heat loss is

$$q_f = M \frac{\cos\text{h}\,(mL) - \left(\dfrac{\theta_L}{\theta_s}\right)}{\sin\text{h}\,(mL)}$$

where $\theta_L = T_L - T_\infty$ = 93°C – 38°C = 55°C and $\theta_s = T_s - T_\infty$ = 204°C – 38°C = 166°C

$$L\,m = L\sqrt{\frac{\bar{h}_c P}{kA}} = L\sqrt{\frac{\bar{h}_c \pi D}{k\dfrac{\pi}{4}D^2}} = L\sqrt{\frac{4\bar{h}_c}{kD}} = 0.3\text{ m}\sqrt{\frac{4\left(17\text{ W/(m}^2\text{ K)}\right)}{4(17\text{ W/(m K)})(0.05\text{ m})}} = 1.687$$

$$M = \sqrt{\bar{h}PkA}\ \ \theta_s = \sqrt{h\frac{\pi^2}{4}D^3k}\ \ \theta_s = \sqrt{\left(17\text{W/(m}^2\text{K)}\right)\frac{\pi^2}{4}(0.05\text{ m})^3\left(43\text{W/(m K)}\right)}\ \ (166°\text{C}) = 78.82\text{ W}$$

$$q_f = 78.82\text{ W}\ \frac{\cosh(1.687) - \dfrac{55}{166}}{\sinh(1.687)} = 74.4\text{ W}$$

COMMENTS

In a real situation the convective heat transfer coefficient will not be uniform over the circumference. It will be higher over the side facing the air stream. But because of the high thermal conductivity, the temperature at any given section will be nearly uniform.

PROBLEM 2.34

Both ends of a 0.6 cm copper U-shaped rod, as shown in the accompanying sketch, are rigidly affixed to a vertical wall, the temperature of which is maintained at 93°C. The developed length of the rod is 0.6 m and it is exposed to air at 38°C. The combined radiative and convective heat transfer coefficient for this system is 34 W/(m^2 K). (a) Calculate the temperature of the midpoint of the rod. (b) What will the rate of heat transfer from the rod be?

GIVEN

- U-shaped copper rod rigidly affixed to a wall
- Diameter (D) = 0.6 cm = 0.006 m
- Developed length (L) = 0.6 m
- Wall temperature is constant at (T_s) = 93°C
- Air temperature (T_∞) = 38°C
- Heat transfer coefficient ($\bar{h}$) = 34 W/(m^2 K)

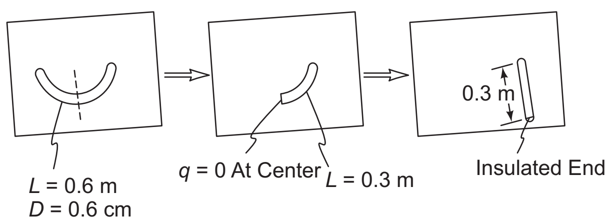
FIND

(a) Temperature of the midpoint (T_{Lf})
(b) Rate of heat transfer from the rod (M)

ASSUMPTIONS

- The system is in steady state
- Variation in the thermal conductivity of copper is negligible
- The U-shaped rod can be approximated by a straight rod of equal length
- Uniform temperature across any section of the rod

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, thermal conductivity of copper (k) = 396 W/(m^2 K) at 64°C

SOLUTION

By symmetry, the conduction through the rod at the center must be zero. Therefore, the rod can be thought of as two pin fins with insulated ends as shown in the sketch above.

(a) From Table 2.1, the temperature distribution for a fin of uniform cross section with an adiabatic tip is

$$\frac{\theta}{\theta_s} = \frac{\cosh[m(L_f - x)]}{\cosh(mL)}$$

where $\theta = T - T_\infty$, $\theta_s = T_s - T_\infty$ and L_f = length of the fin

$$m = \sqrt{\frac{\bar{h}P}{kA}} = \sqrt{\frac{\bar{h}\pi D}{k\left(\frac{\pi}{4}D^2\right)}} = \sqrt{\frac{4\bar{h}}{kD}} = \sqrt{\frac{4\left(34\,\text{W/(m}^2\text{K)}\right)}{(396\,\text{W/(m K)})(0.006\,\text{m})}} = 7.57\ \frac{1}{\text{m}}$$

Evaluating the temperature of the tip of the pin fin

$$\frac{\theta(L_f)}{\theta_s} = \frac{\cosh[m(L_f - L_f)]}{\cosh(m L_f)} = \frac{1}{\cosh(m L_f)}$$

The length of the fin is half of the wire length (L_f = 0.3 m)

$$\frac{\theta(L_f)}{\theta_s} = \frac{T(Lf) - T_\infty}{T_s - T_\infty} = \frac{1}{\cosh\left[7.57\frac{1}{\text{m}}(0.3\text{m})\right]} = 0.205$$

$$T(L_f) = 0.205\,(T_s - T_\infty) + T_\infty = 0.205\,(93°\text{C} - 38°\text{C}) + 38°\text{C} = 49.2°\text{C}$$

The temperature at the tip of the fin is the temperature at the midpoint of the curved rod (49.2°C).

(b) From Table 2.1, the heat transfer from the fin is

$$q_{\text{fin}} = M \tanh\,(m\,L_f)$$

$$\text{where } M = \sqrt{\bar{h} P k A}\;\;\theta_s = \sqrt{\bar{h}\,(\pi D) k \left(\frac{\pi}{4} D^2\right)}\;(T_s - T_\infty)$$

$$M = \sqrt{\frac{\pi}{4}\left(34\,\text{W/(m}^2\text{K)}\right)\left(396\,\text{W/(m K)}\right)(0.006\,\text{m})^3}\;(93°\text{C} - 38°\text{C}) = 4.653\text{ W}$$

$$\therefore q_{\text{fin}} = 4.653\text{ W} \tanh\left(7.57\frac{1}{\text{m}}\right)(0.3\text{ m}) = 4.56\text{ W}$$

The rate of heat transfer from the curved rod is approximately twice the heat transfer of the pin fin

$$q_{\text{rod}} = 2\,q_{\text{fin}} = 2(4.56\text{ W}) = 9.12\text{ W}$$

PROBLEM 2.35

A circumferential fin of rectangular cross section, 3.7 cm *OD* and 0.3 cm thick surrounds a 2.5 cm diameter tube. The fin is constructed of mild steel. Air blowing over the fin produces a heat transfer coefficient of 28.4 W/(m^2 K). If the temperatures of the base of the fin and the air are 260°C and 38°C, respectively, calculate the heat transfer rate from the fin.

GIVEN

- A mild steel circumferential fin of a rectangular cross section on a tube
- Tube diameter (D_t) = 2.5 cm = 0.025 m
- Fin outside diameter (D_f) = 3.7 cm = 0.037 m
- Fin thickness (t) = 0.3 cm = 0.003 m
- Heat transfer coefficient ($\bar{h}_c$) = 28.4 W/(m^2 K)
- Fin base temperature (T_s) = 260°C
- Air temperature (T_∞) = 38°C

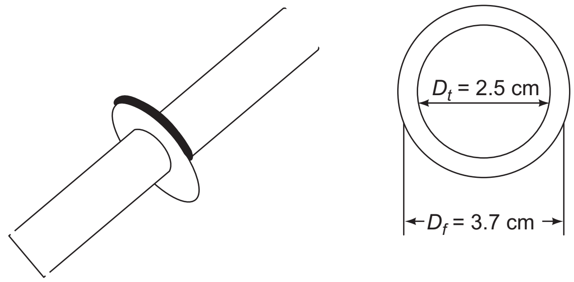
FIND

- The rate of heat transfer from the fin (q_{fin})

ASSUMPTIONS

- The system has reached steady state
- The mild steel is 1% carbon steel
- The thermal conductivity of the steel is uniform
- Radial conduction only (temperature is uniform across the cross section of the fin)
- The heat transfer from the end of the fin can be accounted for by increasing the length by half the thickness and assuming the end is insulated

SKETCH



PROPERTIES AND CONSTANTS

Thermal conductivity of 1% carbon steel (k) = 43 W/(m K) at 20°C

SOLUTION

The rate of heat transfer for the fin can be calculated using the fin efficiency determined from the efficiency graph for this geometry, Figure 2.17.

The length of a fin (L) = $(D_f - D_t)/2$ = 0.006 m

The parameters needed are

$$r_i = \frac{D_t}{2} = 0.125 \text{ m} \qquad r_o = \frac{D_t}{2} + L = 0.125 \text{ m} + 0.006 \text{ m} = 0.0185 \text{ m}$$

$$\left(r_o + \frac{t}{2} - r_i\right)^{\frac{3}{2}} = \left(\frac{2\bar{h}_c}{k\,t(r_o - r_i)}\right)^{\frac{1}{2}} \left(0.0815\,\text{m} + \frac{0.003\,\text{m}}{2} - 0.0125 \text{ m}\right)^{\frac{3}{2}}$$

$$\left(\frac{2\left(28.4\text{w/(m}^2\text{K)}\right)}{(43\text{W/(m K)})\,(0.003 \text{ m})(0.0185\text{m} - 0.0125\,\text{m})}\right)^{\frac{1}{2}} = 0.176$$

$$\frac{\left(r_o + \frac{t}{2}\right)}{r_i} = \frac{0.0185\,\text{m} + 0.0015\,\text{m}}{0.0125 \text{ m}} = 1.6$$

From Figure 2.17, the fin efficiency for these parameters is:

$$\eta_f = 98\%$$

The rate of heat transfer from the fin is

$$q_{\text{fin}} = \eta_f\, \bar{h}_c\, A_{\text{fin}}\, (T_s - T_\infty) = \eta_f\, \bar{h}_c\, 2\pi \left[\left(r_o + \frac{t}{2}\right)^2 - r_i^2\right] (T_s - T_\infty)$$

$$q_{\text{fin}} = (0.98)\, \left(28.4\text{W/(m}^2\text{K)}\right) 2\pi\,[(0.085 \text{ m} + 0.0015 \text{ m})^2 - (0.0125 \text{ m})^2]\, (260°\text{C} - 38°\text{C}) = 9.46 \text{ W}$$

PROBLEM 2.36

A turbine blade 6.3 cm long (see sketch on p. 156), with cross-sectional area $A = 4.6 \times 10^{-4}$ m^2 and perimeter P = 0.12 m, is made of stainless steel (k = 18 W/(m K). The temperature of the root, T_s, is 428°C. The blade is exposed to a hot gas at 871°C, and the heat transfer coefficient h is 454 W/(m^2 K). Determine the temperature of the blade tip and the rate of heat flow at the root of the blade. Assume that the tip is insulated.

GIVEN

- Stainless steel turbine blade
- Length of blade (L) = 6.3 cm = 0.063 m
- Cross-sectional area (A) = 4.6×10^{-4} m^2
- Perimeter (P) = 0.12 m
- Thermal conductivity (k) = 18 W/(m K)
- Temperature of the root (T_s) = 482°C
- Temperature of the hot gas (T_∞) = 871°C
- Heat transfer coefficient ($\bar{h}_c$) = 454 W/(m^2 K)

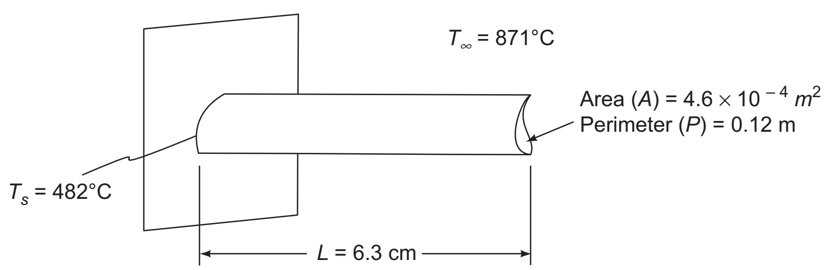
FIND

(a) The temperature of the blade tip (T_L)
(b) The rate of heat flow (q) at the roof of the blade

ASSUMPTIONS

- Steady state conditions prevail
- The thermal conductivity is uniform
- The tip is insulated
- The cross-section of the blade is uniform
- One dimensional conduction

SKETCH



SOLUTION

(a) The temperature distribution in a fin of uniform cross-section with an insulated tip, from Table 2.1, is

$$\frac{\theta}{\theta_s} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

where $$m = \sqrt{\frac{\bar{h}\,P}{k\,A}} = \sqrt{\frac{454\,\text{W/(m}^2\text{K)}(0.12\text{ m})}{18\,\text{W/(m K)}(4.6\times10^{-4}\text{ m}^2)}} = 81.1\,\frac{1}{\text{m}}$$

$$\theta = T - T_\infty$$

At the blade tip, $x = L$, therefore

$$\frac{\theta_L}{\theta_s} = \frac{T_L - T_\infty}{T_s - T_\infty} = \frac{\cosh[m(0)]}{\cosh(mL)} = \frac{1}{\cosh(mL)}$$

$$T_L = T_\infty + \frac{T_s - T_\infty}{\cosh(mL)} = 871°\text{C} + \frac{482^\circ\text{C} - 871^\circ\text{C}}{\cosh\left[\left(81.1\frac{1}{\text{m}}\right)(0.063\,\text{m})\right]} = 866°\text{C}$$

(b) The rate of heat transfer from the fin is given by Table 2.1 to be

$$q = M \tanh(m L)$$

where $$M = \sqrt{\bar{h}_c P k A}\ \theta_s$$

$$M = \sqrt{454\text{W/(m}^2\text{K)}(0.12\text{ m})(18\text{W/(m K)})(4.6\times10^{-4}\text{m}^2)}\ (482°\text{C} - 871°\text{C}) = -261\text{ W}$$

$$\therefore q = (-261\text{ W})\tanh\left[81.1\frac{1}{\text{m}}(0.063\text{ m})\right] = -261\text{ W (out of the blade)}$$

COMMENTS

In a real situation, the heat transfer coefficient will vary over the surface with the highest value near the leading edge. But because of the high thermal conductivity of the blade, the temperature at any section will be esentially uniform.

PROBLEM 2.37

To determine the thermal conductivity of a long, solid 2.5 cm diameter rod, one half of the rod was inserted into a furnace while the other half was projecting into air at 27°C. After steady state had been reached, the temperatures at two points 7.6 cm apart were measured and found to be 126°C and 91°C, respectively. The heat transfer coefficient over the surface of the rod exposed to the air was estimated to be 22.7 W/(m^2 K). What is the thermal conductivity of the rod?

GIVEN

- A solid rod, one half inserted into a furnace
- Diameter of rod (D) = 2.5 cm = 0.25 m
- Air temperature (T_∞) = 27°C
- Steady state has been reached
- Temperatures at two points 7.6 cm apart
  - T_1 = 126°C
  - T_2 = 91°C
- The heat transfer coefficient ($\bar{h}_c$) = 22.7 W/(m^2 K)

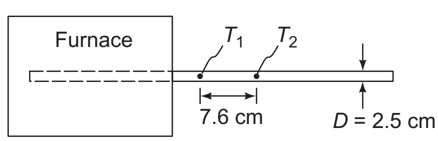
FIND

- The thermal conductivity (k) of the rod

ASSUMPTIONS

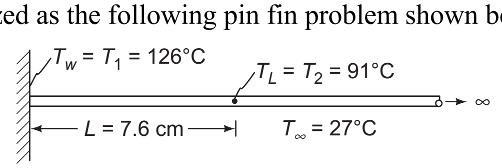
- Uniform thermal conductivity
- One dimensional conduction along the rod
- The rod approximates a fin of infinite length protruding out of the furnace

SKETCH



SOLUTION

This problem can be visualized as the following pin fin problem shown below



The fin is of uniform cross section, therefore Table 2.1 can be used. The temperature distribution for a fin of infinite length, from Table 2.1, is

$$\frac{\theta}{\theta_s} = e^{-mx}$$

$$\text{where } m = \sqrt{\frac{\bar{h}_c P}{kA}} = \sqrt{\frac{\bar{h}_c \pi D}{k \frac{\pi}{2} D^2}} = \sqrt{\frac{4\bar{h}_c}{kD}}$$

Substituting this into the temperature distribution and solving for k

$$\frac{\theta}{\theta_s} = \exp\left(-\sqrt{\frac{4\bar{h}_c}{kD}}\, x\right) \Rightarrow k = \frac{4\bar{h}_c}{D\left(\frac{\ln\left(\frac{\theta}{\theta_s}\right)}{x}\right)^2}$$

at $x = L$

$$\theta_L = T_L - T_\infty = 91°C - 27°C = 64°C$$

$$\theta_s = T_W - T_\infty = 126°C - 27°C = 99°C$$

$$\frac{\theta_L}{\theta_s} = \frac{64}{99} = 0.6465$$

Therefore

$$k = \frac{4\left(22.7 \text{w/(m}^2\text{K)}\right)}{0.025\left[\frac{\ln(0.6465)}{0.076\text{m}}\right]^2} = 110 \text{ W/(m K)}$$

COMMENTS

Note that this procedure can only be used if the assumption of an infinite length fin is valid. Otherwise, the location of the temperature measurements along the fin must be specified to determine the thermal conductivity.

PROBLEM 2.38

> **Heat is transferred from water to air through a brass wall (k = 54 W/(m K)). The addition of rectangular brass fins, 0.08 cm thick and 2.5 cm long, spaced 1.25 cm apart, is contemplated. Assuming a water-side heat transfer coefficient of 170 W/(m^2 K) and an air-side heat transfer coefficient of 17 W/(m^2 K), compare the gain in heat transfer rate achieved by adding fins to: (a) the water side, (b) the air side, and (c) both sides. (Neglect temperature drop through the wall.)**

GIVEN

- A brass wall with brass fins between air and water
- Thermal conductivity of the brass (k) = 54 W/(m K)
- Fin thickness (t) = 0.08 cm = 0.0008 m
- Fin length (L) = 2.5 cm = 0.025 m
- Fin spacing (d) = 1.25 cm = 0.125 m
- Water-side heat transfer coefficient ($\bar{h}_{cw}$) = 170 W/(m^2 K)
- Air-side heat transfer coefficient ($\bar{h}_{ca}$) = 17 W/(m^2 K)

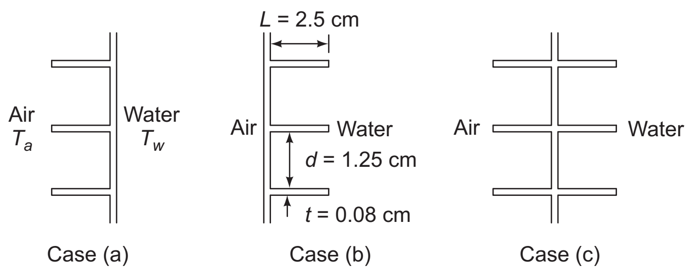
FIND

Compare the heat transfer rate with fins added to
(a) the water side, $q_{(a)}$
(b) the air side, $q_{(b)}$
(c) both sides, $q_{(c)}$

ASSUMPTIONS

- The thermal resistance of the wall is negligible
- Steady state conditions prevail
- Constant thermal conductivity
- One dimensional conduction
- Heat transfer from the tip of the fins is negligible

SKETCH



SOLUTION

The fins are of uniform cross-section, therefore Table 2.1 may be used. To simplify the analysis, the heat transfer from the end of the fin will be neglected. For a fin with adiabatic tip, the rate of heat transfer is

$$q_f = M \tanh(m L)$$

where $$M = \sqrt{\bar{h}_c P k A}\ \theta_s = \sqrt{\bar{h}_c (2w) k (wt)}\ \theta_s = w\ \sqrt{2\bar{h}_c k t}\ \theta_s$$

$$m = \sqrt{\frac{\bar{h}_c P}{kA}} = \sqrt{\frac{\bar{h}_c (2w)}{kwt}} = \sqrt{\frac{2\bar{h}_c}{kt}}$$

The number of fins per square meter of wall is

$$\frac{\text{number of fins}}{\text{m}^2} = \frac{1}{(0.0133\,\text{m/fin})1\text{m width})} = 75.2\ \text{fins/m}^2$$

Fraction of the wall area not covered by fins is

$$\frac{A_{\text{bare}}}{A_n} = \frac{1\text{m}^2 - 75.2(1\text{m width})(0.008\,\text{m})}{\text{m}^2} = 0.939 \approx 0.94$$

The rate of heat transfer from the wall with fins is equal to the sum of the heat transfer from the bare wall and from the fins

$$q = \bar{h}_c\ A_{\text{bare}}\ \theta_s + (\text{number of fins})\ [\text{M} \tanh(m\,L)]$$

$$q = \left[\bar{h}_c A_{\text{bare}} + 75.2\, A_w \frac{M}{\theta_s} \tanh(mL)\right]\theta_s = \frac{\theta_s}{R_c}$$

where A_w is the total base area, i.e., with fins removed.

Therefore, the thermal resistance of a wall with fins based on a unit of base area is

$$R_c = \frac{1}{A_W\left[\bar{h}_c \frac{A_{\text{bare}}}{A_w} + 75.2\frac{M}{\theta_s}\tanh(mL)\right]}$$

For fins on the water side

$$\frac{M_w}{\theta_s} = 1 \text{ m width } \sqrt{170\text{W/(m}^2\text{K)}(2)(54\text{W/(m K)})\ (0.0008\text{ m})} = 3.832\text{ W/K}$$

$$m_{\dot{w}} = \sqrt{\frac{2\left(170\text{W/(m}^2\text{K)}\right)}{54\text{W/(m K)(.0008 m)}}} = 88.72\ \frac{1}{\text{m}}$$

$$\tan\text{h}\ (m_a\, L) = \tanh\left(88.72\frac{1}{\text{m}}\right)\ (0.025\text{ m}) = 0.977$$

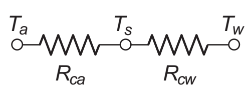
For fins on the air side

$$\frac{M_a}{\theta_s} = 1 \text{ m width } \sqrt{\left(17\text{W(m}^2\text{K)}\right)(2)(54\text{W/(m K)})(0.0008)} = 1.212\text{ W/K}$$

$$m_a = \sqrt{\frac{2\left(17\text{W/(m}^2\text{K)}\right)}{(54\text{W/(m K)})\,(0.0008\text{ m})}} = 28.05\ \frac{1}{\text{m}}$$

$$\tan\text{h}\ m_a\, L = \tan\text{h}\ \left(28.05\frac{1}{\text{m}}\right)\ (0.025\text{ m}) = 0.605$$

The thermal circuit for the problem is



The values of thermal resistances with and without fins are

$$(R_{ca})_{\text{nofins}} = \frac{1}{A_w\bar{h}_{ca}} = \frac{1}{A_w\left(17\text{W/(m}^2\text{K)}\right)} = \frac{1}{A_w}\ 0.0588\ (\text{m}^2\text{ K)/W}$$

$$(R_{cw})_{\text{nofins}} = \frac{1}{A_w\bar{h}_{cw}} = \frac{1}{A_w\left(170\text{ W/(m}^2\text{K)}\right)} = \frac{1}{A_w}\ 0.00588\ (\text{m}^2\text{ K)/W}$$

$$(R_{ca})_{\text{fins}} = \frac{1}{A_w\left[17\text{ W/(m}^2\text{K)(0.94)} + 75.2\text{ m}^{-2}\,(1.212\text{W/K})\,(0.605)\right]} = \frac{1}{A_w}\ 0.0141\ (\text{m}^2\text{ K)/W}$$

$$(R_{cw})_{\text{fins}} = \frac{1}{A_w\left[170\text{ W/(m}^2\text{K)(0.94)} + 75.2\text{ m}^{-2}\,(3.832\text{ W/K})\,(0.977)\right]} = \frac{1}{A_w}\ 0.00227\ (\text{m}^2\text{ K)/W}$$

(a) The rate of heat transfer with fins on the water side only is

$$q_{(a)} = \frac{\Delta T}{(R_{ca})_{\text{no fins}} + (R_{cw})_{\text{fins}}}$$

$$\frac{q_{(a)}}{A_w} = \frac{\Delta T}{(0.0588 + 0.00227)(\text{m}^2\ \text{K})/\text{W}} = 16.4\ \Delta\ T\ \ \text{W}/(\text{m}^2\,\text{K})$$

(b) The rate of heat transfer with fins on the air side only is

$$q_{(b)} = \frac{\Delta T}{(R_{ca})_{\text{fins}} + (R_{cw})_{\text{no fins}}}$$

$$\frac{q_{(b)}}{A_w} = \frac{\Delta T}{(0.0141 + 0.00588)(\text{m}^2\ \text{K})/\text{W}} = 50.1\ \Delta\ T\ \text{W}/(\text{m}^2\,\text{K})$$

(c) With fins on both sides, the rate of heat transfer is

$$q_{(c)} = \frac{\Delta T}{(R_{ca})_{\text{fins}} + (R_{cw})_{\text{no fins}}}$$

$$\frac{q_{(c)}}{A_w} = \frac{\Delta T}{(0.0141 + 0.00227)(\text{m}^2\text{K})/\text{W}} = 61.1\ \Delta\ T\ \text{W}/(\text{m}^2\,\text{K})$$

As a basis of comparison, the rate of heat transfer without fins on either side is:

$$\frac{q}{A_w} = \frac{\Delta T}{(0.0588 + 0.00588)(\text{m}^2\text{W})/\text{K}} = 15.5\ \Delta\ T\ \text{W}/(\text{m}^2\,\text{K})$$

The following percent increase over the no fins case occurs

Case	% Increase
(a) fins on water side	5.8
(b) fins on air side	223
(c) fins on both sides	294

COMMENTS

Placing the fins on the side with the larger thermal resistance, i.e., the air side, has a much greater effect on the rate of heat transfer.

The small gain in heat transfer rate achieved by placing fins on the water side only would most likely not be justified due to the high cost of attaching the fins.

PROBLEM 2.39

The wall of a liquid-to-gas heat exchanger has a surface area on the liquid side of 1.8 m^2 (0.6m $\times$ 3m) with a heat transfer coefficient of 255 W/(m^2 K). On the other side of the heat exchanger wall flows a gas, and the wall has 96 thin rectangular steel fins 0.5 cm thick and 1.25 cm high [k = 3 W/(m K)]. The fins are 3 m long and the heat transfer coefficient on the gas side is 57 W/(m^2 K). Assuming that the thermal resistance of the wall is negligible, determine the rate of heat transfer if the overall temperature difference is 38°C.

GIVEN

- The wall of a heat exchanger has 96 fins on the gas side
- Surface area on the liquid side (A_L) = 1.8 m^2 (0.6 m $\times$ 3 m)
- Heat transfer coefficient on the liquid side (h_{cL}) = 255 W/(m^2 K)
- The wall has 96 thin steel fins 0.5 cm thick and 1.25 cm high
- Thermal conductivity of the steel (k) = 3 W/(m K)
- Fin length (w) = 3 m, Fin height (L) = 1.25 cm = 0.0125 m
- Fin thickness (t) = 0.5 cm = 0.005 m
- Heat transfer coefficient on the gas side (h_{cg}) = 57 W/(m^2 K)
- The overall temperature difference (ΔT) = 38°C

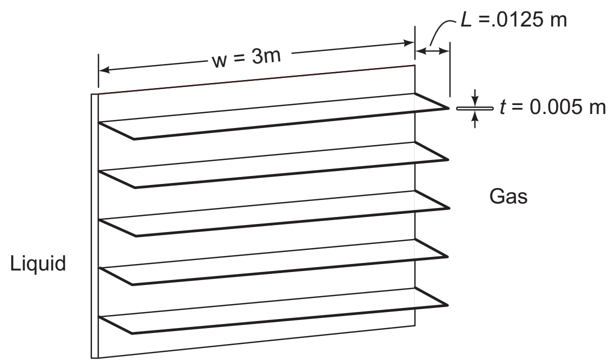
FIND

- The rate of heat transfer (q)

ASSUMPTIONS

- The thermal resistance of the wall is negligible
- The heat transfer through the wall is steady state
- The thermal conductivity of the steel is constant

SKETCH



A Section of the Wall

SOLUTION

The heat transfer from a single fin can be calculated from Table 2.1 for a fin with convection from the tip

$$q_f = M \frac{\sinh(mL) + \left(\frac{h_c}{mk}\right)\cosh(mL)}{\cosh(mL) + \left(\frac{h_c}{mk}\right)\sinh(mL)}$$

where $$m = \sqrt{\frac{\bar{h}_c P}{kA}} = \sqrt{\frac{\bar{h}_c\,(2t+2w)}{k\,(wt)}} = \sqrt{\frac{57\,\text{W/(m}^2\text{K)}(6\,\text{m}+0.01\text{ m})}{3\,\text{W/(m K)}(3\,\text{m})(0.005\text{ m})}} = 87.25\ \frac{1}{\text{m}}$$

$$mL = 87.25\ \frac{1}{\text{m}}\ (0.0125\text{ m}) = 1.091 \text{ and } \frac{\bar{h}_c}{mk} = \frac{57\,\text{W/(m}^2\text{K)}}{87.25\frac{1}{\text{m}}(3\,\text{W/(m K)})} = 0.2178$$

$$M = \sqrt{\bar{h}_c PkA}\ \ \theta_s = \sqrt{\left(57\text{ W/(m}^2\text{K)}\right)(6.01\,\text{m})(3\text{W/(m K)})(3\,\text{m})(0.005\,\text{m})}\ (T_s - T_g) = 3.926\ (T_s - T_g)\text{W/K}$$

$$q_f = \left(3.926(T_s - T_g)\,\text{W/K}\right) \frac{\sinh(1.091) + 0.2178\cos\text{h}(1.091)}{\cosh(1.091) + 0.2178\sin\text{h}(1.091)} = 3.395\ (T_s - T_g)\ \text{W/K}$$

The rate of heat transfer on the gas side is the sum of the convection from the fins and the convection from the bare wall between the fins. The bare area is

$$A_{\text{bare}} = A_{\text{wall}} - \text{(number of fins) (Area of one fin)}$$

$$= 1.8\ \text{m}^2 - \text{(96 fins) [(3 m) (0.005 m)/fin]} = 0.36\ \text{m}^2$$

The total rate of heat transfer to the gas is

$$q_g = q_{\text{bare}} + \text{(number of fins)}\ q_f = \bar{h}_{cg}\, A_{\text{bare}}\,(T_s - T_g) + 96(3.395)\ (T_s - T_g)\ \ \text{W/K}$$

$$q_g = \left[57\ \text{W/(m}^2\text{K)}(0.36\,\text{m}^2) + 96(3.395)\right]\ (T_s - T_g)\ \text{W/K} = 346.4\ (T_s - T_g)\ \text{W/K} = \frac{T_s - T_g}{R_g}$$

The thermal resistance on the gas side is

$$R_g = \frac{1}{346.4\ \text{K/W}} = 0.002887\ \text{K/W}$$

The thermal resistance on the liquid side is

$$R_L = \frac{1}{\bar{h}_{cL} A_w} = \frac{1}{255\,\text{W/(m}^2\text{K)}\,(1.8\ \text{m}^2)} = 0.002179\ \text{K/W}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{tot}}} = \frac{\Delta T}{R_{\text{g}} + R_L} = \frac{38^{\text{o}}\text{C}}{(0.002887 + 0.002179)\text{K/W}} = 7500\ \text{W}$$

COMMENTS

Note that despite the much lower heat transfer coefficient on the gas side, the thermal resistance is no larger than on the liquid side. This is the result of balancing the fin geometries which is a desirable situation from the thermal design perspective. Adding fins on the liquid side would not increase the rate of heat transfer appreciably.

PROBLEM 2.40

The top of a 30 cm I-beam is maintained at a temperature of 260°C, while the bottom is at 93°C. The thickness of the web is 1.25 cm Air at 260°C is blowing along the side of the beam so that $\bar{h}$ = 40 W/(m^2 K). The thermal conductivity of the steel may be assumed constant and equal to 43 W/(m K). Find the temperature distribution along the web from top to bottom and plot the results.

GIVEN

- A steel 30 cm I-beam
- Temperature of the top (T_L) = 260°C
- Temperature of the bottom (T_s) = 93°C
- Thickness of the web (t) = 1.25 cm
- Air temperature (T_∞) = 260°C
- Heat transfer coefficient ($\bar{h}_c$) = 40 W/(m^2 K)
- Thermal conductivity of the steel (k) = 43 W/(m K)

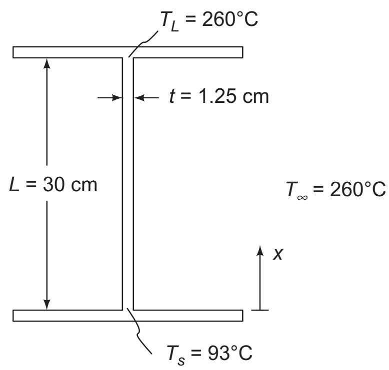
FIND

- The temperature distribution along the web and the plot the results

ASSUMPTIONS

- The thermal conductivity of the steel is uniform
- The beam has reached steady state conditions
- One dimensional through the web
- The beam is very long compared to the web thickness

SKETCH



SOLUTION

The web of the I beam can be thought of as a fin with a uniform rectangular cross section and a fixed tip temperature. From Table 2.1, the temperature distribution along the web is

$$\frac{\theta}{\theta_s} = \frac{\left(\frac{\theta_L}{\theta_s}\right)\sinh(m\,x) + \sinh[m(L-x)]}{\sinh(mL)}$$

where $\theta = T - T_\infty$

$$m = \sqrt{\frac{\bar{h}_c P}{kA}} = \sqrt{\frac{\bar{h}_c 2(w+t)}{kwt}} = \sqrt{\frac{2\bar{h}_c}{kt}} = \sqrt{\frac{2(40\ \text{W}(\text{m}^2\text{K}))}{(43\ \text{W/(m K)})\,(1.25\times 10^{-2}\,\text{m})}} = 12.2\ 1/\text{m}$$

$$mL = 12.2\ \sinh(mL)$$

$$= 3.66$$

$$\theta_s = T_s - T_\infty \qquad\qquad \theta_L - T_L - T_\infty = 0$$

$$= 93°\text{C} - 260°\text{C} = -167°\text{C}$$

Substitute these into the temperature distribution

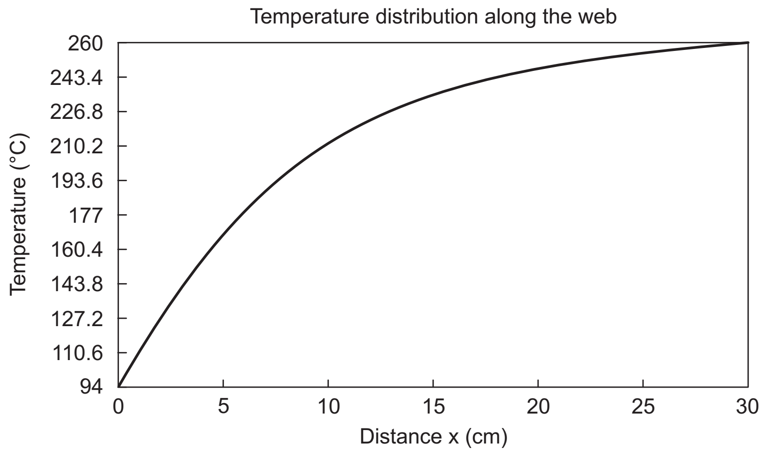
$$\frac{T_x - T_\infty}{\theta_s} = 0.0512\ \sinh\,[12.2\,(0.3 - x)]\ (x \text{ in m})$$

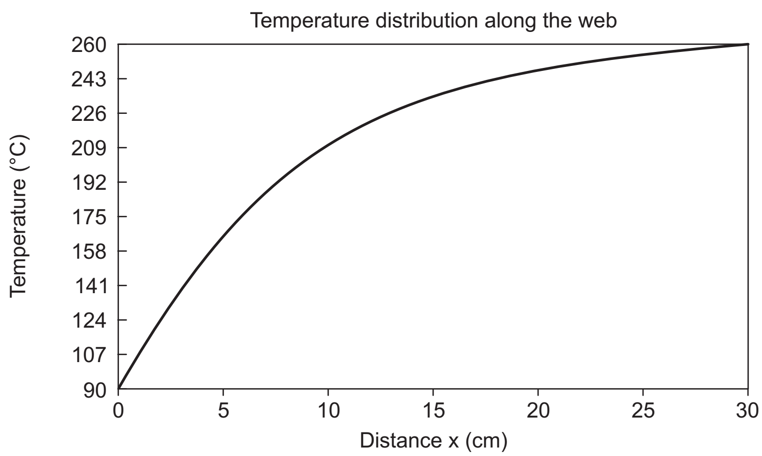
or

$$\frac{T_x - T_\infty}{\theta_s} = 0.0512\ \sinh\left[\frac{12.2}{100}(30 - x)\right]\ (x \text{ in cm})$$

$$\therefore\ T_x = 260 - 8.55\ \sinh\,[0.122\,(30 - x)]\ (x \text{ in cm})$$

This temperature distribution is plotted below





COMMENTS

In a real situation, the heat transfer coefficient is likely to vary with distance and this would require a numerical solution.

PROBLEM 2.41

> **The handle of a ladle used for pouring molten lead is 30 cm long. Originally the handle was made of 1.9 × 1.25 cm mild steel bar stock. To reduce the grip temperature, it is proposed to form the handle of tubing 0.15 cm thick to the same rectangular shape. If the average heat transfer coefficient over the handle surface is 14 W/(m^2 K), estimate the reduction of the temperature at the grip in air at 21°C.**

GIVEN

- A steel handle of a ladle used for pouring molten lead
- Handle length (L) = 30 cm = 0.3 m
- Original handle: 1.9 by 1.25 cm mild steel bar stock
- New handle: tubing 0.15 cm thick with the same shape
- The average heat transfer coefficient ($\bar{h}_c$) = 14 W/(m^2 K)
- Air temperature (T_∞) = 21°C

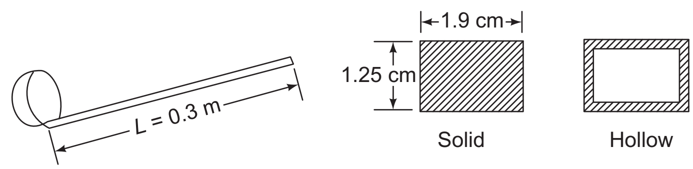
FIND

- The reduction of the temperature at the grip

ASSUMPTIONS

- The lead is at the melting temperature
- The handle is made of 1% carbon steel
- The ladle is normally in steady state during use
- The variation of the thermal conductivity is negligible
- One dimensional conduction
- Heat transfer from the end of the handle can be neglected

SKETCH

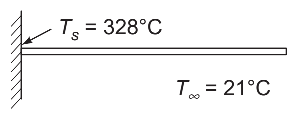


PROPERTIES

From Appendix 2, Tables 10 and 12
Thermal conductivity of 1% carbon steel = 43 W/(m K) at 20°C
Melting temperature of lead (T_s) = 601 K = 328°C

SOLUTION

The ladle handle can be treated as a fin with an adiabatic end as shown below



The temperature distribution in the handle, from Table 2.1 is

$$\frac{\theta}{\theta_s} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

where $\theta = T(x) - T_\infty$ $\qquad \theta_s = T_s - T_\infty = 328°\text{C} - 21°\text{C} = 307°\text{C}$

$$m = \sqrt{\frac{\bar{h}_c P}{kA}}$$

where $P = 2w + 2t = 2(0.019 \text{ m}) + 2(0.0125 \text{ m}) = 0.063 \text{ m}$

The only difference in the two handles is the cross-sectional area
Solid handle

$$A_s = wt = (0.019 \text{ m})\,(0.0125 \text{ m}) = 0.0002375 \text{ m}^2$$

$$m\,L = 0.3 \text{ m}\sqrt{\frac{14 \text{ W/(m}^2\text{K)}(0.063 \text{ m})}{43 \text{ W/(m K)}(0.0002375 \text{ m}^2)}} = 2.788$$

$$\frac{\theta_L}{\theta_s} = \frac{\cosh(0)}{\cosh(2.788)} = 0.1266 \Rightarrow \theta_L = T_L - T_\infty = 0.1226\ \theta_s$$

$$\therefore\ T_L = T_\infty + 0.1266\ \theta_s = 21°\text{C} + 0.1266\ (307°\text{C}) = 60°\text{C}$$

Hollow handle

$$A_H = wt - [w - 2(0.0015\text{ m})]\,[t - 2(0.0015\text{ m})]$$

$$= (0.019\text{ m})\,(0.0125\text{ m}) - (0.016)\,(0.0095\text{ m}) = 0.0000855\text{ m}^2$$

$$m\,L = 0.3\text{ m}\sqrt{\frac{14\text{ W/(m}^2\text{K)}(0.063\text{ m})}{43\text{ W/(m K)}(0.0000855\text{ m}^2)}} = 4.65$$

$$\frac{\theta_L}{\theta_s} = \frac{\cosh(0)}{\cosh(4.647)} = 0.0192$$

$$T_L = T_\infty + 0.01919\ \theta_s = 21°\text{C} + 0.0192\ (307°\text{C}) = 27°\text{C}$$

The temperature of the grip is reduced 33°C by using the hollow handle.

COMMENTS

The temperature of the hollow handle would be comfortable to the bare hand, therefore no insulation is required. This will reduce the cost of the item without reducing utility.

PROBLEM 2.42

A 0.3-cm thick aluminum plate has rectangular fins on one side, 0.16 × 0.6 cm, spaced 0.6 cm apart. The finned side is in contact with low pressure air at 38°C and the average heat transfer coefficient is 28.4 W/(m^2 K). On the unfinned side water flows at 93°C and the heat transfer coefficient is 283.7 W/(m^2 K). (a) Calculate the efficiency of the fins (b) calculate the rate of heat transfer per unit area of wall and (c) comment on the design if the water and air were interchanged.

GIVEN

- Aluminum plate with rectangular fins on one side
- Plate thickness (D) = 0.3 cm = 0.003 m
- Fin dimensions ($t \times L$) = 0.0016 m × 0.006 m
- Fin spacing (s) = 0.006 m apart
- Finned side
  - Air temperature (T_a) = 38°C
  - Heat transfer coefficient ($\bar{h}_a$) = 28.4 W/(m^2 K)
- Unfinned side
  - Water temperature (T_w) = 93°C
  - Heat transfer coefficient ($\bar{h}_w$) = 283.7 W/(m^2 K)

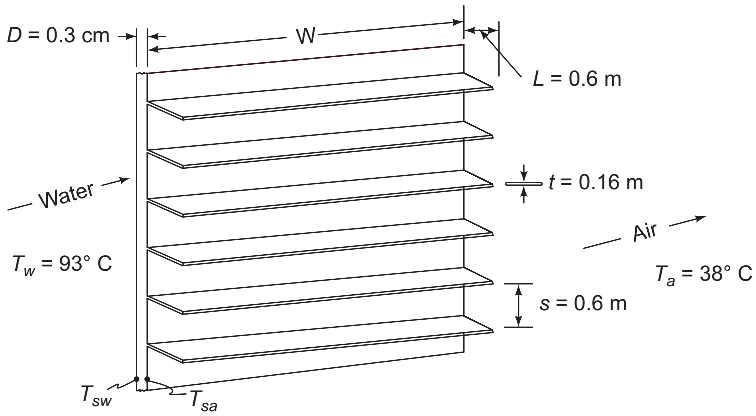
FIND

(a) The fin efficiency (η_f)
(b) Rate of heat transfer per unit wall area (q/A_w)
(c) Comment on the design if the water and air were interchanged

ASSUMPTIONS

- The aluminum is pure
- Width of fins is much longer than their thickness
- The system has reached steady state
- The thermal conductivity of the aluminum is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12

The thermal conductivity of aluminum (k) = 238 W/(m K) at 65°C

SOLUTION

(a) The fin efficiency is defined as the actual heat transfer rate divided by the rate of heat transfer if the entire fin were at the wall temperature. Since the fin is of uniform cross section, Table 2.1 can be used to find an expression for the heat transfer from a fin with a convection from the tip

$$q_f = M\,\frac{\sinh(mL) + \left(\frac{\bar{h}_a}{mk}\right)\cosh(mL)}{\cosh(mL) + (\bar{h}_a mk)\sinh(mL)}$$

where $$m^2 = \frac{\bar{h}_a P}{kA} = \frac{\bar{h}_a 2w}{k(wt)} = \frac{2\bar{h}_a}{kt}$$

$$M = \sqrt{\bar{h}_a PkA}\;\theta_s = w\sqrt{2\bar{h}_a tk}\;\theta_s$$

where $$\theta_s = T_{sa} - T_a$$

If the entire fin were at the wall temperature (T_{sa}) the rate of heat transfer would be

$$q'_f = \bar{h}_a A_f (T_{sa} - T_a) = \bar{h}_a w(2L + t)\,(T_{sa} - T_a)$$

The fin efficiency is

$$\eta_f = \frac{q_f}{q'_f} = \frac{\left[M\,\dfrac{\sinh(mL) + \left(\frac{\bar{h}_a}{mk}\right)\cosh(mL)}{\cosh(mL) + \left(\frac{\bar{h}_a}{mk}\right)\sinh(mL)}\right]}{\bar{h}_a w(2L + t)(T_{sa} + T_a)}$$

$$m = \sqrt{\frac{2\bar{h}_a}{kt}} = \sqrt{\frac{2\left(28.4\text{ W/(m}^2\text{K)}\right)}{238\text{ W/(m K)}(0.0016\text{ m})}} = 12.2\;\frac{1}{\text{m}}$$

$$m\,L = 12.2\;\frac{1}{\text{m}}\;(0.006\text{ m}) = 0.0733$$

$$M = w\,(T_{sa} - T_a)\ \sqrt{2\left(28.4\,\mathrm{W/(m^2K)}\right)(0.0016\ \mathrm{m})\left(238\,\mathrm{W/(m^2K)}\right)}\ = 4.65\ w\,(T_{sa} - T_w)\mathrm{s\ W/(mK)}$$

$$\frac{\bar{h}_a}{mk} = \frac{28.4\ \mathrm{W/(m^2K)}}{12.2\,\frac{1}{\mathrm{m}}(238\mathrm{W/(m\ K)})} = 0.0098$$

$$\eta_f = \frac{4.65\mathrm{W/(m^2K)}\left(\dfrac{\sinh(0.0733) + 0.00977\cosh(0.0733)}{\cosh(0.0733) + 0.00977\sinh(0.0733)}\right)}{28.4\ \mathrm{W/(m^2K)}\ [(2)0.006\ \mathrm{m} + 0.0016\,\mathrm{m}]} = 0.998$$

(b) The heat transfer to the air is equal to the sum of heat transfer from the fins and the heat transfer from the wall area not covered by fins.

The number of fins per meter height is

$$\frac{1\,\mathrm{m}}{0.076\ \mathrm{m/fin}} = 131.6\ \mathrm{fins}$$

The wall area not covered by fins per m^2 of total wall area is

$$A_{\mathrm{bare}} = 1\ \mathrm{m^2} - (131.\ 6\ \mathrm{fins})\ (0.0016\ \mathrm{m/fin})\ \ (1\ \mathrm{m\ width}) = 0.789\ \mathrm{m^2}$$

The surface area of the fins per m^2 of wall area is

$$A_{\mathrm{fins}} = 131.6\ \mathrm{fins}\ (2(0.006\ \mathrm{m}) + 0.0016\ \mathrm{m})\ (1\ \mathrm{m\ width}) = 1.79\ \mathrm{m^2}$$

The rate of heat transfer to the air is

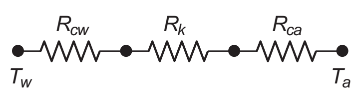
$$q_a = \bar{h}_a\ A_{\mathrm{bare}}\,(T_{sa} - T_a) + \bar{h}_a\ \ \eta_{\mathrm{f}}\,A_{\mathrm{fins}}\,(T_{sa} - T_a)$$

$$q_a = \bar{h}_a\ (A_{\mathrm{bare}} + \eta_f\,A_{\mathrm{fins}})\,(T_{sa} - T_a) = \frac{T_{sa} - T_a}{R_{ca}}$$

Therefore, the resistance to heat transfer on the air side (R_a) is

$$R_{ca} = \frac{1}{\bar{h}_a(A_{\mathrm{bare}} + \eta_f\,A_{\mathrm{fins}})} \approx \frac{1}{\bar{h}_a\,A_{\mathrm{total}}}$$

The thermal circuit for the wall is shown below



The individual resistance based on 1 m^2 of wall area are

$$R_{cw} = \frac{1}{\bar{h}_w A_w} = \frac{1}{238.7\ \mathrm{W/(m^2K)(1\,m^2)}} = 0.00419\ \mathrm{K/W}$$

$$R_k = \frac{D}{kA_w} = \frac{0.003\,\mathrm{m}}{238.7\ \mathrm{W/m\ K(1\,m^2)}} = 0.0000126\ \mathrm{K/W}$$

$$R_{ca} = \frac{1}{\bar{h}_a(A_{\mathrm{bare}} + \eta_f\,A_{\mathrm{fins}})} = \frac{0.003\,\mathrm{m}}{28.4\ \mathrm{W/(m^2K)}\ [0.789\,\mathrm{m^2} + (0.998)(1.79\,\mathrm{m^2})} = 0.0137\ \mathrm{K/W}$$

The rate of heat transfer through the wall is

$$q = \frac{\Delta T}{R_{\mathrm{tot}}} = \frac{T_w - T_a}{R_{cw} + R_k + R_{ca}} = \frac{93^o\mathrm{C} - 38^o\mathrm{C}}{(0.00419 + 0.0000126 + 0.0137)\mathrm{K/W}} = 3072\ \mathrm{W\ (per\ m^2\ of\ wall)}$$

(c) Note that the air side convective resistance is by far the dominant resistance in the problem. Therefore, the fins will enhance the overall heat transfer much less on the water side.

For fins on the water side

$$m = \sqrt{\frac{2\left(283.7\ \text{W/(m}^2\text{K)}\right)}{238\ \text{W/(m K)}(0.0016\ \text{m})}} = 38.6\ \frac{1}{\text{m}} \text{ and } m\,L = 38.6\ \frac{1}{\text{m}}\ (0.006\ \text{m}) = 0.2316$$

$$M = w\,(T_{sw} - T_w)\ \sqrt{2\left(283.7\ \text{W/(m}^2\text{K)}\right)(0.0016\ \text{m})2\left(238\ \text{W/(m K)}\right)} = 14.70\ w\,(T_{sw} - T_\text{w})\text{W/m K}$$

$$\frac{\bar{h}_w}{mk}\ \frac{283.7\ \text{W/(m}^2\text{K)}}{38.6\ \frac{1}{\text{m}}\left(238\ \text{W/(m K)}\right)} = 0.0309$$

$$\eta_f = \frac{14.70\ \text{W/(m K)}\left[\dfrac{\sinh(0.2316) + 0.0309\cosh(0.2316)}{\cosh(0.2316) + 0.0309\sinh(0.2316)}\right]}{283.7\ \text{W/(m}^2\text{K)}\ [2\ (0.006\ \text{m}) + 0.0016\ \text{m}} = 0.978$$

$$q = \frac{T_w - T_a}{\dfrac{1}{\bar{h}_{ca}} + \dfrac{D}{k} + \dfrac{1}{\bar{h}_{cw}(0.089 + \eta 1.79)}} = \frac{93^\text{o}\text{C} - 38^\text{o}\text{C}}{(0.0352 + 0.0000126 + 0.00139)\ (\text{m}^2\text{K})/\text{W}}$$

$$= 1502\ \text{W/m}^2$$

COMMENTS

The fins are most effective in the medium with the lowest heat transfer coefficient.

With no fins, the rate of heat transfer would be 1419 W/m^2. Fins on the water side increase the rate of heat transfer 6%. Fins on the air side increase the rate of heat transfer 116%. Therefore, installing fins on the water side would be a poor design.

PROBLEM 2.43

Compare the rate of heat flow from the bottom to the top in the aluminum structure shown in the sketch with the rate of heat flow through a solid slab. The top is at –10°C, the bottom at 0°C. The holes are filled with insulation which does not conduct heat appreciably.

GIVEN

- The aluminum structure shown in the sketch below
- Temperature of the top (T_T) = – 10°C
- Temperature of the bottom (T_B) = 0°C
- The holes are filled with insulation which does not conduct heat appreciably

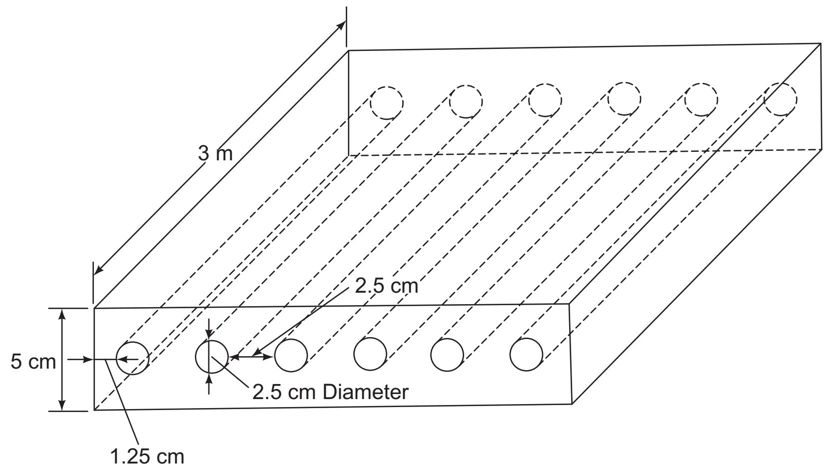
FIND

- Compare the rate of heat flow from the bottom to the top with the rate of heat flow through a solid slab

ASSUMPTIONS

- The structure is in steady state
- Heat transfer through the insulation is negligible
- The thermal conductivity of the aluminum is uniform
- The edges of the structure are insulated
- Two dimensional conduction through the structure

SKETCH

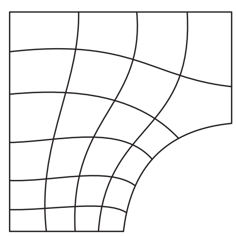


PROPERTIES AND CONSTANTS

The thermal conductivity of aluminum (k) = 236 W/(m K) at 0°C

SOLUTION

Because of the symmetry of the structure, we can draw the flux plot for just one of the twenty-four equivalent sections



(a) The total number of flow lanes in the structure, (M) = (12) (4) = (48). Each flow lane consists of 12 curvilinear squares (6 on top as shown, and 6 on bottom. Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{48}{12} = 4$$

The heat flow per meter, from Equation (2.80), is

$$q = kS\Delta T_{\text{overall}} = 236 \text{ W/m K } (4)\ (0°\text{C} - (-10°\text{C})) = 9440 \text{ W/m}$$

The total rate of heat flow is

$$q_{\text{tot}} = q \text{ (length of structure)} = (9440 \text{ W/m})\ (3 \text{ m}) = 28{,}320 \text{ W}$$

(b) For a solid aluminum plate, the total heat flow from Equation (1.2), is

$$q_{\text{TOT}} = \frac{Ak}{t}\,\Delta T = \frac{(3\,\text{m})(0.3\,\text{m})[236 \text{ W/(m K)}]}{0.05}\ (10 \text{ C}) = 42{,}500 \text{ W}$$

Therefore, the insulation filled tubes reduce the heat transfer rate by 33%.

COMMENTS

The shape factor was determined graphically and can easily be in error by 10%.

Also, the surface temperature will not be uniform in the insulated structure.

PROBLEM 2.44

> **Determine by means of a flux plot the temperatures and heat flow per unit depth in the ribbed insulation shown in the accompanying sketch.**

GIVEN

- The sketch below

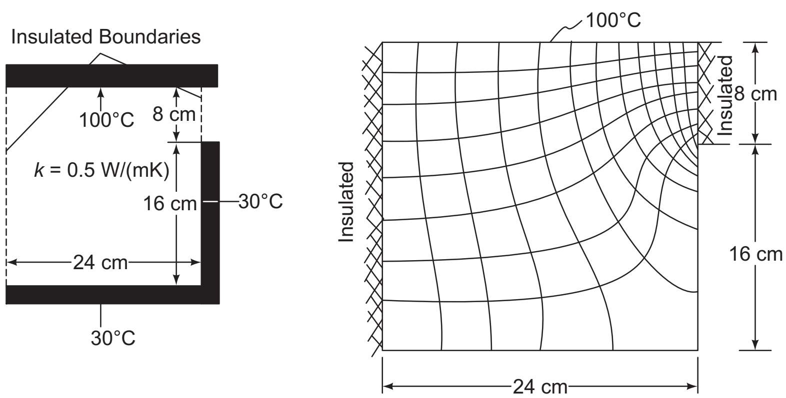
FIND

(a) The temperatures
(b) The heat flow per unit depth

ASSUMPTIONS

- Steady state conditions
- Two dimensional heat flow
- The heat loss through the insulation is negligible
- The thermal conductivity of the material is uniform

SKETCH



SOLUTION

The total number of heat flow lanes (M) = 11
The number of curvilinear squares per lane (N) = 8
Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{11}{8} = 1.38$$

The rate of heat transfer for unit depth is given by Equation 2.80

$$q = kS\Delta T = (0.5 \text{ W/(m K)})\,(1.38)\,(100°\text{C} - 30°\text{C}) = 48.3 \text{ W/m}$$

PROBLEM 2.45

> **Use a flux plot to estimate the rate of heat flow through the object shown in the sketch. The thermal conductivity of the material is 15 W/(m K). Assume no heat is lost from the sides.**

GIVEN

- The shape of object shown in the sketch
- The thermal conductivity of the material (k) = 15 W/(m K)
- The temperatures at the upper and lower surfaces (30°C & 10°C)

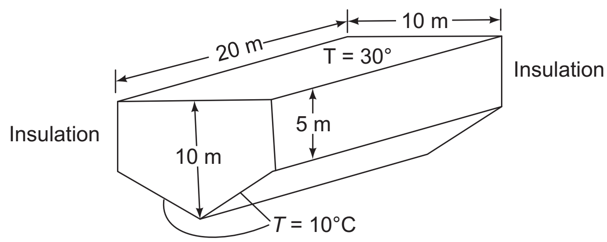
FIND

- The rate of heat flow through the object (By means of a flux plot)

ASSUMPTIONS

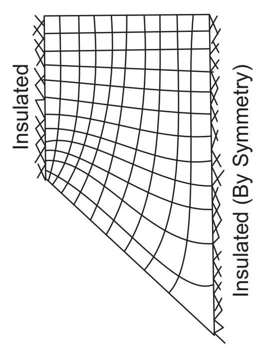
- No heat is lost from the sides and ends
- Uniform thermal conductivity
- Two dimensional conduction
- Steady state

SKETCH



SOLUTION

The flux plot is shown below



The number of heat flow lanes (M) = 2 × 10 = 20
The number of curvilinear squares in each lane (N) = 12
Therefore, the shape factor for this object is

$$S = \frac{M}{N} = \frac{20}{12} = 1.67$$

The rate of heat transfer per unit length from Equation (2.80) is

$$q = kS\Delta T_{\text{overall}} = [15 \text{ W/(m K)}]\,(1.67)\,(20°\text{C}) = 500 \text{ W/m}$$

The total rate of heat transfer is

$$q_{\text{tot}} = q\text{L} = (500 \text{ W/m})\,(20 \text{ m}) = 10{,}000 \text{ W}$$

PROBLEM 2.46

Determine the rate of heat transfer per unit length from a 5-cm-*OD* pipe at 150°C placed eccentrically within a larger cylinder of 85% Magnesia wool as shown in the sketch. The outside diameter of the larger cylinder is 15 cm and the surface temperature is 50°C.

GIVEN

- A pipe placed eccentrically within a larger cylinder of 85% Magnesia wool as shown in the sketch
- Outside diameter of the pipe (D_p) = 5 cm = 0.05 m
- Temperature of the pipe (T_s) = 150°C
- Outside diameter of the larger cylinder (D_o) = 15 cm = 0.15 m
- Temperature of outer pipe (T_o) = 50°C

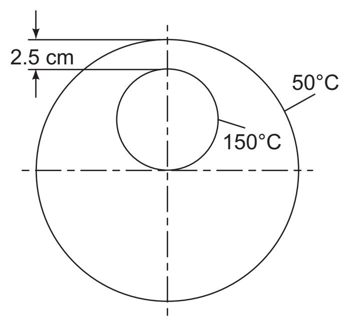
FIND

- The rate of heat transfer per meter length (q)

ASSUMPTIONS

- Two dimensional heat flow (no end effects)
- The system is in steady state
- Uniform thermal conductivity

SKETCH



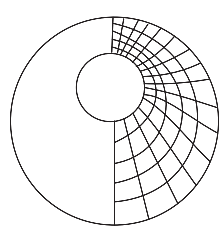
PROPERTIES AND CONSTANTS

From Appendix 2, Table 11

The thermal conductivity of 85% Magnesia wool (k) = 0.059 W/(m K) (at 20°C).

SOLUTION

The rate of heat transfer can be estimated from a flux plot



The number of flow lanes (M) = 2 × 15 = 30

The number of squares per lane (N) = 5

Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{30}{5} = 6$$

Equation (2.80) can be used to find the rate of heat transfer per unit length

$$q = kS\Delta T = kS(T_s - T_o) = [0.059 \text{ W/(m K)}] (6) (150°\text{C} - 50°\text{C}) = 35.4 \text{ W/m}$$

COMMENTS

This problem can also be solved analytically (see Table 2.2)

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{D^2 + d - 4z^2}{2Dd}\right)} = 6.53$$

(z = the distance between the centers of the circular cross sections)

$$\therefore\ q = kS\Delta T = 38.5 \text{ W/m}$$

The answer from the graphical solution is 8% less than the analytical value.

PROBLEM 2.47

> **Determine the rate of heat flow per foot length from the inner to the outer surface of the molded insulation in the accompanying sketch. Use 0.17 W/(m K).**

GIVEN

- The object with a cross section as shown in the sketch below
- The thermal conductivity (k) = 0.17 W/(m K)

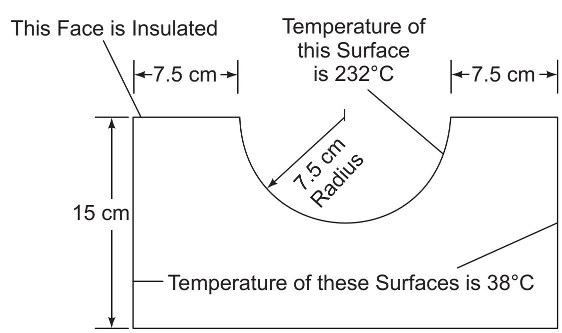
FIND

- The rate of heat flow per foot length from the inner to the outer surface (q)

ASSUMPTIONS

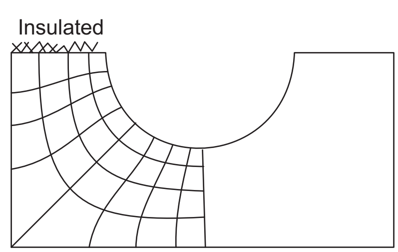
- The system has reached steady state
- The thermal conductivity does not vary with temperature
- Two dimensional conduction

SKETCH



SOLUTION

A flux plot for the object is shown below



The number of heat flow lanes (M) = 2 × 8 = 16

The number of curvilinear squares per lane (N) = 4

Therefore, the shape factor is

$$S = \frac{16}{4} = 4$$

The heat flow per unit length, from Equation (2.80) is

$$q = kS\Delta T_{\text{overall}} = (0.17 \text{ W/(m K)}) (4) (232 - 38) \text{ K} = 111.5 \text{ W/m}$$

COMMENTS

The problem can also be solved analytically. From Table 2.2

$$S = \frac{\pi}{\ln(1.08 \text{ W/D})} = \frac{\pi}{\ln\left(1.08 \frac{30\,\text{cm}}{15\,\text{cm}}\right)} = 4.08$$

$$q = kS\Delta T = 113.7 \text{ W/m}$$

The analytical solution yields a rate of heat flow that is about 2% larger than the value obtained from the flux plot.

PROBLEM 2.48

> **A long 1-cm-diameter electric copper cable is embedded in the center of a 25 cm square concrete block. If the outside temperature of the concrete is 25°C and the rate of electrical energy dissipation in the cable is 150 W per meter length, determine the temperatures at the outer surface and at the center of the cable.**

GIVEN

- A long electric copper cable embedded in the center of a square concrete block
- Diameter of the pipe (D_p) = 1 cm = 0.01 m
- Length of a side of the block = 25 cm = 0.25 m
- The outside temperature of the concrete (T_o) = 25°C
- The rate of electrical energy dissipation ($\dot{Q}_G / L$) = 150 W/m

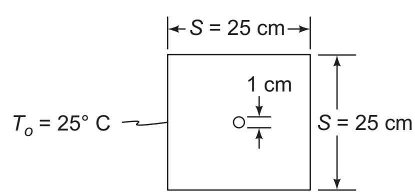
FIND

- The temperatures at the outer surface (T_s) and at the center of the cable (T_c)

ASSUMPTIONS

- Two dimensional, steady state heat transfer
- Uniform thermal conductivities

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11

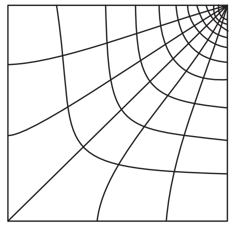
The thermal conductivity of concrete (k_b) = 0.128 W/(m K) at 20°C

From Appendix 2, Table 12

The thermal conductivity of copper (k_c) = 396 W/(m K) at 63°C

SOLUTION

For steady state, the rate of heat transfer through the concrete block must equal the rate of electrical energy dissipation. The heat transfer rate can be estimated with a flux plot of one quarter of the block:



The number of flow lanes (M) = 4 × 6 = 24

The number of squares per lane (N) = 10

Therefore, the shape factor is

$$S = \frac{\text{M}}{\text{N}} = \frac{24}{10} = 2.4$$

The rate of heat flow per unit length is given by Equation (2.80)

$$q = k_b S \Delta T = k_b S (T_s - T_o) = \frac{\dot{Q}_G}{L}$$

Solving for the surface temperature of the cable

$$T_s = T_o + \frac{\left(\frac{\dot{Q}_G}{L}\right)}{k_b S} = 25°\text{C} + \frac{150\text{W/m}}{[0.128\text{W/(m K)}](2.4)} = 513°\text{C}$$

From Equation (2.51) the temperature in the center of the cable is

$$T_c = T_s + \frac{\dot{q}_G r_0^2}{4k_C}$$

Where $\dot{q}_G$ = heat generation per unit volume $\frac{\dot{q}_G}{\pi r_0^2 L}$

$$T_C = T_s + \frac{\left(\frac{\dot{Q}_G}{L}\right)}{4\pi k_C} = 513°\text{C} + \frac{150\text{W/m}}{4\pi(396\text{W/(m K)})} = 513°\text{C} + 0.03°\text{C} \approx 513°\text{C}$$

COMMENT

The thermal conductivity of the cable is quite large and therefore its temperature is essentially uniform.

The analytical solution for this geometry, given in Table 2.2, is

$$S = \frac{2\,\pi}{\text{In}\left(0.8\frac{\text{W}}{\text{D}}\right)} = \frac{2\,\pi}{\text{In}\left(1.08\frac{25\text{ cm}}{1\text{ cm}}\right)} = 1.91$$

This would lead to a cable temperature of 639°C, 20% higher than the flux plot estimate. The high error is probably due to the difficulty in drawing the flux plot close to the cable and may be improved by drawing a larger scale flux plot is geometries that involve tight curves.

PROBLEM 2.49

> **A large number of 3.8 cm-*OD* pipes carrying hot and cold liquids are embedded in concrete in an equilateral staggered arrangement with center line 11.2 cm apart as shown in the sketch. If the pipes in rows A and C are at 16°C while the pipes in rows B and D are at 66°C, determine the rate of heat transfer per meter length from pipe *X* in row *B*.**

GIVEN

- A large number of pipes embedded in concrete as shown below
- Outside diameter of pipes (D) = 3.8 cm
- The temperature of the pipes in rows A and C = 16°C
- The temperature of the pipes in rows B and D = 66°C

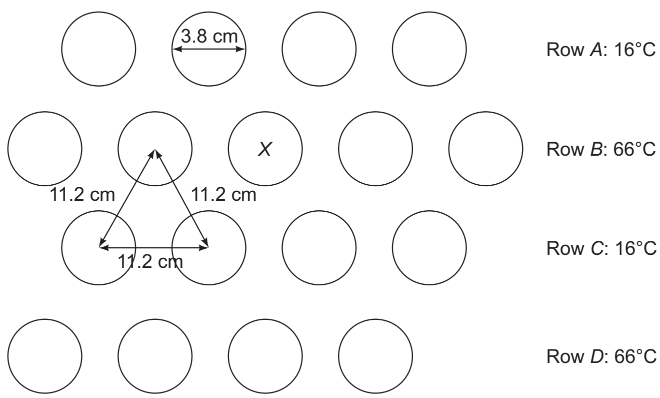
FIND

- The rate of hat transfer per foot length from pipe X in row B

ASSUMPTIONS

- Steady state, two dimensional heat transfer
- Uniform thermal conductivity in the concrete

SKETCH

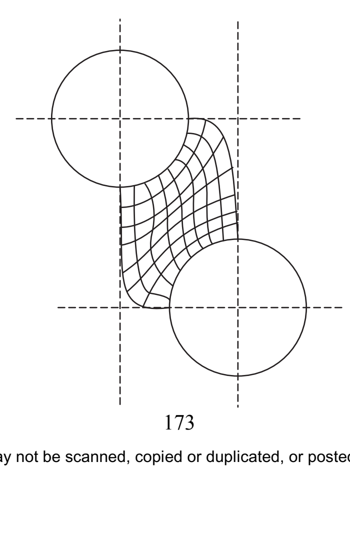


PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
The thermal conductivity of concrete (k_b) = 0.128 W/(m K) at 20°C

SOLUTION

A flux diagram for this problem is shown below



By symmetry, the total heat transfer from the tube X is four times that shown in the flux diagram.

The number of heat flow lanes (M) = 8 × 4 = 32

The number of curvilinear squares per lane (N) = 7

Therefore, the shape factor is

$$S = \frac{\mathrm{M}}{\mathrm{N}} = \frac{32}{7} = 4.6$$

The heat transfer per unit length from Table 2.2, from Equation (2.80) is

$$q\ KS\Delta T_{\text{overall}} = 0.128\ \text{W/(m K)}\ (4.6)\ (66 - 16)$$
$$= 35.3\ \text{W/m}$$

PROBLEM 2.50

A long 1-cm-diameter electric cable is imbedded in a concrete wall (*k* = 0.13 W/(m K)) which is 1 m by 1 m, as shown in the sketch below. If the lower surface is insulated, the surface of the cable is 100°C and the exposed surface of the concrete is 25°C, estimate the rate of energy dissipation per meter of cable.

GIVEN

- A long electric cable imbedded in a concrete wall
- Cable diameter (D) = 1 cm = 0.01 m
- Thermal conductivity of the wall (k) = 0.13 W/(m K)
- Wall dimensions are 1 m by 1 m, as shown in the sketch below
- The lower surface is insulated
- The surface temperature of the cable (T_s) = 100°C
- The temperature of the exposed concrete surfaces (T_o) = 25°C

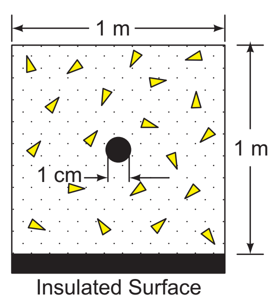
FIND

- The rate of energy dissipation per meter of cable (q/L)

ASSUMPTIONS

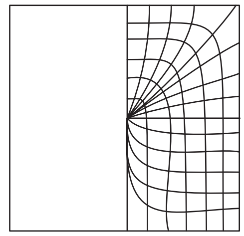
- The system is in steady state
- The thermal conductivity of the wall is uniform
- Two dimensional heat transfer

SKETCH



SOLUTION

By symmetry, only half of the flux plot needs to be drawn



The number of heat flow lanes (M) = 2 × 14 = 28

The number of curvilinear squares per lane (N) = 6

Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{28}{6} = 4.7$$

For steady state, the rate of energy dissipation per unit length in the cable must equal the rate of heat transfer per unit length from the cable which, from Equation (2.80), is

$$q = kS(T_s - T_o) = \big(0.13\ \text{W/(m K)}\ (4.7)\big)\ (100°\text{C} - 25°\text{C}) = 46\ \text{W/m}$$

PROBLEM 2.51

Determine the temperature distribution and heat flow rate per meter length in a long concrete block having the shape shown below. The cross-sectional area of the block is square and the hole is centered.

GIVEN

- A long concrete block having the shape shown below
- The cross-sectional area of the block is square
- The hole is centered

FIND

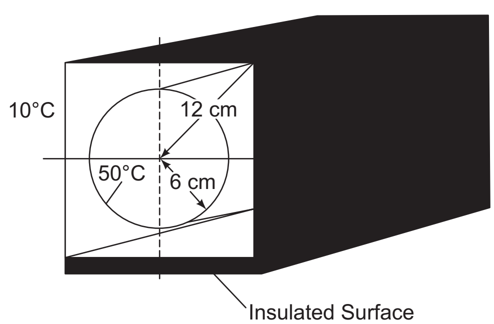
(a) The temperature distribution in the block

(b) The heat flow rate per meter length

ASSUMPTIONS

- The heat flow is two dimensional and in steady state
- The thermal conductivity in the block is uniform

SKETCH



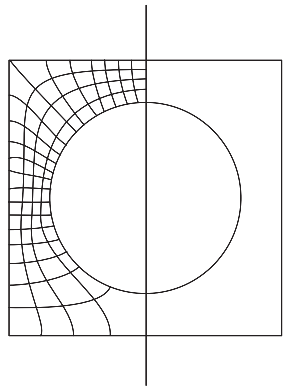
PROPERTIES AND CONSTANTS

From Appendix 2, Table 11

The thermal conductivity of concrete (k_b) = 0.128 W/(m K) at 20°C

SOLUTION

The temperature distribution and heat flow rate may be estimated with a flux plot



(a) The temperature distribution is given by the isotherms in the flux plot.

(b) The number of flow lanes (M) = 2 × 21 = 42

The number of squares per lane (N) = 4

Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{42}{4} = 10.5$$

From Equation (2.80), the rate of heat flow per unit length is

$$q = kS\Delta T = [0.128 \text{ W/(m K)}] (10.5) (40°\text{C}) = 54 \text{ W/m}$$

COMMENTS

If the lower surface were not insulated, the shape factor from Table 2.2, would be

$$S = \frac{2\pi}{\text{In}\left(1.08\frac{\text{W}}{\text{D}}\right)} = 14.8 \Rightarrow q = 75.6 \text{ W/m}$$

The rate of heat transfer with the insulation as calculated with the flux plot is about 29% less than the analytical result without insulation. We would expect a reduction of slightly less than 25%.

PROBLEM 2.52

A 30-cm-*OD* pipe with a surface temperature of 90°C carries steam over a distance of 100 m. The pipe is buried with its center line at a depth of 1 m, the ground surface is –6°C, and the mean thermal conductivity of the soil is 0.7 W/(m K). Calculate the heat loss per day, and the cost, if steam heat is worth \$3.00 per 10^6 kJ. Also, estimate the thickness of 85% magnesia insulation necessary to achieve the same insulation as provided by the soil with a total heat transfer coefficient of 23 W/(m^2 K) on the outside of the pipe.

GIVEN

- A buried steam pipe
- Outside diameter of the pipe (D) = 30 cm = 0.3 m
- Surface temperature (T_s) = 90°C

- Length of pipe (L) = 100 m
- Depth of its center line (Z) = 1 m
- The ground surface temperature (T_g) = –6°C
- The mean thermal conductivity of the soil (k) = 0.7 W/(m K)
- Steam heat is worth \$3.00 per 10^6 kJ
- The heat transfer coefficient (h_c) = 23 W/(m^2 K) for the insulated pipe

FIND

(a) The heat loss per 24 hour day

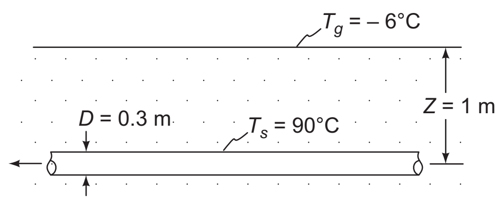
(b) The value of the lost heat

(c) The thickness of 85% magnesia insulation necessary to achieve the same insulation

ASSUMPTIONS

- Steady state conditions
- Uniform thermal conductivity
- Two dimensional heat transfer from the pipe

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11

The thermal conductivity of 85% magnesia (k_i) = 0.059 W/(m K) (at 20°C)

SOLUTION

(a) The shape factor for this problem, from Table 2.2, is

$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{2z}{D}\right)} \quad \text{If} \quad z/L < 1$$

Note that the condition $Z/L << 1$ is satisfied in this problem.

$$S = \frac{2\pi(100\,\text{m})}{\cosh^{-1}\left(\frac{2(1\,\text{m})}{0.3\,\text{m}}\right)} = 243 \text{ m}$$

From Equation (2.80), the rate of heat transfer is

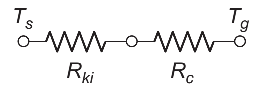
$$q = kS\Delta T = 0.7 \text{ W/(m K) } (243 \text{ m}) (90°\text{C} – (– 6°\text{C}))$$

$$q = 16{,}300 \text{ W } (\text{J/Ws})\left(\frac{\text{kJ}}{1000\,\text{J}}\right)\left(\frac{3600\,\text{s}}{\text{h}}\right)\left(\frac{24\,\text{h}}{\text{day}}\right) = 1.41 \times 10^6 \text{ kJ/Day}$$

(b) The cost of this heat loss is

$$\text{Cost} = \left(1.41 \times 10^6 \text{kJ/day}\right)\left(\frac{\$3.00}{10^6\text{kJ}}\right) = \$4.23\text{/day}$$

(c) The thermal circuit for the pipe covered with insulation is



The rate of heat loss from the pipe is

$$q = \frac{T_s - T_g}{R_{ki} + R_c} = \frac{T_s - T_g}{\frac{1}{2\pi L k_i}\ln\left(\frac{r_o}{r_1} + \frac{1}{2\pi L r_o h_c}\right)} = 16{,}300 \text{ W}$$

$$16{,}300 \text{ W} = \frac{2\pi L(T_s - T_g)}{\frac{1}{k_i}\ln\left(\frac{r_o}{r_i} + \frac{1}{r_o h_c}\right)} = \frac{2\pi L(100\,\text{m})[90°C - (-6°)]}{\frac{1}{0.059\,\text{W/(m K)}}\ln\left(\frac{r_o}{0.15\,\text{m}}\right) + \frac{1}{r_o\left(23\,\text{W/(m}^2\text{K)}\right)}}$$

$$\ln\frac{r_o}{0.15} + 0.00257\,\frac{1}{r_o} = 0.2183$$

By trial and error: $r_o = 0.184$ m

Insulation thickness = $r_o - r_i$ = 0.184 m – 0.15 m = 0.034 m = 3.4 cm

COMMENTS

The value of the heat loss per year is 365 × \$4.23 = \$1544. Hence insulation will pay for itself quite rapidly.

PROBLEM 2.53

Two long pipes, one having a 10-cm-*OD* and a surface temperature of 300°C, the other having a 5-cm-*OD* and a surface temperature of 100°C, are buried deeply in dry sand with their centerlines 15 cm apart. Determine the rate of heat flow from the larger to the smaller pipe per meter length.

GIVEN

- Two long pipes buried deeply in dry sand
- Pipe 1
  - Diameter (D_1) = 10 cm = 0.1 m,
  - Surface temperature (T_1) = 300°C
- Pipe 2
  - Diameter (D_2) = 5 cm = 0.05 m,
  - Surface temperature (T_2) = 100°C
- Spacing between their centerlines (s) = 15 cm = 0.15 m

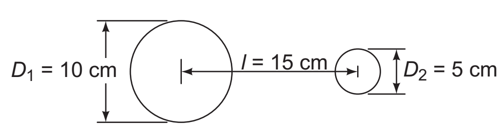
FIND

- The rate of heat flow per meter length (q/L)

ASSUMPTIONS

- The heat flow between the pipes is two dimensional
- The system has reached steady state
- The thermal conductivity of the sand is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11

Thermal conductivity of dry sand (k) = 0.582 W/(m K) at 20°C

SOLUTION

The shape factor for this geometry, from Table 2.2, is

$$S = \frac{2\pi}{\cosh^{-1}\left(\dfrac{L^2 - 1 - r^2}{2r}\right)}$$

where $$r = \frac{r_1}{r_2} = \frac{5\,\text{cm}}{2.5\,\text{cm}} = 2 \text{ and } L = \frac{1}{r_2} = \frac{15\,\text{cm}}{2.5\,\text{cm}} = 6$$

$$\therefore S = \frac{2\pi}{\cosh^{-1}\left(\dfrac{36 - 1 - 4}{4}\right)} = 2.296$$

The rate of heat transfer per unit length is

$$q = Sk\Delta T = (2.296)\ 0.582\ \text{W/(m K)}\ (300°\text{C} - 100°\text{C}) = 267\ \text{W/m}$$

PROBLEM 2.54

A radioactive sample is to be stored in a protective box with 4 cm thick walls having interior dimensions 4 by 4 by 12 cm. The radiation emitted by the sample is completely absorbed at the inner surface of the box, which is made of concrete. If the outside temperature of the box is 25°C, but the inside temperature is not to exceed 50°C, determine the maximum permissible radiation rate from the sample, in watts.

GIVEN

- A radioactive sample in a protective concrete box
- Wall thickness (t) = 4 cm = 0.4 m
- Box interior dimensions: 4 × 4 × 12 cm
- All radiation emitted is completely absorbed at the inner surface of the box
- The outside temperature of the box (T_o) = 25°C
- The maximum inside temperature (T_i) = 50°C

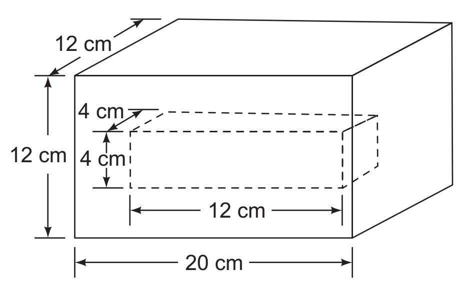
FIND

- The maximum permissible radiation rate from the sample, q (in watts)

ASSUMPTIONS

- The system is in steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
The thermal conductivity of concrete (k_b) = 0.128 W/(m K) at 20°C

SOLUTION

The box consists of 4 wall sections: A = 4 cm × 12 cm
2 wall sections: A = 4 cm × 4 cm
4 edge sections: D = 12 cm long
8 edge sections: D = 4 cm long
8 corner sections: L = 4 cm thick

The shape factors for this geometry (when all interior dimensions are greater than one-fifth of the wall thickness, as in this case) is given on Section 2.5.2 of the text
For the wall sections

$$S_1 = \frac{A}{L} = \frac{(4\,\text{cm})(12\,\text{cm})}{4\,\text{cm}} = 12\text{ m} \quad \text{and} \quad S_2 = \frac{A}{L} = \frac{(4\,\text{cm})(4\,\text{cm})}{4\,\text{cm}} = 4\text{ cm}$$

For the edge sections

$$S_3 = 0.54\,D = 0.54\ (12\text{ cm}) = 6.48\text{ cm} \quad \text{and} \quad S_4 = 0.54\,D = 0.54\ (4\text{ cm}) = 2.16\text{ cm}$$

For the corner sections

$$S_5 = 0.15\,L = 0.15\ (4\text{ cm}) = 0.6\text{ cm}$$

Multiplying each shape factor by the number of elements having that shape factor and summing them

$$S = 4\,S_1 + 2\,S_2 + 4\,S_3 + 8\,S_4 + 8\,S_5$$

$$S = 4\ (12\text{ cm}) + 2(4\text{ cm}) + 4(6.48\text{ cm}) + 8(2.16\text{ cm}) + 8(0.6\text{ cm}) = 104\text{ cm}$$

The rate of heat transfer is

$$q = kS\Delta T = 0.128\text{ W/(m K) (104 cm) (1m/100 cm) (50°C} - \text{25°C) = 3.3 W}$$

COMMENTS

The conductivity of the concrete was evaluated at 20°C while the actual temperature is between 50°C and 25°C. Therefore, the actual rate of heat flow may be slightly different than that calculated, but no better property value is available in the text.

PROBLEM 2.55

A 15 cm-*OD* pipe is buried with its centerline 1.25 m below the surface of the ground (k of soil is 0.35 W/(m K)). An oil having a density of 800 kg/m^3 and a specific heat of 2.1 kJ/(kg K) flows in the pipe at 5.6 liters/s. Assuming a ground surface temperature of 5°C and a pipe wall temperature of 95°C, estimate the length of pipe in which the oil temperature decreases by 5.5°C.

GIVEN

- An oil filled pipe buried below the surface of the ground
- Pipe outside diameter (D) = 15 cm = 0.15 m
- Depth of centerline (z) = 125 cm = 1.25 m
- Thermal conductivity of the soil (k) = 0.35 W/(m K)
- Specific gravity of oil (Sp. Gr.) = 0.8
- Specific heat of oil (c_p) = 2.1 kJ/(kg K)
- Flows rate of oil $\dot{m}$ = 5.6 liters/s
- The ground surface temperature (T_s) = 5°C
- The pipe wall temperature (T_p) = 95°C

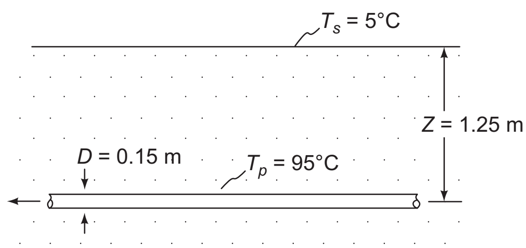
FIND

- The length of pipe (L) in which the oil temperature decreases by 5.5°C

ASSUMPTIONS

- Steady state condition
- Two dimensional heat transfer

SKETCH



SOLUTION

The rate of heat flow from the pipe can be calculated using the shape factor from Table 2.2 for an infinitely long cylinder

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{2Z}{D}\right)} = \frac{2\pi}{\cosh^{-1}\left(\frac{2(1.25)}{0.15}\right)} = 1.79$$

The rate of heat transfer per unit length is given by Equation (2.80)

$$q = kS\Delta T_{\text{overall}} = (0.35 \text{ W/(m K)})\ (1.79)\ (95 - 5)$$
$$= 56.4 \text{ W/m}$$

The total heat loss required to decrease the oil by 5.5°C is

$$qL = \dot{m}\ C_p\ \Delta T = 5.6 \text{ liters/s } (0.8 \text{ kg/liters})\ (2.1 \text{ kJ/(kg K)})\ (5.5°\text{C})$$
$$= 51750 \text{ W}$$

We can estimate the length of pipe in which the oil temperature drops 5.5°C by assuming the rate of heat loss from the pipe per unit length is constant, then:

$$q_t = qL \Rightarrow L = \frac{q_t}{q} = \frac{51750 \text{ W}}{56.4 \text{ W/m}} = 917 \text{ m}$$

COMMENTS

The heat loss from the pipe will actually be less because as the oil temperature and therefore also the pipe temperature decreases with distance from the inlet. This means the length will be slightly longer than the estimate above. If the calculation is based on an arithmetic mean pipe temperature of 90.5°C the estimated length is 966 m about 5% more.

PROBLEM 2.56

A 2.5-cm-*OD* hot steam line at 100°C runs parallel to a 5.0 cm *OD* cold water line at 15°C. The pipes are 5 cm center to center and deeply buried in concrete with a thermal conductivity of 0.87 W/(m K). What is the heat transfer per meter of pipe between the two pipes?

GIVEN

- A hot steam line runs parallel to a cold water line buried in concrete

- Hot pipe outside diameter (D_h) = 2.5 cm = 0.025 m
- Hot pipe temperature (T_h) = 100°C
- Cold pipe outside diameter (D_c) = 5.0 cm = 0.05 m
- Cold pipe temperature (T_c) = 15°C
- Center to center distance between pipes (l) = 5 cm = 0.05 m
- Thermal conductivity of concrete (k) = 0.87 W/(m K)

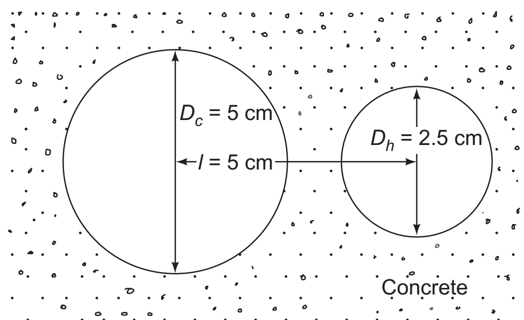
FIND

- The heat transfer per meter of pipe (q/L)

ASSUMPTIONS

- Two dimensional heat transfer between the pipes
- Steady state conditions
- Uniform thermal conductivity

SKETCH



PROPERTIES AND CONSTANTS

Specific heat of water (c_p) = 1 Btu/(lb °F) = 4187 J/(kg K)

SOLUTION

The shape factor for this geometry is in Table 2.2

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{L^2 - 1 - r^2}{2r}\right)}$$

Where $$L = \frac{1}{D_h} = \frac{0.05\,\text{m}}{\cosh^{-1}\left(\frac{0.025\,\text{m}}{2}\right)} = 4 \text{ and } r = \frac{r_c}{r_h} = \frac{D_c}{D_h} = \frac{0.05}{0.025} = 2$$

$$\therefore S = \frac{2\pi}{\cosh^{-1}\left(\frac{16 - 1 - 4}{4}\right)} = 3.763$$

The rate of heat transfer per unit length, from Equation (2.80), is

$$q = kS\Delta T_{\text{overall}} = 0.87 \text{ W/(m K) } (3.763)\ (100°\text{C} - 15°\text{C}) = 278 \text{ W/m}$$

COMMENTS

Normally, the temperature of both fluids will change as heat is transferred between them. Hence, for any appreciable length of pipe, an average temperature difference must be used.

PROBLEM 2.57

Calculate the rate of heat transfer between a 15-cm-*OD* pipe at 120°C and a 10-cm-*OD* pipe at 40°C. The two pipes are 330 m long and are buried in sand [k = 0.33 W/(m K)] 12 m below the surface (T_s = 25°C). The pipes are parallel and are separated by 23 cm (center to center) distance.

GIVEN

- Two parallel pipes buried in sand
- Pipe 1
  - Outside diameter (D_1) = 15 cm = 0.15 m
  - Temperature (T_1) = 120°C
- Pipe 2
  - Outside diameter (D_2) = 10 cm = 0.1 m
  - Temperature (T_2) = 40°C
- Length of pipes (L) = 330 m
- Thermal conductivity of the sand (k) = 0.33 W/(m K)
- Depth below surface (d) = 1.2 m
- Surface temperature (T_s) = 25°C
- Center to center distance between pipes (s) = 23 cm = 0.23 m

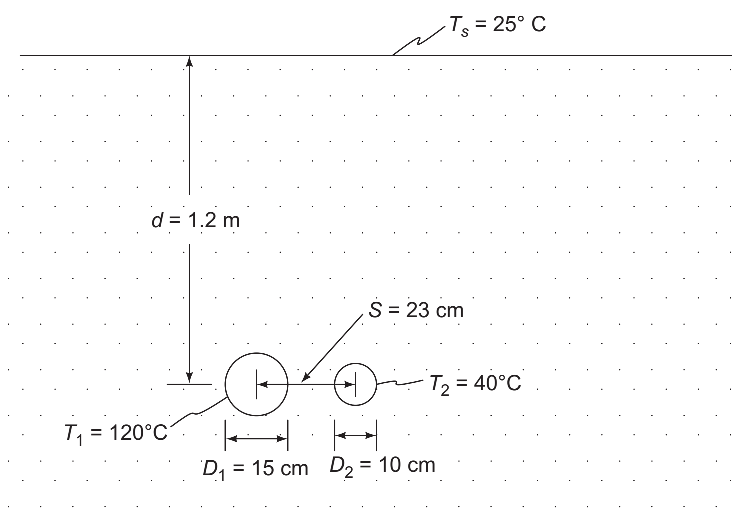
FIND

- The rate of heat transfer between the pipes (q)

ASSUMPTIONS

- The thermal conductivity of the sand is uniform
- Two dimensional, steady state heat transfer

SKETCH



SOLUTION

For the pipe-to-pipe heat transfer, the surface is not important since $Z >> D$. The shape factor for this geometry, from Table 2.2, is

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{L^2 - 1 - r^2}{2r}\right)}$$

$$\text{where} \quad L = \frac{1}{r_2} = \frac{0.23\,\text{m}}{0.05\,\text{m}} = 4.6 \quad \text{and} \quad r = \frac{r_1}{r_2} = \frac{D_1}{D_2} = \frac{15\,\text{m}}{0.1\,\text{m}} = 1.5$$

$$\therefore S = \frac{2\pi}{\cosh^{-1}\left(\frac{(4.6)^2 - 1 - (1.5)^2}{2(1.5)}\right)} = 2.541$$

The rate of heat transfer per unit length is

$$\frac{q}{L} = kS\Delta T = 0.33 \text{ W/(m K) } (2.541)\ (120°\text{C} - 40°\text{C}) = 67 \text{ W/m}$$

For $L =$ 330 m: q = 67 W/m (330 m) = 22,100 W

COMMENTS

Normally, the temperature of both fluids will change as heat is transferred between them. Hence, for any appreciable length of pipe, an average temperature difference must be used.

PROBLEM 2.58

A 0.6-cm-diameter mild steel rod at 38°C is suddenly immersed in a liquid at 93°C with $\bar{h}_c$ = 110 W/(m^2 K). Determine the time required for the rod to warm to 88°C.

GIVEN

- A mild steel rod is suddenly immersed in a liquid
- Rod diameter (D) = 0.6 cm = 0.006 m
- Initial temperature of the rod (T_o) = 38°C
- Liquid temperature (T_∞) = 93°C
- Heat transfer coefficient ($\bar{h}_c$) = 113.5 W/(m^2 K)

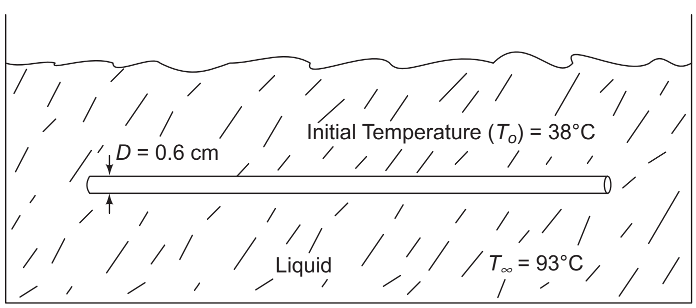
FIND

- The time required for the rod to warm to 88°C

ASSUMPTIONS

- The rod is 1% carbon steel
- Constant thermal conductivity
- End effects are negligible
- The rod is very long compared to its diameter
- There is radial conduction only in the rod

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10: For 1% carbon steel at 20°C:

Thermal conductivity (k) = 43 W/(m K)

Specific heat (c) = 473 J/(kg K)
Density (ρ) = 7801 kg/m^3
Thermal diffusivity (α) = 1.172 × 10^{-5} m^2/s. [$\alpha = k/\rho c$].

SOLUTION

The Biot number is calculated first to check if the internal resistance is negligible

$$B_i = \frac{\bar{h}_c D}{4k} = \frac{\left(110\,\text{W/(m}^2\text{K)}\right)(0.006\,\text{m})}{4\left(43\,\text{W/(m}^2\ \text{K)}\right)} = 0.0038 << 0.1$$

Therefore, the internal resistance of the rod is negligible.
The temperature-time history of the rod, from Equation (2.84) is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{\bar{h}_c A_s}{c\rho V} t\right)$$

$$\frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c \pi DL}{c\rho \frac{\pi}{4} D^2 L} = \frac{4\bar{h}_c}{c\rho D} = \frac{4\left(100\,\text{W/(m}^2\text{K)}\right)(\text{J/Ws})}{(473\,\text{W/kg K})\left(7801\,\text{Kg/m}^3\right)(0.006\,\text{m})} = 0.020\ 1/S$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[-\left(0.020\frac{1}{S}\right) t\right]$$

Solving for the time

$$t = -\,(50.3\text{ s}) \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

The time required to reach 88°C is

$$t = -\,(50.3\text{ s}) \ln\left(\frac{88 - 93}{38 - 93}\right) = 121\text{s}$$

COMMENTS

The analysis has assumed that the heat capacity of the liquid is much larger than that of the rod and thus the liquid temperature remains constant.

PROBLEM 2.59

> **A spherical shell satellite (3-m-*OD*, 1.25-cm-wall thickness, made of stainless steel) reenters the atmosphere from outer space. If its original temperature is 38°C, the effective average temperature of the atmosphere is 1093°C, and the effective heat transfer coefficient is 115 W/(m^2 °C), estimate the temperature of the shell after reentry, assuming the time of reentry is 10 min and the interior of the shell is evacuated.**

GIVEN

- A spherical stainless steel satellite reentering the atmosphere
- Outside diameter (D) = 3 m
- Wall thickness (L) = 1.25 cm = 0.0125 m
- Its original temperature (T_o) = 38°C
- The effective temperature of the atmosphere (T_∞) = 1093°C
- The effective heat transfer coefficient $\bar{h}_c$ = 115 W/(m^2 °C)
- The time of reentry (t_r) = 10 min = 600 s
- The interior of the shell is evacuated

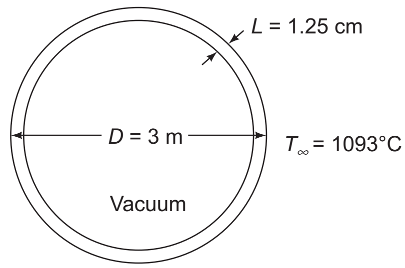
FIND

- The temperature of the shell after reentry (T_f)

ASSUMPTIONS

- Exterior heat transfer is uniform over the shell
- Assume radiation heat transfer is allowed for in the heat transfer coefficient

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, for stainless steel at 20°C

Thermal conductivity (k) = 14.4 W/(m K)
Density (ρ) = 7817 kg/m^3
Specific heat(c) = 461 J/(kg K)

SOLUTION

Since the thickness of the shell is much smaller than the shell radius, the wall can be treated as a plane wall. To estimate the importance of internal thermal resistance, the Biot number is calculated first

$$B_i = \frac{\bar{h}_c L}{k_s} = \frac{[115\,\text{W/(m}^2\,°\text{C)}](0.0125\,\text{m})}{14.4\,\text{W/(m K)}} = 0.099 < 0.1$$

Therefore, the internal resistance is less than 10% of the external resistance and may be neglected. The temperature-time history of the satellite is given by Equation (2.84):

$$\frac{T - T_\infty}{T_s - T_\infty} = \exp\left(-\frac{\bar{h}_c A_s}{c\rho V} t\right) = \exp\,(-\,Bi\,Fo)$$

$$Bi\,Fo = \left(\frac{\bar{h}_c L}{k_s}\right)\left(\frac{\alpha t}{L^2}\right) = \frac{\bar{h}_c A_s}{c\rho V} t = \frac{\bar{h}_c \pi D^2 t}{c\rho \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{D}{2} - L\right)^3\right]}$$

$$Bi\,Fo = \frac{[115\,\text{W/(m}^2\text{K)}](3\,\text{m})^2\,(\text{J/(W s)})\,t}{[461\,\text{J/(kg K)}]\left(7817\,\text{kg/m}^3\right)\frac{4}{3}[(1.5\,\text{m})^3 - (1.5\,\text{m} - 0.0125\,\text{m})^3]}$$

$$= 0.0025t \text{ (t in seconds)}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-0.0025t}$$

$$T = T_\infty + (T_o - T_\infty)e^{-0.0025t}$$

$$T_f = 1093°\text{C} + (38°\text{C} - 1093°\text{C})\,e^{-0.0025(600)} = 868°\text{C}$$

COMMENTS

The analysis has neglected thermodynamic heating during reentry.

PROBLEM 2.60

A thin-wall cylindrical vessel (1 m in diameter) is filled to a depth of 1.2 m with water at an initial temperature of 15°C. The water is well stirred by a mechanical agitator. Estimate the time required to heat the water to 50°C if the tank is suddenly immersed into oil at 105°C. The overall heat transfer coefficient between the oil and the water is 284 W/(m^2 K), and the effective heat transfer surface area is 4.2 m^2.

GIVEN

- A thin wall cylindrical vessel filled with water is suddenly immersed into oil
- Diameter of vessel (D) = 1 m
- Depth of water is vessel = 1.2 m
- Initial temperature (T_o) = 15°C
- Final temperature (T_f) = 50°C
- Oil temperature (T_∞) = 105°C
- The overall heat transfer coefficient between the oil and water $(\bar{h})$ = 284 W/(m^2 K)
- The effective heat transfer surface area (A) = 4.2 m^2

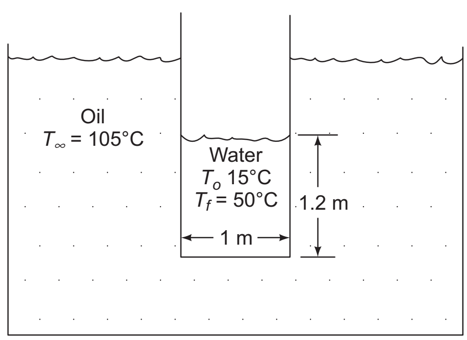
FIND

- The time required to heat the water to 50°C

ASSUMPTIONS

- The thermal capacitance of the cylindrical vessel is negligible
- The temperature of the water is uniform
- The oil temperature remains constant

SKETCH



PROPERTIES AND CONSTANTS

Specific heat of water (c) = 1 Btu/lb = 4187 J/(kg K)

Density of water (ρ) = 1000 kg/m^3

SOLUTION

From Equation (2.83), the temperature-time relationship is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{\bar{h}\,A_s}{c\rho V}t\right)$$

Solving for the time

$$t = \frac{-c\rho V}{\bar{h}\,A_s} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = \frac{-(4187\,\mathrm{J/(kg\,K)})(1000\,\mathrm{kg/m^3})[\pi(0.5\,\mathrm{m})^2(1.2\,\mathrm{m})]}{[284\,\mathrm{W/(m^2K)}](4.2\,\mathrm{m^2})(\mathrm{J/(W\,s)})}\ \ln\left(\frac{50°\mathrm{C} - 105°\mathrm{C}}{15°\mathrm{C} - 105°\mathrm{C}}\right)$$

$$= 1629\ \mathrm{s} = 27\ \mathrm{min}$$

PROBLEM 2.61

A thin-wall jacketed tank, heated by condensing steam at one atmosphere contains 91 kg of agitated water. The heat transfer area of the jacket is 0.9 m^2 and the overall heat transfer coefficient U = 227 W/(m^2 K) based on that area. Determine the heating time required for an increase in temperature from 16°C to 60°C.

GIVEN

- A thin wall jacketed tank, heated by condensing steam
- Steam pressure = one atmosphere
- Mass of water in the tank = 91 kg
- The heat transfer area (A) = 0.9 m^2
- The overall heat transfer coefficient (U) = 227 W/(m^2 K) based on that area
- Temperature increases from 16°C to 60°C

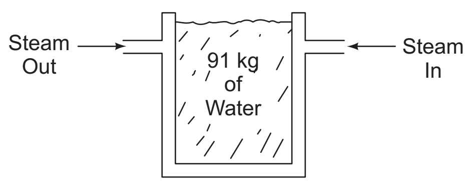
FIND

- Determine the heating time required

ASSUMPTIONS

- Uniform water temperature due to agitation
- Thermal capacitance of the tank wall is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13

The specific heat of water (c) = 1 Btu/lb = 4187 J/(kg K)

Temperature of saturated steam at 1 atmosphere (1.01 × 10^5 ρa) = 100°C

SOLUTION

The temperature-time history for this system is given by Equation (2.83).

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{UA_s}{c\rho V}t\right) = \exp\left(-\frac{UA_s}{c\rho\left(\frac{m}{\rho}\right)}t\right) = \exp\left(-\frac{UA_s}{c\,m}t\right)$$

Solving this expression for the time

$$t = -\frac{\mathrm{cm}}{UA_s}\ln\left(\frac{T_f - T_\infty}{T_o - T_\infty}\right) = -\frac{[4187\,\mathrm{J/(kg\,K)}](91\,\mathrm{kg})((\mathrm{Ws/J}))}{[227\,\mathrm{W/(m^2K)}](0.9\,\mathrm{m^2})}\ln\left(\frac{60°\mathrm{C} - 100°\mathrm{C}}{16°\mathrm{C} - 100°\mathrm{C}}\right) = 1384\ \mathrm{s} = 23\ \mathrm{min}.$$

PROBLEM 2.62

The heat transfer coefficients for the flow of 26.6°C air over a 1.25 cm diameter sphere are measured by observing the temperature-time history of a copper ball of the same dimension. The temperature of the copper ball (c = 376 J/(kg K), ρ = 8928 kg/m^3) was measured by two thermocouples, one located in the center, and the other near the surface. Both of the thermocouples registered, within the accuracy of the recording instruments, the same temperature at a given instant. In one test run, the initial temperature of the ball was 66°C and in 1.15 min, the temperature decreased by 7°C. Calculate the heat transfer coefficient for this case.

GIVEN

- A copper ball with air flowing over it
- Ball diameter (D) = 1.25 cm = 0.0125 m
- Air temperature (T_∞) = 26.6°C
- Specific heat of ball (c) = 376 J/(kg K)
- Density of the ball (P) = 8928 kg/m^3
- Thermocouples in the center and the surface registered the same temperature
- Initial temperature of the ball (T_o) = 66°C
- Lapse time = 1.15 min = 69 s
- The temperature decrease ($T_o - T_f$) = 7°C

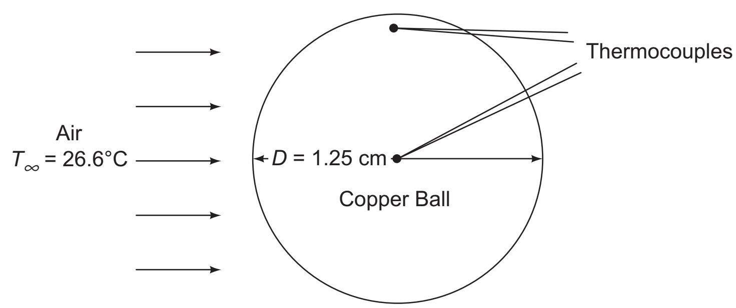
FIND

- The heat transfer coefficient $\left(\bar{h}_c\right)$

ASSUMPTIONS

- The heat transfer coefficient remains constant during the cooling period.

SKETCH



SOLUTION

Since the thermocouples register essentially the same temperature, the internal resistance of the ball is small compared to the external resistance and the ball can be treated with the lumped heat capacity method.

From Equation (2.84) the temperature-time history is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{\bar{h}_c A}{c\rho V} t\right) = \exp\left(-\frac{\bar{h}_c (\pi D^2)}{c\rho\left(\frac{\pi}{6} - D^3\right)} t\right) = \exp\left(\frac{-6\bar{h}_c}{c\rho D} t\right)$$

Solving for the heat transfer coefficient

$$\bar{h}_c = \frac{c\rho D}{6t} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$\bar{h}_c = -\frac{[376\,\text{J/(kg K)}](8928\,\text{kg/m}^3)(0.0125\,\text{m})}{6(69\,\text{s})(\text{J/(Ws)})} \ln\left(\frac{(66°\text{C} - 7°\text{C}) - 26.6°\text{C}}{66°\text{C} - 26.6°\text{C}}\right)$$

$$= 19.8\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

The value is an average over the cooling period.

The procedure described by this problem can be used to evaluate heat transfer coefficients for odd shaped object experimentally.

PROBLEM 2.63

A spherical stainless steel vessel at 93°C contains 45 kg of water initially at the same temperature. If the entire system is suddenly immersed in ice water, determine (a) the time required for the water in the vessel to cool to 16°C, and (b) the temperature of the walls of the vessel at that time. Assume that the heat transfer coefficient at the inner surface is 17 W/(m² K), the heat transfer coefficient at the outer surface is 22.7 W/(m² K), and the wall of the vessel is 2.5 cm thick.

GIVEN

- A spherical stainless steel vessel of water is suddenly immersed in ice water
- Initial temperature of vessel and water (T_i) = 93°C
- Mass of water in the vessel (m) = 45 kg
- The inner heat transfer coefficient $\bar{h}_{ci}$ = 17 W/(m^2 K)
- The outer heat transfer coefficient $\bar{h}_{co}$ = 22.7 W/(m^2 K)
- The vessel wall thickness (L) = 2.5 cm = 0.025 m

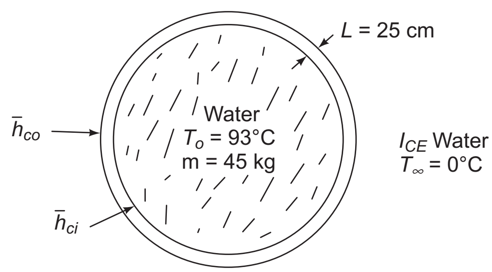
FIND

(a) The time required for the water in the vessel to cool to 16°C
(b) The temperature of the walls of the vessel at that time (T_{sf})

ASSUMPTIONS

- The water in the vessel is well mixed, therefore its temperature is uniform
- The vessel is completely filled with water

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

For stainless steel: The thermal conductivity (k_s) = 14.4 W/(m K)
Density (ρ) = 7817 kg/m^3
Specific heat (c) = 461 J/(kg K)

SOLUTION

If the vessel is completely filled with water

$$V = \frac{m_w}{\rho} = \frac{\pi}{6} D_1{}^3$$

$$D_i = \left(\frac{6\, m_w}{\pi \rho}\right)^{\frac{1}{3}} = \left(\frac{6\,(45\,\text{kg})}{\pi(1000\,\text{kg/m}^3)}\right)^{\frac{1}{3}} = 0.44 \text{ m}$$

$$D_o = D_i + 2\,L = 0.44 \text{ m} + 2\,(0.025 \text{ m}) = 0.49 \text{ m}$$

The internal resistance of the water can be neglected since the water is assumed to be well mixed. The importance of the internal resistance of the vessel wall is indicated by the Biot number of the vessel wall. The characteristic length for the vessel wall is

$$L = \frac{\text{volume}}{\text{Surface area}} = \frac{\frac{\pi}{6}(D_o^3 - D_i^3)}{\pi(D_o^2 + D_i^2)} = \frac{1}{6}\frac{(0.49\,\text{m})^3 - (0.44\,\text{m})^3}{(0.49\,\text{m})^2 + (0.44\,\text{m})^2} = 0.0125 \text{ m}$$

$$\therefore\ Bi = \frac{\bar{h} L}{k_s} = \frac{\frac{1}{2}(\bar{h}_{ci} + \bar{h}_\infty)\, L}{k_s} = \frac{\frac{1}{2}(17 + 22.7)\,[\text{W/(m}^2\text{K)}]\,(0.0125\,\text{m})}{14.4\,\text{W/(m K)}} = 0.017 < 0.1$$

Therefore, the vessel and its contents can be treated as a lumped capacitance and the system approximated two lumped capacitances as covered in Section 2.6.1 of the text.

(a) The temperature-time history of the water in the vessel is given by Equation (2.87)

$$\frac{T_w - T_\infty}{T_0 - T_\infty} = \frac{m_2}{m_2 - m_1} e^{m_1 t} - \frac{m_1}{m_2 - m_1} e^{m_2 t}$$

where T_w = temperature of the water, a function of time

$$m_1 = 0.5\ \{-(k_1 + k_2 + k_3) + [(k_1 + k_2 + k_3)^2 - 4k_1\, k_3]^{0.5}\}$$

$$m_2 = 0.5\ \{-(k_1 + k_2 + k_3) - [(k_1 + k_2 + k_3)^2 - 4k_1\, k_3]^{0.5}\}$$

$$k_1 = \frac{\bar{h}_{ci} A_i}{\rho_w c_w V_i} = \frac{\bar{h}_{ci}\,\pi D_i^2}{\rho_w c_w \frac{\pi}{6} D_i^3} = \frac{6\bar{h}_{ci}}{\rho_w c_w\ D_i} = \frac{6\left(17\,\text{W/(m}^2\text{K)}\right)}{1000\text{kg/m}^3\,(4187\text{J/(kg K)})\,(0.44\,\text{m})} = 5.53 \times 10^{-5}\ 1/\text{s}$$

$$k_2 = \frac{\bar{h}_{ci} A_i}{\rho_s c_s V_s} = \frac{\bar{h}_{ci}\,\pi D_i^2}{\rho_s c_s \frac{\pi}{6}(D_o^3 - D_i^3)} = \frac{\left(17\ \text{W/(m}^2\text{K)}\right)(0.44\,\text{m})^2}{7817\text{kg/m}^3\,(461\text{J/(kg K)})\,(1/6)\,(0.49^3 - 0.044^3)\,\text{m}^3} = 1.69 \times 10^{-4}\ 1/\text{s}$$

$$k_3 = \frac{\bar{h}_{co}\ A_o}{\rho_s c_s V_s} = \frac{\bar{h}_{co}\,\pi D_o^2}{\rho_s c_s \frac{\pi}{6}(D_o^3 - D_i^3)} = \frac{\left(22.7\ \text{W/(m}^2\text{K)}\right)(0.49\,\text{m})^2}{7817\text{kg/m}^3\,(461\ \text{J/(kg/K)})\,(1/6)\,(0.49^3 - 0.044^3)\,\text{m}^3} = 2.79 \times 10^{-4}\ 1/\text{s}$$

$$k_1 + k_2 + k_3 = 5.04 \times 10^{-4}\ \text{s}^{-1}$$

$$4k_1\, k_3 = 6.17 \times 10^{-8}\ \text{s}^{-1}$$

$$m_1 = -\,3.28 \times 10^{-5}\ \text{s}^{-1}$$

$$m_2 = -\,4.71 \times 10^{-4}\ \text{s}^{-1}$$

$$m_2 - m_1 = 4.38 \times 10^{-4}\ \text{s}^{-1}$$

The temperature-time history of the water is

$$\frac{T_w - T_\infty}{T_o - T_\infty} = \frac{-4.71\times10^{-4}}{-4.38\times10^{-4}} e^{\left(-3.28\times10^{-5}\frac{1}{s}\right)t} - \frac{-3.28\times10^{-5}}{-4.38\times10^{-4}} e^{\left(-4.71\times10^{-4}\frac{1}{s}\right)t}$$

For the water to cool to 16°C

$$\frac{16°\text{C} - 0°\text{C}}{93°\text{C} - 0°\text{C}} = 0.1720 = 1.075\, E^{\left(-3.28\times10^{-5}\frac{1}{s}\right)t} - 0.075\, E^{\left(-4.71\times10^{-4}\frac{1}{s}\right)t}$$

By trial and error: t = 55,870 s = 15.5 hours

(b) The energy balance for the fluid is given by Equation (2.86a)

$$-(c\,\rho\,V)_w \frac{dT_w}{dt} = \bar{h}_i\, A_i\, (T_w - T_s)$$

Differentiating the temperature-time history

$$\frac{dT_w}{dt} = (T_0 - T_\infty)\left[\frac{m_1\, m_2}{m_2 - m_1} e^{m_1 t} - \frac{m_1\, m_2}{m_2 - m_1} e^{m_1 t}\right] = (T_0 - T_\infty)\, \frac{m_1 - m_2}{m_2 - m_1}(e^{m_1 t} - e^{m_2 t})$$

Substituting this into the energy balance for the fluid

$$-(c\,\rho\,V)_w (T_0 - T_\infty)\, \frac{m_1\, m_2}{m_2 - m_1}(e^{m_1 t} - e^{m_2 t}) = \bar{h}_{ci}\; A_\text{i}\, (T_w - T_s)$$

$$T_0 = T_w + \frac{(\text{cm})_w}{\bar{h}_{ci}\; A_i}\; (T_0 - T_\infty)\, \frac{m_1\, m_2}{m_2 - m_1}(e^{m_1 t} - e^{m_2 t})$$

$$T_\text{s} = 16°\text{C} + \frac{[4187\text{J/(kg K)}](45\,\text{kg})}{[17\ \text{W/(m}^2\text{K)}]\pi(0.44\,\text{m})^2}(93°\text{C} - 0°\text{C})\ \frac{-3.28\times10^{-5}(1/\text{s})\left(-4.71\times10^{-4}(1/\text{s})\right)}{-4.38\times10^{-4}(1/\text{s})}$$

$$\times\left(e^{-3.28\times10^{-5}(1/\text{s})\ (55870\text{a})} - e^{-4.71\times10^{-4}\ (1/\text{s})\ (55870\text{a})}\right)$$

$$T_s = 6.4\ \text{s}$$

PROBLEM 2.64

A copper wire, 0.8 – mm – *OD*, 5 cm long, is placed in an air stream whose temperature rises at T_{air} = (10 + 14 *t*)°C where t is the time in seconds. If the initial temperature of the wire is 10°C, determine its temperature after 2 s, 10 s and 1 min. The heat transfer coefficient between the air and the wire is 40 W(m^2 K).

GIVEN

- A copper wire is placed in an air stream
- Wire diameter (D) = 0.8 mm = 8 × 10^{-3} m
- Wire length (L) = 5 cm = 5 × 10^{-2} m
- Air stream temperature is: T_{air} = (10 + 14 t)°C
- The initial temperature of the wire (T_o) = 10°C
- The heat transfer coefficient $(\bar{h}_c)$ = 40 W/(m^2 K)

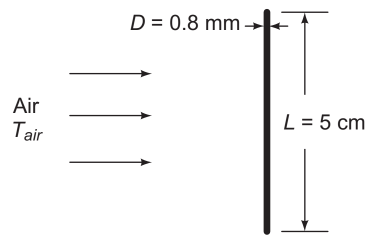
FIND

- The wire temperature after 2 s, 10 s and 1 min

ASSUMPTIONS

- Constant and uniform heat transfer coefficient

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For copper at 127°C

- Thermal conductivity (k) = 391 W/(m K)
- Density (ρ) = 9190 kg/m^3
- Specific heat (c) = 383 J/(kg K)

SOLUTION

The Biot number for this problem is

$$Bi = \frac{\bar{h}_c\, D}{2\, k_c} = \frac{40\ \text{W/(m}^2\text{K)}\,(0.8\times 10^{-3}\ \text{m})}{2\,(391\ \text{W/(m K)})} \simeq 4\times 10^{-5}$$

Therefore the internal resistance of the wire can be neglected.
The temperature-time history of the wire can be calculated from the energy balance, Equation (2.82)

$$-\,c\,\rho\,V\,dT = \bar{h}\ A_s\,(T - T_\infty)\,dt$$

but $T_\infty = T_{\text{air}} = 10 + 14t$

$$\therefore\ -\,c\,\rho\,V\,dT = \bar{h}\ A_s\,(T - 10 + 14t)\,dt$$

Rearranging

$$\frac{dT}{dt} = \frac{\bar{h}\,A_s}{c\rho V}\ (10 + 14t - T)$$

Let $$m = \frac{\bar{h}\,A_s}{c\rho V} = \frac{\bar{h}\,(\pi DL)}{c\rho\left(\frac{\pi}{4}D^2 L\right)} = \frac{4\,\bar{h}}{c\rho D}$$

$$\Rightarrow\qquad m = \frac{4\left(40\ \text{W/(m}^2\text{K)}\right)}{(383\ \text{J/(kg K)})(9190\ \text{kg/m}^3)(0.8\times 10^{-3}\ \text{m})} = 0.057\ 1/\text{s}$$

Thus, the equation becomes

$$\frac{dT}{dt} + m\,T = m\,(10 + 14\,t)$$

This is linear, first order, non-homogeneous differential equation with a homogeneous solution of the form $T = c\, e^{-mt}$ and a particular solution $T = c_o + c_1\, t$. Therefore, the general solution has the form

$$T = c_o + c_1\, t + c_2\, e^{-mt}$$

$$\frac{dT}{dt} = c_1 - c_2\, m\, e^{-mt}$$

Substituting in Equation (1), we get

$$\frac{dT}{dt} + m\,T = c_1 - c_2\, m\, e^{-mt} + m\, c_o + m\, c_1\, t + m\, c_2\, e^{-mt} = m\,(10 + 14\, t)$$

$$\Rightarrow \qquad m\, c_o + c_1 + m\, c_1\, t = 10\, m + 14\, m\, t$$

Equating coefficients on both sides, we get

$$m\, c_1\, t = 14\, mL \;\; \Rightarrow \;\; L_1 = 14$$

$$m\, c_o + c_1 = 10\, m \;\; \Rightarrow \;\; c_o \; \frac{10m - 14}{m} = 10 - \frac{14}{m}$$

Substituting back into the assumed solution yields

$$T = 10 - \frac{14}{m} + 14\, t + c_2\, e^{-mt}$$

Applying the initial condition: $T = 10°\text{C}$ at $t = 0$

$$10 = 10 - \frac{14}{m} + c_2 \;\; \Rightarrow \;\; c_2 = \frac{14}{m}$$

Therefore, the temperature-time history of the wire is

$$T = 10 + 14\, t + \frac{14}{m}\,(e^{-mt} - 1)$$

Since, we have already, calculated before that m = 0.057 1/s,

We can evaluate the temperature of the wire at the requested time instants as follows

$$\text{At} \quad t = 2 \text{ sec},\; T = 10 + 14 \times 2 + \frac{14}{0.057}\,(e^{-0.057 \times 2} - 1) \;\; \Rightarrow \;\; T = 11.54°\text{C}$$

$$\text{At} \quad t = 10 \text{ sec},\; T = 10 + 14 \times 10 + \frac{14}{0.057}\,(e^{-0.057 \times 10} - 1) \;\; \Rightarrow \;\; T = 43.29°\text{C}$$

$$\text{At} \quad t = 1 \text{ min} = 60 \text{ sec},\; T = 10 + 14 \times 60 + \frac{14}{0.057}\,(e^{-0.057 \times 60} - 1) \Rightarrow T = 612.4°\text{C}$$

However, radiation from the wire needs to be accounted well before 60 sec has elapsed.

PROBLEM 2.65

> **A large 2.54-cm.-thick copper plate is placed between two air streams. The heat transfer coefficient on the one side is 28 W/(m^2 K) and on the other side is 57 W/(m^2 K). If the temperature of both streams is suddenly changed from 38°C to 93°C, determine how long it will take for the copper plate to reach a temperature of 82°C.**

GIVEN

- A large copper plate between two air streams whose temperatures suddenly change

- Plate thickness ($2L$) = 2.54 cm = 0.0254 m
- The heat transfer coefficients are
  - $\bar{h}_{c1}$ = 28 W/(m^2 K)
  - $\bar{h}_{c2}$ = 57 W/(m^2 K)
- Air temperature changes from 38°C to 93°C

FIND

- How long it will take for the copper plate to reach a temperature of 82°C

ASSUMPTIONS

- The initial temperature of the plate is 38°C
- The plate can be treated as an infinite slab

SKETCH

PROPERTIES AND CONSTANTS

From Appendix 2, Table 12

For copper Thermal conductivity (k) = 396 W/(m K) at 63°C
Density (ρ) = 8933 kg/m^3
Specific heat (c) = 383 J/kg

SOLUTION

The Biot number for this case, using the larger of the heat transfer coefficients is

$$Bi = \frac{\bar{h}_c\, L}{k} = \frac{[57\text{W/(m}^2\text{K)}](0.0254/2\text{ m})}{396\text{W/(m K)}} = 0.002 << 0.1$$

Therefore, the internal resistance of the slab can be neglected (the temperature of the slab remains uniform) and the temperature-time history can be calculated from an energy balance

Change in internal energy = heat flow from both sides

$$-c\,\rho\,V\,dT = \bar{h}_{c1}\;A\;(T - T_\infty)\,dt + \bar{h}_{c2}\;A\;(T - T_\infty)\,dt$$

$$-c\,\rho\,V\,dT = (\;\bar{h}_{c1} + \bar{h}_{c2}\,)\,A\,(T - T_\infty)\,dt$$

Rearranging

$$\frac{dT}{T - T_\infty} = \frac{d\,(T - T_\infty)}{T - T_\infty} = -\frac{(\bar{h}_{c1} + \bar{h}_{c2})}{c\rho V}dt$$

Integrating between a temperature of T_o at time = 0 to a temperature of T at time = t yields

$$\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right) = \frac{(\bar{h}_{c1} + h_{c2})\;A}{c\rho V}\;t = \frac{(\bar{h}_{c1} + h_{c2})\;A}{c\rho\,(2\;LA)}\;t$$

Solving this for the time

$$t = -\frac{2\;Lc\,\rho}{\bar{h}_{c1} + \bar{h}_{c2}}\;\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = \frac{0.0254 \text{ m} \left(383\text{J/(kg K)}\right)\left((\text{Ws})/\text{J}\right)\left(8933\text{kg/m}^3\right)}{(28+57)\text{W/(m}^2\text{K)}} \ln\left(\frac{82°\text{C} - 93°\text{C}}{30°\text{C} - 93°\text{C}}\right)$$

$$t = 1645 \text{ s} = 27 \text{ min}$$

COMMENTS

Because heat transfer is occurring at both sides of the slab, the characteristic length in the Biot number is approximately half of the slab's thickness. However, since the heat transfer coefficients on the two surfaces are not equal, the center plane is not equivalent to an insulated surface.

PROBLEM 2.66

A 1.4-kg aluminum household iron has a 500 W heating element. The surface area is 0.046 m^2. The ambient temperature is 21°C and the surface heat transfer coefficient is 11 W/(m^2 K). How long after the iron is plugged in will its temperature reach 104°C?

GIVEN

- An aluminum household iron
- Mass of the iron (M) = 1.4 kg
- Power output ($\dot{Q}_G$) = 500 W
- Surface area (A_s) = 0.046 m^2
- The ambient temperature (T_∞) = 21°C
- The heat transfer coefficient ($\bar{h}_c$) = 11 W/(m^2 K)

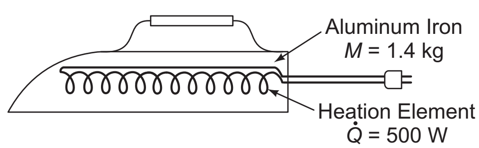
FIND

- How long after the iron is plugged in will its temperature reach 104°C

ASSUMPTIONS

- Constant heat transfer coefficient
- The mass given is for the heated aluminum portion only

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For aluminum
Thermal conductivity (k) = 240 W/(m K) at 127°C
Specific heat (c) = 896 J/(kg K)

SOLUTION

To calculate the Biot number for this problem, we must first calculate the characteristic length

$$L = \frac{\text{Volume}}{\text{Surface area}} = \frac{\frac{M}{\rho}}{A_s} = \frac{M}{\rho A_s} = \frac{1.4 \text{ kg}}{(2702\text{kg/m}^3)(0.046 \text{ m}^2)} = 0.0113 \text{ m}$$

The Biot number is

$$Bi = \frac{\bar{h}_c L}{k} = \frac{[11\ \text{W/(m}^2\ \text{K)}](0.0113\ \text{m})}{240\ \text{W/(m K)}} = 0.0005 < 0.1$$

Therefore, the lumped capacity method may be used. The energy balance for the iron is

Change in internal energy = heat generation – net heat flow from the iron.

$$c\,\rho\,V\,dT = \dot{Q}_G - \bar{h}_c A_s\,(T - T_\infty)\,dt$$

$$\text{Let } \Theta = T - T_\infty \text{ and } m = \frac{\bar{h}_c\ A_s}{c\rho V} = \frac{\bar{h}_c\ A_s}{c\rho\left(\frac{M}{\rho}\right)} = \frac{\bar{h}_c\ A_s}{c M}$$

Then the heat balance can be written

$$\frac{d\Theta}{dt} + m\ \Theta = \frac{\dot{Q}_G}{cM}$$

This is a linear, first order, non-homogeneous differential equation. The solution to the homogeneous equation is $\theta_h = c\ e^{-mt}$ and a particular solution is $\theta_p = c$. The general solution is the sum of the homogeneous and particular solutions

$$\Theta = c_1 + c_2\,e^{-mt}$$

Integrating

$$\frac{d\Theta}{dt} = -\,c_2\,m\,e^{-mt} = -\,m\,(\Theta - c_1) \text{ (From the previous equation)}$$

Substituting this into the heat balance

$$-\,m\,(\Theta - c_1) + m\,\Theta = \frac{\dot{Q}_G}{cM} \Rightarrow c_1 = \frac{\dot{Q}_G}{M\,cM}$$

Applying the initial condition, $\theta = 0$ at $t = 0$ yields

$$-\,c_2\,m = c_1\,m \Rightarrow c_2 = -\,c_1 = \frac{\dot{Q}_G}{M\,cM}$$

Therefore, the temperature-time history of the iron is given by

$$\Theta = \frac{\dot{Q}_G}{m\,cM}\ (1 - e^{-mt})$$

Solving for t

$$t = -\frac{1}{m}\ \ln\left(1 - \frac{\Theta m\ cM}{\dot{Q}_G}\right)$$

$$m = \frac{\bar{h}_c A_s}{cM} = \frac{[11\ \text{W/(m}^2\ \text{K)}](0.046\ \text{m}^2)}{[896\ \text{J/(kg K)}](1.4\ \text{kg})((\text{Ws})/\text{J})} = 4.034 \times 10^{-4}\ \text{s}^{-1}$$

$$t = -\frac{1}{4.034 \times 10^{-4}\,\text{s}^{-1}}\ \ln\left[1 - \frac{(104°\text{C} - 21°\text{C})\left(4.034 \times 10^{-4}\text{s}^{-1}\right)(896\,1/(\text{kg K}))((\text{W s})/\text{J})\ (1.4\ \text{kg})}{500\ \text{W}}\right]$$

$$t = 217\ \text{s} = 3.6\ \text{min}$$

PROBLEM 2.67

Estimate the depth in moist soil at which the annual temperature variation will be 10% of that at the surface.

GIVEN

- Moist soil

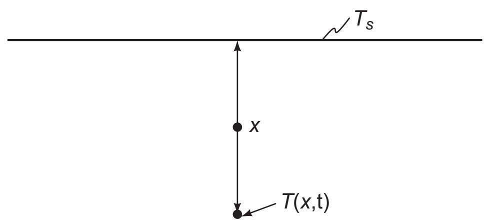
FIND

- The depth in moist soil at which the annual temperature variation will be 10 per cent of that at the surface

ASSUMPTIONS

- Conduction is one dimensional
- The soil has uniform and constant properties
- Annual temperature variation can be treated as a step change in surface temperature with a 6 month response time

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11

For wet soil Thermal conductivity (k) = 2.60 W/(m K) at 20°C

Density (ρ) = 1500 kg/m^3

Thermal diffusivity (α) = 0.0414×10^{-5} m^2/s

SOLUTION

The geometry of this problem is a semi infinite solid as covered in Section 2.6.3. The transient temperature for a change in surface temperature is given by Equation (2.105)

$$\frac{T_{(x,t)} - T_s}{T_i - T_s} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Where T_i is the temperature of the soil until the surface temperature is increased to T_s. For an annual temperature variation of less than 10% of that of the surface

$$T(x, t) - T_i = 0.1\,(T_s - T_i) \text{ at } t = 6 \text{ months}$$

$$T(x, t) = 0.1T_s + 0.9T_i$$

Therefore $\quad T(x, t) - Ts = 0.1T_s + 0.9T_i - T_s = 0.9\,(T_i - T_s)$

$$\frac{T_{(x,t)} - T_s}{T_i - T_s} = 0.9 = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\text{erf}\left(\frac{x}{2\sqrt{[0.0414\times 10^{-5}(\text{m}^2\text{s})](0.5\text{ year})(365\text{ (days/year)})(24\text{ (h/day)})(3600\text{(s/h)})}}\right) = 0.9$$

$$\text{erf}\left(\frac{x}{5.110\text{ m}}\right) = 0.9$$

From Appendix 2, Table 43

$$\text{erf}(1.16) = 0.9$$

$$\therefore \quad \frac{x}{5.110\text{ m}} = 1.16$$

$$x = 6\text{ m}$$

PROBLEM 2.68

A small aluminum sphere of diameter D, initially at a uniform temperature T_o, is immersed in a liquid whose temperature, T_∞, varies sinusoidally according to

$$T_\infty - T_m = A \sin(\omega t)$$

where: T_m = time-averaged temperature of the liquid

A = amplitude of the temperature fluctuation

ω = frequency of the fluctuations

If the heat transfer coefficient between the fluid in the sphere, $\bar{h}_a$, is constant and the system may be treated as a 'lumped capacity,' derive an expression for the sphere temperature as a function of time.

GIVEN

- A small aluminum sphere is immersed in a liquid whose temperature varies sinusoidally
- Diameter of sphere = D
- Liquid temperature variation: $T_\infty - T_m = A \sin(\omega t)$
- The heat transfer coefficient = $\bar{h}_a$ (constant)
- The system may be treated as a 'lumped capacity'

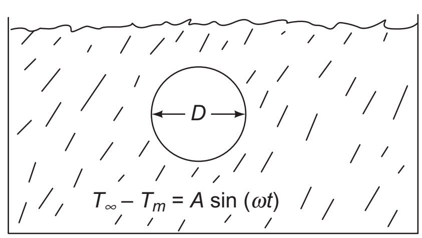
FIND

- An expression for the sphere temperature as a function of time

ASSUMPTIONS

- Constant thermal conductivity

SKETCH



SOLUTION

Let k = thermal conductivity of sphere

ρ = density of sphere

c = specific heat of sphere

An energy balance on the sphere yields

Change in internal energy = heat transfer to liquid

$$\rho c \frac{dT}{dt} = \bar{h}_a A_s (T - T_\infty)$$

$$\frac{dT}{dt} = \frac{h_s A_s}{\rho c V} [T - T_m - A \sin(\omega t)]$$

$$\text{Let } m = \frac{\bar{h}_s A_s}{\rho c V} = \frac{\bar{h}_s \pi d^2}{\rho c \frac{\pi}{6} d^3} = \frac{6 \bar{h}_s}{\rho c D} \text{ and } \Theta = T - T_m$$

$$\frac{d\Theta}{dt} + m\,\Theta = m A_s \sin(\omega t)$$

This is a first order, linear, non-homogeneous differential equation. The general solution is the sum of the homogeneous solution and a particular solution. The homogeneous solution is determined by the characteristic equation, found by substituting $\theta = e^{\lambda t}$ into the homogeneous equation

$$\lambda e^{\lambda t} + m e^{\lambda t} = 0 \qquad (\lambda = -m)$$

The homogeneous solution is $\theta_h = Ce^{-mt.}$

As a particular solution, try $\theta_p = K \cos(\omega t) + M \sin(\omega t)$, substituting θ_p and its derivative into the energy balance

$$-\omega K \sin(\omega t) + M \omega \cos(\omega t) + m K \cos(\omega t) + m M \sin(\omega t) = m A \sin(\omega t)$$

$$(M\omega + mK) \cos(\omega t) - (\omega K - mM) \sin(\omega t) = m A_s \sin(\omega t)$$

$$\therefore M \omega + m K = 0 \Rightarrow M = -\frac{m K}{\omega}$$

$$\text{and } \omega K - m M = -m A_s \quad \Rightarrow \omega K + \frac{m K}{\omega} m = -m A_s$$

$$\therefore K = \frac{M A_s \omega}{\omega^2 + m^2} \text{ and } M = \frac{m^2 A_s}{\omega^2 + m^2}$$

Therefore, the general solution is

$$\Theta = C e^{-mt} + \frac{M A_s}{\omega^2 + m^2} [(-\omega \cos(\omega t) + m \sin(\omega t)]$$

At $t = 0$, $T = T_o$ and $\theta = \theta_o = T_o - T_m$

$$\Theta_o = C - \frac{m A_s \omega}{\omega^2 + m^2} \Rightarrow C = \Theta_o + \frac{m A_s \omega}{\omega^2 + m^2}$$

The dimensionless temperature distribution is

$$\frac{\Theta}{\Theta_o} = \left(1 + \frac{m A_s \omega}{\Theta_o(\omega^2 + m^2)}\right) e^{-mt} + \frac{m A_s}{\omega^2 + m^2} [(m \sin(\omega t) - \omega \cos(\omega t)]$$

PROBLEM 2.69

A wire of perimeter *P* and cross-sectional area A emerges from a die at a temperature *T* above ambient and with a velocity *U*. Determine the temperature distribution along the wire in the steady state if the exposed length downstream from the die is quite long. State clearly and try to justify all assumptions.

GIVEN

- A wire emerging from a die at a temperature (T) above ambient
- Wire perimeter = P
- Cross-sectional area = A
- Wire emerges at a temperature T above ambient
- Wire velocity = U

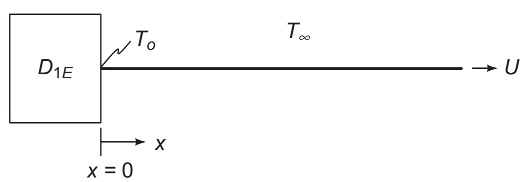
FIND

- The temperature distribution along the wire in the steady state if the exposed length downstream from the die is quite long. State clearly and try to justify all assumptions

ASSUMPTIONS

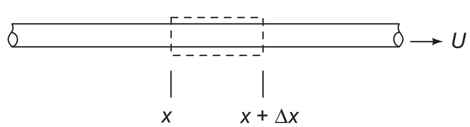
- Ambient temperature is constant at T_∞
- Heat transfer coefficient between the wire and the air is uniform and constant at h_c
- The material properties of the wire are constant
  - Thermal conductivity = k
  - Thermal diffusivity = α
- Axial conduction only
- Wire temperature is uniform at a cross section (negligible internal thermal resistance)

SKETCH



SOLUTION

Consider a control volume around the wire



Performing an energy balance on the control volume

Conduction into volume + Energy carried into the volume by the moving wire = Conduction out of volume + Convection to the environment + Energy carried out of the volume by the moving wire.

$$-kA\left.\frac{dT}{dx}\right|_x + UA\rho c\,T(x) = -kA\left.\frac{dT}{dx}\right|_{x+\Delta x} + \bar{h}_c\, P\,\Delta_x(T - T_\infty) + UA\rho c\,T(x+\Delta x)$$

$$\frac{\left.\frac{dT}{dx}\right|_x - \left.\frac{dT}{dx}\right|_{x+\Delta x}}{\Delta x} = \frac{\frac{\rho c}{k}U[T(x+\Delta x) - T(x)]}{\Delta x} + \frac{\bar{h}_c P}{kA}(T - T_\infty)$$

letting $\Delta x \to 0$

$$\frac{d^2T}{dx^2} = \frac{U}{\alpha}\frac{dT}{dx} + \frac{\bar{h}_c P}{k\,A}\,(T - T_\infty)$$

$$\text{Let } \theta = T - T_\infty \text{ and } m = \frac{\bar{h}_c P}{k\,A} = \frac{\bar{h}_c\,\pi D}{k\frac{\pi}{4}D^2} = \frac{4\,\bar{h}_c}{k\,D}$$

Then

$$\frac{d^2\theta}{dx^2} - \frac{U}{\alpha}\frac{d\theta}{dx} - m\,\theta = 0$$

This is a linear, differential equation with constant coefficients. The solution has the following form

$$\theta = c_1\,e^{s_1 x} + c_2\,e^{s_2 x}$$

Substituting this solution and its derivatives into the differential equation:

$$s_1^{\,2}\,c_1\,e^{s_1 x} + s_2^{\,2}\,c_2\,e^{s_2 x} - \frac{U}{\alpha}\left(s_1\,c_1\,e^{s_1 x} + s_2\,c_2\,e^{s_2 x}\right) - m\,(c_1\,e^{s_1 x} + c_2\,e^{s_2 x}) = 0$$

$$s_1^{\,2} - \frac{U}{\alpha}\,s_1 - m = 0 \Rightarrow s_1 = \frac{1}{2}\left(\frac{U}{\alpha}\sqrt{\left(\frac{U}{\alpha}\right)^2 + 4\text{ m}}\right)$$

$$s_2^{\,2} - \frac{U}{\alpha}\,s_2 - m = 0 \Rightarrow s_2 = \frac{1}{2}\left(\frac{U}{\alpha}\sqrt{\left(\frac{U}{\alpha}\right)^2 + 4\text{ m}}\right)$$

$$\therefore \text{ Let } s_1 = \frac{1}{2}\left(\frac{U}{\alpha} + \sqrt{\left(\frac{U}{\alpha}\right)^2 + 4\text{ m}}\right) \text{ and } s_2 = \frac{1}{2}\left(\frac{U}{\alpha} - \sqrt{\left(\frac{U}{\alpha}\right)^2 + 4\text{ m}}\right)$$

The boundary conditions for the problem are

1. $\theta = \theta_o$ at $x = 0$
2. $\theta \to 0$ at $x \to \infty$

Applying the first boundary condition

$$\theta_o = c_1 + c_2$$

Since, by inspection, s_1 must be positive, for the second boundary condition to be satisfied, the constant c_1 must be zero. Therefore, the temperature distribution in the wire is

$$\theta = \theta_o\,e^{s_2 t}$$

or

$$T = T_\infty + (T_o - T_\infty)\,\exp\left[\frac{x}{2}\left(\frac{U}{\alpha}\sqrt{\left(\frac{U}{\alpha}\right)^2 + 4\text{ m}}\right)\right]$$

PROBLEM 2.70

Ball bearings are to be hardened by quenching them in a water bath at a temperature of 37°C. Suppose you are asked to devise a continuous process in which the balls could roll from a soaking oven at a uniform temperature of 870°C into the water, where they are carried away by a rubber conveyer belt. The rubber conveyor belt would, however, not be satisfactory if the surface temperature of the balls leaving the water is above 90°C. If the surface coefficient of heat transfer between the balls and the water may be assumed

to be equal to 590 W/(m^2 K), (a) find an approximate relation giving the minimum allowable cooling time in the water as a function of the ball radius for balls up to 1.0-cm in diameter, (b) calculate the cooling time, in seconds, required for a ball having a 2.5-cm diameter, and (c) calculate the total amount of heat in watts which would have to be removed from the water bath in order to maintain its temperature uniform if 100,000 balls of 2.5-cm diameter are to be quenched per hour.

GIVEN

- Ball bearings quenched in a water bath
- Water bath temperature (T_∞) = 37°C
- Initial temperature of the balls (T_o) = 870°C
- Final surface temperature of the balls (T_f) = 90°C
- Heat transfer coefficie ($\bar{h}_c$) = 590 W/(m^2 K)

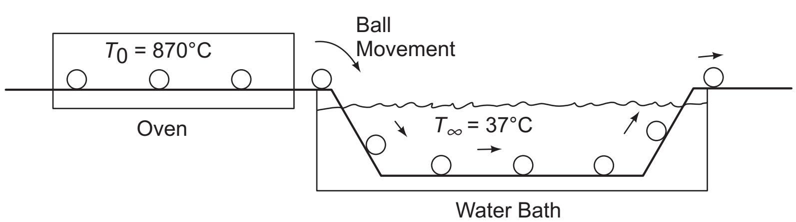
FIND

(a) An approximate relation giving the minimum allowable cooling time in the water as a function of the ball radius for balls upto 1.0 cm in diameter
(b) The cooling time, in seconds, required for a ball having a 2.5 cm diameter
(c) The total amount of heat in watts which would have to be removed from the water bath in order to maintain its temperature uniform if 100,000 balls of 2.5 cm diameter are to be quenched per hour

ASSUMPTIONS

- The ball bearings are 1% carbon steel

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10
For 1% carbon steel Thermal conductivity (k) = 43 W/(m K)
Density (ρ) = 7.801 kg/m^3
Specific heat (c) = 473 J/(kg K)
Thermal diffusivity (α) = 1.172 × 10^{-5} m^2/s

SOLUTION

(a) For 1.0 cm diameter balls

$$Bi = \frac{\bar{h}_c\ r_o}{k} = \frac{[590\,\text{W/(m}^2\ \text{K)}]\,(0.005\,\text{m})}{43\,\text{W/(m K)}} = 0.07 < 0.1$$

Therefore, a lumped capacity method can be used for balls less than 1 cm in diameter. The time temperature history of the ball is given by Equation 2.84

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{h\,A_s}{c\rho V}t} = e^{-\frac{h\,4\pi r_o^2}{c\rho\frac{4}{3}\pi r_o^3}t} = e^{-\frac{3\,h}{c\rho\,r_o}t}$$

Solving for the minimum cooling time

$$t = -\frac{c\rho\, r_o}{3\,h}\ln\left(\frac{T - T_\infty}{T_\text{o} - T_\infty}\right)$$

$$t = -\frac{[473\text{ J/(kg K)}](\text{Ws/J})\left(7801\,(\text{kg/m}^3)\right)r_\text{o}}{3\left(590\text{ W/(m}^2\text{ K)}\right)}\ln\left(\frac{90°\text{C} - 37°\text{C}}{870°\text{C} - 37°\text{C}}\right) = 5743\ r_o\text{ s/m}$$

(b) For balls having a diameter of 2.5 cm

$$Bi = \frac{h_c L}{k} = \frac{[590\text{ W/(m}^2\text{ K)}](0.0125\text{ m})}{43\text{ W/(m K)}} = 0.172 > 0.1$$

Therefore, the internal resistance is significant and a chart solution will be used. From Figure 2.39 for $1/B_i = 5.8$ and $r = r_\text{o}$

$$\frac{T(r_o, t) - T_\infty}{T(0,t) - T_\infty} = 0.92$$

For a final surface temperature ($T\,(r_\text{o}, t)$) of 90°C

$$T(0, t) = T_\infty + \frac{1}{0.92}\ (T(r_\text{o}, t) - T_\infty) = 37°\text{C} = \frac{1}{0.92}\ (90°\text{C} – 37°\text{C}) = 94.6°\text{C}$$

$$\frac{T_{\text{o},t} - T_\infty}{T_\text{o} - T_\infty} = \frac{94.6°\text{C} - 37°\text{C}}{870°\text{C} - 37°\text{C}} = 0.069$$

From Figure 2.39, for $(T_\text{o}, t - T_\infty) / (T_\text{o} - T_\infty) = 0.069$ and $1/Bi = 5.3$: $Fo = 5.3 = \alpha\, t/r_\text{o}^2$

$$t = \frac{Fo\, r_\text{o}^2}{\alpha} = \frac{5.3\,(0.0125\text{ m})^2}{1.172\times10^{-5}(\text{m}^2/\text{s})} = 71\text{ sec}$$

(c) Figure 2.39 can be used to calculate the heat transferred from one ball during the cooling time:

$$(Bi)^2\,Fo = (0.172)^2\,(5.3) – 0.157$$

From Figure 2.39 $Q(t)/Q_i = 0.93$
From Table 2.3

$$Q_i = \rho\, c\,\frac{4}{3}\,\pi\, r_\text{o}^3\ (T_\text{o} - T_\infty) = [7801\,(\text{kg/m}^3)](473(\text{J/kg K}))\frac{4}{3}\,\pi(0.0125\text{ m})^3\,(870°\text{C} – 37°\text{C}) = 25{,}150\text{ J}$$

$$\therefore\ Q(t) = 0.93\,Q_i = 0.93\ (25{,}159\text{ J}) = 23{,}390\text{ J}$$

The amount of heat needed to quench 100,000 balls per hour is

$$q = (\text{Balls/hr})\ (\text{Energy/ball}) = \frac{[100{,}000\,(1/\text{h})](23{,}390\text{ J})}{3600\,(\text{s/h})} = 650{,}000\text{ W}$$

PROBLEM 2.71

Estimate the time required to heat the center of a 1.5-kg roast in a 163°C over to 77°C. State your assumptions carefully and compare your results with cooking instructions in a standard cookbook.

GIVEN

- A roast in an oven
- Mass of the roast (m) = 1.5 kg

- Oven temperature (T_∞) = 163°C
- Final temperature of the roast's center (T_f) = 77°C

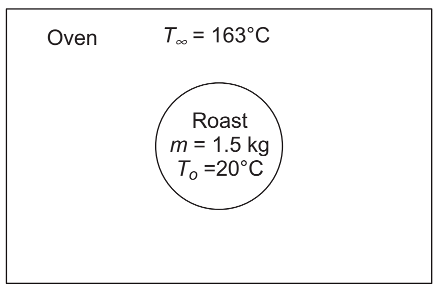
FIND

- The time required to heat the roast

ASSUMPTIONS

- The shape of the roast can be approximated by a sphere
- The roast temperature is initially uniform at (T_o) = 20°C
- The properties of the roast are approximately those of water
- ▪ Thermal conductivity (k) = 0.5 W/(m K)
  ▪ Density (ρ) = 1000 kg/m^3
  ▪ Specific heat = 4000 J/(kg K)
- A uniform heat transfer coefficient of ($\bar{h}_c$) = 18 W/(m^2 K) exists between the roast and the oven air (midline of the range for free convection given in Table 1.4.)

SKETCH



SOLUTION

With the assumptions listed above, the radius of the spherical roast is given by

$$V = \frac{m}{\rho} = \frac{4}{3}\pi r_o^3 \Rightarrow r_o = \left(\frac{3\text{ m}}{4\pi\rho}\right)^{\frac{1}{3}} = \left(\frac{3(1.5\text{ kg})}{4\pi\left(1000\text{ (kg/m}^3)\right)}\right)^{\frac{1}{3}} = 0.071\text{ m}$$

Figure 2.39 can be used to find the Fourier number. To use Figure 2.39, the following parameters (which are listed in Table 2.3) are needed

$$Bi = \frac{\bar{h}_c\, r_o}{k} = \frac{[18\text{ W/(m}^2\text{K)}](0.071\text{ m})}{0.5\text{ W/(m K)}} = 2.56 \Rightarrow \frac{1}{Bi} = 0.391$$

$$\frac{\theta(0,t)}{\theta_o} = \frac{T - T_\infty}{T_o - T_\infty} = \frac{77°\text{C} - 163°\text{C}}{20°\text{C} - 163°\text{C}} = 0.60$$

From Figure 2.39 Fo = 0.2

From Table 2.3 $Fo = \alpha t/r_o^2$

Solving for the time

$$t = \frac{r_o^2\, Fo}{\alpha} = \frac{r_o^2\, Fo\,\rho c}{k}$$

$$t = \frac{(0.071\text{m})^2(0.2)\left(1000\cdot\text{kg/m}^3\right)\left(4000\text{J/(kg K)}\right)}{0.5\text{W/m K}\,(\text{J/W s})}$$

$$t = 8065\text{ s} = 134\text{ min}$$

The Better Homes and Gardens Cookbook recommends cooking a Standing Rib Roast with the oven set at 325°F (163°C) for 27-30 minutes per pound to achieve a center temperature of 170°F (77°C) which is considered well done.

This calculation yielded 134 minutes for 1.5 kg (3.3 lbs) or 40 minutes per pound. The discrepancy is probably due to inaccuracies in the assumed properties of the roast.

PROBLEM 2.72

A stainless steel cylindrical billet [k = 14.4 W/(m K), $\alpha = 3.9 \times 10^{-6}$ m^2/s] is heated to 593°C preparatory to a forming process. If the minimum temperature permissible for forming is 482°C, how long may the billet be exposed to air at 38°C if the average heat transfer coefficient is 85 W/(m^2 K)? The shape of the billet is shown in the sketch.

GIVEN

- A stainless steel cylindrical billet exposed to air
- Thermal conductivity (k) = 14.4 W/(m K)
- Thermal diffusivity (α) = 3.9×10^{-6} m^2/s
- Initial temperature (T_o) = 593°C
- The minimum temperature permissible for forming is 482°C
- Air temperature (T_∞) = 38°C
- Average heat transfer coefficient ($\bar{h}_c$) = 85 W/(m^2 K)

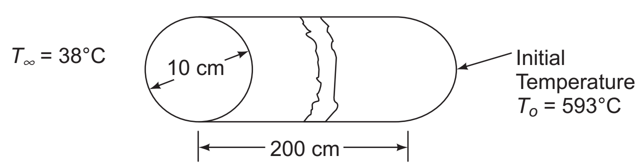
FIND

- How long may the billet be exposed to the air?

ASSUMPTIONS

- End effects are negligible
- Constant heat transfer coefficient
- Conduction in the radial direction only
- Uniform thermal properties

SKETCH



SOLUTION

The Biot number is calculated to determine if internal resistance is significant

$$Bi = \frac{\bar{h}_c r_o}{k} = \frac{[85 \text{ W/(m}^2\text{K)}](0.05 \text{ m})}{14.4 \text{ W/(m K)}} = 0.3 > 0.1$$

Therefore, internal resistance is important, and a chart solution is used.

The chart for this geometry is Figure 2.38. The approach will be as follows:

1. Use the Biot number and the minimum surface temperature given to find $(T_{o,t} - T_\infty)/(T_o - T_\infty)$ from Figure 2.38.
2. Apply $(T_{o,t} - T_\infty)/(T_o - T_\infty)$ and the Biot number to Figure 2.38 to find the Fourier number.
3. Use the Fourier number to find the time it takes for the surface to cool to the given minimum surface temperature.
1. From Figure 2.38, for $r = r_o$ (r/r_o = 1.0) and 1/Bi = 3.33

$$\frac{T(r_o,t)-T_\infty}{T_o-T_\infty} = 0.87$$

The surface temperature must not fall below 482°C

$$\frac{T(r_o,t)-T_\infty}{T_o-T_\infty} = \frac{482°\text{C}-38°\text{C}}{593°\text{C}-30°\text{C}} = 0.80$$

Combining these results

$$\frac{T(0,t)-T_\infty}{T_o-T_\infty} = \frac{\left(\dfrac{T(r_o,t)-T_\infty}{T_o-T_\infty}\right)}{\left(\dfrac{T(r_o,t)-T_\infty}{T(0,t)-T_\infty}\right)} = \frac{0.80}{0.87} = 0.92$$

2. From Figure 2.38, for $1/Bi = 3.33$ and $(T(0, t) - T_{oo})/(T_o - T_{oo}) = 0.92$

$$F_o = \frac{\alpha t}{r_o^2} = 0.2$$

3. Solving for the time

$$t = \frac{Fo\, r_o^2}{\alpha} = \frac{0.2\,(0.05\text{ m})^2}{3.9\times10^{-6}(\text{m}^2/\text{s})} = 128\text{ s} = 2.1\text{ min}$$

PROBLEM 2.73

In the vulcanization of tires, the carcass is placed into a jig, and steam at 149°C is admitted suddenly to both sides. If the tire thickness is 2.5 cm, the initial temperature is 21°C, the heat transfer coefficient between the tire and the steam is 150 W/(m^2 K), and the specific heat of the rubber is 1650 J/(kg K), estimate the time required for the center of the rubber to reach 132°C.

GIVEN

- Tire suddenly exposed to steam on both sides
- Steam temperature (T_∞) = 149°C
- Tire thickness ($2L$) = 2.5 cm = 0.025 m
- Initial tire temperature (T_o) = 21°C
- The heat transfer coefficient (h_c) = 150 W/(m^2 K)
- The specific heat of the rubber (c) = 165 J/(kg K)

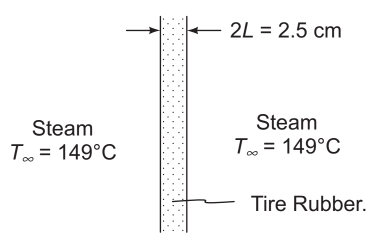
FIND

- The time required for the central layer to reach 132°C

ASSUMPTIONS

- Shape effects are negligible, tire can be treated as an infinite plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
For bana rubber

Thermal conductivity (k) = 0.465 W/(m K) at 20°C
Density (ρ) = 1250 g/m^3

SOLUTION

The significance of the internal resistance is determined from the Biot number

$$Bi = \frac{\bar{h}_c L}{k} = \frac{[150\,\text{W/(m}^2\text{K)}]\left(\frac{0.025}{2}\,\text{m}\right)}{0.465\,\text{W/(m K)}} = 4.0 >> 0.1$$

Therefore, the internal resistance is significant and a chart solution will be used. Figure 2.37 contains the charts for this geometry.

The time required can be calculated from the Fourier number which can be found from Figure 2.37. The centerline at time t must be 132°C, therefore

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{132°\text{C} - 149°\text{C}}{21°\text{C} - 149°\text{C}} = 0.13$$

From Figure 2.37, for $(T(0, t) - T_\infty)/(T_o - T_\infty)$ = 0.13 and $1/Bi$ = 0.25

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c L^2} = 1.32$$

Solving for the time

$$t = \frac{\rho c L^2 Fo}{k} = \frac{[1250\,(\text{kg/m}^3)](1650\,\text{J/(kg K)})\,((\text{W s})/\text{J})\,(0.025/2\ \text{m})^2\,(1.3)}{0.465\,\text{W/(m K)}}$$

$$t = 900\ \text{s} = 15\ \text{min}$$

PROBLEM 2.74

A long copper cylinder 0.6 m in diameter and initially at a uniform temperature of 38°C is placed in a water bath at 93°C. Assuming that the heat transfer coefficient between the copper and the water is 1248 W/(m^2 K), calculate the time required to heat the center of the cylinder to 66°C. As a first approximation, neglect the temperature gradient within the cylinder r/h, then repeat your calculation without this simplifying assumption and compare your results.

GIVEN

- A long copper cylinder is placed in a water bath
- Diameter of cylinder (D) = 0.6 m
- Initial temperature (T_o) = 38°C
- Water bath temperature (T_∞) = 93°C
- The heat transfer coefficient $(\bar{h}_c)$ = 1248 W/(m^2 K)

FIND

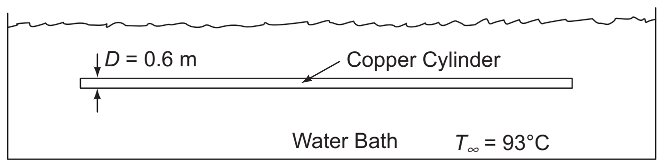
Calculate the time required to heat the center of the cylinder to 66°C assuming

(a) Negligible temperature gradient within the cylinder
(b) Without this simplification, then
(c) Compare your results

ASSUMPTIONS

- Neglect end effects
- Radial conduction only

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For copper

Thermal conductivity (k) = 396 W/(m K) at 63°C
Density (ρ) = 8933 kg/m^3
Specific heat (c) = 383 J/(kg K)
Thermal diffusivity (α) = 1.166×10^{-4} m^2/s

SOLUTION

(a) For a negligible temperature gradient within the cylinder, the temperature-time history is given by Equation (2.84)

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{\bar{h}_c A_s}{c\rho V}t} = e^{-\frac{\bar{h}_c \pi D L}{c\rho \frac{\pi}{4} D^2 L}t} = e^{-\frac{4\bar{h}_c}{c\rho D}t}$$

Solving for the time

$$t = -\frac{c\rho D}{4 h_c} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = \frac{[383\ \text{J/(kg K)}](\text{W s/J})\left(8933(\text{kg/m}^3)\right)(0.6\ \text{m})}{4\left(1248\ \text{W/(m}^2\ \text{K)}\right)} \ln\left(\frac{66°\text{C} - 93°\text{C}}{38°\text{C} - 93°\text{C}}\right)$$

$$t = 293\ \text{sec} = 4.9\ \text{min}$$

(b) The chart method can be used to take the temperature gradient within the cylinder into account. Figure 2.38 contains the charts for a long cylinder.

$$Bi = \frac{\bar{h}_c r_o}{k} = \frac{[1248\ \text{W/(m}^2\text{K)}](0.3\,\text{m})}{396\ \text{W/(m K)}} = 0.95 \Rightarrow \frac{1}{Bi} = 1.1$$

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \left(\frac{66°\text{C} - 93°\text{C}}{38°\text{C} - 93°\text{C}}\right) = 0.49$$

From Figure 2.38, for $1/Bi = 1.1$ and $T(0, t) - T_\infty)/(T_o - T_\infty) = 0.49$

$$Fo = \frac{\alpha t}{r_o^2} = 0.5$$

Solving for the time

$$t = \frac{Fo\, r_o^2}{\alpha} = \frac{0.5(0.3\text{m})^2}{1.166\times 10^{-4}\text{m}^2/\text{s}} = 386 \text{ s} = 6.4 \text{ min}$$

(c) The lumped capacity method (a) underestimates the required time by 24%.

COMMENTS

Since the Biot number is of the order of magnitude of unity, we could not expect that the lumped capacity assumption is valid.

PROBLEM 2.75

A steel sphere with a diameter of 7.6 cm is to be hardened by first heating it to a uniform temperature of 870°C and then quenching it in a large bath of water at a temperature of 38°C. The following data apply

surface heat transfer coefficient $\bar{h}$ = 590 W/(m^2 K)

thermal conductivity of steel = 43 W/(m K)

specific heat of steel = 628 J/(kg K)

density of steel = 7840 kg/m^3

Calculate: (a) time elapsed in cooling the surface of the sphere to 204°C and (b) time elapsed in cooling the center of the sphere to 204°C.

GIVEN

- A steel sphere is quenched in a large water bath
- Diameter (D) = 7.6 cm = 0.076 m
- Initial uniform temperature (T_o) = 870°C
- Water temperature (T_∞) = 38°C
- Surface heat transfer coefficient (h) = 590 W/(m^2 K)
- Thermal conductivity of steel (k) = 43 W/(m K)
- Specific heat of steel (c) = 628 J/(kg K)
- Density of steel (ρ) = 7840 kg/m^3

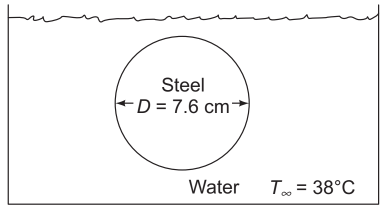
FIND

(a) Time elapsed in cooling the surface of the sphere to 204°C
(b) Time elapsed in cooling the center of the sphere to 204°C

ASSUMPTIONS

- Constant water bath temperature, thermal properties, and transfer coefficient

SKETCH



SOLUTION

The importance of the internal resistance can be determined from the Biot number

$$Bi = \frac{\bar{h}_c r_o}{k} = \frac{[590 \text{ W/(m}^2\text{ K)}]\left(\frac{0.076}{2}\text{ m}\right)}{43 \text{ W/(m K)}} = 0.52 > 0.1$$

Therefore, the internal resistance is significant and a chart solution will be used.
Figure 2.39 contains the charts for this geometry.

(a) From Figure 2.39, for $r = r_o$ and $1/Bi = 1.9$:

$$\frac{T(r_o,t) - T_{oo}}{T(0,t) - T_{oo}} = 0.78$$

Solving for the center temperature

$$T(0, t) = T_\infty + 1.28\ (T(r_o, t) - T_\infty) = 38°\text{C} + 1.28(204°\text{C} - 38°\text{C}) = 251°\text{C}$$

$$\therefore \frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{251°\text{C} - 38°\text{C}}{870°\text{C} - 38°\text{C}} = 0.26$$

From Figure 2.39 for $(T(0, t) - T_\infty)/(T_o - T_\infty) = 0.26$, $1/Bi = 1.9$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k t}{\rho c r_o^2} = 0.8$$

Solving for the time

$$t = \frac{Fo\, \rho c r_o^2}{k} = \frac{0.8\left(7840\text{kg/m}^3\right)\left(628\text{J/(kg K)}\right)\left(\frac{0.076}{2}\text{ m}\right)^2}{43\text{W/(m}^2\text{K)}} = 132 \text{ s} = 2.2 \text{ min}$$

(For the surface temperature to reach 204°C)

(b) For a center temperature of 204°C

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{204°\text{C} - 38°\text{C}}{870°\text{C} - 38°\text{C}} = 0.20$$

From Figure 2.39 for $(T(0, t) - T_\infty)/(T_o - T_\infty) = 0.2$, $1/Bi = 1.9$: $Fo = 1.1$, therefore

$$t = \frac{1.1\left(7840\text{kg/m}^3\right)\left(628\,\text{J/(kg K)}\right)\left(\frac{0.076}{2}\text{ m}\right)^2}{43\text{W/(m}^2\text{ K)}} = 182 \text{ s} = 3.0 \text{ min}$$

(For the center temperature to reach 204°C)

PROBLEM 2.76

A 2.5-cm-thick sheet of plastic initially at 21°C is placed between two heated steel plates that are maintained at 138°C. The plastic is to be heated just long enough for its midplane temperature to reach 132°C. If the thermal conductivity of the plastic is 1.1×10^{-3} W/(m K), the thermal diffusivity is 2.7×10^{-6} m^2/s, and the thermal resistance at the interface between the plates and the plastic is negligible, calculate: (a) the required heating time, (b the temperature at a plane 0.6 cm from the steel plate at the moment the heating is discontinued, and (c) the time required for the plastic to reach a temperature of 132°C 0.6 cm from the steel plate.

GIVEN

- A sheet of plastic is placed between two heated steel plates
- Sheet thickness ($2L$) = 2.5 cm = 0.025 m
- Initial temperature (T_o) = 21°C
- Temperature of steel plates (T_s) = 138°C
- Heat until midplane temperature of sheet (T_c) = 132°C
- The thermal conductivity of the plastic (k) = 1.1×10^{-3} W/(m K)
- The thermal diffusivity (α) = 2.7×10^{-6} m^2/s
- The thermal resistance at the interface between the plates and the plastic is negligible

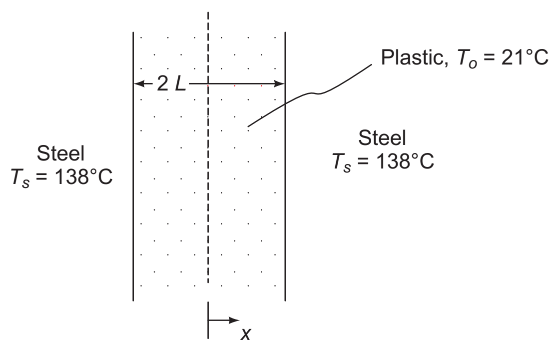
FIND

(a) The required heating time
(b) The temperature at a plane 0.6 cm from the steel plate at the moment the heating is discontinued
(c) The time required for the plastic to reach a temperature of 13°C 0.6 cm from the steel.

ASSUMPTIONS

- The initial temperature of the sheet is uniform
- The temperature of the steel plates is constant
- The thermal conductivity of the sheet is constant

SKETCH



SOLUTION

The chart solutions apply to convective boundary conditions but can be applied to this problem by letting $h_c \to \infty$. Therefore, $1/Bi = 0$.

(a) To find the time required to heat the midplane from 21°C to 132°C, first calculate the coordinate of Figure 2.42

$$\frac{T(0,t)-T_\infty}{T_o - T_\infty} = \frac{132°\text{C} - 138°\text{C}}{21°\text{C} - 138°\text{C}} = 0.0513$$

From Figure 2.42

$$Fo = \frac{\alpha t}{L^2} = 1.3$$

Solving for the time

$$t = \frac{Fo\,L^2}{\alpha} = \frac{1.3\left(\frac{0.025}{2}\,\text{m}\right)^2}{27\times10^{-6}\,\text{m}^2/\text{s}} = 75\ \text{sec}$$

(b) At 0.6 cm from the steel plate

$$x = L - 0.006\ \text{m} = 0.0125\ \text{m} - 0.006\ \text{m} = 0.0065\ \text{m} \Rightarrow \frac{x}{L} = \frac{0.0065\,\text{m}}{0.0125\,\text{m}} = 0.52$$

From Figure 2.42

$$\frac{T(0.0065\,\text{m}, t) - T}{T(0,t) - T_\infty} = 0.70$$

$$T\,(0.0065\ \text{m},\ t) = 0.7\,(T(0,\ t) - T_\infty) + T_\infty = 0.7\,(132°\text{C} - 138°\text{C}) + 138°\text{C} = 133.8°\text{C}$$

(c) When the temperature 0.6 cm from the steel plate is 132°C, the center temperature

$$T(0,\ t) = T_\infty + \frac{1}{0.7}\,(T(0.0065\ \text{m},\ t) - T_\infty) = 138°\text{C} + \frac{1}{0.7}\,(132°\text{C} - 130°\text{C}) = 129.4°\text{C}$$

$$\therefore\ \frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{129.4^\circ\text{C} - 138^\circ\text{C}}{21^\circ\text{C} - 138^\circ\text{C}} = 0.0733$$

From Figure 2.37

$$t = \frac{FoL^2}{\alpha} = \frac{1.15\left(\frac{0.025}{2}\,\text{m}\right)^2}{2.7\times10^{-6}\,\text{m}^2/\text{s}} = 67\ \text{sec}$$

PROBLEM 2.77

A monster turnip (assumed spherical) weighing in at 0.45 kg is dropped into a cauldron of water boiling at atmospheric pressure. If the initial temperature of the turnip is 17°C, how long does it take to reach 92°C at the center? Assume that

$\bar{h}_c$ **= 1700 W/(m^2 K)** $\qquad$ c_ρ **= 3900 J/(kg K)**

k **= 0.52 W/(m K)** $\qquad$ ρ **= 1040 kg/m^3**

GIVEN

- A turnip is dropped into boiling water
- Mass of turnip (M) = 0.45 kg
- Water is boiling at atmospheric pressure
- Initial temperature of the turnip (T_o) = 17°C

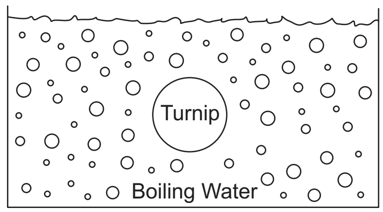
FIND

- Time needed to reach 92°C at the center

ASSUMPTIONS

- Heat transfer coefficient (h_c) = 1700 W/(m^2 K)
- Specific heat ($c\rho$) = 3900 J/(kg K)
- Thermal conductivity (k) = 0.52 W/(m K)
- Density (ρ) = 1040 kg/m^3
- The specific heat of the turnip is constant
- Altitude is sea level, therefore, temperature of boiling water (T_∞) = 100°C
- One dimensional conduction in the radial direction

SKETCH



SOLUTION

The radius of the turnip is given by

$$\text{Volume} = \frac{4}{3}\pi r_o^3 = \frac{M}{\rho} \Rightarrow r_o = \left(\frac{3M}{4\pi\rho}\right)^{\frac{1}{3}} = \left(\frac{3(0.45\,\text{kg})}{4\pi\left(1040\ \text{kg/m}^3\right)}\right)^{\frac{1}{3}} = 0.047\ \text{m}$$

The Biot number is

$$Bi = \frac{\overline{h_c}\, r_o}{k} = \frac{[1700\ \text{W/(m}^2\ \text{K)}]\,(0.047\ \text{m})}{0.52\ \text{W/(m K)}} = 153 > 0.1$$

Therefore, internal resistance is significant and the chart method will be used.

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{92^\circ\text{C} - 100^\circ\text{C}}{17^\circ\text{C} - 100^\circ\text{C}} = 0.096$$

From Figure 2.39, $(T(0,\, t) - T_\infty)/(T_o - T_\infty) = 0.096$ and $1/Bi = 0.0065$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c r_o^2} = 0.25$$

Solving for the time

$$t = \frac{Fo\,\rho c r_o^2}{k} = \frac{0.25\left(1040\ \text{kg/m}^3\right)\left(3900\ \text{J/(kg K)}\right)\left(0.047\ \text{m}\right)^2}{0.52\ \text{W/(m}^2\ \text{K)}}$$

$$t = 4307\ \text{s} = 72\ \text{min} = 1.2\ \text{hours}$$

PROBLEM 2.78

An egg, which for the purposes of this problem can be assumed to be a 5-cm-diameter sphere having the thermal properties of water, is initially at a temperature of 4°C. It is immersed in boiling water at 100°C for 15 min. The heat transfer coefficient from the

water to the egg may be assumed to be 1700 W/(m^2 K). What is the temperature of the egg center at the end of the cooking period?

GIVEN

- An egg is immersed in boiling water
- Initial temperature (T_o) = 4°C
- Temperature of boiling water (T_∞) = 100°C
- Time that the egg is in the water (t) = 15 min. = 900 s
- The heat transfer coefficient (h_c) = 1700 W/(m^2 K)

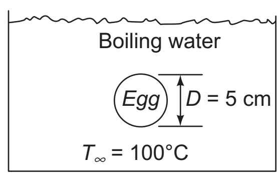
FIND

- The temperature of the egg center at the end of the cooking period

ASSUMPTIONS

- The egg is a sphere of diameter (D) = 5 cm = 0.05 m
- The egg has the thermal properties of water (From Appendix 2, Table 13)
  - Thermal conductivity (k) = 0.682 W/(m K)
  - Density (ρ) = 958.4 kg/m^3
  - Specific Heat (c) = 4211 J/(kg K)

SKETCH



SOLUTION

The Biot number for the egg is

$$Bi = \frac{\overline{h_c}\, r_o}{k} = \frac{[1700\ \text{W/(m}^2\ \text{K)}]\,(0.025\ \text{m})}{0.682\ \text{W/(m K)}} = 62.3 > 0.1$$

Therefore, the internal resistance is significant. Figure 2.39 can be used to solve the problem. The Fourier number at t = 900 s is

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c r_o^2} = \frac{0.682\ \text{W/(m K)}\,(900s)}{[4211\ \text{J/(kg K)}]((\text{W s})/\text{J})\left(958.4\ \text{kg/m}^3\right)} = 0.24$$

From Figure 2.39 for Fo = 0.24 and $1/Bi$ = 0.016

$$\frac{T(0,t) - T_\infty}{T(0,t) - T_\infty} = 0.10 \Rightarrow T(0,\,t) = T_\infty + 0.1(T_o - T_\infty) = 100°\text{C} + 0.1\ (4°\text{C} - 100°\text{C})$$

$$T(0,\,t) = 90.4°\text{C}$$

PROBLEM 2.79

A long wooden rod at 38°C with a 2.5 cm diameter is placed into an airstream at 600°C. The heat transfer coefficient between the rod and air is 28.4 W/(m^2 K). If the ignition temperature of the wood is 427°C, ρ = 800 kg/m^3, k = 0.173 W/(m K), and c = 2500 J/(kg K), determine the time between initial exposure and ignition of the wood.

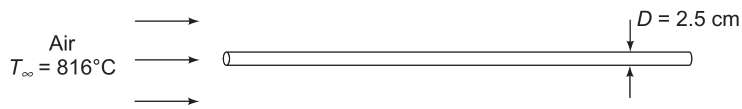
GIVEN

- A long wooden rod is placed into an airstream
- Rod outside diameter (D) = 2.5 cm = 0.025 m
- Initial temperature of the rod (T_o) = 38°C
- Temperature of the airstream (T_∞) = 816°C
- The heat transfer coefficient (h_c) = 28.4 W/(m^2 K)
- The ignition temperature of the wood (T_I) = 427°C
- Density of the rod (ρ) = 800 kg/m^3
- Thermal conductivity (k) = 0.173 W/(m K)
- Specific heat (c) = 2500 J/(kg K)

FIND

- The time between initial exposure and ignition of the wood

SKETCH



SOLUTION

The Biot number for the rod is

$$Bi = \frac{\overline{h_c}\, r_o}{2k} = \frac{[28.4\ \text{W/(m}^2\ \text{K)}]\left(\frac{0.025}{2}\ \text{m}\right)}{0.173\ \text{W/(m K)}} = 2.05 > 0.1$$

$$\frac{1}{Bi} = 0.49$$

Therefore, the internal thermal resistance of the rod is significant and the chart solution of Figure 2.38 will be used. From Figure 2.38 for r/r_o = 1.0 and $1/Bi$ = 0.49

$$\frac{T(r_o,t) - T_\infty}{T(0,t) - T_\infty} = 0.52$$

Solving for the difference between the center and ambient temperatures

$$T(0, t) - T_\infty = \frac{1}{0.52}\ (T(r_o, t) - T_\infty)$$

When the surface temperature of the rod is 427°C

$$T(0, t) - T_\infty = \frac{1}{0.52}\ (427°\text{C} - 600°\text{C}) = -\,333°\text{C}$$

$$\therefore\ \frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{-333^\circ\text{C}}{38^\circ\text{C} - 600^\circ\text{C}} = 0.59$$

From Figure 2.43 for $(T(0, t) - T_\infty)/(T_o - T_\infty)$ = 0.59 and $1/Bi$ = 0.49

$$Fo = \frac{\alpha t}{r_o^2} = 0.2$$

Solving for the time

$$t = \frac{Fo\, r_o^2}{\alpha} = \frac{Fo\rho c r_o^2}{k} = \frac{0.2\left(800\ \text{kg/m}^3\right)\left(2500\ \text{J/(kg K)}\right)\left(\dfrac{0.025}{2}\ \text{m}\right)^2}{0.173\ \text{W/(m}^2\ \text{K)}} = 361\ \text{sec} = 6.0\ \text{min}$$

PROBLEM 2.80

> **In the inspection of a sample of meat intended for human consumption, it was found that certain undesirable organisms were present. In order to make the meat safe for consumption, it is ordered that the meat be kept at a temperature of at least 121°C for a period of at least 20 min during the preparation. Assume that a 2.5-cm-thick slab of this meat is originally at a uniform temperature of 27°C; that it is to be heated from both sides in a constant temperature oven; and that the maximum temperature meat can withstand is 154°C. Assume furthermore that the surface coefficient of heat transfer remains constant and is 10 W/(m^2 K). The following data may be taken for the sample of meat: specific heat = 4184 J/(kg K); density = 1280 kg/m^3; thermal conductivity = 0.48 W/(m K). Calculate the oven temperature and the minimum total time of heating required to fulfill the safety regulation.**

GIVEN

- A slab of meat is heated in constant temperature over
- Meat be kept at a temperature of at least 121°C for a period of at least 20 min during the preparation
- Slab thickness ($2L$) = 2.5 cm = 0.025 m
- Initial uniform temperature (T_o) = 27°C
- The maximum temperature meat can withstand is 154°C
- Specific heat (c) = 4184 J/(kg K)
- Density (ρ) = 1280 kg/m^3
- Thermal conductivity (k) = 0.48 W/(m K)

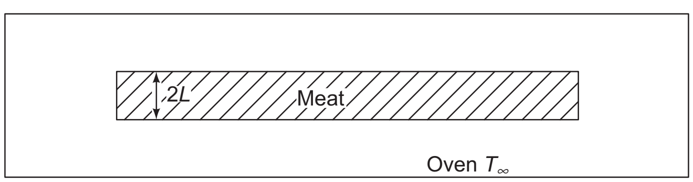
FIND

- The minimum total time of heating required to fulfill the safety regulation

ASSUMPTIONS

- The surface heat transfer coefficient ($\bar{h}_c$)= 10 W/(m K)
- Edge effects are negligible
- One dimensional conduction

SKETCH



SOLUTION

The Biot number for the meat is

$$Bi = \frac{\bar{h}_c L}{k} = \frac{[10\ \text{W/(m}^2\ \text{K)}]\left(\dfrac{0.025}{2}\ \text{m}\right)}{0.48\text{W/(m K)}} = 0.26 > 0.1$$

Therefore, the internal resistance is significant and the transient conduction charts will be used to find a solution.

The highest temperature will occur at the surface of the meat while the lowest will occur at the center of the meat. Therefore, the maximum possible oven temperature (T_∞) can be obtained from Figure 2.37 for $1/Bi = 3.8$; $X = L$

$$\frac{T(L,t) - T_\infty}{T(0,t) - T_\infty} = 0.88$$

$$T_\infty = \frac{0.88(T(0,t) - T_\infty)}{0.9 - 1.0} = \frac{0.88(121^{\circ}\text{C} - 154^{\circ}\text{C})}{-0.1} = 475°\text{C}$$

The actual oven temperature must be less than this so the center temperature can remain above 121°C without the surface temperature exceeding 154°C. The oven temperature and cooking time must be found by iterating the steps below

1. Pick an oven temperature.
2. Use Figure 2.37 to find the Fourier number which determines the time required for the center temperature to reach 121°C.
3. Add 20 min to the time and calculate a new Fourier number.
4. Use the new Fourier number and Figure 2.37 to find the center temperature at the end of the cooking period.
5. Use $(T(r_o, t) - T_\infty)/(T(0, 2t) - T_\infty) = 0.9$ to find the surface temperature at the end of the cooking period.

1. For the first iteration, let the oven temperature (T_∞) = 300°C.
2. $\dfrac{T(0,t) - T_\infty}{T_o - T_\infty} = \dfrac{121^{\circ}\text{C}}{27^{\circ}\text{C}} = 0.656$

From Figure 2.37

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c L^2} = 1.7$$

Solving for the time for the center to reach 121°C:

$$t = \frac{Fo\,\rho c L^2}{k} = \frac{1.7(4187\,\text{kg/m}^3)(1280\,\text{J/(kg K)})(0.0125\,\text{m})^2}{0.48\ \text{W/(m}^2\ \text{K)}} = 2963\ \text{sec}$$

3. After 20 min (1200s) cooking time: $t = 4163$, $F_o = 2.4$.
4. From Figure 2.37 for $F_o = 2.4$, $1/Bi = 3.8$

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = 0.55$$

$$T(0, t) = T_\infty + 0.55\,(T_o - T_\infty) = 300°\text{C} + 0.55\,(27°\text{C} - 300°\text{C}) = 150°\text{C}$$

5. $$\frac{T(L,t) - T_\infty}{T(0,t) - T_\infty} = 0.9$$

$$T(L, t) = T_\infty + 0.9\,(T(0, t) - T_\infty) = 300°\text{C} + 0.9\,(150°\text{C} - 300°\text{C}) = 165°\text{C}$$

Therefore, an oven temperature of 300°C is too high. The following iterations were performed using the same procedure

Oven Temperature	Fo	Time to Reach 121°C	Fo for 20 min	$\frac{T(0,t)-T_\infty}{T_\infty - T_\infty}$	T_o (°C)	T_L (°C)
300°C	1.7	2963 s	2.4	0.55	150	165
200°C	3.2	5578 s	3.9	0.37	136	142
150°C	2.4	4182 s	3.1	0.48	143	156

Therefore, the oven temperature should be set at 250°C and the meat should be heated for a total of 4184 s + 1200 s = 5384 s = 90 min.

PROBLEM 2.81

A frozen-food company freezes its spinach by first compressing it into large slabs and then exposing the slab of spinach to a low-temperature cooling medium. The large slab of compressed spinach is initially at a uniform temperature of 21°C; it must be reduced to an average temperature over the entire slab of –34°C. The temperature at any part of the slab, however, must never drop below –51°C. The cooling medium which passes across both sides of the slab is at a constant temperature of –90°C. The following data may be used for the spinach: density = 80 kg/m^3; thermal conductivity = 0.87 W/(m K); specific heat = 2100 J/(kg K). Present a detailed analysis outlining a method estimate the maximum thickness of the slab of spinach that can be safely cooled in 60 min.

GIVEN

- Large slabs of spinach are exposed to a low-temperature cooling medium
- Initial uniform temperature (T_o) = 21°C
- Average temperature must be reduced to –34°C
- The temperature at any part must never drop below –51°C
- Cooling medium temperature (T_∞) = –90°C
- Density of spinach (ρ) = 80 kg/m^3
- Thermal conductivity (k) = 0.87 W/(m K)
- Specific heat (c) = 2100 J/(kg K)

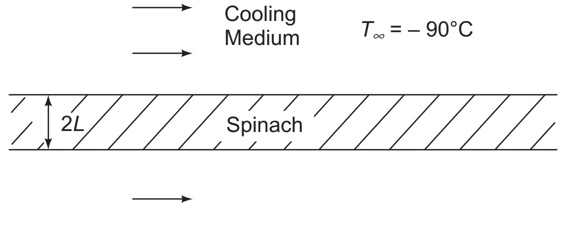
FIND

- Present a detailed analysis outlining a method to estimate the maximum thickness of the slab of spinach that can be safely cooled in 60 min

ASSUMPTIONS

- One dimensional conduction through the slab
- Constant and uniform thermal properties
- The average temperature within the slab is equal to the average of the center and surface temperatures

SKETCH



SOLUTION

For a final average temperature in the slab of –34°C, and a final surface temperature of –51°C, the final center temperature must be

$$T(0,\, t) = 2\, T_{\text{Ave}} - T(L,\, t) = 2(-\,34°\text{C}) + 51°\text{C} = -17°\text{C}$$

Figure 2.37 can be used to find the Biot number for the spinach slab

$$\frac{T(L,t)-T_\infty}{T(0,t)-T_\infty} = \frac{-51^\circ\text{C}-(-90^\circ\text{C})}{-17^\circ\text{C}-(-90^\circ\text{C})} = 0.53$$

From Figure 2.37 $1/Bi = 0.6$.

Figure 2.37 can be used to find the Fourier number

$$\frac{T(0,t)-T_\infty}{T_\infty - T_\infty} = \frac{-17^\circ\text{C}-(-90^\circ\text{C})}{-21^\circ\text{C}-(-90^\circ\text{C})} = 0.66$$

From Figure 2.37 $F_o = 0.4$

$$F\text{o} = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c L^2}$$

Solving for L

$$L = \left(\frac{k\,t}{Fo\,\rho\,c}\right)^{0.5} = \left(\frac{[0.87\,\text{W/(m K)}](\text{J/(W s)})(60\text{ min})(60\text{ s/min})}{0.4\left(80\text{ kg/m}^3\right)(2100\text{ J/(kg K)})}\right)^{0.5} = 0.22\text{ m}$$

The thickness of the slab of spinach that can be cooled in 60 minutes is $2L$ = 0.44 m = 44 cm.

The heat transfer coefficient needed to achieve this cooling can be calculated from the Biot number

$$Bi = \frac{\overline{h_c}\,L}{k} \Rightarrow \overline{h_c} = Bi\,\frac{k}{L} = \frac{1}{0.6}\,\frac{0.87\text{ W/(m K)}}{0.22\text{ m}} = 6.7\ \text{W/(m}^2\text{K)}$$

COMMENTS

The heat transfer coefficients is on the low side of the range for free convection in air (see Table 1.2).

Note that if the heat transfer coefficient is greater than 6.7 W/(m^2 K), the surface temperature of the spinach will drop below –51°C before the average temperature is lowered to –34°C.

PROBLEM 2.82

In the experimental determination of the hat transfer coefficient between a heated steel ball and crushed mineral solids, a series of 1.5% carbon steel balls were heated to a temperature of 700°C and the center temperature-time history of each was measured with a thermocouple while it was cooling in a bed of crushed iron ore, which was placed in a steel drum rotating horizontally at about 30 rpm. For a 5-cm-diameter ball, the time required for the temperature difference between the ball center and the surrounding ore to decrease from 500°C initially to 250°C was found to be 64, 67, and 72 s, respectively, in three different test runs. Determine the average heat transfer coefficient between the ball and the ore. Compare the results obtained by assuming the thermal conductivity to be infinite with those obtained by taking the internal thermal resistance of the ball into account.

GIVEN

- Heat steel balls are put in crushed iron ore
- Balls are 1.5% carbon steel balls
- Initial temperature of balls (T_o) = 700°C
- Ball diameter = 5 cm = 0.05 m
- Temperature difference between the ball center and the ore
- Center temperature of the balls decreases from 500°C to 250°C
- Time taken was found to be 64, 67, and 72 s, respectively, in three different test runs

FIND

The average heat transfer coefficient between the ball and the ore.

Compare the results obtained

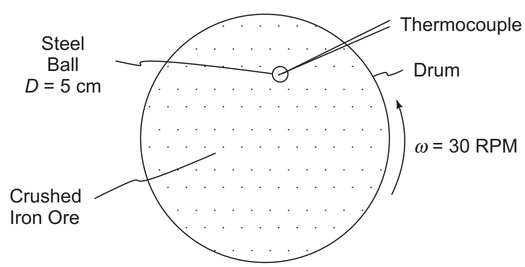
(a) by assuming the thermal conductivity to be infinite with

(b) those obtained by taking the internal thermal resistance of the ball into account

ASSUMPTIONS

- Temperature of the iron ore is uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

For 1.5% carbon steel Thermal conductivity (k) = 36 W/(m K)

Density (ρ) = 7753 kg/m^3

Specific heat (c) = 486 J/(kg K)

Thermal diffusivity (α) = 0.97×10^{-5} m^2/s

SOLUTION

(a) Assuming the internal resistance of the balls is negligible. The temperature-time history is given by Equation (2.89)

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{h_c A_s}{c\rho V}t} = e^{-\frac{h_c \pi D^2}{c\rho \frac{\pi}{4} D^3}t} = e^{-\frac{6 h_c}{c\rho D}t}$$

Solving for the heat transfer coefficient

$$h_c = \frac{c\rho D}{6t} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$h_c = \frac{[486\text{ J/(kg K)}][(\text{Ws})/\text{J}]\left(7753\text{ kg/m}^3\right)(0.05\text{ m})}{6t}\ \ln\left(\frac{250^o\text{C}}{500^o\text{C}}\right) = \frac{21,765}{t}\ \text{Ws/(m}^2\text{ K)}$$

For the three test runs: t = 64 s → h_c = 340 W/(m^2 K)

t = 67 s → h_c = 325 W/(m^2 K)

t = 72 s → h_c = 302 W/(m^2 K)

The average heat transfer coefficient is 322 W/(m^2 K)

(b) The chart method will be used to take the internal thermal resistance into account. Figure 2.44 can be used to determine the Biot number for the balls

$$\frac{T(0,t)-T_\infty}{T_o - T_\infty} = \frac{250^\text{o}\text{C}}{500^\text{o}\text{C}} = 0.5$$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{0.97\times 10^{-5}\text{m}^2/\text{s}(t)}{(0.025\text{ m})^2}$$

For the three test runs: t = 64 s → Fo = 0.99

t = 67 s → Fo = 1.04

t = 72 s → Fo = 1.12

Figure 2.44 is not detailed enough to distinguish between the first two test runs

For the first two runs: F_o = 1.0 → 1/Bi = 4.0 Bi = 0.25

For the third run: F_o = 1.1 → 1/Bi = 4.2 Bi = 0.238

The average Bi number = [2(0.250) = 0.263]/3 = 0.246 = (h_c r_o)/k

Solving for the transfer coefficient

$$h_c = \frac{Bi\,k}{r_o} = \frac{0.246(36\text{W/(m K)})}{0.025\text{ m}} = 354\ \text{W/(m}^2\text{ K)}$$

Neglecting the internal resistance resulted in a calculated heat transfer coefficient 9% lower than using the chart method.

PROBLEM 2.83

A mild-steel cylindrical billet, 25-cm in diameter, is to be raised to a minimum temperature of 760°C by passing it through a 6-m long strip type furnace. If the furnace gases are at 1538°C and the overall heat transfer coefficient on the outside of the billet is 68 W/(m^2 K), determine the maximum speed at which a continuous billet entering at 204°C can travel through the furnace.

GIVEN

- A mild-steel cylindrical billet is passed through a furnace
- Diameter of billet = 25 cm = 0.25 m
- Billet is to be raised to a minimum temperature of 760°C
- Length of furnace = 6 m
- Temperature of furnace gases (T_∞) = 1538°C
- The overall heat transfer coefficient $\left(\bar{h}_c\right)$ = 68 W/(m^2 K)
- Initial temperature of billet (T_o) = 204°C

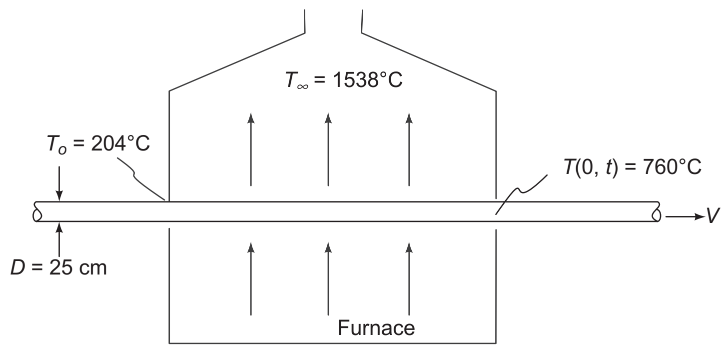
FIND

- The maximum speed at which a continuous billet can travel through the furnace

ASSUMPTIONS

- The heat transfer coefficient is constant
- Billet is 1% carbon steel
- Radial conduction only in the billet, neglect axial conduction

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

For 1% carbon steel Thermal conductivity (k) = 43 W/(m K)
Thermal diffusivity (α) = 1.172×10^{-5} m^2/s

SOLUTION

The Biot number for the billet is

$$Bi = \frac{\bar{h} r_o}{K} = \frac{[68 \text{ W/(m}^2\text{ K)}](0.125\text{ m})}{43 \text{ W/(m K)}} = 0.198 > 0.1$$

Therefore, internal resistance is significant and we cannot use the lumped parameter method, a chart solution must be used.

The billet must obtain a centerline temperature of 760°C, therefore

$$\frac{T(0,t) - T_\infty}{T_i - T_\infty} = \frac{760°\text{C} - 1538°\text{C}}{204°\text{C} - 1538°\text{C}} = 0.583$$

The Fourier number from Figure 2.38 for $1/Bi = 1/0.198$ and $(T(0, t) - T_\infty)/(T_o - T_\infty) = 0.583$ is

$$Fo = \frac{\alpha t}{r_o^2} = 1.4$$

Solving for the time

$$t = \frac{Fo r_o^2}{\alpha} = \frac{1.4(0.125\text{ m})^2}{1.172 \times 10^{-5} \text{ m}^2/\text{s}} = 1866 \text{ s}$$

The maximum speed of the billet is

$$V = \frac{\text{Length of furnace}}{\text{time needed}} = \frac{6\text{ m}}{1866\text{ s}} = 0.0032 \text{ m/s}$$

PROBLEM 2.84

A solid lead cylinder 0.6-m in diameter and 0.6-m long, initially at a uniform temperature of 121°C, is dropped into a 21°C liquid bath in which the heat transfer coefficient $\bar{h}_c$ is 1135 W/(m^2 K). Plot the temperature-time history of the center of this

cylinder and compare it with the time histories of a 0.6 m diameter, infinitely long lead cylinder and a lead slab 0.6-m thick.

GIVEN

- A solid lead cylinder dropped into a liquid bath
- Cylinder diameter (D) = 0.6 m
- Cylinder (L) = 0.6 m
- Initial uniform temperature (T_o) = 121°C
- Liquid bath temperature (T_∞) = 21°C
- Heat transfer coefficient $\left(\bar{h}_c\right)$ = 1135 W/(m^2 K)

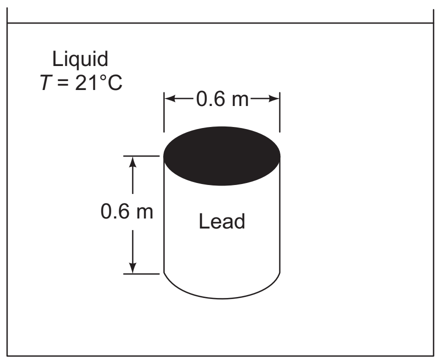
FIND

(a) Plot the temperature-time history of the cylinder center
(b) Compare it with the time history of a 0.6 m diameter, infinitely long lead cylinder
(c) Compare it with the time history of a lead slab 0.6 m thick

ASSUMPTIONS

- Two dimensional conduction within the cylinder
- Constant and uniform properties
- Constant liquid bath temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12

For lead Thermal conductivity (k) = 34.7 W/(m K) at 63°C
Density (ρ) = 11340 kg/m^3
Specific heat (c) = 129 J/(kg K)
Thermal diffusivity (α) = 24.1 × 10^{-6} m^2/s

SOLUTION

The Biot number based on radius is

$$Bi = \frac{\bar{h}_c r_o}{K} = \frac{[1135\,\text{W/(m}^2\text{ K)}](0.3\,\text{m})}{34.7\,\text{W/(m K)}} = 9.81 > 0.1$$

Therefore, internal resistance is significant.

(a) This two-dimensional system required a product solution. From Table 2.4 the product solution is

$$\frac{\theta_p(x,r)}{\theta_o} = P(x)\,C(r)$$

where

$$P(x) = \frac{\theta(x,t)}{\theta_o} \text{ for an infinite plate (Figure 2.37)}$$

$$C(r) = \frac{\theta(r,t)}{\theta_o} \text{ for a long cylinder (Figure 2.38)}$$

Since the length of the cylinder is the same as its diameter, the Biot number based on length is the same as that based on radius

$$\frac{1}{Bi} = \frac{1}{9.81} = 0.102$$

The Fourier number is

$$Fo = \frac{\alpha t}{\left(\frac{L}{2} \text{ or } r_o\right)^2} = \frac{24.1\times10^{-6}\,\mathrm{m}^2/\mathrm{s}(t)}{(0.3\,\mathrm{m})^2} = 0.000268\,t\,\mathrm{s}^{-1}$$

The temperature of the center of the cylinder (x = 0, r = 0) is determined by calculating the Fourier number for that time, finding $P(0)$ on Figure 2.37, finding $C(0)$ on Figure 2.38, and applying

$$\frac{\theta_p(0,0)}{\theta_o} = \frac{T(0,0)-T_\infty}{T_o-T_\infty} = P(x)\,C(r)$$

$$T(0,0) = T_\infty + P(x)\,C(r)\,(T_o - T_\infty)$$

(b) The center temperature for a long cylinder is

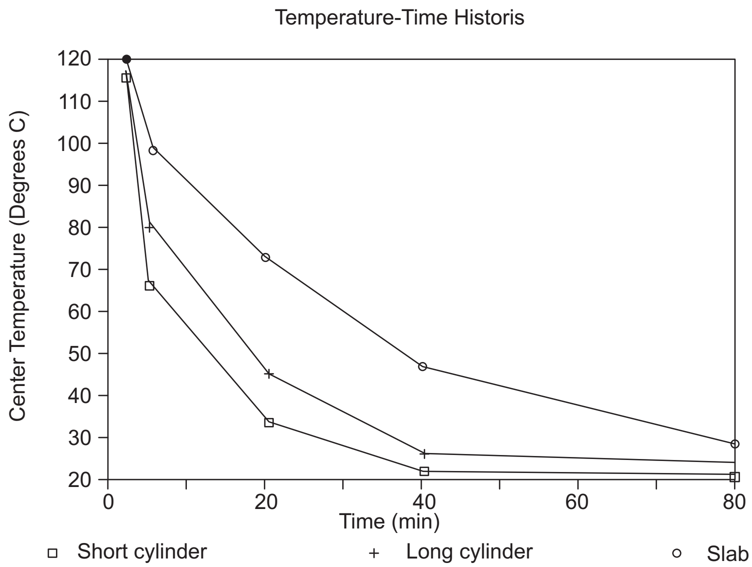
$$T(r = o, t) = T_\infty + C(o)\,(T_o - T_\infty)$$

(c) The center temperature for a slab is

$$T(x = o, t) = T_\infty + P(o)\,(T_o - T_\infty)$$

The temperature-time histories of these three cases are tabulated and plotted below

					$T(0, 0)$ (°C)		
Time(s)	(min)	Fo	$P(0)$	$C(0)$	(a) Short Cylinder	(b) Long Cylinder	(c) Slab
120	2	0.03	0.99	0.95	115	116	120
300	5	0.08	0.78	0.60	68	81	99
1200	20	0.32	0.52	0.24	33	45	73
4800	80	1.28	.075	.033	21	14	29



PROBLEM 2.85

A long 0.6-m-*OD* 347 stainless steel (*k* = 14 W/(m K) cylindrical billet at 16°C room temperature is placed in an oven where the temperature is 260°C. If the average heat transfer coefficient is 170 W/(m^2 K), (a) estimate the time required for the center temperature to increase to 323°C by using the appropriate chart and (b) determine the instantaneous surface heat flux when the center temperature is 232°C.

GIVEN

- A long cylindrical billet placed in an oven
- Billet outside diameter = 0.6 m
- Thermal conductivity (k) = 14 W/(m K)
- Initial temperature (T_i) = 16°C
- Oven temperature (T_∞) = 260°C
- The average heat transfer coefficient $\left(\bar{h}_c\right)$ = 170 W/(m^2 K)
- Center temperature increases to 232°C

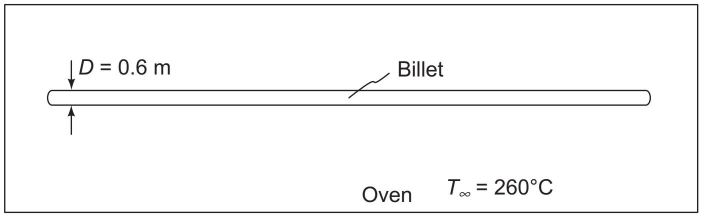
FIND

(a) The time required using the appropriate chart
(b) The instantaneous surface heat fluxes when the center temperature is 232°C

ASSUMPTIONS

- Radial conduction only in billet
- Uniform and constant properties

SKETCH



SOLUTION

(a) The Biot number for the billet is

$$Bi = \frac{\bar{h}_c r_o}{K} = \frac{[170\,\text{W/(m}^2\text{ K)}](0.3\,\text{m})}{14\,\text{W/(m K)}} = 3.643 > 0.1$$

$$\frac{1}{Bi} = 0.275$$

$$\frac{T(0,t_f) - T_\infty}{T_o - T_\infty} = \frac{232°\text{C} - 260°\text{C}}{16°\text{C} - 260°\text{C}} = 0.115$$

From Figure 2.38

$$Fo = \frac{\alpha t}{r_o^2} = 0.65$$

Solving for the time

$$t = \frac{Fo r_o^2}{\alpha} = \frac{0.65(0.3\,\text{m})^2}{0.387 \times 10^{-5}\,\text{m}^2/\text{s}} = 15{,}116\text{ s} = 252\text{ min} = 4.2\text{ hr}$$

(b) The surface temperature is needed to find the surface heat flux. For $1/Bi = 0.275$ and $r = r_o$, from Figure 2.38.

$$\frac{T(r_o, t) - T_\infty}{T(0,t) - T_\infty} = 0.3$$

$$T(r_o, t) = T_\infty + 0.3\,(T(0, t) - T_\infty) = 260°\text{C} + 0.3\,(232°\text{C} - 260°\text{C}) = 251.6°\text{C}$$

The instantaneous surface flux is

$$\frac{q}{A} = \bar{h}\,\,[T_\infty - T(r_o, t)] = 170\;\text{W/(m}^2\text{ K)}\;(251°\text{C} - 260°\text{C}) = 1428\;\text{W/m}^2$$

PROBLEM 2.86

Repeat Problem 2.85(a), but assume that the billet is only 1.2-m long and the average heat transfer coefficient at both ends is 136 W/(m^2 K).

Problem 2.85

A long, 0.6 m *OD* 347 stainless steel (*k* = 14 W/(m K)) cylindrical billet at 16°C room temperature is placed in an over where the temperature is 260°C. If the average heat transfer coefficient is 170 W/(m^2 K), estimate the time required for the center temperature to increase to 232°C by using the appropriate chart.

GIVEN

- A cylindrical billet placed in an over
- Billet outside diameter = 0.6 m
- Thermal conductivity (k) = 14 W/(m K)
- Initial temperature (T_o) = 16°C

- Oven temperature (T_∞) = 260°C
- The average heat transfer coefficient $\left(\bar{h}_{cs}\right)$ = 170 W/(m^2 K)
- Increase of the center temperature is 232°C
- Billet length (2L) = 1.2 m
- Heat transfer coefficient at the ends $\left(\bar{h}_{ce}\right)$ = 136 W/(m^2 K)

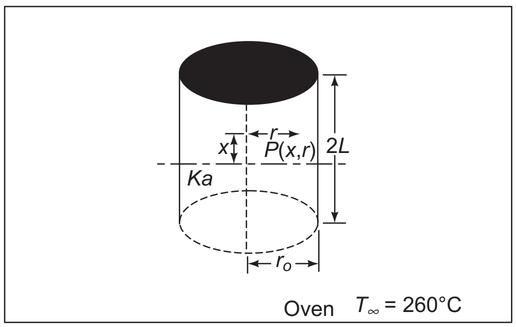
FIND

- The time required using the appropriate charts

ASSUMPTIONS

- Two dimensional conduction within the billet
- Constant and uniform thermal properties
- Constant oven temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10 For Type 304 stainless steel

Thermal diffusivity (α) = 0.387×10^{-5} m^2/s

SOLUTION

From Table 2.4, the solution for this geometry is

$$\frac{\theta_p(x,r)}{\theta_o} = P(x)\,C(r)$$

where

$$P(x) = \frac{\theta(x,t)}{\theta_o} \text{ for an infinite plate (Figure 2.37)}$$

$$C(r) = \frac{\theta(r,t)}{\theta_o} \text{ for a long cylinder (Figure 2.38)}$$

$$\frac{\theta_p(0,0)}{\theta_o} = \frac{T(0,0)-T_\infty}{T_o - T_\infty} = \frac{232°\text{C} - 260°\text{C}}{16°\text{C} - 260°\text{C}} = 0.11 = P(0)\,C(0)$$

For the infinite plate solution

$$(Bi)_x = \frac{\bar{h}_{ce}L}{k} = \frac{[136 \text{ W/(m}^2\text{ K)}](0.6\text{ m})}{14 \text{ W/(m K)}} = 5.83 \Rightarrow \frac{1}{Bi} = 0.17$$

$$Fo = \frac{\alpha t}{L^2} = \frac{0.387\times10^{-5}\,\mathrm{m^2/s}}{(0.6\,\mathrm{m})^2}\,t = 1.075 \times 10^{-5}\ t\ \mathrm{s}^{-1}$$

For the long cylinder solution

$$(Bi)_r = \frac{\bar{h}_{cs} r_o}{k} = \frac{[170\ \mathrm{W/(m^2\ K)}](0.3\,\mathrm{m})}{14\ \mathrm{W/(m\ K)}} = 3.54 \Rightarrow \frac{1}{Bi} = 0.28$$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{0.387\times10^{-5}\,\mathrm{m^2/s}}{(0.3\,\mathrm{m})^2}\,t = 4.3 \times 10^{-5}\ t\ \mathrm{s}^{-1}$$

The time required to reach a product solution of 0.115 is found through trial and error.

Time(s)	(min)	Fo_x	$P(0)$	Fo_r	$C(0)$	$P(0)C(0)$
6,000	100	0.065	0.99	0.26	0.37	0.0366
12,000	200	0.13	0.82	0.52	0.17	0.139
15,000	250	0.16	0.54	0.645	0.10	0.054
13,000	217	0.14	0.60	0.56	0.15	0.090
12,500	208	0.134	0.70	0.538	0.208	0.11

The time required is approximately 208 min or 3.4 hours.

COMMENTS

The uncertainty in the solution is high because of the difficulty reading Figure 2.37 at very low Fourier numbers. For higher accuracy, the differential equations that describe the problem would have to be solved.

PROBLEM 2.87

> **A large billet of steel initially at 260°C is placed in a radiant furnace where the surface temperature is held at 1200°C. Assuming the billet is infinite in extent, compute the temperature at point *P* shown in the accompanying sketch after 25 min has elapsed. The average properties of steel are: *k* = 28 W/(m K), ρ = 7360 kg/m^3, and *c* = 500 J/(kg K).**

GIVEN

- A large billet of steel is placed in a radiant furnace
- Initial temperature (T_o) = 260°C
- Surface temperature of billet in the oven (T_s) = 1200°C
- Lapse time (t) = 25 min = 1500 s
- Thermal conductivity (k) = 28 W/(m K)
- Density (ρ) = 7360 kg/m^3
- Specific heat (c) = 500 J/(kg K)

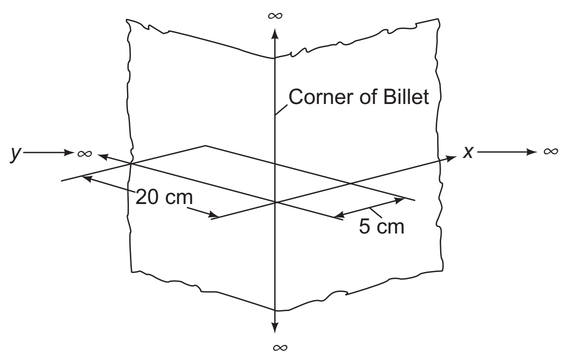
FIND

- The temperature at point *P* shown in the accompanying sketch

ASSUMPTIONS

- The billet infinite in extent

SKETCH



SOLUTION

From Table 2.4, the solution for a one quarter infinite solid is

$$\frac{\theta_p(x,y)}{\theta_o} = \frac{T(x,y,t)-T_s}{T_o-T_s} = S(x)\,S(y)$$

Where $S(x)$ and $S(y)$ are solutions for a semi-infinite solid, which are given for a constant surface temperature by Equation (2.105)

$$\frac{T(x,t)-T_\infty}{T_o-T_\infty} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Therefore, the solution to this problem is

$$\frac{T(x,y,t)-T_\infty}{T_o-T_\infty} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)\ \text{erf}\left(\frac{y}{2\sqrt{\alpha t}}\right)$$

$$T(x, y, t) = T_s + (T_o - T_s)\left[\text{erf}\left(\frac{x}{M}\right)\text{erf}\left(\frac{y}{M}\right)\right]$$

where

$$M = 2\sqrt{\alpha t} = 2\frac{k\,t}{\rho c} = 2\sqrt{\frac{[28\ \text{W/(m K)}](1500\,\text{s})}{(7360\,\text{kg/m}^2)(500\,\text{J/(kg K)})}} = 0.2137$$

$$\therefore\quad T\,(0.05\ \text{m},\ 0.2\ \text{m},\ 1500\ \text{s}) = 1200°\text{C} = (260°\text{C} - 1200°\text{C})\ \text{erf}\left(\frac{0.05\,\text{m}}{0.2137\,\text{m}}\right)\text{erf}\left(\frac{0.2\,\text{m}}{0.2137\,\text{m}}\right)$$

Using Appendix 2, Table 43 for the error function values

$$T\,(0.05\ \text{m},\ 0.2\ \text{m},\ 1500\ \text{s}) = 1200°\text{C} - 940°\ (0.259)\ (0.814) = 1002°\text{C}$$

Chapter 3

PROBLEM 3.1

> **Show that in the limit $\Delta x \to 0$, the difference equation for one-dimensional steady conduction with heat generation, Equation (3.1), is equivalent to the differential equation, Equation (2.27).**

GIVEN

- One dimensional steady conduction with heat generation

SHOW

(a) In the limit of small Δx, the difference equation is equivalent to the differential equation

SOLUTION

From Equation (3.1)

$$T_{i+1} - 2T_i + T_{i-1} = -\frac{\Delta x^2}{k}\dot{q}_{G,i}$$

By definition

$$T_{i-1} = T(x - \Delta x)$$
$$T_i = T(x)$$
$$T_{i+1} = T(x + \Delta x)$$

so we can rewrite Equation (3.1) as follows

$$\frac{T(x+\Delta x) - 2T(x) + T(x-\Delta x)}{\Delta x^2} = -\frac{\dot{q}_G(x)}{k}$$

Now, in the limit $\Delta x \to 0$, from calculus, the left hand side of the above equation becomes $\frac{d^2T}{dx^2}$ so we have

$$k\,\frac{d^2T}{dx^2} = -\dot{q}_G(x)$$

which is equivalent to Equation (2.27).

PROBLEM 3.2

> **'What is the physical significance of the statement that the temperature of each node is just the average of its neighbors if there is no heat generation' [with reference to Equation (3.2)]?**

SOLUTION

The significance is that in regions without heat generation, the temperature profile must be linear. Compare the subject equation with the solution of the differential equation

$$\frac{d^2T}{dx^2} = 0$$

which is $T(x) = a + bx$, which is also linear.

PROBLEM 3.3

Give an example of a practical problem in which the variation of thermal conductivity with temperature is significant and for which a numerical solution is therefore the only viable solution method.

SOLUTION

From Figure 1.6, the thermal conductivity of stainless steel (either 304 or 316) is a fairly strong function of temperature. For example

$$k_{ss\,316}\,(100°\text{C}) = 14.2 \text{ (W/m K)}$$

$$k_{ss\,316}\,(500°\text{C}) = 19.6 \text{ (W/m K)}$$

which is about a 38% difference.

Suppose a stainless steel sheet is to receive a heat treatment that involves heating the sheet to 500°C and then plunging it into a water bath. The water near the sheet would probably boil producing a sheet surface temperature near 100°C while the interior of the sheet would be at 500°C, at least for a short time. One would expect the large variation in thermal conductivity to be important in this type of problem.

PROBLEM 3.4

Discuss advantages and disadvantages of using a large control volume.

SOLUTION

The advantages of a large control volume are

(1) the numerical solution can be carried out quickly
(2) manual calculation for all control volumes are feasible for the purpose of verifying the numerical calculation
(3) energy will be conserved

Disadvantages are

(1) large temperature gradients cannot be accurately represented with large control volumes
(2) it is difficult to accommodate all but rectangular geometries.

PROBLEM 3.5

For one-dimensional conduction, why are the boundary control volumes half the size of interior control volumes?

GIVEN

- One-dimensional conduction

EXPLAIN

(a) Why the boundary control volume is half the size of internal control volumes

SOLUTION

There is a node on the boundary as well as one a distance Δx to the interior of the boundary. Since the interior nodes are centered within a control volume of width Δx, the control volume associated with the first non-boundary node comes within $\Delta x/2$ of the boundary. So, there is a volume of only $\Delta x/2$ left over for the boundary node.

PROBLEM 3.6

Discuss advantages and disadvantages of two methods for solving one-dimensional steady conduction problems.

SOLUTION

The two methods for solving one-dimensional steady conduction problems are matrix inversion and iteration.

Matrix inversion requires that we have some method (usually software) for inverting the matrix or for solving a tridiagonal system of equations. The method is difficult to apply to problems with variable thermal conductivity. If we have access to the required software, the method is simple and fast.

Iteration can handle variable thermal conductivity and does not require software for the inversion of a matrix. In practice, we will likely need to write a program or use a spreadsheet to carry out iteration and it may converge slowly.

PROBLEM 3.7

Solve the system of equations

$$\mathbf{2}\boldsymbol{T_1} + \boldsymbol{T_2} - \boldsymbol{T_3} = \mathbf{30}$$

$$\boldsymbol{T_1} - \boldsymbol{T_2} + \mathbf{7}\boldsymbol{T_3} = \mathbf{270}$$

$$\boldsymbol{T_1} + \mathbf{6}\boldsymbol{T_2} - \boldsymbol{T_3} = \mathbf{160}$$

by Jacobi and Gauss-Seidel iteration. Use as a convergence criterion $|\boldsymbol{T_2^{(p)}} - \boldsymbol{T_2^{(p-1)}}| < \mathbf{0.001}$**. Compare the rate of convergence for the two methods.**

GIVEN

- A system of three equations

FIND

(a) The solution of the system of equation using Jacobi and Gauss-Seidel iteration

SOLUTION

Since we do not know the physical problem these equations originated from, it is difficult to make a good first guess. Let's use 0 for all three temperatures as an initial guess.

If we solve the equations in the order given for T_1, T_2, and T_3 and solve by iteration, we find that the solution is not stable. Let's solve the first equation for T_1, the second for T_3, and the third for T_2. For Jacobi iteration we have

$$T_1^{(p+1)} = \frac{1}{2}(30 - T_2^{(p)} + T_3^{(p)})$$

$$T_3^{(p+1)} = \frac{1}{7}(270 - T_1^{(p)} + T_2^{(p)})$$

$$T_2^{(p+1)} = \frac{1}{6}(160 - T_1^{(p)} + T_3^{(p)})$$

and for Gauss-Seidel iteration we have

$$T_1^{(p+1)} = \frac{1}{2}(30 - T_2^{(p)} + T_3^{(p)})$$

$$T_3^{(p+1)} = \frac{1}{7}(270 - T_1^{(p+1)} + T_2^{(p+1)})$$

$$T_2^{(p+1)} = \frac{1}{6}(160 - T_1^{(p+1)} + T_3^{(p)})$$

The solution was carried out using a spreadsheet as shown on the next page

Problem 3.7 Filename 3_7.WQ1

iteration	T1 (Jacobi)	T2 (Jacobi)	T3 (Jacobi)	T1 (Gauss-Seidel)	T2 (Gauss-Seidel)	T3 (Gauss-Seidel)
0	0.000	0.000	0.000	0.000	0.000	0.000
1	15.000	26.667	38.571	15.000	24.167	39.881
2	20.952	30.595	40.238	22.857	29.504	39.521
3	19.821	29.881	39.949	20.009	29.919	39.987
4	20.034	30.021	40.009	20.034	29.992	39.994
5	19.994	29.996	39.998	20.001	29.999	40.000
6	20.001	30.001	40.000	20.000	30.000	40.000
7	20.000	30.000	40.000	20.000	30.000	40.000
8	20.000	30.000	40.000	20.000	30.000	40.000
9	20.000	30.000	40.000	20.000	30.000	40.000

Applying the criterion that the temperature change per iteration should be less than 0.001, we see that Jacobi iteration requires 7 iterations while Gauss-Seidel iteration requires 6 iterations.

PROBLEM 3.8

Develop the control volume difference equation for one-dimensional steady conduction in a fin with variable cross-sectional area *A*(*x*) and perimeter *P*(*x*). The heat transfer coefficient from the fin to ambient is a constant $\bar{h}_o$ and the fin tip is adiabatic.

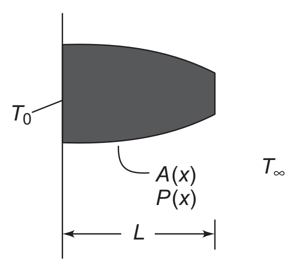
GIVEN

- Fin with variable cross-sectional area and perimeter
- Convection coefficient to ambient is constant, h_o

FIND

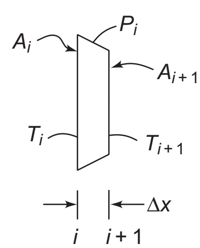
(a) Control volume difference equation

SKETCH



SOLUTION

Consider a control volume as shown below



$1 \le i \le N$

An energy balance on this control volume is expressed by

heat conducted into left face =

heat convected out perimeter + heat conducted out right face

or

$$-kA_i\left(\frac{T_i - T_{i-1}}{\Delta x}\right) = \Delta x\, P_i\, h_o\, (T_i - T_\infty) - kA_{i+1}\left(\frac{T_{i+1} - T_i}{\Delta x}\right)$$

which can be rearranged to give

$$T_{i-1}\, A_i + T_i\left(-A_i - A_{i+1} - \frac{(\Delta x)^2}{k} P_i h_o\right) + T_{i+1}\, A_{i+1} = -\frac{(\Delta x)^2}{k} P_i\, h_o\, T_\infty$$

The boundary conditions can be written as

$$T_1 = T_o$$
$$T_N = T_{N-1}$$

This can be written in the form of a tridiagonal matrix, per Equation (3.10) where the coefficients of the matrix are

$$a_1 = 1 \quad b_1 = 0 \quad d_1 = T_o$$

$$c_i = -A_i \quad a_i = A_i + A_{i+1} + \frac{(\Delta x)^2}{k} P_i\, h_o \quad b_i = -A_{i+1} \quad d_i = \frac{(\Delta x)^2}{k} P_i\, h_o\, T_\infty \quad 1 < i < N$$

$$c_N = -1 \quad a_N = 1 \quad d_N = 0$$

PROBLEM 3.9

Using your results from Problem 3.11 find the heat flow at the base of the fin for the following conditions:

$$\boldsymbol{k = 34\ \text{W/(m K)}}$$
$$\boldsymbol{L = 5\ \text{cm}}$$
$$\boldsymbol{A(x) = 3.23 \times 10^{-4}\left(1 - \frac{1}{3}\sinh\left(\frac{x}{L}\right)\right)\text{m}^2}$$
$$\boldsymbol{P(x)} = [A(x)]^{\frac{1}{2}}$$
$$\boldsymbol{h_o = 110\ \text{W/(m}^2\ \text{K)}}$$
$$\boldsymbol{T_o = 93°\text{C}}$$
$$\boldsymbol{T_\infty = 27°\text{C}}$$

Use a grid spacing of 0.5 cm.

From Problem 3.8: Develop the control volume difference equation for one-dimensional steady conduction in a fin with variable cross-sectional area $A(x)$ and perimeter $P(x)$. The heat transfer coefficient from the fin to ambient is a constant $\bar{h}_o$ and the fin tip is adiabatic.

GIVEN

- A fin with variable cross-sectional area and perimeter

FIND

(a) Heat flow rate for conditions given above

SOLUTION

The number of nodes is $N = 1 + \frac{L}{\Delta x} = 11$. The cross-sectional area at any node is

$$A_i = 3.23 \times 10^{-4} \left(1 - \frac{1}{3}\sinh\left(\frac{(i-1)\,\Delta x}{L}\right)\right) \text{ m}^2$$

and the perimeter at any node is

$$P_i = A_i^{\frac{1}{2}}$$

Heat transfer at the fin root is

$$q_{\text{fin}} = \frac{k}{\Delta x} A_1 (T_1 - T_2)$$

The difference equation as derived in Problem 3.8 is

$$T_{i-1} A_i + T_i \left(-A_i - A_{i+1} - \frac{(\Delta x)^2}{k} P_i h_o\right) + T_{i+1} A_{i+1} = -\frac{(\Delta x)^2}{k} P_i h_o T_\infty$$

The boundary conditions can be written as

$$T_1 = T_o$$

$$T_N = T_{N-1}$$

This can be written in the form of a tridiagonal matrix, per Equation (3.10) where the coefficients of the matrix are

$$a_1 = 1 \quad b_1 = 0 \quad d_1 = T_o$$

$$c_i = -A_i \qquad a_i = A_i + A_{i+1} + \frac{(\Delta x)^2}{k} P_i h_o \qquad b_i = -A_{i+1} \qquad d_i = \frac{(\Delta x)^2}{k} P_i h_o T_\infty \quad 1 < i < N$$

$$c_N = -1 \quad a_N = 1 \quad d_N = 0$$

This set of equations can be easily solved using the matrix inversion function. The temperature values at various nodes are tabulated below:

Node	A(i) sq.m	P(i) m	T(i) K
1	0.000323	0.017961	366
2	0.000312	0.017658	363.5997
3	0.000301	0.017347	361.6232
4	0.00029	0.017025	359.8693
5	0.000278	0.016686	358.3396
6	0.000267	0.016326	357.0372
7	0.000254	0.015941	355.9675
8	0.000241	0.015525	355.139
9	0.000227	0.015069	354.5646
10	0.000212	0.014567	354.2633
11	0.000196	0.014008	354.2633

$\mathbf{Q}_{\text{base}} = 5.2652$ W

MATLAB PROGRAM

```
L    = 0.05;
dx   = 0.5/100;
k    = 34;
Tamb = 300;
h    = 110;
N    = (L-0)/dx

T    = zeros (N+1,1);
A    = zeros (N+1,1);
P    = zeros (N+1,1);

X(1) = 0;
T(1) = 366;
for i = 2: N+1
     x(i) = x(1)+((i-1)*dx);
end
for i=1:N+1
     A(i) = (3.2258/(10^4))*(1-((sinh(x(i)/L))/3));
     P(i) = (A(i)^0.5);
end

a = zeros (N+1); % Lower diagonal of tridiagonal matrix
b = zeros (N+1); % Principal diagonal of tridiagonal matrix
c = zeros (N+1); % Upper diagonal of tridiagonal matrix
d = zeros (N+1); % RHS

     for i = 3: N
       a(i) = A(i);
     end
     for i = 2: N-1
       b(i) = -A(i)-A(i+1)-((h/k)*(dx^2)*P(i));
     end
     b(N) = -A(N)-A(N+1)-((h/k)*(dx^2)*P(N))+A(N+1);
     for i = 2: N-1
       c(i) = A(i+1);
     end
     d(2) = (-(h/k)*(dx^2)*Tamb*P(i))-(A(2)*T(1));
     for i = 3: N
       d(i) = -(h/k)*(dx^2)*Tamb*P(i);
     end

     % TDMA
     for i = 3: N
       b(i) = b(i)-(a(i)*c(i-1)/b(i-1));
       d(i) = d(i)-(a(i)*d(i-1)/b(i-1));
     end
     T(N) = d(N)/b(N);
     for i = N-1:-1:2
       T(i) = (d(i)-(c(i)*T(i+1)))/b(i);
     end
     T(N+1) = T(N);

     qbase = k*A(1)*(T(1)-T(2))/dx
```

The heat loss at the base of the f_{in}

$$q_{\text{base}} = \frac{k}{\Delta x} A_1 (T_1 - T_2)$$

$$= \frac{34(\text{W / mK})}{0.005\ \text{m}} \times (0.000323\ \text{m}^2)(366 - 363.5997)\text{K}$$

$$= 5.2652\ \text{W}$$

PROBLEM 3.10

Consider a pin fin with variable conductivity $k(T)$, constant cross sectional area A_c and constant perimeter, P. Develop the difference equations for steady one-dimensional conduction in the fin and suggest a method for solving the equations. The fin is exposed to ambient temperature T_a through a heat transfer coefficient h. The fin tip is insulated and the fin root is at temperature T_o.

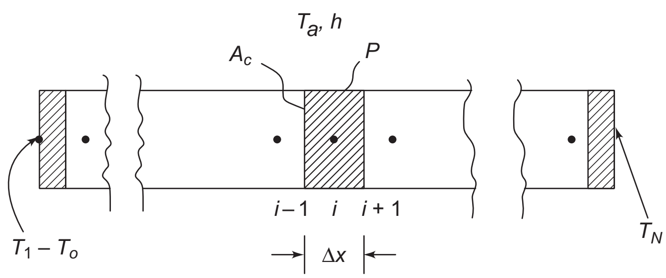
GIVEN

- Fin with variable thermal conductivity, $k(T)$

FIND

(a) Difference equation
(b) Solution method

SKETCH



SOLUTION

For the control volume centered over the interior node i, an energy balance gives

$$A_c\left(\frac{T_{i-1}-T_i}{\Delta x}k_{\text{left}}+\frac{T_{i+1}-T_i}{\Delta x}k_{\text{right}}\right) = h_o\,P\,(T_i - T_a)$$

The thermal conductivities are given in Section 3.2.1

$$k_{\text{left}} = \frac{2k_i\,k_{i-1}}{k_i+k_{i-1}} = \frac{2k(T_i)\,k(T_{i-1})}{k(T_i)+k(T_{i-1})}$$

and

$$k_{\text{right}} = \frac{2k_i\,k_{i+1}}{k_i+k_{i+1}} = \frac{2k(T_i)\,k(T_{i+1})}{k(T_i)+k(T_{i+1})}$$

For the node at the root $T_1 = T_o$.
At the tip, an energy balance gives

$$A_c\,k_N\,\frac{(T_{N-1}-T_N)}{\Delta x} = h_o\,P\,(T_N - T_a)$$

where

$$k_N = \frac{2k(T_N)\,k(T_{N-1})}{k(T_N)+k(T_{N-1})}$$

These equations can be written in tridiagonal form, Equation (3.9)

$$a_i\,T_i = b_i\,T_{i+1} + c_i\,T_{i-1} + d_i$$

where

$$a_1 = 1 \quad b_1 = 0 \quad c_1 = 0 \quad d_1 = T_o$$

For $1 < i < N$

$$a_i = h_o P + \frac{A_c}{\Delta x}(k_{\text{left}} + k_{\text{right}})$$

$$b_i = \frac{A_c}{\Delta x} k_{\text{right}}$$

$$c_i = \frac{A_c}{\Delta x} k_{\text{left}}$$

$$d_i = h_o P T_a$$

and

$$a_N = h_o P + \frac{A_c k_N}{\Delta x}$$

$$b_N = 0$$

$$c_N = \frac{A_c k_N}{\Delta x}$$

$$d_N = h_o P T_a$$

Note that k_{right}, k_{left}, and k_N depend on the nodal temperatures. To solve the system of equations, it will be necessary to

(1) Guess at the nodal temperatures
(2) Calculate the values for k_{right}, k_{left}, and k_N
(3) Calculate the matrix coefficients a_i, b_i, c_i and d_i, $1 \le i \le N$
(4) Solve for the nodal temperatures by inverting the matrix as in Equation (3.10)
(5) Repeat steps 2 through 4 until the nodal temperatures cease to change

PROBLEM 3.11

How would you treat a radiation heat transfer boundary condition for a one-dimensional steady problem? Develop the difference equation for a control volume near the boundary and explain how to solve the entire system of difference equations. Assume that the heat flux at the surface is $q = \varepsilon \sigma (T_s^4 - T_e^4)$ where T_s is the surface temperature and T_e is the temperature of an enclosure surrounding the surface.

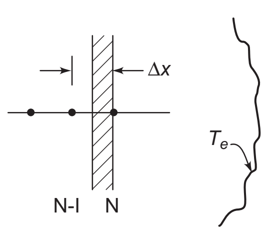
GIVEN

- Radiation boundary condition
- One-dimensional steady conduction

FIND

(a) Difference equation for control volume near surface
(b) Solution method

SKETCH



SOLUTION

An energy balance on the half control volume surrounding the surface node is

$$k\,\frac{\left(T_{N-1}-T_N\right)}{\Delta x} = \varepsilon\,\sigma\,(T_N^{\,4} - T_e^{\,4})$$

The right side of the above equation can be written

$$(T_N - T_e)\,h_r$$

where

$$h_r = \varepsilon\,\sigma\,(T_N^{\,2} + T_e^{\,2})\,(T_N + T_e)$$

The difference equation can be written in the tridiagonal form like Equation (3.9) as follows

$$a_i\,T_i = b_i\,T_{i+1} + c_i\,T_{i-1} + d_i$$

The coefficients for $1 < i < N$ are given just before Equation (3.10). For $i = 1$, the coefficients will depend on the boundary condition at the left boundary. For $i = N$, the coefficients are

$$a_i = h_r + \frac{k}{\Delta x} \quad b_i = 0 \quad c_i = \frac{k}{\Delta x} \quad d_i = h_r\,T_e$$

To solve the set of difference equations, an initial temperature distribution guess will be made. This will allow a determination of all of the coefficients. The tridiagonal matrix can then be solved to get an updated temperature distribution. This distribution will be used to update the coefficients and the procedure will be repeated to convergence.

PROBLEM 3.12

> **How should the control volume method be implemented at an interface between two materials with different thermal conductivities? Illustrate with a steady, one-dimensional example. Neglect contact resistance.**

GIVEN

- Interface between two different materials with different thermal conductivities

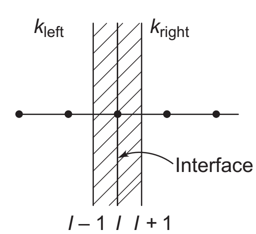
FIND

(a) Difference equation at the interface

ASSUMPTIONS

- No heat generation

SKETCH



SOLUTION

As shown in the sketch, the node at the interface is $i = I$. The thermal conductivity to the left of the interface is k_{left} and on the right side of the interface it is k_{right}. Since there is no contact resistance or heat generation, an energy balance for the control volume that straddles the interface is

$$k_{\text{left}}\,\frac{\left(T_{I-1}-T_I\right)}{\Delta x} = k_{\text{right}}\,\frac{\left(T_I-T_{I+1}\right)}{\Delta x}$$

Simplifying and writing this in the tridiagonal form

$$T_1 (k_{\text{left}} + k_{\text{right}}) = T_{1+1}\, k_{\text{right}} + T_{I-1}\, k_{\text{left}}$$

The above coefficients would be used to write the Ith row of the tridiagonal matrix. The remaining rows for internal nodes would be written as before and those for the boundaries would depend on specified boundary conditions.

PROBLEM 3.13

How would you include contact resistance between the two materials in Problem 3.12? Derive the appropriate difference equations.

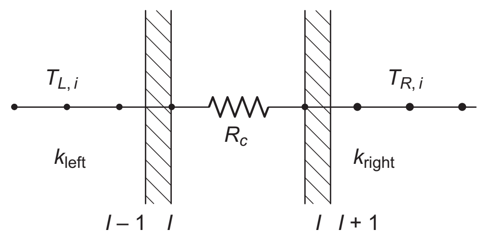
GIVEN

- Interface between two materials with different thermal conductivities and contact resistance at the interface

FIND

(a) The appropriate difference equations

SKETCH



SOLUTION

Let the contact resistance be R_c. The interface is located at node $i = I$. Represent temperatures to the left of the interface with $T_{L,i}$ and to the right of the interface with $T_{R,i}$. Thermal conductivity to the left of the interface is k_{left} and to the right of the interface is k_{right}. We have drawn two half control volumes, one just to the left of the interface and one just to the right of the interface.

An energy balance on the left control volume is

$$k_{\text{left}} \frac{\left(T_{L,I-1} - T_{L,I}\right)}{\Delta x} = \frac{T_{L,I} - T_{R,I}}{R_c}$$

and for the right control volume

$$k_{\text{right}} \frac{\left(T_{R,I+1} - T_{R,I}\right)}{\Delta x} = \frac{T_{R,I} - T_{L,I}}{R_c}$$

Writing these equations in the tridiagonal form we have

$$T_{L,I}\left(\frac{1}{R_c} + \frac{k_{\text{left}}}{\Delta x}\right) = T_{L,I-1}\frac{k_{\text{left}}}{\Delta x} + T_{R,I}\frac{1}{R_c}$$

$$T_{R,I}\left(\frac{1}{R_c} + \frac{k_{\text{right}}}{\Delta x}\right) = T_{L,I}\frac{1}{R_c} + T_{R,I+1}\frac{k_{\text{right}}}{\Delta x}$$

From these equations, the coefficients for the tridiagonal matrix can be defined

$$a_{L,I} = \frac{1}{R_c} + \frac{k_{\text{left}}}{\Delta x} \quad b_{L,I} = \frac{1}{R_c} \quad c_{L,I} = \frac{k_{\text{left}}}{\Delta x} \quad d_{L,I} = 0$$

$$a_{R,\,I} = \frac{1}{R_c} + \frac{k_{\text{right}}}{\Delta x} \quad b_{R,\,I} = \frac{k_{\text{right}}}{\Delta x} \quad c_{R,\,I} = \frac{1}{R_c} \quad d_{R,\,I} = 0$$

The vector of nodal temperatures in Equation (3.10) would be modified to look like

$$\begin{bmatrix} . \\ . \\ T_{L,\,I-1} \\ T_{L,\,I} \\ T_{R,\,I} \\ T_{R,\,I+1} \\ . \\ . \end{bmatrix}$$

The coefficients with subscripts L, I would appear in the row corresponding to $T_{L,\,I}$ and those with subscripts R, I would appear in the row corresponding to $T_{R,\,I}$. Remaining coefficients would be determined as for any other one-dimensional steady problems including those determined by the boundary conditions.

PROBLEM 3.14

A turbine blade 5-cm long, with cross-sectional area A = 4.5 cm^2 and perimeter P = 12 cm, is made of a high-alloy steel [k = 25 W/(m K)]. The temperature of the blade attachement point is 500°C and the blade is exposed to combustion gases at 900°C. The heat transfer coefficient between the blade surface and the combustion gases is 500 W/(m^{2}K). Using the nodal network shown in the accompanying sketch, (a) determine the temperature distribution in the blade, the rate of heat transfer to the blade and the fin efficiency of the blade and, (b) compare the fin efficiency calculated numerically with that calculated by the exact method.

GIVEN

- Turbine blade exposed to combustion gases

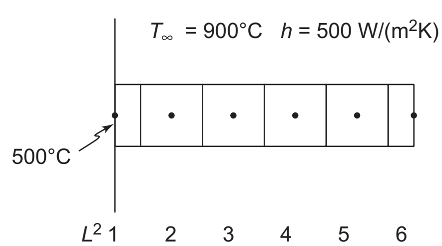
FIND

(a) Blade temperature distribution, heat gain, and fin efficiency
(b) Fin efficiency calculated exactly

ASSUMPTIONS

- The convection coefficient applies at the blade tip

SKETCH



SOLUTION

For the node and control volume arrangement shown in the sketch, we have

$$x_i = \Delta x(i - 1) \quad i = 1, 2, \ldots, N = 6 \quad \Delta x = \frac{L}{N-1}$$

For the control volume at $i = 1$, we have a specified temperature, therefore

$$T_1 = T_{\text{root}}$$

For the interior control volumes, $i = 2, 3, 4, 5$, an energy balance gives

$$kA \left\{ \frac{T_{i+1} - T_i}{\Delta x} + \frac{T_{i-1} - T_i}{\Delta x} \right\} + P\Delta x h\,(T_\infty - T_i) = 0$$

Writing this in the tridiagonal form

$$T_i \left(2 + \frac{P\Delta x^2 h}{kA} \right) = T_{i+1} + T_{i-1} + \frac{P\Delta x^2 h}{kA}\; T_\infty$$

For the control volume at node $i = N$, an energy balance gives

$$kA\; \frac{T_{N-1} - T_N}{\Delta x} + h\,(T_\infty - T_N) \left(P\frac{\Delta x}{2} + A \right) = 0$$

In the tridiagonal form this becomes

$$T_N \left(1 + \frac{h\Delta x}{kA} \left(P\frac{\Delta x}{2} + A \right) \right) = T_{N-1} + \frac{h\Delta x}{kA} \left(P\frac{\Delta x}{2} + A \right) T_\infty$$

Filling in the matrix A coefficients in Equation (3.10) we have

$$a_1 = 1 \quad b_1 = 0 \quad c_1 = 0 \quad d_1 = T_{\text{root}}$$

$$a_i = 2 + \frac{P\Delta x^2 h}{kA} \quad b_i = 1 \quad c_i = 1 \quad d_i = \frac{P\Delta x^2 h}{kA}\; T_\infty \qquad i = 2, 3, 4, 5$$

$$a_N = 1 + \frac{h\Delta x}{kA}\left(P\frac{\Delta x}{2} + A \right) \quad b_N = 0 \quad c_N = 1 \;\; d_N = \frac{h\Delta x}{kA}\left(P\frac{\Delta x}{2} + A \right) T_\infty$$

The matrix can be inverted using a spreadsheet and then the inverse matrix is multiplied by the vector D to give the solution vector T of temperatures.

Heat transfer from the fin is given by the heat loss from the first control volume

$$Q_{\text{fin}} = h\; \frac{\Delta x}{2}\; P\,(T_1 - T_\infty) + \frac{kA}{\Delta x}\,(T_1 - T_2)$$

The spreadsheet is shown below

```
Problem 3.14 Filename: 3_14.WQ1
PROBLM PARAMETERS
==================================
  Ac    =   0.00045  (fin cross sectional area, m^2)
  P     =      0.12  (fin perimeter, m)
  L     =      0.05  (fin length, m)
  h     =       500  (heat transfer coefficient, W/m^2K)
  k     =        25  (fin thermal conductivity, W/mK)
  Troot =       500  (root temperature, deg C)
  Tgas  =       900  (gas temperature, deg C)
  N     =         6  (number of nodes)
  dx    =      0.01  (length of control volume, m)
  K1    =  0.533333  (-)
  K2    =  444.4444  (m^ -2)
  K3    =   0.00105  (m^2)
```

COEFFICIENT MATRIX

1	0	0	0	0	0
-1	2.533333	-1	0	0	0
0	-1	2.533333	-1	0	0
0	0	-1	2.533333	-1	0
0	0	0	-1	2.533333	-1
0	0	0	0	-1	1.466667

INVERSE MATRIX						VECTOR D	VECTOR PRODUCT T
1	0	0	0	0	0	500	500
0.489927	0.489927	0.241149	0.120984	0.065343	0.044552	480	704.0291
0.241149	0.241149	0.610911	0.306492	0.165536	0.112865	480	803.5404
0.120984	0.120984	0.306492	0.655463	0.354014	0.241374	480	851.6065
0.065343	0.065343	0.165536	0.354014	0.731301	0.498614	480	873.8628
0.044552	0.044552	0.112865	0.241374	0.498614	1.021782	420	882.1792

FIN HEAT LOSS --> -349.533 watts

The heat loss from the blade is –349.533 watts, i.e., the fin gains 349.533 watts from the combustion gases.

To determine the fin efficiency of the blade, consider that if the entire blade were at the root temperature, the heat loss would be

$$Q_{I,\max} = (PL + A)\,(T_{\text{root}} - T_\infty)$$

$$Q_{I,\max} = \left(500\,\text{W}/(\text{m}^2\text{K})\right)((0.12\text{ m})\,(0.05\text{ m}) + 0.00045\text{ m}^2)\,(900 - 500)\text{ K}$$

$$= 1290.0\text{ watt}$$

The fin efficiency is therefore

$$\eta_{\text{fin}} = \frac{Q_I}{Q_{I,\max}} = \frac{349.5}{1290.0} = 0.271$$

For the exact solution, use Table 2.1, entry 4 with

$$\mathrm{m} = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{\left(500\,\mathrm{W/(m^2K)}\right)(0.12\,\mathrm{m})}{(25\,\mathrm{W/(m\,K)})\left(0.00045\,\mathrm{m^2}\right)}} = 73.0297\ \mathrm{m}^{-1}$$

$$\mathrm{m}\,L = (73.0297\ \mathrm{m})\ (0.05\ \mathrm{m}) = 3.6514$$

$$\mathrm{M} = \sqrt{hPkA}\ (T_{\mathrm{root}} - T_\infty) = \sqrt{\left(500\,\mathrm{W/(m^2K)}\right)(0.12\,\mathrm{m})(25(\mathrm{W/m\,K})\left(0.00045\,\mathrm{m^2}\right)}$$

$$= 328.633\ \text{watt}$$

giving

$$Q_{\mathrm{fin}} = 328.381\ \text{watt}$$

which is about 6% less than our numerical solution. Presumably, as we increase N, the accuracy would improve.

PROBLEM 3.15

Determine the difference equations applicable to the centerline and at the surface of an axisymmetric cylindrical geometry with volumetric heat generation and convective boundary condition. Assume steady-state conditions.

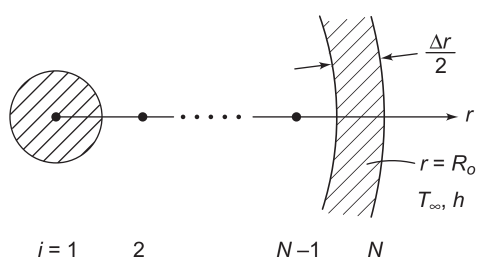
GIVEN

- Axisymmetric, steady, cylindrical geometry with volumetric heat generation and surface convection boundary condition

FIND

(a) Difference equations for the centerline and surface

SKETCH



SOLUTION

The solution to this problem completes the formulation of the cylindrical geometry presented in Section 3.5, with the added constraints of steady state conditions and symmetry.

As in Figure 3.20 and Section 3.5, the radius is given by

$$r = (i - 1)\,\Delta r \qquad i = 1, 2, \ldots N \qquad \Delta r = \frac{R_o}{N-1}$$

Let the convection coefficient be h and ambient temperature be T_∞. The inner surface area per unit length of the shaded control volume is

$$2\pi\left(R_o - \frac{\Delta r}{2}\right)$$

and the outer surface area is

$$2\pi R_o$$

The volume of the control volume per unit length is

$$\pi\left(R_o^2 - \left(R_o - \frac{\Delta r}{2}\right)^2\right) = \pi\left(R_o \Delta r - \frac{\Delta r^2}{4}\right)$$

The energy balance on the control volume gives

$$k\,\frac{T_{N-1} - T_N}{\Delta r}\;2\pi\left(R_o - \frac{\Delta r}{2}\right) + 2\pi R_o h\,(T_\infty - T_N) + \dot{q}_G\,\frac{\Delta r}{2}\left(R_o - \frac{\Delta r}{4}\right) = 0$$

Simplifying and putting into the tridiagonal form

$$T_N\left(\frac{k}{\Delta r}\left(R_o - \frac{\Delta r}{2}\right) + R_o h\right) = T_{N-1}\left(\frac{k}{\Delta r}\left(R_o - \frac{\Delta r}{2}\right)\right) + \left(R_o h T_\infty + \dot{q}_G\,\frac{\Delta r}{2}\left(R_o - \frac{\Delta r}{4}\right)\right)$$

For the control volume for the centerline node, $i = 1$, the volume per unit length is

$$\pi\left(\frac{\Delta r}{2}\right)^2$$

and the surface area per unit length is

$$2\pi\,\frac{\Delta r}{2} = \pi\Delta r$$

The energy balance gives

$$k\,\frac{T_2 - T_1}{\Delta r}\,\pi\Delta r + \dot{q}_G\,\pi\left(\frac{\Delta r}{2}\right)^2 = 0$$

Simplifying and putting into the tridiagonal form

$$T_2 - T_1 + \dot{q}_G\,\frac{\Delta r^2}{4k} = 0$$

COMMENTS

The above two difference equations can be combined with Equation (3.30) to produce the full set of difference equations. The resulting tridiagonal set of equation can be solved just as Equation (3.10). (The steady, axisymmetric version of Equation (3.30) would be used.)

PROBLEM 3.16

> **Determine the appropriate difference equations for an axisymmetric, steady, spherical geometry with volumetric heat generation. Explain how to solve the equations.**

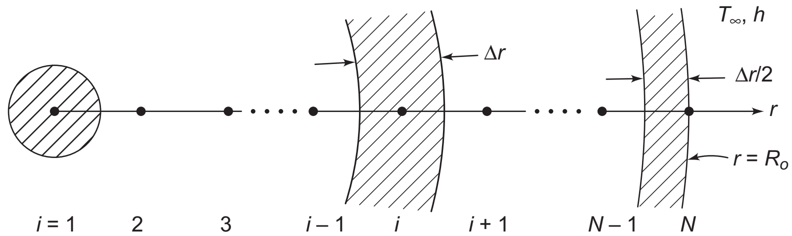
GIVEN

- Axisymmetric, steady, spherical geometry with heat generation

FIND

(a) Difference equations

SKETCH



SOLUTION

We need to perform an energy balance on the three shaded control volumes shown in the text. For the node at the sphere center, $i = 1$

$$\text{Volume} = \frac{4}{3}\pi\left(\frac{\Delta r}{2}\right)^3 = \frac{\pi}{6}\Delta r^3$$

$$\text{Surface} = 4\pi\left(\frac{\Delta r}{2}\right)^2 = \pi\Delta r^2$$

The energy balance is

$$k\,\frac{T_2 - T_1}{\Delta r}\,\pi\Delta r^2 + \dot{q}_G\,\frac{\pi}{6}\,\Delta r^3 = 0$$

In the tridiagonal form

$$T_1\,k\Delta r = T_2\,k\Delta r + \dot{q}_G\,\frac{\Delta r^3}{6}$$

For interior control volumes, $1 < i < N$

$$\text{Volume} = \frac{4}{3}\pi\left[\left(i\Delta r + \frac{\Delta r}{2}\right)^3 - \left(i\Delta r - \frac{\Delta r}{2}\right)^3\right] = \frac{4}{3}\pi\Delta r^3\left(3i^2 + \frac{1}{4}\right) \equiv V_i$$

$$\text{Inner surface area} = 4\pi\left(i\Delta r - \frac{\Delta r}{2}\right)^2 = 4\pi\Delta r^2\left(i - \frac{1}{2}\right)^2 \equiv A_{ii}$$

$$\text{Outer surface area} = 4\pi\left(i\Delta r + \frac{\Delta r}{2}\right)^2 = 4\pi\Delta r^2\left(i + \frac{1}{2}\right)^2 \equiv A_{io}$$

The energy balance is

$$k\,\frac{T_{i-1} - T_i}{\Delta r}\,A_{ii} + k\,\frac{T_{i+1} - T_i}{\Delta r}\,A_{io} + \dot{q}_G\,V_i = 0$$

In the tridiagonal form this becomes

$$T_i\left[\frac{k}{\Delta r}(A_{ii} + A_{io})\right] = T_{i-1}\left[\frac{k}{\Delta r}A_{ii}\right] + T_{i+1}\left[\frac{k}{\Delta r}A_{io}\right] + \dot{q}_G\,V_i$$

For the control volume at the surface of the sphere

$$\text{Volume} = \frac{4}{3}\pi\left[R_o^3 - \left(R_o - \frac{\Delta r}{2}\right)^3\right] \equiv V_o$$

$$\text{Inner surface area} = 4\pi\left(R_o - \frac{\Delta r}{2}\right)^2 \equiv A_{Ni}$$

$$\text{Outer surface area} = 4\pi R_o^2 \equiv A_{No}$$

The energy balance for the surface control volume is

$$k\,\frac{T_{N-1} - T_N}{\Delta r}\,A_{Ni} + A_{No}\,h\,(T_\infty - T_N) + \dot{q}_G\,V_o = 0$$

In the tridiagonal form

$$T_N\left[\frac{k}{\Delta r}A_{Ni} + hA_{No}\right] = T_{N-1}\left[\frac{k}{\Delta r}A_{Ni}\right] + A_{No}\,h + \dot{q}_G\,V_o$$

From the three control volume difference equations given above in the tridiagonal form, we can determine the matrix coefficients

$$a_1 = k\Delta r \quad b_1 = k\Delta r \quad c_1 = 0 \quad d_1 = \dot{q}_G\,\frac{\Delta r^3}{6}$$

$$a_i = \frac{k}{\Delta r}(A_{ii} + A_{io}) \quad b_i = \frac{k}{\Delta r}A_{io} \quad c_i = \frac{k}{\Delta r}A_{ii} \quad d_i = \dot{q}_G\,V_i \quad 1 < i < N$$

$$a_N = \frac{k}{\Delta r}A_{Ni} + hA_{No} \quad b_N = 0 \quad c_N = \frac{k}{\Delta r}A_{Ni} \quad d_N = A_{No}\,h + \dot{q}_G\,V_o$$

To solve this set of equations, we insert these coefficients into the matrix in Equation (3.10) and solve the tridiagonal matrix as was done for other one-dimensional problems.

PROBLEM 3.17

Show that in the limit $\Delta x \to 0$ and $\Delta t \to 0$, the difference Equation (3.12) is equivalent to the differential Equation (2.5).

GIVEN

- The difference equation for one-dimensional transient conduction

SHOW

(a) As Δx and $\Delta t \to 0$, the difference equation is equivalent to the differential equation, Equation (2.5)

SOLUTION

Equation (3.13) is

$$T_{i,\,m+1} = T_{i,\,m} + \frac{\Delta t}{\rho c \Delta x}\left(\frac{k}{\Delta x}\left(T_{i+1,m} - 2T_{i,m} + T_{i-1,m}\right) + \dot{q}_{G\,i,m}\,\Delta x\right)$$

By definition

$$T_{i,\,m} = T(x,\,t)$$

$$T_{i+1,\,m} = T(x + \Delta x,\,t)$$

$$T_{i-1,\,m} = T(x - \Delta x,\,t)$$

$$T_{i,\,m+1} = T(x,\,t + \Delta t)$$

So, the difference equation is equivalent to

$$\rho c \frac{T(x,t+\Delta t)-T(x,t)}{\Delta t} = k \frac{T(x+\Delta x,t)-2T(x,t)+T(x-\Delta x,t)}{\Delta x^2} + \dot{q}_G(x,t)$$

In the limit as $\Delta t \to 0$, from calculus, the left hand side of the above equation becomes

$$\rho c \frac{\partial T}{\partial t}$$

and in the limit as $\Delta x \to 0$, from calculus, the first term on the right hand side of the equation becomes

$$k \frac{\partial^2 T}{\partial x^2}$$

So the equation is equivalent to

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + \dot{q}_G(x,t)$$

which is the same as Equation (2.5).

PROBLEM 3.18

Determine the largest permissible time step for a one-dimensional transient conduction problem to be solved by an explicit method if the node spacing is 1 mm and the material is (a) carbon steel 1C, and (b) window glass. Explain the difference in the two results.

GIVEN

- One-dimensional transient conduction in a 1 mm thickness of carbon steel and window glass

FIND

(a) Largest permissible time step for each material

SOLUTION

(a) From Table 10 in Appendix 2, the thermal diffusivity for carbon steel is $\alpha = 1.172 \times 10^{-5}$ m^2/s. The largest permissible time step is given by Equation (3.14)

$$\Delta t_{max} = \frac{\Delta x^2}{2\alpha} = \frac{(10^{-3}\text{ m})^2}{(2)\left(1.172\times10^{-5}\text{ m}^2/\text{s}\right)} = 0.0427\text{ s}$$

(b) From Table 11 in Appendix 2 the thermal diffusivity for window glass is $\alpha = 0.034\times10^{-5}$ m^2/s. The largest permissible time step is given by Equation (3.14)

$$\Delta t_{max} = \frac{\Delta x^2}{2\alpha} = \frac{(10^{-3}\text{ m})^2}{(2)\left(0.034\times10^{-5}\text{ m}^2/\text{s}\right)} = 1.47\text{ s}$$

Since the heat diffuses much more slowly through the window glass, much larger time steps are allowed.

PROBLEM 3.19

Consider one-dimensional transient conduction with a convective boundary condition in which the ambient temperature near the surface is a function of time. Determine the energy balance equation for the boundary control volume. How would the solution method need to be modified to accommodate this complexity?

GIVEN

- One-dimensional transient conduction where the ambient temperature near the surface is a function of time

FIND

(a) The difference equation for the boundary control volume and explain how to solve the problem

SOLUTION

The difference equation would be derived exactly as Equation (3.17). Assuming we are the boundary in question is the left boundary we would have:

$$T_{1,\, m+1} = T_{1,\, m} = \frac{2\Delta t}{\rho c \Delta x}\left\{ h\left(T_{\infty,m} - T_{1,m}\right) + \dot{q}_{G1,m}\frac{\Delta x}{2} + k\frac{T_{2,m} - T_{1,m}}{\Delta x}\right\}$$

Here, the term T_∞ will depend on the time step m. Since this function of time is presumably known, a marching procedure can be used to solve the set of equations for the whole problem.

PROBLEM 3.20

What are the advantages and disadvantages of using explicit and implicit difference equations?

EXPLAIN

(a) Advantages and disadvantages of explicit and implicit methods

SOLUTION

The explicit method can be solved by marching, which is very simple to implement but the maximum time step is limited by stability considerations. The implicit method forces the use of matrix inversion software to find the solution, but the size of the time step is not limited by stability considerations. (It is limited by accuracy considerations just as it is for any method.)

PROBLEM 3.21

Equation (3.15) is often called the fully-implicit form of the one-dimensional transient conduction difference equation because all quantities in the equation, except for the temperatures in the energy storage term, are evaluated at the new time step, *m* + 1. In an alternate form called Crank-Nicholson, these quantities are evaluated at both time step *m* and *m* + 1 and then averaged. This has the effect of significantly improving accuracy of the numerical solution relative to the fully-implicit from without increasing complexity of the solution method. Derive the one-dimensional transient conduction difference equation in the Crank-Nicholson form.

GIVEN

- One-dimensional transient conduction difference equation in the implicit form

FIND

(a) The Crank-Nicholson form of the difference equation

SOLUTION

We have the explicit difference equation, Equation (3.13)

$$T_{i,\, m+1} = T_{i,\, m} + \frac{\Delta t}{\rho c \Delta x}\left(\frac{k}{\Delta x}\left(T_{i+1,m} - 2T_{i,m} + T_{i-1,m}\right) + \dot{q}_{G\,i,m}\,\Delta x\right)$$

and the implicit difference equation, Equation (3.15)

$$T_{i,\,m+1} = T_{i,\,m} + \frac{\Delta t}{\rho c \Delta x}\left(\frac{k}{\Delta x}\left(T_{i+1,m+1} - 2T_{i,m+1} + T_{i-1,m+1}\right) + \dot{q}_{G\,i,m+1}\,\Delta x\right)$$

Adding these two equations and dividing by 2 gives the desired Crank-Nicholson form of the one-dimensional transient difference equation

$$T_{i,\,m+1} = T_{i,\,m} + \frac{\Delta t}{2\rho c \Delta x}$$

$$\left(\frac{k}{\Delta x}\left(T_{i+1,m} - 2T_{i,m} + T_{i-1,m} + T_{i+1,m+1} - 2T_{i,m+1} + T_{i-1,m+1}\right) + \left(\dot{q}_{G\,i,m} + \dot{q}_{G,i,m+1}\right)\Delta x\right)$$

PROBLEM 3.22

A 3-m-long steel rod (k = 43 W/(mK), α = 1.17 × 10^{-5} m^2/s) is initially at 20°C and insulated completely except for its end faces. One end is suddenly exposed to the flow of combustion gases at 1000°C through a heat transfer coefficient of 250 W/(m^2 K) and the other end is held at 20°C. How long will it take for the exposed end to reach 700°C? How much energy will the rod have absorbed if it is circular in cross section and has a diameter of 3 cm?

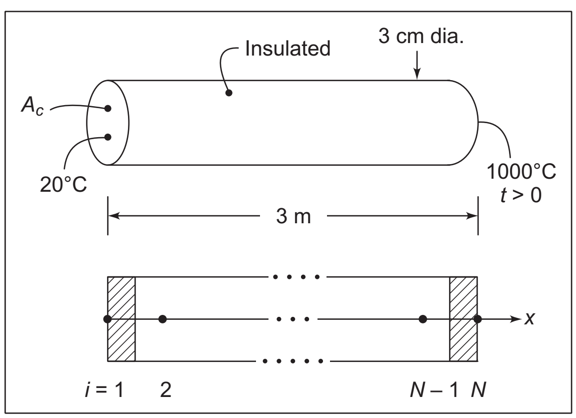
GIVEN

- Steel rod with one end at fixed temperature and the other end exposed to combustion gases

FIND

(a) Time required for the exposed face to reach 700°C
(b) Heat input to the rod

SOLUTION



Control Volume and Node Layout

See the figure to the right for the arrangement of control volumes and nodes and symbol definitions. The nodes are located at

$$x_i = (i-1)\,\Delta x \qquad \Delta x = \frac{L}{(N-1)} \qquad i = 1, 2, \ldots, N$$

and the time steps are given by

$$t_m = m\Delta t \qquad m = 0, 1, 2, \ldots$$

For the half control volume at $i = 1$, the temperature is constant so

$$T_{1,m} = T_{\text{initial}} \qquad m \geq 0$$

For the half control volume at $i = N$, the explicit form of the energy balance is

$$k\,\frac{T_{N-1,m} - T_{N,m}}{\Delta x} + h(T_\infty - T_{N,m}) = \rho\, c\left(\frac{\Delta x}{2}\right)\frac{T_{N,m+1} - T_{N,m}}{\Delta t}$$

Solving for $T_{N,\,m+1}$

$$T_{N,m+1} = T_{N,m} + \frac{2\Delta t}{\rho c \Delta x}\left\{\frac{k}{\Delta x}\left(T_{N-1,m} - T_{N,m}\right) + h\left(T_\infty - T_{N.m}\right)\right\}$$

For all the interior nodes, $i = 2, 3, 4, \ldots N - 1$, the energy balance is

$$\frac{k}{\Delta x}\{(T_{i-1,m} - T_{i,m}) + (T_{i+1,m} - T_{i,m})\} = \rho\, c\Delta x\,\frac{T_{i,m+1} - T_{i,m}}{\Delta t}$$

Solving for $T_{i,\,m+1}$

$$T_{i,m+1} = T_{i,m} + \frac{\alpha\Delta t}{\Delta x^2}\{T_{i-1,m} - 2T_{i,m} + T_{i+1,m}\} \qquad i = 2, 3, \ldots N - 1$$

The heat input to the rod after any time step m is given by

$$Q_{\text{input},\,m} = A_c\,\rho\, c\Delta x\left\{\sum_{i=2}^{N-1}\left(T_{i,m} - T_{i,m=0}\right) + \frac{1}{2}\left(T_{N,m} - T_{N,m=0}\right)\right\}$$

The factor of 1/2 is because the control volume at $i = N$ is $\Delta x/2$ in width.

Since we have chosen an explicit method, we can use the marching procedure as described in Section 3.3.1. Also, the time step Δt is restricted via Equation (3.14). After setting up the computer program to step through the time steps, the energy balance on nodes $i = 1$, 2, and N were checked by hand to insure that the code was correct. The several runs were made with various values of N and Δt to find how large N and how small Δt must be to get an accurate solution. The table below summarizes these runs

N	Δt_{max} (s)	Δt (s)	t_{final} (s)	Q_{input} (J/m^2)
11	3846	10.0	5990	503.72
11	3846	1.0	5993	503.79
21	962	10.0	6350	490.33
41	240	10.0	6440	487.50
81	60	10.0	6460	486.69
81	60	5.0	6455	486.37

Since there is little change between the last 3 runs, the solution is that 6455 seconds are required for the exposed face to reach 700°C and the heat input to the rod is 486.4 joules.

PROBLEM 3.23

A Trombe wall is a masonry wall often used in passive solar homes to store solar energy. Suppose such a wall, fabricated from 20 cm thick solid concrete blocks (k = 0.13 W/(mK), $\alpha = 0.05 \times 10^{-5}$ m²/s is initially at 15°C in equilibrium with the room in which it is located. It is suddenly exposed to sunlight and absorbs 500 W/m² on the exposed face. The exposed face loses heat by radiation and convection to the outside ambient temperature of –15°C through a combined heat transfer coefficient of 10 W/(m² K). The other face of the wall is exposed to the room air through a heat transfer coefficient of 10 W/(m² K). Assuming that the room air temperature does not change, determine the maximum temperature in the wall after 4 hours of exposure and the net heat transferred to the room.

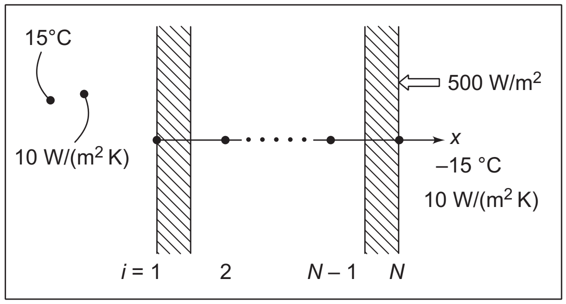
GIVEN

- Trombe wall suddenly exposed to sunlight

FIND

(a) Maximum temperature in the wall after 4 hours
(b) Heat input to the room

SOLUTION



Control Volume and Node Layout

See the figure to the right for the arrangement of control volumes and nodes and symbol definitions. The nodes are located at

$$x_i = (i-1)\Delta x \quad \Delta x = \frac{L}{(N-1)} \quad i = 1, 2, \ldots, N$$

and the time steps are given by

$$t_m = m\Delta t \qquad m = 0, 1, 2, \ldots$$

For the half control volume at $i = 1$, the explicit form of the energy balance is

$$k\,\frac{T_{2,m} - T_{1,m}}{\Delta x} + h\,(T_\infty - T_{1,m}) = \rho\,c\,\frac{\Delta x}{2}\frac{T_{1,m+1} - T_{1,m}}{\Delta t}$$

Solving for $T_{1,\,m+1}$

$$T_{1,\,m+1} = T_{1,\,m} + \frac{2\Delta t}{\rho c \Delta x}\left\{\frac{k}{\Delta x}\left(T_{2,m} - T_{1,m}\right) + h\left(T_\infty - T_{1,m}\right)\right\}$$

where h is the heat transfer coefficient on the room-side of the wall. For the half control volume at $i = N$, the explicit form of the energy balance is

$$k\,\frac{T_{N-1,m} - T_{N,m}}{\Delta x} + q_{abs} = \rho\,c\,\frac{\Delta x}{2}\frac{T_{N,m+1} - T_{N,m}}{\Delta t} + U_o\,(T_{N,\,m} - T_{\text{out}})$$

where U_o is the combined heat transfer coefficient to outside ambient.
Solving for $T_{N,\,m+1}$

$$T_{N,\,m+1} = T_{N,\,m} + \frac{2\Delta t}{\rho c \Delta x}\left\{\frac{k}{\Delta x}\left(T_{N-1,m} - T_{N,m}\right) + q_{abs} - U_o\left(T_{N,m} - T_{\text{out}}\right)\right\}$$

For all the interior nodes, $i = 2, 3, 4, \ldots N-1$, the energy balance is

$$\frac{k}{\Delta x}\,\{(T_{i-1,\,m} - T_{i,\,m}) + (T_{i+1,\,m} - T_{i,\,m})\} = \rho\,c\Delta x\,\frac{T_{i,m+1} - T_{i,m}}{\Delta t}$$

Solving for $T_{i,\, m+1}$

$$T_{i,\, m+1} = T_{i,\, m} + \frac{\alpha \Delta t}{\Delta x^2} \{T_{i-1,\, m} - 2T_{i,\, m} + T_{i+1,\, m}\} \qquad i = 2, 3, \ldots N-1$$

The maximum temperature in the wall at any time step m must be $T_{N,\, m}$.

The heat input to the room after any time step m is given by

$$Q_{\text{input},\, m} = h\, \Delta t \sum_{m=1}^{m_{\text{final}}} (T_{1,m} - T_{\infty}) \left(\text{J/m}^2\right)$$

Since we have chosen an explicit method, we can use the marching procedure as described in Section 3.3.1. Also, the time step Δt is restricted via Equation (3.14). After setting up the computer program to step through the time steps, the energy balance on nodes i = 1, 2, and N were checked by hand to insure that the code was correct.

Then several runs were made with various values of N and Δt to find how large N and how small Δt must be to get an accurate solution. The table below summarizes these runs

N	Δt_{max} (s)	Δt (s)	Q_{input} (J/m^2)	TN (°C)
11	400	100	31838	
11	400	50	31713	
11	400	25	31650	
21	100	25	30892	
31	44	25	30751	
41	25	20	30689	33.29
41	25	5	30666	33.29
41	25	5	30653	33.29
61	11	10	30630	33.29

Since there is little change between the last 4 runs, the solution is that after 4 hours the heat input to the room is 30630 joules per m^2 of wall area and the maximum temperature in the wall is 33.29°C.

PROBLEM 3.24

To more accurately model the energy input from the sun, suppose the absorbed flux in Problem 3.23 is given by

$$q_{abs}(t) = t\,(375 - 46.875\, t)$$

where t is in hours and q_{abs} is in W/m^2. (This time variation of q_{abs} gives the same total heat input to the wall as in Problem 3.23, i.e., 2000 W hr/m^2). Repeat Problem 3.23 with the above equation for q_{abs} in place of the constant value of 500 W/m^2. Explain your results.

From Problem 3.23: A Trombe wall is a masonry wall often used in passive solar homes to store solar energy. Suppose such a wall, fabricated from 200 cm thick solid concrete blocks (k = 0.13 W/(mK), $\alpha = 0.05 \times 10^{-5}$ m^{2}s) is initially at 15°C in equilibrium with the room in which it is located. It is suddenly exposed to sunlight and absorbs 500 W/m^2 on the exposed face. The exposed face loses heat by radiation and convection to the outside ambient temperature of – 15°C through a combined heat transfer coefficient of 10 W/(m^2 K). The other face of the wall is exposed to the room air through a heat transfer coefficient of 10 W/(m^2 K). Assuming that the room air temperature does not change, determine the maximum temperature in the wall after 4 hours of exposure and the net heat transferred to the room.

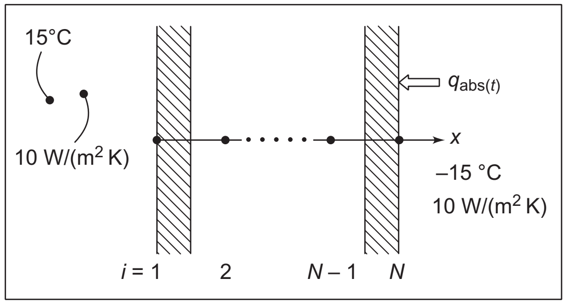
GIVEN

- Trombe wall with specified absorbed solar flux as a function of time

FIND

(a) Maximum temperature in the wall after 4 hours
(b) Heat input to the room

SOLUTION



Control Volume and Node Layout

See the accompanying figure for the arrangement of control volumes and nodes and symbol definitions. The nodes are located as

$$x_i = (i - 1)\,\Delta x$$

$$\Delta x = \frac{L}{(N-1)} \qquad i = 1, 2, \ldots, N$$

and the time steps are given by

$$t_m = m\,\Delta t \qquad m = 0, 1, 2, \ldots$$

For the half control volume at $i = 1$, the explicit form of the energy balance is

$$k\,\frac{T_{2,m} - T_{1,m}}{\Delta x} + h(T_\infty - T_{1,m}) = \rho\, c\,\frac{\Delta x}{2}\frac{T_{1,m+1} - T_{1,m}}{\Delta t}$$

Solving for $T_{1,\,m+1}$

$$T_{1,\,m+1} = T_{1,\,m} + \frac{2\Delta t}{\rho c \Delta x}\left\{\frac{k}{\Delta x}\left(T_{2,m} - T_{1,m}\right) + h\left(T_\infty - T_{1,m}\right)\right\}$$

For the half control volume at $i = N$, the explicit from of the energy balance is

$$k\,\frac{T_{N-1,m} - T_{N,m}}{\Delta x} + q_{\text{abs},m} = \rho\, c\,\frac{\Delta x}{2}\frac{T_{N,m+1} - T_{N,m}}{\Delta x} + U_o\,(T_{N,\,m} - T_{\text{out}})$$

Solving for $T_{N,\,m+1}$

$$T_{N,\,m+1} = T_{N,\,m} + \frac{2\Delta t}{\rho c \Delta x}\left\{\frac{k}{\Delta x}\left(T_{N-1,m} - T_{N,m}\right) + q_{\text{abs},m} - U_o\left(T_{N,m} - T_{\text{out}}\right)\right\}$$

For all the interior nodes, $i = 2, 3, 4, \ldots N - 1$, the energy balance is

$$\frac{k}{\Delta x}\,\{(T_{i-1,\,m} - T_{i,\,m}) + (T_{i+1,\,m} - T_{i,\,m}) = \rho\, c \Delta x\,\frac{T_{i,m+1} - T_{i,m}}{\Delta t}$$

Solving for $T_{i,\,m+1}$

$$T_{i,\,m+1} = T_{i,\,m} + \frac{\alpha \Delta t}{\Delta x^2}\,\{T_{i-1,\,m} - 2T_{i,\,m} + T_{i+1,\,m}\} \qquad i = 2, 3, \ldots N - 1$$

The maximum temperature in the wall at any time step m must be $T_{N,m}$ and the heat input to the room after any time step m is given by

$$Q_{\text{input},\, m} = h\, \Delta t \sum_{m=1}^{m} (T_{1,m} - T_\infty) \left(\frac{\text{J}}{\text{m}^2}\right)$$

Since we have chosen an explicit method, we can use the marching procedure as described in Section 3.3.1. Also, the time step Δt is restricted via Equation (3.14). After setting up the computer program to step through the time steps, the energy balance on nodes i = 1, 2, and N were checked by hand to insure that the code was correct. A run was then made with N = 41, Δt = 5 seconds. The results indicate that the heat input to the room is –1834 joules per m^2 of wall area and the maximum wall temperature is 54.16°C. In comparison with the results from Problem 3.23 where 30630 J/m^2 was delivered to the room, here the room has lost 1834 J/m^2 to the wall. The reason is that for early times, before the absorbed solar flux becomes significant, the wall is losing heat to the outside and is rapidly cooling. The room-side face of the wall dips below the air temperature of 15°C and begins to remove heat from the room. Only at later times does the wall heat up sufficiently to begin transferring heat back to the room. For the short 4 hour run, the net effect is a loss of heat from the room to the wall.

PROBLEM 3.25

> **An interior wall of a cold furnace, initially at 0°C, is suddenly exposed to a radiant flux of 15 kW/m^2 when the furnace is brought on line. The outer surface of the wall is exposed to ambient air at 20°C through a heat transfer coefficient of 10 W/(m^2 K). The wall is 20 cm thick and is made of expanded perlite (k = 0.10 W/(mK), α = 0.03 × 10^{-5} m^2/s) sandwiched between two sheets of oxidized steel. Determine how long after startup will the inner (hot) sheet metal surface get hot enough so that reradiation becomes significant.**

GIVEN

- Furnace wall suddenly exposed to radiant heat flux

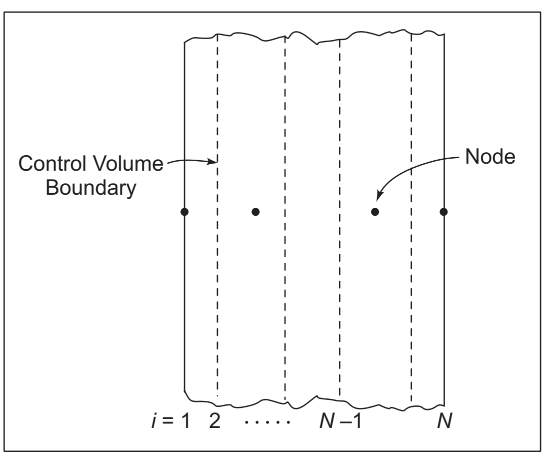
FIND

(a) How long before reradiation from the heated wall becomes significant.

ASSUMPTIONS

(a) Reradiation becomes significant when the reradiated flux from the exposed wall exceeds 10% of the incident radiant flux.
(b) The oxidized surface of the exposed wall is black.

SOLUTION



Control Volume and Node Layout

See the figure to the right for the arrangement of control volumes and nodes and symbol definitions. The nodes are located at

$$x_i = (i-1)\,\Delta x \quad \Delta x = \frac{L}{(N-1)} \qquad i = 1, 2, \ldots, N$$

and the time steps are given by

$$t_m = m\,\Delta t \qquad m = 0, 1, 2, \ldots$$

For the half control volume at $i = 1$, the explicit form of the energy balance is

$$k\,\frac{T_{2,m}-T_{1,m}}{\Delta x} + h(T_\infty - T_{1,m}) = \rho\,c\,\frac{\Delta x}{2}\frac{T_{1,m+1}-T_{1,m}}{\Delta t}$$

Solving for $T_{1,\,m+1}$

$$T_{1,\,m+1} = T_{1,\,m} + \frac{2\Delta t}{\rho c \Delta x}\left\{\frac{k}{\Delta x}\left(T_{2,m}-T_{1,m}\right)+h\left(T_\infty - T_{1,m}\right)\right\}$$

For the half control volume at $i = N$, the explicit form of the energy balance is

$$k\,\frac{T_{N-1,m}-T_{N,m}}{\Delta x} + q_{\text{abs}} = \rho\,c\,\frac{\Delta x}{2}\frac{T_{N,m+1}-T_{N,m}}{\Delta t}$$

Solving for $T_{N,\,m+1}$

$$T_{N,\,m+1} = T_{N,\,m} + \frac{2\Delta t}{\rho c \Delta x}\left\{\frac{k}{\Delta x}\left(T_{N-1,m}-T_{N,m}\right)+q_{\text{abs}}\right\}$$

For all the interior nodes, $i = 2, 3, 4, \ldots N-1$, the energy balance is

$$\frac{k}{\Delta x}\{(T_{i-1,\,m} - T_{i,\,m}) + (T_{i+1,\,m} - T_{i,\,m})\} = \rho\,c\Delta x\,\frac{T_{i,m+1}-T_{i,m}}{\Delta t}$$

Solving for $T_{i,\,m+1}$

$$T_{i,\,m+1} = T_{i,\,m} + \frac{\alpha\Delta t}{\Delta x^2}\{T_{i-1,\,m} - 2T_{i,\,m} + T_{i+1,\,m}\} \qquad i = 2, 3, \ldots N-1$$

Since the exposed wall is black, the reradiated flux from the hot wall is $\sigma T_N^{\,4}$ and the criterion we seek is

$$\sigma T_N^{\,4} \geq 0.1\; q_{\text{abs}}$$

For the given values of problem parameters, this equates to $T_N = 130.3°\text{C}$.

Since we have chosen an explicit method, we can use the marching procedure as described in Section 3.3.1. Also, the time step Δt is restricted via Equation (3.14). After setting up the computer program to step through the time steps, the energy balance on nodes i = 1, 2, and N were checked by hand to insure that the code was correct. Then several runs were made with various values of N and Δt to find how large N and how small Δt must be to get an accurate solution. The table below summarizes these runs

N	Δt_{max} (s)	Δt (s)	t_{final} (s)	T_{max} (C)
21	16.7	0.1	15.25	131.5
41	41.7	0.1	8.0	130.9
61	18.5	0.1	5.6	131.7
81	10.4	0.1	4.4	131.4
161	2.6	0.1	2.7	130.4
321	0.65	0.1	2.2	133.0
641	0.163	0.1	2.0	130.6
1000	0.07	0.05	2.0	131.0

Note that a very large number of nodes is needed because the suddenly imposed flux causes very large temperature gradients in the furnace door. This requires a large number of nodes to accurately depict the temperature profile. The solution is that 2.0 seconds is required before reradiation must be considered.

COMMENTS

The answer given above is conservative because the emissivity of the exposed door surface will be less than 1 and the door will therefore heat up more quickly.

PROBLEM 3.26

A long cylindrical rod, 8 cm in diameter, is initially at a uniform temperature of 20°C. At time $t = 0$, the rod is exposed to an ambient temperature of 400°C through a heat transfer coefficient of 20 W/(m^2 K). The thermal conductivity of the rod is 0.8 W/(mK) and the thermal diffusivity is 3×10^{-6} m^2/s. Determine how much time will be required for the temperature change at the centerline of the rod to reach 93.68% of its maximum value. Use an explicit difference equation and compare your numerical results with a chart solution from Chapter 2.

GIVEN

- Cylindrical rod suddenly exposed to increased ambient temperature

FIND

(a) Time required for the centerline temperature change to reach 93.68% of its maximum value

SOLUTION

Since the rod will eventually reach 400°C, the maximum possible temperature change for any part of the rod is 400 – 20 = 380°C. Taking 93.68% of this temperature difference, we need to find the time such that the centerline temperature is 20 + (0.9368 × 380) = 376°C.

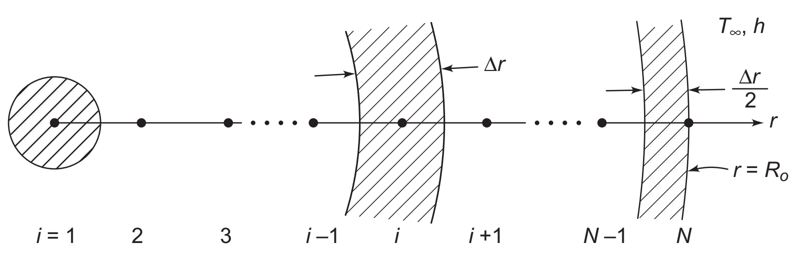
As in Figure 3.20 and Section 3.5, the radius is given by

$$r = (i-1)\,\Delta r \qquad i = 1, 2, \ldots N \qquad \Delta r = \frac{R_o}{N-1}$$

and the time is given by

$$t_m = m\,\Delta t, \qquad m = 0, 1, 2, \ldots$$

Note that since all gradients with respect to the circumferential direction, θ, are zero, the index j is not needed. Let the convection coefficient be h and ambient temperature be T_∞. The following sketch shows the control volumes necessary to solve the problem numerically



Referring to the above sketch, the inner surface area per unit length of the shaded control volume at node $i = N$ is

$$2\pi\left(R_o - \frac{\Delta r}{2}\right)$$

and the outer surface area is

$$2\pi R_o$$

The volume of the control volume per unit length is

$$\pi\left(R_o^{\,2}-\left(R_o-\frac{\Delta r}{2}\right)^2\right) = \pi\left(R_o\Delta r-\frac{\Delta r^2}{4}\right) \equiv V_N$$

The explicit form of the energy balance on the control volume at $i = N$ gives

$$\rho\, c\, V_N\, \frac{T_{N,m+1}-T_{N,m}}{\Delta t} = k\,\frac{\left(T_{N-1,m}-T_{N,m}\right)}{\Delta r}\,2\pi\left(R_o-\frac{\Delta r}{2}\right) + 2\pi R_o\, h\,(T_\infty - T_{N,\,m})$$

Solving for $T_{N,\,m+1}$,

$$T_{N,\,m+1} = T_{N,\,m}\left\{1-\frac{\alpha\,\Delta t 2\pi}{V_N}\left(\frac{R_o}{\Delta r}-\frac{1}{2}\right)-\frac{\Delta t 2\pi R_o h}{\rho c V_N}\right\} + T_{N-1,\,m}\left\{\frac{\alpha\,\Delta t 2\pi}{V_N}\left(\frac{R_o}{\Delta r}-\frac{1}{2}\right)\right\} + \frac{\Delta t 2\pi R_o h T_\infty}{\rho c V_N}$$

For the control volume at the centerline node, $i = 1$, the volume per unit length is

$$\pi\left(\frac{\Delta r}{2}\right)^2 \equiv V_1$$

and the surface area per unit length is

$$2\pi\,\frac{\Delta r}{2} = \pi\Delta r$$

The energy balance on this node is

$$\rho\, c\, V_1\, \frac{T_{1,m+1}-T_{1,m}}{\Delta t} = k\,\frac{T_{2,m}-T_{1,m}}{\Delta r}\,\pi\Delta r$$

Solving for $T_{1,\,m+1}$

$$T_{1,\,m+1} = T_{1,\,m}\left(1-\frac{\alpha\,\Delta t\pi}{V_1}\right) + T_{2,\,m}\,\frac{\alpha\,\Delta t\pi}{V_1}$$

For nodes $1 < i < N$, set the $\frac{\partial}{\partial\theta}$ terms to zero and set $\Delta\,\theta = 2\pi$ in Equation (3.30),

$$\rho\, c\, r\, 2\pi\Delta\, r\frac{T_{i,m+1}-T_{i,m}}{\Delta t} = \frac{k2\pi r}{\Delta r}\left[T_{i-1,m}-2T_{i,m}+T_{i+1,m}+\frac{\Delta r}{2r}\left(T_{i+1,m}-T_{i-1,m}\right)\right]$$

Solving for $T_{i,\,m+1}$

$$T_{i,\,m+1} = T_{i,\,m}\left(1-\frac{2\alpha\,\Delta t}{\Delta r^2}\right) + T_{i+1,\,m}\frac{\alpha\,\Delta t}{\Delta r^2}\left(1+\frac{\Delta r}{2r}\right) + T_{i-1,m}\,\frac{\alpha\,\Delta t}{\Delta r^2}\left(1-\frac{\Delta r}{2r}\right)$$

Note that

$$\frac{\Delta r}{2r} = \frac{1}{2(i-1)}$$

and

$$\frac{R_o}{\Delta r} = N-1$$

Using a time step, Δt, such that

$$\Delta t = \Gamma\,\frac{\Delta r^2}{2\alpha} \text{ where } \Gamma < 1$$

then the explicit solution can be solved by marching and it should be stable. For N = 10 and Γ= 0.5, the centerline temperature is found to exceed 376°C at 994 seconds.

For the chart solution, we refer to Figure 2.38. The Biot number is

$$B_i = \frac{hr_o}{k} = \frac{\left(20\,\text{W}/(\text{m}^2\text{K})\right)(0.04\,\text{m})}{\left(0.8\,\text{W}/(\text{m K})\right)} = 1.0$$

We need to find the abscissa in the figure such that

$$\frac{T(0,t)-T_\infty}{T_i - T_\infty} = \frac{376-400}{20-400} = 0.063$$

For the Biot number calculated above, the abscissa is

$$\frac{\alpha t}{r_o^{\,2}} = 1.78$$

Solving for the time, we find t = 949 seconds, approximately 5% less than the numerical method predicts. Most likely, the difference is due to the precision with which the charts can be read.

PROBLEM 3.27

Develop a reasonable layout of nodes and control volumes for the geometry shown in the sketch below. Provide a scale drawing showing the problem geometry overlaid with the nodes and control volumes.

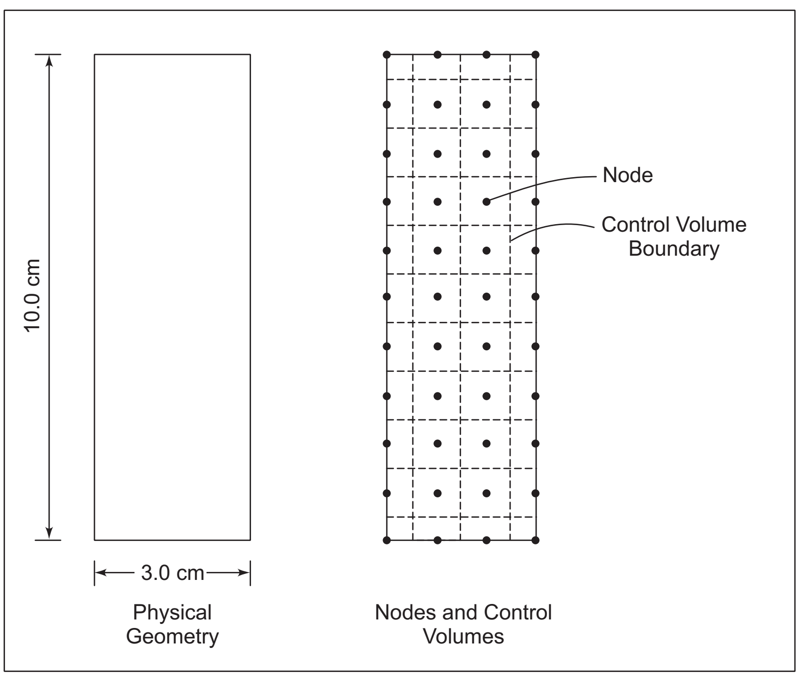
GIVEN

- Rectangular problem geometry

FIND

(a) A reasonable layout of nodes and control volumes

SKETCH



SOLUTION

The largest node spacing divisible into both 3 and 10 is 1 cm. So let's use $\Delta x = \Delta y = 1$ cm. The sketch on the right above shows the resulting placement of nodes and control volume boundaries.

PROBLEM 3.28

> **Develop a reasonable layout of nodes and control volumes for the geometry shown in the sketch below. Provide a scale drawing showing the problem geometry overlaid with the nodes and control volumes. Identify each type of control volume used.**

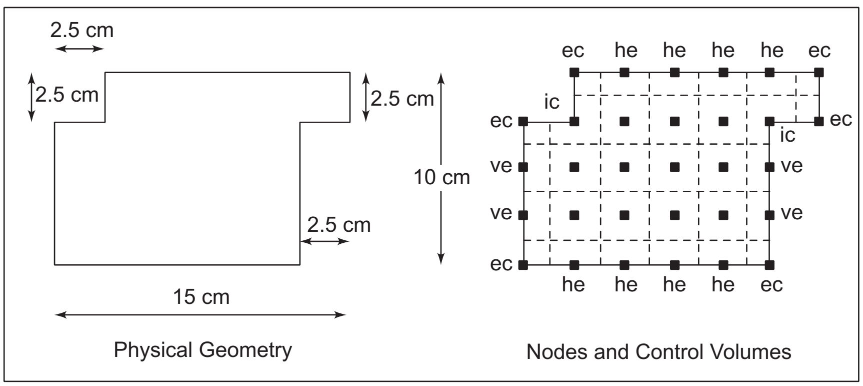
GIVEN

- Rectangular problem geometry with corners removed

FIND

(a) Reasonable layout of nodes and control volumes.
(b) Identify each type of control volume.

SKETCH



SOLUTION

The largest grid spacing for this problem is $\Delta x = \Delta y = 2.5$ cm. If we used a larger node spacing, we could not adequately represent the cutout corners. The right side of the figure shows the resulting placement of nodes and control volumes. The notation for the type of control volumes is: ec = exterior corner, ic = interior corner, he = horizontal edge, ve = vertical edge.

PROBLEM 3.29

> **Determine the temperature at the four nodes shown in the figure. Assume steady conditions and two-dimensional heat conduction. The four faces of the square shape are each at different temperatures as shown.**

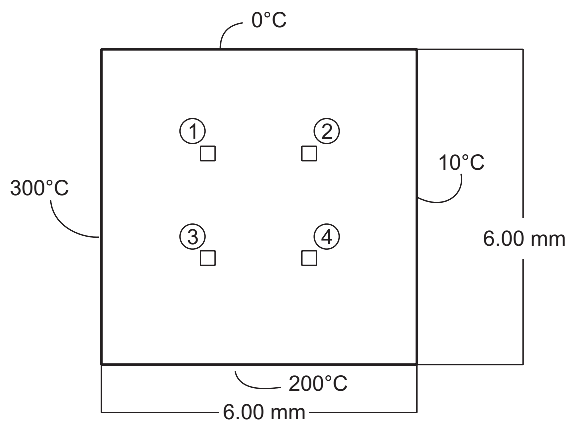
GIVEN

- Square shape with four different face temperatures

FIND

(a) Temperature at four interior nodes

SKETCH



SOLUTION

If the shape is divided into square control volumes then according to Section 3.4.1, the temperature at each node is the average of its four neighbors. The equation for each node is therefore

$$T_1 = \frac{1}{4}(0 + 300 + T_3 + T_2)$$

$$T_2 = \frac{1}{4}(0 + 10 + T_4 + T_1)$$

$$T_3 = \frac{1}{4}(300 + T_1 + T_4 + 200)$$

$$T_4 = \frac{1}{4}(200 + 10 + T_2 + T_3)$$

The equations can be solved by the iterative method. A table showing the calculation for the first 10 iterations is given below. The zero iteration is the initial guess of the temperature at the four nodes.

SOLUTION TO PROBLEM 3.29

iteration	T_1 (°C)	T_2 (°C)	T_3 (°C)	T_4 (°C)
0	150	5	250	100
1	138.75	65.00	187.50	116.25
2	138.13	66.25	188.75	115.63
3	138.75	65.94	188.44	116.25
4	138.59	66.25	188.75	116.09
5	138.75	66.17	188.67	116.25
6	138.71	66.25	188.75	116.21
7	138.75	66.23	188.73	116.25
8	138.74	66.25	188.75	116.24
9	138.75	66.25	188.75	116.25
10	138.75	66.25	188.75	116.25

PROBLEM 3.30

The horizontal cross section of an industrial chimney is shown in the accompanying sketch. Flue gases maintain the interior surface of the chimney at 300°C and the outside is exposed to ambient temperature of 0°C through a heat transfer coefficient of 5 W/(m^2 K). The thermal conductivity of the chimney is *k* = 0.5 W/(mK). For a grid spacing of 0.2 m, determine the temperature distribution in the chimney and the rate of heat loss from the flue gases per unit length of the chimney.

GIVEN

- Chimney with hot flue gases inside, ambient temperature outside

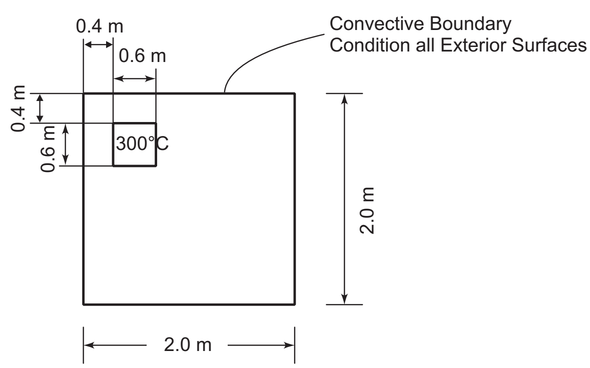
FIND

(a) Temperature distribution in the chimney
(b) Rate of heat loss from the flue gases per unit length

ASSUMPTIONS

- Steady state conditions
- Neglect radiation heat transfer

SKETCH



SOLUTION

Due to a symmetry, only half of the problem geometry needs to be considered. The layout of control volumes and nodes is shown in the figure on the next page. There are a total of 63 control volumes although the temperature at the nodes for 7 of these is specified. So, we need to develop energy balance equations for the remainder.

For shorthand, let's define

$$T \equiv T_{i,j} \quad T_1 \equiv T_{i-1,j} \quad T_r \equiv T_{i+1,j} \quad T_u \equiv T_{i,j+1} \quad T_d \equiv T_{i,j-1}$$

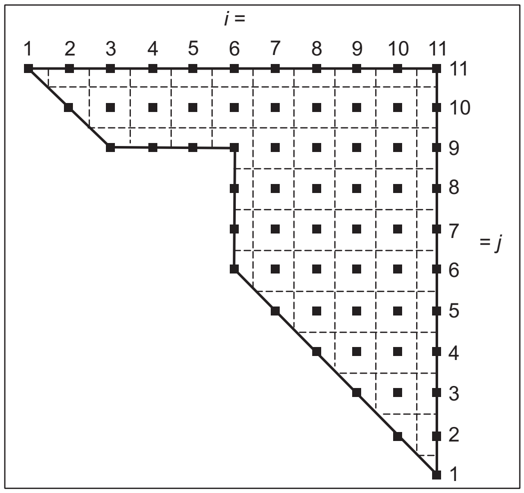
The subscripts in the previous equation stand for left, right, up, and down.

Interior nodes are given by the following indices

i = 3, 4, … 10; j = 10
i = 7, 8, 9, 10; j = 9
i = 7, 8, 9, 10; j = 8
i = 7, 8, 9, 10; j = 7
i = 7, 8, 9, 10; j = 6
i = 8, 9, 10; j = 5
i = 9, 10; j = 4
i = 10; j = 3

and for these control volumes the energy balance equations are

$$T = \frac{1}{4}(T_u + T_d + T_1 + T_r)$$



Control Volume and Node Layout

For the nodes along the top edge (except for the corner nodes) the energy balance gives

$$k\left[\frac{T_1 - T}{\Delta x}\frac{\Delta y}{2} + \frac{T_r - T}{\Delta x}\frac{\Delta y}{2} + \frac{T_d - T}{\Delta y}\Delta x\right] + h\,\Delta x\,(T_\infty - T) = 0$$

or

$$T = \frac{\frac{1}{2}(T_1 + T_r) + T_d + \frac{h\Delta x}{k}T_\infty}{2 + \frac{h\Delta x}{k}} \qquad j = 11 \quad i = 2, 3, \ldots 10$$

For the nodes along the right edge (except for the corner nodes)

$$k\left[\frac{T_1 - T}{\Delta x}\Delta y + \frac{T_u - T}{\Delta y}\frac{\Delta x}{2} + \frac{T_d - T}{\Delta y}\frac{\Delta x}{2}\right] + h\,\Delta y\,(T_\infty - T) = 0$$

or

$$T = \frac{\frac{1}{2}(T_u + T_d) + T_1 + \frac{h\Delta y}{k}T_\infty}{2 + \frac{h\Delta y}{k}} \qquad i = 11 \quad j = 2, 3, \ldots 10$$

Nodes along the diagonal are identified by the following i, j pairs

i =	2	7	8	9	10
j =	10	5	4	3	2

and for these control volumes we have

$$k\left[\frac{T_r - T}{\Delta x}\Delta y + \frac{T_u - T}{\Delta y}\Delta x\right] = 0$$

or

$$T = \frac{1}{2}(T_u + T_r)$$

Nodes along he chimney inner surface are identified by the indices

$j = 9$; $i = 3, 4, 5, 6$ and

$i = 6$; $j = 6, 7, 8$

and for these control volumes

$$T = 300°\text{C}$$

For the corners

$i = 1$; $j = M$

$$k\left[\frac{T_r - T}{\Delta x}\frac{\Delta y}{2}\right] + h\ \frac{\Delta x}{2}\,(T_\infty - T) = 0$$

or

$$T = \frac{T_r + \dfrac{h\Delta x}{k}T_\infty}{1 + \dfrac{h\Delta x}{k}} \qquad j = 11\ i = 1$$

$i = 11$; $j = 11$

$$k\left[\frac{T_1 - T}{\Delta x}\frac{\Delta y}{2} + \frac{T_d - T}{\Delta y}\frac{\Delta x}{2}\right](\Delta x + \Delta y)\,(T_\infty - T) = 0$$

or

$$T = \frac{T_1 + T_d + \dfrac{h}{k}(\Delta x + \Delta y)T_\infty}{2 + \dfrac{h}{k}(\Delta x + \Delta y)} \qquad j = 11\ i = 11$$

$i = N$; $j = 1$

$$k\left[\frac{Tu - T}{\Delta y}\frac{\Delta x}{2}\right] + h\ \frac{\Delta y}{2}\ (T_\infty - T) = 0$$

or

$$T = \frac{T_u + \dfrac{h}{k}\Delta y\,T_\infty}{1 + \dfrac{h}{k}\Delta y} \qquad j = 1\ i = 11$$

This set of difference equations can be solved by iteration. An initial guess of

$$T_{i,j} = \frac{1}{2}\ (0 + 300)°\text{C}$$

for all nodes gives rapid convergence.

The rate of heat loss, q, from the flue gas is equal to the convective loss from the outside surface of the chimney. From symmetry we have

$$q = 2h\left\{\left(T_{1,11} - T_\infty\right)\frac{\Delta x}{2}\left(T_{11,11} - T_\infty\right)\frac{(\Delta x + \Delta y)}{2}\left(T_{1,1} - T_\infty\right)\frac{\Delta y}{2}\right.$$

$$\left. + \sum_{i=2}^{10}\left(T_{i,11} - T_\infty\right)\Delta x + \sum_{j=2}^{10}\left(T_{11,j} - T_\infty\right)\Delta y\right\}$$

Results are given in the following table.

Heat loss from chimney = 923.937002 W/m

Node temperatures, °C

j= \ i=	1	2	3	4	5	6	7	8	9	10	11
11	10.0705	30.2115	48.8663	55.6317	56.3262	51.8848	38.8923	26.6768	17.0475	9.4740	3.0928
10		91.3775	152.5436	169.9305	171.5467	159.9300	116.2884	78.7375	50.1145	27.8258	9.0827
9			300.0000	300.0000	300.0000	300.0000	187.5939	121.8701	76.8473	42.6318	13.9174
8						300.0000	212.2172	144.3014	92.7730	51.9366	16.9928
7						300.0000	216.9733	150.3454	98.0064	55.3487	18.1517
6						300.0000	205.3306	142.1003	93.5585	53.3000	17.5236
5							162.2487	119.1666	80.8271	46.7692	15.4367
4								91.4904	63.8140	37.5129	12.4314
3									45.4255	27.0368	8.9886
2										16.2204	5.4038
1											1.8013

As a check on the above heat loss calculation, we can also calculate the heat loss by determining the heat transferred out of the control volumes at the chimney inner surface. The appropriate equation is

$$q = 2k \left\{ \frac{T_{3,9} - T_{3,10}}{\Delta y} \Delta x + \frac{T_{4,9} - T_{4,10}}{\Delta y} \Delta x + \frac{T_{5,9} - T_{5,10}}{\Delta y} \Delta x + \frac{T_{6,9} - T_{6,10}}{\Delta y} \Delta x \right.$$

$$\left. + \frac{T_{6,9} - T_{7,9}}{\Delta y} \Delta y + \frac{T_{6,8} - T_{7,8}}{\Delta x} \Delta y + \frac{T_{6,7} - T_{7,7}}{\Delta x} \Delta y + \frac{T_{6,6} - T_{7,6}}{\Delta x} \Delta y \right\}$$

The result of this calculation gives

$$q = 923.934 \text{ W/m}$$

which is very close to the value determined via convection at the outer surface.

PROBLEM 3.31

In a long, 30-cm square bar shown in the accompanying sketch, the left face is maintained at 40°C and the top face is maintained at 250°C. The right face is in contact with a fluid at 40°C through a heat transfer coefficient of 60 W/(m^2 K) and the bottom face is in contact with a fluid at 250°C through a heat transfer coefficient of 100 W/(m^2 K). If the thermal conductivity of the bar is 20 W/(mK), calculate the temperature at the 9 nodes shown in the sketch.

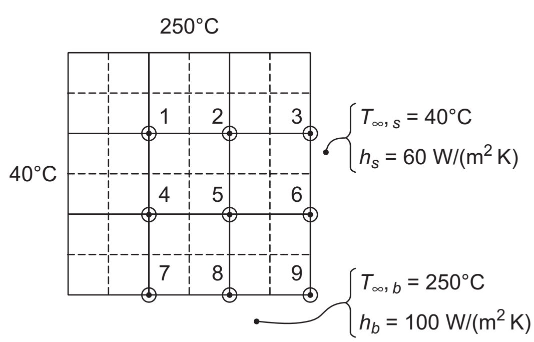
GIVEN

- Square bar with two surfaces at fixed temperature and two surfaces with convective boundary conditions

FIND

(a) Temperature at 9 shown nodes

SKETCH



SOLUTION

Define the following symbols

k	—	thermal conductivity = 20 W/(m K)
$\Delta x = \Delta y$	—	node spacing = 0.1 m
T_T	—	top edge temperature = 250°C
T_L	—	left edge temperature = 40°C
h_s	—	right edge heat transfer coefficient = 60 W/(m^2 K)
$T_{\infty s}$	—	right edge ambient temperature = 40°C
h_b	—	bottom edge heat transfer coefficient = 100 W/(m^2 K)
$T_{\infty b}$	—	bottom edge ambient temperature = 250°C

From Equation (3.23), the temperature at nodes 1, 2, 4, and 5 is just the average of the temperature at the neighbor nodes

$$T_1 = \frac{1}{4} \ (T_L + T_T + T_2 + T_4)$$

$$T_2 = \frac{1}{4} \ (T_T + T_1 + T_3 + T_5)$$

$$T_4 = \frac{1}{4} \ (T_L + T_1 + T_5 + T_7)$$

$$T_5 = \frac{1}{4} \ (T_2 + T_4 + T_6 + T_8)$$

The remaining control volumes have convective boundary conditions and we need to develop individual energy balance equations for each.

For the control volume surrounding node 3

$$k \left\{ \frac{T_r - T_3}{\Delta x} \frac{\Delta x}{2} + \frac{T_2 - T_3}{\Delta x} \Delta x + \frac{T_6 - T_3}{\Delta x} \frac{\Delta x}{2} \right\} + h_s \ (T_{\infty s} - T_3) \ \Delta x = 0$$

which can be solved for T_3 as follows

$$T_3 = \frac{T_T + 2T_2 + T_6 + 2\dfrac{h_s \Delta x}{k} T_{\infty s}}{4 + 2\dfrac{h_s \Delta x}{k}}$$

For the control volume at node 6

$$k \left\{ \frac{T_5 - T_6}{\Delta x} \Delta x + \frac{T_3 - T_6}{\Delta x} \frac{\Delta x}{2} + \frac{T_9 - T_6}{\Delta x} \frac{\Delta x}{2} \right\} + h_s \ (T_{\infty s} - T_c) \ \Delta x = 0$$

or

$$T_6 = \frac{2T_5 + T_3 + T_9 + 2\dfrac{h_s \Delta x}{k} T_{\infty s}}{4 + 2\dfrac{h_s \Delta x}{k}}$$

For the control volume at node 7

$$k \left\{ \frac{T_L - T_7}{\Delta x} \frac{\Delta x}{2} + \frac{T_4 - T_7}{\Delta x} \Delta x + \frac{T_8 - T_7}{\Delta x} \frac{\Delta x}{2} \right\} + h_b \ (T_{\infty b} - T_7) \ \Delta x = 0$$

or

$$T_7 = \frac{T_L + 2T_4 + T_8 + 2\frac{h_b \Delta x}{k} T_{\infty b}}{4 + 2\frac{h_b \Delta x}{k}}$$

For the control volume at node 8

$$k\left\{\frac{T_7 - T_8}{\Delta x}\frac{\Delta x}{2} + \frac{T_5 - T_8}{\Delta x}\Delta x + \frac{T_9 - T_8}{\Delta x}\frac{\Delta x}{2}\right\} + h_b\,(T_{\infty b} - T_8)\,\Delta x = 0$$

or

$$T_8 = \frac{T_7 + 2T_5 + T_9 + 2\frac{h_b \Delta x}{k} T_{\infty b}}{4 + 2\frac{h_b \Delta x}{k}}$$

$$k\left\{\frac{T_6 - T_9}{\Delta x}\frac{\Delta x}{2} + \frac{T_8 - T_9}{\Delta x}\frac{\Delta x}{2}\right\} + h_s(T_{\infty s} - T_9)\frac{\Delta x}{2} + h_b(T_{\infty b} - T_9)\frac{\Delta x}{2} = 0$$

or

$$T_9 = \frac{T_6 + T_8 + \frac{\Delta x}{k}\left(h_s T_{\infty b} + h_b T_{\infty b}\right)}{2 + \frac{\Delta x}{k}\left(h_s + h_b\right)}$$

This set of equations can be solved iteratively. The table below shows the results of the first 25 iterations after which the calculation appears to converge. Values for the 9 nodal temperatures at the zero iteration are the first guess.

				Temperature, °C					
iteration	T1	T2	T3	T4	T5	T6	T7	T8	T9
======	=====	=====	=====	=====	=====	=====	=====	=====	=====
0	40.000	40.000	40.000	40.000	40.000	40.000	40.000	40.000	40.000
1	92.500	105.625	114.185	53.125	59.688	64.687	87.250	99.325	107.504
2	112.188	134.015	131.895	74.781	93.202	97.783	107.778	130.337	130.400
3	124.699	149.949	146.018	91.420	117.372	116.340	120.635	147.156	143.034
4	132.842	161.558	155.099	102.712	131.942	127.395	128.516	157.087	150.529
5	138.568	1678.902	160.695	109.756	140.785	134.086	133.320	163.084	155.061
6	142.165	173.411	164.110	114.067	146.162	138.151	136.244	166.726	157.813
.									
.									
19	147.794	180.423	169.412	120.765	154.500	144.454	140.779	172.371	162.080
20	147.797	180.427	169.415	120.769	154.505	144.458	140.782	172.375	162.083
21	147.799	180.430	169.417	120.772	154.508	144.460	140.784	172.377	162.085
22	147.800	180.431	169.418	120.773	154.510	144.462	140.785	172.378	162.086
23	147.801	180.432	169.419	120.774	154.511	144.462	140.785	172.379	162.086
24	147.802	180.433	169.419	120.775	154.512	144.463	140.786	172.379	162.086
25	147.802	180.433	169.419	120.775	154.513	144.463	140.786	172.380	162.087

PROBLEM 3.32

Repeat Problem 3.31 if the temperature distribution on the top surface of the bar varies sinusoidally from 40°C at the left edge to a maximum of 250°C in the center and back to 40°C at the right edge.

From Problem 3.31: In a long, 30-cm square bar shown in the accompanying sketch, the left face is maintained at 40°C and the top face is maintained at 250°C. The right face is in contact with a fluid at 40°C through a heat transfer coefficient of 60 W/(m^2 K) and the bottom face is in contact with a fluid at 250°C through a heat transfer coefficient of 100 W/(m^2 K). If the thermal conductivity of the bar is 20 W/(mK), calculate the temperature at the 9 nodes shown in the sketch.

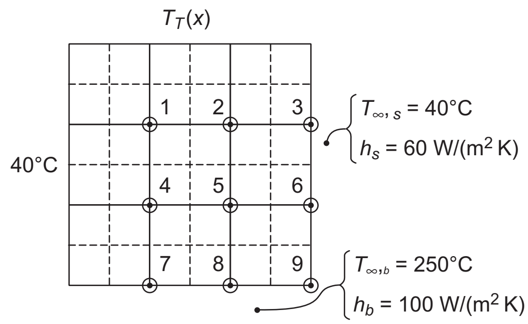
GIVEN

- Square bar with one surface at fixed temperature, one surface with a specified temperature distribution, and two surfaces with convective boundary conditions

FIND

(a) Temperature at 9 shown nodes

SKETCH



SOLUTION

Define the following symbols

k	—	thermal conductivity = 20 W/(mK)
L	—	bar width = 30 cm
$\Delta x = \Delta y$	—	node spacing = 0.1 m
T_L	—	left edge temperature = 40°C
h_s	—	right edge heat transfer coefficient = 60 W/(m^2 K)
$T_{\infty s}$	—	right edge ambient temperature = 40°C
h_b	—	bottom edge heat transfer coefficient = 100 W/(m^2 K)
$T_{\infty b}$	—	bottom edge ambient temperature = 250°C
T_{max}	—	maximum temperature on the top edge = 250°C
T_{min}	—	minimum temperature on the top edge = 40°C

Since the temperature varies sinusoidally across the top surface we have

$$T_T(x) = a + b \sin \frac{x}{L} \pi$$

Since $T(0) = T_{min}$, $T\left(\frac{L}{2}\right) = T_{max}$, $T(L) = T_{min}$ we can solve for a and b giving

$$T_T(x) = T_{min} + (T_{max} - T_{min}) \sin \frac{x}{L} \pi$$

Now, define the temperature at the four nodes on the top edge as

$$T_{T0} \equiv T_T(0) = 40°\text{C} \qquad T_{T1} \equiv T_T\left(\frac{L}{3}\right) = 221.866°\text{C}$$

$$T_{T2} \equiv T_T\left(\frac{2L}{3}\right) = 221.866°\text{C} \qquad T_{T3} \equiv T_T(L) = 40°\text{C}$$

From Equation (3.23), the temperature at nodes 1, 2, 4, and 5 is just the average of the temperature at the neighbor nodes

$$T_1 = \frac{1}{4}\,(T_L + T_{T1} + T_2 + T_4)$$

$$T_2 = \frac{1}{4}\,(T_{T2} + T_1 + T_3 + T_5)$$

$$T_4 = \frac{1}{4}\,(T_L + T_1 + T_5 + T_7)$$

$$T_5 = \frac{1}{4}\,(T_2 + T_4 + T_6 + T_8)$$

The remaining control volumes have convective boundary conditions and we need to develop individual energy balance equations for each.

For the control volume surrounding node 3

$$k\left\{\frac{T_{T3} - T_3}{\Delta x}\frac{\Delta x}{2} + \frac{T_2 - T_3}{\Delta x}\Delta x + \frac{T_6 - T_3}{\Delta x}\frac{\Delta x}{2}\right\} + h_s\,(T_{\infty s} - T_3)\,\Delta x = 0$$

which can be solved for T_3 as follows

$$T_3 = \frac{T_{T3} + 2T_2 + T_6 + 2\dfrac{h_s \Delta x}{k} T_{\infty s}}{4 + 2\dfrac{h_s \Delta x}{k}}$$

For the control volume at node 6

$$k\left\{\frac{T_5 - T_6}{\Delta x}\Delta x + \frac{T_3 - T_6}{\Delta x}\frac{\Delta x}{2} + \frac{T_9 - T_6}{\Delta x}\frac{\Delta x}{2}\right\} + h_s\,(T_{\infty s} - T_6)\,\Delta x = 0$$

or

$$T_6 = \frac{2T_5 + T_3 + T_9 + 2\dfrac{h_s \Delta x}{k} T_{\infty s}}{4 + 2\dfrac{h_s \Delta x}{k}}$$

For the control volume at node 7

$$k\left\{\frac{T_L - T_7}{\Delta x}\frac{\Delta x}{2} + \frac{T_4 - T_7}{\Delta x}\Delta x + \frac{T_8 - T_7}{\Delta x}\frac{\Delta x}{2}\right\} + h_b\,(T_{\infty b} - T_7)\,\Delta x = 0$$

or

$$T_7 = \frac{T_L + 2T_4 + T_8 + 2\dfrac{h_b \Delta x}{k} T_{\infty b}}{4 + 2\dfrac{h_b \Delta x}{k}}$$

For the control volume at node 8

$$k\left\{\frac{T_7 - T_8}{\Delta x}\frac{\Delta x}{2} + \frac{T_5 - T_8}{\Delta x}\Delta x + \frac{T_9 - T_8}{\Delta x}\frac{\Delta x}{2}\right\} + h_b\,(T_{\infty b} - T_8)\,\Delta x = 0$$

or

$$T_8 = \frac{T_7 + 2T_5 + T_9 + 2\dfrac{h_b \Delta x}{k} T_{\infty b}}{4 + 2\dfrac{h_b \Delta x}{k}}$$

Finally, for the control volume at node 9

$$k\left\{\frac{T_6 - T_9}{\Delta x}\frac{\Delta x}{2} + \frac{T_8 - T_9}{\Delta x}\frac{\Delta x}{2}\right\} + h_s(T_{\infty s} - T_9)\frac{\Delta x}{2} + h_b(T_{\infty b} - T_9)\frac{\Delta x}{2} = 0$$

or

$$T_9 = \frac{T_6 + T_8 + \dfrac{\Delta x}{k}\left(h_s T_{\infty s} + h_b T_{\infty b}\right)}{2 + \dfrac{\Delta x}{k}\left(h_s + h_b\right)}$$

This set of equations can be solved iteratively. Here we used a spreadsheet to employ the Gauss-Seidel iteration method. The table below shows the results of the first 25 iterations after which the calculation appears to converge. Values for the 9 nodal temperatures at the zero iteration are the first guess.

	Temperature, °C								
iteration	T1	T2	T3	T4	T5	T6	T7	T8	T9
0	40.000	40.000	40.000	40.000	40.000	40.000	40.000	40.000	40.000
1	85.467	96.833	64.710	51.367	57.050	52.785	86.547	98.129	102.826
2	102.516	111.536	73.882	71.528	83.494	79.934	106.237	125.211	122.195
3	111.232	122.619	84.603	85.241	103.251	95.065	117.139	139.167	132.583
4	117.431	131.788	91.878	94.455	115.119	104.065	123.616	147.287	138.697
5	122.027	137.723	96.415	100.190	122.316	109.510	127.534	152.173	142.387
6	124.945	141.386	99.192	103.699	126.692	112.818	129.914	155.137	144.627
.									
.									
.									
.									
.									
19	129.523	147.090	103.504	109.148	133.476	117.946	133.604	159.730	148.099
20	129.526	147.093	103.507	109.152	133.480	117.949	133.607	159.733	148.101
21	129.528	147.095	103.509	109.154	133.483	117.951	133.608	159.735	148.102
22	129.529	147.097	103.510	109.155	133.484	117.952	133.609	159.736	148.103
23	129.529	147.097	103.510	109.156	133.485	117.953	133.609	159.737	148.103
24	129.530	147.098	103.511	109.156	133.486	117.953	133.610	159.737	148.104
25	129.530	147.098	103.511	109.156	133.486	117.954	133.610	159.737	148.104

COMMENTS

Comparing the results with those from Problem 3.31, we see that the bottom row of temperatures, nodes 7, 8, 9 show that the effect of lower temperatures near the top corners has propagated down through the bar.

PROBLEM 3.33

A 1-cm-thick, 1-m-square steel plate is exposed to sunlight and absorbs a solar flux of 800 W/m^2. The bottom of the plate is insulated, the edges are maintained at 20°C by water-cooled clamps, and the exposed face is cooled by a convection coefficient of 10 W/(m^2 K) to an ambient temperature of 10°C. The plate is polished to minimize reradiation. Determine the temperature distribution in the plate using a node spacing of 20 cm. The thermal conductivity of the steel is 40 W/(m K).

GIVEN

- Square plate with water-cooled edges exposed to solar flux

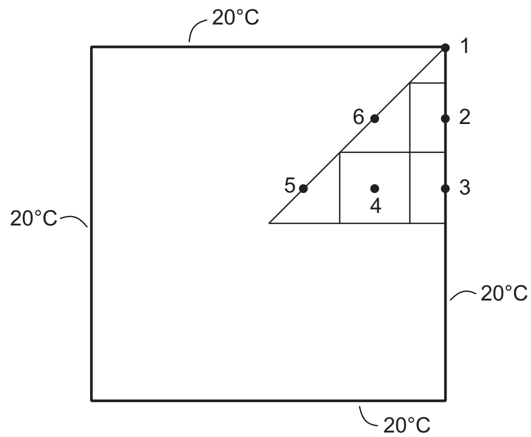
FIND

(a) Temperature distribution in the plate

ASSUMPTIONS

(a) Neglect temperature gradients through the plate thickness

SKETCH



SOLUTION

Because of problem symmetry, we need only consider the 6 nodes in 1/8 th of the square plate as shown in the above sketch. The boundary condition gives us the temperature of nodes 1, 2, and 3 so we only need to perform a heat balance on nodes 4, 5, and 6. Define the following symbols

k	= plate thermal conductivity = 40 W/(m K)
h	= convection coefficient = 10 W/(m^2 K)
T_∞	= ambient temperature = 10°C
Δx	= node spacing = 20 cm = 0.2 m
t	= plate thickness = 1 cm = 0.01 m
q''	= absorbed solar flux = 800 W/m^2
T_{edge}	= specified edge temperature = 20°C

Node 4 transfers heat by conduction with nodes 3, 5, and 6, by convection to ambient, and absorbs the specified solar flux. The energy balance on node 4 is therefore

$$k\,\frac{T_3-T_4}{\Delta x}\,t\,\Delta x + k\,\frac{T_6-T_4}{\Delta x}\,t\,\Delta x + k\,\frac{T_5-T_4}{\Delta x}\,t\,\Delta x + h\,\Delta x^2\,(T_\infty - T_4) + q''\,\Delta x^2 = 0$$

Solving for T_4

$$T_4 = \frac{kt\left(T_3+T_5+T_6\right)+h\Delta x^2 T_\infty + q''\Delta x^2}{3kt+h\Delta x^2}$$

Node 5 transfers heat by conduction with node 4, by convection to ambient, and absorbs the specified flux. The energy balance on node 5 is therefore

$$k\,\frac{T_4-T_5}{\Delta x}\,t\,\Delta x + h\,\frac{\Delta x^2}{2}\,(T_\infty - T_5) + q''\,\frac{\Delta x^2}{2} = 0$$

Solving for T_5

$$T_5 = \frac{2kt\,T_4 + h\Delta x^2 T_\infty + q''\,\Delta x^2}{2kt + h\Delta x^2}$$

Node 6 transfers heat by conduction with nodes 4 and 2, by convection to ambient, and absorbs the specified flux. The energy balance on node 6 is therefore

$$k\,\frac{T_4 - T_6}{\Delta x}\,t\,\Delta x + k\,\frac{T_2 - T_6}{\Delta x}\,t\,\Delta x + h\,\frac{\Delta x^2}{2}\,(T_\infty - T_6) + q''\,\frac{\Delta x^2}{2} = 0$$

Solving for T_6

$$T_6 = \frac{2kt(T_2 + T_4) + h\Delta x^2 T_\infty + q''\,\Delta x^2}{4kt + h\Delta x^2}$$

This set of equations can be solved by iteration. The table below shows the results of Gauss-Seidel iteration. Iteration 0 is the first guess for the temperature at nodes 4, 5, and 6.

	Temperature °C					
iteration	T1	T2	T3	T4	T5	T6
0	20	20	20	50	50	50
1	20.000	20.000	20.000	52.500	65.000	47.000
2	20.000	20.000	20.000	55.500	67.000	48.200
3	20.000	20.000	20.000	56.300	67.533	48.520
4	20.000	20.000	20.000	56.513	67.676	48.605
5	20.000	20.000	20.000	56.570	67.713	48.628
6	20.000	20.000	20.000	56.585	67.724	48.634
7	20.000	20.000	20.000	56.589	67.726	48.636
8	20.000	20.000	20.000	56.591	67.727	48.636
9	20.000	20.000	20.000	56.591	67.727	48.636
10	20.000	20.000	20.000	56.591	67.727	48.636

The solution converges after about 8 iterations giving a peak temperature of 67.727°C at node 5.

PROBLEM 3.34

The plate in Problem 3.33 gradually oxidizes over time so that the surface emissivity increases to 0.5. Calculate the resulting temperature in the plate including radiation heat transfer to the surroundings at the same temperature as the ambient temperature.

From Problem 3.33: A 1-cm-thick, 1-m-square steel plate is exposed to sunlight and absorbs a solar flux of 800 W/m^2. The bottom of the plate is insulated, the edges are maintained at 20°C by water-cooled clamps, and the exposed face is cooled by a convection coefficient of 10 W/(m^2 K) to an ambient temperature of 10°C. The plate is polished to minimize reradiation. Determine the temperature distribution in the plate using a node spacing of 20 cm. the thermal conductivity of the steel is 40 W/(m K).

GIVEN

- Plate in Problem 3.33 oxidizes

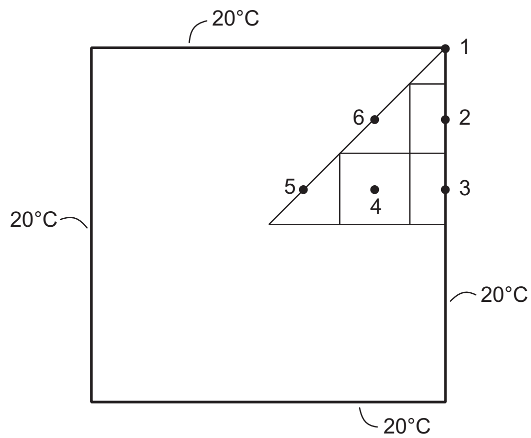
FIND

(a) New temperature distribution considering radiation heat transfer

ASSUMPTIONS

(a) Neglect temperature gradients through the plate thickness

SKETCH



SOLUTION

Addition of radiative heat transfer from the plate can be most easily handled by computing the radiative heat transfer coefficient for each node (using the temperature for the node calculated from the previous iteration) and by then adding this radiative heat transfer coefficient to the convective heat transfer coefficient for the present iteration. The radiative heat transfer coefficient for node *i* is

$$h_{ri} = \varepsilon \sigma (T_i^2 + T_\infty^2)(T_i + T_\infty)$$

The following table gives the results for the Gauss-Seidel iteration. (Recall that temperature must be expressed in Kelvins.)

Iteration	Temperature (K)			Temperature (K)/radiative heat transfer coefficient (W/(m^2 K))		
	T1	T2	T3	T4/hr4	T5/hr5	T6/hr6
0	323	323	323	323	323	323
				3.168	3.168	3.168
1	293.300	293.000	293.000	322.381	330.865	316.622
				3.158	3.299	3.066
2	293.000	293.000	293.000	322.735	330.890	316.820
				3.164	3.299	3.069
3	293.000	293.000	293.000	322.781	330.917	316.836
				3.165	3.300	3.069
4	293.000	293.000	293.000	322.790	330.922	316.839
				3.165	3.300	3.069
5	293.000	293.000	293.000	322.792	330.923	316.840
				3.165	3.300	3.069
6	293.000	293.000	293.000	322.792	330.923	316.840
Node temperature in degrees C						
	20.000	20.000	20.000	49.792	57.923	43.840

The peak temperature has been reduced by about 9.8 K due to radiation.

PROBLEM 3.35

Determine (a) the temperature at the 16 equally spaced points shown in the accompanying sketch to an accuracy of three significant figures and (b) the rate of heat flow per meter thickness. Assume two-dimensional heat flow and k = 1 W/(mK).

GIVEN

- A two-dimensional object with specified surface temperatures

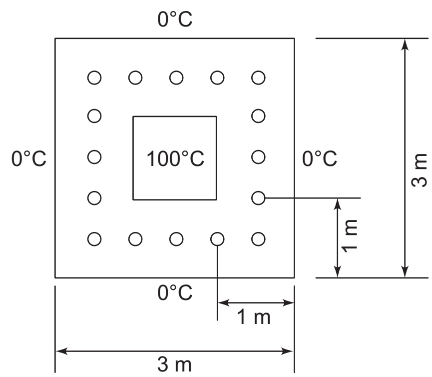
FIND

(a) The temperature at the 16 specified locations
(b) The heat flow per meter thickness

ASSUMPTIONS

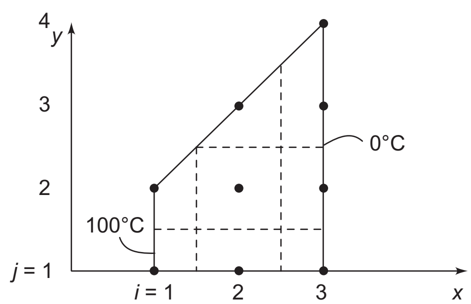
- Steady state

SKETCH



SOLUTION

Because of symmetry, it is only necessary to consider 1/8 th of the figure as shown below



(a) Temperature distribution

There are three nodes remaining for which we must determine the temperature. For these nodes, we need energy balance equations for the control volumes. The control volumes are shown as dashed lines surrounding each node.

For the node at $i = 2, j = 1$

The x axis is a line of symmetry so no heat flows into the control volume across it

$$\frac{\Delta y}{2}(T_{1,1} - T_{2,1}) + \Delta x\,(T_{2,2} - T_{2,1}) + \frac{\Delta y}{2}(T_{3,1} - T_{2,1}) = 0$$

Since we have chosen $\Delta x = \Delta y$, this equation simplifies to

$$T_{2,1} = \frac{1}{4}(T_{1,1} + 2T_{2,2} + T_{3,1})$$

For the node at $i = 2, j = 2$, we use Equation (3.23)

$$T_{2,2} = \frac{1}{4}(T_{1,2} + T_{3,2} + T_{2,1} + T_{2,3})$$

And for the node at $i = 2, j = 3$, we have for energy balance

$$\Delta x\,(T_{2,2} - T_{2,3}) + \Delta y\,(T_{3,3} - T_{2,3}) = 0$$

or

$$T_{2,3} = \frac{1}{2}(T_{2,2} + T_{3,3})$$

Substituting the known boundary temperatures, these equations simplify to

$$4T_{2,1} = 2T_{2,2} + 100 \quad (1)$$

$$4T_{2,2} = T_{2,1} + T_{2,3} + 100 \quad (2)$$

$$2T_{2,3} = T_{2,2} \quad (3)$$

These three equations can be solved by elimination. Substitute Equations (1) and (3) into Equation (2) to give

$$T_{2,2} = 41.666°\text{C}$$

Substitute this result into Equation (3) to get

$$T_{2,3} = 20.833°\text{C}$$

and then from Equation (1) we find

$$T_{2,1} = 45.833°\text{C}$$

(b) Heat flow

The total heat flow for the object can be calculated from

$$Q = 8\left\{k\frac{T_{1,1} - T_{2,1}}{\Delta x}\frac{\Delta y}{2} + k\frac{T_{1,2} - T_{2,2}}{\Delta x}\Delta y\right\}$$

which simplifies to

$$Q = 8k\left\{\frac{1}{2}(T_{1,1} - T_{2,1}) + T_{1,2} - T_{2,2}\right\}$$

$$= 8\ (1\,\text{W/(mK)})\left\{\frac{1}{2}(100 - 45.833) + 100 - 41.666\right\}(\text{K}) = 683.2\ \text{W/m}$$

PROBLEM 3.36

A long steel beam with rectangular cross section of 40 cm by 60 cm is mounted on an insulating wall as shown in the sketch below. the rod is heated by radiant heaters that maintain the top and bottom surfaces at 300°C. A stream of air at 130°C cools the exposed face through a heat transfer coefficient of 20 W/(m^{2}K). Using a node spacing of 1 cm, determine the temperature distribution in the rod and the rate of heat input to the rod. The thermal conductivity of the steel is 40 W/(m K).

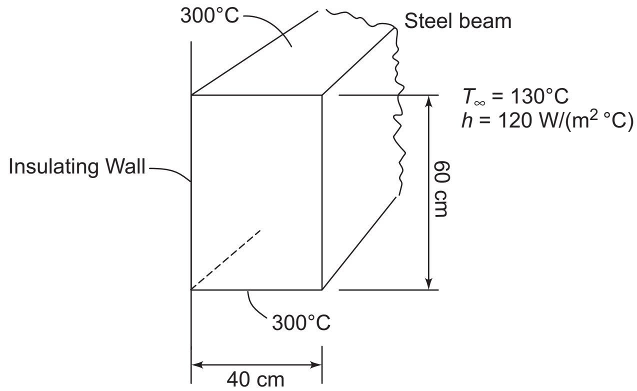
GIVEN

- Rectangular steel beam mounted on an insulating surface, heated top and bottom, with exposed face cooled by an air flow

FIND

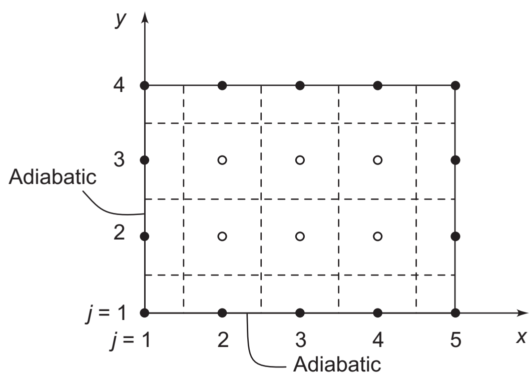
(a) Temperature distribution in the rod
(b) Rate of heat input to the rod

SKETCH



SOLUTION

Since the rod is long, we can consider a two-dimensional solution. By symmetry, the rod can be divided along its horizontal midplane. The sketch below shows the resulting geometry along with the node and control volume locations.



Define the top surface temperature as T_{top} = 300°C, and the ambient temperature as T_∞ = 130°C.

We now need to determine an energy balance for each control volume.

Along the top edge we have

$$T_{i,4} = T_{top} \qquad i = 1, 2, 3, 4, 5$$

For the central nodes we have from Equation (3.23)

$$4\,T_{i,j} = T_{i-1,j} + T_{i+1,j} + T_{i,j+1} + T_{i,j-1} \qquad i = 2, 3, 4 \quad j = 2, 3$$

Along the left edge, an energy balance on the two nodes at j = 2 and 3 gives

$$k\left\{\frac{T_{2,j} - T_{1,j}}{\Delta x}\Delta x + \frac{T_{1,j+1} - T_{1,j}}{\Delta x}\frac{\Delta x}{2} + \frac{T_{1,j-1} - T_{1,j}}{\Delta x}\frac{\Delta x}{2}\right\} = 0 \qquad j = 2, 3$$

or

$$4\,T_{i,j} = 2\,T_{2,j} + T_{1,j+1} + T_{1,j-1} \qquad j = 2, 3$$

Along the bottom edge, an energy balance on the three nodes at i = 2, 3, and 4 gives

$$k\left\{\frac{T_{i,2} - T_{i,1}}{\Delta x}\Delta x + \frac{T_{i-1,1} - T_{i,1}}{\Delta x}\frac{\Delta x}{2} + \frac{T_{i+1,1} - T_{i,1}}{\Delta x}\frac{\Delta x}{2}\right\} = 0 \qquad i = 2, 3, 4$$

or

$$4\,T_{i,1} = 2\,T_{i,2} + T_{i-1,1} + T_{i+1,1} \qquad i = 2, 3, 4$$

For the node at $i = 1$, $j = 1$, an energy balance gives

$$k\left\{\frac{T_{1,2}-T_{1,1}}{\Delta x}\frac{\Delta x}{2}+\frac{T_{2,1}-T_{1,1}}{\Delta x}\frac{\Delta x}{2}\right\} = 0$$

or

$$2\,T_{1,1} = T_{1,2} + T_{2,1}$$

For the node at $i = 5$, $j = 1$, an energy balance gives

$$k\left\{\frac{T_{4,1}-T_{5,1}}{\Delta x}\frac{\Delta x}{2}+\frac{T_{5,2}-T_{5,1}}{\Delta x}\frac{\Delta x}{2}\right\} + h\,\frac{\Delta x}{2}\,(T_\infty - T_{5,1}) = 0$$

or

$$\left(2+\frac{h\Delta x}{k}\right) T_{5,1} = (T_{4,1} + T_{5,2}) + \frac{h\Delta x}{k}\,T_\infty$$

Finally, along the right edge, an energy balance on the two nodes at $j = 2$ and 3 gives

$$k\left\{\frac{T_{4,j}-T_{5,j}}{\Delta x}\Delta x+\frac{T_{5,j-1}-T_{5,j}}{\Delta x}\frac{\Delta x}{2}+\frac{T_{5,j+1}-T_{5,j}}{\Delta x}\frac{\Delta x}{2}\right\} + h\,\Delta x\,(T_\infty - T_{5,j}) = 0 \qquad j = 2, 3$$

or

$$\left(4+\frac{2h\Delta x}{k}\right) T_{5,j} = 2\,T_{4,j} + T_{5,j-1} + T_{5,j+1} + \frac{2h\Delta x}{k}\,T_\infty \qquad j = 2, 3$$

This set of difference equations can be written as a matrix equation as follows

$$AT = C$$

where the coefficient matrix A is given by

–2	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	–4	1	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	–4	1	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	–4	1	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	–2.05	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	–4	2	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	–4	1	0	0	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	–4	1	0	0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	–4	1	0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	2	–4.1	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	–4	2	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0	1	–4	1	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0	0	1	–4	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0	0	0	–1	4	1	0	0	0	1	0
0	0	0	0	0	0	0	0	0	1	0	0	0	2	–4.1	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–1

and the temperature vector T and constant vector C are given by

$$T = \begin{array}{l} T(1,1) \\ T(2,1) \\ T(3,1) \\ T(4,1) \\ T(5,1) \\ T(1,2) \\ T(2,2) \\ T(3,2) \\ T(4,2) \\ T(5,2) \\ T(1,3) \\ T(2,3) \\ T(3,3) \\ T(4,3) \\ T(5,3) \\ T(1,4) \\ T(2,4) \\ T(3,4) \\ T(4,4) \\ T(5,4) \end{array} \qquad C = \begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \\ -6.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ -13 \\ 0 \\ 0 \\ 0 \\ 0 \\ -13 \\ -300 \\ -300 \\ -300 \\ -300 \\ -300 \end{array}$$

Inverting the matrix A with a spreadsheet program and multiplying the constant vector C by the inverted matrix, we get the vector of nodal temperatures

$$T = \begin{array}{l} 295.2854 \\ 294.667 \\ 292.6705 \\ 288.8777 \\ 282.6697 \\ 295.9039 \\ 295.356 \\ 293.5687 \\ 290.0853 \\ 284.0951 \\ 297.6181 \\ 297.2843 \\ 296.1632 \\ 293.7996 \\ 288.9498 \\ 300 \\ 300 \\ 300 \\ 300 \\ 300 \end{array} \quad ^\circ\text{C}$$

To determine the heat flow to the rod, consider the surface of the exposed control volumes. The rate of convective heat transfer from these surfaces must equal the rate of heat input to the rod. Remembering to double the value of account for the symmetry, we have

$$q_{\text{input}} = 2h\Delta x \left(\frac{1}{2}\left(T_{5,4} - T_\infty\right) + \frac{1}{2}\left(T_{5,1} - T_\infty\right) + \left(T_{5,2} - T_\infty\right) + \left(T_{5,3} - T_\infty\right) \right)$$

Inserting the nodal temperatures from the solution vector T given above, we find

$$q_{\text{input}} = 1897 \text{ watts}$$

PROBLEM 3.37

Consider a band-saw blade being used to cut steel bar stock. The blade thickness is 2 mm, its height is 20 mm, and it has penetrated the steel workpiece to a depth of 5 mm (see the accompanying sketch). Exposed surfaces of the blade are cooled by an ambient temperature of 20°C through a convection coefficient of 40 W/(m^{2}K). Thermal conductivity of the blade steel is 30 W/(mK). Energy dissipated by the cutting process supplies a heat flux of 10^4 W/m^2 to the surfaces of the blade that are in contact with the workpiece. Assuming two-dimensional, steady conduction, determine the maximum and minimum temperature in the blade cross section. Use a node spacing of 0.5 mm horizontally and 2 mm vertically.

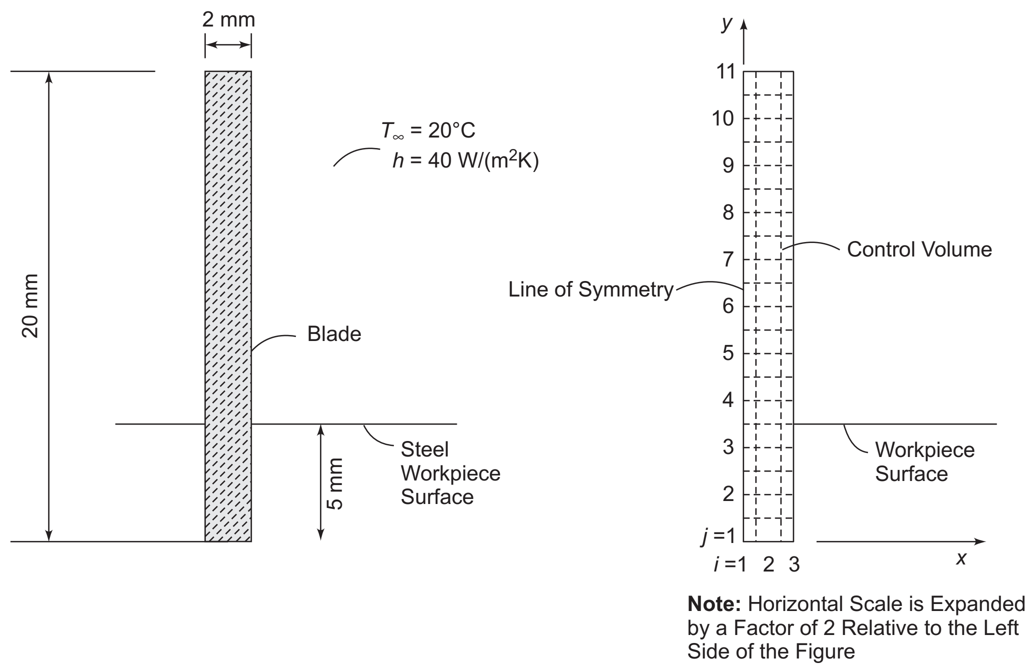
GIVEN

- Band saw blade cutting steel bar stock

FIND

(a) Maximum and minimum temperatures in the blade cross section

SKETCH



SOLUTION

Because of symmetry, we only need to consider half of the geometry as shown in the right side of the sketch. With a node spacing of Δx = 0.5 mm and Δy = 2.0 mm, we have for the number of horizontal and vertical nodes

$$M = \frac{\frac{t}{2}}{\Delta x} + 1 = 3$$

$$N = \frac{H}{\Delta y} + 1 = 11$$

We have 33 control volumes and need to develop an energy balance equation for each. For all the interior nodes

$$\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\Delta y + \frac{T_{i-1,j} - T_{i,j}}{\Delta x}\Delta y + \frac{T_{i,j+1} - T_{i,j}}{\Delta y}\Delta x + \frac{T_{i,j-1} - T_{i,j}}{\Delta x}\Delta x = 0$$

or

$$T_{i,j} = \frac{\frac{\Delta y}{\Delta x}\left(T_{i+1,j} - T_{i-1,j}\right) + \frac{\Delta x}{\Delta y}\left(T_{i,j+1} - T_{i,j-1}\right)}{2\left(\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y}\right)} \qquad i = 2 \quad j = 2, 3 \ldots N-1$$

For all nodes (except the corner nodes) on the left edge

$$\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\,\Delta y + \frac{T_{i,j+1} - T_{i,j}}{\Delta x}\frac{\Delta x}{2} + \frac{T_{i,j-1} - T_{i,j}}{\Delta x}\frac{\Delta x}{2} = 0$$

or

$$T_{i,j} = \frac{\frac{\Delta y}{\Delta x}T_{i+1,j} + \frac{\Delta x}{2\Delta y}\left(T_{i,j+1} - T_{i,j-1}\right)}{\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y}} \qquad i = 1 \qquad j = 2, 3 \ldots \text{N} - 1$$

For $i = 1, j = 1$

$$k\left(\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\frac{\Delta x}{2} + \frac{T_{i,j+1} - T_{i,j}}{\Delta y}\frac{\Delta x}{2}\right) + q''\frac{\Delta x}{2} = 0$$

or

$$T_{i,j} = \frac{\frac{\Delta y}{\Delta x}T_{i+1,j} + \frac{\Delta x}{\Delta y}T_{i,j+1} + q''\frac{\Delta x}{k}}{\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y}} \qquad i = 1 \qquad j = 1$$

For $i = 1, j = N$

$$k\left(\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\frac{\Delta y}{2} + \frac{T_{i,j-1} - T_{i,j}}{\Delta y}\frac{\Delta x}{2}\right) + h\,\frac{\Delta x}{2}\,(T_\infty - T_{i,j}) = 0$$

or

$$T_{i,j} = \frac{\frac{\Delta y}{\Delta x}T_{i+1,j} + \frac{\Delta x}{\Delta y}T_{i,j-1} + h\frac{\Delta x}{k}T_\infty}{\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y} + \frac{h\Delta x}{k}} \qquad i = 1 \qquad j = N$$

For $i = 2, j = N$

$$k\left(\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\frac{\Delta y}{2} + \frac{T_{i-1,j} - T_{i,j}}{\Delta x}\frac{\Delta y}{2} + \frac{T_{i,j-1} - T_{i,j}}{\Delta y}\Delta x\right) + h\,\Delta x\,(T_\infty - T_{i,j}) = 0$$

or

$$T_{i,j} = \frac{\frac{\Delta y}{2\Delta x}\left(T_{i+1,j} + T_{i-1,j}\right) + \frac{\Delta x}{\Delta y}T_{i,j-1} + \frac{h\Delta x}{k}T_\infty}{\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y} + \frac{h\Delta x}{k}} \qquad i = 2 \qquad j = N$$

For $i = 2, j = 1$

$$k\left(\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\frac{\Delta y}{2} + \frac{T_{i-1,j} - T_{i,j}}{\Delta x}\frac{\Delta y}{2} + \frac{T_{i,j+1} - T_{i,j}}{\Delta y}\Delta x\right) + q''\,\Delta x = 0$$

or

$$T_{i,j} = \frac{\dfrac{\Delta y}{2\Delta x}\left(T_{i+1,j} + T_{i-1,j}\right) + \dfrac{\Delta x}{\Delta y} T_{i,j+1} + q'' \dfrac{\Delta x}{k}}{\dfrac{\Delta y}{\Delta x} + \dfrac{\Delta x}{\Delta y}} \qquad i = 2 \qquad j = 1$$

For $i = M$, $j = N$

$$k\left(\frac{T_{i-1,j} - T_{i,j}}{\Delta x}\frac{\Delta y}{2} + \frac{T_{i,j-1} - T_{i,j}}{\Delta y}\frac{\Delta x}{2}\right) + \frac{h}{2}(\Delta x + \Delta y)\,(T_\infty - T_{i,j}) = 0$$

or

$$T_{i,j} = \frac{\dfrac{\Delta y}{\Delta x} T_{i-1,j} + \dfrac{\Delta x}{\Delta y} T_{i,j-1} + \dfrac{h}{k}(\Delta x + \Delta y)\,T_\infty}{\dfrac{\Delta y}{\Delta x} + \dfrac{\Delta x}{\Delta y} + \dfrac{h}{k}(\Delta x + \Delta y)} \qquad i = M \qquad j = N$$

For $i = M$, $j = 1$

$$k\left(\frac{T_{i-1,j} - T_{i,j}}{\Delta x}\frac{\Delta y}{2} + \frac{T_{i,j+1} - T_{i,j}}{\Delta y}\frac{\Delta x}{2}\right) + q''\ \frac{1}{2}(\Delta x + \Delta y) = 0$$

or

$$T_{i,j} = \frac{\dfrac{\Delta y}{\Delta x} T_{i-1,j} + \dfrac{\Delta x}{\Delta y} T_{i,j+1} + \dfrac{q''}{k}(\Delta x + \Delta y)}{\dfrac{\Delta y}{\Delta x} + \dfrac{\Delta x}{\Delta y}} \qquad i = M \qquad j = 1$$

For $i = M$, $j = 2, 3$

$$k\left(\frac{T_{i-1,j} - T_{i,j}}{\Delta x}\Delta y + \frac{T_{i,j+1} - T_{i,j}}{\Delta y}\frac{\Delta x}{2} + \frac{T_{i,j-1} - T_{i,j}}{\Delta y}\frac{\Delta x}{2}\right) + q''\ \Delta y = 0$$

or

$$T_{i,j} = \frac{\dfrac{\Delta y}{\Delta x} T_{i-1,j} + \dfrac{\Delta x}{2\Delta y}\left(T_{i,j+1} + T_{i,j-1}\right) + \dfrac{q''}{k}\Delta y}{\dfrac{\Delta y}{\Delta x} + \dfrac{\Delta x}{\Delta y}} \qquad i = M \quad j = 2, 3$$

Finally, for $i = M$, $j = 4, 5, \ldots N - 1$

$$k\left(\frac{T_{i-1,j} - T_{i,j}}{\Delta x}\Delta y + \frac{T_{i,j+1} - T_{i,j}}{\Delta y}\frac{\Delta x}{2} + \frac{T_{i,j-1} - T_{i,j}}{\Delta y}\frac{\Delta x}{2}\right) + h\,\Delta y\,(T_\infty - T_{i,j}) = 0$$

or

$$T_{i,j} = \frac{\dfrac{\Delta y}{\Delta x} T_{i-1,j} + \dfrac{\Delta x}{2\Delta y}\left(T_{i,j+1} + T_{i,j-1}\right) + \dfrac{h}{k}\Delta y\, T_\infty}{\dfrac{\Delta y}{\Delta x} + \dfrac{\Delta x}{\Delta y} + \dfrac{h}{k}\Delta y} \qquad i = M \quad J = 4, 5 \ldots N - 1$$

This set of equations can be solved iteratively as described in Section 3.4. An initial guess for the temperature distribution is inserted into the right hand side of all the above equations to produce an improved value for $T[i, j]$ for all i and j. These improved values are inserted into the right hand side of the same equations for the next update on $T[i, j]$ and so on. We carried out the iteration until the temperature at $i = 2, j = 1$ changed by less than 10^{-6} °C. The results indicate a maximum temperature of 130.401°C at $i = 3, j = 1$, and a minimum temperature of 108.693°C at $i = 3, j = M$.

PROBLEM 3.38

How would the results of Problem 3.15 be modified if the problem were not axisymmetric?

From Problem 3.15: Determine the difference equations applicable to the centerline and at the surface of the axisymmetric cylindrical geometry with volumetric heat generation and convective boundary condition. Assume steady state conditions.

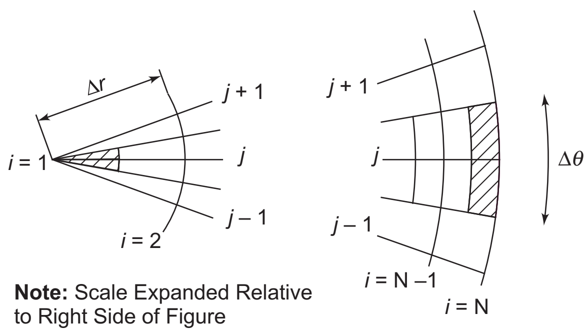
GIVEN

- Non-axisymmetric, steady, cylindrical geometry with heat generation and convective boundary condition.

FIND

(a) Difference equations for the centerline and surface control volumes

SKETCH



SOLUTION

For the control volume at the centerline we have

$$\text{Volume} = \pi\left(\frac{\Delta r}{2}\right)^2\frac{\Delta\theta}{2\pi} = \frac{\Delta r^2\Delta\theta}{8}$$

$$\text{Surface area} = 2\pi\,\frac{\Delta r}{2}\,\frac{\Delta\theta}{2\pi} = \frac{\Delta r\Delta\theta}{2}$$

and the energy balance gives

$$k\,\frac{T_{2,j} - T_{i=1}}{\Delta r}\,\frac{\Delta r\Delta\theta}{2} + \dot{q}_G\,\frac{\Delta r^2\Delta\theta}{8} = 0$$

For the control volume at the surface, we have for the volume per unit length

$$\text{Volume} = \pi\Delta r\left(R_o - \frac{\Delta r}{4}\right)\frac{\Delta\theta}{2\pi} = \Delta r\left(R_o - \frac{\Delta r}{4}\right)\frac{\Delta\theta}{2}$$

and for the surface area (per unit length) of the circumferential face inside R_o

$$\text{Inner circumferential surface area} = 2\pi\left(R_o - \frac{\Delta r}{2}\right)\frac{\Delta\theta}{2\pi} = \left(R_o - \frac{\Delta r}{2}\right)\Delta\theta$$

For the surface area of the circumferential face at R_o we have

$$\text{Outer circumferential surface area} = 2\pi R_o \frac{\Delta\theta}{2\pi} = R_o\,\Delta\theta$$

The surface area per unit length of the radial faces of the control volume are

$$\text{radial surface area} = \frac{\Delta r}{2}$$

The energy balance is

$$k\,\frac{T_{N-1,j}-T_{N,j}}{\Delta r}\left(R_o-\frac{\Delta r}{2}\right)\Delta\theta + k\,\frac{T_{N,j-1}-T_{N,j}}{R_o\Delta\theta}\,\frac{\Delta r}{2} + k\,\frac{T_{N,j+1}-T_{N,j}}{R_o\Delta\theta}\,\frac{\Delta r}{2} + h\,R_o\,\Delta\theta\,(T_\infty - T_{N,j})$$
$$+\ \dot{q}_G\,\Delta r\left(R_o-\frac{\Delta r}{4}\right)\frac{\Delta\theta}{2} = 0$$

The solution of the above set of equations would be carried out in parallel with the method explained in Section 3.4.3.1, for two-dimensional steady problems. The difference equation for the interior nodes given by Equation (3.30) (steady state terms only) would be added to the above difference equations and an iterative solution procedure would be employed to find the solution.

PROBLEM 3.39

> **For the geometry shown in the sketch below, determine the layout of nodes and control volumes. Provide a scale drawing showing the problem geometry overlaid with the nodes and control volumes. Explain how to derive the energy balance equation for all the boundary control volumes.**

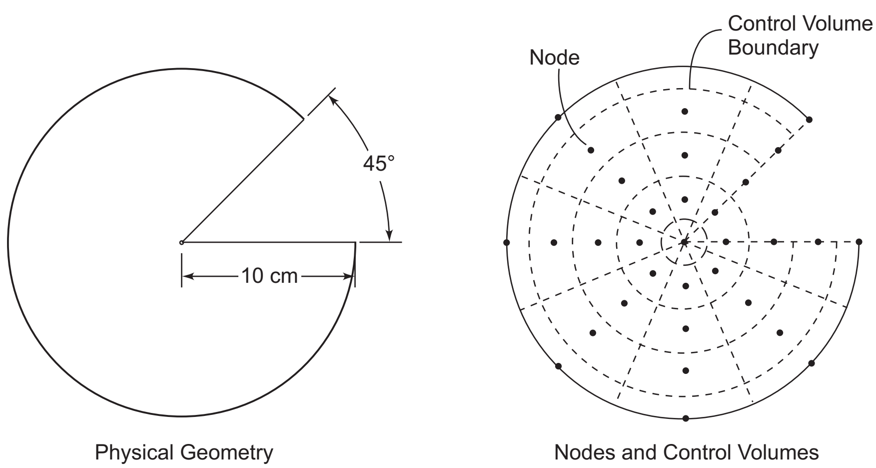
GIVEN

- Cylindrical geometry shown in the figure.

FIND

(a) A reasonable layout of nodes and control volumes

SKETCH



SOLUTION

We do not know what the boundary conditions are so we cannot make any judgment about symmetry. Therefore, we must assume that the problem is axisymmetric. Since R_o = 10 cm, let's use Δr = 2.5 cm giving 5 radial nodes. To accommodate the 45° cutout, let's use a circumferential node spacing, $\Delta\theta$ = 45°. The right side of the sketch shows the resulting layout of nodes and control volumes. Energy balance equations for the control volumes at the circumferential boundary would be derived as described in Section 3.5. For those control volumes, we have conduction from three surrounding control volumes and either convection, specified flux, or a specified temperature at the fourth surface, depending on the boundary condition. For the control volumes along the two radial boundaries, we have conduction from two surrounding control volumes. Treatment of the third surface would depend on the boundary condition.

PROBLEM 3.40

> **Hot flue gases from a combustion furnace flow though a chimney, which is 7 m tall and has a hollow cylindrical cross section with inner diameter d_i = 30 cm and outer diameter d_o = 50 cm. The flue gases flow with an average temperature of T_g = 300°C and convective heat transfer coefficient of h_g = 75 W/(m^2 K). The chimney is made of concrete, which has a thermal conductivity of k = 1.4 W/(m K). It is exposed to outside air that has an average temperature of T_a = 25°C and convective heat transfer coefficient of 15 W/(m^2 K). For steady-state conditions, determine the inner and outer wall temperatures, plot the temperature distribution along the thickness of the chimney wall, and determine the rate of heat loss to outside air from the chimney. Solve the problem by numerical analysis using a nodal network with Δr = 2 cm and $\Delta\theta$ = 10°.**

GIVEN

- Hot flue gas flow in a hollow cylindrical 7 m tall chimney with inner and outer diameters of d_i = 0.3 m and d_o = 0.5 m, and thermal conductivity k = 1.4 W/(m K).
- Average hot gas temperature T_g = 300°C and heat transfer coefficient $\bar{h}_{c,g}$ = 75 W/(m^2 K).
- Outside air average temperature T_a = 25°C and heat transfer coefficient $\bar{h}_{c,a}$ = 15 W/(m^2 K).

FIND

- The temperature distribution in chimney wall and the inside and outside wall temperatures.
- Rate of heat loss from gas to outside air through the chimney.

ASSUMPTIONS

- Steady state conditions, and there is no heat generation in the chimney wall and the conduction along the height of the chimney is negligible.

SOLUTION

For steady-state conditions, the heat conduction equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0$$

which is subject to the following boundary conditions

$$q''=-k\left.\frac{dT}{dr}\right|_{r=r_i}=\bar{h}_{c,g}\left(T_{w,i}-T_g\right) \text{ and } q''=-k\left.\frac{dT}{dr}\right|_{r=r_o}=\bar{h}_{c,a}\left(T_{w,o}-T_a\right)$$

Construct a nodal network with 2 cm spacing in the radial direction in the thickness of the chimney's hollow cylinder (6 nodal points total, including the nodal points on the inner and outer wall) and 10-degree spacing in the angular direction (36 nodal points). This would result in the following form of discretized equation (see Equation 3.36 and simplify it)

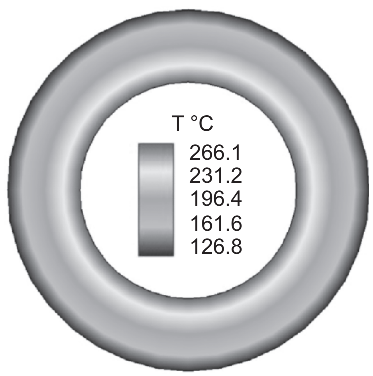
$$\frac{\Delta r}{r\Delta\theta}\left(T_{i,j+1} - 2T_{i,j} + T_{i,j+1}\right) + \frac{r\Delta\theta}{\Delta r}\left(T_{i+1,j} - 2T_{i,j} + T_{i-1,j}\right) + \frac{\Delta\theta}{2}\left(T_{i+1,j} - T_{i-1,j}\right) = 0$$

Though, it may be noted that because of the circular symmetry, this can be solved as a one-dimensional problem without using the angular nodes.

The numerical solution (carried out on MATLAB) yields the following temperature values at the six (6) radial nodes along the wall thickness

Inner Wall Node 1	Node 2	Node 3	Node 4	Node 5	Outer Wall Node 6
266.1°C	235.7°C	208.5°C	183.7°C	160.9°C	126.8°

This temperature distribution is depicted as an isotherm contour plot in the figure below.



Also, the rate of heat loss from the outer wall of the chimney to air can be calculated as

$$q = \bar{h}_{c,a}\,(\pi d_o L)\left(T_{w,o} - T_a\right) = 15(\pi \times 0.5 \times 7)(126.8 - 25) = 16{,}790 \text{ W}$$

PROBLEM 3.41

Show that in the limit $\Delta x \to 0$, $\Delta y \to 0$, $\Delta t \to 0$, the difference Eq. (3.22) is equivalent to the two-dimensional version of the differential Eq. (2.6).

GIVEN

- Difference equation, Equation (3.22)

SHOW

(a) In the limit Δx, Δy, and $\Delta t \to 0$, the difference equation is equivalent to the two-dimensional version of the differential equation, Equation (2.6).

SOLUTION

Equation (3.22) is

$$\frac{T_{i+1,j,m} - 2T_{i,j,m} + T_{i-1,j,m}}{\Delta x^2} + \frac{T_{i,j+1,m} - 2T_{i,j,m} + T_{i,j-1,m}}{\Delta y^2} + \frac{\dot{q}_{G,i,j,m}}{k} = \frac{\rho c}{k}\frac{T_{i,j,m+1} - T_{i,j,m}}{\Delta t}$$

We have by definition

$$T_{i,j,m} = T(x, y, t)$$
$$T_{i+1,j,m} = T(x + \Delta x, y, t)$$
$$T_{i-1,j,m} = T(x - \Delta x, y, t)$$

$$T_{i,j+1,m} = T(x, y + \Delta y, t)$$
$$T_{i,j-1,m} = T(x, y - \Delta y, t)$$
$$T_{i,j,m+1} = T(x, y, t + y\Delta t)$$

so Equation (3.22) is equivalent to

$$\frac{T(x+\Delta x, y,t)-2T(x,y,t)+T(x-\Delta x,y,t)}{\Delta x^2}$$
$$+ \frac{T(x,y+\Delta y,t)-2T(x,y,t)+T(x,y-\Delta y,t)}{\Delta y^2}$$
$$+ \frac{\dot{q}_G(x,y,t)}{k} = \frac{\rho c}{k}\frac{T(x,y,t+\Delta t)-T(x,y,t)}{\Delta t}$$

In the limit $\Delta x \to 0$, from calculus, the first term becomes

In the limit $\Delta y \to 0$, the second term becomes

$$\frac{\partial^2 T}{\partial x^2}$$
$$\frac{\partial^2 T}{\partial y^2}$$

In the limit $\Delta t \to 0$, the right side of the equation becomes

$$\frac{\rho c}{k}\frac{\partial T}{\partial t}$$

so the difference equation becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\dot{q}_G}{k} = \frac{\rho c}{k}\frac{\partial T}{\partial t}$$

which is equivalent to the two-dimensional version of Equation (2.6) as required.

PROBLEM 3.42

Derive the stability criterion for the explicit solution of two-dimensional transient conduction.

GIVEN

- Two-dimensional transient conduction

FIND

(a) The stability criterion for an explicit solution

SOLUTION

From Equation (3.21), the coefficient on the $T_{i,\,j,\,m}$ term is

$$\frac{1}{\alpha\Delta t} - \frac{2}{\Delta x^2} - \frac{2}{\Delta y^2}$$

which must be greater than zero to ensure stability. Therefore

$$\frac{1}{\alpha\Delta t} > 2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)$$

or

$$\Delta t < \frac{1}{2\alpha}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)^{-1}$$

which is the stability criterion as required.

PROBLEM 3.43

Derive Equation (3.27).

GIVEN

- Two-dimensional transient conduction at an inside corner with specified-flux boundary condition

FIND

(a) The control volume energy balance equation, Equation (3.27)

SOLUTION

Referring to Figure 3.14, heat conducted into the control volume is given by

$$k\,\frac{T_{i-1,j,m} - T_{i,j,m}}{\Delta x}\,\Delta y + k\,\frac{T_{i,j+1,m} - T_{i,j,m}}{\Delta y}\,\Delta x$$

$$+\,k\,\frac{T_{i,j-1,m} - T_{i,j,m}}{\Delta y}\,\frac{\Delta x}{2} + k\,\frac{T_{i+1,j,m} - T_{i,j,m}}{\Delta x}\,\frac{\Delta y}{2}$$

The rate of heat generation in the control volume is given by

$$\dot{q}_{G,i,j,m}\,\frac{3}{4}\,\Delta x\Delta y$$

Heat transferred out of the boundaries by the specified fluxes is

$$q''_x\,\frac{\Delta y}{2} - q''_y\,\frac{\Delta x}{2}$$

The rate at which energy is stored in the control volume is given by

$$\rho c\,\frac{T_{i,j,m+1} - T_{i,j,m}}{\Delta t}\,\frac{3}{4}\Delta x\Delta y$$

The sum of the first two equations above must equal the sum of the last two equations above. The coefficients on the individual terms is

$$T_{\text{i, j, m}}: 1 - 2\alpha\Delta t\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right) \quad T_{\text{i}-1,\text{ j, m}}: \frac{4}{3}\frac{\alpha\Delta t}{\Delta x^2} \quad T_{\text{i}+1,\text{ j, m}}: \frac{2}{3}\frac{\alpha\Delta t}{\Delta x^2} \quad T_{\text{i, j}+1,\text{ m}}: \frac{4}{3}\frac{\alpha\Delta t}{\Delta y^2}$$

$$T_{\text{i, j}-1,\text{ m}}: \quad \dot{q}_G: \frac{\Delta t}{\rho c} \quad q''_x: -\frac{2}{3}\frac{\alpha\Delta t}{k\Delta x} \quad q''_y: \frac{2}{3}\frac{\alpha\Delta t}{k\Delta y}$$

which is identical to those in Equation (3.27).

PROBLEM 3.44

Derive the stability criterion for an inside-corner boundary control volume for two-dimensional steady conduction when a convection boundary condition exists.

GIVEN

- Two-dimensional steady conduction at an inside corner with a convection boundary condition

FIND

(a) The stability criterion

SOLUTION

In Equation (3.27) write

$$q''_{x,i,j,m} = h\,(T_{i,j,m} - T_\infty)$$

$$q''_{y,i,j,m} = h\,(T_\infty - T_{i,j,m})$$

to account for the convection boundary condition.

The coefficient on $T_{i,j,m}$ is now

$$1 - 2\alpha\Delta t\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right) + \frac{2}{3}\,\frac{\alpha\Delta t}{k}\left(-\frac{1}{\Delta x} - \frac{1}{\Delta y}\right)$$

$$= 1 - 2a\Delta t\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right) - \frac{2}{3}\,\frac{\alpha h\Delta t}{k}\left(\frac{1}{\Delta x} + \frac{1}{\Delta y}\right)$$

This coefficient must be greater than zero for stability, therefore

$$\Delta t < \frac{1}{2\alpha\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right) + \frac{2}{3}\frac{\alpha h}{k}\left(\frac{1}{\Delta x} + \frac{1}{\Delta y}\right)}$$

Note that as we have seen before, the criterion for a convective boundary condition is more restrictive than other boundary condition stability criteria.

PROBLEM 3.45

A long concrete beam is to undergo a thermal test to determine its loss of strength in the event of a building fire. The beam cross section is triangular as shown in the sketch. Initially, the beam is at a uniform temperature of 20°C. At the start of the test, one of the short faces and the long face are exposed to hot gases at 400°C through a heat transfer coefficient of 10 W/(m^2 K) and the remaining short face is adiabatic. Produce a graph showing the highest and lowest temperatures in the beam as a function of time for the first 1 hour of exposure. For the concrete properties, use *k* = 0.5 W/(mK) and *n* = 5 × 10^{-7} m^2/s. Use a node spacing of 4 cm. and use an explicit difference scheme.

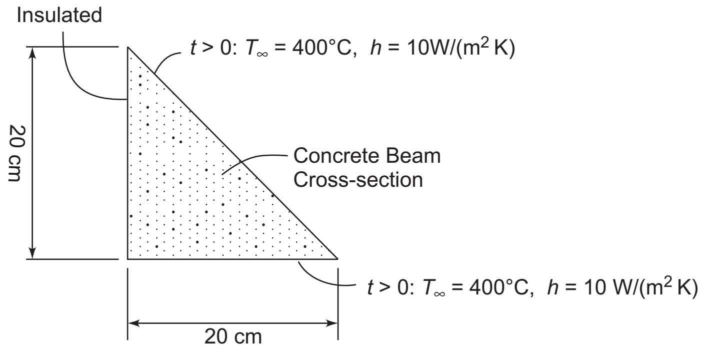
GIVEN

- Concrete beam suddenly exposed to hot gases

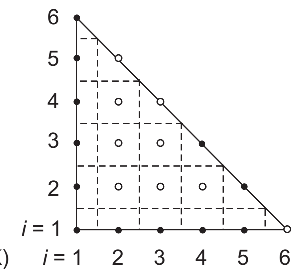
FIND

(a) Highest and lowest temperatures in the beam as a function of time

SKETCH



Problem Definition Sketch



Node and Control Volume Layout

SOLUTION

The arrangement of nodes and control volumes is shown in the figure to right. Examination of this figure reveals that we have 7 unique control volumes. We need to develop an energy balance for each type. To simplify the notation, use the following

$T_o \equiv T_{i,j,k}$	$T_1 \equiv T_{i-1,j,k}$ (left)	$T_r \equiv T_{i+1,j,k}$ (right)	$T_u \equiv T_{i,j+1,k}$ (up)	$T_d \equiv T_{i,j-1,k}$ (down)

and

$$K1 \equiv \frac{\alpha \Delta t}{\Delta x^2} \qquad K2 \equiv \frac{h \Delta t}{\rho c \Delta x}$$

For al interior control volumes: $i = 2$, $j = 2, 3, 4$; $i = 3$, $j = 2, 3$; and $i = 4$, $j = 2$, we have for the energy balance

$$k\{T_1 + T_r + T_u + T_d - 4T_o\} = \rho c\, \Delta x^2 \frac{T_{i,j,k+1} - T_o}{\Delta t}$$

Solving for $T_{i,j,k+1}$

$$T_{i,j,k+1} = T_o + K1\,(T_1 + T_r + T_u + T_d - 4\,T_o)$$

For the bottom row of control volumes (not corners), $j = 1$, $i = 2, 3, 4, 5$, we have

$$k\left\{\frac{T_1 - T_o}{2} + \frac{T_r - T_o}{2} + T_u - T_o\right\} + h\,\Delta x\,(T_\infty - T_o) = \rho c\,\frac{\Delta x^2}{2}\frac{T_{i,j,k+1} - T_o}{\Delta t}$$

Solving for $T_{i,j,k+1}$

$$T_{i,j,k+1} = T_o + 2\,K1\left(\frac{1}{2}(T_1 + T_r) + T_u - 2T_o\right) + 2K2\,(T_\infty - T_o)$$

For the left edge (not corners) $i = 1$, $j = 2, 3, 4, 5$

$$k\left\{\frac{T_d - T_o}{2} + \frac{T_u - T_o}{2} + T_r - T_o\right\} = \rho c\,\frac{\Delta x^2}{2}\frac{T_{i,j,k+1} - T_o}{\Delta t}$$

Solving for $T_{i,j,k+1}$

$$T_{i,j,k+1} = T_o + 2\,K1\left(\frac{1}{2}(T_u + T_d) + T_r - 2T_o\right)$$

For control volumes on the diagonal (i, j) = (2, 5), (3, 4), (4, 3), (5, 2) we have

$$k\ \{T_1 - T_o + T_d - T_o\} + h\sqrt{2}\ \Delta x\ (T_\infty - T_o) = \rho c\ \frac{\Delta x^2}{2}\ \frac{T_{i,j,k+1} - T_o}{\Delta t}$$

Solving for $T_{i,\, j,\, k+1}$

$$T_{i,\, j,\, k+1} = T_o + 2\ K1\ (T_1 + T_d - 2\ T_o) + 2\ \sqrt{2}\ \ K2\ (T_\infty - T_o)$$

Finally, for the corners

$i = 1, j = 1$

$$k\ \left\{\frac{T_u - T_o}{2} + \frac{T_r - T_o}{2}\right\} + h\ \frac{\Delta x}{2}\ (T_\infty - T_o) = \rho\, c\ \frac{\Delta x^2}{4}\ \frac{T_{i,j,k+1} - T_o}{\Delta t}$$

Solving for $T_{i,\, j,\, k+1}$

$$T_{i,\, j,\, k+1} = T_o + 4\ K1\ \left(\frac{1}{2}(T_u + T_r) - T_o\right) + 2\ K2\ (T_\infty - T_o)$$

$i = 6, j = 1$

$$k\ \left\{\frac{T_i - T_o}{2}\right\} + h\ \frac{\Delta x}{2}\ (1 + \sqrt{2}\ )\ (T_\infty - T_o) = \rho\ \mathrm{c}\ \frac{\Delta x^2}{8}\ \frac{T_{i,j,k+1} - T_o}{\Delta t}$$

Solving for $T_{i,\, j,\, k+1}$

$$T_{i,\, j,\, k+1} = T_o + 8\ K1\ \left(\frac{1}{2}(T_i - T_o)\right) + 4\ K2\ (1 + \sqrt{2}\ )\ (T_\infty - T_o)$$

$i = 1, j = 6$

$$k\ \left\{\frac{T_d - T_o}{2}\right\} + h\ \Delta x\ \frac{\sqrt{2}}{2}\ (T_\infty - T_o) = \rho\, c\ \frac{\Delta x^2}{8}\ \frac{T_{i,j,k+1} - T_o}{\Delta t}$$

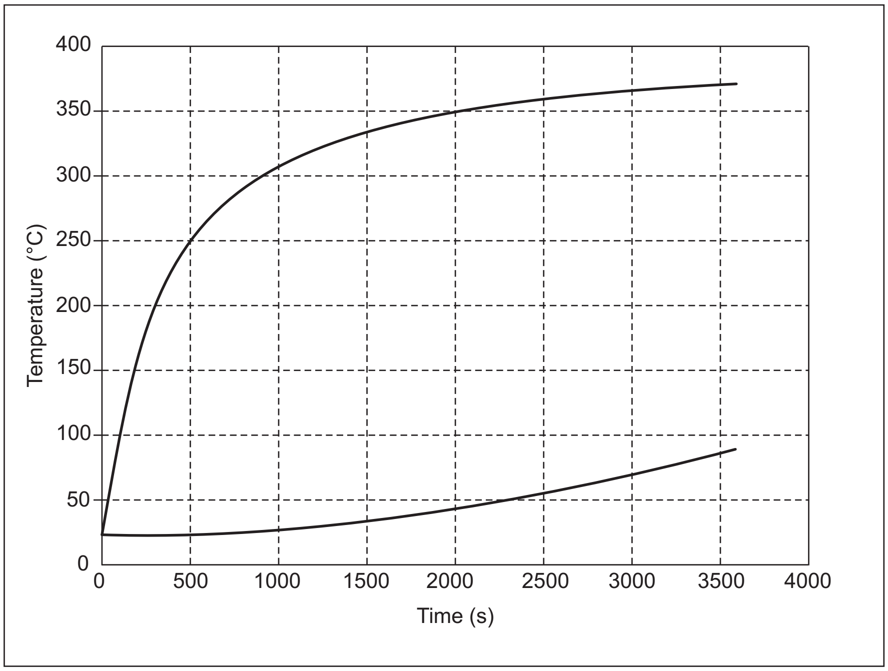
Solving for $T_{i,\, j,\, k+1}$

$$T_{i,\, j,\, k+1} = T_o + 8\ K1\ \left(\frac{1}{2}(T_d - T_o)\right) + 8\ \sqrt{2}\ \ K2\ (T_\infty - T_o)$$

The system of equations can be solved by the marching procedure. We must keep in mind the limitation in Δt given by Equation (3.14) which gives

$$\Delta t_{\max} = 800 \text{ seconds}$$

The equations were solved using Δt = 10 seconds. A check was performed by hand on each of the seven unique control volume energy balances. The maximum temperature occurs at i = 6, j = 1, and the minimum temperature occurs at i = 1, j = 3. The resulting temperature as a function of time is given below.



PROBLEM 3.46

A steel billet is to be heat treated by immersion in a molten salt bath. The billet is 5 cm square and 1 m long. Prior to immersion in the bath, the billet is at a uniform temperature of 20°C. The bath is at 600°C and the heat transfer coefficient at the billet surface is 20 W/(m^{2}K). Plot the temperature at the center of the billet as a function of time. How much time is needed to heat the billet center to 500°C? Use an implicit difference scheme with node spacing of 1 cm. The thermal conductivity of the steel is 40 W/(m K) and the thermal diffusivity is 1 × 10^{-5} m^2/s.

GIVEN

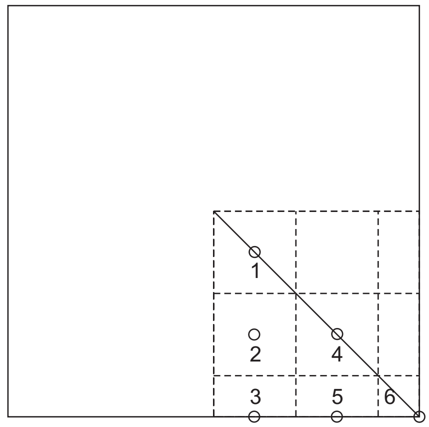
- Steel billet undergoing heat treatment

FIND

(a) Temperature at the center of the billet as a function of time
(b) How much time is needed to heat the center of the billet to 500°C

SOLUTION

The billet can be considered two-dimensional since it is very long. The accompanying sketch shows the geometry.



Allowing for symmetry, we need only consider 6 nodes and control volumes. These are also shown in the sketch. We need to develop a heat balance on each of these control volumes. In the implicit form

Node (1)

$$k\,\frac{T_{2,k+1}-T_{1,k+1}}{\Delta x}\,\Delta x=\rho\,c\,\frac{\Delta x^2}{2}\,\frac{T_{1,k+1}-T_{1,k}}{\Delta t}$$

or

$$T_{1,\,k+1}\left(1+\frac{2\alpha\Delta t}{\Delta x^2}\right)-T_{2,\,k+1}\left(\frac{2\alpha\Delta t}{\Delta x^2}\right)=T_{1,\,k}$$

Node (2)

$$k\left[\frac{T_{1,k+1}-T_{2,k+1}}{\Delta x}\,\Delta x+\frac{T_{4,k+1}-T_{2,k+1}}{\Delta x}\,\Delta x+\frac{T_{3,k+1}-T_{2,k+1}}{\Delta x}\,\Delta x\right]$$

$$=\rho\,c\,\Delta x^2\,\frac{T_{2,k+1}-T_{2,k}}{\Delta t}$$

or

$$T_{2,\,k+1}\left(1+\frac{3\alpha\Delta t}{\Delta x^2}\right)-(T_{1,\,k+1}+T_{3,\,k+1}+T_{4,\,k+1})\left(\frac{\alpha\Delta t}{\Delta x^2}\right)=T_{2,\,k}$$

Node (3)

$$k\left[\frac{T_{2,k+1}-T_{3,k+1}}{\Delta x}\,\Delta x+\frac{T_{5,k+1}-T_{3,k+1}}{\Delta x}\,\frac{\Delta x}{2}\right]+h\,\Delta x\,(T_\infty-T_{3,\,k+1})=\rho\,c\,\frac{\Delta x^2}{2}\,\frac{T_{3,k+1}-T_{3,k}}{\Delta t}$$

or

$$T_{3,\,k+1}\left(1+\frac{3\alpha\Delta t}{\Delta x^2}+\frac{2h\Delta t}{\rho c\Delta x}\right)-T_{2,\,k+1}\left(\frac{2\alpha\Delta t}{\Delta x^2}\right)-T_{5,\,k+1}\left(\frac{\alpha\Delta t}{\Delta x^2}\right)=T_{3,\,k}+\frac{2h\Delta t T_\infty}{\rho c\Delta x}$$

Node (4)

$$k\left[\frac{T_{2,k+1}-T_{4,k+1}}{\Delta x}\,\Delta x+\frac{T_{5,k+1}-T_{4,k+1}}{\Delta x}\right]=\rho\,c\,\frac{\Delta x^2}{2}\,\frac{T_{4,k+1}-T_{4,k}}{\Delta t}$$

or

$$T_{4,\,k+1}\left(1+\frac{4\alpha\Delta t}{\Delta x^2}\right)-T_{2,\,k+1}\left(\frac{2\alpha\Delta t}{\Delta x^2}\right)-T_{5,\,k+1}\left(\frac{2\alpha\Delta t}{\Delta x^2}\right)=T_{4,\,k}$$

Node (5)

$$k\left[\frac{T_{4,k+1}-T_{5,k+1}}{\Delta x}\Delta x+\frac{T_{3,k+1}-T_{5,k+1}}{\Delta x}\frac{\Delta x}{2}+\frac{T_{6,k+1}-T_{5,k+1}}{\Delta x}\frac{\Delta x}{2}\right]$$

$$+\,h\,\Delta x\,(T_\infty - T_{5,\,k+1}) = \rho\, c\,\frac{\Delta x^2}{2}\,\frac{T_{5,k+1}-T_{5,k}}{\Delta t}$$

or

$$T_{5,\,k+1}\left(1+\frac{4\alpha\Delta t}{\Delta x^2}+\frac{2h\Delta t}{\rho c\Delta x}\right)-T_{3,\,k+1}\left(\frac{\alpha\Delta t}{\Delta x^2}\right)-T_{4,\,k+1}\left(\frac{2\alpha\Delta t}{\Delta x^2}\right)$$

$$-\,T_{6,\,k+1}\,\frac{\alpha\Delta t}{\Delta x^2}=T_{5,\,k}+\frac{2h\Delta t T_\infty}{\rho c\Delta x}$$

Node (6)

$$k\,\frac{T_{5,k+1}-T_{6,k+1}}{\Delta x}\frac{\Delta x}{2}+\frac{h\Delta x}{2}\,(T_\infty - T_{6,\,k+1}) = \rho\, c\,\frac{\Delta x^2}{8}\,\frac{T_{6,k+1}-T_{6,k}}{\Delta t}$$

or

$$T_{6,\,k+1}\left(1+\frac{4\alpha\Delta t}{\Delta x^2}+\frac{4h\Delta t}{\rho c\Delta x}\right)-T_{5,\,k+1}\left(\frac{4\alpha\Delta t}{\Delta x^2}\right)=T_{6,\,k}+\frac{4h\Delta t T_\infty}{\rho c\Delta x}$$

The 6 equations for the 6 control volumes can be written in matrix form as follows

$$\begin{bmatrix} 1+2K_1 & -2K_1 & 0 & 0 & 0 & 0 \\ K_1 & 1+3K_1 & -K_1 & -K_1 & 0 & 0 \\ 0 & -2K_1 & 1+3K_1+2K_2 & 0 & -K_1 & 0 \\ 0 & -2K_1 & 0 & 1+4K_1 & -2K_1 & 0 \\ 0 & 0 & -K_1 & -2K_1 & 1+4K_1+2K_2 & -K_1 \\ 0 & 0 & 0 & 0 & -4K_1 & 1+4K_1+4K_2 \end{bmatrix}$$

$$\begin{bmatrix} T_{1,k+1} \\ T_{2,k+1} \\ T_{3,k+1} \\ T_{4,k+1} \\ T_{5,k+1} \\ T_{6,k+1} \end{bmatrix} = \begin{bmatrix} T_{1,k} \\ T_{2,k} \\ T_{3,k} \\ T_{4,k} \\ T_{5,k} \\ T_{6,k} \end{bmatrix} + \begin{bmatrix} \\ \\ 2K \\ \\ 2K \\ 2K \end{bmatrix}$$

In the above matrix, we have used the following notation

$$K_1 = \frac{\alpha\Delta t}{\Delta x^2}$$

and

$$K_2 = \frac{h\Delta t}{\rho c\Delta x}$$

The matrix equation can be written as

$$AT_{k+1} = T_{k+}C$$

For $k = 0$, we know the vector T_k from the initial conditions. Therefore, we know the right hand side of the above equation. Inverting the matrix A and multiplying by both sides of the matrix equation, we have the solution for T_{k+1}

$$T_{k+1} = A^{-1}\,(T_k + C)$$

Incrementing k to $k = 1$, we can then insert the solution for T_1 into the right hand side of the above equation to find T_2 and so forth. This can be implemented fairly easily with a spreadsheet program in two steps. First, the coefficients of the matrix A are determined from the problem parameters. The matrix is then inverted. In the second step, the inverted matrix is repeatedly multiplied by the sum of the two vectors T_k and C. Each time it is multiplied by the sum of these two vectors, the vector T_k is updated with the results. The temperature at node 1 is nearest the center, so it is saved for later plotting. The spreadsheet is shown below.

```
Problem 3_45 Filename: 3_45.WQ1
PROBLEM PARAMETERS
alpha  =   1 K-0.5 m^2/s
dx     =   0.01 m
dt     =   10 sec
h      =   20 W/m^2K
k      =   40 W/mK
rho C  =   4000000 Ws/m^3K
T inf  =   600 C
K1     =   1 (-)
K2     =   0.005 (-)
```

Coefficient Matrix

3	−2	0	0	0	0
−1	4	−1	−1	0	0
0	−2	4.01	0	−1	0
0	−2	0	5	−2	0
0	0	−1	−2	5.01	−1
0	0	0	0	−4	5.02

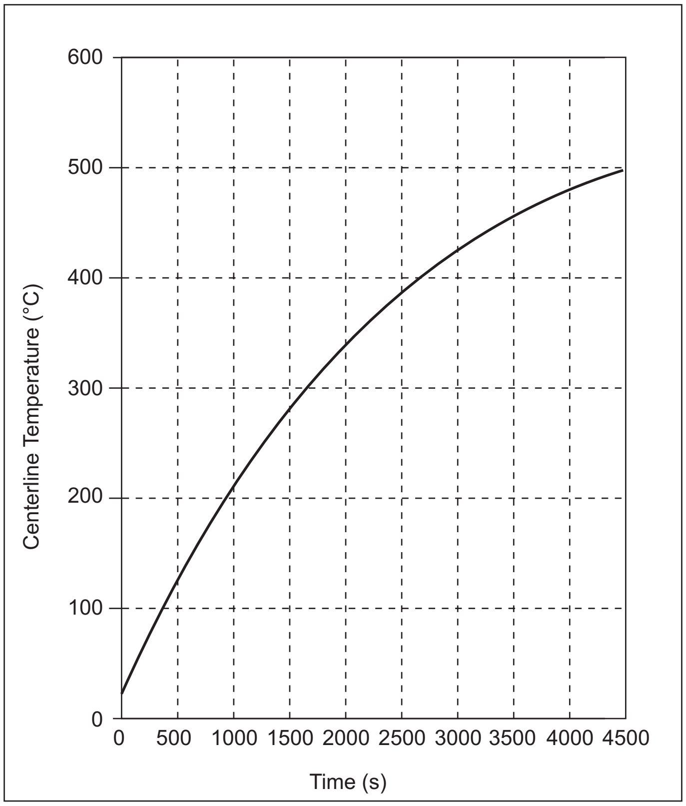
INVERSE MATRIX						VECTOR T(K)	VECTOR C	VECTOR SUM		T(K+1) = INVERSE MATRIX VECTOR SUM
0.435834	0.307502	0.092423	0.086746	0.063115	0.012573	500.356	0	500.3		500.35
0.153751	0.461253	0.138635	0.13012	0.094673	0.018859	500.5545	0	500.55	5	500.554
0.092423	0.27727	0.352369	0.109747	0.135732	0.027038	500.9511	6	506.95	1	500.951
0.086746	0.260239	0.109747	0.323986	0.179844	0.035826	500.7526	0	500.75	6	500.752
0.063115	0.189345	0.135732	0.179844	0.354938	0.070705	501.1484	6	507.14	4	501.148
0.050291	0.150873	0.108153	0.143302	0.282819	0.255542	501.5426	12	513.54	6	501.542

The macro below automatically multiplies the inverse matrix by the “vector sum” and puts the result for T(1,k+1) into the table to the left for plotting

Temperature of a function of time.

Iteration k	time (sec)	T (1, k)
0	0	20
1	10	21.04797
2	20	22.75763
3	30	24.78853
.	.	.
.	.	.
.	.	.
441	4410	499.1603
442	4420	499.3605
443	4430	499.959
444	4440	500.3564

```
{ / Math: MultiplyMatrix}-
{GOTO}
c39-
{END}
{DOWN 2}
{/ Block; Copy}
$1$26-
{IF L26 < 500} {BRANCH E37}
{BEEP 1}
```



Centerline Temperature vs. Time for Problems 3.45

(b) The temperature at the billet centerline exceeds 500°C at 4440 seconds.

PROBLEM 3.47

It has been proposed that highly concentrated solar energy can be used to economically process materials when it is desirable to rapidly heat the material surface without significantly heating the bulk. In one such process for case hardening low-cost carbon steel, the surface of a thin disk is to be exposed to concentrated solar flux. The distribution of absorbed solar flux on the disk is given by

$$q''(r) = q''_{max}\left(1 - 0.9\left(\frac{r}{R_o}\right)^2\right)$$

where r is the distance from the disk axis and q''_{max} and R_o are parameters that describe the flux distribution. The disk diameter is $2R_s$, its thickness is Z_s, its thermal conductivity is k and its thermal diffusivity is α. The disk is initially at temperature T_{init} and at time t = 0 it is suddenly exposed to the concentrated flux. Derive the set of explicit difference equations needed to predict how the disk temperature distribution evolves with time. The edge and bottom surface of the disk are insulated and reradiation from the disk may be neglected.

GIVEN

- Steel disk exposed to concentrated solar flux

FIND

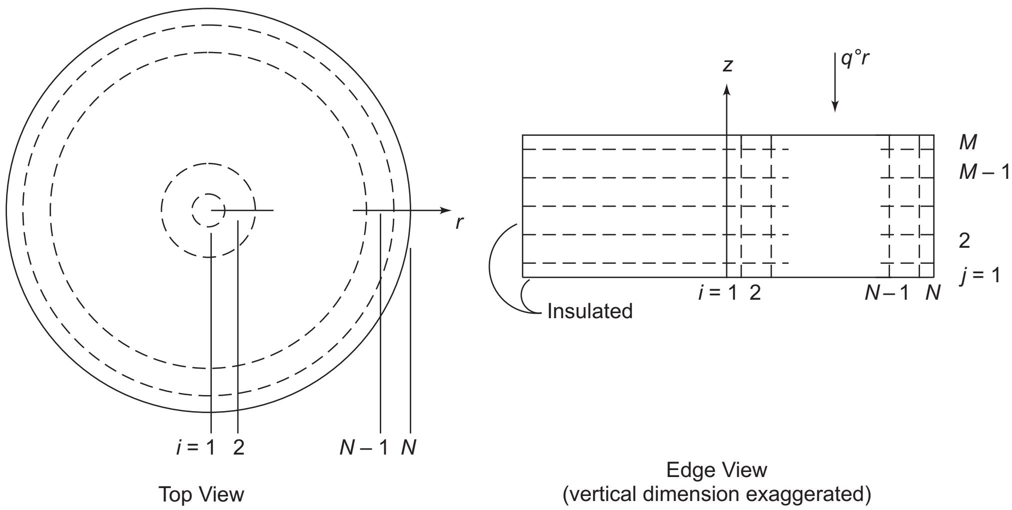
(a) Explicit difference equations that describe evolution of disk temperature

SOLUTION

The problem is a two-dimensional cylindrical geometry in the coordinates r and z. There are no gradients in the circumferential direction. Let there be N radial nodes and M axial nodes as shown in the sketch below. Then the size of the control volumes and the node locations are given by

$$\Delta r = \frac{R_s}{N-1} \qquad r_i = (i-1)\,\Delta r \qquad i = 1, 2, \ldots \text{N}$$

$$\Delta z = \frac{Z_s}{M-1} \qquad z_j = (j-1)\,\Delta z \qquad j = 1, 2, \ldots M$$



There are a total of $N \times M$ control volumes and each has the shape of a ring with rectangular cross-section. We need to develop an energy balance equation for each control volume. First, let us determine the volume and surface area of each control volume since these will be needed in the energy balance equations.

The top or bottom face surface area of each control volume is A_{fi}

$$A_{fi} = \pi \left(\frac{\Delta r}{2}\right)^2 \quad i = 1$$

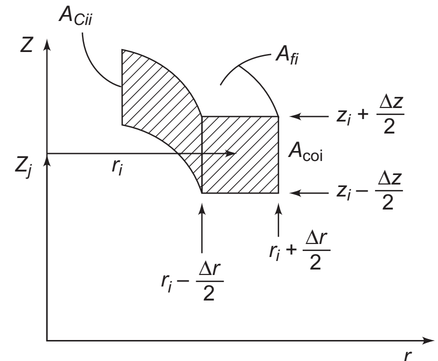
$$A_{fi} = \pi \left(\left(r_i + \frac{\Delta r}{2}\right)^2 - \left(r_i - \frac{\Delta r}{2}\right)^2 \right) = 2\pi \Delta r^2\,(i-1) \qquad i = 2, 3, \ldots N-1$$

$$A_{fi} = \pi \left(R_s^2 - \left(R_s - \frac{\Delta r}{2}\right)^2 \right) = \pi\,\Delta r \left(R_s - \frac{\Delta r}{4}\right) \qquad i = N$$

Now, the volume of each control volume is just

$$V_i = A_{fi}\,\Delta z \qquad i = 1, 2, \ldots N$$

Except for the control volume at node i = 1, each control volume has two curved surfaces, an outer surface and an inner surface, see sketch below.



Cross Section of Typical Control Volume

The surface area of the outer curved surface is

$$A_{\infty i} = 2\pi \left(r_i - \frac{\Delta r}{2}\right) \Delta z = 2\pi \Delta r z \left(i - \frac{1}{2}\right) \qquad i = 1, 2, \ldots N-1$$

$$A_{\infty i} = 2\pi R_s \, \Delta z \qquad i = N$$

The surface area of the inner surface is

$$A_{cii} = 2\pi \left(r_i - \frac{\Delta r}{2}\right) \Delta z = 2\pi \Delta r \Delta z \left(i - \frac{3}{2}\right) \qquad i = 2, 3, \ldots N$$

By definition

$$A_{cii} = 0 \qquad i = 1$$

(In the above notation for A_{cii}, the first i in the subscript refers to the inner curved surface and the second i is the node index.)

The control volumes along the exposed surface absorb solar flux given by the equation in the problem statement. We need to integrate this flux equation over each control volume to determine the solar energy absorbed for each control volume. The following equation expresses this

$$\overline{q_i} \equiv \int_o^{r_i + \frac{\Delta r}{2}} q''(r)\, 2\pi r dr \qquad i = 1$$

$$\overline{q_i} \equiv \int_{r_i + \frac{\Delta r}{2}}^{r_i + \frac{\Delta r}{2}} q''(r)\, 2\pi r dr \qquad i = 2, 3, \ldots N-1$$

$$\overline{q_i} \equiv \int_{r_i - \frac{\Delta r}{2}}^{r_i} q''(r)\, 2\pi r dr \qquad i = N$$

Carrying out the inegration and simplifying we find

$$\overline{q_i} = \frac{\pi}{4} \, q''_{\max} \, \Delta r^2 \left(1 - \frac{0.9}{8}\left(\frac{\Delta r}{R_o}\right)^2\right) \qquad i = 1$$

$$\overline{q_i} = \pi \, q''_{\max} \, \Delta r^2 \left[2(i-1) - \frac{0.9}{8}\left(\frac{\Delta r}{R_o}\right)^2 (4i^3 - 12i^2 + 13i - 5)\right] \qquad i = 2, 3, \ldots N-1$$

$$\overline{q_i} = \pi \, q''_{\max} \, \Delta r^2 \left[N - \frac{5}{4} - \frac{0.9}{8}\left(\frac{\Delta r}{R_o}\right)^2 \left(2N^3 - \frac{15}{2}N^2 + \frac{19}{2}N - \frac{65}{16}\right)\right] i = N$$

We are now in a position to evaluate the energy balance for each control volume. We actually only need to develop 9 unique difference equations. These are for the interior nodes, the nodes at the four corners, and the nodes on the axis, and on the three outer surfaces.

The explicit energy balance equation for all interior nodes is

$$k\left[\frac{T_{i+1,j,k}-T_{i,j,k}}{\Delta r}A_{coi}+\frac{T_{i-1,j,k}-T_{i,j,k}}{\Delta r}A_{cii}+\frac{T_{i,j+1,k}-T_{i,j,k}}{\Delta z}A_{fi}+\frac{T_{i,j-1,k}-T_{i,j,k}}{\Delta z}A_{fi}\right]$$

$$=\rho\,c\,V_i\,\frac{T_{i,j,k+1}-T_{i,j,k}}{\Delta t}$$

Solving for the node temperatures

$$T_{i,j,k+1}=T_{i,j,k}+\frac{\alpha\Delta t}{V_i}\left[\frac{T_{i+1,j,k}-T_{i,j,k}}{\Delta r}A_{coi}+\frac{T_{i-1,j,k}-T_{i,j,k}}{\Delta r}A_{cii}+\right.$$

$$\left.\frac{T_{i,j+1,k}-T_{i,j,k}}{\Delta z}A_{fi}+\frac{T_{i,j-1,k}-T_{i,j,k}}{\Delta z}A_{fi}\right]$$

$$i=2,3,\ldots N-1 \qquad j=2,3,\ldots M-1$$

For the interior nodes along the axis we have

$$k\left[\frac{T_{i+1,j,k}-T_{i,j,k}}{\Delta r}A_{coi}+\frac{T_{i,j+1,k}-T_{i,j,k}}{\Delta z}A_{fi}+\frac{T_{i,j-1,k}-T_{i,j,k}}{\Delta z}A_{fi}\right]=\rho\,c\,V_i\,\frac{T_{i,j,k+1}-T_{i,j,k}}{\Delta t}$$

Solving for the node temperatures

$$T_{i,j,k+1}=T_{i,j,k}+\frac{\alpha\Delta t}{V_i}\left[\frac{T_{i+1,j,k}-T_{i,j,k}}{\Delta r}A_{coi}+\frac{T_{i,j+1,k}-T_{i,j,k}}{\Delta z}A_{fi}+\frac{T_{i,j-1,k}-T_{i,j,k}}{\Delta z}A_{fi}\right]$$

$$i=1 \qquad j=2,3,\ldots \mathrm{M}-1$$

For the node on the top of the axis

$$k\left[\frac{T_{i+1,j,k}-T_{i,j,k}}{\Delta r}A_{coi}+\frac{T_{i,j-1,k}-T_{i,j,k}}{\Delta z}A_{fi}\right]+\overline{q_i}=\rho\,c\,V_\mathrm{i}\,\frac{T_{i,j,k+1}-T_{i,j,k}}{\Delta t}$$

Solving for the node temperature

$$T_{i,j,k+1}=T_{i,j,k}+\frac{\alpha\Delta t}{V_i}\left[\frac{T_{i+1,j,k}-T_{i,j,k}}{\Delta r}A_{coi}+\frac{T_{i,j-1,k}-T_{i,j,k}}{\Delta z}A_{fi}\right]\frac{\overline{q_i}\,\Delta t}{\rho c V_i}$$

$$i=1 \qquad j=M$$

For the node on the bottom of the axis

$$k\left[\frac{T_{i+1,j,k}-T_{i,j,k}}{\Delta r}A_{coi}+\frac{T_{i,j+1,k}-T_{i,j,k}}{\Delta z}A_{fi}\right]=\rho\,c\,V_i\,\frac{T_{i,j,k+1}-T_{i,j,k}}{\Delta t}$$

$$i=1 \qquad j=1$$

For the interior nodes along the outer curved surface

$$k\left[\frac{T_{i-1,j,k}-T_{i,j,k}}{\Delta r}A_{cii}+\frac{T_{i,j+1,k}-T_{i,j,k}}{\Delta z}A_{fi}+\frac{T_{i,j-1,k}-T_{i,j,k}}{\Delta z}A_{fi}\right]=\rho\,c\,V_i\,\frac{T_{i,j,k+1}-T_{i,j,k}}{\Delta t}$$

Solving for the node temperatures

$$T_{i,j,k+1} = T_{i,j,k} + \frac{\alpha\Delta t}{V_i}\left[\frac{T_{i-1,j,k} - T_{i,j,k}}{\Delta r} A_{cii} + \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta z} A_{fi} + \frac{T_{i,j-1,k} - T_{i,j,k}}{\Delta z} A_{fi}\right]$$

$$i = N \qquad j = 2, 3, \ldots M-1$$

For the node on the top of the outer curved surface

$$k\left[\frac{T_{i-1,j,k} - T_{i,j,k}}{\Delta r} A_{cii} + \frac{T_{i,j-1,k} - T_{i,j,k}}{\Delta z} A_{fi}\right] \overline{q_i} = \rho\, c\, V_{\text{i}}\, \frac{T_{i,j,k+1} - T_{i,j,k}}{\Delta t}$$

Solving for the node temperature

$$T_{i,j,k+1} = T_{i,j,k} + \frac{\alpha\Delta t}{V_i}\left[\frac{T_{i-1,j,k} - T_{i,j,k}}{\Delta r} A_{cii} + \frac{T_{i,j-1,k} - T_{i,j,k}}{\Delta z} A_{fi}\right] + \frac{\overline{q_i}\,\Delta t}{\rho c V_i}$$

$$i = N \qquad j = M$$

For the node on the bottom of the outer curved surface

$$k\left[\frac{T_{i-1,j,k} - T_{i,j,k}}{\Delta r} A_{cii} + \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta z} A_{fi}\right] = \rho\, c\, V_i\, \frac{T_{i,j,k+1} - T_{i,j,k}}{\Delta t}$$

Solving for the node temperature

$$T_{i,j,k+1} = T_{i,j,k} + \frac{\alpha\Delta t}{V_i}\left[\frac{T_{i-1,j,k} - T_{i,j,k}}{\Delta r} A_{cii} + \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta z} A_{fi}\right]$$

$$i = N \qquad j = 1$$

For the interior nodes on the bottom surface

$$k\left[\frac{T_{i-1,j,k} - T_{i,j,k}}{\Delta r} A_{cii} + \frac{T_{i+1,j,k} - T_{i,j,k}}{\Delta r} A_{\infty i} + \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta z} A_{fi}\right] = \rho\, c\, V_i\, \frac{T_{i,j,k+1} - T_{i,j,k}}{\Delta t}$$

Solving for the node temperature

$$T_{i,j,k+1} = T_{i,j,k} + \frac{\alpha\Delta t}{V_i}\left[\frac{T_{i-1,j,k} - T_{i,j,k}}{\Delta r} A_{cii} + \frac{T_{i+1,j,k} - T_{i,j,k}}{\Delta r} A_{\infty i} + \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta z} A_{fi}\right]$$

$$i = 2, 3, \ldots N-1 \qquad j = 1$$

Finally, for the interior nodes on the top surface

$$k\left[\frac{T_{i-1,j,k} - T_{i,j,k}}{\Delta r} A_{cii} + \frac{T_{i+1,j,k} - T_{i,j,k}}{\Delta r} A_{\infty i} + \frac{T_{i,j-1,k} - T_{i,j,k}}{\Delta z} A_{fi}\right] + \overline{q_i} = \rho\, c\, V_i\, \frac{T_{i,j,k+1} - T_{i,j,k}}{\Delta t}$$

Solving for the node temperature

$$T_{i,j,k+1} = T_{i,j,k} + \frac{\alpha\Delta t}{V_i}\left[\frac{T_{i-1,j,k} - T_{i,j,k}}{\Delta r} A_{cii} + \frac{T_{i+1,j,k} - T_{i,j,k}}{\Delta r} A_{\infty i} + \frac{T_{i,j-1,k} - T_{i,j,k}}{\Delta z} A_{fi}\right] + \frac{\overline{q_i}\,\Delta t}{\rho c V_i}$$

$$i = 2, 3, \ldots N-1 \qquad j = M$$

PROBLEM 3.48

Solve the set of difference equations derived in Problem 3.46 given the following values of the problem parameters

k = 40.0 W/(mK), disk thermal conductivity

$\alpha = 1 \times 10^{-5}$ m^2/s, disk thermal diffusivity

R_s = 25 mm, disk radius

Z_s = 5 mm, disk thickness

q''_{max} = 3×10^6 W/m^2, peak absorbed flux

R_o = 50 mm, parameter in flux distribution

T_{init} = 20°C, disk initial temperature

Determine the temperature distribution in the disk when the maximum temperature is 300°C.

GIVEN

- Difference equations developed in Problem 3.46 given the following values of the problem parameters

FIND

(a) Disk temperature distribution when the maximum temperature is 300°C

SOLUTION

All 9 difference equations can be written in the form

$$T_{i,j,k+1} = T_{i,j,k}\left(1 - \frac{\alpha\Delta t}{V_i}\left(R_{i,j} + L_{i,j} + U_{i,j} + D_{i,j}\right)\right)$$

$$+ \frac{\alpha\Delta t}{V_i}\left(R_{i,j}\, T_{i+1,j,k} + L_{i,j}\, T_{i-1,j,k} + U_{i,j}\, T_{i,j+1,k} + D_{i,j}\, T_{i,j-1,k}\right) + C_{i,j}$$

Where the coefficients $C_{i,j}$, $R_{i,j}$, $L_{i,j}$, $U_{i,j}$, $D_{i,j}$ are defined in the table below. Note that to use the above general equation for all nodes, we must allow the matrix of node temperatures to extend to indices i = 0, $i = N + 1$, j = 0, and $j = M + 1$. Temperatures at these nodes do not have meaning, but to apply the general difference equation along the axis or on the boundaries, they must be defined. Their values do not matter because the coefficients are set up to account for the special equations on the axis or on the boundaries.

Table of Coefficients for the Difference Equation

$C_{i,j}$	$R_{i,j}$	$L_{i,j}$	$U_{i,j}$	$D_{i,j}$	Applicable Range of i	Applicable Range of j
0	$A_{coi}/\Delta r$	$A_{cii}/\Delta r$	$A_{fi}/\Delta z$	$A_{fi}/\Delta z$	2, 3 … $N-1$	2, 3 … $M-1$
0	$A_{coi}/\Delta r$	0	$A_{fi}/\Delta z$	$A_{fi}/\Delta z$	1	2, 3 … $M-1$
$\frac{\Delta t\,\overline{q_i}}{\rho c V_i}$	$A_{coi}/\Delta r$	0	0	$A_{fi}/\Delta z$	1	M
0	$A_{coi}/\Delta r$	0	$A_{fi}/\Delta z$	0	1	1
0	0	$A_{cii}/\Delta r$	$A_{fi}/\Delta z$	$A_{fi}/\Delta z$	N	2, 3 … $M-1$
$\frac{\Delta t\,\overline{q_i}}{\rho c V_i}$	0	$A_{cii}/\Delta r$	0	$A_{fi}/\Delta z$	N	M
0	0	$A_{cii}/\Delta r$	$A_{fi}/\Delta z$	0	N	1
$\frac{\Delta t\,\overline{q_i}}{\rho c V_i}$	$A_{coi}/\Delta r$	$A_{cii}/\Delta r$	0	$A_{fi}/\Delta z$	2, 3 … $N-1$	M
0	$A_{coi}/\Delta r$	$A_{cii}/\Delta r$	$A_{fi}/\Delta z$	0	2, 3 … $N-1$	1

To maintain a positive coefficient on $T_{i, j, k}$ for stability we must have

$$\frac{\alpha \Delta t}{V_i}(R_{i,j} + L_{i,j} + U_{i,j} + D_{i,j}) < 1 \qquad i = 1, 2, \ldots N \quad j = 1, 2 \ldots M$$

The largest permissible time step Δt is therefore

$$\Delta t_{\max} = \left\{ \frac{V_i}{\alpha\left(R_{i,j} + L_{i,j} + U_{i,j} + D_{i,j}\right)} \right\}\Bigg|_{\max}$$

We will use some fraction of this time step in the program execution.

Note that we must first calculate the coefficients R, L, U, and D before determining $\Delta t_{\max}$. Then we can fill in the coefficients C.

The Pascal program listed below solves the difference equations as described above.

```
Program Prob3_M;       {solution to Problem 3_M}
uses crt, printer;
const   N = 11;        {radial nodes}
        M = 21;        {axial nodes}
        k = 40.0;      {thermal conductivity (W/mK)}
        alpha = 1e–5;  {thermal diffusivity (m^2/s)}
        Rs = 0.025;    {disk radius (m)}
        Zs = 0.005;    {disk thickness (m)}
        qmax = 3.0e6;  {peak flux (W/m^2)}
        Ro = 0.05;     {parameter in flux equation (m)}
        Tinit = 20.0;  {initial temperature (C)}
        Tmax = 300.0;  {maximum desired temperature at top of axis (C)}
var     dr, dz, rhoC, dtMax, dt, time:real;
        i, j : integer;
        C,R,L,U,D : array [1..N,1..M] of real;
        q,V,Aco,Aci,Af : array [1..N] of real;
        Toid,Tnew : array [1..N+1,1..M+1] of real;
begin
        {calculate size of the control volumes}
        dr : = Rs/(N – 1);
        dz : = Zs/(M – 1);
        rhoC: = k/alpha;

        {calculate control volume surface areas and volumes}
        Af[1]  : = pi*dr*dr/4.0;
        Af[N]  : = pi*dr*(Rs – dr/4.0);
        for i  : = 2 to N – 1 do Af[i]  : = 2.0*pi*dr*dr*(i – 1);
        for i  : = 1 to N do V[i] : = Af[i]*dz;

        Aco[N] : = 2.0*pi*Rs*dz;
        for i : = 1 to N – 1 do Aco[i] : = 2.0*pi*dr*dz*(i – 0.5);

        Aci[1] : = 0.0;
        for i : = 2 to N do Aci[i] : = 2.0*pi*dr*dz*(i – 1.5);

        {calculate absorbed flux as function of i}
        q [1] :  = pi/4.0*qmax*dr*dr*(1.0 – 0.9/8.0*sqr(dr/Ro));
        q [N]  : = pi*qmax*dr*dr*(N – 1.25–0.9/8.0*sqr(dr/Ro)*(2.0*N*N*N –7.5*N*N
        + 9.5*N – 65.0/16.0));
        for i :  = 2 to N – 1 do
        q [i]  : = pi*qmax*dr*dr*(2.0*(1 – 1.0) – 0.9/4.0*sqr(dr/Ro)*(4.0*i*i*i – 12.0*i*
        i + 13.0*i – 5.0));

        {fill in the coefficient matrices}

        for i : = 1 to N do    {first, zero all of them out}
        for j : = 1 to M do
        begin
               R[i, j] :  = 0.0;
```

```
        L[i, j] :  = 0.0;
        U[i, j] :  = 0.0;
        D[i, j] :  = 0.0;
        C[i, j] :  = 0.0;
end

for i : = 2 to N – 1 do
for j : = 2 to M – 1 do
begin
        R[i, j]  :  = Aco[i]/dr;
        L[i, j]  :  = Aci[i]/dr;
        U[i, j]  :  = Af[i]/dz;
        D[i, j]  :  = Af[i]/dz;
end;

i : = 1;
for j : = 2 to M – 1 do
begin
        R[i, j] : = Aco[i]/dr;
        U[i, j] : = Af[i]/dz;
        D[i, j] : = Af[i]/dz;
end;

i : = 1;
j : = M;
R[i, j]  : = Aco[i]/dr;
D[i, j]  : = Af[i]/dz;
i : = 1;
j : = 1;
R[i, j]  : = Aco[i]/dr;
U[i, j]  : = Af[i]/dz;

i : = N;
for j : = 2 to M – 1 do
begin
        L[i, j]  : = Aci[i]/dr;
        U[i, j]  : = Af[i]/dz;
        D[i, j]  : = Af[i]/dz;
end;

i : = N;
j : = M;
L[i, j]  : = Aci[i]/dr;
D[i, j]  : = Af[i]/dz;

i : = N;
j : = 1;
L[i, j]   : = Aci[i]/dr;
U[i, j]   : = Af[i]/dz;


j : = M;
for I : = 2 to N – 1 do
begin
        R[i, j]   :  = Aco[i]/dr;
        L[i, j]   :  = Aci[i]/dr;
        D [i, j]  :  = Af[i]/dz;
end;

j : = 1;
for I : = 2 to N – 1 do
begin
        R [i, j]  :  = Aco[i]/dr;
        L [i, j]  :  = Aci[i]/dr;
        U [i, j]  :  = Af[i]/dz;
end;

{find maximum permissible dt}
dtMax : = 0.0;
```

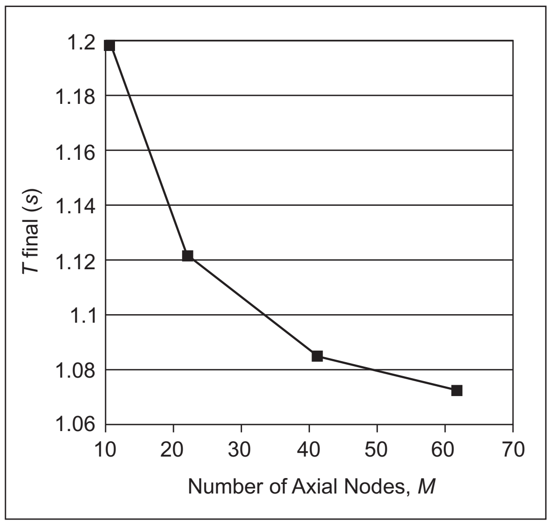
```
for i : = 1 to N do
for j : = 1 to M do
begin
       dt : = V[i]/alpha/(R[i, j] + L[i, j] + U[i, j] + D[i, j]);
       if = dt > dtMax then dtMax : = dt;
end;
dt : = 0.5*dtMax;      {actual value to be used in solution}

{fill in the cij matrix}
for i : = 1 to N do C[i, M] : = dt*q [i]/rhoC/V[i];

{establish the initial conditions}
for i : = 1 to N do
for j : = 1 to M do
Told [i, j] : = Tinit;

{carry out the solution}
time : = 0.0;
repeat
       time: = time + dt;
       writeln (time: 10:5);
       for i: = 1 to N do
       for j: = 1 to M do
       Tnew [i,j] : = Told [i,j]*(1.0 – alpha*dt/V[i]*(R[i,j]+ L[i,j]+ U[i,j]+
       D[i,j])
+ alpha*dt/V[i]*(R[i, j]*Told [i + 1, j] + L[i, j]*Told [i – 1, j] + U[i, j]*Told
[i, j + 1] + D[i, j]*Told [i, j – 1]) + C[i, j];

if Tnew[1, m] > Tmax then     {print out distribution and quit}
begin
       writeln(1st, time : 8 : 4, 'sec dt = ', dt : 15 : 10);
       write(1st,'   ');
       for i : = 1 to N do write (1st, 1 : 10);
       writeln(1st);
       for j : = M downto 1 do
       begin
              write(1st, j : 4);
              for i : = 1 to N do write(1st, Tnew[i, j]; 10 : 5);
              writeln(1st);
       end;
       writeln(1st);
       halt;
end

(otherwise, keep going}
for i : = 1 to N do
for j : = 1 to M do
Told[i, j] ; = Tnew[i, j];

until time < - 1.0;

end.
```

Now, we need to determine the node spacing and Δt required for an accurate solution. As suggested in the text, trial and error is the best method.

First, let's determine the required spatial resolution, that is, the values of N and M. Since the gradients in the radial direction are small compared to those in the axial direction, we don't expect much influence on the results by varying N so we will use $N = 11$. Now, pick values of $M = 11$, 21, 41, and 61. A time step of $\Delta t = 0.0003$ s will give stable results for all these values of M. The table below gives the results for these 4 runs.

M	Time (s) for T_{max} = 300°C	T [1, 1]	T [N, 1]	T [N, M]
11	1.1979	116.29	109.86	281.97
21	1.1223	115.75	109.32	282.05
41	1.0845	115.49	109.06	282.08
61	1.0719	115.40	108.97	282.09

Notice that the temperatures in the table are not significantly affected by M but the time required to reach 300°C is somewhat sensitive. This is displayed in the graph shown to the right. It appears that the time gradually decreases but between M = 41 and M = 61, the graph levels off significantly. At M = 81, the time would probably be somewhat less than that at M = 61 but clearly we have reached the point of diminishing return. We will choose M = 41 as begin a reasonable compromise.



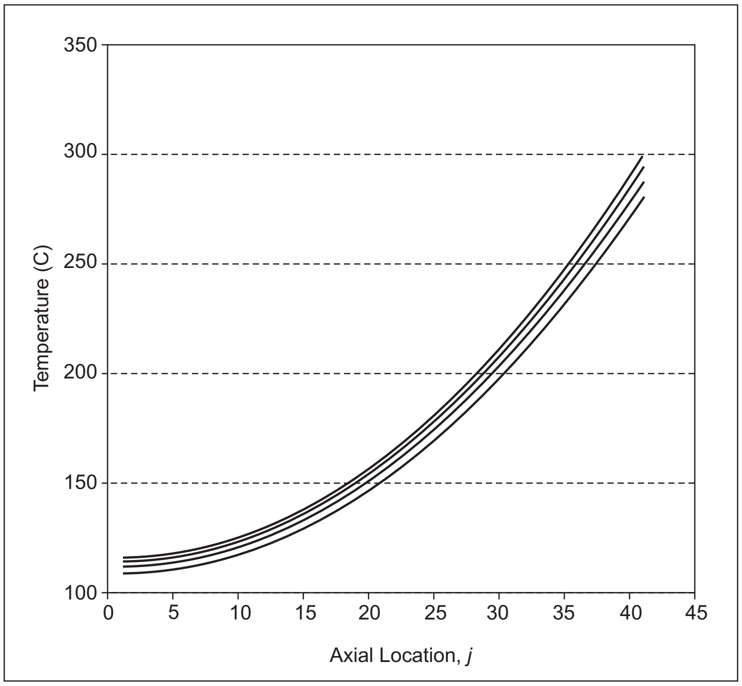
Time Required to Reach a Maximum Temperature of 300°C as a Function of Number of Axial Nodes

Next, we need to determine the appropriate time step, Δt. For N = 11, M = 41, the maximum permissible time step according to the equation given previously is 0.00155 s. In practice, values larger than 1/2 of this maximum result in instability. Running with 0.00015, 0.0003, and 0.0006 seconds, we find

Δt (s)	Time (s) for T_{max} = 300°C	T [1, 1]	T [N, 1]	T [N, M]
0.00015	1.0845	115.488	109.06	282.08
0.0003	1.0845	115.48659	109.06	282.08
0.0006	1.0848	115.52582	109.09	282.13

From the results in the table, it is clear that there is little benefit from a time step of less than 0.0003 s. For a reasonable compromise, choose Δt = 0.0006 s.

Using these choices, M = 41, N = 11, and Δt = 0.0006 s, the results are plotted below



Temperature Distribution along Six Axial Lines

The solution shows that 1.0848 seconds is required for the maximum temperature in the disk to reach 300°C. Furthermore, the graph demonstrates that the temperature gradient axially through the disk is not especially large as was required for the case hardening application. This indicates that the incident flux needs to be increased.

PROBLEM 3.49

Consider two-dimensional steady conduction near a curved boundary. Determine the difference equation for an appropriate control volume near the node (*i, j*). The boundary experiences convective heat transfer through a coefficient *h* to ambient temperature T_a. The surface of the boundary is given by $y_s = f(x)$.

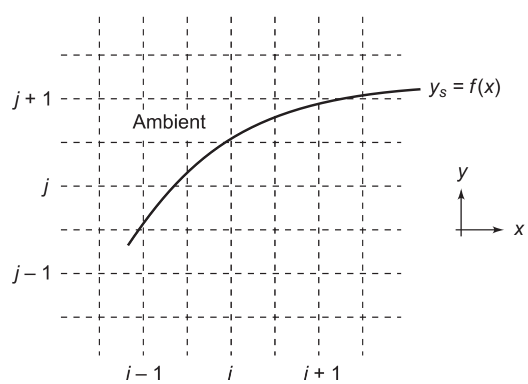
GIVEN

- Two-dimensional steady conduction near a curved surface
- Convective boundary condition
- Curved surface given by $y_s = f(x)$

FIND

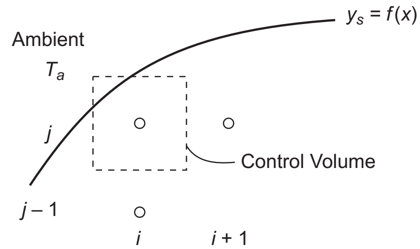
(a) Difference equation for a node *i, j* near the surface

SKETCH



SOLUTION

Consider a control volume for the node i, j as shown below



Heat can flow into or out of the control volume at four surfaces. An energy balance on the control volume is given by

$$k \left\{ \frac{\left(T_{i,j-1} - T_{i,j}\right)}{\Delta y} \Delta x + \frac{\left(T_{i+1,j} - T_{i,j}\right)}{\Delta x} \Delta y \right\} + h_c \{(T_a - T_{i,j})\,\Delta x + (T_a - T_{i,j})\,\Delta y\} = 0$$

PROBLEM 3.50

Derive the control volume energy balance equation for three-dimensional transient conduction with heat generation in a rectangular coordinate system.

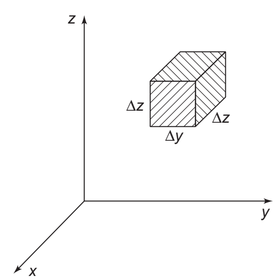
GIVEN

- Three-dimensional transient conduction with heat generation in a rectangular coordinate system

FIND

(a) The control volume energy balance equation

SKETCH



SOLUTION

The control volume, in an x, y, z coordinate system, is shown in the sketch above. Define the nodal indices as follows

$$x = (i - 1)\,\Delta x$$

$$y = (j - 1)\,\Delta y$$

$$z = (1 - 1)\,\Delta z$$

and for simplicity define

$$T \equiv T_{i,j,1,m}$$

Now, heat conducted into the control volume is

$$k\Big(\left(T_{i+1,j,l,m} - T + T_{i-1,j,l,m} - T\right)\frac{\Delta y \Delta z}{\Delta x} + \left(T_{i,j+1,l,m} - T + T_{i,j-1,l,m} - T\right)\frac{\Delta x \Delta z}{\Delta y}$$

$$+\left(T_{i,j,l+1,m}-T+T_{i,j,l-1,m}-T\right)\frac{\Delta x\Delta y}{\Delta z}\Bigg)$$

The heat generated in the control volume is

$$\dot{q}_{G,i,j,1,m}\ \Delta x\ \Delta y\ \Delta z$$

and the rate at which thermal energy is stored in the control volume is

$$\rho\, c\ \Delta x\ \Delta y\ \frac{T_{i,j,l,m+1}-T}{\Delta t}$$

Since the heat conducted into the control volume plus the rate at which heat is generated in the control volume must equal the rate at which energy is stored in the control volume, the difference equation is

$$k\Bigg(\left(T_{i+1,j,l,m}-2T+T_{i-1,j,l,m}\right)\frac{\Delta y\Delta z}{\Delta x}+\left(T_{i,j+1,l,m}-2T+T_{i,j-1,l,m}\right)\frac{\Delta x\Delta z}{\Delta y}$$

$$+\left(T_{i,j,l+1,m}-2T+T_{i,j,l-1,m}\right)\frac{\Delta x\Delta y}{\Delta z}\Bigg)$$

$$+\ \dot{q}_{G,i,j,1,m}\ \Delta x\ \Delta y\ \Delta z=\rho\, c\ \Delta x\ \Delta y\ \Delta z\ \frac{T_{i,j,l,m+1}-T}{\Delta t}$$

Dividing by $k\ \Delta x\ \Delta y\ \Delta z$ we have

$$\frac{T_{i+1,j,l,m}-2T+T_{i-1,j,l,m}}{\Delta x^2}+\frac{T_{i,j+1,l,m}-2T+T_{i,j-1,l,m}}{\Delta y^2}+\frac{T_{i,j,l+1,m}-2T+T_{i,j,l-1,m}}{\Delta z^2}$$

$$+\frac{\dot{q}_{G,i,j,l,m}}{k}=\frac{1}{\alpha}\frac{T_{i,j,l,m+1}-T}{\Delta t}$$

PROBLEM 3.51

Derive the energy balance equation for a corner control volume in a three-dimensional steady conduction problem with heat generation in a rectangular coordinate system. Assume an adiabatic boundary condition and equal node spacing in all three dimensions.

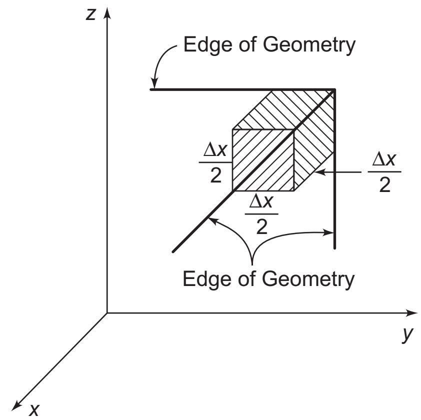
GIVEN

- Three-dimensional steady conduction in a rectangular coordinate system, corner boundary control volume with specified temperature boundary condition

FIND

(a) Energy balance equation for the control volume

SKETCH



SOLUTION

First, define the nodal indices as follows

$$x = (i-1)\,\Delta x \qquad y = (j-1)\,\Delta y \qquad z = (l-1)\,\Delta z$$

and for simplicity, let

$$T \equiv T_{i,j,l,m}$$

where, as usual, m is the time index. Note that the volume of the control volume is

$$\frac{\Delta x \Delta y \Delta z}{8}$$

Referring to the sketch above, we see that there are three surfaces across which heat is transferred by conduction. For these surfaces, the heat transferred into the control volume is

$$k\left\{\frac{T_{i+1,j,l,m}-T}{\Delta x}\frac{\Delta y \Delta z}{4}+\frac{T_{i,j-1,l,m}-T}{\Delta y}\frac{\Delta x \Delta z}{4}+\frac{T_{i,j,l-1,m}-T}{\Delta z}\frac{\Delta y \Delta x}{4}\right\}$$

Heat generation in the control volume is

$$\dot{q}_{G,i,j,l,m}\frac{\Delta x \Delta y \Delta z}{8}$$

and the rate at which energy is stored in the control volume is

$$\rho\, c\,\frac{T_{i,j,l,m+1}-T}{\Delta t}\frac{\Delta x \Delta y \Delta z}{8}$$

The resulting energy balance equation for the control volume is

$$\frac{T_{i+1,j,l,m}+T_{i,j-1,l,m}+T_{i,j,l-1,m}-3T}{4\Delta x^2}+\frac{\dot{q}_{G,i,j,l,m}}{8k}=\frac{1}{8\alpha}\frac{T_{i,j,l,m+1}-T}{\Delta t}$$

PROBLEM 3.52

Determine the stability criterion for an explicit solution of three-dimensional transient conduction in a rectangular geometry.

GIVEN

- Three-dimensional transient conduction in a rectangular geometry

FIND

(a) The stability criterion for an explicit situation

SOLUTION

From the solution of Problem 3.49, the control volume energy balance equation is

$$\frac{T_{i+1,j,l,m} - 2T + T_{i-1,j,l,m}}{\Delta x^2} + \frac{T_{i,j+1,l,m} - 2T + T_{i,j-1,l,m}}{\Delta y^2} + \frac{T_{i,j,l+1,m} - 2T + T_{i,j,l-1,m}}{\Delta z^2}$$

$$+ \frac{\dot{q}_{G,i,j,l,m}}{k} = \frac{1}{\alpha}\frac{T_{i,j,l,m+1} - T}{\Delta t}$$

so the coefficient on T is

$$-2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right) + \frac{1}{\alpha \Delta t}$$

Since this coefficient must be greater than zero to ensure stability, we have

$$\Delta t < \frac{1}{2\alpha}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right)^{-1}$$

Note that this expression is consistent with the extension from one-dimensional to two-dimensional stability criteria.

Chapter 4

PROBLEM 4.1

Evaluate the Reynolds number for flow over a tube from the following data

$$\boldsymbol{D} = \mathbf{6\ cm}$$
$$\boldsymbol{U_\infty} = \mathbf{1.0\ m/s}$$
$$\boldsymbol{\rho} = \mathbf{300\ kg/m^3}$$
$$\boldsymbol{\mu} = \mathbf{0.04\ N\ s/m^2}$$

GIVEN

- D = 6 cm
- U_∞ = 1.0 m/s
- ρ = 300 kg/m^3
- μ = 0.04 N s/m^2

FIND

- The Reynolds Number (Re)

SOLUTION

The Reynolds number, from Table 4.3, is

$$Re = \frac{U_\infty L}{v} = \frac{U_\infty L \rho}{\mu}$$

The Reynolds number based on the tube diameter is

$$Re = \frac{U_\infty D \rho}{\mu} = \frac{(1\,\text{m/s})\,(6\,\text{cm})(1\,\text{m}/(100\,\text{cm}))\left(300\,\text{kg/m}^3\right)}{\left(0.04\,(\text{Ns})/\text{m}^2\right)\left(\text{kg m}/(\text{s}^2\text{N})\right)} = 450$$

PROBLEM 4.2

Evaluate the Prandtl number from the following data:

$$\boldsymbol{c_p} = \mathbf{2.1\ kJ/kg\ K}, \quad \boldsymbol{k} = \mathbf{3.4\ W/mK}, \quad \boldsymbol{\mu} = \mathbf{0.45\ kg/ms}$$

GIVEN

- c_p = 2.1 kJ/kg K
- k = 3.4 W/mK
- μ = 0.45 kg/ms

FIND

- The Prandtl number (P_r)

SOLUTION

Prandtl number is given by Eqn. (4.8)

$$P_r = \frac{c_p \mu}{k} = \frac{(2.1\,\text{kJ/kg K})\,(0.45\,\text{kg/ms})}{(3.4 \times 10^{-3}\ \text{kW/mK})}$$

$$P_r = 278$$

PROBLEM 4.3

Evaluate the Nusselt number for flow over a sphere for the following conditions

$$\boldsymbol{D} = \mathbf{0.15\ m}$$
$$\boldsymbol{k} = \mathbf{0.2\ W/(m\ K)}$$
$$\boldsymbol{h_c} = \mathbf{102\ W/(m^2\ K)}$$

GIVEN

- $D = 0.15$ m
- $k = 0.2$ W/(m K)
- $h_c = 102$ W/(m^2 K)

FIND

- The Nusselt number (Nu)

SOLUTION

The Nusselt number is given by Equation (4.18)

$$Nu = \frac{h_c L}{k} \equiv \frac{h_c D}{k} \text{ if D is characteristic length.}$$

Based on the diameter of the sphere, the Nusselt number is

$$Nu_D = \frac{h_c D}{k} = \frac{\left(102\ \text{W} / (\text{m}^2\ \text{K})\right)(0.15\ \text{m})}{0.2\,\text{W}/(\text{m}\,\text{K})} = 76.5$$

PROBLEM 4.4

Evaluate the Stanton number for flow over a tube from the data below

$$\boldsymbol{D} = \mathbf{10\ cm}$$
$$\boldsymbol{U_\infty} = \mathbf{4\ m/s}$$
$$\boldsymbol{\rho} = \mathbf{13{,}000\ kg/m^3}$$
$$\boldsymbol{\mu} = \mathbf{1 \times 10^{-3}\ N\ s/m^2}$$
$$\boldsymbol{c_p} = \mathbf{140\ J/(kg\ K)}$$
$$\boldsymbol{h_c} = \mathbf{1000\ W/(m^2\ K)}$$

GIVEN

- $D = 10$ cm
- $U_\infty = 4$ m/s
- $\rho = 13{,}000$ kg/m^3
- $\mu = 1 \times 10^{-3}$ N s/m^2
- $c_p = 140$ J/(kg K)
- $\bar{h}_c = 1000$ W/(m^2 K)

FIND

- The Stanton number (St) for flow over a tube

SOLUTION

The Stanton number is given in Table 4.3 as

$$St = \frac{\bar{h}_c}{\rho U_\infty c_p}$$

The Stanton number based on the average heat transfer coefficient is

$$St = \frac{\bar{h}_c}{\rho U_\infty c_p} = \frac{[1000\,\mathrm{W/(m^2K)}]}{13{,}000\,\mathrm{kg/m^3}\,(4\,\mathrm{m/s})(140\,\mathrm{J/(kgK)})(\mathrm{Ws/J})} = 1.37 \times 10^{-4}$$

PROBLEM 4.5

Evaluate the dimensionless groups h_cD/k, $U_\infty D\rho/\mu$, $c_p\,\mu/k$ for water, n-Butyl alcohol, mercury, hydrogen, air, and saturated steam at a temperature of 100°C. Let D = 1 m, U_∞ = 1 m/sec, and h_c = 1 W/(m^2 K).

GIVEN

- D = 1 m
- U_∞ = 1 m/s
- h_c = 1 W/(m^2 K)

FIND

- The dimensionless groups
- $h_c\,D/k$ (Nusselt number)
- $U_\infty D\,\rho/\mu$ (Reynolds number)
- $c_p\,\mu/k$ (Prandtl number)

PROPERTIES AND CONSTANTS

From Appendix 2, at 100°C

Substance	Table Number	Density, ρ (kg/m^3)	Specific Heat, c_p ((J/kg K))	Thermal Conductivity (W/(m K))	Absolute Viscosity $\mu \times 10^6$ (N s/m^2)
Water	13	958.4	4211	0.682	277.5
n-Butyl Alcohol	18	753	3241	0.163	540
Mercury	25	13,385	137.3	10.51	1242
Hydrogen	31	0.0661	14,463	0.217	10.37
Air	27	0.916	1022	0.0307	21.673
Saturated Steam	34	0.5977	2034	0.0249	12.10

SOLUTION

For water at 100°C

$$Nu = \frac{h_cD}{k} = \frac{[1\,\mathrm{W/(m^2\,K)}](1\,\mathrm{m})}{[0.682\,\mathrm{W/(m\,K)}]} = 1.47$$

$$Re_D = \frac{U_\infty D\rho}{\mu} = \frac{(1\,\mathrm{m/s})(1\,\mathrm{m})\left(958.4\,\mathrm{kg/m^3}\right)}{\left(277.5\times10^{-6}\,\mathrm{Ns/m^2}\right)\left(\mathrm{kg\,m/(s^2N)}\right)} = 34 \times 10^6$$

$$Pr = \frac{c_p\mu}{k} = \frac{[4211\,\mathrm{J/(kg\,K)}]\left(277.5\times10^{-6}\,\mathrm{(Ns)/m^2}\right)}{[0.682\,\mathrm{W/(m\,K)}]\left(\mathrm{Ns^2/(kg\,m)}\right)} = 1.71$$

The dimensionaless groups for the other substances can be calculated in a similar manner

Substance	Nu	Re_D	Pr
Water	1.47	3.4×10^6	1.71
n-Butyl Alcohol	6.13	$14. \times 10^6$	10.73
Mercury	0.10	1.1×10^7	0.016
Hydrogen	4.61	6.3×10^3	0.694
Air	32.6	4.2×10^4	0.721
Saturated Steam	40.2	4.9×10^4	0.988

PROBLEM 4.6

Suppose a fluid from the list below flows at 5 m/s over a flat plate 15 cm long. Calculate the Reynolds number at the downstream end of the plate. Indicate if the flow at that point is laminar, transition, or turbulent. Assume all fluids are at 40°C.

(a) Air

(b) CO_2

(c) Water

(d) Engine Oil

GIVEN

- A fluid flows over a flat plate
- Fluid velocity (U_∞) = 5 m/s
- Length of plate (L) = 15 cm = 0.15 m
- Fluid temperature = 40°C

FIND

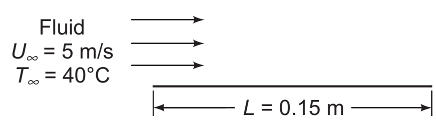
- The Reynolds number at the downstream end of the plate (Re_L) for
  - (a) Air
  - (b) CO_2
  - (c) Water
  - (d) Engine Oil

Indicate if the flow is laminar, transitional, or turbulent

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

At 40°C, the kinematic viscosities of the given fluids are as follows

From Appendix 2, Table 27 for Air (v_a) = 17.6×10^{-6} m^2/s

From Appendix 2, Table 28 for CO_2 (v_c) = 9.07×10^{-6} m^2/s

From Appendix 2, Table 13 for Water (v_w) = 0.658×10^{-6} m^2/s

From Appendix 2, Table 16 for Engine Oil (v_o) = 240×10^{-6} m^2/s

SOLUTION

The Reynolds number, from Table 4.3, is

$$\frac{Re = U_\infty L}{v}$$

The transition from laminar to turbulent flow over a plate occurs at a Reynolds number of about 5×10^5.

For air

$$Re_L = \frac{(5\,\text{m/s})(0.15\,\text{m})}{17.6\times10^{-6}\,\text{m}^2/\text{s}} = 4.3 \times 10^4 \text{ (Laminar)}$$

For CO_2

$$Re_L = \frac{(5\,\text{m/s})(0.15\,\text{m})}{9.07\times10^{-6}\,\text{m}^2/\text{s}} = 8.3 \times 10^4 \text{ (Laminar)}$$

For water

$$Re_L = \frac{(5\,\text{m/s})(0.15\,\text{m})}{0.658\times10^{-6}\,\text{m}^2/\text{s}} = 1.1 \times 10^6 \text{ (Turbulent)}$$

For engine oil

$$Re_L = \frac{(5\,\text{m/s})(0.15\,\text{m})}{240\times10^{-6}\,\text{m}^2/\text{s}} = 3.1 \times 10^3 \text{ (Laminar)}$$

PROBLEM 4.7

Replot the data points of Figure 4.9(b) on log-log paper and find an equation approximating the best correlation line. Compare your results with Figure 4.10. Then, suppose steam at 1 atm and 100°C is flowing across a 5 cm-*OD* pipe at a velocity of 1 m/s. Using the data in Figure 4.10, estimate the Nusselt number, the heat transfer coefficient, and the rate of heat transfer per meter length of pipe if the pipe is at 200°C and compare with predictions from your correlation equation.

GIVEN

- Figure 4.9(b) in text
- Steam flowing across a pipe
- Steam pressure = 1 atm
- Steam temperature (T_s) = 100°C
- Pipe outside diameter (D) = 5 cm = 0.05 m
- Steam velocity (U_∞) = 1 m/s
- Pipe temperature (T_p) = 200°C

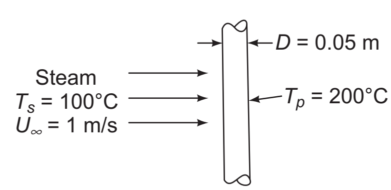
FIND

(a) Replot Figure 4.9(b) on log-log paper and find an equation approximating the best correlation line
(b) Find the Nusselt number (*Nu*), the heat transfer coefficient (*hc*), and the rate of heat transfer per unit length (*q*/*L*) using Figure 4.10
(c) Compare results with your correlated equation

ASSUMPTIONS

- Steady state
- Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 34, for steam at 1 atm and 100°C

Thermal conductivity (k) = 0.0249 W/(m K)

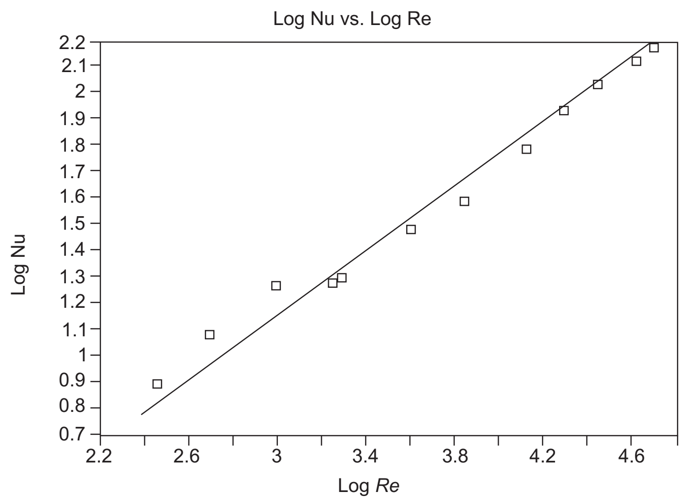
Kinematic viscosity (v) = 20.2×10^{-6} m^2/s

Thermal diffusivity (α) = 0.204×10^{-4} m^2/s

SOLUTION

(a) The data taken from Figure 4.9(b) is shown below and plotted on a log-log scale

Re	*Nu*	Log *Re*	Log *Nu*
240	9	2.38	0.95
500	12	2.70	1.08
1,000	18	3.00	1.26
1,800	19	3.26	1.28
2,000	20	3.30	1.30
4,100	30	3.61	1.48
7,000	39	3.85	1.59
13,500	62	4.13	1.79
20,000	88	4.30	1.94
28,000	110	4.45	2.04
42,000	135	4.62	2.13
50,000	150	4.70	2.18



□ Figure 4.9(b) Variation of Nusselts Number with Reynolds Number on a log-log Scale

Fitting this data with a linear least squares regression yields:

$$\log Nu = 0.615 \log Re_D - 0.687$$

or

$$Nu = 0.21\, Re_D^{0.615}$$

(b) For the given data

$$Re_D = \frac{U_\infty D}{v} = \frac{(1.0\,\text{m/s})(0.05\,\text{m})}{20.2\times10^{-6}\,\text{m}^2/\text{s}} = 2475$$

$$Pr = \frac{v}{\alpha} = \frac{20.2\times10^{-6}\,\text{m}^2/\text{s}}{0.204\times10^{-6}\,\text{m}^2/\text{s}} = 0.990$$

Although Figure 4.10 applies to Reynolds numbers between 3 and 100, we will apply its results to the larger Reynolds number for this case for the purpose of comparison

$$\frac{Nu_D}{Pr^{0.3}} = 0.82\, Re_D^{\,0.4}$$

$$Nu_D = 0.82\, Pr^{0.3}\, Re_D^{\,0.4} = 0.82\ (0.99)_{0.3}\ (2475)^{0.4} = 18.6$$

From Table 4.3

$$Nu_D = \frac{h_c D}{k}$$

$$\therefore \qquad h_c = \frac{Nu_D k}{D} = \frac{18.6\left(0.0249\ \text{W/(m K)}\right)}{0.05\ \text{m}} = 9.27\ \text{W/(m}^2\ \text{K)}$$

The rate of convective heat transfer is given by Equation (1.10)

$$q = h_c\, A\, \Delta T = h_c\, \pi D\, L\, (T_p - T_s)$$

$$\therefore \qquad \frac{q}{L} = h_c\, \pi D\, (T_p - T_s) = \left(9.27\ \text{W/(m}^2\ \text{K)}\right)\ \pi (0.05\ \text{m})\ (200°\text{C} - 100°\text{C}) = 145.6\ \text{W/m}$$

(c) The correlation from part (a) yields

$$Nu = 0.21\ (2475)^{0.615} = 25.7$$

$$h_c = \frac{Nu\, k}{D} = \frac{25.7\left(0.0249\ \text{W/(m K)}\right)}{0.05\ \text{m}} = 12.8\ \text{W/(m}^2\ \text{K)}$$

$$\frac{q}{L} = h_c\, \pi D\, (T_p - T_s) = \left(12.8\ \text{W/(m}^2\text{K)}\right)\ \pi (0.05\ \text{m})\ (200°\text{C} - 100°\text{C}) = 201.0\ \text{W/m}$$

The results obtained from Figure 4.10 are 28% lower than these results.

COMMENTS

The use of Figure 4.10 for a Reynolds number larger than 100 is inappropriate and in this case leads to a significant underestimation of the heat transfer coefficient. On the other hand, the correlation equation we developed from Figure 4.9(b) is strictly valid for air only. Since the Prandtl number for steam is different than that of air, we introduce an (unknown) error in using the data of Figure 4.9(b) to predict heat transfer to steam.

PROBLEM 4.8

The average Reynolds number for air passing in turbulent flow over a 2 m – long flat plate is 2.4 × 10^6. Under these conditions, the average Nusselt number was found to be equal to 4150. Determine the average heat transfer coefficient for an oil having thermal properties similar to those of Table A-17 at 30°C at the same Reynolds number in flow over the same plate.

GIVEN

- Turbulent flow of air over a flat plate
- Average Reynolds number (Re_L) = 2.4 × 10^6
- Plate length (L) = 2 m
- Average Nusselt number (Nu) = 4150

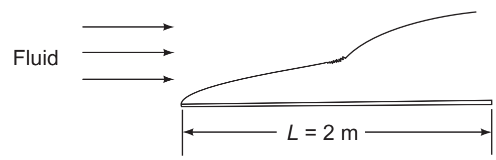
FIND

- Average heat transfer coefficient $\left(\bar{h}_c\right)$ for oil flowing at the same *Re* over the same plate

ASSUMPTIONS

- Steady state
- Fully developed turbulent flow
- Transition from laminar to turbulent flow occurs at $Re_x = 5 \times 10^5$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 17, for oil at 30°C

Thermal Conductivity (k) = 0.11 W/(m K)

Kinematic Viscosity (v) = 15.4×10^{-6} m^2/s

Thermal Diffusivity (α) = 707×10^{-10} m^2/s

SOLUTION

The Prandtl number for the oil is

$$Pr = \frac{v}{\alpha} = \frac{15.4\times10^{-6}\ \text{m}^2/\text{s}}{707\times10^{-10}\ \text{m}^2/\text{s}} = 218$$

The empirical correlation from Table 4.5 can be used to find the Nusselt number for the oil.

$$Nu_L = 0.036\ Pr^{0.33}\ [Re_L^{0.8} - 23{,}200] \text{ for } Re_L > 5 \times 10^5 \text{ and } Pr > 0.5$$

$$Nu_L = 0.036\ (218)^{0.33}\ [(2.4 \times 10^6)^{0.8} - 23{,}200] = 22{,}100$$

$$\bar{h}_c = \frac{Nu_L\, k}{D} = \frac{22{,}100(0.11\,\text{W/(m K)})}{2.0\,\text{m}} = 1216\ \text{W/(m}^2\,\text{K)}$$

PROBLEM 4.9

The dimensionless ratio $U_\infty / \sqrt{Lg}$, called Froude number, is a measure of similarity between an ocean-going ship and a scale model of the ship to be tested in a laboratory water channel. A 150 m long cargo ship is designed to run at 36 kmph, and a 1.5 m geometrically similar model is towed in a water channel to study wave resistance. What should be the towing speed in m s^{-1}?

GIVEN

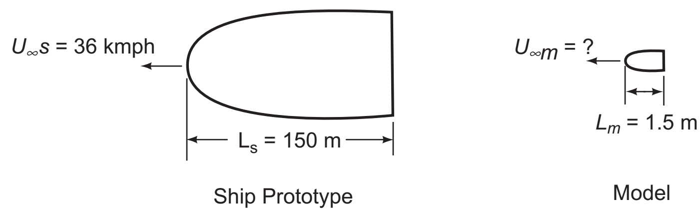
- A ship model and its prototype
- Froude number = $U_\infty / \sqrt{Lg}$ is a measure of similarity
- Ship length (L_s) = 150 m
- Ship speed ($U_\infty s$) = 36 kmph = 10 ms^{-1}
- Model length (L_m) = 1.5 m

FIND

- Model towing speed ($U_{\infty m}$)

ASSUMPTIONS

SKETCH



SOLUTION

For similar wave shape, the Froude number should be the same for the model and the prototype

$$\frac{U_{\infty m}}{\sqrt{L_m g}} = \frac{U_{\infty s}}{\sqrt{L_s g}} \Rightarrow U_{\infty m} = U_{\infty s} \sqrt{\frac{L_m}{L_s}}$$

$$U_{\infty m} = 10 \text{ ms}^{-1} \sqrt{\frac{1.5\text{m}}{150\text{m}}} = 1 \text{ ms}^{-1}$$

PROBLEM 4.10

> **The torque due to the frictional resistance of the oil film between a rotating shaft and its bearing is found to be dependent on the force *F* normal to the shaft, the speed of rotation *N* of the shaft, the dynamic viscosity μ of the oil, and the shaft diameter *D*. Establish a correlation among the variables by using dimensional analysis.**

GIVEN

- The oil film between a rotating shaft and its bearing
- The torque (T) due to frictional resistance is a function of normal force (F), speed of rotation (N), dynamic viscosity (μ), and shaft diameter (D)

FIND

- A correlation among the variables

ASSUMPTIONS

- Steady state

SOLUTION

The Buckingham π Theorem (Sections 4.7.2 and 4.7.3) can be used to find the correlation. The primary dimensions of the variables are listed below

	Variable	Symbol	Dimensions
1.	Normal Force	F	$[M L/t^2]$
2.	Speed of Rotation	N	$[1/t]$
3.	Dynamic Viscosity	μ	$[M/L\, t]$
4.	Shaft Diameter	D	$[L]$
5.	Torque	T	$[M L^2/t^2]$

There are 5 variables and 3 primary dimensions. Therefore, two dimensionless groups are needed to correlate the variables

$$\pi = T^a F^b N^c \mu^d D^e$$

In terms of the primary dimensions

$$[\pi] = \left[\frac{ML^2}{t^2}\right]^a \left[\frac{ML}{t^2}\right]^b \left[\frac{1}{t}\right]^c \left[\frac{M}{Lt}\right]^d \ [L]^e = 0$$

Equation the sum of the exponents of each primary dimension to zero

For μ: $a + b + d = 0$ [1]

For L: $2a + b - d + e = 0$ [2]

For t: $2a + 2b + c + d = 0$ [3]

By inspection of equation [1] and [3]: $c = d$

There are five unknowns but only 3 equations. Therefore, the value of two of the exponents can be chosen for each dimensionless group.

For π_1: Let $a = 1$ and $b = 0$

From equation [1] $d = -1 = c$

From equation [2] $e = -3$

$$\therefore \ \pi_1 = T\,N^{-1}\,\mu^{-1}\,D^{-3} = \frac{T}{N\mu D^3}$$

For π_2: Let $a = 0$ and $b = 1$

From equation [1] $d = -1 = c$

From equation [2] $e = -2$

$$\therefore \ \pi_2 = F\,N^{-1}\,\mu^{-1}\,D^{-2} = \frac{F}{N\mu D^2}$$

From Equation (4.24)

$$\pi_1 = f(\pi_2) \qquad \therefore \ \frac{T}{N\mu D^3} = f\left(\frac{F}{N\mu D^2}\right)$$

PROBLEM 4.11

When a sphere falls freely through a homogeneous fluid, it reaches a terminal velocity at which the weight of the sphere is balanced by the buoyant force and the frictional resistance to the fluid. Make a dimensional analysis of this problem and indicate how experimental data for this problem could be correlated. Neglect compressibility effects and the influence of surface roughness.

GIVEN

- A sphere falling freely through a homogeneous fluid
- Terminal velocity occurs when weight is balanced by buoyant force and friction resistance of the fluid

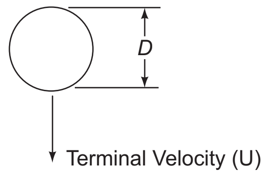
FIND

- Make a dimensional analysis and indicate how data may be correlated

ASSUMPTIONS

- Compressibility effects are negligible
- Influence of surface roughness is negligible

SKETCH



SOLUTION

The variables which must be correlated and their dimensions are shown below

	Variable	Symbol	Dimensions
1.	Acceleration of Gravity	g	$[L/t^2]$
2.	Density Difference	$\rho_s - \rho_f$	$[M/L^3]$
3.	Fluid Density	ρ_f	$[M/L^3]$
4.	Terminal Velocity	U	$[L/t]$
5	Sphere Diameter	D	$[L]$
6.	Fluid Viscosity	μ	$[M/L\ t]$

The density difference was chosen for variable 2 because we anticipate that this difference, rather than the sphere density, will be an important parameter. Clearly, if $\rho_s = \rho_f$, then $U = 0$. The Buckingham π Theorem (Section 4.7.2 and 4.7.3) can be used to correlate the variables. There are 6 variables and 3 primary dimensions. Therefore, 3 dimensionless groups will be found.

$$\pi = g^a\,(\rho_s - \rho_f)^b\,\rho_f^c\,U^d\,D^e\,\mu^f$$

Substituting the primary dimensions into the equation

$$[\pi] = \left[\frac{L}{t^2}\right]^a \left[\frac{M}{L^3}\right]^{b+c} \left[\frac{L}{t}\right]^d [L]^e \left[\frac{M}{Lt}\right]^f = 0$$

Equating the sum of the exponents of each primary dimension to zero:

For M: $b + c + f = 0$ [1]

For L: $a - 3b - 3c + d + e - f = 0$ [2]

For t: $2a + d + f = 0$ [3]

There are 6 unknowns and only 3 equations, therefore, the value of the 3 exponents can be chosen for each π

For π_1, Let $a = 0$, $b = 0$ and $c = 1$

From equation [1] $f = -1$

From equation [3] $d = 1$

From equation [2] $e = 1$

$$\therefore\ \pi_1 = U\,D\,\rho_f\,\mu^{-1} = \frac{UD\rho_f}{\mu} = Re_D$$

For π_2, Let $a = 1$, $b = 1$ and $f = 0$

From equation [1] $c = -1$

From equation [3] $d = -2$

From equation [2] $e = 1$

$$\therefore\ \pi_2 = g\,(\rho_s - \rho_f)\,U^{-2}\,D\,\rho_f^{-1} = \frac{g\left(\rho_s - \rho_f\right)D}{\rho_f U^2} = \frac{g\left(\rho_s - \rho_f\right)\frac{x}{6}D^3}{\frac{4}{3}\left(\frac{1}{2}\rho_f U^2\right)\frac{x}{4}D^4}$$

$$\pi_2 = \frac{\text{Weight of sphere in the fluid}}{\frac{4}{3}\left(\text{Dynamic pressure}\right)\times\left(\text{cross sectional area of sphere}\right)}$$

$$\pi_2 = \frac{3}{4}\frac{(\text{Drag force on sphere})/(\text{Cross sectional area})}{\text{Dynamic pressure}} = \frac{3}{4} C_D \text{ (Drag Coefficient)}$$

For π_3, Let $a = 0$, $b = 1$, and $f = 0$

From equation [1] $c = -1$

From equation [3] $d = 0$

From equation [2] $e = 0$

$$\therefore \quad \pi_3 = (\rho_s - \rho_f)\, \rho_f^{-1} = \frac{(\rho_s - \rho_f)}{\rho_f}$$

But this dimensionless group already appears in π_2. (This redundancy could have been avoided had we chosen the weight of the sphere in the liquid in place of the two variables $(\rho_s - \rho_f)$ and g.) Therefore, the experimental data for this problem could be correlated by

$$C_D = f(Re_D)$$

PROBLEM 4.12

Experiments have been performed on the temperature distribution in a homogeneous long cylinder (0.1 m diameter, thermal conductivity of 0.2 W/(m K) with uniform internal heat generation. By dimensional analysis, determine the relation between the steady-state temperature at the center of the cylinder T_c the diameter, the thermal conductivity, and the rate of heat generation. Take the temperature at the surface as you datum. What is the equation for the center temperature if the difference between center and surface temperature is 30°C when the heat generation is 3000 W/m^3?

GIVEN

- A homogeneous long cylinder with uniform internal heat generation
- Diameter (D) = 0.1 m
- Thermal conductivity (k) = 0.2 W/(m K)
- Difference between surface and center temperature ($T_c - T_s$) = 30°C
- Heat generation rate $(\dot{q})$ = 3000 W/m^3

FIND

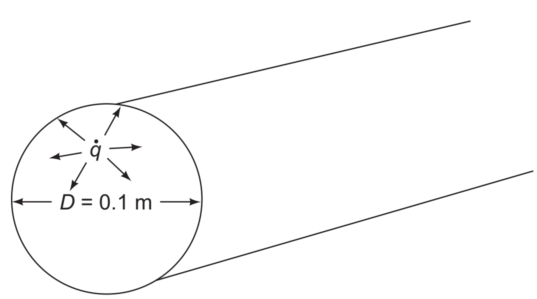
(a) Relation between center temperature (T_c), diameter (D), thermal conductivity (k), and rate of heat generation $(\dot{q})$

(b) Equation for the center temperature for the given data

ASSUMPTIONS

- Steady state
- One dimensional conduction in the radial direction

SKETCH



SOLUTION

(a) The temperature difference is a function of the variable given

$$T_c - T_s = f(D, k, \dot{q})$$

having the following primary dimensions

$$T_c - T_s \rightarrow [T]$$
$$D \rightarrow [L]$$
$$k \rightarrow \left[\frac{ML}{t^3\,T}\right]$$
$$\dot{q} \rightarrow \left[\frac{M}{Lt^3}\right]$$

Let the unknown function be represented by

$$T_c - T_s = A\,D^a\,k^b\,\dot{q}^{\,c}$$

Where A is a dimensionless constant

$$\therefore\ [T] = [L]^a \left[\frac{ML}{t^3T}\right]^b \left[\frac{M}{Lt^3}\right]^c$$

Summing the exponents of each primary dimension

For T: $1 = -b \rightarrow b = -1$

For M: $0 = b + c \rightarrow c = -b = 1$

For L: $0 = a + b - c \rightarrow a = c - b = 2$

For t: $0 = -3b + 3c$

$$\therefore\ T_c - T_s = A\,D^2\,k^{-1}\,\dot{q} = A\,\frac{D^2\dot{q}}{k}$$

The given data can now be used to evaluate the unknown constant

$$A = \frac{k(T_c - T_s)}{D^2\dot{q}} = \frac{(0.2\,\mathrm{W/(m\,K)})(30°\mathrm{C})}{(0.1\,\mathrm{m})^2\left(3000\,\mathrm{W/m^2}\right)} = 0.2$$

The equation for the center temperature is

$$T_c = T_s + 0.2\,\frac{D^2\dot{q}}{k}$$

PROBLEM 4.13

The convection equations relating the Nusselt, Reynolds, and Prandtl numbers can be rearranged to show that for gases, the heat-transfer coefficient h_c depends on the absolute temperature T and the group $\sqrt{U_\infty/x}$. This formulation is of the form $h_{c,x} = CT^n\ \sqrt{U_\infty/x}$, where n and C are constants. Indicate clearly how such a relationship could be obtained for the laminar flow case from $Nu_x = 0.332\ Re_x^{0.5}\ Pr^{0.333}$ for the condition $0.5 < Pr < 5.0$. State restrictions on method if necessary.

GIVEN

- For laminar flow: $Nu_x = 0.332\ Re_x^{\ 0.5}\ Pr^{0.333}$ for $0.5 < Pr < 5.0$

FIND

- Rearrange the given equation to the form

$$h_{c,x} = C\,T^n\sqrt{\frac{U_\infty}{x}}$$

(State restrictions)

ASSUMPTIONS

- Gas behaves as an ideal gas

SOLUTION

From Table 4.3

$$Nu_x = \frac{h_c x}{k} \qquad Re_x = \frac{U_\infty x}{v} \qquad Pr = \frac{c_p \mu}{k}$$

$$\frac{h_c x}{k} = 0.332 \left(\frac{U_\infty \times \rho}{\mu}\right)^{\frac{1}{2}} \left(\frac{c_p \mu}{k}\right)^{\frac{1}{3}}$$

By the ideal gas law

$$\rho = \frac{P}{RT}$$

where p = Pressure

R = Gas constant

T = Absolute temperature

$$\therefore \quad h_c = 0.332 \; c_p^{\frac{1}{3}} \mu^{-\frac{1}{6}} k^{\frac{2}{3}} \left(\frac{p}{R}\right)^{\frac{1}{2}} T^{-\frac{1}{2}} \sqrt{\frac{U_\infty}{x}}$$

$$\therefore \quad C = 0.332 \; c_p^{\frac{1}{3}} \mu^{-\frac{1}{6}} k^{\frac{2}{3}} \left(\frac{p}{R}\right)^{\frac{1}{2}}$$

C is constant if the following restrictions apply

- Constant pressure
- Variation of thermal properties with temperature is negligible

$$h_c = 0.332 \; c_p^{\frac{1}{3}} \mu^{-\frac{1}{6}} k^{\frac{2}{3}} \left(\frac{p}{R}\right)^{\frac{1}{2}} T^{-\frac{1}{2}} \sqrt{\frac{U_\infty}{x}}$$

PROBLEM 4.14

Experimental pressure-drop data obtained in a series of tests in which water was heated while flowing through an electrically heated tube of 1.3 cm ID, 1 m long, are tabulated below

Mass Flow Rate $\dot{m}$ (kg/s)	Fluid Bulk Temp T_b (°C)	Tube Surface Temp T_s (°C)	Pressure Drop with Heat Transfer Δp_{ht} (kPa)
1.37	32	52	67
0.98	45	94	33
0.82	36	104	23
1.39	37	120	58
0.97	42	140	31

Isothermal pressure-drop data for the same tube are given in terms of the dimensionless friction factor $f = (\Delta p/\rho\ \bar{u}^2)\ (2D/L)g_c$ and Reynolds number based on the pipe diameter, $Re_D = \bar{u}\ D/v = 4m/\pi D\mu$ below.

Re_d	1.71×10^5	1.05×10^5	1.9×10^5	2.41×10^5
f	0.0189	0.0205	0.0185	0.0178

By comparing the isothermal with the nonisothermal friction coefficients at similar bulk Reynolds numbers, derive a dimensionless equation for the non-isothermal friction coefficients in the form

$$f = \text{constant} \times Re_d^{\,n} (\mu_s/\mu_b)^m$$

where μ_s **= viscosity at surface temperature**

μ_b **= viscosity at bulk temperature**

n **and** m **= empirical constants.**

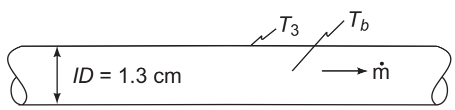
GIVEN

- Water flowing through a tube
- Isothermal and nonisothermal pressure drop data given above
- The dimensionless friction factor (f) = $(\Delta p/\rho\bar{u}^2)\,(2D/L)g_c$
- Reynolds number (Re_D) = $4\,\dot{m}/\pi D\,\mu$
- Inside tube diameter (D) = 1.3 cm
- Tube length (L) = 1 m

FIND

- Dimensionless equation of the form: $f = \text{constant} \times Re_d^{\,n} (\mu_s/\mu_b)^m$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water

Temperature (°C)	Abs. Viscosity, $\mu \times 10^6$ (kg/ms)	Density, ρ (kg/m^3)
32	767.2	
45	605.1	
36	712.7	
37	698.7	
42	646.3	
52	544.2	
94	302.4	
104	272.0	
120	237.6	
140	203.8	
42		1091.4
69.5		1004.6
78.5		999.6
91		991.4

SOLUTION

The exponent n will be determined from the isothermal data by the least squares fit for log Re_D vs. log f

$x = \log Re_D$	$y = \log f$
5.23	–1.724
5.02	–1.688
5.28	–1.733
5.38	–1.750

The least squares straight line fit for the data is

$$\log f = -0.823 - 0.172 \log Re_D$$

or

$$f = 0.149\ Re_D^{-0.1715}$$

The data and straight line fit are shown below

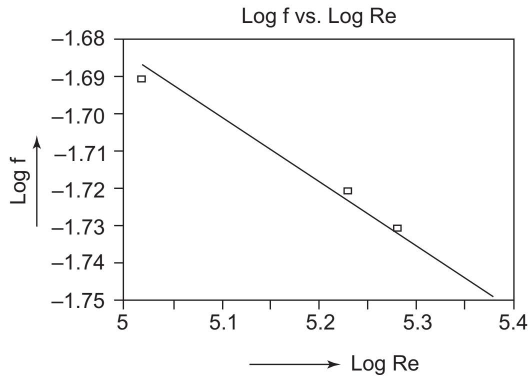


Figure Problem 4.74 (a): Plot of log f with Respect to log Re

Evaluating Re_D (based on the bulk temperature), $0.149\ Re^{-0.1715}$, f (based on the bulk temperature), and μ_s/μ_f for the non-isothermal case

$Re_D \times 10^{-5}$	$0.149\ Re_D^{-0.1715}$	f	μ_s/μ_b
1.73	0.0189	0.0187	0.709
1.55	0.0192	0.0182	0.500
1.11	0.0203	0.0175	0.381
1.91	0.0185	0.0160	0.340
1.45	0.0194	0.0173	0.315

$$\text{Let}\quad y = \log\left(\frac{f}{0.149\,Re_D^{-0.1712}}\right) \text{ and } x = \log\left(\frac{\mu_s}{\mu_b}\right)$$

y	x
–0.0046	–0.150
–0.0232	–0.302
–0.0666	–0.419
–0.0654	–0.469
–0.0520	–0.502

The linear least square fit for this data is

$$y = 0.1649 \times + 0.01976$$

Therefore: $f = 0.156\ Re_D^{-0.1715}\ (\mu_s/\mu_b)^{0.1649}$

The data for x and y and the straight line fit are shown below

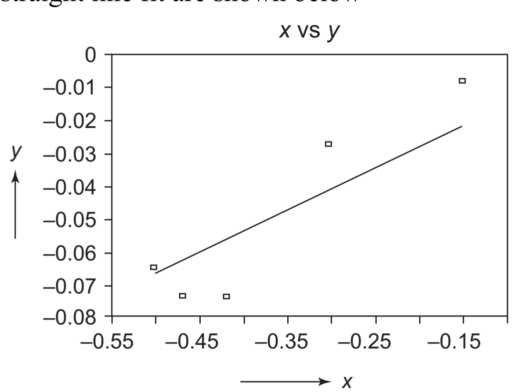


Figure Problem 4.14 (b): Plot of Variables x and y

Comparing the correlation to the experimental data

f, experimental	*f*, correlation	% difference
0.0187	0.0186	–0.31
0.0182	0.0179	–1.6
0.0175	0.0181	3.4
0.0160	0.0162	1.2
0.0173	0.0168	–2.9

PROBLEM 4.15

Tabulated below are some experimental data obtained by passing *n*-butyl alcohol at a bulk temperature of 15°C over a heated flat plate (0.3 m long, 0.9 m wide, surface temperature of 60°C). Correlate the experimental data by appropriate dimensionless numbers and compare the line which best fits the data with equation 4.38.

Velocity (m/s)	**0.089**	**0.305**	**0.488**	**1.14**
Average heat transfer coefficient $\left(\mathrm{W/(m^2\,°C)}\right)$	**121**	**218**	**282**	**425**

GIVEN

- *n*-butyl alcohol flowing over a heated flat plate
- Bulk temperature (T_b) = 15°C
- Plate surface temperature (T_p) = 60°C
- Plate length (L) = 0.3 m
- Plate width (w) = 0.9 m
- The experimental data given above

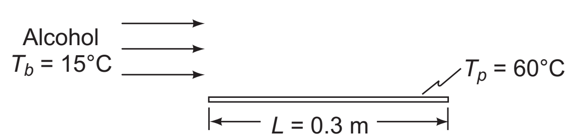
FIND

(a) Correlate the data by appropriate dimensionless numbers
(b) Compare line which best fits the data with Equation 4.38

ASSUMPTIONS

- Steady state
- Alcohol flows parallel to the length of the plate
- Plate temperature is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 18: For *n*-butyl alcohol at the average of the bulk and surface temperatures (known as the film temperature): 37.5°C.

Absolute viscosity (μ) = 1.92×10^{-3} N s/m^2

Thermal conductivity (k) = 0.166 W/(m K)

Density (ρ) = 796 kg/m^3

Prandtl number (Pr) = 29.4

SOLUTION

(a) The relevant variables and their primary dimensions are listed below

Variable	Symbol	Dimensions
Heat transfer coefficient	$\bar{h}_c$	$[M/t^3\ T]$
Velocity	U_∞	$[L/t]$
Length of plate	L	$[L]$
Absolute viscosity	μ	$[M/Lt]$
Thermal conductivity	k	$[ML/t^3\ T]$
Density	ρ	$[M/L^3]$

Note: Specific heat should be included in this list, but we suspect that it will show up as a Prandtl number which is constant for the series of tests performed. Therefore, we can easily extract its contribution. There are 6 variables are 4 primary dimensions, therefore, they can be correlated with two dimensionless groups. These dimensionless groups can determined by the Buckingham π Theorem (Sections 4.7.2 and 4.7.3).

$$\pi = \bar{h}_c^{\,a}\, U_\infty^{\,b}\, L^c\, \mu^d\, k^e\, \rho^f$$

Equating the primary dimensions

$$0 = \left[\frac{M}{t^3 T}\right]^a \left[\frac{L}{t}\right]^b [L]^c \left[\frac{M}{Lt}\right]^d \left[\frac{ML}{t^3 T}\right]^e \left[\frac{M}{L^3}\right]^f$$

Equating the sums of the exponents of each primary dimension

For T: $0 = -a - e$ [1]

For M: $0 = a + d + e + f$ [2]

For t: $0 = -3a - b - d - 3e$ [3]

For L: $0 = b + c - d + e - 3f$ [4]

There are four equations and six unknowns. Therefore, the values of two of the exponents may be chosen for each dimensionless group.

For π_1, Let $f = 1$ $e = 0$

From equation [1]: $a = 0$

From equation [2]: $d = -1$

From equation [3]: $b = 1$

From equation [4]: $c = 1$

$$\therefore \quad \pi^1 = U_\infty L\, \mu^{-1}\, \rho = \frac{\rho u_\infty L}{\mu} = Re_L$$

For π_2, Let $a = 1$ $d = 0$

From equation [1]: $e = -1$

From equation [2]: $f = 0$

From equation [3]: $b = 0$

From equation [4]: $c = 1$

$$\therefore \quad \pi^2 = \bar{h}_c L\, k^{-1} = \frac{\bar{h}_c L}{k} = Nu$$

The range of Prandtl number is insufficient to get a functional relationship, therefore the data can be correlated by the Nusselt number and the Reynolds number:

$$Nu = f(Re)$$

Calculating Re_L and Nu for each data point

U_∞(m/s)	$\bar{h}_c$ $(\text{W}/(\text{m}^2\ \text{K}))$	$Re_L \times 10^{-4}$	$\overline{Nu}$
0.089	121	1.11	218.7
0.305	218	3.79	394.0
0.488	282	6.07	509.6
1.14	425	14.2	768.1

On a log-log plot, these points fall roughly on a straight line

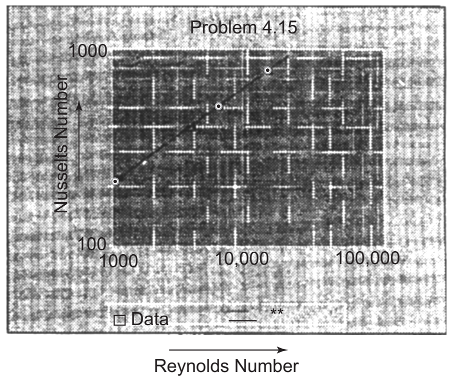


Figure Problem 4.15: Plot of *Nu* vs *Re* on log-log scale

The linear regression gives the following line

$$\log\ \overline{Nu} = 0.494 \log Re_L + 0.339$$

or

$$\overline{Nu} = 2.185\ Re_L^{\ 0.494}$$

(b) For this problem, $Pr = 29.4$. Including this in the correlation

$$\overline{Nu} = 0.708\ Re_L^{0.494}\ Pr^{0.33}$$

Equation 4.38 for laminar flow over a flat plate is

$$\overline{Nu} = 0.664\ Re_L^{0.5}\ Pr^{0.33}$$

which is about 7% less than our experimental data.

PROBLEM 4.16

Tabulated below are reduced test data from measurements made to determine the heat-transfer coefficient inside tubes at Reynolds numbers only slightly above transition and at relatively high Prandtl numbers (as associated with oils). Tests were made in a double-tube exchanger with a counterflow of water to provide the cooling. The pipe used to carry the oils was 1.5 cm *OD*, 18 BWG, 3 m long. Correlate the data in terms of appropriate dimensionless parameters.

Test no.	Fluid	$\bar{h}_c$ (W/m^2 K)	ρu (kg/m^2 S)	c_p (kJ/kg K)	k_f (W/m K)	$\mu_b \times 10^3$ (kg/ms)	$\mu_f \times 10^3$ (kg/ms)
11	10C oil	490	1450	1.971	0.1349	5.63	8.01
19	10C oil	725	2030	1.976	0.1349	5.49	7.85
21	10C oil	1500	3320	2.034	0.1342	3.96	5.75
23	10C oil	810	1445	2.072	0.1337	3.06	4.09
24	10C oil	940	3985	1.896	0.1358	9.82	11.22
25	10C oil	770	1400	2.076	0.1337	3.0	4.81
36	1488 pyranol	800	2425	1.088	0.1273	4.97	6.95
39	1488 pyranol	760	3840	1.088	0.1280	9.5	12.00
45	1488 pyranol	1025	2680	1.088	0.1271	4.25	5.3
48	1488 pyranol	715	5180	1.088	0.1285	16.52	22.00
49	1488 pyranol	600	4370	1.088	0.1285	16.32	18.7

where

h_c = mean surface heat-transfer coefficient based on the mean temperature difference, W/(m^2 K)

ρu = mass velocity, kg/(m^2 s)

c_p = specific heat, kJ/(kg K)

k_f = thermal conductivity, W/(m K) (based on film temperature)

μ_b = viscosity, based on average bulk (mixed mean) temperature, kg/ms

μ_f = viscosity, based on average film temperature, kg/ms

Hint: Start by correlating *Nu* and Re_d irrespective of the Prandtl numbers, since the influence of the Prandtl number on the Nusselt number is expected to be relatively small. By plotting *Nu* vs. *Re* on log-log paper, one can guess the nature of the correlation equation, *Nu* = f_1 (*Re*). A plot of *Nu*/f_1 (*Re*) vs. *Pr* will then reveal the dependence upon *Pr*. For the final equation, the influence of the viscosity variation should also be considered.

GIVEN

- Oil in a counterflow heat exchanger
- Pipe specifications: 1.5 cm *OD*, 18 BWG
- Pipe length (L) = 3 m
- The experimental data above

FIND

- Correlate the data in terms of appropriate dimensionless parameters

ASSUMPTIONS

- The data represents the steady state for each case

PROPERTIES AND CONSTANTS

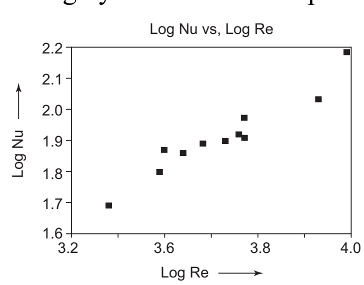
From Appendix 2, Table 42: for 1.5 cm *OD*, 18 BWG tubing, the inside diameter D = 1.34 cm

SOLUTION

The appropriate dimensionless parameters are the average Nusselt number ($Nu = h_c\ D/k_f$). The Reynolds number ($Re_D = \rho u D/\mu_f$) and the Prandtl number ($Pr = C_p\ \mu_f/k$). The values of the dimensionless parameters for each test are listed below.

Test no.	*Nu*	$Re_D \times 10^{-3}$	*Pr*	log *Nu*	log *Re*
11	49.0	2.41	117.9	1.69	3.38
19	72.3	3.46	115.7	1.86	3.54
21	149.9	7.72	87.7	2.18	3.89
23	81.7	4.73	63.7	1.91	3.67
24	93.3	4.75	157.7	1.97	3.68
25	77.4	3.89	75.1	1.89	3.59
36	84.0	4.66	59.7	1.92	3.67
39	79.4	4.27	102.6	1.90	3.63
45	108.4	6.76	45.6	2.03	3.83
48	74.7	3.16	187.2	1.87	3.50
49	62.5	3.13	159.9	1.80	3.49

Plotting log *Nu* vs. log Re_D reveals a roughly linear relationship.



Fitting a least squares regression line to the data

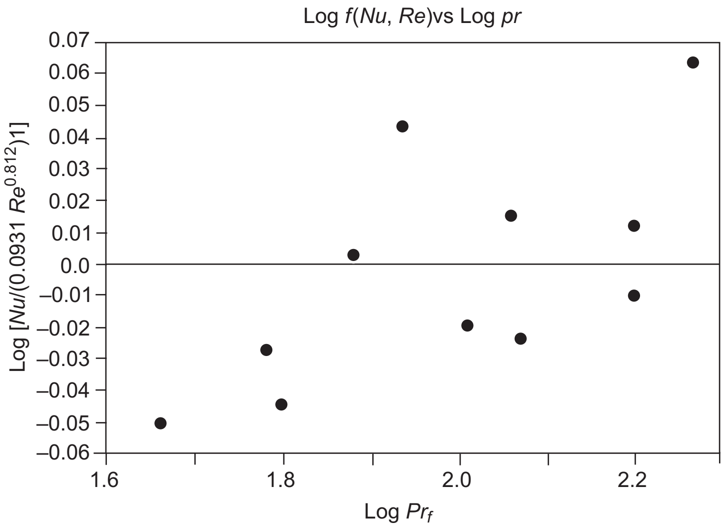
$$\log Nu = -1.0314 + 0.812 \log Re$$

or

$$Nu = 0.0931\, Re^{0.812}$$

The variation of Nu with Pr_f can be determined by plotting log [Nu/(0.0931 $Re^{0.812}$)] vs. log Pr_f.

log [Nu/(0.0931 $Re^{0.812}$)]	log Pr_f
–0.0252	2.07
0.0165	2.06
0.0501	1.94
–0.0405	1.80
0.0156	2.20
0.0047	1.88
–0.0240	1.78
–0.0171	2.01
–0.0438	1.66
0.0624	2.27
–0.0108	2.20



Although there is considerable scatter in this plot, it does follow a trend of increasing log Pr_f with increasing log [Nu/(0.0931 $Re^{0.812}$)] and will be fit with a straight least squares regression line. A least squares fit yields

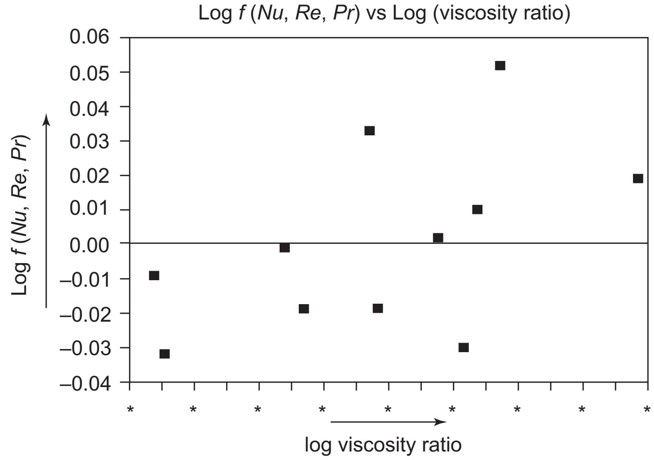
$$\log [Nu/(0.0931\, Re^{0.812})] = -0.2152 + 0.1076 \log Pr_f$$

or

$$Nu = 0.0567\, Re^{0.812}\, Pr_f^{0.108}$$

Plotting log [Nu/0.0567 $Re^{0.812}$ $Pr_f^{0.108}$] vs. log (μ_f/μ_b)

log [Nu/0.0567 $Re^{0.812}$ $Pr_f^{0.108}$]	log (μ_f/μ_b)
–0.0335	0.153
0.0900	0.157
0.0557	0.164
–0.0200	0.127
–0.0064	0.058
0.0175	0.207
0.00041	0.145
–0.0190	0.104
–0.0076	0.098
0.0323	0.124
0.0334	0.061



Fitting these points with a straight least squares regression line

$$\log\left[\frac{Nu}{0.0567\,Re^{0.812}\,Pr_f^{0.108}}\right] = -\,0.0385 + 0.2993\,\log\left(\frac{\mu_f}{\mu_b}\right)$$

or

$$Nu = 0.0519\,Re^{0.812}\,Pr_f^{0.108}\left(\frac{\mu_f}{\mu_b}\right)^{0.2993}$$

Test No.	Experimental Nu	$0.0432\,Re^{0.828}\,Pr_f^{0.118}\left(\frac{\mu_f}{\mu_b}\right)^{0.3128}$
11	49.0	53.8
19	72.2	72.1
21	149.8	134.8
23	81.7	85.3
24	93.2	90.0
25	77.4	78.3
36	83.9	84.8
39	79.4	81.4
45	108.3	107.8
48	74.7	69.0
49	62.5	64.4

PROBLEM 4.17

A turbine blade with a characteristic length of 1 m is cooled in an atmospheric pressure wind tunnel by air at 40°C and a velocity of 100 m/s. At a surface temperature of 500 K, the cooling rate is found to be 10,000 watts. Apply these results to estimate the cooling rate from another turbine blade of similar shape, but with a characteristic length of 0.5 m operating with a surface temperature of 600 K in air at 40°C and a velocity of 200 m/s.

GIVEN

- A turbine blade in a wind tunnel
- Length of blade (L_1) = 1 m
- Air temperature (T_{ai}) = 40°C = 313 K
- Air velocity ($U_{\infty 1}$) = 100 m/s
- Air pressure = 1 atm
- Blade surface temperature (T_s) = 500 K
- Cooling rate (q) = 10,000 W

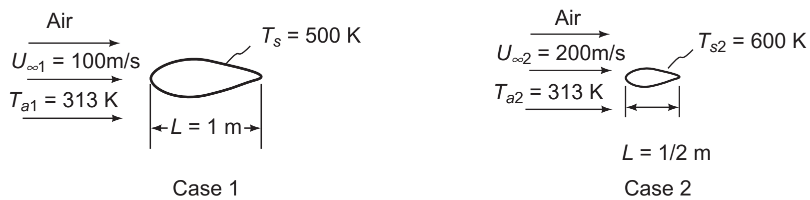
FIND

- Cooling rate from a similar blade with a characteristic length (L_2) of 0.5 m and a surface temperature (T_{s2}) of 600 K and a velocity ($U_{\infty 2}$) of 200 m/s

ASSUMPTIONS

- Steady state for both cases
- Uniform blade surface temperature
- Air temperature is constant and the same in both cases

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the film temperatures

	Case 1 T = 406.5 K	Case 2 T = 456.5 K
Kinematic viscosity, $\nu \times 10^6$ (m^2/s)	27.6	33.2
Thermal conductivity, k (W/(m K))	0.0328	0.0360

SOLUTION

Important variables

	Dimensions
Cooling rate, q	$[M L^2/t^3]$
Length, L	$[L]$
Air–blade Temperatures $(T_s - T_b)$	$[T]$
Air Velocity, U_∞	$[L/t]$
Kinematic Viscosity, ν	$[L^2/t]$
Thermal Conductivity, k	$[M L/t^3 T]$

The (6 – 4 = 2) dimensionless groups can be determined by the Buckingham p theory

$$\pi = q^a L^b (T_s - T_b)^c U_\infty^d \nu^e k^f$$

Equating the primary dimensions

$$0 = \left[\frac{ML^2}{t^3}\right]^a [L]^b [T]^c \left[\frac{L}{t}\right]^d \left[\frac{L^2}{t}\right]^e \left[\frac{ML}{t^3 T}\right]^f$$

For M: $a + f = 0$

For T: $c - f = 0$

For t: $3a + d + e + 3f = 0$

For L: $2a + b + d + 2e + f = 0$

π_1: Let $a = 0$ and $d = 1 \rightarrow f = 0$; $c = 0$; $e = -1$; $b = 1$

$$\pi_1 = \frac{U_\infty L}{\nu} = Re_L$$

π_2: Let $a = 1$ and $d = 0 \rightarrow f = -1$; $c = -1$; $e = 0$; $b = -1$

$$\pi_2 = \frac{q}{L(T_s - T_a)k}$$

$$\therefore \frac{q}{L(T_s - T_a)k} = f(Re_L)$$

Assume that the function has the form

$$\frac{q}{L(T_s - T_a)k} = Re_L^{\,m}$$

The data of the larger blade can be used to evaluate *m*

$$Re_L = \frac{U_\infty L}{\nu} = \frac{100\,\text{m/s}(1.0\,\text{m})}{27.6\times10^{-6}\,\text{m}^2/\text{s}} = 3.62 \times 10^6$$

$$m = \frac{\log\left[\dfrac{q}{L(T_s - T_a)k}\right]}{\log Re_L} = \frac{\log\left[\dfrac{10{,}000\,\text{W}}{1\,\text{m}(500\,\text{K} - 313\,\text{K})(0.0328\,\text{W/(m K)})}\right]}{\log\left(3.62\times10^6\right)} = 0.490$$

$$\therefore \quad q = L\,k\,(T_s - T_a)\,Re_L^{\,0.49}$$

Applying this to the smaller blade

$$Re_L = \frac{U_\infty L}{\nu} = \frac{200\,\text{m/s}(0.5\,\text{m})}{33.2\times10^{-6}\,\text{m}^2/\text{s}} = 3.0 \times 10^6$$

$$q = 0.5\text{ m }(0.0360\,\text{W/(m K)})\ (600\text{ K} - 313\text{ K})\ (3.0 \times 10^6)^{0.49} = 7723\text{ W}$$

PROBLEM 4.18

The drag on an airplane wing in flight is known to be a function of the following quantities

- ρ - **density of air**
- μ - **viscosity of air**
- U_∞ - **free-stream velocity**
- S - **characteristic dimension of the wing**
- τ_s - **shear stress on the surface of the wing**

Show that the dimensionless drag

$$\frac{\tau_s}{\rho U_\infty^{\,2}}$$

can be expressed as a function of the Reynolds number

$$\frac{\rho U_\infty S}{\mu}$$

GIVEN

- An airplane wing in flight
- Drag on wing (D) = $f(\rho, \mu, U_\infty, S, \tau_s)$

FIND

Show that $\dfrac{\tau_s}{\rho U_\infty^{\,2}} = f\left(\dfrac{\rho U_\infty S}{\mu}\right)$

SOLUTION

The relevant variables and their dimensions are shown below

Variable	Symbol	Dimensions
Density	ρ	$[M/L^3]$
Viscosity	μ	$[M/Lt]$
Velocity	U_∞	$[L/t]$
Characteristic Dimensions	S	$[L]$
Shear Stress	τ_s	$[M/Lt^2]$

There are 5 variables and 3 primary dimensions. Therefore, the variables can be correlated with 2 dimensionless groups.

Using the Buckingham π theory (Sections 4.7.2 and 4.7.3)

$$\pi = \rho^a \, \mu^b \, U_\infty^{\,c} \, S^d \, \tau_s^{\,e}$$

In terms of the primary dimensions

$$0 = \left[\frac{M}{L^3}\right]^a \left[\frac{M}{Lt}\right]^b \left[\frac{L}{t}\right]^c [L]^d \left[\frac{M}{Lt^2}\right]^e$$

Equating the sum of the exponents of each primary dimension to zero

For M: $0 = a + b + e$ [1]

For t: $0 = -b - c - 2e$ [2]

For L: $0 = -3a - b + c + d - e$ [3]

Since there are 5 unknowns and only 3 equations, the value two exponents may be chosen for each dimensionless group

For π_1: Let $e = 1$ and $a = -1$

From equation [1]: $b = 0$

From equation [2]: $c = -2$

From equation [3]: $d = 0$

$$\pi_1 = \rho^{-1} \, U_\infty^{-2} \, \tau_s = \frac{\tau_s}{\rho U_\infty^{\,2}}$$

For π_2: Let $a = 1$ and $b = -1$

From equation [1]: $e = 0$

From equation [2]: $c = 1$

From equation [3]: $d = 1$

$$\pi_2 = \rho \, \mu^{-1} \, U_\infty \, S = \frac{\rho U_\infty S}{\mu}$$

As shown in equation (4.24)

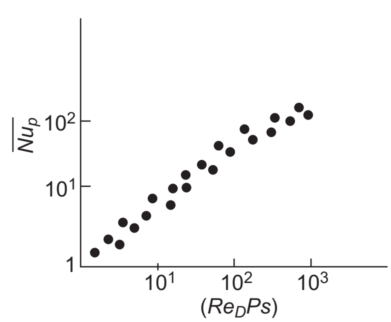
$$\pi_1 = f(\pi_2)$$

or

$$\frac{\tau_s}{\rho U_\infty^{\,2}} = f\left(\frac{\rho U_\infty S}{\mu}\right)$$

PROBLEM 4.19

Suppose that the graph below shows measured values of h_c for air in forced convection over a cylinder of diameter D plotted on a logarithmic graph of Nu_d as a function of $Re_d Pr$.



Write an appropriate dimensionless correlation for the average Nusselt number for these data and state any limitations to your equation.

GIVEN

- Forced convection of air over a cylinder
- Experimental data given above

FIND

- An appropriate dimensionless correlation for the average Nusselt number

SOLUTION

The data lies along an approximately straight line on the log-log graph. Therefore, a straight line fit will be used. Choosing two points on the graph

$$[Nu_D = 1, (Re_D)(Pr) = 1] \text{ and } [Nu_D = 100, (Re_D)(Pr) = 1000]$$

A straight line on the log-log plot is represented by

$$\log(Nu_D) = a \log(Re_D\, Pr) + b$$

Substituting the two points into the equation and solving for a and b

$$\log(1) = a \log(1) + b \rightarrow b = 0$$

$$\log(100) = a \log(1000) + b \rightarrow a = 0.667$$

Therefore

$$\log(Nu_D) = 0.667 \log(Re_D\, Pr)$$

or

$$Nu_D = (Re_D\, Pr)^{0.667}$$

This is based on data in the range $1 < Re_d\, Pr < 10^3$ and is therefore valid only in this range.

PROBLEM 4.20

Engine oil at 100°C flows over and parallel to a flat surface at a velocity of 3 m/s. Calculate the thickness of the hydrodynamic boundary layer at a distance 0.3 m from the leading edge of the surface.

GIVEN

- Engine oil flows over a flat surface
- Engine oil temperature (T_b) = 100°C
- Engine oil velocity (U_∞) = 3 m/s

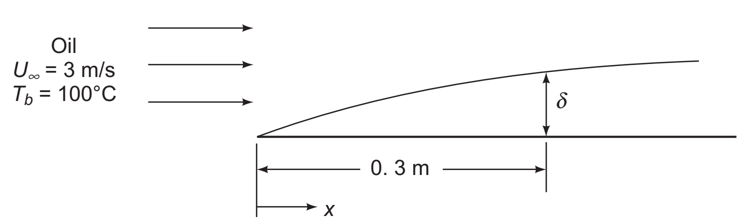
FIND

- The hydrodynamic boundary layer thickness (δ) at a distance 0.3 m from the leading edge

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 16, for engine oil at 100°C

Kinematic viscosity (ν) = 20.3×10^{-6} m²/s

SOLUTION

The local Reynolds 0.3 m from the leading edge based on the bulk fluid temperature is

$$Re_x = \frac{U_\infty x}{\nu} = \frac{3.0\,\text{m/s}(0.3\,\text{m})}{20.3\times10^{-6}\,\text{m}^2/\text{s}} = 4.43 \times 10^4$$

Since $Re_x < 5 \times 10^5$, the boundary layer is laminar. The boundary layer thickness for laminar flow over a flat plate is given by Equation (4.28)

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5(0.3\,\text{m})}{\sqrt{4.43\times10^4}} = 7.1 \times 10^{-3}\ \text{m} = 7.1\ \text{mm}$$

PROBLEM 4.21

Assuming a linear velocity distribution and a linear temperature distribution in the boundary layer over a flat plate, derive a relation between the thermal and hydrodynamic boundary-layer thicknesses and the Prandtl number.

GIVEN

- Boundary layer over a flat plate

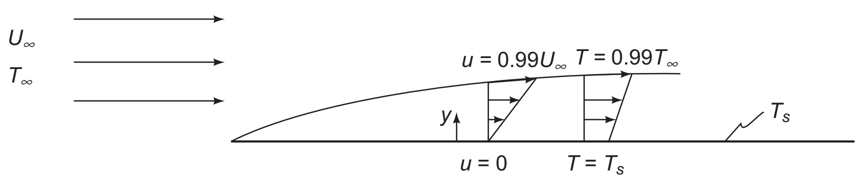
FIND

- A relation between the thermal and hydrodynamic boundary-layer thicknesses and the Prandtl number

ASSUMPTIONS

- Linear velocity and temperature distributions in the boundary layers

SKETCH



SOLUTION

Let Absolute viscosity of the fluid = μ

Plate surface temperature = T_s

Bulk fluid temperature = T_∞

Bulk fluid viscosity = U_∞

Density of the fluid = ρ

Thermal diffusivity of the fluid = α

The linear velocity profile will be used to solve the integral momentum equation first. The integral energy equation will then be solved and combined with the momentum solution.

Linear velocity profile: $u = u_o + ay$

Subject to $u = 0$ at $y = 0 \rightarrow u_o = 0$

$u = 0.99\ U_\infty \approx U_\infty$ at $y = \delta \rightarrow a = U_\infty/\delta$

therefore $u = (U_\infty/\delta)y$

Substituting this into the integral momentum equation for a laminar boundary layer (Equation 4.42)

$$\frac{d}{dx}\int_0^b \rho\left(\frac{U_\infty}{\delta}y\right)\left[U_\infty - \frac{U_\infty}{\delta}y\right]dy = \tau_w = \mu \left.\frac{du}{dy}\right|_{y=0}$$

(The wall shear stress (τ_w) is defined by Equation (4.2))
In this case, du/dy = constant = U_∞/δ
Integrating

$$\frac{d}{dx}\left[\frac{U_\infty\rho}{\delta}\left(\frac{U_\infty}{2}y^2 - \frac{U_\infty}{3\delta}y^3\right)\right]_0^b = \mu\,\frac{U_\infty}{\delta}$$

$$\frac{d}{dx}\left[\frac{U_\infty{}^2\rho\delta}{6}\right] = \mu\,\frac{U_\infty}{\delta}$$

$$\delta\, d\delta = \frac{6\mu}{\rho U_\infty}dx$$

Integrating

$$\frac{1}{2}\delta^2 = \frac{6\mu}{\rho U_\infty}x + c$$

At $x = 0$, $\delta = 0 \rightarrow c = 0$

$$\delta = \left(\frac{12\mu x}{\rho U_\infty}\right)^{\frac{1}{2}} = 3.46\times\left(\frac{\mu}{\rho U_\infty x}\right)^{\frac{1}{2}} = 3.46\times\ Re_x{}^{-\frac{1}{2}}$$

Linear temperature profile: $T = T_o + by$

Subject to $T = T_s$ at $y = 0 \rightarrow T_o = T_s$

$T = 0.99\ T_\infty \approx T_\infty$ at $y = \delta_t \rightarrow b = \dfrac{(T_\infty - T_s)}{\delta_t}$

$$\therefore\quad T = T_s + \frac{T_\infty - T_s}{\delta_t}y$$

Substituting this and the expression for U into the integral energy equation of the laminar boundary layer for low speed flow (Equation 4.44)

$$\frac{d}{dx}\int_0^{\delta_t}\left[T_\infty - \left(T_s + \frac{T_\infty - T_s}{\delta_t}y\right)\right]\left(\frac{U_\infty}{\delta}y\right)dy - a\left[\frac{d}{dy}\left(T_s + \frac{T_\infty - T_s}{\delta_t}y\right)\right]_{y=0} = 0$$

$$\frac{d}{dx}\int_0^{\delta_t}\frac{U_\infty}{\delta}(T_\infty - T_s)\left[y - \frac{1}{\delta_t}y^2\right]dy - \alpha\,\frac{T_\infty - T_s}{\delta_t} = 0$$

Integrating

$$\frac{d}{dx}\left[\frac{U_\infty}{\delta}(T_\infty - T_s)\left(\frac{1}{2}y^2 - \frac{1}{3\delta_t}y^3\right)\right]_0^{\delta_t} = \alpha\,\frac{T_\infty - T_s}{\delta_t}$$

$$\frac{d}{dx}\left[U_\infty\frac{\delta_t^2}{6\delta}\right] = \frac{\alpha}{\delta_t}$$

Let $\quad\zeta = \dfrac{\delta_t}{\delta}$

Then $\frac{d}{dx}\left[U_\infty \frac{\zeta^2 \delta}{6}\right] = \frac{\alpha}{\delta\zeta}$

or $\delta\frac{d\delta}{dx} = \frac{6\alpha}{U_\infty \zeta^3}$ (ζ is independent of x)

Substituting Equation [1] into this expression

$$\frac{6\mu}{\rho U_\infty} = \frac{6\alpha}{U_\infty \zeta^3} \Rightarrow \zeta^3 = \left(\frac{\delta_t}{\delta}\right)^3 = \frac{\alpha\rho}{\mu} = \frac{1}{Pr}$$

$$\frac{\delta}{\delta_t} = Pr^{0.33}$$

PROBLEM 4.22

Air at 20°C flows at 1 m/s between two parallel flat plates spaced 5 cm apart. Estimate the distance from the entrance where the hydrodynamic boundary layers meet.

GIVEN

- Air flows between two parallel flat plates
- Air speed (U_∞) = 1 m/s
- Distance between the plates (D) = 5 cm = 0.05 m
- Air temperature = 20°C

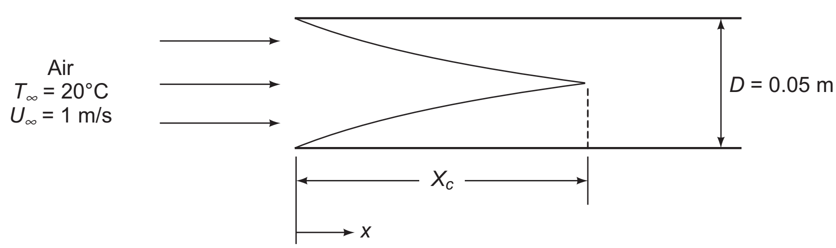
FIND

- The distance from the entrance (X^1) where the boundary layers meet

ASSUMPTIONS

- Steady state
- Laminar flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 20°C

Kinematic viscosity (ν) = 15.7×10^{-6} m²/s

Prandtl number (Pr) = 0.71

SOLUTION

If the boundary layer is laminar, the hydrodynamic boundary layer thickness is given by Equation (4.28)

$$\delta = \frac{5x}{\sqrt{Re_x}} = 5\left(\frac{\nu x}{U_\infty}\right)^{\frac{1}{2}}$$

The boundary layers will meet when $\delta = D/2$

$$\frac{D}{2} = 5\left(\frac{\nu x_c}{U_\infty}\right)^{\frac{1}{2}}$$

Solving for distance x_c

$$x_c = \frac{D^2 U_\infty}{100\,\nu} = \frac{(0.05\,\text{m})^2\,(1\,\text{m/s})}{100\left(15.7\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1.59\text{ m}$$

The Reynolds number at x_c = 1.59 m is

$$Re_{x_c} = \frac{U_\infty x_c}{\nu} = \frac{1\,\text{m/s}\,(1.59\,\text{m})}{15.7\times10^{-6}\,\text{m}^2/\text{s}} = 1.0\times10^5 < 5\times10^5$$

COMMENTS

Since $Re < 5 \times 10^5$, the assumption of a laminar boundary layer is valid. If the Reynolds number were in the turbulent regime, the problem would have to be reworked.

PROBLEM 4.23

A fluid at temperature T_∞ is flowing at a velocity U_∞ over a flat plate which is at the same temperature as the fluid for a distance x_0 from the leading edge, but at a higher temperature T_s beyond this point. Show by means of the integral boundary-layer equations that ζ, the ratio of the thermal boundary-layer thickness to the hydrodynamic boundary-layer thickness, over the heated portion of the plate is approximately

$$\zeta \approx Pr^{-\frac{1}{3}}\left[1-\left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

if the flow is laminar.

GIVEN

- Laminar flow over a flat plate
- Fluid temperature = T_∞
- Fluid velocity = U_∞
- Plate temperature = T_∞ for $x < X_o$
- Plate temperature = T_s for $x > X_o$

FIND

- Show that

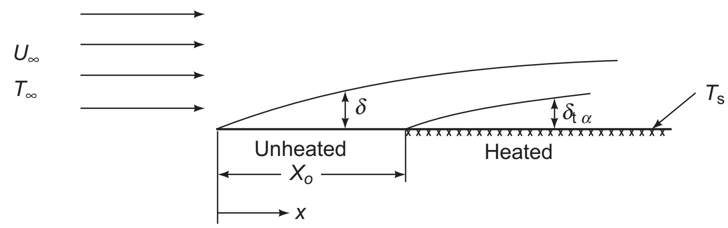
$$\zeta \approx Pr^{-\frac{1}{3}}\left[1-\left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

over the heated portion of the plate

ASSUMPTIONS

- Steady state
- The temperature distribution is a third-order polynomial: $T - T_s = ay + cy^3$
- Property value changes due to the temperature profile do not affect the hydrodynamic boundary layer.

SKETCH



SOLUTION

The velocity and temperature distributions given in Equations (4.46) and (4.53) are valid for this problem

$$\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad \text{for } x > 0$$

$$\frac{T - T_s}{T_\infty - T_s} = \frac{3}{2}\frac{y}{\delta_t} - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 \qquad \text{for } x > x_o$$

The integral energy equation is given by Equation (4.44)

$$\frac{d}{dx}\int_0^{\delta_t}(T_\infty - T)\,u\,dy - \alpha\left(\frac{\partial T}{\partial y}\right)_{y=0} = 0$$

As shown in Section 4.9.1, for the above velocity and temperature distributions

$$\int_0^{\delta_t}(T_\infty - T)\,u\,dy = (T_\infty - T_s)\,U_\infty\,\delta\left(\frac{3}{20}\zeta^2 - \frac{3}{280}\zeta^4\right)$$

where $\zeta = \dfrac{\delta_t}{\delta}$

Also

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = (T_\infty - T_s)\left[\frac{3}{2}\frac{1}{\delta_t} - \frac{3}{2}\frac{1}{\delta_t^3}y^2\right]_{y=0} = \frac{3}{2}\frac{1}{\delta_t}(T_\infty - T_s) = \frac{3}{2}\frac{1}{\zeta\delta}(T_\infty - T_s)$$

Substituting these expressions into the energy equation

$$(T_\infty - T_s)\,U_\infty\,\frac{d}{dx}\left[\delta\left(\frac{3}{20}\zeta^2 - \frac{3}{280}\zeta^4\right)\right] = \frac{3}{2}\frac{\alpha}{\zeta\delta}(T_\infty - T_s)$$

The hydrodynamic boundary layer begins at $X = 0$, but the thermal boundary layer does not begin until $X = X_o$. It will be assumed, therefore, that $\delta_t < \delta \rightarrow \zeta < 1$, therefore, the term 3/280 ζ^4 will be neglected, leaving

$$\frac{3}{20}U_\infty\frac{d}{dx}(\delta\,\zeta^2) = \frac{3}{2}\frac{\alpha}{\zeta\delta}$$

$$\frac{1}{10}U_\infty\,\zeta\,\delta\left(2\delta\zeta\frac{d\zeta}{dx} + \zeta^2\frac{d\delta}{dx}\right) = \alpha$$

$$\frac{1}{10}U_\infty\left(2\delta^2\zeta^2\frac{d\zeta}{dx} + \delta\zeta^3\frac{d\delta}{dx}\right) = \alpha$$

As shown in Equation (4.50)

$$\delta = \sqrt{\frac{280}{13}}\left(\frac{U_\infty x}{\nu}\right)^{-\frac{1}{2}} x$$

$$\therefore \quad \frac{d\delta}{dx} = \frac{1}{2}\sqrt{\frac{280}{13}}\left(\frac{U_\infty x}{\nu}\right)^{-\frac{1}{2}}$$

Substituting these into the energy equation

$$\frac{1}{10} U_\infty \left[\frac{560}{13}\left(\frac{\nu}{U_\infty x}\right) x^2 \zeta^2 \frac{d\zeta}{dx} + \zeta^3 \frac{140}{13}\left(\frac{\nu}{U_\infty x}\right) x\right] = \alpha$$

$$\zeta^3 + 4x\,\zeta^2 \frac{d\zeta}{dx} = \frac{13\alpha}{14\nu} = \frac{13}{14Pr} \approx \frac{1}{Pr}$$

Let $\lambda = \zeta^3 \qquad \therefore \quad \frac{d\lambda}{dx} = 3\zeta^2 \frac{d\zeta}{dx}$

$$\lambda + \frac{4}{3}\times\frac{d\lambda}{dx} = \frac{1}{Pr}$$

The solution to this differential equation is the sum of the homogeneous solution and a particular solution. A particular solution is $\lambda = 1/Pr$. The homogeneous solution can be found by assuming $\lambda = x^m$

$$x^m + \frac{4}{3} x\,(m\,x^{m-1}) = 0$$

$$\left(1 + \frac{4}{3} m\right) x^m = 0$$

$$m = -\frac{3}{4}$$

Therefore, the solution to the differential equation is

$$\lambda = \frac{1}{Pr} + Cx^{-\frac{3}{4}}$$

The constant C can be evaluated by the condition that at $x = x_o$, $\delta_t = 0 \rightarrow \zeta = 0 \rightarrow \lambda = 0$

$$0 = \frac{1}{Pr} + Cx^{-\frac{3}{4}} \Rightarrow C = -\frac{1}{Pr} x_o^{\frac{3}{4}}$$

$$\lambda = \frac{1}{Pr}\left[1 - \left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]$$

$$\zeta = \lambda^{\frac{1}{3}} = Pr^{-\frac{1}{3}}\left[1 - \left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

PROBLEM 4.24

Air 1000°C flows at 2 m/s flows between two parallel flat plates spaced 1 cm apart. Estimate the distance from the entrance where the boundary layers meet.

GIVEN

- Air flows between two parallel flat plates
- Air velocity (U_∞) = 2 m/s
- Plate spacing (S) = 5 cm = 0.05 m

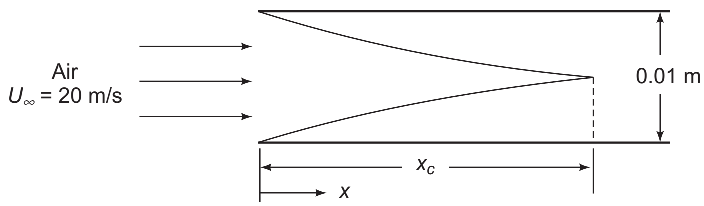
FIND

- The distance from the entrance (x_c) where the boundary layers meet

ASSUMPTIONS

- Steady flow
- Air is dry and at a temperature of 1000°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 1000°C
The kinematic viscosity (ν) = 181×10^{-6} m²/s

SOLUTION

The boundary layers meet when

$$\delta_{x_c} = \frac{1}{2} S = 0.005 \text{ m}$$

Assuming the flow is laminar, the boundary layer thickness is given by Equation (4.28)

$$\delta_x = \frac{5x}{\sqrt{Re_x}} = 5x \left(\frac{\nu}{U_\infty x}\right)^{0.5} \Rightarrow x = \frac{\delta_x{}^2 U_\infty}{25\nu}$$

$$x_c = \frac{(0.005\,\text{m})^2\,(2\,\text{m/s})}{25\left(181\times10^{-6}\,\text{m}^2/\text{s}\right)} = 0.011 \text{ m}$$

Checking the laminar flow assumption
$Re_{xc} < 5 \times 10^5$, therefore, the flow is laminar. The boundary layers meet at x = 0.011 m.

PROBLEM 4.25

Experimental measurements of the temperature distribution in flow of atmospheric pressure air over the wing of an airplane indicate that the temperature distribution near the surface can be approximated by a linear equation

$$(T - T_s) = a\, y\, (T_\infty - T_s)$$

where a **=** a **constant = 2** m^{-1}

T_s **= surface temperature,** K

T_∞ **= free stream temperature,** K

y **= perpendicular distance from surface (mm)**

(a) Estimate the convective heat transfer coefficient if T_s = 50°C and T_∞ = – 50°C.

(b) Calculate the heat flux in W/m^2

GIVEN

- Air flow over an airplane wing
- Temperature distribution is given by the expression above
- Surface temperature (T_s) = 50°C
- Ambient temperature (T_∞) = – 50°C

FIND

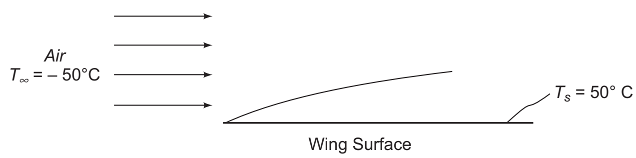
(a) The convective heat transfer coefficient $\left(\bar{h}_c\right)$

(b) The heat flux (q/A) in W/m^2

ASSUMPTIONS

- Steady state conditions
- Uniform surface temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for air at 0°C, the thermal conductivity (k) = 0.0237 W/(m K)

SOLUTION

(a) The heat transfer coefficient is given by Equation (4.1)

$$\bar{h}_c = \frac{-k_f}{T_s - T_\infty} \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

where k_f is the thermal conductivity of the fluid. Evaluate at the average of the bulk fluid temperature and the surface temperature. (This average is called the film temperature).

For this problem

$$\frac{T_s + T_\infty}{2} = \frac{50°\text{C} - 50°\text{C}}{2} = 0°\text{C}; \qquad k_f = 0.0237 \text{ W/(m K)}$$

For the temperature distribution, we find

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = a\,(T_\infty - T_s)$$

$$\therefore \qquad \bar{h}_c = \frac{-k_f}{T_s - T_\infty}\, a\,(T_\infty - T_s) = a\, k_f = 2(1/\text{m})\ \left(0.0237\,\text{W/(m K)}\right) = 0.0474 \text{ W/(m}^2\text{ K)}$$

(b) The rate of heat transfer is given by

$$q = \bar{h}_c A\,(T_s - T_\infty)$$

$$\frac{q}{A} = \bar{h}_c\,(T_s - T_\infty) = \left[0.0474 \text{ W/(m}^2\text{ K)}\right]\ (50°\text{C} + 50°\text{C}) = 4.74 \text{ W/m}^2$$

PROBLEM 4.26

For flow over a slightly curved isothermal surface, the temperature distribution inside the boundary layer δ_t may be approximated by the polynomial

$T(y) = a + by + cy^2 + dy^3$ ($y < \delta_t$) where y is the distance normal to the surface.

(a) By applying appropriate boundary conditions, evaluate the constants a, b, c, and d.

(b) Then obtain a dimensionless relation for the temperature distribution in the boundary layer.

GIVEN

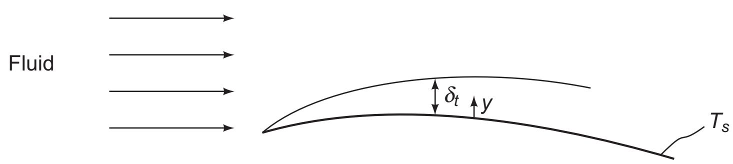
- Flow over a slightly curved isothermal surface
- Polynomial temperature distribution: $T(y) = a + by + cy^2 + dy^3$

FIND

(a) The values for a, b, c, and d

(b) A dimensionless relation for the temperature distribution in the boundary layer

SKETCH



SOLUTION

Let: Bulk fluid temperature = T_∞

Temperature of the surface = T_s

(a) The boundary conditions (BC) are

1. $T = T_s$ at $y = 0$
2. $T = 0.99\, T_\infty \approx T_\infty$ at $y = \delta_t$
3. $\frac{dT}{dy} = 0$ at $y = \delta_t$ (zero heat flux)
4. $\frac{d^2T}{dy^2} = 0$ at $y = 0$ (see Section 4.9.1)

From $B.C.$ 1: $a = T_s$

From $B.C.$ 2: $T_\infty = a + b\delta_t + c\delta_t^2 + d\delta_t^3$

From $B.C.$ 3: $0 = b + 2c\delta_t + 3d\delta_t^2$

From $B.C.$ 4: $0 = c$

Solving this set of 4 equations with 4 unknowns yields

$$a = T_s$$

$$b = \frac{3(T_\infty - T_s)}{2\delta_t}$$

$$c = 0$$

$$d = \frac{T_s - T_\infty}{2\delta_t^3}$$

(b) Substituting the constants into the temperature distribution

$$T = T_s + \frac{3}{2}\frac{T_\infty - T_s}{\delta_t}\,y + \frac{T_s - T_\infty}{2\delta_t^3}\,y^3$$

$$\frac{T - T_s}{T_\infty - T_s} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

Let θ = dimensionless temperature = $\dfrac{T - T_s}{T_\infty - T_s}$

ζ = dimensionless distance = $\dfrac{y}{\delta_t}$

Then

$$\theta = \frac{3}{2}\,\zeta - \frac{1}{2}\,\zeta^3$$

PROBLEM 4.27

The integral method can also be applied to turbulent flow conditions if experimental data for the wall shear stress are available. In one of the earliest attempts to analyze turbulent flow over a flat plate, Ludwig Prandtl proposed in 1921 the following relations for the dimensionless velocity and temperature distributions

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

$$\frac{(T - T_\infty)}{(T_s - T_\infty)} = 1 - \left(\frac{y}{\delta_t}\right)^{\frac{1}{7}} \qquad (T_s > T > T_\infty)$$

From experimental data, an empirical relation relating the shear stress at the wall with boundary layer thickness is

$$\tau_s = \frac{0.023\,\rho U_\infty^2}{Re_\delta^{0.25}} \qquad \text{where } Re_\delta = \frac{U_\infty\,\delta}{\nu}$$

Following the approach outlined in Section 4.9.1 for laminar conditions, substitute the above relations in the boundary layer momentum and energy integral equations and derive equations for:

(a) The boundary layer thickness

(b) The local friction coefficient, and

(c) The local Nusselt number.

Assume $\delta = \delta_t$ and discuss the limitations of your results.

GIVEN

- Turbulent flow over a flat plate
- Velocity and temperature distributions as given above
- Shear stress at the wall as given above

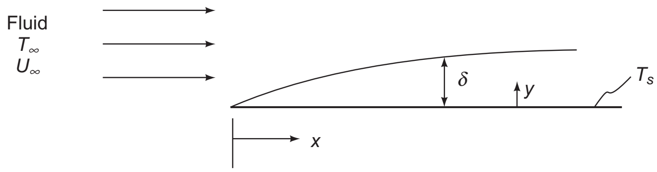
FIND

(a) The boundary layer thickness (δ)
(b) The local friction coefficient (C_f)
(c) The local Nusselt number (Nu_x)

ASSUMPTIONS

- Steady state conditions
- The hydrodynamic and thermal boundary layer thicknesses are equal

SKETCH



SOLUTION

(a) Substituting the relation for the velocity distribution and shear stress at the wall into the integral momentum equation (equation (4.42))

$$\frac{d}{dx}\int_0^{\delta_t} \rho U_\infty \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \left[U_\infty - U_\infty \left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right] dy = \tau_w = 0.023\ \frac{\rho U_\infty^2}{Re_\delta^{\frac{1}{4}}}$$

Integrating

$$\frac{d}{dx}\left[\frac{\rho U_\infty^2}{\delta^{\frac{1}{7}}}\left(\frac{7}{8}\delta^{\frac{8}{7}} - \frac{7}{9\delta^{\frac{1}{7}}}\delta^{\frac{9}{7}}\right)\right] = +\ 0.023\ \rho\ U_\infty^2 \left(\frac{\rho U_\infty \delta}{\mu}\right)^{-\frac{1}{4}}$$

$$\frac{d}{dx}\left(\frac{7}{8}\delta - \frac{7}{9}\delta\right) = 0.023\ \left(\frac{\rho U_\infty}{\mu}\right)^{-\frac{1}{4}} \delta^{-\frac{1}{4}}$$

$$\delta^{\frac{1}{4}}\, d\delta = 0.023\ \left(\frac{72}{7}\right)\left(\frac{\rho U_\infty}{\mu}\right)^{-\frac{1}{4}} dx$$

Integrating

$$\frac{4}{5}\delta^{\frac{5}{4}} = 0.023\ \left(\frac{72}{7}\right)\left(\frac{\rho U_\infty}{\mu}\right)^{-\frac{1}{4}} x + C$$

at $x = 0,\ \delta = o \rightarrow C = 0$

$$\delta = \frac{\left[\frac{5}{4}(0.023)\left(\frac{72}{7}\right)\right]^{\frac{4}{5}} x^{\frac{4}{5}}}{\left(\frac{\rho U_\infty}{\mu}\right)^{\frac{1}{5}}}$$

$$\frac{\delta}{x} = 0.377\ Re_x^{-\frac{1}{5}}$$

(b) Substituting the shear stress at the wall and the expression for δ into Equation (4.51)

$$C_{fx} = \frac{2\tau_s}{\rho U_\infty^2} = \frac{2\left[0.023\,\rho U_\infty^2 \left(\frac{U_\infty \delta}{\nu}\right)^{-\frac{1}{4}}\right]}{\rho U_\infty^2} = 0.046\ \left(\frac{U_\infty}{\nu}\right)^{-\frac{1}{4}} \left(\frac{0.377\,x}{\left(\frac{U_\infty x}{\nu}\right)^{\frac{1}{5}}}\right)^{-\frac{1}{4}}$$

$$C_{fx} = 0.059\ Re^{-\frac{3}{10}}$$

(c) Substituting the velocity and temperature distributions into the integral energy equation, equation (4.44): Note that

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{q_c}{-kA} = -\frac{h_c}{k}(T_s - T_\infty) \text{ and } \delta = \delta_t$$

$$\frac{d}{dx}\int_0^{\delta_t}(T_s - T_\infty)\left[1-\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right]U_\infty\left(\frac{y}{\delta}\right)^{\frac{1}{7}}dy - \alpha\left(\frac{\partial T}{\partial y}\right)_{y=0} = 0$$

$$(T_s - T_\infty)\,U_\infty\,\frac{d}{dx}\int_0^{\delta}\left(\left(\frac{y}{\delta}\right)^{\frac{1}{7}} - \left(\frac{y}{\delta}\right)^{\frac{2}{7}}\right)dy = -\,\alpha\,\frac{h_c}{k}(T_s - T_\infty) = \frac{h_c}{\rho c}(T_s - T_\infty)$$

$$U_\infty\,\frac{d}{dx}\left(\frac{7}{72}\delta\right) = \frac{h_c}{\rho c}$$

From part (a)

$$\delta = 0.377\left(\frac{U_\infty x}{\nu}\right)^{-\frac{1}{5}} x = 0.377\left(\frac{U_\infty}{\nu}\right)^{-\frac{1}{5}} x^{\frac{4}{5}}$$

$$\therefore\quad \frac{d\delta}{dx} = \frac{4}{5}\left(\frac{U_\infty}{\nu}\right)^{-\frac{1}{5}} x^{-\frac{1}{5}} = 0.3016\;Re_x^{-\frac{1}{5}}$$

$$\therefore\quad \frac{h_c}{\rho c} = 0.0293\;U_\infty\;Re_x^{-\frac{1}{5}}$$

$$\frac{h_c x}{k} = Nu = 0.0293\;\frac{U_\infty x}{k}\;Re_x^{-\frac{1}{5}}\;\rho c$$

$$Nu = 0.0293\;\frac{U_\infty x \rho}{\mu}\;Re_x^{-\frac{1}{5}}\;Pr$$

$$Nu \approx 0.0293\;Re_x^{\frac{4}{5}}\;Pr$$

COMMENTS

Note that the assumption that the hydrodynamic and thermal boundary layer thicknesses are equal will only be valid if $Pr \approx 1$.

PROBLEM 4.28

For liquid metals with Prandtl numbers much less the unity, the hydrodynamic boundary layer is much thinner than the thermal boundary layer. As a result, one may assume that the velocity in the boundary layer is uniform [$u = U_\infty$ and $\nu = 0$]. Starting with equation (4.7b), show that the energy equation and its boundary condition are analogous to those for a semi-infinite slab with a sudden change in surface temperature (see Equation (2.105)). Then show that the local Nusselt number is given by

$$Nu_x = 0.565\,(Re_x\,Pr)^{0.5}$$

Compare this relation with the equation for liquid metals in Table 4.5.

GIVEN

- Liquid metal flowing over a flat plate

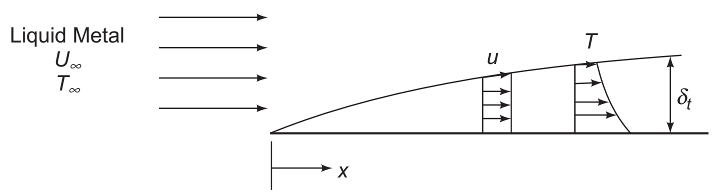
FIND

(a) Show that the energy Equation (4.7b) and its boundary conditions are analogous to those for a semi-infinite slab with sudden change in surface temperature (Equation (2.105))
(b) Show that $Nu_x = 0.564\ (Re_x\ Pr)^{0.5}$
(c) Compare this relation with the equation in Table 4.5 for liquid metals.

ASSUMPTIONS

- Steady state
- Uniform velocity in the boundary layer: $u = U_\infty$, $v = 0$
- Mercury is at room temperature (20°C)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for mercury at 20°C

Kinematic viscosity (ν) = 0.114×10^{-6} m^2/s

Prandtl number (Pr) = 0.0249

SOLUTION

(a) Substituting $u = U_\infty$ and $v = 0$ into the energy Equation (4.7b)

$$U_\infty \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2} \qquad [1]$$

The three dimensional conduction equation is given by Equation (2.6)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_G}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

For a semi-infinite slab with no internal heat generation $q_G = 0$ and the temperature varies only with x and t

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad [2]$$

This is analogous to the energy equation [1] with the following substitutions: $y \to x$ and $x/U_{\infty \to t}$. The boundary conditions for the energy equation and the conduction equation for a semi-infinite slab subjected to a step change in surface temperature are

Energy Equation	Conduction Equation
$T(o, x) = T_w$	$T(o, t) = T_s$
$T(y, o) = T_\infty$	$T(x, o) = T_i$

(b) Both the equations and the boundary conditions are analogous, therefore, the solution for a semi-infinite solid (Equation (2.105)) can be used as a solution to the liquid metal flow problem with the appropriate substitution of variables

$$q_i(x) = \frac{k\,(T_w - T_\infty)}{\sqrt{\dfrac{\pi\alpha x}{U_\infty}}} = k(T_w - T_\infty)\;\frac{1}{\sqrt{\pi}}\left(\frac{U_\infty}{\alpha x}\right)^{\frac{1}{2}}$$

$$Nu_x = \frac{hx}{k} = \frac{\dot{q}\,(x)\,x}{k\,(T_w - T_\infty)} = \frac{1}{\sqrt{\pi}}\left(\frac{U_\infty x}{\alpha}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{\pi}}\left(\frac{U_\infty x}{\nu}\frac{\nu}{\alpha}\right)^{\frac{1}{2}}$$

$$Nu_x = 0.564\;(Re_x Pr)^{\frac{1}{2}}$$

(c) The above equation agrees with the equation given in Table 4.5 for liquid metals.

PROBLEM 4.29

Hydrogen at 15°C and at a pressure of 1 atm is flowing along a flat plate at a velocity of 3 m/s. If the plate is 0.3 m wide and at 71°C, calculate the following quantities at x = 0.3 m and at the distance corresponding to the transition point, i.e., $Re_x = 5 \times 10^5$. (Take properties at 43°C.)

(a) Hydrodynamic boundary layer thickness, in cm.
(b) Thickness of thermal boundary layer, in cm.
(c) Local friction coefficient, dimensionless.
(d) Average friction coefficient, dimensionless.
(e) Drag force, in N.
(f) Local convective-heat-transfer coefficient, in W/(m^2 °C).
(g) Average convective-heat-transfer coefficient, in W/(m^2 °C).
(h) Rate of heat transfer, in W.

GIVEN

- Hydrogen flowing over a flat plate
- Hydrogen temperature (T_∞) = 15°C
- Hydrogen pressure = 1 atm
- Velocity (U_∞) = 3 m/s
- Plate temperature (T_w) = 71°C
- Width of plate = 0.3 m

FIND

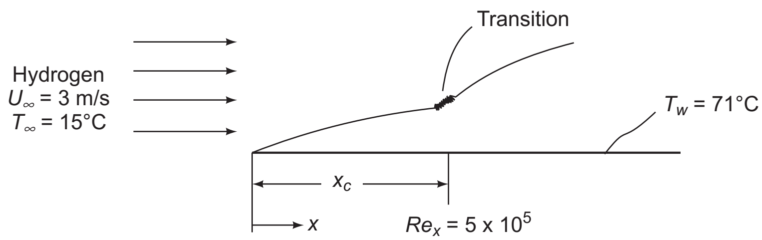
At x = 0.3 m and x_c ($Re_{xc} = 5 \times 10^5$) find

(a) Hydrodynamic boundary layer thickness (δ) in cm
(b) Thickness of thermal boundary layer (δ_t) in cm
(c) Local friction coefficient (C_{fx})
(d) Average friction coefficient (C_f)
(e) Drag force (D) in N
(f) Local convective-heat-transfer coefficient (h_{cx}) in W/(m^2 °C)
(g) Average convective-heat-transfer coefficient (h_c) in W/(m^2 °C)
(h) Rate of heat transfer (q) in W

ASSUMPTIONS

- Steady state
- Constant fluid properties

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 31, for hydrogen at 43°C

Kinematic viscosity (ν) = 119.9 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.703

Density (ρ) = 0.07811 kg/m^3

Thermal conductivity (k) = 0.190 W/(m K)

SOLUTION

Transition to turbulence occurs around $Re_x = (U_\infty x_c)/\nu = 5 \times 10^5$

$$\therefore \qquad x_c = \frac{510^5\,\nu}{U_\infty} = \frac{5\times10^5\left(119.9\times10^{-6}\,\text{m}^2/\text{s}\right)}{3}\text{m/s} = 20.0\text{ m}$$

The Reynolds number at x = 0.3 m is

$$Re_{0.3} = \frac{U_\infty x}{\nu} = \frac{(3\,\text{m/s})(0.3\,\text{m})}{119.9\times10^{-6}\,\text{m}^2/\text{s}} = 7506$$

(a) The hydrodynamic boundary layer thickness is given by Equation (4.28)

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

For x = 0.3 m $\qquad \delta = \dfrac{5(0.03\,\text{m})}{\sqrt{7506}} = 0.017\text{ m} = 1.7\text{ cm}$

For x = 20 m $\qquad \delta = \dfrac{5(20\,\text{m})}{\sqrt{5\times10^5}} = 0.14\text{ m} = 14\text{ cm}$

(b) The thermal boundary layer thickness, from the empirical relation of Equation (4.32) is

$$\delta_t = \frac{\delta}{(Pr)^{\frac{1}{3}}}$$

For x = 0.3 m $\qquad \delta_t = \dfrac{1.7\,\text{cm}}{(0.703)^{\frac{1}{3}}} = 1.91\text{ cm}$

For x = 20 m $\qquad \delta_t = \dfrac{14\,\text{cm}}{(0.703)^{\frac{1}{3}}} = 15.7$ cm

(c) The local friction coefficient is given by Equation (4.30)

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$$

For x = 0.3 m $\qquad C_{fx} = \dfrac{0.664}{\sqrt{7506}} = 0.0077$

For x = 20 m $\qquad C_{fx} = \dfrac{0.664}{\sqrt{5\times 10^5}} = 0.00094$

(d) The average friction coefficient is given by Equation (4.31)

$$\overline{C_f} = \frac{1}{L}\int_0^L C_{fx}\,dx = \frac{1}{L}\left(2L\frac{0.664}{Re_L}\right) = 2\,C_{fL} \text{ (in the laminar regime)}$$

For the plate between $x = 0$ and $x = 0.3$ m $\qquad \overline{C_f} = 2(0.0077) = 0.0154$

For the plate between $x = 0$ and $x = 20$ m $\qquad \overline{C_f} = 2(0.0094) = 0.00188$

(e) The drag force is the product of the wall shear stress (τ_s) and the wall area (A). The wall shear stress is given in terms of the friction coefficient in Equation (4.30)

$$\tau_s = \frac{1}{2}\rho U_\infty^2 C_{fx} \Rightarrow D = \int_0^L w\tau_s\,dx = \frac{1}{2}\rho U_\infty^2 A\,\frac{1}{L}\int_0^L C_{fx}\,dx = \frac{1}{2}\rho A\,U_\infty^2\,C_f$$

For the plate area between $x = 0$ and $x = 0.3$ m

$$D = \frac{1}{2}(0.3\text{ m})\,(0.3\text{ m})\,\left(0.07811\,\text{kg/m}^3\right)\,(3\,\text{m/s})^2\,(0.0154) = 0.00049\text{ N}$$

For the plate area between $x = 0$ and $x = 20$ m

$$D = \frac{1}{2}(0.3\text{ m})\,(20\text{ m})\,\left(0.07811\,\text{kg/m}^3\right)\,(3\,\text{m/s})^2\,(0.00188) = 0.0040\text{ N}$$

(f) The local heat transfer coefficient is given in Equation (4.36)

$$h_{cx} = 0.332\,\frac{k}{x}\,Re_x^{\frac{1}{2}}\,Pr^{\frac{1}{3}}$$

For x = 0.3 m $\quad h_{cx} = 0.332\,\dfrac{[0.19\,\text{W/(m K)}]}{0.3\,\text{m}}\,(7506)^{\frac{1}{2}}\,(0.703)^{\frac{1}{3}} = 16.2\text{ W/(m}^2\text{ K)} = 16.2\text{ W/(m}^2\,°\text{C)}$

For x = 20 m $\quad h_{cx} = 0.332\,\dfrac{[0.19\,\text{W/(m K)}]}{20\,\text{m}}\,\left(5\times 10^5\right)^{\frac{1}{2}}\,(0.703)^{\frac{1}{3}} = 1.98\text{ W/(m}^2\text{ K)} = 1.98\text{ W/(m}^2\,°\text{C)}$

(g) The average heat transfer coefficient is twice the local heat transfer coefficient at the end of the plate length as shown in Equation (4.39)

For the plate area from $x = 0$ to $x = 0.3$ m

$$h_c = 2\left(16.2\,\text{W/(m}^2\,°\text{C)}\right) = 32.4\ \text{W/(m}^2\,°\text{C)}$$

For the plate area from $x = 0$ to $x = 20$ m

$$h_c = 2\left(1.98\,\text{W/(m}^2\,°\text{C)}\right) = 3.96\ \text{W/(m}^2\,°\text{C)}$$

(h) The rate of heat transfer is

$$q = h_c A\,(T_w - T_\infty)$$

For the plate area between $x = 0$ and $x = 0.3$ m

$$q = \left[32.4\ \text{W/(m}^2\,°\text{C)}\right]\ (0.3\ \text{m})\ (0.3\ \text{m})\ (71°\text{C} - 15°\text{C}) = 163\ \text{W}$$

For the plate area between $x = 0$ and $x = 20$ m

$$q = \left[3.96\,\text{W/(m}^2\,°\text{C)}\right]\ (0.3\ \text{m})\ (20\ \text{m})\ (71°\text{C} - 15°\text{C}) = 1330\ \text{W}$$

COMMENTS

Note that the local heat transfer coefficient decreases with distance from the leading edge.

PROBLEM 4.30

Repeat Problem 4.29, parts (d), (e), (g), and (h) for x = 4.0 m and U_∞ = 80 m/s, (a) taking the laminar boundary layer into account and (b) assuming that the turbulent boundary layer starts at the leading edge.

From Problem 4.29: Hydrogen at 15°C and at a pressure of 1 atm is flowing along a flat plate at a velocity of 3 m/s. If the plate is 0.3 m wide and at 71°C, calculate the following quantities: (Take properties at 43°C.)

(d) Rate of heat transfer, in W.

(e) Drag force (D) in N.

(g) Average convective heat transfer coefficient (h_c) in W/(m^2 °C).

(h) Rate of heat transfer (q) in Watts.

GIVEN

- Hydrogen flowing over a flat plate
- Hydrogen temperature (T_∞) = 15°C
- Hydrogen pressure = 1 atm
- Velocity (U_∞) = 80 m/s
- Plate temperature (T_w) = 71°C
- Width of plate = 0.3 m

FIND

Calculate the quantities below for x = 4.0 and

(A) Assuming turbulent boundary layer starts at the leading edge

(B) Taking laminar boundary layer into account

(a) Rate of heat transfer, in W

(b) Drag force (D) in N

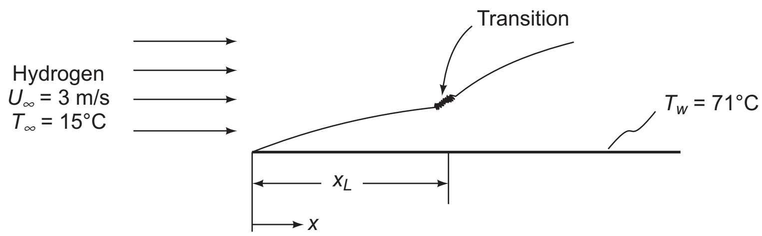
(c) Average convective heat transfer coefficient $\left(\bar{h}_c\right)$ in W/(m^2 °C)

(d) Rate of heat transfer (q) in Watts

ASSUMPTIONS

- Steady state
- Constant fluid properties

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 31, for hydrogen at 43°C

Kinematic viscosity (ν) = 119.9 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.703

Density (ρ) = 0.07811 kg/m^3

Thermal conductivity (k) = 0.190 W/(m K)

SOLUTION

The transition to a turbulent boundary layer occurs at

$$Re_x = \frac{U_\infty x_c}{\nu} = 5 \times 10^5 \Rightarrow x_c = \frac{5\times10^5\,\nu}{U_\infty} = \frac{5\times10^5\left(119.9\times10^{-6}\,\text{m}^2/\text{s}\right)}{80\,\text{m/s}} = 0.75\ \text{m}$$

$$\text{At} \qquad x = 4.0\ \text{m}: Re_x = \frac{80\,\text{m/s}(4.0\,\text{m})}{119.9\times10^{-6}\,\text{m}^2/\text{s}} = 2.67 \times 10^6$$

which is beyond transition and is in the turbulent regime.

(a) The average friction coefficient between $x = 0$ and $x = L = 4.0$ m

(A) Turbulent, Equation (4.78b)

$$\left(\overline{C_f}\right)_T = 0.072\ Re_L^{-\frac{1}{5}} = 0.072\,(2.67\times10^6)^{-\frac{1}{5}} = 3.75 \times 10^{-3}$$

(B) Mixed, Equation (4.80)

$$\left(\overline{C_f}\right)_M = 0.072\left(Re_L^{-\frac{1}{5}} - \frac{0.0464 x_c}{L}\right) = 0.072\left((2.67\times10^6)^{-\frac{1}{5}} - \frac{0.0464(0.75\,\text{m})}{4.0\,\text{m}}\right) = 3.13 \times 10^{-3}$$

(b) The drag force on the plate between $x = 0$ and $x = L$ is given by

$$D = \tau_s A = \overline{C_f}\,\frac{\rho U_\infty^{\ 2}}{2} A$$

(A) Turbulent

$$D_T = \frac{1}{2}(3.75 \times 10^{-3})\left(0.07811\,\text{kg/m}^3\right)(80\,\text{m/s})^2\,(0.3\ \text{m})\,(4.0\ \text{m}) = 1.12\ \text{N}$$

(B) Mixed

$$D_M = \frac{1}{2}(3.13 \times 10^{-3}) \left(0.07811\,\text{kg/m}^3\right)(80\,\text{m/s})^2 \ (0.3\text{ m})\ (4.0\text{ m}) = 0.94\text{ N}$$

(c) The average heat transfer coefficient between $x = 0$ and $x = L = 4.0$ m
(A) Turbulent, Equation (4.82)

$$(h_c)_T = \frac{k}{L} 0.036\ Pr^{\frac{1}{3}} Re_L^{0.8} = \frac{\left[0.19\,\text{W/(m}^2\,°\text{C)}\right]}{4.0\,\text{m}}\ 0.036\ (0.703)^{\frac{1}{3}} (2.67 \times 10^6)^{0.8} = 210.5\text{ W/(m}^2\text{ °C)}$$

(B) Mixed, Equation (4.83)

$$(h_c)_M = \frac{k}{L} 0.036\ Pr^{\frac{1}{3}}\ (Re_L^{0.8} - 23{,}200)$$

$$(h_c)_M = \frac{\left[0.19\,\text{W/(m}^2\,°\text{C)}\right]}{4.0\,\text{m}}\ 0.036\ (0.703)^{\frac{1}{3}}\ [(2.67 \times 10^6)^{0.8} - 23{,}200 = 175.2\text{ W/(m}^2\text{ °C)}$$

(d) The rate of heat transfer

$$q = h_c\, A\, (T_w - T_\infty)$$

(A) Turbulent

$$(q)_T = \left[210.0\text{ W/(m}^2\text{ °C)}\right]\ (0.3\text{ m})\ (4.0\text{ m})\ (71°\text{C} - 15°\text{C}) = 14{,}150\text{ W}$$

(B) Mixed

$$(q)_M = \left[175.2\text{ W/(m}^2\text{ °C)}\right]\ (0.3\text{ m})\ (4.0\text{ m})\ (71°\text{C} - 15°\text{C}) = 11{,}770\text{ W}$$

COMMENTS

Neglecting to take the laminar portion of the boundary layer into account led to a 20% overestimation in the rate of heat transfer from the plate.

PROBLEM 4.31

Determine the rate of heat loss from the wall of a building in a 16 kmph wind blowing parallel to its surface. The wall is 24 m long, 6 m high, its surface temperature is 27°C, and the temperature of the ambient air is 4°C.

GIVEN

- The wall of a building with wind blowing parallel to its surface
- Wind speed (U_∞) = 16 kmph = 4.4 ms^{-1}
- Length of wall (L) = 24 m
- Height of wall (H) = 6 m
- Surface temperature (T_w) = 27°C
- Ambient air temperature (T_∞) = 4°C

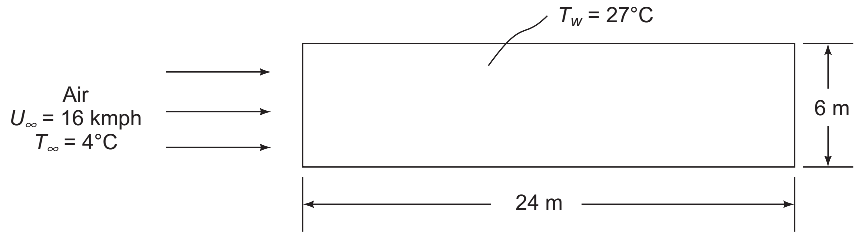
FIND

- The rate of heat loss (q) in W

ASSUMPTIONS

- Steady state
- There is negligible moisture in the air
- The wind blows along the length of the wall and parallel to it
- Radiative loss is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average of the wall and ambient temperatures 15.5°C

Kinematic viscosity (ν) = 1.5×10^{-6} m²/s

Thermal conductivity (k) = 0.025 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

Transition from laminar to turbulent boundary layer occurs at

$$Re_x = \frac{U_\infty x_c}{\nu} = 5 \times 10^5 \quad \Rightarrow x_c = \frac{5\times10^5 (1.5\times10^{-5}\,\text{m}^2/\text{s})}{4.4\,\text{m/s}} = 1.7\text{ m}$$

Therefore, the boundary layer will be mixed and the average convective heat transfer coefficient is given by Equation (4.83)

$$h_c = \frac{k}{L} 0.036 \; Pr^{\frac{1}{3}} (Re_1{}^{0.8} - 23{,}200)$$

where $$Re_L = \frac{U_\infty L}{\nu} = \frac{4.4\text{ m/s (24 m)}}{1.5\times10^{-5}\,\text{m}^2/\text{s}} = 7.04 \times 10^6$$

$$\therefore \quad h_c = 0.036 \; \frac{0.025\text{ W/(m K)}}{24\text{ m}} (0.71)^{\frac{1}{3}} \; [(7.04 \times 10^6)^{0.8} - 23{,}200] = 8.29\text{ W/(m}^2\text{ K)}$$

The rate of convective heat loss from the wall is

$$q = h_c A (T_w - T_\infty) = (8.29\text{ W/(m}^2\text{ K)) (6 m) (24 m) (27°C} - \text{4°C)}$$
$$= 27.46\text{ kW}$$

COMMENTS

Treating the whole boundary layer as turbulent, (Equation (4.82)) would lead to a rate of heat loss 8% higher than the mixed boundary layer solution shown above.

PROBLEM 4.32

A spacecraft heat exchanger is to operate in a nitrogen atmosphere at a pressure of about 10^4 N/m² and 38°C. For a flat-plate heat exchanger designed to operate on earth, in air at one atmosphere and 38°C in turbulent flow, estimate the ratio of heat-transfer coefficients on the earth to that in nitrogen, assuming forced circulation cooling of the flat plate surface at the same velocity in both cases.

GIVEN

- Flat plate heat exchangers in turbulent flow in air and nitrogen
- Nitrogen pressure = 10^4 N/m²
- Nitrogen and air temperature (T_∞) = 38°C
- Air pressure = 1 atm = 101,300 N/m²

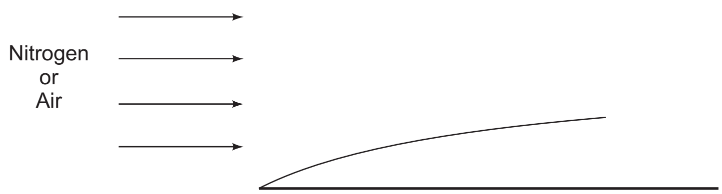
FIND

- Ratio of the heat transfer coefficients

ASSUMPTIONS

- Forced circulation coding of the plate surfaces at the same velocity in both cases
- Steady state
- Moisture in the air is negligible
- The laminar portion of the boundary layer is negligible
- Variation of Nitrogen properties with pressure is negligible
- Nitrogen behaves as an ideal gas

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 38°C

Kinematic viscosity (ν_a) = 17.4×10^{-6} m^2/s

Thermal conductivity (k_a) = 0.0264 W/(m K)

Prandtl number (Pr_a) = 0.71

From Appendix 2, Table, for nitrogen at 38°C and 1 atm

Density (ρ_1) = 1.110 kg/m^3

Absolute viscosity (μ_n) = 18.3×10^{-6} kg/m s

Thermal conductivity (k_a) = 0.02699 W/(m K)

Prandtl number (Pr_n) = 0.711

The nitrogen density at $p = 10^4$ Pa can be determined as follows

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2} \Rightarrow \rho_2 = \rho_1 \frac{P_2}{P_1} = 1.110 \text{ kg/m}^3 \left(\frac{10^4}{101,300}\right) = 0.1096 \text{ kg/m}^3$$

Therefore, the kinematic viscosity of the nitrogen is

$$\nu = \frac{\mu}{\rho} = \frac{18.3\times10^{-6}\,\text{kg/ms}}{0.1096\,\text{kg/m}^3} = 167 \times 10^{-6} \text{ m}^2\text{/s}$$

SOLUTION

The average heat transfer coefficient for a turbulent boundary layer is given by Equation (4.82)

$$h_c = 0.036\ \frac{k}{L}\ Pr^{\frac{1}{3}}\, Re_L^{\,0.8} = 0.036\ \frac{k}{L}\ Pr^{\frac{1}{3}} \left(\frac{U_\infty L}{\nu}\right)^{0.8}$$

The ratio of the heat transfer coefficient in air to the heat transfer coefficient in nitrogen is

$$\frac{h_{ca}}{h_{cn}} = \frac{k_a}{k_n}\left(\frac{Pr_a}{Pr_n}\right)^{\frac{1}{3}}\left(\frac{\nu_n}{\nu_a}\right)^{0.8} = \frac{0.0264}{0.027}\left(\frac{0.71}{0.711}\right)^{\frac{1}{3}}\left(\frac{167\times10^{-6}}{17.4\times10^{-6}}\right)^{0.8} = 5.67$$

The heat transfer coefficient in air is 6 times greater than that in the low pressure nitrogen.

PROBLEM 4.33

A heat exchanger is under development for purposes of heating liquid mercury. The exchanger can be visualized as a 15 cm long and 0.3 m wide flat plate. If the plate is maintained at 70°C and the mercury flows parallel to the short side at 15°C and a velocity of 0.3 m/s find

(a) The local friction coefficient at the middle point of the plate, and the total drag force on the plate.

(b) The temperature of the mercury at a point 10 cm from the leading edge and 1.25 mm from the surface of the plate.

(c) The Nusselt number at the end of the plate.

GIVEN

- Mercury flowing over a flat plate
- Temperature of mercury (T_∞)) = 15°C
- Velocity ($U_{\infty)}$) = 0.3 m/s
- Plate length (L) = 15 cm = 0.15 m
- Plate width = 0.3 m
- Plate surface temperature (T_s) = 70°C

FIND

(a) Local friction coefficient (C_{fx}) at the middle point of the plate (x = 7.5 cm) and the total drag force (D) on the plate

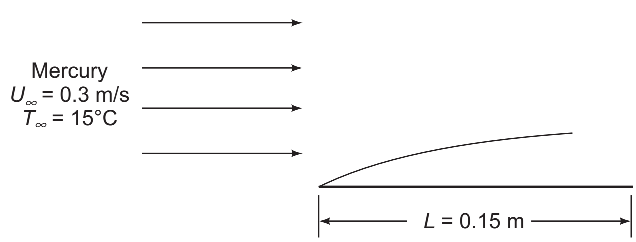
(b) Temperature of the mercury 10 cm from the leading edge (x = 0.1 m) and (1.25 mm) from the surface of the plate

(c) The Nusselt number (Nu_L) at the end of the plate

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for mercury at the average of T_∞ and T_s (42.5°C)

Thermal conductivity (k) = 9.33 W/(m K)

Kinematic viscosity (ν) = 1.044 × 10^{-7} m^2/s

Prandtl number (Pr) = 0.0216

Density (ρ) = 13900 kg/m^3

SOLUTION

The Reynolds number at the end of the plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(0.3\text{ m/s})\,(0.15\text{ m})}{1.044\times10^{-7}\text{ m}^2/\text{s}} = 4.3\times10^5 < 5\times10^5$$

Therefore, the boundary layer is laminar over the entire plate.

(a) The local friction coefficient for a laminar boundary layer is given by Equation (4.30)

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$$

At $x = 7.5$ cm: $Re_x = \dfrac{U_\infty x}{\nu} = \dfrac{(0.3\,\text{m/s})\,(0.075\,\text{m})}{1.044 \times 10^{-7}\,\text{m}^2/\text{s}} = 2.15 \times 10^5$

$$\therefore \quad C_{fx} = \frac{0.664}{\sqrt{2.15 \times 10^5}} = 1.42 \times 10^{-3}$$

The total drag force on the plate is the product of the average shear stress and the area. The shear stress is related to the friction coefficient by Equation (4.30)

$$D = \tau_s A = \frac{1}{2} \rho U_\infty^2 \overline{C_f} \; A = \rho U_\infty^2 C_{fL} A = \rho U_\infty^2 \frac{0.664}{\sqrt{Re_L}} A$$

$$D = 13900 \text{ kg/m}^3 \, (0.3 \text{ m/s})^2 \frac{0.664}{\sqrt{4.3 \times 10^5}} \, (0.15 \text{ m}) \, (0.3 \text{ m}) = 0.057 \text{ N}$$

(b) The laminar thermal boundary layer thickness is given by Equations (4.32) and (4.28)

$$\delta_{th} = \frac{\delta}{Pr^{\frac{1}{3}}} = \frac{5x}{Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}}$$

At $x = 0.1$ m: $Re_x = \dfrac{(0.3\,\text{m/s})\,(0.1\,\text{m})}{1.044 \times 10^{-7}\,\text{m}^2/\text{s}} = 2.87 \times 10^5$

$$\delta_{th} = \frac{5(0.1\,\text{m})}{(2.87 \times 10^5)^{\frac{1}{2}} (0.0216)^{\frac{1}{3}}} \; 3.3 \text{ mm}$$

Therefore, a point 1.25 mm from the plate surface is outside of the thermal boundary layer and the temperature of the mercury is the bulk temperature 15°C.

(c) The local Nusselt number for a laminar boundary layer is given by Equation (4.37)

$$Nu_{x=L} = 0.332 \; Pr^{\frac{1}{3}} Re_L^{\frac{1}{2}} = 0.332 \; (0.0216)^{\frac{1}{3}} (4.3 \times 10^5)^{\frac{1}{2}} = 61.4$$

PROBLEM 4.34

Water at a velocity of 2.5 m/s flows parallel to a 1-m-long horizontal, smooth and thin flat plate. Determine the local thermal and hydrodynamic boundary-layer thicknesses, and the local friction coefficient, at the midpoint of the plate. What is the rate of heat transfer from the plate to the water per unit width of the plate, if the surface temperature is kept uniformly at 150°C, and the temperature of the main water stream is 15°C?

GIVEN

- Water flows over a smooth and thin flat plate
- Water velocity (U_∞) = 2.5 m/s
- Length of plate (L) = 1 m
- Surface temperature (T_s) = 150°C
- Water temperature (T_∞) = 15°C

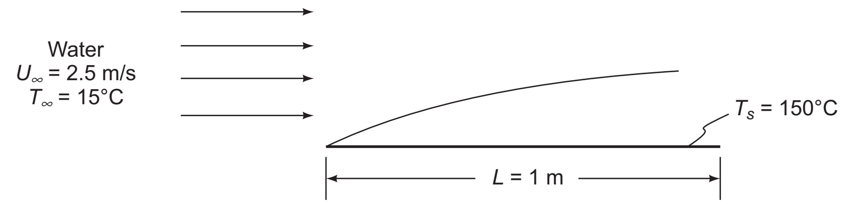
FIND

(a) Local thermal and hydrodynamic boundary layer thicknesses (δ, δ_{th}) and the local friction coefficient (C_{fx}) at the midpoint of the plate (x = 0.5 m)
(b) Heat transfer from the plate per unit width (q/w)

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the average of T_∞ and T_s (83°C)

Kinematic viscosity (ν) = 0.343 × 10^{-6} m^2/s

Thermal conductivity (k) = 0.675 W/(m K)

Prandtl number (Pr) = 2.08

SOLUTION

The Reynolds number at x = 0.5 m is

$$Re_x = \frac{U_\infty x}{\nu} = \frac{2.5(\text{m/s})(0.5\,\text{m})}{0.343\times10^{-6}\,\text{m}^2/\text{s}} = 3.46 \times 10^6 > 5 \times 10^5$$

Therefore, the boundary layer is turbulent. The hydrodynamic boundary layer thickness for a turbulent boundary layer is given by Equation (4.79)

$$\delta_x = 0.37 \left(\frac{\nu}{U_\infty}\right)^{\frac{1}{5}} x^{\frac{4}{5}} = 0.37 \left[\frac{(0.343\times10^{-6}\,\text{m}^2/\text{s})}{2.5\,\text{m/s}}\right]^{\frac{1}{5}} (0.5\,\text{m})^{\frac{4}{5}} = 0.0092\text{ m} = 9.1\text{mm}$$

The thermal boundary layer thickness for a turbulent boundary layer is also given by Equation (4.79)

$$\delta_{thx} = \delta_x = 9.1\text{ mm}$$

The local friction factor for a turbulent boundary layer is given by the empirical Equation (4.78a) (for $5 \times 10^5 < Re < 10^7$)

$$C_{fx} = 0.0576\ Re^{-\frac{1}{5}} = 0.0576\ (3.64\times10^6)^{-\frac{1}{5}} = 2.81 \times 10^{-3}$$

(b) The heat transfer coefficient for a mixed boundary layer with transition at $Re = 5 \times 10^5$ is given by Equation (4.83)

$$h_c = \frac{k}{L} 0.036\ Pr^{\frac{1}{3}}\ [Re_L^{0.8} - 23{,}200] \qquad \text{where: } Re_L = \frac{2.5\,\text{m/s}(1\,\text{m})}{0.343\times10^{-6}\,\text{m}^2/\text{s}} = 7.28 \times 10^6$$

$$h_c = \frac{(0.675\,\text{W/(m K)})}{1\,\text{m}} 0.036\,(2.08)^{\frac{1}{3}}\ [(7.28 \times 10^6)^{0.8} - 23{,}200] = 8560\text{ W/(m}^2\text{ K)}$$

The rate of heat transfer from the plate is

$$q = h_c A (T_s - T_\infty)$$

$$\therefore \quad \frac{q}{w} = h_c L (T_s - T_\infty) = \left(8560\,\text{W}/(\text{m}^2\text{K})\right) (1 \text{ m}) (150°\text{C} - 15°\text{C}) = 1.16 \times 10^6 \text{ W/m}$$

COMMENTS

Treating the entire boundary layer as turbulent would lead to an overestimation of the rate of heat transfer of about 12%.

PROBLEM 4.35

A thin, flat plate is placed in an atmospheric pressure air stream flowing parallel to it at a velocity of 5 m/s. The temperature at the surface of the plate is maintained uniformly at 200°C, and that of the main air stream is 30°C. Calculate the temperature and horizontal velocity at a point 30 cm from the leading edge and 4 mm above the surface of the plate.

GIVEN

- A thin plate in an air stream at atmospheric pressure
- Air velocity (U_∞) = 5 m/s
- Plate surface (T_s) = 200°C (uniform)
- Air temperature (T_∞) = 30°C

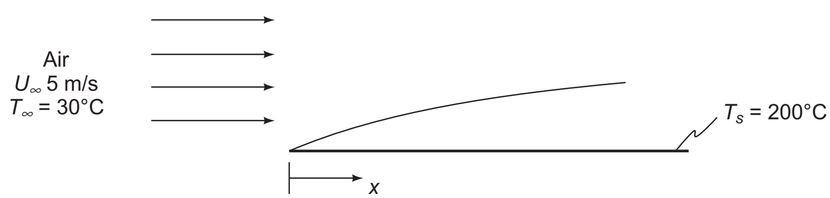
FIND

- The air temperature and horizontal velocity at x = 30 cm = 0.3 m and 4 mm = 0.004 m above the plate

ASSUMPTIONS

- Steady state
- Moisture in the air is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average of T_s and T_∞ (115°C)

Kinematic viscosity (ν) = 25.4×10^{-6} m²/s

Prandtl number (Pr) – 0.71

SOLUTION

The Reynolds number at x = 0.3 m is

$$Re_x = \frac{U_\infty x}{\nu} = \frac{(5\text{ m/s})(0.3\text{m})}{25.4\times10^{-6}\text{ m}^2/\text{s}} = 5.91 \times 10^4 < 5 \times 10^5$$

Therefore, the boundary layer is laminar. The laminar hydrodynamic boundary layer thickness is given by Equation (4.28)

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5(0.3\text{m})}{\sqrt{5.91\times10^4}} = 0.0062 \text{ m}$$

The thermal boundary layer thickness is given in Equation (4.32)

$$\delta_{th} = \frac{\delta}{Pr^{\frac{1}{3}}} = \frac{0.0062\,\text{m}}{(0.71)^{\frac{1}{3}}} = 0.007\text{ m}$$

Therefore, the point of interest is within both the hydrodynamic and thermal boundary layers. Figures 4.11 and 4.13 can be used to find the velocity and temperature at $x = 0.3$ m, $y = 0.004$ m. The abscissa of Figure 4.11 is

$$\frac{y}{x}\sqrt{Re_x} = \frac{0.004\,\text{m}}{0.3\,\text{m}}\sqrt{5.91\times10^4} = 3.24$$

From Figure 4.11

$$\frac{u}{U_\infty} \approx 0.87$$

$$\therefore \quad u = 0.87\,U_\infty = 0.82\,(5\,\text{m/s}) = 4.4\text{ m/s}$$

The abscissa for Figure 4.13 is

$$\frac{y}{x}\sqrt{Re_x}\,Pr^{\frac{1}{3}} = 3.24\,(0.71)^{\frac{1}{3}} = 2.89$$

From Figure 4.13

$$\therefore \quad T = T_s + 0.78\,(T_\infty - T_s) = 30°\text{C} + 0.78\,(200°\text{C} - 30°\text{C}) = 163°\text{C}$$

PROBLEM 4.36

The surface temperature of a thin, flat plate located parallel to an air stream is 90°C. The free stream velocity is 60 m/s and the temperature of the air is 0°C. The plate is 60 cm wide and 45 cm long in the direction of the air stream. Neglecting the end effect of the plate and assuming that the flow in the boundary layer changes abruptly from laminar to turbulent at a transition Reynolds number of $Re_{tr} = 4 \times 10^5$, find

(a) the average heat transfer coefficient in the laminar and turbulent regions

(b) the rate of heat transfer for the entire plate, considering both sides

(c) the average friction coefficient in the laminar and turbulent regions

(d) the total drag force

Also plot the heat transfer coefficient and local friction coefficient as a function of the distance from the leading edge of the plate.

GIVEN

- Air flow over a flat plate
- Plate surface temperature (T_s) = 90°C
- Air velocity (U_∞) = 60 m/s
- Air temperature (T_∞) = 0°C
- Plate length (L) = 45 cm = 0.45 m
- Plate width = 60 cm = 0.6 m

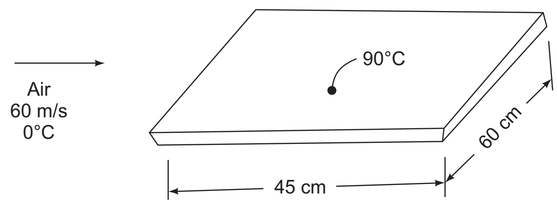
FIND

(a) The average heat transfer coefficient in the laminar (h_{cL}) and turbulent (h_{cT}) regions
(b) The rate of heat transfer for the entire plate, considering both sides (both sides)
(c) The average friction coefficient in the laminar (C_{fL}) and turbulent (C_{fT}) regions
(d) The total drag force (D)
(e) Plot the heat transfer coefficient (h_{cx}) and local friction coefficient (C_{fx}) as a function of the distance from the leading edge (x).

ASSUMPTIONS

- Steady state
- End effect of the plate is negligible
- Boundary layer changes from laminar to turbulent at $Re = 4 \times 10^5$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2*,* Table 27, for dry air at the average of T_s and T_∞ (45°C)

Kinematic viscosity (ν) = 18.1×10^{-6} m^2/s

Thermal conductivity (k) = 0.0269 W/(m K)

Prandtl number (Pr) = 0.71

Density (ρ) = 1.075 kg/m^3

SOLUTION

The transition to turbulence occurs at

$$Re_x = \frac{U_\infty x_c}{\nu} = 4 \times 10^5 \Rightarrow x_c = \frac{4\times10^5\,\nu}{U_\infty} = \frac{4\times10^5\left(18.1\times10^{-6}\,\text{m}^2/\text{s}\right)}{60} = 0.121\text{ m}$$

The Reynolds number at the end of the plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(60\,\text{m/s})(0.45\,\text{m})}{18.1\times10^{-6}\text{m}^2/\text{s}} = 1.49 \times 10^6$$

(a) For the laminar region, h_{cx} is given by Equation (4.56) and the average heat transfer coefficient is

$$h_{cL} = \frac{1}{x_c}\int_0^{x_c} \frac{k}{x} 0.33 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}\, dx = \frac{k}{x_c} 0.66\, Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

$$h_{cL} = \frac{[0.0269\,\text{W/(m K)}]}{0.121\,\text{m}}\, 0.66\ (4\times10^5)^{\frac{1}{2}} (0.71)^{\frac{1}{3}} = 82.8\text{ W/(m}^2\text{ K)}$$

For the turbulent region, h_{cx} is given by Equation (4.81)

$$h_{cT} = \frac{1}{L-x_c}\int_{x_c}^{L} \frac{k}{x} 0.0288\, Re_x^{0.8}\, Pr^{\frac{1}{3}}\, dx = \frac{k}{L-x_c}\ 0.036\ (Re_L^{0.8} - Re_{x_c}^{0.8})\, Pr^{\frac{1}{3}}$$

$$h_{cT} = \frac{[0.0269\,\text{W/(m K)}]}{0.45\,\text{m} - 0.121\,\text{m}}\ 0.036\ [(1.49\times10^6)^{0.8} - (4\times10^5)^{0.8}]\ (0.71)^{\frac{1}{3}} = 148.6\text{ W/(m}^2\text{ K)}$$

(b) The total heat transfer is the sum of the heat transfer from both regions.

$$q = q_{\text{Lam}} + q_{\text{Turb}} = (h_{cL} A_L + h_{cT} A_T)\,(T_s - T_\infty)$$

$$q = \left[\left(82.8\,\text{W/(m}^2\text{K)}\right)(0.121\,\text{m})(0.6\,\text{m}) + \left(148.6\,\text{W/(m}^2\text{K)}\right)(0.45\,\text{m} - 0.121\,\text{m})(0.6\,\text{m})\right](90°\text{C} - 0°\text{C})$$

$$q = 3131\text{ W}$$

For both sides $\quad q_{\text{Total}} = 2\,q = 6362$ W

(c) The average friction coefficient in the laminar region is given by Equation (4.31)

$$\overline{C_{fL}} = 1.33\, Re_{x_c}^{-\frac{1}{2}} = 1.33\,(4\times10^{5})^{-\frac{1}{2}} = 0.00210$$

The local friction coefficient in the turbulent region is given by Equation (4.78a). The average friction coefficient in the turbulent region is

$$\overline{C_{fL}} = \frac{k}{L-x_c}\int_{x_c}^{L} C_{fx}\,dx = \frac{1}{L-x_c}\int_{x_c}^{L} 0.0576\left(\frac{U_\infty x}{\nu}\right)^{-\frac{1}{3}} dx = \frac{0.0576}{L-x_c}\left(\frac{U_\infty}{\nu}\right)^{-\frac{1}{5}} \frac{5}{4}\left(L^{\frac{4}{5}} - x_c^{\frac{4}{5}}\right)$$

$$\overline{C_{fL}} = \frac{0.0576}{0.45\,\text{m}-0.121\,\text{m}}\left(\frac{(60\,\text{m/s})}{18.1\times10^{-6}\,\text{m}^2/\text{s}}\right)^{-\frac{1}{5}} 1.25\,[(0.45\text{ m})^{0.8} - (0.121)^{0.8}] = 0.00373$$

(d) The drag force is

$$D = \tau_s A \qquad \text{where: } \tau_s = \overline{C_f}\,\frac{1}{2}\rho\, U_\infty^2\, A_s \qquad \text{from Equation (4.13)}$$

For both sides of the plate

$$D = \rho\, U_\infty^2 \left(\overline{C_{fL}}\, A_L + \overline{C_{fT}}\, A_T\right)$$

(e) For the laminar region, $0 < x < 0.121$ m, Equation (4.56) gives the heat transfer coefficient

$$h_{cx} = \frac{k}{x} 0.33\, Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{[0.0269\,\text{W/(m K)}]}{x}\, 0.33\left(\frac{(60\,\text{m/s})}{18.1\times10^{-6}}\right)^{\frac{1}{2}} x^{\frac{1}{2}}\,(0.71)^{\frac{1}{3}} = \left(14.41\,\text{W/m}^{\frac{3}{2}}\,\text{K}\right) x^{-\frac{1}{2}}$$

For the turbulent region, $0.121\text{ m} < x < 0.45$ m, from Equation (4.81)

$$h_{cx} = \frac{k}{x} 0.0288\; Pr^{\frac{1}{3}}\, Re_x^{0.8} = \left(113.7\,\text{W/(m}^2\,\text{K)}\right) x^{-0.2}$$

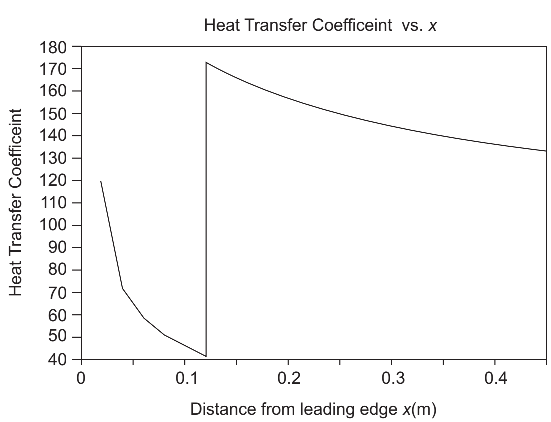
The friction coefficient for the laminar region (Equation (4.30)) is

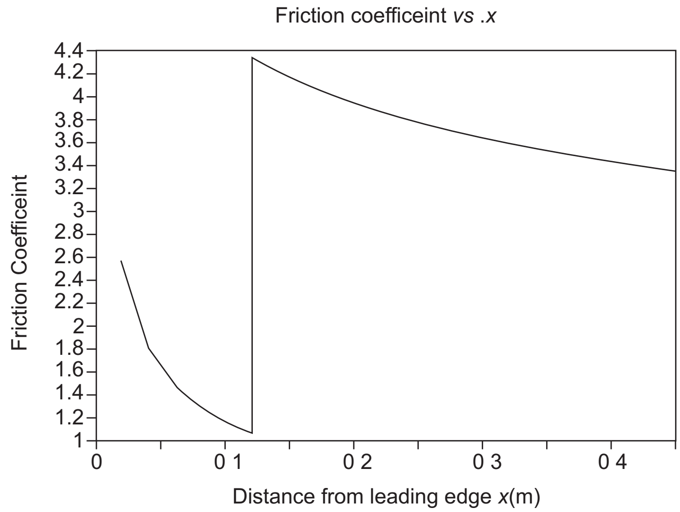
$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} = 3.65\times10^{-4}\; x^{-\frac{1}{2}}$$

For the turbulent region (Equation (4.78a))

$$C_{fx} = 0.0576\; Re^{-\frac{1}{5}} = 2.859\times10^{-3}\; x^{-\frac{1}{5}}$$

The variations of the heat transfer coefficient and friction coefficient with distance from the leading edge are plotted below





PROBLEM 4.37

The wing of an airplane has a polished aluminum skin. At a 1500 m altitude, it absorbs 100 W/m^2 by solar radiation. Assuming that the interior surface of the wing's skin is well insulated and the wing has a chord of 6 m length, i.e., *L* = 6 m, estimate the equilibrium temperature of the wing at a flight speed of 150 m/s at distances of 0.1 m, 1 m, and 5 m from the leading edge. Discuss the effect of a temperature gradient along the chord.

GIVEN

- Airplane wing with polished aluminum skin
- Altitude = 1500 m
- Absorbed solar radiation (q_{sol}/A) = 100 W/m^2
- Cord length of wing (L) = 6 m
- Flight speed (U_∞) = 150 m/s

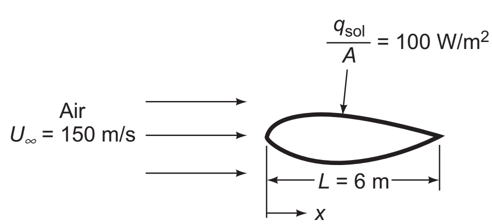
FIND

(a) Equilibrium temperature at x = 0.1 m, 1 m, and 5 m
(b) Discuss the effect of temperature gradient along the wing

ASSUMPTIONS

- Steady state
- Inside surface of wing is well insulated, so heat loss from the inner surface is negligible
- Radiative loss from the wing surface is negligible
- Flight speed given is air speed not ground speed
- Variation of air properties with pressure is negligible
- Neglect aerodynamic heating

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 37 at 1500 m altitude the air temperature (T_∞) = 5°C and the density of the air (ρ) = 1.06 kg/m^3

From Appendix 2, Table 27, for dry air at 1 atm and 5°C

Absolute viscosity (μ) = 17.7 × 10^{-6} N s/m^2

Thermal conductivity (k) = 0.0273 W/(m K)

Prandtl number (Pr) = 0.71

The kinematic viscosity at 1500 m is: $\nu = \mu/\rho$ = 16.7 × 10^{-6} m^2/s

SOLUTION

(a) The Reynolds numbers at the desired locations are

$$\text{At } x = 0.1 \text{ m: } Re_x = \frac{U_\infty x}{\nu} = \frac{(150\,\text{m/s})(0.1\,\text{m})}{16.7\times10^{-6}\,\text{m}^2/\text{s}} = 0.89 \times 10^6$$

$$\text{At } x = 1 \text{ m: } Re_x = \frac{(150\,\text{m/s})(1\,\text{m})}{16.7\times10^{-6}\,\text{m}^2/\text{s}} = 8.98 \times 10^6$$

$$\text{At } x = 5 \text{ m: } Re_x = \frac{(150\,\text{m/s})(5\,\text{m})}{16.7\times10^{-6}\,\text{m}^2/\text{s}} = 4.49 \times 10^7$$

The boundary layer is turbulent at all these locations. For a turbulent boundary layer, the local heat transfer coefficient is given by Equation (4.81)

$$h_{cx} = \frac{k}{x}\, 0.0288\, Re_x^{\,0.8}\, Pr^{\frac{1}{3}}$$

$$\text{At } x = 0.1 \text{ m: } h_{cx} = \frac{[0.0237\,\text{W/(m K)}]}{0.1\,\text{m}}\, 0.0288\, (0.89 \times 10^6)^{0.8}\, (0.71)^{\frac{1}{3}} = 350 \text{ W/(m}^2\text{ K)}$$

$$\text{At } x = 1 \text{ m: } h_{cx} = \frac{[0.0237\,\text{W/(m K)}]}{1\,\text{m}}\, 0.0288\, (9.98 \times 10^6)^{0.8}\, (0.71)^{\frac{1}{3}} = 222 \text{ W/(m}^2\text{ K)}$$

$$\text{At } x = 5 \text{ m: } h_{cx} = \frac{[0.0237\,\text{W/(m K)}]}{5\,\text{m}}\, 0.0288\, (4.49 \times 10^7)^{0.8}\, (0.71)^{\frac{1}{3}} = 161 \text{ W/(m}^2\text{ K)}$$

The local convective heat loss from the wing must equal the radiative heat gain for equilibrium to exist

$$\frac{q_{cx}}{A} = h_{cx}\,(T_s - T_\infty) = \frac{q_{\text{sol}}}{A}$$

Solving for the wing surface temperature

$$T_s = T_\infty + \frac{1}{h_{cx}}\frac{q_{\text{sol}}}{A}$$

$$\text{At } x = 0.1 \text{ m: } T_s = 5°\text{C} + \frac{1}{[350\,\text{W/(m}^2\text{ K)}]}\left(100\,\text{W/m}^2\right) = 5.29°\text{C}$$

$$\text{At } x = 1 \text{ m: } T_s = 5°\text{C} + \frac{1}{[222\,\text{W/(m}^2\text{K)}]}\left(100\,\text{W/m}^2\right) = 5.45°\text{C}$$

$$\text{At } x = 5 \text{ m: } T_s = 5°\text{C} + \frac{1}{[161\,\text{W/(m}^2\text{K)}]}\left(100\,\text{W/m}^2\right) = 5.62°\text{C}$$

In all three cases, the film temperature is very nearly 5°C, so our choice of 5°C for calculating the air properties is justified.

(b) Conduction along the aluminum skin will effectively smooth out these small temperature differences.

PROBLEM 4.38

An aluminum cooling fin for a heat exchanger is situated parallel to an atmospheric pressure air stream. The fin is 0.075 m high, 0.005 m thick, and 0.45 m in the flow direction. Its base temperature is 88°C, and the air is at 10°C. The velocity of the air is 27 m/s. Determine the total drag force and the total rate of heat transfer from the fin to the air.

GIVEN

- Air flow over a heat exchanger fin
- Fin length (L) = 0.45 m
- Fin height (w) = 0.075
- Fin thickness = 0.005 m
- Fin base temperature (T_b) = 88°C
- Air temperature (T_∞) = 10°C
- Air velocity (U_∞) = 27 m/s

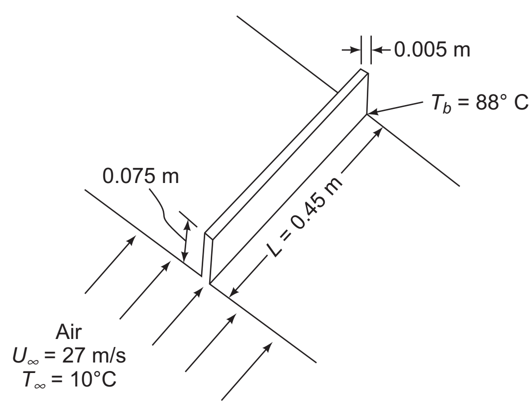
FIND

(a) The total drag force (D) on the fin
(b) The total rate of heat transfer (q) from the fin to the air

ASSUMPTIONS

- Steady state
- Edge effects are negligible
- Both sides of the fin are exposed to the air
- Transition to a turbulent boundary layer occurs at
- $Re_x = 5 \times 10^5$
- Fin thickness is negligible
- Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average of T_b and T_∞ (49°C)

Kinematic viscosity (ν) = 18.4×10^{-6} m^2/s

Thermal conductivity (k) = 0.0271 W/(m K)

Density (ρ) = 1.061 kg/m^3

Prandtl number (Pr) = 0.71

From Appendix 2, Table 12, for aluminum at the average of T_b and T_∞ (49°C)

Thermal conductivity (k) = 238 W/(m K)

SOLUTION

The Reynolds number at the end of the fin is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(27\,\text{m/s})\,(0.45\,\text{m})}{18.4\times10^{-6}\,\text{m}^2/\text{s}} = 6.60\times10^5 > 5\times10^5$$

The boundary layer is turbulent at the end of the fin. The transition to turbulence occurs at

$$Re_{x_c} = \frac{U_\infty x_c}{\nu} = 5\times10^5 \Rightarrow x_c = \frac{5\times10^5\ \nu}{U_\infty} = \frac{5\times10^5\left(18.4\times10^{-6}\,\text{m}^2/\text{s}\right)}{27}\frac{\text{m}}{\text{s}} = 0.341\ \text{m}$$

(a) The average friction factor (C_f) for a mixed boundary layer is given by Equation (4.80)

$$\overline{C_f} = 0.072\left(\text{Re}_L^{-\frac{1}{5}}\frac{0.0464\,x_c}{L}\right)$$

$$= 0.072\left[(6.6\times10^5)^{-\frac{1}{5}} - \frac{0.0464\,(0.341\,\text{m})}{0.45\,\text{m}}\right] = 0.00240$$

The drag force on both sides of the plate (using Equation (4.13) for the shear stress at the wall) is

$$D = 2\ \tau_s A = C_f\,\rho\,U_\infty^2\,A = 0.0024\ \left(1.061\,\text{kg/m}^3\right)(27\,\text{m/s})^2\,(0.075\ \text{m})\,(0.45\ \text{m}) = 0.063\ N$$

(b) The average heat transfer coefficient (h_c) for a mixed boundary layer is given in Equation (4.83)

$$h_c = \frac{k}{L}\ 0.036\ Pr^{\frac{1}{3}}\ [Re_L^{0.8} - 23{,}200] = \frac{(0.0271\,\text{W/(m K)})}{0.45\,\text{m}}\ 0.036\ (0.71)^{\frac{1}{3}}\ [(6.6\times10^5)^{0.8} - 23{,}200\}$$

$$h_c = 42.7\ \text{W/(m}^2\ \text{K)}$$

The rate of heat transfer from a fin of uniform cross section and convection from the tip is given in Table 2.1

$$q = M\ \frac{\sinh(mL_f) + \left(\dfrac{h}{m k_a}\right)\cosh(mL_f)}{\cosh(mL_f) + \left(\dfrac{h}{m k_a}\right)\sinh(mL_f)}$$

$$\text{where } m \equiv \sqrt{\frac{\overline{h}_c P}{k_a A_c}}$$

P = perimeter = 2(0.45 m + 0.005 m) = 0.91 m

A_c = cross sectional area = (0.005 m) (0.45 m) = 0.00225 m^2

L_f = 0.075 m

$$\therefore\ m = \sqrt{\frac{\left(42.7\,\text{W/(m}^2\,\text{K)}\right)(0.91\,\text{m})}{(238\,\text{W/(m K)})(0.00225\,\text{m}^2)}} = 8.52\ \frac{1}{\text{m}}$$

$$m\,L_f = 8.52\ \frac{1}{\text{m}}\ (0.075\ \text{m}) = 0.639$$

$$M = (T_b - T_\infty)\sqrt{\bar{h}_c P k_a A_c} = (88°\text{C} - 10°\text{C})\sqrt{(42.7\,\text{W/(m}^2\text{K)})(0.91\,\text{m})(238\,\text{W/(m K)})(0.0025\,\text{m}^2)}$$

$$M = 356 \text{ W}$$

$$\frac{\bar{h}_c}{m k_a} = \frac{(42.7\,\text{W/(m}^2\,\text{K)})}{8.52\frac{1}{3}(238\,\text{W/(m K)})} = 0.0211$$

$$\therefore q_f = 356 \text{ W} \frac{\sinh(0.639) + 0.0211\cosh(0.639)}{\cosh(0.639) + 0.0211\sinh(0.639)} = 206 \text{ W}$$

COMMENTS

If the entire fin was assumed to be at the base temperature, the rate of heat transfer from the fin would be about 225 W, about 9% higher than calculated above. The high conductivity of the fin material makes this installation very thermally efficient, i.e., $\eta_f = 91\%$.

PROBLEM 4.39

Air at 320 K with a free stream velocity of 10 m/s is used to cool small electronic devices mounted on a printed circuit board as shown in the sketch below. Each device is 5 mm × 5 mm square in plane-form and dissipates 60 milliwatts. A turbulator is located at the leading edge to trip the boundary layer so that it will become turbulent. Assuming that the lower surface of the electronic devices are insulated, estimate the surface temperature at the center of the fifth device on the circuit board.

GIVEN

- Air flows over small electronic devices
- Air temperature (T_∞) = 320 K
- Air velocity (U_∞) = 10 m/s
- Dimensions of each device = 5 mm × 5 mm = .005 m × .005 m
- Power dissipation per device ($\dot{q}_G$) = 60 milliwatts = 0.06 W
- There is a turbulator at the leading edge

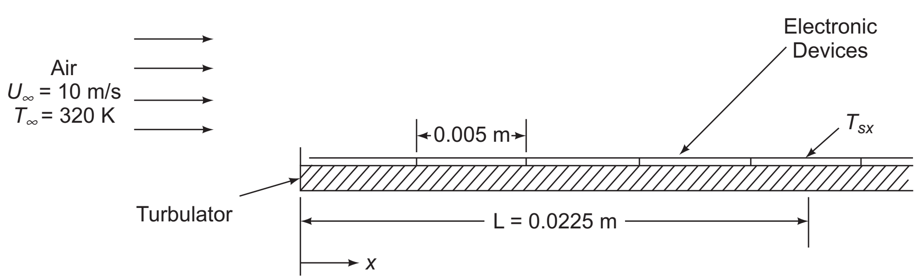
FIND

- The surface temperature (T_{sx}) at the center of the fifth device

ASSUMPTIONS

- Steady state
- Lower surface of the devices is insulated (negligible heat loss)
- The devices are placed edge-to-edge on the board
- The boundary layer is turbulent from the leading edge on
- The bulk fluid temperature is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average of 320 K

Kinematic viscosity (ν) = 18.2×10^{-6} m^2/s

Thermal conductivity (k) = 0.0270 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

The center of the fifth chip is 0.0225 m from the leading edge. The Reynolds number at this point is

$$Re_x = \frac{U_\infty x}{\nu} = \frac{(10\,\text{m/s})(0.0225\,\text{m})}{18.2\times10^{-6}\,\text{m}^2/\text{s}} = 1.24 \times 10^4$$

Although this would normally be a laminar boundary layer, in this case, it will be turbulent due to the turbulator at the leading edge. For a turbulent boundary layer, the local heat transfer coefficient is given by Equation (4.81)

$$h_{cx} = \frac{k}{x}\, 0.0288\, Re_x^{0.8}\, Pr^{\frac{1}{3}} = \frac{(0.0270\,\text{W/(m K)})}{0.0225\,\text{m}}\, 0.0288\, (1.24 \times 10^4)^{0.8}\, (0.71)^{\frac{1}{3}} = 57.9\ \text{W/(m}^2\ \text{K)}$$

For steady state, the rate of convective heat flux at x = 0.0225 m must equal the rate of heat generation per unit surface area

$$\frac{q_{cx}}{A} = h_{cx}\,(T_{sx} - T_\infty) = \frac{q_G}{A}$$

Solving for the surface temperature

$$T_{sx} = T_\infty + \frac{1}{h_{cx}}\frac{q_G}{A} = 320\ \text{K} + \frac{1}{57.9\,\text{W/(m}^2\,\text{K)}}\left(0.06\ \text{W/chip}\frac{1\,\text{chip}}{(0.005\,\text{m})\,(0.005\,\text{m})}\right)$$

$$= 361\ \text{K} = 88°\text{C}$$

The film temperature is therefore (320 K + 361 K)/2 = 341 K. Performing another iteration using air properties evaluated at 341 K yields the following results

$$\nu = 20.2 \times 10^{-6}\ \text{m}^2/\text{s}$$

$$k = 0.0285\ \text{W/(m K)}$$

$$Pr = 0.71$$

$$Re_x = 11{,}117$$

$$h_{cx} = 56.1$$

$$T_{sx} = 363\ \text{K} = 90°\text{C}$$

PROBLEM 4.40

The average friction coefficient for flow over a 0.6 m-long plate is 0.01. What is the value of the drag force in N per m width of the plate for the following fluids: (a) air at 15°C, (b) steam at 100°C and atmospheric pressure, (c) water at 40°C, (d) mercury at 100°C, and (e) *n*-Butyl alcohol at 100°C?

GIVEN

- Flow over a plate
- Friction coefficient (C_f) = 0.01
- Length of plate (L) = 0.6 m

FIND

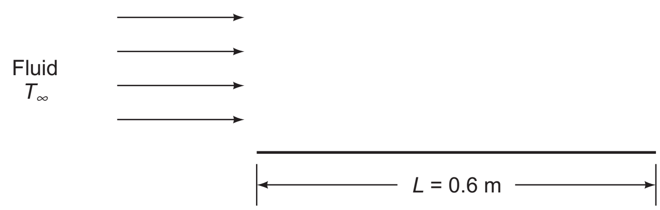
The value of the drag force (D) in N per meter width of the plate for

(a) Air at 15°C
(b) Steam at 100°C and atmospheric pressure
(c) Water at 40°C
(d) Mercury at 100°C
(e) *N*-Butyl alcohol at 100°C

ASSUMPTIONS

- Steady state
- Fully developed turbulent flow

SKETCH



PROPERTIES AND CONSTANTS

The following information is from Appendix 2

Substance	Table Number	Temperature (°C)	Kinematic Viscosity $\nu \times 10^6$ m²/s	Density, ρ kg/m³
(a) Air	27	1 5	15.3	1.19
(b) Steam	34	100	20.2	0.5977
(c) Water	13	40	0.658	992.2
(d) Mercury	25	100	0.0928	13.385
(e) *n*-Butyl Alcohol	18	100	0.69	751

SOLUTION

Assuming the boundary layer is laminar, the average friction is given by Equation (4.31)

$$\bar{C}_f = 1.33 \left(\frac{U_\infty L}{\nu}\right)^{-\frac{1}{2}} \Rightarrow U_\infty = \left(\frac{1.33}{\bar{C}_f}\right)^2 \frac{\nu}{L}$$

Therefore, the Reynolds number at the end of the plate is

$$Re_1 = U_\infty \frac{L}{\nu} = \left[\left(\frac{1.33}{\bar{C}_f}\right)^2 \frac{\nu}{L}\right]\frac{L}{\nu} = \left(\frac{1.33}{0.01}\right)^2 = 1.77 \times 10^4 < 5 \times 10^5$$

Therefore, the assumption that the boundary layer is laminar is valid.
The drag force on the plate is

$$D = \bar{\tau}_w A$$

The wall shear stress (τ_w) is related to the friction coefficient by Equation (4.13)

$$\bar{\tau}_w = \bar{C}_f \frac{1}{2}\rho U_\infty^2$$

$$\therefore D = \bar{C}_f \frac{1}{2}\rho\left[\left(\frac{1.33}{\bar{C}_f}\right)^2 \frac{\nu}{L}\right]^2 A_s = \frac{1.565}{\bar{C}_f^{\,3}}\rho\left(\frac{\nu}{L}\right)^2 L\, w$$

$$\frac{D}{w} = \frac{1.565}{\overline{C}_f^3}\frac{\rho}{L}v^2$$

$$\text{(a) Air: } \frac{D}{w} = \frac{1.565}{(0.01)^3}\left(\frac{1.190\,\text{kg/m}^3}{0.6\,\text{m}}\right)\left(15.3\times10^{-6}\,\text{m}^2/\text{s}\right)^2 = 7.3 \times 10^{-4}\text{ N/m}$$

$$\text{(b) Steam: } \frac{D}{w} = \frac{1.565}{(0.01)^3}\left(\frac{0.5977\,\text{kg/m}^3}{0.6\,\text{m}}\right)\left(20.2\times10^{-6}\,\text{m}^2/\text{s}\right)^2 = 6.4 \times 10^{-4}\text{ N/m}$$

$$\text{(c) Water: } \frac{D}{w} = \text{s}\frac{1.565}{(0.01)^3}\left(\frac{992.2\,\text{kg/m}^3}{0.6}\right)\left(0.658\times10^{-6}\,\text{m}^2/\text{s}\right)^2 = 1.1 \times 10^{-3}\text{ N/m}$$

$$\text{(d) Mercury: } \frac{D}{w} = \frac{1.565}{(0.01)^3}\left(\frac{13{,}385\,\text{kg/m}^3}{0.6\,\text{m}}\right)\left(0.0928\times10^{-6}\,\text{m}^2/\text{s}\right)^2 = 3.0 \times 10^{-4}\text{ N/m}$$

$$\text{(e) Alcohol: } \frac{D}{w} = \frac{1.565}{(0.01)^3}\left(\frac{751\,\text{kg/m}^3}{0.6\,\text{m}}\right)\left(0.69\times10^{-6}\,\text{m}^2/\text{s}\right)^2 = 9.3 \times 10^{-4}\text{ N/m}$$

PROBLEM 4.41

A thin, flat plate 15 cm square is tested for drag in a wind tunnel with air at 30 m/s, 100 kPa, and 16°C flowing across and parallel to the top and bottom surfaces. The observed total drag force is 0.06 N. Using the definition of friction coefficient, Equation (4.13), and the Reynolds analogy, calculate the rate of heat transfer from this plate when the surface temperature is maintained at 120°C.

GIVEN

- Air flow over the top and bottom of a thin plate
- Plate dimensions = 0.15 m × 0.15 m
- Air speed (U_∞) = 30 m/s
- Air pressure = 100 kPa
- Air temperature (T_∞) = 16°C
- Total drag force (D) = 0.06 N
- Surface temperature (T_s) = 120°C

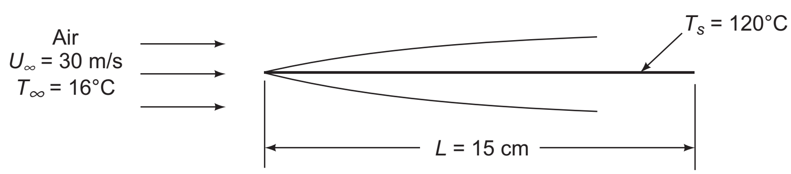
FIND

- The rate of heat transfer (q)

ASSUMPTIONS

- Steady state
- Constant and uniform plate temperature
- Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for air at the average of T_s and T_∞ (68.0°C)

Kinematic viscosity (ν) = 2.96×10^{-5} m^2/s
Thermal conductivity (k) = 0.0342 W/(m K)
Density (ρ) = 0.833 kg/m^3
Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number at the end of the plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(30\,\text{m/s})(0.15\,\text{m})}{2.96\times 10^{-5}\,\text{m}^2/\text{s}} = 1.52 \times 10^5 \text{ (Laminar)}$$

Equation (4.13)

$$\bar{C}_f = \frac{2\,\tau_s}{pU_\infty^2}$$

The total drag force on both sides of the plate is

$$D = 2\,A\,\tau_s \quad \Rightarrow \quad \tau_s = \frac{D}{2A}$$

where A = the area of one side of the plate.

$$\bar{C}_f = \frac{D}{\rho A U_\infty^2}$$

The Reynolds analogy, corrected for Prandtl numbers other than unity, is given in Equation (4.40)

$$Nu_x = \frac{h_{cx}\,x}{k} = \frac{C_{fx}}{2} Re_x Pr^{\frac{1}{3}}$$

$$\frac{\nu}{U_\infty k\,Pr^{\frac{1}{3}}}\;h_{cx}\,x = \frac{C_{fx}}{2}$$

Averaging this over the length of the plate yields

$$\frac{\nu}{U_\infty k\,Pr^{\frac{1}{3}}}\frac{1}{L}\int_0^L h_{cx}\,dx = \frac{1}{2L}\int_0^L C_{fx}\,dx$$

$$\frac{\nu}{U_\infty k\,Pr^{\frac{1}{3}}}\;h_c = \frac{1}{2}C_f$$

$$\therefore \qquad \bar{h}_c = \frac{k}{L}\left(\frac{C_t}{2}\right)\left(\frac{U_\infty L}{\nu}\right)Pr^{\frac{1}{3}} = \frac{k}{L}\left(\frac{D}{2\rho A U_\infty^2}\right)Re_L Pr^{\frac{1}{3}}$$

$$\bar{h}_c = \frac{(0.0342\,\text{W/(m K)})}{0.15\,\text{m}}\;\frac{0.06\,\text{N}}{2(0.833\,\text{kg/m}^3)(0.15\,\text{m})^2\,(30\,\text{m/s})^2}\;(1.52\times 10^5)\,(0.71)^{\frac{1}{3}}$$

$$= 55 \text{ W/(m}^2\text{ K)}$$

The rate of heat transfer from both sides of the plate is

$$q = 2\,\bar{h}_c\,A\,(T_s - T_\infty) = 2(55\text{ W/(m}^2\text{ K))}(0.15\text{ m})^2\,(120°\text{C} - 16°\text{C}) = 257.4\text{ W}$$

COMMENTS

If radiation is included, assuming the plate behaves as a blackbody, and is totally enclosed by the wind tunnel which behaves as a blackbody at 60°C, the rate of heat transfer would be

$$q = q_c + q_r = q_c + A\,\sigma(T_s^4 - T_\infty^4)$$

$$q = 257\text{ W} + (0.15\text{ m})^2\,(5.67 \times 10^{-8}\text{ W/(m}^2\text{ K))}[(393\text{ K})^4 - (289\text{ K})^4]$$

$$= 257.4\text{ W} + 21.5\text{ W}$$

$$q = 278.9\text{ W}$$

(9% higher than the results neglecting radiation)

PROBLEM 4.42

A thin, flat plate 15 cm square is suspended from a balance into a uniformly flowing stream of engine oil in such a way that the oil flows parallel to and along both surfaces of the plate. The total drag on the plate is measured and found to be 55.5 N. If the oil flows at the rate of 15 m/s and at a temperature of 45°C, calculate the heat-transfer coefficient using the Reynolds analogy.

GIVEN

- Engine oil flowing along a thin flat plate
- Plate dimensions = 15 cm × 15 cm = 0.15 m × 0.15 m
- Engine oil velocity (U_∞) = 15 m/s
- Engine oil temperature (T_∞) = 45°C
- Total drag force (D) 55.5 N

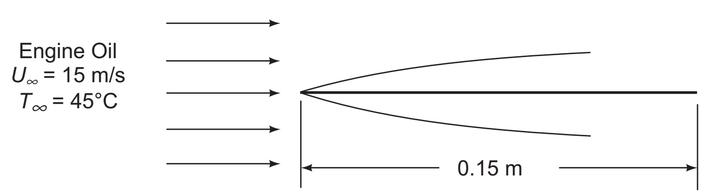
FIND

- The heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- Constant fluid properties
- Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 16, for unused engine oil at (45°C):

Kinematic viscosity (ν) = 201×10^{-6} m²/s
Thermal conductivity (k) = 0.143 W/(m K)
Density (ρ) = 873.1 kg/m³
Prandtl number (Pr) = 24.2

SOLUTION

The Reynolds number is

$$Re_L = \frac{U_\infty\, L}{\nu} = \frac{(15\,\text{m/s})\,(0.15\,\text{m})}{201 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.12 \times 10^4 \text{ (Laminar)}$$

The friction coefficient is defined in Equation (4.13)

$$\overline{C}_f = \frac{2\,\tau}{\rho U_\infty^2}$$

The drag force on both sides of the plate is

$$D = 2\,A\,\tau \quad \Rightarrow \quad \tau = \frac{D}{2\,A}$$

Where A = the area of one side of the plate.

$$\therefore\ \overline{C}_f = \frac{D}{\rho\,A U_\infty^2}$$

The Reynolds analogy relates the heat transfer coefficient and the friction coefficient in Equation (4.40) (corrected for Prandtl numbers other than unity)

$$Nu_x = \frac{h_{cx}x}{k} = \frac{1}{2}\ C_{fx}\,Re_x\ Pr^{\frac{1}{3}}$$

$$\frac{\nu}{U_\infty k\,Pr^{\frac{1}{3}}}\ h_{cx} = \frac{C_{fx}}{2}$$

Averaging this over the length of the plate yields:

$$\frac{\nu}{U_\infty k\,Pr^{\frac{1}{3}}}\frac{1}{L}\int_0^L h_{cx}\,dx = \frac{1}{2\,L}\int_0^L C_{fx}\,dx$$

$$\frac{\nu}{U_\infty k\,Pr^{\frac{1}{3}}}\ h_c = \frac{1}{2}\,C_f$$

$$\therefore\quad \overline{h}_c = \frac{k}{L}\left(\frac{C_f}{2}\right)\left(\frac{U_\infty\,L}{\nu}\right)Pr^{\frac{1}{3}} = \frac{k}{L}\left(\frac{D}{2\rho\,AU_\infty^2}\right)\ Re_L\ Pr^{\frac{1}{3}}$$

$$\overline{h}_c = \frac{(0.143\,\mathrm{W/(m\,K)})(55.5\,\mathrm{N})\left(\mathrm{kg\,m/(Ns^2)}\right)}{2\,(0.15\,\mathrm{m})\left(873.1\,\mathrm{kg/m^3}\right)(0.15\,\mathrm{m})\,(0.15\,\mathrm{m})(15\,\mathrm{m/s})^2}\ (1.12\times10^4)\,(24.2)^{\frac{1}{3}}$$

$$= 194\ \mathrm{W/(m^2\ K)}$$

COMMENTS

Since the plate is submerged in the engine oil, the assumption that radiative heat transfer is negligible is valid. If the plate were in a gas, this assumption may not be valid. (See Problem 4.41.)

PROBLEM 4.43

For a study on global warming, an electronic instrument has to be designed to map and the CO_2 absorption characteristics of the Pacific Ocean. The instrument package resembles a flat plate with a total (upper and lower) surface area of 2 m^2. For safe operation, its surface temperature must not exceed the ocean temperature by 2°C. To monitor the temperature of the instrument package, which is towed by a ship moving at 20 m/s, the tension in the towing cable is measured. If the tension is 400 N, calculate the maximum permissible heat generation rate from the instrument package.

GIVEN

- A flat plate towed through water
- Total surface area (A) = 2 m^2
- Speed through the water (U_∞) = 20 m/s
- Towing cable tension (T) = 400 N
- Maximum plate surface temperature – ocean temperature (ΔT_{max}) = 2°C

FIND

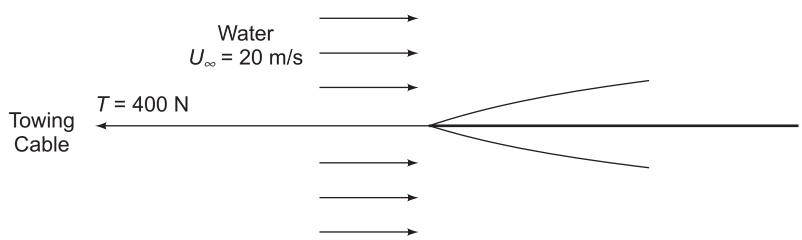
- The maximum permissible heat generation rate $(\dot{q}_G)$

ASSUMPTIONS

- Steady state
- Edge effects are negligible
- Effects due to the towing cable are negligible
- The speed given is speed relative to the water
- The water temperature is about 20°C
- The length of the plate in the direction of motion is not known

SKETCH

The plate can be visualized as stationary with the water moving over it



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 20°C

Kinematic viscosity (ν) = 1.006 × 10^{-6} m^2/s

Thermal conductivity (k) = 0.597 W/(m K)

Density (ρ) = 998.2 kg/m^3

Prandtl number (Pr) = 7.0

SOLUTION

The Drag force on the plate is equal to the tension on the cable

$$T = D = \tau_s A \quad \Rightarrow \quad \tau_s = \frac{T}{A}$$

The friction coefficient is defined in Equation (4.13) as

$$\overline{C_f} = \frac{2\tau_s}{\rho U_\infty^2} = \frac{2T}{\rho A U_\infty^2} = \frac{2\left(400\,(\text{kg m/s}^2)\right)}{\left(998.2\,(\text{kg/m}^3)\right)(2\text{ m}^2)(20\,\text{m/s})^2} = 1.002 \times 10^{-3}$$

The maximum possible heat generation rate is equal to the rate of heat transfer at the maximum permissible temperature difference

$$q_G = q_c = h_c\,\Delta T_{max}$$

(a) Assuming the boundary layer is laminar, Equations (4.38) and (4.31) give the average heat transfer coefficient and friction coefficient

$$h_c = \frac{k}{L}\ 0.664\left(\frac{U_\infty L}{\nu}\right)^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

$$C_f = 1.33\left(\frac{U_\infty L}{\nu}\right)^{-\frac{1}{2}}$$

These equations can be combined to eliminate the length of the plate (L) which is not known

$$h_c = 0.664\ \frac{C_f}{1.33\left(\frac{U_\infty L}{\nu}\right)^{-\frac{1}{2}}}\frac{k}{L}\left(\frac{U_\infty L}{\nu}\right)^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{1}{2}\ C_f k\ \frac{U_\infty}{\nu} Pr^{\frac{1}{3}}$$

(b) Assuming the boundary layer is turbulent and the laminar region can be neglected, the heat transfer coefficient can be taken from Equation (4.82) and the friction coefficient from Equation (4.78b)

$$h_c = \frac{k}{L}\ 0.036\ Pr^{\frac{1}{3}}\left(\frac{U_\infty L}{\nu}\right)^{0.8}$$

$$C_f = 0.072\left(\frac{U_\infty L}{\nu}\right)^{-0.2}$$

Combining these to eliminate L

$$\bar{h}_c = \frac{1}{2}\bar{C}_f k \frac{U_\infty}{\nu} Pr^{\frac{1}{3}}$$

This relationship is valid for the average heat transfer and friction coefficients for both laminar and turbulent boundary layers. Therefore, regardless of Reynolds number

$$\bar{h}_c = \frac{1}{2}\ (1.002\times10^{-3})\ (0.597\,\text{W/(m K)})\left(\frac{20\,\text{m/s}}{1.006\times10^{-6}\ \text{m}^2/\text{s}}\right)(7.0)^{\frac{1}{3}} = 1.14\times10^{4}\ \text{W/(m}^2\,\text{K)}$$

$$\therefore\quad \dot{q}_G = 1.14\times10^{1}\ \text{W/(m}^2\ \text{K)}\ (2\ \text{m}^2)\ (2°\text{C}) = 4.55\times10^{4}\ \text{W} = 45.5\ \text{kW}$$

PROBLEM 4.44

For flow of gas over a flat surface that has been artificially roughened by sand-blasting, the local heat transfer by convection can be correlated by the dimensionless reaction

$$Nu_x = 0.05\ Re_x^{0.9}$$

(a) Derive a relationship for the average heat transfer coefficient in flow over a plate of length *L*.

(b) Assuming the analogy between heat and momentum transfer to be valid, derive a relationship for the local friction coefficient.

(c) Assuming the gas to be air at a temperature at 400 K flowing at a velocity of 50 m/s, estimate the heat flux 1 m from the leading edge for a plate surface temperature of 300 K.

GIVEN

- Gas flow over a roughened flat surface
- Local Nusselt number $Nu_x = 0.05\ Re_x^{0.9}$

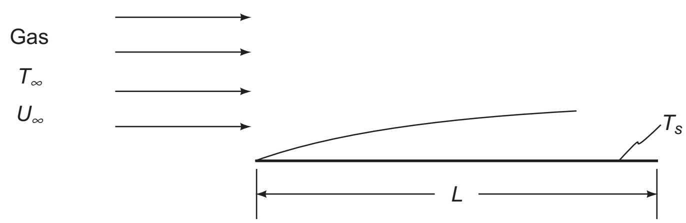
FIND

(a) Average heat transfer coefficient $(\bar{h}_c)$ for a plate of length L
(b) The local friction coefficient (C_{fx})
(c) If gas is air at temperature (T_∞) = 400 K and velocity (U_∞) = 50 m/s, estimate flux (q_x/A) at 1m from the leading edge for a plate surface temperature (T_s) = 300 K.

ASSUMPTIONS

- Steady state
- she analogy between heat and momentum transfer is valid

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the film temperature of 350 K

Thermal conductivity (k) = 0.0292 W/(m K)

Kinematic viscosity (ν) = 21.4×10^{-6} m^2/s

SOLUTION

(a) The Nusselt number is defined in Table 4.3.

$$Nu = \frac{h_c D}{k} = 0.05\ Re_x^{0.9} = 0.05\left(\frac{U_\infty x}{\nu}\right)^{0.9}$$

$$\therefore \quad h_{cx} = 0.05\ k\left(\frac{U_\infty}{\nu}\right)^{0.9} x^{-0.1}$$

The average heat transfer coefficient is obtained by integrating the local heat transfer coefficient between $X = 0$ and $X = L$

$$h_c = \frac{1}{L}\int_0^L 0.05\ k\left(\frac{U_\infty}{\nu}\right)^{0.9} x^{-0.1}\,dx = 0.05\ k\left(\frac{U_\infty}{\nu}\right)^{0.9}\frac{1}{0.9}L^{-0.1}$$

$$h_c = 0.056\ \frac{k}{L}\ Re_L^{0.9}$$

(b) The relationship between the heat transfer and friction coefficients is given in Equation (4.77)

$$\frac{C_{fx}}{2} = \frac{Nu_x}{Re_x Pr^{\frac{1}{3}}} = \frac{0.05\,Re_x^{0.9}}{Re_x Pr^{\frac{1}{3}}}$$

$$C_{fx} = 0.1\ Re_x^{-0.1}\ Pr^{-\frac{1}{3}}$$

(c) From part (a)

$$h_{cx} = 0.05\ (0.0291\,\text{W/(m K)})\left(\frac{50\,\text{m/s}}{21.2\times10^{-6}\,\text{m}^2/\text{s}}\right)(1\text{ m})^{-0.1} = 791\ \text{W/(m}^2\text{ K)}$$

The heat flux is

$$\frac{q_x}{A} = h_{cx}\,(T_s - T_\infty) = 791\,\left(791\ \text{W/(m}^2\ \text{K)}\right)(400\ \text{K} - 300\ \text{K}) = 79100\ \text{W/m}^2 = 79.1\ \text{kW/m}^2$$

PROBLEM 4.45

When viscous dissipation is appreciable, the conservation of energy equation 4.6 in the text must take into account the rate at which mechanical energy is irreversibly converted to thermal energy due to viscous effects in the fluid. This gives rise to an additional term, ϕ, on the right-hand side, the viscous dissipation where

$$\frac{\phi}{\mu} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2$$

Apply the resulting equation to laminar flow between two infinite parallel plates, with the upper plate moving at a velocity U. Assuming the constant physical properties [ρ, c_p, k, μ], obtain expressions for the velocity and temperature distributions. Compare the solutions with the dissipation term included with the results when dissipation is neglected. Find the plate velocity required to produce a 1 K temperature rise in nominally 40°C air relative to the case where dissipation is neglected.

GIVEN

- Laminar flow between two infinite parallel plates
- Upper plate moves at a velocity U_∞
- The viscous dissipation term given above must be used in the conservation of energy equation

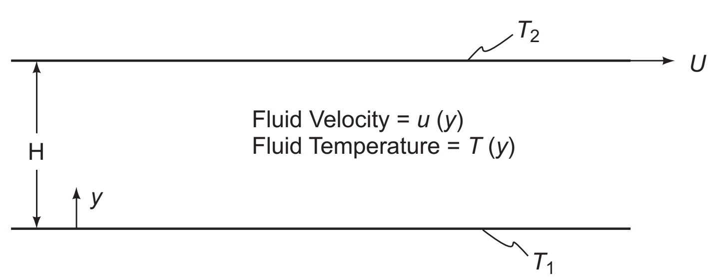
FIND

(a) Expression for velocity and temperature distributions
(b) Compare these to solutions without the dissipation term
(c) Plate velocity that gives a 1 K rise in 40°C air relative to the case without the dissipation term

ASSUMPTIONS

- Steady state
- Constant physical properties
- The plates are at constant temperatures, T_1, T_2

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 40°C

Thermal conductivity (k) = 0.0265 W/(m K)

Absolute viscosity (μ) = 19.1×10^{-6} N s/m^2

SOLUTION

(a) Including the viscous dissipation term in Equation (4.6)

$$\rho\, c_p \left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$+\,\mu\left\{\left(\frac{\partial u}{dy} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2\right\}$$

Eliminating the terms which are zero for this case

$$0 = k\,\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2$$

(Note that since the left side of Equation (4.6) drops out completely, d^2T/dy^2 is multiplied by k and not by α-see Section 4.4.) For this case, the conservation of momentum Equation (4.5) reduces to:

$$0 = \frac{d^2u}{dy^2}$$

The boundary conditions for these equations are

1. $T = T_1$, $u = 0$ at $y = 0$
2. $T = T_2$, $u = U$ at $y = H$

Integrating the momentum equation twice yields

$$\frac{du}{dy} = c_1 \qquad u(y) = c_1\,y + c_2$$

Applying the first boundary condition: $c_2 = 0$

Applying the second boundary condition: $c_1 = U/H$

Therefore, the velocity distribution between the plates is

$$u(y) = U\,\frac{y}{H} \quad\Rightarrow\quad \frac{du}{dy} = \frac{U}{H}$$

Substituting this into the energy equation yields

$$0 = k\,\frac{d^2T}{dy^2} + \mu\left(\frac{U}{H}\right)^2 \quad \text{or} \quad \frac{d^2T}{dy^2} = -\frac{\mu}{k}\left(\frac{U}{H}\right)^2$$

Integrating twice

$$\frac{dT}{dy} = -\frac{\mu U^2}{k\,H^2}\,y + c_1 \qquad T(y) = -\frac{\mu U^2}{2k\,H^2}\,y_2 + c_1\,y + c_2$$

Applying the first boundary condition: $c_2 = T_1$

Applying the second boundary condition: $c_1 = (T_2 - T_1)/H + (U)/(2\,k\,H)$

Therefore, the temperature distribution is

$$T(y) = -\frac{\mu U^2}{2k\,H^2}\,y^2 + \left[\frac{T_2 - T_1}{H} + \frac{\mu U^2}{2k\,H}\right]y + T_1$$

$$T(y) = T_1 + (T_2 - T_1)\,\frac{y}{H} + \frac{\mu U^2}{2k}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

(b) When dissipation is neglected, the momentum equation, and therefore, the velocity distribution, remain unchanged. Without viscous dissipation, the energy equation is

$$0 = \frac{d^2T}{dy^2}$$

Integrating twice

$$\frac{dT}{dy} = c_1 \qquad T(y) = c_1\, y + c_2$$

From the first boundary condition: $c_2 = T_1$
From the second boundary condition: $c_1 = (T_2 - T_1)/H$

$$\therefore\ T(y) = T_1 + \frac{T_2 - T_1}{H}\ y$$

Including the viscous dissipation term leads to an increase in temperature of

$$\frac{\mu U^2}{2k}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

at each distance y from the lower plate.

(c) This temperature increase is a maximum at

$$\frac{d}{dy}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right] = 0 \quad \Rightarrow \quad y = \frac{H}{2}$$

At this point, the temperature increase is

$$\Delta T = \frac{\mu U^2}{8k} \quad \text{So} \quad U = \sqrt{\frac{8k\,\Delta T}{\mu}}$$

For $\Delta T = 1$ K

$$U = \sqrt{\frac{8\,(0.0265\,\text{W/(m K)})\,(1\text{K})}{19.1\times10^{-6}\ \text{Ns/m}^2\ (\text{Ws/Nm})}} = 105\ \text{m/s}$$

PROBLEM 4.46

A journal bearing may be idealized as a flat plate with another flat plate moving parallel to the first and the space between the two filled by an incompressible fluid. Consider such a bearing with the stationary and moving plates at 10°C and 20°C respectively, the distance between them 3 mm, the speed of the moving plate 5 m/s, and engine oil between the plates.

(a) Calculate the heat flux to the upper and lower plates, and

(b) Determine the maximum temperature of the oil.

GIVEN

- Journal bearing: Two flat plates, one stationary, one moving with oil between them
- Stationary plate temperature (T_s) = 10°C
- Moving plate temperature (T_m) = 20°C
- Distance between plate (H) = 3 mm = 0.003 m
- Speed of the moving plate (U_p) = 5 m/s

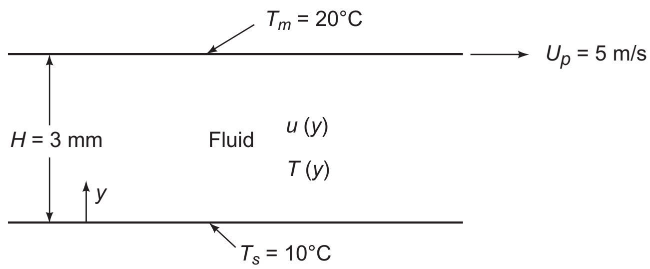
FIND

(a) Heat flux (q/A) for the plates
(b) The maximum temperature of the oil

ASSUMPTIONS

- Steady state
- Constant physical properties
- Negligible edge effects
- Oil is incompressible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 16, for engine oil at 15°C

Thermal conductivity (k) = 0.145 W/(m K)
Absolute viscosity (μ) = 1.561 (Ns)/m^2
Prandtl number (Pr) = 196

SOLUTION

(a) The temperature distribution for this geometry was derived in Problem 4.45

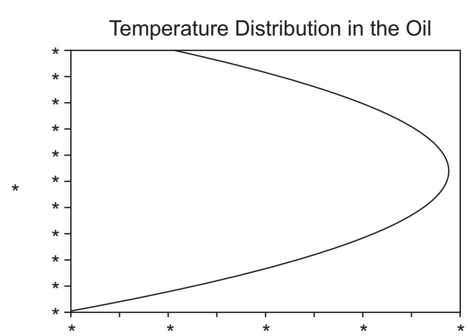
$$T(y) = T_s + (T_m - T_s)\ \frac{y}{H} + \frac{\mu U^2}{2k}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

For this case

$$T(y) = 10°\text{C} + (10°\text{C})\ \frac{y}{H} + \frac{1.561(\text{Ns})/\text{m}^2\,(5\,\text{m/s})^2}{2(0.145\,\text{W/(m K)})((\text{Nm})/(\text{Ws}))}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

$$T(y) = 10°\text{C} + (10°\text{C})\ \frac{y}{H} + (134.6\ \text{K})\ \left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

This is plotted below



The first derivative of the temperature is

$$\frac{dT}{dy} = \frac{T_m - T_s}{H} + \frac{\mu U^2}{2k}\left[\frac{1}{H} - 2\frac{y}{H^2}\right]$$

The heat flux at the top plate ($y = H$) is

$$\frac{q}{A} = -k\left.\frac{dT}{dy}\right|_{y=H} = -\frac{k(T_m - T_s)}{H} - \frac{1}{2}\mu U_p^2\left(\frac{1}{H} - \frac{2}{H}\right)$$

$$\frac{q}{A} = \frac{1}{H}\left[\frac{1}{2}\mu U_p^2 - k(T_m - T_s)\right]$$

$$\frac{q}{A} = \frac{1}{0.003\,\text{m}}\left[\frac{1}{2}\left(1.561\,(\text{Ns})/\text{m}^2\right)(5\,\text{m/s})^2\,((\text{Ws})/(\text{Nm})) - 0.145\,\text{W/(m K)}\,(20°\text{C} - 10°\text{C})\right]$$

$$\frac{q}{A} = 6020\ \text{W/m}^2\ \text{(into the plate)}$$

The heat flux at the bottom plate ($y = 0$) is

$$\frac{q}{A} = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\left[\frac{T_m - T_s}{H} + \frac{\mu U_p^2}{2kH}\right] = -\frac{1}{H}\left[\frac{1}{2}\mu U_p^2 + k(T_m - T_s)\right]$$

$$\frac{q}{A} = \frac{1}{0.003\,\text{m}}\left[\frac{1}{2}\left(1.56\,(\text{Ns})/\text{m}^2\right)(5\,\text{m/s})^2\,((\text{Ws})/(\text{Nm})) + 0.145\,\text{W/(m K)}\,(20°\text{C} - 10°\text{C})\right]$$

$$\frac{q}{A} = -6980\ \text{W/m}^2\ \text{(out of the plate)}$$

(b) The maximum temperature occurs where the first derivative is zero

$$0 = \frac{T_m - T_s}{H} + \frac{\mu U_p^2}{2k}\left[\frac{1}{H} - \frac{2y}{H^2}\right] \Rightarrow \frac{y}{H} = \frac{1}{2} + \frac{k(T_m - T_s)}{\mu U_p^2}$$

$$\frac{y}{H} = \frac{1}{2} + \frac{0.145\,\text{W/(m K)}\,(20°\text{C} - 10°\text{C})}{1.561\,(\text{Ns})/\text{m}^3\,(5\,\text{m/s})^2\,((\text{Ws})/(\text{Nm}))} = 0.537$$

$$y_{\max} = 0.537(3\ \text{mm}) = 1.61\ \text{mm}$$

Checking the second derivative

$$\frac{d^2T}{dy^2} = -\frac{\mu U_p^2}{kH}$$

This is negative throughout the region, therefore, $T(y)$ is concave down throughout the region and the temperature at $y = 1.6$ mm is indeed the maximum temperature. These calculations are verified by the graph of $T(y)$. Inserting $y_{\max}$ into the expression for $T(y)$ yields: $T_{\max}$ = T(0.00161 mm) = 48.8°C.

COMMENTS

The difference in heat fluxes at the plate $\Delta q''$ = 6980 W/m^2 – 6020 W/m^2 = 920 W/m^2 must equal the heat dissipated within the oil.

PROBLEM 4.47

A journal bearing has a clearance of 0.5 mm. The journal has a diameter of 100 mm and rotates at 3600 rpm within the bearing. The journal is lubricated by an oil having a density of 800 kg/m^3, a viscosity of 0.01 kg/ms, and a thermal conductivity of 0.14 W/(m K). If the bearing surface is at 60°C, determine the temperature distribution in the oil film assuming that the journal surface is insulated.

GIVEN

- A journal bearing
- Diameter (D) = 100 mm = 0.1 m
- Clearance (H) = 0.5 mm = 0.0005 m
- Rotational speed (ω) = 3600 rpm
- Oil properties
  - Density (ρ) = 800 kg/m^3
  - Viscosity (μ) = 0.01 kg/ms
  - Thermal conductivity (k) = 0.14 W/(m K)
- Temperature of bearing surface (T_b) = 60°C

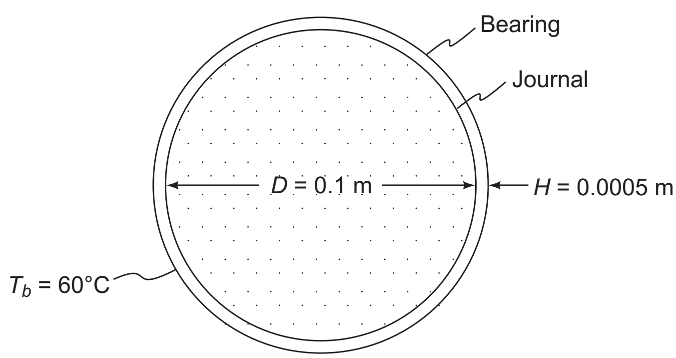
FIND

(a) Temperature distribution in the oil film

ASSUMPTIONS

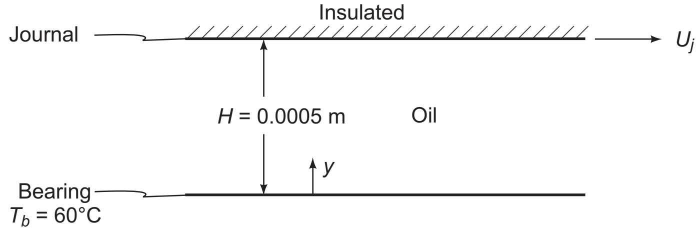
- Steady state
- Uniform and constant bearing surface temperature
- Constant fluid properties
- The journal surface is insulated (negligible heat transfer)

SKETCH



SOLUTION

Since the clearance is small compared to the bearing diameter, the bearing may be idealized as parallel flat plates with oil between them, one stationary and one moving



(a) As shown in Problem 4.44, the velocity distribution for this geometry is linear

$$u(y) = U_j \frac{y}{H} \quad \text{where: } U_j = \frac{D}{2}\ \omega = \frac{1}{2}\ (0.1\text{ m})$$

$$\left(3600 \frac{\text{rotations}}{\text{s}}\right)\left(\frac{2\times\text{rad}}{\text{rotation}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 18.85\text{ m/s}$$

For this geometry, the energy Equation (4.6) with viscous dissipation reduces to

$$k\ \frac{d^2t}{dy^2} = -\mu\left(\frac{du}{dy}\right)^2 = -\mu\left(\frac{U_j}{H}\right)^2$$

With boundary conditions: at $y = 0$ $T = T_b$

at $y = H$ $dT/dy = 0$ (insulated)

Integrating

$$k\ \frac{dt}{dy} = -\mu\left(\frac{U_j}{H}\right)^2 y + c_1$$

Applying the second boundary condition

$$c_1 = \frac{\mu U_j^2}{H} \Rightarrow k\ \frac{dt}{dy} = \mu\ \frac{U_j^2}{H}\left(1 - \frac{y}{H}\right)$$

Integrating

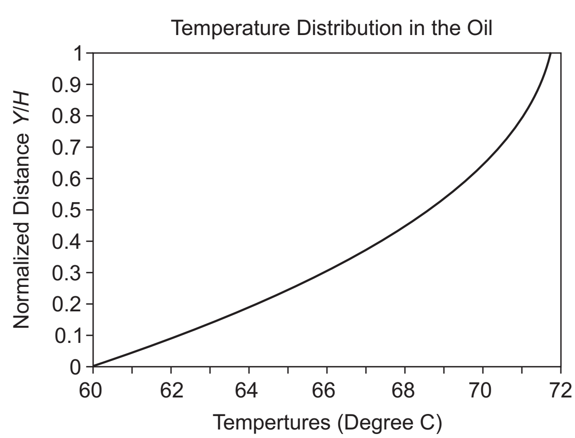
$$kT = \mu\ \frac{U_j^2}{H}\left(y - \frac{y^2}{2H}\right) + c_2 = \mu\ U_j^2\left[\frac{y}{H} - \frac{1}{2}\left(\frac{y}{H}\right)^2\right] + c_2$$

Applying the first boundary condition $c_2 = kT_b$

$$T = T_b + \frac{\mu U_j^2}{k}\left[\frac{y}{H} - \frac{1}{2}\left(\frac{y}{H}\right)^2\right]$$

$$T = 60°\text{C} + \frac{0.01\,\text{kg/sm}(18.85\,\text{m/s})^2}{0.14\,\text{W/(m K)}\left((\text{kg m}^2)/(\text{Ws}^3)\right)}\left[\frac{y}{H} - \frac{1}{2}\left(\frac{y}{H}\right)^2\right] = 60°\text{C} + (25.38\text{ K})\left[\frac{y}{H} - \frac{1}{2}\left(\frac{y}{H}\right)^2\right]$$

This is shown graphically below



PROBLEM 4.48

A journal bearing has a clearance of 0.5 mm. The journal has a diameter of 100 mm and rotates at 3600 rpm within the bearing. The journal is lubricated by an oil having a density of 800 kg/m^3, a viscosity of 0.01 kg/(ms), and a thermal conductivity of 0.14 W/(m K). Both the journal and the bearing temperatures are maintained at 60°C. Calculate the rate of heat transfer from the bearing and the power required for rotation per unit length.

GIVEN

- A journal bearing
- Diameter (D) = 100 mm = 0.1 m
- Clearance (H) = 0.5 mm = 0.0005 m
- Rotational speed (ω) = 3600 rpm
- Oil properties
  - Density (ρ) = 800 kg/m^3
  - Viscosity (μ) = 0.01 kg/(ms)
  - Thermal conductivity (k) = 0.14 W/(m K)
- Temperature of both surface (T_b) = 60°C

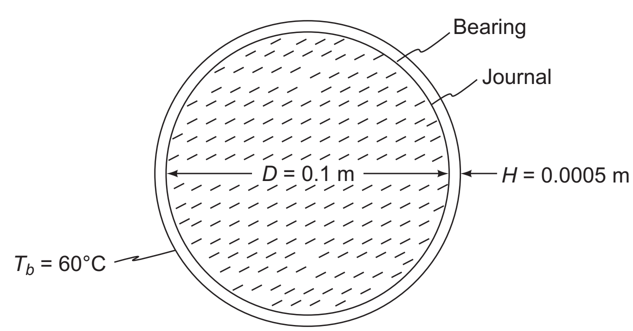
FIND

(a) The rate of heat transfer from the bearing
(b) The power required for rotation per unit length

ASSUMPTIONS

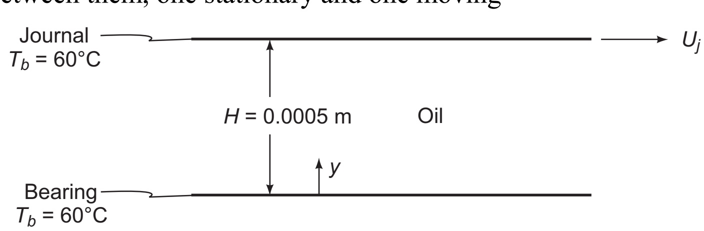
- Steady state
- Uniform and constant surface temperatures
- Constant fluid properties

SKETCH



SOLUTION

Since the clearance is small compared to the bearing diameter, the bearing may be idealized as parallel flat plates with oil between them, one stationary and one moving



The temperature distribution for this geometry was derived in Problem 4.45

$$T(y) = T_b + (T_j - T_b)\ \frac{y}{H} + \frac{m\,U_j^2}{2k}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

From Problem 4.47: U_j = 18.85 m/s

$$T(y) = 60°\text{C} + \frac{0.1\,\text{kg/(ms)}(18.85\,\text{m/s})^2}{2(0.14\,\text{W/(m K)})\left((\text{kg m}^2)/(\text{Ws})^2\right)}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

$$T(y) = 60°\text{C} + (126.9\text{ K})\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

The rate of heat transfer to the bearing is given by Fourier's Law

$$q = -kA\ \frac{dt}{dy}\Big|_{y=0} = -k\,[\pi(D+2H)\,L]\left[\frac{\mu U_j^2}{kH}\left(1 - \frac{y}{H}\right)\right]_{y=0}$$

$$\frac{q}{L} = -\pi\ \mu\ U_j^2\left(\frac{D}{H} + 2\right)$$

$$\frac{q}{L} = -\pi\,(0.01\,\text{kg/ms})(18.85\,\text{m/s})^2\left(\frac{0.1\text{ m} + 0.001\text{ m}}{0.0005\text{ m}}\right)\left((\text{W s}^2)/(\text{kg m}^2)\right) = -2235\text{ W/m}$$

(into the bearing)

The power required to turn the journal is the product of the drag force on the journal and the journal speed

$$P = F\,U_j = \tau_w\,A\,U_j = \pi D\,L\,U_j\ \tau_w$$

The shear stress (τ) is given by Equation (4.2)

$$\frac{P}{L} = \pi D\,U_j\left(\mu\frac{du}{dy}\right)_{y=H} = \pi D\,U_j\,\mu\ \frac{U_j}{H} = \frac{1}{H}\ \pi D\,\mu\,U_j^2$$

$$\frac{P}{L} = \frac{1}{0.0005\text{ m}}\ \pi(0.1\text{ m})(0.01\,\text{kg/(ms)})(18.85\,\text{m/s})^2\left((\text{Ws}^3)/(\text{kg m}^2)\right) = 2233\text{ W/m}$$

COMMENTS

Note that for conservation of energy, the power required (P/L) should be equal to the heat loss (q/L). The slight difference (0.09%) in the results is due to the difference in surface area of journal and the bearing which was not incorporated into the analysis.

PROBLEM 4.49

A refrigeration truck is traveling at 130 kmph on a desert highway where the air temperature is 50°C. The body of the truck may be idealized as a rectangular box, 3 m wide, 2.1 m high, and 6 m long, at a surface temperature of 10°C. Assume that the heat transfer from the front and back of the truck may be neglected, that the stream does not separate from the surface, and that the boundary layer is turbulent over the whole surface. If, for every 3600 W of heat loss, one ton capacity of the refrigerating unit is necessary, calculate the required tonnage of the refrigeration unit.

GIVEN

- A refrigeration truck traveling on a desert highway
- Speed of truck (U_∞) = 130 kmph = 36.0 m/s
- Air temperature (T_∞) = 50°C
- Truck may be idealized as a box: 3 m wide, 2.1 m high, 6 m long
- Truck surface temperature (T_s) = 10°C
- One ton of refrigeration unit is needed for every 3600 W of heat loss

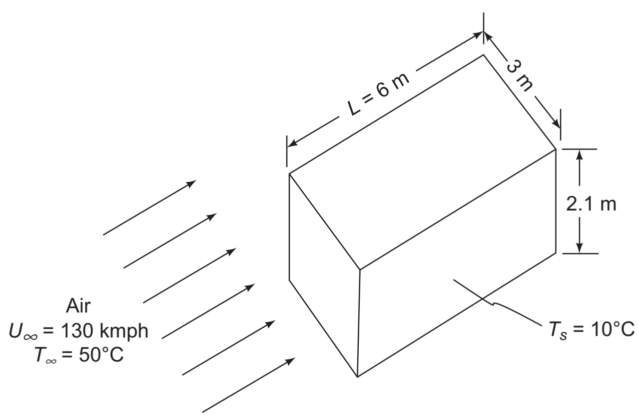
FIND

- The required tonnage of the refrigeration unit

ASSUMPTIONS

- The heat transfer from the front and back of the truck is negligible
- Air stream does not separate from the surface
- The boundary layer is turbulent over the whole surface
- Moisture of the air is negligible
- Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average of T_∞ and T_s (30°C)

Kinematic viscosity (ν) = 16.7×10^{-6} m^2/s

Thermal conductivity (k) = 0.0258 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

The sides of the truck can be visualized as flat plates with air flowing over them. The Reynolds number at the back of the truck is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(36.0\,\text{m/s})(6\,\text{m})}{16.7 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.28 \times 10^7$$

The total area of the sides, top, and bottom of the truck is

$$A = 2\,(6\text{ m})\,(3\text{ m}) + 2\,(6\text{ m})\,(2.1\text{ m}) = 61.2\text{ m}^2$$

The average heat transfer coefficient over the truck with a turbulent boundary layer is given by Equation (4.82)

$$h_c = \frac{k}{L}\ 0.036\ Pr^{\frac{1}{3}}\ Re_L^{0.8}$$

$$h_c = \frac{0.0258\ \text{W/(m K)}}{6\ \text{m}}\ 0.036\ (0.71)^{\frac{1}{3}}\ (1.28 \times 10^7)^{0.8} = 67.0\ \text{W/(m}^2\ \text{K)}$$

The rate of convective heat transfer to the truck is

$$q = h_c\, A\, (T_s - T_\infty) = 67.0\ \text{W/(m}^2\ \text{K)}\ (61.2\ \text{m}^2)\ (50°\text{C} - 10°\text{C}) = 1.64 \times 10^5\ \text{W/m}$$

The tonnage required to cool the truck is

$$\text{Tonnage} = 1.64 \times 10^5\ \text{W} \left(\frac{1\ \text{ton}}{3600\ \text{W}}\right) = 45.5\ \text{tons}$$

COMMENTS

Solar gain may increase the required tonnage on a sunny day depending on the emissivity of the truck surface. Including the laminar portion of the boundary layer would result in an average heat transfer coefficient of 64 W/(m^2 K) and a 5% decrease in the calculated required tonnage.

PROBLEM 4.50

At the equator, where the sun at noon is approximately overhead, a near optimum orientation for a flat plate solar hot water heater is in the horizontal position. Suppose a 4 m × 4 m solar collector for domestic hot water use is mounted on a horizontal roof as shown in the attached sketch. The surface temperature of the glass cover is estimated to be 40°C, and air at 20°C is blowing at a velocity of 24 kmph over the roof. Estimate the heat loss by convection from the collector to the air when the collector is mounted

(a) at the leading edge of the roof [L_c = 0] and,

(b) at a distance of 10 m from the leading edge.

GIVEN

- A solar collector on a flat, horizontal roof
- Collector area ($L \times w$) = 4 m × 4 m

- Glass cover surface temperature (T_s) = 40°C
- Air temperature (T_∞) = 20°C
- Air velocity (U_∞) = 24 kmph = 6.7 m/s

FIND

The heat loss by convection (q_c) from the collector when it is mounted

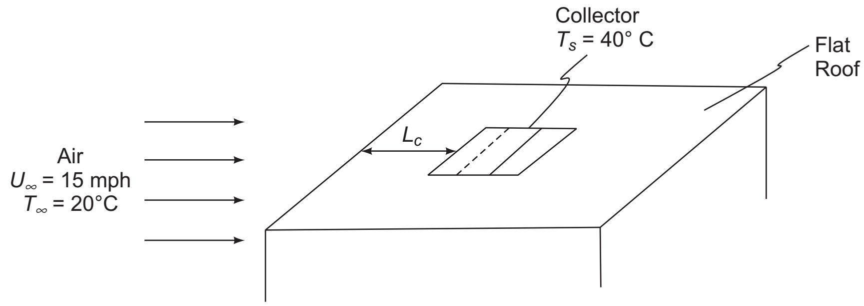
(a) at the leading edge of the roof (L_c = 0)

(b) at a distance of 10 m from the leading edge (L_c = 10 m)

ASSUMPTIONS

- Steady state
- The collector surface is connected smoothly to the roof surface
- Uniform collector surface temperature
- Moisture in the air is negligible
- Neglect radiation heat transfer losses

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the film temperature (30°C)

Kinematic viscosity (ν) = 16.7×10^{-6} m^2/s

Thermal conductivity (k) = 0.0258 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

(a) For $L_c = 0$, the Reynolds number at the trailing edge of the collector is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(6.7\text{ m/s})(4\text{m})}{16.7\times10^{-6}\text{ m}^2\text{/s}} = 1.61 \times 10^6 > 5 \times 10^5$$

(Turbulent)

The average convective heat transfer coefficient over the collector with a mixed boundary layer is given by Equation (4.83)

$$h_c = \frac{k}{L}\ 0.036\ Pr^{\frac{1}{3}}\ [Re_L^{0.8} - 23{,}200]$$

$$h_c = \frac{0.0258\text{ W/(m K)}}{4\text{ m}}\ 0.036\ (0.71)^{\frac{1}{3}}\ [(1.61 \times 10^6)^{0.8} - 23{,}200] = 14.3\text{ W/m}^2\text{ K}$$

The rate of convective heat loss from the collector is:

$$q = h_c A (T_s - T_\infty) = 14.3\ \left(14.3\text{ W/(m}^2\text{ K)}\right)\ (4\text{ m})\ (4\text{ m})\ (40°\text{C} - 20°\text{C}) = 4572\text{ W/m}$$

(b) For $L_c = 10$ m, the boundary layer will be turbulent over the entire collector surface. The average heat transfer coefficient over the collector can be calculated by integrating the local heat transfer coefficient, Equation (4.81), between L_c and $L_c + L$ and dividing by the length of the collector L

$$h_c = \frac{1}{L}\int_{L_c}^{L+L_c} h_{cx}\,dx = \frac{1}{L}\int_{L_c}^{L+L_c} \frac{k}{x}\ 0.0288\,Pr^{\frac{1}{3}}\left(\frac{U_\infty x}{\nu}\right)^{0.8} dx$$

$$= 0.0288\ \frac{k}{L} Pr^{\frac{1}{3}}\left(\frac{U_\infty}{\nu}\right)^{0.8}\ \int_{L_c}^{L+L_c} x^{-0.2}$$

$$h_c = 0.0288\ \frac{k}{L} Pr^{\frac{1}{3}}\left(\frac{U_\infty}{\nu}\right)^{0.8}\ 1.25\ [(L + L_c)^{0.8} - L_c^{0.8}]$$

$$h_c = \quad 0.036\ \frac{0.0258\text{ W/(m K)}}{4\text{ m}}(0.71)^{\frac{1}{3}}\left[\frac{6.7\text{ m/s}}{16.7\times10^{-6}\text{ m}^2\text{/s}}\right]^{0.8}\ [(14\text{ m})^{0.8} - (10\text{ m})^{0.8}] = 12.3\,\overline{\pi}$$

The rate of convective heat loss from the collector is

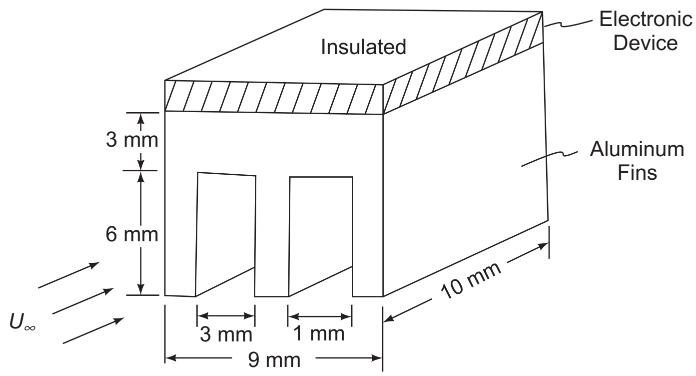
$$q = h_c A (T_s - T_\infty) = [12.3 \text{ W/(m}^2 \text{ K)}] (4 \text{ m}) (4 \text{ m}) (40°\text{C} - 20°\text{C}) = 3928 \text{ W/m}$$

COMMENTS

Note that placing the collector 10 m from the leading edge of the roof lowers the rate of convective loss by about 14% because the local convective coefficient is largest at the leading edge.

PROBLEM 4.51

An electronic device is to be cooled by air flowing over aluminum fins attached to its lower surface as shown



U_∞ = 10 m/s

T_∞ Air = 20°C

The device dissipates 5 W and the thermal contact resistance between the lower surface of the device and the upper surface of the cooling fin assembly is 0.1 cm^2 K/W. If the device is at a uniform temperature and insulated at the top, estimate that temperature under steady state.

GIVEN

- Electronic device attached to aluminum fins as shown above
- Air velocity (U_∞) = 10 m/s
- Air temperature (T_∞) = 20°C
- The device temperature is uniform
- Top is insulated
- Heat dissipation from the device ($\dot{q}_G$) = 5 W
- Contact resistance between the device and the fins (R_i) = 0.1 cm^2 K/W

FIND

- The steady state temperature of the device (T_{device})

ASSUMPTIONS

- Fin material is pure aluminum
- Heat transfer is one dimensional through the 3 mm thickness of aluminum
- Heat loss through the insulation is negligible
- Convection from the fins may be approximated as parallel flow over a flat plate
- Heat loss from the edges of the device is negligible
- Heat loss from the front and back edges of the fins is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, for aluminum at 127°C: Thermal conductivity (k_a) = 240 W/(m K)
From Appendix 2, Table 27, for dry air at 20°C

Kinematic viscosity (ν) = 15.7×10^{-6} m^2/s
Thermal conductivity (k) = 0.0251 W/(m K)
Prandtl number (Pr) = 0.71

SOLUTION

The thermal circuit for heat flow from the device to the air is shown below

The Reynolds number at the trailing edge of the device is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(10\,\text{m/s})\,(0.01\,\text{m})}{15.7\times 10^{-6}\,\text{m}^2/\text{s}} = 6.37 \times 10^3 \text{ (Laminar)}$$

The average heat transfer coefficient over the aluminum fins is given by Equation (4.38)

$$h_c = \frac{k}{L}\ 0.664\ Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{0.0251\,\text{W/(m K)}}{0.01\,\text{m}}\ 0.664\,(6.37\times 10^3)^{\frac{1}{2}}(0.71)^{\frac{1}{3}} = 118.7 \text{ W/(m}^2\text{ K)}$$

The rate of heat transfer from a single fin with convection over the tip is given in Table 2.1

$$q = M\ \frac{\sinh(m\,L_f) + \left(\dfrac{h}{m\,k_a}\right)\cosh(m\,L_f)}{\cosh(m\,L_f) + \left(\dfrac{h}{m\,k_a}\right)\sinh(m\,L_f)} \qquad (L_f = 0.006 \text{ m})$$

where $L_f = 0.006$ m and $m = \sqrt{\dfrac{h_c\,P}{k_a\,A_c}}$

P = fin perimeter = 20 mm = 0.02 m (Neglecting front and back surfaces.)
A_c = fin cross sectional area = (0.01 m) (0.001 m) = 1×10^{-5} m^2

$$\therefore\ m = \sqrt{\frac{118.7\,\text{W/(m}^2\,\text{K)}\,(0.02\,\text{m})}{240\,\text{W/(m K)}\,(1\times 10^{-5}\,\text{m}^2)}} = 31.4 \text{ m}^{-1}$$

$$m\,L_f = \left(31.4\frac{1}{\text{m}}\right)\ (0.006 \text{ m}) = 0.189$$

$$M = (T_s - T_\infty)\ \sqrt{\bar{h}_c P\, k_a A_c}$$

$$M = (T_s - T_\infty)\ \sqrt{118.7\,\text{W/(m}^2\,\text{K)}\,(0.02\,\text{m})\,(240\,\text{W/(m K)})\,(1\times 10^{-5}\,\text{m}^2)} = 0.076\,(T_s - T_\infty) \text{ W/K}$$

$$\frac{\bar{h}_c}{m\,k_a} = \frac{118.7\,\text{W/(m}^2\,\text{K)}}{31.4\,\text{m}^{-1}\,(240\,\text{W/(m K)})} = 0.0158$$

The rate of heat transfer from a single fin is

$$q_f = 0.076\ (T_s - T_\infty)\ \text{W/K}\ \frac{\sinh(0.189) + 0.0158\cosh(0.189)}{\cosh(0.189) + 0.0158\sinh(0.189)} = 0.0153\ (T_s - T_\infty)\ \text{W/K}$$

The total heat transfer is the sum of the heat transfer from the aluminum base not covered by the fins and the heat transfer from three fins. This must equal the heat generation rate

$$q = h_c A_b (T_s - T_\infty) + 3\ [0.0153(R_s - T_\infty)\,\text{W/K}] = \dot{q}_G$$

Where $A_b = 2(0.003\text{ m})\ (0.01\text{ m}) = 0.6 \times 10^{-4}\text{ m}^2$
Solving for T_s

$$T_s = T_\infty + \frac{\dot{q}_G}{h_c A_b + 0.046\,\text{W/K}} = 20°\text{C} + \frac{5\text{ W}}{118.7\,\text{W/(m}^2\text{ K)}(0.6\times 10^{-4}\text{m}^2) + 0.046\,\text{W/K}} = 114°\text{C}$$

The device temperature is given by

$$T_{\text{device}} = T_s + \dot{q}_G\ (R_{\text{contact}} + R_{\text{aluminum}})$$

where

$$\text{R}_{\text{aluminum}} = \frac{t}{A\,k_a} = \frac{0.003\text{ m}}{(0.01\text{m})\,(0.009\text{m})(240\,\text{W/(m K)})} = 0.1389\text{ K/W}$$

$$R_{\text{contact}} = \frac{R_i}{A} = \frac{0.1(\text{cm}^2\text{ K})/\text{W}}{(1\text{cm})(0.9\text{cm})} = 0.1111\text{ K/W}$$

$$\therefore\ T_{\text{device}} = 114°\text{C} + 5\text{ W}\ (0.1111 + 0.1389)\text{ K/W} = 115°\text{C}$$

PROBLEM 4.52

An array of sixteen silicon chips arranged in 2 rows are insulated at the bottom and cooled by air flowing in forced convection over the top. The array can be located either with its long side or its short side facing the cooling air. If each chip is 10 mm × 10 mm in surface area and dissipates the same power, calculate the rate of maximum power dissipation permissible for both possible arrangements if the maximum permissible surface temperature of the chips is 100°C. What would be the effect of a turbulator on the leading edge to trip the boundary layer into turbulent flow? The air temperature is 30°C and its velocity is 25 m/s.

GIVEN

- An array of sixteen silicon chips cooled by air flowing over the top
- Bottom is insulated
- Dimensions of each chip = 10 mm × 10 mm = 0.01 m × 0.01 m
- Array is 2 rows, 8 chips per row
- Each chip dissipates the same power ($\dot{q}_G / A$)
- Maximum surface temperature (T_s) = 100°C
- Air temperature (T_∞) = 30°C
- Air velocity (U_∞) = 25 m/s

FIND

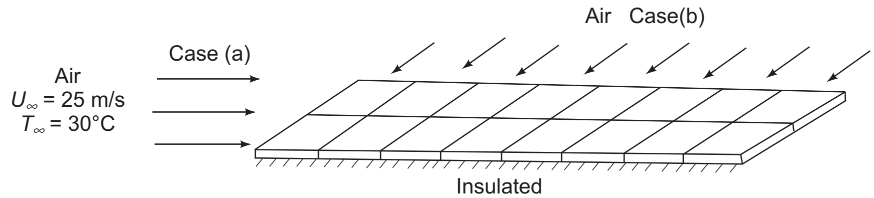
The maximum power dissipation permissible (q_G/A) for

(a) The long side facing the air flow
(b) The short side facing the air flow
(c) The effect of a turbulator on the leading edge

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27 for dry air at the film temperature (65°C)

Kinematic viscosity (ν) = 19.9×10^{-6} m²/s

Thermal conductivity (k) = 0.0283 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

(a) With the long edge facing the air flow, L = (2) (.01 m) = 0.02 m

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(25\,\text{m/s})(0.02\,\text{m})}{19.9\times10^{-6}\,\text{m}^2/\text{s}} = 2.51 \times 10^4 < 5 \times 10^5 \text{ (Laminar)}$$

The local heat transfer coefficient for laminar flow is given by Equation (4.37)

$$h_{cx} = \frac{k}{x}\ 0.332\ Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

The transfer coefficient decreases with increasing x and it will be minimum at $x = L$. The permissible heat generation rate for a given maximum surface temperature will therefore be limited to the heat flux from the plate at $x = L$

$$\left(\frac{q_G}{A}\right)_{\max} = \left(\frac{q_G}{A}\right)_{x=L} = h_{cL}\,(T_s - T_\infty) = \frac{k}{L}\ 0.332\ Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}}\ (T_s - T_\infty)$$

$$\left(\frac{q_G}{A}\right)_{\max} = \frac{0.0283\,\text{W/(m K)}}{0.02\,\text{m}}\ 0.332\ (2.5\times10^4)^{\frac{1}{2}}(0.71)^{\frac{1}{3}}(100°\text{C} - 30°\text{C}) = 4650\ \text{W/m}^2$$

$$= 0.465\ \text{W/chip}$$

(b) With the short edge facing the air flow, L = (8) (0.01 m) = 0.08 m

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(25\,\text{m/s})(0.08\,\text{m})}{19.9\times10^{-6}\,\text{m}^2/\text{s}} = 1.01 \times 10^5 \text{ (still laminar)}$$

As shown in part (a)

$$\left(\frac{q_G}{A}\right)_{\max} = \frac{0.0283\,\text{W/(m K)}}{0.08\,\text{m}}\ 0.332\ (1.01\times10^5)^{\frac{1}{2}}\ (0.71)^{\frac{1}{3}}\,(100°\text{C} - 30°\text{C}) = 2330\ \text{W/m}^2$$

$$= 0.233\ \text{W/chip}$$

(c) Assuming the boundary layer is turbulent over the entire array, the local heat transfer coefficient is given by Equation (4.81)

$$h_{cx} = 0.0288\ \frac{k}{x} Re_x^{\,0.8} Pr^{\frac{1}{3}}$$

Therefore, the maximum heat generation rate is

$$\left(\frac{\dot{q}_G}{A}\right)_{\max} = 0.0288\ \frac{k}{L}\ Re_L^{\,0.8}\ Pr^{\frac{1}{3}}\ (T_s - T_\infty)$$

For the long edge facing the air flow

$$\left(\frac{\dot{q}_G}{A}\right)_{\max} = 0.0288\ \frac{0.0283\,\text{W/(m K)}}{0.02\,\text{m}}\ (2.5\times10^4)^{0.5}\ (0.71)^{\frac{1}{3}}\,(100°\text{C} - 30°\text{C})$$

$$\left(\frac{\dot{q}_G}{A}\right)_{\max} = 0.842\ \frac{\text{W}}{\text{chip}}$$

For the short edge facing the air flow

$$\left(\frac{\dot{q}_G}{A}\right)_{\max} = 0.0288\ \frac{0.0283\,\text{W/(m K)}}{0.08\,\text{m}}\ (1.01\times10^5)^{0.8}\ (0.71)^{\frac{1}{3}}\,(100°\text{C} - 30°\text{C})$$

$$\left(\frac{\dot{q}_G}{A}\right)_{\max} = 0.641\ \text{W/chip}$$

COMMENTS

Orienting the short edge rather than the long edge into the air flow allows about a doubling of the heat generation rate for the laminar case and about a 31% increase in the turbulent case. Note that the heat transfer coefficient decreases less rapidly with x for a turbulent boundary layer than it does for a laminar boundary layer.

The turbulator allows an increase in the heat generation rate of about 80% for the long edge oriented towards the air flow.

PROBLEM 4.53

The air conditioning system in a new Chevrolet Van for use in desert climates is to be sized. The system is to maintain an interior temperature of 20°C when the van travels at 100 km/h through dry air at 30°C at night. If the top of the van can be idealized as a flat plate 6 m long and 2 m wide, and the sides as flat plates 3 m tall and 6 m long, estimate the rate of which heat must be removed from the interior to maintain the specified comfort conditions. Assume the heat transfer coefficient on the inside of the van wall is 10 W/(m^2 K).

GIVEN

- A Chevrolet van traveling through dry air
- Interior van temperature (T_i) = 20°C
- Velocity (U_∞) = 100 km/h
- Air temperature (T_∞) = 30°C
- Top dimensions = 6 m long, 2 m wide
- Side dimensions = 6 m long, 3 m wide

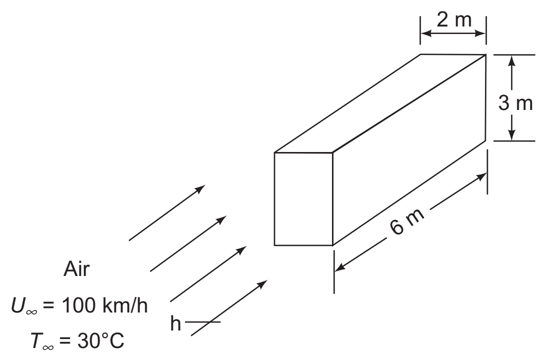
FIND

- The rate of which heat must be removed (q)

ASSUMPTIONS

- Heat gain from the front, back, and bottom of the van is negligible
- Radiative heat transfer is negligible
- Van walls are insulated
- Thermal resistance of the sheet metal van walls is negligible
- The interior heat transfer coefficient $(\bar{h}_c)$ = 10 W/(m^2 K)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at film temperature (25°C)

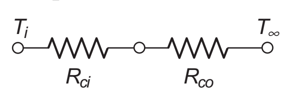
Kinematic viscosity (ν) = 16.2×10^{-6} m^2/s

Thermal conductivity (k) = 0.0255 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

The thermal circuit for the van walls and top is shown below



The average heat transfer coefficient on the outside of the van (h_{co}) can be calculated by treating the top and sides as flat plates and length (L) = 6 m

$$Re_L = \frac{U_\infty\, L}{\nu} = \frac{100\,\text{km/h}(1000\,\text{m/km})(6\,\text{m})}{16.2\times 10^{-6}\,\text{m}^2/\text{s}(3600\,\text{s/h})} = 1.03 \times 10^7 > 5 \times 10^5$$

(Turbulent)

For a mixed boundary layer, Equation (4.83) gives the average heat transfer coefficient on the outside of the van

$$h_{co} = 0.036 \frac{k}{L} Pr^{\frac{1}{3}} [Re_L^{0.8} - 23{,}200]$$

$$h_{co} = 0.036 \frac{(0.0255\,\text{W/(mK)})}{6\,\text{m}} (0.71)^{\frac{1}{3}} [(1.03 \times 10^7)^{0.8} - 23{,}200] = 52.5 \text{ W/(m}^2\text{ K)}$$

The value of the thermal resistances are

Outside

$$R_{co} = \frac{1}{h_o A_o} = \frac{1}{(52.5\,\text{W/(m}^2\text{ K)})[2(3\,\text{m})(6\,\text{m}) + 2\,\text{m}(6\,\text{m})]} = 0.00040 \text{ K/W}$$

Inside

$$R_{ci} = \frac{1}{h_i A_i} = \frac{1}{10\,\text{W/(m}^2\text{ K)}[2(3\,\text{m})(6\,\text{m}) + 2\,\text{m}(6\,\text{m})]} = 0.00208 \text{ K/W}$$

The rate at which heat must be removed is equal to the convective heat gain

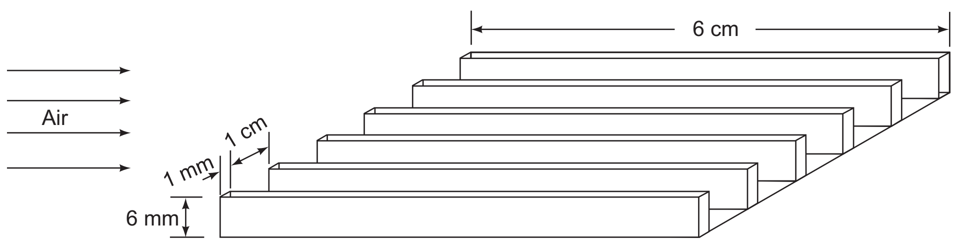
$$q = \frac{\Delta T}{R_{co} - R_{ci}} = \frac{30°\text{C} - 20°\text{C}}{(0.0004 + 0.002.8)\,\text{K/W}} = 4032 \text{ W}$$

COMMENTS

Radiative heat transfer may not be negligible depending on the color of the van and the temperature of the night sky.

PROBLEM 4.54

To cool an electronic device, six identical aluminum fins, as shown in the figure below, are attached. Cooling air is available at a velocity of 5 m/s from a fan at 20°C. If the average temperature at the base of a fin is not to exceed 100°C, estimate the maximum permissible power dissipation for the device.



GIVEN

- Aluminum fins with air flowing over them
- Air velocity (U_∞) = 5 m/s
- Air temperature (T_∞) = 20°C
- Maximum average temperature of the fin base (T_s) = 100°C

FIND

- The maximum permissible power dissipation q

ASSUMPTIONS

- Steady state

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the film temperature (60°C)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Thermal conductivity (k) = 0.0279 W/(m K)

Prandtl number (Pr) = 0.071

From Appendix 2, Table 12, for aluminum at 100°C

Thermal conductivity (k_{al}) = 239 W/(m K)

SOLUTION

The Reynolds number at the trailing edge of the fins is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(5\,\text{m/s})(0.06\,\text{m})}{19.4 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.55 \times 10^4 < 1 \times 10^5 \text{ (Laminar)}$$

The average transfer coefficient for a laminar boundary layer from Equation (4.38) is

$$h_c = \frac{k}{L}\, 0.664\, Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{(0.0279\,\text{W/(m K)})}{0.06\,\text{m}}\, 0.664\,(1.55 \times 10^4)^{\frac{1}{2}} (0.71)^{\frac{1}{3}} = 34.3 \text{ W/(m}^2\text{ K)}$$

The maximum permissible heat generation is equal to the sum of the heat loss from the fins and heat loss from the wall area between the fins. The heat loss from a single fin is given in Table 2.1

$$q_f = M\, \frac{\sinh(m L_f) + (h/m k_a)\cosh(m L_f)}{\cosh(m L_f) + (h/m k_a)\sinh(m L_f)}$$

$$\text{where } L_f = 0.006 \text{ m and m} = \sqrt{\frac{h_c P}{k_a A_c}}$$

P = fin perimeter = 2(0.06 m) + 2(0.001 m) = 0.122 m

A_c = fin cross sectional area = (0.001 m) (0.06 m) = 6×10^{-5} m^2

$$\therefore\ m = \sqrt{\frac{34.3\,\text{W/(m}^2\text{ K)}(0.122\,\text{m})}{239\,\text{W/(m K)}(6 \times 10^{-5}\,\text{m}^2)}} = 17.1 \text{ m}^{-1}$$

$$m L_f = 17.1 \text{ m}^{-1} (0.006 \text{ m}) = 0.10$$

$$M = (T_b - T_\infty)\sqrt{h_c P k_a A_c}$$

$$M = (100°\text{C} - 20°\text{C})\sqrt{34.3\,\text{W/(m}^2\text{K)}(0.122\,\text{m})(239\,\text{W/(m K)})(6 \times 10^{-5}\,\text{m}^2)} = 19.6 \text{ W}$$

$$\frac{h_c}{m k_a} = \frac{(34.3\,\text{W/(m}^2\text{ K)})}{17.1\,\text{m}^{-1}(239\,\text{W/(m K)})} = 0.0084$$

$$\therefore\ q_f = 19.6 \text{ W}\ \frac{\sinh(0.1) + 0.0084\cosh(0.1)}{\cosh(0.1) + 0.0084\sinh(0.1)} = 1.94 \text{ W}$$

Summing the heat loss from the six fins and the five wall areas

$$q = 6q_f + 5q_w = 6q_f + 5h_c A_w (T_s - T_\infty)$$

$$q = 6\,(1.94 \text{ W}) + 5\left(34.3 \frac{\text{W}}{\text{m}^2\text{K}}\right)(0.01 \text{ m})(0.06 \text{ m})(100°\text{C} - 20°\text{C})$$

$$= 11.6 \text{ W} + 8.2 \text{ W} = 19.8 \text{ W}$$

COMMENTS

The fins account for about 60% of the total heat transfer. The rate of heat transfer without the fins would be about 9.2 W—less than half of that with the fins. If the entire fin temperature was assumed to be at the base temperature, the calculated heat loss rate would be 21.0 W—about 4% higher than that calculated above. This means that the fin efficiency is very high and L_f could therefore be increased to increase the heat dissipation quite effectively.

PROBLEM 4.55

A row of 25 square computer chips each 10 × 10 mm in size and 1 mm thick and spaced 1 mm apart is mounted on an insulating plastic substrate as shown below. The chips are to be cooled by nitrogen flowing along the length of the row at –40°C and atmospheric pressure to prevent their temperature from exceeding 30°C. The design is to provide for a dissipation rate of 30 milliwatts per chip. Estimate the minimum free stream velocity required to provide safe operating conditions for every chip in the array.

GIVEN

- A row of 25 square computer chips with nitrogen flowing over them
- Chip dimensions = 10 mm × 10 mm = 0.01 m × 0.01 m
- Chip thickness = 1 mm = 0.001 m
- Maximum temperature of the chips (T_s) = 30°C
- Nitrogen temperature (T_∞) = – 40°C
- Heat dissipation = 30 m W/chip = 0.03 W/chip

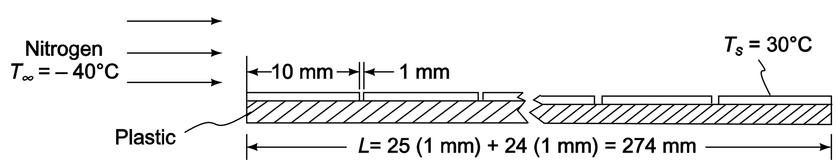
FIND

- The minimum free stream velocity (U_∞)

ASSUMPTIONS

- Steady state
- Heat transfer from the edge of the chips is negligible
- The chips trip the boundary layer into turbulence
- Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 32, for nitrogen at the film temperature (– 5°C)

Kinematic viscosity (ν) = 22.5×10^{-6} m²/s

Thermal conductivity (k) = 0.03106 W/(m K)

Prandtl number (Pr) = 0.698

SOLUTION

The heat flux from the chips is

$$\frac{\dot{q}_G}{A} = \frac{0.03\,\text{W/chip}}{(0.01)^2\ \text{m}^2/\text{chip}} = 300\ \text{W/m}^2$$

The local heat transfer coefficient for a turbulent boundary layer is given by Equation (4.81)

$$h_{cx} = 0.0288\ \frac{k}{x} Pr^{\frac{1}{3}} \left(\frac{U_\infty x}{\nu}\right)^{0.8}$$

Since h_{cx} decreases with increasing x, the lowest heat transfer coefficient occurs at the trailing edge of the array. Therefore, the minimum velocity needed o keep the trailing edge of the array at 30°C will be determined by conditions at $x = L$. The convective heat flux at the trailing edge is

$$\frac{q_c}{A} = h_{cL}\,(T_s - T_\infty) = \frac{q_G}{A}$$

$$\frac{q_G}{A} = 0.0288\ \frac{k}{L} Pr^{\frac{1}{3}} \left(\frac{U_\infty L}{\nu}\right)^{0.8} (T_s - T_\infty)$$

Solving for free stream velocity

$$U_\infty = 84.29\ \frac{\nu}{L}\left[\frac{q_G}{A} Pr^{-\frac{1}{3}} \frac{L}{k\,(T_s - T_\infty)}\right]^{1.25}$$

$$U_\infty = 84.29\ \frac{22.5\times 10^{-6}\ \mathrm{m^2/s}}{0.274\,\mathrm{m}}$$

$$\left[300\,\mathrm{W/m^2}(0.698)^{-\frac{1}{3}} \frac{0.274\,\mathrm{m}}{(0.03106\,\mathrm{W/(m\,K)})(30°\mathrm{C} + 40°\mathrm{C})}\right]^{1.25} = 0.75\ \mathrm{m/s}$$

The Reynolds number at the trailing edge for this velocity is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(0.75\,\mathrm{m/s})(0.274\,\mathrm{m})}{22.5\times 10^{-6}\ \mathrm{m^2/s}} = 9133$$

COMMENTS

Assuming the boundary layer is laminar would lead to higher U_∞, (1.35 m/s).

PROBLEM 4.56

It has been proposed to tow icebergs from the polar region to the Middle East in order to supply potable water to arid regions there. A typical iceberg suitable for towing should be relatively broad and flat. Consider an iceberg 0.25 km thick and 1 km square. This iceberg is to be towed at 1 km/h over a distance of 6000 km through water whose average temperature during the trip is 8°C. Assuming that the interaction of the iceberg with its surrounding can be approximated by the heat transfer and friction at its bottom surface, calculate the following parameters:

(a) The average rate at which ice will melt at the bottom surface.

(b) The power required to tow the iceberg at the designated speed.

(c) If towing energy costs are approximately 50 cents per kilowatt hour of power and the cost of delivering water at the destination can also be approximated by the same figure, calculate the cost of fresh water.

The latent heat of fusion of the ice is 334 kJ/kg and its density is 900 kg/m^3.

GIVEN

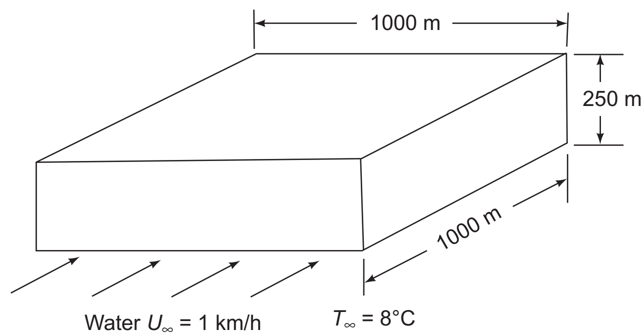
- An ice sheet being towed through water
- Ice sheet dimensions: 1 km × 1 km × 0.25 km = 1000 m × 1000 m × 250 m
- Towing speed (U_∞) = 1 km/h = 1000 m/h
- Distance towed = 6000 km = 6×10^6 m
- Average water temperature (T_∞) = 8°C
- Towing cost = $.50/kWh
- Latent heat of fusion of the ice (h_{sf}) = 334 kJ/kg
- Density of he ice (ρ_i) = 900 kg/m^3

FIND

(a) Average melt rate (m) at the bottom surface
(b) Power (P) required to tow the iceberg
(c) Cost of delivered water (= towing cost)

ASSUMPTIONS

- Heat transfer and friction of the sides of the iceberg are negligible
- Properties of sea water are the same as fresh water sketch



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the film temperature (4°C)

Kinematic viscosity (ν) = 1.586×10^{-6} m^2/s
Thermal conductivity (k) = 0.566 W/(m K)
Density (ρ_w) = 1000 kg/m^3
Prandtl number (Pr) = 11.9

SOLUTION

(a) The Reynolds number at the trailing edge of the iceberg is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(1000\,\text{m/h})(1000\,\text{m})}{(1.586 \times 10^{-6}\,\text{m}^2/\text{s})\,(3600\,\text{s/h})} = 1.75 \times 10^8 > 5 \times 10^5$$

Therefore, the flow is turbulent. The Reynolds number is large enough that the laminar region of the boundary layer will be neglected. The average heat transfer coefficient over the iceberg bottom is given by Equation (4.82)

$$h_c = 0.036\,\frac{k}{L} Pr^{\frac{1}{3}}\,Re_L^{0.8} = 0.036\,\frac{(0.566\,\text{W/(m K)})}{1000\,\text{m}}(11.9)^{\frac{1}{3}}\,(1.75 \times 10^8)^{0.8} = 182.8\ \text{W/(m}^2\,\text{K)}$$

The average rate of convective heat transfer from the bottom of the iceberg is

$$q = \bar{h}_c A (T_s - T_\infty) = \left(182.8 \text{ W/(m}^2\text{ K)}\right) (1000 \text{ m}) (1000 \text{ m}) (8°\text{C} - 0°\text{C}) = 1.46 \times 10^9 \text{ W}$$

This will cause the ice to melt at an average rate (m) given by

$$m = \frac{q}{h_{sf}} = \frac{1.46 \times 10^9 \text{ W}(\text{J}/(\text{Ws}))}{334 \text{ kJ/kg}(1000 \text{ J/kJ})} = 4370 \text{ kg/s}$$

(b) The power required to tow the iceberg is the product of the drag force on the bottom of the iceberg and the towing speed

$$P = D\, U_\infty = \tau_s A\, U_\infty$$

but by definition [Equation (4.13)]

$$C_f = \frac{2\,\tau_s}{\rho_w U_\infty^2} \quad \Rightarrow \quad \tau_s = \frac{1}{2}\, \rho_w U_\infty^2\, C_f$$

The friction coefficient for turbulent flow, $5 \times 10^5 < Re < 10^7$, is given by Equation (4.78b). Although this relation has not been verified for $Re > 10^7$, it will be extrapolated to $Re = 1.75 \times 10^8$ for this problem

$$C_f = 0.072\ Re_L^{-\frac{1}{5}} \quad \Rightarrow \quad P = 0.036\ Re_L^{-\frac{1}{5}}\ \rho_w A\, U_\infty^3$$

$$P = 0.036\,(1.75 \times 10^8)^{-0.2} \left(1000 \text{ kg/m}^3\right) (1000 \text{ m}) (1000 \text{ m}) \left[\frac{1000 \text{ m/h}}{3600 \text{ s/h}}\right]^3$$

$$= 1.73 \times 10^4\ \frac{\text{kg m}}{\text{s}^2}\frac{\text{m}}{\text{s}} = 17.3 \text{ W}$$

(c) The cost of towing (C) per unit mass of delivered ice is

$$\text{Cost} = \frac{\text{Total cost}}{\text{Mass delivered}} = \frac{\text{(Towing power) (Towing time) (Towing energy cost)}}{\text{Initial mass} - \text{(Rate of melting) (Towing time)}}$$

$$\text{Towing time} = \frac{6000 \text{ km}}{1 \text{ km/h}} = 6000 \text{ h}$$

$$\text{Initial mass} = \text{(Volume } (\rho_i) = (1000 \text{ m})(1000 \text{ m})(250 \text{ m}) \left(900 \text{ kg/m}^3\right) = 2.25 \times 10^{11} \text{ kg}$$

$$\therefore \text{ Cost} = \frac{17.3 \text{ kW}\,(6000 \text{ h})\,(\$.50/\text{kWh})}{2.25 \times 10^{11} \text{ kg} - (4370 \text{ kg/s})(6000 \text{ h})(3600 \text{ s/h})} = 4 \times 10^{-7}\ \frac{\$}{\text{kg}}$$

COMMENTS

About 42% of the ice melts during the journey. There are 3.79 kg of water in a gallon, therefore, the transportation costs are $1 for every 6.6 million gallons of water.

PROBLEM 4.57

In a manufacturing operation, a long strip sheet of metal is transported on a conveyor at a velocity of 2 m/s while a coating on its top surface is to be cured by radiant heating. Suppose that infrared lamps mounted above the conveyor provide a radiant flux of 2500 W/m^2 on the coating. The coating absorbs 50% of the incident radiant flux, has an emissivity of 0.5, and radiates to surrounding at a temperature of 25°C. In addition, the coating also loses heat by convection though a heat transfer coefficient between both the upper and lower surface and the ambient air which may be assumed to be at the same temperature as the environment. Estimate the temperature of the coating under steady state conditions.

GIVEN

- A long strip of sheet metal on a conveyor
- Velocity (U_∞) = 2 m/s
- Radiant flux on upper surface (q_{lamps}/A) 2500 W/m^2
- Coating absorbs 50 of incident radiant flux, absorptivity (α) = 0.5
- Surroundings temperature (T_∞) = 25°C = 298 K
- Emissivity of coating (ε) = 0.5

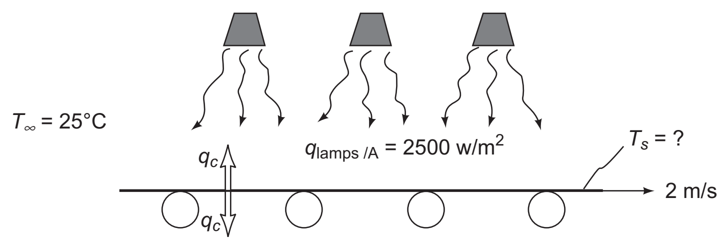
FIND

- The temperature of the coating (T_s)

ASSUMPTIONS

- Steady state
- The thermal resistance of the sheet metal is negligible
- The surroundings behave as a blackbody
- The ambient air is still
- The ambient temperature is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 25°C (as a first guess)

Kinematic viscosity (ν) = 16.2×10^{-6} m^2/s

Thermal conductivity (k) = 0.0255 W/(m K)

Prandtl number (Pr) = 0.71

From Appendix 1, Table 5,

The Stephen-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The length of metal strip (L_c) needed to reach the critical Reynolds number is given by

$$Re_L \frac{U_\infty L_c}{\nu} = 5 \times 10^5 \Rightarrow L_c = \frac{5\times10^5 \, \nu}{U_\infty} = \frac{5\times10^5 \left(16.2\times10^{-6}\ \text{m}^2/\text{s}\right)}{2}\ \text{m/s} = 4.1\text{m}$$

Assuming the metal strip is less than 4.1 m, the flow will be laminar. The estimate of surface temperature will be based on the average convective heat transfer coefficient in the laminar region which is given by Equation (4.38)

$$h_c = \frac{k}{L} 0.664\, Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{\left(0.0255\,\text{W/(m K)}\right)}{4.1\,\text{m}} 0.664\,(5\times10^5)^{\frac{1}{2}}\ (0.71)^{\frac{1}{3}} = 2.61\ \text{W/(m}^2\ \text{K)}$$

By the conservation of energy, of steady state

$$\alpha\left(\frac{q_{\text{lamps}}}{A}\right) = \frac{q_r}{A} + 2\,\frac{q_c}{A}$$

$$\text{where } \frac{q_r}{A} = \varepsilon\,\sigma(T_s^4 - T_\infty^4) \text{ [Equation (1.17)]}$$

$$\frac{q_c}{A} = h_c\,(T_s - T_\infty) \text{ [Equation (1.10)]}$$

$$\therefore\ \alpha\left(\frac{q_{\text{lamps}}}{A}\right) = \varepsilon\,\sigma\ \ (T_s^4 - T_\infty^4) + 2\,h_c\,(T_s - T_\infty)$$

$$0.5\left(2500\,\text{W/m}^2\right) = 0.5\left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)[T_s^4 - (298\text{ K})^4] + 2\left(2.61\,\text{W/(m}^2\,\text{K)}\right)(T_s - 298\text{ K})$$

Checking the units for consistency, then dropping them for clarity

$$2.835\times10^{-8}\,T_s^4 + 5.22\,T_s - 3029 = 0$$

Solving by trial and error

$$T_s = 417\text{ K} = 144°\text{C}$$

COMMENTS

Note that absolute temperatures must by used in the radiation equation.

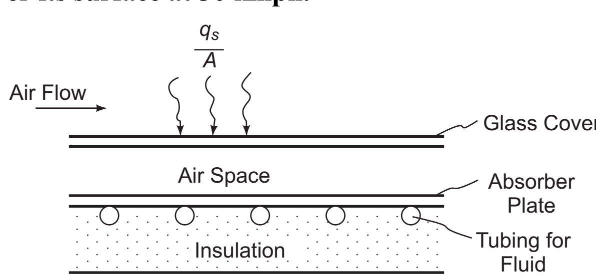
Heat loss from the sheet is nearly equally divided between radiation and convection.

The air properties where taken at the ambient temperature. The estimate could be improved by evaluating the properties at the corrected film temperature, (25°C + 144°C)/2 = 135°C and calculating a new steady state surface temperature.

Because of the small forced convection component in this problem, natural convection from the metal strip may be important. Natural convection will be covered in Chapter 5.

PROBLEM 4.58

A 2 m by 2 m flat plate solar collector for domestic hot water heating is shown schematically in the sketch. Solar radiation at a rate of 750 W/m² is incident on the glass cover which transmits 90% of the incident flux. Water flows through the tubes soldered to the backside of the absorber plate, entering with a temperature of 25°C. The Glass cover has a temperature of 27°C in the steady state and radiates heat with an emissivity of 0.92 to the sky at –50°C. In addition, the glass cover losses heat by convection to air at 20°C flowing over its surface at 30 kmph.



(a) **Calculate the rate at which heat is collected by the working fluid, i.e., the water in the tubes, per unit area of the absorber plate.**

(b) **Calculate the collector efficiency η_c defined as the ratio of useful energy transferred to the water in the tubes to the solar energy incident on the collector cover plate.**

(c) **Calculate the outlet temperature of the water if its flow rate through the collector is 0.02 kg/s. The specific heat of the water is 4179 J/(kg K).**

GIVEN

- A flat plate solar collector with air flowing over it
- Collector dimensions = 2 m × 2 m
- Incident solar flux = 750 W/m^2
- Glass cover transmits 90% of solar flux
- Water enters tubes at a temperature (T_{wi} = 25°C)
- Glass cover steady state temperature (T_s) = 27°C = 300 K
- Emissivity of glass cover (ε) = 0.92
- Sky temperature ($T_{\infty r}$) = –50°C = 223 K
- Ambient air temperature ($T_{\infty c}$) = 20°C = 293 K
- Air speed (U_∞) = 8.33 m/s
- Water flow rate (m) = 0.02 kg/s
- Specific heat of water (c_p) = 4179 J/(kg K)

FIND

(a) Heat flux to the water q_w/A

(b) Collector efficiency (η_c) = $\dfrac{\text{energy to the water}}{\text{incident solar energy}}$

(c) Outlet temperature of the water (T_{wo})

ASSUMPTIONS

- Steady State
- Radiative heat transfer between the absorber and the glass plate is negliagible
- The absorber plate absorbs all the incident solar radiation
- Radiative heat transfer from the absorber plate, through the glass to the sky, is negligible
- Heat transfer through the back and sides of the collector is negligible
- Solar radiation blocked by the collector frame is negligible
- Glass cover and absorber temperatures are uniform
- The solar energy absorbed by the glass is negligible
- The sky behaves as a blackbody

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the film temperature (23.5°C)

Kinematic viscosity (ν) = 16.0 × 10^{-6} m^2/s

Thermal conductivity (k) = 0.0295 W/(m K)

Prandtl number (Pr) = 0.71

From Appendix 1, Table 5

The Stephan-Boltzmann Constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The glass cover absorbs solar energy and also absorbs energy from the absorber plate. The cover loses heat to ambient by convection to the air and by reradiation. An energy balance on the glass cover will allow us to determine the rate of heat transfer from the absorber plate to the glass cover plate. The absorber plate absorbs solar energy but loses some of this to the glass cover plate as described above. Therefore, an energy balance on the absorber plate will allow us to determine the rate at which energy is absorbed by the absorber plate. Based on the assumptions listed above, this will equal the rate at which energy is delivered to the water.

(a) Energy balance on the glass cover plate

Radiative flux to the sky (q_r/A) + Convective flux to the air (q_c/A) = Net energy gain from the absorber plate (q_{A-G}/A) + Solar energy absorbed (q_s/A)

where $$\frac{q_r}{A} = \varepsilon\,\sigma(T_s^4 - T_{\infty r}^4) \qquad \text{[Equation (1.17)]}$$

$$\frac{q_c}{A} = \bar{h}_c\,(T_s - T_{\infty c}) \qquad \text{[Equation (1.10)]}$$

$$\frac{q_s}{A} = (0.1)\left(750\,\text{W/m}^2\right) = 75\ \text{W/m}^2$$

$$\frac{q_{A-G}}{A} = \varepsilon\,\sigma(T_s^4 - T_{\infty r}^4) + h_c\,(T_s - T_{\infty c}) - q_s/A$$

The Reynolds number at the trailing edge of the glass cover plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{8.33\ \text{m/s (2 m)}}{(16.2\times10^{-6}\text{m}^2/\text{s})} = 1.03\times10^6 > 5\times10^5$$

Therefore, the boundary layer is mixed and the average convective heat transfer coefficient is given by Equation (4.83)

$$h_\infty = 0.036\,\frac{k}{L}\,Pr^{\frac{1}{3}}\,[Re_L^{0.8} - 23{,}200]$$

$$h_c = 0.036\,\frac{(0.0295\,\text{W/(m K)})}{2\,\text{m}}\,(0.71)^{\frac{1}{3}}\,[(1.03\times10^6)^{0.8} - 23{,}200] = 19.8\ \text{W/(m}^2\ \text{K)}$$

$$\frac{q_{A-G}}{A} = 0.92\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)[(300\ \text{K})^4 - (223\ \text{K})^4] + 19.8\ \text{W/(m}^2\ \text{K)}$$

$$(300\ \text{K} - 293\ \text{K}) - 75\ \text{W/(m}^2\ \text{K)}$$

$$\frac{q_{A-G}}{A} = (294 + 138.6 - 75)\ \text{W/m}^2 = 357.6\ \text{W/m}^2$$

Energy balance on the absorber plate

heat flux to the water (q_w/A) = solar gain (0.9 q_s/A) heat flux to glass cover (q_{A-G}/A)

$$\frac{q_w}{A} = 0.9\left(750\,\text{W/m}^2\right) - 357.6\ \text{W/m}^2 = 317.4\ \text{W/m}^2$$

(b) Collector efficiency

$$\eta_c = \frac{\frac{q_w}{A}}{\frac{q_s}{A}} = \frac{317.4}{750} = 0.423 = 42.3\%$$

(c) $$q_w = m\,c_p\,(T_{wo} - T_{wi}) = \left(\frac{q_w}{A}\right)A$$

Solving for the water outlet temperature

$$T_{wo} = T_{wi} + \left(\frac{q_w}{A}\right)\frac{A}{m\,c_p} = 25°\text{C} + \left(317.4\,\text{W/m}^2\right)\frac{(2\,\text{m})(2\,\text{m})}{(0.02\,\text{kg/s})(4179\,\text{J/(kg K)})} = 40.2°\text{C}$$

PROBLEM 4.59

A 2.5 cm-diam, 15 cm-long transite rod (k = 0.97 W/(m K), 1647 kg/m^3, 837 J/(kg K) on the end of a 2.5 cm-diam wood rod at a uniform temperature of 100°C is suddenly placed into a 16°C, 30 m/s air stream flowing parallel to the axis of the rod. Estimate the average center line temperature of the transite rod 8 min after cooling starts. Assume radial heat conduction, but include radiation losses, based on an emissivity of 0.90, to black surroundings at air temperature.

GIVEN

- Transite rod on the end of a wood rod with air flowing parallel to the axis
- Transite properties
  - Thermal conductivity (k_t) = 0.97 W/(m K)
  - Density (ρ_t) = 1647 kg/m^3
  - Specific heat (c_t) = 837 J/(kg K)
- Rod diameter (D) = 2.5 cm
- Transite rod length = 15 cm
- Initial temperature (T_o) = 100°C = 373 K
- Air temperature (T_∞) = 16°C = 289 K
- Air speed (U_∞) = 30 m/s
- Rod emissivity (ε) = 0.90

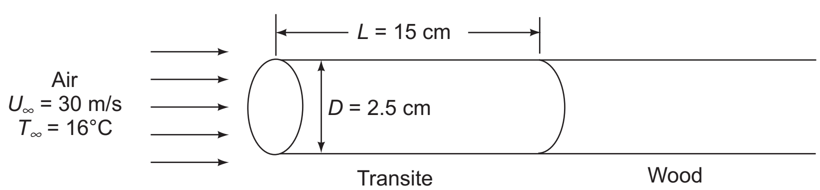
FIND

- The average temperature of the center of the rod after 8 min.

ASSUMPTIONS

- Radial heat conduction only – neglect end effects
- Wood rod acts as an insulator and only provides support for the transite
- Surroundings behave as a black body at the ambient temperature
- Convective heat transfer can be approximated as a flat plate
- Constant thermal properties of the rod and the air

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the initial film temperature (58°C)

Kinematic viscosity (ν) = 1.885 × 10^{-5} m^2/s

Thermal conductivity (k) = 0.0277 W/m K

Prandtl number (Pr) = 0.71

From Appendix 1, Table 5

The Stephen-Boltzmann Constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The convective heat transfer coefficient between the rod and the air will be determined by treating the rod as a flat plate.

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(30\text{ m/s})\,(0.15\text{ m})}{(1.885\times10^{-5}\text{ m}^2\text{/s})} = 2.39 \times 10^5 \text{ (Laminar)}$$

From Equation (4.38) for a laminar boundary layer

$$h_c = \frac{k}{L} 0.664\, Re_L^{\frac{1}{2}}\, Pr^{\frac{1}{3}} = \frac{(0.0277\text{ W/(m K)})}{(0.15\text{ m})}\, 0.664\,(2.39\times10^5)^{\frac{1}{2}}\,(0.71)^{\frac{1}{3}}$$

$\Rightarrow \qquad h_c = 53.5\text{ W/(m}^2\text{ K)}$

The overall heat transfer coefficient satisfies the following equation

$$\frac{q_{\text{total}}}{A} = h_t\,(T_o - T_\infty) = \frac{q_c}{A} + \frac{q_r}{A} = h_c\,(T_o - T_\infty) + \varepsilon\,\sigma\,(T_o^{\,4} - T_\infty^{\,4})$$

$$h_t = \frac{h_c\left(T_o - T_\infty\right) + \varepsilon\sigma\left(T_o^{\,4} - T_\infty^{\,4}\right)}{T_o - T_\infty}$$

$$h_t = \frac{(53.5\text{ W/(m}^2\text{K))}\,(100°\text{C} - 16°\text{C}) + 0.9\,(5.67\times10^{-8}\text{ W/(m}^2\text{K}^4))\,(373^4 - 289^4)}{100°\text{C} - 16°\text{C}}$$

$$= 61\text{ W/(m}^2\text{ K)}$$

The Biot number for the rod is given in Table 2.3

$$B_i = \frac{h_c r_o}{k_t} = \frac{(61\text{ W/(m}^2\text{K))}\,(1.25\times10^{-2}\text{ m})}{0.97\text{ W/(m K)}} = 0.79 > 0.1$$

Therefore, internal resistance is significant and a chart solution must be used: Figure 2.38 applies to a long cylinder. At $t = 8$ min, the Fourier number is

$$Fo = \frac{\alpha t}{r_o^{\,2}} = \frac{k_t t}{\rho_t c_t r_o^{\,2}} = \frac{(0.97\text{ W/(m K)}\,(4805)}{(1647\text{ kg/m}^3)\,(837\text{ J/(kg K)}\,(1.25\times10^{-2}\text{m})^2} = 2.16$$

From Figure 2.38 for $F_o = 2.15$ and $1/Bi = 1.27$

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = 0.07$$

Solving for the centerline temperature at $t = 8$ min ($T_{(0,t)}$)

$$T(0,t) = T_\infty + 0.07\,(T_o - T_\infty) = 16°\text{C} + 0.07\,(100°\text{C} - 16°\text{C}) = 21.9°\text{C}$$

COMMENTS

Note that absolute temperatures must be used in radiative equations. The overall heat transfer coefficient is actually decreasing slightly as the rod cools. At 8 min, the rod surface temperature would be about 68.8°C leading to a heat transfer coefficient of 10.2 Btu/(h ft^2°F).

PROBLEM 4.60

> **A highly polished chromium flat plate is placed in a high-speed wind tunnel to simulate flow over the fuselage of a supersonic aircraft. The air flowing in the wind tunnel is at a temperature of 0°C, a pressure of 3500 N/m^2, and a velocity parallel to the plate of 800 m/s. What temperature is the adiabatic wall temperature in the laminar region and how long is the laminar boundary layer?**

GIVEN

- High speed air flow over a flat plate
- Air temperature (T_∞) = 0°C = 273 K
- Air pressure = 3500 N/m^2
- Air velocity (U_∞) = 800 m/s

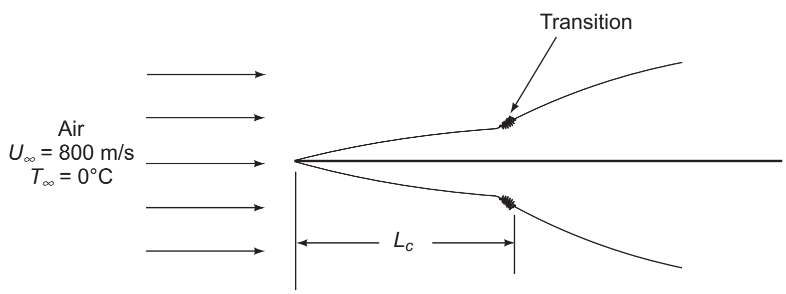
FIND

(a) Adiabatic wall temperature (T_{as})
(b) Length of laminar boundary layer (L_c)

ASSUMPTIONS

- Steady state
- Transition to turbulence occurs at $Re_x^\bullet = 10^5$
- The air behaves as an ideal gas
- Radiation heat transfer is negligible because of the low emissivity of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at one atmosphere and at the bulk temperature
(0°C) Prandtl number (Pr) = 0.71
Specific heat (C_p) = 1011 J/kg K
From Appendix 1, Table 5 g_c = 1.000 kg m/N s^2 (by definition)

SOLUTION

(a) The stagnation temperature is given by Equation (4.91)

$$T_o = T_\infty + \frac{U_\infty^{\,2}}{2g_cC_p} = 273\text{ K} + \frac{(800\,\text{m/s})^2}{2\big(1\,(\text{kg m})/(\text{Ns}^2)\big)(1011\,\text{J/(kg K)})((\text{Nm})/\text{J})} = 273\text{ K} + 317\text{ K} = 590\text{ K}$$

The recovery factor in the laminar region is $Pr^{1/2}$. The adiabatic surface temperature is given by Equation (4.94)

$$\frac{T_{as} - T_\infty}{T_o - T_\infty} = r = Pr^{\frac{1}{2}}$$

$$T_{as} = T_\infty + Pr^{\frac{1}{2}}\,(T_o - T_\infty) = 273\text{ K} + (0.71)^{\frac{1}{2}}\,(590\text{ K} - 273\text{ K}) = 540\text{ K}$$

(b) The reference temperature (T), which must be used in evaluating the Reynolds number, is given by Equation (4.97)

$$T^\bullet = T_\infty + 0.5\,(T_s - T_\infty) + 0.22\,(T_{as} - T_\infty)$$

A surface temperature must be assumed to evaluate T. Assuming $T_s = T_\infty = 273$ K

$$T^\bullet = 273\text{ K} + 0.5\ (0) + 0.22\ (540\text{ K} - 273\text{ K}) = 332\text{ K}$$

The length of the laminar boundary layer (L_c) is given by

$$Re^\bullet_{L_c} = \frac{U_\infty L_c \rho^\bullet}{\mu^\bullet} = 10^5 \quad \Rightarrow \quad L_c = \frac{10^5\ \mu^\bullet}{U_\infty \rho^\bullet}$$

The density at the given pressure and reference temperature can be determined from the ideal gas law

$$\rho^\bullet = \frac{P}{R_a T^\bullet} \qquad \text{where} \qquad R_a = \text{The gas constant for air} = 287\text{ J/(kg K)}$$

$$\rho^\bullet = \frac{3500\,\text{N/m}^2}{(287\,\text{J/(kg K)})(\text{Nm/J})(332\,\text{K})} = 0.0367\text{ kg/m}^3$$

From Appendix 2, Table 27, for dry air at $T^\bullet = 332$ K, the absolute viscosity (μ) = 19.3×10^{-6} N s/m^2

$$\therefore \quad L_c = \frac{10^5\left(19.3\times10^{-6}\right)\left(\text{kg m/(s}^2\text{N)}\right)}{(800\,\text{m/s})\left(0.0367\,\text{kg/m}^3\right)} = 0.066\text{m} = 6.6\text{ cm}$$

COMMENTS

For a more accurate estimation of the length of the laminar region, the average heat transfer coefficient from Equation (4.99) can be used to find the surface temperature. The surface temperature can be used to generate a new reference temperature which is used to find the length L_c. This procedure would be repeated until the value of L_c converges.

PROBLEM 4.61

Air at a static temperature of 21°C and a static pressure of 0.7 kPa (abs.) flows at zero angle of attack over a thin electrically heated flat plate at a velocity of 240 m/s. If the plate is 10 cm long in the direction of flow and 0.6 m the direction normal to the flow, determine the rate of electrical heat dissipation necessary to maintain the plate at an average temperature of 55°C.

GIVEN

- High speed air flow over a heated flat plate
- Air static temperature (T_A) = 21°C
- Air static pressure (P) = 0.7 kPa (abs.)
- Air velocity (U_∞) = 240 m/s
- Plate length (L) = 0.1 m
- Plate width (w) = 0.6 m
- Average plate temperature (T_s) = 55°C

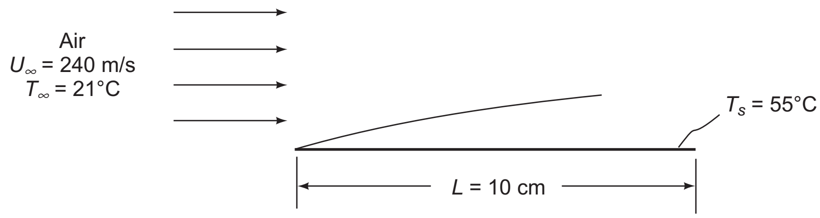
FIND

- The rate of electrical heat dissipation $(\dot{q}_G)$ to maintain the specified plate temperature

ASSUMPTIONS

- Steady state
- Air behaves as an ideal gas
- Air flows on one side of the plate only
- Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the free-stream temperature of 21°C

Specific heat (c_p) = 1013 J/(kg K)

Prandtl number (Pr) = 0.71

SOLUTION

The stagnation (T_o) and adiabatic surface (T_{as}) temperatures must be calculated to find the reference temperature (T). From Equation (4.91)

$$T_o = T_\infty + \frac{U_\infty{}^2}{2g_c C_p} = 21°\text{C} + \frac{(240\text{ m/s})^2}{2(9.81\text{ m/s}^2)\,(1013\text{ J/(kg K)}\,(0.10\text{ kg m/J})} = 50°\text{C}$$

Assuming the flow is laminar, $r = Pr^{1/2}$ and the adiabatic surface temperature is given by Equation (4.94)

$$T_{as} = T_\infty + Pr^{\frac{1}{2}}(T_o - T_\infty) = 21°\text{C} + (0.71)^{\frac{1}{2}}(50°\text{C} - 21°\text{C}) = 45.5°\text{C}$$

From Equation (4.97)

$$T^\bullet = T_\infty + 0.5\,(T_s - T_\infty) + 0.22\,(T_{as} - T_\infty)$$

$$T^\bullet = 21°\text{C} + 0.5\,(55°\text{C} - 21°\text{C}) + 0.22\,(45.5°\text{C} - 21°\text{C}) = 43.4°\text{C} = 316.4\text{ K}$$

The density of air at the reference temperature can be calculated from the ideal gas law

$$\rho^\bullet = \frac{P}{R_a T^\bullet} \quad \text{where} \quad R_a = \text{The gas constant for air} = 287\text{ J/(kg K)}$$

$$\rho^\bullet = \frac{(700\text{ Pa})}{(287\text{ J/(kg K)})\,(0.1\text{ kg m/J})\,(316.4\text{ K})} = 0.0\ 77\text{ kg/m}^3$$

From Appendix 2, Table 27, for dry air at the reference temperature (43.4°C), the absolute viscosity $\left(\mu^\bullet\right)$ = 1.94 × 10^{-4} kg/ms, the Prandtl number $\left(Pr^\bullet\right)$ = 0.71

The Reynolds number at the trailing edge of the plate is

$$Re^\bullet_{L_c} = \frac{U_\infty L_c \rho^\bullet}{\mu^\bullet} = \frac{(240\text{ m/s})\,(0.1\text{ m})\,(0.077\text{ kg/m}^3)}{1.94\times10^{-4}\text{ kg/ms}} = 9525 < 10^5$$

Therefore, the laminar flow assumption is valid.

The average heat transfer coefficient over the plate can be calculated by averaging Equation (4.99)

$$h_c = \int_0^L 0.332\,c_p\,\rho^\bullet\,U_\infty (Re_x^\bullet)^{-\frac{1}{2}}(Pr^\bullet)^{-\frac{2}{3}}\,dx = 0.664\,c_p\,\rho^\bullet\,U_\infty (Re_L^\bullet)^{-\frac{1}{2}}(Pr^\bullet)^{-\frac{2}{3}}$$

$$h_c = 0.664\,(1013\text{ J/(kg K)})\,(0.077\text{ kg/m}^3)\,(240\text{ m/s})\,(9525)^{-\frac{1}{2}}\,(0.71)^{-\frac{2}{3}}$$

$$= 16\text{ W(m}^2\text{ K)}$$

The electrical heat dissipation required is equal to the convective heat transfer rate

$$q_G = q_c = h_c A (T_s - T_{as})$$

$$= 16 \text{ W/(m}^2 \text{ K) (0.1 m) (0.6 m) (55°C} - \text{45.5°C)}$$

$$= 9.1 \text{ W}$$

PROBLEM 4.62

Heat rejection from high-speed racing automobiles is a problem because the required heat exchangers generally create additional drag. For a car to be tested at the Bonneville Salt Flats, it has been proposed to integrate heat rejection into the skin of the vehicle. Preliminary tests are to be performed in a wind tunnel on a flat plate without heat rejection. Atmospheric air in the tunnel is at 10°C and flows at 250 ms^{-1} over the 3 m long thermally nonconducting flat plate. What is the plate temperature 1 m downstream from the leading edge? How much does this temperature differ from that which exists 0.005 m from the leading edge?

GIVEN

- High speed air flow over a thermally nonconducting flat plate
- Plate length (L) = 3 m
- Air temperature (T_∞) = 10°C
- Air speed (U_∞) = 250 m/s
- Air pressure = 1 atmosphere

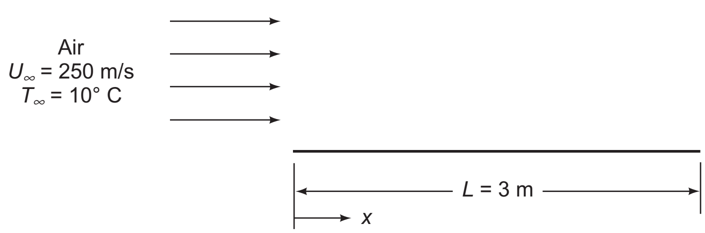
FIND

(a) The plate temperature (T_s) at x = 1 m
(b) Temperature difference between x = 1 m and x = 0.005 m

ASSUMPTIONS

- Steady state
- Air is on only one side of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 10°C

Kinematic viscosity (ν) = 14.8 × 10^{-6} m^2/s

Prandlt number (Pr) = 0.71

Specific heat (c_p) = 1011 J/(kg K)

SOLUTION

Since the surface is nonconducting, its surface temperature is equal to the adiabatic surface temperature (T_{as})

At x = 1 m: $$Re_x = \frac{U_\infty x}{\nu} = \frac{(250\,\text{m/s})(1\text{m})}{14.8\times10^{-6}\,\text{m}^2/\text{s}} = 1.69 \times 10^7 \text{ (Turbulent)}$$

$$\text{At} \quad x = 0.005 \text{ m:} \quad Re_x = \frac{U_\infty x}{\nu} = \frac{(250\,\text{m/s})(1\,\text{m})}{14.8\times 10^{-6}\,\text{m}^2/\text{s}} = 8.45 \times 10^4 \text{ (Laminar)}$$

The stagnation temperature is given by Equation (4.91)

$$T_o = T_\infty + \frac{U_\infty^{\,2}}{2g_c c_p} = 10°\text{C} + \frac{(250\,\text{m/s})^2}{2\left(1\,\text{kg m/(N s}^2)\right)(1011\,\text{J/(kg K)})((\text{N m})/\text{J})} = 41°\text{C}$$

The adiabatic surface temperature is given by Equation (4.93)

$$T_{as} = T_{oc} + r\,(T_o - T_\infty)$$

(a) For the turbulent region, $r = Pr^{1/3}$

$$\therefore \quad \text{At} \quad x = 1 \text{ m: } T_{as} = 10°\text{C} + (0.71)^{\frac{1}{3}}\,(41°\text{C} - 10°\text{C}) = 38°\text{C}$$

(b) For the laminar region, $r = pr^{1/2}$

$$\therefore \quad \text{At} \quad x = 0.005\text{m: } T_{as} = 10°\text{C} + (0.71)^{\frac{1}{2}}\,(41°\text{C} - 10°\text{C}) = 36°\text{C}$$

The temperature difference between x =1 m and x = 0.005 m is 2°C.

PROBLEM 4.63

Air at 15°C and 0.01 atmospheres pressure flows over a thin flat strip of metal, 0.1 m long in the direction of flow, at a velocity of 250 m/s. Determine (a) the surface temperature of the plate at equilibrium and (b) the rate of heat removal required per meter width if the surface temperature is to be maintained at 30°C.

GIVEN

- High speed air flow over a thin flat strip of metal
- Air temperature (T_∞) = 15°C = 288 K
- Air pressure (P) = 0.01 atm = 1013 N/m^2
- Metal strip length (L) = 0.1 m
- Air velocity (U_∞) = 250 m/s

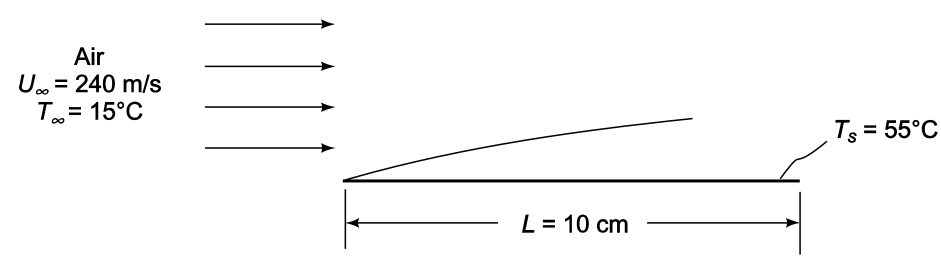
FIND

(a) Equilibrium surface temperature (T_s)
(b) Rate of heat removal per unit width (q/w) for T_s = 30°C

ASSUMPTIONS

- Air flows over one side of the strip only
- Air behaves as an ideal gas

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 15°C

Absolute viscosity (μ) = 18.0×10^{-6} N s/m^2

Prandtl number (Pr) = 0.71

Specific heat (c_p) = 1012 J/(kg K)

SOLUTION

(a) The density of air at the given pressure and temperature can be calculated from the ideal gas law

$$\rho = \frac{P}{R_a T} \qquad \text{where} \qquad R_a = \text{The gas constant for air} = 287 \text{ J/(kg K)}$$

$$\rho = \frac{1013\,\text{N/m}^2}{(287\,\text{J/(kg K)})(\text{N m/J})(288\,\text{K})} = 0.0123 \text{ kg/m}^3$$

$$Re_L = \frac{U_\infty L \rho}{\mu} = \frac{(250\,\text{m/s})(0.1\,\text{m})\left(0.0123\,\text{kg/m}^3\right)}{\left(18.0\times10^{-6}\,\text{N s/m}^2\right)\left(\text{kg m/(N s}^2)\right)} = 1.69 \times 10^4 \text{ (Laminar)}$$

The stagnation temperature is given by Equation (4.91)

$$T_o = T_\infty + \frac{U_\infty{}^2}{2 g_c c_p} = 15°\text{C} + \frac{(250\,\text{m/s})^2}{2\left(1\,\text{kg m/(N s}^2)\right)(1012\,\text{J/(kg K)})(\text{N m/J})} = 46°\text{C}$$

At equilibrium with no heat removal, the surface temperature is equal to the adiabatic surface temperature given by Equation (4.93) with $r = Pr^{1/2}$.

$$T_{as} = T_\infty + Pr^{\frac{1}{2}} (T_o - T_\infty) = 15°\text{C} + (0.71)^{\frac{1}{2}} (46°\text{C} - 15°\text{C}) = 41°\text{C}$$

(b) The reference temperature, Equation (4.97), must be used for the non-adiabatic case

$$T^\bullet = T_\infty + 0.5\,(T_s - T_\infty) + 0.22\,(T_{as} - T_\infty)$$

$$T^\bullet = 15°\text{C} + 0.5\,(30°\text{C} - 15°\text{C}) + 0.22\,(41°\text{C} - 15°\text{C}) = 28°\text{C} = 301 \text{ K}$$

From Appendix 2, Table 27, for dry air at 28°C

Absolute Viscosity $\left(\mu^\bullet\right)$ = 18.6×10^{-6} N s/m^2

Prandtl number $\left(Pr^\bullet\right)$ = 0.71

The density, from the ideal gas law

$$\rho^\bullet = \frac{1013\,\text{N/m}^2}{(287\,\text{J/(kg K)})(\text{N m/J})(301\,\text{K})} = 0.0117 \text{ kg/m}^3$$

$$Re_L{}^\bullet = \frac{U_\infty L_c \rho^\bullet}{\mu^\bullet} = \frac{(250\,\text{m/s})(0.1\,\text{m})\left(0.0117\,\text{kg/m}^3\right)}{\left(18.0\times10^{-6}\,\text{N s/m}^2\right)\left(\text{kg m/(N s}^2)\right)} = 1.57 \times 10^4 \text{ (Laminar)}$$

Averaging Equation (4.99) over the length of the plate yields

$$h_c = \frac{1}{L}\int_0^L 0.332\, c_p\, \rho^\bullet U_\infty (Re_x{}^\bullet)^{-\frac{1}{2}} (Pr^\bullet)^{-\frac{2}{3}}\, dx = 0.664\, c_p\, \rho^\bullet U_\infty (Re_L{}^\bullet)^{-\frac{1}{2}} (Pr^\bullet)^{-\frac{2}{3}}$$

$$h_c = 0.664(1013\,\text{J/(kg K)})\left(0.0117\,\text{kg/m}^3\right)(250\,\text{m/s})(\text{N m/J})(1.57\times10^4)^{-\frac{1}{2}}(0.71)^{-\frac{2}{3}} = 19.7 \text{ W/(m}^2\text{ K)}$$

The heat removal rate must equal the rate of heat gain from the air to maintain a constant surface temperature

$$\frac{q}{A} = h_c\,(T_s - T_{as}) \Rightarrow \text{q/W} = h_c L\,(T_s - T_{as}) = 19.7\ \text{W/(m}^2\,\text{K)}\ (0.1\ \text{m})\ (30°\text{C} - 43°\text{C}) = -25.6\ \text{W/m}$$

The negative sign indicates heat gained by the plate.

PROBLEM 4.64

> **A flat plate is placed in a supersonic wind tunnel with air flowing over it at a Mach number of 2.0, a pressure of 25,000 N/m^2, and an ambient temperature of –15°C. If the plate is 30 cm long in the direction of flow, calculate the cooling rate per unit area that is required to maintain the plate temperature below 120°C.**

GIVEN

- High speed air flow over a flat plate
- Mach number (M) = 2.0
- Air pressure (P) = 25,000 N/m^2
- Ambient temperature (T_∞) = –15°C = 258 K
- Plate length (L) = 30 cm = 0.3 m

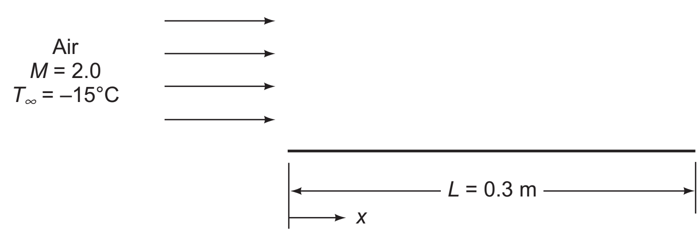
FIND

- Cooling rate per unit area (q_c/A) to keep plate temperature (T_s) below 120°C

ASSUMPTIONS

- Steady state
- Air behaves as an ideal gas
- Negligible radiative heat transfer
- Air flows over only one side of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Section 4.13, the specific heat ration for air (γ) = 1.4. The gas constant for air (R_a) = 287 J/(kg K).

SOLUTION

The acoustic velocity (a) is given by Equation (4.89)

$$a_\infty = \sqrt{\gamma\, R_a\, T_\infty} = \sqrt{1.4(287\,\text{J/(kg K)})\,(\text{N m/J})\left(\text{kg m/(N s}^2)\right)258\,\text{K}} = 322\ \text{m/s}$$

$$U_\infty = M\, a_\infty = 2.0\ (322\,\text{m/s}) = 644\ \text{m/s}$$

The stagnation temperature, from Equation (4.92)

$$T_o = T_\infty\left[1 + \frac{\gamma - 1}{2} M^2\right] = 258\ \text{K}\left[1 + \frac{1.4 - 1}{2}\,4.0\right] = 464\ \text{K} = 191°\text{C}$$

Assuming the flow is turbulent, $r = Pr^{1/3}$ and the adiabatic surface temperature (T_{as}) is given by Equation (4.93)

$$T_{as} = T_\infty + Pr^{\frac{1}{3}} (T_o - T_\infty) = -15°\text{C} + (0.71)^{\frac{1}{3}} [191°\text{C} - (-15°\text{C})] = 169°\text{C}$$

The reference temperature ($T^\bullet$) is given by Equation (4.97)

$$T^\bullet = T_\infty + 0.5 (T_s - T_\infty) + 0.22 (T_{as} - T_\infty)$$

$$T^\bullet = -15°\text{C} + 0.5 (120°\text{C} + 15°\text{C}) + 0.22 (169°\text{C} + 15°\text{C}) = 93°\text{C} = 366 \text{ K}$$

From Appendix 2, Table 27, for dry air at the reference temperature (93°C)

Absolute viscosity ($\mu^\bullet$) = 21.36×10^{-6} Ns/m^2

Prandtl number ($Pr^\bullet$) = 0.71 Specific heat (c_p) = 1021 J/(kg J)

The density can be calculated using the ideal gas law

$$\rho^\bullet = \frac{P}{R_a T^\bullet} \quad \text{where } R_a = \text{The gas constant for air} = 287 \text{ J/(kg J)}$$

$$\rho^\bullet = \frac{25{,}000 \text{ N/m}^2}{287 \text{ J/(kg J)} (\text{N m/J}) (366 \text{ K})} = 0.238 \text{ kg/m}^3$$

$$\text{At } L = 0.3 \text{ m } Re_L^\bullet = \frac{U_\infty L \rho^\bullet}{\mu^\bullet} = \frac{(644 \text{ m/s})(0.3 \text{ m})\left(0.238 \text{ kg/m}^3\right)}{\left(21.36 \times 10^{-6} \text{ N s/m}^2\right)\left(\text{kg m/(N s}^2)\right)} = 2.15 \times 10^6$$

Therefore, the assumption of turbulence is valid and the average heat transfer coefficient, neglecting the laminar portion of the boundary layer, can be calculated by averaging Equation (4.100) from $x = 0$ to $x = L$

$$h_c = \frac{1}{L}\int_0^L 0.0288\, c_p\, \rho^\bullet\, U_\infty\, (Re_x^\bullet)^{-0.2}\, (Pr^\bullet)^{-\frac{2}{3}} dx = 0.036\, c_p\, \rho^\bullet\, U_\infty\, (Re_L^\bullet)^{-0.2} (Pr^\bullet)^{-\frac{2}{3}}$$

$$h_c = 0.036\, (1021 \text{ J/(kg K)}) \left(0.238 \text{ kg/m}^3\right) (644 \text{ m/s}) (\text{N m/J})\, (2.15 \times 10^6)^{-0.2} (0.71)^{-\frac{2}{3}} = 383 \text{ W/(m}^2 \text{ K)}$$

The rate of cooling must equal the rate of heat loss by convection, given by Equation (4.98)

$$\frac{q_c}{A} = h_c (T_s - T_{as}) = 383 \text{ W/(m}^2 \text{ K)} (120°\text{C} - 169°\text{C}) = -18{,}770 \text{ W/m}^2$$

The negative sign indicates heat is being transferred to the plate from the air.

COMMENTS

The length of the laminar boundary layer is determined by

$$Re_{L_c}^\bullet = \frac{U_\infty L_c \rho^\bullet}{\mu^\bullet} = 10^5 \quad \Rightarrow \quad L_c = \frac{10^5 \mu^\bullet}{U_\infty \rho^\bullet} = 0.014 \text{ m} << 0.3 \text{ m}$$

Therefore, neglecting the laminar region does not introduce significant error.

PROBLEM 4.65

A satellite reenters the earth's atmosphere at a velocity of 2700 m/s. Estimate the maximum temperature the heat shield would reach if the shield material is not allowed to ablate and radiation effects are neglected. The temperature of the upper surface of the atmosphere –50°C.

GIVEN

- High speed air flow over a satellite
- Velocity (U_∞) = 2700 m/s
- Air temperature (T_∞) = – 50°C

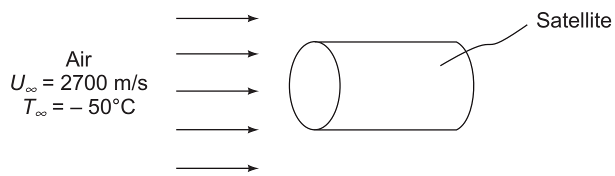
FIND

- Maximum heat shield temperature (T_s)

ASSUMPTIONS

- Radiative heat transfer is negligible
- Shield material does not ablate
- Shield can be approximated as a flat plate
- Boundary layer is turbulent

SKETCH



PROPERTIES AND CONSTANTS

Extrapolating from Appendix 2, Table 27, for dry air at –50°C

Prandtl number (Pr) = 0.71

Specific heat = 1000 J/(kg K)

SOLUTION

The stagnation temperature is given by Equation (4.91)

$$T_o = T_\infty + \frac{U_\infty^2}{2\, g_c\, c_p} = -50°\text{C} + \frac{(2700\,\text{m/s})^2}{2\big(1\,\text{kg m/(N s}^2)\big)\big(1000\,\text{J/(kg K)}\big)\big(\text{N m/J}\big)} = 3595°\text{C}$$

With no ablation or heat removal, the surface temperature of the satellite will be the adiabatic surface temperature given in Equation (4.93) where $r = Pr^{1/3}$ for turbulent flow

$$T_{as} = T_\infty + Pr^{\frac{1}{3}}\,(T_o - T_\infty) = -50°\text{C} + (0.71)^{\frac{1}{3}}\,(3295°\text{C} + 50°\text{C}) = 3200°\text{C}$$

PROBLEM 4.66

A scale model of an airplane wing section is tested in a wind tunnel at a Mach number of 1.5. The air pressure and temperature in the test section are 20,000 N/m^2 and –30°C, respectively. If the wing section is to be kept at an average temperature of 60°C, determine the rate of cooling required. The wing model may be approximated by a flat plant of 0.3 length in the flow direction.

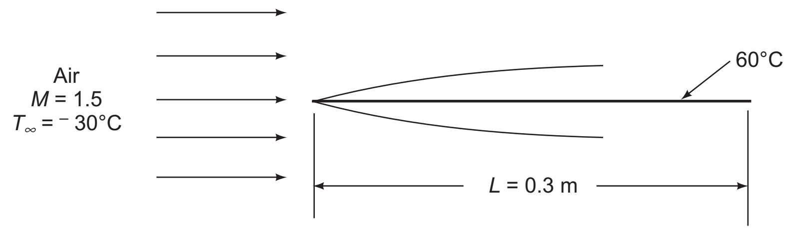
GIVEN

- High speed air flow over an airplane wing section
- Mach number (M_∞) = 1.5
- Air pressure (P) = 20,000 N/m^2
- Air temperature = –30°C = 243 K
- Average wing surface temperature = 60°C = 333 K
- Wing may be approximated as a flat of length (L) = 0.3 m

FIND

- The cooling rate (q/A) required

SKETCH



PROPERTIES AND CONSTANTS

Extrapolating from Appendix 2, Table 27, for dry air at the ambient temperature,

The Prandtl number (Pr) = 0.71.

From Section 4.13, the specific heat ratio (γ) = 1.4

The gas constant for air (R_a) = 287 J/(kg K)

SOLUTION

The stagnation temperature is given by Equation (4.92)

$$T_o = T_\infty \left[1+\frac{\gamma-1}{2}M^2\right] = 243\text{ K}\left[1+\frac{1.4-1}{2}(1.5)^2\right] = 352\text{ K} = 179°\text{C}$$

The air speed (U_∞) is calculated from

$$U_\infty = M_\infty a_\infty = M_\infty \sqrt{\gamma R_a T_\infty} = 1.5\sqrt{1.4(287\,\text{J/(kg K)})(\text{N m/J})(\text{kg m/(N s}^2))243\,\text{K}}$$

$$= 469\text{ m/s}$$

The adiabatic surface temperature, from Equation (4.93) is

$$T_{as} = T_\infty + r\,(T_o - T_\infty)$$

Assuming the boundary layer is turbulent, $r = Pr^{1/3}$

$$T_{as} = T_\infty + Pr^{\frac{1}{3}}\,(T_o - T_\infty) = 243\text{ K} + (0.71)^{\frac{1}{3}}\,(352\text{ K} - 243\text{ K}) = 340\text{ K}$$

The reference temperature is given by Equation (4.97)

$$T^\bullet = T_\infty + 0.5\,(T_s - T_\infty) + 0.22\,(T_{as} - \text{T}_\infty)$$

$$T^\bullet = 243\text{ K} + 0.5\,(333\text{ K} - 243\text{ K}) + 0.22\,(340\text{ K} - 243\text{ K}) = 309\text{ K}$$

From Appendix 2, Table 27, for dry air at 309 K

Absolute viscosity ($\mu^\bullet$) = 18.9×10^{-6} Ns/m^2

Prandtl number ($Pr^\bullet$) = 0.71

Specific heat ($C_p^\bullet$) = 1014 J/(kg K)

The density of the air can be calculated from the ideal gas law

$$\rho^\bullet = \frac{P}{R_a T^\bullet}$$ Where: R_a = The gas constant for air 287 J/(kg K)

$$\rho^{\bullet} = \frac{20{,}000\,\text{N/m}^2}{287\,\text{J/(kg K)}\,(\text{N m/J})\,(309\,\text{K})} = 0.226\ \text{kg/m}^3$$

$$\text{At } L = 0.3\ \text{m } Re_L^{\bullet} = \frac{U_\infty\, L\rho^{\bullet}}{\mu^{\bullet}} = \frac{(469\,\text{m/s})(0.3\,\text{m})\left(0.226\,\text{kg/m}^3\right)}{\left(18.9\times10^{-6}\,\text{N s/m}^2\right)\left(\text{kg m/(Ns}^2)\right)} = 1.68\times10^6$$

Therefore, the assumption that the boundary layer is turbulent is valid.

The average heat transfer coefficient over the wing can be calculated by averaging equation (4.100), assuming constant thermal properties:

$$h_c = \frac{1}{L}\int_0^L 0.0288\, c_p\, \rho^{\bullet}\, U_\infty\, (Re_x^{\bullet})^{-0.2} (Pr^{\bullet})^{-\frac{2}{3}}\, dx = 0.036\, c_p\, \rho^{\bullet}\, U_\infty\, (Re_L^{\bullet})^{-0.2} (Pr^{\bullet})^{-\frac{2}{3}}$$

$$h_c = 0.036\ (1014\,\text{J/kg})\ \left(0.226\,\text{kg/m}^2\right)\ (469\,\text{m/s})\ (\text{N m/J})\ (1.68\times10^6)^{-0.2}\,(0.71)^{-\frac{2}{3}}$$

$$= 277\ \text{W/(m}^2\ \text{K)}$$

The rate of heat transfer from the wing is given by Equation (4.98)

$$\frac{q_c}{A} = h_c\,(T_s - T_{as}) = 277\ \text{W/(m}^2\ \text{K)}\ (333\ \text{K} - 340\ \text{K}) = -1940\ \text{W/m}^2$$

The negative sign indicates that heat is being transferred from the air to the wing. Therefore, 1940 W/m^2 must be removed to maintain the wing at 60°C.

Chapter 5

PROBLEM 5.1

Show that the coefficient of thermal expansion for an ideal gas is 1/*T*, where *T* is the absolute temperature.

GIVEN

- Ideal gas
- Absolute temperature = T

FIND

- Show that the thermal expansion coefficient (β) = $1/T$

SOLUTION

The volumetric thermal expansion coefficient is defined as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

For an ideal gas

$$pV = mRT \Rightarrow \frac{m}{V} = \rho = \frac{p}{RT}$$

where
p = pressure
V = volume
m = mass
R = gas constant

For a constant pressure

$$\frac{\partial \rho}{\partial T} = -\frac{p}{RT^2}$$

$$\therefore \beta = -\frac{1}{\left(\frac{p}{RT}\right)}\left(-\frac{p}{RT^2}\right) = \frac{1}{T}$$

PROBLEM 5.2

Calculate the coefficient of thermal expansion, β, for saturated water at 403 K from its definition and property values in Appendix 2, Table 13. Then compare your results with the value in the table.

GIVEN

- Saturated water at 400 K

FIND

- The thermal expansion coefficient, β, from its definition

SOLUTION

The coefficient of thermal expansion is defined as the change of density with temperature at constant pressure

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

It can be approximated as

$$\beta \cong -\frac{1}{\left(\frac{\rho_1 + \rho_2}{2}\right)}\left(\frac{\rho_1 - \rho_2}{T_1 - T_2}\right)$$

For comparison with Appendix 2, Table 13, let T_1 = 393 K, T_2 = 413 K: From the table, ρ_1 = 943.5 Kg/m^3, ρ_2 = 926.3 Kg/m^3.

$$\beta \cong -\frac{1}{\left(\frac{943.5+926.3}{2}\right)\text{kg/m}^3}\left[\frac{(943.5+926.3)\,\text{kg/m}^3}{(393\,\text{K}-413\,\text{K})}\right] = 9.19 \times 10^{-4}\ \text{K}^{-1}$$

The table lists the thermal expansion coefficient at the average of these temperatures (403 K) to be 9.1 $\times 10^{-4}$ 1/K— a difference of about 1%.

PROBLEM 5.3

Calculate the coefficient of thermal expansion, β, from its definition for steam at 450°C and pressures of 0.1 atm and 10 atm from standard steam tables. Then compare your results with the value obtained by assuming that steam is a perfect gas and explain the difference.

GIVEN

- Steam
- Temperature = 450°C = 723 K

FIND

The coefficient of thermal expansion at 0.1 atm and 10 atm from

(a) Standard Steam tables
(b) Perfect Gas Law

PROPERTIES AND CONSTANTS

From steam tables

Temperature (K)	Pressure (Atm)	Density (kg/m^3)
673	10	3.262
773	10	2.824
673	0.1	0.03219
773	0.1	0.02803

SOLUTION

(a) The thermal expansion coefficient is given by

$$\beta = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \equiv -\frac{1}{\left(\frac{\rho_1+\rho_2}{2}\right)}\left(\frac{\rho_1-\rho_2}{T_1-T_2}\right)$$

At 10 Atm.: $\beta \cong -\dfrac{1}{\left(\dfrac{3.262+2.824}{2}\right)\text{kg/m}^3}\left[\dfrac{(3.262-2.824)\ \text{kg/m}^3}{(673\,\text{K}-773\,\text{K})}\right] = 1.439\times10^{-3}\ \text{K}^{-1}$

At 0.1 Atm

$$\beta \cong -\frac{1}{\left(\frac{0.03219+0.02803}{2}\right)\text{kg/m}^3}\left[\frac{(0.03219-0.02803)\ \text{kg/m}^3}{(673\,\text{K}-773\,\text{K})}\right] = 1.381\times10^{-3}\ \text{K}^{-1}$$

(b) For an ideal gas

$$\rho_1 T_1 = \rho_2 T_2 \Rightarrow \rho_1 = \rho_2\frac{T_2}{T_1}$$

$$\therefore\quad \beta \cong -\frac{1}{\frac{\rho_2}{2}\left(\frac{T_2}{T_1}+1\right)}\ \frac{\rho_2\left(\frac{T_2}{T_1}-1\right)}{T_1-T_2} = \frac{1}{\left(\frac{T_2+T_1}{2}\right)} = \frac{1}{T_{ave}}$$

At T_{ave} = 723 K

$$\beta \cong \frac{1}{723\,\text{K}} = 1.383\times10^{-3}\ \frac{1}{\text{K}}\ \text{(independent of pressure)}$$

COMMENTS

The result of part (b) correlates better with ρ calculated in part (a) at 0.1 atm than at 10 atm because steam more closely resembles an ideal gas at 0.1 atm than at 10 atm.

PROBLEM 5.4

A long cylinder of 0.1 m diameter has a surface temperature of 400 K. If it is immersed in a fluid at 350 K, natural convection will occur as a result of the temperature difference. Calculate the Grashof and Rayleigh numbers that will determine the Nusselt number if the fluid is

(a) Nitrogen (b) Air (c) Water

(d) Oil (e) Mercury

GIVEN

- A long cylinder immersed in fluid
- Diameter (D) = 0.1 m
- Surface temperature (T_s) = 400 K
- Fluid temperature (T_∞) = 350 K

FIND

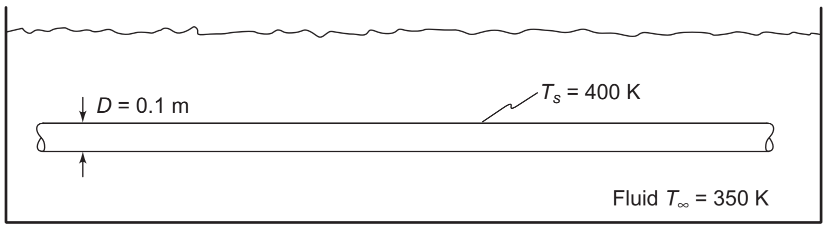
The Grashof number (*Gr*) and the Rayleigh number (*Ra*) if the fluid is

(a) Nitrogen (b) Air (c) Water
(d) Oil (e) Mercury

ASSUMPTIONS

- The cylinder is in a horizontal position (the characteristic length is the diameter of the cylinder)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 32, 27, 13, 16 and *25*, at the mean temperature of 375 K

Fluid	Nitrogen	Air	Water	Oil	Mercury
Table number	32	27	13	16	25
Thermal expansion coefficient, β (1/K)	0.00271	0.00268	0.00075	—	—
Kinematic viscosity, × 106 m^2/s	23.21	23.67	0.294	20.3	0.0928
Prandtl number (*Pr*)	0.697	0.71	1.75	2.76	0.0162

Fluid	Oil		Mercury	
Temperature (K)	353	393	323	423
Density, ρ (kg/m^3)	852.0	829.0	13,506	13.264

The thermal expansion coefficients for oil and mercury can be estimated from

$$\beta \cong -\frac{2}{(\rho_1 + \rho_2)}\left(\frac{\rho_1 - \rho_2}{T_1 - T_2}\right)$$

For oil at 373 K $\beta \approx 0.00068$ 1/K
For mercury at 373 K $\beta \approx 0.00018$ 1/K

SOLUTION

The Grashof number based on the cylinder diameter is

$$Gr_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^3}$$

For nitrogen

$$Gr_D = \frac{(9.8\ \text{m/s}^2)\left(0.00271\ \text{K}^{-1}\right)(400\ \text{K} - 350\ \text{K})(0.1\ \text{m})^3}{\left(23.21\times 10^{-6}\ \text{m}^2/\text{s}\right)^2} = 2.46 \times 10^6$$

The Rayleigh number is defined as

$$Ra_D = Gr_D\, Pr = 2.46 \times 10^6\ (0.697) = 1.72 \times 10^6 \text{ (for Nitrogen)}$$

Calculating the Gr_D and Ra_D in a similar manner for the other fluids

Fluid	Gr_D	Ra_D
Nitrogen	2.46×10^6	1.72×10^6
Air	2.34×10^6	1.66×10^6
Water	4.25×10^9	7.44×10^9
Oil	8.08×10^5	2.23×10^6
Mercury	1.02×10^{10}	1.66×10^8

PROBLEM 5.5

For the conditions given in Problem 5.4, determine the Nusselt Number and the heat transfer coefficient from Fig. 5.3.

From Problem 5.4: A long cylinder of 0.1 m diameter has a surface temperature of 400 K. If it is immersed in a fluid at 350 K, natural convection will occur as a result of the temperature difference. Calculate the Grashof and Rayleigh numbers that will determine the Nusselt number if the fluid is

(a) Nitrogen **(b) Air** **(c) Water**

(d) Oil **(d) Mercury**

GIVEN

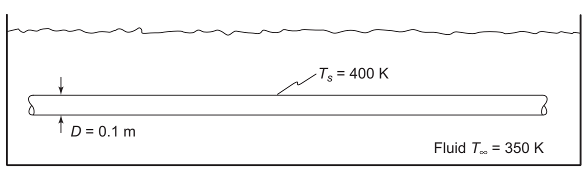
- A long cylinder immersed in a fluid
- Diameter (D) = 0.1 m
- Surface temperature (T_s) = 400 K
- Fluid temperature (T_∞) = 350 K

Fluid	Gr_D	Ra_D
Nitrogen	2.46×10^6	1.72×10^6
Air	2.34×10^6	1.66×10^6
Water	4.25×10^9	7.44×10^9
Oil	8.08×10^5	2.23×10^6
Mercury	1.02×10^{10}	1.66×10^8

FIND

- The Nusselt number (Nu) and heat transfer coefficient (h_c) for each fluid from Figure 5.5

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 32, 27, 13, 16 and 25, at the mean temperature of 375 K

Fluid	Thermal Conductivity, k (W/(m K))
Nitrogen	0.0316
Air	0.0307
Water	0.682
Oil	0.137
Mercury	10.51

SOLUTION

From Problem 5.4, the Rayleigh number for nitrogen is 1.72×10^6. The abscissa of Figure 5.3 is the base 10 log of the Rayleigh number

$$\text{Log}\,(1.72 \times 10^6) = 6.24$$

From Figure 5.3: $\log\left(\overline{Nu}_D\right) \approx 1.25 \rightarrow \overline{Nu}_D = 17.78$

$$\therefore\ \bar{h}_c = \frac{k}{D}\,\overline{Nu}_D = \frac{\left(0.03156\ \text{W/(m K)}\right)}{0.1\,\text{m}}\ 17.78 = 5.61\ \text{W/(m}^2\text{K)}$$

Following a similar procedure for the other fluids yields the following results

Fluid	log (Ra_D)	log ($\overline{Nu}_D$)	($\overline{Nu}_D$)	$\bar{h}_c$ (W/(m^2 K))
Nitrogen	6.24	1.25	17.78	5.61
Air	6.22	1.25	17.78	5.46
Water (extrapolating)	9.87	2.2	158	1081
Oil	6.34	1.28	19.1	26.1
Mercury	8.22	1.75	56.2	5910

PROBLEM 5.6

An empirical equation proposed for the heat transfer coefficient in natural convection from long vertical cylinders to air at atmospheric pressure is

$$\bar{h}_c = \frac{536.5\,(T_s - T_\infty)^{0.33}}{T}$$

Where T = the film temperature = 1/2 ($T_s + T_\infty$) and T is in the range 0 to 200°C

The corresponding equation in dimensionless form is

$$\frac{(\bar{h}_c\ L)}{k} = C(GrPr)^m$$

By comparing the two equations, determine those values of C, m and n in the second equation that will give the same results as the first equation.

GIVEN

- Empirical equations shown above

FIND

- Values of C, m and n

ASSUMPTIONS

- Air behaves as an ideal gas

SOLUTION

$$\frac{\bar{h}_c\ L}{k} = C(Gr_L\ Pr)^m$$

But $$Gr_L = \frac{g\,\beta\,(T_s - T_\infty)\,L^3}{\nu^2} \quad \text{and} \quad Pr = \frac{c_p\,\mu}{k}$$

$$\therefore\ \frac{\bar{h}_c\,L}{k} = C\left[\frac{g\,\beta\,(T_s - T_\infty)\,L^3\,c_p\,\mu}{\nu^2 k}\right]^m$$

But $$\nu = \frac{\mu}{\rho} \text{ and for an ideal gas: } \beta \cong \frac{1}{T} \text{ and } \rho = \frac{P}{RT}$$

$$\therefore\ \bar{h}_c = C\,\frac{k}{L}\left[\frac{g\,P^2 (T_s - T_\infty)\,L^3\,c_p}{T^3\,\mu\,R^2\,k}\right]^m$$

Equating this to the empirical equation

$$\bar{h}_c = (Ck^{1-m}\,g^m\,p^{2m}\,\mu^{-m}\,R^{-2m}\,c_p^{\,m})\,(T_s - T_\infty)^m\,L^{3m-1}\,T^{-3m} = 536.5\,(T_s - T_\infty)^{0.33}\,T^{-1}$$

The exponents of the variables must be the same, so

For $(T_s - T_\infty)$ $\quad m = 0.33$

For L $\quad 3m - 1 = 0$

For T $\quad -3m = -1$

The value of the constant C is determined by

$$C\ k^{\frac{2}{3}}\,g^{\frac{1}{3}}\,p^{\frac{2}{3}}\,\mu^{-\frac{1}{3}}\,R^{-\frac{2}{3}}\,c_p^{\frac{1}{3}} = 536.5$$

From Appendix 2, Table 27 for dry air at 100°C and one atmosphere

$k = 0.0307$ W/(m K)

$\mu = 21.673 \times 10^{-6}$ N s/m^2

$c_p = 1022$ J/(kg K)

The gas constant for air (R) = 287 J/(kg K)

The absolute pressure of one atmosphere (P) = 101,000 N/m^2.

$$\therefore\ C\left(0.0307\text{W/(m K)}\right)^{\frac{2}{3}}\left(9.8\text{ m/s}^2\right)^{\frac{1}{3}}\left(101{,}000\text{ N/m}^2\right)^{\frac{2}{3}}$$

$$\left(21.673\times10^{-6}\text{ (Ns)/m}^2\right)^{-\frac{1}{3}}\left(287\text{ J/(kg K)}\right)^{-\frac{2}{3}}\left(1022\text{ J/(kg K)}\right)^{\frac{1}{3}} = 536.5$$

$$C = 0.142$$

The non-dimensional empirical equation is

$$Nu_L = 0.142\ (Gr_L\,Pr)^{\frac{1}{2}}$$

COMMENTS

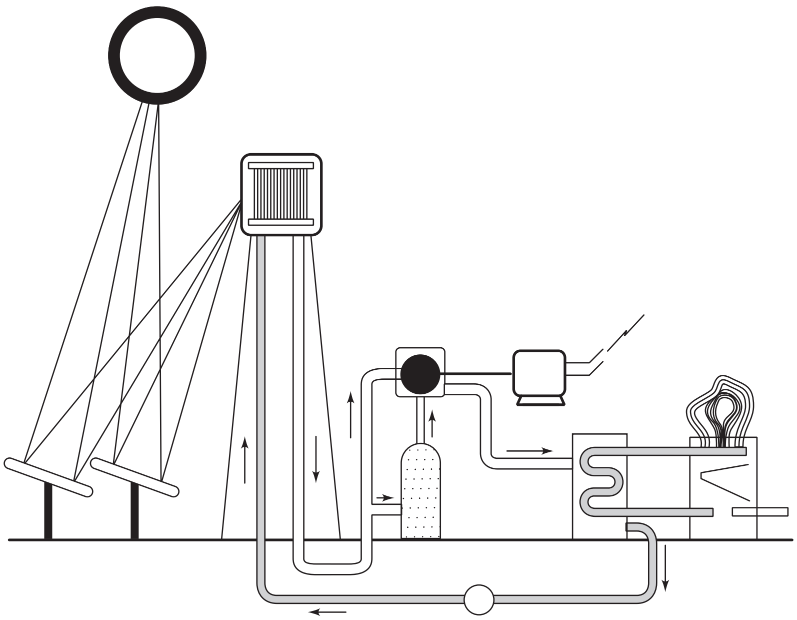
The units of the constant in the empirical equation must be W/(m^2 K$^{1/3}$).

The non-dimensional empirical equation closely resembles that given by Equation (5.13) for turbulent natural convection from vertical cylinders.

PROBLEM 5.7

'Solar One' is the first large-scale (10 MW electric) solar-thermal electric-power-generating plant in the U.S. It is located near Barstow, CA. A schematic diagram of the receiver and tower is shown below (the heliostat, i.e., mirror field, is not shown). The

receiver may be treated as a cylinder 7 m in diameter and 13.5 m tall. At the design operating conditions, the average outer surface temperature of the receiver is about 675°C and ambient air temperature is about 40°C. Estimate the rate of heat loss, in MW, from the receiver—via natural convection only—for the temperatures given. What are other mechanisms by which heat may be lost from the receiver?



GIVEN

- A vertical cylinder in air
- Height of cylinder (L) = 13.5 m
- Diameter (D) = 7 m
- Surface temperature (T_s) = 675°C
- Ambient air temperature (T_∞) = 40°C

FIND

(a) The rate of convective heat loss (q_c) in MW
(b) What other mechanisms for heat loss exist?

ASSUMPTIONS

- Air is still
- Surface temperature is uniform and constant

PROPERTIES AND CONSTANTS

From Appendix 2, Tables 27, for dry air at the mean temperature of 357.5°C

Thermal expansion coefficient (β) = 0.00160 1/K

Thermal conductivity (k) = 0.0461 W/(m K)

Kinematic viscosity (ν) = 58.1×10^{-6} m^2/s

Prandtl number (Pr) = 0.72

SOLUTION

The Grashof number is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)\,L^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.0016\,1/\text{K})(675°\text{C} - 40°\text{C})(13.5\,\text{m})^3}{\left(58.1\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 7.26\times10^{12}$$

Therefore, the flow is turbulent.

Equation (5.13) gives the Nusselt number for a turbulent boundary layer

$$Nu_L = 0.13\ (Gr_L\,Pr)^{\frac{1}{3}} = 0.13\ [7.26\times10^{12}(0.72)]^{\frac{1}{3}} = 2256$$

$$\therefore\ \bar{h}_c = \frac{k}{L}\ Nu_L = \frac{\left(0.0461\text{ W/(m K)}\right)}{13.5\,\text{m}}\ 2256 = 7.70\ \text{ W/(m}^2\text{ K)}$$

(a) The rate of convective heat transfer is

$$q_c = \bar{h}_c\,\pi D\,L\,(T_s - T_\infty) = \left(7.7\text{ W/(m}^2\text{ K)}\right)\pi(7\text{ m})\,(13.5\text{ m})\,(675°\text{C} - 40°\text{C})\left(10^{-6}(\text{MW})/\text{W}\right) = 1.45\text{ MW}$$

(b) Other mechanisms for heat transfer from the surface are

1. Radiation to the surroundings.
2. Conduction to the interior of the cylinder where the heat can be removed by a working fluid.
3. Conduction to the support structure.
4. Forced convection to the ambient air when breezes occur.

PROBLEM 5.8

Compare the rate of heat loss from a human body with the typical energy intake from consumption of food (1033 kcal/day). Model the body as a vertical cylinder 30 cm in diameter and 1.8 m high in still air. Assume the skin temperature is 2°C below normal body temperature. Neglect radiation, transpiration cooling (sweating), and the effects of clothing.

GIVEN

- Human body idealized as a vertical cylinder in still air
- Diameter (D) = 30 cm = 0.3 m
- Height (L) = 1.8 m
- Skin temperature (T_s) = 2°C below normal body temperature (37°C) = 35°C

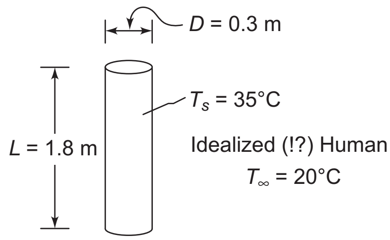
FIND

- Heat loss (q) and compare to consumption of food 1300 kcal/day

ASSUMPTIONS

- Steady state
- Radiation, transpiration cooling, and clothing effects are negligible
- Ambient air temperature (T_∞) = 20°C
- Heat loss from the top of the cylinder is small compared to that from the sides

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27*,* for dry air at the mean temperature of 27.5°C

Thermal expansion coefficient (β) = 0.00333 1/°C

Thermal conductivity (k) = 0.0257 W/(m K)

Kinematic viscosity (ν) = 16.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Grashof number based on the height is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00333\ 1/\text{K})(35°\text{C} - 20°\text{C})(1.8\,\text{m})^3}{\left(16.4\times 10^{-6}\ \text{m}^2/\text{s}\right)^2} = 1.06 \times 10^{10} > 10^9$$

Therefore, the flow is turbulent.

For turbulent boundary layer, the average heat transfer coefficient is given by Equation (5.13)

$$h_c = 0.13\ \frac{k}{L}\ (Gr_L\,Pr)^{\frac{1}{3}} = 0.13\,\frac{\left(0.0256\ \text{W/(m K)}\right)}{1.8\,\text{m}}[1.06\times 10^{10}(0.71)]^{\frac{1}{3}} = 3.62\ \text{W/(m}^2\ \text{K)}$$

The rate of convective heat loss from the sides of the cylinder is

$$q_c = h_c\,\pi D\,L\,(T_s - T_\infty) = \left(3.62\ \text{W/(m}^2\ \text{K)}\right)\pi\ (0.3\ \text{m})\ (1.8\ \text{m})\ (35°\text{C} - 20°\text{C}) = 92.2\ \text{W}$$

$$\text{Food consumption} = 1033\,\text{(kcal)/day}\ \left(1000\ \text{cal/(kcal)}\right)\ (4.1868\ \text{J/cal})\left(\frac{1\ \text{day}}{24\ \text{h}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)(\text{Ws/J}) = 50.1\text{W}$$

COMMENTS

The heat loss calculated for the idealized human is about 46% greater than the average food consumption. This point out the importance of clothing.

PROBLEM 5.9

An electric room heater has been designed in the shape of a vertical cylinder 2 m tall and 30 cm in diameter. For safety, the heater surface cannot exceed 35°C. If the room air is 20°C, find the power rating of the heater in watts.

GIVEN

- An electric heater in the shape of a vertical cylinder
- Heater height (H) = 2 m
- Heater diameter (D) = 30 cm = 0.3 m
- Room air temperature (T_∞) = 20°C = 293 K

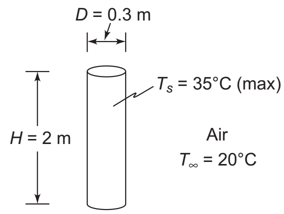
FIND

- The power rating of the heater (q) in watts

ASSUMPTIONS

- Radiation is negligible
- Heat transfer from the top and bottom of the tank is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 27.5°C

Thermal expansion coefficient (β) = 0.00332 1/K
Thermal conductivity (k) = 0.0256 W/(m K)
Kinematic viscosity (ν) = 16.4×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

When the heater surface temperature is 35°C, the Grashof number for the heater sides is

$$Gr_H = \frac{g\,\beta (T_s - T_\infty) H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00332\ 1/\text{K})(35°\text{C} - 20°\text{C})(2\ \text{m})^2}{\left(16.4\times 10^{-6}\ \text{m}^2/\text{s}\right)^2} = 1.45 \times 10^{10} > 10^9$$

Therefore, the flow is turbulent.

The average heat transfer coefficient is given by Equation (5.13)

$$h_c = 0.13\ \frac{k}{L}(Gr_L\,Pr)^{\frac{1}{3}} = 0.13\ \frac{\left(0.0256\ \text{W/(m K)}\right)}{2\,\text{m}}\ [1.45\times 10^{10}(0.71)]^{\frac{1}{3}} = 3.62\ \ \text{W/(m}^2\ \text{K)}$$

The power rating of the heater must equal the rate of heat transfer from the heater

$$q = h_c\,\pi D\,L\,(T_s - T_\infty) = \left(3.62\ \text{W/(m}^2\ \text{K)}\right)\ \pi(0.3\ \text{m})\,(2\ \text{m})\,(35°\text{C} - 20°\text{C}) = 102\ \text{W}$$

COMMENTS

This heater would probably not suffice for most applications. Either the surface temperature needs to be raised, of the size of the heater needs to be increased.

PROBLEM 5.10

Consider a design for a nuclear reactor using natural-convection heating of liquid bismuth. The reactor is to be constructed of parallel vertical plates 1.8 m tall and 1.2 m wide, in which heat is generated uniformly. Estimate the maximum possible heat dissipation rate from each plate if the average surface temperature of the plate is not to exceed 870°C and the lowest allowable bismuth temperature is 315°C.

GIVEN

- Vertical plates with uniform heat generation in bismuth
- Plate height (L) = 1.8 m

- Plate width (w) = 1.2 m
- Maximum average surface temperature (T_s) = 870°C
- Minimum bismuth temperature (T_∞) = 315°C

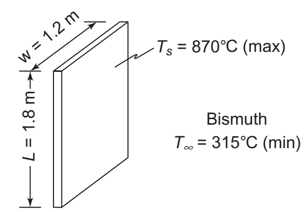
FIND

- Maximum possible heat dissipation rate (q) from each plate

ASSUMPTIONS

- Steady state
- Free convection only
- Edge effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 24, for bismuth at the mean temperature of 593°C (mean temperature)

Thermal conductivity (k) = 15.6 W/(m K)

Kinematic viscosity (ν) = 3.5×10^{-4} m^2/s

Prandtl number (Pr) = 0.010

Also Density at 540°C = 3020 kg/m^3

Density at 650°C = 2980 kg/m^3

To find the thermal expansion coefficient (β)

$$\beta = \frac{2}{(\rho_{540} + \rho_{650})}\left(\frac{\rho_{540} - \rho_{650}}{110}\right) = 1.21 \times 10^{-5} \text{ 1/K}$$

SOLUTION

The Grashof number based on the vertical length of the plate is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)\,L^3}{\nu^2} = \frac{(9.81\text{ m/s}^2)(1.21\times10^{-5}\text{1/K})(870-315)\text{K}\,(1.8\text{ m})^3}{(3.5\times10^{-4}\text{ m}^2/\text{s})^2} = 3.14 \times 10^6$$

The average Nusselt number for a vertical plate in liquid metal for $Gr < 10^9$ is given by Equation (5.12c)

$$\overline{Nu}_L = \frac{\bar{h}_c L}{k} = 0.68\ (Gr_L\,Pr^2)^{\frac{1}{4}}$$

Solving for the heat transfer coefficient

$$\bar{h}_c = 0.68\ \frac{k}{L}(Gr_L Pr^2)^{\frac{1}{4}}$$

$$\bar{h}_c = 0.68\ \frac{15.6\text{ W/(m K)}}{1.8\text{ m}}\ [3.14\times10^6 \times (0.01)^2]^{\frac{1}{4}} = 24.8\text{ W/(m}^2\text{ K)}$$

The maximum rate of heat transfer from both sides of the plate is given by Equation (1.10)

$$q = \bar{h}_c \; A \, \Delta T = 24.8 \, [2 \, (1.8 \text{ m}) \, (1.2 \text{ m})] \, [870 - 315]\text{K}$$
$$= 5.95 \times 10^4 \text{ W}$$
$$= 59.5 \text{ kW}$$

PROBLEM 5.11

A mercury bath at 60°C is to be heated by immersing cylindrical electric heating rods, each 20 cm tall and 2 cm in diameter. Calculate the maximum electric power rating of a typical rod if its maximum surface temperature is 140°C.

GIVEN

- Cylindrical heating rods in a mercury bath
- Mercury temperature (T_∞) = 60°C
- Rod diameter (D) = 2 cm = 0.02 m
- Rod height (L) = 20 cm = 0.2 m
- Maximum surface temperature (T_s) = 140°C

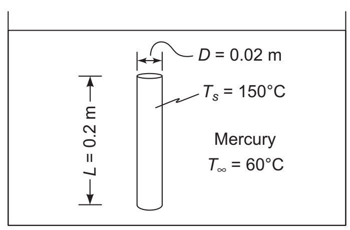
FIND

- The maximum electric power rating $(\dot{q}_e)$ of a rod

ASSUMPTIONS

- Steady state
- The rods are in a vertical position

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for mercury at the mean temperature of 100°C

Thermal conductivity (k) = 10.51 W/(m K)

Kinematic viscosity (ν) = 0.093×10^{-6} m^2/s

Prandtl number (Pr) = 0.0162

Also Density at 50°C (ρ_{50}) = 13,506 kg/m^3

Density at 15°C (ρ_{150}) = 13,264 kg/m^3

To find the thermal expansion coefficient (β)

$$\beta = \left(\frac{2}{\rho_{50} + \rho_{150}}\right)\left(\frac{\rho_{50} - \rho_{150}}{100°\text{C}}\right) = 1.81 \times 10^{-4} \text{ 1/K}$$

SOLUTION

The Grashof number at the top of the cylinder is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)\,L^3}{\nu^2} = \frac{(9.8 \text{ m/s}^2)\left(1.81 \times 10^{-4} \text{ 1/K}\right)(140°\text{C} - 60°\text{C})(0.2 \text{ m})^3}{\left(0.093 \times 10^{-6} \text{m}^2\text{/s}\right)^2} = 1.31 \times 10^{11} > 10^9$$

Therefore, the boundary layer is turbulent and the average heat transfer coefficient is given by Equation (5.13)

$$\bar{h}_c = 0.13\ \frac{k}{L}(Gr_L Pr)^{\frac{1}{3}} = 0.13\ \frac{\left(10.5\ \text{W/(m K)}\right)}{0.2\ \text{m}}[1.31\times10^{11}(0.0162)]^{\frac{1}{3}} = 8776\ \text{W/(m}^2\ \text{K)}$$

The maximum electric power rating of a rod is equal to the maximum rate of heat transfer from a rod

$$\dot{q}_c = q_c = \bar{h}_c\ \pi D\ L\ (T_s - T_\infty) = \left(8776\ \text{W/(m}^2\ \text{K)}\right)\pi(0.02\ \text{m})(0.2\ \text{m})\ (140°\text{C} - 60°\text{C}) = 8823\ \text{W}$$

PROBLEM 5.12

> **An electric heating blanket is subjected to an acceptance test. It is to dissipate 400 W on the high setting when hanging in air at 20°C. If the blanket is 1.3 m wide: (a) what is the length required if its average temperature at the high setting is to be 40°C, and (b) if the average temperature at the low setting is to be 30°C, what rate of dissipation would be possible?**

GIVEN

- An electric blanket hanging in air
- Heat dissipation rate (q_h) = 400 W
- Air temperature (T_∞) = 20°C
- Blanket width (w) = 1.3 m
- Average temperatures
  - High (T_{sh}) = 40°C
  - Low (T_{s1}) = 30°C

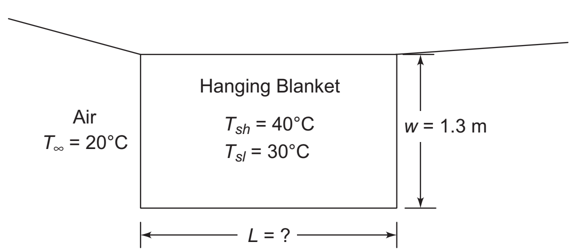
FIND

(a) The length of the blanket (L)
(b) Heat dissipation rate on the low setting (q_1)

ASSUMPTIONS

- Air is still
- Moisture in the air has a negligible effect
- Blanket is hung vertically with its 1.3 m sides vertical

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27 for dry air at the mean temperatures for the two settings

Mean Temperature (°C)	30°C	25°C
Thermal expansion coefficient, β (1/K)	0.00330	0.00336
Thermal conductivity, k (W/(m K))	0.0258	0.0255
Kinematic viscosity, $\nu \times 10^6$ (m²/s)	16.7	16.2
Prandtl number, Pr	0.71	0.71

SOLUTION

(a) On the high setting, the Grashof number for the blanket is

$$Gr_w = \frac{g\,\beta(T_s - T_\infty)\,w^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.0033\,1/\text{K})(40°\text{C} - 20°\text{C})(1.3\,\text{m})^3}{\left(16.7\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 5.09\times10^9$$

Therefore, the boundary layer is turbulent and the natural convection heat transfer coefficient is given by Equation (5.13)

$$\bar{h}_{ch} = 0.13\,\frac{k}{w}(Gr_w Pr)^{\frac{1}{3}} = 0.13\,\frac{\left(0.0258\text{ W/(m K)}\right)}{1.3\,\text{m}}[5.09\times10^9(0.71)]^{\frac{1}{3}} = 3.96\ \text{W/(m}^2\text{ K)}$$

The rate of heat transfer from both sides of the blanket is

$$q_h = \bar{h}_{ch}\ A\ (T_{sh} - T_\infty) = \bar{h}_{ch}\ (2Lw)\ (T_{sh} - T_\infty)$$

Solving for the length of the blanket

$$L = \frac{q_h}{2\,\bar{h}_{ch}\ w(T_{sh} - T_\infty)} = \frac{400\,\text{W}}{2\left(3.96\text{ W/(m}^2\text{ K)}\right)(1.3\,\text{m})(40°\text{C} - 20°\text{C})} = 1.94\text{ m}$$

(b) The Grashof number for the blanket on the low setting is

$$Gr_w = \frac{g\,\beta(T_s - T_\infty)\,w^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.00336\,1/\text{K})(30°\text{C} - 20°\text{C})(1.3\,\text{m})^3}{\left(16.2\times10^{-6}\ \text{m}^2/\text{s}\right)^2} = 2.76\times10^9$$

This is still turbulent, therefore

$$\bar{h}_{cl} = 0.13\,\frac{\left(0.0255\text{ W/(m K)}\right)}{1.3\text{ m}}[2.76\times10^9(0.71)]^{\frac{1}{3}} = 3.19\ \text{W/(m}^2\text{K)}$$

The heat dissipation rate possible is equal to the rate of convection from both sides

$$q_1 = \bar{h}_{cl}\ 2Lw\ (T_{sl} - T_\infty) = \left(3.19\text{ W/(m}^2\text{ K)}\right)\ (2)\ (1.94\text{ m})\ (1.3\text{ m})\ (30°\text{C} - 20°\text{C}) = 161\text{ W}$$

PROBLEM 5.13

An aluminum sheet, 0.4 m tall, 1 m long, and 0.002 m thick is to be cooled from an initial temperature of 150°C to 50°C by immersing it suddenly in water at 20°C. The sheet is suspended from two wires at the upper corners.

(a) Determine the initial and the final rate of heat transfer from the plate.

(b) Estimate the time required.

(Hint: Note that in laminar natural convection, $h \approx \Delta T^{0.25}$)

GIVEN

- A vertical aluminum sheet in water
- Plate dimensions: height (H) = 0.4 m, length (L) = 1 m, thickness (s) = 0.002 m
- Initial plate temperature (T_{si}) = 150°C
- Water temperature (T_∞) = 20°C
- Final plate temperature (T_{sf}) = 50°C

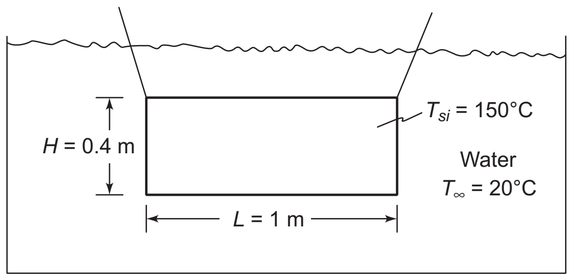
FIND

(a) The initial and final heat transfer rates
(b) The time required

ASSUMPTIONS

- Constant and uniform water temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, for aluminum at the mean temperature of 100°C

Thermal conductivity (k_{al}) = 238 W/(m K)

Density (ρ) = 2702 kg/m^3

Specific heat (c) = 896 J/(kg J)

From Appendix 2, Table 13, for water at the mean temperatures:

Mean Temperature (°C)	85°C	35°C
Thermal expansion coefficients, β (1/k)	0.00066	0.00034
Thermal conductivity, k (W/(m K))	0.675	0.624
Kinematic viscosity, $\nu \times 10^6$ (m^2/s)	0.337	0.725
Prandtl number, Pr	2.04	4.8

SOLUTION

(a) The Grashof number based on the height of the plate is

Initial

$$Gr_H = \frac{g\,\beta\,(T_s - T_\infty)\,H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00066\ 1/\text{K})(150°\text{C} - 20°\text{C})(0.4\,\text{m})^3}{\left(0.337\times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 4.74\times 10^{11}\ \text{(Turbulent)}$$

Final

$$Gr_H = \frac{(9.8\ \text{m/s}^2)(0.00034\,1/\text{K})(50°\text{C} - 20°\text{C})(0.4\,\overline{\text{m}})^3}{\left(0.725\times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.22\times 10^{10}$$

(Turbulent)

The average heat transfer coefficient from a vertical plate with a turbulent boundary layer is given by Equation (5.13)

$$\bar{h}_c = 0.13\ \frac{K}{H}(Gr_L Pr)^{\frac{1}{3}}$$

Initial

$$\bar{h}_{ci} = 0.13\ \frac{\left(0.675\ \text{W/(m K)}\right)}{0.4\,\text{m}}[4.74\times 10^{11}(2.04)]^{\frac{1}{3}} = 2169\ \ \text{W/(m}^2\text{K)}$$

Final

$$\bar{h}_{cf} = 0.13\ \frac{\left(0.624\ \text{W/(m K)}\right)}{0.4\,\text{m}}[1.22\times10^{10}(4.8)]^{\frac{1}{3}} = 787\ \ \text{W/(m}^2\text{K)}$$

The rate of convective heat transfer from the plate is given by Equation (1.10)

$$q_c = \bar{h}_c\ A\,\Delta\,T$$

Initial

$$q_{ci} = \left(2169\ \text{W/(m}^2\text{K)}\right)\ [2\ (0.4\ \text{m})\ (1\ \text{m})]\ (150°\text{C} - 20°\text{C}) = 2.26\times10^5\ \text{W}$$

Final

$$q_{cf} = \left(787\ \text{W/(m}^2\text{K)}\right)\ [2\ (0.4\ \text{m})\ (1\ \text{m})]\ (50°\text{C} - 20°\text{C}) = 1.89\times10^4\ \text{W}$$

(b) The initial Biot number for the aluminum sheet is

$$Bi = \frac{\bar{h}_{ci}\,S}{2\,K_{al}} = \frac{\left(2169\ \text{W/(m}^2\text{K)}\right)(0.002\ \text{m})}{2\left(238\text{W/(m K)}\right)} = 0.009 << 0.1$$

Therefore, the internal thermal resistance of the aluminum sheet is negligible during the entire cool down and the temperature-time history of the sheet is given by Equation (2.84)

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{-\bar{h}_c\ A_s}{c\,\rho V}\,t\right)$$

Solving for the time

$$t = \frac{c\,\rho V}{\bar{h}_c\ A_s}\ \ln\left(\frac{T_o - T_\infty}{T - T_\infty}\right) \cong \frac{c\,\rho s}{2\,\bar{h}_c}\ \ln\left(\frac{T_o - T_\infty}{T - T_\infty}\right)$$

Using the average of the initial and final heat transfer coefficients

$$t = \frac{\left(896\ \text{J/(kg K)}\right)\left(2702\ \text{kg/m}^2\right)(0.002\,\text{m})}{2\left(1478\ \text{W/(m}^2\text{K)}\right)\left(\text{J/(W s)}\right)}\ \ln\left(\frac{150°\text{C} - 20°\text{C}}{50°\text{C} - 20°\text{C}}\right) = 2.4\ \text{s}$$

PROBLEM 5.14

A 0.1 cm thick flat copper plate, 2.5 m × 2.5 m square is to be cooled in a vertical position. The initial temperature of the plate is 90°C with the ambient fluid at 30°C. The fluid medium is either atmospheric air or water.

(a) Calculate the Grashof numbers
(b) Determine the initial heat transfer coefficient
(c) Calculate the initial rate of heat transfer by convection
(d) Estimate the initial rate of temperature change for the plate

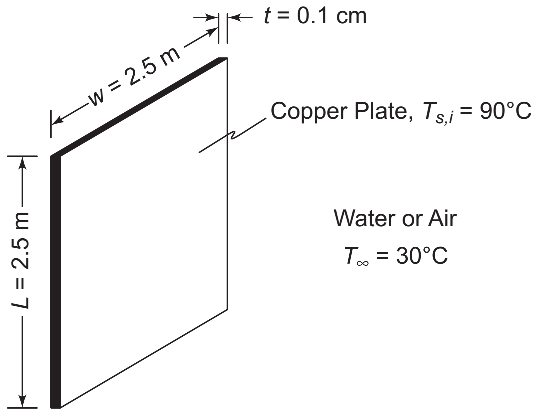
GIVEN

- A vertical flat copper plate in either air or water
- Plate thickness (t) = 0.1 cm = 0.001 m
- Plate dimensions ($L \times w$) = 2.5 m × 2.5 m
- Initial plate temperature ($T_{s,i}$) = 90°C
- Ambient fluid temperature (T_∞) = 30°C

FIND

(a) The Grashof number (Gr)
(b) The rate of heat transfer by convection (q_c)
(c) The initial rate of temperature change $(dT/dt)_{t=0}$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, for copper

Density (ρ_c) = 8933 kg/m^3

Specific heat (c_c) = 383 J/(kg K)

From Appendix 2, Tables 13 and 27

	Water at 60°C	Air at 60°C
Thermal conductivity, k (W/(m K))	0.657	0.0279
Thermal expansion coefficient, β (1/K)	0.00052	0.003
Kinematic Viscosity $\nu \times 10^6$ m^2/s	0.480	19.4
Prandtl number, Pr	3.02	0.71

SOLUTION

(a) The Grashof number is defined as

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2}$$

For water

$$Gr_L = \frac{(9.8\text{ m/s}^2)(0.00052\text{ 1/K})(90\text{°C} - 30\text{°C})(2.5\text{ m})^3}{\left(4.480\times10^{-6}\text{m}^2/\text{s}\right)^2} = 2.07\times10^{13}$$

For air

$$Gr_L = \frac{(9.8\text{ m/s}^2)(0.003\text{ 1/K})(90\text{°C} - 30\text{°C})(2.5\text{ m})^3}{\left(19.4\times10^{-6}\text{m}^2/\text{s}\right)^2} = 7.32\times10^{10}$$

(b) The average heat transfer coefficient is given by equation (5.13) (turbulent)

$$\bar{h}_c = 0.13\,\frac{k}{L}(Gr_L\,Pr)^{\frac{1}{3}}$$

For water

$$\bar{h}_c = 0.13\ \frac{\left(0.657\ \text{W/(m K)}\right)}{2.5\,\text{m}}[2.07\times10^{13}(3.02)]^{\frac{1}{3}} = 1356\ \ \text{W/(m}^2\text{K)}$$

For air

$$\bar{h}_c = 0.13\ \frac{\left(0.0279\ \text{W/(m K)}\right)}{2.5\,\text{m}}[7.32\times10^{10}(0.71)]^{\frac{1}{3}} = 5.4\ \ \text{W/(m}^2\text{K)}$$

The rate of convective heat transfer is given by Equation (1.10)

$$q_c = \bar{h}_c\ A\ \Delta T$$

For water

$$q_c = \left(1356\ \text{W/(m}^2\text{K)}\right)\ [2\ (2.5\ \text{m})^2]\ (90°\text{C} - 30°\text{C}) = 1.017\times10^6\ \text{W}$$

For air

$$q_c = \left(5.4\ \text{W/(m}^2\text{K)}\right)\ [2\ (2.5\ \text{m})^2]\ (90°\text{C} - 30°\text{C}) = 4050\ \text{W}$$

(c) Since the sheet is very thin and the thermal conductivity of copper is very high, it is safe to assume that the Biot number is less than 0.1 for both cases. The initial rate of temperature change is given by

$$\left(\frac{dT}{dt}\right)_{t=0} = \frac{q_c}{mc} = \frac{q_c}{\rho V c} = \frac{q_c}{\rho c L W t}$$

For water

$$\left(\frac{dT}{dt}\right)_{t=0} = \frac{1.017\times10^6\ \text{W}\left(\text{J/(W s)}\right)}{(8933\ \text{kg/m}^3)\left(383\ \text{J/(kg K)}\right)(2.5\,\text{m})(2.5\,\text{m})(0.001\,\text{m})} = 47.6\ \text{K/s}$$

For air

$$\left(\frac{dT}{dt}\right)_{t=0} = \frac{4050\ \text{W}\left(\text{J/(W s)}\right)}{(8933\ \text{kg/m}^3)\left(383\ \text{J/(kg K)}\right)(2.5\,\text{m})(2.5\,\text{m})(0.001\,\text{m})} = 0.19\ \text{K/s}$$

COMMENTS

The initial cooling rate in water is about 250 times that in air.

PROBLEM 5.15

A laboratory apparatus is used to maintain a horizontal slab of ice at – 2.2°C so that specimens can be prepared on the surface of the ice and kept close to 0°C. If the ice is 10 cm by 3.8 cm and the laboratory is kept at 16°C, find the cooling rate in watts that the apparatus must provide to the ice.

GIVEN

- A slab of ice in a laboratory
- Ice temperature (T_i) = – 2.2°C
- Ice dimensions: 10 cm by 3.8 cm
- Ambient temperature (T_∞) = 16°C

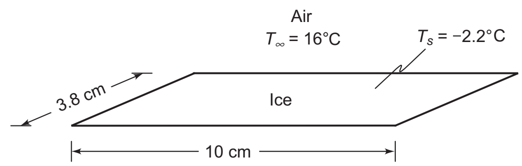
FIND

- The cooling rate (q) in watts

ASSUMPTIONS

- Air in the laboratory is still
- Effects of sublimation are negligible
- Effects of moisture in the air are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 6.9°C

Thermal expansion coefficient (β) = 3.57×10^{-3} 1/K
Thermal conductivity (k) = 0.024 W/(m K)
Kinematic viscosity (ν) = 1.424×10^{-8} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

The characteristic length for the ice is

$$L = \frac{A}{P} = \frac{(0.1\text{ m})\,(0.038\text{ m})}{2(0.1\text{m} + 0.038\text{m})} = 0.014\text{ m}$$

The Grashof and Rayleigh numbers based on this length are

$$Gr_L = \frac{g\,\beta(T_\infty - T_i)\,L^3}{\nu^2}$$

$$= \frac{(9.81\,\text{m/s}^2) \times (3.57 \times 10^{-3})\,1/\text{K}\ (16 + 2.2)\text{K}\,(0.014\,\text{m})^3}{(1.424 \times 10^{-8}\,\text{m}^2/\text{s})^2}$$

$$= 8.6 \times 10^9$$

$$Ra_L = 8.6 \times 10^9\ (0.71) = 6.12 \times 10^9$$

Equation (5.17) may be used to find the Nusselt number.

$$Nu_L = 0.27\ Ra_L^{\frac{1}{4}} = 0.27\ (6.12 \times 10^9)^{\frac{1}{4}} = 75.5$$

$$\bar{h}_c = Nu_L\,\frac{k}{L} = 75.5\ \frac{0.024\,\text{W/(m K)}}{0.014\,\text{m}}$$

$$= 129.4\ \text{W/(m}^2\text{ K)}$$

The cooling load is

$$q = \bar{h}_c\ A\,(T_\infty - T_i) = 129.4\ \text{W/(m}^2\text{ K)}\ (0.1\text{ m})\,(0.038\text{ m})\,(16 + 2.2)$$

$$= 8.95\text{ W}$$

PROBLEM 5.16

An electronic circuit board is the shape of a flat plate 0.3 m × 0.3 m in plan-form and dissipates 15 W. It may be placed in operation on an insulated surface in a horizontal position or at an angle of 45 degrees to horizontal, both in still air at 25°C. If the circuit would fail above 60°C, determine if the two proposed installations are safe.

GIVEN

- A flat plate with insulated back, horizontal or at an angle of 45 degrees in still air
- Plate size ($s \times s$) = 0.3 m × 0.3 m

- Heat generation rate (q_G) = 15 W
- Air temperature (T_∞) = 25°C
- Maximum plate temperature (T_s) = 60°C

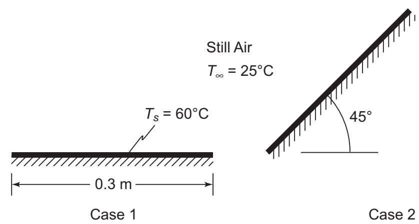
FIND

- If the two plate positions are safe

ASSUMPTIONS

- Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 43°C

Thermal expansion coefficient (β) = 0.00316 1/K

Thermal conductivity (k) = 0.0267 W/(m K)

Kinematic viscosity (ν) = 17.9×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

For the horizontal case, the characteristic length

$$L = \frac{A}{P} = \frac{s^2}{4s} = \frac{s}{4} = \frac{0.3\,\text{m}}{4} = 0.075\text{ m}$$

The Grashof and Rayleigh numbers for the flat case at the maximum operating temperature are

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.00316\text{ 1/K})(60°\text{C} - 25°\text{C})(0.075\,\text{m})^3}{\left(17.9\times10^{-6}\text{m}^2\text{/s}\right)^2} = 1.83 \times 10^6$$

$$Ra_L = Gr_L\,Pr = 1.83 \times 10^6\,(0.71) = 1.30 \times 10^6$$

For the inclined case, the characteristic length is the length of the inclined side (0.3 m), the Grashof number for the inclined case is

$$Gr_L = \frac{(9.8\text{ m/s}^2)(0.00316\text{ 1/K})(60°\text{C} - 25°\text{C})(0.3\,\text{m})^3}{\left(17.9\times10^{-6}\text{m}^2\text{ /s}\right)^2} = 1.17 \times 10^8$$

Case 1: The average Nuselt number is given by Equation (5.15)

$$\overline{Nu} = 0.54\ Ra_L^{\frac{1}{4}} = 0.54\ (1.30\times10^6)^{\frac{1}{4}} = 18.23$$

$$\bar{h}_c = \overline{Nu}\,\frac{k}{L} = 18.23\,\frac{\left(0.0267\text{ W/(m K)}\right)}{0.075\,\text{m}} = 6.49\text{ W/(m}^2\text{K)}$$

The rate of heat transfer from the plate at T_s = 60°C is

$$q = \bar{h}_c\ A\ (T_s - T_\infty) = \left(6.49\ \text{W/(m}^2\text{K)}\right)\ (0.3\ \text{m})^2\ (60°\text{C} - 25°\text{C}) = 26.3\ \text{W}$$

Since this is larger than the heat generation rate, the actual surface temperature will be less than 60°C. Case 1 configuration is safe.

Case 2

$$Gr_L\, Pr \cos\theta = 1.17 \times 10^8\ (0.71)\cos(45°) = 5.87 \times 10^7$$

Therefore, the average heat transfer coefficient is given by Equation (5.14)

$$\bar{h}_c = 0.56\ \frac{k}{L}\ (Gr_L Pr \cos\theta)^{\frac{1}{4}} = 0.56\ \frac{\left(0.0267\ \text{W/(m K)}\right)}{0.3\,\text{m}} (5.87 \times 10^7)^{\frac{1}{4}} = 4.36\ \ \text{W/(m}^2\text{K)}$$

The rate of heat transfer when T_s = 60°C is

$$q = \left(4.36\ \text{W/(m}^2\text{K)}\right)\ (0.3\ \text{m})^2\ (60°\text{C} - 25°\text{C}) = 17.7\ \text{W}$$

Since this is also greater than the heat generation rate, the actual temperature will be less than 60°C. Therefore, Case 2 is also safe.

PROBLEM 5.17

Cooled air is flowing through a long sheet metal air conditioning duct, 0.2 m high and 0.3 m wide. If the duct temperature is 10°C and passes through a crawl space under a house at 30°C, estimate

(a) The heat transfer rate to the cooled air per meter length of duct.

(b) The additional air conditioning load if the duct is 20 m long.

(c) Discuss qualitatively the energy conservation if the duct were insulated with glass wool.

GIVEN

- An air conditioning duct in a crawl space
- Duct height (H) = 0.2 m
- Duct width (w) = 0.3 m
- Duct temperature (T_s) = 10°C
- Ambient temperature (T_∞) = 30°C

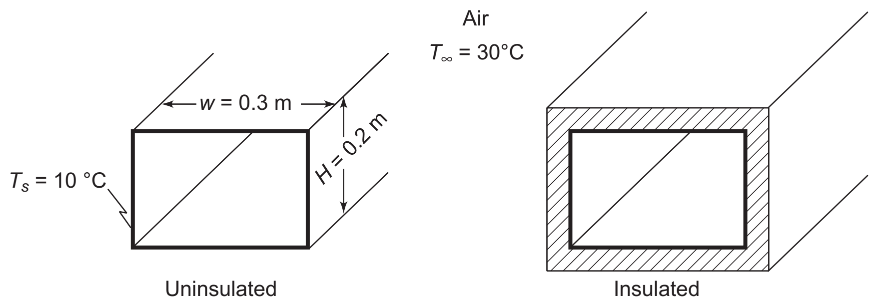
FIND

(a) The heat transfer rate per meter length (q_c/L) to the cooled air in the duct
(b) The additional air conditioning load (q_{20}) if the duct length (L) = 20 m
(c) Discuss qualitatively the energy conservation if the duct were insulated with glass wool

ASSUMPTIONS

- Ambient air is still
- Duct temperature is constant and uniform
- Radiation is negligible
- Edge effects are negligible
- No condensation on the duct surface

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 20°C

Thermal expansion coefficient (β) = 0.00341 1/K

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

(a) The duct can be thought of as two vertical and two horizontal cooled flat plates.
For the sides

$$Gr_H = \frac{g\,\beta(T_s - T_\infty)\,H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00341\ \text{1/K})(30\,°\text{C} - 10\,°\text{C})(0.2\,\text{m})^3}{\left(15.7\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 2.17\times10^7 < 10^9$$

So the flow is laminar.

For the top and bottom, the characteristic length (L_c) is given by

$L_c = A/P = Lw/(2L + 2w)$. Since $L >> w$: $L_c \approx w/2 = 0.15$ m

$$Gr_{Lc} = \frac{g\,\beta(T_s - T_\infty)\,L_c^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00341\ \text{1/K})(30\,°\text{C} - 10\,°\text{C})(0.15\,\text{m})^3}{\left(15.7\times10^{-6}\ \text{m}^2/\text{s}\right)^2} = 9.15\times10^6 < 10^7$$

The heat transfer coefficient for the vertical sides of the duct is given by Equation (5.12a)

$$\bar{h}_{cs} = 0.68\ Pr^{\frac{1}{2}}\,\frac{Gr_H^{\frac{1}{4}}}{(0.952+Pr)^{\frac{1}{4}}}\,\frac{k}{H} = 0.68\ (0.71)^{\frac{1}{4}}\,\frac{(2.17\times10^7)^{\frac{1}{4}}}{(0.952+0.71)^{\frac{1}{4}}}\,\frac{\left(0.0251\ \text{W/(m K)}\right)}{0.2\,\text{m}} = 4.32\ \text{W/(m}^2\text{K)}$$

The top is a cooled surface facing upward. The heat transfer coefficient from the top is the same as that for a heated surface facing downward and given by Equation (5.17)

$$\bar{h}_{ct} = 0.27\ \frac{k}{L_c}\,(Gr_L Pr)^{\frac{1}{4}} = 0.27\ \frac{0.0251\ \text{W/(m K)}}{0.15\,\text{m}}\,[9.15\times10^6\ (0.71)]^{\frac{1}{4}} = 2.28\ \text{W/(m}^2\text{K)}$$

The heat transfer coefficient for the bottom, a cooled surface facing downward, is given by Equation (5.15) since $Ra_L < 10^7$

$$\bar{h}_{cb} = 0.54\ \frac{k}{w}\,(Gr_{L_c} Pr)^{\frac{1}{4}} = 0.54\ \frac{\left(0.0251\ \text{W/(m K)}\right)}{0.15\,\text{m}}\,[9.15\times10^6\ (0.71)]^{\frac{1}{4}} = 4.56\ \text{W/(m}^2\text{K)}$$

The total convective heat transfer to the duct is

$$q_c = [2\bar{h}_{cs} H L + (\bar{h}_{ct} + \bar{h}_{cb}) w L] \; (T_\infty - T_s)$$

$$\frac{q_c}{L} = \left[2\left(4.32\ \text{W/(m}^2\text{K)}\right)(0.2\,\text{m}) + \left(2.28\ \text{W/(m}^2\text{K)} + 4.56\ \text{W/(m}^2\text{K)}\right)0.3\,\text{m}\right](30°\text{C} - 10°\text{C}) = 75.6\ \text{W/m}$$

(b) For a 20 m long duct

$$q_c = \left(\frac{q_c}{L}\right) L = 75.6\ \text{W/m}\ (20\ \text{m}) = 1512\ \text{W}$$

(c) The addition of insulation to the outer surface of the duct will have several effects
1. It will increase the outer surface temperature of the duct and decrease the duct wall temperature.
2. The higher surface temperature will lower the natural convection heat transfer coefficient because the temperature difference between the duct and the ambient air will be reduced.
3. The lower convective heat transfer coefficient and the additional conductive thermal resistance of the insulation will lead to a decrease in the rate of heat transfer to the air in the duct. This will reduce the load on the air conditioning system assuming that the crawl space is not to be intentionally cooled.

PROBLEM 5.18

Solar radiation at 600 W/m^2 is absorbed by a black roof inclined at 30°C as shown. If the underside of the roof is well insulated, estimate the maximum roof temperature in 20°C air.

GIVEN

- Inclined roof, well insulated on the underside
- Incline angle (θ) = 30 degrees
- Air temperature = 20°C
- Solar radiation absorbed (q_s) = 600 W/m^2

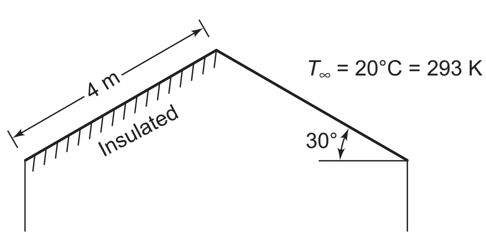
FIND

- The maximum roof temperature

ASSUMPTIONS

- The roof behaves as a black body (ε = 1.0)
- The sky behaves as a black body at 0 K

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5,
The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4).

SOLUTION

The maximum roof temperature will occur when the air is quiescent. Since the air properties must be evaluated at the mean of the ambient and surface temperatures, as iterative procedure must be used.

Iteration #1

Let T_s = 60°C = 333 K

From Appendix 2, Table 27, for dry air at the mean temperature of 40°C

Thermal expansion coefficient (β) = 0.00319 1/K

Thermal conductivity (k) = 0.0265 W/(m K)

Kinematic viscosity (ν) = 17.6×10^{-6} m²/s

Prandtl number (Pr) = 0.71

The Grashof number is

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00319\ 1/\text{K})\ (333\,\text{K} - 293\,\text{K})(4\,\text{m})^3}{\left(17.6\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 2.58 \times 10^{11}$$

$$Gr_L\, Pr \cos\theta = 2.58 \times 10^{11}\ (0.71)\cos(30°) = 1.59 \times 10^{11}$$

The average convective heat transfer coefficient for this geometry is given by Equation (5.14). Although $Gr_L\ Pr\ \cos\ \theta$ is slightly larger than 10^{11}, Equation (5.14) will be extrapolated by this problem

$$\bar{h}_c = 0.56\ \frac{k}{L}\,(Gr_L Pr\cos\theta)^{\frac{1}{4}} = 0.56\ \frac{\left(0.0265\ \text{W/(m K)}\right)}{4\,\text{m}}(1.59\times10^{11})^{\frac{1}{4}} = 2.34\ \text{W/(m}^2\ \text{K)}$$

For steady state, the solar gain must equal the convective and radiative losses

$$\frac{q_s}{A} = \bar{h}_c\ (T_s - T_\infty) + \sigma T_s^4$$

$$600\ \text{W/m}^2 = \left(2.34\ \text{W/(m}^2\text{K)}\right)\ (T_s - 293\ \text{K}) + 5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)\ (T_s^4)$$

Checking the units then eliminating them for clarity

$$5.67 \times 10^{-8}\ T_s^4 + 2.34\ T_s - 1286 = 0$$

By trial and error: T_s = 314 K.

Repeating this procedure for another iteration

T_s = 314 K	$Gr_L\ Pr \cos\ \theta = 9.58 \times 10^{10}$
T_{mean} = 304 K = 30°C	
β = 0.0033 1/K	h_c = 2.01 W/(m² K)
k = 0.0258 W/(m² K)	T_s = 315 K
$\nu = 16.7 \times 10^{-6}$ m²/s	
PR = 0.71	

The maximum roof temperature: (T_s) = 315 K = 42°C

COMMENTS

The procedure converges quickly because of the 1/4 power in the Nusselt number correlation.

PROBLEM 5.19

A 1 m square copper plate is placed horizontally on 2 m high legs. The plate has been coated with a material that provides a solar absorptance of 0.9 and an infrared emittance of 0.25. If

the air temperature is 30°C, determine the equilibrium temperature on an average clear day in which the solar radiation incident on a horizontal surface is 850 W/m^2.

GIVEN

- A horizontal copper plate in air
- Plate dimensions ($s \times s$) = 1 m × 1 m
- Solar absorptance (α_s) = 0.9
- Infrared emittance (ε) = 0.25
- Air temperature (T_∞) = 30°C = 303 K
- Incident solar radiation (q_s/A) = 850 W/m^2

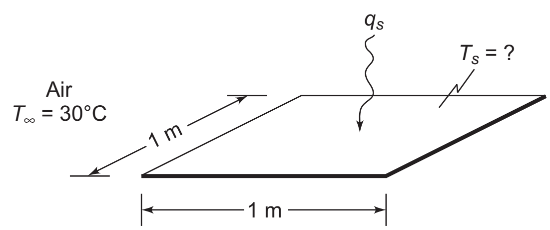
FIND

- Equilibrium Temperature (T_s)

ASSUMPTIONS

- The sky behaves as a black body at 0 K
- The effect of the legs is negligible
- Air is quiescent
- Radiative heat transfer from the bottom of the plate is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5,

The Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4).

SOLUTION

Since the air properties must be evaluated at the mean of the surface and ambient temperatures, an iterative process must be used

1. Guess at the surface temperature.
2. Evaluate the air properties and calculate the Grashof number.
3. Use the appropriate correlation to find the average convective heat transfer coefficients on the top and bottom of the plate.
4. Calculate a new surface temperature.

This process must be repeated until the temperature converges within an acceptable tolerance.

Iteration #1

1. Let T_s = 90°C = 363 K
2. From Appendix 2, Table 27, for dry air at he mean temperature of (60°C)
   Thermal expansion coefficient (β) = 0.00300 1/K
   Thermal conductivity (k) = 0.0279 W/(m K)
   Kinematic viscosity (ν) = 19.4 × 10^{-6} m^2/s
   Prandtl number (Pr) = 0.71

The characteristic length of the plate (L) = A/P = (1 m^2)/(4 m) = 0.25 m
The Grashof and Rayleigh numbers based on the characteristic length are

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)\,L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\,(0.003\ 1/\text{K})\,(90°\text{C} - 30°\text{C})\,(0.25\,\text{m})^3}{\left(19.4\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 7.32\times10^7$$

$$Ra_L = Gr_L\,Pr = 7.32\times10^7\,(0.71) = 5.20\times10^7$$

3. For the top of the plate, Equation (5.16) gives the average Nusselt number

$$\overline{Nu} = 0.15\ Ra^{\frac{1}{3}} = 0.15\ (5.20\times10^7)^{\frac{1}{3}} = 56.00$$

$$\bar{h}_{ct} = \overline{Nu}\,\frac{k}{L} = 56.00\ \frac{\left(0.0279\ \text{W/(m K)}\right)}{0.25\,\text{m}} = 6.25\ \text{W/(m}^2\ \text{K)}$$

For the bottom of the plate, Equation (5.17) gives the average Nusselt number

$$\overline{Nu} = 0.27\ (Ra_L)^{\frac{1}{4}} = 0.27\ (5.20\times10^7)^{\frac{1}{4}} = 22.9$$

$$\bar{h}_{cb} = \overline{Nu}\,\frac{k}{L} = 22.9\ \frac{\left(0.0279\ \text{W/(m K)}\right)}{0.25\,\text{m}} = 2.56\ \text{W/(m}^2\ \text{K)}$$

4. For equilibrium, the rate of solar gain must equal the total rate of convective heat transfer from top and bottom and radiative heat transfer from the top surface.

$$\alpha\left(\frac{q_s}{A}\right) = (\bar{h}_{ct} + \bar{h}_{cb})\ (T_s - T_\infty) + \frac{1}{2}\ \varepsilon\,\sigma(T_s^{\,4} - T_{\text{sky}}^{\,4})$$

$$0.9\ \left(850\ \text{W/m}^2\right) = [(6.25 + 2.56)\ \text{W/(m}^2\ \text{K)}]\ (T_s - 303\ \text{K}) + 0.25\left(5.67\times10^{-8}\,\text{W/(m}^2\ \text{K}^4)\right)(T_s^4 - 0)$$

Checking the units then eliminating them for clarity

$$1.43\times10^{-8}\ T_s^4 + 8.81\ T_s - 3434 = 0$$

By trial and error: T_s = 362 K = 89°C.

Since this is very close to the initial guess, another iteration is not necessary. The equilibrium temperature is about 89°C.

COMMENTS

The coating on the plate is called a selective surface and is often used in solar applications for decreasing reradiation losses from absorbers.

PROBLEM 5.20

A 2.5 × 2.5 m steel sheet 1.5 mm thick is removed from an annealing oven at a uniform temperature of 425°C and placed in a large room at 20°C in a horizontal position. (a) Calculate the rate of heat transfer from the steel sheet immediately after its removal from the furnace, considering both radiation and convection. (b) Determine the time required for the steel sheet to cool to a temperature of 60°C. Hint: This will require numerical integration.

GIVEN

- Horizontal steel sheet in air
- Sheet dimensions = 2.5 m × 2.5 m × 0.0015 m
- Sheet initial temperature (T_{si}) = 425°C = 698 K
- Air temperature (T_∞) = 20°C = 293 K

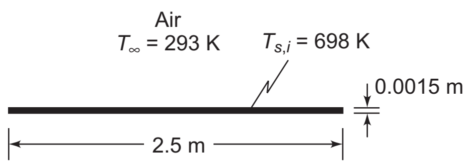
FIND

(a) The initial rate of heat transfer (q)
(b) The time required for the sheet to cool to 60°C (333 K)

ASSUMPTIONS

- The room behaves as a black body at T_∞
- The steel sheet behaves as a black body (ε = 1.0)
- Heat transfer takes place from both top and bottom of the sheet
- The steel is 1% carbon steel
- Heat transfer from the edges of the plate is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5,
The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Appendix 2, Table 10, for 1% carbon steel
Specific heat (c_s) = 473 J/(kg K)
Density (ρ_s) = 7801 kg/m^3
From Appendix 2, Table 27, for dry air at the mean temperature of 496 K (223°C)
Thermal expansion coefficient (β) = 0.00203 1/K
Thermal conductivity (k) = 0.0384 W/(m K)
Kinematic viscosity (ν) = 38.7×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

(a) The characteristic length for this geometry is

$$L = \frac{A}{P} = \frac{S^2}{4S} = \frac{S}{4} = \frac{2.5\,\text{m}}{4} = 0.625\text{ m}$$

The Grashof and Rayleigh numbers are

$$Gr_L = \frac{g\,\beta\,(T_s - T_\infty)\,L^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.00203\text{ 1/K})\,(698\text{K} - 293\text{K})\,(0.625\text{ m})^3}{\left(38.7\times10^{-6}\text{ m}^2/\text{s}\right)} = 1.31 \times 10^9$$

$$Ra_L = Gr_L\,Pr = 1.31 \times 10^9\,(0.71) = 9.33 \times 10^8$$

The average Nusselt number on the bottom of the plate is given by Equation (5.17)

$$\overline{Nu} = 0.27\,(Ra_L)^{\frac{1}{4}} = 0.27\,(9.33\times10^8)^{\frac{1}{4}} = 47.2$$

$$\bar{h}_{ct} = \overline{Nu}\,\frac{k}{L} = 47.2\,\frac{\left(0.0384\text{ W/(m K)}\right)}{0.625\,\text{m}} = 2.90\text{ W/(m}^2\text{ K)}$$

The average Nusselt number on the top of the plate is given by Equation (5.16)

$$\overline{Nu} = 0.15\ Ra^{\frac{1}{3}} = 0.15\ (9.33\times10^{8})^{\frac{1}{3}} = 146.6$$

$$\bar{h}_c = \overline{Nu}\,\frac{k}{L} = 146.6\ \frac{\left(0.0384\ \text{W/(m K)}\right)}{0.625\,\text{m}} = 9.01\ \ \text{W/(m}^2\ \text{K)}$$

The total rate of heat transfer is the sum of the convective and radiative components

$$q_{\text{total}} = (\bar{h}_{ct} + \bar{h}_{cb})\ A\ (T_s - T_\infty) + 2\ A\ \sigma(T_s^{\,4} - T_\infty^{\,4})$$

$$q_{\text{total}} = \left[(9.01 + 2.90)\ \text{W/(m}^2\text{K)}\right]\ (2.5\ \text{m})^2\ (698\ \text{K} - 293\ \text{K}) + 2\ (2.5\ \text{m})^2$$

$$\left(5.67\times10^{-8}\text{W/(m}^2\ \text{k}^4)\right)\ [(698\ \text{K})^4 - (293\ \text{K})^4]$$

$$q_{\text{total}} = 1.94\times10^5\ \text{W}$$

(b) As the plate cools, the rate of heat transfer will decrease. The cooling time will be estimated by calculating a new sheet temperature and heat transfer every time period.

The Biot number for the sheet is

$$Bi = \frac{\bar{h}\,\text{s}}{2\,k} = \frac{\left(9.01\ \text{W/(m}^2\text{K)}\right)\ (0.0015\,\text{m})}{2\left(0.0384\ \text{W/(m K)}\right)} = 0.176$$

This is slightly above 0.1. For a first order approximation, we can neglect the thermal resistance in the plate. For the first 20 second interval

$$\text{Total energy loss, } q_{\text{total}}\ (20\text{s}) = \text{m c } \Delta\text{T} = V\,\rho\,c\ (T_{s,i} - T_{s,\,20})$$

Solving for temperature after 20 sec

$$T_{s,20} = T_{s,i} - \frac{q_{\text{total}}\ (20\,\text{s})}{V\,\rho c} = 698\ \text{K} - \frac{1.94\times10^5\ \text{W}\,(\text{J/(W s)})\ (20\,\text{s})}{0.0015\,\text{m}\,(2.5\,\text{m})^2\left(7801\ \text{kg/m}^3\right)\left(473\ \text{J/(kg K)}\right)} = 586\ \text{K}$$

This temperature is then used to calculate new transfer coefficients and heat transfer rates as shown above. This procedure is followed until the temperature of the plate is 333 K.

Time (s)	20	40	80	120
T_s (K)	586	528	451	411
T_{mean} (K)	440	411	372	352
β (1/K)	0.00230	0.00244	0.00268	0.00284
k (W/m K)	0.0349	0.0332	0.0307	0.0292
$\nu\times10^6$ (m²/s)	31.5	28.3	23.6	21.4
Pr	0.71	0.71	0.71	0.71
$Ra_L\times10^{-9}$	1.15	1.22	1.29	1.24
h_{cb} (W/(m² K))	2.78	2.67	2.51	2.37
h_{ct} (W/(m² K))	8.78	8.50	8.02	7.52
$q_{\text{total}}\times10^{-4}$ (W)	9.99	6.65	3.46	2.24

Time (s)	200	300
T_s (K)	359	330
T_{mean}	326	
β (1/K)	0.00306	
k (W/m K)	0.0267	
$\nu \times 10^6$ (m^2/s)	17.9	
Pr	0.71	
$Ra_L \times 10^{-9}$	1.07	
h_{cb} ((W/(m^2 K))	2.09	
h_{ct} ((W/(m^2 K))	6.56	
$q_{total} \times 10^{-4}$ (W)	1.01	

Interpolating between 200 and 300 seconds

The time required to reach 333 K is approximately 290 seconds = 4.8 min.

PROBLEM 5.21

A thin electronic circuit board, 0.1 m by 0.1 m in size, is to be cooled in air at 25°C. The board is placed in a vertical position and the back side is well insulated. If the heat dissipation is uniform at 200 W/m^2, determine the average temperature of the surface of the board cover.

GIVEN

- Vertical circuit board in air
- Back is well insulated
- Board dimensions ($L \times H$) = 0.1 m × 0.1 m
- Air temperature (T_∞) = 25°C
- Uniform heat dissipation rate ($\dot{q}_G / A$) = 200 W/m^2

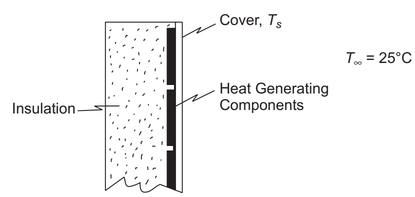
FIND

- The average temperature of the surface of the board (T_s)

ASSUMPTIONS

- Ambient air is still
- The board has reached steady state
- Radiation is negligible

SKETCH



SOLUTION

Since the fluid properties must be evaluated at the average of the surface and ambient temperatures, an iterative procedure is required to calculate the average surface temperature of the cover. For the first iteration, let T_s = 55°C

From Appendix 2, Table 27, for dry air at the mean temperature of 40°C

Thermal expansion coefficient (β) = 0.00319 1/K

Thermal conductivity (k) = 0.0265 W/(m K)

Kinematic viscosity (ν) = 17.6×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Grashof number is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.00319\text{ 1/K})(55\,°\text{C} - 25\,°\text{C})(0.1\text{m})^3}{\left(17.6 \times 10^{-6}\text{m}^2/\text{s}\right)} = 3.03 \times 10^6 < 10^9$$

For a laminar boundary layer, Equation (5.12a) gives the average heat transfer coefficient

$$\bar{h}_c = 0.68\ Pr^{\frac{1}{4}} \frac{Gr_H^{\frac{1}{4}}}{(0.952 + Pr)^{\frac{1}{4}}} \frac{k}{H} = 0.68\ (0.71)^{\frac{1}{4}} \frac{(3.03 \times 10^6)^{\frac{1}{4}}}{(0.952 + 0.71)^{\frac{1}{4}}} \frac{\left(0.0265\text{ W/(m K)}\right)}{0.1\text{m}} = 6.07\text{ W/(m}^2\text{ K)}$$

The rate of heat generation must equal the rate of convection heat transfer for steady state

$$\frac{\dot{q}_G}{A} = \frac{q_c}{A} = \bar{h}_c\ (T_s - T_\infty)$$

Solving for the surface temperature

$$T_s = T_\infty + \left(\frac{\dot{q}_G}{A}\right) = 25°\text{C} + \frac{200\text{ (W/m}^2)}{6.07\text{ W/(m}^2\text{K)}} = 57.9°\text{C}$$

For the second try, let T_s = (55°C + 57.9°C/2 = 56.5°C, i.e. halfway between the first guess and the result of the first interation. This gives T_{mean} = 40.7°C so the property values will change very little. For the second iteration.

T_s = 56.5°C

Mean Temp. = 40.7°C

β = 0.00319 1/K

k = 0.0265 W/(m K)

ν = 17.6×10^{-6} (m^2/s)

Pr = 0.71

Gr_L = 3.17×10^6

$\bar{h}_c$ = 6.15 W/(m^2 K)

T_s = 57.5°C

Therefore, the average surface temperature is about 58°C.

PROBLEM 5.22

A plot of coffee has been allowed to cool to 17°C. If the electrical coffee maker is turned back on, the hot plate on which the pot rests is brought up to 70°C immediately and held at that temperature by a thermostat. Consider the pot to be a vertical cylinder 130 mm in diameter and the depth of coffee in the pot to be 100 mm. Neglect heat losses from the sides and top of the pot. How long will it take before the coffee is drinkable (50°C)? How much did it cost to heat the coffee if electricity costs $0.05 per kilowatt-hour?

GIVEN

- Coffee pot, idealized as a vertical cylinder, on a hot plate
- Initial temperature of the pot and coffee ($T_{s,i}$) = 17°C
- Hot plate temperature (T_{hp}) = 70°C (constant)
- Pot diameter (D) = 130 mm = 0.13 m
- Depth of coffee (δ) = 100 mm = 0.1 m

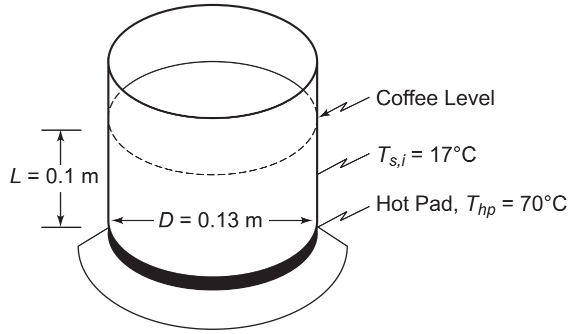
FIND

(a) Time for the coffee to reach 50°C
(b) Cost to heat the coffee if electricity costs \$0.05/kWh

ASSUMPTIONS

- Heat losses from the sides and the top are negligible
- All energy from the hot plate goes into the coffee
- Internal resistance of the coffee is negligible
- Thermal resistance of the bottom of the pot is negligible
- Coffee has the thermal properties of water
- Variation of the thermal properties of the coffee with temperature can be neglected

SKETCH



PROPERTIES AND CONSTANTS

The relevant thermal properties will be evaluated using the average coffee temperature of (17°C + 50°C)/2 = 33.5°C.

From Appendix 2, Table 13, for water

At 33.5°C Density (ρ) = 994.6 kg/m^3
Specific Heat (c) = 4175 J/(kg K)

At the mean temperature of (33.5°C + 70°C)/2 = 51.8°C

Thermal expansion coefficient (β_c) = 0.00047 1/K
Thermal conductivity (k_c) = 0.648 W/(m K)
Kinematic viscosity (ν_c) = 0.549 × 10^{-6} m^2/s
Prandtl number (Pr_c) = 3.5

SOLUTION

(a) The heat transfer coefficient from between the hot plate and the coffee can be evaluated by treating the coffee volume as a horizontal water layer heated from below. The Rayleigh number is

$$Ra_\delta = \frac{g\,\beta(T_{hp} - T_c)\,\delta^3 Pr_c}{\nu_c^2} = \frac{(9.8\ \text{m/s}^2)(0.00047\ 1/\text{K})(70\,°\text{C} - 33.5\,°\text{C})\,(0.1\text{m})^3(3.5)}{\left(0.549\times 10^{-6}\ \text{m}^2/\text{s}\right)^2} = 1.95 \times 10^9$$

The Nusselt number for this geometry is given by Equation (5.30b)

$$Nu_\delta = 1 + 1.44\left[1 - \frac{1708}{Ra_\delta}\right]\left[\left(\frac{Re_\delta}{5830}\right)^{\frac{1}{3}} - 1\right] + 2.0\left[\frac{Re_\delta^{\frac{1}{3}}}{140}\right]^{\left[1 - \ln\left(Ra_\delta^{\frac{1}{3}}/140\right)\right]}$$

where the notation [ ] indicates that if the quantity inside the bracket is negative, the quantity is to be taken as zero.

$$Nu_\delta = 1 + 1.44\left[1 - \frac{1708}{1.95\times10^9}\right] + \left[\left(\frac{1.95\times10^9}{5830}\right)^{\frac{1}{3}} - 1\right] + 2.0\left[\frac{(1.95\times10^9)^{\frac{1}{3}}}{140}\right]^{\left[1-\ln((1.95\times10^9)^{\frac{1}{3}}/140)\right]}$$

$$Nu_\delta = 71.0$$

$$h_c = Nu_\delta \frac{k_c}{\delta} = 71.0 \frac{\left(0.648\ \text{W/(m K)}\right)}{0.1\text{m}} = 460\ \text{W/(m}^2\ \text{K)}$$

The time required for heating can be calculated from Equation (2.84), solving for the time

$$t = \frac{c\rho V}{h_c A} \ln\left(\frac{T - T_{hp}}{T_o - T_{hp}}\right) = -\frac{c\rho \frac{\pi}{4} D^2 \delta}{h_c \frac{\pi}{4} D^2} \ln\left(\frac{T - T_{hp}}{T_o - T_{hp}}\right) = -\frac{c\rho\delta}{h_c} \ln\left(\frac{T - T_{hp}}{T_o - T_{hp}}\right)$$

$$t_f = \frac{\left(4175\ \text{J/(kg K)}\right)\left(994.6\ \text{kg/m}^3\right)(0.1\text{m})}{\left(460\ \text{W/(m}^2\text{K)}\right)\left(\text{J/(W s)}\right)} \ln\left[\frac{50°\text{C} - 70°\text{C}}{17°\text{C} - 70°\text{C}}\right] = 880\ \text{s} = 14.7\ \text{min}$$

(b) The total heat transfer from the hot plate during this time is

$$E = \int_0^{t_1} q_t\, dt = \int_0^{t_1} \bar{h}_c A_{\text{bottom}} [T_{hp} - T(t)]\, dt = h_c \frac{\pi}{4} D^2 \int_0^{t_1} [T_{hp} - T(t)]\, dt$$

From Equation (2.84)

$$T_{hp} - T(t) = (T_{hp} - T_{si}) \exp\left(-\frac{\bar{h}_c t}{c\rho\delta}\right)$$

Therefore

$$\int_0^{t_1} [T_{hp} - T(t)]\, dt = (T_{hp} - T_{si}) \int_0^{t_1} \exp\left(-\frac{\bar{h}_c t}{c\rho\delta}\right) dt = (T_{hp} - T_{si}) \left(-\frac{c\rho\delta}{h_c}\right)\left[\exp\left(-\frac{\bar{h}_c t_f}{c\rho\delta}\right) - 1\right]$$

$$\therefore E = -\frac{\pi}{4} D^2 c\, \rho\, \delta (T_{hp} - T_{si}) \left[\exp\left(-\frac{\bar{h}_c t_f}{c\rho\delta}\right) - 1\right]$$

$$E = -\frac{\pi}{4}(0.13\ \text{m})^2 \left(4175\ \text{J/(kg K)}\right)\left(994.6\ \text{kg/m}^3\right)(0.1\ \text{m})$$

$$(70°\text{C} - 17°\text{C})\left[\exp\left(-\frac{\left(460\ \text{W/(m}^2\text{K)}\right)(880\text{s})}{\left(4175\ \text{J/(kg K)}\right)\left(994.6\ \text{kg/m}^3\right)(0.1\text{m})}\right) - 1\right]$$

$$E = 181{,}916\ \text{J} \left(\frac{\text{h}}{3600\text{s}}\right)\left((\text{Ws})/\text{J}\right)\left(\frac{\text{kW}}{1000\ \text{W}}\right) = 0.051\ \text{kWh}$$

$$\text{Cost} = E\left(\frac{\$0.05}{\text{kWh}}\right) = (0.051\ \text{kWh})\left(\frac{\$0.05}{\text{kWh}}\right) = \$0.003$$

COMMENTS

The power consumption of the hot plat is about 12.5 watts.

The cost estimate neglects all losses from the hot plate to the ambient air.

PROBLEM 5.23

A laboratory experiment has been performed to determine the natural-convection heat transfer correlation for a horizontal cylinder of elliptical cross section in air. The cylinder is 1 m long, has a hydraulic diameter of 1 cm, a surface area of 0.0314 m^2, and is heated internally by electrical resistance heating. Recorded data include power dissipation, cylinder surface temperature, and ambient air temperature. The power dissipation has been corrected for radiation effects:

$T_s - T_\infty$ (°C)	q (W)
15.2	4.60
40.7	15.76
75.8	34.29
92.1	43.74
127.4	65.62

Assume that all air properties may be evaluated at 27°C and determine the constants in the correlation equation: $Nu = C\,(Gr\,Pr)^m$

GIVEN

- A horizontal, elliptical cylinder in air
- Hydraulic diameter (D_h) = 1 cm = 0.01 m
- Length (L) = 1 m
- Cylinder surface area (A_s) = 0.0314 m^2
- Experimental data for ($T_s - T_\infty$) and q shown above

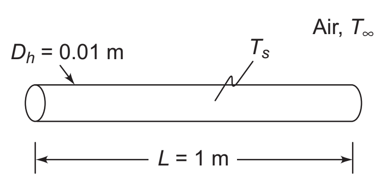
FIND

- The constants in the correlation equation $Nu = C(Gr\,Pr)^m$

ASSUMPTIONS

- All air properties may be evaluated at 27°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 27°C

Thermal expansion coefficient (β) = 0.00333 1/K

Thermal conductivity (k) = 0.0256 W/(m K)

Kinematic viscosity (ν) = 16.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Nusselt and Grashof number for the points are given by the following equation

$$\overline{Nu}_D = \frac{h_c D_h}{k} = \frac{D_h}{k}\left[\frac{q}{A_s\,(T_s - T_\infty)}\right]$$

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)\,D_h^3}{\nu^2}$$

Tabulating these and their logarithms for the experimental data

$\overline{Nu}_D$	$(Gr_D\,Pr) \times 10^{-3}$	$\log \overline{Nu}_D$	$\log (Gr_D\,Pr)$
3.76	1.31	0.575	3.11
4.81	3.51	0.682	3.55
5.62	6.53	0.750	3.81
5.91	7.93	0.772	3.90
6.40	10.98	0.806	4.04

Performing a least squares fit on the data yields

$$\log \overline{Nu}_D = 0.250 \log (Gr_D\,Pr) - 0.204$$

or

$$\overline{Nu}_D = 0.63\,(Gr_D\,Pr)^{0.25}$$

PROBLEM 5.24

A long, 2-cm-*OD* horizontal copper pipe carries dry saturated steam at 1.2 atm absolute pressure. The pipe is contained within an environmental testing chamber in which the ambient air pressure can be adjusted from 0.5 to 2.0 atm, absolute while the ambient air temperature is held constant at 20°C. What is the effect of this pressure change on the rate of condensate flow per meter length of pipe? Assume that the pressure change does not affect the absolute viscosity, thermal conductivity, or specific heat of the air.

GIVEN

- A long horizontal copper pipe carrying saturated steam within an environmental testing chamber
- Outside diameter (D) = 2 cm = 0.02 m
- Steam pressure = 1.2 atm
- Ambient pressure range (P) = 0.5 to 2 atm
- Ambient air temperature (T_∞) = 20°C

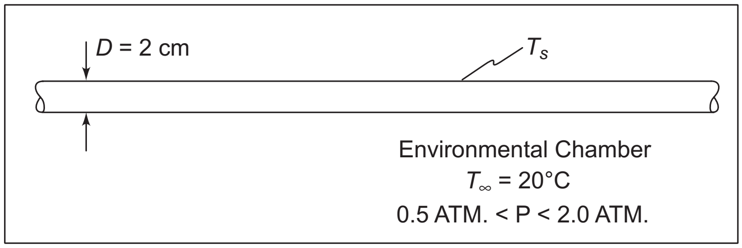
FIND

- Effect of ambient pressure change on rate of condensate flow per meter length of pipe

ASSUMPTIONS

- Pressure change has no effect on absolute viscosity, thermal conductivity, or specific heat of the air
- Air is still
- Chamber temperature is held constant while pressure is changed
- Convective thermal resistance on the inside of the pipe is negligible
- Thermal resistance of the copper pipe is negligible
- The air behaves as an ideal gas

SKETCH



PROPERTIES AND CONSTANTS

From standard steam tables: For saturated steam at 1.2 atm (0.12 MP_a) the heat of vaporization (h_{fg}) = 2238 kJ/kg, and the temperature (T_s) = 105°C.

From Appendix 2, Table 27, for dry air at the mean temperature of 62.5°C and one atmosphere

Thermal expansion coefficient (β) = 0.00298 1/K

Thermal conductivity (k) = 0.0281 W/(m K)

Absolute viscosity (μ) = 20.02×10^{-6} (N s)/m^2

Prandtl number (Pr) = 0.71

Density (ρ) = 1.018 kg/m^3

For an ideal gas

$$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} \Rightarrow \rho_2 = \frac{P_2}{P_1}\,\rho_1$$

$$\text{At } P = \text{ 0.5 atm: } \rho = \frac{0.5}{1}\left(1.018 \text{ kg/m}^3\right) = 0.509 \text{ kg/m}^3 \quad \Rightarrow \quad \nu = \frac{\mu}{\rho} = 3.93 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{At } P = \text{ 2.0 atm: } \rho = \frac{2.0}{1}\left(1.018 \text{ kg/m}^3\right) = 2.036 \text{ kg/m}^3 \quad \Rightarrow \quad \nu = \frac{\mu}{\rho} = 9.83 \times 10^{-6} \text{ m}^2/\text{s}$$

SOLUTION

The Grashof number based on the pipe diameter is

$$Gr_D = \frac{g\,\beta (T_s - T_\infty)\,D^3}{\nu^2}$$

At 0.5 atm

$$Gr_D = \frac{(9.8 \text{ m/s}^2)(0.00298 \text{ 1/K})\,(105\text{°C} - 20\text{°C})\,(0.02 \text{ m})^3}{\left(3.93 \times 10^{-5} \text{m}^2/\text{s}\right)^2} = 1.29 \times 10^4$$

At 2.0 atm

$$Gr_D = \frac{(9.8 \text{ m/s}^2)(0.00298 \text{ 1/K})\,(105\text{°C} - 20\text{°C})\,(0.02 \text{ m})^3}{\left(9.83 \times 10^{-6} \text{m}^2/\text{s}\right)^2} = 2.05 \times 10^5$$

The Nusselt number for a horizontal cylinder is given by Equation (5.20). (All requirements are satisfied at both pressures.)

$$\overline{Nu}_D = 0.53\;(Gr_D\,Pr)^{\frac{1}{4}}$$

At 0.5 atm

$$\overline{Nu}_D = 0.53\ [1.29 \times 10^4\ (0.71)]^{\frac{1}{4}} = 5.18$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 5.18 \frac{(0.0281\ \text{W/(m K)})}{0.2\,\text{m}} = 7.28\ \text{W/(m}^2\text{K)}$$

At 2.0 atm

$$\overline{Nu}_D = 0.53\ [2.05 \times 10^5\ (0.71)]^{\frac{1}{4}} = 10.35$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{L} = 10.35 \frac{(0.0281\,\text{W/(m K)})}{0.02\,\text{m}} = 14.54\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer per meter length of pipe is

$$\frac{q_c}{L} = \bar{h}_c\ \pi D\,(T_s - T_\infty)$$

At 0.5 atm

$$\frac{q_c}{L} = (7.28\,\text{W/(m}^2\text{K)})\ (\pi)\ (0.02\ \text{m})\ (105°\text{C} - 20°\text{C}) = 38.9\ \text{W/m}$$

At 2.0 atm

$$\frac{q_c}{L} = (14.54\,\text{W/(m}^2\text{K)})\ (\pi)\ (0.02\ \text{m})\ (105°\text{C} - 20°\text{C}) = 77.7\ \text{W/m}$$

It is clear that raising the ambient pressure from 0.5 atm to 2.0 atm will double the flow of condensation $(\dot{m}_c)$ per meter of pipe

At 0.5 atm

$$\frac{\dot{m}_c}{L} = \frac{\frac{q_c}{L}}{h_{fg}} = \frac{38.9\ \text{W/m}}{(2238\ \text{kJ/kg})\,(1000\ \text{J/k J})\,((\text{W s})/\text{J})} = 1.74 \times 10^{-5}\ \text{kg/s} = 1.04\ \text{g/min}$$

At 2.0 atm

$$\frac{\dot{m}_c}{L} = \frac{77.7\ \text{W/m}}{2238\ \text{kJ/kg}\,(1000\ \text{J/(k J)})\,(\text{Ws/J})} = 3.47 \times 10^{-5}\ \text{kg/s} = 2.08\ \text{g/min}$$

PROBLEM 5.25

Compare the rate of condensate flow from the pipe in Problem 5.24 (air pressure = 2.0 atm) with that for a 3.89-cm-*OD* pipe and 2.0 atm air pressure. What is the rate of condensate flow if the 2 cm pipe is submerged in a 20°C constant-temperature water bath?

From Problem 5.24: Long, 2-cm-*OD* horizontal copper pipe carries dry saturated steam at 1.2 atm absolute pressure. The pipe is contained within an environmental testing chamber in which the ambient air pressure can be adjusted form 0.5 to 2.0 atm, absolute while the ambient air temperature is held constant at 20°C. Assume that the pressure change does not affect the absolute viscosity, thermal conductivity, or specific heat of the air.

GIVEN

- A long horizontal copper pipe carrying saturated steam within an environmental testing chamber or a water bath
- Steam pressure = 1.2 atm
- Ambient pressure (P) = 2 atm
- Ambient air or water temperature (T_∞) = 20°C

FIND

Rate of condensate flow for

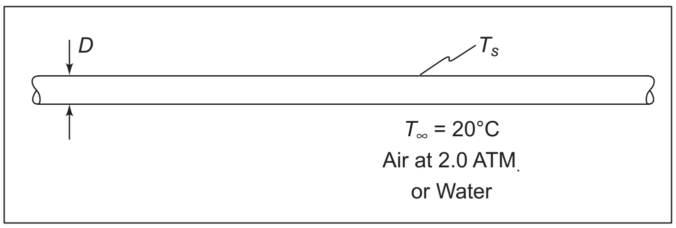
(a) Diameter (D) = 3.89 cm = 0.0389 m   Fluid is air at 2.0 atm

(b) Diameter (D) = 2 cm = 0.02 mFluid is water at T_∞ = 20°C

ASSUMPTIONS

- Pressure change has no effect on absolute viscosity, thermal conductivity, or specific heat of the air
- Air is still
- Convective thermal resistance on the inside of the pipe is negligible
- Thermal resistance of the copper pipe is negligible
- The air behaves as an ideal gas

SKETCH



PROPERTIES AND CONSTANTS

From standard steam tables: For saturated steam at 1.2 atm (0.12 MP_a), the heat of vaporization (h_{fg}) = 2238 kJ/kg, and the temperature (T_s) = 105°C.

From Appendix 2, Table 27, for dry air at the mean temperature of 62.5°C and one atmosphere

Thermal expansion coefficient (β) = 0.00298 1/K

Thermal conductivity (k) = 0.0281 W/(m K)

Prandtl number (Pr) = 0.71

From Problem 5.24: at P = 2.0 Atm, Kinematic viscosity (ν) = 9.83 × 10^{-6} N s/m^2

From Appendix 2, Table 13, for water at the mean temperature of 62.5°C and one atmosphere

β = 0.00053 1/K k = 0.659 W/(m K)

ν = 0.461 × 10^{-6} m^2/s Pr = 2.89

SOLUTION

The Grashof number is

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)\,D^3}{\nu^2}$$

Case (a)

$$Gr_D = \frac{(9.8\text{ m/s}^2)(0.00298\text{ 1/K})\,(105\,°\text{C} - 20\,°\text{C})\,(0.0389\text{ m})^3}{\left(9.83\times10^{-6}\text{m}^2/\text{s}\right)^2} = 1.51\times10^6$$

Case (b)

$$Gr_D = \frac{(9.8\text{ m/s}^2)(0.00053\text{ 1/K})\,(105\,°\text{C} - 20\,°\text{C})\,(0.02\text{ m})^3}{\left(0.461\times10^{-6}\text{m}^2/\text{s}\right)^2} = 1.66\times10^7$$

Both cases fall within the range of requirements for the use of Equation (5.20)

$$\overline{Nu_D} = 0.53\ (Gr_D\,Pr)^{\frac{1}{4}}$$

Case (a)

$$\overline{Nu_D} = 0.53\ [1.51\times10^6\ (0.71)]^{\frac{1}{4}} = 17.1$$

$$\bar{h}_c = \overline{Nu_D}\,\frac{k}{D} = 17.1\ \frac{(0.0281\ \text{W/(m K)})}{0.0389\,\text{m}} = 12.3\ \ \text{W/(m}^2\text{K)}$$

Case (b)

$$\overline{Nu_D} = 0.53\ [1.66\times10^7\ (2.89)]^{\frac{1}{4}} = 44.1$$

$$\bar{h}_c = \overline{Nu_D}\,\frac{k}{D} = 44.1\ \frac{(0.659\ \text{W/(m K)})}{0.02\ \text{m}} = 1453\ \ \text{W/(m}^2\text{K)}$$

The condensate flow rate per meter of pipe is given by

$$\frac{\dot{m}_c}{L} = \frac{\frac{q_c}{L}}{h_{gf}} = \frac{\bar{h}_c\,\pi D\,(T_s - T_\infty)}{h_{fg}}$$

Case (a)

$$\frac{\dot{m}_c}{L} = \frac{(12.3\ \text{W/(m}^2\text{K)})\ \pi\,(0.0389\,\text{m})\ (105\,°\text{C} - 20°\text{C})}{2238\ \text{k J/kg}(1000\ \text{J/k J})(\text{W s/J})} = 5.7\times10^{-5}\ \text{kg/s}\ = 3.42\ \text{g/min}$$

Case (b)

$$\frac{\dot{m}_c}{L} = \frac{(1453\ \text{W (m}^2\text{K)})\ \pi\,(0.02\,\text{m})\ (105\,°\text{C} - 20°\text{C})}{2238\ \text{k J/kg}(1000\ \text{J/(k J)})(\text{W s/J})} = 3.47\times10^{-3}\ \text{kg/s}\ = 208\ \text{g/min}$$

COMMENTS

The rate of condensate flow from Problem 5.24 with a 2 cm diameter pipe in air at 2.0 atm. is 2.1 g/min. A change in the fluid from air to water leads to a much larger increase in the rate of condensate flow (100 times) than an increase in the pipe diameter to 3.89 cm (1.6 times).

PROBLEM 5.26

A thermocouple (0.8 mm *OD*) is located horizontally in a large enclosure whose walls are at 37°C. The enclosure is filled with a transparent quiescent gas which has the same properties as air. The electromotive force (emf) of the thermocouple indicates a temperature of 230°C. Estimate the true gas temperature if the emissivity of the thermocouple is 0.8.

GIVEN

- Horizontal thermocouple in a large enclosure
- Thermocouple outside diameter (D) = 0.8 mm = 0.0008 m
- Enclosure wall temperature (T_e) = 37°C = 310 K
- Gas is enclosure is quiescent and has the same properties as air
- Thermocouple reading (T_{tc}) = 230°C = 503 K
- Thermocouple emissivity (ε) = 0.8

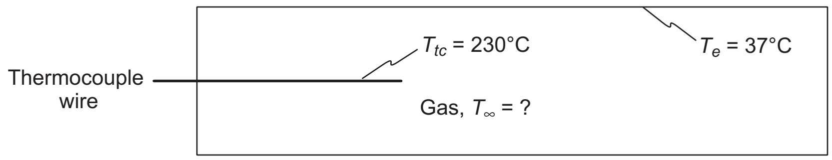
FIND

- True gas temperature (T_∞)

ASSUMPTIONS

- Enclosure behaves as a black body
- Conduction along the thermocouple out of the enclosure is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5 The Stephan-Boltzmann Constant (σ) = 5.67×10^{-8} W/(m^2 K^4).

SOLUTION

An iterative procedure is required. For the first iteration, let T_∞ = 300°C = 573 K.
From Appendix 2, Table 27, for dry air at the mean temperature of 265°C

Thermal expansion coefficient (β) = 0.00188 1/K
Thermal conductivity (k) = 0.0408 W/(m K)
Kinematic viscosity (ν) = 44.4×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

The Grashof number based on the thermocouple diameter is

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00188\ 1/\text{K})(300°\text{C} - 230°\text{C})(0.0008\,\text{m})^3}{\left(44.4\times10^{-6}\text{m}^2/\text{s}\right)^2} = 0.335$$

The Rayleigh number is

$$Ra_D = Gr_D\,(Pr) = (0.335)\ (0.71) = 0.238$$

log Ra_D = – 0.624
From Figure 5.3 log Nu ≈ – 0.05 → Nu = 0.89

$$h_c = Nu\ \frac{k}{D} = 0.89\ \frac{\left(0.0408\ \text{W}/(\text{m K})\right)}{0.0008\,\text{m}} = 45\ \ \text{W}/(\text{m}^2\text{K})$$

For steady state, the rate of convection to the thermocouple must equal the rate of radiation from the thermocouple

$$h_c\,A\,(T_\infty - T_{tc}) = \varepsilon\,\sigma A\,(T_{tc}^{\,4} - T_e^{\,4})$$

Solving for the gas temperature

$$T_\infty = T_{tc} + \frac{\varepsilon\sigma}{h_c}\ (T_{tc}^{\,4} - T_e^{\,4})$$

$$T_\infty = 503\ \text{K} + \frac{0.8\left(5.67\times10^{-8}\ \text{W}/(\text{m}^2\text{K}^4)\right)}{\left(45\ \text{W}/(\text{m}^2\text{K})\right)}\ [(503\ \text{K})^4 - (310\ \text{K})^4] = 559\ \text{K}$$

Using this as the beginning of another iteration

T_{mean} = 259°C
β = 0.00190 1/K
k = 0.0405 W/(m K)
ν = 43.5×10^{-6} m^2/s

$Pr = 0.71$

$Ra_D = 0.20$

$h_c = 43$ W/(m^2 K)

$T_\infty = 561$ K

Therefore, the true gas temperature is about 560 K = 287°C.

PROBLEM 5.27

Only 10 per cent of the energy dissipated by the tungsten filament of an incandescent lamp is in the form of useful visible light. Consider a 100 W lamp with a 10 cm spherical glass bulb. Assuming an emissivity of 0.85 for the glass and ambient air temperature of 20°C, what is the temperature of the glass bulb?

GIVEN

- A spherical glass light bulb in air
- Bulb power consumption (P) = 100 W
- 10% of energy is in the form of visible light
- Diameter (D) = 10 cm = 0.1 m
- Bulb emissivity (ε) = 0.85
- Ambient temperature (T_∞) = 20°C = 293 K

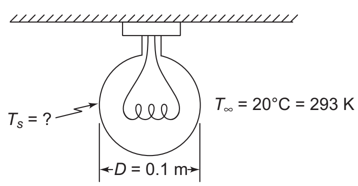
FIND

- The temperature of the glass bulb (T_s)

ASSUMPTIONS

- Ambient air is till
- The bulb has reached steady state
- The surrounding behave as a black body at T_∞

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5 the Stephan-Boltzmann constant (σ) = 5.7×10^{-8} W/(m^2 K^4).

SOLUTION

The rate of heat transfer by convection and radiation from the bulb must equal the rate of heat generation.

$$q_c + q_r = \pi D^2 \left[\bar{h}_c \ (T_s - T_\infty) + \varepsilon \, \sigma (T_s^4 - T_\infty^4) \right] = 0.9 \ (100 \text{ W}) = 90 \text{ W}$$

Since the fluid properties depend on the surface temperature, an iterative procedure must be used. For the first iteration, let T_s = 100°C = 373 K.

From Appendix 2, Table 27, for dry air at the mean temperature of 60°C

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The characteristic length for a 3-D body is given by

$$L^+ = \frac{A}{\left(\frac{4\,A_{\text{Horz}}}{\pi}\right)^{\frac{1}{2}}} = \frac{\pi D^2}{\left(\frac{4\left(\frac{\pi}{4}D^2\right)}{\pi}\right)^{\frac{1}{2}}} = \pi D = \pi\ (0.1\text{ m}) = 0.314\text{ m}$$

The Grashof and Rayleigh numbers are

$$Gr_{L+} = \frac{g\,\beta(T_s - T_\infty)\,(L^+)^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.003\,1/\text{K})(100°\text{C} - 20°\text{C})(0.314\text{ m})^3}{\left(19.4\times10^{-6}\text{m}^2/\text{s}\right)^2} = 1.94\times10^8$$

$$Ra_{L+} = Gr_{L+} + Pr = 1.94\times10^8\ (0.71) = 1.38\times10^8$$

Equation (5.25) correlates data for 3-D bodies including spheres for $200 < Ra_{L+} < 1.5\times10^9$

$$Nu^+ = 5.75 + 0.75\left[\frac{Ra^+}{F\,(Pr)}\right]^{0.252}$$

$$\text{where}\quad F(Pr) = \left[1+\left(\frac{0.49}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}} = \left[1+\left(\frac{0.49}{0.71}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}} = 2.88$$

$$\therefore\ Nu^+ = 5.75 + 0.75\left[\frac{1.38\times10^8}{2.88}\right] = 70.4$$

$$\bar{h}_c = Nu^+\,\frac{k}{L^+} = 70.4\,\frac{\left(0.0279\text{ W/(m K)}\right)}{0.314\text{ m}} = 6.26\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer by convection and radiation must equal the heat generation rate

$$q_c + q_r = \pi(0.1\text{ m})^2\left[6.26\text{ W/(m}^2\text{K)}(T_s - 293\text{K}) + 0.85\left(5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\right)(T_s^4 - (293\text{K})^4)\right] = 90\text{ W}$$

Checking the units then eliminating them for clarity

$$0.197\ T_s + 1.514\times10^{-9}\ T_s^4 - 158.8 = 0$$

By trial and error: T_s = 460 K = 187°C.
The results of further iterations are tabulated below

Iteration #	2	3
T_s (K)	459	457
T_{mean} (K)	376	375
β (1/K)	0.00266	0.00270
k (W/(m^2 K))	0.0309	0.0308
$\nu\times10^6$ (m^2/s)	24.0	23.8
Pr	0.71	0.71
$Ra^+\times10^{-8}$	1.86	1.68
h_c (W/(m^2 K))	7.43	7.23
T_s (°C)	181	182

The bulb temperature, therefore, is approximately 182°C.

COMMENTS

Note that radiative transfer accounts for about 66% of the total heat transfer from the bulb.

PROBLEM 5.28

A sphere 20 cm in diameter containing liquid air (–140°C) is covered with 5 cm thick glass wool (50 kg/m^3 density) with an emissivity of 0.8. Estimate the rate of heat transfer to the liquid air from the surrounding air at 20°C by convection and radiation. How would you reduce the heat transfer?

GIVEN

- A sphere containing liquid air covered with glass wool
- Sphere diameter (D_s) = 20 cm = 0.2 m
- Liquid air temperature (T_a) = – 140°C = 133 K
- Surrounding air temperature (T_∞) = 20°C = 293 K
- Insulation thickness (s) = 5 cm = 0.05 m
- Insulation emissivity (ε) = 0.8

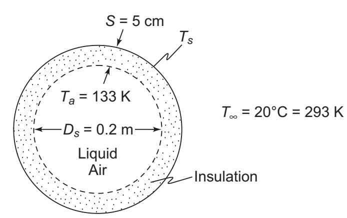
FIND

- Rate of heat transfer from liquid air to surrounding air (q)
- How can this be reduced?

ASSUMPTIONS

- Steady state conditions
- The surroundings behave as a black body enclosure at T_∞
- Surrounding air is still
- Thermal resistance of the convection inside the sphere and of the container wall are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4).
From Appendix 2, Table 11, the thermal conductivity of glass wool (k_i) = 0.037 W/(m K).

SOLUTION

The natural convection heat transfer coefficient on the exterior of the insulation depends on the exterior temperature of the insulation (T_s), an iterative procedure is therefore required. For the first iteration, let T_s = – 20°C (253 K)

From Appendix 2, Table 27, for dry air at the mean temperature of 0°C

Thermal expansion coefficient (β) = 0.00366 1/K

Thermal conductivity (k) = 0.0237 W/(m K)

Kinematic viscosity (ν) = 13.9×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The characteristic length for the sphere is

$$L^+ = \frac{A}{\left(\frac{4\,A_{\text{Horz}}}{\pi}\right)^{\frac{1}{2}}} = \frac{\pi D_i^2}{D_i} = \pi D_i = \pi\ (D_s + 2s) = \pi\ [0.2 \text{ m} + 2(0.05 \text{ m})] = 0.942 \text{ m}$$

The Grahsof and Rayleigh numbers based on this length are

$$Gr_{L+} = \frac{g\,\beta(T_s - T_\infty)\,(L^+)^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.00366\text{ 1/K})(20°\text{C} - 20°\text{C})(0.942\text{ m})^3}{\left(13.9\times10^{-6}\text{m}^2/\text{s}\right)^2} = 6.21\times10^9$$

$$Ra_{L+} = Gr_{L+}\,Pr = 6.21\times10^9\,(0.71) = 4.41\times10^9$$

Although the empirical relation of Equation (5.25) extends only to $Ra^+ = 1.5 \times 10^9$, it will be extrapolated here to estimate the Nusselt number

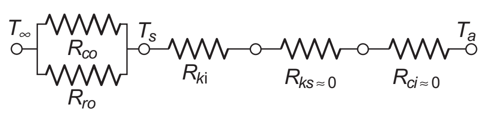
$$Nu^+ = 5.75 + 0.75\left[\frac{Ra^+}{F\,(Pr)}\right]^{0.252}$$

where
$$F(Pr) = \left[1 + \left(\frac{0.49}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}} = \left[1 + \left(\frac{0.49}{0.71}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}} = 2.88$$

$$\therefore\ Nu^+ = 5.75 + 0.75\left[\frac{4.41\times10^9}{2.88}\right]^{0.252} = 160.5$$

$$\bar{h}_c = Nu^+\frac{k}{L^+} = 160.5\,\frac{\left(0.0237\text{ W/(m K)}\right)}{0.942\text{ m}} = 4.04\ \text{W/(m}^2\text{K)}$$

The thermal circuit for the sphere is shown below



where R_{ci} = interior convective resistance (negligible)
R_{ks} = conductive resistance of the container (negligible)
R_{ki} = conductive resistance of the insulation
R_{co} = exterior convective resistance
R_{ro} = exterior radiative resistance

From Equation (2.48)

$$R_{ki} = \frac{r_o - r_i}{4\pi k_i\,r_o\,r_i} \quad \text{where } r_o = \frac{D_s}{2} + s = 0.1\text{ m} + 0.05\text{ m} = 0.15\text{ m and } r_i = \frac{D_s}{2} = 0.1\text{ m}$$

$$\therefore\ R_{ki} = \frac{0.15\text{m} - 0.1\text{m}}{4\pi\left(0.037\text{ W/(m K)}\right)(0.15\text{m})(0.1\text{m})} = 7.17\ \text{K/W}$$

From Equation (1.14)

$$R_{co} = \frac{1}{\bar{h}_c\,A} = \frac{1}{\bar{h}_c\,4\pi r_o^2} = \frac{1}{\left(4.04\text{ W/(m}^2\text{K)}\right)4\pi(0.15\text{m})^2} = 0.875\ \text{K/W}$$

The exterior radiative resistance is

$$R_{ro} = \frac{T_\infty - T_s}{4\pi r_o^2\,\varepsilon\sigma\,(T_\infty^4 - T_s^4)} = \frac{293\text{K} - 253\text{K}}{4\pi(0.15\text{m})^2\,(0.8)\left(5.67\times10^{-8}\text{W/(m}^2\text{K}^4)\right)[(293\text{K})^4 - (253\text{K})^4]}$$

$$R_{ro} = 0.953\ \text{K/W}$$

The net resistance for the thermal network is $R_t = R_{ki} + R_o$ where

$$R_o = \frac{R_{co}\ R_{ro}}{R_{co} + R_{ro}} = \frac{(0.875\,\text{K/W})\,(0.953\,\text{K/W})}{0.875\,\text{K/W} + 0.953\,\text{K/W}} = 0.46 \text{ K/W}$$

$$R_t = 7.17 \text{ K/W} = 0.46 \text{ K/W} = 7.63 \text{ K/W}$$

The rate of heat transfer is given by

$$q = \frac{T_\infty - T_a}{R_t} = \frac{293\,\text{K} - 133\,\text{K}}{7.63\,\text{K/W}} = 20.97 \text{ W}$$

The accuracy of the insulation surface temperature guess can be checked from

$$q = \frac{T_\infty - T_{so}}{R_o} = \frac{293\,\text{K} - 253\,\text{K}}{0.46\,\text{K/W}} = 86.9 \text{ W} > 20.97 \text{ W}$$

Therefore, we need to reduce T_{so}. However, notice that nearly 94% of the total thermal resistance is due to the insulation. This means that adjusting T_{so} has little effect on the total rate of heat transfer. It also means that the heat gain by the liquid air can be most easily reduced by increasing the thickness of insulation, selecting an insulation with lower thermal conductivity, or both.

PROBLEM 5.29

A 2-cm-*OD* bare aluminum electric power transmission line with an emissivity of 0.07 carries 500 amps at 400 kV. The wire has an electrical resistivity of 1.72 micro-ohms cm^2/cm at 20°C and is suspended horizontally between two towers separated by 1 km. Determine the surface temperature of the transmission line if the air temperature is 20°C. What fraction of the dissipated power is due to radiation heat transfer?

GIVEN

- An aluminium electric power transmission line suspended horizontally
- Emissivity (ε) in air = 0.3
- Line diameter (D) = 2 cm = 0.02 m
- Current (I) = 500 amp
- Voltage (V) = 400 kV
- Electrical resistivity (ρ_e) = 1.72 Ω cm^2/cm at 20°C
- Space between towers (L) = 1 km
- Air temperature (T_∞) = 20°C = 293 K

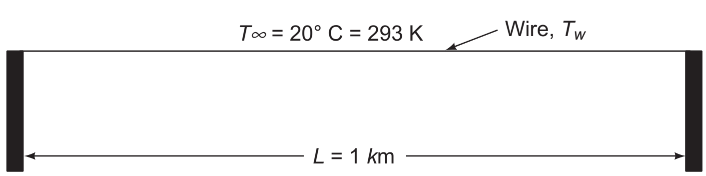
FIND

(a) Surface temperature of wire (T_w)
(b) Fraction of dissipated power due to radiation

ASSUMPTIONS

- Steady state
- The wire radiates to the surroundings which behave as a black body enclosure at T_∞

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/m^2 K^4.

SOLUTION

The power dissipation is given by Ohm's Law

$$P = I^2 R_c = I^2 \frac{\rho}{A_c} = \frac{4 I^2 \rho}{\pi D^2} = \frac{4\,(500\,\text{Amps})^2 \left(1.72\times10^{-6}\,\text{ohm cm}^2/\text{cm}\right)}{\pi\,(2\,\text{cm})^2}$$

$$= 0.1368\ \text{W/cm} = 13.68\ \text{W/m}$$

This must equal the rate of heat transfer by convection and radiation per meter

$$P = \pi D\,[h_c\,(T_w - T_\infty) + \varepsilon_w\,\sigma(T_w^{\,4} - T_\infty^{\,4})]$$

Since h_c varies with T_w, an iterative procedure must be used. For the first iteration, let T_w = 60°C.

From Appendix 2, Table 27, for dry air at the mean temperature of 40°C

Thermal expansion coefficient (β) = 0.00319 1/K

Thermal conductivity (k) = 0.0265 W/(m K)

Kinematic viscosity (ν) = 17.6 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Grashof number based on the wire diameter is

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00319\ 1/\text{K})\ (60\,°\text{C} - 20\,°\text{C})\ (0.02\,\text{m})^3}{\left(17.6\times10^{-6}(\text{m}^2/\text{s})\right)^2} = 3.23\times10^4$$

The Nusselt number for this geometry and Grashof number is given by Equation (5.20)

$$Nu_D = 0.53\ (Gr_D Pr)^{\frac{1}{4}} = 0.53\ [3.23\times10^4(0.71)]^{\frac{1}{4}} = 6.52$$

$$h_c = Nu_D\,\frac{k}{D} = 6.52\,\frac{\left(0.0265\ \text{W/(m K)}\right)}{0.02\,\text{m}} = 8.64\ \text{W/(m}^2\ \text{K)}$$

$$\therefore\ P = 13.68\ \text{W/m} = \pi\,(0.02\ \text{m})$$

$$\left[8.64\ \text{W/(m}^2\ \text{K)}\ (T_w - 293\,\text{K}) + 0.07\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)[T_w^{\,4} - (293\,\text{K})^4]\right]$$

Checking the units then eliminating them for clarity

$$3.97\times10^{-9}\ T_w^{\,4} + 8.64\ T_w - 2778 = 0$$

By trial and error T_w = 317 K = 44°C

Performing further iterations

Iteration #	2	3
T_w (°C)	44	46.6
Mean Temp. (°C)	32	33.33
β (1/K)	0.00328	0.00326
k W/(m K)	0.0259	0.0260
$\nu\times10^6$ (m^2/s)	16.8	17.0
Pr	0.71	0.71
$Gr_D\times10^{-4}$	2.19	2.35
Nu_D	5.92	6.02
h_c (W/(m^2 K))	7.66	7.83
T_w (°C)	47	46

The equilibrium surface temperature is 46°C.

(b) The rate of heat transfer by convection is

$$\frac{q_c}{L} = h_c\,\pi D\,(T_w - T_\infty) - 7.83\ \text{W/(m}^2\text{K)}\ \pi\ (0.02\ \text{m})\ (46°\text{C} - 20°\text{C}) = 12.79\ \text{W/m}$$

The rate of heat transfer by radiation is

$$\frac{q_r}{L} = \varepsilon\,\sigma\,\pi D(T_s^4 - T_\infty^4) = 0.07\ \left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\ \pi\ (0.02\ \text{m})$$

$$[(319\ \text{K})^4 - (293\ \text{K})^4] = 0.74\ \text{W}$$

As a check on the results

$$\frac{q_c}{L} + \frac{q_r}{L} = 12.79\ \text{W/m} + 0.74\ \text{W/m} = 13.53\ \text{W/m} \cong P$$

The fraction of the power dissipation by radiation is

$$\frac{\frac{q_r}{L}}{P} = \frac{0.74}{13.53} = 0.055 = 5.5\%$$

PROBLEM 5.30

An 20 cm *OD* horizontal steam pipe carries 1.66 kg/min dry saturated steam at 120°C. If ambient air temperature is 20°C, determine the rate of condensate flow at the end of 3 m of pipe. Use an emissivity of 0.85 for the pipe surface. If it is desired to keep heat losses below 1 percent of the rate of energy transport by the steam, what thickness of fiberglass insulation is required? The rate of energy transport by the steam is the heat of condensation of the steam flow. The heat of vaporization of the steam is 2210 kJ/kg.

GIVEN

- A horizontal steam pipe in air
- Pipe outside diameter (D) = 20 cm = 0.2 m
- Mass flow rate of steam (ms) = 1.66 kg/min = 0.0276 kg/s
- Steam temperature (T_s) = 393 K
- Ambient air temperature (T_∞) = 293 K
- Emissivity of pipe surface (ε) = 0.85
- Heat of vaporization (h_{fg}) = 2210 kJ/kg

FIND

(a) Rate of condensate flow (mc) at the end of 3 m of pipe.

(b) Thickness of fiberglass insulation (S) to keep loss below 1% of the energy transport by steam

ASSUMPTIONS

- Steady state
- Air is still
- Thermal resistance of the convection in the pipe and of the pipe wall are negligible
- The surroundings behave as an enclosure at T_∞
- Insulation is foil covered, its emissivity ≈ 0.0

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

From Appendix 2, Table 11, Thermal conductivity of fiberglass (ki) = 0.035 W/(m K)

From Appendix 2, Table 27, for dry air at the mean temperature of 70°C

Thermal expansion coefficient (β) = 0.0029 1/K
Thermal conductivity (k) = 0.0287 W/(m K)
Kinematic viscosity (ν) = 2.02×10^{-5} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

(a) The Grashof number for the uninsulated pipe is

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2}\ \frac{(9.81\,\text{m/s}^2 \times 2.9\times10^{-3}\,\text{1/K})(120-20)°\text{C}\,(0.2\,\text{m})^3}{(2.02\times10^{-5}\,\text{m}^2/\text{s})^2} = 5.58 \times 10^7$$

The rate of radiative heat less from the pipe surface

$$q_\text{r} = \pi DL\ \sigma\,\varepsilon\,(T_s^{\,4} - T_\infty^{\,4})$$
$$= \pi \times (0.2\text{ m})\ 3\text{ m} \times 5.67 \times 10^{-8}\text{ W/(m}^2\text{ K}^4)\ (0.85)\ [393^4 - 293^4]$$
$$= 1497\text{ W}$$

The convective heat transfer from the pipe surface can be found as

$$Nu_D = 0.53\ (Gr_D Pr)^{\frac{1}{4}} = 42.05$$

$$\therefore \qquad h_c = Nu_D \frac{K}{D} = 42.05 \times \frac{0.0287\,\text{W/(m K)}}{0.2\,\text{m}} = 6.03\text{ W/(m}^2\text{ K)}$$

Hence

$$q_c = \pi DL\ h_c\ (T_s - T_\infty)$$
$$= \pi\,(0.2\text{ m})\ 3\text{ m}\ (6.03\text{ W/(m}^2\text{ K))}\ (100\text{ K})$$
$$= 1136\text{ W}$$

$\therefore$ Total heat transfer

$$q_\text{total} = q_r + q_c = 1497\text{ W} + 1136\text{ W}$$

$$\Rightarrow \qquad q_\text{total} = 2633\text{ W}$$

If the rate of condensate flow at the end of pipe length be $\dot{m}_\text{cond}$,

then $$\underset{\text{(kg/s)}}{\dot{m}_\text{cond}} \times 2210 \times 10^3\text{ J/kg} = 2633\text{ W}$$

$$\Rightarrow \qquad \dot{m}_\text{cond} = 1.19 \times 10^{-3}\text{ kg/s} = 1.19\text{ g/s}$$

In presence of insulation, the maximum heat less equals 1% of energy transported by steam.

$$q = h_c\ \pi\,(D + 2s)\ L(T_{si} - T_\infty) = \frac{T_{si} - T_\infty}{R_k} = 0.01 \times \frac{1.66}{60}\text{ kg/s} \times 2210 \times 10^3\text{ J}$$
$$= 611\text{ W}$$

where $$R_k = \frac{\ln\left(\dfrac{D+2s}{D}\right)}{2\pi\ Lk_i}$$

Rearranging to eliminate the insulation thickness

$$s = \frac{q}{2h_c \pi L(T_{si} - T_\infty)} - \frac{D}{2} - q = \frac{2\pi \, Lk_i \, (T_{si} - T_\infty)}{\ln\left(\dfrac{h_c \, \pi DL(T_{si} - T_\infty)}{q}\right)}$$

Since h_c depends on the insulation surface temperature T_{si}, an iterative procedure must be used.
For the first iteration, let T_{si} = 49°C.
From Appendix 2, Table 27, for dry air at the mean temperature of 34.5°C

Thermal expansion coefficient (β) = 3.25 × 10^{-3} 1/K
Thermal conductivity (k) = 0.026 W/(m K)
Kinematic viscosity (ν) = 1.68 × 10^{-5} m^2/s
Prandtl number (Pr) = 0.71

Assuming the insulation is thin compared to the pipe radius

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.81\,\text{m/s}^2)(3.25\times10^{-3}\,\text{1/K})(49-20)\text{K}\,(0.2\,\text{m})^3}{(1.68\times10^{-5})^2}$$

$$= 2.6 \times 10^7$$

Now $$h_c = 0.53\;(Gr_D Pr)^{\frac{1}{4}}\,\frac{K}{D}$$

$$\Rightarrow \qquad h_c = 0.53\;(2.6\times10^7 \times 0.71)^{\frac{1}{4}}\;0.026\text{ W, }0.2\text{ m} = 4.51\text{ W/(m}^2\text{ K)}$$

$$\Rightarrow \qquad 611\text{ W} = \frac{2\pi\;(3\text{ m})\;(0.026\text{ W/(m K)})\;(T_{si}\text{ - }20)\text{K}}{\ln\left[\dfrac{(4.51\,\text{W/(m}^2\text{K))}\,\pi\,(0.2\,\text{m})\,(3\,\text{m})\,(T_{si}-20)}{611}\right]}$$

By trial and error, it gives T_{si} = 86.5°C

The surface temperature of the insulation is about 86°C

$$\therefore\; s = \left(\frac{611\,\text{W}}{2\,(4.51\,\text{W/(m}^2\text{K))}\,\pi\,(3\,\text{m})\,(86.5°\text{C} - 20°\text{C})} - \frac{0.20}{2}\right)\text{m}$$

$$\Rightarrow \qquad s = 0.8\text{ cm}$$

PROBLEM 5.31

A long steel rod (2 cm in diameter, 2 m long) has been heat-treated and quenched to a temperature of 100°C in an oil bath. In order to cool the rod further it is necessary to remove it from the bath and expose it to room air. Will the faster cool-down result from cooling the cylinder in the vertical or horizontal position? How long will the two methods require to allow the rod to cool to 40°C in 20°C air?

GIVEN

- A long steel rod in air
- Diameter (D) = 2 cm = 0.02 m
- Length (L) = 2 m
- Initial temperature ($T_{s,i}$) = 100°C
- Air temperature (T_∞) = 20°C

FIND

(a) Is it faster to cool the rod vertically or horizontally?
(b) Time for rod to cool to 40°C in each position

ASSUMPTIONS

- Steel is 1% carbon

PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, for 1% carbon steel

Thermal conductivity (ks) = 43 W/(m K)
Specific heat (c) = 473 J/(kg K)
Density (ρ) = 7801 Kg/m^3

From Appendix 2, Table 27, for dry air at the initial mean temperature of 60°C

Thermal expansion coefficient (β) = 0.003 1/K
Thermal conductivity (k) = 0.0279 W/(m K)
Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

As the temperature of the rod decreases, the heat transfer coefficient will also decrease. Therefore, a rough numerical integration will be used to estimate the cooling time.

Note that the air properties must be evaluated at each step.

Time (min)	5	10	15	20	30	40	42	60	76
Vertical h_c (W/(m^2 K))	5.96	5.80	5.65	5.50	5.34	5.06	4.78	4.29	
DT (°C)	7.8	6.8	6.0	5.3	9.4	7.3	11.6	5.7	
New T_s (°C)	92.2	85.4	79.4	74.0	64.6	57.3	45.7	40.0	
Horizontal h_c (W/(m^2 K))	10.15	9.73	9.37	9.02	8.67	8.02	7.54		
DT (°C)	13.2	10.6	8.6	7.0	11.5	7.6	1.4		
New T_s (°C)	86.8	76.2	67.6	60.6	49.1	41.5	40.1		

Cooling time: about 76 minutes in the vertical position, about 42 minutes in the horizontal position

An alternate method of solution uses the average heat transfer coefficients and the time-temperature history given by Equation (2.84). Evaluating the heat transfer coefficients when the rod has reached 40°C

Vertical: h_{cv}, final = 4.22 W/(m^2 K) $\rightarrow$ hcv, ave = 5.09 W/(m^2 K)
Horizontal: h_{ch}, final = 7.77 W/(m^2 K) $\rightarrow$ h_{ch}, ave = 8.96 W/(m^2 K)

The time required for the rod to cool to the temperature T_f is calculated by rearranging Equation (2.84)

Similarly for the horizontal position: $t = 2854\ s = 48$ min.

This more approximate technique yields cooling times about 10-14% greater than the numerical technique shown above.

PROBLEM 5.32

In petroleum processing plants, it is often necessary to pump highly viscous liquids such as asphalt through pipes. In order to keep pumping costs within reason, the pipelines are electrically heated to reduce the viscosity of the asphalt. Consider a 15-cm-*OD* uninsulated pipe and an ambient temperature of 20°C. How much power per meter of pipe length is necessary to maintain the pipe at 50°C? If the pipe is insulated with 5 cm of fiberglass insulation, what is the power requirement?

GIVEN

- An electrically heated pipe
- Diameter (D) = 15 cm = 0.15 m
- Pipe surface temperature (T_{sp}) = 50°C

FIND

(a) Power per meter (q_e/L) required with no insulation
(b) Power per meter required with 5 cm (0.05 m) of fiberglass insulation

ASSUMPTIONS

- The pipe is horizontal and in quiescent air
- Radiative heat transfer is negligible
- No heat is transferred to the fluid in the pipe

PROPERTIES AND CONSTANTS

From Appendix 2, Table *27*, for dry air at the mean temperature of 35°C

Thermal expansion coefficient (β) = 0.00325 1/K
Thermal conductivity (k) = 0.0262 W/(m K)
Kinematic viscosity (ν) = 17.1×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

From Appendix 2, Table 11, the thermal conductivity of fiberglass (k_{fg}) = 0.035 W/(m K)

SOLUTION

The correlation for the average heat transfer coefficient for this geometry is given by Equation (5.20). (Note that the criteria of 103 < GrD < 10^9 and Pr > 0.5 is satisfied.)

The thermal properties needed to evaluate h_c must be calculated at the mean of T_{si} and T_∞. Therefore, an iterative process is required.

For iteration #1, let T_{si} = 35°C

From Appendix 2, Table 27, for dry air at the mean temperature of 27.5°C

Thermal expansion coefficient (β) = 0.00333 1/K
Thermal conductivity (k) = 0.0256 W/(m K)
Kinematic viscosity (ν) = 16.4×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

Equation (5.20) gives the heat transfer coefficient

Performing further iterations using the same Procedure

Iteration #	2	3
T_{si} (°C)	23.9	25.2
Mean Temp. (°C)	22.0	22.6
β (1/K)	0.00339	0.00338
k (W/(m K))	0.0252	0.0253
$\nu \times 10^6$ (m^2/s)	15.9	15.9
Pr	0.71	0.71
$Gr_D \times 10^{-6}$	8.01	10.6
h_c (W/(m^2 K))	2.61	2.81
$L\ R_c$ (m K)/W	0.488	0.452
T_{si} (°C)	25.2	24.9

COMMENTS

The insulation has reduced the rate of heat loss by 84%.

PROBLEM 5.33

Estimate the rate of convective heat transfer across a 1 m tall double-pane window assembly in which the outside pane is at 0°C and the inside pane is at 20°C. The panes are spaced 2.5 cm apart. What is the thermal resistance ('*R*' value) of the window if the rate of radiative heat flux is 84 W/m^2?

GIVEN

- Double-pane window assembly
- Height (H) = 1 m
- Spacing (δ) = 2.5 cm = 0.025 m
- Pane temperatures
  - Inside (T_i) = 20°C
  - Outside (T_o) = 0°C
- Radiative heat flux (q_r/A) = 84 W/m^2

FIND

(a) The rate of convective heat transfer (q_c/A)
(b) The thermal resistance (R)

ASSUMPTIONS

- Steady state
- Conduction through the window frame is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 10°C

Thermal expansion coefficient (β) = 0.00354 1/K
Thermal conductivity (k) = 0.0244 W/(m K)
Kinematic viscosity (ν) = 14.8 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

(a) The aspect ratio for the window is

$$\frac{H}{\delta} = \frac{1\,\text{m}}{0.025\,\text{m}} = 40$$

The Grashof and Rayleigh numbers based on the spacing are

$$Ra_\delta = Gr_\delta Pr = 4.95 \times 10^4\,(0.71) = 3.51 \times 10^4$$

$$Gr_\delta = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)(0.00354\,1/\text{K})(20°\text{C} - 0°\text{C})(0.025\,\text{m})^3}{\left(14.8\times10^{-6}\ \text{m}^2/\text{s}\right)^2} = 4.95 \times 10^4$$

$$h_c = Nu_\delta \frac{k}{\delta} = 1.89\ \frac{\left(0.0244\ \text{W/(m K)}\right)}{0.025\ \text{m}} = 1.85\ \ \text{W/(m}^2\text{K)}$$

$$Nu_\delta \quad = 0.42\, Ra_\delta^{0.25}\, Pr^{0.012} \left(\frac{H}{\delta}\right)^{-0.3} = 0.42\,(3.51 \times 10^4)^{0.25}\,(0.71)^{0.012}\,(40)^{-0.3} = 1.89$$

The Nusselt number for an enclosed space with H/δ= 40 and $10^9 < Ra_\delta 10^7$ is given by Equation (5.29a)

The rate of heat transfer by convection is given by

$$\frac{q_c}{A} = h_c \left(T_i - T_o = \left(1.85 \text{ W/(m}^2\text{K)}\right) (20°\text{C} - 0°\text{C}\right) = 37.0 \text{ W}$$

(b)The R value must satisfy the following equation

$$\frac{q_{\text{total}}}{A} = \frac{T_i - T_o}{R} = \frac{q_c}{A} + \frac{q_r}{A} - R = \frac{T_i - T_o}{\frac{q_c}{A} + \frac{q_r}{A}} = \frac{20°C - 0°C}{(37 + 84)\text{ W/m}^2} = 0.165 \text{ m}^2\text{K/W}$$

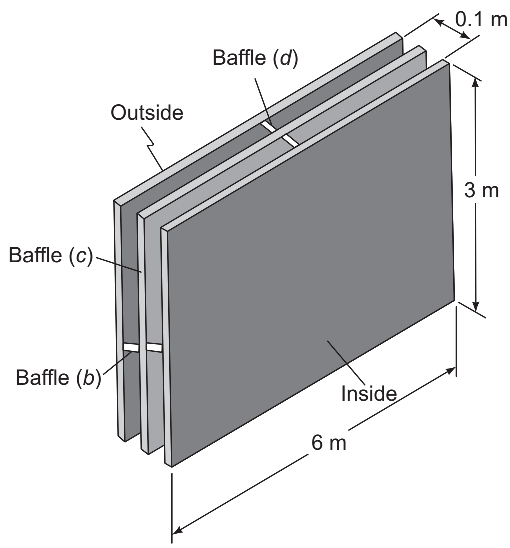
The R value is usually expressed in English units

$$\left(0.165 \text{ m}^2\text{K/W}\right) \left(\frac{0.5275 \text{ h°F/Btu}}{\text{K/W}}\right) \left(10.764 \text{ ft}^2/\text{m}^2\right) = 0.94 \text{ h ft}^2\text{°F/Btu}$$

The R value is approximately 1.

PROBLEM 5.34

An architect is asked to determine the heat loss through a wall of a building constructed as shown in the sketch. If the wall spacing is 10 cm, the inner surface is at 20°C and the outer surface is at – 8°C with air between, (a) estimate the heat loss by natural convection. Then determine the effect of placing a baffle (b) horizontally at the mid-height of the vertical section (*B*), (c) vertically at the center of the horizontal section (*C*), and (d) vertically half-way between the two surfaces (*D*).



GIVEN

- Air filled wall construction as shown above
- Inner wall temperature (T_i) = 20°C
- Outer wall temperature (T_o) = – 8°C
- Wall spacing (δ) = 10 cm = 0.1 m
- Wall height (L) = 3 m
- Wall width (w) = 6 m

FIND

The rate of heat loss by natural convection (q_c) for the wall

(a) without baffles
(b) with a horizontal baffle at a mid-height of the wall-baffle *B*
(c) with a vertical baffle at the center of the horizontal section-baffle *C*
(d) with a vertical baffle midway between the walls-baffle *D*

ASSUMPTIONS

- Wall temperatures are constant and uniform
- Steady state conditions
- Baffle thickness is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 6°C

Thermal expansion coefficient (β) = 0.00359 1/K

Thermal conductivity (k) = 0.0241 W/(m K)

Kinematic viscosity (ν) = 14.4×10^{-6} m²/s

Prandtl number (Pr) = 0.71

SOLUTION

(a) The Grashof and Rayleigh numbers based on the space between the walls (d) are

$$Ra_\delta = Gr_\delta Pr = 4.75 \times 10^6\ (0.71) = 3.37 \times 10^6$$

$$Gr_\delta = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00359\ 1/\text{K})\ (20°\text{C} + 8°\text{C})(0.1\text{m})^3}{\left(14.4 \times 10^{-6}\text{m}^2/\text{s}\right)^2} = 4.75 \times 10^6$$

The aspect ratio (L/δ) = (3 m)/(0.1 m) = 30

The correlation for this geometry is given by Equation (5.29a)

$$Nu_\delta = 0.42\ Ra_\delta^{0.25}\ Pr^{0.012}\left(\frac{L}{\delta}\right)^{-0.3} = 0.42(3.37 \times 10^6)^{0.25}\ (0.71)^{0.012}\ (30)^{-0.3} = 6.46$$

$$h_c = Nu_\delta \frac{k}{\delta} = 6.46\ \frac{(0.0241\ \text{W/(m K)})}{0.1\text{m}} = 1.56\ \ \text{W/(m}^2\text{K)}$$

The rate of heat loss is

$$q = h_c\, A\, (T_i - T_o) = \left(1.56\ \text{W/(m}^2\text{K)}\right)\ (3\ \text{m})\ (6\ \text{m})\ (20°\text{C} + 8°\text{C}) = 786\ \text{W}$$

$$h_c = Nu\delta \frac{k}{\delta} = 7.95\ \frac{(0.0241\ \text{W/(m K)})}{0.1\text{m}} = 1.92\ \ \text{W/(m}^2\text{K)}$$

(b) With baffles at mid-height, the Rayleigh number is unchanged, but L = 1.5 m, L/δ= (1.5 m)/(0.1 m) = 15

$$Nu_\delta = 0.42\ (3.37 \times 10^6)^{0.25}\ (0.71)^{0.012}\ (15)^{-0.3} = 7.95$$

$$q = \left(1.92\ \text{W/(m}^2\text{K)}\right)\ (3\ \text{m})\ (6\ \text{m})\ (20°\text{C} + 8°\text{C}) = 966\ \text{W}$$

These baffles actually increase the rate of heat transfer by 23%.

(c) The temperature of the vertical baffles is assumed to be approximately equal to the average of the wall temperatures (6°C). From Appendix 2, Table 27, for dry air at the mean temperatures for the two enclosed spaces

Mean Temperature (°C)	– 1°C (estimated) 13°C	
β (1/K)	0.00365	0.00350
k (W/(m K))	0.0236	0.0246
N × 10^6 (m²/s)	13.5	15.1
Pr	0.71	0.71

The Rayleigh numbers for the two sections are

$$Ra_\delta = Gr_\delta Pr = \frac{g\beta(T_s - T_\infty)\left(\frac{\delta}{2}\right)^3 Pr}{\nu^2}$$

For the inside section

$$Ra_\delta = \frac{(9.8 \text{ m/s}^2)(0.0035 \text{ 1/K})(20°\text{C} - 6°\text{C})(0.05 \text{ m})^3(0.71)}{\left(15.1 \times 10^{-6} \text{m}^2/\text{s}\right)^2} = 1.87 \times 10^5$$

For the outside section

$$Ra_\delta = \frac{(9.8 \text{ m/s}^2)(0.00365 \text{ 1/K})(6°\text{C} + 8°\text{C})(0.05 \text{ m})^3(0.71)}{\left(13.5 \times 10^{-6} \text{m}^2/\text{s}\right)^2} = 2.44 \times 10^5$$

$$R_{ci} = \frac{1}{h_c A} = \frac{1}{\left(1.25 \text{ W/(m}^2\text{K)}\right)(3 \text{ m})(6 \text{ m})} = 0.0443 \text{ K/W}$$

The aspect ratio is $L/\delta = 3/0.05 = 60$

$$Nu_\delta = 0.42\ (1.87 \times 10^5)^{0.25}\ (0.71)^{0.012}\ (60)^{-0.3} = 2.55$$

$$h_c = 2.55\ \frac{\left(0.0246 \text{ W/(m K)}\right)}{0.05 \text{ m}} = 1.25 \text{ W/(m}^2\text{K)}$$

Although this is beyond the range of the correlation, Equation (5.29a) will be used to estimate the Nusselt numbers

$$R_{ci} = \frac{1}{h_c A} = \frac{1}{\left(1.28 \text{ W/(m}^2\text{K)}\right)(3 \text{ m})(6 \text{ m})} = 0.0432 \text{ K/W}$$

Inside section

$$h_c = 2.72\ \frac{\left(0.0236 \text{ W/(m K)}\right)}{0.05 \text{ m}} = 1.28 \text{ W/(m}^2\text{K)}$$

Outside section

$$Nu_\delta = 0.42\ (2.44 \times 10^5)^{0.25}\ (0.71)^{0.012}\ (60)^{-0.3} = 2.72$$

These two thermal resistances are in series: therefore, the total resistance is their sum and the rate of heat transfer through the wall is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_i - T_o}{R_{ci} + R_{co}} = \frac{20°\text{C} + 8°\text{C}}{0.0443 \text{ K/W} + 0.0432 \text{ K/W}} = 319.8 \text{ W}$$

This represents a 59% decrease in rate of heat transfer from the unbaffled case.

(d) Since the width of the enclosed space does not enter into the calculation of the heat transfer coefficient, this baffle will have no effect on the rate of heat transfer.

PROBLEM 5.35

A flat plate solar collector of 3 m × 5 m area has an absorber plate that is to operate at a temperature of 70°C. To reduce heat losses, a glass cover is placed 0.05 m from the absorber and its operating temperature is estimated at 35°C. Determine the rate of heat loss from the absorber if the 3 m edge is tilted at angles of inclination from the horizontal of 0°, 30°, and 60°.

GIVEN

- A flat plate solar collector
- Area = 3 m × 5 m
- Absorber temperature (T_a) = 70°C
- Glass cover temperature (T_c) = 30°C
- Distance between absorber and cover (δ) = 0.05 m

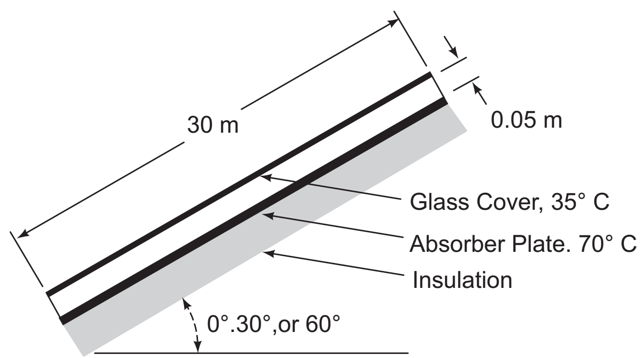
FIND

Heat loss by natural convection from the absorber of angles (θ) of (a) 0°, (b) 30°, and (c) 60° from the horizontal

ASSUMPTIONS

- The space is air filled

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 52.5°C

Thermal expansion coefficient (β) = 0.00307 1/K
Thermal conductivity (k) = 0.0274 W/(m K)
Kinematic viscosity (ν) = 18.7×10^{-6} m²/s
Prandtl number (Pr) = 0.71

SOLUTION

The Grashof and Rayleigh numbers for this geometry are

$$Gr_\delta = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00307\ 1/\text{K})(70°\text{C} - 35°\text{C})(0.05\,\text{m})^3}{(18.7\times10^{-6}\,\text{m}^2/\text{s})^2} = 3.76 \times 10^5$$

$$Ra_\delta = Gr_\delta Pr = 3.76 \times 10^5\ (0.71) = 2.67 \times 10^5$$

The heat transfer coefficient is given by Equation (5.31), where the quantities enclosed by [ ] are to be set to zero if they are negative: At $\theta = 0°$.

Since the aspect ratio (L/δ) = 3/0.05 = 60, the critical angle is 70°

$$h_c = 2.75\ \text{W/(m}^2\text{K)}$$

The rate of natural convective heat transfer is given by

$$q_c = h_c\, A(T_a - T_c) = \left(2.75\ \text{W/(m}^2\text{K)}\right)\ (3\ \text{m})\ (5\ \text{m})\ (70°\text{C} - 35°\text{C}) = 1444\ \text{W/m}$$

Performing a similar calculation for the other angles yields the following results

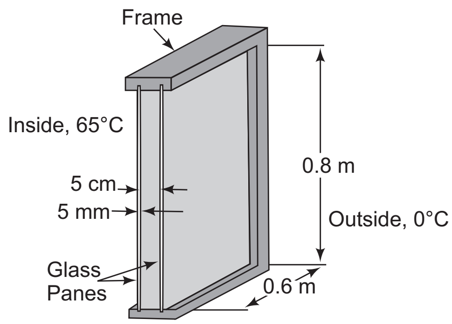
Angle, θ (degrees)	h_c (W/(m^2 K))	q_c (W)
0	2.75	1444
30	2.65	1390
60	2.32	1221

COMMENTS

Heat transfer by radiation will also be significant in this case.

PROBLEM 5.36

Determine the rate of heat loss through a double glazed window, as shown in the sketch, if the inside room temperature is 65°C and the average outside air is 0°C during December. Neglect the effect of the window frame.



If the house is electrically heated at a cost of \$0.06/(kW hr), estimate the savings achieved with a double glazed compared to a single glazed window during December.

GIVEN

- A double glazed window as shown
- Inside room temperature ($T_{\infty i}$) = 65°C
- Outside air temperature ($T_{\infty o}$) = 0°C
- Cost of heating = \$0.06/(kW hr)

FIND

(a) The rate of heat loss through a double glazed window
(b) The saving of double glazing over single glazing

ASSUMPTIONS

- Saving can be based on steady state analysis
- Inside and outside air is still
- Radiative heat transfer is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 11, thermal conductivity of window glass (kg) = 0.81 W/(m K)

SOLUTION

A single glazed window will be analyzed first

$$Gr_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00311\ 1/\text{K})(65°\text{C} - 32.5°\text{C})(0.8\,\text{m})^3}{\left(18.4\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.50\times10^9$$

Since the temperature of the glass is unknown, an iterative procedure is required. For the first iteration, let T_g = 32.5°C.

From Appendix 2, Table 27, for dry air

Mean Temperature (°C)	16.3	48.8
Thermal expansion coefficient, β (1/K)	0.00346	0.00311
Thermal conductivity, k (W/(m K))	0.0248	0.0271
Kinematic viscosity, $\nu \times 10^6$ (m²/s)	15.4	18.4
Prandtl Number, Pr	0.71	0.71

The Grashof number based on the window height is

Inside

$$Gr_H = \frac{g\beta(T_s - T_\infty) H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00311\ 1/\text{K})(65°\text{C} - 32.5°\text{C})(0.8\,\text{m})^3}{\left(18.4\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.50 \times 10^9$$

Outside

$$Gr_H = \frac{g\beta(T_s - T_\infty) H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00346\ 1/\text{K})(32.5°\text{C} - 0°\text{C})(0.8\,\text{m})^3}{\left(15.4\times10^{-6}\ \text{m}^2/\text{s}\right)^2} = 2.38 \times 10^9$$

Since $Gr_H > 10^9$, the Nusselt numbers are given by Equation (5.13)

$$Nu_L = 0.13\ (Gr_L Pr)^{\frac{1}{3}}$$

Inside

$$Nu_L = 0.13\left[2.38\times10^9(0.71)\right]^{\frac{1}{3}} = 154.8$$

$$h_c = Nu_L\ \frac{k}{L} = 154.8\ \frac{(0.0248\ \text{W/(m K)})}{0.8\ \text{m}} = 4.80\ \text{W/(m}^2\text{K)}$$

Outside

$$Nu_L = 0.13\left[2.38\times10^9(0.71)\right]^{\frac{1}{3}} = 154.8$$

$$h_c = Nu_L\ \frac{k}{L} = 154.8\ \frac{(0.0248\ \text{W/(m K)})}{0.8\text{m}} = 4.80\ \text{W/(m}^2\text{K)}$$

The rate of convection inside and outside must be the same

$$h_{ci}\, A\ (T_{\infty i} - T_g) = h_{co}\, A\ (T_g - T_{\infty o})$$

Solving for the glass temperature

$$T_g = \frac{h_{ci}\, T_{\infty i} + h_{co}\, T_{\infty o}}{h_{ci} + h_{co}} = \frac{\left(4.5\ \text{W/(m}^2\ \text{K)}\right)(65°\text{C}) + \left(4.8\ \text{W/(m}^2\ \text{K)}\right)(0°\text{C})}{(4.5 + 4.8)\ \text{W/(m}^2\text{K)}} = 31.5°\text{C}$$

This is close enough to the initial guess that the heat transfer coefficients will not be re-calculated. The thermal circuit for the single glazed window is shown below

where

$$R_{co} = \frac{1}{h_{co}A} = \frac{1}{\left(4.8\ \text{W/(m}^2\text{K)}\right)(0.8\,\text{m})(0.6\,\text{m})} = 0.434\ \text{K/W}$$

$$R_k = \frac{t_g}{k_g A} = \frac{0.005\,\text{m}}{\left(0.81\ \text{W/(m K)}\right)(0.8\,\text{m})(0.6\,\text{m})} = 0.0129\ \text{K/W}$$

$$R_{ci} = \frac{1}{h_{ci} A} = \frac{1}{\left(4.5\ \text{W/(m}^2\text{K)}\right)(0.8\,\text{m})(0.6\,\text{m})} = 0.463\ \text{K/W}$$

The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{\infty i} - T_{\infty o}}{R_{co} + R_k + R_{ci}} = \frac{65°\text{C} - 0°\text{C}}{(0.434 + 0.0129 + 0.463)\,\frac{\text{K}}{\text{W}}} = 71.4\ \text{W}$$

For the double glazed case, the temperature of the inside and outside panes must be estimated for the first iteration. Let T_{go} = 16°C and T_{gi} = 49°C (by symmetry).

From Appendix 2, Table 27, for dry air

	Outside	Enclosure	Inside
Mean Temperature (°C)	8	32.5	57
β (1/K)	0.00356	0.00327	0.00303
k (W/(m K))	0.0243	0.0260	0.0277
$\nu \times 10^6$ (m²/s)	14.6	16.9	19.1
Pr	0.71	0.71	.071

The Grashof numbers are

Inside

$$Gr_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00303\ 1/\text{K})(65°\text{C} - 49°\text{C})(0.8\,\text{m})^3}{\left(19.1\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 6.67\times10^8$$

Outside

$$Gr_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00356\ 1/\text{K})(16°\text{C} - 0°\text{C})(0.8\,\text{m})^3}{\left(14.6\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.34\times10^9$$

Enclosed space

$$Gr_\delta = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00327\ 1/\text{K})(49°\text{C} - 16°\text{C})(0.5\,\text{m})^3}{\left(16.9\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 4.63\times10^5$$

$$Ra_\delta = Gr_\delta Pr = 4.63\times10^5\,(0.71) = 3.29\times10^5$$

Using the correlation in Equation (5.13)

Inside

$$Nu_L = 0.13\ (Gr_L Pr)^{\frac{1}{3}} = 0.13\left[6.67\times10^8(0.71)\right]^{\frac{1}{3}} = 101$$

$$h_{ci} = Nu_L\,\frac{k}{L} = 101\,\frac{\left(0.0277\ \text{W/(m K)}\right)}{0.8\ \text{m}} = 3.51\ \text{W/(m}^2\text{K)}$$

Outside

$$Nu_L = 0.13\left[1.34\times10^9(0.71)\right]^{\frac{1}{3}} = 128$$

$$h_{co} = Nu_L\,\frac{k}{L} = 128\,\frac{\left(0.0243\ \text{W/(m K)}\right)}{0.8\ \text{m}} = 3.88\ \text{W/(m}^2\text{K)}$$

Enclosed space: H/δ= 0.8 m/0.05 m = 16. Although Pr < 1, the correlation in Equation (5.29a) will be applied to estimate the Nusselt number for the enclosed space

$$Nu_\delta = 0.42\, Ra_\delta^{0.25}\, Pr^{0.012} \left(\frac{H}{\delta}\right)^{-0.3} = 0.42\,(3.29 \times 10^5)^{0.25}\,(0.71)^{0.012}\,(16)^{-0.3} = 4.36$$

$$H_{c\delta} = Nu_\delta \frac{k}{L} = 4.36\, \frac{(0.0260\ \text{W/(m K)})}{0.05\ \text{m}} = 2.27\ \text{W/(m}^2\text{K)}$$

The thermal circuit for the double glazed window is shown below

The thermal resistance of the glass (Rk_g) is the same as the single glazed case. The remaining thermal resistances are

$$R_{ci} = \frac{1}{h_{ci}A} = \frac{1}{(3.51\ \text{W/(m}^2\ \text{K)})(0.8\,\text{m})\ (0.6\,\text{m})} = 0.594\ \text{K/W}$$

$$R_{c\delta} = \frac{1}{h_{c\delta}A} = \frac{1}{(2.27\ \text{W/(m}^2\text{K)})(0.8\,\text{m})\ (0.6\,\text{m})} = 0.918\ \text{K/W}$$

$$R_{co} = \frac{1}{h_{co}A} = \frac{1}{(3.88\ \text{W/(m}^2\text{K)})(0.8\,\text{m})\ (0.6\,\text{m})} = 0.537\ \text{K/W}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{\infty i} - T_{\infty o}}{R_{ci} + R_c\delta + R_{co} + 2R_{\text{kg}}} = \frac{65°\text{C} - 0°\text{C}}{[0.594 + 0.918 + 0.537 + 2\,(0.0129)]\,\text{K/W}} = 31.3\ \text{W}$$

The inner and outer glass temperatures can be checked by the convection equations for those surfaces

Inside

$$q = h_{ci}\, A\, (T_i - T_{gi})$$

$$T_{gi} = T_i - \frac{q}{h_{ci}A} = 65°\text{C} - \frac{31.3\ \text{W}}{(3.51\ \text{W/(m}^2\text{K)})(0.8\,\text{m})(0.6\,\text{m})} = 46.4°\text{C}$$

Outside

$$T_{go} = T_o + \frac{q}{h_{co}A} = 0°\text{C} + \frac{31.3\ \text{W}}{(3.88\ \text{W/(m}^2\text{K)})(0.8\,\text{m})\ (0.6\,\text{m})} = 16.8°\text{C}$$

These are close enough to the initial guesses that another iteration is not warranted.

The savings of the double glazed over the single glazed window are

$$\text{Savings} = (q_{\text{double}} - q_{\text{single}})\ (\text{Cost of heating})$$

$$\text{Savings} = (71.4\ \text{W} - 31.3\ \text{W}) \left(\frac{\$0.06}{\text{kWh}}\right) \left(\frac{\text{kW}}{1000\,\text{W}}\right) (24\ \text{h/day}) = \frac{\$0.06}{\text{day}}$$

This one small double glazed window saves 6 cents per day.

COMMENTS

The two surfaces of each pane of glass will actually be at a slightly different temperature. This can be neglected because the convective resistance are an order of magnitude greater than the conductive resistance of the glass.

PROBLEM 5.37

Calculate the rate of heat transfer between a pain of concentric horizontal cylinders 20 mm and 126 mm in diameter. The inner cylinder is maintained at 37°C and the outer cylinder is maintained at 17°C.

GIVEN

- Concentric cylinders
- Smaller diameter (D_i) = 20 mm = 0.02 m
- Larger diameter (D_o) = 126 mm = 0.126 m
- Inner cylinder temperature (T_i) = 37°C
- Outer cylinder temperature (T_o) = 17°C

FIND

The rate of heat transfer (q)

ASSUMPTIONS

- Steady state
- The space between the cylinders is filled with air
- Radiative heat transfer is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 27°C

Thermal expansion coefficient (β) = 0.00333 1/K
Thermal conductivity (k) = 0.0256 W/(m K)
Kinematic viscosity (ν) = 16.4×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

$$b = \frac{D_o - D_i}{2} = \frac{0.126\,\text{m} - 0.02\,\text{m}}{2} = 0.053\text{ m}$$

The Grashof number based on the space between the cylinders is

$$Gr_b = \frac{g\beta(T_s - T_\infty)\,b^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.00333\text{ 1/K})(37°\text{C} - 17°\text{C})(0.053\,\text{m})^3}{\left(16.4\times10^{-6}\text{ m}^2/\text{s}\right)^2} = 3.61 \times 10^5$$

The Rayleigh number is

$$Ra_b = Gr_b\,Pr = 3.61 \times 10^5\,(0.71) = 2.57 \times 10^5$$

To use the correlation given in Equation (5.33), the following criteria must be satisfied

$$10 \le \left[\frac{\ln\left(\dfrac{D_o}{D_i}\right)}{b^{\frac{3}{4}}\left(\dfrac{1}{D_i^{\frac{3}{5}}} + \dfrac{1}{D_o^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]^4 Ra_b < 10^7$$

$$\left[\frac{\ln\left(\dfrac{0.126}{0.02}\right)}{(0.053)^{\frac{3}{4}}\left(\dfrac{1}{(0.02)^{\frac{3}{5}}} + \dfrac{1}{(0.126)^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]^4 (2.57 \times 10^5) = (0.6196)^4\,(2.57 \times 10^5) = 3.79 \times 10^4$$

The effective thermal conductivity of the air in the gap between the cylinders is given by Equation (5.33)

$$k_{\text{eff}} = 0.386\, k \left[\frac{\ln\left(\frac{D_o}{D_i}\right)}{b^{\frac{3}{4}} \left(\frac{1}{D_i^{\frac{3}{5}}} + \frac{1}{D_o^{\frac{3}{5}}} \right)^{\frac{5}{4}}} \right]^4 \left(\frac{Pr}{0.861 + Pr} \right)^{\frac{1}{4}} Ra_b^{\frac{1}{4}}$$

$$k_{\text{eff}} = 0.386\ \left(0.0386\ \text{W/(m K)}\right)\ [0.6196] \left(\frac{0.71}{0.861 + 0.71} \right)^{\frac{1}{4}} \left(2.57 \times 10^5\right)^{\frac{1}{4}} = 0.113\ \text{W/(m K)}$$

The rate of heat transfer is given by Equation (2.38)

$$q_k = \frac{T_o - T_i}{R_{th}}$$

where R_{th} is given by substituting k_{eff} for k in Equation (2.39)

$$R_{th} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\Pi \text{L} \text{k}_{\text{eff}}} = \frac{\ln\left(\frac{0.126}{0.02}\right)}{2\Pi \text{L}\left(0.113\ \text{W/(m}^2\ \text{K)}\right)} = 2.59\ \frac{1}{L}\ \text{mK/W}$$

$$\therefore\ q_k = \frac{37°\text{C} - 17°\text{C}}{2.59\ \frac{1}{L}\ \text{(mK/W)}} \Rightarrow \frac{q_k}{L} = 7.72\ \text{W/m}$$

PROBLEM 5.38

Two long concentric horizontal aluminum tubes of 0.2 m and 0.25 m diameter are maintained at 300 K and 400 K respectively. The space between the tubes is filled with nitrogen. If the surfaces of the tubes are polished to prevent radiation, estimate the rate of heat transfer for gas pressure of (a) 10 atm and (b) 0.1 atm in the annulus.

GIVEN

- Two concentric horizontal aluminum tubes with nitrogen between them
- Diameters:
  - $D_i = 0.2$ m
  - $D_o = 0.25$ m
- Temperatures:
  - $T_i = 300$ K
  - $T_o = 400$ K
- Surface of tubes is polished

FIND

The rate of heat transfer for
(a) Pressure (p_a) = 10 atm
(b) Pressure (p_b) = 0.1 atm

ASSUMPTIONS

- Steady state conditions
- Radiative heat transfer is negligible
- Only the density of the nitrogen is affected by the pressure

PROPERTIES AND CONSTANTS

From Appendix 2, Table 32, for Nitrogen at one atmosphere and the mean temperature of 350 K

Thermal expansion coefficient (β) = 0.00292 1/K
Thermal conductivity (k) = 0.02978 W/(m K)
Absolute viscosity (μ) = 19.91×10^{-6} (Ns)/m^2
Density (ρ) = 0.9980 Kg/m^3
Prandtl number (Pr) = 0.702

Correcting the density for Pressure

(a) At p_a = 10 atm

$$\frac{\rho_a}{\rho} = \frac{p_a}{1} \text{ atm} = 10 \quad \rho_a = 9.98 \text{ kg/m}^3 - \nu_a = \frac{\mu}{\rho_a} = 1.99 \times 10^{-6} \text{ m}^2/\text{s}$$

(b) At p_b = 0.1 atm

$$\frac{\rho_b}{\rho} = \frac{p_b}{1} \text{ atm} = 0.1 \qquad \rho_a = 0.0998 \text{ kg/m}^3 - \nu_b = \frac{\mu}{\rho_b} = 199 \times 10^{-6} \text{ m}^2/\text{s}$$

SOLUTION

The gap between the cylinders (b) = $(D_o - D_i)/2$ = 0.025 m

The Grashof and Rayleigh numbers based on the gap between the cylinders (b) are

Case (a)

$$Gr_b = \frac{g\beta(T_s - T_\infty)\,b^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.00292\text{ 1/K})(400\text{ K} - 300\text{ K})(0.025\text{ m})^3}{(1.99\times10^{-6}\text{ m}^2/\text{s})^2} = 1.13 \times 10^7$$

$$Ra_b = Gr_b\,Pr = 1.13 \times 10^7\,(0.702) = 7.93 \times 10^6$$

Case (b)

$Gr_b = 1.13 \times 10^3$ $Ra_b = 793$

To use the correlation of Equation (5.33), the following criteria must be met

For case (b)

$$10 \le \left[\frac{\ln\left(\frac{D_o}{D_i}\right)}{b^{\frac{3}{4}}\left(\frac{1}{D_i^{\frac{3}{5}}} + \frac{1}{D_o^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]^4 Ra_b < 10^7$$

$$\left[\frac{\ln\left(\frac{0.25}{0.2}\right)}{(0.025)^{\frac{3}{4}}\left(\frac{1}{(0.2)^{\frac{3}{5}}} + \frac{1}{(0.25)^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]^4 (793) = (0.4839)^4\,(793) = 59.8$$

For case (a): $(0.4838)^4\,(7.93 \times 10^6) = 4.35 \times 10^5$

Therefore, the condition is met for both cases.

The effective thermal conductivity of the gap is given by Equation (5.33)

$$k_{\text{eff}} = 0.386\, k \left[\frac{\ln\left(\dfrac{D_o}{D_i}\right)}{b^{\frac{3}{4}}\left(\dfrac{1}{D_i^{\frac{3}{5}}} + \dfrac{1}{D_o^{\frac{3}{5}}}\right)^{\frac{5}{4}}} \right] \left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} Ra_b^{\frac{1}{4}}$$

Case(a)

$$k_{\text{eff}} = 0.386\ \left(0.02978\ \text{W/(mK)}\right)\ [0.4839]\ \left(\frac{0.702}{0.861 + 0.702}\right)^{\frac{1}{4}} (793\times10^6)^{\frac{1}{4}} = 0.242\ \ \text{W/(mK)}$$

Case(b)

$$k_{\text{eff}} = 0.386\ (0.02978\ \text{W/(mK)})\ \ [0.4839]\ \left(\frac{0.702}{0.861 + 0.702}\right)^{\frac{1}{4}} (793)^{\frac{1}{4}} = 0.0242\ \ \text{W/(mK)}$$

The rate of heat transfer is given by Equations (2.38) and (2.39)

$$q = \frac{\Delta T}{R_{th}} = \frac{2\pi L k_{\text{eff}} (T_o - T_i)}{\ln\left(\dfrac{r_o}{r_i}\right)}$$

$$\frac{q}{L} = \frac{2\pi(400\,\text{K} - 300\,\text{K})}{\ln(0.25/0.2)}\ k_{\text{eff}} = (2815.7\text{K})\ k_{\text{eff}}$$

Case (a) q/L = 681 W/m   Case (b): q/L = 68.1 W/m

PROBLEM 5.39

A solar collector design consists of several parallel tubes each enclosed concentrically in an outer tube which is transparent to solar radiation. The tubes are thin walled with diameter of the inner and outer cylinders of 0.10 and 0.15 m respectively. The annular space between the tubes is filled with air at atmospheric pressure. Under operating condition the inner and outer tube surface temperatures are 70°C and 30°C respectively.

(a) What is the convective heat loss per meter of tube length?
(b) If the emissivity of the outer surface of the inner tube is 0.2 and the outer cylinder behaves as though it were a black body, estimate the radiation loss.
(c) Discuss design options for reducing the total heat loss.

GIVEN

- Thin walled concentric tubes with air atmospheric pressure between them
- Inner tube diameter (D_i) = 0.1 m
- Outer tube diameter (D_o) = 0.15 m
- Inner tube temperature (T_i) = 70°C = 343 K
- Outer tube temperature (T_o) = 30°C = 303 K
- Outer surface emissivity of inner tube (ε) = 0.2

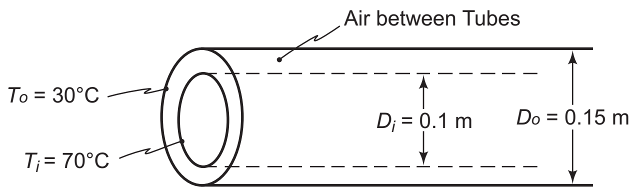
FIND

(a) The convective loss pe meter of tube (q_c/L)
(b) The radiative loss (q_r/L)
(c) Discuss design options for reducing the total heat loss

ASSUMPTIONS

- Steady state
- Tubes are horizontal

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Appendix 2, Table 27, for dry air at the mean temperature of 50°C

Thermal expansion coefficient (β) = 0.00310 1/K
Thermal conductivity (k) = 0.0272 W/(m K)
Kinematic viscosity (ν) = 18.5×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

(a) The characteristic length for the Problem is the air gap

$$b = \frac{D_o - D_i}{2} = \frac{0.15\,\text{m} - 0.1\,\text{m}}{2} = 0.025 \text{ m}$$

The Rayleigh number based on the characteristic length is

$$Ra_b = Gr_b\,Pr = \frac{g\,\beta(T_s - T_\infty)\,b^3 Pr}{\nu^2}$$

$$= \frac{(9.8\text{ m/s}^2)(0.0031\text{ 1/K})(70°\text{C} - 30°\text{C})(0.025\,\text{m})^3(0.71)}{\left(18.5\times10^{-6}\text{m}^2/\text{s}\right)^2} = 3.94 \times 10^4$$

The correlation for this geometry is given in Equation (5.33). Its use is restricted to the following condition

$$10 \le \left[\frac{\ln\left(\dfrac{D_o}{D_i}\right)}{b^{\frac{3}{4}}\left(\dfrac{1}{D_i^{\frac{3}{5}}} + \dfrac{1}{D_o^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]^4 Ra_b < 10^7$$

$$\left[\frac{\ln\left(\frac{0.15}{0.1}\right)}{(0.025)^{\frac{3}{4}}\left(\frac{1}{(0.1)^{\frac{3}{5}}}+\frac{1}{(0.15)^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]^4 (3.94 \times 10^4) = (0.556)^4\,(3.94 \times 10^4) = 3.77 \times 10^3$$

Therefore, the condition is met.
The effective thermal conductivity of the gap is

$$k_{\text{eff}} = 0.386\,k\left[\frac{\ln\left(\frac{D_o}{D_i}\right)}{b^{\frac{3}{4}}\left(\frac{1}{D_i^{\frac{3}{5}}}+\frac{1}{D_o^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]\left(\frac{Pr}{0.861+Pr}\right)^{\frac{1}{4}} Ra_b^{\frac{1}{4}}$$

$$k_{\text{eff}} = 0.386\,\big(0.0272\ \text{W/(m K)}\big)\ [0.556]\left(\frac{0.71}{0.861+0.71}\right)^{\frac{1}{4}}(3.94\times10^4)^{\frac{1}{4}} = 0.0674\ \text{W/(m K)}$$

The convective heat transfer per unit length across the gap is given by Equation (2.38) and (2.39)

$$\frac{q_c}{L} = \frac{(T_i - T_o)\,2\pi k_{\text{eff}}}{\ln\left(\frac{D_o}{D_i}\right)} = \frac{(70°\text{C} - 30°\text{C})\,2\pi\big(0.0674\ \text{W/(m K)}\big)}{\ln(0.15/0.1)} = 41.8\ \text{W/m}$$

(b) Since the inner tube is completely surrounded by the outer tube, the radiative heat transfer is given by Equation (1.17)

$$\frac{q_r}{L} = \pi D_i\,\varepsilon\,\sigma(T_i^4 - T_o^4) = \pi(0.1\ \text{m})\,(0.2)\big(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\big)\,[(343\ \text{K})^4 - (303\ \text{K})^4] = 19.3\ \text{W}$$

The total rate of heat transfer is the sum of the convective and radiative components

$$\frac{q_{\text{total}}}{L} = \frac{q_c}{L} + \frac{q_r}{L} = 41.8\ \text{W/m} + 19.3\ \text{W/m} = 61.1\ \text{W/m}$$

(c) Evacuating the space between the tubes would eliminate the convective heat transfer and thereby reduce the total rate of heat transfer by 67%. The heat loss could be further decreased by decreasing the emissivity of both cylinders.

PROBLEM 5.40

Liquid oxygen at – 183°C is stored in a thin walled spherical container with an outside diameter of 2 m. This container is surrounded by another sphere of 2.5 m inside diameter to reduce heat loss. The inner spherical surface has an emissivity of 0.05 and the outer sphere is black. Under normal operation the space between the spheres is evacuated. But due to an accident a leak developed in the outer sphere and the space is filled with air at one atm. If the outer sphere is at 25°C, compare the heat losses before and after the accident.

GIVEN

- A sphere filled with liquid oxygen surrounded by a larger sphere
- Sphere diameters:
  - $D_i = 2$ m
  - $D_o = 2.5$ m
- Emissivity of inner sphere (ε) = 0.05
- Outer sphere temperature (T_o) = 25°C = 298 K
- Liquid oxygen temperature (T_i) = –183° = 90 K
- Outer sphere is black

FIND

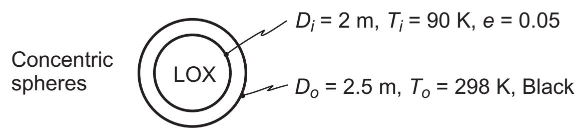
The rate of heat loss with

(a) A vacuum between the spheres
(b) Air at 1 atm between the spheres

ASSUMPTIONS

- Steady state
- The internal convective resistance and the resistance of the inner sphere wall are negligible

SKETCH



PROPERTIES AND CONSTANTS

The thermal expansion coefficient (β) ≈ $1/T$ = 1/(194 K) = 0.0052 1/K

Extrapolating from Appendix 2, Table 27, for dry air at the mean temperature of –79°C from values at 0°C and 20°C

Thermal conductivity (k) = 0.018 W/(m K)
Kinematic viscosity (ν) = 6.8×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) With the space evacuated, there will only be radiative heat transfer as given by Equation (1.17)

$$q_r = A_i\, \varepsilon_i\, \sigma (T_o^4 - T_i^4) = \pi D_i^2\, \varepsilon_1\, \sigma (T_o^4 - T_i^4)$$

$$q_r = \pi (2\text{m})^2\, (0.05)\left(5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\right) [(298\ \text{K})^4 - (90\text{K})^4] = 278.6\ \text{W}$$

The characteristic length for the problem is: $b = (D_o - D_i)/2$ = (2.5 m – 2.0 m)/2 = 0.25 m

The Rayleigh number is

$$Ra_b = Gr_b\, Pr = \frac{g\beta (T_o - T_i)\, b^3 Pr}{\nu^2} = \frac{(9.8\,\text{m/s}^2)(0.0052\ \text{1/K})(298\,\text{K} - 90\,\text{K})(0.25\,\text{m})^3 (0.71)}{\left(6.8 \times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 2.54 \times 10^9$$

(b) The following criteria must be satisfied to use Equation (5.34) for the convective heat transfer

$$10 \le \left[\frac{b}{(D_o - D_i)^4 \left(D_i^{-\frac{7}{5}} + D_o^{-\frac{7}{5}} \right)^5} \right] Ra_b < 10^7$$

$$\left[\frac{0.25\,\text{m}}{[(2\,\text{m})(2.5\,\text{m})]^4\left((2.5\,\text{m})^{-\frac{7}{5}}+(2.5\,\text{m})^{-\frac{7}{5}}\right)^5}\right]2.54\times10^9 = (0.0032879)\,(2.54\times10^9) = 8.36\times10^6$$

Therefore, the condition is met.

The effective thermal conductivity of the air space is

$$k_{\text{eff}} = 0.74\,k\left[\frac{b^{\frac{1}{4}}}{D_oD_i\left(D_i^{-\frac{7}{5}}+D_o^{-\frac{7}{5}}\right)^{\frac{5}{4}}}\right]Ra_b^{\frac{1}{4}}\left(\frac{Pr}{0.861+Pr}\right)^{\frac{1}{4}}$$

$$k_{\text{eff}} = 0.74\,(0.018\ \text{W/(mK)})\left[\frac{(0.25\,\text{m})^{\frac{1}{4}}}{(2\,\text{m})(2.5\,\text{m})\left((2\,\text{m})^{-\frac{7}{5}}+(2.5)^{-\frac{7}{5}}\right)^{\frac{5}{4}}}\right](2.54\times10^9)^{\frac{1}{4}}\left(\frac{0.71}{0.861+0.71}\right)^{\frac{1}{4}}$$

$$k_{\text{eff}} = 0.059\ \ \text{W/(m K)}$$

The total rate of heat transfer will be the sum of the convective and radiative heat transfer

$$q = q_c + q_r = \frac{T_o - T_i}{R_{\text{eff}}} + q_r$$

where R_{eff} is given by Equation (2.48)

$$R_{\text{eff}} = \frac{r_o - r_i}{4\pi k_{\text{eff}}\,r_or_i} = \frac{D_o - D_i}{2\pi k_{\text{eff}}\,D_oD_i} = \frac{0.5\,\text{m}}{2\pi(0.059\ \text{W/(m K)})(2\,\text{m})(2.5\,\text{m})} = 0.270\ \text{K/W}$$

$$q = \frac{298\text{K} - 90\text{K}}{0.270\ \text{K/W}} + 278.6\ \text{W} = 771.1\ \text{W} + 278.6\text{W} = 1050\ \text{W}$$

The leak causes the rate of heat loss to increase 3.8 times

COMMENTS

The rate of convective heat transfer is about 73% of the total rate of heat transfer.

PROBLEM 5.41

The surfaces of two concentric spheres having radii of 75 and 100 mm are maintained at 325 K and 275 K, respectively.

(a) If the space between the spheres is filled with nitrogen at 5 atm, estimate the convection heat transfer rate.

(b) If both sphere surfaces are black, estimate the total rate of heat transfer between them.

(c) Suggest ways to reduce the heat transfer.

GIVEN

- Concentric spheres with nitrogen between them
- Nitrogen pressure (p) = 5 atm
- Sphere radii
  - Inner sphere (r_i) = 75 mm = 0.075 m
  - Outer sphere (r_o) = 100 mm = 0.1 m
- Sphere temperatures
  - Inner sphere (T_i) = 325 K
  - Outer sphere (T_o) = 275 K
- Both spheres are black

FIND

(a) Convective heat transfer (q_c)
(b) Total heat transfer (q_{total})
(c) Suggest ways to reduce the heat transfer

ASSUMPTIONS

- Sphere temperatures are constant and uniform
- Only the density of the nitrogen is affected significantly by pressure
- The nitrogen behaves as an ideal gas

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann consant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Appendix 2, Table 32, for Nitrogen at atmospheric pressure and the mean temperature of 300 K

Thermal expansion coefficient (β) = 0.00333 1/K
Thermal conductivity (k) = 0.02620 W/(m K)

Absolute viscosity (μ) = 17.84×10^{-6} N s/m^2
Density (ρ) = 1.142 kg/m^3
Prandtl number (Pr) = 0.713

The density of the nitrogen at 5 atm can be calculated from the ideal gas law

$$\rho_2 = \frac{p_2}{p_1}\,\rho_1 = \frac{5\,\text{atm}}{1\,\text{atm}}\left(1.1421\,\text{kg/m}^3\right) = 5.7105\ \text{kg/m}^3$$

$$\therefore \quad \text{The kinematic viscosity } (\nu) = \frac{\mu}{\rho} = \frac{17.84\times10^{-6}\,(\text{Ns})/\text{m}^2}{5.7105\,\text{kg/m}^3} = 3.124\ 10^{-6}\ \text{m}^2/\text{s}$$

SOLUTION

(a) The effective thermal conductivity of the nitrogen is given by Equation (5.34)

$$k_{\text{eff}} = 0.74\,k\left[\frac{b^{\frac{1}{4}}}{D_o D_i\left(D_i^{-\frac{7}{5}} + D_o^{-\frac{7}{5}}\right)^{\frac{5}{4}}}\right] Ra_b^{\frac{1}{4}}\left(\frac{Pr}{0.861+Pr}\right)^{\frac{1}{4}}$$

where $b = r_o - r_i$ = 25 mm = 0.025 m

$$Ra_b = Gr_b\,Pr = \frac{g\beta(T_i - T_o)\,b^3 Pr}{\nu^2} = \frac{(9.8\,\text{m/s}^2)(0.00333\ 1/\text{K})(325\,\text{K} - 275\,\text{K})(0.025\,\text{m})^3(0.713)}{\left(3.124\times10^{-6}\,\text{m}^2/\text{s}\right)^2}$$

$$Ra_b = 1.86 \times 10^6$$

The following condition must be met to use the above correlation

$$10 \le \left[\frac{b}{(D_o D_i)^4 \left(D_i^{-\frac{7}{5}} + D_o^{-\frac{7}{5}} \right)^5} \right] Ra_b < 10^7$$

$$\left[\frac{0.025\,\text{m}}{[(0.2\,\text{m})(0.15\,\text{m})]^4 \left((0.15\,\text{m})^{-\frac{7}{5}} + (0.2\,\text{m})^{-\frac{7}{5}} \right)^5} \right] 1.86 \times 10^6 = 7.59 \times 10^3$$

Therefore, the condition is met.

$$k_{\text{eff}} = 0.74 \left(0.0262\ \text{W/(m K)}\right) \left[\frac{(0.025\,\text{m})^{\frac{1}{4}}}{(0.2\,\text{m})(0.15\,\text{m})\left((0.15\,\text{m})^{-\frac{7}{5}} + (0.2\,\text{m})^{-\frac{7}{5}}\right)^{\frac{5}{4}}} \right] (1.86 \times 10^6)^{\frac{1}{4}} \left(\frac{0.713}{0.861 + 0.713} \right)^{\frac{1}{4}}$$

$$k_{\text{eff}} = 0.148\ \text{W/(m K)}$$

The thermal resistance of the nitrogen is given by Equation (2.48)

$$R_{\text{eff}} = \frac{r_o - r_i}{4\pi k_{\text{eff}}\, r_o\, r_i} = \frac{0.1\,\text{m} - 0.075\,\text{m}}{4\pi\left(0.148\ \text{W/(m K)}\right)(0.1\,\text{m})(0.075\,\text{m})} = 1.792\ \text{K/W}$$

The rate of convective heat transfer is given by

$$q_c = \frac{\Delta T}{R_{\text{eff}}} = \frac{325\,\text{K} - 275\,\text{K}}{1.792\ \text{K/W}} = 27.9\ \text{W}$$

(b) The radiative heat transfer from a black body to a black body enclosure is given by Equation (1.16)

$$q_r = A_1\, \sigma(T_1^4 - T_2^4) = 4\,\pi r_i\, \sigma(T_1^4 - T_2^4)$$

$$q_r = 4\,\pi(0.075\ \text{m})^2 \left(5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)\right) ((325\text{K})^4 - (275\text{K})^4) = 21.8\ \text{W}$$

The total rate of heat transfer is the sum of the radiative and convective heat transfer

$$q_{\text{total}} = q_r + q_c = 21.8\ \text{W} + 27.9\ \text{W} = 49.7\ \text{W}$$

(c) The rate of heat transfer could be reduced in several ways, including

- Coating the spheres to reduce their emissivity, thereby decreasing the rate of radiative heat transfer.
- Partially or totally evacuating the space between the spheres to decrease the rate of convective heat transfer

PROBLEM 5.42

Estimate the rate of heat transfer from one side of a 2 m diameter disk rotating at 600 rev/min in 20°C air, if its surface temperature is 50°C.

GIVEN

- A disk rotating in air
- Diameter (D) = 2 m
- Rotational speed (ω) = 600 rev/min
- Air temperature (T_∞) = 20°C
- Surface temperature (T_s) = 50°C

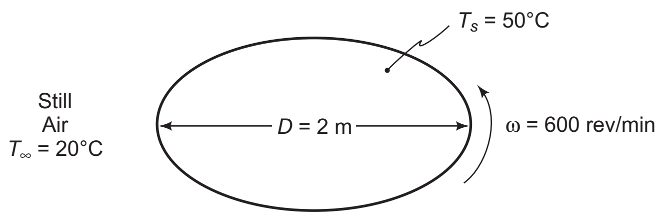
FIND

- The rate of heat transfer from one side (q)

ASSUMPTIONS

- The heat transfer has reached steady state
- The disk is horizontal
- Air is still

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 35°C

Thermal expansion coefficient (β) = 0.00325 1/K
Thermal conductivity (k) = 0.0262 W/(m K)
Kinematic viscosity (ν) = 17.1×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

The rotational Reynolds number for the disk is

$$Re_\omega = \frac{\omega D^2}{\nu} = \frac{(600\ \text{rev/min})(2\pi\ \text{rad/rev})(2\,\text{m})^2}{17.1\times10^{-6}\,\text{m}^2/\text{s}(60\ \text{s/min})} = 1.47 \times 10^7 > 10^6 \text{ (turbulent)}$$

The critical Reynolds number is given in Section 5.4

$$Re_\omega = 1 \times 10^6 = \frac{4 r_c^2\,\omega}{\nu} \quad \Rightarrow \quad r_c = \sqrt{\frac{(1\times10^6)\,\nu}{4\omega}}$$

$$= \sqrt{\frac{1\times10^6\left(17.1\times10^{-6}\,\text{m}^2/\text{s}\right)(60\ \text{s/min})}{4(600\ \text{rev/min})(2\pi\ \text{rad/rev})}} = 0.26 \text{ m}$$

The average heat transfer coefficient is given by Equation (5.38)

$$\bar{h}_c = \frac{k}{r_o}\left\{0.36\left(\frac{\omega r_o^2}{\nu}\right)^{\frac{1}{2}}\left(\frac{r_c}{r_o}\right)^2 + 0.015\left(\frac{\omega r_o^2}{\nu}\right)^{0.8}\left(1-\left(\frac{r_c}{r_o}\right)^{2.6}\right)\right\}$$

Since $\omega D^2/\nu = 1.47 \times 10^7$, $\omega r_o^2/\nu = 3.67 \times 10^6$ and $r_c/r_o = 0.26$. So

$$\bar{h}_c = \frac{(0.0262\ \text{W/(m K)})}{1\text{m}}\left[(0.36)(3.67\times10^6)^{\frac{1}{2}}(0.26)^2 + (0.015)(3.67\times10^6)^{0.8}(1-(0.26)^{2.6})\right]$$

$$= 69.3 \text{ W/(m}^2\text{K)}$$

The rate of heat transfer is

$$q = h_c\, A\, (T_s - T_\infty) = h_c\, \pi r_o^2\, (T_s - T_\infty) = \left(69.3\ \text{W/(m}^2\text{K)}\right)\ \pi (1\ \text{m})^2\ (50°\text{C} - 20°\text{C}) = 6535\ \text{W}$$

PROBLEM 5.43

A sphere 0.1 m diameter is rotating at 20 RPM in a large container of CO_2 at atmospheric pressure. If the sphere is at 60°C and the CO_2 at 20°C, estimate the rate of heat transfer.

GIVEN

- A rotating sphere in carbon dioxide at atmospheric pressure
- Diameter (D) = 0.1 m
- Speed of rotation (ω) = 20 rev/min
- Sphere temperature (T_s) = 60°C
- CO_2 temperature (T_∞) = 20°C

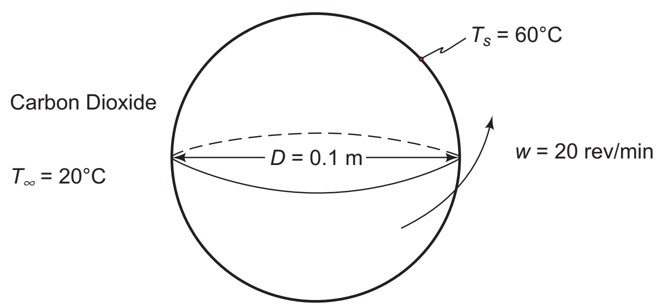
FIND

- The rate of heat transfer

ASSUMPTIONS

- Steady state conditions
- The carbon dioxide is still
- Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for CO_2 at the mean temperature of 40°C

Thermal expansion coefficient (β) = 0.00319 1/K

Thermal conductivity (k) = 0.0176 W/(m K)

Kinematic viscosity (ν) = 9.0×10^{-6} m^2/s

Prandtl number (Pr) = 0.77

SOLUTION

Converting the rotational speed to radians per second

$$\omega = \frac{(20\,\text{rev/min})(2\pi\,\text{rad/rev})}{60\ \text{s/min}} = 2.09\ \frac{1}{\text{s}}$$

The rotational Reynolds number for the sphere is

$$\omega = \frac{(20\ \text{rev/min})(2\pi\ \text{rad/rev})}{60\ \text{s/min}} = 2.09\ \frac{1}{\text{s}}$$

All requirements are met for the correlation presented in Equation (5.41)

$$\overline{Nu}_D = 0.43\, Re_\omega^{0.5}\, Pr^{0.4} = 0.43\,(2322)^{0.5}\,(0.77)^{0.4} = 18.67$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 18.67 \frac{0.0176 \text{ W/(m K)}}{0.1\text{m}} = 3.29 \text{ W/(m}^2\text{K)} \qquad (5.38)$$

The rate of heat transfer by natural convection is given by

$$q_c = \bar{h}_c\, A\,(T_s - T_\infty) = \bar{h}_c\, \pi D^2\,(T_s - T_\infty) = \left(3.29 \text{ W/(m}^2\text{ K)}\right)\, \pi (0.1 \text{ m})^2\,(60°\text{C} - 20°\text{C}) = 4.13 \text{ W}$$

PROBLEM 5.44

A mild steal (1% carbon), 2 cm *OD* shaft, rotating in 20°C air at 20,000 rev/min, is attached to two bearings 0.7 m apart. If the temperature at the bearings is 90°C, determine the temperature distribution along the shaft. Hint: Show that for the high rotational speeds equation (5.35) approaches: $\overline{Nu}_D = 0.086\,(\pi D^2 \omega/\nu)^{0.7}$

GIVEN

- A mild steel shaft rotating in air between two bearings
- Shaft diameter (D) = 2 cm = 0.02 m
- Rotational speed (ω) = 20,000 rev/min
- Air temperature (T_∞) = 20°C
- Length of shaft (L) = 0.7 m
- Bearing temperatures (T_b) = 90°C

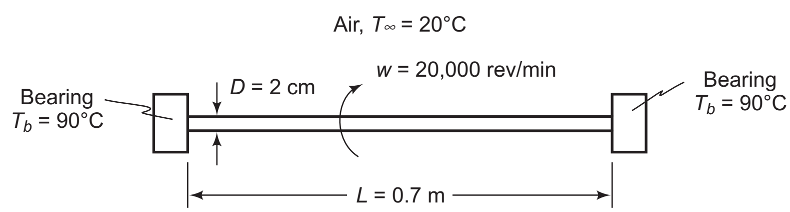
FIND

- The temperature distribution along the shaft

ASSUMPTIONS

- The rod has reached steady state
- Radiation is negligible
- The shaft is horizontal

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, thermal conductivity of 1% carbon steel (k_s) = 43 W/(m K)

SOLUTION

The Nusselt number for this geometry is given by Equation (5.35)

$$\overline{Nu}_D = 0.11\,(0.5\, Re_\omega^2 + Gr_D\, Pr)^{0.35}$$

where

$$Re_\omega = \frac{\pi \omega D^2}{\nu}$$

Evaluating the air properties at the mean of the air and bearing temperatures (55°C); from Appendix 2, Table 27

Thermal expansion coefficient (β) = 0.00305 1/K

Thermal conductivity (k) = 0.0276 W/(m K)

Kinematic viscosity (ν) = 19.0×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The rotational Reynolds number is

$$Re_\omega = \frac{\pi(2000\ \text{rev/min})(2\pi\ \text{rad/rev})(0.02\,\text{m})^2}{19.0\times10^{-6}\,\text{m}^2/\text{s}(60\ \text{s/min})} = 1.39 \times 10^5$$

$$Gr_D\,Pr = \frac{g\,\beta(T_b - T_\infty)D^3\,Pr}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00305\ 1/\text{K})\ (90\,°\text{C} - 20\,°\text{C})(0.02\,\text{m})^3(0.71)}{\left(19.0\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 3.29 \times 10^4$$

For this problem, 0.5 $Re_\omega^2 >> Gr_D\,Pr$ because of the high rotational speed, therefore. $Gr_D\,Pr$ can be neglected and the Nusselt number is given by

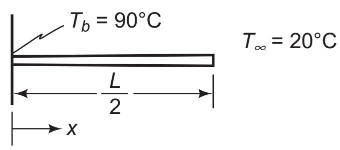
$$\overline{Nu}_D = 0.11\ (0.5\ Re_\omega^{2})^{0.35} = 0.0863\ 1Re_\omega^{0.7}$$

Based on the average of the air and bearing temperatures

$$\overline{Nu}_D = 0.0863\ (1.39 \times 10^5)^{0.7} = 344$$

$$\bar{h}_c = \overline{Nu}_D\ \frac{k}{D} = 344\ \frac{0.0276\ \text{W/(m K)}}{0.02\,\text{m}} = 474\ \ \text{W/(m}^2\text{K)}$$

By symmetry, the axial conduction at the center of the shaft must be zero and the shaft can be treated as two pin fins with adiabatic tips as shown below



The temperature distribution for this configuration is given in Table 2.1

$$\frac{T - T_\infty}{T_b - T_s} = \frac{\cosh\,[m(L_f - x)]}{\cosh\,(m L_f)} \qquad \left(L_f = \frac{\text{L}}{2}\right)$$

$$\text{where m} = \sqrt{\frac{h_c P}{k_s A_c}} = \sqrt{\frac{h_c\,\pi D}{k_s \pi / 4\ D^2}} = \sqrt{\frac{4 h_c}{D k_s}} = \sqrt{\frac{4\left(474\ \text{W/(m}^2\text{K)}\right)}{0.02\,\text{m}(43\ \text{W/(m K)})}} = 47.0\ 1/\text{m}$$

$$T = T_\infty + (T_b - T_s)\left[\frac{\cosh\,[m(L_f - x)]}{\cosh\,(m L_f)}\right] = 20°\text{C} + (90°\text{C} - 20°\text{C})\left[\frac{\cosh\,[47.0\ 1/\text{m}(0.35\,\text{m} - x)]}{\cosh[(47.0\ 1/\text{m})(0.35\,\text{m})]}\right]$$

$$T = 20°\text{C} + (1.0 \times 10^{-5}\,°\text{C}\ \cosh\,(16.5 - 47.0\,x)$$

The average rod temperature is given by

$$T_{\text{ave}} = \frac{1}{\frac{L}{2}}\int_0^{L/2} T\,dx$$

Let $A = 1.0 \times 10^{-5}$ °C and $y = 16.5 - 47.0\,x$ then: $dy = -\,47.0\,dx$

$$\text{when } x = \frac{L}{2},\ y = 0$$

$$\text{when } x = 0,\ y = 16.5$$

$$T_{ave} = \frac{-2}{47.0L}\int_{16.5}^{0} [20 + A\cosh(y)]\,dy = \frac{-2}{47.0L}\,[20\,y + A\sinh(y)]^{0}_{16.5}$$

$$T_{ave} = \frac{-2}{47.0L}\,[-20(16.5) - (1.0\times10^{-5})\sinh(16.5)] = 24.5°\text{C}$$

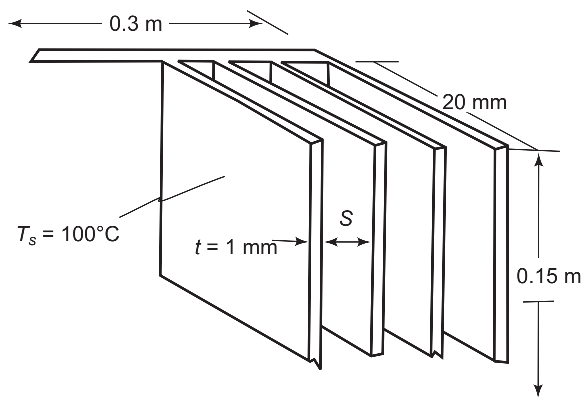
Using the mean of the air temperature and the average shaft temperature to evaluate the air properties and re-evaluating the temperature profile

$$T_{mean} = 22.2°\text{C}$$
$$k = 0.0253 \text{ W/(m K)}$$
$$\nu = 15.9\times10^{-6} \text{ m}^2\text{/s}$$
$$Re_{\omega} = 1.6\times10^{5}$$
$$h_c = 491 \text{ W/(m}^2\text{ K)}$$
$$m = 47.8 \text{ 1/m}$$
$$T = 20°\text{C} + (7.59\times10^{-6}\,°\text{C})\cosh(16.7 - 47.8\,x)$$
$$T_{ave} = 24.0°\text{C}$$

where x = distance in meters from a bearing up to $L/2$.

PROBLEM 5.45

An electronic device is to be cooled in air at 20°C by an array of equally spaced vertical rectangular fins as shown in the sketch below. The fins are made of aluminum and their average temperature, T_s, is 100°C.



Estimate **(a) The optimum spacing, *s***
(b) The number of fins
(c) The rate of heat transfer from one fin
(d) The total rate of heat dissipation
(e) Is the assumption of a uniform fin temperature justified?

GIVEN

- Electronic device with vertical aluminum fins in air
- Air temperature (T_∞) = 20°C
- Average fin temperature (T_s) = 100°C

FIND

(a) The optimum spacing (s)
(b) The number of fins
(c) The rate of heat transfer from one fin
(d) The total rate of heat dissipation
(e) Is the assumption of a uniform fin temperature justified?

ASSUMPTIONS

- Steady state
- Uniform fin temperature
- The air is still
- Heat transfer from the top and bottom of the fins is negligible
- The heat transfer coefficient on the wall area between the fins is approximately the same as on the fins

PROPERTIES AND CONSTANTS

From Appendix 2, Table 12

For aluminum: Thermal conductivity (k_{al}) = 239 W/(m K) at 100°C

From Appendix 2, Table 27, for dry air at the mean temperature of 60°C

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m²/s

Prandtl number (Pr) = 0.71

SOLUTION

The Grashof number for the fins, based on vertical height of the fin (L) is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.003\ \text{1/K})\,(100\,°\text{C} - 20\,°\text{C})\,(0.15\,\text{m})^3}{\left(19.4\times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 2.11 \times 10^7$$

Therefore, the Rayleigh number is

$$Ra_L = Gr_L\,Pr = 2.11 \times 10^7\,(0.71) = 1.50 \times 10^7$$

(a) The optimum fin spacing (s) is given by Equation (5.56a)

$$s = \frac{2.7}{P^{0.25}}$$

$$\text{where}\quad P = \frac{Ra_L}{L^4} = \frac{1.50\times 10^7}{(0.15\,\text{m})^4} = 2.96 \times 10^{10}\ \text{1/m}^4$$

therefore, $s = 0.0065$ m = 6.5 mm

(b) Let n = the number of fins on the device, then

$$n\,t + (n-1)\,s = 0.3\ \text{m}$$

$$n = \frac{0.3\,\text{m} + s}{s + t} = \frac{0.3\,\text{m} + 0.0065\,\text{m}}{0.0065\,\text{m} + 0.001\,\text{m}} = 40.9$$

40 fins will fit on the device with optimum spacing.

(c) The average heat transfer coefficient over a fin is given in Table 5.1

$$h_c = \frac{k}{s}\left[\frac{576}{P^2\,s^8} + \frac{2.873}{P^{\frac{1}{2}}\,s^2}\right]^{-\frac{1}{2}}$$

$$h_c = \frac{\left(0.0279\ \text{W/(m K)}\right)}{0.0065\,\text{m}}\left[\frac{576}{\left(2.96\times 10^{10}\,\text{1/m}^4\right)^2(0.0065\,\text{m})^3} + \frac{2.87}{\left(2.96\times 10^{10}\,\text{1/m}^4\right)^{\frac{1}{2}}(0.0065\,\text{m})}\right]^{-\frac{1}{2}}$$

$$h_c = 5.78\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer from a single fin is

$$q_f = h_c\, A_f\, (T_s - T_\infty) = 5.78\ \text{W/(m}^2\text{K)}\ [0.15\ \text{m}\ (0.041\ \text{m})]\ (100°\text{C} - 20°\text{C}) = 2.84\ \text{W}$$

(d) The total rate of heat dissipation is the sum of the heat transfer from the fins and the heat transfer from the wall area between the fins

$$q_{\text{total}} = \nu\, q_f + (\nu - 1)\, h_c\, A_w\, (T_s - T_\infty)$$

$$q_{\text{total}} = 40\ (2.84\ \text{W}) + 39\ \left(5.78\ \text{W/(m}^2\text{K)}\right)\ (0.15\ \text{m})(0.0065\ \text{m})\ (100°\text{C} - 20°\text{C})$$

$$q_{\text{total}} = 113.6\ \text{W} + 17.6\ \text{W} = 131.2\ \text{W}$$

(e) From Table 2.1, if the heat transfer from the tips of the fins is neglected, the temperature distribution along each fin is

$$\frac{T(x) - T_\infty}{T(0) - T_\infty} - \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

The temperature change along the fin is

$$\frac{T(0) - T(L)}{T(0) - T_\infty} = 1 - \frac{1}{\cosh(mL)}$$

$$\text{where } m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{5.78\ \text{W/(m}^2\text{K)}[2(0.15\,\text{m} + 0.001\,\text{m})]}{0.001\,\text{m}\,(0.15\,\text{m})\left(239\ \text{W/(m K)}\right)}} = 6.98\ \text{m}^{-1} L = 0.02\ \text{m}$$

$$\frac{T(0) - T(L)}{T(0) - T_\infty} = 1 - \frac{1}{\cosh[(6.98\,\text{m}^{-1})(0.02\,\text{m})]} = 0.966$$

Therefore, the assumption of an isothermal fin is justified.

PROBLEM 5.46

Consider a vertical 20 cm tall flat plate at 120°C suspended in a fluid at 100°C. If the fluid is being forced past the plate from above, estimate the fluid velocity for which natural convection becomes negligible (less than 10%) in: (a) mercury (b) air (c) water.

GIVEN

- A vertical flat plate suspended in a fluid
- Plate temperature (T_s) = 120°C
- Fluid temperature (T_∞) = 100°C
- Fluid is being forced past the plate from above
- Plate height (H) = 20 cm = 0.2 m

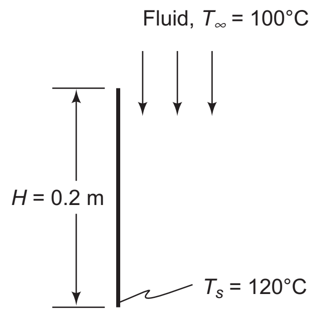
FIND

- The fluid velocity (U_∞) for which natural convection has a less than 10% effect in (a) mercury (b) air (c) water

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 25, 27 and 13

Fluid (at 110°C)	Mercury	Air	Water
Thermal expansion coefficient, β (1/K)	0.000182	0.00262	0.00080
Kinematic viscosity, $\nu \times 10^6$ m²/s	0.0913	24.8	0.269
Prandtl number, Pr	0.016	0.71	1.59

SOLUTION

From Equation (5.46), for laminar forced convection over a flat plate, the effect of buoyancy will be less than 10% if

$$Gr_H < 0.150\, Re_H^2 \Rightarrow \frac{g\,\beta(T_s - T_\infty)\,H^3}{\nu^2} < 0.150\left(\frac{U_\infty H}{\nu}\right)^2$$

Solving for the fluid velocity

$$U_\infty > [6.67\, g\,\beta(T_s - T_\infty)\,H]^{\frac{1}{2}}$$

$$U_\infty > \left[6.67\left(9.8\ \text{m/s}^2\right)\left(\beta\ (1/\text{K})\right)(120°\text{C} - 100°\text{C})\,(0.2\ \text{m})^{\frac{1}{2}}\right] = 16.17\ \beta^{\frac{1}{2}}\ \text{m/s}$$

(a) For mercury: $U_\infty < 16.17\,(0.000182)^{\frac{1}{2}} = 0.22$ m/s

(b) For air: $U_\infty < 16.17\ (0.00262)^{\frac{1}{2}} = 0.83$ m/s

(c) For water: $U_\infty < 16.17\ (0.0008)^{\frac{1}{2}} = 0.46$ m/s

The Reynolds numbers for these fluid velocities are

(a) For mercury: $Re_H = \dfrac{(0.22\ \text{m/s})\,(0.2\,\text{m})}{0.0913\times10^{-6}\,\text{m}^2/\text{s}} = 4.82 \times 10^5$

(b) For air: $Re_H = \dfrac{(0.83\ \text{m/s})(0.2\,\text{m})}{24.8\times10^{-6}\,\text{m}^2/\text{s}} = 6.69 \times 10^3$

(c) For water: $Re_H = \dfrac{(0.46\ \text{m/s})(0.2\,\text{m})}{0.269\times10^{-6}\,\text{m}^2/\text{s}} = 3.42 \times 10^5$

These Reynolds numbers are all within the laminar regime (mercury is approaching the transition to turbulence). Therefore, the use of Equation (5.46) was valid.

PROBLEM 5.47

Suppose a thin vertical flat plate. 60 cm high and 40 cm wide, is immersed in a fluid flowing parallel to is surface. If the plate is at 40°C and the fluid at 10°C, estimate the Reynolds number at which buoyancy effects are essentially negligible for heat transfer from the plate if the fluid is: (a) mercury, (b) air, and (c) water. Then calculate the corresponding fluid velocity for the three fluids.

GIVEN

- A thin flat plate immersed in a fluid flowing parallel to its surfaces
- Plate height (H) = 60 cm = 0.6 m
- Plate width (w) = 40 cm = 0.4 m
- Plate temperature (T_s) = 40°C
- Fluid temperature (T_∞) = 10°C

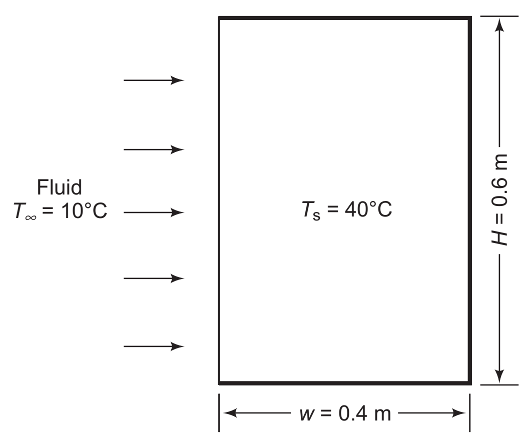
FIND

- The Reynolds number and corresponding fluid velocity (U_∞) for buoyancy effects to be negligible, if the fluid is (a) mercury, (b) air, (c) water

ASSUMPTIONS

- Steady state conditions

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 25, 27 and 13 at the mean temperature of 25°C

Fluid	Mercury	Air	Water
Thermal expansion coefficient, β (1/K)	—	0.00336	0.000255
Kinematic viscosity, $\nu \times 10^6$ m²/s	0.112	16.2	0.884
Density, ρ (kg/m³):	13,628 (0°C) 13,506 (50°C)		

The thermal expansion coefficient of mercury can be estimated from

$$\beta \cong \frac{2}{\rho_0 + \rho_{50}} = \left(\frac{\rho_0 - \rho_{50}}{273\,\mathrm{K} - 323\,\mathrm{K}}\right) = \frac{2}{(13,658 + 13,506)\ \mathrm{kg/m^3}} \left(\frac{(13,658 - 13,506)\ \mathrm{kg/m^3}}{273\,\mathrm{K} - 323\,\mathrm{K}}\right)$$

$$= 0.00018\ 1/\mathrm{K}$$

SOLUTION

The Grashof number based on height is

$$Gr_H = \frac{g\,\beta(T_s - T_\infty)\,H^3}{\nu^2}$$

For mercury

$$Gr_s = \frac{(9.8\ \text{m/s}^2)(0.00018\ \ 1/\text{K})\,(40°\text{C} - 10°\text{C})\,(0.6\,\text{m})^3}{\left(0.112\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 9.11\times10^{11}$$

For this geometry, the ratio that must be satisfied for the natural convection to have an essentially negligible effect is given at the end of Section 5.5 as

$$\frac{Gr_H}{Re_w^2} < 0.7 \Rightarrow Re_w = \frac{U_\infty\, w}{\nu} > 1.20\ \ Gr_H^{\frac{1}{2}}$$

For mercury

$$Re_w > 1.20(9.11\times10^{11})^2 = 1.15\times10^6$$

$$\therefore\ U_\infty = Re_w\,\frac{\nu}{w} = 1.15\times10^6\ \frac{0.112\times10^{-6}\,\text{m}^2/\text{s}}{0.4\,\text{m}} = 0.321\ \text{m/s}$$

Applying a similar analysis to the other fluids yields the following results

Fluid	Mercury	Air	Water
Gr_H	9.11×10^{11}	8.13×10^{8}	2.07×10^{10}
Re_w	1.15×10^{6}	3.42×10^{4}	1.72×10^{5}
U_∞ (m/s)	0.32	5.54	0.382

PROBLEM 5.48

A vertical isothermal plate 30 cm high is suspended in an atmosphere air stream flowing at 2 m/s in a vertical direction. If the air is at 16°C, estimate the plate temperature for which the natural-convection effect on the heat transfer coefficient will be less than 10 per cent.

GIVEN

- A vertical isothermal plate is an atmospheric air stream
- Plate height (L) = 30 cm = 0.3m
- Air velocity (U_∞) = 2 m/s (vertically)
- Air temperature (T_∞) = 16°C

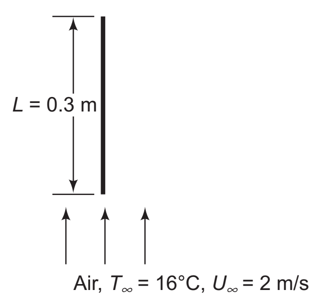
FIND

- The plate temperature (T_s) for which natural convection effect on the heat transfer coefficient will be less than 10%.

ASSUMPTIONS

- Steady State

SKETCH



SOLUTION

The average of the air and plate surfaces must be used to evaluate the fluid properties. Since the surface temperature is not known, the problem will first be solved by guessing the plate surface temperature. This temperature will be used to evaluate fluid properties. The resulting plate temperature will be used to update the fluid properties. For the first iteration, let T_s = 30°C. Therefore, the fluid properties will be evaluated at (30°C + 16°C)/2 = 23°C. From Appendix 2, Table 27

Thermal expansion coefficient (β) = 0.00338 1/K

Kinematic viscosity (ν) = 16.0×10^{-6} m^2/s

The Reynolds number for the top of the plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(2\text{ m/s})(0.3\text{ m})}{16.0\times 10^{-6}\text{m}^2/\text{s}} = 3.75 \times 10^4 < 5 \times 10^5 \text{ (laminar)}$$

By Equation (5.46), the natural convection effect will be less then 10% when

$$Gr_L < 0.150\, Re_L^2 \;\Rightarrow\; \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} < 0.150\left(\frac{U_\infty L}{\nu}\right)^2$$

Solving for the surface temperature

$$T_s < T_\infty + \frac{0.150\, U_\infty^2}{L\, g\, \beta} = 16°\text{C} + \frac{0.15(2\text{ m/s})^2}{(0.3\text{ m})(9.8\text{ m/s}^2)(0.00338\text{ 1/K})} = 76.4°\text{C}$$

Re-evaluating the thermal equation coefficient at the mean temperature of 46.2°C

β = 0.00313 1/K

$$T_s = 16°\text{C} + \frac{0.15(2\text{ m/s})^2}{0.3\text{ m}\left(9.8\text{ m/s}^2\right)(0.00313\text{ 1/K})} = 81.2°\text{C}$$

Performing one more iteration

At T_{avg} = 48.6°C, β = 0.00311 1/K

$$T_s = 16°\text{C} + \frac{0.15(2\text{ m/s})^2}{0.3\text{ m}\left(9.8\text{ m/s}^2\right)(0.00311\text{ 1/K})} = 81.6°\text{C}$$

For all surface temperatures

$$T_s \leq 81.6°\text{C}$$

natural convection heat transfer will contribute less than 10% to the total heat transfer.

PROBLEM 5.49

A horizontal disk 1 m in diameter rotates in air at 25°C. If the disk is at 100°C, estimate the RPM at which natural convection for a stationary disk becomes less than 10% of the heat transfer for a rotating disk.

GIVEN

- A rotating horizontal disk in air
- Diameter (D) = 1 m
- Air temperature (T_∞) = 25°C
- Disk temperature (T_s) = 100°C

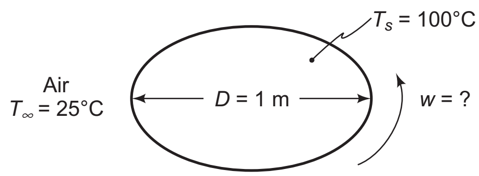
FIND

- The rotational speed (ω) at which natural convection becomes less than 10% of the thermal effects of rotation

ASSUMPTIONS

- Steady state conditions

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 62.5°C

Thermal expansion coefficient (β) = 0.00297 1/K

Thermal conductivity (k) = 0.0281 W/(m K)

Kinematic viscosity (ν) = 19.7×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The characteristic length for free convection from the stationary disk is

$$L_c = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4} = 0.25 \text{ m}$$

The Rayleigh number is

$$Ra_{Lc} = Gr_{Lc}\,Pr = \frac{g\,\beta(T_s - T_\infty)\,L_c^3\,Pr}{\nu^2}$$

$$= \frac{(9.8 \text{ m/s}^2)(0.00297 \text{ 1/K})(100°\text{C} - 25°\text{C})(0.25 \text{ m})^3(0.71)}{\left(19.7 \times 10^{-6} \text{m}^2/\text{s}\right)^2} = 6.24 \times 10$$

The Nusselt number for a static disk is given by Equation (5.16)

$$Nu_{Lc} = 0.15\ Ra_{Lc}^{\frac{1}{3}} = 0.15\ (6.24 \times 10^7)^{\frac{1}{3}} = 59.50$$

$$h_{\text{stat}} = Nu_{Lc}\frac{k}{L_c} = 59.50\ \frac{\left(0.0281 \text{ W/(m K)}\right)}{0.25 \text{ m}} = 6.69 \text{ W/(m}^2\text{K)}$$

Assuming the rotational speed is high enough to product turbulent flow, the Nusselt number is given by Equation (5.38)

$$\bar{h}_c = \frac{k}{r_o}\left\{0.36\left(\frac{\omega r_o^2}{\nu}\right)^{\frac{1}{2}}\left(\frac{r_c}{r_o}\right)^2 + 0.015\left(\frac{\omega r_o^2}{\nu}\right)^{0.8}\left(1 - \left(\frac{r_c}{r_o}\right)^{2.6}\right)\right\}$$

where $\frac{4 r_c^2 \omega}{\nu} = 10^6$

Since

$$h_{\text{rot}} = 10 \times 6.69 = 66.9 \ \text{W/(m}^2\text{K)}$$

we have

$$\frac{h_{\text{rot}}\, r_o}{k} = 1190$$

This can be written as

$$0.36\left(\frac{Re}{4}\right)^{\frac{1}{2}} \frac{Re_c}{Re} + 0.015\left(\frac{Re}{4}\right)^{0.8}\left(1 - \left(\frac{Re_c}{Re}\right)^{1.3}\right) = 1190$$

By trial and error: $Re = 5.64 \times 10^6$ (which is turbulent) and $\omega = 111$ rad/s = 1060 rpm.
Note that $r_c = 0.211$ m.

PROBLEM 5.50

The refrigeration system for an indoor ice rink is to be sized by an HVAC contractor. The refrigeration system has a COP (coefficient of performance) of 0.5. The ice surface is estimated to be –2°C and the ambient air is 24°C. Determine the size of the refrigeration system in kW required for a 110 m diameter circular ice surface.

GIVEN

- Round ice rink
- Diameter (D) = 110 m
- Ice surface temperature (T_s) = – 2°C
- Air temperature (T_∞) = 24°C
- COP of refrigeration system = 0.5

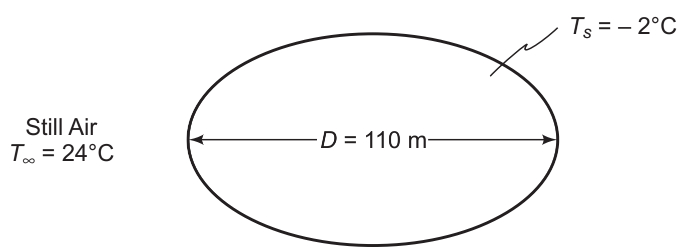
FIND

- Size of the refrigeration system required

ASSUMPTIONS

- Air is quiescent
- The effects of sublimation are negligible
- Radiation heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 11°C

Thermal expansion coefficient (β) = 0.00352 1/K
Thermal conductivity (k) = 0.0245 W/(m K)
Kinematic viscosity (ν) = 14.9×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

The characteristic length (L) for the ice rink is

$$L = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4} = \frac{110\,\text{m}}{4} = 27.5 \text{ m}$$

The grashof and Rayleigh numbers are

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)\left(0.00352\ (1/\text{K})\right)(24°\text{C} + 2°\text{C})(27.5\,\text{m})^3}{\left(14.9\times10^{-6}\text{m}^2/\text{s}\right)^2} = 8.40 \times 10^{13}$$

$$Ra_L = Gr_L\, Pr = 8.4 \times 10^{13}\ (0.71) = 5.97 \times 10^{13}$$

Although this is beyond the range of available horizontal plate correlations, the correlation will be extended to estimate the Nusselt number for the ice rink. The correlation for a cooled surface facing downward is Equation (5.16)

$$\overline{Nu}_L = 0.15\ Ra_L^{\frac{1}{3}} = 0.15\ (5.97\times10^{13})^{\frac{1}{3}} = 5862$$

$$\bar{h}_c = \overline{Nu}_L\ \frac{k}{L} = 5862\ \frac{\left(0.0245\ \text{W/(m K)}\right)}{27.5\,\text{m}} = 5.22\ \ \text{W/(m}^2\text{K)}$$

The rate of heat transfer to the rink is

$$q_c = \bar{h}_c\ A\ (T_\infty - T_s) = \bar{h}_c\ \frac{\pi}{4}\ D^2\ (T_s - T_\infty)$$

$$q_c = \left(5.22\ \text{W/(m}^2\text{K)}\right)\ \frac{\pi}{4}\ (110\text{ m})^2\ (24°\text{C} + 2°\text{C}) = 1.29 \times 10^6\text{ W} = 1290\text{ kW}$$

The size of the refrigeration unit (q_{ref}) is

$$q_{\text{ref}} = \frac{q_c}{\text{COP}} = \frac{1290\,\text{kW}}{0.5} = 2580\text{ kW}$$

PROBLEM 5.51

A 0.15 m square circuit board is to be cooled in a vertical position as shown. The board is insulated on one side while on the other, 100 closely spaced square chips are mounted, each of which dissipated 0.06 W of heat. The board is exposed to air at 25°C and the maximum allowable chip temperature is 60°C. Investigate the following cooling options

(a) Natural convection

(b) Air cooling with upward flow at a velocity of 0.5 m/s

(c) Air cooling with downward flow at the same velocity as (b)

GIVEN

- Square vertical circuit board insulated on one side, chips on the other side
- Length of each side (L) = 0.15 m
- Heat dissipation per chip (q) = 0.06 W
- Number of chips (N) = 100
- Ambient air temperature (T_∞) = 25°C
- Maximum allowable chip temperature (T_s) = 60°C

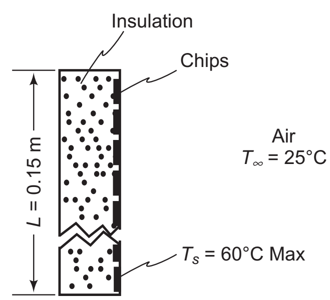
FIND

Investigate the following cooling options
(a) Natural convection
(b) Forced air cooling with an upward air velocity (U_∞) = 0.5 m/s
(c) Forced air cooling with a downward air velocity (U_∞) = 0.5 m/s

ASSUMPTIONS

- Steady state
- Uniform surface temperature
- Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperature of 42.5°C

Thermal expansion coefficient (β) = 0.00317 1/K
Thermal conductivity (k) = 0.0267 W/(m K)
Kinematic viscosity (ν) = 17.8×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

The rate of heat generation per unit area is

$$\frac{q_g}{A} = \frac{N\,q}{L^2} = \frac{100(0.06\,\text{W})}{(0.15\,\text{m})^2} = 266.7\ \text{W/m}^2$$

(a) The Grashof number is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\big(0.00317\ (1/\text{K})\big)(60\,°\text{C} - 25°\text{C})(0.15\,\text{m})^3}{\left(17.8\times 10^{-6}\text{m}^2/\text{s}\right)^2} = 1.16\times 10^7$$

The Nusselt number for natural convection is given by Equation (5.12b)

$$(Nu_L)_{\text{free}} = 0.68\ Pr^{\frac{1}{2}}\ \frac{Gr_L^{\frac{1}{4}}}{(0.952 + Pr)^{\frac{1}{4}}} = 0.68\ (0.71)^{\frac{1}{2}}\ \frac{(1.16\times 10^7)^{\frac{1}{4}}}{(0.952+0.71)^{\frac{1}{4}}} = 29.44$$

$$(h_c)_{\text{free}} = Nu_L\ \frac{k}{L} = 29.44\ \frac{\big(0.0267\ \text{W/(m K)}\big)}{0.15\,\text{m}} = 5.24\ \text{W/(m}^2\text{K)}$$

The rate of convective heat transfer must equal the rate of heat generation if

$$\frac{q_c}{A} = h_c\,(T_s - T_\infty) = \big(5.24\ \text{W/(m}^2\text{K)}\big)\ (60°\text{C} - 25°\text{C}) = 183.4\ \text{W/m}^2 < \frac{q_g}{A}$$

Since this is lower than the heat generation rate, the actual surface temperature will be higher than the maximum of 60°C.

Therefore, natural convection alone will not keep the chips cool enough.

(b) The Reynolds number for U_∞ = 0.5 m/s is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(0.5\text{ m/s})(0.15\text{ m})}{17.8\times 10^{-6}\text{m}^2/\text{s}} = 4.21 \times 10^3 < 5 \times 10^5 \text{ (laminar)}$$

From Equation (5.45) the relative importance of natural and forced convection is indicated by the following ratio

$$\frac{Gr_L}{Re_L^{\,2}} = \frac{1.16\times 10^7}{(4.21\times 10^3)^2} = 0.65$$

Since $(Gr_L/Re_L^2) \approx 1$, natural and forced convection are of the same order of magnitude. The average Nusselt number can be estimated from Equation (5.48)

$$Nu = \left[(Nu_{\text{forced}})^3 + (Nu_{\text{free}})^3\right]^{\frac{1}{3}}$$

In this case, the natural convective flow is in the same direction as the forced convection flow; therefore, the plus sign is appropriate.

The forced convection Nusselt number is given by Equation (4.38)

$$(Nu_L)_{\text{forced}} = 0.664\ Re_L^{\frac{1}{2}}\ Pr^{\frac{1}{3}} = 0.664\ (4.21\times 10^3)^{\frac{1}{2}}\ (0.71)^{\frac{1}{3}} = 38.44$$

$$\therefore\ N_u = [(38.44)^3 + (29.44)^3]^{\frac{1}{3}} = 43.50$$

$$h_c = Nu_L\ \frac{k}{L} = 43.50\ \frac{(0.0267\ \text{W/(m K)})}{0.15\text{ m}} = 7.74\ \text{W/(m}^2\text{K)}$$

$$\frac{q_c}{A} = h_c\,(T_s - T_\infty) = \left(7.74\text{ W/(m}^2\text{K)}\right)\ (60°\text{C} - 25°\text{C}) = 271.0\text{ W/m}^2 > \frac{q_g}{A}$$

Therefore, this configuration is adequate to keep the chip surface temperature below 60°C.

(c) In this configuration, the free convective flow opposes the forced convection

$$N_u = [(Nu_{\text{forced}})^3 - (Nu_{\text{free}})^3]^{\frac{1}{3}} = [(38.44)^3 - (29.44)^3]^{\frac{1}{3}} = 31.51$$

$$h_c = Nu_L\ \frac{k}{L} = 31.51\ \frac{(0.0267\ \text{W/(m K)})}{0.15\text{ m}} = 5.61\ \text{W/(m}^2\text{K)}$$

$$\frac{q_c}{A} = h_c\,(T_s - T_\infty) = \left(5.16\text{ W/(m}^2\text{K)}\right)\ (60°\text{C} - 25°\text{C}) = 196.3\ \text{W/m}^2 < \frac{q_g}{A}$$

Therefore, this configuration will not keep the chips cool enough.

PROBLEM 5.52

A gas-fired industrial furnace is used to generate steam. The furnace is a 3 m cubic structure and the interior surfaces are completely covered with boiler tubes transporting pressurized wet steam at 150°C. It is desired to keep the furnace losses to 1% of the total heat input of 1 MW. The outside of the furnace can be insulated with a blanket-type mineral wool insulation [*k* = 0.13 W/(m °C)], which is protected by a polished metal sheet outer shell. Assume the floor of the furnace is insulated. What is the temperature of the metal shell sides? What thickness of insulation is required?

GIVEN

- An insulated cubic furnace with steam filled tubes on the inner walls
- Steam temperature (T_{st}) = 150°C
- Length of a side of the furnace (L) = 3 m
- Thermal conductivity of mineral wool insulation (k_i) = 0.13 W/(m°C)
- Insulation is protected by metal sheet outer shell
- Furnace losses (q_c) = 1% of total heat input
- Total heat input (q_{in}) = 1 MW = 106 W

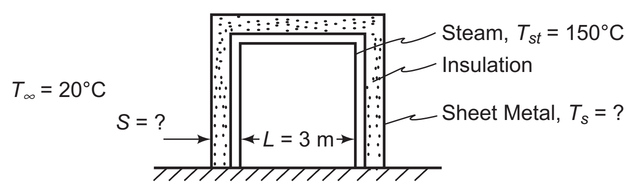
FIND

(a) Temperature of the metal sheel sides (T_s)
(b) The thickness of insulation (s) required

ASSUMPTIONS

- Steady state operation
- Thermal resistance of the convection within the steam pipes, the steam pipe walls, the furnace walls, and the metal shell negligible compared to that of the insulation
- Air outside the furnace is still
- The floor is well insulated —heat loss is negligible
- Temperature of the metal shell is uniform
- Ambient temperature (T_∞) = 20°C (293 K)
- Edge effects are negligible
- The emissivity of the polished metal shell (ε) = 0.05 (see Table 9.2)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann Constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) Since the Grashof number on the outside of the metal shell will depend on the temperature of the metal shell, an iterative procedure is required. For the first iteration, let T_s = 100°C (373 K).

From Appendix 2, Table 27, for dry air at the mean temperature of 60°C

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Grashof number for the four sides of the furnace, assuming the insulation thickness is small compared to 3 m, is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.003\ 1/\text{K})(100°\text{C} - 20°\text{C})(3\,\text{m})^3}{\left(19.4 \times 10^{-6}\text{m}^2/\text{s}\right)^2} = 1.69 \times 10^{11}$$

The heat transfer from the furnace will be calculated by treating the sides as vertical flat plates and the top as a horizontal flat plate facing upward. From Equation (5.13), the heat transfer coefficient for the sides is

$$\bar{h}_{cs} = 0.13\ \frac{k}{L}\ (Gr_L\, Pr)^{\frac{1}{3}} = 0.13\ \frac{\left(0.0279\ \text{W/(m K)}\right)}{3\,\text{m}}\ [1.69\times 10^{11}(0.71)]^{\frac{1}{3}} = 5.96\ \text{W/(m}^2\text{K)}$$

The characteristic dimension for the top of the furnace (L_c) is

$$L_c = \frac{A}{P} = \frac{L^2}{4L} = \frac{L}{4} = 0.75\ \text{m}$$

The Grashof and Rayleigh numbers based on this dimension are

$$Gr_{Lc} = \frac{(9.8\ \text{m/s}^2)\left(0.00188\ (1/\text{K})\right)(100°\text{C} - 20°\text{C})(0.75\,\text{m})^3}{\left(19.4\times 10^{-6}\text{m}^2/\text{s}\right)^2} = 2.64\times 10^9\ \text{(turbulent)}$$

$$Ra_{Lc} = Gr_{Lc}\, Pr = 2.64\times 10^9\,(0.71) = 1.87\times 10^9$$

The average Nusselt number is given by Equation (5.16)

$$\overline{Nu}_{Lc} = 0.15\ Ra_{Lc}^{\frac{1}{3}} = 0.15\ (1.87\times 10^9)^{\frac{1}{3}} = 184.9$$

$$\bar{h}_{ct} = \overline{Nu}_{L_c}\ \frac{k}{L_c} = 184.9\ \frac{\left(0.0279\ \text{W/(m K)}\right)}{0.75\,\text{m}} = 6.88\ \text{W/(m}^2\text{K)}$$

The rate of convection and radiation must be 1% of the total heat input

$$q_c = q_r = (\bar{h}_{cs}\ A_s + \bar{h}_{ct}\ A_t)\,(T_s - T_\infty) + \varepsilon\,\sigma A\,(T_s^4 - T_\infty^4) = 0.01\ \text{q}_{\text{in}}$$

where A_s = the area of the sides = $4(3\ \text{m})^2 = 36\ \text{m}^2$
A_t = the area of the top = $(3\ \text{m})^2 = 9\ \text{m}^2$
$A = A_s + A_t = 45\ \text{m}^2$

$$\left[\left(5.96\,\text{W/(m}^2\text{K)}\right)(36\,\text{m}^2) + \left(6.88\ \text{W/(m}^2\text{K)}\right)(9\,\text{m}^2)\right]h\ \ (T_s - 293\ \text{K}) + 0.05$$
$$\left(5.67\times 10^{-8}\,\text{W/(m}^2\text{K}^4)\right)\ (45\ \text{m}^2)\,(T_s^4 - (293\ \text{K})^4)$$
$$= 0.01\ (10^6\ \text{W})$$

By trial and error: T_s = 327 K = 54°C

Following the same procedure for other iterations

Iteration #	2	3	4
T_s (°C)	51	64	60
Mean Temp. (°C)	35.5	42	40
β (1/K)	0.00324	0.00317	0.00319
k (W/(m K))	0.0262	0.0266	0.0265
$\nu\times 10^6$ (m²/s)	17.2	17.8	17.6
Pr	0.71	0.71	0.71
h_{cs} (W/(m² K))	4.54	5.02	4.89
h_{ct} (W/(m² K))	5.23	5.79	5.65
T_s (°C)	64	60	61

Therefore, the surface temperature (T_s) ≈ 61°C

(b) The rate of conductive heat transfer through the insulation must also be 1% of the input heat

$$q_k = \frac{Ak_i}{S}\,(T_{st} - T_s) = 0.01\ q_{\text{in}}$$

Solving for the insulation thickness

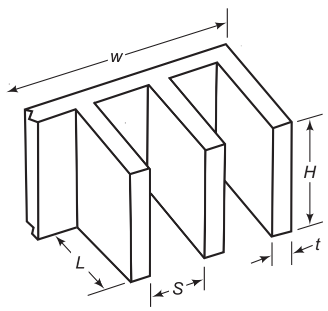
$$s = \frac{A k_i}{0.01 q_{\text{in}}} (T_{st} - T_s) = \frac{45\,\text{m}^2\left(0.13\ \text{W/(m K)}\right)}{0.01(10^6\,\text{W})} (150°\text{C} - 61°\text{C}) = 0.052\text{m} = 5.2\text{ cm}$$

COMMENTS

The insulation thickness is small compared to the length of a side of the furnace, therefore, neglecting the edge effects or effect on the exterior surface area should not introduce appreciable error.

PROBLEM 5.53

An electronic device is to be cooled by natural convection in atmospheric air at 20°C. The device generates internally 50 W and only one of its external surfaces is suitable for attaching fins. The surface available for attaching cooling fins is 0.15 m tall and 0.4 m wide. The maximum length of a fin perpendicular to the surface is limited to 0.02 m and the temperature at the base of the fin is not to exceed 70°C in one design and 100°C in another.



Design an array of fins spaced at a distance (*s*) from each other so that the boundary layers will not interfere will each other appreciable and maximum rate of heat dissipation is approached. For the evaluation of this spacing, assume that the fins are at a uniform temperature. Then select a thickness (*t*) that will provide good fin efficiency and ascertain which base temperature is feasible.

(For complete thermal analyses see ASME J. Heat Transfer, 1977, p. 369, *J*. Heat Transfer, 1979, p. 569, and *J*. Heat Transfer, 1984, p. 116.)

GIVEN

- An electronic device with vertical aluminum fins in air
- Air temperature (T_∞) = 20°C
- Heat generation ($\dot{q}_G$) = 50 W
- Height of surface (H) = 0.15 m
- Width of surface (w) = 0.4 m
- Maximum fin length (L_f) = 0.02 m
- Maximum base temperatures:
  - T_{b1} = 70°C
  - T_{b2} = 100°C
- Fin spacing = s
- Fin thickness = t

FIND

(a) Fin spacing such that the boundary layers do not interfere

(b) Select a fin thickness that gives a good fin efficiency and ascertain which base temperature is feasible

ASSUMPTIONS

- The fins are at a uniform temperature equal to the base temperature

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the mean temperatures for each of the base temperatures

Mean Temperature (°C)	45°C	60°C
Thermal expansion coefficient, β (1/K)	0.00314	0.003
Thermal conductivity, k (W/(m K))	0.0269	0.0279
Kinematic viscosity, $\nu \times 10^{-6}$ (m²/s)	18.1	19.4
Prandtl number, Pr	0.71	0.71

From Appendix 2, Table 12, the thermal conductivity of aluminum in the range of 70 to 100°C (k_a) = 240 (W/(m K))

SOLUTION

(a) The boundary layer thickness on a vertical flat plate is given by Equation (5.11b)

$$\delta(x) = 4.3 \times \left[\frac{Pr + 0.56}{Pr^2 + Gr_x}\right]^{\frac{1}{4}}$$

The fin spacing (s) must be twice the boundary thickness at the top of the fin ($x = H$) to avoid boundary layer interference

$$s = 2\ \delta(H) = 8.6\ H\ \left[\frac{Pr + 0.56}{Pr^2 + Gr_H}\right]^{\frac{1}{4}}$$

where $Gr_H = \dfrac{g\,\beta(T_s - T_\infty)H^3}{\nu^2}$

For T_b = 70°C

$$Gr_H = \frac{(9.8\ \text{m/s}^2)(0.00314\ (1/\text{K}))(70°\text{C} - 20°\text{C})(0.15\,\text{m})^3}{\left(18.1 \times 10^{-6}\text{m}^2/\text{s}\right)^2} = 1.59 \times 10^7$$

$$s = 8.6\ (0.15\ \text{m})\left[\frac{0.71 + 0.56}{0.71^2(1.59 \times 10^7)}\right]^{\frac{1}{4}} = 0.026\text{m} = 2.6\ \text{cm}$$

For T_b = 100°C

$$Gr_\text{H} = \frac{(9.8\ \text{m/s}^2)(0.003\ (1/\text{K}))(100°\text{C} - 20°\text{C})(0.15\,\text{m})^3}{\left(19.4 \times 10^{-6}\text{m}^2/\text{s}\right)^2} = 2.11 \times 10^7$$

$$s = 8.6\ (0.15\ \text{m})\left[\frac{0.71 + 0.56}{0.71^2(2.11 \times 10^7)}\right]^{\frac{1}{4}} = 0.024\text{m} = 2.4\ \text{cm}$$

Let the fin spacing (s) = 2.5 cm.
The average Nusselt number of the fins is given by Equation (5.12b)

$$\overline{Nu}_H = 0.68 \; Pr^{\frac{1}{2}} \; \frac{Gr_H^{\frac{1}{4}}}{(0.952 + Pr)^{\frac{1}{4}}}$$

For T_b = 70°C: $\overline{Nu}_H = 0.68 \; (0.71)^{\frac{1}{2}} \; \frac{(1.59 \times 10^7)^{\frac{1}{4}}}{(0.952 + 0.71)^{\frac{1}{4}}} = 31.87$

$$\bar{h}_c = \overline{Nu}_H \frac{k}{H} = 31.87 \frac{(0.0269 \text{ W/(m K)})}{0.15 \text{ m}} = 5.71 \text{ W/(m}^2\text{K)}$$

For T_b = 100°C: $\overline{Nu}_H = 0.68 \; (0.71)^{\frac{1}{2}} \; \frac{(2.11 \times 10^7)^{\frac{1}{4}}}{(0.952 + 0.71)^{\frac{1}{4}}} = 34.20$

$$\bar{h}_c = \overline{Nu}_H \frac{k}{H} = 34.2 \frac{(0.0269 \text{ W/(m K)})}{0.15 \text{ m}} = 6.36 \text{ W/(m}^2\text{K)}$$

(b) The fin efficiency is approximated by Equation (2.67).
For a 'good' fin efficiency, let η_f = 0.99

$$0.99 = \frac{\tanh\sqrt{W}}{\sqrt{W}} \Rightarrow \qquad W = 0.0289$$

$$\text{where } W = \frac{2\bar{h}_c L_c^2}{k_a t} \qquad\qquad L_c = L_f + \frac{t}{2}$$

For T_b = 70°C: $W = \frac{2(5.71 \text{ W/(m}^2\text{K)})(0.02 \text{ m} + t/2)^2}{(240 \text{ W/(m K)})\, t} = 0.0289 \Rightarrow t = 0.0007 \text{ m} = 0.7 \text{ mm}$

For T_b = 100°C

$$W = \frac{2(6.36 \text{ W/(m}^2\text{K)})\left(\frac{0.02 \text{ m} + t}{2}\right)^2}{(240 \text{ W/(m K)})\, t} = 0.0289 \quad \Rightarrow t = 0.0008 \text{ m} = 0.8 \text{ mm}$$

Let t = 0.75 mm for either case.
The number of fins on the device (N) is given by

$$N\,t + (N - 1)\,s = w \quad \Rightarrow \; N = \frac{w+s}{t+s} = \frac{0.4 \text{ m} + 0.025 \text{ m}}{0.00075 \text{ m} + 0.025 \text{ m}} = 16.5$$

There will be 17 fins. The surface area of the fins and wall area between them is

$$A = H\,[N(2L_f + t) + (N - 1)s] = 0.15\text{m}[17(0.04\text{m} + 0.00075\text{m}) + 16(0.025\text{m})] = 0.164\text{m}^2$$

The rate of heat transfer is

$$q = hc\,A\,(T_b - T_\infty)$$

For T_b = 70°C $\qquad q = 5.71 \text{ W/(m}^2\text{ K)} \; (0.164\text{m}^2)\,(70°\text{C} - 20°\text{C}) = 46.8 \text{ W} < q_G$

For T_b = 100°C $\qquad q = 6.36 \text{ W/(m}^2\text{ K)}\,(0.164\text{m}^2)\,(100°\text{C} - 20°\text{C}) = 83.4 \text{ W} > q_G$

The 100°C base temperature is feasible; the 70°C base temperature is not.

COMMENTS

The optimum spacing from Equation (5.56a) is 0.0017 m for t = 0.00075 m indicating the surface area gained outweighs the reduction in heat transfer due to the interference of the boundary layers. The rate of heat transfer with this spacing and a base temperature of 70°C is 47.4 W. Still not quite adequate.

Chapter 6

PROBLEM 6.1

To measure the mass flow rate of a fluid in a laminar flow through a circular pipe, a hot wire type velocity meter is placed in the center of the pipe. Assuming that the measuring station is far from the entrance of the pipe, the velocity distribution is parabolic, or

$$\frac{u(r)}{U_{max}} = \left[1 - \left(\frac{2r}{D}\right)^2\right]$$

where U_{max} **is the centerline velocity (**$r = 0$**)**

r **is the radial distance from the pipe centerline**

D **is the pipe diameter.**

(a) Derive an expression for the average fluid velocity at the cross-section.

(b) Obtain an expression for the mass flow rate.

(c) If the fluid is mercury at 30°C, D **= 10 cm, and the measured value of** U_{max} **is 0.2 cm/s, calculate the mass flow rate from the measurement.**

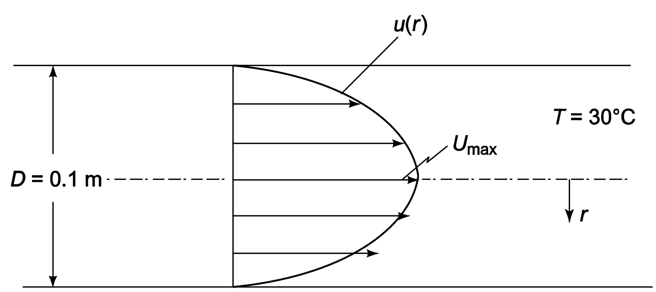
GIVEN

- Fully developed flow of mercury through a circular pipe
- Parabolic velocity distribution: $u(r)/U_{max} = 1 - (2r/D)^2$
- Mercury temperature (T) = 30°C
- Pipe diameter (D) = 10 cm = 0.1 m
- Measured center velocity (U_{max}) = 0.2 cm/s = 0.002 m/s

FIND

(a) An expression for the average fluid velocity ($\bar{u}$)
(b) An expression for the mass flow rate ($\dot{m}$)
(c) The value of the mass flow rate ($\dot{m}$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for Mercury at 30°C: Density (ρ) = 13,555 kg/m^3

SOLUTION

(a) The average fluid velocity is calculated as follows

$$\bar{u} = \frac{1}{r_o}\int_o^{r_o} u(r)\,dr \qquad \text{where } r_o = \frac{D}{2}$$

$$\bar{u} = \frac{U_{max}}{r_o} \int_o^{r_o} \left[1 - \left(\frac{r}{r_o} \right)^2 \right] dr = \frac{U_{max}}{r_o} \left(r - \frac{1}{3} \frac{r^3}{r_o^2} \right) \Bigg|_0^{r_o}$$

$$\bar{u} = U_{max} \left(1 - \frac{1}{3} \right) = \frac{2}{3} U_{max}$$

(b) The mass flow rate is given by

$$\dot{m} = \bar{u} A_c \rho = \frac{2}{3} U_{max} (\pi r_o^2) \rho$$

$$\dot{m} = \frac{2}{3} \pi U_{max} r_o^2 \rho$$

(c) Inserting the values of these quantities into this expression

$$\dot{m} = \frac{2}{3} \pi \ (0.002 \text{ m/s}) \ (0.05 \text{ m})^2 \ (13{,}555 \text{ kg/m}^3) = 0.14 \text{ kg/s}$$

PROBLEM 6.2

Nitrogen at 30°C and atmospheric pressure enter a triangular duct 0.02 m on each side at a rate of 4×10^{-4} kg/s. If the duct temperature is uniform at 200°C, estimate the bulk temperature of the nitrogen 2 m and 5 m from the inlet.

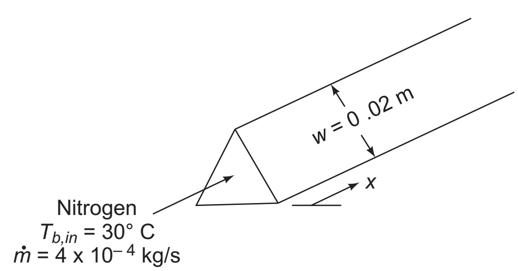
GIVEN

- Atmospheric nitrogen flowing through a triangular duct
- Bulk inlet temperature ($T_{b,\text{in}}$) = 30°C
- Width of each side of the duct (w) = 0.02 m
- Mass flow rate ($\dot{m}$) = 4×1^{-4} kg/s
- Duct temperature (T_s) = 200°C (uniform)

FIND

- The bulk temperature (a) 2 m from the inlet and, (b) 5 m from the inlet

SKETCH



SOLUTION

(a) Assuming the outlet temperature is 70°C, then the average bulk temperature is 50°C
From Appendix 2, Table 32, for nitrogen

Specific heat (c_p) = 1042 J/(kg K)
Thermal conductivity (k) = 0.0278 W/(m K)
Absolute viscosity (μ) = 18.79×10^{-6} (Ns)/m^2
Prandtl Number (Pr) = 0.71

The hydraulic diameter of the duct is

$$D_H = \frac{4 A_c}{P} = \frac{4\left[\frac{1}{2} \text{W} \sqrt{\text{W}^2 - \left(\frac{\text{W}}{2} \right)^2} \right]}{3\text{W}} = \frac{4(1.73 \times 10^{-4} \text{ m}^2)}{3(0.02 \text{ m})} = 0.0115 \text{ m}$$

$$\therefore\ Re_D = \frac{U_\infty D\rho}{\mu} = \frac{\dot{m}\,D_H}{A_c\,\mu} = \frac{\left(4\times10^{-4}\ \text{kg/s}\right)(0.0115\,\text{m})}{1.73\times10^{-4}\text{m}^2\left(18.79\times10^{-6}\ \text{Ns/m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 1415$$

The length from the entrance at which the velocity and temperature profiles become fully developed can be obtained from Equations (6.7) and (6.8)

$$x_{fd} = 0.05\ D_H\ Re_D = 0.05\ (0.0115\ \text{m})(1415) = 0.81\ \text{m}$$

$$x_{ft,T} = 0.05\ D_H\ Re_D\ \text{Pr} = 0.05\ (0.0115\ \text{m})\ (1415)(0.71) = 0.58\ \text{m}$$

Therefore, the flow is fully developed over most of the duct length.

From Table 6.1, for fully developed flow in triangular cross-section duct: $\overline{Nu}_D = 2.47$

$$\therefore\ \bar{h}_c = \overline{Nu}_D\ \frac{k}{D_H} = 2.47\,\frac{(0.0278\ \text{W/(m K)})}{0.0115\,\text{m}} = 5.98\ \text{W/(m}^2\ \text{K)}$$

Rearranging Equation (6.36)

$$T_{b,\text{out}} = T_s + (T_{b,\text{in}} - T_s)\ \exp\left(-\frac{PLh_c}{\dot{m}\,c_p}\right)$$

$$T_{b,\text{out}} = 200°\text{C} + (30°\text{C} - 200°\text{C})\ \exp\left(-\frac{3(0.02\,\text{m})(2\,\text{m})\left(5.98\ \text{W/(m}^2\text{K)}\right)}{\left(4\times10^{-4}\ \text{kg/s}\right)(1042\ \text{J/(kg K)})(\text{Ws/J})}\right) = 170°\text{C}$$

With this outlet temperature, the average bulk temperature will be 100°C. This is far enough from the initial guess that another iteration is warranted

$c_p = 1045$ J/(kg K) $\qquad \bar{h}_c = 6.75$ W/(m^2 K)

$k = 0.0314$ W/(m K) $\qquad T_{b,\text{out}} = 176$°C

The bulk temperature at $x = 2$ m is 176°C.

(b) The same procedure can be used to find the bulk temperature at $x = 5$ m. Let $T_{b,\text{out}} = 190$°C.
Average bulk temperature = 110°C

$c_p = 1045$

$k = 0.0321$ W/(m K)

$\bar{h}_c = 6.90$ W/(m^2 K)

$T_{b,\text{out}} = 199$°C

The bulk temperature a $x = 5$ m is about 199°C.

PROBLEM 6.3

Air at 30°C enters a rectangular duct 1 m long and 4 mm by 16 mm in cross-section at a rate of 0.0004 kg/s. If a uniform heat flux of 500 W/m^2 is imposed on both of the long sides of the duct, calculate (a) the air outlet temperature (b) the average duct surface temperature, and (c) the pressure drop.

GIVEN

- Air flowing through a rectangular duct
- Inlet bulk air temperature ($T_{b,\text{in}}$) = 30°C
- Duct length (L) = 1 m
- Duct height (H) 4 mm = 0.004 m
- Duct width (w) = 16 mm = 0.016 m

- Air mass flow rate ($\dot{m}$) = 0.0004 kg/s
- Uniform heat flux (q/A) = 500 W/m^2 on the long sides

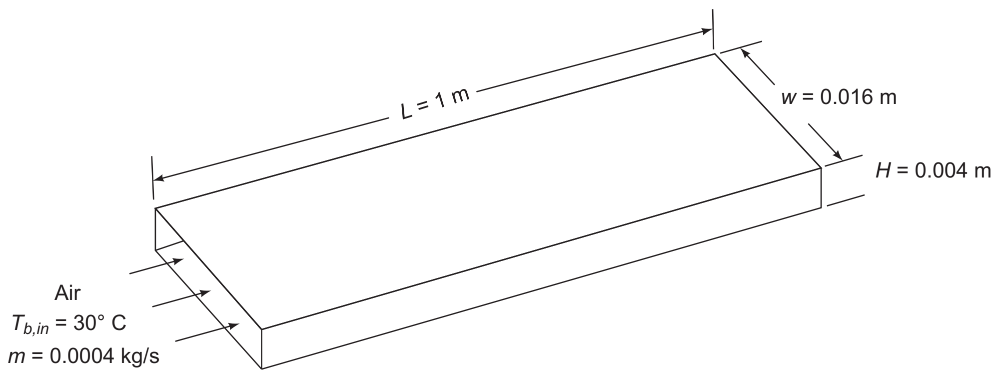
FIND

(a) Air outlet temperature ($T_{b,\text{out}}$)
(b) The average duct surface temperature (T_s)
(c) The pressure drop (Δp)

ASSUMPTIONS

- The short sides of the duct are insulated
- Entrance effects are negligible

SKETCH



SOLUTION

(a) The total rate of heat transfer to the air

$$q = \left(\frac{q}{A}\right) A = \left(\frac{q}{A}\right) 2\text{ L w} = \left(500\text{ W/(m}^2\text{K)}\right)(2)(1\text{ m})(0.016\text{ m}) = 16\text{ W}$$

$$q = \dot{m}\ c_p\ \Delta T = \dot{m}\ c_p\ (T_{b,\text{out}} - T_{b,\text{in}}) \Rightarrow T_{b,\text{out}} = T_{b,\text{in}} + \frac{q}{\dot{m}\, c_p}$$

The specific heat (c_p) ≈ 1000 J/(kg K), therefore, $T_{b,\text{out}}$ ≈ 70°C. From Appendix 2, Table 27, the specific heat at the approximate average bulk temperature of 50°C is 1016 J/(kg K).

$$\therefore\ T_{b,\text{out}} = 30°\text{C} + \frac{16\,\text{W}}{(0.0004\,\text{kg/s})(1016\,\text{J/(kg K)})((\text{Ws})/\text{J})} = 69.4°\text{C}$$

(b) The average duct surface temperature is given by

$$\frac{q}{A} = h_c\,(T_s - T_{b,\text{ave}}) \Rightarrow T_s = T_{b,\text{ave}} + \frac{q}{A\,h_c} = \frac{T_{b,\text{in}} + T_{b,\text{out}}}{2} + \frac{q}{A\,h_c}$$

The heat transfer coefficient can be obtained from the proper correlation.
The hydraulic diameter of the duct is

$$D_H = \frac{4A}{P} = \frac{4\,\text{w H}}{2(L+H)} = \frac{4(0.016\,\text{m})(0.004\,\text{m})}{2(0.02\,\text{m})} = 0.0064\text{ m}$$

$$\frac{L}{D_H} = \frac{1\,\text{m}}{0.0064\,\text{m}} = 156$$

Therefore, entrance effects will be neglected.

From Appendix 2, Table 27, for dry air at the average bulk temperature of 49.7°C

Thermal conductivity (k) = 0.0272 W/(m K)

Absolute viscosity (μ) = 19.503 × 10^{-6} (Ns)/m^2

Density (ρ) = 1.015 kg/m^3

The Reynolds number is

$$Re_D = \frac{U_\infty D \rho}{\mu} = \frac{\dot{m} D_H}{\text{w} H \mu} = \frac{(0.0004\,\text{kg/s})}{(0.016\,\text{m})(0.004\,\text{m})\left(19.503\times10^{-6}(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 2051 < 2100$$

Therefore, the flow is laminar.

The Nusselt number for this geometry is given in Table 6.1

For $\frac{2b}{2a} = \frac{0.008}{0.032} = 0.25,\ \overline{Nu}_D = \overline{Nu}_{H_2} = 2.93$

$$\therefore\ h_c = Nu_D \frac{k}{D_H} = 2.93\ \frac{(0.0272\ \text{W/(m K)})}{0.0064\,\text{m}} = 12.5\ \text{W/(m}^2\ \text{K)}$$

The average surface temperature is

$$T_s = \frac{30°\text{C} + 69.4°\text{C}}{2} + \frac{\left(500\ \text{W/m}^2\right)}{\left(12.5\ \text{W/(m}^2\ \text{K)}\right)} = 90°\text{C}$$

From Table 6.1 for $2b/2a = 1/4$, $fRe_D = 72.93$

$$\therefore f = \frac{72.93}{\text{Re}_D} = \frac{72.93}{2051} = 0.0356$$

The pressure drop is given by Equation (6.13)

$$\Delta p = f\,\frac{L}{D_H}\,\frac{\rho U^2}{2g_c} = f\,\frac{L}{D_H}\,\frac{1}{2g_c\rho}\left(\frac{\dot{m}}{\text{w H}}\right)^2$$

$$\Delta p = 0.0356\ \frac{1\,\text{m}}{0.0064\,\text{m}}\ \frac{1}{2\left((\text{kg m})/(\text{Ns}^2)\right)\left(1.059\,\text{kg/m}^3\right)}\left(\frac{0.0004\,\text{kg/s}}{(0.016\,\text{m})(0.004\,\text{m})}\right)^2 = 102\ \text{Pa}$$

PROBLEM 6.4

Engine oil flows at a rate of 0.5 kg/s through a 2.5 cm *ID* tube. The oil enters 25°C while the tube wall is at 100°C. (a) If the tube is 4 m long. Determine whether the flow is fully developed. (b) Calculate the heat transfer coefficient.

GIVEN

- Engine oil flows through a tube
- Mass flow rate ($\dot{m}$) = 0.5 kg/s
- Inside diameter (D) = 2.5 cm = 0.025 m
- Oil temperature at entrance (T_i) = 25°C
- Tube surface temperature (T_s) = 100°C
- Tube length (L) = 4 m

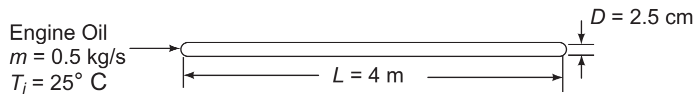
FIND

(a) Is flow fully developed?
(b) The heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 16, for unused engine oil at the initial temperature of 25°C

Density (ρ) = 885.2 kg/m^3
Thermal conductivity (k) = 0.145 W/(m K)
Absolute viscosity (μ) = 0.652 (Ns)/m^2
Prandtl number (Pr) = 85.20
Specific heat (c) = 1091 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V D \rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.5\,\text{kg/s})}{\pi(0.025\,\text{m})\left(0.652\,\text{Ns/m}^2\right)\left(\text{kg m/(Ns}^2)\right)} = 39.1$$

Therefore, the flow is laminar.

(a) The entrance length at which the velocity profile approaches its fully developed shape is given by Equation (6.7)

$$\frac{x_{fd}}{D} = 0.05\, Re_D \Rightarrow x_{fd} = 0.05\, D\, Re_D = 0.05\ (0.025\ \text{m})\ (39.1) = 0.049\ \text{m} = 4.9\ \text{cm}$$

Therefore, the velocity profile is fully developed for 98.8% of the tube length.

The entrance length at which the temperature profile approaches its fully developed shape is given by Equation (6.8)

$$\frac{x_{fd}}{D} = 0.05\, Re_D\, Pr \Rightarrow x_{fd} = 0.05\, D\, Re_D\, Pr = 0.05(0.025\ \text{m})\ (39.1)\ (8520) = 416\ \text{m}$$

Therefore, the temperature profile is not fully developed.

(b) Since the velocity profile is fully developed but the temperature profile is not, Figure 6.10 will be used to estimate the Nusselt number

$$\frac{Re_D Pr D}{L} \times 10^{-2} = \frac{(39.1)(85.20)(0.025\,\text{m})}{4\,\text{m}} \times 10^{-2} = 0.208$$

Using the 'parabolic velocity' curve of Figure 6.12, $Nu_D \approx 4.8$

$$h_c = Nu_D \frac{k}{D} = 4.8\ \frac{(0.145\ \text{W/(m K)})}{0.025\,\text{m}} = 27.8\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

The rate of heat transfer calculated with the heat transfer coefficient at the inlet is

$$q_{\max} = h_c\, \pi D\, L\, (T_s - T_b) = \left(27.8\ \text{W/(m}^2\text{K)}\right)\ \pi(0.025\ \text{m})\ (4\ \text{m})\ (100°\text{C} - 25°\text{C}) = 656\ \text{W}$$

The outer temperature (T_o) is given by

$$q_{\max} = \dot{m}\, c\, (T_{o,\max} - T_i)$$

$$T_o - T_i \le \frac{q_{\max}}{\dot{m} c} = \frac{656\,\text{W}\,(\text{J/(Ws)})}{(0.5\,\text{kg/s})(1091\,\text{J/(kg K)})} = 1.2°\text{C}$$

This small temperature change does not warrant another iteration. If the temperature change was larger, the fluid properties would need to be re-evaluated at the average bulk temperature and a new heat transfer coefficient calculate.

PROBLEM 6.5

The equation

$$\overline{Nu} = \frac{\bar{h}_c D}{k} = \left[3.65 + \frac{0.0668\left(\frac{D}{L}\right) RePr}{1 + 0.04\left[\left(\frac{D}{L}\right) RePr\right]^{\frac{2}{3}}}\right] = \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

was recommended by H. Hausen (Zeitschr. Ver. Deut. Ing., Belherft No. 4, 1943)

for forced-convection heat transfer in fully developed laminar flow through tubes. Compare the values of the Nusselt number predicted by Hausen's equation for *Re* = 1000, *Pr* = 1, and *L*/*D* = 2, 10 and 100, respectively, with those obtained from two other appropriate equations or graphs in the text.

GIVEN

- Fully developed laminar flow through a tube
- The Nusselt number correlation shown above
- Reynolds number (*Re*) = 1000
- Prandtl number (*Pr*) = 1
- Length divided by diameter (*L*/*D*) = 2, 10, or 100

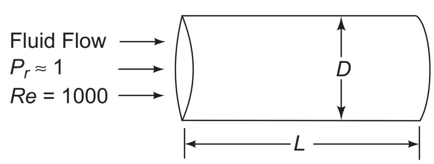
FIND

- The Nusselt number (*Nu*) from the above correlation and two others from the text

ASSUMPTIONS

- $\mu_b / \mu_s \approx 1.0$
- Constant wall temperature

SKETCH



SOLUTION

Using the Hausen correlation and *L*/*D* = 2

$$\overline{Nu} = \frac{\bar{h}_c D}{k} = \left[3.65 + \frac{0.0668\left(\frac{1}{2}\right)(1000)(1)}{1 + 0.04\left[\left(\frac{1}{2}\right)(1000)(1)\right]^{\frac{2}{3}}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 13.1 \left(\frac{\mu_b}{\mu_s}\right)^{0.14} \approx 13.1$$

Similarly for the other cases $\quad$ For $\frac{L}{D} = 10 \;\rightarrow\; \overline{Nu} \approx 7.2$

For $\frac{L}{D} = 100 \;\rightarrow\; \overline{Nu} \approx 4.2$

Figure 6.10 can also be used to estimate the Nusselt number. The velocity entrance region for this calculated for Equation (6.7)

$$\frac{x_{fd}}{D} = 0.05\ Re_D = 0.05\ (1000) = 50$$

The thermal entrance for this problem can be calculated from Equation (6.8)

$$\frac{x_{fd,T}}{D} = 0.05\ Re_D\ Pr = 0.05\ (1000)(1) = 5$$

Therefore, in the first case, the temperature and velocity profiles are not fully developed and the 'short duct approximation' curve will be used

$$Re_D\ Pr\ \frac{D}{L} \times 10^{-2} = 1000\ (1)\left(\frac{1}{2}\right) \times 10^{-2} = 5$$

From Figure 6.12, $Nu \approx 14$

For $\frac{L}{D} = 10$

$$Re_D\ Pr\ \frac{D}{L} \times 10^{-2} = 1000\ (1)\left(\frac{1}{10}\right) \times 10^{-2} = 0.1$$

From Figure 6.12, for a parabolic velocity distribution, $Nu \approx 7.5$

For $\frac{L}{D} = 100$

$$Re_D\ Pr\ \frac{D}{L} \times 10^{-2} = 1000\ (1)\left(\frac{1}{100}\right) \times 10^{-2} = 0.1$$

From Figure 6.12 for a parabolic velocity distribution, $Nu \approx 4.1$

Finally, the Sieder and Tate correlations contained in Equation (6.40) can be applied (since $Pr = 1$ implies that the fluid is a liquid)

$$Nu_{D_H} = 1.86\left(Re_D Pr \frac{D}{L}\right)^{0.33}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\text{For } \frac{L}{D} = 2\quad Nu_{D_H} = 1.86\left[1000\ (1)\left(\frac{1}{2}\right)\right]^{0.33}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 14.8\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \approx 14.8$$

Similarly for $\frac{L}{D} = 10 \rightarrow Nu \approx 8.6$ $\quad \frac{L}{D} = 100 \rightarrow Nu \approx 4.0$

Tabulating the results

	Nusselt Numbers, *Nu*		
L/*D*	2	10	100
Hausen Correlation	13.1	7.2	4.2
Figure 6.10	14	7.5	4.1
Sieder and Tate Correlation	14.8	8.6	4.0
Average	14.0	7.8	4.1
Maximum % Variation from Average	6%	10%	2%

COMMENTS

The agreement among the three correlations is within the accuracy of empirical correlations.

PROBLEM 6.6

Air at an average temperature of 150°C flows through a short square duct 10 × 10 × 2.25 cm at a rate of 15 kg/h. The duct wall temperature is 430°C. Determine the average heat transfer coefficient, using the duct equation with appropriate *L*/*D* correction. Compare your results with flow-over-flat-plate relations.

GIVEN

- Air flowing through a short square duct
- Average air temperature (T_a) = 150°C
- Duct dimensions = 10 × 10 × 2.25 cm = 0.1 × 0.1× 0.0225 m
- Duct wall surface temperature (T_s) = 430°C
- Mass flow rate ($\dot{m}$) = 15 kg/h

FIND

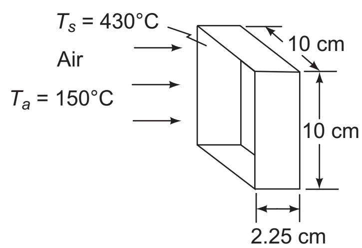
The average heat transfer coefficient ($\bar{h}_c$) using

(a) The duct equation with appropriate L/D correction
(b) The flow-over-flat-plate relation

ASSUMPTIONS

- Constant and uniform duct wall temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average temperature of 150°C

Thermal conductivity (k) = 0.0339 W/(m K)
Absolute viscosity (μ_b) = 23.683 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71

At the surface temperature of 430°C, the absolute viscosity (μ_s) = 33.66 × 10^{-6} (Ns)/m^2.

SOLUTION

The hydraulic diameter of the duct is

$$D_H = \frac{4\,A_c}{P} = \frac{4(0.1\,\text{m})(0.1\,\text{m})}{4(0.1\,\text{m})} = 0.1 \text{ m}$$

The Reynolds number is

$$Re_{D_H} = \frac{V D_H \rho}{\mu} = \frac{\dot{m} D_H}{A_c\,\mu} = \frac{(15\,\text{kg/h})(0.1\,\text{m})}{(0.1\,\text{m})(0.1\,\text{m})\left(23.683\times10^{-6}\,(\text{N s})/\text{m}^2\right)(3600\,\text{s/h})\left((\text{kg m})/(\text{N s}^2)\right)} = 1760$$

Therefore, the flow is laminar.

(a) Using the Hausen correlation, Equation (6.39) to estimate the Reynolds number with $D/L = D_H/L$ = 10/2.25 = 4.44

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = \left[3.66 + \frac{0.0668 Re_{D_H} Pr\left(\frac{D}{L}\right)}{1 + 0.045\left[Re_{D_H} Pr\left(\frac{D}{L}\right)\right]^{0.66}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\overline{Nu}_{D_H} = \left[3.66 + \frac{0.0668(1760)(0.71)(4.44)}{1+0.045[(1760)(0.71)(4.44)]^{0.66}}\right]\left(\frac{23.683}{33.666}\right)^{0.14} = 28.3$$

$$\bar{h}_c = \overline{Nu}_{D_H}\frac{k}{D_H} = 28.3\ \frac{(0.0339\ \text{W/(m K)})}{0.1\ \text{m}} = 9.59\ \text{W/(m}^2\ \text{K)}$$

(b) Applying the flow-over-flat-plate relation of Equation (6.38)

$$\overline{Nu}_{D_H} = \frac{Re_{D_H} Pr}{4}\frac{D_H}{L}\ln\left[\frac{1}{1-\dfrac{2.654}{Pr^{0.167}\left[Re_{D_H} Pr\left(\frac{D_H}{L}\right)\right]^{0.5}}}\right]$$

$$\overline{Nu}_{D_H} = \frac{(1760)(0.71)}{4}(4.44)\ \ln\left[\frac{1}{1-\dfrac{2.654}{(0.71)^{0.167}[1760(0.71)(4.44)]^{0.5}}}\right] = 53.3$$

$$\bar{h}_c = \overline{Nu}_{D_H}\frac{k}{D_H} = 53.3\ \frac{(0.0339\ \text{W/(m K)})}{0.1\ \text{m}} = 18.1\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

The flat plate estimate is almost twice the previous estimate based on flow through a short duct. It should be noted that the flow-over-flat-plate relation is only applicable in the following range: [Re_D Pr (D/L)] from 100 to 1500. For this problem, $Re_D\ Pr\ D/L$ = 5548.

PROBLEM 6.7

Water enters a double pipe heat-exchanger at 60°C. The water flows on the inside through a copper tube 2.54 cm (1 in) *ID* at a velocity of 2 cm/s. Steam flows in the annulus and condenses on the outside of the copper tube at a temperature of 80°C. Calculate the outlet temperature of the water if the heat exchanger is 3m long.

GIVEN

- Water flow through a tube in a double pipe heat-exchanger
- Water entrance temperature ($T_{b,\text{in}}$) = 60°C
- Inside tube diameter (D) = 2.54 cm = 0.0254 m
- Water velocity (V) = 2 cm/s = 0.02 m/s
- Steam condenses at (T_s) = 0.80°C on the outside of the pipe
- Length of heat exchanger (L) = 3 m

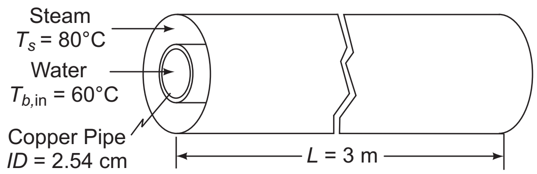
FIND

- Outlet temperature of the water ($T_{b,\text{out}}$)

ASSUMPTIONS

- Steady state
- Thermal resistance of the copper pipe is negligible
- Pressure in the annulus is uniform therefore, T_s is uniform
- Heat transfer coefficient of the condensing steam is large (see Table 1.4) so its thermal resistance can be neglected
- Outside surface of the heat exchanger is insulated

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the inlet temperature of 60°C

Specific heat (c) = 4182 J/(kg K)
Thermal conductivity (k) = 0.657 W/(m K)
Kinematic viscosity (ν) = 0.480×10^{-6} m^2/s
Prandtl number (Pr) = 3.02
Density (ρ) = 982.8 kg/m^3

The absolute viscosity is $\mu_b = 484 \times 10^{-6}$ (Ns)/m^2 at 60°C
$\mu_s = 357 \times 10^{-6}$ (Ns)/m^2 at 80°C

SOLUTION

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(0.02\text{ m/s})\,(0.0254\text{ m})}{0.480\times10^{-6}\text{ m}^2/\text{s}} = 1058 \text{ (Laminar)}$$

The thermal entrance length is given by Equation (6.8)

$$\frac{x_{fd}}{D} = 0.05\, Re_D\, Pr = 0.05\,(1058)\,(3.02) = 159.8 \rightarrow x_{fd} = 159.8\,(0.0254\text{m}) = 4.06\text{m} > L$$

Therefore, the flow is not fully developed and the Sieder and Tale correlation, Equation (6.40) will be used

$$\overline{Nu}_{D_H} = 1.86\left(Re_D\, Pr\,\frac{D}{L}\right)^{0.33}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\overline{Nu}_{D_H} = 1.86\left[1058\,(3.02)\left(\frac{0.0254}{3\text{m}}\right)\right]^{0.33}\left(\frac{484}{357}\right)^{0.14} = 5.76$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 5.76\,\frac{(0.657\text{ W/(m K)})}{0.0254\text{ m}} = 149.1 \text{ W/(m}^2\text{ K)}$$

The outlet temperature is given by Equation (6.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s} = \exp\left(-\frac{P L \bar{h}_c}{\dot{m}\, c_p}\right) = \exp\left(-\frac{(\pi D) L \bar{h}_c}{\rho V\left(\frac{\pi}{4} D^2\right) c_p}\right)$$

Solving for the bulk water outlet temperature

$$T_{b,\text{out}} = T_s + (T_{b,\text{in}} - T_s)\exp\left(-\frac{P\,\bar{h}_c\, L}{\rho V D\, c_p}\right)$$

$$T_{b,\text{out}} = 80°\text{C} + (60°\text{C} - 80°\text{C})\exp$$

$$\left(-\frac{4\left(149.1\text{ W/(m}^2\text{ K)}\right)(3\text{m})}{\left(982.8\text{ kg/m}^3\right)(0.02\text{ m/s})(0.0254\text{ m})\left(4182\text{ J/(kg K)}\right)(\text{Ws/J})}\right) = 71.5°\text{C}$$

Performing a second iteration using the water properties at the average temperature of 66°C

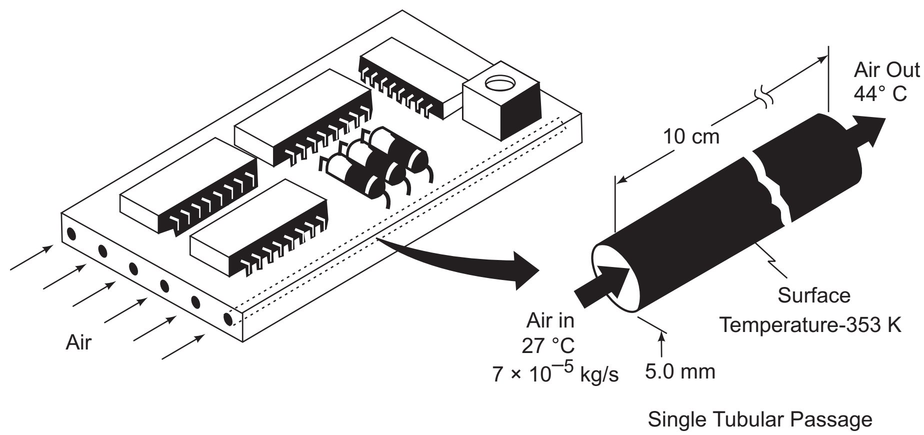
c = 4186 J/(kg K)	$\nu = 0.434 \times 10^{-6}$ m²/s	Re_D = 1171
ρ = 988.1 kg/m³	Pr = 2.71	$\overline{Nu}_D$ = 5.68
k = 0.662 W/(m K)	$\mu_b = 440.9 \times 10^{-6}$ (Ns)/m²	$\bar{h}_c$ = 147.9 W/(m² K)
		$T_{b,\text{out}}$ = 71.4°C

COMMENTS

The negligible change of $T_{b,\text{out}}$ in the second iteration could be expected because the changes in the water properties are small.

PROBLEM 6.8

An electronic device is cooled by passing air at 27°C through six small tubular passages in parallel drilled through the bottom of the device as shown below. The mass flow rate per tube is 7×10^{-5} kg/s.



Heat is generated in the device resulting in approximately uniform heat flux to the air in the cooling passage. To determine the heat flux, the air outlet temperature is measured and found to be 77°C. Calculate the rate of heat generation, the average heat transfer coefficient, and the surface temperature of the cooling channel at the center and at the outlet.

GIVEN

- Air flow through small tubular passages as shown above
- Air temperature
  - Entrance ($T_{b,\text{in}}$) = 27°C
  - Exit ($T_{b,\text{out}}$) = 77°C
- Mass flow rate per passage ($\dot{m}$) = 7×10^{-5} kg/s
- Number of passages (N) = 6

FIND

(a) The rate of heat generation ($\dot{Q}_G$)
(b) The average heat transfer coefficient ($\bar{h}_c$)
(c) Cooling channel surface temperature at the center ($T_{s,c}$)
(d) Cooling channel surface temperature at the outlet ($T_{s,\text{out}}$)

ASSUMPTIONS

- Steady state
- Uniform heat generation
- Uniform heat flux to the air
- Viscosity variation is negligible
- Heat transfer coefficient is approximately constant axially

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average bulk temperature of 52°C

Specific heat (c) = 1016 J/(kg K)
Thermal conductivity (k) = 0.0273 W/(m K)
Absolute viscosity (μ) = 19.593×10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number is

$$Re_D = \frac{V D \rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4\left(7 \times 10^{-5}\ \text{kg/s}\right)}{\pi(0.005\,\text{m})\left(19.593\times10^{-6}\ (\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 910 \text{ (Laminar)}$$

The thermal entrance length is given by Equation (6.8)

$$\frac{x_{fd}}{D} = 0.05\, Re_D\, Pr = 0.05\ (910)\ (0.71) = 32.3 \rightarrow x_{fd} = 32.3\ (0.005\text{ m}) = 0.16\text{ m} > L$$

Therefore, the temperature profile is not fully developed.

(a) The total rate of heat generation can be obtained by an energy balance

$$\dot{q}_G = N\,\dot{m}_{\text{total}}\ c(T_{b,\text{out}} - T_{b,\text{in}}) = 6\ \left(7 \times 10^{-5}\ \text{kg/s}\right)\ (1016\ \text{J/(kg K)})\ (\text{W s/J})\ (77°\text{C} - 27°\text{C}) = 21.3\text{ W}$$

(b) The Nusselt number for this geometry with uniform heat flux and fully developed flow is given Table 6.1 as $\overline{Nu}$ = 4.364. Since no correction for entrance effect in a tube with uniform heat flux boundary is given in the text, the fully developed value will be used.

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 4.36\ \frac{(0.0273\,\text{W/(m K)})}{0.005\,\text{m}} = 23.8\ \text{W/(m}^2\ \text{K)}$$

(c) The surface temperature at the center is the average surface temperature (T_s) given by

$$q = \bar{h}_c\ 6\ \pi D\, L\ (T_s - T_{b,\text{ave}}) = \dot{q}_G$$

Solving for the duct surface temperature

$$T_s = \frac{\dot{q}_G}{\bar{h}_c 6\pi DL} + \frac{T_{b,\text{in}} + T_{b,\text{out}}}{2} = \frac{21.3\,\text{W}}{\left(23.8\,\text{W/(m}^2\text{K)}\right) 6\pi(0.005\,\text{m})\ (0.1\,\text{m})} + \frac{27\,°\text{C} + 77\,°\text{C}}{2} = 147°\text{C}$$

(d) The heat flux to be air is

$$\frac{q}{A} = \frac{\dot{q}_G}{A} = \frac{\dot{q}_G}{6\pi DL} = \frac{21.3\,\text{W}}{6\pi(0.005\,\text{m})\ (0.1\,\text{m})} = 2260\ \text{W/m}^2$$

The surface temperature at the outlet is given be

$$\frac{q}{A} = h_{cL}\,(T_{s,\text{out}} - T_{b,\text{out}}) \rightarrow T_{s,\text{out}} = \frac{q}{A}\frac{1}{h_{cL}} = T_{b,\text{out}}$$

$$\therefore\ (T_{s,\text{out}})_{\max} = \frac{\left(2260\ \text{W/m}^2\right)}{\left(23.8\ \text{W/(m}^2\text{K)}\right)} + 77°\text{C} = 172°\text{Cs}$$

PROBLEM 6.9

Unused engine oil with a 100°C inlet temperature flows at a rate of 250 g/sec through a 5.1-cm-*ID* pipe that is enclosed by a jacket containing condensing steam at 150°C. If the pipe is 9 m long, determine the outlet temperature of the oil.

GIVEN

- Unused engine oil flows through a pipe enclosed by a jacket containing condensing steam.
- Oil flow rate, $\dot{m}$ = 250 g/s = 0.25 kg/s.
- Oil inlet temperature, $T_{b,\text{in}}$ = 100°C.
- Inner or inside diameter of pipe in which oil flows, D = 5.1 cm = 0.051 m.
- Length of heated pipe (heated by condensing steam) in which oil flows, L = 9 m.
- Temperature of condensing steam, T_s = 150°C.

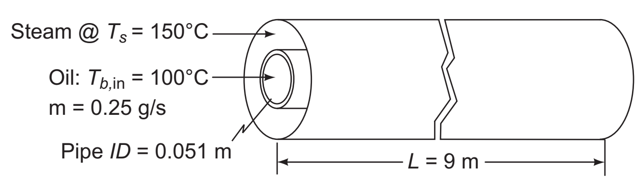
FIND

- Temperature of oil, $T_{b,\text{out}}$, at the outlet of the 9 m long heated pipe.

ASSUMPTIONS

- Steady-state flow of oil and its heating by the condensing steam in the outer jacket.
- The temperature of condensing steam is constant and uniform across the length of pipe.
- The thermal resistance of the pipe is negligible, and hence the inside surface temperature of the pipe is $T_w = T_s$, this represents a uniform pipe surface temperature condition.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 17, for unused engine oil at $T_{b,\text{in}}$ = 100°C

Density, ρ = 840.0 kg/m^3

Thermal conductivity, k = 0.137 W/(m K)

Absolute viscosity, $\mu_b = 17.1 \times 10^{-3}$ (Ns)/m^2

Prandtl number, Pr = 276

Specific heat, c_p = 2219 J/(kg K)

At the pipe surface temperature of 150°C, the absolute viscosity $\mu_s = 5.52 \times 10^{-3}$ (Ns)/m^2

SOLUTION

The Reynolds number for oil flow inside the pipe is

$$\mathrm{Re}_D = \frac{\rho VD}{\mu_b} = \frac{4\dot{m}}{\pi D \mu_b} = \frac{4\times 0.25}{\pi \times 0.051 \times 0.017} = 367.1 \Rightarrow \text{Laminar flow}$$

The thermal entrance length is given by Equation (6.8) for laminar flow, and it can be calculated as

$$x_{fd} = 0.05D\,\mathrm{Re}_D\,\mathrm{Pr} = 0.05\times 0.051\times 367\times 276 = 258\text{ m} \gg L = 9\text{ m}$$

Hence, the temperature profile is NOT fully developed, or the flow is thermally developing.

Because there is a large variation in the oil viscosity at the pipe wall temperature and the bulk temperature, the effect of property (viscosity) variation has to be considered. From Section 6.3.3 either the Hausen correlation of Equation (6.41) or the Sieder and Tate correlation of Equation (6.42) could be used because (μ_b/μ_s) = 3.1 (< 9.75; the limit for Equation (6.42) to calculate the Nusselt number. Thus, using the more simpler Sieder and Tate correlation

$$\overline{\mathrm{Nu}}_D = 1.86\left(\frac{\mathrm{Re}_D\,\mathrm{Pr}\,D}{L}\right)\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left(\frac{367\times 276\times 0.051}{9}\right)\left(\frac{0.0171}{0.00552}\right)^{0.14} = 1251$$

$$\Rightarrow \quad \bar{h}_c = \overline{\mathrm{Nu}}_D\frac{k}{D} = 1251\frac{0.137}{0.051} = 3361\ \text{W/(m}^2\text{ K)}$$

The outlet temperature can now be calculated by Equation (6.36) as

$$\frac{\Delta T_{out}}{\Delta T_{in}} = \exp\left(-\frac{\bar{h}_c PL}{\dot{m}c_p}\right) \quad \Rightarrow \quad T_{b,out} = T_s + \left(T_{b,in} - T_s\right)\exp\left(-\frac{\bar{h}_c PL}{\dot{m}c_p}\right)$$

$$\therefore \quad T_{b,\,out} = 150 + (100-150)\exp\left(-\frac{3361\times\pi\times 0.051\times 9}{0.25\times 2219}\right) = 149.9 \approx 150°\text{C}$$

COMMENTS

The oil flow attains the tube wall (or the condensing steam) temperature at the outlet of the 9-m-long pipe. Also, because of the 50°C temperature difference between the inlet and the outlet, the above calculation should be repeated after evaluating the properties at the average temperature between the inlet and outlet.

PROBLEM 6.10

Determine the rate of heat transfer per foot length to a light oil flowing through a 1-in.-*ID*, 0.6 m copper tube at a velocity of 0.03 m/s. The oil enters the tube at 15°C and the tube is heated by steam condensing on its outer surface at atmospheric pressure with a heat transfer coefficient of 11.3 kW/(m² K). The properties of the oil at various temperatures are listed in the accompanying tabulation

***T*(°C)**	**15**	**30**	**40**	**65**	**100**
ρ (kg/m³)	**938.8**	**938.8**	**922.3**	**906**	**889.4**
***c* (kJ/(kg K))**	**1.8**	**1.84**	**1.92**	**2.0**	**2.13**
***k* (W/(m K))**	**0.133**	**0.133**	**0.131**	**0.129**	**0.128**
μ (kg/ms)	**0.089**	**0.0414**	**0.023**	**0.00786**	**0.0033**
Pr	**1204**	**573**	**338**	**122**	**55**

GIVEN

- Oil flowing through a copper tube with atmospheric pressure steam condensing on the outer surface
- Oil properties listed above
- Inside diameter (D) = 2.5 cm
- Tube length (L) = 0.6 m
- Oil velocity (V) = 0.03 m/s
- Inlet oil temperature ($T_{b,\text{in}}$) = 16°C
- Heat transfer coefficient on outside of pipe ($\bar{h}_{c,o}$)= 11.3 × 10^3 W/(m^2 K)

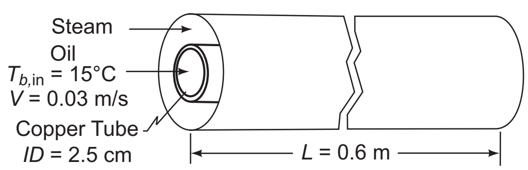
FIND

- The rate of heat transfer (q) to the oil

ASSUMPTIONS

- Steady state
- The thermal resistance of the copper tube is negligible
- Constant wall temperature
- The tube wall is thin

SKETCH



PROPERTIES AND CONSTANTS

At atmospheric pressure, steam condenses at a temperature (T_s) of 100°C.

SOLUTION

The Reynolds number for the oil flowing through the pipe is

$$Re_D = \frac{V\,D\,\rho}{\mu}$$

Using the oil properties at the inlet temperature of 15°C

$$Re_D = \frac{(0.03\,\text{m/s})(2.5\times10^{-2}\,\text{m})(938.8\,\text{kg/m})}{(0.089\,\text{kg/ms})} = 7.91 \text{ (Laminar)}$$

The thermal entrance length is given by Equation (6.8)

$$\frac{x_{fd}}{D} = 0.05\,Re_D\,Pr = 0.05\,(7.91)\,(1204) = 476 \Rightarrow x_{fd} = 476\,(0.025\text{ m}) = 11.9\text{ m} >> \text{L}$$

Therefore, the temperature profile is not fully developed and the Hausen correlation of Equation (6.39) will be used (assuming the wall temperature ≈ T_s for μ_s)

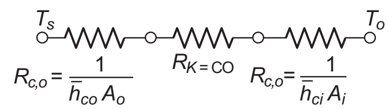
$$\overline{Nu} = \left[3.66 + \frac{0.0668\,Re_{D_H}\,Pr\left(\frac{D}{L}\right)}{1+0.045\left[Re_{D_H}\,Pr\left(\frac{D}{L}\right)\right]^{0.66}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\overline{Nu} = \left[3.66 + \frac{0.0668\,(7.91)\,(1204)\left(\dfrac{2.5\times10^{-2}}{0.6}\right)}{1+0.045\left[(7.91)\,(1204)\left(\dfrac{0.025}{0.6}\right)\right]^{0.66}} \right] \left(\frac{0.089}{0.0033}\right)^{0.14}$$

$$= \left[3.66 + \frac{26.51}{2.335}\right](1.586) \ = 23.8$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 23.8 \ \frac{0.133\,\text{W/(m K)}}{0.025\,\text{m}} = 126.6\ \text{W/(m}^2\ \text{K)}$$

The thermal circuit for heat flow from the steam to the oil is shown below



If the tube wall is thin, $A_o \approx A_i = \pi DL = \pi(0.025\ \text{m})(0.6\ \text{m}) = 0.0471\ \text{m}^2$ and the thermal resistance is

$$A\,R_{co} = \frac{1}{11.3\,\text{kW/(m}^2\text{K)}} = 8.85 \times 10^{-5}\ (\text{m}^2\ \text{K})/\text{W}$$

$$A\,R_{co} = \frac{1}{126.6\,\text{W/(m}^2\text{K)}} = 7.9 \times 10^{-3}\ (\text{m}^2\ \text{K})/\text{W}$$

$$A\,R_{\text{total}} = A\,R_{co} + A\,R_{ci} = (0.0885 + 7.9) \times 10^{-3}\ (\text{m}^2\text{K})/\text{W} = 7.99 \times 10^{-3}\ (\text{m}^2\ \text{K})/\text{W}$$

The outlet temperature can be calculated by replacing h_cA by $1/AR_{\text{total}}$ in Equation (6.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s} = \exp\left(-\frac{P\,L}{(A\,R_{\text{total}})\,\dot{m}\,c_p}\right) = \exp\left(-\frac{\pi DL}{(A\,R_{\text{total}})\,(\rho VA)\,c_p}\right)$$

$$T_{b,\text{out}} = T_s + (T_{b,\text{in}} - T_s)\ \exp\left(-\frac{4\,L}{(A\,R_{\text{total}})\,D\rho VA\,c_p}\right)$$

$$T_{b,\text{out}} = 100 + (15 - 100)$$

$$\exp\left[-\frac{4\,(0.6\,\text{m})}{(7.99\times10^{-3}\ (\text{m}^2\text{K})/\text{W})\,(0.025\,\text{m})\,(938.8\ \text{kg/m}^3)\,(0.03\ \text{m/s})\,(1800\,\text{J/kg K})}\right]$$

$$\Rightarrow \qquad T_{b,\text{out}} = 100 - 85\ \exp\left[-\frac{2.4}{10.126}\right] = 32.9°\text{C}$$

The mean temperature of oil = $\dfrac{15 + 32.9}{2} \simeq 24°\text{C}$. Hence the properties used above may change significantly at this temperature.

This is a significant change in the oil temperature and warrants another iteration using the properties of the oil at the average bulk temperature of 24°C. Interpolating the oil properties from the given data

$\rho = 938.8\ \text{kg/m}^3$ $\qquad Re = 11.93$

$c = 1840\ \text{J/(kg K)}$ $\qquad \overline{Nu}_D = 21.6$

$k = 0.133\ \text{W/(m K)}$ $\qquad A\,R_{\text{total}} = 8.15 \times 10^{-3}\ ((\text{m}^2\ \text{K})/\text{W})$

$\mu_b = 0.059\ \text{kg/ms}$ $\qquad T_{b,\text{out}} = 32.3°\text{C}$

$Pr = 816$

The rate of heat transfer is given by Equation (6.37) substituting $1/A\,R_{\text{total}}$ for h_c

$$q_c = \frac{A}{A\,R_{\text{total}}}\left[\frac{\Delta T_{\text{out}}}{\ln\dfrac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}}}\right]$$

$$\Rightarrow \qquad q_c = \frac{0.0471}{8.15\times10^{-3}}\left[\frac{(100-32.3)-(100-15)}{\ln\left(\dfrac{100-32.3}{100-15}\right)}\right] = 438\text{ W}$$

COMMENTS

Note that 99% of the thermal resistance is on the inside of the pipe.

PROBLEM 6.11

Calculate the Nusselt number and the convection heat transfer coefficient by three different methods for water at a bulk temperature of 32°C flowing at a velocity of 1.5 m/s through a 2.54-cm-*ID* duct with a wall temperature of 43°C. Compare the results.

GIVEN

- Water flowing through a duct
- Bulk water temperature (T_b) = 32°C
- Water velocity (V) = 1.5 m/s
- Inside diameter of duct (D) = 2.54 cm = 0.0254 m
- Duct wall surface temperature (T_s) = 43°C

FIND

Use three different methods to find

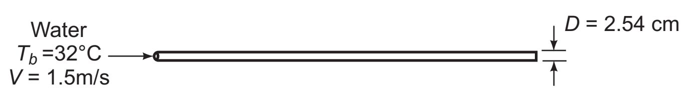
(a) The Nusselt number (Nu_D)

(b) The convective heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- Fully developed, incompressible flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at a reference temperature equal to the bulk temperature (T_{ref} = 32°C)

Thermal conductivity (k) = 0.619 W/(m K)

Kinematic viscosity (ν) = 0.773×10^{-6} m^2/s

Prandtl number (Pr) = 5.16

Absolute viscosity (μ_b) = 763×10^{-6} (Ns)/m^2

At the surface temperature of 43°C: Absolute viscosity (μ_s) = 626.3×10^{-6} (Ns)/m^2

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_{co}\,D}{\nu} = \frac{(1.5\text{ m/s})\,(0.0254\text{ m})}{0.773\times10^{-6}\text{ m}^2/\text{s}} = 4.93\times10^4 > 2000$$

Therefore, the flow is turbulent.

(a) Therefore, the flow is turbulent. Three different correlations that can be used to calculate the Nusselt number are contained in Table 6.3

1. The Dittus-Boelter Equation (6.63)
2. The Sieder and Tate Equation (6.64)
3. The Petukhov-Popov Equation (6.66)

1. $\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n$ where $n = 0.4$ for heating

$$\overline{Nu}_D = 0.023\,(4.93 \times 10^4)^{0.8}\,(5.16)^{0.4} = 252$$

2. $$\overline{Nu}_D = 0.027 Re_D^{0.8} Pr^{0.3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027\,(4.93 \times 10^4)^{0.8}\,(5.16)^{0.3} \left(\frac{763}{626.3}\right)^{0.14} = 257$$

3. $$\overline{Nu}_D = \frac{\left(\frac{f}{8}\right) Re_D Pr}{K_1 + K_2 \left(\frac{f}{8}\right)^{\frac{1}{2}} \left(Pr^{\frac{2}{3}} - 1\right)}$$

where $f = (1.82 \log Re_D - 1.64)^{-2} = [1.82 \log(4.93 \times 10^4) - 1.64]^{-2} = 0.0210$

$K_1 = 1 + 3.4 f = 1 + 3.4(0.0210) = 1.071$

$$K_2 = 11.7 + \frac{1.8}{\left(Pr^{\frac{2}{3}}\right)} = 11.7 + \frac{1.8}{\left[(5.16)^{\frac{2}{3}}\right]} = 12.30$$

$$\overline{Nu}_D = \frac{\left(\frac{0.0210}{8}\right)(4.93 \times 10^4)\,(5.16)}{1.071 + 12.30\left(\frac{0.0210}{8}\right)^{\frac{1}{2}} \left[(5.16)^{\frac{2}{3}} - 1\right]} = 288$$

(b) The heat transfer coefficient is given by

1. $$\bar{h}_c = 252\, \frac{(0.619\ \text{W}/(\text{m K}))}{0.0254\ \text{m}} = 6141\ \text{W}/(\text{m}^2\ \text{K})$$

2. $$\bar{h}_c = 257\, \frac{(0.619\ \text{W}/(\text{m K}))}{0.0254\ \text{m}} = 6263\ \text{W}/(\text{m}^2\ \text{K})$$

3. $$\bar{h}_c = 288\, \frac{(0.619\ \text{W}/(\text{m K}))}{0.0254\ \text{m}} = 7019\ \text{W}/(\text{m}^2\ \text{K})$$

COMMENTS

The Nusselt numbers vary by about 8% around the average value of 266. This is within the accuracy of empirical correlations.

PROBLEM 6.12

Atmospheric pressure air is heated in a long annulus (25 cm *ID*, 38 cm *OD*) by steam condensing at 149°C on the inner surface. If the velocity of the air is 6 m/s and its bulk temperature is 38°C, calculate the heat transfer coefficient.

GIVEN

- Atmospheric flow through an annulus with steam condensing in inner tube
- Diameters
  - Inside (D_i) = 25 cm = 0.25 m
  - Outside (D_o) = 38 cm = 0.38 m

- Steam temperature (T_s) = 149°C
- Air velocity (V) = 6 m/s
- Air bulk temperature (T_b) = 38°C

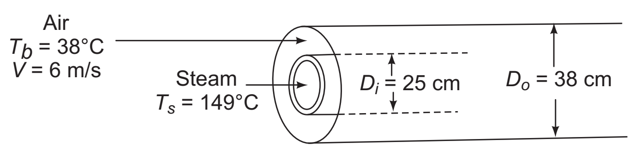
FIND

- The heat transfer coefficient ($\bar{h}_c$)

ASSUMPTIONS

- Steady state
- Steam temperature is constant and uniform
- Heat transfer to the outer surface is negligible
- Air temperature given is the average air temperature
- Thermal resistance of inner tube wall and condensing steam is negligible (Inner tube wall surface temperature = T_s)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 38°C

Density (ρ) = 1.099 kg/m^3
Thermal conductivity (k) = 0.0264 W/(m K)
Absolute viscosity (μ_b) = 19.0 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71

At the surface temperature of 149°C μ_s = 23.7 × 10^{-6} (Ns)/m^2

SOLUTION

As shown in Equation (6.3), the hydraulic diameter of the annulus is given by

$$D_H = D_o - D_i = 0.38 \text{ m} - 0.25 \text{ m} = 0.13 \text{ m}$$

The Reynolds number based on this diameter is

$$Re_D = \frac{V D_H \rho}{\mu} = \frac{(6\text{ m/s})\,(0.13\text{m})\left(1.099\text{kg/m}^3\right)}{19.035\times10^{-6}\left((\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)} = 4.50 \times 10^4 \text{ (Turbulent)}$$

Applying the Seider-Tale correlation of Equation (6.64)

$$\overline{Nu}_D = 0.027\, Re_D^{0.8}\, Pr^{0.3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027\,(4.50 \times 10^4)^{0.8}\,(0.71)^{0.3} \left(\frac{19.0}{23.7}\right)^{0.14} = 125$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 125 \frac{(0.0264\text{ W/(m K)})}{0.13\text{m}} = 25.4 \text{ W/(m}^2\text{ K)}$$

PROBLEM 6.13

If the total resistance between the steam and the air (including the pipe wall and scale on the steam side) in Problem 6.12 is 0.05 m^2 K/W, calculate the temperature difference between the outer surface of the inner pipe and the air. Show the thermal circuit.

From Problem 6.12: In a long annulus (25 cm *ID*, 38 cm *OD*), atmospheric air is heated by steam condensing at 149°C on the inner surface. The velocity of the air is 6 m/s and its bulk temperature is 38°C.

GIVEN

- Atmospheric flow through an annulus with steam condensing in inner tube
- Diameters Inside
  - (D_i) = 25 cm = 0.25 m
  - Outside (D_o) = 38 cm = 0.38 m
- Steam temperature (T_s) = 149°C
- Air velocity (V) = 6 m/s
- Total resistance between the steam and air ($A_t R_{\text{tot}}$) = 0.05 (m^2 K)/W
- Air bulk temperature (T_b) = 38°C
- From Problem 6.12 heat transfer coefficient on the outer surface of the inner pipe ($\bar{h}_c$) = 25.4 W/(m^2 K)

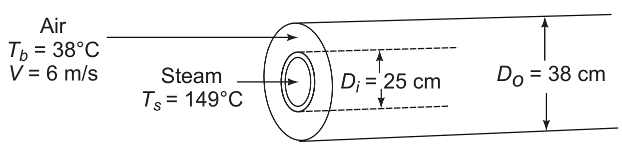
FIND

- The temperature difference between the outer surface of the inner pipe and the air (ΔT)

ASSUMPTIONS

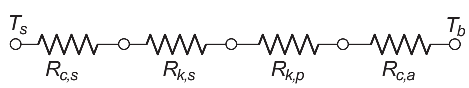
- Steady state
- Steam temperature is constant and uniform
- Heat transfer to the outer surface is negligible
- Air temperature given is the average air temperature
- Thermal resistance of inner tube wall and condensing steam is negligible (Inner tube wall surface temperature = T_s)

SKETCH



SOLUTION

The thermal circuit for the heat transfer between the steam and the air is shown below



where $R_{c,s}$ = Convective thermal resistance on the steam side

$R_{k,s}$ = Conductive thermal resistance of scaling on the steam side

$R_{k,p}$ = Conductive thermal resistance of the pipe wall

$R_{c,a}$ = Convective thermal resistance on the air side = $1/A_t \bar{h}_c$

$R_{\text{Tot}} = R_{c,s} + R_{k,s} + R_{k,p} + R_{c,a}$

$$R_{ca} = \frac{1}{A_t \bar{h}_c} \rightarrow A_t R_{ca} = \frac{1}{\bar{h}_c} = \frac{1}{\left(25.4 \text{ W/(m}^2\text{K)}\right)} = 0.0394 \text{ (m}^2 \text{ K)/W}$$

The total rate of heat transfer must equal the rate of convective heat transfer from the pipe wall to the air

$$\frac{T_s - T_b}{R_{\text{total}}} = \frac{\Delta T}{R_{ca}}$$

$$\Delta T = \frac{R_{ca}}{R_{\text{total}}}(T_s - T_b) = \frac{A_t R_{ca}}{A_t R_{\text{total}}}(T_s - T_b) = \left(\frac{0.0394}{0.05}\right)(149°\text{C} - 38°\text{C}) = 87.4°\text{C}$$

COMMENTS

Note that 79% of the thermal resistance is the convective resistance on the air side.

PROBLEM 6.14

Atmospheric air at a velocity of 61 m/s and a temperature of 16°C enters a 0.61-m-long square metal duct of 20 × 20 cm cross section. If the duct wall is at 149°C, determine the average heat transfer coefficient. Comment briefly on the L/D_h effect.

GIVEN

- Atmospheric air flow through a square metal duct
- Air velocity (V) = 61 m/s
- Inlet air temperature ($T_{b,\text{in}}$) = 16°C
- Duct dimensions: 20 cm × 10 cm × 0.61 m = 0.2 m × 0.2 m × 0.61 m
- Duct wall surface temperature (T_s) = 149°C

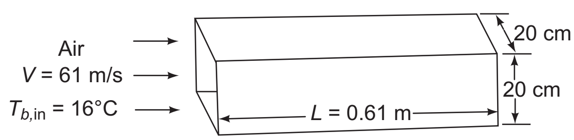
FIND

- The average heat transfer coefficient ($\bar{h}_c$)

ASSUMPTIONS

- Steady state
- Constant and uniform wall surface temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the inlet temperature of 16°C

Density (ρ) = 1.182 kg/m^3
Thermal conductivity (k) = 0.0248 W/(m K)
Absolute viscosity (μ_b) = 18.08 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71
Specific heat (c) = 1012 J/(kg K)

At the wall temperature of 149°C μ_s = 23.8 × 10^{-6} (Ns)/m^2

SOLUTION

The hydraulic diameter of the duct is given by Equation (6.2)

$$D_H = \frac{4A_c}{P} = \frac{4(0.2\,\text{m})(0.2\,\text{m})}{4(0.2\,\text{m})} = 0.2\text{ m} \Rightarrow = \frac{L}{D_H} = \frac{0.61\,\text{m}}{0.2\,\text{m}} = 3.05$$

The Reynolds number based on the hydraulic diameter is

$$Re_D = \frac{V D_H \rho}{\mu} = \frac{(61\,\text{m/s})\,(0.2\,\text{m})\left(1.182\,\text{kg/m}^3\right)}{18.08\times10^{-6}\left((\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 7.97\times10^5\text{ (Turbulent)}$$

Using the Sieder-Tate correlation of Equation (6.64) with the hydraulic diameter

$$\overline{Nu}_{D_H} = 0.027\, Re_{D_H}{}^{0.8}\, Pr^{0.3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$= 0.027(7.97 \times 10^5)^{0.8}\,(0.71)^{0.3} \left(\frac{18.08}{23.8}\right)^{0.14} = 1235$$

$$\bar{h}_c = \overline{Nu}_{D_H} \frac{k}{D_H} = 1235\, \frac{(0.0248\ \text{W/(m K)})}{0.2\ \text{m}} = 153\ \text{W/(m}^2\ \text{K)}$$

Note that since $2 < L/D_H < 20$, the heat transfer coefficient will be corrected using Equation (6.68) although this is strictly applicable only to circular ducts

$$\frac{\overline{Nu}}{\overline{Nu}_{fd}} = \frac{\bar{h}_{c,L}}{\bar{h}_c} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24/Re_D^{0.23} = 24/(7.97 \times 10^5)0.23 = 1.054$

$b = 2.08 \times 10^{-6}\, Re_D - 0.815 = 2.08 \times 10^{-6}\,(7.97 \times 10^5) - 0.815 = 0.843$

$$\bar{h}_{c,L} = \left(153\ \text{W/(m}^2\text{K)}\right)\ [1 + 1.054\,(3.05)^{0.843} = 3.70] = 566\ \text{W/(m}^2\ \text{K)}$$

The air properties at the inlet temperature were used in the calculation. This may lead to significant errors if the air temperature rises appreciably within the duct, therefore, the outlet air temperature will be calculated. The outlet temperature can be calculated using Equation (6.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{Ts - T_{b,in}} = \exp\left(-\frac{P\,L\,\bar{h}_{c,L}}{\dot{m}\,c_p}\right) = \exp\left(-\frac{\bar{h}_{c,L}\,P\,L}{A_c\,\rho V\,c}\right)$$

$$T_{b,\text{out}} = T_s - (T_s - T_{b,\text{in}})\ \exp\left(-\frac{\bar{h}_{c,L}\,P\,L}{A_c\,\rho V\,c}\right)$$

$$T_{b,\text{out}} = 149°\text{C} - (149°\text{C} - 16°\text{C})$$

$$\exp\left(-\frac{\left(566\,\text{W/(m}^2\,\text{K)}\right)4\,(0.2\,\text{m})(0.61\,\text{m})}{(0.2\,\text{m})^2\left(1.182\,\text{kg/m}^3\right)(61\,\text{m/s})(1012\,\text{J/(kg K)})((\text{Ws})/\text{J})}\right) = 28°\text{C}$$

Therefore, the average air temperature is about 22°C. The difference in air properties at 22°C and 16°C is not great enough to justify another iteration.

COMMENTS

Note that the average heat transfer coefficient in the duct is greater than that in a long duct due to the *L*/*D* effect. The heat transfer coefficient is largest at the entrance. This is analogous to flow over a flat plate as discussed in Chapter 4.

PROBLEM 6.15

Compute the average heat transfer coefficient h_c for 10°C water flowing at 4 m/s in a long, 2.5-cm-*ID* pipe (surface temperature 40°C) by three different equations and compare your results. Also determine the pressure drop per meter length of pipe.

GIVEN

- Water flowing through a pipe
- Water temperature (T_b) = 10°C
- Water velocity (V) = 4 m/s
- Inside diameter of pipe (D) = 2.5 cm = 0.025 m
- Pipe surface temperature (T_s) = 40°C

FIND

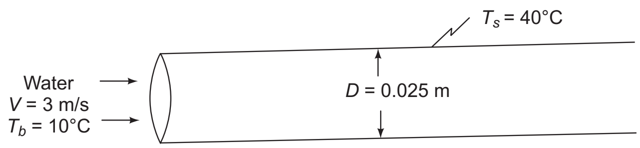
(a) The average heat transfer coefficient ($\bar{h}_c$) by 3 different equations.

(b) The pressure drop per meter length ($\Delta p/L$)

ASSUMPTIONS

- Steady state
- Uniform and constant wall surface temperature
- Pipe wall is smooth
- Fully developed flow ($L/D > 60$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 10°C

Density (ρ) = 999.7 kg/m^3
Thermal conductivity (k) = 0.577 W/(m K)
Kinematic viscosity (ν) = 1.300×10^{-6} m^2/s
Prandtl number (Pr) = 9.5
Absolute viscosity (μ_b) = 1296×10^{-6} (Ns)/m^2

At the surface temperature of 40°C $\mu_s = 658 \times 10^{-6}$ (Ns)/m^2

SOLUTION

The Reynolds number for this problem is

$$Re_D = \frac{V\,D}{\nu} = \frac{(4\text{ m/s})\,(0.025\text{m})}{1.3\times10^{-6}\text{ m}^2/\text{s}} = 7.69 \times 10^4 \text{ (Turbulent)}$$

(a)

1. Using the Dittus-Boelter correlation of Equation (6.63)

$$Nu_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$Nu_D = 0.023\,(7.69 \times 10^4)^{0.8}\,(9.5)^{0.4} = 458.8$$

$$h_c = Nu_D\,\frac{k}{D} = 458.8\,\frac{(0.577\text{ W/(m K)})}{0.025\text{m}} = 10{,}590\text{ W/(m}^2\text{ K)}$$

2. Using the Sieder-Tale correlation of Equation (6.64)

$$Nu_D = 0.027\, Re_D^{0.8}\, Pr^{0.3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027\,(7.69 \times 10^4)^{0.8}\,(9.5)^{0.3}\left(\frac{1296}{658}\right)^{0.14} = 472.9$$

$$h_c = Nu_D\,\frac{k}{D} = 472.9\,\frac{(0.577\text{ W/(mK)})}{0.025\text{m}} = 10{,}914\text{ W/(m}^2\text{ K)}$$

3. Using the Petukhov-Popov correlation of Equation (6.66)

$$Nu_D = \frac{\left(\frac{f}{8}\right) Re_D Pr}{K_1 + K_2 \left(\frac{f}{8}\right)^{\frac{1}{2}} (Pr^{\frac{2}{3}} - 1)}$$

where $f = (1.82 \log(Re_D) - 1.64)^{-2} = (1.82 \log(7.69 \times 10^4) - 1.64)^{-2} = 0.0190$

$$K_1 = 1 + 3.4\,f = 1 + 3.4(0.019) = 1.065$$

$$K_2 = 11.7 + \frac{1.8}{\left(Pr^{\frac{1}{3}}\right)} = 11.7 + \frac{1.8}{\left(9.5^{\frac{1}{3}}\right)} = 12.55$$

$$Nu_D = \frac{\left(\frac{0.019}{8}\right)(7.69 \times 10^4)(9.5)}{1.065 + 12.55\left(\frac{0.019}{8}\right)^{\frac{1}{2}}\left[(9.5)^{\frac{2}{3}} - 1\right]} = 543$$

$$h_c = Nu_D \frac{k}{D} = 543 \frac{(0.577\ \text{W/(mK)})}{0.025\,\text{m}} = 12{,}530\ \text{W/(m}^2\ \text{K)}$$

(b) The friction factor correlation of Equation (6.54) is good only for $1 \times 10^5 < Re_D$. Therefore, the friction factor will be estimated from the bottom curve of Figure 6.18: For $Re = 7.69 \times 10^4$, $f \approx 0.0188$ (Note that this is in good agreement with the friction factor, f in the Petukhov-Popov correlation).

The pressure drop per unit length can be calculated from Equation (6.13)

$$\Delta \frac{P}{D} = \frac{f}{D}\frac{\rho V^2}{2 g_c} = \frac{0.0188}{0.025\,\text{m}} \frac{999.7\ \text{kg/m}^3\ (4\,\text{m/s})^2}{2\left((\text{N m}^2)/\text{Pa}\right)\left((\text{kg m})/(\text{N s}^2)\right)} = 6014\ \text{Pa}$$

COMMENTS

The heat transfer coefficients vary around the average of 11,345 W/(m^2 K) by a maximum of 10%. This is within the accuracy of empirical correlations.

PROBLEM 6.16

Water at 80°C is flowing through a thin copper tube (15.2 cm *ID*) at a velocity of 7.6 m/s. The duct is located in a room at 15°C and the heat transfer coefficient at the outer surface of the duct is 14.1 W/(m^2 K). (a) Determine the heat transfer coefficient at the inner surface. (b) Estimate the length of duct in which the water temperature drops 1°C.

GIVEN

- Water flowing through a thin copper tube in a room
- Water temperature (T_b) = 80°C
- Inside diameter of tube (D) = 15.2 cm = 0.152 cm
- Water velocity (V) = 7.6 m/s
- Room air temperature (T_∞) = 15°C
- Outer surface heat transfer coefficient ($\bar{h}_{co}$) = 14.1 W/(m^2 K)

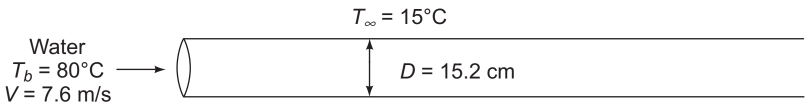
FIND

(a) The heat transfer coefficient at the inner surface ($\bar{h}_{ci}$)
(b) Length of duct (L) for temperature drop of 1°C

ASSUMPTIONS

- Steady state
- Thermal resistance of the copper tube is negligible
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 80°C

Density (ρ) 971.6 kg/m^3
Thermal conductivity (k) = 0.673 W/(m K)
Absolute viscosity (μ) = 356.7 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 2.13
Specific heat (c) = 4194 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V\,D\,\rho}{\mu} = \frac{(7.6\text{ m/s})\,(0.152\text{ m})\left(971.6\text{ kg/m}^3\right)}{356.7\times10^{-6}\left((\text{N s})/\text{m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)} = 3.15\times10^6 \text{ (Turbulent)}$$

(a) Applying the Dittus-Boelter correlation of Equation (6.63)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\,(3.15\times10^6)^{0.8}\,(2.13)^{0.3} = 4555$$

$$\bar{h}_{ci} = \overline{Nu}_D\,\frac{k}{D} = 4555\,\frac{(0.673\text{ W/(m K)})}{0.152\text{ m}} = 20{,}170\text{ W/(m}^2\text{ K)}$$

(b) Since the pipe wall is thin, $A_o = A_i$ and the overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{\bar{h}_{ci}} + \frac{1}{\bar{h}_{co}} = \left(\frac{1}{20{,}170}+\frac{1}{14.1}\right)\text{(m}^2\text{ K)/W} = 0.071\text{ (m}^2\text{ K)/W} \Rightarrow U = 14.1\text{ W/(m}^2\text{ K)} = \bar{h}_{co}$$

The length can be calculated using Equation (6.63)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_{co}}{T_{b,\text{in}} - T_{co}} = \exp\left(\frac{-P\,L\,\bar{h}_{co}}{\dot{m}\,c_p}\right) = \exp\left[\frac{-\pi\,D\,L\,\bar{h}_{co}}{\frac{\pi}{4}\,D^2\rho V c_p}\right]$$

Solving for the length

$$L = -\frac{D\,\rho V\,c_p}{4\bar{h}_c}\ln\left[\frac{T_{b,\text{out}} - T_{co}}{T_{b,\text{in}} - T_{co}}\right]$$

$$L = -\frac{(0.152\text{ m})\left(971.6\text{ kg/m}^3\right)(7.6\text{ m/s})(4194\text{ J/(kg K)})((\text{W s})/\text{J})}{4\left(14.11\text{ W/(m}^2\text{ K)}\right)}\ \ln\left(\frac{79\,°\text{C} - 15\,°\text{C}}{80\,°\text{C} - 15\,°\text{C}}\right) = 1294\text{ m}$$

For these conditions, it would take over a kilometer for a 1°C temperature drop. This is largely the result of the small natural convection heat transfer coefficient over the outer surface.

PROBLEM 6.17

Mercury at an inlet bulk temperature of 90°C flows through a 1.2-cm-*ID* tube at a flow rate of 4535 kg/h. This tube is part of a nuclear reactor in which heat can be generated uniformly at any desired rate by adjusting the neutron flux level. Determine the length of tube required to raise the bulk temperature of the mercury to 230°C without generating any mercury vapor, and determine the corresponding heat flux. The boiling point of mercury is 355°C.

GIVEN

- Mercury flow in a tube
- Inlet bulk temperature ($T_{b,\text{in}}$) = 90°C
- Inside tube diameter (D) = 1.2 cm = 0.012 m
- Flow rate ($\dot{m}$) = 4535 kg/h = 1.26 kg/s
- Outlet bulk temperature ($T_{b,\text{out}}$) = 230°C
- Boiling point of mercury = 355°C

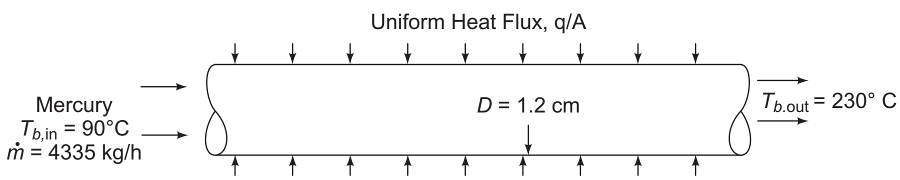
FIND

(a) The length of tube (L) required to obtain $T_{b,\text{out}}$ without generating mercury vapor
(b) The corresponding heat flux (q/A)

ASSUMPTIONS

- Steady state
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for mercury at the average bulk temperature of 160°C

Density (ρ) = 13,240 kg/m^3
Thermal conductivity (k) 11.66 W/(m K)
Absolute viscosity (μ) = 11.16 × 10^{-4} (Ns)/m^2
Prandtl number (Pr) = 0.0130
Specific heat (c) = 140.6 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V D \rho}{\pi} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(1.26\,\text{kg/s})}{\pi(0.012\,\text{m})\left(11.16\times10^{-4}\,(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 1.2 \times 10^2 \text{ (Turbulent)}$$

$$Re_D\, Pr = 1.2 \times 10^5\,(0.013) = 1557 > 100$$

Therefore, Equation (6.76) can be applied to calculate the Nusselt Number

$$\overline{Nu}_D = 4.82 + 0.0185(Re_D\, Pr)^{0.827} = 4.82 + 0.0185\,(1557)0.827 = 12.9$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 12.9\,\frac{(11.66\,\text{W/(mK)})}{0.012\,\text{m}} = 1.25 \times 10^4\ \text{W/(m}^2\ \text{K)}$$

(b) The maximum allowable heat flux is determined by the outlet conditions. The outlet wall temperature must not be higher than the mercury boiling point

$$\frac{q}{A} = (T_{\text{wall,max}} - T_{b,\text{out}})\ \bar{h}_c = (355°\text{C} - 230°\text{C})\ \left(1.25 \times 10^4\ \text{W/(m}^2\ \text{K)}\right) = 1.57 \times 10^2\ \text{W/(m}^2\ \text{K)}$$

(a) The length of the tube required can be calculated from the following

$$q = \dot{m}\, c\ (T_{b,\text{out}} - T_{b,\text{in}}) = \frac{q}{A}\ (\pi D\, L)$$

Solving for the length

$$L = \frac{\dot{m}\, c\ (T_{b,\text{out}} - T_{b,\text{in}})}{\frac{q}{A}\pi D} = \frac{(1.26\,\text{kg/s})(140.6\,\text{J/(kg K)})\ (230\,°\text{C} - 90\,°\text{C})}{\left(1.57 \times 10^6\ \text{W/(m}^2\text{K)}\right)(\text{J/(Ws)})\ \pi\ (0.012\,\text{m})} = 0.419\ \text{m}$$

COMMENTS

Note that L/D = 0.419 m/0.012 m = 35 > 30, therefore, the assumption of fully developed flow and use of Equation (6.76) is valid.

PROBLEM 6.18

> **Exhaust gases having properties similar to dry air enter a thin-walled cylindrical exhaust stack at 800 K. The stack is made of steel and is 8 m tall and 0.5 m inside diameter. If the gas flow rate is 0.5 kg/s and the heat transfer coefficient at the outer surface is 16 W/(m^2 K), estimate the outlet temperature of the exhaust gas if the ambient temperature is 280 K.**

GIVEN

- Gas flow through a vertical cylindrical thin-walled steel exhaust stack
- Gas properties are similar to dry air
- Gas entrance temperature ($T_{b,\text{in}}$) = 800 K
- Length of stack (L) = 8 m
- Diameter of stack (D) = 0.5 m
- Mass flow rate ($\dot{m}$) = 0.5 kg/s
- Heat transfer coefficient on the outer surface ($\bar{h}_{c,o}$) = 16 W/(m^2 K)
- Ambient temperature (T_∞) 280 K

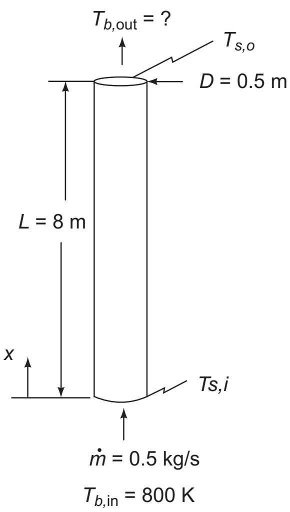
FIND

- The outlet temperature of the exhaust gas ($T_{b,\text{out}}$)

ASSUMPTIONS

- Radiation heat transfer is negligible
- Natural convection can be neglected
- The inlet to the stack is sharp-edged
- Thermal resistance of the stack wall is negligible

SKETCH



SOLUTION

For this problem, neither the heat flux nor the surface temperature will be constant. However, the ambient temperature will be constant, therefore, Equation (6.33) can be applied by replacing the surface temperature (T_s) with the constant ambient temperature (T_∞) and replacing h_c with U where

$$U = \text{Overall heat transfer coefficient} = \frac{1}{\dfrac{1}{\bar{h}_{co}} + \dfrac{1}{\bar{h}_{ci}}}$$

This results in the following version of Equation (6.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_{co}}{T_{b,\text{in}} - T_{co}} = \exp\left(-\frac{P\,L\,U}{\dot{m}\,c_p}\right) = \exp\left(-\frac{U\,\pi\,D\,L}{\dot{m}\,c_p}\right)$$

$$\therefore\ T_{b,\text{out}} = T_\infty + (T_{b,\text{in}} - T_\infty)\exp\left(-\frac{U\,\pi\,D\,L}{\dot{m}\,c_p}\right)$$

The internal heat transfer coefficient and the average fluid properties will depend on the outlet bulk fluid temperature, therefore, an iterative procedure is required. For the first iteration, let $T_{b,\text{out}}$ = 500 K. From Appendix 2, Table 27, for dry air at the average bulk temperature of 650 K

Specific Heat (c_p) = 1056 J/(kg K)
Thermal conductivity (k) = 0.0472 W/(m K)
Absolute viscosity (μ) = 31.965 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71

The Reynolds number for flow in the stack is

$$Re_D = \frac{U_{co}\,D\,\rho}{\mu} = \frac{4\,\dot{m}}{\pi\,D\,\mu} = \frac{4(0.5\,\text{kg/s})}{\pi(0.5\,\text{m})\left(31.965\times10^{-6}\,(\text{Ns})/\text{m}^2\right)\left(\text{kg m}/(\text{Ns}^2)\right)} = 3.98\times10^4\ (\text{Turbulent})$$

L/D = (8 m)/(0.5) = 16

Since 2 < L/D < 20, the flow will not be fully developed, therefore, the correlation of Molki and Sparrow for sharp-edged inlets Equation (6.68), will be used to correct the correlation of Dittus-Boelter, Equation (6.63)

$$\overline{Nu}_{fd} = 0.023\,Re_D^{0.8}\,Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

and $$\overline{Nu} = \overline{Nu}_{fd} \left[1 + a\left(\frac{L}{D}\right)^b\right]$$

where $a = 24/Re_D^{0.23} = 24/(3.98 \times 10^4)^{0.23} = 2.10$

$b = 2.08 \times 10^{-6}\, Re_D - 0.815 = 2.08 \times 10^{-6}\,(3.98 \times 10^4) - 0.815 = -0.732$

$$\overline{Nu} = 0.023\,(3.98 \times 10^4)^{0.8}\,(0.71)^{0.4}\,[1 + 2.10(16)^{-0.732}] = 122.5$$

$$\bar{h}_c = \overline{Nu}\,\frac{k}{D} = 122.5\,\frac{(0.0472\,\text{W/(m K)})}{0.5\,\text{m}} = 11.6\ \text{W/(m}^2\ \text{K)}$$

$$U = \frac{1}{\left(\frac{1}{16} + \frac{1}{11.6}\right)(\text{m}^2\text{K})/\text{W}} = 6.7\ \text{W/(m}^2\ \text{K)}$$

$$\therefore\ T_{b,\text{out}} = 280\ \text{K} + (800\ \text{K} - 280\ \text{K})\ \exp\left(-\frac{\left(6.7\,\text{W/(m}^2\text{K)}\right)\pi\,(0.5\,\text{m})\,(8\,\text{m})}{(0.5\,\text{kg/s})(1056\,\text{J/(kg K)})((\text{Ws})/\text{J})}\right) = 723\ \text{K}$$

Another iteration using the same procedure yields

Average bulk temperature = 762 K
Specific Heat (c_p) = 1074 J/(kg K)
Thermal conductivity (k) = 0.0534 W/(m K)
Absolute viscosity (μ) = 35.460 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.72
Reynolds number (Re_D) = 3.59 × 10^4
Heat transfer coefficient ($\bar{h}_{ci}$) = 12.1 W/(m^2 K)
Outlet temperature ($T_{b,\text{out}}$) = 722 K

The outlet gas temperature = 722 K

PROBLEM 6.19

Water at an average temperature of 27°C is flowing through a smooth 5.08-cm-*ID* pipe at a velocity of 0.91 m/s. If the temperature at the inner surface of the pipe is 49°C, determine (a) the heat transfer coefficient, (b) the rate of heat flow per meter of pipe, (c) the bulk temperature rise per meter, and (d) the pressure drop per meter.

GIVEN

- Water flowing through a smooth pipe
- Average water temperature (T_w) = 27°C
- Pipe inside diameter (D) = 5.08 cm = 0.0508 m
- Water velocity (V) = 0.91 m/s
- Inner surface temperature of pipe (T_s) = 49°C

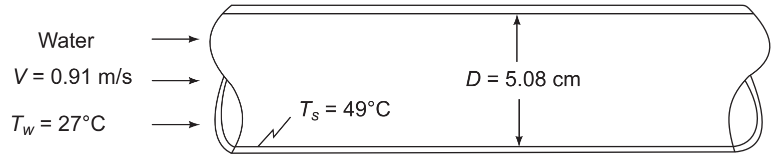
FIND

(a) The heat transfer coefficient ($\bar{h}_c$)
(b) The rate of heat flow per meter of pipe (q/L)
(c) The bulk temperature rise per meter of pipe ($\Delta T_w/L$)
(d) The pressure drop per meter of pipe ($\Delta p/L$)

ASSUMPTIONS

- Steady state
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 27°C

Density (ρ) = 996.5 kg/m^3
Specific Heat (c_p) = 4178 J/(kg K)
Thermal conductivity (k) = 0.608 W/(m K)
Absolute viscosity (μ) = 845.3 × 10^{-6} (Ns)/m^2
Kinematic viscosity (ν) = 0.852 × 10^{-6} m^2/s
Prandtl number (Pr) = 5.8

At the surface temperature of 49°C
Absolute viscosity (μ_s) = 565.1 × 10^{-6} (Ns)/m^2

SOLUTION

The Reynolds number for this flow is

$$Re_D = \frac{V\,D}{\nu} = \frac{(0.91\,\text{m/s})(0.0508\,\text{m})}{0.852\times10^{-6}\,\text{m}^2/\text{s}} = 5.42\times10^4 > 2000$$

Therefore, the flow is turbulent.

The variation in property values is accounted for by using Equation (6.64) to calculate the Nusselt number

$$\overline{Nu}_D = 0.027 Re_D^{0.8}\,Pr^{0.3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027(5.42\times10^4)^{0.8}\,(5.8)^{0.3}\left(\frac{845.3}{565.1}\right)^{0.14} = 296$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 296\,\frac{(0.608\,\text{W/(m K)})}{0.0508\,\text{m}} = 3543\ \text{W/(m}^2\ \text{K)}$$

(b) The rate of convective heat transfer is given by

$$q = \bar{h}_c\,A_t\,(T_s - T_w) = h_c\,\pi D\,L\,(T_s - T_w)$$

$$\frac{q}{L} = \left(3543\ \text{W/(m}^2\text{K)}\right)\ \pi(0.0508\ \text{m})\ (49°\text{C} - 27°\text{C}) = 12{,}438\ \text{W/m}$$

(c) This rate of heat transfer will lead to a temperature rise in the water given by

$$q = \dot{m}\,c_p\,\Delta T_w = \left(\rho V \frac{\pi}{4} D^2\right) c_p\,\Delta T_w$$

$$\therefore\ \frac{\Delta T_w}{L} = \frac{4}{\rho V\,\pi D^2 c_p}\left(\frac{q}{L}\right)$$

$$\frac{\Delta T_w}{L} = \frac{4}{\left(996.5\,\text{kg/m}^3\right)(0.91\,\text{m/s})\,\pi\,(0.0508\,\text{m})^2\,(4178\,\text{J/(kg K)})((\text{Ws})/\text{J})}\,(12{,}438\ \text{W/m}) = 1.6\ \text{K/m}$$

(d) From Table 6.4, the friction factor for fully developed turbulent flow through smooth tubes is given by Equation (6.59)

$$f = 0.184\, Re_D^{-0.2} = 0.184\,(5.42 \times 10^4)^{-0.2} = 0.0208$$

The pressure drop is given by Equation (6.13)

$$\frac{\Delta p}{L} = \frac{f \rho V^2}{2D} = \frac{0.0208\left(996.5\,\text{kg/m}^3\right)(0.91\,\text{m/s})^2}{2\,(0.0508\,\text{m})\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(\text{N}/(\text{Pa m}^2)\right)} = 169 \text{ Pa/m}$$

PROBLEM 6.20

An aniline-alcohol solution is flowing at a velocity of 3 m/s through a long, 2.5 cm-*ID* thin-wall tube. On the outer surface of the tube, steam is condensing at atmospheric pressure, and the tube-wall temperature is 100°C. The tube is clean, and there is no thermal resistance due to a scale deposit on the inner surface. Using the physical properties tabulated below, estimate the heat transfer coefficient between the fluid and the pipe by means of Equations (6.63) and (6.64), and compare the results. Assume that the bulk temperature of the aniline solution is 20°C and neglect entrance effects.

Physical properties of the aniline solution

Temperature (°C)	Viscosity (kg/ms)	Thermal Conductivity (W/(m K))	Specific Gravity	Specific Heat (kJ/(kg K))
20	0.0051	0.173	1.03	2.09
60	0.0014	0.169	0.98	2.22
100	0.0006	0.164		2.34

GIVEN

- An aniline-alcohol solution flowing through a thin-walled tube
- Tube is clean with no scaling on inner surface
- Velocity (V) = 3 m/s
- Inside diameter of tube (D) = 2.5×10^{-2} m
- Tube wall surface temperature (T_s) = 100°C
- Solution has the properties listed above
- Solution bulk temperature (T_b = 20°C)

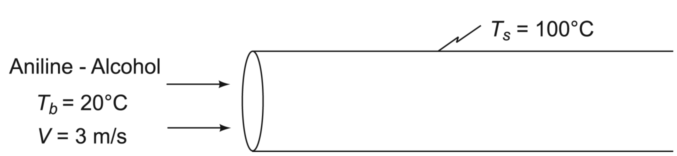
FIND

- The heat transfer coefficient ($\bar{h}_c$) using: (a) Equation (6.63) (b) Equation (6.64)

ASSUMPTIONS

- Steady state
- Entrance effects are negligible
- Thermal resistance of the tube is negligible
- Tube wall temperature is constant and uniform
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

The density of water ≈ 1000 kg/m^3

SOLUTION

The kinematic viscosity (ν) of the solution at the bulk temperature is

$$\nu = \frac{\mu}{\rho} = \frac{\mu}{(s.g.)\rho_{\text{water}}} = \frac{(0.0051\,\text{kg/ms})}{(1.03)(1000\,\text{kg/m}^3)} = 4.95 \times 10^{-6}\ \text{m}^2/\text{s}$$

The Prandtl number is

$$Pr = \frac{c_p\,\mu}{k} = \frac{(2090\,\text{J/(kg K)})(0.0051\,\text{kg/ms})}{(0.173\,\text{W/(m K)})} = 61.6$$

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(3\,\text{m/s})(2.5\times10^{-2}\,\text{m})}{4.95\times10^{-6}\,\text{m}^2/\text{s}} = 15150\ \text{(Turbulent)}$$

(a) Applying the Dittus-Boelter correlation of Equation (6.63)

$$\overline{Nu}_D = 0.023\,Re_D^{0.8}\,Pr^n \quad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(15150)^{0.8}\,(61.6)^{0.4} = 264$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 264\,\frac{0.173\,\text{W/(m K)}}{2.5\times10^{-2}\,\text{m}} = 1827\ \text{W/(m}^2\ \text{K)}$$

(b) Using the Sieder-Tate correlation of Equation (6.64)

$$\overline{Nu}_D = 0.027 Re_D^{0.8}\,Pr^{0.3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027(15150)^{0.8}\,(61.6)^{0.3}\left(\frac{5.1}{0.6}\right)^{0.14} = 277$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 277\,\frac{0.173\,\text{W/(m K)}}{2.5\times10^{-2}\,\text{m}} = 1917\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

These estimates vary by about 3% around an average value of 1880 W/(m^2 K). But the Sieder-Tate correlation is more applicable in this case because it takes the large variation of the viscosity with temperature into account.

Note that the above correlations require that all properties (except μ_s) be evaluated at the bulk temperature.

PROBLEM 6.21

In a refrigeration system, brine (10 per cent NaCl) by weight having a viscosity of 0.0016 (Ns)/m^2 and a thermal conductivity of 0.85 W/(m K) is flowing through a long 2.5-cm-*ID* pipe at 6.1 m/s. Under these conditions, the heat transfer coefficient was found to be 16,500 W/(m^2 K). For a brine temperature of –1°C and a pipe temperature of 18.3°C, determine the temperature rise of the brine per meter length of pipe if the velocity of the brine is doubled. Assume that the specific heat of the brine is 3768 J/(kg K) and that its density is equal to that of water.

GIVEN

- Brine flowing through a pipe
- Brine properties
  - Viscosity (μ) = 0.0016 Ns/m^2
  - Thermal conductivity (k) = 0.85 W/(m K)
  - 10% NaCl by weight
  - Specific heat (c) = 3768 J/(kg K)
- Pipe inside diameter (D) = 2.5 cm = 0.025 m
- Brine velocity (V) = 6.1 m/s
- Heat transfer coefficient ($\bar{h}_c$) = 16,500 W/(m^2 K)
- Brine temperature (T_b) = –1°C
- Pipe temperature (T_s) = 18.3°C

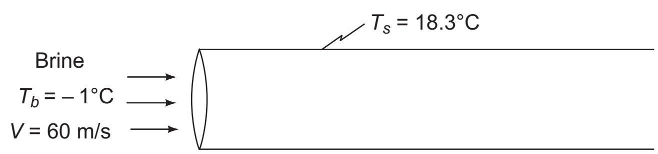
FIND

- Temperature rise of the brine per meter length ($\Delta T_b/m$) if the velocity is doubled (V = 12.2 m/s)

ASSUMPTIONS

- Steady state
- Fully developed flow
- Constant and uniform pipe wall temperature
- Density of the brine is the same as water density

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, density (ρ) of water ≈ 1000 kg/m^3

SOLUTION

The Reynolds number at the original velocity is

$$Re_D = \frac{V D \rho}{\mu} = \frac{(6.1\,\text{m/s})(0.025\,\text{m})\left(1000\,\text{kg/m}^3\right)}{0.0016\,(\text{Ns})/\text{m}^2\left((\text{kg m})/(\text{s}^2\text{N})\right)} = 95{,}313 \text{ (Trubulent)}$$

The thermal conductivity of the fluid can be calculated from the given heat transfer coefficient using the Dittus-Boelter correlation of Equation (6.63)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$k = \frac{\bar{h}_c D}{0.023 Re_D^{0.8}\left(\frac{c\mu}{k}\right)^{0.4}}$$

$$k = \left[\frac{\bar{h}_c D}{0.023 Re_D^{0.8}(c\mu)^{0.4}}\right]^{\frac{1}{0.6}}$$

$$k = \left[\frac{0.025\,\text{m}\left(16{,}500\,\text{W/(m}^2\text{K)}\right)}{0.023(95{,}313)^{0.8}\left[(3768\,\text{J/(kg K)})\left(0.0016\,(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)((\text{Ws})/\text{J})^{0.4}\right]} \right]^{\frac{1}{0.6}}$$

$$= 0.852 \text{ W/(m K)}$$

The Prandtl number is

$$Pr = \frac{c\,\mu}{k} = \frac{(3768\,\text{J/(kg K)})\left(0.0016\,(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)}{(0.852\,\text{W/(m K)})\,(\text{J/(Ws)})} = 7.08$$

The Reynolds number for the new velocity is twice the original Reynolds number: Re_D = 190,626. From Equation (6.63): For fully developed flow

$$\overline{Nu}_D = 0.023\ (190{,}626)^{0.8}\ (7.08)^{0.4} = 843$$

$$\bar{h}_c = \overline{Nu}_D\ \frac{k}{D} = 843\ \frac{(0.852\,\text{W/(m K)})}{0.025\,\text{m}} = 28{,}733\ \text{W/(m}^2\ \text{K)}$$

The temperature after one meter is given by Equation (6.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_{co}}{T_{b,\text{in}} - T_{co}} = \exp\left(-\frac{P\,L\,\bar{h}_c}{\dot{m}\,c}\right) = \exp\left(-\frac{4\,\bar{h}_c\,L}{\rho V\,D\,c}\right)$$

$$\frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s} = \exp\left(-\frac{4\left(38{,}733\,\text{W/(m}^2\text{K)}\right)(1\,\text{m})}{\left(1000\,\text{kg/m}^3\right)(12.2\,\text{m/s})\,(0.025\,\text{m})\,(3768\,\text{J/(kg K)})\,((\text{Ws})/\text{J})}\right)$$

$$= 0.9048 \text{ (per m length)}$$

$$\Delta T_b = T_{b,\text{out}} - T_{b,\text{in}} = \left[\left(\frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s}\right)\left(T_{b,\text{in}} - T_s\right) + T_s\right] - T_{b,\text{in}}$$

$$\Delta T_b = [0.9043\ (–1°\text{C} – 18.3°\text{C}) + 18.3°\text{C}] + 1°\text{C} = 1.84°\text{C per meter length}$$

PROBLEM 6.22

Derive an equation of the form $h_c = f(T, D, V)$ for turbulent flow of water through a long tube in the temperature range between 20° and 100°C.

GIVEN

- Turbulent water flow through a long tube
- Water temperature range (T_b) = 20°C to 100°C

FIND

- An expression of the form $\bar{h}_c = f(T, D.\ V)$

ASSUMPTIONS

- Steady state
- Variation of properties with temperature can be approximated with a power law
- Fully developed flow
- Water is being heated

SKETCH

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water

Temperature (°C)	20	100
Temperature (K)	293	373
Density, ρ (kg/m^3)	998.2	958.4
Thermal conductivity, k $(\mathrm{W/(m\,K)})$	0.597	0.682
Absolute viscosity, $((\mathrm{Ns})/\mathrm{m}^2)$	993×10^{-6}	277.5×10^{-6}
Prandtl number, Pr	7.0	1.75

SOLUTION

Applying the Dittus-Boelter expression of Equation (6.63) for the Nusselt number

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023 \left(\frac{\rho D V}{\mu}\right)^{0.8} Pr^n$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 0.023 \frac{\rho^{0.8} Pr^{0.4}\,\mathrm{k}}{\mu^{0.8}} D^{-0.2}\ V^{0.8}$$

To put this in the required form, the fluid properties must be expressed as a function of temperature. Assuming the power law variation

$$\text{Property} = \mathrm{AT^R}$$

where A and n are constant evaluated from the property values.

For density $\quad \rho(293) = 998.2\ \mathrm{kg/m^3} = A(293)n$

$\rho(373) = 958.2\ \mathrm{kg/m^3} = A(373)n$

Solving these simultaneously

$$A = 2613 \qquad n = -0.1694$$

Therefore, $\rho(T) = 2613\ T^{-0.1694}$

Applying a similar analysis for the remaining properties yields the following relationships

$k\,(T) = 0.02605\ T^{0.5514}$

$\mu\,(T) = 1.058 \times 1010\ T^{-5.281}$

$Pr\,(T) = 1.026 \times 1015\ T^{-5.7426}$

Substituting these into the expression for the heat transfer coefficient

$$\bar{h}_c = 0.023 \frac{(2612 T^{-0.1694})^{0.8} (1.026 \times 10^{15} T^{-5.7426})^{0.4} (0.02605 T^{0.5514})}{(1.058 \times 10^{10}\ T^{-5.281})^{0.8}} D^{-0.2}\ V^{0.8}$$

$$\bar{h}_c = 0.0031\ T^{2.34} D^{-0.2}\ V^{0.8}$$

COMMENTS

Note that in equations of the type derived, the coefficient has definite dimensions. Hence, the use of such equations is limited to the conditions specified and are not recommended.

PROBLEM 6.23

The intake manifold of an automobile engine can be approximated as a 4 cm *ID* tube, 30 cm in length. Air at a bulk temperature of 20°C enters the manifold at a flow rate of 0.01 kg/s. The manifold is a heavy aluminum casting and is at a uniform temperature of 40°C. Determine the temperature of the air at the end of the manifold.

GIVEN

- Air flow through a tube
- Tube inside diameter (D) = 4 cm = 0.04 m
- Tube length (L) = 30 cm = 0.30 m
- Inlet bulk temperature ($T_{b,\text{in}}$) = 20°C
- Air flow rate ($\dot{m}$) = 0.01 kg/s
- Tube surface temperature (T_s) = 40°C

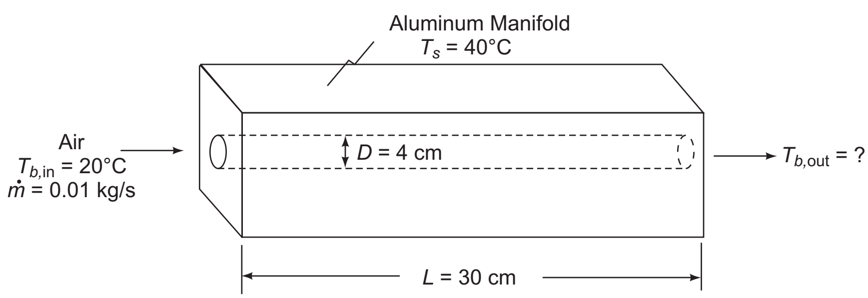
FIND

- Outlet bulk temperature ($T_{b,\text{out}}$)

ASSUMPTIONS

- Steady state
- Constant and uniform tube surface temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the inlet bulk temperature of 20°C

Thermal conductivity (k) = 0.0251 W/(m K)
Absolute viscosity (μ) = 18,240 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71
Specific heat (c) = 1012 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V D \rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.01\,\text{kg/s})}{\pi(0.04\,\text{m})\left(18.240\times10^{-6}\,(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)}$$

$$= 17{,}451 \text{ (Turbulent)}$$

The Nusselt number for fully developed flow can be estimated from the Dittus-Boelter correlation of Equation (6.36)

$$Nu_{fd} = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$Nu_{fd} = 0.023\,(17{,}451)^{0.8}\,(0.71)^{0.4} = 49.62$$

$$h_{c,fd} = Nu_{fd}\,\frac{k}{D} = 49.62\,\frac{(0.0251\,\text{W/(m K)})}{0.04\,\text{m}} = 31.14\ \text{W/(m}^2\ \text{K)}$$

Since L/D = 30cm/4 cm = 7.5 < 60, the flow is not fully developed and the fully developed heat transfer coefficient must be corrected using Equation (6.68)

$$\frac{Nu}{Nu_{fd}} = \frac{h_{c,L}}{h_{c,fd}} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24/Re_D^{0.23} = 24/(17{,}451)^{0.23} = 2.538$

$b = 2.08 \times 10^{-6}\, Re_D - 0.815 = 2.08 \times 10^{-6}\,(17{,}451) - 0.815 = -\,0.7787$

$$h_{c,L} = \left(31.14\ \text{W/(m}^2\text{K)}\right)\left[1 + 2.538\left(\frac{30\,\text{cm}}{4\,\text{cm}}\right)^{-0.7787}\right] = 47.60\ \text{W/(m}^2\ \text{K)}$$

Applying Equation (6.36) to determine the outlet air temperature

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s} = \exp\left(-\frac{P\,L\,h_c}{\dot{m}\,c}\right) = \exp\left(-\frac{h_c\,\pi\,D\,L}{\dot{m}\,c}\right)$$

$$T_{b,\text{out}} = T_s - (T_s - T_{b,\text{in}})\,\exp\left(-\frac{h_c\,\pi\,D\,L}{\dot{m}\,c}\right)$$

$$T_{b,\text{out}} = 40°\text{C} - (40°\text{C} - 20°\text{C})\,\exp\left(-\frac{\left(47.60\,\text{W/(m}^2\text{K)}\right)\pi\,(0.04\,\text{m})\,(0.3\,\text{m})}{(0.01\,\text{kg/s})\,(1012\,\text{J/(kg K)})\,((\text{Ws})/\text{J})}\right) = 23.2°\text{C}$$

COMMENTS

The rise in air temperature is not large enough to require another iteration using new air properties at the average bulk air temperature.

PROBLEM 6.24

High-pressure water at a bulk inlet temperature of 93°C is flowing with a velocity of 1.5 m/s through a 0.015-m-diameter tube, 0.3 m long. If the tube wall temperature is 204°C, determine the average heat transfer coefficient and estimate the bulk temperature rise of the water.

GIVEN

- Water flowing through a tube
- Bulk inlet water temperature ($T_{b,\text{in}}$) = 93°C
- Water velocity (V) = 1.5 m/s
- Tube diameter (D) = 0.015 m
- Tube length (L) = 0.3 m
- Tube surface temperature (T_s) = 204°C

FIND

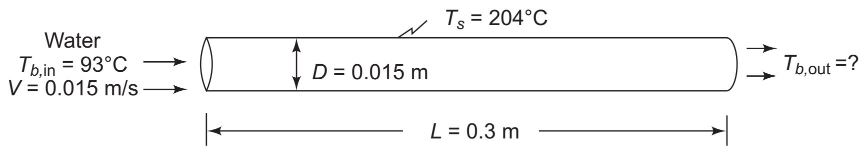
(a) The average heat transfer coefficient ($\bar{h}_{c,L}$)

(b) The bulk temperature rise of the water (ΔT_b)

ASSUMPTIONS

- Steady state
- Constant and uniform tube temperature
- Pressure is high enough to supress vapor generation.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the inlet bulk temperature of 93°C

Density (ρ) = 963.0 kg/m^3
Thermal conductivity (k) = 0.679 W/(m K)
Kinematic viscosity (ν) = 0.314 × 10^{-6} m^2/s
Prandtl number (Pr) = 1.88
Specific heat (c) = 4205 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(1.5\,\text{m/s})(0.015\,\text{m})}{0.314\times 10^{-6}\,\text{m}^2/\text{s}} = 71{,}656 \text{ (Turbulent)}$$

(a) The Nusselt number for fully developed flow can be estimated from the Dittus-Boelter correlation of Equation (6.36)

$$\overline{Nu}_{fd} = 0.023\, Re_D^{0.8}\, Pr^n \quad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_{fd} = 0.023\,(71{,}656)^{0.8}\,(1.88)^{0.4} = 226.8$$

$$\bar{h}_{c,fd} = \overline{Nu}_{fd}\,\frac{k}{D} = 226.8\,\frac{(0.679\,\text{W/(m K)})}{0.015\,\text{m}} = 10{,}265 \text{ W/(m}^2\text{ K)}$$

Since L/D = 0.3/0.015 = 20 < 60, the heat transfer coefficient must be corrected by Equations (6.68) and (6.69). Since L/D is at the upper end of the range for (6.68) and the lower end of the range for (6.69), the average of the two equations will be used.

From (6.68)

$$\frac{\overline{Nu}}{\overline{Nu}_{fd}} = \frac{\bar{h}_{c,L}}{\bar{h}_c} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24\, Re_D^{0.23} = 24/(71{,}656)^{0.23} = 1.834$

$b = 2.08 \times 10^{-6}\, Re_D - 0.815 = 2.08 \times 10^{-6}\,(71{,}656) - 0.815 = -\,00.666$

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + 1.834\left(\frac{0.3}{0.015}\right)^{-0.666} = 1.249$$

From (6.69)

$$\frac{\overline{Nu}}{\overline{Nu}_{fd}} = \frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + \left(\frac{6D}{L}\right) = 1 + \left(\frac{6(0.015)}{0.3}\right) = 1.300$$

The average of the two values is $\bar{h}_c, \bar{L} / \bar{h}_{c,fd} = 1.27$

$$\therefore \quad \bar{h}_{c,L} = 1.27\left(10{,}265\,\text{W/(m}^2\text{K)}\right) = 13{,}037\ \text{W/(m}^2\ \text{K)}$$

(b) The bulk temperature can be calculated from Equations (6.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}} = \exp\left(-\frac{P L \bar{h}_c}{\dot{m} c_p}\right) = \exp\left(-\frac{4 \bar{h}_c L}{\rho V D c}\right)$$

$$T_{b,\text{out}} = T_s - (T_s - T_{b,\text{in}}) \exp = \left(-\frac{4 \bar{h}_c L}{\rho V D c}\right)$$

$$T_{b,\text{out}} = 204\ °\text{C} - (204°\text{C} - 93°\text{C})$$

$$\exp\left(-\frac{4\left(13{,}037\,\text{W/(m}^2\text{K)}\right)(0.3\,\text{m})}{\left(963.0\,\text{kg/m}^3\right)(1.5\,\text{m/s})(0.015\,\text{m})(4205\,\text{J/(kg K)})((\text{Ws})/\text{J})}\right) = 111°\text{C}$$

The bulk temperature rise is

$$\Delta T_b = T_{b,\text{out}} - T_{b,\text{in}} = 111°\text{C} - 93°\text{C} = 18°\text{C}$$

PROBLEM 6.25

Suppose an engineer suggests that air is to be used instead of water in the tube of Problem 6.24 and the velocity of the air is to be increased until the heat transfer coefficient with the air equals that obtained with water at 1.5 m/s. Determine the velocity required and comment on the feasibility of the engineer's suggestion. Note that the speed of sound in air at 100°C is 387 m/s.

From Problem 6.24: Water at a bulk inlet temperature of 93°C is flowing with a velocity of 1.5 m/s through a 0.015-m-diameter tube, 0.3 m long. If the tube wall temperature is 204°C, determine the average heat transfer coefficient and estimate the bulk temperature rise of the water.

GIVEN

- Air flow through a tube
- Bulk inlet air temperature ($T_{b,\text{in}}$) = 93°C
- Tube diameter (D) = 0.015 m
- Tube length (L) = 0.3 m
- Tube surface temperature (T_s) = 204°C
- From Problem 6.23: $\bar{h}_{c,L}$ = 13,037 W/(m^2 K)

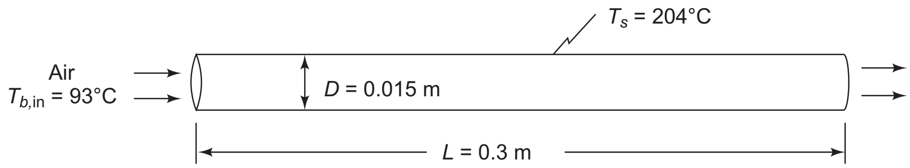
FIND

- The velocity (V) required to obtain $\bar{h}_{c,L}$ = 13,037 W/(m^2 K)

ASSUMPTIONS

- Steady state
- Constant and uniform tube temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the inlet bulk temperature of 93°C

Thermal conductivity (k) = 0.0302 W/(m K)

Kinematic viscosity (ν) = 22.9×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The flow must be turbulent, therefore, the heat transfer coefficient of the fully developed case must be 13,037 W/(m^2 K) as shown in Problem 6.24. Therefore, the Nusselt number is

$$\overline{Nu}_{fd} = \frac{\bar{h}_{c,fd} D}{k} = \frac{(13{,}037\,\text{W/(m}^2\,\text{K)})(0.015\,\text{m})}{(0.0302\,\text{W/(m K)})} = 6475$$

Applying the Dittus-Boelter correlation of Equation (6.63)

$$\overline{Nu}_{fd} = 0.023\, Re_D^{0.8}\, Pr^n = 5099 \qquad \text{where } n = 0.4 \text{ for heating}$$

Solving for the Reynolds number

$$Re_D = \frac{V D}{\nu} = \left(\frac{\overline{Nu}_{fd}}{0.023\,Pr^{0.4}}\right)^{1.25} = \left(\frac{6475}{0.023(0.71)^{0.4}}\right)^{1.25} = 7.70 \times 10^6$$

Solving for the velocity

$$V = Re_D \frac{\nu}{D} = 7.70 \times 10^6 \frac{(22.9\times10^{-6}\,\text{m}^2/\text{s})}{0.015\,\text{m}} = 11{,}749 \text{ m/s}$$

This velocity is obviously unrealistic because it corresponds to a Mach number of 30. Under such conditions when the speed of sound is reached, a shock wave will form and choke the flow.

PROBLEM 6.26

Atmospheric air at 10°C enters a 2 m long smooth rectangular duct with a 7.5 cm × 15 cm cross-section. The mass flow rate of the air is 0.1 kg/s. If the sides are at 150°C, estimate (a) the heat transfer coefficient, (b) the air outlet temperature, (c) the rate of heat transfer, and (d) the pressure drop.

GIVEN

- Atmospheric air flow through a rectangular duct
- Inlet bulk temperature ($T_{b,\text{in}}$) = 10°C
- Duct length (L) = 2 m
- Cross-section = 7.5 cm × 15 cm = 0.075 m × 0.15 m
- Mass flow rate (m) = 0.1 kg/s
- Duct surface temperature (T_s) = 150°C

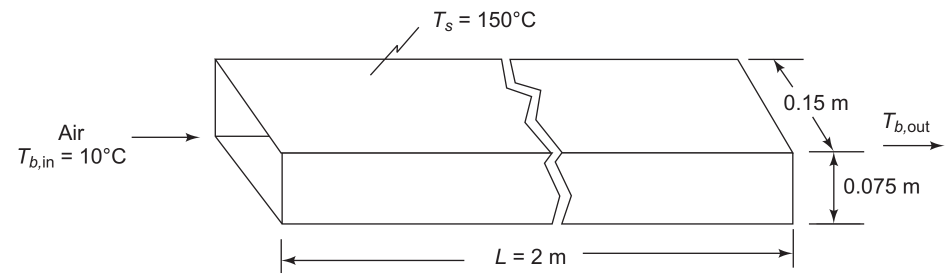
FIND

(a) The heat transfer coefficient $\left(\bar{h}_c\right)$
(b) The air outlet temperature ($T_{b,\text{out}}$)
(c) The rate of heat transfer (q)
(d) The pressure drop (Δp)

ASSUMPTIONS

- Steady state
- The duct is smooth

SKETCH



SOLUTION

The hydraulic diameter of the duct is

$$D_H = \frac{4A_c}{P} = \frac{4(0.15\,\text{m})(0.075\,\text{m})}{2(0.15\,\text{m})(0.075\,\text{m})} = 0.10\text{m}$$

For the first iteration, let $T_{b,\text{out}}$ = 50°C. For dry air at the average bulk temperature of 30°C

Density (ρ) = 1.128 kg/m^3

Thermal conductivity (k) = 0.0258 W/(m K)

Absolute viscosity (μ) = 18.68 × 10^{-6} (Ns)/m^2

Prandtl number (Pr) = 0.71

Specific heat (c_p) = 1013 J/(kg K)

$$Re_D = \frac{VD_H}{\nu} = \frac{\dot{m}D_H}{A\mu} = \frac{(0.1\,\text{kg/s})(0.10\,\text{m})}{(0.15\,\text{m})(0.075\,\text{m})\left(18.68\times10^{-6}\,(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)}$$

$$= 47{,}585 > 10{,}000 \text{ (Turbulent)}$$

$$\frac{L}{D_H} = \frac{2\,\text{m}}{0.1\,\text{m}} = 20$$

(a) Therefore, entrance effects may be significant — the correction Equations (6.68) and (6.69) will be applied to the Dittus Boelter correlation, Equation (6.63).

From Equation (6.63)

$$\overline{Nu}_{fd} = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_{fd} = 0.023(47{,}585)^{0.8}\,(0.71)^{0.4} = 111$$

$$\bar{h}_{c,fd} = \overline{Nu}_{fd}\,\frac{k}{D} = 111\,\frac{(0.0258\,\text{W/(m K)})}{0.1\,\text{m}} = 28.56\ \text{W/(m}^2\ \text{K)}$$

Since L/D = 20 is on the low end of the range of Equation (6.69) and the high end of the range for Equation (6.68), the average of these two corrections will be applied

From Equation (6.68)

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24\ Re^{-0.23} = 24(47{,}585)^{-0.23} = 2.02$

$b = 2.08 \times 10^{-6}\ Re - 0.815 = 2.08 \times 10^{-6}\ (47{,}585) - 0.815 = -0.716$

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + 2.02\left(\frac{2}{0.1}\right)^{-0.716} = 1.24$$

From Equation (6.69)

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + \left(6\frac{D}{L}\right) = 1 + \frac{6(0.1)}{2} = 1.3$$

The average of the two values is $\bar{h}_{c,L} / \bar{h}_{c,fd} = 1.27$

$$\therefore \quad \bar{h}_{c,L} = 1.27\left(28.56\,\text{W/(m}^2\text{K)}\right) = 36.22\ \text{W/(m}^2\ \text{K)}$$

(b) The outlet temperature is found by rearranging Equation (6.63)

$$T_{b,\text{out}} = T_s - (T_{b,\text{in}} - T_s)\ \exp\left(-\frac{PL\bar{h}_c}{\dot{m}c}\right)$$

$$T_{b,\text{out}} = 150°\text{C} - (10°\text{C} - 150°\text{C})\ \exp\left(-\frac{2(0.075\,\text{m}+0.15\,\text{m})(2\,\text{m})\left(36.32\,\text{W/(m}^2\text{K)}\right)}{(0.1\,\text{kg/s})(1013\,\text{J/(kg K)})((\text{Ws})/\text{J})}\right) = 48°\text{C}$$

No further iteration is needed since the result is close to the initial guess.

(c) The rate of heat transfer is given by

$$q = \dot{m}\ c\Delta T_b = (0.1\,\text{kg/s})\ (1013\,\text{J/(kg K)})\ (48°\text{C} - 10°\text{C}) = 3849\ \text{W}$$

(d) The friction can be estimated from the lowest line for Figure 6.18: Re = 47,585 $\rightarrow f \approx 0.021$. The pressure drop is given by Equation (6.13)

$$\text{o}\Delta p = f\frac{L}{D_H}\frac{\rho V^2}{2} = f\frac{L}{D_H}\frac{\rho\left(\frac{4\dot{m}}{\rho\pi D_H^2}\right)^2}{2} = f\frac{L}{D_H}\frac{8}{\rho}\left(\frac{\dot{m}}{\pi D_H^2}\right)^2$$

So

$$\Delta p = 0.021\left(\frac{2\ \text{m}}{0.1\,\text{m}}\right)\left(\frac{8}{1.128\,\text{kg/m}^3}\right)\left(\frac{0.1\,\text{kg/s}}{\pi(0.1\,\text{m})^2}\right)^2\left((\text{Pa m}^2)/\text{N}\right)\left((\text{s}^2\text{N})/(\text{kg m})\right) = 30.2\ \text{Pa}$$

PROBLEM 6.27

Air at 16°C and atmospheric pressure enters a 1.25-cm-*ID* tube at 30 m/s. For an average wall temperature of 100°C, determine the discharge temperature of the air and the pressure drop if the pipe is (a) 10 cm long and (b) 102 cm long.

GIVEN

- Atmospheric air flowing through a tube
- Entering air temperature ($T_{b,\text{in}}$) = 16°C
- Tube inside diameter (D) = 1.25 cm = 0.0125 m
- Air velocity (V) = 30 m/s
- Average wall surface temperature (T_s) = 100°C

FIND

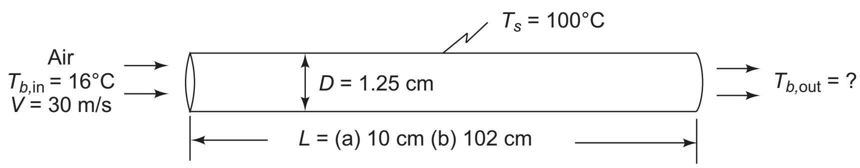
The discharge temperature ($T_{b,\text{out}}$) and the pressure drop (Δp) if the pipe length (L) is

(a) 10 cm (0.1 m)
(b) 102 cm (1.02 m)

ASSUMPTIONS

- Steady state
- The tube is smooth

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the entering bulk temperature of 16°C

Density (ρ) = 1.182 kg/m^3
Thermal conductivity (k) = 0.0248 W/(m K)
Kinematic viscosity (ν) = 15.3 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71
Specific heat (c) = 1012 J/(kg K)

SOLUTION

The discharge temperature will first be calculated using air properties evaluated at the entering temperature and will then be recalculated using the average bulk air temperature of the first iteration to evaluate the air properties.

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(30\,\text{m/s})(0.0125\,\text{m})}{15.3\times10^{-6}\,\text{m}^2/\text{s}} = 24{,}510 \text{ (Turbulent)}$$

(a) L/D = 0.1 m/0.0125 m = 8 < 20. Therefore, the flow is not fully developed and the heat transfer coefficient will have to be corrected with Equation (6.68). The Dittus-Boelter correlation of Equation (6.63) will be used to calculate the fully developed Nusselt number

$$\overline{Nu}_{fd} = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_{fd} = 0.023\,(24{,}510)^{0.8}\,(0.71)^{0.4} = 65.11$$

$$\bar{h}_{c,fd} = \overline{Nu}_{fd}\,\frac{k}{D} = 65.11\,\frac{(0.0248\,\text{W/(m K)})}{0.0125\,\text{m}} = 129.2 \text{ W/(m}^2\text{ K)}$$

Applying Equation (6.68)

$$\frac{\overline{Nu}}{\overline{Nu}_{fd}} = \frac{\bar{h}_{c,L}}{\bar{h}_c} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24/Re_D^{0.23} = 24/(24{,}510)^{0.23} = 2.347$

$b = 2.08 \times 10^{-6}\, Re_D - 0.815 = 2.08 \times 10^{-6}\,(24{,}510) - 0.815 = -\,0.7640$

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + 2.347\left(\frac{0.1}{0.0125}\right)^{-0.7640} = 1.480$$

$$\therefore \quad \bar{h}_{c,L} = 1.480\left(129.2\,\text{W/(m}^2\text{K)}\right) = 191.1\ \text{W/(m}^2\ \text{K)}$$

The outlet temperature is given be Equation (6.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}} = \exp\left(-\frac{PL\bar{h}_c}{\dot{m}c_p}\right) = \exp\left(-\frac{4\bar{h}_c L}{\rho VDc}\right)$$

$$T_{b,\text{out}} = T_s - (T_s - T_{b,\text{in}})\exp\left(-\frac{4\bar{h}_c L}{\rho VDc}\right)$$

$$T_{b,\text{out}} = 100°\text{C} - (100°\text{C} - 16°\text{C})\exp\left(-\frac{4\left(191.1\,\text{W/(m}^2\,\text{K)}\right)(0.1\,\text{m})}{\left(1.182\,\text{kg/m}^3\right)(30\,\text{m/s})(0.0125\,\text{m})(1012\,\text{J/(kg K)})((\text{Ws})/\text{J})}\right)$$

$$= 29.2°\text{C}$$

Performing another iteration

$T_{b,\text{avg}} = 22.6°\text{C}$	$c = 1012$ J/(kg K)
$\rho = 1.155$ kg/m^3	$Re = 23{,}584$
$k = 0.0253$ W/(m K)	$\overline{Nu}_{fd} = 63.1$
$\nu = 15.9 \times 10^{-6}$ m^2/s	$\bar{h}_{c,L} = 189.4$
$Pr = 0.71$	$T_{b,\text{out}} = 29.3°\text{C}$ ($T_{b,\text{avg}} = 22.7°\text{C}$)

(b) The friction factor, from Equation (6.59) is

$$f = \frac{0.184}{Re_D^{0.2}} = \frac{0.184}{(23{,}584)^{0.2}} = 0.0246$$

$$\Delta p = f\,\frac{L}{D_H}\,\frac{\rho V^2}{2g_c} = 0.0246\left(\frac{0.1}{0.0125}\right)\frac{\left(1.155\,\text{kg/m}^3\right)(30\,\text{m/s})^2}{2\left(\text{N/(m}^2\text{Pa)}\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)} = 102.1\ \text{Pa}$$

For L = 1.02 m, L/D = 1.02m/0.0125 m = 81.6 > 60. Therefore, the analysis is the same as above except that the L/D correction of Equation (6.68) does not need to be applied. From the first iteration, the heat transfer coefficient ($h_{c,fd}$) = 129.2 W/(m^2 K).

$$\therefore\ T_{b,\text{out}} = 100°\text{C} - (100°\text{C} - 16°\text{C})\exp\left(-\frac{4\left(129.2\,\text{W/(m}^2\text{K)}\right)(1.02\,\text{m})}{\left(1.182\,\text{kg/m}^3\right)(30\,\text{m/s})(0.0125\,\text{m})(1012\,\text{J/(kg K)})((\text{Ws})/\text{J})}\right)$$

$$T_{b,\text{out}} = 74.1°\text{C}$$

Performing another iteration

$T_{b,\text{avg}} = 45.0°\text{C}$	$c = 1015$ J/(kg K)
$\rho = 1.075$ kg/m^3	$Re_D = 20{,}718$
$k = 0.0270$ W/(m K)	$\overline{Nu}_D = 56.9$
$\nu = 18.1 \times 10^{-6}$ m^2/s	$\bar{h}_c = 123.0$ W/(m^2 K)
$Pr = 0.71$	$T_{b,\text{out}} = 75.4°\text{C}$ ($T_{b,\text{avg}} = 45.7°\text{C}$)

From Equation (6.59) $f = \dfrac{0.184}{(20{,}178)^{0.2}} = 0.0252$

From Equation (6.13) $\Delta p = 0.0252\left(\dfrac{0.2}{0.0125}\right)\dfrac{\left(1.075\,\text{kg/m}^3\right)(30\,\text{m/s})^2}{2\left(\text{N}/(\text{m}^2\text{Pa})\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)} = 995.0\ Pa$

COMMENTS

Note that by increasing the length of the pipe by a factor of 10 leads to a temperature rise increase of about 350% and a pressure drop increase of about 875%

PROBLEM 6.28

The equation

$$Nu = 0.116\,(Re^{\frac{2}{3}} - 125)\,Pr^{\frac{1}{3}}\left[1 + \left(\frac{D}{L}\right)^{\frac{2}{3}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

has been proposed by Hausen for the transition range (2300 < *Re* < 8000) as well as for higher Reynolds numbers. Compare the values of Nu predicated by Hausen's equation for *Re* = 3000 and *Re* = 20,000 at *D*/*L* = 0.1 and 0.01 with those obtained from appropriate equations or charts in the text. Assume the fluid is water at 15°C flowing through a pipe at 100°C.

GIVEN

- Water flowing through a pipe
- The Hausen correlation given above
- Water temperature = 15°C
- Pipe temperature = 100°C

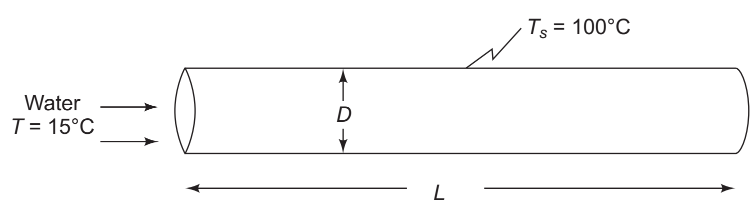
FIND

- The Nusselt number using the Hausen correlation and appropriate equations and charts in the text for *Re* = 3000 and 20,000 and *D*/*L* = 0.1 and 0.01

ASSUMPTIONS

- Steady state
- Constant and uniform pipe temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 15°C

Absolute viscosity (μ_b) = 1136×10^{-6} (Ns)/m^2

Prandtl number (Pr) = 8.1

At 100°C $\mu_s = 277.5 \times 10^{-6}$ (Ns)/m^2

SOLUTION

For Re = 3000, D/L = 0.1 the flow is in the transition region. In addition, L/D = 10. Therefore, the flow is not fully developed. The "short duct approximation" curve of Figure 6.12 in the text will be used to estimate the Nusselt number

$$Re_D\, Pr \frac{D}{L} \times 10^{-2} = 3000(8.1)\,(0.1) \times 10^{-2} = 24.3$$

From Figure 6.12, Nu_D= 23.

For Re = 3000, D/L = 0.01, the flow is fully developed and the Nusselt number will be estimated by the laminar correlation of Sieder-Tate, Equation (6.40)

$$Nu_D = 1.86\left(Re_D\; Pr \frac{D}{L}\right)^{0.33} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$Nu_D = 1.86\;[3000(8.1)\;(0.01)]^{0.33} \left(\frac{1136}{277.5}\right)^{0.14} = 13.88$$

For Re = 20,000, D/L = 0.1, the flow is turbulent, not fully developed. The fully developed Nusselt number can be estimated from Equation (6.63)

$$Nu_D = 0.023\; Re_D^{0.8}\; Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$Nu_D = 0.023\;(20{,}000)^{0.8}\;(8.1)^{0.4} = 146.5$$

Correcting this for the entrance effect using Equation (6.68)

$$\frac{Nu}{Nu_{fd}} = \frac{h_{c,L}}{h_c} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24/Re_D^{0.23} = 24/(20{,}000)^{0.23} = 2.459$

$b = 2.08 \times 10^{-6}\; Re - 0.815 = 2.08 \times 10^{-6}\;(20{,}000) - 0.815 = -\,0.7734$

$$\frac{Nu}{Nu_{fd}} = 1 + 2.459\;(10)^{-0.7734} = 1.41$$

$$\therefore \qquad Nu = 1.41\;(146.5) = 206.6$$

For Re = 20,000, D/L = 0.01, the entrance effect can be neglected Nu_D = 146.5

The Hausen correlation yields

$$\overline{Nu} = 0.116\;\left((3000)^{\frac{2}{3}} - 125\right)\;(8.1)^{\frac{1}{3}}\left[1+(0.1)^{\frac{2}{3}}\right]\left(\frac{1136}{277.5}\right)^{0.14} = 28.63$$

Applying the Hausen correlation to the remaining cases and comparing them to the results from the text yields the following

Case	1	2	3	4
Re	3000	3000	20,000	20,000
D/L	0.1	0.01	0.1	0.01
$\overline{Nu}$ from text	23	13.88	206.6	146.5
$\overline{Nu}$ from Hausen	28.63	24.65	211.0	181.7
Percent Difference	20%	44%	12%	14%

COMMENTS

Note that the large difference in Case 2 is probably due to the use of a laminar correlation from the text when the flow is transitional. There are large variations is flow and heat transfer in this regime and it is usually avoided by good designers.

PROBLEM 6.29

Water at 20°C enters a 1.91 cm *ID*, 57 cm long tube at a flow rater of 3 gm/s. The tube wall is maintained at 30°C. Determine the water outlet temperature. What error in the water temperature results if natural convection effects are neglected?

GIVEN

- Water flowing through a tube
- Entering water temperature ($T_{b,\text{in}}$) = 20°C
- Tube inside diameter (D) = 1.91 cm = 0.0191 m
- Tube length (L) = 57 cm = 0.57 m
- Mass flow rate (m) = 3 gm/s = 0.003 kg/s
- Tube wall surface temperature (T_s) = 30°C

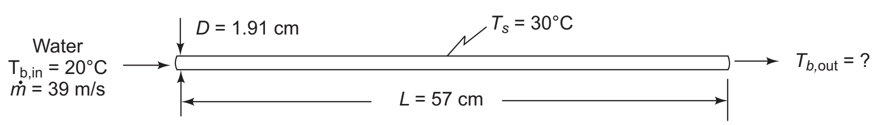
FIND

(a) The water outlet temperature ($T_{b,\text{out}}$)
(b) Percent error in water temperature rise if natural convection is neglected

ASSUMPTIONS

- Steady state
- Tube temperature is uniform and constant
- The tube is horizontal

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 20°C

Specific heat (c) = 4182 J/(kg K)

Thermal conductivity (k) = 0.597 W/(m K)

Kinematic viscosity (ν) = 1.006×10^{-6} m^2/s

Prandtl number (Pr) = 7.0

Absolute viscosity (μ_b) = 993×10^{-6} (Ns)/m^2

Thermal expansion coefficient (β) = 2.1×10^{-4} 1/K

At 30°C $\mu_s = 792 \times 10^{-6}$ (Ns)/m^2

SOLUTION

The Reynolds number is

$$Re_D = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.003\,\text{kg/s})}{\pi(0.0191\,\text{m})\left(993\times10^{-6}\,(\text{N s})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 210.4 \text{ (Laminar)}$$

The Graetz number is

$$Gz = \frac{\pi}{4} Re_D\, Pr \frac{D}{L} = \frac{\pi}{4}\ (201.4)\ (7.0) \left(\frac{0.0191\,\text{m}}{0.57\,\text{m}}\right) = 37.10$$

The Grashof number (from Table 4.3) will be based on the diameter since the tube is horizontal

$$Gr_D = \frac{g\beta(T_s - T_b)D^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)(2.1\times10^{-4}\,1/\text{K})(30°\text{C} - 20°\text{C})(0.0191\,\text{m})^3}{(1.006\times10^{-6}\,(\text{Ns})/\text{m}^2)^2} = 1.42\times10^5$$

$$Gr_D\, Pr \frac{D}{L} = 1.42\times10^5\ (7.0) \left(\frac{0.0191\,\text{m}}{0.57\,\text{m}}\right) = 3.3\times10^4$$

For this value and $Re_D = 200$, Figure 6.12a indicates that the flow is in the 'mixed convection laminar flow' region.

$$\overline{Nu}_D = 1.75 \left(\frac{\mu_b}{\mu_s}\right)^{0.14} \left[Gz + 0.12\,(Gz\ Gr_D^{\frac{1}{3}}\, Pr^{0.36})^{0.88}\right]^{\frac{1}{3}}$$

(a) The Nusselt number can be estimated using Equation (6.44)

$$\overline{Nu}_D = 1.75 \left(\frac{\mu_b}{\mu_s}\right)^{0.14} \left[Gz + 0.12\,(Gz\ Gr_D^{\frac{1}{3}}\, Pr^{0.36})^{0.88}\right]^{\frac{1}{3}}$$

$$\overline{Nu}_D = 1.75 \left(\frac{933}{792}\right)^{0.14} \left(37.1 + 0.12\left[(37.1)\ (1.42\times10^5)^{\frac{1}{3}}\ (7)^{0.36}\right]^{0.88}\right)^{\frac{1}{3}} = 10.7$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 10.7 \frac{(0.597\,\text{W/(m K)})}{0.0191\,\text{m}} = 334\ \text{W/(m}^2\ \text{K)}$$

The outlet temperature can be calculated from Equation (6.63)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}} = \exp\left(-\frac{PLh_c}{\dot{m}c_p}\right) = \exp\left(-\frac{h_c \pi DL}{\dot{m}c}\right)$$

$$T_{b,\text{out}} = T_s - (T_s - T_{b,\text{in}})\ \exp\left(-\frac{h_c \pi DL}{\dot{m}c}\right)$$

$$T_{b,\text{out}} = 30°\text{C} - (30°\text{C} - 20°\text{C})\ \exp\left(-\frac{(334\,\text{W/(m}^2\text{K)})\,\pi(0.0191\,\text{m})(0.57\,\text{m})}{(0.003\,\text{kg/s})(4182\,\text{J/(kg K)})((\text{Ws})/\text{J})}\right)$$

$$T_{b,\text{out}} = 26°\text{C}$$

The average bulk temperature is 23°C. Another iteration is therefore not warranted because the change in property values will not affect the result appreciably.

(b) Natural convection can be neglected by applying Equation (6.40) for the Nusselt number

$$Nu_D = 1.86 \left(Re_D\ Pr \frac{D}{L}\right)^{0.33} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$Nu_D = 1.86 \left[201.4(7.0)\left(\frac{0.0191\,\text{m}}{0.57\,\text{m}}\right)\right]^{0.33} \left(\frac{933}{792}\right)^{0.14} = 6.8$$

$$h_c = Nu_D \frac{k}{D} = 6.8 \frac{(0.597\,\text{W/(m K)})}{0.0191\,\text{m}} = 212.3\ \text{W/(m}^2\ \text{K)}$$

$$T_{b,\text{out}} = 30° - (30°\text{C} - 20°)\exp\left(-\frac{(212.3\,\text{W/(m}^2\text{K)})\,\pi(0.0191\,\text{m})(0.57\,\text{m})}{(0.003\,\text{kg/s})(4182\,\text{J/(kg K)})((\text{Ws})/\text{J})}\right)$$

$$T_{b,\text{out}} = 24.4°\text{C}$$

The error in outlet temperature is

$$\text{Error} = 26 - 24.4 = 1.6°\text{C}$$

PROBLEM 6.30

A solar thermal central receiver generates heat by focusing sunlight with a field of mirrors on a bank of tubes through which a coolant flows. Solar energy absorbed by the tubes is transferred to the coolant which can then deliver useful heat to a load. Consider a receiver fabricated from multiple horizontal tubes in parallel. Each tube is 1 cm *ID* and 1 m long. The coolant is molten salt which enters the tubes at 370°C. Under start-up conditions, the salt flow is 10 gm/s in each tube and the net solar flux absorbed by the tubes is 10^4 W/m^2. The tube wall material will tolerate temperatures up to 600°C. Will the tubes survive start-up? What is the salt outlet temperature?

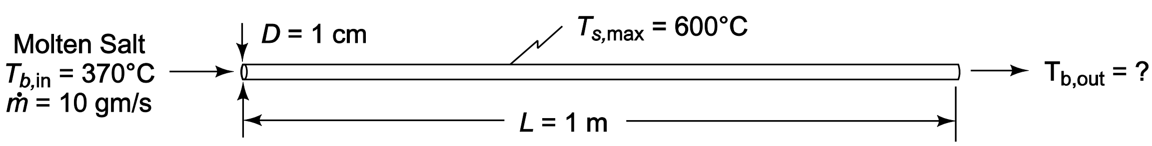
GIVEN

- Molten salt flowing through a horizontal tube that is absorbing solar energy
- Tube inside diameter (D) = 1 cm = 0.01 m
- Tube length (L) = 1 m
- Entering salt temperature ($T_{b,\text{in}}$) = 370°C
- Start-up mass flow rate (m) = 10 gm/s = 0.01 kg/s
- Net solar energy absorbed by the tube (q_s) = 10^4 W/m^2
- Maximum tube wall temperature (T_s) = 600°C

FIND

(a) Salt outlet temperature ($T_{b,\text{out}}$)
(b) Will the tubes survive start-up?

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 23, for molten salt at 370°

Specific heat (c) = 1629 J/(kg K)

SOLUTION

(a) By the conservation of energy

$$q_s A = \dot{m}\, c(T_{b,\text{out}} - T_{b,\text{in}})$$

$$\therefore \quad T_{b,\text{out}} = T_{b,\text{in}} + \frac{q_s(\pi D L)}{\dot{m}c} = 370°\text{C} + \frac{(10^4\,\text{W/m}^2)\,\pi(0.01\,\text{m})(1.0\,\text{m})}{(0.01\,\text{kg/s})(1629\,\text{J/(kg K)})((\text{Ws})/\text{J})} = 389°\text{C}$$

Evaluating the molten salt properties at the average bulk temperature of 380°C

Density (ρ) = 1849 kg/m^3

Absolute viscosity (μ) = 1970 × 10^{-6} (Ns)/m^2

Thermal expansion coefficient (β) = 3.55 × 10^{-4} 1/K

Thermal conductivity (k) = 0.516 W/(m K)

Kinematic viscosity (ν) = 1.065 × 10^{-6} m^2/s

Prandtl number (Pr) = 6.18

(b) The Reynolds number is

$$Re_D = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.01\,\text{kg/s})}{\pi(0.01\,\text{m})(1970\times10^{-6}\,(\text{Ns})/\text{m}^2)((\text{kg m})/(\text{Ns}^2))} = 646 \text{ (Laminar)}$$

With laminar flow and the high temperature differences possible, natural convection may be important. Since we do not know the tube wall temperature needed to evaluate the Grashof number, an iterative procedure must be used. For the first iteration, let the average tube wall temperature be 10°C above the average bulk salt temperature (T_s = 390°C).

From Appendix 2, Table 23, at the tube temperature of 390°C μ_s = 1882 × 10^{-6} (Ns)/m^2

The Graetz number is

$$Gz = \frac{\pi}{4} Re_D\, Pr \frac{D}{L} = \frac{\pi}{4}\ (646.3)(6.18)\left(\frac{0.01\,\text{m}}{1\,\text{m}}\right) = 31.37$$

The Grashof number based on the diameter, from Table 4.3 is

$$Gr_D = \frac{g\beta(T_s - T_b)D^3}{\nu^2} = \frac{(9.8\,\text{m}^2/\text{s})(3.55\times10^{-4}\,1/\text{K})(390°\text{C}-380°\text{C})(0.01\,\text{m})^3}{(1.065\times10^{-6}\,(\text{Ns})/\text{m}^2)^2} = 3.07\times10^4$$

$$Gr_D\, Pr \frac{D}{L} = 3.07\times10^4\ (6.18)\ (0.01) = 1.9\times10^3$$

For this value and Re_D = 6.5 × 102, Figure 6.12a indicates the flow is in the mixed convection regime, therefore, Equation (6.46) will be used to estimate the Nusselt number. Note that this will be a rough estimate since Equation (6.46) is technically only for isothermal tubes.

$$\overline{Nu}_D = 1.75\left(\frac{\mu_b}{\mu_s}\right)^{0.14}\left[Gz + 0.12\,(Gz\, Gr_D^{\frac{1}{3}}\, Pr^{0.36})^{0.88}\right]^{\frac{1}{3}}$$

$$\overline{Nu}_D = 1.75\ \left(\frac{1970}{1882}\right)^{0.14}\left(31.37.1+0.12\left[(31.37)\ (3.07\times10^4)^{\frac{1}{3}}\,(6.18)^{0.36}\right]^{0.88}\right)^{\frac{1}{3}} = 8.76$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 8.76\,\frac{(0.516\,\text{W}/(\text{m K}))}{0.01\,\text{m}} = 452\ \text{W}/(\text{m}^2\ \text{K})$$

The rate of heat transfer to the molten salt is

$$q_c = \bar{h}_c A_t\,(T_s - T_b) = q_s''\,A_t$$

$$\therefore \qquad T_s - T_b = \frac{q_s''}{h_c} = \frac{10^4\ \text{W/m}^2}{452\,\text{W}/(\text{m}^2\text{K})} = 22.1°\text{C}$$

Further iterations are necessary. However, the fluid properties will not change appreciably. Therefore, Re_D, Pr, and Gz will not change.

Iteration #	2	3
T_s (°C)	402	401
$\mu_s \times 106$	1791	1798
$T_s - T_b$ (°C)	22.1	20.7
$Gr_D \times 10^{-4}$	6.78	6.35
$\overline{Nu}_D$	9.36	9.31
$\bar{h}_c$ $\left(\text{W}/(\text{m}^2\ \text{K})\right)$	483	481
$T_s - T_b$ (°C)	20.7	20.8

The maximum tube wall temperature is therefore

$$T_{b,\text{out}} + (T_s - T_b) = 389°\text{C} + 21°\text{C} = 410°\text{C}$$

which is well below the tube melting point. The tube will have no problems surviving the start-up in good shape.

PROBLEM 6.31

Determine the heat transfer coefficient for liquid bismuth flowing through an annulus (5 cm *ID*, 6.1 cm *OD*) at a velocity of 4.5 m/s. The wall temperature of the inner surface is 427°C and the bismuth is at 316°C. It may be assumed that heat losses from the outer surface are negligible.

GIVEN

- Liquid bismuth flowing through an annulus
- Annulus diameters
  - D_i = 5 cm = 0.05 m
  - D_o = 6.1 cm = 0.061 m
- Bismuth velocity (V) = 4.5 m/s
- Temperature
  - Inner wall surface (T_si) = 427°C
  - Bismuth (T_b) = 316°C

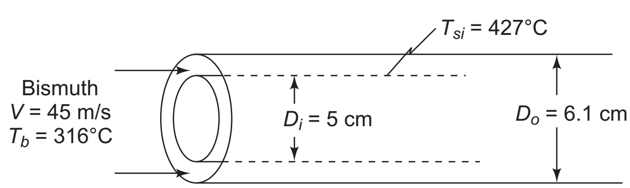
FIND

- The heat transfer coefficient

ASSUMPTIONS

- Steady state
- Heat losses from outer surfaces are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 24, for bismuth at the bulk temperature of 316°C

Thermal conductivity (k) = 16.44 W/(m K)

Kinematic viscosity (ν) = 1.57 × 10^{-7} m²/s

Prandtl number (Pr) = 0.014

SOLUTION

The hydraulic diameter for the annual is given by Equation (6.3)

$$D_H = D_o - D_i = 0.061 \text{ m} - 0.05 \text{ m} = 0.011 \text{ m}$$

The Reynolds number based on the hydraulic diameter is

$$Re_{D_H} = \frac{VD_H}{\nu} = \frac{(4.5\,\text{m/s})(0.011\,\text{m})}{1.57\times10^{-7}\,\text{m}^2/\text{s}} = 3.15 \times 10^5$$

For liquid metals, the Nusselt number is given by Equation (6.75)

$$\overline{Nu}_{D_H} = 0.625\,(Re_{D_H}\;Pr)^{0.4} = 0.625\,[3.15 \times 10^5\,(0.014)]^{0.4} = 17.9$$

$$\bar{h}_c = \overline{Nu}_{D_H}\,\frac{k}{D_H} = 17.9\,\frac{(16.44\,\text{W/(m K)})}{0.011\,\text{m}} = 26{,}800\ \text{W/(m}^2\ \text{K)}$$

PROBLEM 6.32

Mercury flows inside a copper tube 9 m long with a 5.1 cm inside diameter at an average velocity of 7 m/s. The temperature at the inside surface of the tube is 38°C uniformly throughout the tube, and the arithmetic mean bulk temperature of the mercury is 66°C. Assuming the velocity and temperature profiles are fully developed, calculate the rate of heat transfer by convection for the 9 m length by considering the mercury as (a) an ordinary liquid and (b) liquid metal. Compare the results.

GIVEN

- Mercury flows inside a copper tube
- Tube length (L) = 9 m
- Inside diameter (D) = 5.1 cm = 0.051 m
- Average mercury velocity (V) = 7 m/s
- Tube inside surface temperature (T_s) = 38°C (uniform)
- Bulk temperature of mercury (T_b) = 66°C

FIND

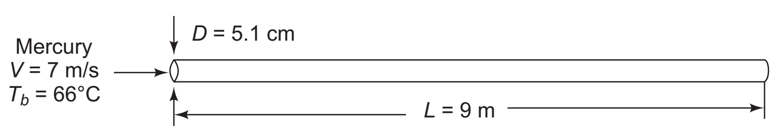
The rate of heat transfer for the 9 m length considering mercury as

(a) an ordinary liquid, and
(b) a liquid metal

ASSUMPTIONS

- Steady state
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for mercury at the bulk temperature of 66°C

Thermal conductivity (k) = 9.76 W/(m K)

Kinematic viscosity (ν) = 0.1004×10^{-6} m²/s

Prandtl number (Pr) = 0.0193

SOLUTION

The Reynolds number is

$$Re_D = \frac{VD}{\nu} = \frac{(7\,\text{m/s})(0.051\,\text{m})}{\left(0.1004\times10^{-6}\,\text{m}^2/\text{s}\right)} = 3.6 \times 10^6$$

(a) The mercury will be treated as an ordinary liquid by applying the Dittus-Boelter Equation (6.63)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\,(3.6 \times 10^6)^{0.8}\,(0.0193)^{0.3} = 1229$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 1229 \frac{(9.76\,\text{W/(m K)})}{0.051\,\text{m}} = 2.35 \times 10^5 \text{ W/(m}^2\text{ K)}$$

The rate of heat transfer is

$$q = \bar{h}_c A\,(T_b - T_s) = \bar{h}_c\, \pi D L\,(T_b - T_s)$$

$$q = \left(2.35\times10^5\,\text{W/(m}^2\text{K)}\right) \pi (0.051\text{ m})\,(9\text{ m})\,(66°\text{C} - 38°\text{C}) = 9.5 \times 10^6 \text{ W}$$

(b) For liquid metals and a constant surface temperature boundary, the Nusselt number is given by Equation (6.78)

$$\overline{Nu}_D = 5.0 + 0.025\,(Re_D\, Pr)^{0.8} = 5.0 + 0.025\,[(3.6 \times 10^6)\,(0.0193)]^{0.8} = 191$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 191 \frac{(9.76\,\text{W/(m K)})}{0.051\,\text{m}} = 3.65 \times 10^4 \text{ W/(m}^2\text{ K)}$$

$$q = \left(3.65\times10^4\,\text{W/(m}^2\text{K)}\right) \pi (0.051\text{ m})\,(9\text{ m})\,(66°\text{C} - 38°\text{C}) = 1.47 \times 10^6 \text{ W}$$

COMMENTS

Applying the Dittus-Boelter equation, which is valid for $Pr > 0.5$ only, to mercury ($Pr \approx 0.02$) leads to a 648% overestimation in the rate of heat transfer to the pipe. This shows that application of empirical outside the limits of experimental verification can lead to serious errors.

PROBLEM 6.33

> **A heat exchanger is to be designed to heat a flow of molten bismuth from 377°C to 477°C. The heat exchanger consists of a 50 mm *ID* tube with surface temperature maintained uniformly at 500°C by an electrical heater. Find the length of the tube and the power required to heat 4 kg/s and 8 kg/s of bismuth.**

GIVEN

- Molten Bismuth flows through a tube
- Bismuth temperature: Inlet ($T_{b,\text{in}}$) = 377°C Outlet ($T_{b,\text{out}}$) = 477°C
- Tube inside diameter (D) = 50 mm = 0.05 m
- Surface temperature (T_s) = 500°C

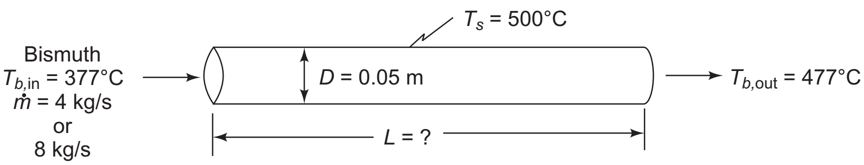
FIND

- The length (L) tube and power required (q) to heat 4 kg/s and 8 kg/s of bismuth

ASSUMPTIONS

- Steady state
- Uniform and constant surface temperature
- Losses from the heater are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 24, for Bismuth at the average bulk temperature of 427°C

Specific heat (c) = 150 J/(kg K)

Thermal conductivity (k) = 15.58 W/(m K)

Absolute viscosity (μ) = 13.39 × 10^{-4} (Ns)/m^2

Prandtl number (Pr) = 0.013

SOLUTION

At $\dot{m}$ = 4 kg/s the Reynolds number is

$$Re = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(4\,\text{kg/s})}{\pi(0.05\,\text{m})\left(13.39\times10^{-4}\,(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 76{,}070$$

$Re\,Pr$ = 76,070 (0.013) = 989

Therefore, Equation (6.78) can be used. The resulting L/D should be greater than 30

$$\overline{Nu}_D = 5.0 + 0.025\,(Re_D\,Pr)^{0.8} = 5.0 + 0.025\,(989)^{0.8} = 11.23$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 11.23\,\frac{(15.58\,\text{W/(m K)})}{0.05\,\text{m}} = 3498\ \text{W/(m}^2\ \text{K)}$$

Equation (6.36) can be used to find the length required

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}} = \exp\left(-\frac{PL\bar{h}_c}{\dot{m}c_p}\right) = \exp\left(-\frac{\bar{h}_c\pi DL}{\dot{m}c}\right)$$

$$L = -\frac{\dot{m}c}{\bar{h}_c\pi D}\ln\left(\frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}}\right) = -\frac{(4\,\text{kg/s})(150\,\text{J/(kg K)})((\text{Ws})/\text{J})}{\left(3498\,\text{W/(m}^2\text{K)}\right)\pi(0.05\,\text{m})}\ln\left(\frac{500°\text{C} - 477°\text{C}}{500°\text{C} - 377°\text{C}}\right) = 1.83\ \text{m}$$

$$L/D = (1.83\ \text{m})/(0.05\ \text{m}) = 37$$

Repeating the analysis for $\dot{m}$ = 8 kg/s yields the following

Re = 152,140

$RePr$ = 1978

$\overline{Nu}_D$ = 15.84

$\bar{h}_c$ = 4935 W/(m^2 K)

L = 2.60 m

$\frac{L}{D}$ = 52

PROBLEM 6.34

Liquid sodium is to be heated from 500 K to 600 K by passing it at a flow rate of 5.0 kg/s through a 5 cm *ID* tube whose surface is maintained at 620 K. What length of tube is required?

GIVEN

- Liquid sodium flow in a tube
- Bulk temperatures
  - Inlet ($T_{b,\text{in}}$) = 500 K
  - Outlet ($T_{b,\text{out}}$) = 600 K
- Inside tube diameter (D) = 5 cm = 0.05 m
- Tube surface temperature (T_s) = 620 K
- Mass flow rate (m) = 5.0 kg/s

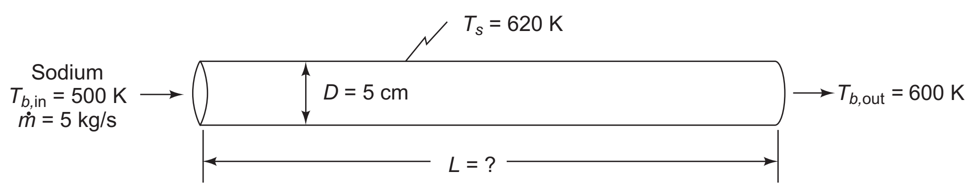
FIND

- The length of tube (L) required

ASSUMPTIONS

- Surface temperature is constant and uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for liquid sodium at the average bulk temperature of 550 K

Specific heat (c) = 1322 J/(kg K)

Thermal conductivity (k) = 76.9 W/(m K)

Absolute viscosity (μ) = 3.67×10^{-4} (Ns)/m^2

Prandtl number (Pr) = 0.0063

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D \rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(5\,\text{kg/s})}{\pi(0.05\,\text{m})\left(3.67\times10^{-4}\,(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 3.47 \times 10^5$$

$$Re_D\, Pr = 3.47 \times 10^5 \,(0.0063) = 2186$$

This is within the range of Equation (6.78)

$$\overline{Nu}_D = 5.0 + 0.025(Re_D\, Pr)^{0.8} = 5.0 + 0.025(2186)^{0.8} = 16.7$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 16.7 \frac{(76.9\,\text{W/(m K)})}{0.05\,\text{m}} = 2.57 \times 10^4 \text{ W/(m}^2\text{ K)}$$

Solving Equation (6.36) for the length

$$L = \frac{\dot{m} c_p}{\bar{h}_c \pi D} \ln\left(\frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}}\right) = \frac{(5\,\text{kg/s})(1322\,\text{J/(kg K)})}{\left(2.57\times10^4\,\text{W/(m}^2\text{K)}\right)\pi(0.05\,\text{m})(\text{J/(Ws)})} \ln\left(\frac{620\,\text{K} - 600\,\text{K}}{620\,\text{K} - 500\,\text{K}}\right) = 2.93 \text{ m}$$

Note that L/D = 2.93 m/0.05 m = 58.6 > 30. Therefore, use of Equation (6.78) is valid.

PROBLEM 6.35

A 2.54-cm-*OD*, 1.9-cm-*ID* steel pipe carries dry air at a velocity of 7.6 m/s and a temperature of –7°C. Ambient air is at 21°C and has a dew point of 10°C. How much insulation with a conductivity of 0.18 W/(m K) is needed to prevent condensation on the exterior of the insulation if h = 2.4 W/(m² K) on the outside?

GIVEN

- Dry air flowing through an insulated steel pipe
- Pipe diameters
  - Inside (D_i) = 1.9 cm = 0.019 m
  - Outside (D_o) = 2.54 cm = 0.0254 m
- Air velocity (V) = 7.6 m/s
- Air temperature (T_a) = –7°C
- Ambient temperature (T_∞) = 21°C
- Ambient dew point (T_{dp}) = 10°C
- Thermal conductivity of insulation (k_I) = 0.18 W/(m K)
- Heat transfer coefficient on exterior $\left(\bar{h}_{c\infty}\right)$ = 2.4 W/(m² K)

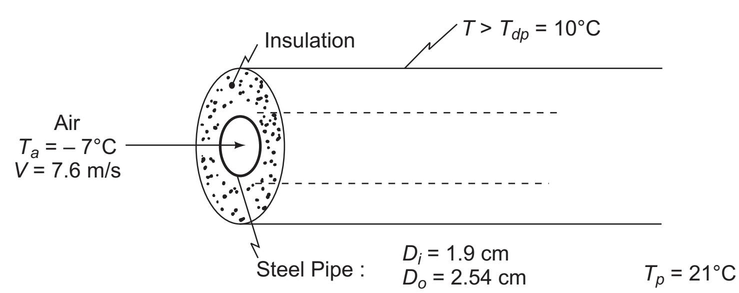
FIND

- Thickness of insulation (t) required to prevent codensation

ASSUMPTIONS

- Steady state
- Flow is fully developed
- Pipe surface temperature can be considered uniform and constant
- Radiation heat transfer to the insulation is negligible or included in $\bar{h}_{c\infty}$
- Pipe is 1% carbon steel

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at –7°C by extrapolation

Thermal conductivity (k) = 0.0232 W/(m K)

Kinematic viscosity (ν) = 13.3 × 10⁻⁶ m²/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 10, the thermal conductivity of 1% carbon steel (ks) = 52 W/(m K)

SOLUTION

Interior heat transfer coefficient $\left(\bar{h}_{ca}\right)$

The Reynolds number for the air flow is

$$Re_D = \frac{VD_i}{\nu} = \frac{(7.6\,\text{m/s})(0.019\,\text{m})}{\left(13.3\times10^{-6}\,\text{m}^2/\text{s}\right)} = 10{,}860 \text{ (Turbulent)}$$

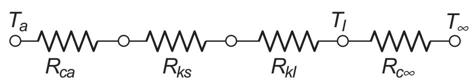
Applying Equation (6.63)

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\ (10{,}860)^{0.8}\ (0.71)^{0.4} = 33.95$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D_i} = 33.95 \frac{(0.0232\ \text{W/(m K)})}{0.019\ \text{m}} = 41.45\ \text{W/(m}^2\ \text{K)}$$

Thermal circuit



where

$$R_{ca} = \frac{1}{\bar{h}_{ca}A_i} = \frac{1}{\bar{h}_{ca}\pi D_i L} = \frac{1}{(41.45\ \text{W/(m}^2\text{K)})\ \pi(0.019\ \text{m})L} = \left(0.404\frac{1}{L}\right)\ \text{(m K)/W}$$

$$R_{ks} = \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{2.54}{1.9}\right)}{2\pi L(52\ \text{W/(m K)})} = \left(0.00089\frac{1}{L}\right)\ \text{(m K)/W}$$

$$R_{kI} = \frac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L k_I} = \frac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L(0.18\ \text{W/(m K)})} = \left(0.884\frac{\ln\left(\frac{D_I}{D_o}\right)}{L}\right)\ \text{(m K)/W}$$

$$R_{c\infty} = \frac{1}{\bar{h}_{c\infty}A_I} = \frac{1}{\bar{h}_{c\infty}\pi D_I L} = \frac{1}{(2.4\ \text{W/(m}^2\text{K)})\ \pi D_I L} = \left(0.1326\frac{1}{D_I L}\right)\ \text{(m}^2\ \text{K)/W}$$

The heat transfer from T_∞ to T_I and from T_I to T_a will be equated

$$\frac{T_\infty - T_I}{R_{c\infty}} = \frac{T_I - T_a}{R_{kI} + R_{ks} + R_{ca}}$$

$$\frac{D_I(T_\infty - T_I)}{0.1326\,(\text{m}^2\text{K})/\text{W}} = \frac{T_I - T_a}{\left(0.00089 + 0.884\ln\left(\frac{D_I}{D_o} + 0.404\right)\right)(\text{m}^2\text{K})/\text{W}}$$

$$D_I\left(6.67\ 1/\text{m}\ \ln\left[\frac{D_I}{(0.0254\ \text{m})}\right] + 3.05\ 1/\text{m}\right) = \frac{T_I - T_a}{T_\infty - T_I} = \frac{10°\text{C} + 7°\text{C}}{21°\text{C} - 10°\text{C}} = 1.545$$

By trial and error: $D_I = 0.117$ m = 11.7 cm

Therefore, the insulation thickness must be greater than

$$t > \frac{D_I - D_o}{2} = \frac{11.7\ \text{cm} - 2.54\ \text{cm}}{2} = 4.6\ \text{cm}$$

PROBLEM 6.36

A double-pipe heat exchanger is used to condense steam at 7370 N/m^2. Water at an average bulk temperature of 10°C flows at 3.0 m/s through the inner pipe, which is made of copper and has a 2.54-cm *ID* and a 3.05-cm *OD*. Steam at its saturation temperature flows in the annulus formed between the outer surface of the inner pipe and an outer pipe of 5.08-cm-*ID*. The average heat transfer coefficient of the condensing steam is 5700 W/(m^2 K), and the thermal resistance of a surface scale on the outer surface of the copper pipe is 0.000118 (m^2 K)/W. (a) Determine the overall heat transfer coefficient between the steam and the water based on the outer area of the copper pipe and sketch the thermal circuit. (b) Evaluate the temperature at the inner surface of the pipe. (c) Estimate the length required to condense 45 gm/s of steam. (d) Determine the water inlet and outlet temperatures.

GIVEN

- Double-pipe heat exchanger, steam in annulus, and water in inner pipe.
- Steam is condensing at a pressure of 7370 N/m^2.
- Average bulk water temperature, T_b = 10°C, and water velocity, V = 3.0 m/s.
- Copper inner pipe
  - Inside diameter, $D_{p,i}$ = 0.0254 m
  - Outside diameter, $D_{p,o}$ = 0.0305 m.
- Outer pipe inside diameter, D_o = 0.0508 m
- Heat transfer coefficient of condensing steam, $\bar{h}_{c,s}$ = 5700 W/(m^2 K)
- Thermal resistance of scale on outside of copper pipe, ($AR_{k,s}$) = 0.000118 (m^2 K)/W

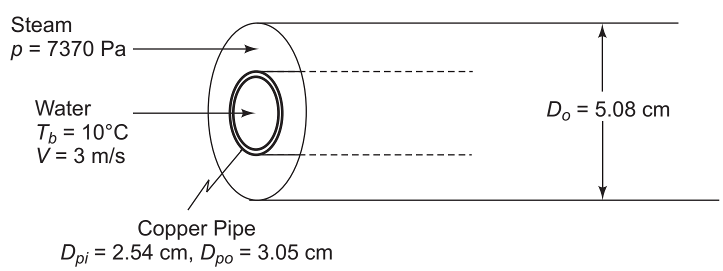
FIND

(a) Overall heat transfer coefficient, U_o.
(b) Temperature of inner surface of the pipe, T_{wi}.
(c) The length, L, required to condense 0.45 kg/s of steam.
(d) Water inlet and outlet temperatures, $T_{w,in}$ and $T_{w,out}$.

ASSUMPTIONS

- Steady-state.
- Constant steam temperature during condensation.
- The flow is fully developed, and copper tube is made of pure copper.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for steam at 7370 N/m^2, the saturation temperature T_s = 40°C, and the heat of vaporization, h_{fg} = 2406 kJ/kg.

From Appendix 2, Table 12, for copper at ~ 40°C (~ steam temperature) the thermal conductivity k_c = 398 W/(m K).

From Appendix 2, Table 13, for water at average bulk temperature of 10°C

Density, $\rho = 999.7$ kg/m^3

Thermal conductivity, $k = 0.577$ W/(m K)

Absolute viscosity, $\mu_b = 1.296 \times 10^{-3}$ (Ns)/m^2

Prandtl number, Pr = 9.5

Specific heat, $c_p = 4195$ J/(kg K)

SOLUTION

The Reynolds number for water flow inside the pipe is

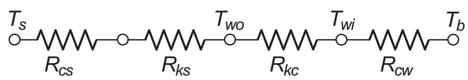
$$\mathrm{Re}_D = \frac{\rho V D_{p,i}}{\mu_b} = \frac{999.7 \times 3.0 \times 0.0254}{0.001296} = 58{,}779 \Rightarrow \text{turbulent flow}$$

Using the simpler Dittus-Boelter correlation for turbulent pipe flow, Equation (6.60), the average Nusselt number and hence the heat transfer coefficient for water flow can be calculated as

$$\overline{\mathrm{Nu}}_D = 0.023\,\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.4} = 0.023(58{,}779)^{0.8}(9.5)^{0.4} = 370$$

$$\Rightarrow \quad \bar{h}_{c,w} = \overline{\mathrm{Nu}}_D \frac{k}{D_{p,i}} = 370 \frac{0.577}{0.0254} = 8405 \text{ W/(m}^2\text{ K)}$$

The thermal circuit for heat flow from the steam to the water can be sketched as follows



Here, considering the pipe length to by L, each of the four resistances can be calculated (see Chapter 1, Section 1.6.3, and Chapter 2, Section 2.3.2, for respective definitions) as follows

$$R_{c,s} = \frac{1}{\bar{h}_{c,s} \pi D_{p,o} L} = \frac{1}{5700 \times \pi \times 0.0305 \times L} = \left(\frac{0.00183}{L}\right) \text{ (m K)/W}$$

$$R_{k,s} = \frac{A R_{k,s}}{\pi D_{p,o} L} = \frac{0.000118}{\pi \times 0.0305 \times L} = \left(\frac{0.00123}{L}\right) \text{ (m K)/W}$$

$$R_{k,c} = \frac{\ln\left(\dfrac{D_{p,o}}{D_{p,i}}\right)}{2\pi k_c L} = \frac{\ln\left(\dfrac{0.0305}{0.0254}\right)}{2\pi \times 398 \times L} = \left(\frac{0.000073}{L}\right) \text{ (m K)/W}$$

$$R_{c,w} = \frac{1}{\bar{h}_{c,w} \pi D_{p,i} L} = \frac{1}{8405 \times \pi \times 0.0254 \times L} = \left(\frac{0.00149}{L}\right) \text{ (m K)/W}$$

(a) The overall heat transfer coefficient based on the outer area of the copper pipe is

$$U_o = \frac{1}{A_o R_{total}} = \frac{1}{\pi D_{p,o} L \left(R_{c,s} + R_{k,s} + R_{k,c} + R_{c,w}\right)}$$

$$\therefore \quad U_o = \frac{1}{\pi(0.0305)(0.00183 + 0.00123 + 0.000073 + 0.00149)} = 2257 \text{ W/(m}^2\text{ K)}$$

(b) The temperature of the inner surface of the pipe can be calculated by equating the rate of heat transfer between steam and water to the rate of convection to the water

$$q = U_o\left(\pi D_{p,o} L\right)(T_s - T_b) = \bar{h}_{c,w}\left(\pi D_{p,i} L\right)(T_{wi} - T_b)$$

$$\therefore \quad T_{wi} = T_b + \frac{U_o D_{p,o}(T_s - T_b)}{\bar{h}_{c,w} D_{p,i}} = 10 + \frac{2257 \times 0.0305(40-10)}{8405 \times 0.0254} = 19.7°\text{C}$$

(c) The length L can now be determined from the rate of heat transfer needed to condense 0.45 kg/s of steam as follows

$$q = \dot{m} h_{fg} = U_o\left(\pi D_{p,o} L\right)(T_s - T_b)$$

$$\therefore \quad L = \frac{\dot{m} h_{fg}}{U_o \pi D_{p,o}(T_s - T_b)} = \frac{0.45 \times 2406 \times 1000}{2257 \times \pi \times 0.0305(40-10)} = 167 \text{ m}$$

(d) Recognizing that with steam condensation on the outside of the copper pipe its surface temperature would be nearly constant and uniform, and hence the inlet and outlet temperatures for water flow can be calculated from Equation (6.36) as follows

$$\frac{T_{wi} - T_{w,out}}{T_{w,i} - T_{w,in}} = \exp\left(-\frac{\bar{h}_{c,w}\pi D_{p.i} L}{\dot{m}_w c_p}\right)$$

$$\text{and} \quad T_b = \frac{T_{w,in} + T_{w,out}}{2} \quad \Rightarrow \quad T_{w,in} = 2T_b - T_{w,out}$$

Thus, if the water mass flow rate, $\dot{m}_w$ is known then both $T_{w,in}$ and $T_{w,out}$ can be calculated.

PROBLEM 6.37

Assume that the inner cylinder in Problem 6.31 is a heat source consisting of an aluminum-clad rod of uranium, 5-cm-*OD* and 2 m long. Estimate the heat flux that will raise the temperature of the bismuth 40°C and the maximum center and surface temperatures necessary to transfer heat at this rate.

From Problem 6.31: Determine the heat transfer coefficient for liquid bismuth flowing through an annulus (5-cm-*ID*, 6.1-cm-*OD*) at a velocity of 4.5 m/s. The bismuth is at 316°C. It may be assumed that heat losses from the outer surface are negligible.

GIVEN

- Liquid bismuth flowing through an annulus
- Annulus inside diameter (D_i) = 5 cm = 0.05 m
- Annulus outside diameter (D_o) = 6.1 cm = 0.061 m
- Bismuth velocity (V) = 4.5 m/s
- Bismuth temperature (T_b) = 316°C
- Inner cylinder is an aluminum clad uranium heat source
- Cylinder length (L) = 2 m
- From Problem 6.31: $\bar{h}_c$ = 26,800 W/(m^2 K)

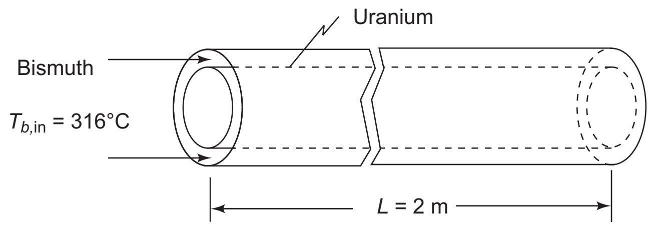
FIND

(a) The heat flux (Q_G/A_t) necessary to raise the bismuth temperature 40°C, and
(b) The maximum center ($T_{u,o}$) and surface ($T_{u,ro}$) temperatures of the uranium

ASSUMPTIONS

- Steady state
- The Bismuth temperature given in Problem 6.31 is the bulk Bismuth temperature
- Thermal resistance of the aluminum is negligible
- Thickness of the aluminum is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2*,* Table 12, for uranium

Thermal conductivity (k_u) = 36.4 W/(m K) at 427°C

From Appendix 2, Table 24, for Bismuth at 316°C

Specific heat (c_b) = 144.5 J/(kg K)

Density (ρ) = 10,011 kg/m^3

SOLUTION

(a) The rate of heat transfer required to raise the Bismuth by 40°C is

$$q = \dot{m}\, c_b \Delta T_b = \rho\, V A_c\, c_b\, \Delta T_b = \rho\, V \frac{\pi}{4} (D_o^2 - D_i^2)\, c_b\, \Delta T_b$$

$$q = 10{,}011\ \text{kg/m}^3 (4.5\,\text{m/s}) \frac{\pi}{4} [(0.061\ \text{m})^2 - (0.05\ \text{m})^2](144.5\,\text{J/(kg K)})(40°\text{C})\ ((\text{Ws})/\text{J})$$

$$= 2.50 \times 10^5\ \text{W}$$

Therefore, the average heat flux is

$$\frac{\dot{Q}_G}{A_t} = \frac{q}{A_t} = \frac{q}{\pi D_i L} = \frac{2.50\times10^5\ \text{W}}{\pi(0.05\,\text{m})(2\,\text{m})} = 7.95 \times 10^5\ \text{W/m}^2$$

The temperature difference between the uranium and bismuth (ΔT_{ub}) required to transfer this heat can be calculated from

$$\frac{q}{A_t} = \bar{h}_c\, \Delta T_{ub} \Rightarrow \Delta T_{ub} = \frac{q}{A_t h_c} = \frac{(7.95\times10^5\,\text{W/m}^2)}{(26{,}500\,\text{W/(m}^2\text{K))}} = 29.7\ \text{K}$$

The maximum uranium surface temperature will occur at the outlet where the bismuth temperature is $T_{b,\max}$ = 316°C + 0.5(ΔT_b) = 336°C

$$T_{u,ro,\max} = T_{b,\max} + \Delta T_{ub} = 336°\text{C} + 29.7\ \text{K} \approx 366°\text{C}$$

The rate of internal heat generation per unit volume is

$$\dot{q}_G = \frac{\dot{Q}_G}{\text{Volume}} = \frac{q}{\frac{\pi}{4} D_i^2 L} = \frac{2.50\times10^5\ \text{W}}{\frac{\pi}{4}(0.05\,\text{m})^2 (2\,\text{m})} = 6.37 \times 10^7\ \text{W/m}^3$$

The maximum temperature at the center of the uranium is given by Equation (2.51)

$$T_{u,o,\max} = T_{u,ro,\max} + \frac{\dot{q}_G r_o^2}{4\,k_u} = 366°\text{C} + \frac{(6.37\times10^7\ \text{W/m}^3)(0.05\,\text{m}/2)^2}{4(36.4\,\text{W/(m K)})} = 639°\text{C}$$

At the inlet

$$T_{u,ro} = (T_b - 0.5\ \Delta T_b) + \Delta T_{ub} + \frac{\dot{q}_G r_o^2}{4\,k_u}$$

$$T_{u,ro} = [316°\text{C} - 0.5(40°\text{C})] + 29.7°\text{C} + \frac{(6.37\times10^7\ \text{W/m}^3)(0.05\,\text{m}/2)^2}{4(36.4\,\text{W/(m K)})} = 599°\text{C}$$

Therefore, the average uranium temperature is approximately

$$T_{u,\text{ave}} = \frac{\left(\frac{366+639}{2} + \frac{329+599}{2}\right)}{2} = 483°\text{C}$$

Repeating the calculation using the thermal conductivity of uranium evaluated at this temperature yields the following result

$T_{u,\text{ave}} = 483°\text{C}$

$k_u = 37.7$ W/(m K)

$T_{u,o,\max} = 630°\text{C}$

$T_{u,ro}$ (inlet) = 590°C

$T_{u,\text{ave}} = 478°\text{C}$ (Convergence)

PROBLEM 6.38

Evaluate the rate of heat loss per meter from pressurized water flowing at 200°C through a 10-cm-*ID* pipe at a velocity of 3 m/s. The pipe is covered with a 5-cm-thick layer of 85% magnesia wool which has an emissivity of 0.5. Heat is transferred to the surroundings at 20°C by natural convection and radiation. Draw the thermal circuit and state all assumptions.

GIVEN

- Pressurized water flowing through an insulated pipe
- Water temperature (T_w) = 200°C = 493 K
- Pipe inside diameter (D_i) = 10 cm = 0.1 m
- Water velocity (V) = 3 m/s
- Magnesia wool insulation thickness (t) = 5 cm = 0.05 m
- Emissivity of the wool insulation (ε) = 0.5
- Temperature of the surroundings (T_∞) = 20°C = 293 K

FIND

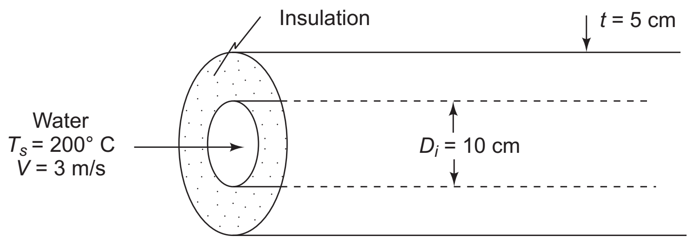
- The rate of heat loss per meter (q/L)

ASSUMPTIONS

- Steady state
- Pipe surface temperature can be considered constant and uniform
- Surroundings behave as a black body
- Pipe is horizontal
- Thermal resistance of the pipe is negligible

- Ambient air is still
- Pipe thickness is negligible
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11, the thermal conductivity of 85% magnesia (k_I) = 0.059 W/(m K) at 20°C.
From Appendix 2, Table 13, for water at 200°C

Thermal conductivity (k_w) = 0.665 W/(m K)
Kinematic viscosity (ν_w) = 0.160×10^{-6} m²/s
Prandtl number (Pr_w) = 0.95

From Appendix 1, Table 5, the Stephan Boltzmann constant (σ) = 5.67×10^{-8} W/(m² K⁴).

SOLUTION

Heat transfer coefficient on the water side:
The Reynolds number of the water flow is

$$Re_D = \frac{V\,D}{\nu_w} = \frac{(3\,\text{m/s})(0.1\,\text{m})}{\left(0.16\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1.875 \times 10^6 \text{ (Turbulent)}$$

Applying Equation (6.63)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\,(1.875 \times 10^6)^{0.8}\,(0.95)^{0.3} = 2363$$

$$\overline{h}_{cw} = \overline{Nu}_D \frac{k_w}{D_i} = 2363 \frac{(0.665\,\text{W/(m K)})}{0.1\,\text{m}} = 15{,}713 \text{ W/(m}^2\text{ K)}$$

Heat transfer coefficient on the air side:
The natural convection heat transfer coefficient on the outside of the insulation is a function of the exterior temperature of the insulation (T_I). For a first iteration, let $T_I = T_\infty + 20° = 40°$C. Evaluating the air properties from Appendix 2, Table 27, at the film temperature of 30°C

Thermal expansion coefficient (β) = 0.0033 1/K
Thermal conductivity (k_a) = 0.0258 W/(m K)
Kinematic viscosity (ν_a) = 16.7×10^{-6} m²/s
Prandtl number (Pr_a) = 0.71

The Grashof number is

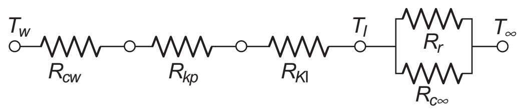
$$Gr_D = \frac{g\beta(T_I - T_\infty)\,D_I^3}{\nu_a^2} = \frac{\left(9.8\,\text{m/s}^2\right)(0.0033\,1/\text{K})\,(20\,°\text{C})(0.2\,\text{m})^3}{\left(16.7\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1.855 \times 10^7$$

Applying Equation (5.20)

$$\overline{Nu}_D = 0.53\ (Gr_D Pr)^{\frac{1}{4}} = 0.53\left[1.855\ \times\ 10^7\ (0.71)\right]^{\frac{1}{4}} = 31.93$$

$$\bar{h}_{ca} = \overline{Nu}_D \frac{k_a}{D_I} = 31.93\ \frac{(0.0258\,\text{W/(m K)})}{0.2\text{ m}} = 4.12\ \text{W/(m}^2\text{ K)}$$

The thermal circuit for this problem is shown below



where

$$R_{cw} = \frac{1}{\bar{h}_{cw}\,A_w} = \frac{1}{\bar{h}_{cw}\,\pi\,D_w\,L} = \frac{1}{\left(15{,}713\,\text{W/(m}^2\text{K)}\right)\pi\,(0.1\,\text{m})L} = \left(0.000203\,\frac{1}{L}\right)\ \text{(m K)/W}$$

R_{kp} = Thermal résistance of the pipe wall ≈ 0

$$R_{kI} = \frac{\ln\left(\frac{D_I}{D_i}\right)}{2\,\pi\,L\,k_I} = \frac{\ln\left(\frac{0.2}{0.1}\right)}{2\pi\,L\,(0.059\,\text{W/(m K)})} = \left(1.870\,\frac{1}{L}\right)\ \text{(m K)/W} \quad \text{From Equation (2.39)}$$

R_r = Radiative resistance

R_{cw} = Natural convective resistance

The insulation temperature (T_I) can be determined by equating the heat transfer between T_w and T_I to that from T_I to T_∞

$$\frac{T_w - T_I}{R_{cw} - R_{kI}} = q_{ca} + q_{ra} = A_I\,[\,\bar{h}_{ca}\,(T_I - T_\infty) + \varepsilon\,\sigma\,(T_I^{\,4} - T_\infty^{\,4})]$$

$$\frac{473\text{ K} - T_I}{(0.000203{+}1.87)\left(\frac{1}{L}\right)((\text{m K})/\text{W})} = \pi\,(0.2\text{ m})\,L$$

$$\left[\left(4.12\ \text{W/(m}^2\text{K)}\right)(T_I - 293\,\text{K}) + 0.5\left(5.67\times 10^{-8}\ \text{W/(m}^2\text{K}^4)\right)[T_I^4 - (293\,\text{K})^4]\right]$$

Checking the units then eliminating them for clarity

$$1.79\times 10^{-8}\ T_I^4 + 3.124\ T_I - 1142.7 = 0$$

By trial and error: T_I = 312 K = 39°C

Further iterations are not required. The rate of heat loss can be calculated from

$$q = \frac{T_w - T_I}{R_{cw} - R_{kI}} = \frac{473\,\text{K} - 312\,\text{K}}{1.87\left(\frac{1}{L}\right)((\text{m K})/\text{W})} = 86.1\ \text{W/m}$$

COMMENTS

Note that the convective resistance in the turbulent water is negligible compared to that of the insulation.

PROBLEM 6.39

In a pipe-within-a-pipe heat exchanger, water is flowing in the annulus and an aniline-alcohol solution having the properties listed in Problem 6.20 is flowing in the central pipe. The inner pipe is 1.3 cm *ID*, 1.6 cm *OD*, and the *ID* of the outer pipe is 1.9 cm. For a water bulk temperature of 27°C and an aniline bulk temperature of 60°C, determine the overall heat transfer coefficient based on the outer diameter of the central pipe and the frictional pressure drop per unit length of the water and the aniline for the following volumetric flow rates, (a) water rate 0.06 litres/sec, aniline rate 0.06 litres/sec, (b) water rate 0.6 litres/sec, aniline rate 0.06 litres/sec, (c) water rate 0.06 litres/sec, aniline rate 0.6 litres/sec, and (d) water rate 0.6 litres/sec, aniline rate 0.6 litres/sec (*L*/*D* = 400).

Physical properties of the aniline solution

Temperature (°C)	Viscosity (kg/ms)	Thermal Conductivity (W/(m K))	Specific Gravity	Specific Heat kJ/(kg K)
20	0.0051	0.173	1.03	2.09
60	0.0014	0.169	0.98	2.22
100	0.0006	0.164		2.34

GIVEN

- Pipe-within-a-pipe heat exchanger with water in the annulus and aniline-alcohol solution in the inner pipe
- Solution properties listed above
- Pipe diameters
  - Inner pipe: D_{ii} = 1.3 cm, D_i = 1.6 cm
  - Inside of outer pipe: D_o = 1.9 cm
- Bulk temperatures
  - Water (T_w) = 27°C
  - Aniline (T_a) = 60°C
- L/D = 400

FIND

- The overall heat transfer coefficient (U) based on D_i and the pressure drop (Δp) for the following volumetric flow rates ($\dot{V}$)

Case	(a)	(b)	(c)	(d)
Water flow rate (L/s)	0.06	0.6	0.06	0.6
Aniline flow rate (L/s)	0.06	0.06	0.6	0.6

ASSUMPTIONS

- Steady state
- Thermal resistance of the pipe is negligible
- Nusselt number can be estimated from correlations for constant and uniform surface temperature
- The effect of viscosity variation is negligible
- The tubes are smooth
- Fully developed flow (L/D = 400)

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 27°C

Density (ρ) = 999 kg/m^3
Thermal conductivity (k_w) = 0.61 W/(m K)
Kinematic viscosity (ν_w) = 8.41×10^{-7} m^2/s
Prandtl number (Pr_w) = 5.87

SOLUTION

Water side heat transfer coefficient:

The Reynolds number on the water side is

$$Re_D = \frac{V\,D_{ii}}{\nu_w} = \frac{\dot{V}\,D_{ii}}{A_c\,\nu_w} = \frac{4\dot{V}}{\pi\,D_{ii}\,\nu_w}$$

For $\dot{V}$ = 1 gal/min

$$Re_D = \frac{4\times(0.06\times10^{-3}\ \text{m}^3/\text{s})}{\pi(1.3\times10^{-2}\ \text{m})(8.41\times10^{-7}\ \text{m}^2/\text{s})} = 6990 \text{ (Turbulent)}$$

Applying Equation (6.63)

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\ (6990)^{0.8}\ (5.87)^{0.4} = 55.56$$

$$\bar{h}_{cw,1} = \overline{Nu}_D\ \frac{k}{D_{ii}} = 55.56\ \frac{0.61\ \text{W/(m K)}}{(1.3\times10^{-2}\ \text{m})} = 2607\ \text{W/(m}^2\ \text{K)}$$

For V = 0.6 m/s, $Re \simeq 0.70 \times 10^5$ (Turbulent)

$$\overline{Nu}_D = 0.023\ (0.70\times10^5)^{0.8}\ (5.87)^{0.4} = 351$$

$$\bar{h}_{cw,10} = 351\ \frac{0.61\ \text{W/(m K)}}{(1.3\times10^{-2}\ \text{m})} = 16.47\ \text{kW/(m}^2\ \text{K)}$$

Aniline side heat transfer coefficient:

The hydraulic diameter of the annulus, from Equation (6.3) is

$$D_H = D_o - D_i = 1.9\ \text{cm} - 1.6\ \text{cm} = 0.3\ \text{cm} = 3\times10^{-3}\ \text{m}$$

From the given properties; for aniline at 60°C

Density, $\rho = \rho_{H2O}(\text{s.g.}) = 998\ \text{kg/m}^3\ (0.98) = 978\ \text{kg/m}^3$

Kinematic viscosity, $\nu_a = \frac{\mu}{\rho} = \frac{0.0014\ \text{kg/ms}}{978\ \text{kg/m}^3} = 1.431\times10^{-5}\ \text{m}^2/\text{s}$

Prandtl number, $Pr = \frac{c\,\mu}{k} = \frac{(2.22\ \text{kJ/kg K})(0.0014\ \text{kg/ms})}{(0.169\ \text{W/(m K)})} = 18.4$

for ($\dot{V}$) = 0.06 L/s

$$Re_{D_H} = \frac{V\,D_H}{\nu_a} = \frac{\dot{V}\,D_H}{A_c\,\nu_a} = \frac{4\dot{V}\,D_H}{\pi(D_o^2 - D_i^2)\,\nu_a}$$

$$Re_{D_H} = \frac{4(0.06\times10^{-3}\ \text{m}^3/\text{s})(3\times10^{-3}\ \text{m})}{\pi(1.9^2 - 1.6^2)\times10^{-4}\ \text{m}^2\times1.431\times10^{-5}\ \text{m}^2} = 1526 \text{ (Laminar)}$$

From Table (6.2): For D_i/D_o = 1.6/1.9 = 0.84: $\overline{Nu}_D \approx 5.15$

$$\bar{h}_{ca,1} = \overline{Nu}_{D_H}\ \frac{k}{D_H} = 5.15\ \frac{(0.169\ \text{W/(m K)})}{3\times10^{-3}\ \text{m}} = 290\ \text{W/(m}^2\ \text{K)}$$

For $\dot{V}$ = 0.6 L/s $\quad Re$ = 15260 (Turbulent)

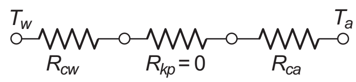
Applying Equation (6.63)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\,(15260)^{0.8}\,(18.4)^{0.3} = 122.5$$

$$\bar{h}_{ca,10} = 122.5 \frac{0.169\ \text{W/(m K)}}{3\times10^{-3}\ \text{m}} = 6800\ \text{W/(m}^2\ \text{K)} = 6.8\ \text{kW/(m}^2\ \text{K)}$$

Overall heat transfer coefficient:
The thermal circuit for the problem is shown below



where

$$R_{cw} = \frac{1}{\bar{h}_{cw}\, A_w} = \frac{1}{\bar{h}_{cw}\, \pi D_{ii}\, L}$$

$$R_{kp} \approx 0$$

$$R_{ca} = \frac{1}{\bar{h}_{ca}\, A_a} = \frac{1}{\bar{h}_{ca}\, \pi D_i\, L}$$

The overall heat transfer coefficient is

$$U A_{\text{ref}} = \frac{1}{R_{cw} + R_{ca}} \quad \text{where } A_{\text{ref}} = \pi D_i L$$

$$\therefore\ U = \frac{1}{D_i \left(\dfrac{1}{\bar{h}_{cw,1}\, D_{ii}} + \dfrac{1}{\bar{h}_{ca,1}\, D_i} \right)}$$

For case (a)

$$\Rightarrow \qquad U = \left[1.6\times10^{-2}\ \text{m} \left(\frac{1}{2607\ \text{W/(m}^2\text{K)}\,(1.3\times10^{-2}\ \text{m})} + \frac{1}{290\ \text{W/(m}^2\text{K)}\,(1.6\times10^{-2}\ \text{m})} \right) \right]^{-1}$$

$$= 255\ \text{W/(m}^2\ \text{K)}$$

Substituting the appropriate convective heat transfer coefficients into the above equations yields the following overall heat transfer coefficient for the remaining cases

(b) $U = 277$ W/(m^2 K)
(c) $U = 1623$ W/(m^2 K)
(d) $U = 4487$ W/(m^2 K)

Friction factors and pressure drop:
For the turbulent cases, the friction factor is given by Equation (6.59)

$$f = 0.184\, Re_p^{-0.2}$$

For water with $\dot{V} = 0.06$ L/s: $f_{w,1} = 0.184\,(6990)^{-0.2} = 0.0313$

For water with $\dot{V} = 0.6$ L/s: $f_{w,10} = 0.184\,(69900)^{-0.2} = 0.0197$

For the aniline solution with $\dot{V} = 0.6$ L/s: $f_{a,10} = 0.184\,(15260)^{-0.2} = 0.0268$

For the aniline solution with $\dot{V}$ = 0.06 L/s, the flow is laminar and the friction factor is given by Table (6.2): $f\,Re_{D_H} = 95.7 \rightarrow f_{a,1} = 0.0594$

The pressure drop is given by Equation (6.13)

$$\Delta p = f\,\frac{L}{D}\,\frac{\rho V^2}{2g_c} = \frac{f\,\rho V^2}{2\,A_c^2} = \frac{L}{D}$$

For the water, $A_c = (\pi/4)D_{ii}^{\,2}$

For the aniline solution, $A_c = (\pi/4)(D_o^2 - D_i^2)$

For the water with $\dot{V}$ = 0.06 L/s

$$\Delta p = \frac{0.0313(400)(999\,\text{kg/m}^3)(0.06\times10^{-3}\,\text{m}^3/\text{s})^2}{2\left(\dfrac{\pi}{4}\times(1.3\times10^{-2}\,\text{m})^2\right)^2} = 1.28\text{ kPa}$$

Similarly for the other cases

For water, $\dot{V}$ = 10 gpm: Δp = 81.3 kPa

For aniline solution $\dot{V}$ = 0.6 L/s: Δp = 6.17 kPa

For aniline solution $\dot{V}$ = 0.06 L/s: Δp = 280 kPa

Tabulating all the results

Case	(a)	(b)	(c)	(d)
Water flow rate (L/s)	0.06	0.6	0.06	0.6
Aniline flow rate (L/s)	0.06	0.06	0.6	0.6
Overall heat transfer coef. W/(m^2 K)	255	277	1623	4487
Water pressure drop (kPa)	1.28	81.3	1.28	81.3
Aniline pressure drop (kPa)	6.17	280	6.17	280

COMMENTS

Note that the flow rate of the aniline solution has a greater effect on the overall heat transfer coefficient than that of the water because the aniline flow changes from laminar to turbulent, whereas the water flow is turbulent at both flow rates.

PROBLEM 6.40

A plastic tube of 7.6-cm *ID* and 1.27 cm wall thickness having a thermal conductivity of 1.7 W/(m K), a density of 2400 kg/m^3, and a specific heat of 1675 J/(kg K) is cooled from an initial temperature of 77°C by passing air at 20°C inside and outside the tube parallel to its axis. The velocities of the two air streams are such that the coefficients of heat transfer are the same on the interior and exterior surfaces. Measurements show that at the end of 50 min, the temperature difference between the tube surfaces and the air is 10 percent of the initial temperature difference. It is proposed to cool a tube of a similar material having an inside diameter of 15 cm and a wall thickness of 2.5 cm from the same initial temperature, also using air at 20°C and feeding to the inside of the tube the same number of kilograms of air per hour that was used in the first experiment. The air-flow rate over the exterior surfaces will be adjusted to give the same heat transfer coefficient on the outside as on the inside of the tube. It may be assumed that the air-flow rate is so high that the temperature rise along the axis of the tube may be neglected. Using the experience gained initially with the 4.5-cm tube, estimate how long it will take to cool the surface of the larger tube to 27°C under the conditions described. Indicate all assumptions and approximations in your solution.

GIVEN

- Air flow inside and outside a plastic tube

Case 1

- Tube 1 inside diameter (D_{1i}) = 7.6 cm = 0.076 m
- Tube 1 wall thickness (S_1) = 1.27 cm = 0.0127 m
- Plastic properties
  - Thermal conductivity (k_p) = 1.7 W/(m K)
  - Density (ρ) = 2400 kg/m^3
  - Specific heat (c) = 1675 J/(kg K)
- Tube initial temperature (T_{ti}) = 77°C
- Air temperature (T_a) = 20°C
- After 10 min: ($T_t - T_a$) = 10% of initial ($T_t - T_a$)

Case 2

- Tube 2 inside diameter (D_{2i}) = 15 cm = 0.15 m
- Tube 2 wall thickness (S_2) = 2.5 cm = 0.025 m
- Same initial temperature and air temperature as Case 1
- Same interior air flow rate as Case 1
- Air velocities are such that heat transfer coefficients inside and outside are equal

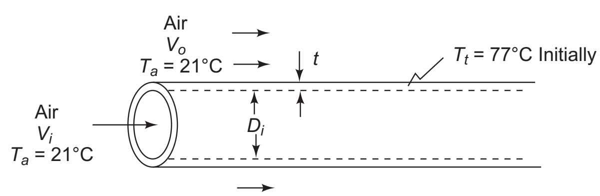
FIND

- Time for T_t to reach 27°C in Case 2

ASSUMPTIONS

- Temperature rise along the tube is negligible
- Tube may be treated as a lumped capacitance (This will be checked)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 20°C

Density (ρ) = 1.164 kg/m^3
Thermal conductivity (k) = 0.0251 W/(m K)
Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71
Absolute viscosity (μ) = 18.24 × 10^{-6} (Ns)/m^2

SOLUTION

Case 1

Assuming the tube can be treated as a lumped capacitance: Equation (2.84) can be applied

$$\ln\left(\frac{T_{tf} - T_a}{T_{ti} - T_a}\right) = -\frac{\bar{h}_c A_s}{c\rho V} t = -\frac{\bar{h}_c \pi (D_i + D_o) L}{c\rho\left[\frac{\pi}{4}(D_o^2 - D_i^2) L\right]} t$$

where $D_o = D_i + 2s$ = 0.076 m + 2(0.0127 m) = 0.1014 m

Solving for the heat transfer coefficient

$$\bar{h}_c = -\frac{c\rho(D_o^2 - D_i^2)}{4\,(D_i + D_o)t}\ \ln\left(\frac{T_{tf} - T_a}{T_{ti} - T_a}\right)$$

$$\bar{h}_c = -\frac{(1675\,\text{J/(kg K)})(2400\,\text{kg/m}^3)\,[(0.1014\,\text{m})^2 - (0.076\,\text{m})^2]}{4(0.076\,\text{m} + 0.1014\,\text{m})(50\,\text{min})(60\,\text{s/min})(\text{J/(Ws)})}\ \ln(0.10) = 19.6\ \text{W/(m}^2\ \text{K)}$$

Checking the lumped capacity assumption, the Biot number should be based on half of the tube wall thickness since convection occurs equally on the inside and outside of the tube

$$B_i = \frac{\bar{h}_c\, s}{2\,k_s} = \frac{(19.6\,\text{W/(m}^2\text{K)})(0.0127\,\text{m})}{2\,(1.7\,\text{W/(m K)})} = 0.07 < 0.1$$

Therefore, the lumped capacity assumption is valid. Assuming the air flow is turbulent and applying Equation (6.63) to determine the Reynolds number for the interior air flow

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\therefore\ Re_D = \left[\frac{\bar{h}_c D_i}{0.023\,\text{Pr}^{0.4}\,\text{K}}\right]^{1.25} = \left[\frac{(19.6\,\text{W/(m}^2\text{K)})(0.076\,\text{m})}{0.023(0.71)^{0.4}(0.0251\,\text{W/(m K)})}\right]^{1.25} = 21{,}825\ \text{(Turbulent)}$$

Therefore, the air velocity is

$$V = \frac{Re_D\,\nu}{D_i} = \frac{21{,}825(15.7\times10^{-6}\,\text{m}^2/\text{s})}{0.076\,\text{m}} = 4.51\ \text{m/s}$$

The mass flow rate is

$$\dot{m} = V\rho A_c = V\rho\frac{\pi}{4}D_i^2 = 4.51\ \text{m/s}(1.1641\,\text{kg/m}^3)\ \frac{\pi}{4}(0.076\ \text{m})^2 = 0.024\ \text{kg/s}$$

Case 2

Applying the mass flow rate to Case 2

$$Re_D = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.024\,\text{kg/s})}{\pi(0.15\,\text{m})(18.24\times10^{-6}\,(\text{Ns})/\text{m}^2)(\text{kg m/(Ns}^2))} = 11{,}169\text{(Turbulent)}$$

Applying Equation (6.63)

$$\overline{Nu}_D = 0.023\ (11{,}169)^{0.8}\ (0.71)^{0.4} = 34.72$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 34.72\ \frac{(0.0251\ \text{W/(m K)})}{0.15\,\text{m}} = 5.81\ \text{W/(m}^2\ \text{K)}$$

The Biot number is

$$Bi = \frac{\bar{h}_c\, s}{2\,k_s} = \frac{(5.81\,\text{W/(m}^2\text{K)})(0.025\,\text{m})}{2\,(1.7\,\text{W/(m K)})} = 0.04 < 0.1$$

Therefore, the internal thermal resistance can be neglected and Equation (2.84) can be applied. Solving for the time

$$t = -\frac{c\rho(D_o^2 - D_i^2)}{4(D_i + D_o)\,\bar{h}_c}\ln\left(\frac{T_{tf} - T_a}{T_{ti} - T_a}\right) = -\frac{c\rho(D_o - D_i)}{4\,\bar{h}_c}\ln\left(\frac{T_{tf} - T_a}{T_{ti} - T_a}\right) = -\frac{c\rho(2s)}{4\,\bar{h}_c}\ln\left(\frac{T_{tf} - T_a}{T_{ti} - T_a}\right)$$

$$t = -\frac{(1675\,\text{J/(kg K)})(2400\,\text{kg/m}^3)\,2\,(0.025\,\text{m})}{4(5.81\,\text{W/(m}^2\text{K)})(\text{J/(W s)})}\ln\left(\frac{27°\text{C} - 20°\text{C}}{77°\text{C} - 20°\text{C}}\right)$$

t = 18,137 s = 302 min ≈ 5 hours

PROBLEM 6.41

Exhaust gases having properties similar to dry air enter an exhaust stack at 800 K. The stack is made of steel and is 8 m tall and 0.5 m *ID*. The gas flow rate is 0.5 kg/s and the ambient temperature is 280 K. The outside of the stack has an emissivity of 0.9. If heat loss from the outside is by radiation and natural convection, calculate the gas outlet temperature.

GIVEN

- Exhaust gas flow through a steel stack
- Exhaust gas has the properties of dry air
- Entering exhaust temperature ($T_{b,\text{in}}$) = 800 K
- Stack height (L) = 8 m
- Stack diameter (D) = 0.5 m
- Gas flow rate ($\dot{m}$)= 0.5 kg/s
- Ambient temperature (T_∞) = 280 K
- Stack emissivity (ε) = 0.9

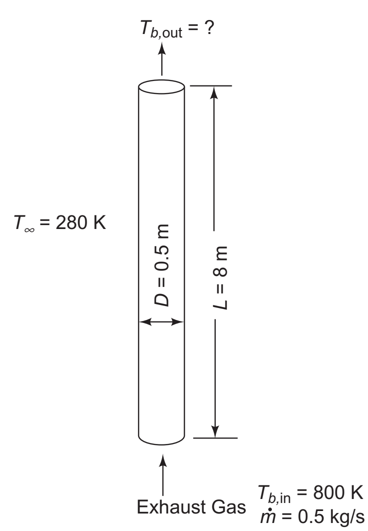
FIND

- The outlet gas temperature ($T_{b,\text{out}}$)

ASSUMPTIONS

- Steady state
- The surrounding behave as a black body enclosure at the ambient temperature
- Thermal resistance of the duct is negligible
- Duct thickness is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 800 K

Density (ρ) = 0.433 kg/m^3
Thermal conductivity (k) = 0.0552 W/(m K)
Kinematic viscosity (ν) = 86.4 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.72
Specific heat (c) = 1079 J/(kg K)

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

Interior convection:

The Reynolds number is

$$Re_D = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.5\,\text{kg/s})}{\pi(0.5\,\text{m})\left(86.4\times10^{-6}\,(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 14{,}740$$

Applying Equation (6.63)

$$\overline{Nu}_D = 0.023\; Re_D^{0.8}\; Pr^n \qquad \text{where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\;(14{,}740)^{0.8}\;(0.72)^{0.3} = 45.0$$

$$\bar{h}_{\text{cfi}} = \overline{Nu}_D \frac{k}{D} = 45.0\;\frac{(0.0552\,\text{W/(m K)})}{0.5\,\text{m}} = 4.97\ \text{W/(m}^2\ \text{K)}$$

For the first iteration, let the duct temperature (T_d) equal the average of the exhaust and ambient temperatures = 540 K. Then the interior film temperature is 670 K. From Appendix 2, Table 27, for dry air at 670 K

Thermal expansion coefficient (β) = 0.00149 1/K
Thermal conductivity (k) = 0.0485 W/(m K)
Kinematic viscosity (ν) 64.6 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.72

The Grashof number is

$$Gr_L = \frac{g\beta(T_I - T_\infty)L^3}{\nu_a^2} = \frac{(9.8\,\text{m/s}^2)(0.00149\,(1/\text{K}))(800\,\text{K} - 540\,\text{K})(8\,\text{m})^3}{(64.6\times10^{-6}\,\text{m}^2/\text{s})^2} = 4.66\times10^{11}$$

$$\frac{Gr}{Re^2} = \frac{4.66\times10^{11}}{(14{,}740)^2} = 2143$$

Therefore, natural convection cannot be neglected.

The interior natural convection heat transfer coefficient will be estimated using the vertical plant correlation of Equation (5.13)

$$\overline{Nu}_L = 0.13\;(Gr_L Pr)^{\frac{1}{3}} = 0.13\left[4.66\times10^{11}\;(0.72)\right]^{\frac{1}{3}} = 903$$

$$\bar{h}_{\text{cni}} = \overline{Nu}_L \frac{k}{L} = 903\;\frac{(0.0485\,\text{W/(m K)})}{8\,\text{m}} = 5.47\ \text{W/(m}^2\ \text{K)}$$

Combining the natural forced coefficients using Equation (5.49)

$$\bar{h}_{ci} = \left(\bar{h}_{\text{cfi}}^{\,3} + \bar{h}_{\text{cfn}}^{\,3}\right)^{\frac{1}{3}} = \left[(4.97)^3 + (5.47)^3\right]^{\frac{1}{3}} = 6.59\ \text{W/(m}^2\ \text{K)}$$

Exterior convection:

Air properties at the exterior film temperature of 410 K are

Thermal expansion coefficient (β) = 0.00247 1/K
Thermal conductivity (k) = 0.033 W/(m K)
Kinematic viscosity (ν) = 28.0×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

$$Gr_L = \frac{\left(9.8\,\text{m/s}^2\right)\left(0.00247\,(1/\text{K})\right)(540\,\text{K} - 280\,\text{K})(8\,\text{m})^3}{\left(28.0\times10^{-6}\,\text{m}^2/\text{s}\right)} = 4.11 \times 10^{12}$$

$$\overline{Nu}_L = 0.13\left[4.11\times10^{12}\ (0.71)\right]^{\frac{1}{3}} = 1857$$

$$\bar{h}_{co} = 1857\ \frac{(0.033\,\text{W/(m K)})}{8\,\text{m}} = 7.66\ \text{W/(m}^2\ \text{K)}$$

Duct temperature:

The rate of convection to the duct interior must equal the sum of convection and radiation from the exterior

$$\bar{h}_{ci}\,A_t\,(T_b - T_d) = \bar{h}_{co}\,A_t\,(T_d - T_\infty) + A_t\,\varepsilon\,\sigma\,(T_d^{\,4} - T_\infty^{\,4})$$

$$\left(6.59\ \text{W/(m}^2\text{K)}\right)(800\ \text{K} - T_d) = \left(7.66\ \text{W/(m}^2\text{K)}\right)(T_d - 280\ \text{K}) + 0.9\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\left[T_d^{\,4} - (280\ \text{K})^4\right]$$

$$5.13 \times 10^{-8}\ T_d^{\,4} + 14.25\ T_d - 7730 = 0$$

By trial and error: T_d = 425 K

Using the duct temperature to estimate the rate of heat transfer

$$q = \bar{h}_{ci}\ \pi D\,L\,(T_b - T_d) = \left(6.59\ \text{W/(m}^2\text{K)}\right)\ \pi\,(0.5\ \text{m})\ (8\ \text{m})\ (800\ \text{K} - 425\ \text{K}) = 3.11 \times 10^4\ \text{W}$$

The temperature rise of the exhaust gas is

$$\Delta T_b = \frac{q}{\dot{m}c} = \frac{3.11\times10^4\,\text{W}}{(0.5\,\text{kg/s})(1079\,\text{J/(kg K)})((\text{Ws})/\text{J})} = 57.5\ \text{K}$$

$$T_{b,\text{out}} = T_{b,\text{in}} - \Delta T_b = 800\ \text{K} - 57.5\ \text{K} = 742\ \text{K}$$

The average bulk temperature is 721 K. This is close enough to the first iteration value that another iteration is not necessary.

PROBLEM 6.42

A 3.05 m long vertical cylindrical exhaust duct from a commercial laundry has an *ID* of 15.2 cm. Exhaust gases having physical properties approximating those of dry air enter at 316°C. The duct is insulated with 10.2 cm of rock wool having a thermal conductivity of: k = 0.7 + 0.016 T (where T is in °C and k in W/(m K).

If the gases enter at a velocity of 0.61 m/s, calculate

(a) The rate of heat transfer to quiescent ambient air at 15.6 °C.

(b) The outlet temperature of the exhaust gas.

Show your assumptions and approximations.

GIVEN

- Exhaust gases flowing through an insulated long vertical cylindrical duct
- Exhaust gases have the physical properties of dry air
- Duct length (L) = 3.05 m
- Duct inside diameter (D) = 15.2 cm = 0.152 m
- Entering exhaust temperature ($T_{b,\text{in}}$)
- Rock wool insulation thickness (s) = 10.2 cm = 0.102 m
- Thermal conductivity of insulation (k) = 0.7 + 0.016 T (T is in °C and k in W/(m K))
- Exhaust velocity (V) = 0.61 m/s
- Gas inlet temperature ($T_{b,\text{in}}$) = 316°C

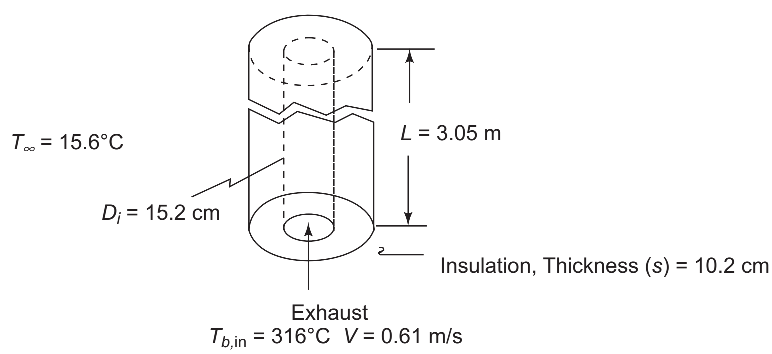
FIND

(a) Rate of heat transfer to ambient air at (T_∞) = 15.6°C
(b) Outlet exhaust gas temperature ($T_{b,\text{out}}$)

ASSUMPTIONS

- Steady state
- Thermal resistance of the duct wall is negligible
- Heat transfer by radiation is negligible
- Natural convection on the inside of the duct can be approximated by natural convection from a vertical plate
- The interior heat transfer coefficient can be accurately estimated using uniform surface temperature correlations
- The ambient air is still

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the inlet temperature of 316°C

Thermal expansion coefficient (β) = 0.00175 1/K
Thermal conductivity (k) = 0.0438 W/(m K)
Kinematic viscosity (ν) = 51.7×10^{-6} m^2/s
Prandtl number (Pr) = 0.71
Density (ρ) = 0.582 kg/m^3
Specific heat (c) = 1049 J/(kg K)
Absolute viscosity (μ_b) = 28.869×10^{-6} Ns/m^2

SOLUTION

The expression for thermal conductivity is given as

$$k = 0.7 + 0.016\ T\ (T \text{ in °C},\ k \text{ in W/(m K)})$$

Interior convection:

The Reynolds number for the exhaust flow is

$$Re_D = \frac{V\,D}{\nu} = \frac{(0.61\,\text{m/s})(0.152\,\text{m})}{\left(51.7\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1793 \text{ (Laminar)}$$

For the first iteration, let the duct wall temperature (T_d) = 300°C and the insulation surface temperature (T_I) = 20°C. From Appendix, Table 27, the absolute viscosity at T_d = 300°C is $\mu_s = 29.332 \times 10^{-6}$ (Ns)/m^2. Applying Equation (6.39)

$$\overline{Nu}_D = \left[3.66 + \frac{0.0668\,Re_D Pr\left(\frac{D}{L}\right)}{1+0.045\left[Re_D Pr\left(\frac{D}{L}\right)\right]^{0.66}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\overline{Nu}_D = \left[3.66 + \frac{0.0668(1793)(0.71)\left(\frac{0.152}{3.05}\right)}{1+0.045\left[(1793)(0.71)\left(\frac{0.152}{3.05}\right)\right]^{0.66}}\right]\left(\frac{28.869}{29.332}\right)^{0.14} = 6.17$$

$$\bar{h}_{c,\text{forced}} = \overline{Nu}_D\,\frac{k}{D_i} = 6.17\,\frac{(0.0438\,\text{W/(m K)})}{0.152\,\text{m}} = 1.77 \text{ W/(m}^2\text{ K)}$$

The interior Grashof number based on the duct length is

$$Gr_L = \frac{g\beta(T_{b,\text{in}} - T_d)L^3}{\nu_a^2} = \frac{\left(9.8\,\text{m/s}^2\right)(0.00171(1/\text{K}))(316°\text{C} - 300°\text{C})(3.05\,\text{m})^3}{\left(51.7\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 2.85 \times 10^9$$

$$\frac{Gr_L}{Re_D^2} = \frac{2.85\times10^9}{(1793)^2} = 885$$

Therefore, natural convection on the inside of the duct cannot be ignored. The natural convection Nusselt number will be estimated with Equation (5.13)

$$\overline{Nu}_L = 0.13\,(Gr_L Pr)^{\frac{1}{3}} = 0.13\left[2.85\times10^9\,(0.71)\right]^{\frac{1}{3}} = 164.4$$

$$\bar{h}_{c,\text{natural}} = \overline{Nu}_L\,\frac{k}{L} = 164.4\,\frac{(0.0438\,\text{W/(m K)})}{3.05\,\text{m}} = 2.36 \text{ W/(m}^2\text{ K)}$$

Combining the free and forced convection using Equation (5.49)

$$\bar{h}_{ci} = \left(\bar{h}_{ci}^3 + \bar{h}_{cn}^3\right)^{\frac{1}{3}} = \left[(1.77)^3 + (2.36)^3\right]^{\frac{1}{3}} = 2.65 \text{ W/(m}^2\text{ K)}$$

Exterior convection:

The Grashof number on the exterior of the insulation is

$$Gr_L = \frac{g\beta(T_1 - T_\infty)L^3}{\nu_a^2}$$

For the film temperature of 17.8°C

$\beta = 0.00344$ 1/K

$\nu = 15.5 \times 10^{-6}$ m^2/s

$$Pr = 0.71$$

$$k = 0.0249 \text{ W/(m K)}$$

$$Gr_L = \frac{(9.8\,\text{m/s}^2)(0.00344\,(1/\text{K}))(20°\text{C} - 15.6°\text{C})(3.05\,\text{m})^3}{(15.5 \times 10^{-6}\,\text{m}^2/\text{s})^2} = 1.75 \times 10^{10}$$

The Nusselt number is given by Equation (5.13)

$$\overline{Nu}_L = 0.13\ (Gr_L Pr)^{\frac{1}{3}} = 0.13\left[1.75 \times 10^{10}(0.71)\right]^{\frac{1}{3}} = 301.2$$

$$\bar{h}_{co} = \overline{Nu}_L\ \frac{k}{L} = 301.2\ \frac{(0.0249\,\text{W/(m K)})}{3.05\,\text{m}} = 2.46 \text{ W/(m}^2\text{ K)}$$

Conduction through the insulation:

The rate of heat transfer through the insulation is

$$q = -k\,A\ \frac{dT}{dr}$$

$$\frac{q}{2\pi L}\int_{r_i}^{r_o}\frac{dr}{r} = \int_{T_d}^{T_i} k\,dT = -\int_{T_d}^{T_i}(0.7 + 0.016\,T)\,dt$$

where T_I = exterior insulation temperature

T_d = duct wall temperature = interior insulation temperature

$$\frac{q}{2\pi L}\ln\left(\frac{r_o}{r_i}\right) = -\,0.7\,(T_I - T_d) - \frac{0.016}{2}\,(T_I^2 - T_d^2)$$

$$q = \frac{2\pi L}{\ln\left(\frac{r_o}{r_i}\right)}\,[0.7\,(T_d - T_I) + 0.008\,(T_d^2 - T_I^2)] = \frac{T_d - T_I}{R_k}$$

$$\therefore\ \frac{1}{R_k} = \frac{2\pi L}{\ln\left(\frac{r_o}{r_i}\right)}\left[0.7 + 0.008\left(\frac{T_d^2 - T_I^2}{T_d - T_I}\right)\right] = \frac{2\pi L}{\ln\left(\frac{r_o}{r_i}\right)}\,[0.7 + 0.008\,(T_D + T_I)]$$

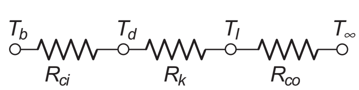
where $r_i = D_i/2 = (0.0152\text{ m})/2 = 0.076\text{ m}$

$r_o = r_i + s = 0.076\text{ m} + 0.102\text{ m} = 0.178\text{ m}$

$$\frac{1}{R_k} = \frac{2\pi(3.05\,\text{m})}{\ln\left(\frac{0.178}{0.076}\right)}\,[0.7 + 0.008\,(300°\text{C} + 20°)] = 73.4 \text{ W/K}$$

$$R_k = 0.0136 \text{ K/W}$$

The thermal circuit for the problem is shown below



$$R_{ci} = \frac{1}{\bar{h}_{ci}A_i} = \frac{1}{\bar{h}_{ci}\pi D_i L} = \frac{1}{(2.65\,\text{W/(m}^2\text{K)})\,\pi(0.152\,\text{m})(3.05\,\text{m})} = 0.259 \text{ K/W}$$

$$R_{co} = \frac{1}{\bar{h}_{co}\pi D_o L} = \frac{1}{(2.46\,\text{W/(m}^2\text{K)})\,\pi[0.152\,\text{m} + 2(0.102\,\text{m})](3.05\,\text{m})} = 0.119 \text{ K/W}$$

The total rate of heat transfer is

$$q = \frac{T_b - T_\infty}{R_{ci} + R_k + R_{co}} = \frac{316\,°\text{C} - 15.6\,°\text{C}}{(0.259 + 0.0136 + 0.119)\,\text{K/W}} = 767\ \text{W}$$

Calculating a new duct wall temperature and insulation temperature

$$q = \frac{T_b - T_d}{R_{ci}} \Rightarrow T_d = T_b - q\,R_{ci} = 316°\text{C} - 767\ \text{W}\,(0.259\ \text{K/W}) = 117°\text{C}$$

$$q = \frac{T_I - T_\infty}{R_{co}} \Rightarrow T_I = T_\infty + q\,R_{co} = 15.6°\text{C} + 767\ \text{W}\,(0.119\,\text{K/W}) = 107°\text{C}$$

$$\Delta T_{\text{bulk}} = \frac{q}{\dot{m}\,c} = \frac{q}{\frac{\pi}{4} D_i^2 \rho V c} = \frac{767\,\text{W}}{\frac{\pi}{4}(0.152\,\text{m})^2\left(0.582\,\text{kg/m}^3\right)(0.61\,\text{m/s})(1049\,\text{J/(kg K)})((\text{Ws})/\text{J})} = 113°\text{C}$$

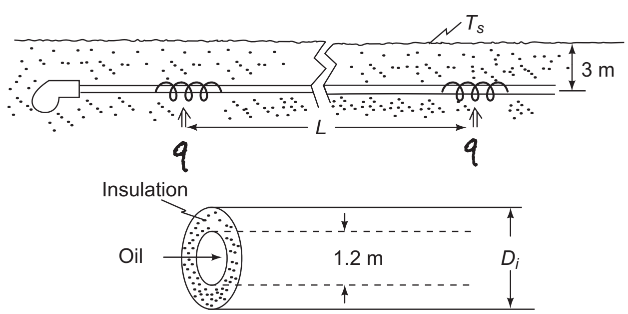
The third and fourth iterations are nearly converged.

Therefore, the rate of heat transfer is about 767 watts.

The outlet exhaust gas temperature = $T_{b,\text{in}} - \Delta T_b$ = 316°C – 113°C = 203°C

PROBLEM 6.43

A long 1.2 m *OD* pipeline carrying oil is to be installed in Alaska. To prevent the oil from becoming too viscous for pumping, the pipeline is buried 3 m below ground. The oil is also heated periodically at pumping stations as shown schematically below



The oil pipe is to be covered with insulation having a thickness *t* and a thermal conductivity of 0.05 W/(m K). It is specified by the engineer installing the pumping station that the temperature drop of the oil in a distance of 100 km should not exceed 5°C when the soil surface temperature T_s = – 40°C. The temperature of the pipe after each heating is to be 120°C and the flow rate is 500 kg/s. The properties of the oil being pumped are given below

Density (ρ_{oil}) = 900 kg/m^3

Thermal conductivity (k_{oil}) = 0.14 W/(m K)

Kinematic viscosity (ν_{oil}) = 8.5 × 10^{-4} m^2/s

Specific heat (c_{oil}) = 2000 J/(kg K)

The soil under arctic conditions is dry (Table 11, k_s = 0.35 W/(m K)).

Estimate the thickness of insulation necessary to meet the specifications of the pumping engineer.

Calculate the required rate of heat transfer to the oil at each heating point.

Calculate the pumping power required to move the oil between two adjacent heating stations.

GIVEN

- An insulated underground oil pipeline
- Pipe outside diameter (D_{po}) = 1.2 m
- Depth to centerline (Z) = 3 m
- Insulation thickness = t
- Insulation thermal conductivity (k_i) = 0.05 W/(m K)
- For L = 100 km = 100,000 m, Maximum ΔT_b = 5°C when ground surface temp. (T_s) = – 40°C
- Oil temperature after heating ($T_{b,\text{in}}$) = 120°C
- Mass flow rate ($\dot{m}$) = 500 kg/s
- Fluid properties listed above
- Soil thermal conductivity (k_s) = 0.35 W/(m K)

FIND

(a) The thickness of insulation (t) required
(b) The required rate of heat transfer to the oil at each heating point (q_h)
(c) The pumping power required

ASSUMPTIONS

- Constant thermal properties
- Uniform ground surface temperature
- Flow is fully developed
- The thermal resistance of the pipe is negligible
- The thickness of the pipe is negligible compared to the diameter

SOLUTION

The interior heat transfer coefficient can be evaluated from correlations. The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{4\dot{m}}{\pi D \nu \rho} = \frac{4(500\,\text{kg/s})}{\pi(1.2\,\text{m})(900\,\text{kg/m}^3)(8.5\times10^{-4}\,\text{m}^2/\text{s})} = 693 \text{ (Laminar)}$$

Since the oil bulk temperature is to drop only 5°C, for practical purposes, the pipe is isothermal. Therefore, for fully developed flow: $\overline{Nu}_D$ = 3.66

$$\bar{h}_c = \overline{Nu}_D \frac{k_{\text{oil}}}{D} = 3.66\,\frac{(0.14\,\text{W/(m K)})}{1.2\,\text{m}} = 0.427 \text{ W/(m}^2\text{ K)}$$

(a) A heat balance on an element of the oil yields

$$dq = \dot{m}\, c_p dT_b$$

The rate of heat flow from the element is

$$dq = U\,(T_b - T_s) \quad \text{where } U = \frac{1}{R_{\text{total}}} = \frac{1}{R_c + R_{ki} + R_{ks}}$$

where R_c = interior convective resistance = $\dfrac{1}{\bar{h}_c\, A} = \dfrac{1}{\bar{h}_c\, \pi D_{po}\, dx}$

$$R_{ki} = \text{conductive resistance of the insulation} = \frac{\ln\left(\dfrac{D_i}{D_{po}}\right)}{2\pi k_i\, dx}$$

$$R_{ks} = \text{conductive resistance of the soil} = \frac{1}{k_s S}$$

The shape factor (S) is given in Table 2.2

$$S = \frac{2\pi\, dx}{\cosh^{-1}\left(\frac{2Z}{D_i}\right)}$$

$$\therefore\ R_{\text{total}} = \frac{1}{dx}\left[\frac{1}{\bar{h}_c \pi D_{po}} + \frac{\ln\left(\frac{D_i}{D_{po}}\right)}{2\pi k_i} + \frac{\cos^{-1}\left(\frac{2Z}{D_i}\right)}{2\pi k_s}\right]$$

$$\text{Let } U' = \frac{1}{dx R_{\text{total}}} = \frac{U}{dx} \text{ then } dq = U'(T_b - T_s)\, dx = \frac{U}{dx}$$

$$\frac{dT_b}{T_b - T_s} = \frac{U'}{\dot{m} c_p} dx$$

Integrating

$$\int_{T_{b,\text{in}}}^{T_{b,\text{out}}} \frac{1}{T_b - T_s} dT_b = -\int_0^L \frac{U'}{\dot{m}\, c_p}\, dx$$

$$\ln\left(\frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s}\right) = -\frac{U' L}{\dot{m}\, c_p}$$

Solving for the overall heat transfer coefficient

$$U' = \frac{\dot{m}\, c_p}{L} \ln\left(\frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s}\right) = -\frac{(500\,\text{kg/s})(2000\,\text{J/(kg K)})((\text{Ws})/\text{J})}{100{,}000\,\text{m}} \ln = \left(\frac{115\,°\text{C} + 40\,°\text{C}}{120\,°\text{C} + 40\,°\text{C}}\right)$$

$$= 0.317\ \text{W/(m K)}$$

$$\frac{1}{U'} = \frac{1}{\bar{h}_c \pi D_{po}} + \frac{\ln\left(\frac{D_i}{D_{po}}\right)}{2\pi k_i} + \frac{\cosh^{-1}\left(\frac{2Z}{D_i}\right)}{2\pi k_s}$$

$$\frac{1}{(0.317\,\text{W/(m K)})} = \frac{1}{\left(0.427\,\text{W/(m}^2\text{K)}\right)\pi(1.2\,\text{m})} + \frac{\ln\left(\frac{D_i}{1.2\text{m}}\right)}{2\pi(0.05\,\text{W/(m K)})} + \frac{\cosh^{-1}\left(\frac{2\,(3\,\text{m})}{D_i}\right)}{2\pi(0.35\,\text{m W/(m K)})}$$

checking the units then eliminating them for clarity

$$5.571 = 7.01\, n\left(\frac{D_i}{1.2\ \text{m}}\right) + \cosh^{-1}\left(\frac{6}{D_i}\right)$$

by trial and error: $D_i = 2.06$ m

$$t = \frac{(D_i - D)}{2} = \frac{[(2.06\ \text{m}) - (1.2\ \text{m})]}{2} = 0.43\ \text{m} = 43\ \text{cm}$$

(b) The rate of heating required at each pumping station is

$$q = \dot{m} c_p\, \Delta T = (500\ \text{kg/s})(2000\,\text{J/(kgK)})\,(5°\text{C})((\text{Ws})/\text{J}) = 5 \times 10^6\ \text{W} = 5\ \text{MW}$$

(c) The pumping power P, equals the product of the volumetric flow rate and the pressure drop, or

$$P = \dot{m}_{\Delta p}$$

Incorporating Equation (6.13) for the pressure drop and Equation (6.18) for the friction factor

$$P = \left(\frac{\dot{m}}{\rho}\right)\frac{64}{Re_d}\frac{L}{D}\frac{\rho U^2}{2g_c} = 32\,\frac{\dot{m}L}{Dg_c Re_D}\left(\frac{4\dot{m}}{\rho\pi D^2}\right)^2 = \frac{512}{\pi^2}\frac{L\dot{m}^3}{g_c Re_D \rho^2 D^5}$$

$$P = \frac{512}{\pi^2}\frac{100{,}000\,\text{m}(500\,\text{kg/s})^3}{\left((\text{kg m})/(\text{s}^2\text{N})\right)(693)\left(900\,\text{kg/m}^3\right)^2(1.2\,\text{m})^5}\left((\text{Ws})/(\text{Nm})\right)$$

$$= 1.46 \times 10^6\ \text{W} = 1.46\ \text{MW}$$

PROBLEM 6.44

Show that for fully developed laminar flow between two flat plates spaced 2*a* apart, the Nusselt number based on the 'bulk mean' temperature and the passage spacing is 4.12 if the temperature of both walls varies linearly with the distance *x*, i.e., $\partial T/\partial x$ = C. The 'bulk mean' temperature is defined as

$$T_b = \frac{\int_{-a}^{a} u(y)\,T(y)\,dy}{\int_{-a}^{a} u(y)\,dy}$$

GIVEN

- Fully developed laminar flow between two flat plates
- Spacing = $2a$
- $\partial T/\partial x$ = C
- Bulk mean temperature as defined above

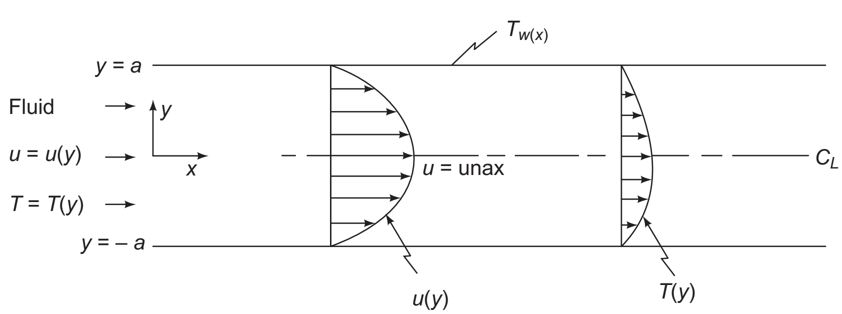
FIND

- Show that the Nusselt number based on the bulk mean temperature = 4.12

ASSUMPTION

- Steady state
- Constant and uniform property values
- Fluid temperature varies linearly with x
  (This corresponds to a constant heat flux boundary)

SKETCH



SOLUTION

The solution will progress as follows

1. Derive the temperature distribution in the fluid.
2. Use the temperature distribution to obtain an expression for the bulk mean temperature.
3. Use the bulk mean temperature to derive the Nusselt number.

Beginning with the laminar flow energy equation of Equation (4.7b)

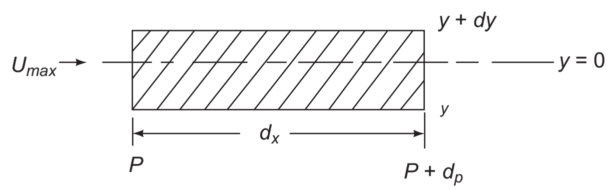
$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

v = component of the velocity in the Y direction = 0

$$\therefore u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}$$

Note that $\partial T/\partial x$ = constant by assumption.

The velocity profile $u(y)$ must be substituted into this equation before the equation can be solved for the temperature distribution. The velocity profile can be derived by considering a differential element of fluid of width w as shown below



A force balance on this element yields

$$2\,w\,y\;[p-(p+dp)] = 2\,\tau\,\mathrm{w}\,dx = \mu\,\frac{\partial u}{\partial y} - \mu\left(\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y}dy\right) w\,dx$$

Since the flow is fully developed

$$\frac{dp}{dx} = \mu \frac{d^2 u}{dy^2}$$

Integrating with respect to y

$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + \mathrm{C}$$

This is subject to the following boundary conditions

$u = u_{\max}$ at $y = 0$ therefore, $\mathrm{C} = u_{\max}$

$u = 0$ at $y = + a$ therefore, $u_{\max} = -\dfrac{a^2}{2\mu}\dfrac{dp}{dx}$

Therefore, the velocity distribution is

$$u = u_{\max}\left[1-\left(\frac{y}{a}\right)^2\right]$$

Substituting this into the energy equation

$$\frac{\partial^2 T}{\partial y^2} = \frac{u_{\max}}{\alpha}\frac{\partial T}{\partial x}\left[1-\left(\frac{y}{a}\right)^2\right]$$

$$\text{Let } z = \frac{u_{\max}}{\alpha}\frac{\partial T}{\partial x} \quad \text{(a constant)}$$

$$\text{Let } z = \frac{u_{\max}}{\alpha}\frac{\partial T}{\partial x} \quad \text{(a constant)}$$

Subject to the boundary conditions

$$\frac{\partial T}{\partial y} = 0 \text{ at } y = 0 \text{ (by symmetry)}$$

$$T = T_w \text{ at } y = + a$$

Integrating once

$$\frac{\partial T}{\partial y} = z\left(y - \frac{1}{3}\frac{y^3}{a^2}\right) + C_1$$

Applying the first boundary condition, $C_1 = 0$

Integrating again

$$T = z\left(\frac{1}{2}y^2 - \frac{1}{12}\frac{y^4}{a^2}\right) + C_2$$

Applying the second boundary condition

$$T_w = z\left(\frac{1}{2}a^2 - \frac{1}{12}a^2\right) + C_2 \Rightarrow C_2 = T_w - \frac{5}{12}\,z\,a^2$$

Therefore, the temperature distribution is

$$T(x,y) = T_w(x) - \frac{5}{12}\,z\,a^2 + \frac{z}{2}\,y^2 - \frac{z}{12}\frac{y^4}{a^2}$$

The bulk mean temperature is defined as

$$T_b = \frac{\int_{-a}^{a} u(y)T(y)\,dy}{\int_{-a}^{a} u(y)\,dy}$$

Solving the numerator of this expression

$$\int_{-a}^{a} u(y)T(x,y)dy = \int_{-a}^{a} u_{\max}\left[1-\left(\frac{y}{a}\right)^2\right]\left[T_w(x) - \frac{5}{12}za^2 + \frac{z}{2}y^2 - \frac{z}{12a^2}y^4\right]dy$$

$$\int_{-a}^{a} u(y)T(x,y)dy = u_{\max}\left[2a\left(T_w(x) - \frac{5}{12}za^2\right) + \frac{z}{3}a^3 - \frac{z}{30}a^3 - \frac{2}{3}a\left(T_w(x) - \frac{5}{12}za^2\right) - \frac{z}{5}a^3 + \frac{z}{42}a^3\right]$$

$$\int_{-a}^{a} u(y)T(x,y)dy = u_{\max}\left[\frac{4}{3}a\left(T_w(x) - \frac{5}{12}za^2\right) + \frac{13}{105}za^3\right]$$

The denominator is

$$\int_{-a}^{a} u_{\max}\left[1-\left(\frac{y}{a}\right)^2\right]dy = u_{\max}\left[2a-\frac{2}{3}a\right] = \frac{4}{3}u_{\max}\,a$$

$$\therefore\ T_b = \frac{\frac{4}{3}a\left(T_w(x)-\frac{5}{12}za^2\right)+\frac{13}{105}za^3}{\frac{4}{3}a}$$

$$T_b - T_w = \frac{13}{105}\left(\frac{3}{4}\right) za^2 - \frac{5}{12}\ za^2 = -\frac{34}{105}\,za^2$$

The rate of heat transfer is given by

$$\frac{q}{A} = \bar{h}_c\,(T_b - T_w) = -k\,\frac{\partial T}{\partial y}\Big|_{y=a}$$

$$\text{where } \frac{\partial T}{\partial y}\Big|_{y=a} = z\,a - \frac{z}{3}\frac{a^3}{a^2} = \frac{2}{3}\,a\,z$$

$$\therefore\ \bar{h}_c = \frac{-k\frac{\partial T}{\partial y}\Big|_{y=a}}{T_b - T_w} = \frac{-k\left(\frac{2}{3}\,az\right)}{-\frac{34}{105}\,za^2} = \frac{210}{51}\left(\frac{k}{2}\,a\right)$$

$$\overline{Nu} = \frac{\bar{h}_c L}{k} = \frac{\bar{h}_c\,2a}{k} = \frac{210}{51} = 4.12$$

PROBLEM 6.45

Repeat Problem 6.44 but assume that one wall is insulated while the temperature of the other walls increases linearly with *x*.

From Problem 6.44: For fully developed laminar flow between two flat plates spaced 2*a* apart, find the Nusselt number based on the 'bulk mean' temperature if the temperature of both walls varies linearly with the distance *x*, i.e., ∂*T*/∂*x* = C. The 'bulk mean' temperature is defined as

$$T_b = \frac{\int_{-a}^{a} u(y)\,T(y)\,dy}{\int_{-a}^{a} u(y)\,dy}$$

GIVEN

- Fully developed laminar flow between two flat plates
- Spacing = 2*a*
- $\partial T/\partial x$ = C
- Bulk mean temperature as defined above
- One wall is insulated
- The temperature of the other wall increases linearly with *x*

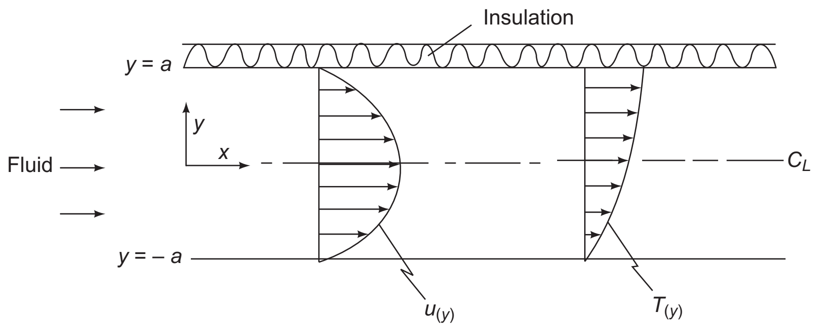
FIND

- The Nusselt number based on the bulk mean temperature (*Nu*)

ASSUMPTIONS

- Steady state
- Constant and uniform property values
- Fluid temperature varies linearly with *x*
  (This corresponds to a constant heat flux boundary)

SKETCH



SOLUTION

The velocity profile derived in the solution to Problem 6.44 remains unchanged

$$u = u_{max}\left[1-\left(\frac{y}{a}\right)^2\right]$$

As does the energy equation

$$\frac{\partial^2 T}{\partial y^2} = z\left[1-\left(\frac{y}{a}\right)^2\right]$$

$$\text{where} \quad z = \frac{u_{max}}{\alpha}\frac{\partial T}{\partial x} \quad \text{(a constant)}$$

The new boundary conditions are

$$\frac{\partial T}{\partial y} = 0 \text{ at } y = a \text{ (due to the insulation)}$$

$$T = T_w(x) \text{ at } y = -a$$

Integrating the energy equation once

$$\frac{\partial T}{\partial y} = z\left(y - \frac{1}{3}\frac{y^3}{a^2}\right) + C_1$$

Applying the first boundary condition

$$0 = za - \frac{za^3}{3a^2} + C_1 \Rightarrow C_1 = -\frac{2}{3}za$$

Integrating the energy equation again

$$T = -\frac{2}{3}zay + \frac{1}{2}zy^2 - \frac{z}{12a^2}y^4 + C_2$$

Applying the second boundary condition

$$T_w(x) = -\frac{2}{3}\, z\,a^2 + \frac{1}{2}\, z\,a^2 - \frac{z}{12a^2}\, a^4 + C_2 \Rightarrow C_2 = T_w(x) + \frac{1}{4}\, za^2$$

Therefore, the temperature distribution is

$$T(x,y) = T_w(x) + \frac{1}{4}\, za^2 - \frac{2}{3}\, zay + \frac{1}{2}\, zy^2 - \frac{z}{12a^2}\, y^4$$

The numerator of the bulk mean temperature expression is

$$\int_{-a}^{a} u(y)\,T(x,y)\,dy = \int_{-a}^{a} u_{\max}\left[1-\left(\frac{y}{a}\right)^2\right]\left[\left(T_w(x)+\frac{1}{4}za^2\right)-\frac{2}{3}zay+\frac{1}{2}zy^2-\frac{z}{12a^2}y^4\right]dy$$

$$= u_{\max}\left[2a\left(T_w(x)+\frac{1}{4}za^2\right)+\frac{1}{3}za^3-\frac{1}{30}za^3-\frac{2}{3}a\left(T_w(x)+\frac{1}{4}za^2\right)-\frac{1}{5}za^3+\frac{1}{42}za^3\right]$$

$$\int_{-a}^{a} u(y)\,T(x,y)\,dy = u_{\max}\left[\frac{4}{3}a\left(T_w(x)+\frac{1}{4}za^2\right)+\frac{13}{105}za^3\right]$$

The denominator of the bulk mean temperature is

$$\int_{-a}^{a} u_{\max}\left[1-\left(\frac{y}{a}\right)^2\right]dy = u_{\max} = \left[2a-\frac{2}{3}a\right]\frac{4}{3}\, u_{\max}\, a$$

$$\therefore\ T_b = \left(T_w(x)+\frac{1}{4}za^2\right) + \left(\frac{3}{4}\right)\frac{13}{105}\, za^2 = T_w(x) + \frac{57}{210}\, za^2$$

$$T_w(x) - T_b = -\frac{57}{210}\, za^2$$

$$\text{At } z = -a:\ \frac{\partial T}{\partial y} = z(-a) - \frac{z}{3a^2}\,(-a)^3 - \frac{2}{3}\, za = -\frac{4}{3}\, za$$

$$\therefore\ \bar{h}_c = \frac{-k\left.\dfrac{\partial T}{\partial y}\right|_{y=-a}}{T_b - T_w} = \frac{-k\left(-\dfrac{4}{3}az\right)}{\dfrac{57}{210}za^2} = \frac{560}{57}\left(\frac{k}{2}a\right)$$

By definition

$$Nu = \frac{\bar{h}_c L}{k} = \frac{\bar{h}_c\, 2a}{k} = \frac{560}{57} = 9.82$$

PROBLEM 6.46

For fully turbulent flow in a long tube of diameter D, develop a relation between the ratio $(L/\Delta T)/D$ in terms of flow and heat transfer parameters, where $L/\Delta T$ is the tube length required to raise the bulk temperature of the fluid by ΔT. Use Equation 6.63 for fluids with Prandtl number of the order of unity or larger and Equation 6.75 for liquid metals.

GIVEN

- Fully developed turbulent flow in a long tube
- Diameter = D
- $L/\Delta T$ = Tube length required to raise the bulk temperature by ΔT

FIND

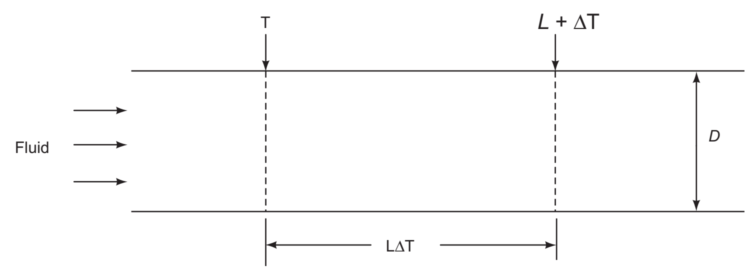
A relationship for $(L/\Delta T)/D$ in terms of flow and heat transfer parameters using

(a) Equation 6.63 for fluids with $Pr \approx 1$
(b) Equation 6.75 for liquid metals

ASSUMPTIONS

- Steady state
- Constant fluid properties
- Uniform wall temperatures

SKETCH



SOLUTION

Let

k = the thermal conductivity of the fluid
μ = the absolute viscosity of the fluid
c = the specific heat of the fluid
V = the velocity of the fluid
ρ = the density of the fluid
T_b = Average bulk fluid temperature
T_w = wall temperature

(a) Using Equation (6.63) for the Nusselt number

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 0.023 \frac{k}{D} Re_D^{0.8}\, Pr^{0.4}$$

The rate of heat transfer to the fluid must equal the energy needed to raise the temperature of the fluid by ΔT

$$q = \bar{h}_c\, \pi D\, L\, (T_b - T_w) = \dot{m}\, c\, \Delta T$$

$$\frac{L}{\Delta T} = \frac{\dot{m} c}{\bar{h}_c \pi D (T_b - T_w)} = \frac{\rho V \frac{\pi}{4} D^2 C}{0.023 \frac{k}{D} Re_D^{0.8} Pr^{0.4} \pi D (T_b - T_w)}$$

$$\text{But } Re_D = \frac{V D \rho}{\mu} \text{ and } Pr = \frac{c\mu}{k}$$

$$\therefore\ \frac{L}{\Delta T} = 10.87\ \frac{\rho V D^2 C}{k\left(\frac{V\rho D}{\mu}\right)^{0.8}\left(\frac{c\mu}{k}\right)^{0.4}(T_b - T_w)}$$

$$\frac{\frac{L}{D}}{\Delta T}\quad 10.87\,\rho^{0.2}\,V^{0.2}\,D^{0.2}\,c^{0.6}\,\mu^{0.4}\,k^{-0.6}\,(T_b - T_w)^{-1}$$

Checking the units

$$\left[\frac{\frac{L}{D}}{\Delta T}\right] = \left[\text{kg/m}^3\right]^{0.2}[\text{m/s}]^{0.2}\,[\text{m}]^{0.2}\,[\text{J/(kgK)}]^{0.6}\left[(\text{Ns})/\text{m}^2\right]^{0.4}[\text{W/(mK)}]^{-0.6}\ [\text{K}]^{-1}$$

$$[(\text{Ws})/\text{J}]^{0.6}\left[\text{kg m}/(\text{s}^2\text{N})\right]^{0.4} = [1/\text{K}]$$

(b) From Equation (6.75)

$$\bar{h}_c = 0.625\ \frac{k}{D}\ Re_D^{\ 0.4}\ Pr^{0.4}$$

$$\frac{L}{\Delta T} = \frac{\rho V \frac{\pi}{4} D^2 c}{0.625\frac{k}{D}\,Re_D^{0.4} Pr^{0.4}\pi D\,(T_b - T_w)}$$

$$\therefore\ \frac{L}{\Delta T} = 0.40\ \frac{\rho V D^2 c}{k\left(\frac{V\rho D}{\mu}\right)^{0.4}\left(\frac{c\mu}{k}\right)^{0.4}(T_b - T_w)}$$

$$\frac{\frac{L}{D}}{\Delta T} = 0.40\,\rho^{0.6}\,V^{0.6}\,D^{0.6}\,c^{0.6}\,k^{-0.6}\,(T_b - T_w)^{-1}$$

PROBLEM 6.47

Water in turbulent flow is to be heated in a single-pass tubular heat exchanger by steam condensing on the outside of the tubes. The flow rate of the water, its inlet and outlet temperatures, and the steam pressure are fixed. Assuming that the tube wall temperature remains constant, determine the dependence of the total required heat exchanger area on the inside diameter of the tubes.

GIVEN

- Water in turbulent flow in tubes with steam condensing on the outside
- Water flow rate, inlet and outlet temperatures, and steam pressure are fixed

FIND

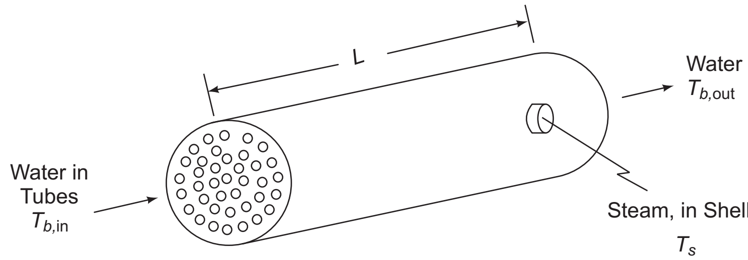
- At = $f(D)$ where A_t = Total heat exchanger area
  D = Inside diameter of the tubes

ASSUMPTIONS

- Steady state
- Fully developed flow
- Tube wall temperature remains constant

- The heat exchanger is designed such that the flow is fully developed turbulent flow
- Thermal resistance of the condensing steam is negligible
- Thermal resistance of the water pipe is negligible

SKETCH



SOLUTION

Let

N = The number of tubes
$\dot{m}$ = Mass flow rate of the water
$T_{b,\text{in}}$ = Water inlet bulk temperature
$T_{b,\text{out}}$ = Water outlet bulk temperature
$T_{b,\text{avg}}$ = Average of water inlet and outlet bulk temperatures
T_s = Saturation temperature of the steam
k = Thermal conductivity of water evaluated at T_b
ρ = Density of water evaluated at T_b
μ = Absolute viscosity of water evaluated at T_b
Pr = Prandtl number of water evaluated at T_b
c = Specific heat of water evaluated at T_b

The Nusselt number on the inside of the tubes is given by Equation (6.63)

$$Nu_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$h_c = Nu_D \frac{k}{D} = 0.023 \frac{k}{D} Re_D^{0.8}\, Pr^{0.4} = 0.023 \frac{k}{D} \left(\frac{V D \rho}{\mu}\right)^{0.8} Pr^{0.4}$$

$$h_c = 0.023 \frac{k}{D} \left(\frac{4\left(\frac{\dot{m}}{N}\right)}{\pi D \mu}\right)^{0.8} Pr^{0.4} = 0.0279\, k\, D^{-1.8} \left(\frac{\dot{m}}{N \mu}\right)^{0.8} Pr^{0.4}$$

The heat transfer by convection to the water must equal the energy required to raise the water temperature by the given amount

$$h_c\, A_t\, (T_s - T_{b,\text{ave}}) = \dot{m}\, c\, (T_{b,\text{out}} - T_{b,\text{in}})$$

$$A_t = \frac{\dot{m} c}{h_c} \frac{T_{b,\text{out}} - T_{b,\text{in}}}{T_s - T_{b,\text{ave}}} = \frac{\dot{m} c}{0.0279\, k\, D^{-1.8} \left(\frac{\dot{m}}{\text{N} \mu}\right)^{0.8} Pr^{0.4}} \frac{T_{b,\text{out}} - T_{b,\text{in}}}{T_s - T_{b,\text{ave}}}$$

$$A_t = 35.8 \frac{\dot{m}^{0.2} \mu^{0.8} N^{0.8}}{k\, Pr^{0.4}} \left(\frac{T_{b,\text{out}} - T_{b,\text{in}}}{T_s - T_{b,\text{ave}}}\right) D^{1.8}$$

$$A_t \propto D^{1.8}$$

Checking the units

$$[A_t] = [\text{kg/s}]^{0.2} \left[(\text{Ns})/\text{m}^2\right]^{0.8} [\text{J/(kg K)}][\text{W/(m K)}]^{-1} [(\text{Ws})/\text{J}]^{0.6}\ [\text{m}]^{1.8} = [\text{m}]^2$$

COMMENTS

The tube diameter must not become so large that the water flow becomes laminar.

PROBLEM 6.48

The following thermal-resistance data were obtained on a 5000 m^2 condenser constructed with 2.5 cm-*OD* brass tubes, 7.2 m long, 1.2 mm wall thickness, at various water velocities inside the tubes [Trans. ASME, vol. 58, *p*. 672, 1936].

$1/U_o \times 10^3$ (Km^2/W)	Water Velocity (m/s)	$1/U_o \times 10^3$ (km^2/W)	Water Velocity (m/s)
0.364	2.11	0.544	0.90
0.373	1.94	0.485	1.26
0.391	1.73	0.442	2.06
0.420	1.50	0.593	0.87
0.531	0.89	0.391	1.91
0.368	2.14		

Assuming that the heat transfer coefficient on the steam side is 11.3 kW/(m^2 K) and the mean bulk water temperature is 50°C, determine the scale resistance.

GIVEN

- Water flowing inside a brass tube condenser
- Total transfer are (A_t) = 5000 m^2
- Tube outside diameter (D) = 2.5 cm
- Tube length (L) = 7.2 m
- Tube wall thickness (t) = 1.2 mm
- Heat transfer coefficient on the steam side ($\bar{h}_{cs}$) = 11.3 kW/(m^2 K)
- Mean bulk water temperature = 50°C
- Thermal resistance data shown above

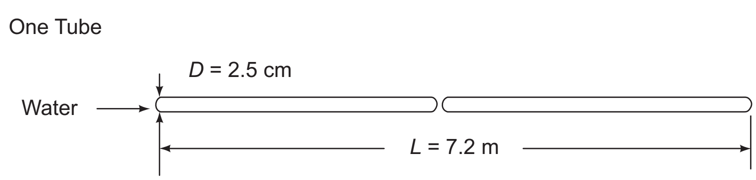
FIND

- The scale resistance ($A_t R_{ks}$)

ASSUMPTIONS

- Data were taken at steady state
- The tube temperature can be considered uniform and constant
- Condenser surface area is based on the tube outside diameter

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 50°C

Thermal conductivity (k) = 0.65 W/(m K)
Kinematiuc viscosity (ν) = 5.46 × 10^{-7} m^2/s
Prandtl number (Pr) = 3.55

From Appendix 2, Table 10, the thermal conductivity of brass (k_b) = 111 W/(m K)

SOLUTION

The inside tube diameter is $D_i = D_o - 2t$ = 2.5 cm – 0.24 cm = 2.26 cm

The maximum and minimum velocities in the given data are 0.87 m/s and 2.14 m/s. These correspond to the following Reynolds numbers

$$Re_{\min} = \frac{VD}{\nu} = \frac{(0.87\,\text{m/s})(2.26\times10^{-2}\,\text{m})}{5.46\times10^{-7}\,\text{m}^2/\text{s}} = 36{,}000$$

$$Re_{\max} = \frac{VD}{\nu} = \frac{(2.14\,\text{m/s})(2.26\times10^{-2}\,\text{m})}{5.46\times10^{-7}\,\text{m}^2/\text{s}} = 88{,}580$$

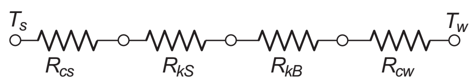
Therefore, the flow is turbulent in all cases. Applying Equation (6.63) to the minimum Re case

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(36{,}000)^{0.8}\,(3.55)^{0.4} = 168.6$$

$$\bar{h}_{cw} = \overline{Nu}_D \frac{k}{D} = 168.6\frac{(0.65\,\text{W/(m K)})}{2.26\times10^{-2}\,\text{m}} = 4850\ \text{W/(m}^2\ \text{K)}$$

The thermal circuit for this problem is shown below



$$A_t\, R_{cs} = \frac{1}{\bar{h}_{cs}} = \frac{1}{11.3\times10^3\ \text{W/(m}^2\text{K)}} = 8.85\times10^{-5}\ (\text{K m}^2)/\text{W}$$

$A_t\, R_{ks}$ = scaling resistance

For one tube

$$A_t\, R_{kB} = \pi D\, L\ \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_s}$$

$$A_t\, R_{kB} = \frac{\pi(2.5\times10^{-2}\,\text{m})(7.2\,\text{m})\ln\left(\frac{2.5}{2.26}\right)}{2\pi(7.2\,\text{m})(111\,\text{W/(m K)})} = 1.13\times10^{-5}\ (\text{K m}^2)/\text{W}$$

For the minimum *Re* case

$$A_r\, R_{cw} = \frac{1}{\bar{h}_{cw}} = \frac{1}{48.50\,\text{W/(m}^2\text{K)}} = 2.06\times10^{-4}\ (\text{K m}^2)/\text{W}$$

These resistances are in series, therefore

$$A_r\, R_{\text{total}} = \frac{1}{U_o} = A_t\,(R_{cs} + R_{ks} + R_{kB} + R_{cw})$$

$$\therefore\ A_t\, R_{ks} = \frac{1}{U_o} - A_t\,(R_{cs} + R_{kB} + R_{cw})$$

For the minimum *Re* case (from the given table)

$$A_t\, R_{ks} = 0.593\times10^{-3}\ (\text{K m}^2)/\text{W} - (0.0885 + 0.0113 + 0.206)\times10^{-3}\ (\text{K m}^2)/\text{W}$$

$$A_t\, R_{ks} = 2.33\times10^{-4}\ (\text{K m}^2)/\text{W}$$

Repeating this method for the rest of the data given

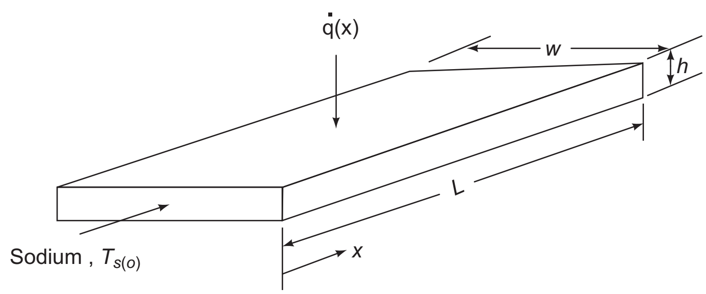
Water Velocity (m/s)	$\overline{h}_{cw}$ W/(m^2 K)	$A_t R_{ks} \times 10^4$ (Km^2)/W
2.11	9850	1.43
1.94	9210	1.45
1.73	8405	1.50
1.50	7500	1.64
0.89	4940	1.99
2.14	9965	1.48
0.90	4950	2.13
1.26	6522	2.02
2.06	9665	2.11
0.87	4850	2.33
1.91	9100	1.58
		Average: 1.79×10^{-4} (K m^2)/W

COMMENTS

The standard deviation in the scale resistance is 24%.

PROBLEM 6.49

A nuclear reactor has rectangular flow channels with a large aspect ratio (w/h)>>1



Heat generation is equal from the upper and lower surface and uniform at any value of *x*. However, the rate varies along the flow path of the sodium coolant according to

$$q''(x) = q_o'' \sin(\pi x/L)$$

Assuming that entrance effects are negligible so that the convection heat transfer coefficient is uniform

(a) Obtain an expression for the variation of the mean temperature of the sodium, $T_m(x)$.

(b) Derive a relation for the surface temperature of the upper and lower portion of the channel, $T_s(x)$.

(c) Determine the distance x_{max} at which $T_s(x)$ is maximum.

GIVEN

- Sodium flow through a rectangular flow channel with a large aspect ratio
- Heat generation from each surface (upper and lower): $q''(x) = q_o'' \sin(\pi x/L)$

FIND

(a) An expression for the variation of the mean sodium temperature, $T_m(x)$
(b) A relationship for the upper and lower surface temperature $T_s(x)$
(c) The distance x_{max} at which $T_s(x_{max})$ is maximum

ASSUMPTIONS

- Entrance effects are negligible
- The convective heat transfer coefficient is uniform
- Steady state

SOLUTION

The hydraulic diameter for the duct is

$$D_H = \frac{4\,A_c}{P} = \frac{4wh}{2w+2h} = 2h$$

(a) In steady state, all of the heat generation must be removed by the sodium, Therefore, the heat transfer to an element of sodium in the duct is

$$dq = 2\,q''\,w\,dx = 2\,q''_o \sin\left(\frac{\pi x}{L}\right)\,w\,dx$$

This will lead to a rise in temperature in the sodium according to

$$dq = \dot{m}\,c\,dT_m = 2\,q''_o \sin\left(\frac{\pi x}{L}\right)\,w\,dx$$

$$\frac{dT_m}{dx} = \frac{2q''_o w}{\dot{m}c}\,\sin\left(\frac{\pi x}{L}\right)$$

Integrating

$$\int_{T_{m,\text{in}}}^{T_m(x)} dT_m = T_m(x) - T_{m,\text{in}} = \frac{2q''_o w}{\dot{m}c}\int_0^x \sin\left(\frac{\pi x}{L}\right)dx$$

$$T_m(x) = T_{m,\text{in}} + \frac{2q''_o wL}{\pi\dot{m}c}\left[1-\cos\left(\frac{\pi x}{L}\right)\right]$$

(b) The rate of heat transfer from both surfaces must equal the rate of heat generation

$$q_{cx} = \dot{q}\,(x) \Rightarrow \quad 2\,h_c\,w\,dx\,(T_s - T_m) = 2\,q''_o\,\sin\left(\frac{\pi x}{L}\right)w\,dx$$

Solving for the surface temperature

$$T_s = T_m + \frac{q''_o}{h_c}\sin\left(\frac{\pi x}{L}\right)$$

$$T_s = T_{m,\text{in}} + \frac{2q''_o wL}{\pi\dot{m}c}\left[1-\cos\left(\frac{\pi x}{L}\right)\right] + \frac{q''_o}{h_c}\,\sin\left(\frac{\pi x}{L}\right)$$

Assuming the flow is fully developed and approximating the heat flux as uniform, the Nusselt number, from Table 6.1, is 8.235. Therefore, $h_c = 8.235\,k/D_H$.

$$T_s = T_{m,\text{in}} + \frac{2q''_o wL}{\pi\dot{m}c}\left[1-\cos\left(\frac{\pi x}{L}\right)\right] + \frac{q''_o D_H}{16.47\,k}\,\sin\left(\frac{\pi x}{L}\right)$$

(c) The maximum occurs when the first derivative of the expression for T_s is zero

$$\frac{dT_s}{dx} = \frac{2q''_o\, w}{\dot{m}c} \sin\left(\frac{\pi x}{L}\right) - \frac{q''_o\, \pi D_H}{16.47\, k\, L} \cos\left(\frac{\pi x}{L}\right) = 0$$

$$\frac{\sin\left(\frac{\pi x}{L}\right)}{\cos\left(\frac{\pi x}{L}\right)} = \frac{\pi \dot{m} c D_H}{16.47\, k\, 2\, w\, L}$$

$$x_{\max} = \frac{L}{\pi} \text{ Arctan} \left(\frac{\pi \dot{m} c h}{16.47\, k w L}\right)$$

Chapter 7

PROBLEM 7.1

Determine the heat transfer coefficient at the stagnation point and the average value of the heat transfer coefficient for a single 5-cm-*OD*, 60-cm-long tube in cross-flow. The temperature of the tube surface is 260°C, the velocity of the fluid flowing perpendicularly to the tube axis is 6 m/s, and its temperature is 38°C. The following fluids are to be considered (a) air, (b) hydrogen, and (c) water.

GIVEN

- A single tube in cross-flow
- Tube outside diameter (D) = 5 cm = 0.05 m
- Tube length (L) = 60 cm = 0.6 cm
- Tube surface temperature (T_s) = 260°C
- Fluid velocity (V) = 6 m/s
- Fluid temperature (T_b) = 38°C

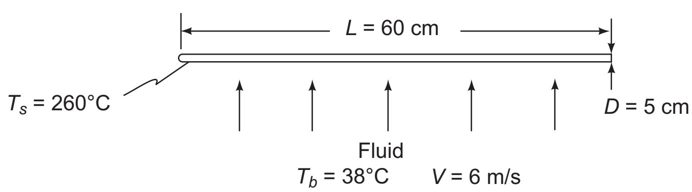
FIND

1. The heat transfer coefficient at the stagnation point (h_{co})
2. The average heat transfer coefficient $\left(\bar{h}_c\right)$ for the following fluids
   (a) air, (b) hydrogen, and (c) water.

ASSUMPTIONS

- Steady state
- Turbulence level of the free stream approaching the tube is low

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the bulk temperature of 38°C

Thermal conductivity (k) = 0.0264 W/(m K)

Kinematic viscosity (ν) = 17.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

and the Prandtl number at the surface temperature

(Pr_s) = 0.71.

From Appendix 2, Table 31, for hydrogen

Thermal conductivity (k) = 0.187 W/(m K)

Kinematc viscosity (ν) = 116.6×10^{-6} m^2/s

Prandtl number (Pr) = 0.704

Prandtl number at the surface temperature (Pr_s) = 0.671

From Appendix 2, Table 13, for water

Thermal conductivity (k) = 0.629 W/(m K)

Kinematic viscosity (ν) = 0.685×10^{-6} m^2/s

Prandtl number (Pr) = 4.5

Prandtl number at the surface temperature (Pr_s) = 0.86

SOLUTION

For air as the fluid

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(6\,\text{m/s})(0.05\,\text{m})}{\left(17.4\times10^{-6}\text{m}^2/\text{s}\right)} = 17{,}241$$

The heat transfer coefficient at the stagnation point can be calculated by applying Equation (7.2) at $\theta = 0$

$$h_\infty = 114\frac{k}{D}Re_D^{0.5}Pr^{0.4} = 1.14\frac{(0.0264\,\text{W/(m K)})}{0.05\,\text{m}}(17{,}241)^{0.5}(0.71)^{0.4} = 68.9\ \text{W/(m}^2\,\text{K)}$$

The average Nusselt number is given by Equation (7.3)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C Re_D{}^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

For $Re_D = 17{,}241$

$C = 0.26$ $\quad m = 0.6$

For $Pr = 0.71$

$n = 0.37$

$$\overline{Nu}_D = 0.26(17{,}241)^{0.6}(0.71)^{0.37}\left(\frac{0.71}{0.71}\right)^{0.25} = 79.8$$

$$\bar{h}_c = \overline{Nu}_D\frac{k}{D} = 79.8\,\frac{(0.0264\,\text{W/(m K)})}{0.05\,\text{m}} = 42.1\ \text{W/(m}^2\text{K)}$$

Using the properties listed above and applying the methodology above to the other fluids yields the following results

Fluid	Re	h_{co} (W/(m^{2}K))	$\bar{h}_c$ (W/(m^{2}K))
Air	17,241	68.9	42.1
Hydrogen	2572	187.9	96.1
Water	438,000	17,322	20,900

COMMENTS

Since the Reynolds number for water is much higher than the air or hydrogen transition from a laminar to a turbulent boundary layer occurs sooner and the flow over most of the cylinder surface is turbulent. Hence the average heat transfer coefficient over the surface is higher than the heat transfer coefficient at the stagnation point.

PROBLEM 7.2

A mercury-in-glass thermometer at 40°C (*OD* = 1 cm) is inserted through duct wall into a 3 m/s air stream at 66°C. Estimate the heat transfer coefficient between the air and the thermometer.

GIVEN

- Thermometer in an air stream
- Thermometer temperature (T_s) = 40°C
- Thermometer outside diameter (D) = 1 cm = 10^{-2} m

- Air velocity (V) = 3 m/s
- Air temperature (T_b) = 66°C

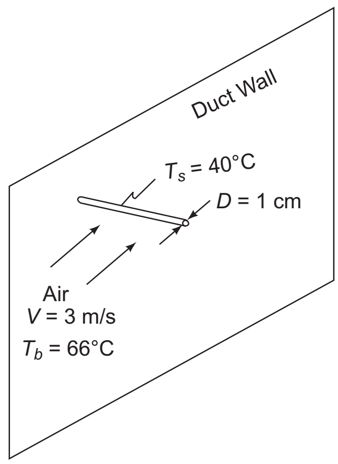
FIND

- The heat transfer coefficient $\left(\bar{h}_c\right)$

ASSUMPTIONS

- Steady state
- Turbulence in the free stream approaching the thermometer is low
- Effect of the duct walls in negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the bulk temperature of 66°C

Thermal conductivity (k) = 0.0282 W/(m K)

Kinematic viscosity (ν) = 1.961×10^{-5} m²/s

Prandtl number (Pr) = 0.71

At the thermometer surface temperature of 40°C, the Prandtl number (Pr_s) = 0.71

SOLUTION

The Reynolds number for this case is

$$Re_D = \frac{V\,D}{\nu} = \frac{3\,\text{m/s}\,(10^{-2}\,\text{m})}{1.961\times 10^{-5}\,\text{m}^2/\text{s}} = 1530$$

The Nusselt number is given by Equation (7.3)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\,Re_D{}^m\,Pr^n\left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where $C = 0.26$

$m = 0.6$

$n = 0.37$

$$\overline{Nu}_D = 0.26(1530)^{0.6}\,(0.71)^{0.37}\left(\frac{0.71}{0.71}\right)^{0.25} = 18.65$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 18.65 \frac{0.0282\,\text{W/(m K)}}{1\times 10^{-2}\,\text{m}} = 52.6\ \text{W/(m}^2\ \text{K)}$$

PROBLEM 7.3

Steam at 1 atm and 100°C is flowing across a 5-cm-*OD* tube at a velocity of 6 m/s. Estimate the Nusselt number, the heat transfer coefficient, and the rate of heat transfer per meter length of pipe if the pipe is at 200°C.

GIVEN

- Steam flowing across a tube
- Steam pressure = 1 atm
- Steam bulk temperature (T_b) = 100°C
- Tube outside diameter (D) = 5 cm = 0.05 m
- Steam velocity (V) = 6 m/s
- Pipe surface temperature (T_s) 200°C

FIND

(a) The Nusselt number $\left(\overline{Nu}_D\right)$

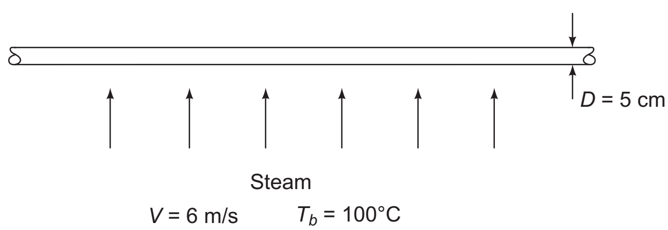
(b) The heat transfer coefficient $\left(\bar{h}_c\right)$

(c) The rate of heat transfer per unit length (q/L)

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 34, for steam at 100°C and 1 atm

Thermal conductivity (k) = 0.0249 W/(m K)

Kinematic viscosity (ν) = 20.2×10^{-6} m^2/s

Prandtl number (Pr) = 0.987

At the tube surface temperature of 200°C, the Prandtl number of the steam (Pr_s) = 1.00

SOLUTION

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(6\,\text{m/s})(0.05\,\text{m})}{\left(20.2\times 10^{-6}\,\text{m}^2/\text{s}\right)} = 1.49 \times 10^4$$

(a) The Nusselt number for this geometry is given by Equation (7.3)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\,Re_D{}^m\,Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

For $Re = 1.49 \times 10^4$

$C = 0.26 \qquad m = 0.6 \qquad n = 0.37$

$$\overline{Nu}_D = 0.26(1.49 \times 10^4)^{0.6}\,(0.987)^{0.37} \left(\frac{0.987}{1.00}\right)^{0.25} = 82.2$$

(b)

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 82.2 \frac{(0.0249\,\text{W/(m K)})}{0.05\,\text{m}} = 40.9\ \text{W/(m}^2\text{K)}$$

(c) The rate of heat transfer by convection from the tube is

$$q = \bar{h}_c\, A_t\,(T_s - T_b) = \bar{h}_c\, \pi DL\,(T_s - T_b)$$

$$\frac{q}{L} = (40.9\text{W/(m}^2\text{K)})\ \pi(0.05\text{ m})\ (200°\text{C} - 100°\text{C}) = 642\ \text{W/m}$$

PROBLEM 7.4

An electrical transmission lin of 1.2 cm diameter carries a current of 200 Amps and has a resistance of 3 × 10^{-4} ohm per meter of length. If the air around this line is at 16°C, determine the surface temperature on a windy day, assuming a wind blows across the line at 33 km/h.

GIVEN

- An electrical transmission line on a windy day
- Line outside diameter (D) = 12 cm = 0.012 m
- Current (I) = 200b Amps
- Resistance per unit length (R_e/L) = 3 × 10^{-4} ohm/m
- Air temperature (T_b) = 16°C
- Air velocity (V) = 33 km/h = 9.17 m/s

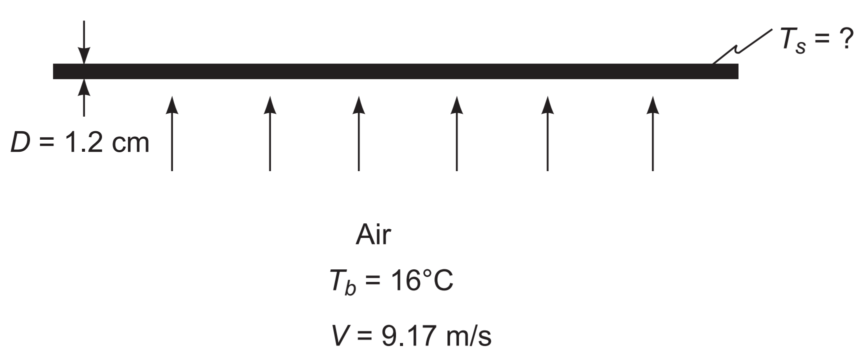
FIND

- The line surface temperature (T_s)

ASSUMPTIONS

- Steady state conditions
- Air flow approaching line has low free-stream turbulence

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 16°C

Thermal conductivity (k) = 0.0248 W/(m K)

Kinematic viscosity (ν) = 15.3×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(9.17\,\text{m/s})(0.012\,\text{m})}{\left(15.3\times 10^{-6}\,\text{m}^2/\text{s}\right)} = 7192$$

The Nusselt number is given by Equation (7.3). The variation of the Prandtl number with temperature is small enough to be neglected for air

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\,Re_D^{\,m}\,Pr^n$$

For $Re_D = 7192$

$C = 0.26$ $m = 0.6$ $n = 0.37$

$$\overline{Nu}_D = 0.26(7192)^{0.6}\,(0.71)^{0.37} = 47.2$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 47.2\,\frac{(0.0248\,\text{W/(m K)})}{0.012\,\text{m}} = 97.5\ \text{W/(m}^2\ \text{K)}$$

The rate of heat transfer by convection must equal the energy dissipation

$$\bar{h}_c\,\pi D\,L\,(T_s - T_b) = I^2\,Re$$

Solving for the tube surface temperature

$$T_s = \frac{I^2\left(\frac{R_e}{L}\right)}{\bar{h}_c \pi D} + T_b = \frac{(200\,\text{A})^2\left(3\times 10^{-4}\,\text{Ohm/m}\right)\left(\text{W/(A}^2\text{Ohm)}\right)}{\left(97.5\,\text{W/(m}^2\text{K)}\right)\pi(0.012\,\text{m})} + 16°\text{C} = 19.3°\text{C}$$

COMMENTS

It is assumed that the thermal conductivity is high and thus the surface temperature is approximately uniform.

PROBLEM 7.5

Derive an equation in the form $\bar{h}_c = f(T, D, U_\infty)$ for flow of air over a long horizontal cylinder for the temperature range 0°C to 100°C, using Equation (7.3) as a basis.

GIVEN

- Flow over a long horizontal cylinder
- Air temperature range is 0°C < T < 100°C

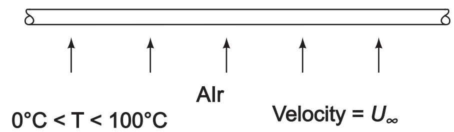
FIND

- An equation in the form $\bar{h}_c = f(T, D, U_\infty)$ based on Equation (7.3)

ASSUMPTIONS

- Steady state
- Prandtl number variation is negligible

SKETCH



SOLUTION

From Appendix 2, Table 27, for dry air the Prandtl number is constant (Pr = 0.71) for the given temperature range. From Equation (7.3), neglecting the variation of Prandtl number term

$$\bar{h}_c = C\frac{k}{D}\ Re_D^{\ m}\ Pr^n$$

where n = 0.37 for air and C and m are given in Table 7.1.

To obtain the desired functional relationship, the kinematic viscosity (ν) and thermal conductivity (k) must be expressed as a function of temperature.

From Appendix 2, Table 27

T(°C)	$\nu \times 10^6$ (m²/s)	k (W/(m K))
0	13.9	0.0237
20	15.7	0.0251
40	17.6	0.0265
60	19.4	0.0279
80	21.5	0.0293
100	23.6	0.0307

Plotting these data, we see that the relationship is nearly linear in both cases. Therefore, a linear least squares regression line will be fit to the data

$$\nu = 1.38 \times 10^{-5} + 9.67 \times 10^{-8}\ T \quad (\nu \text{ in m}^2\text{/s}, T \text{ in °C})$$

$$k = 0.0237 + 7.0 \times 10^{-5}\ T \quad (k \text{ in W/(m K)}, T \text{ in °C})$$

Therefore

$$\bar{h}_c = \text{C}\frac{0.0237 + 7\times10^{-5}T}{D}\left(\frac{U_\infty D}{1.38\times10^{-5} + 9.67\times10^{-8}T}\right)^m (0.71)^{0.37}$$

$$\bar{h}_c = 0.881\ C\ U_\infty^{\ m}\ D^{m-1}\frac{0.0237 + 7\times10^{-5}T}{(1.38\times10^{-5} + 9.67\times10^{-8}T)^m}$$

where T is in °C

$\bar{h}_c$ is in W/(m² K)

and C and M are given in Table 7.1 as a function of Reynolds number

PROBLEM 7.6

Repeat Problem 7.5 for water in the temperature range 10°C to 40°C. From Problem 7.5: Derive an equation in the form $\bar{h}_c = f(T, D, U_\infty)$ for flow over a long horizontal cylinder using Equation (7.3) as a basis.

GIVEN

- Water flow over a long horizontal cylinder
- Water temperature range is 10°C < T < 40°C

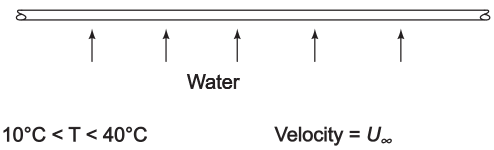
FIND

- An equation in the form $\bar{h}_c = f(T, D, U_\infty)$ based on Equation (7.3)

ASSUMPTIONS

- Steady state
- Temperature difference between water and the cylinder is small enough that the Prandtl number variation is negligible
- The density of water can be considered constant

SKETCH



SOLUTION

Equation (7.3) neglecting the Prandtl number variation

$$\bar{h}_c = C\frac{k}{D} Re^m\, Pr^n = \frac{k}{D}\left(\frac{U_\infty D\rho}{\mu}\right)^m \left(\frac{c\mu}{k}\right)^n$$

Where C and m are given in Table 7.1 and n = 0.37. Since Pr < 10 for the given temperature range. From Appendix 2, Table 13, for water

T (°C)	k (W/(m K))	$\mu \times 10^6$ (Ns/m^2)
10	0.577	1296
15	0.585	1136
20	0.597	993
25	0.606	880.6
30	0.615	792.4
35	0.624	719.8
40	0.633	658.0

Over the given temperature range, the density of water varies only 0.8%. Therefore, the density will be considered constant at its average value: ρ = 996 kg/m^3. Likewise, the variation in specific heat is only 0.5% and its average value is c = 4185 J/(kg K).

Applying a linear least squares regression to k vs. T yields

$$k = 0.588 + 1.89 \times 10^{-3}\, T.\ (T \text{ in °C},\ k \text{ in W/(m K)}).$$

Applying a linear least squares regression on log (μ) vs. log (T) yields

$$\log(\mu) = -2.375 - 0.493 \log(T)$$

$$\mu = 0.0042\, T^{-0.493}$$

Substituting these into the expression for h_c

$$\bar{h}_c = \text{C}(0.558 + 1.89 \times 10^{-3}T)^{(1-0.37)}\, D^{m-1} U_\infty^{\,m} \left(996\,\text{kg/m}^3\right)^m$$

$$(0.0042T^{-0.493})^{(0.37-m)} (4185\,\text{J/(kg K)})^{0.37}$$

$$\bar{h}_c = 21.88(996)^m\, C\, U_\infty^{\,m}\, D^{(m-1)}(0.558 + 1.89 \times 10^{-3}\, T)^{0.63}(0.0042\, T^{-0.493})^{(0.37-m)}$$

where $\overline{h}_c$ is in W/(m K) T is in °C

U_∞ is in m/s D is in m

and C and m are given in Table 7.1 as function of Re

PROBLEM 7.7

The Alaska Pipeline carries 230 million liters per day of crude oil from Prudhoe Bay to Valdez covering a distance of 1280 kilometers. The pipe diameter is 1.2 m and it is insulated with 10 cm of fiberglass covered with steel sheeting. Approximately half of the pipeline length is above ground, nominally running in the north-south direction. The insulation maintains the outer surface of the steel sheeting at approximately 10°C. If the ambient temperature averages 0°C and prevailing winds are 2 m/s from the northeast, estimate the total rate of heat loss from the above-ground portion of the pipeline.

GIVEN

- Fiberglass insulated pipe with air flow at 45° to its axis
- Insulation is covered with sheet steel
- Length of pipe above ground (L) = $\frac{1280}{2}$ = 640 kilometers
- Pipe diameter (D_p) = 1.2 m
- Insulation thickness (t) = 10 cm
- Sheet steel temperature (T_s) = 10°C
- Average ambient air temperature (T_∞) = 0°C
- Average air velocity (U_∞) = 2 m/s

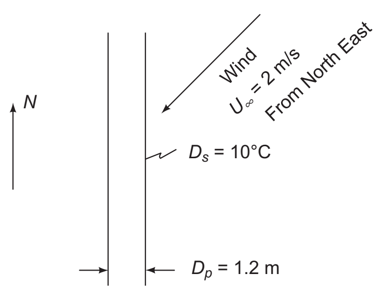
FIND

- The total rate of heat loss from the above ground portion of the pipe (q)

ASSUMPTIONS

- Thermal resistance of the sheet steel as well as contact resistance can be neglected

SKETCH



Plan View

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 0°C

Thermal conductivity (k) = 0.0237 W/(m K)

Kinematic viscosity (ν) = 13.9×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The outside diameter of the insulated pipe is

$$D = D_p + 2t = 1.2 \text{ m} + 2 \times 0.1 = 1.4 \text{ m}$$

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{2\text{ m/s}(1.4\text{ m})}{13.6\times10^{-6}\text{ m}^2/\text{s}} = 2.06\times10^5$$

Since the air flow is not perpendicular to the pipe axis, Groehn's correlation, Equation (7.4), must be used

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = 0.206\,Pr^{0.36}\,Re_N^{\;0.63}$$

where $Re_N = Re_D \sin\theta = 2.06\times10^5 \sin(45°) = 1.45\times10^5$

$$\overline{Nu}_D = 0.206\,(0.71)^{0.36}\,(1.45\times10^5)^{0.63} = 325$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 325\,\frac{0.0237\text{ W/(m K)}}{1.4\text{ m}} = 5.5\text{ W/(m}^2\text{K)}$$

The total rate of heat transfer is give by

$$q = \bar{h}_c A_t\,(T_s - T_\infty)$$

where A_t = the total transfer area = $\pi DL = \pi(1.4\text{ m})\,(640\times10^3\text{ m}) = 2.81\times10^6\text{ m}^2$

$$q = (5.5\text{ W/(m}^2\text{K))}\,(2.81\times10^6\text{ m}^2)\,(10°\text{C} - 0°\text{C}) = 1.54\times10^8\text{ W} = 154\text{ MW}$$

COMMENTS

The calculation has assumed that there is no significant interaction between the ground and the pipe.

PROBLEM 7.8

An engineer is designing a heating system which consists of multiple tubes placed in a duct carrying the air supply for a building. She decides to perform preliminary tests with a single copper tube, 2 cm o.d., carrying condensing steam at 100°C. The air velocity in the duct is 5 m/s and its temperature is 20°C. The tube can be placed normal to the flow, but it may be advantageous to place the tube at an angle to the air flow since additional heat transfer surface area will result. It the duct width is 1 m, predict the outcome of the planned tests.

GIVEN

- A copper tube carrying condensed steam in an air duct
- Tube outside diameter (D) = 2 cm = 0.02 m
- Steam temperature (T_s) = 100°C
- Air velocity (U_∞) = 5 m/s
- Air temperature (T_∞) = 20°C
- Duct width (w) = 1 m

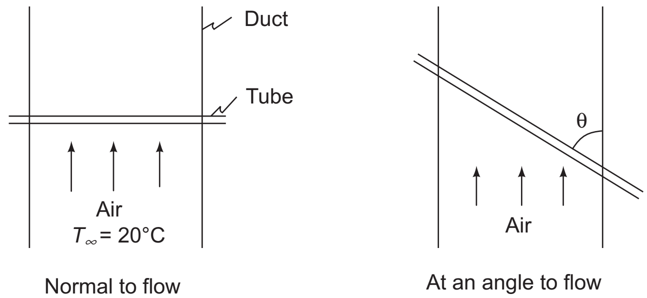
FIND

- Is it more advantageous to have the tubes normal to the air flow or at some angle to the air flow?

ASSUMPTIONS

- Steady state
- Air velocity in the duct is uniform
- Thermal resistance due to steam condensing is negligible
- Thermal resistance of the tube wall is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 20°C

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number based on the tube diameter is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(5\,\text{m/s})(0.02\,\text{m})}{(15.7\times10^{-6}\,\text{m}^2/\text{s})} = 6369$$

For the perpendicular position, the tube length (L) = w = 1 m and the Nusselt number can be calculated using Equation (7.4) with θ = 90°

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = 0.206\,Pr^{0.36}\,Re_N^{0.63}$$

where $Re_N = Re_D \sin(\theta) = Re_D$ for $\theta = 90°$

$$\overline{Nu}_D = 0.206\,(0.71)^{0.36}\,(6369)^{0.63} = 45.4$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 45.4\,\frac{(0.0251\,\text{W/(m K)})}{0.02\,\text{m}} = 57.0\ \text{W/(m}^2\ \text{K)}$$

The rate of heat transfer is

$$q = \bar{h}_c\,\pi D\,L\,(T_s - T_\infty) = (57.0\,\text{W/(m}^2\text{K)})\,\pi\,(0.02\ \text{m})(1\ \text{m})(100°\text{C} - 20°\text{C}) = 287\ \text{W}$$

For the angled position, the tube length (L) = $w/\sin\theta$. Applying Equation (7.4)

$$\overline{Nu}_D = 0.206\,(0.71)^{0.36}\,(6369\,\sin\theta)^{0.63} = 45.38\,(\sin\theta)^{0.63}$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 45.38\,(\sin\theta)^{0.63}\,\frac{(0.0251\,\text{W/(m K)})}{0.02\,\text{m}} = 56.95\,(\sin\theta)^{0.63}\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer is

$$q = \bar{h}_c\,\pi D\,L\,(T_s - T_\infty) = \bar{h}_c\,\pi D\,\frac{\text{w}}{\sin\theta}\,(T\text{s} - T_\infty)$$

$$q = (56.95(\sin\theta)^{0.63}\ \text{W/(m}^2\text{K)})\ \pi\,(0.02\ \text{m})\ \frac{1\,\text{m}}{\sin\theta}\,(100°\text{C} - 20°\text{C}) = 286.3\,(\sin\theta)^{-0.37}\ \text{W}$$

The engineer will find that the rate of heat transfer will increase because the heat transfer coefficient decreases with $(\sin\theta)^{0.63}$ but the area increases with $1/\sin\theta$. Therefore, the rate of heat transfer increases with $1/(\sin\theta)^{0.37}$.

PROBLEM 7.9

A long hexagonal copper extrusion is removed from a heat-treatment oven at 400°C and immersed into a 50°C air stream flowing perpendicular to its axis at 10 m/s. Due to oxidation, the surface of the copper has an emissivity of 0.9. The rod is 3 cm across opposing flats, has a cross-sectional area of 7.79 cm^2, and a perimeter of 10.4 cm. Determine the time required for the center of the copper to cool to 100°C.

GIVEN

- A long hexagonal copper extrusion in an air stream flowing perpendicular to its axis
- Initial temperature (T_o) = 400°C
- Air temperature (T_∞) = 50°C
- Air velocity (V_∞) = 10 m/s
- Surface emissivity (ε) = 0.9
- Distance across the flats (D) = 3 cm = 0.03 m
- Cross sectional area of the extrusion (A_c) = 7.79 cm^2 = 7.79×10^{-4} m^2
- Perimeter of the extrusion (P) = 10.4 cm = 0.104 m

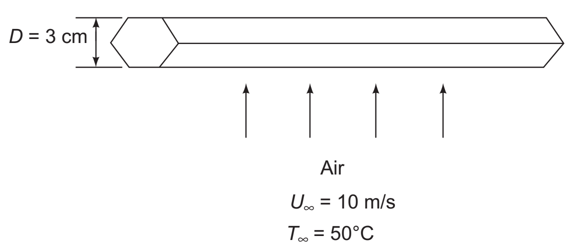
FIND

- The time (t) required for the center of the copper to cool to 100°C

ASSUMPTIONS

- Variations of the copper properties with temperature are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average of the initial and final film temperature of 150°C

Thermal conductivity (k_a) = 0.0339 W/(m K)

Kinematic viscosity (ν) = 29.6×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

For Appendix 2, Table 12, for copper

Thermal Conductivity (k) = 386 W/(m K) at 250°C

Density (ρ) = 8933 kg/m^3 at 20°C

Specific heat (c) = 383 J/(kg K) at 20°C

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(10\,\text{m/s})(0.02\,\text{m})}{(29.6\times10^{-6}\ \text{m}^2/\text{s})} = 10{,}135$$

The Nusselt number for non-circular cross sections in gases by Equation (7.6)

$$\overline{Nu}_D = B\, Re_D^{\,n}$$

where D, B, and n are given by Table 7.2 $B = 0.138$, $n = 0.638$

$$\overline{Nu}_D = 0.138\ (10{,}135)^{0.638} = 49.6$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 49.6 \frac{(0.0339\ \text{W/(m K)})}{0.03\,\text{m}} = 56.0\ \text{W/(m}^2\ \text{K)}$$

The characteristic length for determining the Biot number of the rod is defined in Section 2.6.1 as

$$L_c = \frac{\text{volume}}{\text{surface area}} = \frac{LA_c}{LP} = \frac{A_c}{P} = \frac{7.79\times10^{-4}\,\text{m}^2}{0.104\,\text{m}} = 0.0075\ \text{m}$$

The Biot Number, from Table 4.3, is

$$Bi = \frac{\bar{h}_c L_c}{k_c} = \frac{\left(56\,\text{W/(m}^2\text{K)}\right)(0.0075\,\text{m})}{(386\,\text{W/(m K)})} = 0.0011 << 0.1$$

Therefore, the internal thermal resistance of the extrusion may be neglected and lumped parameters may be applied. An energy balance on the extrusion, including radiation, yields the following

$$q = PL[\,\bar{h}_c\,(T - T_\infty) + \varepsilon\,\sigma\,(T^4 - T_\infty{}^4)] = -\rho\, A_c\, L_c \frac{dT}{dt}$$

This equation must be solved numerically

$$\frac{dT}{dt} = -\frac{P}{\rho A_c c}[\,\bar{h}_c\,(T - T_\infty) + \varepsilon\,\sigma\,(T^4 - T_\infty{}^4)] = -\frac{P}{\rho L_c c}[\,\bar{h}_c\,(T - T_\infty) + \varepsilon\,\sigma\,(T^4 - T_\infty{}^4)]$$

$$\frac{dT}{dt} = \frac{-1}{\left(8933\ \text{kg/m}^3\right)(0.0075\,\text{m})(383\ \text{(Ws)/(kg K)})}$$

$$\left[\left(56.0\ \text{W/(m}^2\ \text{K)}\right)(T - T_\infty) + 0.9\left(5.67\times10^{-8}\ \text{W/(m}^2\ \text{K}^4)\right)[T^4 - T_\infty{}^4]\right]$$

$$\frac{dT}{dt} = -\,0.0022(T - T_\infty) - 1.9887\times10^{-12}\,(T^4 - T_\infty{}^4)\ \text{K / s}$$

This can be solved numerically using a finite difference method

$$\Delta T = T(t + \Delta T) - T(t) = -\Delta t\{0.0022[T(t) - T_\infty] + 1.9887\times10^{-12}[T(t)^4 - T_\infty{}^4]\}$$

$T_\infty = 323$ K, Let $\Delta t = 30$ seconds

t (s)	T (K)		t (s)	T (K)
0	673		390	431
30	638		420	422
60	608		450	415
90	582		480	407
120	559		510	401
150	538		540	395
180	519		570	389
210	503		600	384
240	488		630	380
270	474		660	375
300	462	$\Delta t = 14$ s	674	373.3
330	451			
360	440			

$t = 674$ s = 11.2 minutes

PROBLEM 7.10

Repeat Problem 7.9 if the extrusion cross-section is elliptical, major axis normal to the air flow and same mass per unit length. The major axis of the elliptical cross-section is 5.46 cm and its perimeter is 12.8 cm.

From Problem 7.9: A long copper extrusion is removed from a heat-treatment oven at 400°C and immersed into a 50°C air stream flowing at 10 m/s velocity. Due to oxidation, the surface of the copper has an emissivity of 0.9. Determine the time required for the center of the copper to cool to 100°C.

GIVEN

- A long elliptical copper extrusion in an air stream
- Initial temperature (T_o) = 400°C
- Air Temperature (T_∞) = 50°C
- Air velocity (V_∞) = 10 m/s
- Surface emissivity (ε) = 0.9
- Elliptical cross-section with major axis normal to the air flow
- Length of the major axis of the ellipse (D) = 5.46 cm = 0.0546 m
- Perimeter of ellipse (P) = 12.8 cm = 0.128 m
- Same mass per unit length as Problem 7.9

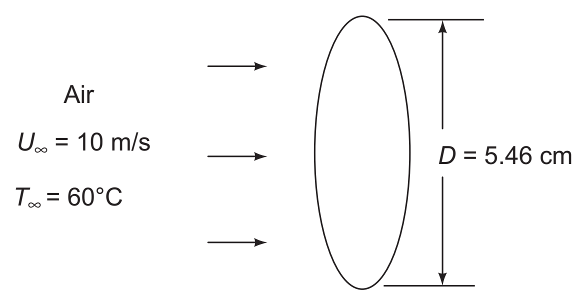
FIND

- The time (t) required for the center of the copper to cool to 100°C

ASSUMPTIONS

- Air flow is perpendicular to the axis of the extrusion
- Variation of the copper properties with temperature is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average of the initial and final film temperature of 150°C

Thermal conductivity (k_a) = 0.0339 W/(m K)

Kinematic viscosity (ν) = 29.6 × 10^{-69} m^2/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 12, for copper

Thermal conductivity (k) = 386 W/(m K) at 250°C

Density (ρ) = 8933 kg/m^3 at 20°C

Specific heat (c) = 383 J/(kg K) at 20°C

SOLUTION

Since the density of the extrusion in this problem is the same as the previous problem, the same mass per unit length implies the same cross-section area

$$A_{c,\text{ellipse}} = A_{c,\text{hexagon}} = 7.79 \text{ cm}^2 = 7.79 \times 10^{-4} \text{ m}^2$$

Following the same procedure as the solution to Problem 7.9

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(10\,\text{m/s})(0.0546\,\text{m})}{(29.6\times10^{-6}\,\text{m}^2/\text{s})} = 18{,}446$$

The Nusselt number for non-circular cross sections in gases is given by Equation (7.6)

$$\overline{Nu}_D = B\, Re_D{}^n$$

where D, B, and n are given by Table 7.2 $B = 0.085$, $n = 0.804$

(Although the Reynolds number for this case is slightly out of range of Equation (7.6), it will be applied to estimate the Nusselt number)

$$\overline{Nu}_D = 0.085\ (18{,}446)^{0.804} = 229$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 229 \frac{(0.0339 \text{ W/(m K)})}{0.0546\,\text{m}} = 142 \text{ W/(m}^2\text{ K)}$$

The characteristic length for determining the Biot number of the rod is defined in Section 2.6.1 as

$$L_c = \frac{\text{volume}}{\text{surface area}} = \frac{LA_c}{LP} = \frac{A_c}{P} = \frac{7.79\times10^{-4}\,\text{m}^2}{0.128\,\text{m}} = 0.0061 \text{ m}$$

The Biot number, from Table 4.3, is

$$Bi = \frac{\bar{h}_c L_c}{k_c} = \frac{(142\,\text{W/(m}^2\text{K)})(0.0061\,\text{m})}{(386\,\text{W/(m K)})} = 0.0022 << 0.1$$

Therefore, the internal thermal resistance of the extrusion may be neglected and lumped parameters may be applied. An energy balance on the extrusion, including radiation, yields the following

$$\frac{dT}{dt} = -\frac{1}{\rho L_c c}\,[\,\bar{h}_c\,(T - T_\infty) + \varepsilon\,\sigma\,(T^4 - T_\infty{}^4)]$$

$$\frac{dT}{dt} = \frac{-1}{(8933\,\text{kg/m}^3)(0.0061\,\text{m})(383\,(\text{W s})/(\text{kg K}))}$$

$$\left[(142\,\text{W/(m}^2\text{ K)})(T - T_\infty) + 0.9(5.67\times10^{-8}\,\text{W/(m}^2\text{ K}^4))[T^4 - T_\infty{}^4]\right]$$

$$\frac{dT}{dt} = -\,0.0068\ (T - T_\infty) - 2.445 \times 10^{-12}\ (T^4 - T_\infty{}^4) \text{ K/s}$$

This can be solved numerically using a finite difference method

$$\Delta T = T(t + \Delta T) - T(t) = -\Delta t\{0.0068[T(t) - T_\infty] + 2.445 \times 10^{-12}\ [T(t)^4 - T_\infty{}^4]\}$$

$T_\infty = 323$ K, Let $\Delta t = 30$ seconds

t (s)	T (K)
0	673
30	587
60	525

	90	479
	120	444
	150	418
	180	397
Let Δt = 26 seconds	206	383
Let Δt = 19 seconds	225	375
	$t \approx 225$ s = 3.75 minutes	

COMMENTS

The elliptical extrusion cools more quickly due to both higher convection heat transfer coefficient and more surface area.

PROBLEM 7.11

> **Calculate the rate of heat loss from a human body at 37°C in an air stream of 5 m/s, 35°C. The body can be modeled as a cylinder 30 cm in diameter, 1.8 m high. Compare your results with those for natural convection from a body and with the typical energy intake from food, 1033 kcal/day (Problem 5.8).**

GIVEN

- Human body modeled as a cylinder in an air stream
- Body surface temperature (T_s) = 37°C
- Air velocity (V_∞) = 5 m/s
- Air temperature (T_∞) = 35°C
- Cylinder diameter (D) = 30 cm = 0.3 m
- Cylinder height (H) = 1.8 m

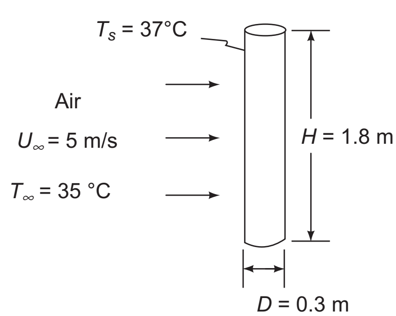
FIND

(a) The heat loss from the idealized human body
(b) Compare with the free convection results of Problem 5.8 and with the typical food consumption rate of 1033 kcal/day

ASSUMPTIONS

- Air velocity is perpendicular to the axis of the cylinder
- Air flow approaching cylinder is laminar
- Heat transfer from the ends can be neglected

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 35°C

Thermal conductivity (k) = 0.0262 W/(m K)

Kinematic viscosity (ν) = 17.1×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At the surface temperature of 37°C Pr_s = 0.71

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(5\,\text{m/s})(0.3\,\text{m})}{\left(17.1\times10^{-6}\,\text{m}^2/\text{s}\right)} = 87{,}719$$

$$\frac{L}{D} = \frac{1.8\,\text{m}}{0.3\,\text{m}} = 6$$

(a) Since $L/D > 4$, its effect on the Nusselt number is negligible and Equation (7.3) may be applied

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\,Re_D{}^m\,Pr^n\left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where $n = 0.37$ and, from Table 7.1: $C = 0.26$ $m = 0.6$

$$\overline{Nu}_D = 0.26\,(87{,}719)^{0.6}\,(0.71)^{0.37}\,(1) = 212$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 212\,\frac{(0.0262\,\text{W/(m K)})}{0.3\,\text{m}} = 18.6\,\text{W/(m}^2\text{K)}$$

The rate of heat transfer is

$$q = \bar{h}_c\,\pi D\,L\,(T_s - T_\infty) = \left(18.6\,\text{W/(m}^2\text{K)}\right)\pi(0.3\text{ M})(1.8\text{ m})(37°\text{C} - 35°\text{C}) = 63.1\text{ W}$$

(b) From Problem 5.8 for natural convection

$$q_{\text{natural}} = 92.2\text{ W}$$

This result is 46% higher than that calculated above. Note that the ambient air temperature in Problem 5.8 is 20°C. The natural convection heat transfer coefficient for that problem was 3.6 W/m^2 K which is only 19% of the value calculated above for forced convection.

The rate of food consumption is

$$\text{Food consumption} = 1033\,\text{kcal/day}(1000\,\text{cal/kcal})\,(4.1868\,\text{J/cal})\left(\frac{1\,\text{day}}{24\,\text{hr}}\right)\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right)((\text{Ws})/\text{J}) = 50.1\text{W}$$

This heat transfer rate is 21% lower than that calculated in part (a).

PROBLEM 7.12

A nuclear reactor fuel rod is a circular cylinder 6 cm in diameter. The rod is to be tested by cooling it with a flow of sodium at 205°C and a velocity of 5 cm/s Perpendicular to its axis. If the rod surface is not to exceed 300°C, estimate the maximum allowable power dissipation in the rod.

GIVEN

- Cylinder in a cross flow of liquid sodium
- Cylinder diameter (D) = 6 cm = 0.06 m
- Sodium temperature (T_∞) = 205°C
- Sodium velocity (U_∞) = 5 cm/s = 0.05 m/s
- Maximum rod surface temperature (T_s) = 300°C

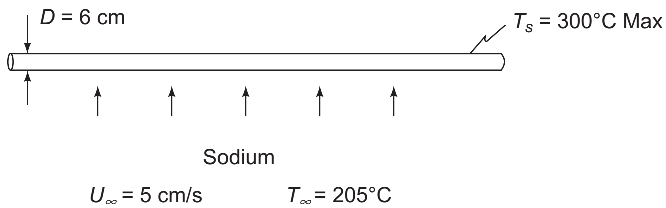
FIND

- The maximum allowable power dissipation $(\dot{q}_G)$

ASSUMPTIONS

- Steady state
- Turbulence in the sodium flow approaching the rod is low
- Heat generation per unit volume in the rod is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for sodium at 205°C

Thermal conductivity (k) = 80.3 W/(m K)

Kinematic viscosity (ν) = 4.6×10^{-7} m^2/s

Prandtl number (Pr) = 0.0072

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(0.05\,\text{m/s})(0.06\,\text{m})}{\left(4.6\times10^{-7}\text{m}^2/\text{s}\right)} = 6522$$

$Re_D\, Pr$ = 6522 (0.0072) = 47.0

Therefore, Equation (7.7) may be applied

$$\overline{Nu}_D = 1.125\,(Re_D\,Pr)^{0.413} = 1.125\,(47.0)^{0.413} = 5.52$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 5.52\,\frac{(80.3\,\text{W/(m K)})}{0.06\,\text{m}} = 7381\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer at the maximum surface temperature is

$$q = \bar{h}_c A_t\,(T_s - T_\infty) = \bar{h}_c\,\pi D L\,(T_s - T_\infty)$$

$$\frac{q}{L} = \left(7381\,\text{W/(m}^2\text{K)}\right)\pi(0.06\text{ m})(1\text{ m})(300°\text{C} - 205°\text{C}) = 1.32 \times 10^5\ \text{W/m}$$

The maximum rate of heat generation per unit volume of the rod is

$$\dot{q}_G = \frac{q}{\text{volume}} = \frac{q}{\frac{\pi}{4}D^2L} = \frac{4}{\pi D^2}\left(\frac{q}{L}\right) = \frac{4}{\pi(0.06\,\text{m})^2}\left(1.32\times10^5\,\text{W/m}\right) = 4.67 \times 10^7\ \text{W/m}^3$$

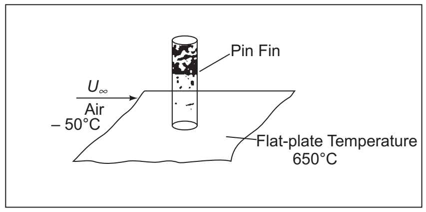
COMMENTS

If the rate of heat generation exceeds the value calculated, the surface temperature will rise to dissipate the energy. Also, nonuniform heat generation can lead to hot spots as will variations in the local value of the heat transfer coefficient around the circumference (see equation (7.2)).

PROBLEM 7.13

A stainless steel pin fin 5 cm long, 6 mm *OD*, extends from a flat plate into a 175 m/s air stream as shown in the accompanying sketch. (a) Estimate the average heat transfer

coefficient between air and the fin. (b) Estimate the temperature at the end of the fin. (c) Estimate the rate of heat flow from the fin.



GIVEN

- A stainless steel pin fin in an air stream
- Pin length (L) = 5 cm = 0.05 m
- Pin diameter (D) = 6 mm = 0.006 m
- Air velocity (U_∞) = 175 m/s

FIND

(a) The average heat transfer coefficient ($\bar{h}_c$)
(b) The temperature of the end of the fin (T_L)
(c) The rate of heat flow from the fin (q_f)

ASSUMPTIONS

- Steady state
- Air approaching the fin has negligible turbulence
- Radiative heat transfer is negligible
- Steel is type 304
- Steel properties are uniform

PROPERTIES AND CONSTANTS

Extrapolating from Appendix 2, Table 27, for dry air at –50°C

Thermal conductivity (k) = 0.0202 W/(m K)

Kinematic viscosity (ν) = 9.3×10^{-6} m²/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 10, for Type 304 stainless steel k_s = 14.4 W/(m K) at 20°C
(Note that figure 1.6 shows very little increase in k for stainless steel in the range of 300°C to 700°C.)

SOLUTION

(a) The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(175\,\text{m/s})(0.006\,\text{m})}{\left(9.3\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1.13 \times 10^5$$

$$\frac{L}{D} = \frac{0.05\,\text{m}}{0.006\,\text{m}} = 8.33 > 4$$

Therefore, Equation (7.3) and Table 7.1 may be used. (Note that $Pr/Pr_s = 1$)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\, Re_D^{\,m}\, Pr^n \qquad c = 0.26 \qquad m = 0.6 \qquad n = 0.37$$

$$\overline{Nu}_D = 0.26\,(1.13 \times 10^5)^{0.6}\,(0.71)^{0.37} = 247$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 247\,\frac{(0.0202\,\text{W/(m K)})}{0.006\,\text{m}} = 829\ \text{W/(m}^2\text{K)}$$

(b) From Table 2.1, for a fin of uniform cross-section with convection at the tip, the temperature distribution is

$$\frac{T - T_\infty}{T_s - T_\infty} = \frac{\cosh[m(L-x)] + \left(\frac{\bar{h}_c}{\text{m k}}\right)\sinh[m(L-x)]}{\cosh(m\,L) + \left(\frac{\bar{h}_c}{\text{m k}}\right)\sinh[m\,L]}$$

where

$$m = \sqrt{\frac{\bar{h}_c P}{k_s A_c}} = \sqrt{\frac{\bar{h}_c \pi D}{k_s \frac{\pi}{4} D^2}} = \sqrt{\frac{4\bar{h}_c}{k_s D}} = \sqrt{\frac{4\left(829\,\text{W/(m}^2\,\text{K)}\right)}{(14.4\,\text{W/(m K)})(0.006\,\text{m})}} = 196.3\ \text{1/m}$$

$$m\,L = 196.3\ \text{1/m}\ (0.05\ \text{m}) = 9.81$$

$$\frac{\bar{h}_c}{\text{m K}} = \frac{\left(829\,\text{W/(m}^2\text{K)}\right)}{(196.3(1/\text{m}))(14.4\,\text{W/(m K)})} = 0.2943$$

At $x = L$

$$\frac{T - T_\infty}{T_s - T_\infty} = \frac{\cosh(0) + 0.2943\sinh(0)}{\cosh(9.81) + 0.2943\sinh(9.81)} = 0.000085$$

$$\therefore \quad T = 0.000085\ (T_s - T_\infty) + T_\infty = 0.000085\ (650°\text{C} - 50°\text{C}) - 50°\text{C} = -\ 49°\text{C}$$

The tip temperature is practically the same as the ambient temperature.

(c) The rate of heat transfer, from Table 2.1 is

$$q_f = M\,\frac{\sinh(m\,L) + \left(\frac{\bar{h}_c}{\text{m k}}\right)\cosh(m\,L)}{\cosh(m\,L) + \left(\frac{\bar{h}_c}{\text{m k}}\right)\sinh(m\,L)}$$

where $M = \sqrt{\bar{h}_c P k_s A_a}\ (T_s - T_\infty) = \sqrt{\bar{h}_c \frac{\pi^2}{4} D^3 k_s}\ \ (T_s - T_\infty)$

$$M = \sqrt{\left(829\,\text{W/(m}^2\text{K)}\right)\frac{\pi^2}{4}(0.006\,\text{m})^3\,(14.4\,\text{W/(m K)})}\ (650°\text{C} + 50°\text{C}) = 55.94$$

$$q_f = 55.94\ \text{W}\ \frac{\sinh(9.81) + 0.2943\cosh(9.81)}{\cosh(9.81) + 0.2943\sinh(9.81)} = 55.9\ \text{W}$$

COMMENTS

These results should be considered an estimate due to uncertainty in the air properties.

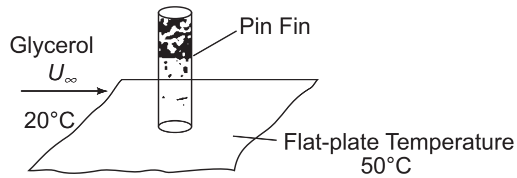
Also, due to the presence of the surface from which the fin protrudes, the flow is not uniform as assumed by Equation (7.3), therefore, the heat transfer coefficient may vary.

PROBLEM 7.14

Repeat Problem 7.13 with glycerol at 20°C flowing over the fin at 2 m/s. The plate temperature is 50°C.

From Problem 7.13: A stainless steel pin fin 5 cm long, 6-mm-*OD*, extends from a flat plate into a 175 m/s glycerol stream as shown in the accompanying sketch.

(a) Estimate the average heat transfer coefficient between glycerol and the fin. (b) Estimate the temperature at the end of the fin. (c) Estimate the rate of heat flow from the fin.



GIVEN

- A stainless steel pin fin in an air stream
- Pin length (L) = 5 cm = 0.05 m
- Pin diameter (D) = 6 mm = 0.006 m
- Glycerol velocity (U_∞) = 2 m/s
- Glycerol temperature (T_∞) = 20°C
- Plate temperature (T_p) = 50°C

FIND

(a) The average heat transfer coefficient ($\bar{h}_c$)
(b) The temperature of the end of the fin (T_L)
(c) The rate of heat flow from the fin (q_f)

ASSUMPTIONS

- Steady state
- Turbulence in the glycerol approaching the fin is low
- Radiative heat transfer is negligible
- Steel is type 304
- Steel properties are uniform
- Variation of the thermal properties of glycerol and steel with temperature is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 21, for glycerol at 20°C

Thermal conductivity (k) = 0.285 W/(m K)

Kinematic viscosity (ν) = 1175 × 10^{-6} m^2/s

Prandtl number (Pr) = 12,609

From Appendix 2, Table 10, for type 304 stainless steel

$$k_s = 14.4 \text{ W/(m K) at } 20°\text{C}$$

SOLUTION

(a) The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(2\,\text{m/s})(0.006\,\text{m})}{(1175\times10^{-6}\text{m}^2/\text{s})} = 10.21$$

Therefore, Equation (7.3) and Table 7.1 may be used. (Note that $Pr/Pr_s = 1$)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\, Re_D^{\,m}\, Pr^n \qquad c = 0.75 \qquad m = 0.4 \qquad n = 0.36$$

$$\overline{Nu}_D = 0.75(10.21)^{0.4}\,(12{,}609)^{0.36} = 56.88$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 56.88\,\frac{(0.285\,\text{W/(m K)})}{0.006\,\text{m}} = 2701\ \text{W/(m}^2\text{K)}$$

(b)

$$m = \sqrt{\frac{4\bar{h}_c}{k_s D}} = \sqrt{\frac{4\left(2701\,\text{W/(m}^2\text{K)}\right)}{(14.4\,\text{W/(m K)})(0.006\,\text{m})}} = 354\ 1/\text{m}$$

$$m\,L = 354\ 1/\text{m}\ \ (0.05\ \text{m}) = 17.7$$

$$\frac{\bar{h}_c}{\text{m K}} = \frac{\left(2701\,\text{W/(m}^2\text{K)}\right)}{354\,1/\text{m}(14.4\,\text{W/(m K)})} = 0.53$$

At $x = L$

$$\frac{T - T_\infty}{T_s - T_\infty} = \frac{\cosh(0) + 0.53\sinh(0)}{\cosh(17.7) + 0.53\sinh(17.7)} = 2.69 \times 10^{-8}$$

Therefore, the tip temperature is practically the same as the ambient qlycerol temperature.

(c) The rate of heat transfer, from Table 2.1 is

$$M = \sqrt{\left(2701\,\text{W/(m}^2\text{K)}\right)\frac{\pi^2}{4}(0.006\,\text{m})^3\,(14.4\,\text{W/(m K)})}\ (50°\text{C} - 20°\text{C}) = 4.32$$

$$q_f = 4.32\ \text{W}\ \frac{\sinh(17.7) + 0.53\cosh(17.7)}{\cosh(17.7) + 0.556\sinh(18.54)} = 4.32\ \text{W}$$

PROBLEM 7.15

Water at 180°C and at 3 m/s enters a bare, 15-m-long, 2.5-cm wrought iron pipe, if air at 10°C flows perpendicular to the pipe at 12 m/s, determine the outlet temperature of the water. (Note that the temperature difference between the air and the water varies along the pipe.)

GIVEN

- Wrought-iron pipe with water flow inside and perpendicular air flow outside
- Water entrance temperature ($T_{W,\text{in}}$) = 180°C
- Water velocity (V_W) = 3 m/s
- Pipe length (L) = 15 m
- Pipe diameter (D) = 2.5 cm = 0.025 m
- Air temperature (T_a) = 10°C
- Air velocity (V_a) = 12 m/s

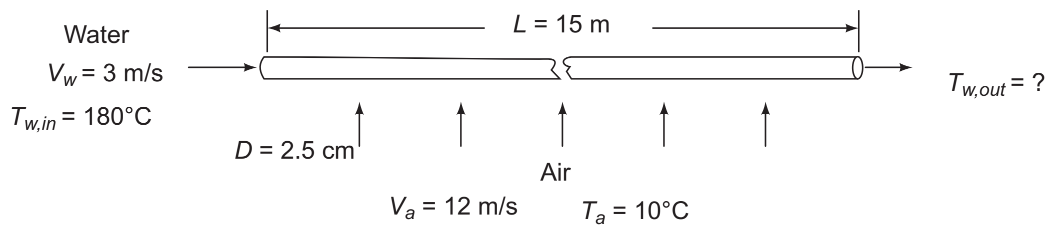
FIND

- Outlet temperature of the water ($T_{W,\text{out}}$)

ASSUMPTIONS

- Steady state
- Air flow approaching pipe is negligible
- Thermal resistance of the pipe is negligible
- The pipe thickness can be neglected

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 10°C

Thermal conductivity (k_a) = 0.0244 W/(m K)

Kinematic viscosity (ν_a) = 17.8×10^{-6} m^2/s

Prandtl number (Pr_a) = 0.71

From Appendix 2, Table 13, for water at the entrance temperature of 180°C

Thermal conductivity (k_w) = 0.673 W/(m K)

Kinematic viscosity (ν_w) = 0.173×10^{-6} m^2/s

Prandtl number (Pr_w) = 1.01

Density (ρ_w) = 886.6 kg/m^2

Specific Heat (c) = 4396 J/(kg K)

SOLUTION

Air Side:

The Reynolds number on the air side is

$$(Re_D)_{\text{air}} = \frac{V_a D}{\nu_a} = \frac{(12\,\text{m/s})(0.025\,\text{m})}{(17.8\times10^{-6}\,\text{m}^2/\text{s})} = 16{,}853$$

The Nusselt number is given by Equation 7.3 and Table 7.1

$$\left(\overline{Nu}_D\right)_{\text{air}} = \frac{\left(\bar{h}_c\right)_{\text{air}} D}{k_a} = C\, Re_D^{\,m}\, Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where C = 0.26, m = 0.6, and n = 0.37.

Note that the Prandtl number of air does not change appreciably between the air and water temperatures. Therefore, $Pr/Pr_s = 1$.

$$\left(\overline{Nu}_D\right)_{\text{air}} = 0.26\,(16{,}853)^{0.6}\,(0.71)^{0.36} = 78.7$$

$$\left(\bar{h}_c\right)_{\text{air}} = \left(\overline{Nu}_D\right)_{\text{air}} \frac{k_a}{D} = 78.7\,\frac{(0.0244\,\text{W/(m K)})}{0.025\,\text{m}} = 76.8\ \text{W/(m}^2\text{K)}$$

Water Side:

The Reynolds number based on the inlet properties is

$$Re_D = \frac{V_w D}{\nu_w} = \frac{(3\,\text{m/s})(0.025\,\text{m})}{(0.173\times10^{-6}\,\text{m}^2/\text{s})} = 4.33 \times 10^5 \text{ (Turbulent)}$$

Applying Equation (6.63)

$$\left(\overline{Nu}_D\right)_{\text{water}} = \frac{\left(\bar{h}_c\right)_{\text{water}} D}{k_w} = 0.023\; Re_D^{\,0.8}\; Pr_n \qquad n = 0.3 \text{ for cooling}$$

$$\left(\overline{Nu}_D\right)_{\text{water}} = 0.023\;(4.33 \times 10^5)^{0.8}\;(1.01)^{0.3} = 746$$

$$\left(\bar{h}_c\right)_{\text{water}} = \left(\overline{Nu}_D\right)_{\text{water}} \frac{k_w}{D} = 746\;\frac{(0.673\,\text{W/(m K)})}{0.025\,\text{m}} = 20{,}078\;\;\text{W/(m}^2\text{K)}$$

The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{\left(\bar{h}_c\right)_{\text{air}}} + \frac{1}{\left(\bar{h}_c\right)_{\text{water}}} = \frac{1}{(76.8\,\text{W/(m}^2\text{K))}} + \frac{1}{(20{,}078\,\text{W/(m}^2\text{K))}} = 0.0131\;\;(\text{m}^2\text{K})/\text{W}$$

$$U = 76.6\,\text{W/(m}^2\text{K)}$$

Let's assume that the water temperature changes little from the pipe inlet to outlet. Since the air temperature is constant and uniform, the heat transfer from the water is then analogous to the uniform surface temperature analysis of Section 6.2.2 and Equation (6.36) may be applied

$$\frac{T_{w,\text{out}} - T_a}{T_{w,\text{in}} - T_a} = \exp\left(-\frac{UPL}{\dot{m}\,c}\right) = \exp\left(-\frac{U\pi DL}{\frac{\pi}{4} V_w D^2 \rho_w c}\right) = \exp\left(-\frac{4UL}{V_w D \rho_w c}\right)$$

Solving for the water outlet temperature

$$T_{w,\text{out}} = T_a + (T_{w,\text{in}} - T_a)\exp\left(-\frac{4UL}{V_w D \rho_w c}\right)$$

$$T_{w,\text{out}} = 10°\text{C} + (180°\text{C} - 10°\text{C})\exp\left[-\frac{4(76.6\,\text{W/(m}^2\text{K)})(15\,\text{m})}{(3\,\text{m/s})(0.025\,\text{m})(886.6\,\text{kg/m}^3)(4396\,\text{J/(kg K)})((\text{Ws})/\text{J})}\right]$$

$$T_{w,\text{out}} = 177°\text{C}$$

Therefore, the assumption that the water changes little from pipe inlet to outlet is valid.

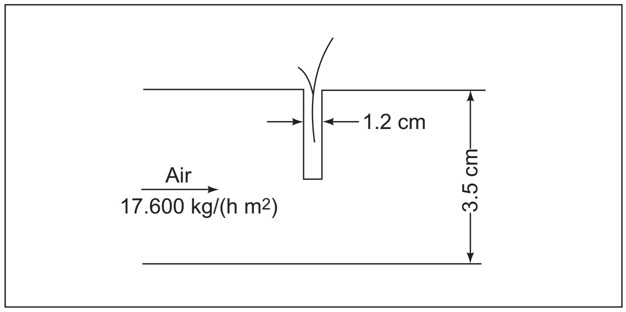
COMMENTS

The average water temperature is 178.5°C. This is not different enough from the inlet temperature to justify another iteration using the water properties at the average water temperature.

Note that the convective thermal resistance of the air is 99.6% of the total thermal resistance.

PROBLEM 7.16

> **The temperature of air flowing through a 25-cm-diameter duct whose inner walls are at 320°C is to be measured with a thermocouple soldered in a cylindrical steel wall of 1.2 cm *OD*, whose exterior is oxidized as shown in the accompanying sketch. The air flows normal to the cylinder at a mass velocity of 17,600 kg/(h m^2). If the temperature indicated by the thermocouple is 200°C, estimate the actual temperature of the air.**



GIVEN

- Cylindrical thermocouple wall in an air duct
- Duct diameter (D_d) = 25 cm = 0.25 m
- Duct wall temperature (T_{ds}) = 320°C =m 593 K
- Wall outside diameter (D_w) = 1.2 cm = 0.012 m
- Exterior of wall is oxidized
- Air mass velocity $(\dot{m}/A)$ = 17,600 kg/(h m^2)
- Thermocouple indicated temperature (T_{tc}) = 200°C = 473 K

FIND

- Air temperature (T_∞)

ASSUMPTIONS

- Steady state
- Thermal resistance between the thermocouple and the wall exterior surface is negligible
- Inside of duct behaves as a black body enclosure
- Conduction to the thermocouple wall from the duct wall can be neglected

PROPERTIES AND CONSTANTS

From Appendix 2, Table 7, the emissivity of oxidized steel (ε) = 0.94.
From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

An iterative solution must be used since the rate of heat transfer will depend on the air properties which are a function of the unknown air temperature. Heat is transferred by radiation from the duct wall to the thermocouple wall and from the thermocouple wall to the air. Therefore, the air temperature will be lower then the thermocouple reading. The rate of heat transfer from the wall to the thermocouple must equal that from the thermocouple to the air

$$\bar{h}_c A\,(T_{tc} - T_a) = \sigma\,\varepsilon\,A\,(T_{ds}^{\;4} - T_{tc}^{\;4})$$

Solving for the air temperature

$$T_a = T_{tc} - \frac{\sigma\,\varepsilon}{\bar{h}_c}\,(T_{ds}^{\;4} - T_{tc}^{\;4})$$

For the first iteration, let T_a = 150°C. From Appendix 2, Table 27, for air at 150°C

Density (ρ) = 0.820 kg/m^3

Thermal conductivity (k) = 0.0339 W/(m K)

Kinematic viscosity (ν) = 29.6×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At the wall temperature of 200°C Pr_s = 0.71

The air velocity (U_∞) is

$$U_\infty = \frac{\text{m}}{A\rho} = \frac{\left(17600\,\text{kg/(h m}^2)\right)}{\left(0.820\,\text{kg/m}^3\right)(3600\,\text{s/h})} = 5.96\ \text{m/s}$$

The Reynolds number based on the well diameter is

$$Re_D = \frac{U_\infty D_w}{\nu} = \frac{(5.96\,\text{m/s})(0.012\,\text{m})}{\left(29.6\times10^{-6}\,\text{m}^2/\text{s}\right)} = 2417$$

The Nusselt number is given by Equation (7.3) and Table 7.1

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\, Re_D^{\,m}\, Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25} \quad \text{where } C = 0.026 \quad m = 0.6 \quad n = 0.37$$

$$\overline{Nu}_D = 0.26\ (2417)^{0.6}\ (0.71)^{0.36}\ (1) = 24.54$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 24.54\ \frac{(0.0339\,\text{W/(m K)})}{0.012\,\text{m}} = 69.3\ \text{W/(m}^2\text{K)}$$

The air temperature is

$$T_a = 200°\text{C} - \frac{\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)(0.94)}{\left(69.3\,\text{W/(m}^2\text{K)}\right)}\ [(593\ \text{K})^4 - (473\ \text{K})^4] = 143°\text{C}$$

The original guess for T_a is close to the above value. Another iteration using air properties at 143°C would not significantly improve the result.

PROBLEM 7.17

Develop an expression for the ratio of the rate of heat transfer to water at 40°C from a thin flat strip of width $\pi D/2$ and length L at zero angle of attack and a tube of the same length and diameter D in cross-flow with its axis normal to the water flow in the Reynolds number range between 50 and 1000. Assume both surface are at 90°C.

GIVEN

- Water flowing over a thin flat strip at zero angle of attack or a tube in crossflow
- Water temperature (T_∞) = 40°C
- Tube diameter = D
- Strip width = $\pi D/2$
- Tube and strip length = L
- Reynolds number: $50 < Re < 1000$

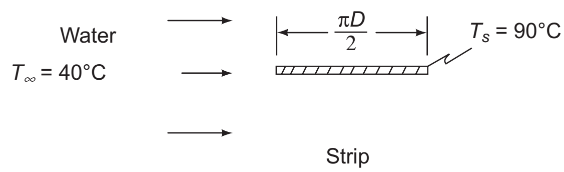
FIND

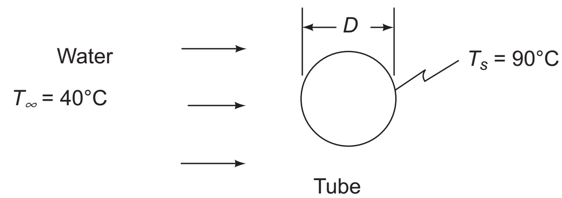
- The ratio of the heat transfer from the strip and that from the cylinder. (q_s/q_t)

ASSUMPTIONS

- Steady state for both cases
- The tube and strip temperatures (T_s) are 90°C

SKETCH





PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 40°C: Prandtl number (Pr) = 4.3
At the surface temperature of 90°C: Pr_s = 1.94

SOLUTION

Note that the heat transfer area (πD) is the same in both cases.

Thin Strip:

The flow over the thin strip is laminar for the Reynolds number given. The Nusselt number is given by Equation (4.38)

$$\overline{Nu_L} = 0.664\ Re_L^{\frac{1}{2}}\ Pr^{\frac{1}{3}}$$

Tube:

The Nusselt number for the tube is given by Equation (7.3) and Table 7.1

$$\overline{Nu_D} = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

Re_D	C	m
1 – 40	0.75	0.4
$40 - 1 \times 10^3$	0.51	0.5
$1 \times 10^3 - 2 \times 10^5$	0.26	0.6
$2 \times 10^5 - 1 \times 10^6$	0.076	0.7

Since the transfer areas and temperature differences are the same, the ratio of the rates of heat transfer is equal to the ratio of the heat transfer coefficients. The heat transfer rate from the strip will be 64% of that from the tube with the same Reynolds number.

PROBLEM 7.18

Repeat Problem 7.17 for air flowing over the same two surfaces in the Reynolds number range between 40,000 and 200,000. Neglect radiation.

From Problem 7.17: Develop an expression for the ratio of the rate of heat transfer to air at 40°C from a thin flat strip of width $\pi D/2$ and length L at zero angle of attack and a tube of the same length and diameter D in cross-flow with its axis normal to the flow. Assume both surfaces are at 90°C.

GIVEN

- Air flowing over a thin flat strip at zero angle of attack or a tube in crossflow
- Air temperature (T_∞) = 40°C

- Tube diameter = D
- Strip width = $\pi D/2$
- Tube and strip length = L
- Reynolds number : 40,000 < Re < 200,000

FIND

- The ratio of the heat transfer from the strip and that from the cylinder. (q_s/q_t)

ASSUMPTIONS

- Radiative heat transfer is negligible
- Steady state for both cases
- The tube and strip temperatures (T_s) are 90°C

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 40°C

Kinematic viscosity (ν) = 17.6×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

at 90°C Pr_s = 0.71

SOLUTION

This solution follows the same procedure as the solution to Problem 7.17

Applying Equation (4.38)

$$\overline{Nu}_L = 0.664\ Re_L^{\frac{1}{2}}\ Pr^{\frac{1}{3}}$$

For the tube, from Equation (7.3) and Table 7.1

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

Re_D	C	m
1 – 40	0.75	0.4
40 – 1×10^3	0.51	0.5
1×10^3 – 2×10^5	0.26	0.6
2×10^5 – 1×10^6	0.076	0.7

but $Pr = Pr_s$. The heat transfer rate from the strip will be 165% of that from the tube with the same Reynolds number.

PROBLEM 7.19

The instruction manual for a hot-wire anemometer states that 'roughly speaking, the current varies as the one-fourth power of the average velocity at a fixed wire resistance'. Check this statement, using the heat transfer characteristics of thin wire in air and water.

GIVEN

- A thin current carrying wire in an air or water stream

FIND

- Show that the current (I) varies as the one-fourth power of the fluid velocity (V_∞) at a fixed resistance (Re_I)

ASSUMPTIONS

- Radiative heat transfer is negligible

SOLUTION

This solution follows the same procedure as the solution to Problem 7.17

Holding the wire resistance constant has the effect of holding the wire temperature (T_s) and therefore, the fluid properties constant. T_s, T_b, A_t, and Re_I are constant.

According to Equation (7.3)

$$\overline{Nu_D} = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

for 40 < *Re* < 1000 m = 0.5 → *I a U*∞1/4

Since the wire diameter is typically a few microns, we expect the Reynolds number to be very low. Therefore, from Table 7.1 m = 0.4 to 0.5 and m/2 = 0.2 to 0.25

Re_D	C	m
1 – 40	0.75	0.4
40 – 1 × 10^3	0.51	0.5
1 × 10^3 – 2 × 10^5	0.26	0.6
2 × 10^5 – 1 × 10^6	0.076	0.7

PROBLEM 7.20

A hot-wire anemometer is used to determine the boundary layer velocity profile in the air flow over a scale model of an automobile. The hot-wire is held in a traversing mechanism that moves the wire in a direction normal to the surface of the model. The hot-wire is operated at constant temperature. The boundary layer thickness is to be defined as the distance from the model surface at which the velocity is 90% of the free stream. If the probe current is low when the hot-wire is held in the free stream velocity, U_∞, What current will indicate the edge of the boundary layer? Neglect radiation heat transfer from the hot-wire and conduction from the ends of the wire.

GIVEN

- Thin, electrically-heated constant-temperature wire in air flow near an automobile model
- Boundary layer thickness ° point when velocity (U_y) = 90% free stream velocity V_o
- Probe current at $U_\infty = I_o$

FIND

- Probe current at edge of the boundary layer (I_b)

ASSUMPTIONS

- Radiation is negligible
- Conduction from the ends of the hot-wire is negligible
- Reynolds number is small

SOLUTION

Restating the desire result: What is the current of $V = 0.9\ V_\infty$ in terms of the current I_o at V_o? Since the diameter of the wire will be very small, the Reynolds number will be small. For 1 < *Re* < 40 the Nusselt number, use Equation (7.3) and Table 7.1

$$\overline{Nu_D} = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

Re_D	C	m
1 – 40	0.75	0.4
40 – 1 × 10^3	0.51	0.5
1 × 10^3 – 2 × 10^5	0.26	0.6
2 × 10^5 – 1 × 10^6	0.076	0.7

U_y is the air velocity at a distance y from the model surface.

The rate of electrical energy dissipation must equal the rate of convective heat transfer. The electrical resistance of the wire (Re_I) is a function of the wire temperature only and is therefore constant in this case.

For $U_y = 0.9\ U_\infty$

$$I = I_o\ (0.9)0.2$$

$$I = 0.979\ I_o$$

The current will be 0.979 I_o at the edge of the boundary layer.

PROBLEM 7.21

A platinum hot-wire anemometer operated in the constant-temperature mode has been used to measure the velocity of a helium stream. The wire diameter is 20 m, its length is 5 mm, and it is operated at 90°C. The electronic circuit used to maintain the wire temperature has a maximum power output of 5 watts and is unable to accurately control the wire temperature if the voltage applied to the wire is less than 0.5 volt. Compare the operation of the wire in the helium stream at 20°C and 10 m/s with operation in air and water at the same temperature and velocity. The electrical resistance of the platinum at 90°C is 21.6 W-cm.

GIVEN

- A constant temperature platinum hot-wire in a stream of helium
- Wire diameter = 20 m = 20×10^{-6} m
- Wire length (L) = 5 mm = 0.005 m
- Wire temperature (T_w) = 90°C
- Maximum electric power to wire ($P_{\max}$) = 5 W
- Minimum voltage ($V_{\min}$) = 0.5 V
- Helium temperature (T_∞) = 20°C
- Helium velocity (U_∞) = 10 m/s
- Resistivity (r_e) = 21.6 W cm = 21.6×10^{-8} W m

FIND

- Compare the operation of the wire in helium to that in air and water

ASSUMPTIONS

- Radiation is negligible

PROPERTIES AND CONSTANTS

From Appendix 2

Fluid	Helium	Air	Water
Table number	30	27	13
Thermal conductivity at 20°C, k (W/(mK))	0.1471	0.0251	0.597
Kinematic viscosity at 20°C, $\times 10^6$ (m^2/s)	122.2	15.7	1.006
Prandtl number at 20°C, Pr	0.70	0.71	7.0
Prandtl number at 90°C, Pr_s	0.71	0.71	1.94

SOLUTION

The Nusselt number is given by Equation (7.3) and Table 7.1

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

Re_D	C	m

1 – 40	0.75	0.4
$40 - 1 \times 10^3$	0.51	0.5
$1 \times 10^3 - 2 \times 10^5$	0.26	0.6
$2 \times 10^5 - 1 \times 10^6$	0.076	0.7

Helium: $C = 0.75$ $m = 0.4$ $n = 0.37$

Air: $C = 0.75$ $m = 0.4$ $n = 0.37$

Water: $C = 0.51$ $m = 0.5$ $n = 0.37$

The rate of convective heat transfer must equal the electrical power dissipated. Therefore, the power capabilities of the unit are sufficient for these conditions in helium and air but not in water. The voltage for the case with air is too low for the device. Therefore, the device will perform adequately only for the helium flow under these conditions.

PROBLEM 7.22

A hot-wire anemometer consists of a 5 m diameter platinum wire, 5 mm long.

The probe is operated at constant current of 0.03 amp. The electrical resistivity of platinum is 17 W cm at 20°C and increases by 0.385% per °C.

(a) If the voltage across the wire is 1.75 Volts, determine the velocity of the air flowing across it and the wire temperature if the free-stream air temperature is 20°C.

(b) What is the wire temperature and voltage if the air velocity is 10 m/s?

Neglect radiation and conduction heat transfer from the wire.

GIVEN

- A hot wire in air
- Wire diameter (D) = 5 m = 5×10^{-6} m
- Wire length (L) 5 mm = 0.005 m
- Current (I) = 0.03 A (constant)
- Electrical resistivity (r_{el}) = 17 W cm = 17×10^{-8} W m at 20°C and increases 0.385% per °C.
- Air temperature (T_∞) = 20°C

FIND

(a) The air velocity (U_∞) and the wire temperature (T_w) if the voltage across the wire (V_{el}) =1.75V

(b) The wire temperature (T_w) and voltage (V_{el}) if the air velocity (U_∞) = 10 m/s

ASSUMPTIONS

- Radiative heat transfer is negligible
- Variation of Prandtl number with temperature id negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 20°C

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At 90°C: $Pr = 0.71$

SOLUTION

The electrical resistivity of the wire as a function of temperature is

$$\rho_{el} = \rho_{el,20}\,[1 + 0.00385\,(T_w - 20°\text{C})] \qquad (T_w \text{ in °C})$$

The electrical resistance of the wire is

$$R_{el} = \frac{\rho_{el} L}{A_c} = \frac{4\rho_e L}{\pi D^2} = \frac{4(0.005\,\text{m})}{\pi(5\times10^{-6}\,\text{m})^2}[17 \times 10^{-8}\ \Omega\text{m}]\ [1 + 0.00385\ (T_w - 20°\text{C})]$$

$$R_{el} = 43.29\ \Omega\ [1 + 0.00385\ (T_w - 20°\text{C})]$$

(a) The voltage across the wire is given by

$$V_{el} = IR_{el} = I(43.29\ \Omega)\ [1 + 0.00385\ (T_w - 20°\text{C})]$$

Solving for the wire temperature

$$T_w = \frac{1}{0.00385}\left[\frac{V_{el}}{I(43.29\,\Omega)} - 1\right] + 20°\text{C}$$

$$= \frac{1}{0.00385}\left[\frac{1.75\ \text{volt}}{0.03\,\text{A}(43.29\,\Omega)} - 1\right] + 20°\text{C} = 110°\text{C}$$

The rate of convective heat transfer for the wire must equal the electrical power dissipated.

$$\bar{h}_c\, \pi D\, L\ (T_w - T_\infty) = V_{el} I$$

$$\bar{h}_c = \frac{V_{el} I}{\pi D L (T_w - T_\infty)} = \frac{1.75\,\text{volt}\,(0.03\,\text{A})(\text{W}/(\text{volt A}))}{\pi(5\times10^{-6}\,\text{m})(0.005\,\text{m})(110°\text{C} - 20°\text{C})} = 7427\ \text{W}/(\text{m}^2\text{K})$$

Assuming that $1 < Re < 40$, and neglecting variation of Prandtl number, Equation (7.3) and Table 7.1 give the heat transfer coefficient as

$$\bar{h}_c = 0.75\frac{k}{D}\left(\frac{U_\infty D}{\nu}\right)^{0.4} Pr^{0.37}$$

Solving for the air velocity

$$U_\infty = \left[\frac{D\bar{h}_c}{0.75\,k} Pr^{-0.37}\left(\frac{\nu}{D}\right)^{0.4}\right]^{2.5}$$

$$U_\infty = \left[\frac{(5\times10^{-6}\,\text{m})(7427\,\text{W}/(\text{m}^2\text{K}))}{0.75(0.0251\,\text{W}/(\text{m K}))}(0.71)^{-0.37}\left(\frac{15.7\times10^{-6}\,\text{m}^2/\text{s}}{5\times10^{-6}\,\text{m}}\right)^{0.4}\right]^{2.5} = 23.6\ \text{m/s}$$

Note that $Re_D = \frac{U_\infty D}{\nu} = \frac{(23.6\,\text{m/s})(5\times10^{-6}\,\text{m})}{(15.7\times10^{-6}\,\text{m}^2/\text{s})} = 7.5$ which is in the assumed range.

(b) The Reynolds number at $U_\infty = 10$ m/s is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(10\,\text{m/s})(5\times10^{-6}\,\text{m})}{(15.7\times10^{-6}\,\text{m}^2/\text{s})} = 3.18$$

$$\overline{Nu}_D = 0.75\ (3.18)^{0.4}\ (0.71)^{0.37} = 1.05$$

$$\bar{h}_c = \overline{Nu}_D\frac{k}{D} = 1.05\frac{(0.0251\,\text{W}/(\text{m K}))}{5\times10^{-6}\,\text{m}} = 5271\ \text{W}/(\text{m}^2\text{K})$$

Balancing the rate of heat transfer and electrical power dissipation

$$\bar{h}_c\, \pi D\, L\ (T_w - T_\infty) = I^2 R_{el} = I^2\ (43.29\ \Omega)\ [1 + 0.00385\ (T_w - 20°\text{C})]$$

$$(5271\,\text{W}/(\text{m}^2\text{K}))\ \pi(5\times10^{-6}\ \text{m})\ (0.005\text{m})\ (T_w - 20°\text{C}) = (0.03\ \text{A})^2\ (43.29\ \Omega)\ [1 + 0.00385\ (T_w - 20°\text{C})]$$

$$(0.000414\ \text{W/K})\ (T_w - 20°\text{C}) = (0.0390\ \text{W})\ [1 + 0.00385\ (T_w - 20°\text{C})]$$

By trial and error T_w = 168°C

$$V_{el} = IR_{el} = (0.03\text{A})\ (43.29\ \Omega)\ [1 + 0.00385(168°\text{C} - 20°\text{C})] = 2.04\ \text{Volts}$$

COMMENTS

Some heat transfer correlations require that air properties be evaluated at the film temperature. This type of correlation would make the calculation of velocity from a given voltage much more difficult since the film temperature changes with the wire temperature. In this case, operation in the constant temperature mode is much simpler because the film temperature is fixed.

PROBLEM 7.23

A 2.5 cm sphere is maintained at 50°C in an air stream or a water stream, both at 20°C and 2 m/s velocity. Compare the rate of heat transfer and the drag on the sphere for both fluids.

GIVEN

- A sphere in an air stream or a water stream
- Sphere diameter (D) = 2.5 cm = 0.025 m
- Sphere temperature (T_s) = 50°C
- Fluid temperature (T_f) = 20°C
- Fluid velocity (U_∞) = 2 m/s

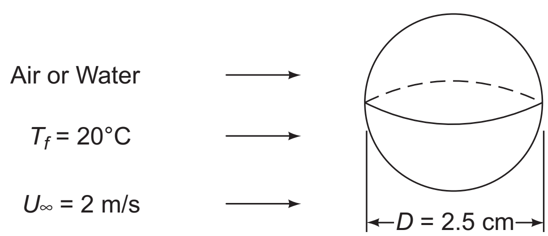
FIND

- The rate of heat transfer (q) and the drag force

ASSUMPTIONS

- Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2

Fluid	Air	Water
Table Number	27	13
Density at 20°C, ρ (kg/m^3)	1.164	998.2
Thermal conductivity at 20°C, k (W/(m K))	0.0251	0.597
Kinematic Viscosity at 20°C, $\nu \times 10^6$ (m^2/s)	15.7	1.006
Prandtl number at 20°C, Pr	0.71	7.0
Absolute viscosity at 20°C, $\mu_\infty \times 10^6$ $((\text{Ns})/\text{m}^2)$	18.240	993
Absolute viscosity at 50°C, $\mu_\infty \times 10^6$ $((\text{Ns})/\text{m}^2)$	19.515	555.1

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu}$$

For air

$$Re_D = \frac{(2\,\text{m/s})(0.025\,\text{m})}{(15.7\times10^{-6}\,\text{m}^2/\text{s})} = 3185$$

For water

$$Re_D = \frac{(2\,\text{m/s})(0.025\,\text{m})}{(1.006\times10^{-6}\,\text{m}^2/\text{s})} = 49{,}702$$

Equation (7.11) can be applied to both cases

$$Nu_D = 2 + (0.4\,Re_D^{0.5} + 0.06\,Re_D^{0.67})\,Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{0.25}$$

For air

$$Nu_D = 2 + (4.0\,(3185)^{0.5} + 0.06\,(3185)^{0.67})\,(0.71)^{0.4}\left(\frac{18.240}{19.515}\right)^{0.25} = 32.8$$

$$h_c = Nu_D\,\frac{k}{D} = 32.8\,\frac{(0.0251\,\text{W/(m K)})}{0.025\,\text{m}} = 32.9\ \text{W/(m}^2\text{K)}$$

For water

$$Nu_D = 2 + (0.4\,(49{,}702)^{0.5} + 0.06\,(49{,}702)^{0.7})\,(7.0)^{0.4}\left(\frac{993}{555.1}\right)^{0.25} = 438$$

$$h_c = Nu_D\,\frac{k}{D} = 438\,\frac{(0.597\,\text{W/(m K)})}{0.025\,\text{m}} = 10{,}469\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer is

$$q = h_c\,A\,\Delta T = h_c\,\pi D^2\,(T_s - T_\infty)$$

For air

$$q = (32.9\,\text{W/(m}^2\text{K)})\,\pi(0.025\text{ m})^2\,(50°\text{C} - 20°\text{C}) = 1.9\text{ W}$$

For water

$$q = (10{,}469\,\text{W/(m}^2\text{K)})\,\pi(0.025\text{ m})^2\,(50°\text{C} - 20°\text{C}) = 617\text{ W}$$

The total drag coefficient can be read from Figure 7.7 and is defined in Section 7.2 as

$$C_D = \frac{\text{Drag force}}{\left(\frac{\rho U_\infty^2}{2}\right)\left(\frac{\pi D^2}{4}\right)} \Rightarrow \text{Drag force} = \frac{1}{8}\,C_D\,\rho\,U_\infty^2\,\pi D^2$$

For air, From Figure 7.7, $C_D = 0.4$

$$\text{Drag force} = \frac{1}{8}(0.4)\,(1.164\,\text{kg/m}^3)\,(2\,\text{m/s})^2\,\pi(0.025\text{ m})^2\,((\text{Ns}^2)/(\text{kg m})) = 0.00046\text{ N}$$

For water, From Figure 7.7, $C_D = 0.5$

$$\text{Drag force} = \frac{1}{8}\ (0.5)\ \left(998.2\,\text{kg/m}^3\right)\ (2\,\text{m/s})^2\ \pi(0.025\text{m})^2\ \left((\text{Ns}^2)/(\text{kg m})\right) = 0.49\ \text{N}$$

COMMENTS

Note that the heat transfer increases by a factor of 324 in water while the drag force increases by a factor of 1065.

PROBLEM 7.24

Compare the effect of forced convection on heat transfer from an incandescent lamp, Problem 5.27. What will the glass temperature be for air velocities of 0.5, 1, 2, and 4 m/s?

From Problem 5.27: Only ten percent of the energy dissipated by the tungsten filament of an incandescent lamp is in the form of useful visible light. Consider a 100 W lamp with a 10 cm spherical glass bulb. Assuming an emissivity of 0.85 for the glass and ambient air temperature of 20°C, what is the temperature of the glass bulb?

GIVEN

- A spherical glass light bulb in air
- Bulb power consumption (P) = 100 W
- 10% of energy is in the form of visible light
- Diameter (D) = 10 cm = 0.1 m
- Bulb emissivity (ε) = 0.85
- Ambient air temperature (T_∞) = 20°C = 293 K

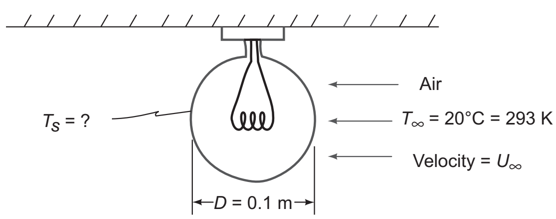
FIND

- The glass temperature (T_s) for air velocities (U_∞) of 0.5, 1, 2, and 4 m/s

ASSUMPTIONS

- The bulb has reached steady state
- The surrounding behave as a black body at T_∞

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4).

From Appendix 2, Table 27, for dry air at 20°C

Thermal conductivity (k) = 0.025 W/(m K)

Kinematic viscosity (ν) = 15.7×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu}$$

For $\quad U_\infty = 0.5$ m/s $\qquad Re_d = \dfrac{(0.5\,\text{m/s})(0.1\,\text{m})}{\left(15.7\times10^{-6}\,\text{m}^2/\text{s}\right)} = 3185$

The Nusselt number is given by Equation (7.9)

$$Nu_D = 0.37\, Re_D^{0.6}$$

$$h_c = Nu_D \frac{k}{D} = 0.37\, \frac{k}{D}\, Re_D^{0.6}$$

For $U_\infty = 0.5 \frac{\text{m}}{\text{s}}$

$$h_c = 0.37\, \frac{(0.0251\,\text{W/(m K)})}{0.1\,\text{m}}\, (3185)^{0.6} = 11.74\ \text{W/(m}^2\text{K)}$$

The rate of convective and radiative heat loss must equal the rate of heat generation

$$q_c + q_r = \pi D^2\, [h_c\, (T_s - T_\infty) + \varepsilon\, \sigma (T_s^4 - T_\infty^4))] = 0.9\ (100\ \text{W}) = 90\ \text{W}$$

For $U_\infty = 0.5$ m/s

$$q_c + q_r = \pi (0.1)^2 \left[\left(11.74\,\text{W/(m}^2\text{K)}\right)\left(T_s - 293\,\text{K}\right) + 0.85\left(5.67\times 10^{-8}\,\text{W/(m}^2\text{K}^4)\right)\left[T_s^4 - (293\,\text{K})^4\right]\right] = 90\ \text{W}$$

Checking the units, then eliminating them for clarity

$$1.514 \times 10^{-9}\, T_s^4 + 0.3688\, T_s - 209.2 = 0$$

By trial and error: $T_s = 429$ K = 156°C

Following the same procedure for the other air velocities yields the following results

Velocity, U_∞ (m/s)	Heat transfer coefficient, h_c (W/m^2 K))	Glass Temperature, T_s (°C)
0.5	11.74	141
1.0	17.79	130
2.0	26.96	104
4.0	40.87	81

PROBLEM 7.25

An experiment was conducted in which the heat transfer from a sphere in sodium was measured. The sphere, 1.27 cm in diameter was pulled through a large sodium bath at a given velocity while an electrical heater inside the sphere maintains the temperature at a set point. The following table gives the results of the experiment

Run #	**1**	**2**	**3**	**4**	**5**
Velocity (m/s)	**3.44**	**3.14**	**1.56**	**3.44**	**2.16**
Sphere Surface Temp (°C)	**478**	**434**	**381**	**350**	**357**
Sodium Bath Temp (°C)	**300**	**300**	**300**	**200**	**200**
Heater Temp (°C)	**486**	**439**	**385**	**357**	**371**
Heat Flux × 10^{-6} (W/m^2)	**14.6**	**8.94**	**3.81**	**11.7**	**8.15**

Determine how well the above data is predicated by the appropriate correlation given in the text. Express your results in terms of the percent difference between the experimentally determined Nusselt number and that from the equation.

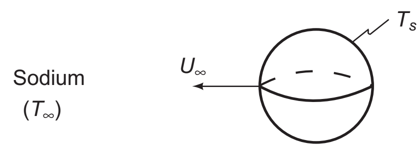
GIVEN

- A sphere is pulled through a sodium bath at a given velocity
- Sphere diameter = 1.27 cm = 0.0127 m
- Sphere temperature is kept constant
- Experimental data given above

FIND

- The standard deviation between the data and the appropriate correlation

SKETCH



SOLUTION

The Correlation of the heat transfer rate will be illustrated with Run #1. The film temperature (T_f) = (T_s + T_∞)/2 = (478°C + 300°C)/2 = 389°C.

From Appendix 2, Table 26, for sodium at 389°C

Thermal conductivity (k) = 71.6 W/(m K)

Kinematic viscosity (ν) = 3.08×10^{-7} m²/s

Prandtl number (Pr) = 0.0050

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(3.44\,\text{m/s})(0.0127\,\text{m})}{\left(2.75\times10^{-7}\,\text{m}^2/\text{s}\right)} = 1.42 \times 10^5$$

The Nusselt number for spheres in liquid metals for $3.6 \times 10^4 < Re_D < 2 \times 10^5$ is given by Equation (7.14)

$$\overline{Nu}_D = 2 + 0.386\,(RePr)^{\frac{1}{2}} = 2 + 0.386\left[1.42\times10^5(0.005)\right]^{\frac{1}{2}} = 12.28$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 12.28\,\frac{(71.6\,\text{W/(m K)})}{0.0127\,\text{m}} = 69{,}230\ \text{W/(m}^2\text{K)}$$

The heat flux from the sphere is

$$\frac{q}{A} = \bar{h}_c\,(T_s - T_\infty) = \left(69{,}230\,\text{W/(m}^2\,\text{K)}\right)\ (478°\text{C} - 300°\text{C}) = 1.23 \times 10^7\,\text{W/m}^2$$

Similarly for the other test runs

Run #	1	2	3	4	5
Film Temp. (°C)	389	367	341	275	279
k (W/(m K))	71.6	72.6	73.8	77.0	76.8
$\nu \times 10^7$ (m²/s)	3.08	3.19	3.42	3.99	3.96
Pr	0.0050	0.0052	0.0055	0.0063	0.0063
$Re_D \times 10^{-5}$	1.42	1.25	0.579	1.09	0.693
$\bar{h}_c$ (W/(m² K))	69,230	67,690	51,649	73,463	60,868
$q/A \times 10^{-6}$ (W/m²)	12.3	9.07	4.18	11.0	9.56
exper. $q/A \times 10^{-6}$ (W/m²)	14.6	8.94	3.18	11.7	8.15
Percent difference (%)	–15.8	+1.5	+9.7	–5.9	–17.3

PROBLEM 7.26

A copper sphere initially at a uniform temperature of 132°C is suddenly released at the bottom of a large bath of bismuth at 500°C. The sphere diameter is 1 cm and it rises through the bath at 1 m/s. How far will the sphere rise before its center temperature is 300°C? What is its surface temperature at that point? (The sphere has a thin nickel plating to protect the copper from the bismuth.)

GIVEN

- A copper sphere with a thin nickel plating rising through a bath of bismuth
- Initial copper temperature (T_o) = 132°C (uniform)
- Bismuth temperature (T_∞) = 500°C
- Ascent velocity (U_∞) = 1 m/s
- Sphere diameter (D) = 1 cm = 0.01 m

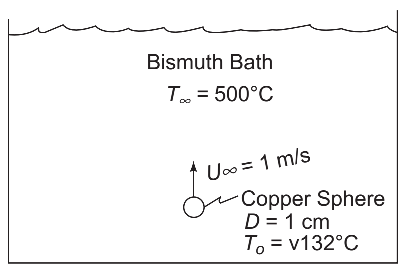
FIND

(a) Distance sphere will rise before its center temperature, $T(o,t)$ = 300°C
(b) The sphere surface temperature at that time, $T(r_o,t)$

ASSUMPTIONS

- Thermal resistance of the nickel plating is negligible
- Thermal properties of the copper can be considered uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 24, for Bismuth at the initial film temperature of 316°C

Thermal conductivity (k_b) = 16.44 W/(m K)

Kinematic viscosity (ν) = 1.57×10^{-7} m^2/s

Prandtl number (Pr) = 0.014

From Appendix 2, Table 12, for copper

Thermal conductivity (k_c) = 388 W/(m K) at its mean temperature of 216°C

Specific heat (c) = 383 J/(kg K) at 20°C

Density (ρ) = 8933 kg/m^3 at 20°C

Thermal diffusivity (α) = 116.6×10^{-6} m^2/s

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(1\,\text{m/s})(0.01\,\text{m})}{\left(1.57\times10^{-7}\,\text{m}^2/\text{s}\right)} = 6.37 \times 10^4$$

Applying Equation (7.14)

$$\overline{Nu}_D = 2 + 0.386\ (RePr)^{\frac{1}{2}} = 2 + 0.386\left[6.37\times10^4(0.014)\right]^{\frac{1}{2}} = 13.52$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 13.53\ \frac{(16.44\,\text{W/(m K)})}{0.01\,\text{m}} = 2.22 \times 10^4\ \text{W/(m}^2\ \text{K)}$$

(a) The Biot number for the sphere is

$$Bi = \frac{\bar{h}_c r}{k_s} = \frac{\left(2.22\times10^4\ \text{W/(m}^2\text{K)}\right)(0.005\,\text{m})}{(388\,\text{W/(m K)})} = 0.287 > 0.1$$

Therefore, internal thermal resistance is significant and the chart solutions of Figure 2.39 must be used.

$$\frac{T(0,t)-T_\infty}{T_o - T_\infty} = \frac{300°\text{C} - 500°\text{C}}{132°\text{C} - 500°\text{C}} = 0.543$$

$$\text{and} \quad \frac{1}{Bi} = \frac{1}{0.287} = 3.48$$

From Figure 2.39

$$Fo = \frac{\alpha t}{r_o^{\,2}} = 0.75$$

$$\therefore \quad t = 0.75\,\frac{r_o^{\,2}}{\alpha} = 0.75\,\frac{(0.005\,\text{m})^2}{\left(116.6\times10^{-6}\ \text{m}^2/\text{s}\right)} = 0.16\ \text{s}$$

The distance (x) the sphere will rise during this time is

$$x = U_\infty t = 1\ \text{m/s}\ (0.16\text{s}) = 0.16\ \text{m} = 16\ \text{cm}$$

(b) The surface temperature can be determined from Figure 2.39

$$\frac{1}{Bi} = 3.48 \text{ and } \frac{r}{r_o} = 1 \qquad \Rightarrow \frac{T(r_o,t)-T_\infty}{T(0,t)-T_\infty} = 0.84$$

$$T(r_o,t) = 0.86\ (300°\text{C} - 500°\text{C}) + 500°\text{C} = 332°\text{C}$$

PROBLEM 7.27

A spherical water droplet of 1.5 mm diameter is freely falling in atmospheric air. Calculate the average convection heat transfer coefficient when the droplet has reached its terminal velocity. Assume that the water is at 50°C and the air is at 20°C. Neglect mass transfer and radiation.

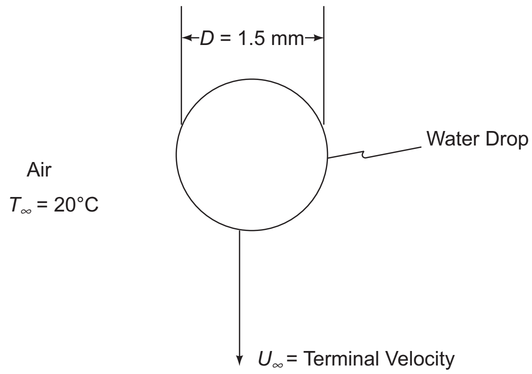
GIVEN

- A spherical water droplet freely falling in atmospheric air
- Drop diameter (D) = 1.5 mm = 0.0015 m
- Water drop temperature (T_d) = 50°C
- Air temperature (T_∞) = 20°C

FIND

- The average heat transfer coefficient at terminal velocity ($\bar{h}_c$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 20°C

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Density (ρ_a) = 1.164 kg/m^3

From Appendix 2, Table 13, for water at 50°C

Density (ρ_w) = 988.1 kg/m^3

SOLUTION

The weight of the water droplet is

$$W = (\text{mass})\, g_g = (\text{Volume})\, \rho_w\, g_g = \frac{\pi}{6} D^3\, \rho_w\, g_g = \frac{\pi}{6} (0.0015\text{m})^3 \left(988.1\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)$$

$$W = 1.713 \times 10^{-5}\ (\text{kg m})/\text{s}^2 = 1.713 \times 10^{5}\ \text{N}$$

Terminal velocity occurs when the droplet's weight is balance by the viscous drag force which is given in Section 7.2 and Figure 7.7.

$$W = C_D \left(\frac{\rho_a U_\infty{}^2}{2}\right)\left(\frac{\pi D^2}{4}\right)$$

Solving for the velocity

$$U_\infty = \left(\frac{8\,W}{C_D \pi D^2 \rho_a}\right)^{\frac{1}{2}} = \left[\frac{8\left(1.13\times10^{-5}\,(\text{kg m})/\text{s}^2\right)}{C_D \pi (0.0015\,\text{m})^2 \left(1.164\,\text{kg/m}^3\right)}\right]^{\frac{1}{2}}$$

$$U_\infty = (4.081\,\text{m/s})\, C_D{}^{-\frac{1}{2}}$$

But C_D is a function of U_∞ through the Reynolds number and Figure 7.7 by trial and error

U_∞ (m/s)	Re_D	C_D	4.081 $C_D^{-1/2}$ (m/s)
10	955	0.44	6.15
6.0	588	0.55	5.50
5.5	525	0.59	5.3
5.3	506	0.60	5.3

Terminal velocity ≈ 5.3 m/s

The Nusselt number is given by Equation (7.12) as

$$\overline{Nu}_D = 2 + \left(\frac{Re_D}{4} + 3\times10^{-4}\, Re_D^{\,1.6}\right)^{\frac{1}{2}}$$

$$\overline{Nu}_D = 2 + \left(\frac{506}{4} + 3\times10^{-4}(506)^{1.6}\right)^{\frac{1}{2}} = 13.53$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 13.53\ \frac{(0.0251\,\text{W/(m K)})}{0.0015\,\text{m}} = 226\ \text{W/(m}^2\text{K)}$$

COMMENT

In this solution, the effect of evaporation has been neglected.

PROBLEM 7.28

> **In a lead-shot tower, spherical 0.95-cm-diameter *BB* shots are formed by drops of molten lead which solidify as they descend in cooler air. At the terminal velocity, i.e., when the drag equals the gravitational force, estimate the total heat transfer coefficient if the lead surface is at 171°C, the surface of the lead has an emissivity of 0.63, and the air temperature is 16°C. Assume C_D = 0.75 for the first trial calculation.**

GIVEN

- Spherical lead-shot falling through the air at terminal velocity
- Shot diameter (D) = 0.95 cm = 0.0095 m
- Lead surface temperature (T_s) = 171°C = 494 K
- Lead surface emissivity (ε) = 0.63
- Air temperature (T_∞) = 16°C 289 K
- Assume C_D = 0.75 for the first trial calculation

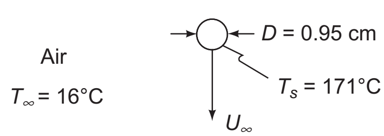
FIND

- The total average heat transfer coefficient (h_{total})

ASSUMPTIONS

- The surroundings act as a black body enclosure at T_∞

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 16°C

Thermal conductivity (k) = 0.0248 W/(m K)

Kinematic viscosity (ν) = 15.3 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Density (ρ) = 1.182 kg/m^3

From Appendix 2, Table 12, the density of lead (ρ_L) = 11,340 kg/m^3

From Appendix 1, Table 15, the Stephan-Boltzmann constant(σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The weight of the lead shot is

$$W = (\text{mass})g_g = (\text{Volume})\,\rho\, g_g = \frac{\pi}{6} D^3\, \rho\, g_g = \frac{\pi}{6}\ (0.0095\text{m})^3\ \left(11{,}340\,\text{kg/m}^3\right)\ \left(9.81\,\text{m/s}^2\right)$$

$$W = 0.0499\ (\text{kg m})/\text{s}^2 = 0.0499\ \text{N}$$

Terminal velocity occurs when the weight is balanced by the drag force which is given Section 7.2

$$W = \text{Drag Force} = C_D \left(\frac{\rho_a U_\infty^{\,2}}{2}\right)\left(\frac{\pi D^2}{4}\right)$$

Solving for the terminal velocity

$$U_\infty = \left(\frac{8W}{C_D \pi D^2 \rho_a}\right)^{\frac{1}{2}} = \left[\frac{8\left(0.0499\,(\text{kg m})/\text{s}^2\right)}{C_D \pi (0.0095\,\text{m})^2\,(1.182\,\text{kg/m})}\right]^{\frac{1}{2}}$$

$$U_\infty = (34.51\,\text{m/s})\ C_D^{\,-\frac{1}{2}}$$

Using the recommended drag coefficient for the first iteration

$$U_\infty = 34.51\ \text{m / s}\,(0.75)^{-\frac{1}{2}} = 39.9\ \text{m/s}$$

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(39.9\,\text{m/s})(0.0095\,\text{m})}{\left(15.3\times10^{-6}\ \text{m}^2/\text{s}\right)} = 24{,}745$$

From Figure 7.7, for Re_D = 24,745, C_D = 0.47

Repeating this procedure for further iterations

Iteration #	2	3
C_D	0.47	0.48
U_∞ (m/s)	50.34	49.81
Re_D	31,259	30,931
C_D (from Figure 7.7)	0.48	0.48

The Nusselt number is given by Equation (7.12)

$$\overline{Nu}_D = 2 + \left(\frac{Re_D}{4} + 3\times10^{-4}\, Re_D^{\,1.6}\right)^{\frac{1}{2}}$$

$$\overline{Nu}_D = 2 + \left(\frac{30{,}391}{4} + 3\times10^{-4}(30{,}391)^{1.6}\right)^{\frac{1}{2}} = 113$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 113\,\frac{(0.0248\,\text{W/(m K)})}{0.0095\,\text{m}} = 295\ \text{W/(m}^2\text{K)}$$

The total rate of heat transfer can be used to calculate the total heat transfer coefficient as follows

$$q_{\text{total}} = \bar{h}_{\text{total}}\ A\ (T_s - T_\infty) = q_c + q_r = \bar{h}_c A\ (T_s - T_\infty) + \varepsilon\, \sigma A\ (T_s^{\,4} - T_\infty^{\,4})$$

$$\bar{h}_{\text{total}} = \bar{h}_c + \frac{\varepsilon\sigma\left(T_s^{\,4} - T_\infty^{\,4}\right)}{\left(T_s - T_\infty\right)}$$

$$\bar{h}_{\text{total}} = \left(295\,\text{W/(m}^2\text{K)}\right) + \frac{0.63\left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)[(494\,\text{K})^4 - (289\,\text{K})^4]}{(494\,\text{K} - 289\,\text{K})}$$

$$\bar{h}_{\text{total}} = \left((295 + 9.1)\text{W/(m}^2\text{K)}\right) = 304\ \text{W/(m}^2\text{K)}$$

COMMENTS

97% of the heat transfer is due to convection.

PROBLEM 7.29

A copper sphere 2.5 cm in diameter is suspended by a fine wire in the center of an experimental hollow cylindrical furnace whose inside wall is maintained uniformly at 430°C. Dry air at a temperature of 90°C and a pressure of 1.2 atm is blown steadily through the furnace at a velocity of 14 m/s. The interior surface of the furnace wall is black. The copper is slightly oxidized, and its emissivity is 0.4. Assuming that the air is completely transparent to radiation, calculate for the steady state (a) the convective heat transfer coefficient between the copper sphere and the air, and (b) the temperature of the sphere.

GIVEN

- A copper sphere suspended in a furnace with air flowing over it
- Sphere diameter (D) = 2.5 cm = 0.025 m
- Sphere emissivity (ε) = 0.4
- Furnace wall temperature (T_w) = 430°C = 703 K
- Air temperature (T_a) = 90°C = 363 K
- Air pressure (p_a) = 1.2 atm
- Air velocity (U_∞) = 14 m/s

FIND

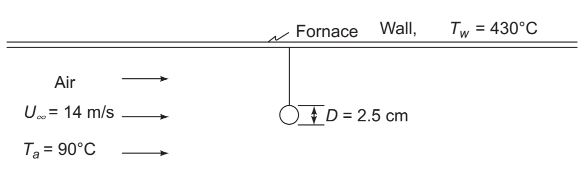
(a) The convective heat transfer coefficient $\left(\bar{h}_c\right)$

(b) The temperature of the sphere (T_s)

ASSUMPTIONS

- Steady state
- The air behaves as an ideal gas
- The furnace can be treated as a black body enclosure
- Only the density of the air varies with temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 90°C and 1 atm pressure

Density (ρ) = 0.942 kg/m^3

Thermal conductivity (k) = 0.0300 W/(m K)

Absolute viscosity (μ) = 21.232 × 10^{-6} (Ns)/m^2

Prandtl number (pr) = 0.71

From Appendix 1, Table 15, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The density of the air at 1.2 atm can be calculated from Boyle's law

$$\frac{P_1}{P_{1.2}} = \frac{\rho_1}{\rho_{1.2}} \Rightarrow \rho_{1.2} = \rho_1 \frac{P_{1.2}}{P_1} = 0.942 \text{ kg/m}^3 \, \frac{1.2 \text{ atm}}{1 \text{ atm}} = 1.130 \text{ kg/m}^3$$

The kinematic viscosity is

$$\nu = \frac{\mu}{\rho} = \frac{\left(21.232\times10^{-6}\,(\text{N s})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)}{\left(1.130\text{ kg/m}^3\right)} = 18.8 \times 10^{-6} \text{ m}^2/\text{s}$$

(a) The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(14\text{ m/s})(0.025\text{ m})}{\left(18.8\times10^{-6}\text{ m}^2/\text{s}\right)} = 18{,}617$$

The convective Nusselt number can be estimated using Equation (7.12)

$$\overline{Nu}_D = 2 + \left(\frac{Re_D}{4} + 3\times10^{-4} Re_D^{1.6}\right)^{\frac{1}{2}}$$

$$\overline{Nu}_D = 2 + \left(\frac{18{,}617}{4} + 3\times10^{-4}(18{,}617)^{1.6}\right)^{\frac{1}{2}} = 83.8$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 83.8 \frac{(0.03\text{ W/(m K)})}{0.025\text{ m}} = 100.6 \text{ W/(m}^2\text{K)}$$

(b) In steady state, the sphere temperature will be between T_a and T_w and the convective loss to the air must equal the radiative gain from the furnace walls

$$\bar{h}_c A\,(T_s - T_\infty) = \varepsilon\,\sigma(T_\infty^4 - T_s^4)$$

$$\left(100.6 \text{ W/(m}^2\text{K)}\right)\,(T_s - 363\text{ K}) = 0.4\,\left(5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\right)\,[(703\text{ K})^4 - T_s^4]$$

Checking the units, then eliminating them for clarity

$$2.28\times10^{-8}\,T_s^4 + 100.6\,T_s - 42{,}087 = 0$$

By trial and error: T_s = 412 K = 139°C

PROBLEM 7.30

A method for measuring the convective heat transfer from spheres has been proposed. A 20 m diameter copper sphere with an embedded electrical heater is to be suspended in a wind tunnel. A thermocouple inside the sphere measures the sphere surface temperature. The sphere is supported in the tunnel by a type 304 stainless steel tube 5 mm outside diameter 3 mm inside diameter and 20 cm long. The steel tube is attached to the wind tunnel wall in such a way that no heat is transferred through the wall. For this experiment, examine the magnitude of the correction that must be applied to the sphere heater power to account for conduction along the support tube. The air temperature is 20°C and the desired range of Reynolds numbers is 10^3 to 10^5.

GIVEN

- A heater copper sphere supported by a steel tube in a wind tunnel
- Sphere diameter (D_s) = 20 mm = 0.02 m
- Tube diameters
  - Outside (D_{to}) = 5 mm = 0.005 m
  - Inside (D_{ti}) = 3 mm = 0.003 m
- Tube length (L) = 20 cm = 0.2 m
- There is no heat transfer between the tube and the wall
- Air temperature (T_∞) = 20°C
- Reynolds number range: $10^3 < Re_{Ds} < 10^5$

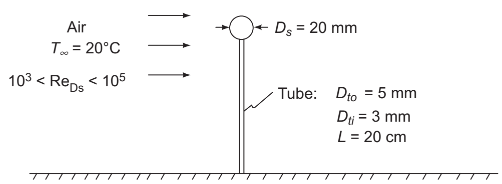
FIND

- The correction to the heater power to account for conduction along the support tube

ASSUMPTIONS

- Steady state
- Contact resistance between the sphere and the tube is negligible
- The effect of the boundary layer near the wind tunnel wall on the heat transfer from the tube is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 20°C

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 10, for type 304 stainless steel: k_s = 14.4 W/(m K)

SOLUTION

Equation (7.9) can be used to estimate the heat transfer coefficient on the sphere

$$\bar{h}_{cs} = 0.37 \frac{k}{D_s} Re_{D_s}^{0.6}$$

At $Re_{D_s} = 10^3$

$$\bar{h}_{cs} = 0.37 \frac{(0.0251\,\text{W/(m K)})}{0.02\,\text{m}} (10^3)^{0.6} = 29.3\ \text{W/(m}^2\text{K)}$$

At $Re_{D_s} = 10^5$

$$\bar{h}_{cs} = 0.37 \frac{(0.0251\,\text{W/(m K)})}{0.02\,\text{m}} (10^5)^{0.6} = 464\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer from the sphere, neglecting the influence of the tube, would be

$$q_s = \bar{h}_{cs}\, \pi D_s^2 (T_s - T_\infty) = 0.37 \frac{k}{D} Re_{D_s}^{0.6}\ \pi D_s^2 (T_s - T_\infty)$$

The heat transfer coefficient on the tube can be calculated using Equation 7.3 and Table 7.1 (Note that $Pr/Pr_s \approx 1$ for air in the anticipated temperature range)

$$\bar{h}_{ct} = 0.26 \frac{k}{D_{to}} Re_{D_{to}}^{0.6} Pr^{0.37} = 0.26 \frac{k}{D_{to}} Re_{D.}^{0.6} \left(\frac{D_{to}}{D_s}\right)^{0.6} Pr^{0.37}$$

At $R_{D.} = 10^3$ $\bar{h}_{ct} = 0.26 \frac{(0.0251\,\text{W/(m K)})}{0.005\,\text{m}} (10^3)^{0.6} \left(\frac{0.005\,\text{m}}{0.02\,\text{m}}\right)^{0.6} (0.71)^{0.37} = 31.6\,\text{W/(m}^2\text{K)}$

At $R_{D.} = 10^5$ $\bar{h}_{ct} = 501\,\text{W/(m}^2\text{K)}$

The tube can be modeled as a fin of uniform cross-section with an adiabatic tip protruding from the sphere. The cross-sectional area of the tube (A_f) and perimeter of the air (P) are

$$A_f = \frac{\pi}{4}(D_{to}^2 - D_{ti}^2) = \frac{\pi}{4} [(0.005\text{ m})^2 - (0.003)^2] = 1.256 \times 10^{-5}\text{ m}^2$$

$$P = \pi D_{to} = \pi(0.005\text{ m}) = 0.0157\text{ m}$$

The rate of heat transfer from the tube for a given sphere temperature is given Table 2.1 as

$$q_t = M \tanh(mL)$$

where $$m = \sqrt{\frac{\bar{h}_{ct} P}{k_s A_f}}$$

$$M = \sqrt{\bar{h}_{ct} P k_s A_f}\ (T_s - T_\infty)$$

The fraction correction to the power data due to the tube is

$$\frac{q_t}{q_s} = \frac{\sqrt{\bar{h}_{ct} P k_s A_f}\ \tanh\left(L\sqrt{\frac{\bar{h}_{ct} P}{k_s A_f}}\right)}{\bar{h}_{cs} \pi D_s^2}$$

At $Re_{DS} = 10^3$

$$\sqrt{\bar{h}_{ct} P k_s A_f} = \sqrt{(31.6\,\text{W/(m}^2\text{K)})(0.0157\,\text{m})(14.4\,\text{W/(m K)})(1.256\times10^{-5}\,\text{m}^2)} = 0.0095\text{ W/K}$$

$$\frac{q_t}{q_s} = \frac{(0.0095\,\text{W/K})\tanh\left((0.2\,\text{m})\sqrt{\frac{(31.6\,\text{W/(m}^2\text{K)})(0.0157\,\text{m})}{(14.4\,\text{W/(m K)})(1.256\times10^{-5}\,\text{m}^2)}}\right)}{(29.3\,\text{W/(m K)})\ \pi(0.02\,\text{m})^2} = 0.258 = 25.8\%\text{ correction}$$

At $Re_{D.} = 10^5$

$$\frac{q_t}{q_a} = 0.065 = 6.5\%\text{ correction}$$

PROBLEM 7.31

Estimate (a) the heat transfer coefficient for a spherical fuel droplet injected into a diesel engine at 80°C and 90 m/s. The oil droplet is 0.025 mm in diameter, the cylinder pressure

is 4800 kPa, and the gas temperature is 944 K. (b) Estimate the time required to heat the droplet to its self-ignition temperature of 300°C.

GIVEN

- An oil droplet injected into a diesel engine
- Initial droplet temperature (T_o) = 80°C
- Injection velocity (U_d) = 90 m/s
- Droplet diameter (D) = 0.025 mm = 2.5×10^{-5} m
- Cylinder pressure = 4800 kPa
- Gas temperature (T_∞) = 944 K = 671°C

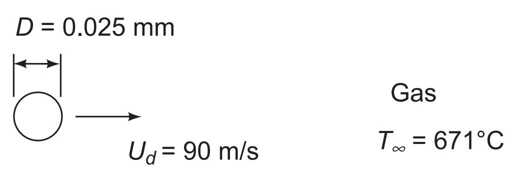
FIND

(a) The heat transfer coefficient ($\bar{h}_c$)
(b) The time (t) required for the drop to reach 300°C

ASSUMPTIONS

- Radiative heat transfer is negligible
- The gas has the properties of air and behaves as an ideal gas
- Only the density of the gas is affected by pressure
- Variation of the thermal conductivity of the oil with temperature is negligible
- Fuel has the same properties as unused engine oil

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 671°C and 1 atm (101 kPa) pressure

Density (ρ) = 0.382 kg/m^3
Thermal conductivity (k) = 0.0616 W/(m K)
Absolute viscosity (μ) = 40.121×10^{-6} Ns/m^2
Prandtl number (Pr) = 0.73

Extrapolating from Appendix 2, Table 16, for unused engine oil at the average temperature of 190°C

Density (ρ_o) = 789.4 kg/m^3
Thermal conductivity (k_o) = 1.131 W/(m K)
Specific heat (c) = 2615 J/(kg K)

SOLUTION

The density of the air at 4800 kPa can be calculated from Boyle's law

$$p_1 V_1 = p_2 V_2 \Rightarrow \frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} \Rightarrow \rho_2 = \rho_1 \frac{P_2}{P_1} = \left(0.382\,\text{kg/m}^3\right) \frac{4800\,\text{kPa}}{101\,\text{kPa}} = 18.15\,\text{kg/m}^3$$

The Reynolds number is

$$Re_D = \frac{U_d D \rho}{\mu} = \frac{(90\,\text{m/s})\left(2.5\times10^{-5}\,\text{m}\right)\left(18.15\,\text{kg/m}^3\right)}{\left(40.121\times10^{-6}\,(\text{Ns})/\text{m}^2\right)} = 1018$$

(a) The Nusselt number can be calculated from Equation (7.12)

$$\overline{Nu}_D = 2 + \left(\frac{Re_D}{4} + 3\times10^{-4}\, Re_D{}^{1.6}\right)^{\frac{1}{2}}$$

$$\overline{Nu}_D = 2 + \left(\frac{1018}{4} + 3\times10^{-4}(1018)^{1.6}\right)^{\frac{1}{2}} = 18.55$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 18.55 \frac{(0.0616\,\mathrm{W/(m\,K)})}{2.5\times10^{-5}\,\mathrm{m}} = 4.57\times10^4\ \mathrm{W/(m^2K)}$$

(b) The Biot number for the droplet is

$$Bi = \frac{\bar{h}_c D}{2k_s} = \frac{(4.57\times10^4\ \mathrm{W/(m^2K)})(2.5\times10^{-5}\,\mathrm{m})}{2(0.130\,\mathrm{W/(m\,K)})} = 4.40 >> 0.1$$

Therefore, the internal resistance of the oil drop cannot be neglected and the chart solution of Figure 2.39 must be used. Assuming the heat transfer coefficient is constant, the ratio of the rate of heat transfer at time t and initially is

$$\frac{Q(t)}{Q_i} = \frac{T(r_o,t)-T_\infty}{T_o - T_\infty} = \frac{300°\mathrm{C} - 671°\mathrm{C}}{80°\mathrm{C} - 671°\mathrm{C}} = 0.628$$

From Figure 2.39, for $Bi = 4.40$

$$(Bi)^2\, Fo = \frac{\bar{h}_c{}^2 \alpha\, t}{k_o{}^2} = \frac{\bar{h}_c{}^2\, t}{k_o \rho c} = 2.8$$

Solving for the time

$$t = \frac{2.8\, k_o{}^2 \rho c}{\bar{h}_c{}^2} = \frac{2.8(0.13\,\mathrm{W/(m\,K)})(789.4\,\mathrm{kg/m^3})(2615\,\mathrm{J/(kg\,K)})((\mathrm{W\,s})/\mathrm{J})}{(4.57\times10^4\ \mathrm{W/(m^2K)})^2}$$

$$t = 3.6\times10^{-6}\ \mathrm{s} = 360\ \mu\mathrm{s}$$

PROBLEM 7.32

Heat transfer from an electronic circuit board is to be determined by placing a model for the board in a wind tunnel. The model is a 15 cm square plate with embedded electrical heaters. The wind from the tunnel air is delivered at 20°C. Determine the average temperature of the model as a function of power dissipation for an air velocity of 2.5 and 10 m/s. The model is pitched 30°C and yawed 10° with respect to the flow direction. The surface of the model acts as a blackbody.

GIVEN

- A Model square electronic circuit board pitched 30° and yawed 10° in a wind tunnel
- Length of a side (L) = 15 cm = 0.15 m
- Air temperature (T_∞) = 20°C = 293 K
- Air velocity (U_∞) = 2.5 m/s or 10 m/s

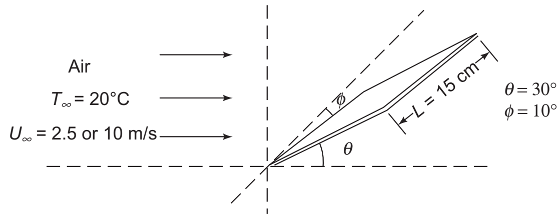
FIND

- The average surface temperature of the model (T_s) as a function of power dissipation ($\dot{Q}$)

ASSUMPTIONS

- Steady state
- The surface of the model acts as a black body (ε = 1.0)
- The wind tunnel acts as a black body enclosure at the air temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 20°C

Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s

Prandtll number (Pr) = 0.71

Density (ρ) = 1.164 kg/m^3

Specific heat (c) = 1012 J/(kg K)

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The Reynolds number is

For U_∞ = 2.5 m/s

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(2.5\,\text{m/s})(0.15\,\text{m})}{(15.7\times10^{-6}\,\text{m}^2/\text{s})} = 2.39 \times 10^4$$

For U_∞ = 10 m/s

$$Re_L = \frac{(10\,\text{m/s})(0.15\,\text{m})}{(15.7\times10^{-6}\,\text{m}^2/\text{s})} = 9.55 \times 10^4$$

The pitch and yaw angles as well as the Reynolds numbers fall within the range of Equation (7.18)

$$\left(\frac{\bar{h}_c}{c\rho U_\infty}\right) Pr^{\frac{2}{3}} = 0.930\ Re_L^{-\frac{1}{2}}$$

For U_{inf} = 2.5 m/s

$$\bar{h}_c = 0.930\ c\ \rho\ U_\infty\ Pr^{-\frac{2}{3}} Re_L^{-\frac{1}{2}} = 0.930(1012\,\text{J/(kg K)})\ (1.164\,\text{kg/m}^3)$$

$$(2.5\,\text{m/s})\ (2.39\times10^4)^{-\frac{1}{2}}\ (0.71)^{-\frac{2}{3}}\ ((\text{Ws})/\text{J})$$

$$\bar{h}_c = 22.3\ \text{W/(m}^2\text{K)}$$

For U_{inf} = 10 m/s

$$\bar{h}_c = 44.5\ \text{W/(m}^2\text{K)}$$

The rate of power dissipation is the sum of the convective and radiative losses

$$\dot{Q} = \bar{h}_c\ 2L^2\ (T_s - T_\infty) + \sigma 2L^2\ (T_s^4 - T_\infty^4)$$

At U_{inf} = 2.5 m/s

$$\dot{Q} = \left(22.3\,\text{W/(m}^2\text{K)}\right) 2(0.15\text{m})^2 \,(T_s - 293\text{ K}) + \left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right) 2(0.15\text{m})^2\,[T_s^4 - (293\text{ K})^4]$$

Checking the units, then eliminating for clarity

$$2.565 \times 10^{-9}\, T_s^4 + 1.0035\, T_s - 312.8 = \dot{Q}$$

($\dot{Q}$ in watts, T_s in K)

Similarly for U_∞ = 10 m/s

$$2.565 \times 10^{-9}\, T_s^4 + 2.0025\, T_s - 605.5 = \dot{Q}$$

($\dot{Q}$ in watts, T_s in K)

PROBLEM 7.33

An electronic circuit contains a power resistor that dissipates 1.5 watts. The designer wants to modify the circuitry in such a way that it will be necessary for the resistor to dissipate 2.5 watts. The resistor is in the shape of a disk 1 cm in diameter and 0.6 mm thick. Its surface is aligned with a cooling air flow at 30°C and 10 m/s velocity. The resistor lifetime becomes unacceptable if its surface temperature exceeds 90°C. Is it necessary to replace the resistor for the new circuit?

GIVEN

- A heat generating resistor disk with its surface aligned with a cooling airflow
- Heat generation rate ($\dot{Q}_G$) = 2.5 W
- Disk diameter (D) = 1 cm = 0.01 m
- Disk thickness (t) = 0.6 mm = 0.0006 m
- Air temperature (T_∞) = 30°C
- Air velocity (U_∞) = 10 m/s
- Maximum surface temperature (T_s) = 90°C

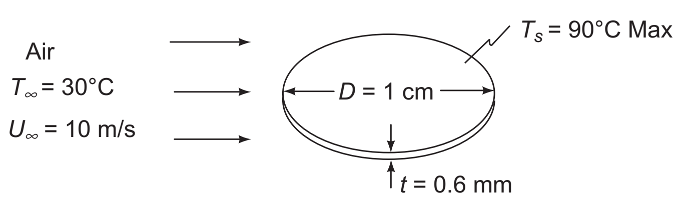
FIND

- Is it necessary to replace the resistor?

ASSUMPTIONS

- Steady state
- Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the maximum film temperature of 60°C

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number based on the diameter is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(10\,\text{m/s})(0.01\,\text{m})}{(19.4\times10^{-6}\,\text{m}^2/\text{s})} = 5155$$

$$\frac{t}{D} = \frac{0.0006\,\text{m}}{0.01\,\text{m}} = 0.06$$

The Nusselt number for the geometry is given by Equation (7.19)

$$\overline{Nu}_D = 0.591\ Pr^{\frac{1}{3}}\,Re_D^{0.564} = 0.591\ (0.71)^{\frac{1}{3}}\ (5155)^{0.564} = 65.42$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 65.42\,\frac{(0.0279\,\text{W/(m K)})}{0.01\,\text{m}} = 182.5\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer at the maximum surface temperature of 90°C is

$$q = \bar{h}_c A\,(T_s - T_\infty) = \bar{h}_c\left[2\left(\frac{\pi}{4}D^2\right) + \pi D\,t\right](T_s - T_\infty)$$

$$q = (182.5\,\text{W/(m}^2\text{K)})\left[\frac{\pi}{2}(0.01)^2 + \pi\ (0.01\ \text{m})\ (0.0006\ \text{m})\right](90°\text{C} + 30°\text{C}) = 1.93\ \text{W} < \dot{Q}$$

Therefore, the surface temperature must be greater than 90°C to dissipate the required 2.5 Watts. The resistor must be replaced.

PROBLEM 7.34

> **Suppose the resistor in Problem 7.33 is rotated so that its axis is aligned with the flow. What is the maximum permissible power dissipation?**
>
> **From Problem 7.33: An electronic circuit contains a power resistor that dissipates 1.5 watts. The designer wants to modify the circuitry in such a way that it will be necessary for the resistor to dissipate 2.5 watts. The resistor is in the shape of a disk 1 cm in diameter and 0.6 mm thick. Its axis aligned with a cooling air flow at 30°C and 10 m/s velocity. The resistor lifetime becomes unacceptable if its surface temperature exceeds 90°C. Is it necessary to replace the resistor for the new circuit?**

GIVEN

- A heat generating resistor disk with its axis aligned with a cooling airflow
- Heat generation rate ($\dot{Q}_G$) = 2.5 W
- Disk diameter (D) = 1 cm = 0.01 m
- Disk thickness (t) = 0.6 mm = 0.0006 m
- Air temperature (T_∞) = 30°C
- Air velocity (U_∞) = 10 m/s
- Maximum surface temperature (T_s) = 90°C

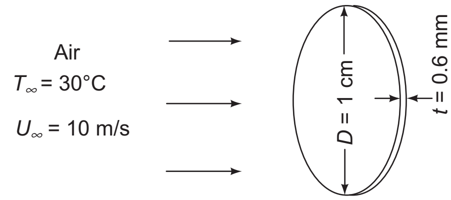
FIND

- The maximum permissible power dissipation ($\dot{Q}_G$)

ASSUMPTIONS

- Heat transfer from the edges is negligible
- The heat flux is equal from both faces

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the maximum film temperature of 60°C

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 27, for dry air at the free stream temperature of 30°C

Thermal conductivity (k) = 0.0258 W/(m K)

Kinematic viscosity (ν) = 16.7×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number based on the film temperature is unchanged from Problem 7.33 $Re_{Df} = 5155$

The Reynolds number based on the free stream conditions is $Re_{Dfs} = 5988$

The Nusselt number for the upstream face of the disk is given by Equation (7.17) using the free stream properties

$$\overline{Nu}_D = 1.05\ Re_{Dfs}^{\frac{1}{2}}\ Pr^{0.36} = 1.05\,(5988)^{\frac{1}{2}}\,(0.71)^{0.37} = 71.58$$

$$\bar{h}_{cu} = \overline{Nu}_D\ \frac{k}{D} = 71.58\,\frac{(0.0258\,\text{W/(mK)})}{0.01\,\text{m}} = 184.7\ \text{W/(m}^2\text{K)}$$

The Nusselt number for the downstream face can be estimated from Equation (7.15) (using the properties at film temperature) because the flow behind the disk will be separated and the separated region behind a normal flat plate will be similar

$$\overline{Nu}_D = 0.20\ Re_{Df}^{\frac{2}{3}} = 0.20\ (5155)^{\frac{2}{3}} = 59.69$$

$$\bar{h}_{cd} = \overline{Nu}_D\ \frac{k}{D} = 59.69\ \frac{(0.0279\,\text{W/(m K)})}{0.01\,\text{m}} = 166.5\ \text{W/(m}^2\text{K)}$$

The maximum power dissipation for the whole chip is

$$\dot{Q}_{\text{max}} = \left(\bar{h}_{cd} + \bar{h}_{cu}\right)\ \frac{\pi}{4} D^2\,(T_s - T_\infty)$$

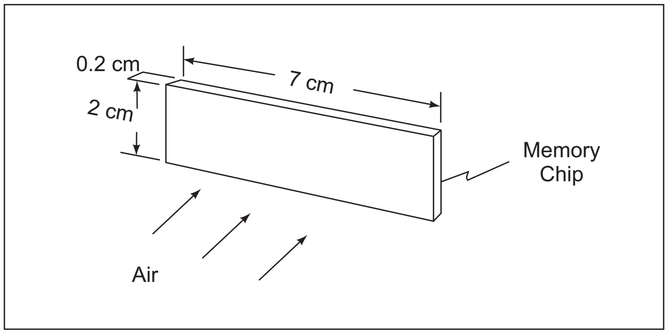
$$\dot{Q}_{\text{max}} = \left(166.5\,\text{W/(m}^2\text{K)}\right) + \left(184.7\,\text{W/(m}^2\text{K)}\right)\ \frac{\pi}{4}(0.01\ \text{m})^2\,(90°\text{C} - 30°\text{C}) = 1.65\ \text{W}$$

COMMENTS

The circuit designer cannot solve the problem by reorienting the resistor. The upstream face of the disk has about the same heat transfer coefficient as the surface of the disk aligned with the flow. However, the downstream face has significantly lower heat transfer coefficient because it is in a separated flow regime.

PROBLEM 7.35

To decrease the size of personal computer mother boards, designers have turned to a more compact method of mounting memory chips on the board. The single in-line memory modules, as they are called, essentially mount the chips on their edges so that their thin dimension is horizontal, as shown in the sketch below. For memory chips operating at 90°C, determine their maximum power dissipation if they are cooled by an air stream at 60°C and velocity of 10 m/s.



GIVEN

- Computer memory chip in an air stream as shown above
- Chip temperature (T_s) = 90°C
- Air temperature (T_∞) = 60°C
- Air velocity (U_∞) = 10 m/s

FIND

- The maximum power dissipation ($\dot{Q}_G$)

ASSUMPTIONS

- Radiative heat transfer is negligible
- Use of Equation (7.18) for a non-square surface will not introduce significant error
- Heat transfer from all four edges of the chip is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the free stream temperature of 60°C

Density (ρ) = 1.025 kg/m^3

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Specific heat (c) = 1017 J/(kg K)

From Appendix 2, Table 27, for dry air at the film temperature of 75°C

Thermal conductivity (k_f) = 0.0290 W/(m K)

Kinematic viscosity (ν_f) = 21.0 × 10^{-6} m^2/s

Prandtl number (Pr_f) = 0.71

SOLUTION

Front of chip:

The Reynolds number for the front of the chip will be based on a characteristic length that is equal to the length of the side of a square with the same area and on the properties evaluated at the free stream temperature.

$$L_{\text{eq}} = \sqrt{(7\,\text{cm})(2\,\text{cm})} = 3.74 \text{ cm} = 0.0374 \text{ m}$$

$$Re_{L_{\text{eq}}} = \frac{U_\infty L_{\text{eq}}}{\nu} = \frac{(10\,\text{m/s})(0.0374\,\text{m})}{\left(19.4\times10^{-6}\,\text{m}^2/\text{s}\right)} = 19{,}278$$

Applying Equation (7.18) as the only correlation available to estimate the heat transfer coefficient on the front of the chip

$$\bar{h}_{cf} = 0.930\; c\,\rho\,U_\infty Pr^{-\frac{2}{3}}\; Re_L^{-\frac{1}{2}}$$

$$= 0.930\,(1017\,\text{J/(kg K)})\left(1.025\,\text{kg/m}^3\right)(10\,\text{m/s})\;(19{,}278)^{-\frac{1}{2}}\,(0.71)^{-\frac{2}{3}}\;((\text{Ws})/\text{J})$$

$$\bar{h}_{cf} = 87.7\;\text{W/(m}^2\text{K)}$$

Back of the chip:

The Reynolds number based on the properties evaluated at the film temperature is

$$Re_{L_{eq}} = \frac{U_\infty L_{eq}}{\nu} = \frac{(10\,\text{m/s})(0.0374\,\text{m})}{\left(21.0\times10^{-6}\,\text{m}^2/\text{s}\right)} = 17{,}809$$

Applying Equation (7.15) (using the properties evaluated film temperature) to estimate the Nusselt number on the back of the chip

$$\overline{Nu}_D = 0.20\,Re_D^{\frac{2}{3}} = 0.20\;(17{,}809)^{\frac{2}{3}} = 136.3$$

$$\bar{h}_{cb} = \overline{Nu}_{L_{eq}}\,\frac{k}{L_{eq}} = 143.8\,\frac{(0.0290\,\text{W/(m K)})}{0.0374\,\text{m}} = 105.7\;\text{W/(m}^2\text{K)}$$

The maximum power dissipation is equal to the total rate of heat transfer

$$\dot{Q}_G = q_c = \left(\bar{h}_{cf} + \bar{h}_{cb}\right) A_{\text{frontal}}\,(T_s - T_\infty)$$

$$\dot{Q}_G = \left((87.7 + 105.7)\,\text{W/(m}^2\,\text{K)}\right)(0.07\text{ m})\,(0.02\text{ m})\,(90°\text{C} - 60°\text{C})$$

$$\dot{Q}_G = 8.1\text{ W}$$

COMMENTS

The rate of heat transfer would be greater if the long axis of the chip were aligned with the air velocity.

PROBLEM 7.36

A long, half-round cylinder is placed in an air stream with its flat face down-stream. An electrical resistance heater inside the cylinder maintains the cylinder surface temperature at 50°C. The cylinder diameter is 5 cm, the air velocity is 31.8 m/s, and the air temperature is 20°C. Determine the power input of the heater per unit length of cylinder. Neglect radiation heat transfer.

GIVEN

- A long, half-round cylinder in an air stream with its flat surface downstream
- Cylinder surface temperature (T_s) = 50°C

- Cylinder diameter (D) = 5 cm = 0.05 m
- Air velocity (U_∞) = 31.8 m/s
- Air temperature (T_∞) = 20°C

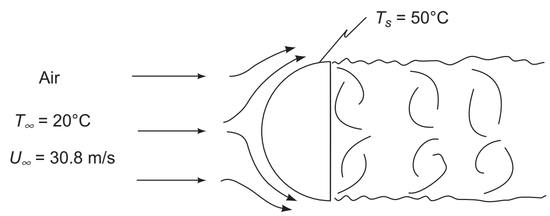
FIND

- Power input to the heater ($\dot{Q}_G/L$)

ASSUMPTIONS

- Steady state
- Radiative heat transfer is negligible
- Flow separates at the cylinder edges as shown below

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the film temperature of 35°C

Thermal conductivity (k) = 0.0262 W/(m K)
Kinematic viscosity (ν) = 17.1×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

At the surface temperature of 50°C: Pr_s = 0.71

SOLUTION

The Reynolds number based on the film temperature is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(31.8\,\text{m/s})(0.05\,\text{m})}{(17.1\times10^{-6}\,\text{m}^2/\text{s})} = 9.30 \times 10^4$$

This is within the range of the correlation of Equation (7.16), for the downstream surface

$$\overline{Nu_D} = 0.16\ Re_D^{\frac{2}{3}} = 0.16\ (9.30\times10^4)^{\frac{2}{3}} = 328.4$$

$$\bar{h}_{cd} = \overline{Nu_D}\,\frac{k}{D} = 328.4\ \frac{(0.0262\,\text{W/(m K)})}{0.05\,\text{m}} = 172.1\ \text{W/(m}^2\text{K)}$$

For the half-round upstream face, the average Nusselt number can be estimated by numerically integrating the curve of Figure 7.8 for Re_D = 101,300

$$\overline{Nu_D} = \frac{1}{90°}\int_0^{90°} Nu_{D\theta}\,d\theta = \frac{1}{90°}\ (\text{Area under the } Nu_{D\theta} \text{ Vs } \theta \text{ curve from } \theta = 0 \text{ to } 90°)$$

Performing the integration by the trapezoidal rule

$$\overline{Nu_D} = \frac{1}{90°}\,[20°\,(320) + 20°\,(310) + 20°\,(280) + 20°(190) + 10°\,(150)] = 260$$

$$\bar{h}_c = \overline{Nu_D}\,\frac{k}{D} = 260\ \frac{(0.0262\,\text{W/(m K)})}{0.05\,\text{m}} = 136.2\ \text{W/(m}^2\text{K)}$$

The power input to the heater must equal the rate of convective heat loss

$$Q_G = (\bar{h}_{cd}\ A_d + \bar{h}_{cu}\ A_u)\,(T_s - T_\infty) = \left[\bar{h}_{cd}\,(DL) + \bar{h}_{cu}\left(\frac{\pi}{2}DL\right)\right](T_s - T_\infty)$$

$$\frac{Q_G}{L} = \left(\bar{h}_{cd} + \frac{\pi}{2}\bar{h}_{cu}\right) D(T_s - T_\infty) = \left[\left(172.1 + \frac{\pi}{2}(136.2)\right)\mathrm{W/(m^2K)}\right]\ (0.05\ \mathrm{m})$$

$$(50°\mathrm{C} - 20°\mathrm{C}) = 579\ \ \mathrm{W/m}$$

COMMENT

The use of Equation 7.3 and Table 7.1 rather than Figure 7.8 to estimate the upstream heat transfer coefficient leads to a transfer coefficient 11% lower and a final result 6% lower than those presented above.

PROBLEM 7.37

> **One method of storing solar energy for use during cloudy days, or at night, is to store it in the form of sensible heat in a rock bed. Suppose such a rock bed has been heated to 70°C and it is desired to heat a stream of air by blowing it through the bed. If the air inlet temperature is 10°C and the mass velocity of the air in the bed is 0.5 kg/(s m^2), how long must the bed be in order for the initial outlet air temperature to be 60°C? Assume that the rocks are spherical, 2 cm in diameter, and that the bed void fraction is 0.5. Hint: The surface area of the rocks per unit volume of the bed is (6/D_p)(1 – ε).**

GIVEN

- Packed bed of rocks with air blowing through it
- Initial temperature of rocks (T_r) = 70°C
- Inlet air temperature ($T_{a,\mathrm{in}}$) = 10°C
- Mass velocity of air ($\dot{m}/A$) = 0.5 kg/(s m^2)
- Outlet air temperature ($T_{a,\mathrm{out}}$) = 65°C
- Surface area per unit volume (A_s/V) = $(6/D_p)(1 - \varepsilon)$

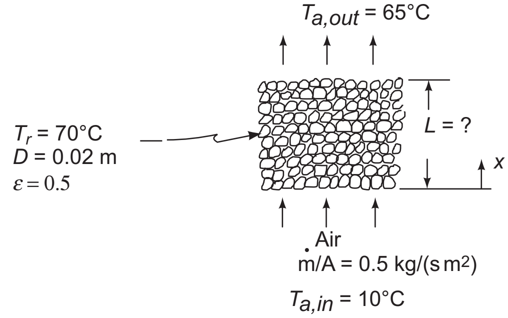
FIND

- Length of bed required (L)

ASSUMPTIONS

- Rocks are spherical with diameter (D) = 2 cm = 0.02 m
- Void fraction of the bed (ε) = 0.5
- Rock temperature remains practically constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average temperature of 37.5°C

Density (ρ) = 1.101 kg/m^3

Thermal conductivity (k) = 0.0263 W/(m K)

Kinematic viscosity (ν) = 17.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Specific heat (c_{pa}) = 10147 J/(kg K)

SOLUTION

The Whitaker definition of the Reynolds number is

$$Re_{D_p} = \frac{D_p U_s}{\nu(1-\varepsilon)}$$

where $$D_p = \frac{6\,(\text{volume})}{\text{surface area}} = \frac{6\left(\frac{\pi}{6}D^3\right)}{\pi D^2} = D \text{ (for spherical packing)}$$

$$U_s = \frac{m}{A\rho} = \frac{\left(0.5\,\text{kg/(s m}^2)\right)}{\left(1.101\,\text{kg/m}^3\right)} = 0.454 \text{ m/s}$$

$$\therefore \quad Re_{D_p} = \frac{(0.02\,\text{m})(0.45\,\text{m/s})}{\left(17.4\times10^{-6}\,\text{m}^2/\text{s}\right)(1-0.5)} = 1044$$

The heat transfer coefficient is given by Equation (7.20)

$$\frac{\bar{h}_c D_p}{k} = \frac{1-\varepsilon}{\varepsilon}\left(0.5\,Re_{D_p}^{\frac{1}{2}} + 0.2\,Re_{D_p}^{\frac{2}{3}}\right) Pr^{\frac{1}{3}}$$

$$\frac{\bar{h}_c D_p}{k} = \frac{1-0.5}{0.5}\left[0.5(1044)^{\frac{1}{2}} + 0.2(1044)^{\frac{2}{3}}\right](0.71)^{\frac{1}{3}} = 32.77$$

$$\bar{h}_c = 32.77\,\frac{(0.0263\,\text{W/(m K)})}{0.02\,\text{m}} = 43.1\ \text{W/(m}^2\text{K)}$$

A local energy balance on the air flow through the bed yields

$$\frac{\dot{m}}{A}\,c_{pa}\,[T(x+\Delta x) - T(x)] = \bar{h}_c\,\frac{A_s}{V}\,dx\,[T_r - T(x)]$$

where $$\frac{A_s}{V} = \frac{6}{D_p}(1-\varepsilon) = \frac{6}{0.02\,\text{m}}(1-0.5) = 150\,\frac{1}{\text{m}}$$

$$\left(0.5\,\text{kg/(m}^2\text{s)}\right)\,(1014\,(\text{W s})/(\text{kg K}))\,[T(x+\Delta x) - T(x)] = \left(43.1\,\text{W/(m}^2\text{K)}\right)\,(150\,1/\text{m})\,dx\,[T_r - T(x)]$$

Checking the units, then eliminating them for clarity

$$T(x+\Delta x) = T(x) + 12.75\,\Delta x\,[T_r - T(x)]$$

Let Δx = 1 cm = 0.01 m

Applying the above equation iteratively until the temperature reaches 65°C yields the following results

	x (m)	$T(x)$ (°C)
	0.00	10.00
	0.01	17.65
	0.02	24.32
	0.03	30.14
	0.04	35.22
	0.05	39.66
	0.06	43.53
	0.07	46.91
	0.08	49.85
Let Δx = 0.02 m	0.10	54.99
	0.12	58.81
	0.14	61.66
	0.16	63.79
	0.18	65.31
	L = 0.18 m = 18 cm	

PROBLEM 7.38

Suppose the rock bed in Problem 7.37 has been completely discharged and the entire bed is at 10°C. Hot air at 90°C and 0.2 m/s is then used to recharge the bed. How long will it take until the first rocks are back up to 70°C and what is the total heat transfer from the air to the bed?

From Problem 7.37: One method of storing solar energy for use during cloudy days, or at night, is to store it in the form of sensible heat in a rock bed. Assume that the rocks are spherical, 2 cm in diameter, and that the bed void fraction is 0.5. Hint: The surface area of the rocks per unit volume of the bed is $(6/D_p)(1-\varepsilon)$.

GIVEN

- Packed bed of rocks with air blowing through it
- Inlet air temperature ($T_{a,\text{in}}$) = 90°C
- Mass velocity of air ($\dot{m}/A$) = 0.5 kg/s m^2
- Surface area per unit volume (A_s/V) = $(6/D_p)(1-\varepsilon)$
- Length of bed (from solution to Problem 7.37) (L) = 19 cm = 0.19 m
- Initial temperature of rocks ($T_{r,i}$) = 10°C

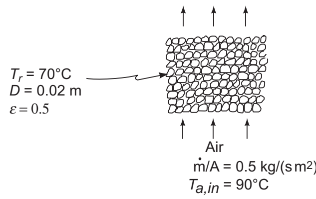
FIND

(a) The time required (t) for $T_{r,\max}$ = 70°C
(b) The rate of heat transfer (q) for that time

ASSUMPTIONS

- Rocks are spherical with diameter (D) = 2 cm = 0.02 m
- Void fraction of the bed (ε) = 0.5
- The rock has the density and thermal conductivity of granite
- The specific heat of the rock is approximately the same as brick or concrete: c = 840 J/(kg K)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 90°C

Density (ρ) = 0.942 kg/m^3

Thermal conductivity (k) = 0.0300 W/(m K)

Kinematic viscosity (ν) = 22.6 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 11, for granite

Density (ρ) = 2750 kg/m^3

Thermal conductivity (k_r) = 3.0 W/(m K)

SOLUTION

The air velocity is given by

$$U_s = \frac{\dot{m}}{A\rho} = \frac{\left(0.5\,\text{kg}/(\text{m}^2\text{s})\right)}{\left(0.942\,\text{kg}/\text{m}^3\right)} = 0.531\ \text{m/s}$$

$$\therefore \qquad Re_{D_p} = \frac{D_p U_s}{\nu(1-\varepsilon)} = \frac{(0.02\,\text{m})(0.531\,\text{m/s})}{\left(22.6\times10^{-6}\ \text{m}^2/\text{s}\right)(1-0.5)} = 940$$

Applying Equation 7.20

$$\frac{\bar{h}_c D_p}{k} = \frac{1-\varepsilon}{\varepsilon}\left(0.5\,Re_{D_p}^{\frac{1}{2}} + 0.2\,Re_{D_p}^{\frac{2}{3}}\right) Pr^{\frac{1}{3}}$$

$$\frac{\bar{h}_c D_p}{k} = \frac{1-0.5}{0.5}\left[0.5(940)^{\frac{1}{2}} + 0.2(940)^{\frac{2}{3}}\right](0.71)^{\frac{1}{3}} = 30.8$$

$$\bar{h}_c = 30.8\,\frac{(0.0300\,\text{W}/(\text{m K}))}{0.02\,\text{m}} = 46.2\ \text{W}/(\text{m}^2\text{K})$$

The upstream rocks in the bed will heat up most quickly because they are exposed to air at the inlet temperature of 90°C. The Biot number for a rock is

$$Bi = \frac{\bar{h}_c D}{2k_r} = \frac{\left(46.2\,\text{W}/(\text{m}^2\text{K})\right)(0.02\,\text{m})}{2(3.0\,\text{W}/(\text{m K}))} = 0.15 > 0.1$$

Therefore, the internal thermal resistance of the rocks cannot be neglected and the chart solution of Figure 2.39 must be used.

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{70°\text{C} - 90°\text{C}}{10°\text{C} - 90°\text{C}} = 0.25$$

$$\frac{1}{Bi} = \frac{1}{0.15} = 6.66$$

From Figure 2.39

$$Fo = \frac{\alpha t}{r_o^2} = 3$$

Solving for the time

$$t = \frac{Fo\, r_o^2}{\alpha} = \frac{Fo\, r_o^2 \rho_r c}{k_r} = \frac{3(0.1\,\text{m})^2 \left(2750\,\text{kg/m}^3\right)(840\,\text{J/(kg K)})((\text{W s})/\text{J})}{(3.0\,\text{W/(m K)})}$$

$$t = 23{,}100 \text{ s} = 6.4 \text{ hours}$$

It will take 6.4 hours for the center of the upstream rocks to reach 70°C.

COMMENTS

Since an average solar radiation is available for more than 6 hours in sunny climates, the rock storage appears to be sized properly

PROBLEM 7.39

An automotive catalytic convertor is a packed bed in which a platinum catalyst is coated on the surface of small alumina spheres. A metal container holds the catalyst pellets and allows engine exhaust gases to flow through the bed of pellets. The catalyst must be heated by the exhaust gases to 300°C before the catalyst can help combust unburned hydrocarbons in the gases. The time required to achieve this temperature is critical because unburned hydrocarbons emitted by the vehicle during a cold start comprise a large fraction of the total emissions from the vehicle during an emission test. A fixed volume of catalyst is required but the shape of the bed can be modified to increase the heat-up rate. Compare the heat-up time for a bed 5 cm diameter and 20 cm long with one 10cm diameter and 5 cm long. The catalyst pellets are spherical, 5 mm diameter, have a density of 2 g/cm^3, thermal conductivity of 12 W/(m K) and specific heat of 1100 J/(kg K). The packed-bed void fraction is 0.5. Exhaust gas from the engine is at a temperature of 400°C, a flow rate of 6.4 gm/s, and has the properties of air.

GIVEN

- A packed bed catalytic converter comprised of platinum coated alumina spheres with exhaust gases flowing through them
- Two possible bed geometries
  - Case *a*: Diameter (D_b) = 5 cm = 0.05 m; Length (L) = 20 cm = 0.2 m
  - Case *b*: Diameter (D_b) = 10 cm = 0.1 m; Length (L) = 5 cm = 0.05 m
- Sphere density (ρ_p) = 2 g/cm^3 = 2000 kg/m^3
- Sphere diameter (D_p) = 5 mm = 0.005 m
- Sphere thermal conductivity (k_p) = 12 W/(m K)
- Void fraction (ε) = 0.5
- Sphere specific heat (c_p) = 1100 J/(kg K)
- Engine exhaust gas temperature (T_g) = 400°C
- Engine exhaust mass flow rate ($\dot{m}$) = 6.4 g/s = 0.0064 kg/s
- Engine exhaust has the properties of air

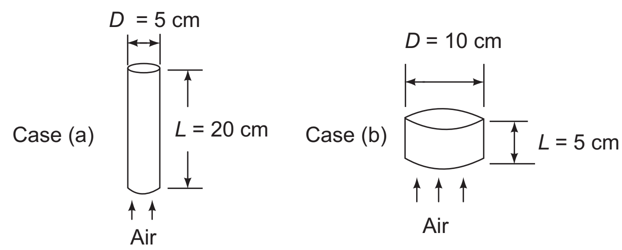
FIND

- The heat-up time (t) for the pellet surface temperature (T_p) to reach 300°C

ASSUMPTIONS

- The initial temperature of the bed (T_o) = 20°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 400°C

- Density (ρ) = 0.508 kg/m^3
- Thermal conductivity (k) = 0.0485 W/(m K)
- Absolute viscosity (μ) = 32.754 × 10^{-6} (Ns)/m
- Prandtl number (Pr) = 0.72

SOLUTION

The Reynolds number is

$$Re_{D_p} = \frac{D_p U_s}{\nu(1-\varepsilon)} = \frac{4\,\text{m}\,D_p}{\pi D_b{}^2 \mu(1-\varepsilon)}$$

Case (a) $$Re_{D_p} = \frac{4(0.0064\,\text{kg/s})(0.005\,\text{m})}{\pi(0.05\,\text{m})^2 \left(32.754\times10^{-6}(\text{N s})/\text{m}\right)\left((\text{kg m})/(\text{N s}^2)\right)(1-0.05)} = 995$$

Case (b) $Re_{D_p} = 249$

Applying Equation (7.20)

$$\frac{\bar{h}_c D_p}{k} = \frac{1-\varepsilon}{\varepsilon}\left(0.5\,Re_{D_p}{}^{\frac{1}{2}} + 0.2\,Re_{D_p}{}^{\frac{2}{3}}\right) Pr^{\frac{1}{3}}$$

Case (a) $$\bar{h}_{ca} = \frac{(0.0485\,\text{W/(m K)})}{0.005\,\text{m}}\left(\frac{1-0.5}{0.5}\right)\left[0.5(995)^{\frac{1}{2}} + 0.2(995)^{\frac{2}{3}}\right](0.71)^{\frac{1}{3}} = 310.4\ \text{W/(m}^2\,\text{K)}$$

Case (b) $\bar{h}_{cb}$ = 137.4 W/(m^2 K)

The Bio number for case (a) is

$$Bi = \frac{\bar{h}_c D_p}{2k_p} = \frac{\left(310.4\,\text{W/(m}^2\text{K)}\right)(0.005\,\text{m})}{2(12\,\text{W/(m K)})} = 0.065 < 0.1$$

Therefore, the internal resistance of the spheres is negligible in both cases and the lumped parameter analysis of Section 2.6.1 can be used. The upstream portion of the bed will reach 300°C first. Since they will be continuously exposed to 400°C air at a constant heat transfer coefficient, Equation (2.84) may be applied. Solving Equation 2.84 for time

$$t = -\frac{c_p \rho_p V}{\bar{h}_c A_s}\ln\left(\frac{T-T_g}{T_o - T_g}\right) \quad \text{where} \quad \frac{V}{A_s} = \frac{\frac{\pi}{6}D_p{}^3}{\pi D_p{}^2} = \frac{D_p}{6}$$

For case (a)

$$t = -\frac{(1100\,\text{J/(kg K)})(2000\,\text{kg/m}^3)}{(310.4\,\text{W/(m}^2\text{K)})\,(\text{J/(Ws)})}\left(\frac{0.005\,\text{m}}{6}\right)\ln\left(\frac{300°\text{C} - 400°\text{C}}{20°\text{C} - 400°\text{C}}\right) = 7.9\text{ s}$$

For case (b) t = 17.8 s

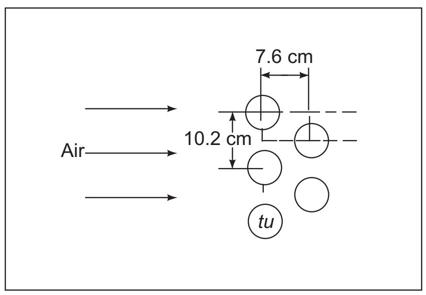
COMMENTS

The short configuration takes more than twice as long to heat up because of the lower heat transfer coefficient due to the lower gas velocity through the bed.

Once the front row of the bed reaches 300°C, catalytic combustion will occur and quickly heat the rest of the packed bed.

PROBLEM 7.40

Determine the average heat transfer coefficient for air at 60°C flowing at a velocity of 1 m/s over a bank of 6-cm-*OD* tubes arranged as shown in the accompanying sketch. The tube-wall temperature is 117°C.



GIVEN

- Air flow through the tube bank shown
- Air temperature (T_a) = 60°C
- Air velocity (U_s) = 1 m/s
- Tube outside diameter (D) = 6 cm = 0.06 m
- Tube wall temperature (T_w) = 117°C

FIND

- The average heat transfer coefficient ($\bar{h}_c$)

ASSUMPTIONS

- Steady state

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 60°C

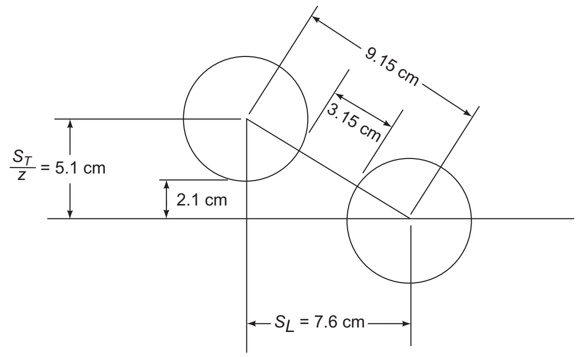
Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At T_w: Pr_s = 0.71

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{max} = U_s \frac{5.1}{2.1} = 2.43 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_{max} D}{\nu} = \frac{(2.43 \text{ m/s})(0.06 \text{ m})}{(19.4 \times 10^{-6} \text{ m}^2/\text{s})} = 7515 \qquad \text{(Transition region)}$$

$$\frac{S_T}{S_L} = \frac{10.2 \text{ cm}}{7.6 \text{ cm}} = 1.34 < 2$$

Therefore, Equation (7.30) is applicable

$$\overline{Nu}_D = 0.35 \left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.60} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.35(1.34)^{0.2} (7515)^{0.60} (0.71)^{0.36} (1) = 69.4$$

However, this Nusselt is applicable only to tube banks of ten or more rows. Since there are only tow rows in this case, the average Nusselt number can be estimated by multiplying the above result by 0.75 as shown in Table 7.3.

$$\overline{Nu}_D = 0.75 \,(69.4) = 52.05$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 52.05 \frac{(0.0279 \text{ W/(m K)})}{0.06 \text{ m}} = 24.2 \text{ W/(m}^2\text{K)}$$

PROBLEM 7.41

Repeat Problem 7.40 for a tube bank in which all of the tubes are spaced with their centerlines 7.5 cm apart.

From Problem 7.40: Determine the average heat transfer coefficient for air at 60°C flowing at a velocity of 1 m/s over a bank of 6-cm-*OD* tubes. The tubewall temperature is 117°C.

GIVEN

- Air flow through a tube bank
- Tube spacing (S) = 7.5 cm = 0.075 m
- Air temperature (T_a) = 60°C
- Air velocity (U_s) = 1 m/s

- Tube outside diameter (D) = 6 cm = 0.06 m
- Tube wall temperature (T_w) = 117°C

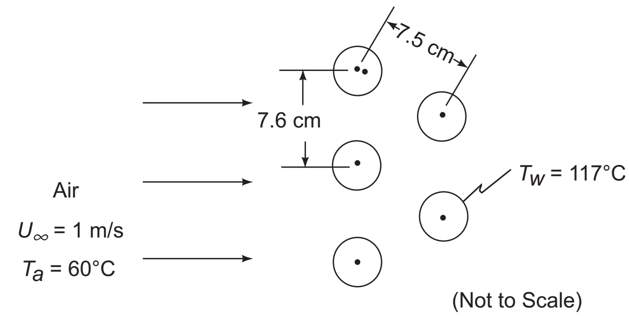
FIND

- The average heat transfer coefficient ($\bar{h}_c$)

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at 60°C

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At T_w

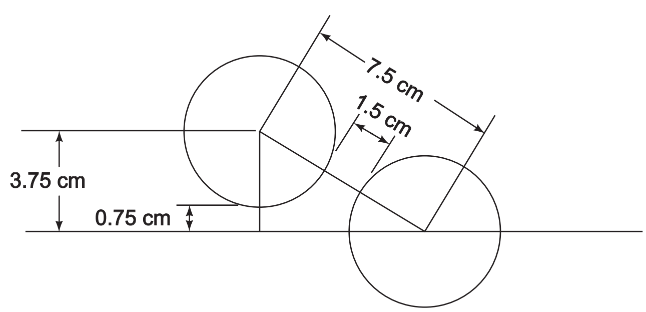
Pr_s = 0.71

SOLUTION

The longitudinal pitch for this case is given by

$$S_T^2 + S_L^2 = S_L^2 \Rightarrow S_L = \sqrt{S_L^2 - S_T^2} = \sqrt{(7.5\,\text{m})^2 - [(7.5\,\text{m})/2]^2} = 6.50 \text{ cm} = 0.065 \text{ m}$$

$$\frac{S_T}{S_L} = \frac{7.5\,\text{m}}{6.5\,\text{m}} = 1.15$$



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_s \frac{3.75}{0.75} = 5.0 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_{\max} D}{\nu} = \frac{(5.0\,\text{m/s})(0.06\,\text{m})}{\left(19.4\times10^{-6}\,\text{m}^2/\text{s}\right)} = 15{,}464 \qquad \text{(Transition region)}$$

Applying Equation (7.30)

$$Nu_D = 0.35\left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{\,0.60}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.35\ (1.15)^{0.2}\ (15{,}464)^{0.60}\ (0.71)^{0.36}\ (1) = 103.8$$

Adjusting the Nusselt number for the first two rows

$$Nu_D = 0.75\ (103.6) = 77.70$$

$$h_c = Nu_D \frac{k}{D} = 77.70 \frac{(0.0279\,\text{W/(m K)})}{0.06\,\text{m}} = 36.1\ \text{W/(m}^2\text{K)}$$

COMMENT

The change in the geometry from Problem 7.40 lead to a 50% increase in the heat transfer coefficient.

PROBLEM 7.42

> **Carbon dioxide gas at 1 atmosphere pressure it to be heated from 25°C to 75°C by pumping it through a tube bank at a velocity of 4 m/s. The tubes are heated by steam condensing within them at 200°C. The tubes are 10 mm outside diameter, are in an in–line arrangement, have a longitudinal spacing of 15 mm and a transverse spacing of 17 mm. If 13 tube rows are required, what is the average heat transfer coefficient and what is the pressure drop of the carbon dioxide?**

GIVEN

- In-line tube bank: condensing steam inside, CO_2 outside
- CO_2 temperatures
  - In: ($T_{g,\text{in}}$) = 25°C
  - Out: ($T_{g,\text{out}}$) = 75°C
- CO_2 velocity (U_s) = 4 m/s
- Steam temperature (T_s) = 200°C
- Tube outside diameter (D) = 10 mm = 0.01 m
- Longitudinal spacing (S_L) = 15 mm = 0.015 m
- Transverse spacing (S_T) = 17 mm = 0.017 m
- Number of tubes (N) = 13

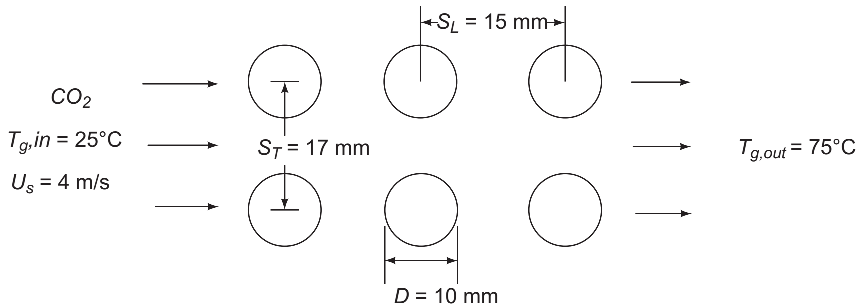
FIND

(a) The average heat transfer coefficient (h_c)
(b) The CO_2 pressure drop (Δp)

ASSUMPTIONS

- Steady state
- The thermal resistances of the condensing steam and the tube walls are negligible (tube wall temperature = T_s)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for carbon dioxide at the average temperature of 50°C

Density (ρ) = 1.6772 kg/m^3

Thermal conductivity (k) = 0.01836 W/(m K)

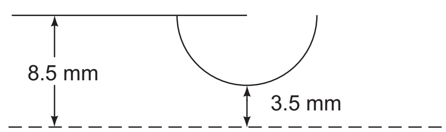
Kinematic viscosity (ν) = 9.64 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.763

Specific heat (c) = 884 J/(kg K)

At the tube surface temperature of 200°C: Pr_s = 0.712

SOLUTION



The maximum CO_2 velocity is

$$U_{\max} = U_s \frac{8.5\,\text{mm}}{3.5\,\text{mm}} = 9.71 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_s D}{\nu} = \frac{(9.71\,\text{m/s})(0.01\,\text{m})}{\left(9.64\times10^{-6}\,\text{m}^2/\text{s}\right)} = 10{,}077 \qquad \text{(Transition regime)}$$

$$\frac{S_T}{S_L} = \frac{17\,\text{mm}}{15\,\text{mm}} = 1.133$$

(a) The Nusselt number for this geometry is obtained from Equation (7.29)

$$\overline{Nu}_D = 0.27\, Re_D^{0.63}\, Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.27\,(10{,}077)^{0.63}\,(0.763)^{0.36} \left(\frac{0.763}{0.712}\right)^{0.25} = 82.9$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 82.9 \frac{(0.01863\,\text{W/(m K)})}{0.01\,\text{m}} = 154.5 \text{ W/(m}^2\text{K)}$$

(No correction is needed since $N > 10$)

(b) From Equation (7.37)

$$\Delta p = f \frac{\rho U^2_{\max}}{2} N$$

The pressure drop coefficient (f) is contained in Figure 7.25

$$\left(\frac{S_T}{D}-1\right)\left(\frac{S_L}{D}-1\right) = \left(\frac{17\,\text{mm}}{10\,\text{mm}}-1\right)\left(\frac{15\,\text{mm}}{10\,\text{mm}}-1\right) = 0.35 \quad \Rightarrow x = 3$$

For $S_L/D = 1.5$ and $Re_D = 10^4$, from Figure 7.25: $f/x \approx 0.5$

$$f = 0.5\ (3) = 1.5$$

$$\Delta p = 1.5\,\frac{(1.6772\,\text{kg/m}^3)(9.7\,\text{m/s})^2}{2}(13)\left((\text{N s}^2)/(\text{kg m})\right)\left((\text{Pa m}^2)/\text{N}\right) = 1.5 \times 10^3\ \text{Pa}$$

PROBLEM 7.43

Estimate the heat transfer coefficient for liquid sodium at 540°C flowing over a 10-row staggered-tube bank of 2.5 cm diameter tubes arranged in an equilateral-triangular array with a 1.5 pitch-to-diameter ratio. The entering velocity is 0.6 m/s, based on the area of the shell, and the tube surface temperature is 200°C. The outlet sodium temperature is 310°C.

GIVEN

- Liquid sodium flowing over an equilateral staggered tube bank
- Sodium temperatures
  - $T_{s,\text{in}} = 540°\text{C}$
  - $T_{s,\text{out}} = 310°\text{C}$
- Number of tube rows (N) = 10
- Tube diameter (D) = 2.5 cm
- Pitch to diameter ratio (S/D) = 1.5
- Entering velocity (U_s) = 0.6 m/s
- Tube surface temperature (T_t) = 200°C

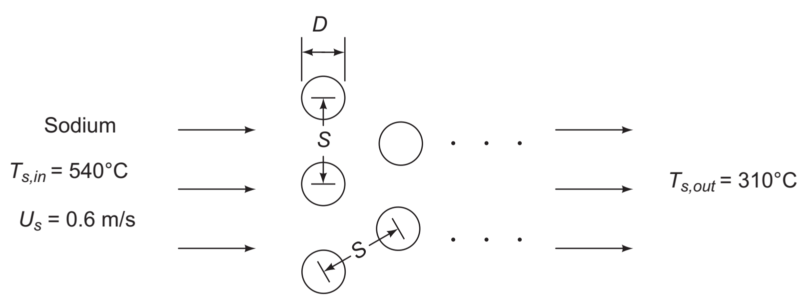
FIND

- Estimate the heat transfer coefficient ($\bar{h}_c$)

ASSUMPTIONS

- Steady state
- The correlation of Equation 7.38 can be applied to a pitch-to-diameter ratio of 1.5
- Tube wall temperature is uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for sodium at the average temperature of 425°C

Density (ρ) = 871.2 kg/m^3

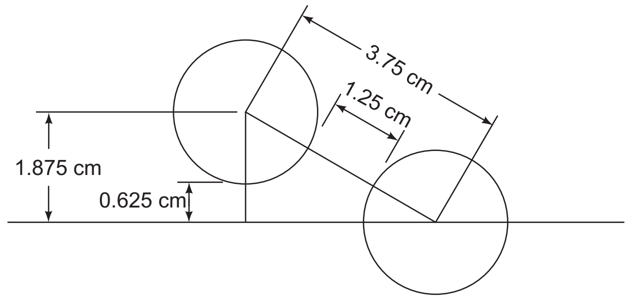
Specific heat (c) = 1285 J/(kg K)

Thermal conductivity (k) = 70 W/(m K)

Kinematic viscosity (ν) = 2.863×10^{-7} m^2/s

Prandtl number (Pr) = 0.0047

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{max} = 0.6 \text{ m/s} \frac{1.875 \text{cm}}{0.625 \text{cm}} = 1.8 \text{ m/s}$$

The Reynolds number for the tube bank is

$$Re_D = \frac{U_s D}{\nu} = \frac{1.8 \text{m/s} (2.5 \times 10^{-2} \text{ m})}{2.863 \times 10^{-7} \text{ m}^2/\text{s}} = 1{,}57{,}200$$

The Reynolds number is out of the range of applicability of Equation (7.38). However, it is the only correlation available for liquid metals in this geometry

$$\overline{Nu}_D = 4.03 + 0.228\,(Re_D\,Pr)^{0.67} = 4.03 + 0.228[1{,}57{,}200\,(0.0047)]^{0.67} = 23.1$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 23.2 \frac{70 \text{ W/(m K)}}{2.5 \times 10^{-2} \text{ m}} = 64960 \text{ W/(m}^2 \text{ K)}$$

$$\simeq 65 \text{ kW/(m}^2 \text{ K)}$$

No correction is needed since $N \geq 10$.

PROBLEM 7.44

Liquid mercury at a temperature of 315°C flows at a velocity of 10 cm/s over a staggered bank of 5/8-in. 16 BWG stainless steel tubes, arranged in an equilateral triangular array with a pitch-to-diameter ratio of 1.375. If water at 2 atm pressure is being evaporated inside the tubes, estimate the average rate of heat transfer to the water per meter length of the bank, if the bank is 10 rows deep and has 60 tubes in it. The boiling heat transfer coefficient is 20,000 W/(m^2 K).

GIVEN

- Liquid mercury flow over an equilaterally staggered tube bank
- Inlet mercury temperature ($T_{m,\text{in}}$) = 315°C
- Mercury velocity (U_s) = 10 cm/s = 0.1 m/s
- Tubes are 5/8 in., 26 BWG stainless steel
- Pitch to diameter ratio (S/D) = 1.375
- Water at 2 atm pressure is being evaporated within the tubes

- Number of rows of tubes (N) = 10
- Number of tubes (N_t) = 60
- The boiling heat transfer coefficient ($\bar{h}_b$) = 20,000 W/(m^2 K)

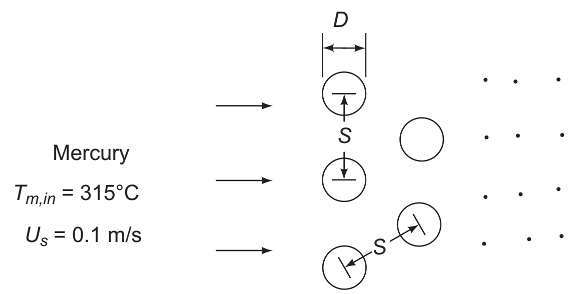
FIND

- The average rate of heat transfer per meter length of the bank (q/L)

ASSUMPTIONS

- Steady state
- Tubes are type 304 stainless steel
- Temperature change of the mercury across the tube bank is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for mercury at the inlet temperature of 315°C

Density (ρ) = 12,847 kg/m^3

Thermal conductivity (k) = 14.02 W/(m K)

Kinematic viscosity (ν) = 0.0673×10^{-6} m^2/s

Prandtl number (Pr) = 0.0083

Specific heat (c) = 134.0 J/(kg K)

From Appendix 2, Table 13, the saturation temperature of water at 2 atm (2.02×10^5 Pa) is T_w = 120°C

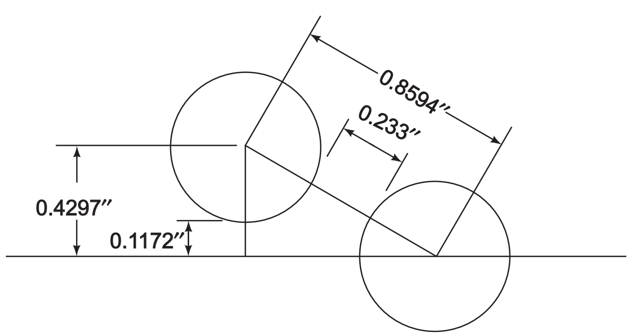
From Appendix 2, Table 42, for 5/8 in. 16 BWG tubes

Outside diameter (D_o) 5/8 in. 0.0159 m

Inside diameter (D_i) = 0.495 in. = 0.0126 m

From Appendix 2, Table 10, the thermal conductivity of type 304 stainless steel (k_s) = 14.4 W/(m K) at 20°C

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_s \frac{0.4297}{0.1172} = 0.37 \text{ m/s}$$

The Reynolds number is

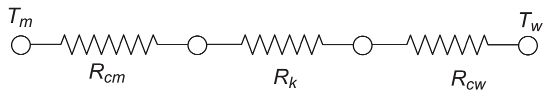
$$Re_D = \frac{U_s D}{\nu} = \frac{(0.37\,\text{m/s})(0.0159\,\text{m})}{\left(0.0673\times10^{-6}\,\text{m}^2/\text{s}\right)} = 87{,}414$$

Applying the correlation of Equation (7.38)

$$\overline{Nu}_D = 4.03 + 0.228\ (Re_D\,Pr)^{0.67} = 4.03 + 0.228\ [87{,}414(0.0083)]^{0.67} = 22.8$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D_o} = 22.8 \frac{(14.02\,\text{W/(m K)})}{0.0159\,\text{m}} = 20{,}147\ \ \text{W/(m}^2\text{K)}$$

The thermal circuit for the problem is shown below



where

R_{cw} = Thermal resistance of the boiling water

$$= \frac{1}{\bar{h}_b A_i} = \frac{1}{\bar{h}_b N_t \pi D_i L} = \frac{1}{\left(20{,}000\,\text{W/(m}^2\text{K)}\right)60\pi(0.0126\,\text{m})\,L} = \left(2.11\times10^{-5}(\text{m K})/\text{W}\right)\left(\frac{1}{L}\right)$$

R_k = Conductive resistance of the tube wall

$$R_k = \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{0.0159\,\text{m}}{0.0126\,\text{m}}\right)}{2\pi L(14.4\,\text{W/(m K)})} = (0.00257\,(\text{m K})/L)\left(\frac{1}{L}\right)$$

R_{cm} = Convective resistance of the mercury side

$$R_{cm} = \frac{1}{\bar{h}_c A_i} = \frac{1}{\bar{h}_c N_t \pi D_o L} = \frac{1}{\left(20{,}147\,\text{W/(m}^2\text{K)}\right)(60)\,\pi(0.0159\,\text{m})\,L} = \left(1.656\times10^{-5}(\text{m K})/L\right)\left(\frac{1}{L}\right)$$

$$R_{\text{total}} = R_{cw} + R_k + R_{cm} = (2.11\times10^{-5} + 0.00257 + 1.656\times10^{-5})((\text{m K})/L)\left(\frac{1}{L}\right)$$

$$= (0.00259(\text{m K})/\text{W})\left(\frac{1}{L}\right)$$

The rate of heat transfer to the steam is

$$\frac{q}{L} = \frac{\Delta T}{L R_{\text{total}}} = \frac{315°\text{C} - 120°\text{C}}{(0.00259\,(\text{m K})/\text{W})} = 75{,}300\ \text{W/m}$$

COMMENT

Note that the thermal resistance of the tube wall is 99% of the total resistance.

PROBLEM 7.45

Compare the rate of heat transfer and the pressure drop for an in-line and a staggered arrangement of a tube bank consisting of 300 tubes, 1.8 m long and 2.5 cm *OD*. The tubes are to be arranged in 15 rows with longitudinal and transverse spacing of 5 cm The tube

surface temperature is 95°C and water at 35°C is flowing at a mass rate of 5400 kg/s over the tubes.

GIVEN

- Water flowing over an in-line and a staggered tube bank
- Number of tubes (N_t) = 300
- Length of tubes (L) = 1.8 m
- Tube outside diameter (D) = 2.5 cm
- Number of rows (N) = 15
- Normal and parallel spacing = 5 cm
- Tube surface temperature (T_t) = 95°C
- Water inlet temperature ($T_{w,\text{in}}$) = 35°C
- Mass flow rate of water ($\dot{m}$) = 5400 kg/s

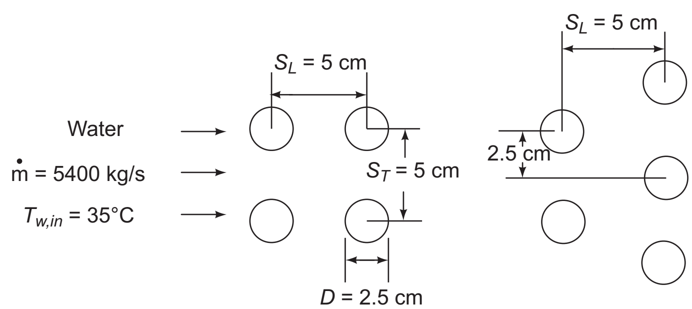
FIND

(a) Compare the rate of heat transfer (q) and
(b) The pressure drop (Δp) for the two configurations

ASSUMPTIONS

- Steady state
- Tube temperature is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 13, for water at the inlet temperature of 35°C

Density (ρ) = 1000 kg/m^3

Thermal conductivity (k) = 0.63 W/(m K)

Absolute viscosity (μ) = 6.92×10^{-4} kg/s

Prandtl number (Pr) = 4.5

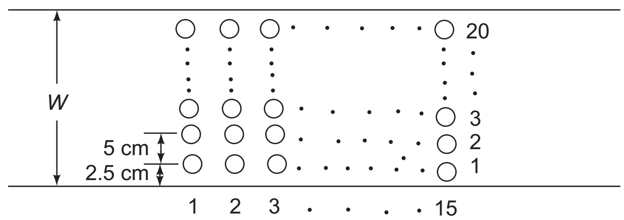
Specific heat (c) = 4174 J/(kg K)

At the tube temperature of 95°C

Pr_s = 1.88

SOLUTION

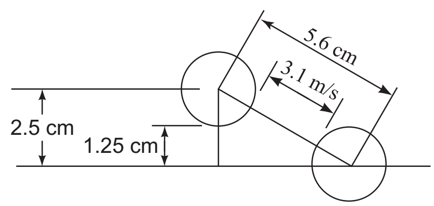
The water velocity can be calculated with the help of the sketch below.



W = 19(5 cm) + 5 cm = 100 cm = 1 m

Therefore, the water velocity is

$$U_s = \frac{\dot{m}}{\rho A} = \frac{5400\,\text{kg/s}}{1000\,\text{kg/m}^3\,(1\,\text{m})(1.8\,\text{m})} = 3\text{ m/s}$$



Therefore, for the staggered configuration, the minimum free area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_s \frac{2.5\,\text{cm}}{1.25\,\text{cm}} = 6\text{ m/s}$$

This is also the maximum velocity for the in-line case.

The Reynolds number for either case is

$$Re_D = \frac{U_{\max} D \rho}{\mu} = \frac{(6\,\text{m/s})(2.5\times10^{-2}\,\text{m})(1000\,\text{kg/m}^3)}{6.92\times10^{-4}\,\text{m/s}} = 2.17\times10^5$$

(Turbulent)

(a) The Nusselt number for the in-line case is given by Equation (7.32)

$$\overline{Nu}_D = 0.021\, Re_D^{0.84}\, Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.021(2.17\times10^5)^{0.84}\,(4.5)^{0.36}\left(\frac{4.5}{1.88}\right)^{0.25} = 1363$$

$$\left(\bar{h}_c\right)_{IL} = \overline{Nu}_D \frac{k}{D} = 1363 \frac{0.63\,\text{W/(m K)}}{2.5\times10^{-2}\,\text{m}} = 34.35\text{ kW/m}^2$$

The Nusselt number for the staggered case is given by Equation (7.33)

$$\overline{Nu}_D = 0.022\, Re_D^{0.84}\, Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.022\,(2.17\times10^5)^{0.84}\,(4.5)^{0.36}\left(\frac{4.5}{1.88}\right)^{0.25} = 1427$$

$$\left(\bar{h}_c\right)_{ST} = \overline{Nu}_D \frac{k}{D} = 1427 \frac{0.63\,\text{W/(m K)}}{2.5\times10^{-2}\,\text{m}} = 35.96\text{ kW/m}^2$$

These heat transfer coefficient differ by only 5%, therefore, the rate of heat transfer for the two tube banks will be nearly equal.

(b) These pressure drop is given by Equation (7.37)

$$\Delta p = f\,\frac{\rho (U_{\max})^2}{2} N, \text{ where } N \text{ is number of rows of tubes}$$

In-line bank:

$$\left(\frac{S_T}{D_o} - 1\right)\left(\frac{S_L}{D_o} - 1\right) = 1$$

From Figure 7.25: $x = 1$, $S_L/D = 2$, $f/x \approx 0.19 \rightarrow f = 0.19$

$$(\Delta p)_{IL} = 0.19\,\frac{(1000\,\text{kg/m}^3)(6\,\text{m/s})^2\,(15)}{2} = 51300 \text{ Pa} = 51.3 \text{ kPa}$$

Staggered tube bank:

From Figure 7.26: x = 1.1. $S_T/D = 2$, $f/x \approx 0.16 \rightarrow f = 0.18$

$$(\Delta p)_{ST} = (51.3 \text{ kPa}) \left(\frac{0.18}{0.19}\right) = 48.6 \text{ kPa}$$

For nearly the same rate of heat transfer, the staggered bank has slightly lower pressure drop.

COMMENTS

These results are only true because the flow is turbulent. Greater difference would occur at lower Reynolds numbers.

PROBLEM 7.46

> **Consider a heat exchanger consisting of 12.5 mm outside diameter copper tubes in a staggered arrangement with transverse spacing 25 mm, longitudinal spacing 30 mm, and 9 tubes in the longitudinal direction. Condensing steam at 150°C flows inside the tubes. The heat exchanger is used to heat a stream of air flowing at 5 m/s from 20°C to 32°C. What is the average heat transfer coefficient and pressure drop for the tube bank?**

GIVEN

- Staggered copper tube bank with condensing steam inside tubes and air flowing over the outside
- Tube outside diameter (D) = 12.5 mm = 0.0125 m
- Transverse spacing (S_T) = 25 mm = 0.025 m
- Longitudinal spacing (S_L) = 30 mm = 0.03 m
- Number of rows of tubes (N) = 9
- Steam temperature (T_s) = 150°C
- Air velocity (U_s) = 5 m/s
- Air temperature. $T_{a,\text{in}}$ = 20°C
  $T_{a,\text{out}}$ = 32°C

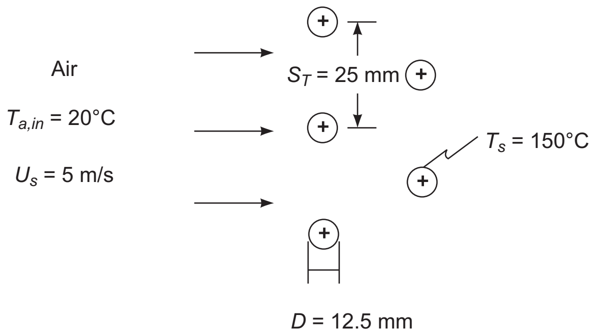
FIND

(a) The average heat transfer coefficient (h_c)
(b) The pressure drop (Δp)

ASSUMPTIONS

- Steady state
- Thermal resistance of the condensing steam and the copper tube is negligible. Therefore, the tube surface temperature = T_s

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average temperature of 26°C

Density (ρ) = 1.157 kg/m^3

Thermal conductivity (k) = 0.0251 W/(m K)

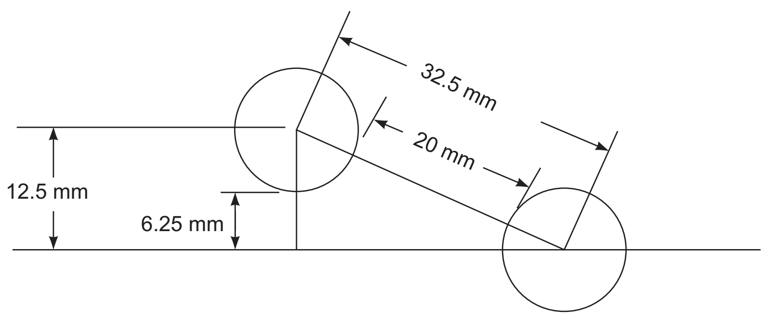
Kinematic viscosity (ν) = 15.9×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Specific heat (c) = 1012 J/(kg K)

At the tube temperature of 150°C: Pr_s = 0.71

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_s \frac{12.5}{6.25} = 10 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_s D}{\nu} = \frac{(10\,\text{m/s})(0.0125\,\text{m})}{\left(15.9\times10^{-6}\,\text{m}^2/\text{s}\right)} = 7862 \qquad \text{(Transition regime)}$$

$$\frac{S_T}{S_L} = \frac{25\,\text{mm}}{30\,\text{mm}} = 0.833$$

(a) Applying equation (7.30)

$$\overline{Nu}_D = 0.35\left(\frac{S_T}{S_L}\right) 0.2\; Re_D^{0.60}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.35\;(0.833)^{0.2}\;(7862)^{0.60}\;(0.71)^{0.36}\;(1) = 64.9$$

However, this Nusselt is applicable only to tube banks of ten or more rows. Since there are only 9 rows in this case, the average Nusselt number can be estimated by multiplying the above result by 0.99 as shown in Table 7.3.

$$\overline{Nu}_D = 0.99\ (64.9) = 64.3$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 64.3 \frac{(0.0251\,\text{W/(m K)})}{0.0125\,\text{m}} = 129\ \text{W/(m}^2\text{K)}$$

(b) The pressure drop is given by Equation (7.37)

$$\Delta p = f \frac{\rho (U_{\max})^2}{2} N$$

From Figure 7.26 For $\frac{S_T}{S_L} = 0.833$, $Re = 7862 \rightarrow x = 1$

For $\frac{S_D}{D} = \frac{25}{12.5} = 2 \rightarrow \frac{f}{x} = 0.4 \rightarrow f = 0.4$

$$\Delta p = \frac{(0.4)\left(1.157\,\text{kg/m}^3\right)(10\,\text{m/s})}{2} (9)\left((\text{N s}^2)/(\text{kg m})\right)\left((\text{Pa m}^2)/\text{N}\right) = 208\ \text{Pa}$$

Chapter 8

PROBLEM 8.1

In a heat exchanger, air flows over brass tubes of 1.8 cm *ID* and 2.1 cm *OD* that contain steam. The convective heat-transfer coefficients on the air and steam sides of the tubes are 70 W/(m^2 K) and 210 W/(m^2 K), respectively. Calculate the overall heat transfer coefficient for the heat exchanger (a) based on the inner tube area, (b) based on the outer tube area.

GIVEN

- Air flow over brass tubes containing steam
- Tube diameters
  - Inside (D_i) = 1.8 cm = 0.018 m
  - Outside (D_o) = 2.1 cm = 0.021 m
- Convective heat transfer coefficients
  - Air side $\left(\bar{h}_o\right)$ = 70 W/(m^2 K)
  - Steam side $\left(\bar{h}_i\right)$ = 210 W/(m^2 K)

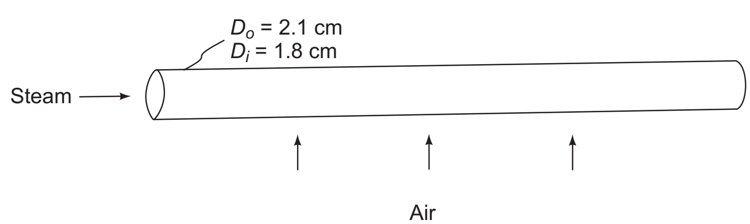
FIND

- The overall heat transfer coefficient for the heat exchanger based on (a) the inner tube area (U_i) and (b) the outer tube area (U_o)

ASSUMPTIONS

- The heat transfer coefficients are uniform over the transfer surfaces

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, the thermal conductivity of brass at 20°C (k_b) = 111 W/(m K)

SOLUTION

(a) The overall heat transfer coefficient based on the inner area is given by Equation (8.3)

$$U_i = \frac{1}{\left(\dfrac{1}{\bar{h}_i}\right) + \left[\dfrac{A_i \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L}\right] + \left(\dfrac{A_i}{A_o \bar{h}_o}\right)}$$

where A_i = inside area = $\pi D_i L$

A_o = outside area = $\pi D_o L$

$$\therefore \quad U_i = \frac{1}{\left(\dfrac{1}{\bar{h}_i}\right) + \left[\dfrac{D_i \ln\left(\frac{D_o}{D_i}\right)}{2k}\right] + \left(\dfrac{D_i}{D_o \bar{h}_o}\right)}$$

$$U_i = \frac{1}{\frac{1}{(210\ \text{W/(m}^2\ \text{K)})} + \left[\frac{(0.018\,\text{m})\ln\left[\frac{0.021\,\text{m}}{0.018\,\text{m}}\right]}{2(111\text{W/(m K)})}\right] + \left(\frac{0.018\,\text{m}}{0.021\,\text{m}(70\ \text{W/(m}^2\ \text{K)})}\right)} = 58.8\ \text{W/(m}^2\ \text{K)}$$

(b) The overall heat transfer coefficient based on the outer area is given by Equation (8.2)

$$U_o = \frac{1}{\left(\frac{A_o}{A_i \bar{h}_i}\right) + \left[\frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L}\right] + \left(\frac{1}{\bar{h}_o}\right)} = \frac{1}{\left(\frac{D_o}{D_i \bar{h}_i}\right) + \left[\frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\frac{0.021\,\text{m}}{(0.018\,\text{m})(210\ \text{W/(m}^2\ \text{K)})} + \left[\frac{(0.021\,\text{m})\ln\left[\frac{0.021\,\text{m}}{0.018\,\text{m}}\right]}{2\pi(111\ \text{W/(m K)})}\right] + \frac{1}{(70\ \text{W/(m}^2\ \text{K)})}} = 50.4\ \text{W/(m}^2\ \text{K)}$$

PROBLEM 8.2

Repeat Problem 8.1 but assume that a fouling factor on the inside of the tube of 0.00018 (m^2 K)/W has developed during operation.

From Problem 8.1: In a heat exchanger, air flows over brass tubes of 1.8 cm *ID* and 2.1 cm *OD* that contain steam. The convective heat-transfer coefficients on the air and steam sides of the tubes are 70 W/(m^2 K) and 210 W/(m^2 K), respectively. Calculate the overall heat transfer coefficient for the heat exchanger (a) based on the inner tube area, (b) based on the outer tube area.

GIVEN

- Air flow over brass tube containing steam
- Tube diameters
  - Inside (D_i) = 1.8 cm = 0.018 m
  - Outside (D_o) = 2.1 cm = 0.021 m
- Convective heat transfer coefficients
  - Air side $(\bar{h}_o)$ = 70 W/(m^2 K)
  - Steam side $(\bar{h}_i)$ = 210 W/(m^2 K)
- Fouling factor on the inside of the tube (R_d) = 0.0018 (m^2 K)/W

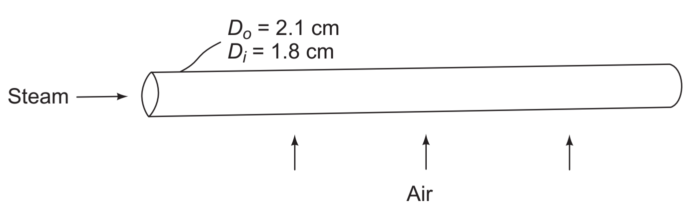
FIND

- The overall heat transfer coefficient for the heat exchanger based on (a) the inner tube area (U_i) and (b) the outer tube area (U_o)

ASSUMPTIONS

- The heat transfer coefficients are uniform over the transfer surfaces

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, the thermal conductivity of brass (k_b) = 111 W/(m K)

SOLUTION

From the solution to Problem 8.1 $\quad U_i$ = 58.76 W/(m^2 K)

U_o = 50.36 W/(m^2 K)

without fouling. The overall heat transfer coefficient with fouling (U_d) can be calculated by rearranging Equation (8.4)

$$U_d = \frac{1}{R_d + \frac{1}{U}}$$

(a) Based on the inner tube area

$$U_{di} = \frac{1}{R_{di} + \frac{1}{U_i}} = \frac{1}{\left(0.0018\,(\mathrm{m^2\,K})/\mathrm{W}\right) + \frac{1}{\left(58.8\ \mathrm{W/(m^2\,K)}\right)}} = 53.1\ \mathrm{W/(m^2\,K)}$$

(b) To base the overall heat transfer coefficient on the outer tube area, the fouling factor must also be based on the outer tube area

$$R_{do} = \frac{A_o}{A_i} R_{di} = \frac{D_o}{D_i} R_{di}$$

$$U_{do} = \frac{1}{R_{do} + \frac{1}{U_o}} = \frac{1}{\frac{D_o}{D_i} R_{do} + \frac{1}{U_o}} = \frac{1}{\frac{2.1\,\mathrm{m}}{1.8\,\mathrm{m}}\left(0.0018\,(\mathrm{m^2\,K})/\mathrm{W}\right) + \frac{1}{\left(50.4\ \mathrm{W/(m^2\,K)}\right)}} = 45.5\ \mathrm{W/(m^2\,K)}$$

PROBLEM 8.3

A light oil flows through a copper tube of 2.6 cm *ID* and 3.2 cm *OD*. Air is flowing over the exterior of the tube. The convection heat transfer coefficient for the oil is 120 W/(m^2 K) and for the air is 35 W/(m^2 K). Calculate the overall heat transfer coefficient based on the outside area of the tube (a) considering the thermal resistance of the tube, (b) neglecting the resistance of the tube.

GIVEN

- Air flow over a copper tube with oil flow within the tube
- Tube diameters
  - Inside (D_i) = 2.6 cm = 0.026 m
  - Outside (D_o) = 3.2 cm = 0.032 m
- Convective heat transfer coefficients
  - Oil $\left(\bar{h}_i\right)$ = 120 W/(m^2 K)
  - Air $\left(\bar{h}_o\right)$ = 35 W/(m^2 K)

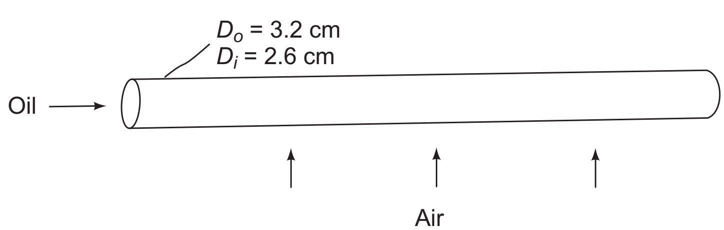
FIND

- The overall heat transfer coefficient based on the outside tube area (U_o), (a) considering the thermal resistance of the tube, and (b) neglecting the tube resistance

ASSUMPTIONS

- Uniform heat transfer coefficients
- Variation of thermal properties is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) (at 127°C)

SOLUTION

(a) The overall heat transfer coefficient based on the outer tube area is given by Equation (8.2)

$$U_o = \frac{1}{\left(\dfrac{A_o}{A_i \bar{h}_i}\right) + \left[\dfrac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L}\right] + \left(\dfrac{1}{\bar{h}_o}\right)}$$

where A_i = inside area = $\pi D_i L$

A_o = outside area = $\pi D_o L$

$$\therefore \quad U_o = \frac{1}{\left(\dfrac{D_o}{D_i \bar{h}_i}\right) + \left[\dfrac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k}\right] + \left(\dfrac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\dfrac{0.032\,\text{m}}{(0.026\,\text{m})\left(120\ \text{W/(m}^2\ \text{K)}\right)} + \left[\dfrac{(0.032\,\text{m})\ln\left[\frac{0.032\,\text{m}}{0.026\,\text{m}}\right]}{2(392\ \text{W/(m K)})}\right] + \dfrac{1}{35\ \text{W/(m}^2\ \text{K)}}}$$

$$U_o = \frac{1}{(0.01026 + 0.0000085 + 0.02857)\ (\text{m}^2\ \text{K})/\text{W}} = 25.8\ \text{W/(m}^2\ \text{K)}$$

(b) The thermal resistance of the tube wall can be neglected by eliminating the bracketed term in the denominators of the expressions above

$$U_o = \frac{1}{(0.01026 + 0.02857)\ (\text{m}^2\ \text{K})/\text{W}} = 25.8\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

- Neglecting the thermal resistance of the tube wall has a negligible effect on the overall heat transfer coefficient.
- The thermal resistance on the air side is 74% of the overall thermal resistance.

PROBLEM 8.4

Repeat Problem 8.3, but assume that fouling factor of 0.0009 (m^2 K)/W on the inside and 0.0004 (m^2 K)/W on the outside respectively have developed.

From Problem 8.3: A light oil flows through a copper tube of 2.6 cm *ID* and 3.2 cm *OD*. Air is flowing over the exterior of the tube. The convection heat transfer coefficient for the oil is 120 W/(m^2 K) and for the air is 35 W/(m^2 K). Calculate the overall heat transfer coefficient based on the outside area of the tube (a) considering the thermal resistance of the tube, (b) neglecting the resistance of the tube.

GIVEN

- Air flow over a copper tube with oil flow within the tube
- Tube diameters
  - Inside (D_i) = 2.6 cm = 0.026 m
  - Outside (D_o) = 3.2 cm = 0.032 m

- Convective heat transfer coefficients
  - Oil $(\bar{h}_i)$ = 120 W/(m^2 K)
  - Air $(\bar{h}_o)$ = 35 W/(m^2 K)
- Fouling factors
  - Inside (R_{di}) = 0.0009 (m^2 K)/W
  - Outside (R_{do}) = 0.0004 (m^2 K)/W

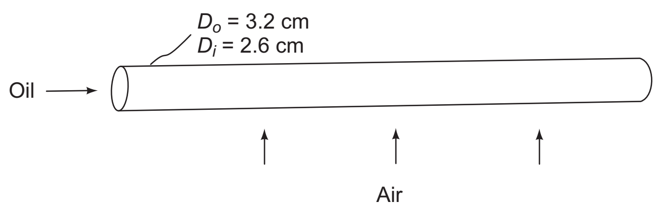
FIND

- The overall heat transfer coefficient based on the outside tube area (U_o), (a) considering the thermal resistance of the tube, and (b) neglecting the tube resistance

ASSUMPTIONS

- Uniform heat transfer coefficients
- Variation of thermal properties is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) (at 127°C)

SOLUTION

From the solution to Problem 8.3 with or without tube wall resistance

$$U_o = 25.75 \text{ (m}^2\text{ K)/W}$$

(a) The overall heat transfer with fouling can be calculated by rearranging Equation (8.4)

$$U_d = \frac{1}{R_d + \frac{1}{U}}$$

where R_d = the total fouling factor

Based on the outside tube area

$$R_d = R_{do} + \frac{A_o}{A_i} R_{di} = R_{do} + \frac{D_o}{D_i} R_{di}$$

$$R_d = \left(0.0004 \text{ (m}^2\text{K)/W}\right) + \left(\frac{3.2\text{ m}}{2.6\text{ m}}\right)\left(0.0009 \text{ (m}^2\text{K)/W}\right) = 0.001508 \text{ (m}^2\text{ K)/W}$$

$$U_d = \frac{1}{\left(0.001508 \text{ (m}^2\text{ K)/W}\right) + \frac{1}{\left(25.8 \text{ W/(m}^2\text{ K)}\right)}} = 24.8 \text{ W/(m}^2\text{ K)}$$

(b) The tube wall resistance is negligible as shown in the solution to Problem 8.3.

COMMENTS

The given fouling factors lead to a 4% decrease in the overall heat transfer coefficient based on the outer tube wall area.

PROBLEM 8.5

Water flowing in a long aluminum tube is to be heated by air flowing perpendicular to the exterior of the tube. The *ID* of the tube is 1.85 cm and its *OD* is 2.3 cm. The mass flow rate of the water through the tube is 0.65 kg/s and the temperature of the water in the tube averages 30°C. The free stream velocity and ambient temperature of the air are 10 m/s and 120°C, respectively. Estimate the overall heat transfer coefficient for the heat exchanger using appropriate correlations from previous chapters. State all your assumptions.

GIVEN

- Air flowing perpendicular to the exterior of an aluminum tube with water flowing within the tube
- Tube diameters
  - Inside (D_i) = 1.85 cm = 0.0185 m
  - Outside (D_o) = 2.3 cm = 0.023 m
- Mass flow rate of water (m_w) = 0.65 kg/s
- Average temperature of the water (T_w) = 30°C
- Air free stream velocity (V_a) = 10 m/s
- Air temperature (T_a) = 120°C

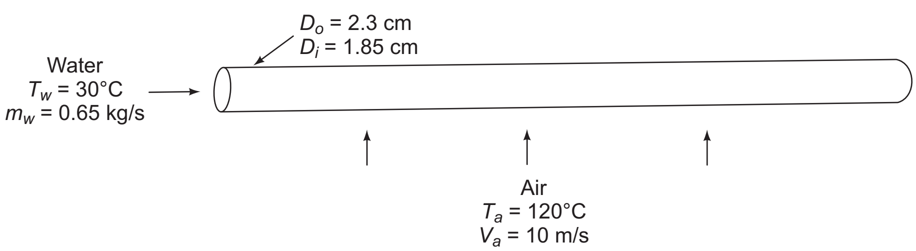
FIND

- The overall heat transfer coefficient (U)

ASSUMPTIONS

- Steady state
- The variation of the Prandtl number of air with temperature is negligible
- The aluminum is pure

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, thermal conductivity of aluminum (k_{al}) = 238 W/(m K) at 75°C

From Appendix 2, Table 13, for water at 30°C

Density (ρw) = 996 kg/m^3

Thermal conductivity (k_w) = 0.615 W/(m K)

Absolute viscosity (μ_w) = 792×10^{-6} (Ns)/m^2

Prandtl number (Pr_w) = 5.4

From Appendix 2, Table 27, for dry air at 120°C

Thermal conductivity (k_a) = 0.0320 W/(m K)

Kinematic viscosity (ν) = 26.0×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number on the water side is

$$Re_D = \frac{V_w D_i}{\nu} = \frac{4\dot{m}_w}{\pi D_i \mu} = \frac{4(0.65\ \text{kg/s})}{\pi(0.0185\,\text{m})\left(792\times10^{-6}\ (\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 56{,}500\ \text{(Turbulent)}$$

The Nusselt number on the water side is given by Equation (6.63)

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\ (56{,}500)^{0.8}\ (5.4)^{0.4} = 286$$

$$\bar{h}_i = \overline{Nu}_D\ \frac{k_w}{D_i} = 286\,\frac{(0.615\ \text{W/(m K)})}{0.0185\,\text{m}} = 9508\ \text{W/(m}^2\ \text{K)}$$

The Reynolds number on the air side is

$$Re_D = \frac{V_a D_o}{\nu_a} = \frac{(10\ \text{m/s})(0.023\,\text{m})}{\left(26\times10^{-6}\ \text{m}^2/\text{s}\right)} = 8846$$

Applying Equation (7.3) and Table 7.1 but neglecting the Prandtl number variation

$$\overline{Nu}_D = 0.26\ Re_D^{0.6}\ Pr^{0.36} = 0.26\ (8846)^{0.6}\ (0.71)^{0.36} = 53.64$$

$$\bar{h}_o = \overline{Nu}_D\ \frac{k_a}{D_o} = 53.64\,\frac{(0.0320\ \text{W/(m K)})}{0.023\,\text{m}} = 74.63\ \text{W/(m}^2\ \text{K)}$$

The overall heat transfer coefficient based on the outer tube diameter is given by Equation (8.2)

$$U_o = \frac{1}{\left(\dfrac{A_o}{A_i \bar{h}_i}\right) + \left[\dfrac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L}\right] + \left(\dfrac{1}{\bar{h}_o}\right)}$$

where A_i = inside area = $\pi D_i L$

A_o = outside area = $\pi D_o L$

$$\therefore \qquad U_o = \frac{1}{\left(\dfrac{D_o}{D_i \bar{h}_i}\right) + \left[\dfrac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k}\right] + \left(\dfrac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\dfrac{0.023\,\text{m}}{(0.0185\,\text{m})\left(9508\ \text{W/(m}^2\ \text{K)}\right)} + \left[\dfrac{(0.023\,\text{m})\ln\left(\frac{0.023\,\text{m}}{0.0185\,\text{m}}\right)}{2\left(238\ \text{W/(m K)}\right)}\right] + \dfrac{1}{74.6\ \text{W/(m}^2\ \text{K)}}} = 73.8\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

The air side thermal resistance accounts for 99% of the total resistance. The water side convective resistance and the conductive resistance of the tube are of the same order of magnitude.

PROBLEM 8.6

Hot water is used to heat air in a double pipe heat exchanger. If the heat transfer coefficients on the water side and on the air side are 550 W/(m^2 K) and 55 W/(m^2 K), respectively, calculate the overall heat transfer coefficient based on the outer diameter. The heat exchanger pipe is 5 cm, schedule 40, made of steel (*k* = 54 W/m K), with water inside. Express your answer in W/m^2 °C.

GIVEN

- A double pipe heat exchanger with water in inner tube and air in the annulus
- Heat transfer coefficients
  - Water side $(\bar{h}_i)$ = 550 W/(m^2 K)
  - Air side $(\bar{h}_o)$ = 55 W/(m^2 K)
- Inner pipe: 5 cm, schedule 40, made of steel
- Pipe thermal conductivity (k) = 54 W/(m K)

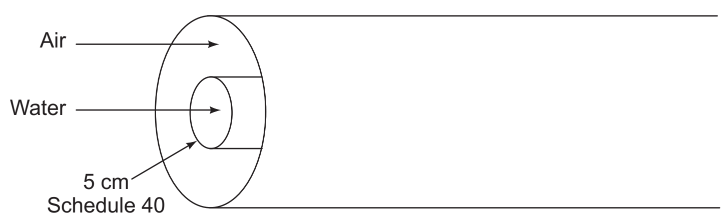
FIND

- The overall heat transfer coefficient based on the outer diameter (U_o) in W/m°C

ASSUMPTIONS

- Uniform heat transfer coefficients

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 41, for 5 cm, schedule 40 pipe

- Outside diameter (D_o) = 5.98 cm
- Wall thickness (t) = 0.385 cm

SOLUTION

Inside diameter (D_i) = $D_o - 2t$ = 5.98 – 2(0.385) = 5.21 cm

The overall heat transfer coefficient based on the outer tube diameter is given by Equation (8.2)

$$U_o = \frac{1}{\left(\frac{A_o}{A_i \bar{h}_i}\right) + \left[\frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

where A_i = inside area = $\pi D_i L$

A_o = outside area = $\pi D_o L$

$$\therefore \quad U_o = \frac{1}{\left(\frac{D_o}{D_i \bar{h}_i}\right) + \left[\frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\dfrac{5.98\,\text{cm}}{(5.21\,\text{cm})(550\,\text{W/(m}^2\text{K))}} + \left[\dfrac{5.98\times10^{-2}\,\text{m}\ln\left(\frac{5.98\,\text{cm}}{5.21\,\text{cm}}\right)}{2(54\,\text{W/(m K))}}\right] + \dfrac{1}{55\,\text{W/(m}^2\text{K)}}}$$

$$\Rightarrow \qquad U_o = \frac{1}{2.087\times10^{-3}\,(\text{K m}^2)/\text{W} + 0.076\times10^{-3}\,(\text{K m}^2)/\text{W} + 0.01818\,(\text{K m}^2)/\text{W}}$$

$$\Rightarrow \qquad U_o = 49.16\ \text{W/(m}^2\ \text{K)}$$

PROBLEM 8.7

Repeat Problem 8.6, but assume that a fouling factor of 0.173 (m^2 K)/kW based on the tube outside diameter has developed over time.

From Problem 8.6: Hot water is used to heat air in a double pipe heat exchanger. If the heat transfer coefficients on the water side and on the air side are 550 W/(m^2 K) and 55 W/(m^2 K), respectively, calculate the overall heat transfer coefficient per unit length based on the outer diameter. The heat exchanger pipe is 5 cm, schedule 40, made of steel (*k* = 54 W/(m K)), with water inside.

GIVEN

- A double pipe heat exchanger with water in inner tube and air in the annulus
- Heat transfer coefficients
  - Water side $(\bar{h}_i)$ = 550 W/(m^2 K)
  - Air side $(\bar{h}_o)$ = 55 W/(m^2 K)
- Inner pipe: 5 cm, schedule 40, made of steel
- Pipe thermal conductivity (k) = 54 W/(m K)
- Fouling factor (R_d) = 0.173×10^{-3} (m^2 K)/W, based on the tube outside area

FIND

- The overall heat transfer coefficient based on the outer diameter (U_o) in W/m°C

ASSUMPTIONS

- Uniform heat transfer coefficients
- The given fouling factor is based on the outer tube diameter (D_o)

SOLUTION

From the solution to Problem 8.6: U_o = 49.16 W/(m^2 K) without the fouling factor. The overall heat transfer coefficient with the fouling factor (U_d) can be found by rearranging Equation (8.4)

$$U_d = \frac{1}{R_d + \dfrac{1}{U}} = \frac{1}{0.173\times10^{-3}\,(\text{m}^2\text{K})/\text{W} + \dfrac{1}{49.16}(\text{m}^2\text{K})/\text{W}}$$

$$\Rightarrow \qquad U_d = 48.75\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

The inclusion of the fouling factor reduces the overall heat transfer coefficient by around 1%.

PROBLEM 8.8

The heat transfer coefficient on the inside of a copper tube (1.9 cm *ID* and 2.3 cm *OD*) is 500 W/(m^2 K) and 120 W/(m^2 K) on the outside, but a deposit with a fouling factor of 0.009 (m^2 K)/W (based on the tube outside diameter) has built up over time. Estimate the percent increase in the overall heat transfer coefficient if the deposit were removed.

GIVEN

- Heat transfer through a copper tube
- Heat transfer coefficients
  - Inside $(\bar{h}_i)$ = 500 W/(m^2 K)
  - Outside $(\bar{h}_o)$ = 120 W/(m^2 K)
- Tube diameters
  - Inside (D_i) = 1.9 cm = 0.019 m
  - Outside (D_o) = 2.3 cm = 0.023 m
- Fouling factor (R_d) = 0.009 (m^2 K)/W

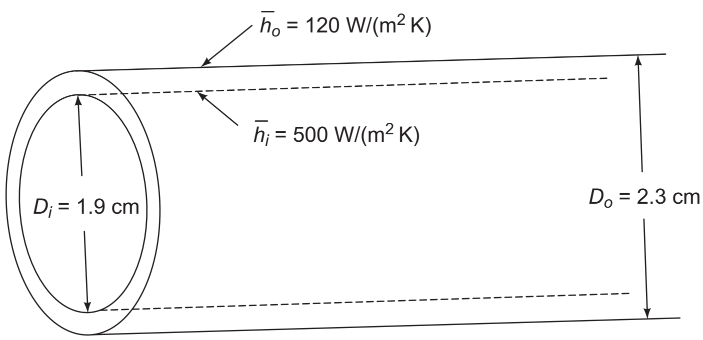
FIND

- Percent increase in the overall heat transfer coefficient if the deposit were removed

ASSUMPTIONS

- Constant thermal conductivity properties
- The copper is pure

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) at 127°C

SOLUTION

The overall heat transfer coefficient without fouling based on the outside tube area is given by Equation (8.2)

$$U_o = \frac{1}{\left(\frac{A_o}{A_i \bar{h}_i}\right) + \left[\frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L}\right] + \left(\frac{1}{\bar{h}_o}\right)} = \frac{1}{\left(\frac{D_o}{D_i \bar{h}_i}\right) + \left[\frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\frac{0.023\,\text{m}}{(0.019\,\text{m})(500\ \text{W/(m}^2\ \text{K)})} + \left[\frac{(0.023\,\text{m})\ln\left(\frac{0.023\,\text{m}}{0.019\,\text{m}}\right)}{2(392\ \text{W/(m K)})}\right] + \frac{1}{(120\ \text{W/(m}^2\ \text{K)})}} = 92.9\ \text{W/(m}^2\ \text{K)}$$

The overall heat transfer coefficient with fouling is given by Equation (8.4)

$$U_d = \frac{1}{R_d + \frac{1}{U_o}} = \frac{1}{(0.009\ (\text{m}^2\text{K})/\text{W}) + \frac{1}{(92.9\ \text{W/(m}^2\text{K)})}} = 50.6\ \text{W/(m}^2\text{K)}$$

The percent increase is

$$\frac{U_o - U_d}{U_d} \times 100 = \frac{92.9 - 50.6}{50.6} \times 100 = 84\%$$

PROBLEM 8.9

> **In a shell-and-tube heat exchanger with $\bar{h}_i = \bar{h}_o = 5600$ W/(m^2 K) and negligible wall resistance, by what percent would the overall heat transfer coefficient (based on the outside area) change if the number of tubes was doubled? The tubes have an outside diameter of 2.5 cm and a tube wall thickness of 2 mm. Assume that the flow rates of the fluids are constant, the effect of temperature on fluid properties is negligible, and the total cross sectional area of the tubes is small compared to the flow area of the shell.**

GIVEN

- Shell and tube heat exchanger
- Heat transfer coefficients: $\bar{h}_i = \bar{h}_o = 5600$ W/(m^2 K)
- Negligible wall resistance
- Tube outside diameter (D_o) = 2.5 cm = 0.025 m
- Tube wall thickness (t) = 2 mm = 0.002 m

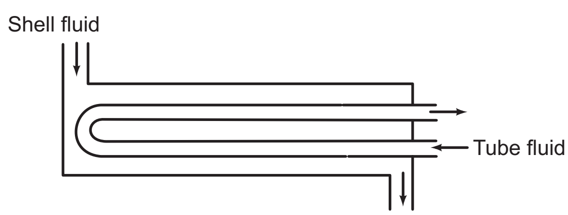
FIND

- The percent change in the overall heat transfer coefficient if the number of tubes is doubled

ASSUMPTIONS

- Flow rates of the fluids are constant
- The effect of temperature of fluid properties is negligible
- The fluid flow is turbulent in the tubes (this is consistent with the high heat transfer coefficients)

SKETCH



SOLUTION

The inside diameter (D_i) = $D_o - 2t$ = 2.5 cm – 2(0.2cm) = 2.1 cm = 0.021 m

The original overall heat transfer coefficient is given by Equation (8.2). Neglecting wall resistance

$$\frac{1}{U_o} = \frac{A_o}{A_i \bar{h}_i} + \frac{1}{\bar{h}_o} = \frac{D_o}{D_i \bar{h}_i} + \frac{1}{\bar{h}_o} = \left(\frac{2.5}{2.1}\right) \frac{1}{\left(5600\ \text{W/(m}^2\text{K)}\right)} + \frac{1}{\left(5600\ \text{W/(m}^2\text{K)}\right)}$$

$$U_o = 2557\ \text{W/(m}^2\ \text{K)}$$

Since the total cross sectional area of the tubes is small compared to the flow area of the shell, doubling the number of tubes will have little effect on the shell-side fluid velocity. Therefore, the shell side heat transfer coefficient will not change. Doubling the number of tubes with the same mass flow rate cuts the fluid velocity in the tubes in half. For turbulent flow in tube (from Section 6.5)

$$\bar{h}_c \propto Re_D^{\,0.8} \quad \Rightarrow \bar{h}_c \propto V^{\,0.8}$$

$$\frac{\bar{h}_{c,\text{new}}}{\bar{h}_{c,\text{orig}}} = \left(\frac{V_{\text{new}}}{V_{\text{orig}}}\right)^{0.8} = (0.5)^{0.8} = 0.574$$

$$\bar{h}_{c,\text{new}} = 0.574\left(5600\ \text{W/(m}^2\text{K)}\right) = 3214\ \text{W/(m}^2\text{K)}$$

$$\therefore\ \frac{1}{U_{o,\text{new}}} = \left(\frac{2.5}{2.1}\right)\frac{1}{\left(3214\ \text{W/(m}^2\text{K)}\right)} + \frac{1}{\left(5600\ \text{W/(m}^2\text{K)}\right)} = 1822\ \text{W/(m}^2\text{K)}$$

$$\%\ \text{Change} = \frac{U_o - U_{o,\text{new}}}{U_o} \times 100 = \frac{2557 - 1822}{2557} \times 100 = 29\%\ \text{Decrease}$$

PROBLEM 8.10

Water at 27°C enters a No. 18 BWG 1.6 cm condenser tube made of nickel chromium steel (*k* = 26 W/(m K)) at a rate of 0.32 L/s. The tube is 3 m long and its outside is heated by steam condensing at 49°C. Under these conditions, the average heat-transfer coefficient on the water side is 9.9 kW/(m^2 K). The heat transfer coefficient on the steam side may be taken as 11.3 kW/(m^2 K). On the interior of the tube, however, there is a scale forming with a thermal conductance equivalent to 5.6 kW/(m^2 K). (a) Calculate the overall heat transfer coefficient *U* per square foot of exterior surface area after the scale has formed, and (b) calculate the exit temperature of the water.

GIVEN

- Water flow in nickel chromium steel condenser tube
- Water flow rate $(\dot{v})$ = 0.32 L/s
- Tube: 1.6 cm, No. 18 BWG
- Inlet water temperature ($T_{w,\text{in}}$) = 27°C
- Steel thermal conductivity (k) = 26 W/(m K)
- Tube length (L) = 3 m
- Steam temperature (T_s) = 49°C
- Water side heat transfer coefficient $(\bar{h}_i)$ = 9.9 kW/(m^2 K)
- Steam side heat transfer coefficient $\bar{h}_o$ = 11.3 kW/(m^2 K)
- Interior scaling conductance ($1/R_i$) = 5.6 kW/(m^2 K)

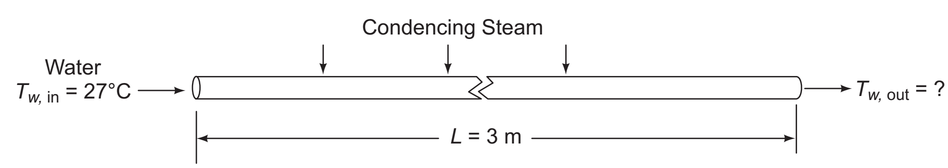
FIND

(a) The overall heat transfer coefficient (U_o) based on exterior surface area
(b) Water exit temperature ($T_{w,\text{out}}$)

ASSUMPTIONS

- Negligible scaling on the outside of the tube (R_o = 0)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 42, for 5/8 in No. 18 BWG Tubing D_o = 1.57 cm
D_i = 1.32 cm

From Appendix 2, Table 13, for water at an estimated average temperature of 38°C

Density (ρ) = 1012 kg/m^3

Specific heat (c_p) = 4174 J/(kg K)

SOLUTION

(a) The overall heat transfer coefficient is given by Equation (8.6) (R_o = 0)

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i}$$

$$R_k = \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi L k} = \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k}$$

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{D_o}{D_i}\left[\frac{1}{\left(\frac{1}{R_i}\right)} + \frac{1}{\bar{h}_i}\right]$$

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{D_o}{D_i}\left[\frac{1}{R_i} + \frac{1}{h_i}\right]$$

$$\Rightarrow U_o = \frac{1}{\frac{1}{11.3\times 10^3 \text{ W/(m}^2\text{K)}} + \frac{1.57\times 10^{-2}\text{ m}\ln\left(\frac{1.57\text{ cm}}{1.32\text{ cm}}\right)}{2(26\text{ W/(m K)})} + \frac{1.57\text{ cm}}{1.32\text{ cm}}\left[\frac{1}{5.6\times 10^3\text{ W/(m}^2\text{K)}} + \frac{1}{(9.9\times 10^3\text{ W/(m}^2\text{K))}}\right]}$$

$$\Rightarrow \quad U_o = \frac{10^3}{(0.0885 + 0.0523 + 0.3325)}\text{ W/(m}^2\text{ K)}$$

$$\Rightarrow \quad U_o = 2110\text{ W/(m}^2\text{ K)}$$

(b) The heat capacity rate of the steam is essentially infinite. The heat capacity rate of the water is

$$C_w = \dot{m}_w c_p = \dot{v}\,\rho\, c_p = 0.32\times 10^{-3}\text{ m}^3\text{/s} \times 1012\text{ kg/m}^3 \times 4174\text{ J/(kg K)}$$

$$\Rightarrow \quad C_w = 1352\text{ W/K}$$

$$\frac{C_{\min}}{C_{\max}} = 0$$

The number of transfer units (NTU) is

$$NTU = \frac{U_o A_o}{C_{\min}} = \frac{U_o \pi D_o L}{C_{\min}} = \frac{2110\text{ W/(m}^2\text{K)}\,\pi(1.57\times 10^{-2}\text{ m})3\text{ m}}{1352\text{ W/K}}$$

$$\Rightarrow \quad NTU = 0.23$$

For $C_{\min}/C_{\max}$ = 0, parallel and counter flow heat exchangers have the same effectiveness and Equation (8.25) reduces to

$$\varepsilon = 1 - e^{-NTU} = 1 - e^{-0.23} = 0.205$$

From Equation (8.21b) ($C_c = C_{\min}$)

$$\varepsilon = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}}$$

$$T_{w,\text{out}} = T_{w,\text{in}} + \varepsilon \ (T_s - T_{w,\text{in}}) = 27°\text{C} + 0.205\ (49 - 27)°\text{C}$$

$$\Rightarrow \qquad T_{w,\text{out}} = 31.5°\text{C}$$

The mean water temperature is 29.2°C. The error due to determining the water density and specific heat at 38°C is negligible.

PROBLEM 8.11

Water is heated by hot air in a heat exchanger. The flow rate of the water is 12 kg/s and that of the air is 2 kg/s. The water enters at 40°C and the air enters at 460°C. The overall heat transfer coefficient of the heat exchanger is 275 W/(m^2 K) based on a surface area of 14 m^2. Determine the effectiveness of the heat exchanger if it is (a) parallel-flow type or a (b) cross-flow type (both fluids unmixed). Then calculate the heat transfer rate for the two types of heat exchangers described and the outlet temperatures of the hot and cold fluids for the conditions given.

GIVEN

- Water heated by air in a heat exchanger
- Water flow rate $(\dot{m}_w)$ = 12 kg/s
- Air flow rate $(\dot{m}_a)$ = 2 kg/s
- Inlet temperatures
  - Water ($T_{w,\text{in}}$) = 40°C
  - Air ($T_{a,\text{in}}$) = 460°C
- Overall heat transfer coefficient (U) = 275 W/(m^2 K)
- Transfer area (A) = 14 m^2

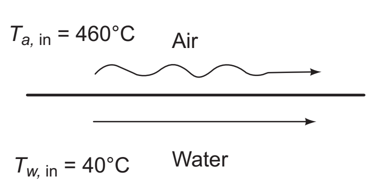
FIND

(a) The effectiveness (e)
(b) The heat transfer rate (q)
(c) The outlet temperature ($T_{w,\text{out}}$, $T_{a,\text{out}}$) for: 1. parallel-flow 2. cross-flow

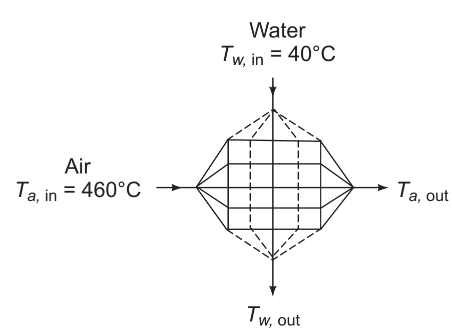
ASSUMPTIONS

- Steady state

SKETCH



1. Parallel Flow



2. Cross flow

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, the specific heat of dry air (c_{pa}) = 1059 J/(kg K) at 400°C
From Appendix 2, Table 13, the specific heat of water (c_{pw}) = 4178 J/(kg K) at 50°C

SOLUTION

The heat capacity rates are

For air $C_a = \dot{m}_a c_{pa} = (2\text{ kg/s})(1059\text{ J/(kg K)}) = 2118\text{ W/K}$

For water $C_w = \dot{m}_w c_{pw} = (12\text{ kg/s})(4178\text{ J/(kg K)}) = 50{,}136\text{ W/K}$

(a) The effectiveness for parallel-flow geometry is given by Equation 8.25

$$\text{E} = \frac{1-\exp\left[-\left(1+\frac{C_{\min}}{C_{\max}}\right)\frac{UA}{C_{\min}}\right]}{1+\frac{C_{\min}}{C_{\max}}}$$

$$\text{E} = \frac{1-\exp\left[-\left(1+\frac{2118}{50{,}136}\right)\frac{(275\text{ W/(m}^2\text{ K)})(14\text{ m}^2)}{2118\text{ W/K}}\right]}{1+\frac{2118}{50{,}136}} = 0.815$$

For cross-flow, the effectiveness can be taken from Figure 8.20

$$\frac{C_{\min}}{C_{\max}} = \frac{2188}{50{,}136} = 0.042$$

$$NTU = \frac{UA}{C_{\min}} = \frac{((275\text{ W/(m}^2\text{K)})(14\text{ m}^2)}{(2118\text{ W/(m}^2\text{K)})} = 1.82$$

From Figure 8.20: $e \approx 0.83$

(b) The rate of heat transfer is

$$q = \text{E}\, C_{\min}\,(T_{h,\text{in}} - T_{c,\text{in}})$$

For parallel-flow

$$q = 0.815\,(2118\text{ W/K})\,(460°\text{C} - 40°\text{C}) = 7.25 \times 10^5\text{ W} = 725\text{ kW}$$

For cross-flow

$$q = 0.83\,(2118\text{ W/K})\,(460°\text{C} - 40°\text{C}) = 7.38 \times 10^5\text{ W} = 738\text{ kW}$$

(c) The outlet temperatures can be calculated from

$$q = \dot{m}\, c_p\, \Delta T = C\,(T_{\text{out}} - T_{\text{in}}) \Rightarrow T_{\text{out}} = T_{\text{in}} + q/C$$

Parallel-flow

Water $T_{w,\text{out}} = 40°\text{C} + \dfrac{7.25\times10^5\text{ W}}{50{,}136\text{ W/K}} = 54°\text{C}$

Air $T_{a,\text{out}} = 460°\text{C} - \dfrac{7.25\times10^5\text{ W}}{2118\text{ W/K}} = 118°\text{C}$

Cross-flow

Water $T_{w,\text{out}} = 40°\text{C} + \dfrac{7.38\times10^5\ \text{W}}{50{,}136\ \text{W/K}} = 55°\text{C}$

Air $T_{a,\text{out}} = 460°\text{C} - \dfrac{7.38\times10^5\ \text{W}}{2118\ \text{W/K}} = 112°\text{C}$

COMMENTS

The cross-flow arrangement improves the heat transfer rate by 1.8%.

PROBLEM 8.12

Exhaust gases from a power plant are used to preheat air in a cross-flow heat exchanger. The exhaust gases enter the heat exchanger at 450°C and leave at 200°C. The air enters the heat exchanger at 70°C, leaves at 250°C, and has a mass flow rate of 10 kg/s. Assume the properties of the exhaust gases can be approximated by those of air. The overall heat transfer coefficient of the heat exchanger is 154 W/(m^2 K). Calculate the heat exchanger surface area required if (a) the air is unmixed and the exhaust gases are mixed and (b) both fluids are unmixed.

GIVEN

- Cross-flow heat exchanger – exhaust gas to air
- Exhaust temperatures
  - $T_{e,\text{in}} = 450°\text{C}$
  - $T_{e,\text{out}} = 200°\text{C}$
- Air temperatures
  - $T_{a,\text{in}} = 70°\text{C}$
  - $T_{a,\text{out}} = 250°\text{C}$
- Air flow rate $(\dot{m}_a)$ = 10 kg/s
- Overall heat transfer coefficient (U) = 154 W/(m^2 K)

FIND

The heat exchanger surface area (A) if

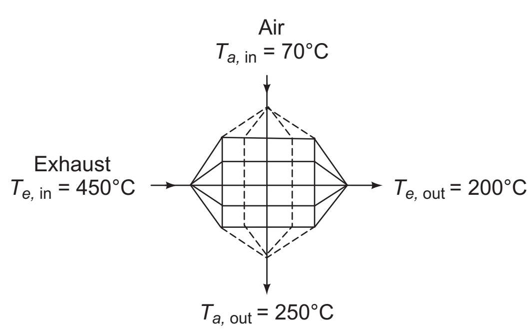
(a) Air is unmixed, exhaust is mixed

(b) Both are unmixed

ASSUMPTIONS

- The properties of the exhaust gases can be approximated by those of air
- The air is in the tube

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, the specific heat of air at the mean temperature of 160°C (c_{pa}) = 1030 J/(kg K)

SOLUTION

For counterflow, from Figure 8.12 $\Delta T_a = T_{e,\text{in}} - T_{a,\text{out}} = 450°\text{C} - 250°\text{C} = 200°\text{C}$

$\Delta T_b = T_{e,\text{out}} - T_{a,\text{in}} = 200°\text{C} - 70°\text{C} = 130°\text{C}$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{200°\text{C} - 130°\text{C}}{\ln\left(\frac{200}{130}\right)} = 162°\text{C}$$

(a) For cross-flow with the exhaust mixed, the *LMTD* must be modified according to Figure 8.15

$$P = \frac{T_{a,\text{out}} - T_{a,\text{in}}}{T_{e,\text{in}} - T_{a,\text{in}}} = \frac{250 - 70}{450 - 70} = 0.47$$

$$Z = \frac{T_{e,\text{in}} - T_{e,\text{out}}}{T_{a,\text{out}} - T_{a,\text{in}}} = \frac{450 - 200}{250 - 70} = 1.4$$

From Figure 8.15, $F = 0.76$

$$\therefore \quad \Delta T_{\text{mean}} = F(LMTD) = 0.76\ (162°\text{C}) = 123°\text{C}$$

The rate of heat transfer is

$$q = UA\ \Delta T_{\text{mean}} = \dot{m}_a\, c_{pa}\, (T_{a,\text{out}} - T_{a,\text{in}})$$

Solving for the heat exchanger surface area

$$A = \frac{\dot{m}_a c_{pa} (T_{a,\text{out}} - T_{a,\text{in}})}{U \Delta T_{\text{mean}}} = \frac{(10\ \text{kg/s})(1030\ \text{J/(kg K)})(250°\text{C} - 70°\text{C})}{\left(154\ \text{W/(m}^2\text{K)}\right)(123°\text{C})(\text{J/(Ws)})} = 98\ \text{m}^2$$

(b) For both fluids unmixed, the *LMTD* must be corrected using Figure 8.16: $F = 0.86$

$$\therefore \quad \Delta T_{\text{mean}} = 0.86\ (162°\text{C}) = 139°\text{C}$$

$$A = \frac{(10\ \text{kg/s})(1030\ \text{J/(kg K)})(250°\text{C} - 70°\text{C})}{\left(154\ \text{W/(m}^2\text{K)}\right)(139°\text{C})(\text{J/(Ws)})} = 86\ \text{m}^2$$

COMMENTS

The required transfer area is 15% smaller when both fluids are unmixed in this case.

PROBLEM 8.13

A shell-and-tube heat exchanger has one shell pass and four tube passes. The fluid in the tubes enters at 200°C and leaves at 100°C. The temperature of the fluid entering the shell is 20°C and is 90°C as it leaves the shell. The overall heat transfer coefficient based on the surface area of 12 m^2 is 300 W/(m^2 K). Calculate the heat transfer rate between the fluids.

GIVEN

- A shell-and-tube heat exchanger with one shell pass and four tube passes
- Temperature of fluid in tube
  - $T_{t,\text{in}} = 200°\text{C}$
  - $T_{t,\text{out}} = 100°\text{C}$
- Temperature of fluid in shell
  - $T_{s,\text{in}} = 20°\text{C}$
  - $T_{s,\text{out}} = 90°\text{C}$
- Overall heat transfer coefficient (U) = 300 W/(m^2 K)
- Surface area (A) = 12 m^2

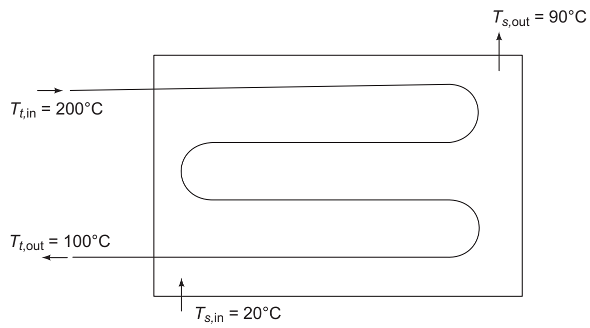
FIND

- The heat transfer rate (q)

ASSUMPTIONS

- Steady state
- Exchanger geometry is counterflow as shown in Figure 8.13

SKETCH



SOLUTION

A log-mean temperature difference for this counterflow arrangement is

$$LMTD = \Delta T = \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)}$$

From Figure 8.12 $\qquad \Delta T_a = T_{t,\text{in}} - T_{s,\text{out}} = 200°\text{C} - 90°\text{C} = 110°\text{C}$

$$\Delta T_b = T_{t,\text{out}} - T_{s,\text{in}} = 100°\text{C} - 20°\text{C} = 80°\text{C}$$

$$\Delta T = \frac{110°\text{C} - 80°\text{C}}{\ln\left(\dfrac{110}{80}\right)} = 94.2°\text{C}$$

This value must be modified to account for the shell-and-the geometry using Figure 8.13

$$P = \frac{T_{t,\text{out}} - T_{t,\text{in}}}{T_{s,\text{in}} - T_{t,\text{in}}} = \frac{100-200}{20-200} = 0.556$$

$$Z = \frac{T_{s,\text{in}} - T_{s,\text{out}}}{T_{t,\text{out}} - T_{t,\text{in}}} = \frac{20-90}{100-200} = 0.7$$

From Figure 8.13, $F = 0.85$

$$\therefore \qquad \Delta T_{\text{mean}} = F(\Delta T) = 0.85\ (94.2°\text{C}) = 80.1°\text{C}$$

The heat transfer rate is given by

$$q = U A\, \Delta T_{\text{mean}} = \left(300\ \text{W/(m}^2\ \text{K)}\right)(12\ \text{m}^2)\ (80.1°\text{C}) = 2.88 \times 10^5\ \text{W}$$

COMMENTS

Since ΔT_a is less than 50% greater than ΔT_b, the mean temperature difference may be used in place of the LMTD without introducing significant error. Use of the mean temperature difference in this case leads to a heat transfer rate of 2.907×10^5 W (1% higher).

PROBLEM 8.14

Oil $\left(c_p = 2.1\ \text{kJ/(kg K)}\right)$ is used to heat water in a shell and tubeheat exchanger with a single shell and two tube passes. The overall heat transfer coefficient is 525 W/(m^2 K). The mass flow rates are 7 kg/s for the oil and 10 kg/s for the water. The oil and water

enter the heat exchanger at 240°C and 20°C, respectively. The heat exchanger is to be designed so that the water leaves the heat exchanger with a minimum temperature of 80°C. Calculate the heat transfer surface area required to achieve this temperature.

GIVEN

- Oil heats water in a heat exchanger with one shell pass and two tube passes
- Oil specific heat (c_{po}) = 2.1 kJ/(kg K) = 2100 J/(kg K)
- Overall heat transfer coefficient (U) = 525 W/(m^2 K)
- Oil mass flow rate $(\dot{m}_o)$ = 7 kg/s
- Water mass flow rate $(\dot{m}_w)$ = 10 kg/s
- Inlet temperatures: Oil ($T_{o,\text{in}}$) = 240° Water ($T_{w,\text{in}}$) = 20°C
- Minimum water outlet temperature ($T_{w,\text{out}}$) = 80°C

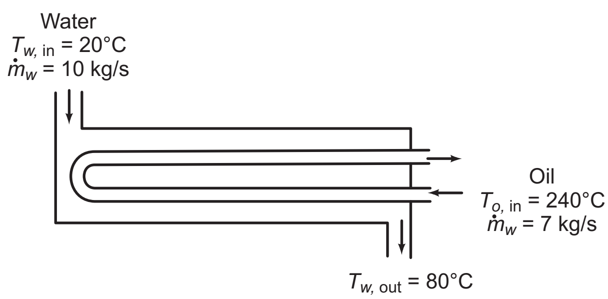
FIND

- The heat transfer area (A) required

ASSUMPTIONS

- Oil is in the tubes

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the average temperature of 50°C (c_{pw}) = 4178 J/(kg K)

SOLUTION

The heat capacity rates are

$$C_o = \dot{m}_o\, c_{po} = (7\text{ kg/s})\left(2100\text{ J/(kg K)}\right) = 14{,}700\text{ W/K}$$

$$C_w = \dot{m}_w\, c_{pw} = (10\text{ kg/s})\left(4178\text{ J/(kg K)}\right) = 41{,}780\text{ W/K}$$

$$\frac{C_{\min}}{C_{\max}} = \frac{14{,}700}{41{,}780} = 0.35$$

The effectiveness required to achieve $T_{w,\text{out}}$ = 80°C is

$$\text{E} = \frac{C_w\left(T_{w,\text{out}} - T_{w,\text{in}}\right)}{C_{\min}\left(T_{o,\text{in}} - T_{w,\text{in}}\right)} = \frac{\left(41{,}780\text{ J/(kg K)}\right)\left(80°\text{C} - 20°\text{C}\right)}{\left(14{,}700\text{ J/(kg K)}\right)\left(240°\text{C} - 20°\text{C}\right)} = 0.775$$

The number of transfer units, NTU, from Figure 8.19: NTU = $(U\,A)/C_{\min} \approx 2.5$

$$\therefore \qquad A = \text{NTU}\frac{C_{\min}}{U} = 2.5\,\frac{(14{,}700\text{ W/K})}{\left(525\text{ W/(m}^2\text{ K)}\right)} = 70\text{ m}^2$$

PROBLEM 8.15

A shell-and-tube heat exchanger with two tube passes and a single shell pass is used to heat water by condensing steam in the shell. The flow rate of the water is 15 kg/s and it is heated from 60 to 80°C. The steam condenses at 140°C and the overall heat transfer coefficient of the heat exchanger is 820 W/(m^2 K). If there are 45 tubes with an OD of 2.75 cm, calculate the length of tubes required.

GIVEN

- Shell-and-tube heat exchanger with two tube passes and one shell pass
- Water in tubes, condensing steam in shell
- Water flow rate $(\dot{m}_w)$ = 15 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 60°C
  - $T_{w,\text{out}}$ = 80°C
- Steam temperature (T_s) = 140°C
- Overall heat transfer coefficient (U) = 820 W/(m^2 K)
- Number of tubes (N) = 45
- Tube outside diameter (D) = 2.75 cm = 0.0275 m

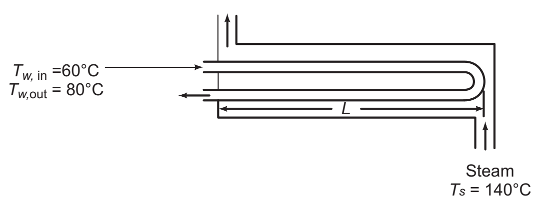
FIND

- The length of tubes (L)

ASSUMPTIONS

- Counterflow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, the specific heat of water at the mean temperature of 70°C (c_{pw}) = 4188 J/(kg K)

SOLUTION

For counterflow, from Figure 8.12

$$\Delta T_a = T_s - T_{w,\text{out}} = 140°\text{C} - 80°\text{C} = 60°\text{C}$$
$$\Delta T_b = T_s - T_{w,\text{in}} = 140°\text{C} - 60°\text{C} = 80°\text{C}$$

Since ΔT_a is less than 50% greater than ΔT_b, the mean temperature may be used

$$\Delta T_{\text{mean}} = \frac{1}{2}(80°\text{C} - 60°\text{C}) = 70°\text{C}$$

The rate of heat transfer is given by

$$q = UA\,\Delta T_{\text{mean}} = \dot{m}_w\, c_{pw}\,(T_{w,\text{out}} - T_{w,\text{in}})$$

Solving for the transfer area

$$A = \frac{\dot{m}_w c_{pw}\left(T_{w,\text{out}} - T_{w,\text{in}}\right)}{U\Delta T_{\text{mean}}} = \frac{(15\,\text{kg/s})(4188\,\text{J/(kg K)})(80°\text{C} - 60°\text{C})}{\left(820\,\text{W/(m}^2\text{K)}\right)(70°\text{C})(\text{J/(Ws)})} = 21.9\ \text{m}^3$$

The outside area of the tubes is

$$A = (2\ \text{passes})\,N\,\pi D\,L \;\Rightarrow\; L = \frac{A}{2N\pi D} = \frac{21.9\,\text{m}^2}{2(45)\pi(0.0275\,\text{m})} = 2.8\ \text{m}$$

If each tube is bent in half to create the two tube passes, than the total tube length is 2 L = 5.6 m

PROBLEM 8.16

Benzene flowing at 12.5 kg/s is to be cooled continuously from 80°C to 54°C by 10 kg/s of water available at 15.5°C. Using Table 8.5, estimate the surface area required for (a) cross-flow with six tube passes and one shell pass with neither of the fluids mixed and (b) a counterflow exchanger with one shell pass and eight tube passes with the colder fluid inside tubes.

GIVEN

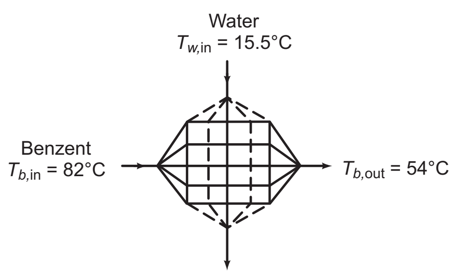
- Benzene coded by water in a heat exchanger
- Benzene flow rate $(\dot{m}_b)$ = 12.5 kg/s
- Water flow rate $(\dot{m}_w)$ = 10 mg/s
- Benzene temperatures
  - $(T_{b,\text{in}})$ = 82°C
  - $(T_{b,\text{out}})$ = 54°C
- Water inlet temperature $(T_{w,\text{in}})$ = 15.5°C

FIND

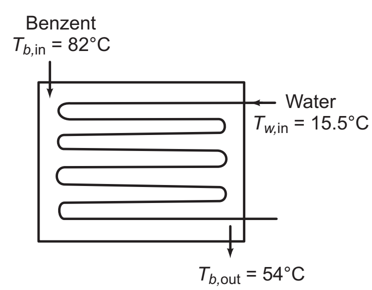
The surface area required for

(a) Cross-flow, 6 tube passes, 1 shell pass, both unmixed
(b) Counterflow, 8 tube passes, 1 shell pass, water in tubes

SKETCH



Case (a)



Case (b)

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at 20°C (c_{pw}) = 4182 J/(kg K)
From Appendix 2, Table 20, the specific heat of Benzene at 68°C (c_{pb}) = 1926 J/(kg K)

SOLUTION

The heat capacity rates are

$$C_b = \dot{m}_b\, c_{pb} = (12.5\ \text{kg/s})(1926\ \text{J/(kg K)}) = 24{,}075\ \text{W/K}$$

$$C_w = \dot{m}_w\, c_{pw} = (10\ \text{kg/s})(4182\ \text{J/(kg K)}) = 41{,}820\ \text{W/K}$$

$$\frac{C_{\min}}{C_{\max}} = \frac{24,075}{41,820} = 0.576$$

The rate of heat transfer is

$$q = \text{E}\ C_{\min}\,(T_{b,\text{in}} - T_{w,\text{in}}) = \dot{m}_b\, c_{pb}\, \Delta T_b = C_{\min}\,(T_{b,\text{in}} - T_{b,\text{out}})$$

Solving for the effectiveness of the heat exchanger

$$\text{E} = \frac{T_{b,\text{in}} - T_{b,\text{out}}}{T_{w,\text{in}} - T_{w,\text{in}}} = \frac{82°\text{C} - 54°\text{C}}{82°\text{C} - 15.5°\text{C}} = 0.42$$

(a) For unmixed cross-flow, the number of transfer units is given by Figure 8.20: NTU ≈ 0.7.
By definition

$$\text{NTU} = \frac{UA}{C_{\min}} \Rightarrow A = \text{NTU}\frac{C_{\min}}{U}$$

The overall heat transfer coefficient (U), from Table 8.5 is in the range of 280-850 W/(m^2 K) between water and organic solvents. Therefore, use U = 565 + 50% W/(m^2 K).

$$\therefore \qquad A = 0.7\frac{(24,075 \text{ W/K})}{\left((565+50\%)\text{ W/(m}^2\text{K)}\right)} = 30 \text{ m}^2 + 50\%$$

(b) NTU for counterflow from Figure 8.19 is NTU ≈ 0.75

$$\therefore \qquad A = 0.75\frac{(24,075 \text{ W/K})}{\left((565+50\%)\text{ W/(m}^2\text{K)}\right)} = 32 \text{ m}^2 + 50\%$$

PROBLEM 8.17

Water entering a shell-and-tube heat exchanger at 35°C is to be heated to 75°C by an oil. The oil enters at 110°C and leaves at 75°C. The heat exchanger is arranged for counterflow with the water making one shell pass and the oil two tube passes. If the water flow rate is 68 kg per minute and the overall heat transfer coefficient is estimated from Table 8.1 to be 320 W/(m^2 K), calculate the required heat exchanger area.

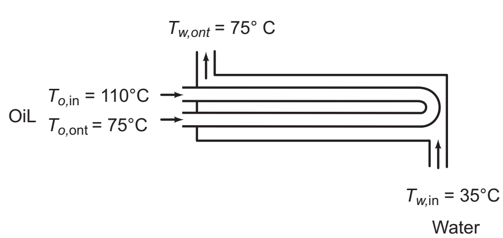
GIVEN

- Shell in tube counterflow heat exchanger water in shell, oil in tubes
- One shell pass, two tube passes
- Water temperatures
  - $T_{w,\text{in}}$ = 35°C
  - $T_{w,\text{out}}$ = 75°C
- Oil temperatures
  - $T_{o,\text{in}}$ = 110°C
  - $T_{w,\text{out}}$ = 75°C
- Water flow rate $(\dot{m}_n)$ = 68 kg/min = 1.133 kg/s
- Overall heat transfer coefficient (U) = 320 W/(m^2 K)

FIND

- The required area (A)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the average temperature of 55°C (c_{pw}) = 4180 J/(kg K)

SOLUTION

For counterflow from Figure 8.12 $\Delta T_a = T_{t,\text{in}} - T_{s,\text{out}}$ = 110°C – 75°C = 35°C

$\Delta T_b = T_{t,\text{out}} - T_{s,\text{in}}$ = 75°C – 35°C = 40°C

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)} = \frac{35°\text{C} - 40°\text{C}}{\ln\left(\dfrac{35}{40}\right)} = 37.4°\text{C}$$

The *LMTD* must be modified according to Figure 8.13

$$P = \frac{T_{o,\text{out}} - T_{o,\text{in}}}{T_{w,\text{in}} - T_{o,\text{in}}} = \frac{75-110}{35-110} = 0.47$$

$$Z = \frac{T_{w,\text{in}} - T_{w,\text{out}}}{T_{o,\text{out}} - T_{o,\text{in}}} = \frac{35-75}{75-110} = 1.14$$

From Figure 8.13: $F \approx 0.80$

$$\therefore \qquad \Delta T_{\text{mean}} = F(LMTD) = 0.80\ (37.4°\text{C}) = 29.9°\text{C}$$

The rate of heat transfer is

$$q = U\,A\,\Delta T_{\text{mean}} = \dot{m}_w\, c_{pw}\, (T_{w,\text{out}} - T_{w,\text{in}})$$

Solving for the transfer area

$$A = \frac{\dot{m}_w c_{pw}\left(T_{w,\text{out}} - T_{w,\text{in}}\right)}{U\Delta T_{\text{mean}}} = \frac{(1.133\text{ kg/s})(4180\text{ J/(Kg K)})(75°\text{C} - 35°\text{C})}{\left(320\text{ W/(m}^2\text{K)}\right)(29.9°\text{C})(\text{J/(W s)})} = 19.8\text{ m}^2$$

PROBLEM 8.18

Starting with a heat balance, show that the effectiveness for a counterflow arrangement is

$$\mathbf{E} = \frac{1 - \exp\left[-\left(\dfrac{1 - C_{\min}}{C_{\max}}\right) NTU\right]}{1 - \left(\dfrac{C_{\min}}{C_{\max}}\right)\exp\left[-\left(\dfrac{1 - C_{\min}}{C_{\max}}\right) NTU\right]}$$

GIVEN

- Counterflow heat exchanger

FIND

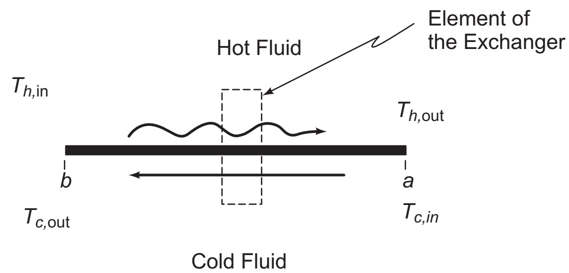
- Show that the effectiveness is

$$\mathrm{E} = \frac{1 - \exp\left[-\left(\dfrac{1 - C_{\min}}{C_{\max}}\right) NTU\right]}{1 - \left(\dfrac{C_{\min}}{C_{\max}}\right)\exp\left[-\left(\dfrac{1 - C_{\min}}{C_{\max}}\right) NTU\right]}$$

ASSUMPTIONS

- Heat loss to surroundings in negligible

SKETCH



SOLUTION

A heat balance on an element of the heat exchanger yields

$$dq = -C_h\, dT_h = C_c\, dT_c = U(T_h - T_c)\, dA$$

Rearranging

$$-\frac{C_h}{C_c}\frac{dT_h}{(T_h - T_c)} = -\frac{dT_c}{(T_h - T_c)} = \frac{U}{C_c}\, dA$$

Note that if $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$

then $\frac{C_1}{C_2} = \frac{A_1 + B_1}{A_2 + B_2}$

Therefore

$$\frac{U}{C_c}\, dA = \frac{-(dT_h - dT_c)}{\frac{C_c}{C_h}(T_h - T_c) - (T_h - T_c)} = \frac{d(T_h - T_c)}{\left(1 - \frac{C_c}{C_h}\right)(T_h - T_c)}$$

$$\frac{1}{T_h - T_c}\, d(T_h - T_c) = \frac{U}{C_c}\left(1 - \frac{C_c}{C_h}\right) dA$$

Integrating from *a* to *b*

$$\ln\left[\frac{T_{h,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{out}}}\right] = \frac{UA}{C_c}\left(1 - \frac{C_c}{C_h}\right)$$

But, from Equation (8.21)

$$\text{E} = \frac{C_h}{C_{\min}}\frac{T_{h,\text{out}} - T_{h,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{C_c}{C_{\min}}\frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}}$$

From which

$$T_{h,\text{out}} = T_{h,\text{in}} - \text{E}\ (T_{h,\text{in}} - T_{c,\text{in}})$$
$$T_{c,\text{out}} = T_{c,\text{in}} + \text{E}\ (T_{h,\text{in}} - T_{c,\text{in}})$$

Therefore

$$T_{h,\text{out}} - T_{c,\text{in}} = T_{h,\text{in}} - \text{E}\ (T_{h,\text{in}} - T_{c,\text{in}})\left(\frac{C_{\min}}{C_h}\right) - T_{c,\text{in}} = \left(1 - E\frac{C_{\min}}{C_h}\right)(T_{h,\text{in}} - T_{c,\text{in}})$$

$$T_{h,\text{in}} - T_{c,\text{out}} = T_{h,\text{in}} - T_{c,\text{in}} - \text{E}\ (T_{h,\text{in}} - T_{c,\text{in}})\ \frac{C_{\min}}{C_c} = \left(1 - E\frac{C_{\min}}{C_c}\right)(T_{h,\text{in}} - T_{c,\text{in}})$$

Substituting these into the energy balance

$$\ln\left[\frac{1-E\dfrac{C_{\min}}{C_h}}{1-E\dfrac{C_{\min}}{C_c}}\right] = \frac{UA}{C_c}\left(1-\frac{C_c}{C_h}\right) \Rightarrow \frac{1-E\dfrac{C_{\min}}{C_h}}{1-E\dfrac{C_{\min}}{C_c}} = \exp\left[\frac{UA}{C_c}\left(1-\frac{C_c}{C_h}\right)\right]$$

Solving for effectiveness

$$1-E\frac{C_{\min}}{C_h}\ \exp\left[\left(1-\frac{C_c}{C_h}\right)\frac{UA}{C_c}\right] = 1-E\frac{C_{\min}}{C_c}$$

$$E\frac{C_{\min}}{C_c} - E\frac{C_{\min}}{C_h}\ \exp\left[-\left(1-\frac{C_c}{C_h}\right)\frac{UA}{C_c}\right] = \exp\left[-\left(1-\frac{C_c}{C_h}\right)\frac{UA}{C_c}\right]$$

$$\text{E} = \frac{1-\exp\left[-\left(1-\dfrac{C_c}{C_h}\right)\dfrac{UA}{C_c}\right]}{\dfrac{C_{\min}}{C_c}-\dfrac{C_{\min}}{C_h}\exp\left[-\left(1-\dfrac{C_c}{C_h}\right)\dfrac{UA}{C_c}\right]}$$

Define $NTU = UA/C_{\min}$

Case (a) If $C_c = C_{\min} \rightarrow C_h = C_{\max}$

Then:

$$\text{E} = \frac{1-\exp[-\left(\dfrac{1-C_{\min}}{C_{\max}}\right)NTU]}{1-\left(\dfrac{C_{\min}}{C_{\max}}\right)\exp\left[-\left(\dfrac{1-C_{\min}}{C_{\max}}\right)NTU\right]}$$

Case (b) If $C_c = C_{\max} \rightarrow C_h = C_{\min}$

$$\text{E} = \frac{1-\exp\left[-\left(1-\dfrac{C_{\max}}{C_{\min}}\right)\dfrac{UA}{C_{\max}}\right]}{\dfrac{C_{\max}}{C_{\min}}-\exp\left[-\left(1-\dfrac{C_{\max}}{C_{\min}}\right)\dfrac{UA}{C_{\max}}\right]}$$

Multiplying the numerator and denominator by

$$\exp\left[\left(1-\frac{C_{\max}}{C_{\min}}\right)\frac{UA}{C_{\max}}\right] = \exp\left[\left(1-\frac{C_{\max}}{C_{\min}}\right)\left(\frac{C_{\max}}{C_{\min}}\right)NTU\right] = \exp\left[\left(1-\frac{C_{\max}}{C_{\min}}\right)NTU\right]$$

yields the following result

$$\text{E} = \frac{\exp\left[-\left(\dfrac{1-C_{\max}}{C_{\min}}\right)NTU\right]-1}{\dfrac{C_{\min}}{C_{\max}}\exp\left[-\left(\dfrac{1-C_{\max}}{C_{\min}}\right)NTU\right]-1}$$

$$\text{E} = \frac{1-\exp\left[-\left(\dfrac{1-C_{\min}}{C_{\max}}\right)NTU\right]}{1-\left(\dfrac{C_{\min}}{C_{\max}}\right)\exp\left[-\left(\dfrac{1-C_{\min}}{C_{\max}}\right)NTU\right]}$$

PROBLEM 8.19

In a tubular heat exchanger with two shell passes and eight tube passes, 12.6 kg/s of water are heated in the shell from 80°C to 150°C. Hot exhaust gases having roughly the same physical properties as air enter the tubes at 340°C and leave at 180°C. The total surface, based on the outer tube surface, is 930 m^2. Determine (a) the log-mean temperature difference if the heat exchanger were simple counterflow type, (b) the correction factor *F* for the actual arrangement, (c) the effectiveness of the heat exchanger, (d) the average overall heat transfer coefficient.

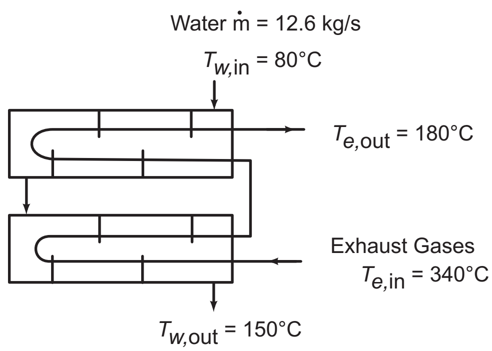
GIVEN

- A tubular heat exchanger with 2 shell passes and 8 tube passes
- Water in the shell, exhaust gases in the tubes
- Exhaust gases have roughly the same physical properties as air
- Water flow rate $(\dot{m}_w)$ = 12.6 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 80°C
  - $T_{w,\text{out}}$ = 150°C
- Exhaust temperatures
  - $T_{e,\text{in}}$ = 340°C
  - $T_{e,\text{out}}$ = 180°C
- Surface are (A) = 930 m^2

FIND

(a) *LMTD* if the exchanger is a simple counterflow type
(b) Correction *F* for the actual arrangement
(c) Effectiveness (e)
(d) The overall heat transfer coefficient (U)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the mean temperature of 115°C (c_{pw}) = 4224 J/(kg K).

SOLUTION

(a) From Figure 8.12 for simple counterflow $\Delta T_a = T_{e,\text{in}} - T_{w,\text{out}} = 340 - 150 = 190°\text{C}$

$\Delta T_b = T_{e,\text{out}} - T_{w,\text{in}} = 180 - 80 = 100°\text{C}$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{190°\text{C} - 100°\text{C}}{\ln\left(\frac{190}{100}\right)} = 140.2°\text{C}$$

(b) The *LMTD* must be modified as shown in Figure 8.14

$$P = \frac{T_{e,\text{out}} - T_{e,\text{in}}}{T_{w,\text{in}} - T_{e,\text{in}}} = \frac{180-340}{80-340} = 0.62$$

$$Z = \frac{T_{w,\text{in}} - T_{w,\text{out}}}{T_{e,\text{out}} - T_{e,\text{in}}} = \frac{80-150}{180-340} = 0.43$$

From Figure 8.14 $F = 0.96$

(c) To find the effectiveness, the heat capacity rate of the exhaust must first be determined. An energy balance yields

$$C_e\,(T_{e,\text{in}} - T_{e,\text{out}}) = C_w\,(T_{w,\text{out}} - T_{w,\text{in}})$$

where $C_w = \dot{m}_w\, c_{pw} = 12.6$ kg/s (4224 J/(kg K)) = 53.22 kW/K

$$C_h = C_e = C_w \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_{e,\text{in}} - T_{e,\text{out}}} = 53220\left(\frac{150-80}{340-180}\right) = 23.3\ \text{kW/K} = C_{\min}$$

The effectiveness is given by Equation (8.21a)

$$\mathrm{E} = \frac{C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)}{C_{\min}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)} = \frac{T_{e,\text{in}} - T_{e,\text{out}}}{T_{e,\text{in}} - T_{w,\text{out}}} = \frac{340-180}{340-150} = 0.84 = 84\ \%$$

(d) The rate of heat transfer is

$$q = U\,A\,\Delta T_{\text{mean}} = U\,A\,F\,(LMTD) = C_w\,(T_{w,\text{out}} - T_{w,\text{in}})$$

Solving for the overall heat transfer coefficien

$$U = \frac{C_w\left(T_{w,\text{out}} - T_{w,\text{in}}\right)}{AF\left(LMTD\right)} = \frac{53220\ \text{W/K}\,(150-80)\text{K}}{930\,\text{m}^2\,(0.96)\,(140.2\,\text{K})} = 29.7\ \text{W/(m}^2\ \text{K)}$$

PROBLEM 8.20

In gas turbine recuperators, the exhaust gases are used to heat the incoming air and $C_{\min}/C_{\max}$ is therefore approximately equal to unity. Show that for this case $e = NTU/(1 + NTU)$ for counterflow and $e = 1/2\,(1 - e^{-2NTU})$ for parallel flow.

GIVEN

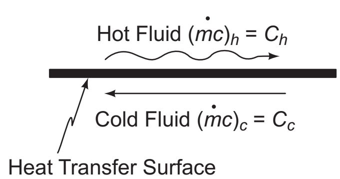
- Gas turbine recuperator
- $C_{\min}/C_{\max} \approx 1$

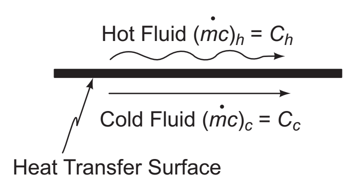
FIND

Show that (a) $e = NTU/(1 + NTU)$ for counterflow

(b) $e = 1/2\,(1 - e^{-2NTU})$ for parallel flow

SKETCH





SOLUTION

(a) From the solution of Problem 8.19: for counterflow

$$\mathrm{E} = \frac{1-\exp[-(1-C^*)\,NTU]}{1-C^*\exp[-(1-C^*)\,NTU]}$$

where $C^* = C_{\min}/C_{\max}$ For $C^* = 1$, e is undefined

Applying L'Hopital's rule

$$\mathrm{E}_{C^* \to 1} = \lim_{C^* \to 1} \frac{f(C^*)}{g(C^*)} = \lim_{C^* \to 1} \frac{f'(C^*)}{g''(C^*)}$$

$$\mathrm{E}_{C^* \to 1} = \lim_{C^* \to 1} \frac{-NTU \exp[-(1-C^*)NTU]}{-C^*\,NTU \exp[-(1-C^*)NTU] - \exp[-(1-C^*)NTU]}$$

$$\mathrm{E} = \frac{NTU}{1+NTU}$$

(b) For parallel flow, the effectiveness is given by Equation (8.25)

$$\mathrm{E} = \frac{1-\exp[-\left(\frac{1-C_{\min}}{C_{\max}}\right)NTU]}{1+\left(\frac{C_{\min}}{C_{\max}}\right)}$$

For $C_{\min}/C_{\max} = 1$

$$\mathrm{E} = \frac{1-\exp(-2NTU)}{2} = \frac{1}{2}(1 - e^{-2NTU})$$

PROBLEM 8.21

In a single-pass counterflow heat exchanger, 4536 kg/h of water enter at 15°C and cool 9071 kg/h of an oil having a specific heat of 2093 J/(kg °C) from 93 to 65°C. If the overall heat transfer coefficient is 284 W/(m^2 °C), determine the surface area required.

GIVEN

- Oil and water in a single-pass counterflow heat exchanger
- Water flow rate $(\dot{m}_w)$ = 4536 kg/h = 1.26 kg/s
- Oil flow rate $(\dot{m}_o)$ = 9071 kg/h = 2.52 kg/s
- Inlet temperatures
  - Water ($T_{w,\text{in}}$) = 15°C
  - Oil ($T_{o,\text{in}}$) = 93°C
- Oil outlet temperature ($T_{o,\text{out}}$) = 65°C
- Oil specific heat (c_{po}) = 2093 J/(kg °C)
- Overall heat transfer coefficient (U) = 284 W/(m^2 °C)

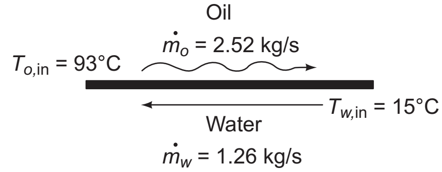
FIND

- The surface area (A) required

ASSUMPTIONS

- Steady state
- Constant thermal properties

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at 20°C (c_{pw}) = 4182 J/(kg K)

SOLUTION

The outlet temperature of the water can be determined from an energy balance

$$\dot{m}_w c_w (T_{w,\text{out}} - T_{w,\text{in}}) = \dot{m}_o c_o (T_{o,\text{in}} - T_{o,\text{out}})$$

$$T_{w,\text{out}} = T_{w,\text{in}} + \frac{\dot{m}_o c_{po}}{\dot{m}_w c_{pw}} (T_{o,\text{in}} - T_{o,\text{out}})$$

$$T_{w,\text{out}} = 15°\text{C} + \frac{(2.52 \text{ Kg/s})(2093 \text{ J/(Kg K)})}{(1.26 \text{ Kg/s})(4182 \text{ J/(Kg K)})} (93°\text{C} - 65°\text{C}) = 43°\text{C}$$

From Figure 8.12 $\Delta T_a = T_{o,\text{in}} - T_{w,\text{out}} = 93°\text{C} - 43°\text{C} = 50°\text{C}$

$\Delta T_b = T_{o,\text{out}} - T_{w,\text{in}} = 65°\text{C} - 15°\text{C} = 50°\text{C}$

Therefore, $\Delta T_{\text{mean}} = 50°\text{C}$

The rate of heat transfer is

$$q = U A \Delta T_{\text{mean}} = \dot{m}_o c_{po} (T_{o,\text{in}} - T_{o,\text{out}})$$

$$\therefore \quad A = \frac{\dot{m}_o c_{po} \left(T_{o,\text{in}} - T_{o,\text{out}}\right)}{U \Delta T_{\text{mean}}} = \frac{(2.52 \text{ Kg/s})(2093 \text{ J/(kg k)})(93°\text{C} - 65°\text{C})}{\left(284 \text{ W/(m}^2\text{K)}\right)(50°\text{C})(\text{J/(W s)})} = 10.4 \text{ m}^2$$

PROBLEM 8.22

A steam-heated single-pass tubular preheater is designed to raise 5.6 kg/s of air from 20°C to 75°C, using saturated steam at 26 bar (abs). It is proposed to double the flow rate of air and, in order to be able to use the same heat exchanger and achieve the desired temperature rise, it is proposed to increase the steam pressure. Calculate the steam pressure necessary for the new conditions and comment on the design characteristics of the new arrangement.

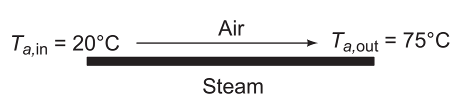
GIVEN

- Steam-heated single-pass tubular preheated heating air
- Air flow rate $(\dot{m}_a)$ = 5.6 kg/s
- Air temperature
  - $T_{a,\text{in}}$ = 20°C
  - $T_{a,\text{out}}$ = 75°C
- Saturated steam pressure (p_s) = 26 bar = 2.59×10^6 N/m^2

FIND

- The steam pressure necessary for a double $\dot{m}_a$ with the same temperature rise

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the saturation temperature (T_s) of steam at 2.59×10^6 N/m^2 = 194°C

SOLUTION

The heat capacity rate of the steam is virtually infinite, therefore, C_{min}/C_{max} = 0. The effectiveness must be determined using Equation (8.21b)

$$\text{E} = \frac{C_c}{C_{min}} \frac{T_{a,\text{out}} - T_{a,\text{in}}}{T_s - T_{a,\text{in}}} \qquad \text{where } C_c = C_a = C_{min}$$

$$\therefore \text{E} = \frac{75 - 20}{194 - 20} = 0.32$$

Examination of $e = e\,(NTU)$ when $C_{min}/C_{max} = 0$ in Figures 8.17 and 8.18 or in Equation (8.25) and the solution of Problem 8.19 reveals that $e = e\,(NTU)$ is the same of both counterflow and parallel flow when $C_{min}/C_{max} = 0$. For $e = 0.32$, Figure 8.17 gives $NTU = (U\,A)/C_{min} \approx 0.5$.

Doubling the flow rate of air doubles its heat capacity rate (C_{min}). Therefore, the NTU is halved. For the new flow rate: $NTU = 0.25$.

From Figure 8.17 for $NTU = 0.25$, $C_{min}/C_{max} = 0 \rightarrow e = 0.2$

The rate of heat transfer, from Equation (8.22) is

$$q = e\, C_{min}\,(T'_s - T_{s,\text{in}}) = \dot{m}_a\; c_{pa}\,(T_{a,\text{out}} - T_{a,\text{in}}) = C_{min}\,(T_{a,\text{out}} - T_{a,\text{in}})$$

Solving for the steam temperature required.

$$T'_s = T_{a,\text{in}} + \frac{T_{a,\text{out}} - T_{a,\text{in}}}{E} = 20 + \frac{75 - 20}{0.2} = 295°\text{C} \qquad \text{(very high)}$$

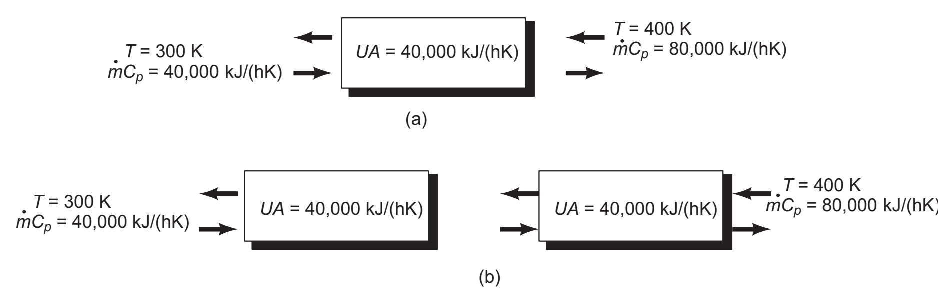
The steam temperature required for the doubled air flow rate is 295°C. From Appendix 2, Table 13, for T_s = 295°C, the saturation pressure = 5.3248×10^5 N/m^2 = 77.2 psia.

COMMENTS

Pressure increases rapidly with steam temperature, therefore, the solution is only practical if the equipment is designed to operate safely at high pressure. This corresponds to a steam pressure of 80 bar

PROBLEM 8.23

A heat exchanger performs as shown below in the Figure A for safety reasons. An engineer suggests that it would be wise to double the heat transfer area so as to double the heat transfer rate. The suggestion is made to add a second, identical exchanger as shown in Figure B. Evaluate this suggestion, i.e., show whether or not the heat transfer rate would double.



GIVEN

Case 1

Heat exchanger as shown above.

- Overall heat transfer coefficient times the transfer area (UA) = 40,000 kJ/(hr K)
- Heat capacity rates
  - C_h = 80,000 kJ/(h K)
  - C_c = 40,000 kJ/(h K)
- Entering temperatures
  - T_b,in = 400 K
  - T_c,in = 300 K

Case 2

Two of the same heat exchanges as shown above in Figure B.

FIND

- Does the heat transfer rate double?

ASSUMPTIONS

- Heat exchangers are simple counterflow geometry

SOLUTIONS

Case 1

$C_{\min}/C_{\max}$ = (40,000)/(80,000) = 0.5

The number of transfer units is: $NTU = (UA)/C_{\min}$ = (40,000)/(40,000) = 1.0

From Figure 8.18: e_1 = 0.54

Case 2

For this case, the transfer area is doubled, therefore

$$UA_{\text{total}} = 2(UA) = 80{,}000 \text{ kJ/(h K)}$$

$$NTU = (80{,}000)/(40{,}000) = 2.0$$

From Figure 8.18: e_2 = 0.78

Applying Equation 8.22

$$\frac{q_2}{q_1} = \frac{E_2\, C_{\min}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)}{E_1\, C_{\min}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)} = \frac{E_2}{E_1} = \frac{0.78}{0.54} = 1.44$$

The rate of heat transfer does not double. It is increased by only 44%.

PROBLEM 8.24

> **In a single-pass counterflow heat exchanger, 1.25 kg/s of water enter at 15°C and cool 2.5 kg/s of an oil having a specific heat of 2093 J/(kg K) from 95°C to 65°C. If the overall heat transfer coefficient is 280 W/(m^2 K), determine the surface area required.**

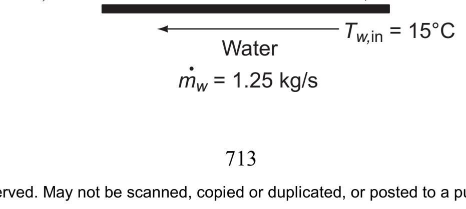
GIVEN

- Water cooling oil in a single-pass counterflow heat exchanger
- Water flow rate $(\dot{m}_w)$ = 1.25 kg/s
- Oil flow rate $(\dot{m}_o)$ = 2.5 kg/s
- Oil specific heat (c_{po}) = 2.1 kJ/(kg K)
- Water inlet temperature ($T_{w,\text{ in}}$) = 15°C
- Oil temperature
  - $T_{o,\text{ in}}$ = 95°C
  - $T_{o,\text{ out}}$ = 65°C
- Overall heat transfer coefficient (U) = 280 W/(m^2 K)

FIND

- The surface area (A) required

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water (c_{pw}) ≈ 4186 J/(kg K)

SOLUTION

The heat capacity rates are

$$\text{where } C_w = \dot{m}_w\, c_{pw} = 1.25 \text{ kg/s } (4186 \text{ J/(kg K)}) = 5232 \text{ W/K}$$

$$\text{where } C_o = \dot{m}_o\, c_{po} = 2.5 \text{ kg/s } (2093 \text{ J/(kg K)}) = 5232 \text{ W/K}$$

Therefore, $C_{min}/C_{max} = 1.0$

The outlet temperature of the water can be determined from an energy balance

$$q = C_o\,(T_{o,\text{in}} - T_{o,\text{out}}) = C_w\,(T_{w,\text{out}} - T_{w,\text{in}})$$

$$T_{w,\text{out}} = T_{w,\text{in}} + (T_{o,\text{in}} - T_{o,\text{out}}) = 15 + (95 - 65) = 45°\text{C}$$

From Figure 8.12, for counterflow $\Delta T_a = T_{o,\text{in}} - T_{w,\text{out}} = 95 - 45 = 50°\text{C}$

$\Delta T_b = T_{o,\text{out}} - T_{w,\text{in}} = 65 - 15 = 50°\text{C}$

(Note that $\Delta T_a = \Delta T_b$ because $C_w = C_o$)

Therefore, $\Delta T_{means} = 50°\text{C}$

The rate of heat transfer is

$$q = U\,A\,\Delta T_{mean} = C_o\,(T_{o,\text{in}} - T_{o,\text{out}})$$

Solving for the transfer area

$$A = \frac{C_o(T_{o,\text{in}} - T_{o,\text{out}})}{U\,\Delta T_{\text{mean}}} = \frac{5232\,\text{W/(m K)}\,(95-65)}{280\,\text{W/(m}^2\text{K)}\,(50\,\text{K})} = 11.2 \text{ m}^2$$

PROBLEM 8.25

Determine the outlet temperature of oil in Problem 8.24 for the same initial temperatures of the fluids if the flow arrangement is one shell pass and two tube passes, but with the same total area and average overall heat transfer coefficient as the unit in Problem 8.24.

From Problem 8.24*:* In a heat exchanger, 1.25 kg/s of water enter at 15°C and cool 2.5 kg/s of an oil having a specific heat of 2093 J/(kg K) from 95°C to 65°C. If the overall heat transfer coefficient is 280 W/(m² K), determine the surface area required.

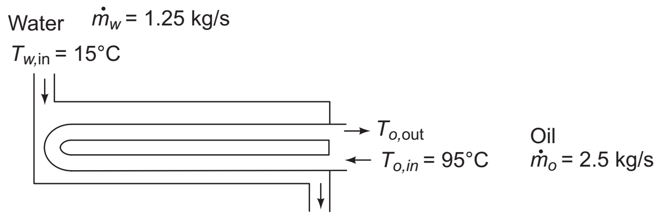
GIVEN

- Water cooling oil in a tube and shell heat exchanger
- Water flow rate $(\dot{m}_w)$ = 1.25 kg/s
- Oil flow rate $(\dot{m}_o)$ = 2.5 kg/s
- Oil specific heat (c_{po}) = 2093 J/(kg K)
- Water inlet temperature: ($T_{w,\text{in}}$) = 15°C
- Oil inlet temperature: $T_{o,\text{in}}$ = 95°C
- Overall heat transfer coefficient (U) = 280 W/(m^2 K)
- One shell and two tube passes
- Same surface area as Problem 8.24: $A = 11.2 \text{ m}^2$

FIND

- The oil outlet temperature ($T_{o,\text{out}}$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water (c_{pw}) ≈ 4186 J/(kg K)

SOLUTION

From the solution to Problem 8.24 $C_w = C_o$ = 5232 W/K

C_{min}/C_{max} = 1.0

The number of transfer units is

$$NTU = \frac{UA}{C_{min}} = \frac{(280\,\text{W/(m}^2\text{K))}(11.2\,\text{m}^2)}{5282\text{ W/K}} = 0.59$$

From Figure 8.19: $e \approx 0.34$

From Equation (8.21a) (for C_h/C_{min} = 1.0)

$$\text{E} = \frac{T_{o,\text{in}} - T_{o,\text{out}}}{T_{o,\text{in}} - T_{w,\text{in}}}$$

$$T_{o,\text{ out}} = T_{o,\text{ in}} - \text{E}\,(T_{o,\text{ in}} - T_{w,\text{ on}}) = 95°\text{C} - 0.34\,(95°\text{C} - 15°\text{C}) = 67.8°\text{C}$$

COMMENTS

The outlet oil temperature is approximately the same as the previous problem because the effectiveness is not improved significantly by an additional pass for small values of *NTU*.

PROBLEM 8.26

Carbon dioxide at 427°C is to be used to heat 12.6 kg/s of pressurized water from 37°C to 148°C while the gas temperature drops 204°C. For an overall heat transfer coefficient of 57 W/(m^2 K), compute the required area of the exchanger in square feet for (a) parallel flow, (b) counterflow, (c) a 2-4 reversed current exchanger, and (d) crossflow, gas mixed.

GIVEN

- CO_2 heating water in a heat exchanger
- CO_2 temperatures
  - $T_{g,\text{in}}$ = 427°C
  - $T_{g,\text{out}}$ = 427 – 204 = 223°C
- Water temperatures
  - $T_{w,\text{in}}$ = 37°C
  - $T_{w,\text{out}}$ = 148°C
- Water flow rate ($\dot{m}_w$) = 12.6 kg/s
- Overall heat transfer coefficient (U) = 57 W/(m^2 K)

FIND

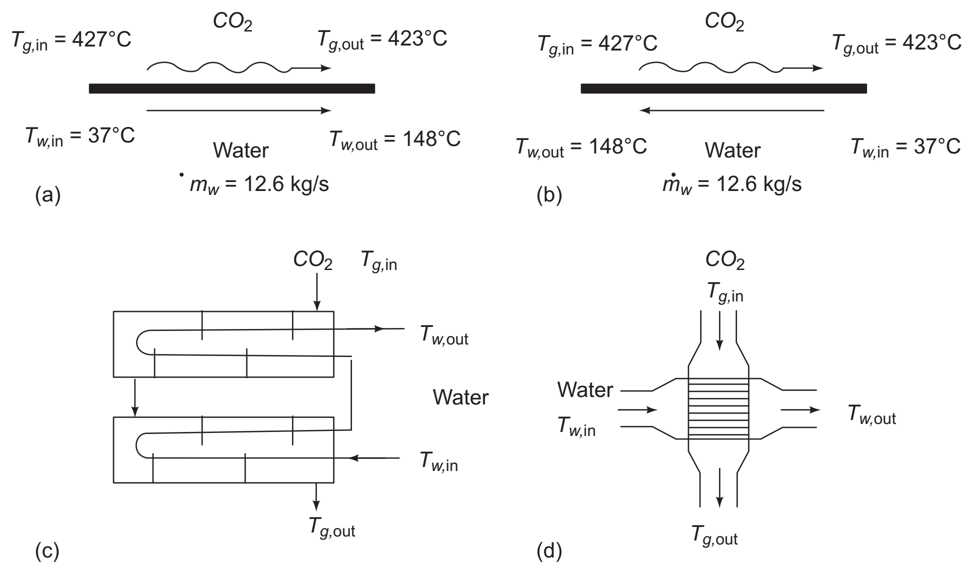
The area (A) required for:

(a) parallel-flow (c) A 2- 4 reversed current exchanger
(b) counterflow (d) Crossflow, gas mixed

ASSUMPTIONS

- In configuration (c), the gas is in the shell side

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the mean temperature of 92°C (c_{pw}) = 4205 J/(kg K)

SOLUTION

(a) For parallel flow, from Figure 8.11

$$\Delta T_a = T_{g,\text{ in}} - T_{w,\text{in}} = 427°\text{C} - 37°\text{C} = 390°\text{C}$$
$$\Delta T_b = T_{g,\text{out}} - T_{w,\text{out}} = 223°\text{C} - 148°\text{C} = 75°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\dfrac{\Delta T_a}{\Delta T_b}} = \frac{390°\text{C} - 75°\text{C}}{\ln\left(\dfrac{390}{75}\right)} = 191°\text{C}$$

The rate of heat transfer is

$$q = U\,A\,(LMTD) = \dot{m}_w\; c_{pw}\,(T_{w,\text{out}} - T_{w,\text{in}})\text{s}$$

Solving for the transfer area

$$A = \frac{\dot{m}_w C_{pw}(Tw,\text{out} - T_{w,\text{in}})}{U\,(\text{LMTD})} = \frac{(12.6\text{ kg/s})(4205\text{ J/(kg K)})(148°\text{C} - 37°\text{C})}{\left(57\text{ W/(m}^2\text{K)}\right)(191°\text{C})(\text{J/(Ws)})}$$

$$= 541\text{ m}^2 = 5827\text{ ft}^2$$

(b) For counterflow, from Figure 8.12

$$\Delta T_a = T_{g,\text{in}} - T_{w,\text{out}} = 427°\text{C} - 148°\text{C} = 279°\text{C}$$
$$\Delta T_b = T_{g,\text{out}} - T_{w,\text{in}} = 223°\text{C} - 37°\text{C} = 186°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\dfrac{\Delta T_a}{\Delta T_b}} = \frac{279°\text{C} - 186°\text{C}}{\ln\left(\dfrac{279}{186}\right)} = 229°\text{C}$$

Similarly

$$A = \frac{(12.6\text{ kg/s})(4205\text{ J/(kg K)})(148°\text{C} - 37°\text{C})}{\left(57\text{ W/(m}^2\text{K)}\right)(229°\text{C})(\text{J/(Ws)})} = 450\text{ m}^2 = 4850\text{ ft}^2$$

(c) The counterflow *LMTD* must be corrected using Figure 8.14 for this configuration

$$P = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_{g,\text{in}} - T_{w,\text{in}}} = \frac{148 - 37}{427 - 37} = 0.28$$

$$= \frac{T_{g,\text{in}} - T_{g,\text{out}}}{T_{w,\text{out}} - T_{w,\text{in}}} = \frac{427 - 223}{148 - 37} = 1.84$$

From Figure 8.14 $F \approx 0.97$

$$\therefore \Delta T_{\text{mean}} = F(\Delta T) = 0.97\ (229°\text{C}) = 222°\text{C}$$

$$A = \frac{(12.6\ \text{kg/s})(4205\ \text{J/(kg K)})(148°\text{C} - 37°\text{C})}{(57\ \text{W/(m}^2\text{K)})(222°\text{C})(\text{J/(Ws)})} = 465\ \text{m}^2 = 5003\ \text{ft}^2$$

(d) The counterflow *LMTD* must be modified using Figure 8.15 for crossflow, gas mixed.
For $P = 0.28$, $Z = 1.84 \rightarrow F = 0.93$

$$\therefore \Delta T_{\text{mean}} = F(\Delta T) = 0.93\ (229°\text{C}) = 213°\text{C}$$

$$A = \frac{(12.6\ \text{kg/s})(4205\ \text{J/(kg K)})(148°\text{C} - 37°\text{C})}{(57\ \text{W/(m}^2\text{K)})(213°\text{C})(\text{J/(Ws)})} = 484\ \text{m}^2 = 5210\ \text{ft}^2$$

COMMENTS

A simple counterflow heat exchanger requires the least transfer area for this case.

PROBLEM 8.27

An economizer is to be purchased for a power plant. The unit is to be large enough to heat 7.5 kg/s of pressurized water from 71 to 182°C. There are 26 kg/s of flue gases (c_p = 1000 J/(kg K)) available at 426°C. Estimate (a) the outlet temperature of the flue gases, (b) the heat transfer area required for a counterflow arrangement if the overall heat transfer coefficient is 57 W/(m^2 K).

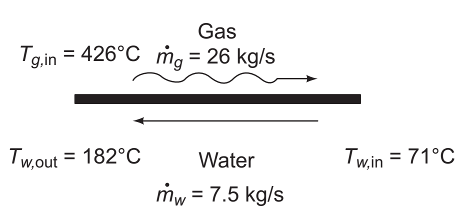
GIVEN

- Counterflow heat exchanger - flue gases heating water
- Water flow rate $(\dot{m}_w)$ = 7.5 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 71°C
  - $T_{w,\text{out}}$ = 182°C
- Gas flow rate ($\dot{m}_g$) = 26 kg/s
- Gas specific heat (c_{pg}) = 1000 J/(kg K)
- Gas inlet temperature ($T_{g,\text{in}}$) = 426°C
- Overall heat transfer coefficient (U) = 57 W/(m^2 K)

FIND

(a) Outlet gas temperature $T_{g,\text{out}}$
(b) Heat transfer area (A) required

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the mean temperature of 126.5°C (c_{pw}) = 4240 J/(kg K)

SOLUTION

The heat capacity rates are

$$C_w = \dot{m}_w \ c_{pw} = 7.5 \text{ kg/s}\left(4240 \text{ J/(kg K)}\right) = 31{,}801 \text{ W/K}$$

$$C_g = \dot{m}_g \ c_{pg} = 26 \text{ kg/s}\left(1000 \text{ J/(kg K)}\right) = 26{,}000 \text{ W/K}$$

(a) A heat balance yields

$$C_g \ (T_{g,\text{in}} - T_{g,\text{out}}) = C_w \ (T_{w,\text{out}} - T_{w,\text{in}})$$

Solving for the outlet gas temperature

$$T_{g,\text{out}} = T_{g,\text{in}} - \frac{C_w}{C_g} \ T_{w,\text{out}} - T_{w,\text{in})} = 426°\text{C} - \frac{31{,}801}{26{,}000} \ (182°\text{C} - 71°\text{C}) = 290°\text{C}$$

(b) From Figure 8.12 for counterflow $\Delta T_a = T_{g,\text{in}} - T_{w,\text{out}} = 426°\text{C} - 182°\text{C} = 244°\text{C}$

$\Delta T_b = T_{g,\text{out}} - T_{w,\text{in}} = 290°\text{C} - 71°\text{C} = 219°\text{C}$

$$\Delta T_{\text{mean}} = LMTD = \frac{\Delta T_a - \Delta T_b}{\dfrac{\Delta T_a}{\Delta T_b}} = \frac{244°\text{C} - 219°\text{C}}{\ln\left(\dfrac{244}{219}\right)} = 231°\text{C}$$

The rate of heat transfer is

$$q = U \, A \, \Delta T_{\text{mean}} = C_w(T_{w,\text{out}} - T_{w,\text{in}})$$

$$A = \frac{C_w(T_{w,\text{out}} - T_{w,\text{in}})}{U \Delta T_{\text{mean}}} = \frac{(31{,}801 \text{ W/K})(182°\text{C} - 71°\text{C})}{\left(57 \text{ W/(m}^2\text{K)}\right)(231°\text{C})} = 268 \text{ m}^2$$

PROBLEM 8.28

Water is heated while flowing through a pipe by steam condensing on the outside of the pipe. (a) Assuming a uniform overall heat transfer coefficient along the pipe, derive an expression for the water temperature as a function of distance from the entrance. (b) For an overall heat transfer coefficient of 570 W/(m^2 K), based on the inside diameter of 5 cm, a steam temperature of 104°C, and water-flow rate of 0.063 kg/s, calculate the length required to raise the water temperature from 15.5 to 65.5°C.

GIVEN

- Water flowing through a pipe steam condensing on the outside

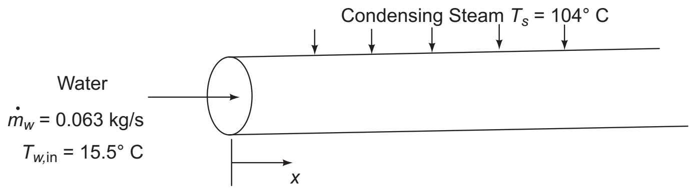
FIND

(a) An expression for the water temperature as a function of distance from the entrance, $T_w\,(x)$
(b) For Overall heat transfer coefficient (U) = 570 W/(m^2 K)
Inside diameter (D) = 5 cm = 0.05 m
Steam temperature (T_s) = 104°C
Water flow rate ($\dot{m}_w$) = 0.063 kg/s
Water temperatures: $T_{w,\text{in}}$ = 15.5°C $T_{w,\text{out}}$ = 65.5°C
Find the length (L) required

ASSUMPTIONS

- A uniform overall heat transfer coefficient

SKETCH

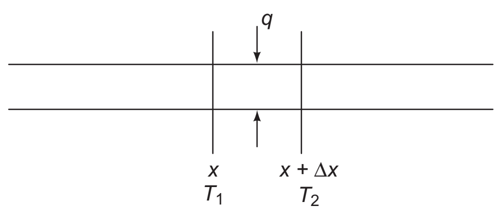


PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the mean temperature of 40°C (c_{pw}) = 4175 J/(kg K)

SOLUTION

(a) Consider an element of the exchanger as shown below



An energy balance on the element yields

$$\dot{m}_w \; c_{pw} \, \Delta T = U \, A \; [T_s - T(x)]$$

where $$\Delta T = T_2 - T_1 = \left(T_1 + \frac{dT}{dx} \Delta x \right) - T_1 = \frac{dT}{dx} \; \Delta x$$

and $$A = \pi D \, \Delta x$$

As $\Delta x \to 0$, $T_1 \to T$

$$\therefore \qquad \dot{m}_w \; c_{pw} \; \frac{dT}{dx} = U \, \pi D \, (T_s - T)$$

$$\left(\frac{1}{T - T_s} \right) dT = - \frac{U \pi D}{\dot{m}_w \, c_{pw}} \, dx$$

Integrating from 0 to X

$$\ln \left(\frac{T_w(x) - T_s}{T_{w,\text{in}} - T_s} \right) = - \frac{U \pi D}{\dot{m}_w \, c_{pw}} \, x$$

$$T_w(x) = T_s + (T_{w,\text{in}} - T_s) \exp \left(- \frac{U \pi D x}{\dot{m}_w c_{pw}} \right)$$

(b) Solving for the distance X

$$x = - \frac{\dot{m}_w C_{pw}}{U \pi D} \; \ln \left(\frac{T_w(x) - T_s}{T_{w,\text{in}} - T_s} \right)$$

$$L = - \frac{(0.063 \text{ kg/s})(4175 \text{ J/(kg k)})}{(570 \text{ W/(m}^2\text{K)})(\text{J/(Ws)}) \, \pi (0.05 \text{ m})} \; \ln \left(\frac{65.5°\text{C} - 104°\text{C}}{15.5°\text{C} - 104°\text{C}} \right) = 2.45 \text{ m}$$

PROBLEM 8.29

At a rate of 0.32 liters/s, water at 27°C enters a No. 18 BWG 1.6 cm a condenser tube made of nickel chromium steel (*k* = 26 W/(m K)). The tube is 3 m long and its outside is heated by steam condensing at 50°C. Under these conditions, the average heat transfer coefficient on the water side is 10 kW/(m^2 K), and the heat transfer coefficient on the steam side may be taken as 11.3 kW/(m^2 K). On the interior of the tube, however, there is a scale having a thermal conductance equivalent to 5.6 kW/(m^2 K). (a) Calculate the overall heat transfer coefficient *U* per square meter of exterior surface area. (b) Calculate the exit temperature of the water.

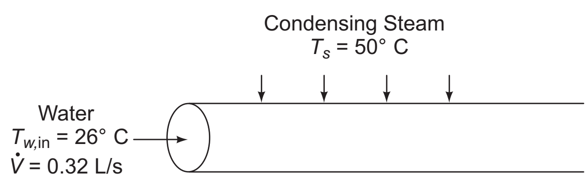
GIVEN

- A nickel chromium steel condenser tube with water inside and condensing steam outside
- Water flow rate $\dot{v}$ = 0.32 liters/s
- Water inlet temperature ($T_{w,\text{in}}$) = 27°C
- Tube: No. 18 BWG 1.6 cm
- Steel thermal conductivity (k_{st}) = 26 W/(m K)
- Tube length (L) = 3 m
- Steam temperature (T_s) = 50°C
- Average water- side transfer coefficient $(\bar{h}_i)$ = 10 kW/(m^2 K)
- Average steam transfer coefficient $(\bar{h}_o)$ = 11.3 kW/(m^2 K)
- Interior scaling conductance ($1/R_i$) = 5.6 kW/(m^2 K)

FIND

(a) The overall heat transfer coefficient (U) based on exterior surface area
(b) The outlet water temperature ($T_{w,\text{in}}$)

SKETCH



PROPERTIES AND CONSTANT

From Appendix 2, Table 42, for No. 18 BWG 5/8 in tube

Inside Diameter (D_i) = 1.32 cm

Outside Diameter (D_o) = 1.56 cm

From Appendix 2, Table 13, the density of water at 26°C (ρ) = 998 kg/m^3 = 0.998 kg/L; the specific heat (c_{pw}) = 4186 J/(kg K)

SOLUTION

(a) The overall heat transfer coefficient can be calculated from Equation (8.5)

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i\,A_o}{A_i} + \frac{A_o}{A_i\,\bar{h}_i}$$

where R_o = exterior scaling resitance = 0

$A_o = \pi D_o L$

$A_i = \pi D_i L$

$$R_k = \frac{A_o \ln\left(\frac{D_o}{D_i}\right)}{2\pi L k} = \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k}$$

$$\frac{1}{U_o} = \frac{1}{11300\,(\text{m}^2\text{K})/\text{W}} + \frac{1.56\times10^{-2}\ \text{m}\ \ln\left(\frac{1.56\,\text{cm}}{1.32\,\text{cm}}\right)}{2(26\ \text{W/(m K)})} + \frac{1.56}{1.32}$$

$$\frac{1}{5600\ \text{W/(m}^2\text{K)}} + \frac{1.56}{1.32}\frac{1}{10000\ \text{W/(m}^2\text{K)}}$$

$\Rightarrow$ $U_o = 2365\ \text{W/(m}^2\ \text{K)}$

(b) The number of transfer units is

$$NTU = \frac{UAc}{C_{\min}} = \frac{U\,\pi D_o L}{\dot m_w\, c_{pw}} = \frac{U\,\pi D_o L}{\dot v\, \rho c_{pw}}$$

$$NTU = \frac{(2365\ \text{W/(m}^2\text{K)})\,\pi(1.56\times10^{-2}\ \text{m})(3\,\text{m})}{(0.32\,\text{L/s})(0.998\,\text{kg/L})(4186\,\text{J/(kg K)})} = 0.26$$

Since the heat capacity rate of the steam is essentially infinite, $C_{\min}/C_{\max} = 0$.

From Figure 8.17 or 8.18 (both are the same for $C_{\min}/C_{\max} = 0$): $e \approx 0.20$

From Equation (8.12b)

$$\text{E} = \frac{C_c}{C_{\min}}\frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}} = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}}$$

$$T_{w,\text{out}} = T_{w,\text{in}} + \text{E}\,(T_s - T_{w,\text{in}}) = 27°\text{C} + 0.20\ (50°\text{C} - 27°\text{C}) = 31.6°\text{C}$$

PROBLEM 8.30

It is proposed to preheat the water for a boiler with flue gases from the boiler stack. The flue gases are available at 150°C, at the rate of 0.25 kg/s and specific heat of 1000 J/(kg K). The water entering the exchanger at 15°C at the rate of 0.05 kg/s is to be heated at 90°C. The heat exchanger is to be of the reversed current type, one shell pass and 4 tube passes. The water flows inside the tubes which are made of copper (2.5 cm-*ID*, 3.0 cm-*OD*). The heat transfer coefficient at the gas side is 115 W/(m² K), while the heat transfer coefficient on the water side is 1150 W/(m² K). A scale on the water side offers an additional thermal resistance of 0.002 (m² K)/W. (a) Determine the overall heat transfer coefficient based on the outer tube diameter. (b) Determine the appropriate mean temperature difference for the heat exchanger. (c) Estimate the required tube length. (d) What would be the outlet temperature and the effectiveness if the water flow rate would be doubled, giving a heat transfer coefficient of 1820 W/(m² K)?

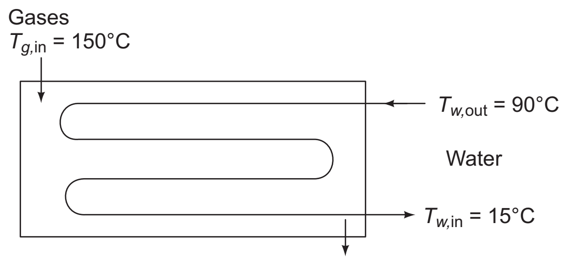
GIVEN

- Reverse current heat exchanger - 1 shell pass , 4 tube passes
- Water in tubes, flue gases in shell
- Copper tubes
  - Inside diameter (D_i) = 2.5 cm = 0.025 m
  - Outside diameter (D_o) = 3.0 cm = 0.03 m
- Specific heat of gases (c_{pg}) = 1000 J/(kg K)
- Gas inlet temperature
  - $T_{g,\text{in}} = 150°\text{C}$
- Water temperatures
  - $T_{w,\text{in}} = 15°\text{C}$
  - $T_{w,\text{out}} = 90°\text{C}$
- Gas flow rate $(\dot m_g)$ = 0.25 kg/s
- Water flow rate $(\dot m_w)$ = 0.05 kg/s
- Tubes are copper
- Gas side heat transfer coefficient $(\bar h_c)$ = 115 W/(m² K)
- Water side heat transfer coefficient $(\bar h_i)$ = 1150 W/(m² K)
- Scaling resistance on the water side (R_i) = 0.002 (m² K)/W

FIND

(a) The overall heat transfer coefficient (U_o) based on the outside tube diameter
(b) The appropriate mean temperature difference (ΔT_{mean})
(c) The required tube length (L)
(d) The outlet temperature and effectiveness if the water flow rate were doubled, making $\bar{h}_i = 1820 \text{ W/(m}^2\text{ K)}$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the average temperature of 52.5°C (c_{pw}) = 4179 J/kg K

From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) at 127°C

SOLUTION

(a) The overall heat transfer coefficient is given by Equation (8.5)

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i}$$

where $R_o = 0$

$A_o = \pi D_o L$

$A_i = \pi D_i L$

$$R_k = \frac{A_o \ln\left(\frac{D_o}{D_i}\right)}{2\pi k L} = \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k}$$

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + R_i \frac{D_o}{D_i} + \frac{D_o}{D_i \bar{h}_i}$$

$$\frac{1}{U_d} = \frac{1}{\left(115 \text{ W/(m}^2\text{K)}\right)} + \frac{0.03 \text{ m} \ln\left(\frac{3}{2.5}\right)}{2\left(392 \text{ W/(m K)}\right)} + \left(0.002 \text{ (m}^2\text{ K)/W}\right) \frac{3}{2.5} + \frac{3 \text{cm}}{\left(1150 \text{ W/(m}^2\text{K)}\right)\left(2.5 \text{cm}\right)}$$

$$U_d = 82.3 \text{ W/(m}^2\text{ K)}$$

(b) The outlet temperature of the gases can be determined from an energy balance

$$\dot{m}_s \; c_{pg} \left(T_{g,\text{in}} - T_{g,\text{out}}\right) = \dot{m}_w \; c_{pw} \left(T_{w,\text{out}} - T_{w,\text{in}}\right)$$

$$T_{g,\text{out}} = T_{g,\text{in}} - \frac{\dot{m}_w c_{pw}}{\dot{m}_g c_{pg}} \left(T_{w,\text{out}} - T_{w,\text{in}}\right)$$

$$T_{g,\text{out}} = 150°\text{C} - \frac{\left(0.05 \text{ kg/s}\right)\left(4179 \text{ J/(kg K)}\right)}{\left(0.25 \text{ kg/s}\right)\left(100 \text{ J/(kg K)}\right)} \left(90°\text{C} - 15°\text{C}\right) = 87°\text{C}$$

From Figure 8.12 for a simple counterflow heat exchanger

$$\Delta T_a = T_{g,\text{in}} - T_{w,\text{out}} = 150°\text{C} - 90°\text{C} = 60°\text{C}$$

$$\Delta T_b = T_{g,\text{out}} - T_{w,\text{in}} = 87°\text{C} - 15°\text{C} = 72°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\dfrac{\Delta T_a}{\Delta T_b}} = \frac{60°\text{C} - 72°\text{C}}{\ln\left(\dfrac{60}{72}\right)} = 66°\text{C}$$

This must be corrected using Figure 8.13

$$P = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_{g,\text{in}} - T_{w,\text{in}}} = \frac{90-15}{150-15} = 0.56$$

$$Z = \frac{T_{g,\text{in}} - T_{g,\text{out}}}{T_{w,\text{out}} - T_{w,\text{in}}} = \frac{150-87}{90-15} = 0.84$$

From Figure 8.13, $F = 0.78$

$$\therefore \Delta T_{\text{mean}} = F(LMTD) = 0.78\ (66°\text{C}) = 51°\text{C}$$

(c) The rate of heat transfer is

$$q = U A_o \Delta T_{\text{means}} = \dot{m}_w\ c_{pw}\ (T_{w,\text{out}} - T_{w,\text{in}})$$

$$L = \frac{\dot{m}_w C_{pw}}{\pi D_o U \Delta T_{\text{mean}}}(T_{w,\text{out}} - T_{w,\text{in}}) = \frac{(0.05\text{ kg/s})(4179\text{ J/(kg K)})}{(0.03\text{ m})(82.3\text{ W/(m}^2\text{K)})(51°\text{C})}\ (90°\text{C} - 15°\text{C}) = 39.6\text{ m}$$

Length of each tube pass = $L/4$ = 9.9 m

(d) For a doubled water flow rate, h_i = 1820 W/(m^2 K) similarly to part (a)

$$\frac{1}{U_d} = \frac{1}{(115\text{ W/(m}^2\text{ K)})} + \frac{0.03\text{ m}\ln\left(\dfrac{3}{2.5}\right)}{2(392\text{ W/(m}^2\text{K)})} + (0.002\ (\text{m}^2\text{K})/\text{W})\left(\frac{3}{2.5}\right) + \frac{3\text{ cm}}{(1820\text{ W/(m}^2\text{ K)})(2.5\text{ cm})}$$

$$U_d = 85.0\text{ W/(m}^2\text{ K)}$$

The heat capacity rates are

$$C_w = \dot{m}_w\ c_{pw} = (0.1\text{ kg/s})(4179\text{ J/(kg K)}) = 418\text{ W/K}$$

$$C_g = \dot{m}_g\ c_{pg} = (0.25\text{ kg/s})(1000\text{ J/(kg K)}) = 250\text{ W/K}$$

$$C_{\min}/C_{\max} = 250/418 = 0.60$$

The number of transfer units is

$$NTU = \frac{UA_o}{C_{\min}} = \frac{U\pi D_o L}{C_{\min}} = \frac{(85.0\text{ W/(m}^2\text{K)})\pi(0.03\text{ m})(39.6\text{ m})}{(250\text{ W/(m}^2\text{K)})} = 1.27$$

From Figure 8.19 $e \approx 0.57$

The outlet temperature can be calculated from Equation (8.21b)

$$\text{E} = \frac{C_w \, T_{w,\text{out}} - T_{w,\text{in}}}{C_{\min} \, T_{g,\text{in}} - T_{w,\text{in}}}$$

$$T_{w,\text{out}} = T_{w,\text{in}} + \text{E} \, \frac{C_{\min}}{C_w} \, (T_{g,\text{in}} + T_{w,\text{in}}) = 15°\text{C} + 0.57\left(\frac{250}{418}\right)(150°\text{C} - 15°\text{C}) = 61°\text{C}$$

PROBLEM 8.31

> **Water is to be heater from 10 to 30°C, at the rate of 300 kg/s by atmospheric pressure steam in a single-pass shell-and-tube heat exchanger consisting of 1-in schedule 40 steel pipe. The surface coefficient on the steam side is estimated to be 11,350 W/(m^2 K). A pump is available which can deliver the desired quantity of water provided the pressure drop through the pipes does not exceed 103.4 kPa. Calculate the number of tubes in parallel and the length of each tube necessary to operate the heat exchanger with the available pump.**

GIVEN

- Single-pass shell-and-tube heat exchanger
- Water is heated by atmosphere steam
- Water temperatures
  - $T_{w,\text{in}} = 10°\text{C}$
  - $T_{w,\text{out}} = 30°\text{C}$
- Water flow rate $(\dot{m}_w)$ = 300 kg/s
- Inner tube: 1 in schedule 40 steel pipe
- Maximum water pressure drop (Δ_p) = 15 psi = 103.4 kPa
- Steam side heat transfer coefficient $(\bar{h}_o)$ = 11,350 W/(m^2 K)

FIND

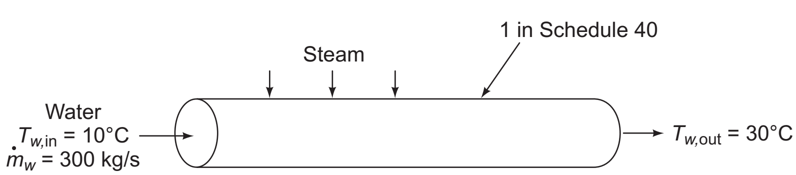
(a) The number of tubes in parallel (N)

(b) The length of each tube (L)

ASSUMPTIONS

- The tube is smooth
- The tube is 1% carbon steel
- Uniform pipe surface temperature
- Fully developed flow in pipe
- Water flow is turbulent to insure good heat transfer

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 41, for 1″ schedule 40 pipe

Inside diameter (D_i) = 1.049 in = 0.0266 m

Outside diameter (D_o) = 1.315 in = 0.0334 m

From Appendix 2, Table 13, the saturation temperature of steam at 1 atm = 100°C

From Appendix 2, Table 13, for water at the average temperature of 20°C

Absolute viscosity (μ) = 993 × 10^{-6} N s/m^2

Density (ρ) = 998.2 kg/m^3

Specific heat (c_{pw}) = 4182 J/(kg K)

Thermal conductivity (k_w) = 0.597 W/(m K)

Prandtl number (Pr) = 7.0

From Appendix 2, Table 10, the thermal conductivity of 1% carbon steel (k_s) = 43 W/(m K).

SOLUTION

The Reynolds number for the water flow through the pipes is

$$Re_D = \frac{VD_i}{\nu} = \frac{4\dot{m}}{N\pi D_i\mu} = \frac{4(300\text{ kg/s})}{N\pi(0.0266\text{ m})\left(993\times10^{-6}(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = (1.45\times10^7)\ \frac{1}{N}$$

The friction factor for turbulent flow through smooth tubes for $10^5 < Re < 10^6$ is given by Equation (6.59)

$$f = 1.84\ Re^{-0.2} = 1.84\ (1.45\times10^7\ N^{-1})^{-0.2} = 0.068\ N^{0.2}$$

The pressure drop through the tube is given by Equation (6.13)

$$\Delta p = f\,\frac{L}{D_i}\,\frac{\rho V^2}{2g_c} = 1.84\left(\frac{4\dot{m}}{N\pi D_i\mu}\right)^{-0.2}\frac{L}{D_i}\,\frac{\text{p}}{2g_c}\left(\frac{4\dot{\text{m}}}{4\pi \text{D}_\text{i}^2\rho}\right) = 11.16\left(\frac{\dot{m}}{\pi}\right)^{1.8}\frac{\mu^{0.2}}{\rho D^{4.8}}LN^{-1.8}$$

$$\frac{L}{N^{1.8}} = 0.0896\ \Delta p\left(\frac{\pi}{\dot{m}}\right)^{1.8}\frac{\rho D^{4.8}}{\mu^{0.2}} = 0.0896\ (103400\text{ N/m}^2)\left((\text{kg m})/(\text{s}^2\text{N})\right)$$

$$\left(\frac{\pi}{300\text{ kg/s}}\right)^{1.8}\frac{(993\text{ kg/s})(0.0266\text{ m})^{48}}{\left[993\times10^{-6}\left((\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)\right]}$$

$$\frac{L^{1.8}}{N} = 8.26\times10^{-4}\text{ m} \rightarrow L = (2.75\times10^{-4})\ N^{1.8}$$

The heat capacity rate of the condensing steam is essentially infinite, therefore, $C_{\min}/C_{\max}$ = 0. The effectiveness, from Equation (8.21b)is

$$\text{E} = \frac{C_w}{C_{\min}}\,\frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}} = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}} = \frac{30°\text{C} - 10°\text{C}}{1000°\text{C} - 10°\text{C}} = 0.22$$

From Figure 8.17 or 8.18, $NTU \approx 0.25$

$$NTU = \frac{UA}{C_{\min}} = \frac{U_o A_o}{\dot{m}_w c_{pw}} = \frac{U_o N\pi D_o L}{\dot{m}_w c_{pw}}$$

The overall heat transfer coefficient (U_o) is given by Equation (8.2)

$$U_O = \frac{1}{\left(\frac{A_o}{A_i h_i}\right) + \left[A_o \ln\left(\frac{\frac{r_o}{r_i}}{2k_s L}\right)\right] + \left(\frac{1}{\bar{h}_o}\right)} = \frac{1}{\left(\frac{D_o}{D_i h_i}\right) + \left[D_o \ln\left(\frac{\frac{D_o}{D_i}}{2k_s L}\right)\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

For fully developed turbulent flow with constant surface temperature, the Nusselt number in the pipe is given by Equation (6.63)

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\,Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$\bar{h}_i = \overline{Nu}_D \frac{k_w}{D_i} = 0.023 \frac{k_w}{D_i}\left(\frac{4\dot{m}}{N\pi D_i}\right)^{0.8} Pr^{0.4}$$

Substituting these and $L = 2.75 \times 10^{-4} N^{1.8}$ m into the expression for NTU

$$NTU = \frac{N\pi D_o\left(2.75\times10^{-4}\,\text{m}\right)N^{1.8}}{\left(\dot{m}_w c_{pw}\right)\left[\dfrac{D_o}{D_i\left[\left(\dfrac{0.023\,k_w}{D_i}\right)\left(1.45\times10^7 N^{-1}\right)^{0.8}(Pr)^{0.4}\right]} + \left(D_o \ln\dfrac{\frac{D_o}{D_i}}{2k_s L}\right) + \left(\dfrac{1}{\bar{h}_o}\right)\right]}$$

$$0.25 = 300\ \text{kg/s}\,(4182\ \text{J/(kg K)})$$

$$\frac{\pi(0.0334\,\text{m})\left(2.75\times10^{-4}\,\text{m}\right)\text{N}^{1.8}}{(300\ \text{kg/s})(4182\ \text{J/(kg k)})\left[\left(\dfrac{0.03340}{0.0266}\right)\dfrac{1}{(0.023)\dfrac{0.597\ \text{W/(mK)}}{0.0266\,\text{m}}\left(\dfrac{1.45\times10^7}{\text{N}}\right)^{0.8}7^{0.4}} + \dfrac{\left(0.0334\ln\left(\dfrac{0.0034}{0.0026}\right)\right)}{2(43\ \text{W/(mK)})} + \dfrac{1}{\left(11{,}350\ \text{W/(m}^2\text{K)}\right)}\right]}$$

Canceling all units

$$0.25 = \frac{2.30\times10^{11} N^{2.8}}{2.084\times10^{-6} N^{0.8} + 1.77\times10^{-4}}$$

By trial and error, $N = 220$

Therefore

$$L = 2.75 \times 10^{-4}\,(220)^{1.8} = 4.5\ \text{m}$$

PROBLEM 8.32

Water flowing at a rate of 12.6 kg/s is to be cooled from 90 to 65°C by means of an equal flow rate of cold water entering at 40°C. The water velocity with the such that the overall coefficient of heat transfer U is 2300 W/(m^2 K). Calculate the square meters of heat-exchanger surface needed for each of the following arrangements: (a) parallel flow, (b) counterflow, (c) a multi-pass heat exchanger with the hot water making one pass through a well-baffled shell and the cold water making two passes through the tubes, and (d) a crossflow heat exchanger with both sides unmixed.

GIVEN

- Warm water cooled by cold water in a heat exchanger
- Both flow rates ($\dot{m}_c = \dot{m}_w$) = 12.6 kg/s
- Water temperatures
  - $T_{h,\text{in}}$ = 90°C
  - $T_{h,\text{out}}$ = 65°C
  - $T_{c,\text{in}}$ = 40°C
- Overall heat transfer coefficient (U) = 2300 W/(m^2 K)

FIND

The transfer area (A) for

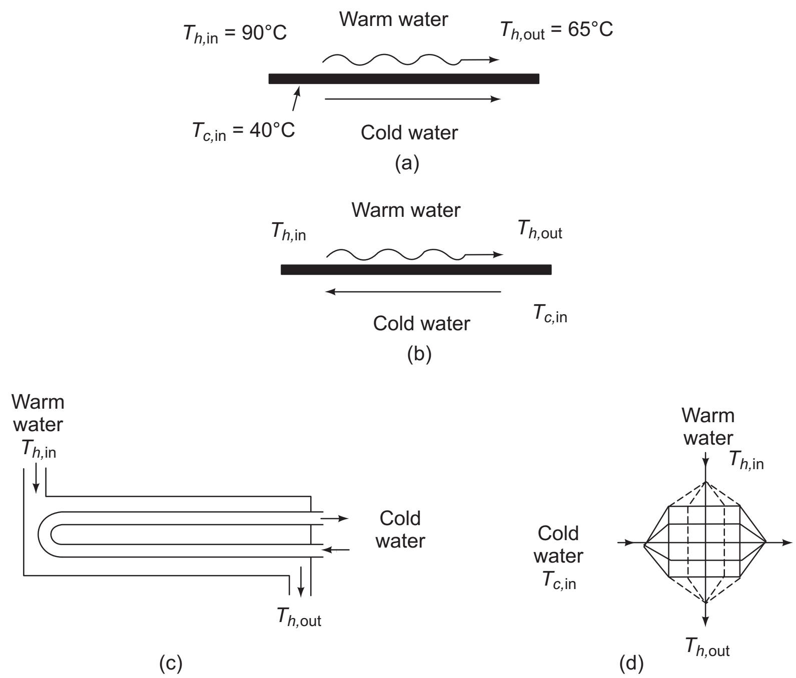
(a) Parallel flow (c) Tube-and-shell; 1 hot shell pass, 2 cold tube passes

(b) Counterflow (d) Crossflow - both unmixed

ASSUMPTIONS

- The specific heat is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, the specific heat of water in the temperature range of interest (c_p) = 4187 J/(kg K)

SOLUTION

$$\text{Since} \quad \dot{m}_h = \dot{m}_c \text{ and } c_{ph} = c_{pc} \Rightarrow \frac{C_{\min}}{C_{\max}} = 1.0$$

$$\text{Also} \quad \Delta T_h = \Delta T_c \Rightarrow T_{c,\text{out}} = T_{c,\text{in}} + \Delta T_h = 40°\text{C} + 25°\text{C} = 65°\text{C}$$

The effectiveness of the heat exchanger, from Equation (8.21a) is

$$\text{E} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{90°\text{C} - 65°\text{C}}{90°\text{C} - 40°\text{C}} = 0.50$$

(a) Figure 8.16 shows that infinite NTU would be required to reach e = 0.5 for $C_{\min}/C_{\max} = 1$ with a parallel flow configuration. Therefore, parallel flow is not practical. For $C_{\min}/C_{\max} = 1$, Equation (8.25) reduces to

$$\text{E} = \frac{1}{2}(1 - e^{-2NTU}) \quad \text{For } \text{E} = 0.5\text{: } e^{-NTU} = 0 \; NTU \to \infty$$

However, for practical purposes, e = 0.5 at NTU = 2.5

$$\therefore A = NTU\ \frac{C_{\min}}{U} = 2.5\ \frac{(52{,}756 \text{ W/K})}{\left(2300 \text{ W/(m}^2\text{K)}\right)} = 57.3 \text{ m}^3$$

(b) From Figure (8.18), for e = 0.5 and $C_{\min}/C_{\max}$ = 1.0: NTU = 1.1

$$NTU = \frac{UA}{C_{\min}} \text{ where } C_{\min} = \dot{m}\ \ 8\ c_p = (12.6 \text{ kg/s})(4187 \text{ J/(kg K)}) = 52.756 \text{ W/K}$$

Solving for the area

$$A = NTU\ \frac{C_{\min}}{U} = 1.1\ \frac{(52{,}756 \text{ W/K})}{\left(2300 \text{ W/(m}^2\text{K)}\right)} = 25.2 \text{ m}^2$$

(c) From Figure (8.19), NTU = 1.3

$$A = 1.3\ \frac{(52{,}756 \text{ W/K})}{\left(2300 \text{ W/(m}^2\text{K)}\right)} = 29.8 \text{ m}^2$$

(d) From Figure (8.20), NTU = 1.2

$$A = 1.2\ \frac{(52{,}756 \text{ W/K})}{\left(2300 \text{ W/(m}^2\text{K)}\right)} = 27.5 \text{ m}^2$$

PROBLEM 8.33

Water flowing at a rate of 10 kg/s through 50 double-pass tubes in a shell and tube heat exchanger heats air that flows through the shell side. The length of the brass tubes is 6.7 m and they have an outside diameter of 2.6 cm and an inside diameter of 2.3 cm. The heat transfer coefficient of the water and air are 470 W/(m^2 K) and 210 W/(m^2 K), respectively. The air enters the shell at a temperature of 15°C and a flow rate of 1.6 kg/s. The temperature of the water as it enters the tubes is 75°C. Calculate (a) the heat exchanger effectiveness, (b) the heat transfer rate to the air, and (c) the outlet temperature of the air and water.

GIVEN

- Shell-and-tube heat exchanger - one shell, two tube passes
- Water in brass tubes, air in shell
- Water flow rate $(\dot{m}_w)$ = 10 kg/s
- Number of double passes (N) = 50
- Tube length (L) = 6.7 m
- Tube diameters
  - D_o = 2.6 cm = 0.026 m
  - D_i = 2.3 cm = 0.023 m
- Heat transfer coefficient
  - Water $(\bar{h}_i)$ = 470 W/(m^2 K)
  - Air $(\bar{h}_o)$ = 210 W/(m^2 K)
- Air inlet temperature ($T_{a,\text{in}}$) = 15°C
- Air flow rate $(\dot{m}_a)$ = 1.6 kg/s
- Water inlet temperature ($T_{w,\text{in}}$) = 75°C

FIND

(a) Effectiveness (e)

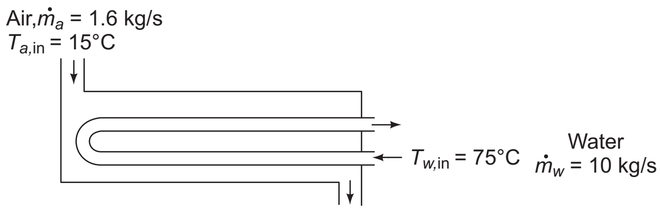
(b) The heat transfer rate (q)

(c) Outlet temperatures ($T_{a,\text{out}}$, $T_{w,\text{out}}$)

ASSUMPTIONS

- Tube length includes both passes

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at 75°C (c_{pw}) = 4190 J/(kg K)
From Appendix 2, Table 27, the specific heat of air at 15°C (c_{pa}) = 1012 J/(kg K)
From Appendix 2, Table 10, the thermal conductivity of brass (k_b) = 111 W/(m K)

SOLUTION

The heat capacity rates are

$$C_w = \dot{m}_w\ c_{pw} = (10\text{ kg/s})\ (4190\text{ J/(kg K)}) = 41{,}900\text{ W/K}$$

$$C_a = \dot{m}_a\ c_{pa} = (1.6\text{ kg/s})\ (1012\text{ J/(kg K)}) = 1619\text{ W/K}$$

$$\frac{C_{\min}}{C_{\max}} = \frac{1619}{41{,}900} = 0.039$$

(a) The overall heat transfer coefficient is given by Equation (8.2)

$$U_o = \frac{1}{\left(\dfrac{A_o}{A_i\bar{h}_i}\right) + \left(A_o \ln\left(\dfrac{\frac{r_o}{r_i}}{2\pi kL}\right)\right) + \left(\dfrac{1}{\bar{h}_o}\right)} = \frac{1}{\left(\dfrac{D_o}{D_i\bar{h}_i}\right) + \left(D_o \ln\left(\dfrac{\frac{D_o}{D_i}}{2\pi kL}\right)\right) + \left(\dfrac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\dfrac{(0.026\text{ m})}{(0.023\text{m})\left(470\text{ W/(m}^2\text{K)}\right)} + \dfrac{(0.026\text{m})(\ln(0.026\text{m}/0.023\text{m}))}{2(111\text{ W/(m K)})} + \dfrac{1}{\left(210\text{ W/(m}^2\text{K)}\right)}} = 139\text{ W/(m}^2\text{ K)}$$

The transfer area is $A = N(\pi D_o L) = 50[\pi(0.026\text{ m})(6.7\text{ m})] = 27.36\text{ m}^2$
The Number of transfer units is

$$NTU = \frac{UA}{C_{\min}} = \frac{\left(139\text{ W/(m}^2\text{K)}\right)(27.36\text{ m}^2)}{\left(1619\text{ W/(m}^2\text{K)}\right)} = 2.35$$

From Figure 8.19 for $C_{\min}/C_{\max}$ = 0.04 and NTU = 2.35, e ≈ 0.88 = 88%
(b) The rate of heat transfer is given by Equation (8.22)

$$q = \text{E}\ C_{\min}\ (T_{h,\text{in}} - T_{c,\text{in}}) = 0.88\ \left(1619\text{ W/(m}^2\text{K)}\right)\ (75°\text{C} - 15°\text{C}) = 85{,}480\text{ W}$$

(c) For the water

$$q = C_w\ (T_{w,\text{in}} - T_{w,\text{out}})$$

$$\therefore\quad T_{w,\text{out}} = T_{w,\text{in}} - \frac{q}{C_w} = 75°\text{C} - \frac{85{,}480\text{ W}}{(41{,}90\text{ (W/K)})} = 73°\text{C}$$

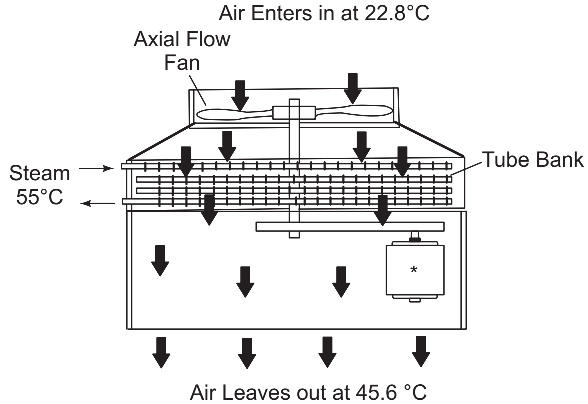
For the air

$$q = C_a\,(T_{a,\text{out}} - T_{a,\text{in}})$$

$$\therefore \qquad T_{a,\text{out}} = T_{a,\text{in}} + \frac{q}{C_a} = 15°\text{C} + \frac{85{,}480\,\text{W}}{(1619\,(\text{W/K}))} = 68°\text{C}50$$

PROBLEM 8.34

An air-cooled low-pressure-steam condenser is shown below.



The tube bank is four rows deep in the direction of air flow. There are 80 tubes total. The tubes have *ID* = 2.2 cm and *OD* 2.5 cm and are 9 m long. The tubes have circular fins on the outside. The tube-plus-fin area is 16 times the bare tube area (i.e., the fin area is 15 times the bare tube area, neglect the tube surface covered by fins). The fin efficiency is 0.75. Air flows past the outside of the tubes. On a particular day, the air enters at 22.8°C and leaves at 45.6°C. The air flow rate is 3.4×10^5 kg/h.

The steam temperature is 55°C and has a condensing coefficient of 10^4 W/(m^2 K). The steam-side fouling coefficient is 104 W/(m^{2}K). The tube wall conductance per unit area is 10^5 W/(m^{2}K). The air-side fouling resistance is negligible. The air-side-film heat transfer coefficient is 285 W/(m^{2}K). (Note this value has been corrected for the number of transverse tube rows.)

(a) What is the log-mean temperature difference between the two streams?
(b) What is the rate of heat transfer?
(c) What is the rate of steam condensation?
(d) Estimate the rate of steam condensation if there were no fins.

GIVEN

- The condenser shown above
- Number of tubes (N) = 80
- Number of rows (N_r) = 4 (in air flow direction)
- Tube diameters
  - D_i = 2.2 cm = 0.022 m
  - D_o = 2.5 cm = 0.025 m
- Tube length (L) = 9 m
- Air temperature
  - $T_{a,\text{in}}$ = 22.8°C
  - $T_{a,\text{out}}$ = 45.6°C
- Air flow rate $(\dot{m}_a)$ = 3.4×10^5 kg/h = 94.4 kg/s
- Steam temperature = 55°C (constant)
- Fin area = 15 (tube area)
- Fin efficiency (η_f) = 0.75

- Steam side
  - Transfer coefficient $\left(\bar{h}_i\right)$ = 10^4 W/(m^2 K)
  - Fouling coefficient ($1/R_i$) = 10^4 W/(m^2 K)
- Tube wall conductance per unit area ($1/R_k$) = 10^5 W/(m^2 K)
- Air side: Transfer coefficient $\left(\bar{h}_o\right)$ = 285 W/(m^2 K)
- Fouling resistance on the air side is negligible

FIND

(a) The log-mean temperature difference (*LMTD*)
(b) The rate of heat transfer (q)
(c) The rate of steam condensation $\left(\dot{m}_c\right)$
(d) Estimate the rate of steam condensation if there were no fins $\left(\dot{m}_{c2}\right)$

ASSUMPTIONS

- Air side transfer coefficient is the same with or without fins
- Tube surface covered by the fins is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average temperature of 34.2°C, the specific heat (c_{pa}) = 1013 J/(kg K)

From Appendix 2, Table 13, for steam at a saturation temperature of 55°C, the heat of vaporization (h_{fg}) = 2600 (kJ/kg)

SOLUTION

(a) From Figure 8.10

$$\Delta T_a = T_s - T_{a,\text{in}} = 55°\text{C} - 22.8°\text{C} = 32.2°\text{C}$$
$$\Delta T_b = T_s - T_{a,\text{out}} = 55°\text{C} - 45.6°\text{C} = 9.4°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{32.2°\text{C} - 9.4°\text{C}}{\ln\left(\frac{32.2}{9.4}\right)} = 18.5°\text{C}$$

(b) The overall heat transfer coefficient is given by Equation (8.6) for the base tube area

$$\frac{1}{U_{d,\text{bare}}} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + 0 + \frac{1}{\left(\frac{1}{R_k}\right)} + \frac{D_o}{D_i}\left(\frac{1}{\left(\frac{1}{R_i}\right)} + \frac{1}{\bar{h}_i}\right)$$

$$\frac{1}{U_{d,\text{bare}}} = \frac{1}{\left(285\ \text{W/(m}^2\text{K)}\right)} + \frac{1}{\left(10^5\ \text{W/(m}^2\text{K)}\right)} + \left(\frac{2.5}{2.2}\right)\left(\frac{1}{10^4\left(\text{W/(m}^2\text{ K)}\right)} + \frac{1}{10^4\left(\text{W/(m}^2\text{ K)}\right)}\right)$$

$$U_{d,\text{bare}} = 267\ \text{W/(m}^2\text{ K)}$$

The rate of heat transfer for the bare tubes above is

$$q_b = U A\,(LMTD) = U_d(N \pi D_o L)(LMTD)$$
$$q_b = \left(267\ \text{W/(m}^2\text{K)}\right) 80\,\pi (0.025\ \text{m})\,(9\ \text{m})\,(18.5°\text{C}) = 2.79 \times 10^5\ \text{W}$$

If the entire fin area was at the same temperature as the exterior of the tube wall, the rate of heat transfer from the fin would be

$$Q'_f = U A_{\text{fin}}\,(LMTD) = U_d\,(15\,A_t)\,LMTD = 15\,q_b$$

The actual rate of heat transfer from the fins is

$$q_f = \eta_f\, q'_f = 15\,\eta_f\, q_b$$

The total rate of heat transfer is

$$q = q_b + q_f = q_b\,(1 + 15\eta_f) = 2.79 \times 10^5\,[1 + 15(0.75)] = 3.42 \times 10^6\ \text{W}$$

(c) The rate of condensation is

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{3.42\times10^6\ \text{W}\,(\text{J}/(\text{Ws}))}{(2600\ \text{kJ/kg})(1000\ \text{J/kJ})} = 1.32\ \text{kg/s}$$

(d) If there were no fins, the rate of heat transfer would be that from the bare tube alone. Therefore

$$\dot{m}_{c2} = \frac{q_b}{h_{fg}} = \frac{2.79\times10^5\ \text{W}\,(\text{J}/(\text{W s}))}{(2600\ \text{kJ/kg})(1000\ \text{J/kJ})} = 0.11\ \text{kg/s}$$

COMMENTS

The rate of condensate flow without the fins is only 8% of that with fins.

PROBLEM 8.35

Design (i.e., determine the overall area and a suitable arrangement of shell and tube passes) for a tubular-feed water heater capable of heating 2,300 kg/h of water from 21 to 90°C. The following specification are given (a) saturated steam at 920 kPa absolute pressure is condensing on the outer tube surface, (b) heat transfer coefficient on steam side is 6800 W/(m^2 K), (c) tubes are of copper, 2.5 cm, 2.3 cm *ID*, 24 m long, and (d) water velocity is 0.8 m/s.

GIVEN

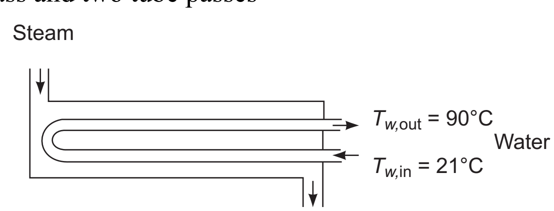
- A tubular-feed water heater, condensing saturated steam on outside
- Water flow rate $(\dot{m}_w)$ = 2300 kg/h = 0.639 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 21°C
  - $T_{w,\text{out}}$ = 90°C
- Steam temperature = 920 kPa absolute
- Heat transfer coefficient on steam side $(\bar{h}_o)$ = 6800 W/(m^2 K)
- Tubes are copper
  - D_i = 2.3 cm = 0.023 m
  - D_o = 2.5 cm = 0.025 m
- Water velocity (V) = 0.8 m/s

FIND

- The transfer area (A) and a suitable arrangement of shell and tube passes

SKETCH

Assuming a single shell pass and two tube passes



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the thermal conductivity of copper at 127°C (k) = 392 W/(m K)

From Appendix 2, Table 13, the saturation temperature of steam at 920 kPa (T_s) = 176°C

From Appendix 2, Table 13, for water at the average temperature of 56°C

Density (ρ) = 984.9 kg/m^3

Specific area (c_{pw}) = 4181 J/(kg K)

Thermal conductivity (k_w) = 0.653 W/(m K)
Kinematic viscosity (ν) = 0.510 × 10^6 m^2/s
Prandtl number (Pr) = 3.23

SOLUTION

The Reynolds number for the water flow is

$$Re_D = \frac{VD_i}{\nu} = \frac{(0.8\text{ m/s})(0.023\text{ m})}{(0.510\times10^{-6}\text{ m}^2/\text{s})} = 3.61 \times 10^4$$

For turbulent flow in a tube, the Nusselt number is given by Equation (6.63)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(6.31 \times 10^6)^{0.8}\,(3.23)^{0.4} = 163$$

$$\bar{h}_i = \overline{Nu}_D \frac{k_w}{D} = 163\frac{(0.653\text{ W/(m K)})}{0.023\text{ m}} = 4617\text{ W/(m}^2\text{K)}$$

The overall heat transfer coefficient is given by Equation (8.2)

$$U_o = \frac{1}{\left(\frac{A_o}{A_i\bar{h}_i}\right) + \left(A_o \ln\left(\frac{\frac{r_o}{r_i}}{2\pi kL}\right)\right) + \left(\frac{1}{\bar{h}_o}\right)} = \frac{1}{\left(\frac{D_o}{D_i\bar{h}_i}\right) + \left(D_o \ln\left(\frac{\frac{D_o}{D_i}}{2k}\right)\right) + \left(\frac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\frac{(0.025\text{ m})}{(0.023\text{ m})(4617\text{ W/(m}^2\text{ K)})} + \frac{(0.025\text{ m})\left(\ln\frac{0.025}{0.023}\right)}{2(392\text{ W/(m K)})} + \frac{1}{(6800\text{ W/(m}^2\text{ K)})}} = 2596\text{ W/(m}^2\text{K)}$$

From Figure 8.9 $\quad \Delta T_a = T_s - T_{w,\text{in}} = 176°\text{C} - 21°\text{C} = 155°\text{C}$

$$\Delta T_b = T_s - T_{w,\text{out}} = 176°\text{C} - 90°\text{C} = 86°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{155°\text{C} - 86°\text{C}}{\ln\left(\frac{155}{86}\right)} = 117°\text{C}$$

The *LMTD* must be corrected using Figure (8.12)

$$P = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_{s,\text{in}} - T_{w,\text{in}}} = \frac{90-21}{176-21} = 0.445$$

$$Z = \frac{T_{s,\text{in}} - T_{s,\text{out}}}{T_{w,\text{out}} - T_{w,\text{in}}} = 0$$

From Figure (8.12), F = 1.0. Due to the constant temperature of the condensing steam, this arrangement is as effective as a pure counterflow heat exchanger.

The rate of heat transfer is given by

$$q = U_o\, A_o\, (LMTD) = \dot{m}_w\, c_{pw}\, (T_{w,\text{in}} - T_{w,\text{out}})$$

Solving for the outer tube area required

$$A_o = \frac{\dot{m}_w c_{pw}\left(T_{w,\text{in}} - T_{w,\text{out}}\right)}{U_o\,(LMTD)} = \frac{(0.639\text{ kg/s})(4181\text{ J/(kg K)})(90°\text{C} - 21°\text{C})}{\left(2596\text{ W/(m}^2\text{ K)}\right)(177°\text{C})(\text{J/(Ws)})} = 0.61\text{ m}^2$$

The number of tubes (N) required for the water to have the given velocity and flow rate is given by

$$\dot{m}_w = V\,\rho\,A_{\text{flow}} = V\,\rho\,N\frac{\pi}{4}D_i^2$$

$$N = \frac{4\dot{m}_w}{V\rho\pi D_i^2} = \frac{4(0.639\ \text{kg/s})}{(0.8\ \text{m/s})(984.9\ \text{kg/m}^3)\,\pi(0.023\,\text{m})^2} = 1.95 \approx 2$$

If there are two tubes each making two passes, the length of each pass (L_p) is determined from

$$A_o = (2\ \text{tubers})\ (2\ \text{passes})\ L_p\ \pi D_o$$

$$L_p = \frac{A_o}{4\pi D_o} = \frac{0.61\,\text{m}^2}{4\pi(0.025\,\text{m})} = 1.94\ \text{m}$$

The tube length is 2 L_p = 3.88 m

The tube length is 2 L_p = 3.88 m

Heat exchanger specifications:

– Shell and tube design with two tubes
– One shell pass, two tube passes
– Length of each tube pass = 1.87 m
– Length of each tube = 3.88 m

PROBLEM 8.36

Two engineers are having an argument about the efficiency of a tube-side multipass heat exchanger compared to a similar exchanger with a single tube-side pass. Mr. Smith claims that for a given number of tubes are rate of heat transfer, more area is required in a two-pass exchanger than in a one-pass, because the effective temperature difference is less. Mr. Jones, on the other hand, claims that because the tube-side velocity and hence coefficient is higher, less area is required in a two-pass exchanger.

With the conditions given below, which is correct? Which case would you recommend, of what changes in the exchanger would you recommend?

Exchanger specifications
– 200 tube passes total
– 1 inch O.D copper tubes, 16 B.W.G.
Tube-side fluid
Water entering at 16°C, leaving at 28°C, with a rate of 225,000 kg/h.
Shell-side fluid
Mobiltherm 600, entering at 50°C, leaving at 33°C.
Shell side coefficient = 1700 W/(m^2 K)

GIVEN

- Tube and shell heat exchanger - water in tubes, Mobiltherm 600 in shell
- Number of tube passes (N_p) = 200
- Tubes are 1 in copper 16 B.W.G.
- Water flow rate $(\dot{m}_w)$ = 225,000 kg/h = 62.5 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 16°C
  - $T_{w,\text{out}}$ = 28°
- Mobiltherm temperatures
  - $T_{m,\text{in}}$ = 50°C
  - $T_{m,\text{out}}$ = 33°C
- Shell side heat transfer coefficient $(\bar{h}_o)$ = 1700 W/(m^2 K)

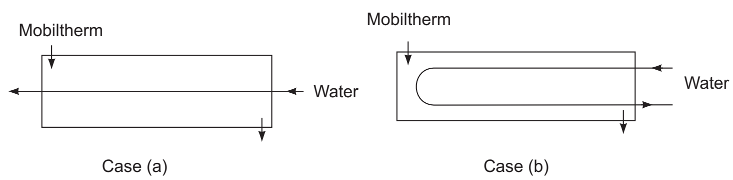
FIND

- Which required less transfer area: (a) single tube pass (b) Two tube passes?

ASSUMPTIONS

- Thermal resistance of copper tube wall is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 42, for 1 in 16 B.W.G. tubes, the diameters are

D_i = 0.870 in = 0.0221 m

D_o = 1.0 in = 0.0254 m

From Appendix 2, Table 13, for water at the average temperature of 22°C

Thermal conductivity (k) = 0.601 W/(m K)

Kinematic viscosity (ν) = 0.957×10^{-6} m^2/s

Prandtl number (Pr) = 6.6

Density (ρ) = 998 kg/m^3

Specific heat (c_{pw}) = 4180 J/(kg K)

From Appendix 2, Table 22, the specific heat of Mobiltherm 600 at its average temperature of 42°C (c_{pm}) = 1654 J/(kg K)

SOLUTION

For case (a) number of flow passages (N) = Total passes/(passes per tube) = 200/1 = 200.

For case (b) N = 200/2 = 100.

The water velocity (V) is determined by

$$V = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho N\left(\frac{\pi}{4}\right) D_i^2} = \frac{4\dot{m}}{N\pi D_i^2}$$

Case (a)

$$V_a = \frac{4(62.5 \text{ kg/s})}{200\left(998 \text{ kg/m}^3\right)\pi(0.0221 \text{ m})^2} = 0.816 \text{ m/s}$$

Case (b)

$$V_b = 2\, V_a = 2(0.816 \text{ m/s}) = 1.63 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{VD_i}{\nu}$$

Case (a)

$$Re_{Da} = \frac{(0.0816 \text{ m/s})(0.0221 \text{ m})}{\left(0.957 \times 10^{-6} \text{ m}^2/\text{s}\right)} = 18{,}851$$

Case (b)

$$Re_{Db} = 2\ Re_{Da} = 37{,}701$$

The Nusselt number for turbulent flow in a tube is given by Equation (6.63)

$$Nu_D = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

Case (a)

$$\overline{Nu}_{Da} = 0.023\ (18{,}851)^{0.8}\ (6.6)^{0.4} = 129$$

$$\bar{h}_{ia} = \overline{Nu}_{Da}\ \frac{k}{D_i} = 129\,\frac{(0.601\,\text{W/(m K)})}{0.0221\,\text{m}} = 3502\ \text{W/(m}^2\ \text{K)}$$

Case (b)

$$\overline{Nu}_{Db} = 0.023\ (37{,}701 \times 10^6)^{0.8}\ (6.6)^{0.4} = 224$$

$$\bar{h}_{ib} = 224\,\frac{(0.601\ \text{W/(m K)})}{0.0221\,\text{m}} = 6108\ \text{W/(m}^2\text{K)}$$

The overall heat transfer coefficient, neglecting the tube wall resistance is

$$\frac{1}{U_o} = \frac{D_o}{D_i \bar{h}_i} + \frac{1}{\bar{h}_o}$$

Case (a)

$$\frac{1}{U_o} = \left(\frac{0.0254}{0.0221}\right) \frac{1}{(3502\ \text{W/(m}^2\text{K)})} + \frac{1}{(1700\ \text{W/(m}^2\text{K)})} \Rightarrow U_o = 1091\ \text{W/(m}^2\text{K)}$$

Case (b)

$$\frac{1}{U_o} = \left(\frac{0.0254}{0.0221}\right) \frac{1}{(6108\ \text{W/(m}^2\text{K)})} + \frac{1}{(1700\ \text{W/(m}^2\ \text{K)})} \Rightarrow U_o = 1288\ \text{W/(m}^2\text{K)}$$

The Log-mean temperature difference for counterflow, from Figure 8.9 is

$$\Delta T_a = T_{m,\text{in}} - T_{w,\text{out}} = 50°\text{C} - 28°\text{C} = 22°\text{C}$$

$$\Delta T_b = T_{m,\text{out}} - T_{w,\text{in}} = 33°\text{C} - 16°\text{C} = 17°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{22°\text{C} - 17°\text{C}}{\ln\left(\frac{22}{17}\right)} = 19.4°\text{C}$$

For case (a): $\Delta T_{\text{mean}} = LMTD = 19.4°\text{C}$

For case (b): The *LMTD* must be corrected using Figure 8.13

$$P = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_{m,\text{in}} - T_{w,\text{in}}} = \frac{28-16}{50-16} = 0.35$$

$$Z = \frac{T_{m,\text{in}} - T_{m,\text{out}}}{T_{w,\text{out}} - T_{w,\text{in}}} = \frac{50-33}{28-16} = 1.41$$

From Figure 8.13, $F = 0.91$

$$\therefore \qquad \Delta T_{\text{mean}} = F(LMTD) = 0.91\ (19.4°\text{C}) = 17.7°\text{C}$$

The rate of heat transfer is

$$q = U_o\, A_o\, \Delta T_{\text{mean}} = \dot{m}_w\, c_{pw}(T_{w,\text{out}} - T_{w,\text{in}})$$

$$\therefore \qquad A_o = \frac{\dot{m}_w c_{pw}\left(T_{w,\text{out}} - T_{w,\text{in}}\right)}{U_o \Delta T_{\text{mean}}}$$

Case (a)

$$A_o = \frac{(62.5\text{ kg/s})(4180\text{ J/(kg K)})(28°\text{C} - 16°\text{C})}{\left(1091\text{ W/(m}^2\text{K)}\right)(19.4°\text{C})(\text{J/(W s)})} = 148\text{ m}^2$$

Case (b)

$$A_o = \frac{(62.5\text{ kg/s})(4180\text{ J/(kg K)})(28°\text{C} - 16°\text{C})}{\left(1288\text{ W/(m}^2\text{K)}\right)(17.7°\text{C})(\text{J/(W s)})} = 138\text{ m}^2$$

COMMENTS

For these operating conditions, the double-pass heat exchanger requires about 8% less area because although the mean temperature difference for the double pass is 9% less than that for the single pass, the overall heat transfer coefficient is 18% greater.

PROBLEM 8.37

A horizontal shell-and-tube heat exchanger is used to condense organic vapors. The organic vapors condense on the outside of the tubes. Water is used as the cooling medium on the inside of the tubes. The condenser tubes are 1.9 cm *O.D.*, 1.6 cm *ID* copper tubes, 2.4 m in length. There are a total of 768 tubes.

The water makes four passes through the exchanger.

Test data obtained when the unit was first placed into service are as follows

Water rate = 3700 l/min

Inlet water temperature = 29°C

Outlet water temperature = 49°C

Organic-vapor condensation temperature = 118°C

After 3 months of operation, another test, made under the same conditions as the first, i.e., same water rate and inlet temperature and same condensation temperature, showed that the exit water temperature was 46°C.

(a) What is the tube-side-fluid (water) velocity?

(b) What is the effectiveness, *e*, of the exchanger at the time of the first and second test.

(c) Assuming no changes in either the inside transfer coefficient on the condensing coefficient and negligible shell-side fouling, and no fouling at the time of the first test, estimate the tube-side fouling coefficient at the time of the second test.

GIVEN

- A shell-and-tube exchanger, organic vapors condensing in shell, water in copper tubes
- Tube diameters
  - $D_o = 1.9$ cm = 0.019 m
  - $D_i = 1.6$ cm = 0.016 m
- Tube length (L) = 2.4 m
- Number of tubes (N) = 768

- Number of tube passes (N_p) = 4
- Water flow rate $(\dot{v}_w)$ = 3700 1/min = 3.7 m^3/min
- Water temperatures
  - $T_{w,\text{in}}$ = 29°C
  - $T_{w,\text{out}}$ = 49°C
- Organic vapor condensation temperature (T_c) = 118°C
- After 3 months: $T_{w,\text{out}}$ = 46°C

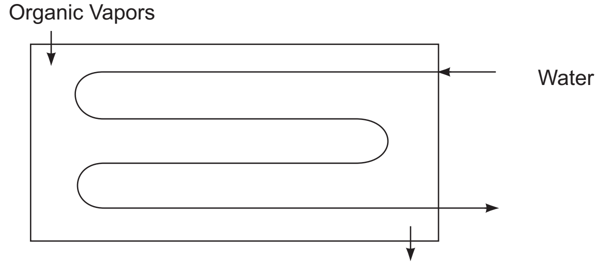
FIND

(a) Water velocity (V_w)
(b) The effectiveness (e) at the time of both tests
(c) Fouling coefficient ($1/R_i$) at the time of the second test

ASSUMPTIONS

- No fouling at the time of the first test
- No change in the inside and outside heat transfer coefficients
- Negligible shell-side fouling
- Length given is the length of one tube - all four passes

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 40°C

Density (ρ) = 992 kg/m^3
Specific heat (c_{pw}) = 4175 J/(kg K)
Thermal conductivity (k) = 0.633 W/(m K)
Kinematic viscosity (ν) = 0.658×10^{-6} m^2/s
Prandtl number (Pr) = 4.3

From Appendix 2, Table 12, the thermal conductivity of copper (k_c) = 392 W/(m K) at 127°C.

SOLUTION

(a) The water velocity is

$$V_w = \frac{\dot{v}_w}{A_{\text{flow}}} = \frac{\dot{v}_w}{N \frac{\pi}{4} D_i^2} = \frac{4\left(3.7 \text{ m}^3/\text{min}\right)}{(768)\,\pi (0.016\,\text{m})^2 (60\,\text{s/min})} = 0.40 \text{ m/s}$$

(b) The heat capacity rate of the condensing vapor is essentially infinite. The heat capacity rate of the water is

$$C_w = \dot{m}_w c_{pw} = \dot{v}_w \rho\, c_{pw} = \frac{(3.7 \text{ m}^3/\text{min})}{(60\,\text{s/min})}\left(992 \text{ kg/m}^3\right)\left(4175 \text{ J/(kgK)}\right) = 255{,}450 \text{ W/K}$$

From Equation (8.12b) with $C_c = C_{\min}$

$$\mathrm{E} = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_c - T_{w,\text{in}}}$$

No scaling $$\mathrm{E} = \frac{49-29}{118-29} = 0.22 = 22\%$$

With scaling $$\mathrm{E} = \frac{46-29}{118-29} = 0.19 = 19\%$$

(c) For $C_{\min}/C_{\max} = 0$, the effectiveness of parallel and counterflow exchangers is the same and Equation (8.25) reduces to

$$\mathrm{E} = 1 - e^{-NTU} \Rightarrow NTU = \frac{U_o A_o}{C_{\min}} = -\ln(1 - \mathrm{E})$$

Solving for the overall heat transfer coefficient

$$U_o = -\frac{C_{\min}}{A_o}\ln(1 - \mathrm{E}) = -\frac{C_{\min}}{N\pi D_o L}\ln(1 - \mathrm{E})$$

No scaling

$$U_o = -\frac{(255{,}450\ \text{W/K})}{(768)\,\pi\,(0.019\,\text{m})\,(2.4\,\text{m})}\ \ln(1 - 0.22) = 577\ \text{W/(m}^2\text{K)}$$

Similarly for scaling $U_o = 489\ \text{W/(m}^2\ \text{K)}$
From Equation 8.4

$$R_D = \frac{1}{U_d} - \frac{1}{U} = \frac{1}{\left(489\ \text{W/(m}^2\text{K)}\right)} - \frac{1}{\left(577\ \text{W/(m}^2\text{K)}\right)} = 0.000312\ \text{W/(m}^2\text{K)}$$

$$\frac{1}{R_D} = 3206\ \ \text{W/(m}^2\text{K)}$$

PROBLEM 8.38

A shell-and-tube heat exchanger is to be used to cool 25.2 kg/s of water from 38°C to 32°C. The exchanger has one shell-side pass and two tube side passes. The hot water flows through the tubes and the cooling water flows through the shell. The cooling water enters at 24°C and leaves at 32°C. The shell-side (outside) heat transfer coefficient is estimated to be 5678 W/(m^2 K).

Design specifications require that the pressure drop through the tubes be as close to 13.8 kPa as possible and that the tubes be 18 BWG copper tubing 1.24 mm wall thickness, and each pass is 4.9 m long. Assume that the pressure losses at the inlet and outlet are equal to one and one half of a velocity heat $\rho V^2/g_c$, respectively.

For these specifications, what tube diameter and how many tubes are needed?

GIVEN

- A water-to-water shell-and-tube exchanger, hot water in tubes, cooling water in shell
- One shell and two tube passes
- Hot water flow rate $(\dot{m}_h)$ = 25.2 kg/s
- Water temperatures
  - Hot: $T_{h,\text{in}}$ = 38°C $T_{h,\text{out}}$ = 32°C
  - Cold: $T_{c,\text{in}}$ = 24°C $T_{c,\text{out}}$ = 32°C

- Shell-side transfer coefficient $\left(\bar{h}_o\right)$ = 5678 W/(m^2 K)
- Pressure drop (Δp) = 13.8 kPa
- Tube wall thickness (t) = 1.24 mm = 0.00124 m
- Tube length per pass (L_p) = 4.9 m

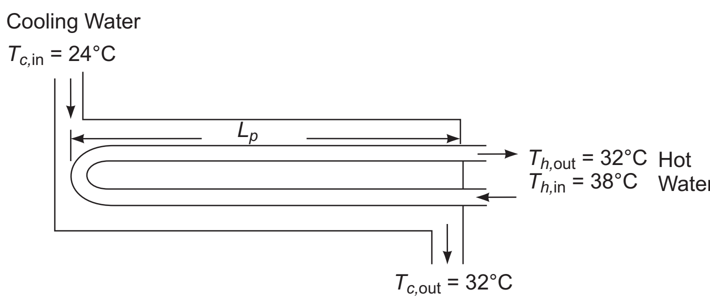
FIND

- The tube diameter (D_o) and number of tubes (N)

ASSUMPTIONS

- Pressure losses at inlet and outlet (Δp_{ii}) = 1.5 ($\rho V^2/g_c$)
- Variation of thermal properties with temperature is negligible
- Fouling resistance is negligible
- Thermal resistance of the tube walls is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 30°C

Density (ρ) = 995.7 kg/m^3

Specific heat (c_p) = 4176 J/(kg K)

Thermal conductivity (k) = 0.615 W/(m K)

Kinematic viscosity (ν) = 0.805 × 10^{-6} m^2/s

Prandtl number (Pr) = 5.4

SOLUTION

From Figure 8.9 $\quad \Delta T_a = T_{h,\text{in}} - T_{c,\text{out}} = 38°\text{C} - 32°\text{C} = 6°\text{C}$

$$\Delta T_b = T_{h,\text{out}} - T_{c,\text{in}} = 32°\text{C} - 24°\text{C} = 8°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{6°\text{C} - 8°\text{C}}{\ln\left(\frac{6}{8}\right)} = 7°\text{C}$$

This must be corrected using Figure 8.13

$$P = \frac{T_{h,\text{out}} - T_{h,\text{in}}}{T_{c,\text{in}} - T_{h,\text{in}}} = \frac{32-38}{24-38} = 0.43$$

$$Z = \frac{T_{c,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{h,\text{in}}} = \frac{24-32}{32-38} = 1.33$$

From Figure 8.13, F = 0.78

$$\therefore \quad \Delta T_{\text{mean}} = F(LMTD) = 0.78\ (7°\text{C}) = 5.5°\text{C}$$

An iterative solution is required. For a first guess, let the tubing be 1″ *OD*. From Appendix 2, Table 42, for 1″ BWG 18 tubing: D_i = 0.902 in = 0.0229 m; D_o = 0.0254 m. Assume from Table 8.1 that the overall heat transfer coefficient U_o = 1700 W/(m^2 K). The rate of heat transfer is

$$q = U_o A_o \Delta T_{\text{mean}} = \dot{m}_h c_{ph} (T_{h,\text{in}} - T_{h,\text{out}})$$

$$U_o [N \pi D_o (2L_p)] \Delta T_{\text{mean}} = \dot{m}_h c_{ph} (T_{h,\text{in}} - T_{h,\text{out}})$$

$$N = \frac{\dot{m}_h c_{ph}}{U_o \pi D_o 2L_p} \frac{T_{h,\text{in}} - T_{h,\text{out}}}{\Delta T_{\text{mean}}} = \frac{(25.2\text{ kg/s})(4176\text{ J/(kg K)})}{\left(1700\text{ W/(m}^2\text{K)}\right)\pi(0.0254\text{ m})2(4.9\text{ m})} \frac{38°\text{C} - 32°\text{C}}{55°\text{C}} = 86\text{ tubes}$$

The water velocity in the tubes for 86 tubes is

$$V = \frac{\dot{m}}{\rho A_i} = \frac{4\dot{m}}{\rho N \pi D_i^2} = \frac{4(25.2\text{ kg/s})}{\left(995.7\text{ kg/m}^3\right)(86)\pi(00229\text{ m})^2} = 0.715\text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{VD_i}{\nu} = \frac{(0.715\text{ m/s})(0.0229\text{ m})}{\left(0.805\times10^{-6}\text{ m}^2/\text{s}\right)} = 20{,}326\text{ (Turbulent)}$$

From Equation (6.63)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\, (20{,}326)^{0.8}\, (5.4)^{0.4} = 248$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 248 \frac{(0.615\text{ W/(m K)})}{0.0229\text{ m}} = 6655\text{ W/(m}^2\text{K)}$$

The overall heat transfer coefficient, neglecting fouling and tube wall thermal resistance, from Equation (8.2), is

$$\frac{1}{U_o} = \frac{1}{\bar{h}_o} + \frac{D_o}{D_i}\frac{1}{\bar{h}_i} = \frac{1}{\left(5678\text{ W/(m}^2\text{ K)}\right)} + \left(\frac{254}{229}\right)\frac{1}{\left(6655\text{ W/(m}^2\text{K)}\right)}$$

$$U_o = 2917\text{ W/(m}^2\text{ K)}$$

The pressure drop through the tube is obtained by adding the inlet and outlet pressure drops to Equation 6.13

$$\Delta p = \left(f\frac{L}{D_i} + 1.5\right)\frac{\rho V^2}{2g_c} \qquad \text{(where } L = 2\, L_p = 9.8\text{ m)}$$

The friction factor *f*, is given by Equation (6.59) for turbulent flow

$$f = \frac{0.184}{Re_D^{0.2}} = \frac{0.184}{(20{,}326)^{0.2}} = 0.0253$$

$$\Delta p = \left((0.0243)\left(\frac{9.8\text{ m}}{0.0229\text{ m}}\right) + 1.5\right)\frac{\left(995.7\text{ kg/m}^3\right)(0.715\text{ m/s})^2}{1}\left((\text{s}^2\text{N})/(\text{kg m})\right)$$

$$= 6275\text{ N/m}^2 = 6.3\text{ kPa}$$

There is about half of the required pressure drop. Therefore, smaller tubes should be used. For a second iteration, let the tubes be 3/4” 18 BWG tubes

From Appendix 2, Table 42

$$D_i = 1.66\text{ cm} = 0.0166\text{ m} \qquad D_o = 1.19\text{ cm} = 0.019\text{ m}$$

Following the same procedure shown above but using U_o = 2000 W/(m^2 K) yields

N = 98 tubes

V = 1.19 m/s

$\bar{h}_i$ = 5452 W/(m^2 K)

U_o = 2582 W/(m^2 K)

Δp = 22.4 kPa

Performing the procedure for the same tubes but using the U_o derived above $\left(2502 \text{ W/(m}^2 \text{ K)}\right)$ yields

N = 76

V = 1.53 m/s

This will give an even higher pressure drop, therefore, use the 1″ 18 BWG tubes.

PROBLEM 8.39

A shell-and-tube heat exchanger with the characteristics given below is to be used to heat 27,000 kg/h of water before it is sent to a reaction system. Saturated steam at 2.36 atm absolute pressure is available as the heating medium and will be condensed without subcooling on the outside of the tubes. From previous experience, the steam-side condensing coefficient may be assumed constant and equal to 11,300 W/(m^2 K).

If the water enters at 16°C, at what temperature will it leave the exchanger? Use reasonable estimates for fouling coefficients.

Exchanger specifications

- **Tubes – 2.5 cm *OD*, 2.3 cm *ID*, horizontal copper tubes in six vertical rows**
- **Tube length = 2.4 m**
- **Total number of tubes = 52**
- **Number of tube-side passes = 2**

GIVEN

- Shell-and-tube heat exchanger - water in copper tubes, saturated steam is shell
- Water flow rate $(\dot{m}_w)$ = 27,000 kg/h = 7.5 kg/s
- Steam pressure = 2.36 atm = 239 kPa
- Steam-side coefficient $(\bar{h}_o)$ = 11,300 W/(m^2 K)
- Water entrance temperature: $T_{w,\text{in}}$ = 16°C
- Tube diameters
  - D_o = 2.5 cm = 0.025 m
  - D_i = 2.3 cm = 0.023 m
- Tube length (L) = 2.4 m
- Number of tubes (N) = 52
- Number of tube passes = 2

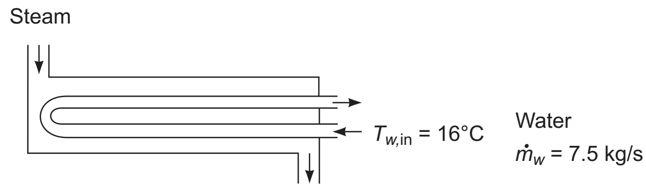
FIND

- The water exit temperature ($T_{w,\text{out}}$)

ASSUMPTIONS

- Length given is total tube length for both passes

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the temperature of saturated steam at 239 kPa (T_a) = 125°C
From Appendix 2, Table 13, for water at 20°C

Thermal conductivity (k) = 0.597 W/(m K)

Kinematic viscosity (ν) = 1.006×10^{-6} m^2/s

Prandtl number (Pr) = 7.0

Density (ρ) = 998.2 kg/m^3

Specific heat (c_p) = 4182 J/(kg K)

From Appendix 2, Table 12, the thermal conductivity of copper (k_c) = 392 W/(m K) at 127°C

SOLUTION

Tube side transfer coefficient
The water velocity is

$$V = \frac{\dot{m}_w}{\rho A_{\text{flow}}} = \frac{4\dot{m}}{\rho N \pi D_i^2} = \frac{4(7.5\ \text{kg/s})}{(998.2\ \text{kg/m}^3)(52)\pi(0.023\ \text{m})^2} = 0.348\ \text{m/s}$$

The Reynolds number is

$$Re_D = \frac{VD_i}{\nu} = \frac{(0.348\ \text{m/s})(0.023\ \text{m})}{(1.006\times10^{-6}\ \text{m}^2/\text{s})} = 7956\ \text{(Turbulent)}$$

From Equation (6.63)

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\ (7956)^{0.8}\ (7.0)^{0.4} = 66.1$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 66.1 \frac{(0.597\ \text{W/(m K)})}{0.023\ \text{m}} = 1716\ \text{W/(m}^2\text{K)}$$

From Table 8.2: A reasonable fouling factor on the water side (R_i) ≈ 0.0002 (m^2 K)/W and on the steam side (R_o) ≈ 0.00009 (m^2 K)/W.
The overall heat transfer coefficient is given by Equation (8.5)

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + R_o + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k_c} + \frac{D_o}{D_i}\left(R_i + \frac{1}{\bar{h}_i}\right)$$

$$\frac{1}{U_d} = \frac{1}{(11{,}300\ \text{W/(m}^2\text{K)})} + (0.00009\ (\text{m}^2\ \text{K})/\text{W}) + \frac{(0.025\ \text{m})\ln(25/23)}{2(392\ \text{W/(m K)})}$$

$$+\left(\frac{25}{23}\right)\left(\left(0.0002\,(\text{m}^2\text{K})/\text{W}\right)+\frac{1}{\left(1716\ \text{W}/(\text{m}^2\text{K})\right)}\right)$$

$$U_d = 969\ \text{W}/(\text{m}^2\ \text{K})$$

The heat capacity rate of the condensing steam is essentially infinite. The heat capacity rate of the water is

$$C_w = \dot{m}_w\, c_{pw} = (7.5\ \text{kg/s})\ \left(4182\ \text{J}/(\text{kg}\,\text{K})\right)\ = 31{,}365\ \text{W/K}$$

The number of transfer units is

$$NTU = \frac{U_o A_o}{C_{\min}} = \frac{U_o N \pi D_o L}{C_w} = \frac{\left(969\ \text{W}/(\text{m}^2\ \text{K})\right)(52)\,\pi\,(0.025\,\text{m})\,(2.4\,\text{m})}{31{,}365\ \text{W/K}} = 0.30$$

For $C_{\min}/C_{\max} = 0$, $NTU = 0.30$. From Figure 8.19, $e = 0.24$

The outlet temperature can be calculated from Equation (8.21b). Note: $C_c = C_{\min}$.

$$\text{E} = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}}$$

$$T_{w,\text{out}} = T_{w,\text{in}} + \text{E}\ (T_s - T_{w,\text{in}}) = 16°\text{C} + 0.24\ (125°\text{C} - 16°\text{C}) = 42°\text{C}$$

PROBLEM 8.40

Determine the appropriate size of a shell-and-tube heat exchanger with two tube passes and one shell pass to heat 8.82 kg/s of pure ethanol from 15.6 to 60°C. The heating medium is saturated steam at 152 kPa condensing on the outside of the tubes with a condensing coefficient of 15,000 W/(m^{2}K). Each pass of the exchanger has 50 copper tubes with an *OD* of 1.91 cm and a wall thickness of 0.211 cm. For the sizing, assume the header cross-sectional area per pass is twice the total inside tube cross-sectional area. The ethanol is expected to foul the inside of the tubes with a fouling coefficient of 5678 W/(m^2 K).

After the size of the heat exchanger, i.e., the length of the tubes, is known, estimate the frictional pressure drop using the inlet loss coefficient of unity. Then estimate the pumping power required with a pump efficiency of 60% and the pumping cost per year with $0.10 per kw-hr.

GIVEN

- Shell-and-tube heat exchanger, ethanol in copper tubes, steam in shell
- One shell pass and two tube passes
- Ethanol flow rate $\dot{m}_e$ = 8.82 kg/s
- Ethanol temperatures
  - $T_{e,\text{in}}$ = 15.6°C
  - $T_{e,\text{out}}$ = 60°C
- Steam pressure = 152 kPa
- Number of tubes (N) = 50
- Tube outside diameter (D_o) = 1.91 cm = 0.0191 m
- Tube wall thickness (t) = 0.211 cm = 0.00211 m
- Header area per pass = 2 (total inside cross-sectional area)
- Tube side fouling coefficient ($1/R_i$) = 5678 W/(m^2 K)
- Shell-side transfer coefficient $\left(\bar{h}_o\right)$ = 15,000 W/(m^2 K)

FIND

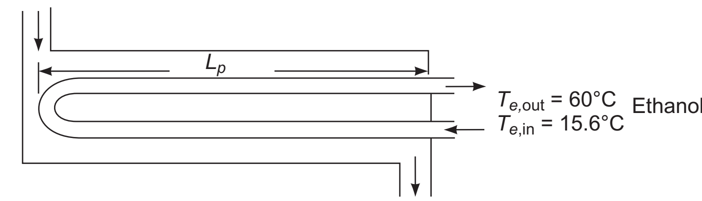
(a) Size: length of one pass (L_p)
(b) The frictional pressure drop (Δp)
(c) The pumping power required (P_p) with a pump efficiency (η_p) = 60%
(d) Pumping cost per year for energy cost of $0.10/kw-hr

ASSUMPTIONS

- The variation of thermal properties with temperature is negligible
- Shell side fouling is negligible

- The tubes are smooth
- Entrance pressure drop effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the temperature of saturated steam at 152 kPa (T_s) = 110°C
From Appendix 2, Table 21, for ethanol (ethy1 alcohol) at 20°C

Density (ρ) = 790 kg/m^3

Thermal conductivity (k) = 0.182 W/(m K)

Absolute viscosity (μ) = 12.0×10^{-4} (Ns)/m^2

Prandtl number (Pr) = 16.29

Specific heat (c_p) = 2470 J/(kg K)

From Appendix 2, Table 12, the thermal conductivity of copper (k_c) = 392 W/(m K) at 127°C

SOLUTION

The inside diameter of the tubes is

$$D_i = D_o - 2t = 1.91 \text{ cm} - 2(0.211 \text{ cm}) = 1.49 \text{ cm} = 0.0149 \text{ m}$$

The Reynolds number for the ethanol flow is

$$Re_D = \frac{VD_i}{\nu} = \frac{4\dot{m}}{N\pi D_i \mu} = \frac{4(8.82 \text{ kg/s})}{(52)\pi(0.0149\text{ m})\left(12.0\times10^{-4} \text{ (N s)/m}^2\right)\left(\text{(kg m)/(N s}^2)\right)} = 12{,}078 \text{ (Turbulent)}$$

From Equation (6.63)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\, (12{,}078)^{0.8}\, (16.29)^{0.4} = 129.4$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 129.4 \frac{(0.182 \text{ W/(mK)})}{0.0149\text{ m}} = 1581 \text{ W/(m}^2\text{K)}$$

The overall heat transfer coefficient with fouling is given by Equation (8.5)

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + 0 + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k_c} + \frac{D_o}{D_i}\left(R_i + \frac{1}{\bar{h}_i}\right)$$

$$\frac{1}{U_d} = \frac{1}{\left(15{,}000\ \text{W/(m}^2\text{K)}\right)} + \frac{(0.0191\,\text{m})\ln\left(\frac{191}{149}\right)}{2\left(392\ \text{W/(m K)}\right)} + \left(\frac{191}{149}\right)\left(\frac{1}{\left(5678\ \text{W/(m}^2\text{K)}\right)} + \frac{1}{\left(1581\ \text{W/(m}^2\text{K)}\right)}\right)$$

$$U_d = 901\ \text{W/(m}^2\ \text{K)}$$

The heat capacity rate of the steam is essentially infinite. The heat capacity rate of the ethanol is

$$C_e = \dot{m}_e\, c_p = (8.82\ \text{kg/s})\left(2470\ \text{J/(kg K)}\right) = 21{,}785\ \text{W/K}$$

From Figure 8.9

$$\Delta T_a = T_s - T_{e,\text{out}} = 110°\text{C} - 15.6°\text{C} = 94.4°\text{C}$$

$$\Delta T_b = T_s - T_{e,\text{in}} = 110°\text{C} - 60°\text{C} = 50°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{94.4°\text{C} - 50°\text{C}}{\ln\left(\frac{94.4}{50}\right)} = 69.9°\text{C}$$

Because $Z = 0$; $F = 1$ and $\Delta T_{\text{mean}} = LMTD$.

The rate of heat transfer is

$$q = U_o A_o \Delta T_{\text{mean}} = C_e\,(T_{e,\text{out}} - T_{e,\text{in}})$$

$$U_o\,(N \pi D_o L)\,\Delta T_{\text{mean}} = C_e\,(T_{e,\text{out}} - T_{e,\text{in}})$$

Solving for the length

$$L = \frac{C_e}{U_o N \pi D_o}\frac{T_{e,\text{out}} - T_{e,\text{in}}}{\Delta T_{\text{mean}}} = \frac{\left(21{,}785\ \text{W/K}\right)}{\left(901\ \text{W/(m}^2\text{K)}\right)(52)\,\pi(0.0191\,\text{m})}\frac{60°\text{C} - 15.6°\text{C}}{69.9°\text{C}} = 4.92\ \text{m}$$

$$L_p = \frac{4.92\ \text{m}}{2} = 2.46\ \text{m}$$

The effectiveness, from Equation (8.21b) is

$$\text{E} = \frac{T_{e,\text{out}} - T_{e,\text{in}}}{T_s - T_{e,\text{in}}} = \frac{60°\text{C} - 15.6°\text{C}}{110°\text{C} - 15.6°\text{C}} = 0.47$$

From Figure 8.19, $NTU \approx 0.7$

$$NTU = \frac{U_o A_o}{C_{\min}} = \frac{U_o N \pi D_o L}{C_{\min}}$$

Solving for the length

$$L = \frac{NTU\ C_{\min}}{U_o N \pi D_o} = \frac{0.7\left(21{,}785\ \text{W/K}\right)}{\left(901\ \text{W/(m}^2\text{K)}\right)(52)\,\pi(0.0191\,\text{m})} = 5.42\ \text{m}$$

The length of one pass = L/(# of passes) = (5.42 m)/2 = 2.71 m

This method relies on reading the low end of Figure 8.18 and is probably less accurate than the *LMTD* method.

(b) From Equation (6.13) the pressure drop is

$$\Delta p = f\frac{L}{D_i}\frac{\rho V^2}{2g_c} = f\frac{L}{D_i}\frac{\rho}{2g_c}\left(\frac{4\dot{m}_e}{N\pi\rho D_i^2}\right)^2$$

Where the friction factor, f, is given for turbulent flow by Equation (6.59)

$$f = 0.184\ Re_D^{-0.2} = 0.184\ (12{,}078)^{-0.2} = 0.0281$$

$$\Delta p = 0.0281\ \frac{4.92\ \text{m}}{0.0149\ \text{m}}\ \frac{\left(790\ \text{kg/m}^3\right)}{2}\left(\frac{4(8.82\ \text{kg/s})}{52\pi\left(790\ \text{kg/m}^3\right)(0.0149\ \text{m})^2}\right)^2\ \left((\text{s}^2\ \text{N})/(\text{kg m})\right) = 5557\ \text{N/m}^2$$

(c) The pumping power required is

$$P_p = \frac{\dot{v}\Delta p}{\eta_p} = \frac{\dot{m}_e}{\eta_p \rho}\Delta p = \frac{(8.82\ \text{kg/s})}{0.6\left(790\ \text{kg/m}^3\right)}\ 5557\ \text{N/m}^2\ \left((\text{W s})/(\text{N m})\right) = 103\ \text{W}$$

(d) The cost to run the pump is

$$\text{Cost} = \left(\frac{\$0.10}{\text{kWh}}\right)(103\ \text{W})\left(\frac{1\,\text{kW}}{1000\ \text{W}}\right)\left(\frac{24\ \text{h}}{\text{day}}\right)\left(\frac{365\ \text{days}}{\text{year}}\right) = \$91/\text{year}$$

PROBLEM 8.41

A counterflow regenerator is used in a gas turbine power plant to preheat the air before it enters the combustor. The air leaves the compressor at a temperature of 350°C. Exhaust gas leaves the turbine at 700°C. The mass flow rates of air and gas are 5 kg/s. Take the c_p of air and gas to be equal to 1.05 kJ/(kg K). Determine the required heat transfer area as a function of the regenerator effectiveness, if the overall heat transfer coefficient is 75 W/(m^2 K).

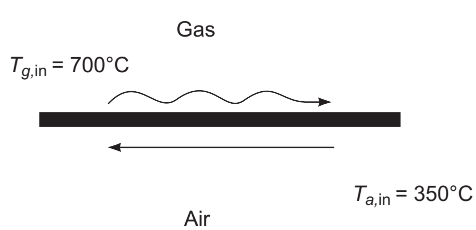
GIVEN

- Counterflow air-to-gas heat exchanger
- Entering temperatures
  - $T_{a,\text{in}} = 350°\text{C}$
  - $T_{g,\text{in}} = 700°\text{C}$
- Mass flow rates: $\dot{m}_a = \dot{m}_g = 5$ kg/s
- Specific heats: $c_{pa} = c_{pg} = 1.05$ kJ/(kg K) = 1050 J/(kg K)
- Overall heat transfer coefficient (U) = 75 W/(m^2 K)

FIND

- The heat transfer area (A) as a function of the effectiveness (e)

SKETCH



SOLUTION

From Equations (8.22) and (8.16)

$$q = E\ C_{\min}\,(T_{g,\text{in}} - T_{a,\text{in}}) = U\,A\,\Delta T \Rightarrow A = \frac{E C_{\min}\left(T_{h,\text{in}} - T_{a,\text{in}}\right)}{U\Delta T}$$

Since $m_a\ c_{pa} = m_g\ c_{pg}$, the temperature difference between the gas and air remain constant and $\Delta T = T_{g,\text{in}}\ T_{a,\text{out}}$. The heat capacity rates are equal, therefore, Equation (8.21b) reduces to

$$E = \frac{T_{a,\text{out}} - T_{a,\text{in}}}{T_{g,\text{in}} - T_{a,\text{in}}} \Rightarrow T_{a,\text{out}} = T_{a,\text{in}} + E\,(T_{g,\text{in}} - T_{a,\text{in}})$$

$$\therefore \qquad \Delta T = T_{g,\text{in}} - T_{a,\text{in}} - E\,(T_{g,\text{in}} - T_{a,\text{in}}) = (T_{g,\text{in}} - T_{a,\text{in}})\,(1 - E)$$

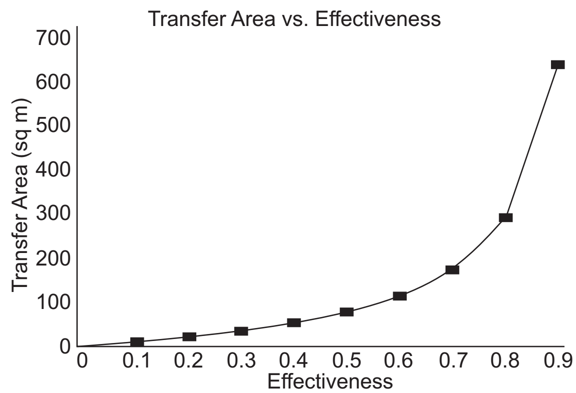
Substituting this into the expression for area

$$A = \frac{E C_{\min}}{U(1-E)} = \frac{E \dot{m}\, c_p}{U(1-E)}$$

$$A = \left(\frac{E}{1-E}\right) \frac{(5\text{ kg/s})(1050\text{ J/(kg K)})}{\left(75\text{ W/(m}^2\text{ K)}\right)(\text{J/(W s)})} = 70\text{ m}^2 \left(\frac{E}{1-E}\right)$$

This is tabulated and plotted below

e	*A* (m²)
0	0
0.1	7.8
0.2	17.5
0.3	30
0.4	47
0.5	70
0.6	105
0.7	163
0.8	280
0.9	630
1.0	∞



COMMENTS

This problem can also be solved by calculating the number of transfer units for a given area then reading the effectiveness off Figure 8.18.

PROBLEM 8.42

> **Determine the heat transfer area requirements of Problem 8.41 if a 1-2 shell and tube, an unmixed crossflow, and a parallel flow heat exchanger are used, respectively.**
>
> **From Problem 8.41: A regenerator is used in a gas turbine power plant to preheat the air before it enters the combustor. The air leaves the compressor at a temperature of 350°C. Exhaust gas leaves the turbine at 700°C. The mass flow rates of air and gas are 5 kg/s. Take the c_p of air and gas to be equal to 1.05 kJ/(kg K). Determine the required heat transfer area as a function of the regenerator effectiveness, if the overall heat transfer coefficient is 75 W/(m² K).**

GIVEN

- An air-to-gas heat exchanger
- Entering temperatures
  - $T_{a,\text{in}} = 350°\text{C}$
  - $T_{g,\text{in}} = 700°\text{C}$

- Mass flow rates: $m_a = m_g$ = 5 kg/s
- Specific heats: $c_{pa} = c_{pg}$ = 1.05 kJ/(kg K) = 1050 J/(kg K)
- Overall heat transfer coefficient (U) = 75 W/(m^2 K)

FIND

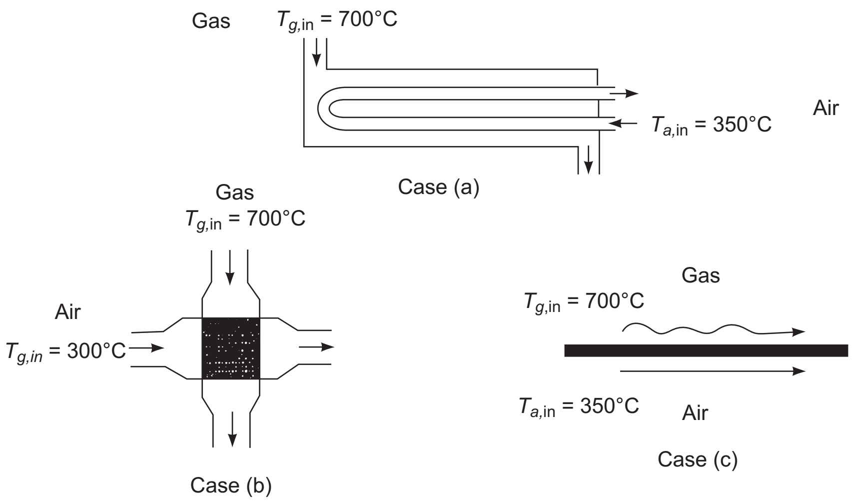
The heat transfer area (A) as a function of the effectiveness (e) for

(a) A 1-2 shell and tube heat exchanger
(b) An unmixed crossflow heat exchanger
(c) A parallel flow heat exchanger

ASSUMPTIONS

- For case (a) the air is in the tubes

SKETCH



SOLUTION

As shown in the solution to Problem 8.41 for counterflow

$$\Delta T = (T_{g,\text{in}} - T_{a,\text{in}})(1 - e)$$

This must be corrected for case (a) and (b) by Figures 8.13 and 8.16 where

$$P = \frac{T_{a,\text{out}} - T_{a,\text{in}}}{T_{g,\text{in}} - T_{a,\text{in}}}$$

From Equation (8.21b)

$$T_{a,\text{out}} = T_{a,\text{in}} + e\,(T_{g,\text{in}} - T_{a,\text{in}})$$

Therefore

$$P = e$$

Since $C_g = C_a$, $Z = 1$

The solution for parts (a) and (b) are the same as for Problem 8.41 expect that the mean temperature (ΔT) must be multiplied by the factor F with the following results

$$A = \frac{E\dot{m}_w c_p}{UF(1-E)} = \frac{70\,\text{m}^2}{F}\left(\frac{E}{1-E}\right)$$

where F is from Figure 8.13 for part (a) and from Figure 8.16 for part (b) where $P = e$ and $Z = 1.0$.

(c) For parallel flow

$$\Delta T = LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)}$$

where

$$\Delta T_a = T_{g,\text{in}} - T_{a,\text{in}}$$

$$\Delta T_b = T_{g,\text{out}} - T_{a,\text{out}} = [T_{g,\text{in}} - E\,(T_{g,\text{in}} - T_{a,\text{in}})] - [T_{a,\text{in}} + E\,(T_{g,\text{in}} - T_{a,\text{in}})]$$

$$= (T_{g,\text{in}} - T_{a,\text{in}})(1 - 2E)$$

$$\Delta T_a - \Delta T_b = (T_{g,\text{in}} - T_{a,\text{in}})(2E)$$

$$\frac{\Delta T_a}{\Delta T_b} = \frac{1}{1-2E}$$

$$\therefore \quad LMTD = \frac{2E\left(T_{g,\text{in}} - T_{a,\text{in}}\right)}{\ln\left(\frac{1}{1-2E}\right)}$$

From Equation (8.22) and (8.16)

$$q = E\,C_{\min}\,(T_{g,\text{in}} - T_{a,\text{in}}) = U A \frac{2E\left(T_{g,\text{in}} - T_{a,\text{in}}\right)}{\ln\left(\frac{1}{1-2E}\right)}$$

$$A = \frac{C_{\min}}{2U} \ln\left(\frac{1}{1-2E}\right) = \frac{(5\text{ kg/s})(1050\text{ J/(kg K)})}{2\left(75\text{ W/(m}^2\text{K)}\right)} \ln\left(\frac{1}{1-2E}\right) = 35\text{ m}^2 \ln\left(\frac{1}{1-2E}\right)$$

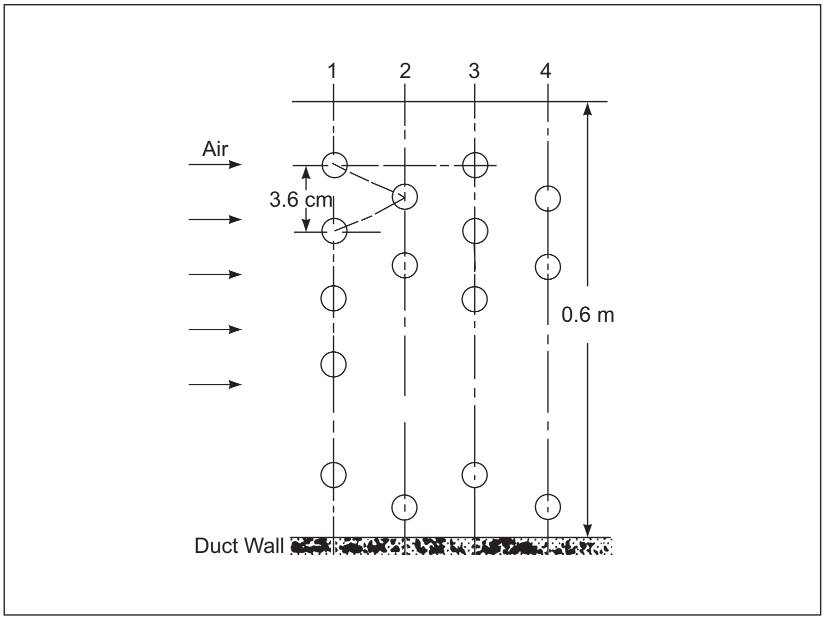
Tabulating these results

e	$F(a)$*	$F(b)$**	$A(a)$ (m^2)	$A(b)$ (m^2)	$A(c)$ (m^2)
0	1.0	1.0	0	0	0
0.1	1.0	1.0	7.7	7.7	7.7
0.2	0.99	0.98	17.9	17.9	17.9
0.3	0.97	0.97	30.4	30.9	32.1
0.4	0.92	0.94	50.7	49.6	56.3
0.5	0.8	0.91	87.5	76.9	∞
0.57	0.5	0.86	186	108	
0.6	NA	0.84		125	
0.7	NA	0.70		233	
0.8	NA	0.5		560	
0.9	NA	NA			
1.0	NA	NA			

* From Figure 8.13
** From Figure 8.16
NA - Data not available from the figures

PROBLEM 8.43

A small space heater is constructed of 1.25 cm, 18-gauge brass tubes, 0.6 m long. The tubes are arranged in equilateral, staggered triangles on 3.6 cm centers, four rows of 15 tubes each. A fan blows 0.95 m^3/s of atmospheric pressure air at 21°C uniformly over the tubes (see sketch). Estimate: (a) heat transfer rate (b) exit temperature of the air (c) rate of steam condensation, assuming that saturated steam at 15 kPa inside the tubes as the heat source. State your assumptions. Work parts *a*, *b*, and *c* of this problem by two methods. First use the *LMTD*, which requires a trial-and-error or graphical solution then use the effectiveness methods.



GIVEN

- A small heater made of 4 rows of 15 tubes each as shown above
- Tubes: 1.25 cm., 18 gauge brass
- Tube length (L) = 0.6 m
- Distance between tube centers (S_T) = 3.75 cm
- Air flow rate $(\dot{V})$ = 0.95 m^3/s
- Air inlet temperature ($T_{a,\text{in}}$) = 21°C
- Saturated steam inside the tubes at pressure (p_s) = 15 kPa g = 115 kPa
- Duct width (w) = 0.6 m

FIND

Using both the *LMTD* method and *e* method find: (a) The heat transfer rate (q)
(b) Air exit temperature ($T_{a,\text{out}}$)
(c) Rate of steam condensation $(\dot{m}_c)$

PROPERTIES AND CONSTANTS

From Appendix 2, Table 42, 18 gauge tubes have a wall thickness (t) = 0.125 cm

From Appendix 2, Table 10, the thermal conductivity of brass (k_b) = 111 W/(m K).

From Appendix 2, Table 13, the temperature of saturated steam at 115 kPa: T_s = 102.8°C and the heat of evaporation (h_{fg}) = 2265 kJ/kg.

From Appendix 2, Table 27, for air at an estimated mean temperature of 40°C

Specific heat (c_{pa}) = 1013 J/(kg K)

Kinematic viscosity (ν) = 1.73×10^{-5} m^2/s

Prandtl number = 0.71

Thermal conductivity (k_a) = 0.026 W/(m K)

Density (ρ) = 1.122 kg/m^3

At the steam temperature of 102.8°C, Pr_s = 0.71.

SOLUTION

The tube diameters are $D_o = 1.25$ cm

$D_i = D_o - 2t = 1$ cm

From Table 8.1, the heat transfer coefficient for the condensing steam (h_i) ≈ 5000 – 30,000 W/(m^{2}K), $h_i = 17000$ W/(m^2 K).

The heat transfer coefficient for the air flow over the tube bank can be calculated as shown in Chapter 7. The Reynolds number for this geometry is

$$Re_D = \frac{V_{\max} D}{\nu} = \frac{\dot{v}_a}{A_{\min}} \frac{D_o}{\nu} = \frac{\dot{v}_a D_o}{[16(S_T - D_o) + D_o] L \nu}$$

$$Re_D = \frac{(0.95\ \text{m}^3/\text{s})(1.25 \times 10^{-2}\ \text{m})}{[16(3.75 \times 10^{-2} - 1.25 \times 10^{-2})\ \text{m} + 1.25 \times 10^{-2}\ \text{m}]\ 0.6\,\text{m}(1.73 \times 10^{-5}\ \text{m}^2/\text{s})}$$

⇒ $Re_D = 2770$ (Transition Regime)

The Nusselt number is given by Equation (7.30)

$$\overline{Nu}_D = 0.35 \left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.6}\, Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where S_L = Longitudinal spacing = $\frac{1}{2}\sqrt{(2S_T)^2 - S_T^2} = 3.25$ cm

$$\therefore \quad \overline{Nu}_D = 0.35 \left(\frac{3.75}{6.5}\right)^{0.2} (2770)^{0.6}\, (0.71)^{0.36} = 32.2$$

$$\therefore \quad \bar{h}_o = \overline{Nu}_D \frac{k_a}{D_o} = 32.2 \frac{0.026\ \text{W/(m K)}}{1.25 \times 10^{-2}\ \text{m}} = 66\ \text{W/(m}^2\ \text{K)}$$

The overall heat transfer coefficient is given by Equation (8.2)

$$\frac{1}{U_o} = \frac{A_o}{A_i \bar{h}_i} + \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{\bar{h}_o} = \frac{D_o}{D_i \bar{h}_i} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{1}{\bar{h}_o}$$

$$\Rightarrow \quad \frac{1}{U_o} = \frac{1.25\,\text{cm}}{1\,\text{cm}(17000\ \text{W/(m}^2\text{K)})} + \frac{(1.25 \times 10^{-2}\ \text{m}) \ln\left(\frac{1.25\,\text{cm}}{1\,\text{cm}}\right)}{2 \times (0.026\ \text{W/(m K)})} + \frac{1}{66\ \text{W/(m}^2\text{K)}}$$

$$\Rightarrow \quad U_o = 65.6\ \text{W/(m}^2\ \text{K)}$$

LMTD method

From Figure 8.9 $\Delta T_a = T_s - T_{a,\text{in}} = 102.8 - 21 = 81.8°\text{C}$

$\Delta T_b = T_s - T_{a,\text{out}}$

To find the *LMTD*, the air outlet temperature must be known, therefore, an iterative solution is required. For the first iteration, let $T_{a,\text{out}} = 38°\text{C}$.

$$\Delta T_b = 102.8 - 38 = 64.8°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{81.8°\text{C} - 64.8°\text{C}}{\ln\left(\frac{81.8}{64.8}\right)} = 73°\text{C}$$

The total transfer area is

$$A = (\text{Number of tubes})\, \pi D_o L = (4)(15)\, \pi\, (0.0125 \text{ m})\, (0.6 \text{ m}) = 1.41 \text{ m}^2$$

The rate of heat transfer is given by Equation (8.16)

$$q = U A\, \Delta T = U A\, (LMTD) = 65.6 \text{ W/(m}^2 \text{ K)}\, (1.41 \text{ m}^2)\, (73°\text{C}) = 6752 \text{ W}$$

The outlet air temperature can be calculated from

$$q = \dot{m}_a\, c_{pa}\, (T_{a,\text{out}} - T_{a,\text{in}}) = \dot{v}_a\, \rho\, c_{pa}\, (T_{a,\text{out}} - T_{a,\text{in}})$$

$$\therefore \qquad T_{a,\text{out}} = T_{a,\text{in}} + \frac{q}{\dot{v}_a \rho c_{pa}} = 21°\text{C} + \frac{6752 \text{ W}}{(0.95 \text{ m}^3/\text{s})(1.122 \text{ kg/m}^3)(1013 \text{ J/(kg K)})}$$

$$\Rightarrow \qquad T_{a,\text{out}} = 27.25°\text{C}$$

Following a similar procedure for a second iteration yields

Mean air temperature = 24°C

$\rho = 1.18$ kg/m^3

$c_{pa} = 1008$ J/(kg K)

$LMTD = 62°$C

(a) $q = 7329$ W

(b) $T_{a,\text{out}} = 27.4°$C

The rate of steam condensation is given by

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{7329 \text{ W}}{2265 \times 10^3 \text{ J/kg}} = 3.23 \times 10^{-3} \text{ kg/s}$$

The effectiveness method

The heat rate of the steam is essentially infinite. The heat rate of the air is

$$C_a = \dot{m}_a\, c_{pa} = \dot{v}_a\, \rho_a\, c_{pa} = (0.95 \text{ m}^3/\text{s})\, (1.18 \text{ kg/m}^3)\, 1008 \text{ J/(kg K)}$$

$$\Rightarrow \qquad C_a = 1130 \text{ W/K}$$

Now $$\frac{C_{\text{mixed}}}{C_{\text{unmixed}}} = 0$$

The number of transfer units is

$$NTU = \frac{U A}{C_{\min}} = \frac{(65.6 \text{ W/(m}^2\text{K)})(1.41 \text{m}^2)}{1130 \text{ W/K}} = 0.082$$

For cross flow, one fluid mixed and other fluid unmixed (steam), from Figure 8.21, $e \approx 0.06$.

(a)

$$q = e\, C_{\min}\, (T_{h,\text{in}} - T_{c,\text{in}}) = 0.06\, (1130 \text{ W/K})\, (102.8°\text{C} - 21°\text{C}) = 5546 \text{ W}$$

Applying Equation (8.21b)

$$e = \frac{C_a}{C_{\min}} \frac{T_{a,\text{out}} - T_{a,\text{in}}}{T_s - T_{a,\text{in}}}$$

$$\therefore \qquad T_{a,\text{out}} = T_{a,\text{in}} + e\, (T_s - T_{a,\text{in}}) = 21 + 0.06\, (102.8 - 21)°\text{C} = 25.9°\text{C}$$

(c) The steam condensation rate is

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{5546 \text{ W}}{2265 \times 10^3 \text{ J/kg}} = 2.45 \times 10^{-3} \text{ kg/s}$$

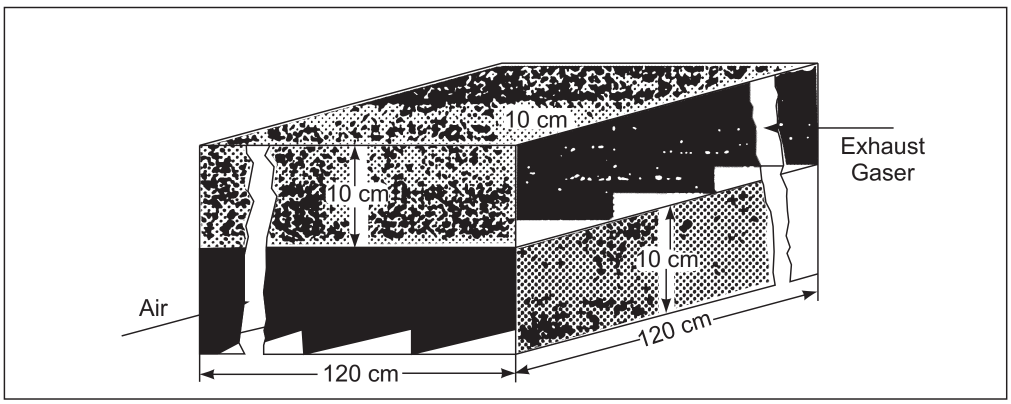
COMMENTS

Although the effectiveness method is more direct for this type of problem, its accuracy is poor due to the low value of *NTU*. The *LMTD* method requires an iterative procedure but gives much better accuracy.

The heat transfer coefficient for the condensing steam will be discussed in more detail in Chapter 10. For this problem, the thermal resistance of the condensing steam is less than 1% of the total thermal resistance, therefore, a rough estimate is adequate.

PROBLEM 8.44

A one-tube pass cross-flow heat exchanger is considered for recovering energy from the exhaust gases of a turbine-driven engine. The heat exchanger is constructed of flat plates, forming an egg-crate pattern as shown in the sketch below. The velocities of the entering air (10°C) and exhaust gases (425°C) are both equal to 61 m/s. Assuming that the properties of the exhaust gases are the same as those of the air, estimate for a path length of 1.2 m the overall heat transfer coefficient *U*, neglecting the thermal resistance of the intermediate metal wall. Then determine the outlet temperature of the air, comment on the suitability of the proposed design, and if possible, suggest improvements. State your assumptions.



GIVEN

- The heat exchanger shown above
- Air and exhaust velocities ($V_a = V_e$) = 61 m/s
- Inlet temperatures
    - Air ($T_{a,\text{in}}$) = 10°C
    - Exhaust ($T_{e,\text{in}}$) = 425°C
- Path length (L) = 1.2 m

FIND

(a) The overall heat transfer coefficient (U)
(b) The outlet temperature of the air ($T_{a,\text{out}}$)

ASSUMPTIONS

- Steady state
- Exhaust gas properties are the same as air
- Thermal resistance of the metal walls is negligible
- Thermal properties can be evaluated at the average temperature
- Heat losses from the exterior walls are negligible
- Dividing walls within one side of the exchanger do not participate in the heat transfer process

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for air at the average inlet temperature of about 200°C

- Density (ρ) = 0.723 kg/m^3
- Thermal conductivity (k) = 0.0370 W/(m K)
- Kinematic viscosity (ν) = 35.5 × 10^{-6} m^2/s
- Prandtl number (Pr) = 0.71
- Specific heat (c_p) = 1035 J/(kg K)

SOLUTION

(a) The Reynolds number at the flow is

$$Re_{D_h} = \frac{V D_h}{\nu}$$

$$\text{where} \quad D_h = \text{Hydraulic diameter} = \frac{4\,A}{P} = \frac{4\,(0.1\,\text{m})^2}{4\,(0.1\,\text{m})} = 0.1 \text{ m}$$

$$Re_{D_h} = \frac{(61\,\text{m/s})(0.1\,\text{m})}{\left(35.5\times10^{-6}\ \text{m}^2/\text{s}\right)} = 1.72 \times 10^5 \text{ (Turbulent)}$$

The Nusselt number for turbulent flow through ducts is given by Equation (6.63)

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \text{ where } n = 0.4 \text{ for heating, 0.3 for cooling}$$

For the air being heated

$$\overline{Nu}_D = 0.023\ (1.72 \times 10^5)^{0.8}\ (0.71)^{0.4} = 309.5$$

$$\bar{h}_a = \overline{Nu}_D\ \frac{k}{D_h} = 309.5\ \frac{(0.0370\ \text{W/(mK)})}{0.1\,\text{m}} = 114.5\ \text{W/(m}^2\ \text{K)}$$

For the exhaust being cooled

$$\overline{Nu}_D = 0.023\ (1.72 \times 10^5)^{0.8}\ (0.71)^{0.3} = 320.3$$

$$\bar{h}_e = 118.5\ \text{W/(m}^2\ \text{K)}$$

The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{\bar{h}_e} + \frac{1}{\bar{h}_a} = \frac{1}{\left(114.5\ \text{W/(m}^2\text{K)}\right)} + \frac{1}{\left(118.5\ \text{W/(m}^2\text{K)}\right)}$$

$$U = 58.2\ \text{W/(m}^2\ \text{K)}$$

(b) The heat capacity of both fluids is

$$C = \dot{m}\ c_p = V\,\rho\,A_c\,c_p = (60\ \text{m/s})\left(0.723\ \text{kg/m}^3\right)(1.2\ \text{m})\ (0.1\ \text{m})\ \left(118.5\ \text{W/(m}^2\text{K)}\right)\ \left((\text{Ws})/\text{J}\right)\ = 5477\ \text{W/K}$$

The number of transfer units is

$$NTU = \frac{U\,A_t}{C_{\min}} = \frac{\left(58.2\ \text{W/(m}^2\text{K)}\right)(1.2\,\text{m})(1.2\,\text{m})}{\left(5477\ \text{W/(m}^2\text{K)}\right)} = 0.015$$

From Figure 8.20, e ≈ 1%

Rearranging Equation (8.21b) (C_{min}/C_{max} = 1)

$$T_{a,\text{out}} = T_{a,\text{in}} + \text{E}\ (T_{e,\text{in}} - T_{a,\text{in}}) = 10°\text{C} + 0.01\ (425°\text{C} - 10°\text{C}) = 14°\text{C}$$

COMMENTS

The accuracy of the air outlet temperature is low because the effectiveness is very low and difficult to read on Figure 8.20. Greater accuracy could be achieved by using the *LMTD* and iterating.

The small effectiveness is due to the small *NTU*. The *NTU* can be increased by using small ducts to increase the overall heat transfer coefficient or redesign the exchanger to increase the transfer area.

PROBLEM 8.45

A shell-and-tube counterflow heat exchanger is to be designed for heating an oil from 27°C to 82°C. The heat exchanger has two tube passes and one shell pass. The oil is to pass through 0 K schedule 40 pipes at a velocity of 1 ms^{-1} and steam is to condense at 102°C on the outside of the pipes. The specific heat of the oil is 1800 J/(kg K) and its mass density is 925 kg/m^3. The steam-side heat transfer coefficient is approximately 10 kW/(m^2 K), and the thermal conductivity of the metal of the tubes is 30 W/ (m K). The results of previous experiments giving the oil-side heat transfer coefficients for the same pipe size at the same oil velocity as those to be used in the exchanger are shown below

	Δ*T* (°C)		**75**	**64**	**53**	**42**	**20**	
T_{oil} (°C)	**27**	**38**	**49**	**60**	**71**	**82**		
	h_{c1} (W/(m^2 K))		**80**	**85**	**100**	**140**	**250**	**540**

(a) Find the overall heat transfer coefficient *U*, based on the outer surface area at the point where the oil is 38°C (b) Find the temperature of the inside surface of the pipe when the oil temperature is 38°C (c) Find the required length of the tube bundle.

GIVEN

- A shell-and-tube counterflow heat exchanger - oil in tubes, steam is shell
- Oil temperatures
  - $T_{o,\text{in}}$ = 27°C
  - $T_{o,\text{out}}$ = 82°C
- Tubes: 1.5 in schedule 40 pipes
- Oil velocity (V_o) = 1 m/s
- Steam temperature (T_s) = 102°C
- Oil specific heat (c_{po}) = 1800 J/(kg K)
- Oil density (ρ) = 925 kg/m^3
- Steam side heat transfer coefficient ($\bar{h}_s$) = 10 kW/(m^2 K)
- Thermal conductivity of the tube material (k_t) = 30 W/(m K)
- Experimental data above was taken at the same oil velocity

FIND

(a) The overall heat transfer coefficient (U_o) at the point where the oil is 38°C
(b) The inside pipe surface temperature (T_{wi}) when the oil temperature is 38°C
(c) The required length of the tube bundle

PROPERTIES AND CONSTANTS

From Appendix 2, Table 41, for 1.5 in schedule 40 pipe

D_i = 4 cm

D_o = 4.8 cm

SOLUTION

(a) The overall heat transfer coefficient based on the outside tube area is given by Equation (8.2)

$$\frac{1}{U_o} = \frac{A_o}{A_i \bar{h}_i} + \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{\bar{h}_o} = \frac{D_o}{D_i \bar{h}_i} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{1}{\bar{h}_s}$$

$$\frac{1}{U_o} = \frac{4.8\,\text{cm}}{4\,\text{cm} \times 85\,\text{W/(m}^2\text{K)}} + \frac{4.8 \times 10^{-2}\,\text{m} \ln\left(\frac{4.8\,\text{cm}}{4\,\text{cm}}\right)}{2\,(30\,\text{W/(m K)})} + \frac{1}{10000\,\text{W/(m}^2\text{K)}}$$

$$\frac{1}{U_o} = (0.0141 + 0.00015 + 10^{-5})\ (\text{m}^2\ \text{K})/\text{W}$$

$$U_o = 70.2\ \text{W/(m}^2\ \text{K)}$$

(b) The rate of heat transfer from the oil to the inner pipe surface must equal the rate of heat transfer between the oil and the steam.

$$\bar{h}_{ci}\ A_i\ (T_{wi} - T_o) = U\,A_o\ (T_s - T_o)$$

$$T_{wi} = T_o + \frac{D_o U}{D_i \bar{h}_{ci}} (T_s - T_o) = 38°\text{C} + \left(\frac{4.8}{4}\right)\left(\frac{70.2}{85}\right)(102°\text{C} - 38°\text{C}) = 101.4°\text{C}$$

(c) The heat capacity rate of the steam is essentially infinite, therefore, $C_{\min}/C_{\max} = 0$

The effectiveness is given by Equation (8.12b) ($C_c = C_{\min}$).

$$\text{E} = \frac{C_c}{C_{\min}} = \frac{U_{o,\text{out}} - T_{o,\text{in}}}{T_s - T_{o,\text{in}}} = \frac{82 - 27}{102 - 27} = 0.733$$

From Figure 8.19 $NTU = 1.4$

$$NTU = \frac{U_o A_o}{C_{\min}} = \frac{U_o A_o}{\dot{m}_o C_{po}} = \frac{U_o A_o}{V_o \rho A_c c_{po}} = \frac{U_o \pi D_o L}{V_o \rho \frac{\pi}{4} D_i^2 c_{po}}$$

$$\therefore L = \frac{NTU\, V_o \rho c_{po} D_i^2}{4 U_o D_o} = \frac{1.4 \times (1\ \text{m/s})(925\ \text{kg/m}^3)(1800\ \text{J/(kg K)})(0.04\,\text{m})^2}{4(70.2\ \text{W/(m}^2\text{K)})(0.048\,\text{m})} = 276\ \text{m}$$

The length of each pass of a double tube pass would need to be $L/2 = 138$ m.

PROBLEM 8.46

A shell-and-tube heat exchanger in an ammonia plant is preheating 1132 cubic meters of atmospheric pressure nitrogen per hour from 21 to 65°C using steam condensing at 138,000 N/m^2. The tube in the heat exchanger have an inside diameter of 2.5 cm. In order to change from ammonia synthesis to methanol synthesis, the same heater is to be used to preheat carbon monoxide from 21 to 77°C, using steam condensing at 241,000 N/m^2. Calculate the flow rate which can be anticipated from this heat exchanger in kg of carbon monoxide per second.

GIVEN

- Shell-and-tube heat exchanger - nitrogen in tubes, condensing steam in shell
- Nitrogen volumetric flow rate $(\dot{V}_n)$ = 1132 m^3/h = 0.3144 m^3/s

- Nitrogen temperatures
  - $T_{n,\text{in}} = 21°C$
  - $T_{n,\text{out}} = 65°C$
- Steam pressure = 138,000 N/m^2
- Tube inside diameter (D_i) = 2.5 cm
- Same heat exchanger is then used with carbon monoxide:
- Carbon monoxide temperatures
  - $T_{c,\text{in}} = 21°C$
  - $T_{c,\text{out}} = 77°C$
- New steam pressure = 241 N/m^2

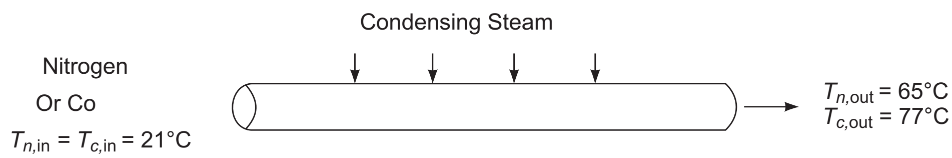
FIND

- The flow rate of carbon dioxide ($\dot{m}_c$)

ASSUMPTIONS

- Two or a multiple of two shell passes
- Thermal resistance of the condensing steam and the tube wall are a small fraction of the total thermal resistance

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the saturation temperature of steam (T_s) is

$T_s = 107°C$ at 138,000 N/m^2

$T_s = 125°C$ at 241,000 N/m^2

32, the saturation temperature of steam (T_s) is

From Appendix 2, Table 32, for nitrogen at the average temperature of 43°C

Density (ρ_n) = 1.096 kg/m^3

Specific heat (c_{pn}) = 1042 J/(kg K)

Thermal conductivity (k_n) = 0.02734 W/(m K)

Absolute viscosity (μ_n) = 18.5×10^{-6} (Ns)/m^2

From Appendix 2, Table 29, for the CO at its average temperature of 49°C

Specific heat (c_{pc}) = 1042 J/(kg K)

Thermal conductivity (k_c) = 0.0268 W/(m K)

Absolute viscosity (μ_c) = 18.83×10^{-6} (N s)/m^2

SOLUTION

The data with nitrogen will be used to calculate the overall heat transfer coefficient which will then be modified and applied to the carbon monoxide case.

With nitrogen

The heat capacity rate of the steam is essentially infinite, therefore, $C_{\min}/C_{\max} = 0$.

Applying Equation (8.22b) $C_{\min} = C_c$

$$\mathrm{E} = \frac{T_{n,\text{out}} - T_{n,\text{in}}}{T_s - T_{n,\text{in}}} = \frac{65-21}{107-21} = 0.51$$

From Figure 8.20, $NTU = U\,A/C_{\min} = 0.75$

The heat capacity rate of the nitrogen is

$$C_n = C_{\min} = \dot{m}_n\ c_{pn} = \dot{v}_n\ \rho_n\, c_{pn} = (0.3144\ \text{m}^3/\text{s})\left(1.096\ \text{kg/m}^3\right)\ (1042\ \text{J/(kg K)})\ \left((\text{Ws})/\text{J}\right) = 359.1\ \text{W/K}$$

$$\therefore\ U A = NTU\,(C_{\min}) = 0.75\ (359.1\ \text{W/K}) = 269.3\ \text{W/K}$$

With CO

Assuming that the flow of either gas is turbulent, the overall heat transfer coefficient is

$$U_O \propto h_i \propto k\,Re^{0.8} \qquad \text{Since } Pr \approx 0.71 \text{ for either gas}$$

$$\text{so } U_o \propto k\left(\frac{\dot{m}}{\mu}\right)^{0.8}$$

Since the properties of the two gases are very close, $U_c \approx U_n$

The effectiveness of the heat exchanger with the carbon monoxide is

$$\text{E} = \frac{T_{n,\text{out}} - T_{n,\text{in}}}{T_s - T_{n,\text{in}}} = \frac{77°\text{C} - 21°\text{C}}{125°\text{C} - 21°\text{C}} = 0.54 \approx \text{E}_n$$

$$\therefore\ \dot{m}_c \approx \dot{m}_n = \rho\ \dot{v}_n = (1.096\ \text{kg/m}^3)\ \left(0.3144\ \text{m}^3/\text{s}\right) = 0.34\ \text{kg/s}$$

PROBLEM 8.47

In an industrial plant a shell-and-tube heat exchanger is heating pressurized dirty water at the rate of 38 kg/s from 60 to 110°C by means of steam condensing at 115°C on the outside of the tubes. The heat exchanger has 500 steel tubes (*ID* = 1.6 cm, *OD* = 2.1 cm) in a tube bundle which is 9 m long. The water flows through the tubes while the steam condenses in the shell. If it may be assumed that the thermal resistance of the scale on the inside pipe wall is unaltered when the mass rate of flow is increased and that changes in water properties with temperature are negligible, estimate (a) the heat transfer coefficient on the water side and (b) the exit temperature of the dirty water if its mass rate of flow is doubled.

GIVEN

- Shell-and-tube heat exchanger - dirty water in steel tubes, steam condensing in shell
- Water flow rate $(\dot{m}_w)$ = 38 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 60°C
  - $T_{w,\text{out}}$ = 110°C
- Steam temperature (T_s) = 115°C
- Number of tubes (N) = 500
- Tube diameters
  - D_i = 1.6 cm = 0.016 m
  - D_o = 2.1 cm = 0.021 m
- Tube bundle length (L) = 9 m

FIND

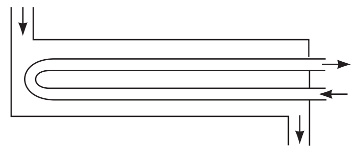
(a) The heat transfer coefficient on the water side $(\bar{h}_i)$

(b) The exit temperature of the dirty water ($T_{w,\text{out}}$) if the mass flow rate $(\dot{m}_w)$ is doubled

ASSUMPTIONS

- The thermal resistance of the scale in the pipe is unaltered when the mass flow rate is increased
- Changes in water properties with temperature are negligible
- Two, or a multiple of two, passes
- The dirty water has the same thermal properties as clean water

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the average temperature of 85°C

Specific heat (c_{pw}) = 4198 J/(kg K)

Thermal conductivity (k) = 0.675 W/(m K)

Absolute viscosity (μ) = 337 × 10^{-6} (N s)/m^2

Prandtl number (Pr) = 2.04

From Appendix 2, Table 10, the thermal conductivity of 1% carbon steel (k_s) = 43 W/(m K) (at 20°C)

SOLUTION

(a) The Reynolds number for flow in the tubes is

$$Re_D = \frac{VD_i}{\nu} = \frac{4\dot{m}}{N\pi D_i \mu} = \frac{4\,(38\text{ kg/s})}{(500)\,\pi\,(0.016\text{ m})\left(337\times10^{-6}\,(\text{Ns})/\text{m}^2\right)\left((\text{kg m})/(\text{Ns}^2)\right)} = 17{,}946 \text{ (Turbulent)}$$

Applying Equation (6.63) for turbulent flow in tubes

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(17{,}946)^{0.8}\,(2.04)^{0.4} = 77.4$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D} = 77.4\,\frac{(0.675\text{ W/(m K)})}{0.016\text{ m}} = 3265\ \text{W/(m}^2\text{K)}$$

(b) The scaling resistance can be calculated from the water temperature data
From Figure (8.9)

$$\Delta T_a = T_s - T_{w,\text{in}} = 115°\text{C} - 60°\text{C} = 55°\text{C}$$

$$\Delta T_b = T_s - T_{w,\text{out}} = 115°\text{C} - 110°\text{C} = 5°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)} = \frac{55°\text{C} - 5°\text{C}}{\ln\left(\dfrac{55}{5}\right)} = 21°\text{C}$$

This must be corrected for use in a shell-and-tube heat exchanger according to Figure 8.13. But since Z = 0 for condensers, F = 1 and $\Delta T_{\text{mean}} = LMTD$, the rate of heat transfer is

$$q = U_o A_o \Delta T_{\text{mean}} = \dot{m}_w\, c_{pw}\,(T_{w,\text{out}} - T_{w,\text{in}})$$

$$\therefore\ U_o = \frac{\dot{m}_w c_{pw}}{2N\pi D_o L \Delta T_{\text{mean}}}\,(T_{w,\text{out}} - T_{w,\text{in}})$$

$$\therefore\ U_o = \frac{(38\text{ kg/s})(4198\text{ J/(kg)})((\text{Ws})/\text{J})}{2\,(500)\,\pi\,(0.021\text{ m})(9\text{ m})(21°\text{C})}(110°\text{C} - 60°\text{C}) = 640\text{ W/(m}^2\text{ K)}$$

Applying Equation (8.5) (R_o = 0)

$$\frac{1}{U_o} = \frac{1}{\bar{h}_o} + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + R_k + \frac{R_i D_o}{D_i} + \frac{D_o}{D_i \bar{h}_i}$$

Solving for the sum of the scaling, conductive, and outer convective resistances

$$\frac{1}{\bar{h}_o} + R_k + \frac{R_i D_o}{D_i} = \frac{1}{U_o} - \frac{D_o}{D_i \bar{h}_i} = \frac{1}{(640\ \text{W/(m}^2\text{K))}} - \left(\frac{2.1}{1.6}\right) \frac{1}{(3265\ \text{W/(m}^2\text{K))}} = 0.000116\ (\text{m}^2\text{K})/\text{W}$$

For a double flow rate, the Reynolds number is doubled: Re_D = 35,592

$$\therefore\ \overline{Nu}_D = 0.023\ (35{,}892)^{0.8}\ (2.04)^{0.4} = 135$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D} = 135 \frac{(0.675\ \text{W/(m K)})}{0.016\,\text{m}} = 5685\ \ \text{W/(m}^2\text{K)}$$

The new overall heat transfer coefficient is

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\ (17{,}946)^{0.8}\ (2.04)^{0.4} = 77.4$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D} = 77.4 \frac{(0.675\ \text{W/(m K)})}{0.016\,\text{m}} = 3265\ \ (\text{m}^2\ \text{K})/\text{W}$$

From Figure 8.19, e = 1

From Equation (8.12b)

$$T_{w,\text{out}} = T_{w,\text{in}} + \text{E}\ (T_s - T_{w,\text{in}}) = 60°\text{C} + 1\ (115°\text{C} - 60°\text{C}) = 115°\text{C}$$

PROBLEM 8.48

Liquid benzene (specific gravity = 0.86) is to be heated in a counterflow concentric-pipe heat exchanger from 30 to 90°C. For a tentative design, the velocity of the benzene through the inside pipe (*ID* = 2.7 cm; *OD* = 3.3 cm) can be taken as 8 m/s. Saturated process steam at 1.38 × 10^6 N/m^2 is available for heating. Two methods of using this steam are proposed (a) Pass the process steam directly through the annulus of the exchanger; this would require that the letter be designed for the high pressure. (b) Throttle the steam adiabatically to 138,000 N/m^2 before passing it through the heater. In both cases, the operation would be controlled so that saturated vapor enters and saturated water leaves the heater. As an approximation, assume that for both cases the heat transfer coefficient for condensing steam remains constant at 12,800 W/(m^2 K), that the thermal resistance of the pipe wall is negligible, and that the pressure drop for the steam is negligible. If the inside diameter of the other pipe is 5 cm, calculate the mass rate of flow of steam (kg/s per pipe) and the length of heater required for each arrangement.

GIVEN

- A concentric pipe, counterflow heat exchanger - benzene in inner tube; saturated steam in annulus
- Specific gravity of benzene (s.g.) = 0.86
- Benzene temperatures
  - $T_{b,\text{in}}$ = 30°C
  - $T_{b,\text{out}}$ = 90°C
- Pipe diameters
  - D_{ii} = 2.7 cm = 0.027 m
  - D_{io} = 3.3 cm = 0.033 m
  - D_o = 5 cm = 0.05 m
- Benzene velocity (V_b) = 2.5 m/s
- Saturated steam pressure = 1.38 × 10^6 N/m^2
- Saturated vapor enters condenser and saturated water leaves

FIND

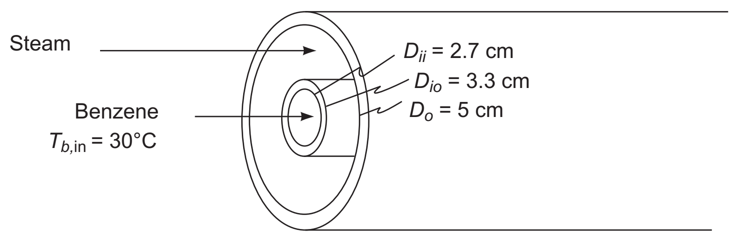
The mass flow rate of steam $(\dot{m}_s)$ and the length of the heater (L) for

(a) Passing steam directly through condenser, and
(b) Steam throttled adiabatically to 138,000 N/m^2 before the heater

ASSUMPTIONS

- The heat transfer coefficient on the steam side $(\bar{h}_o)$ = 12,800 W/(m^2 K)
- The thermal resistance of the pipe wall is negligible
- The pressure drop for the steam is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the saturation temperature and heat of vaporization of steam at

1.38×10^6 N/m^2 (T_{sa}) = 194°C h_{fga} = 1963 kJ/kg
1.38×10^5 N/m^2 (T_{sb}) = 108°C h_{fgb} = 2236 kJ/kg

From Appendix 2, Table 20, for benzene at the average temperature of 60°C

Specific heat (c_p) = 1908 J/(kg K)
Thermal conductivity (k) = 0.149 W/(m K)
Kinematic viscosity (ν) = 0.485×10^{-6} m^2/s
Prandtl number (Pr) = 4.6
Density (ρ) = 859 kg/m^3

SOLUTION

The Reynolds number of the benzene flow is

$$Re_D = \frac{V_b D_{ii}}{\nu} = \frac{(2.5\text{ m/s})(0.027\text{ m})}{\left(0.485\times10^{-6}\text{ m}^2/\text{s}\right)} = 1.39 \times 10^5 \text{ (Turbulent)}$$

The Nusselt number can be calculated using Equation (6.63)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(1.39 \times 10^5)^{0.8}\,(4.6)^{0.4} = 551$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 551 \frac{(0.149\text{ W/(m K)})}{0.027\text{ m}} = 3044 \text{ W/(m}^2\text{ K)}$$

The overall heat transfer coefficient, neglecting wall resistance is

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + \frac{D_o}{D_i \bar{h}_i} = \frac{1}{\left(12{,}800\text{ W/(m}^2\text{K)}\right)} + \left(\frac{3.3}{2.7}\right)\frac{1}{\left(3041\text{ W/(m}^2\text{K)}\right)}$$

$$U_o = 2085 \text{ W/(m}^2\text{ K)}$$

(a) T_s = 194°C and C_{min}/C_{max} = 0

From Equation (8.21b) ($C_c = C_{min}$)

$$\text{E} = \frac{T_{b,\text{out}} - T_{b,\text{in}}}{T_s - T_{b,\text{in}}} = \frac{90-30}{194-30} = 0.37$$

From Figure 8.17 or 8.18 NTU = 0.5

$$NTU = \frac{UA}{C_{\min}} = \frac{U_o \pi D_{io} L}{\dot{m}_b c_p} = \frac{U_o \pi D_{io} L}{\rho V_b \frac{\pi}{4} D_{ii}^2 c_p}$$

$$\therefore \quad L = \frac{NTU \, \rho V_b D_{ii}^2 c_p}{4 U_o D_{io}} = \frac{0.5\left(859\text{ kg/m}^3\right)(2.5\text{ m/s})(0.027\text{ m})^2 (1908\text{ J/(kg K)})}{4\left(2085\text{ W/(m}^2\text{K)}\right)(\text{J/(Ws)})(0.033\text{ m})} = 5.4\text{ m}$$

The rate of heat transfer is

$$q = \text{E}\, C_{\min} (T_s - T_{b,\text{in}}) = \dot{m}_s \; h_{\text{fga}}$$

$$\dot{m}_s = \frac{E C_{\min}}{h_{fga}} (T_s - T_{b,\text{in}}) = \frac{E \rho V_o \frac{\pi}{4} D_{ii}^2 c_p}{h_{fga}} (T_s - T_{b,\text{in}})$$

$$\dot{m}_s = \frac{0.37\left(859\text{ kg/m}^3\right)(2.5\text{ m/s})(0.027)^2 (1908\text{ J/(kg K)})}{(1963\text{ kJ/kg})(1000\text{ J/(kJ)})} (194°\text{C} - 30°\text{C}) = 0.073\text{ kg/s}$$

(b) T_s = 108°C C_{min}/C_{max} = 0 e = (90 – 30)/(108 – 30) = 0.77

From Figure 8.17 or 8.18 $NTU \approx 1.5$

If U_o remains the same and C_{min} is the same as case (a), then

$$L = \frac{NTU\,(\text{a})}{NTU\,(\text{b})} L\,(\text{a}) = \frac{1.5}{0.5} (5.4\text{ m}) = 16.2\text{ m}$$

$$\dot{m}_s = \frac{0.77\left(859\text{ kg/m}^3\right)(2.5\text{ m/s})\frac{\pi}{4}(0.027)^2 (1908\text{ J/(kg K)})}{(2236\text{ kJ/kg})(1000\text{ J/(kJ)})} (108°\text{C} - 30°\text{C}) = 0.063\text{ kg/s}$$

COMMENTS

Throttling the steam reduces the flow rate of steam required by 14% but increases the size of the condenser by 300%. Economic considerations are needed to choose between the two options.

PROBLEM 8.49

Calculate the overall heat transfer coefficient and the rate of heat flow from the hot gasses to the cold air in the cross flow tube-bank of heat exchanger shown in the accompanying illustration for the following operating conditions

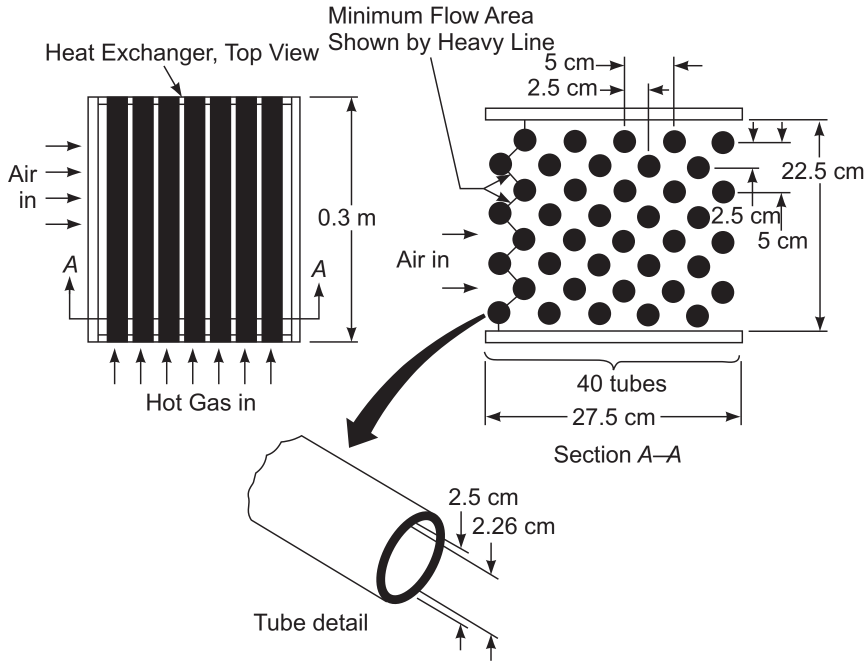
Air flow rate = 0.4 kg/s.

Hot gas flow rate = 0.65 kg/s.

Temperature of hot gasses entering exchanger = 870°C.

Temperature of cold air entering exchanger = 40°C.

Both gases are approximately at atmospheric pressure.



GIVEN

- The crossflow tube bank heat exchanger shown above
- Air flow rate $(\dot{m}_a)$ = 0.4 kg/s
- Gas flow rate $(\dot{m}_g)$ = 0.65 kg/s
- Entrance temperatures
  - Air ($T_{a,\text{in}}$) = 40°C
  - Gas ($T_{g,\text{in}}$) = 870°C
- Both gases are at 1 atm pressure

FIND

(a) The overall heat transfer coefficient
(b) The rate of heat transfer (q)

ASSUMPTIONS

- The hot gases have the same thermal properties as air
- No scaling
- Thermal resistance of the tube walls can be neglected

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the entering temperatures

Temperature	40°C	870°C
Thermal conductivity, k (W/(m K))	0.0263	0.074
Specific heat, c_p (J/(kg K))	1013.5	1122
Density, ρ (kg/m^3)	1.13	0.321
Absolute viscosity, μ (kg/ms)	1.92×10^{-5}	4.56×10^{-5}
Prandtl number	0.71	0.73

SOLUTION

(a) Heat transfer coefficient inside tubes $(\bar{h}_i)$

The Reynolds number in the tubes is

$$Re_D = \frac{V_g D_i}{\nu_g} = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4(0.65\ \text{kg/s})}{\pi(2.26\times10^{-2}\ \text{m})(4.56\times10^{-5}\ \text{kg/ms})} = 8\times10^5\ \text{(Turbulent)}$$

The Nusselt number for turbulent flow in a tube is given by Equation (6.63)

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr_n \text{ where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\ (8\times10^5)^{0.80}\ (0.73)^{0.3} = 1104.6$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 1104.6\ \frac{0.074\ \text{W/(m K)}}{2.26\times10^{-2}\ \text{m}} = 3617\ \text{W/(m}^2\ \text{K)}$$

Heat transfer coefficient outside the tubes $(\bar{h}_o)$

The velocity of the air based on the minimum flow area is

$$V_{\max} = \frac{\dot{m}}{\rho A_{\min}}$$

From the sketch

$$A_{\min} = [7(S'_L - D_o) + 2.5\ \text{cm}]L \quad \text{where} \quad S'_L = \sqrt{(2.5)^2 + (2.5)^2} = 3.53\ \text{cm}$$

$$\therefore A_{\min} = [7(3.53 - 2.5) + 2.5]\times10^{-2}\ \text{m}\times0.3\ \text{m} = 0.0291\ \text{m}^2$$

$$\therefore V_{\max} = \frac{(0.4\ \text{kg/s})}{(1.13\ \text{kg/m}^3)(0.0291\text{m}^2)} = 12.16\ \text{m/s}$$

The Reynolds number based on the minimum flow area is

$$Re_D = \frac{V_{\max} D}{\nu} = \frac{12.16\text{m/s}\ (2.5\times10^{-2}\ \text{m})(1.13\text{kg/m}^3)}{(1.92\times10^{-5}\ \text{kg m/s})} = 1.79\times10^4\ \text{(Turbulent)}$$

Applying Equation (7.34)

$$Nu_D = 0.019\ Re_D^{0.84} = 0.019\ (1.79\times10^4)^{0.84} = 71$$

$$h_o = Nu_D \frac{k}{D_o} = 71\frac{0.0263\ \text{W/(m K)}}{(2.5\times10^{-2}\ \text{m})} = 74.7\ \text{W/(m}^2\ \text{K)}$$

The overall heat transfer coefficient, neglecting the thermal resistance of the tube wall, is

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + \frac{A_o}{A_i\bar{h}_i} = \frac{1}{\bar{h}_o} + \frac{D_o}{D_i\bar{h}_i} = \frac{1}{(74.7\ \text{W/(m}^2\ \text{K)})} + \frac{2.5}{2.26}\frac{1}{(3617\ \text{W/(m}^2\ \text{K)})}$$

$$\Rightarrow \quad U_o = 73\ \text{W/(m}^2\ \text{K)}$$

(b) The heat capacity rates are

$$C_a = \dot{m}_a\ c_{pa} = (0.4\ \text{kg/s})\ (1013.5\ \text{J/(kg K)}) = 405\ \text{W/K}$$

$$C_g = \dot{m}_g\ c_{pg} = (0.65\ \text{kg/s})\ (1122\ \text{J/(kg K)}) = 729.3\ \text{W/K}$$

$$\frac{C_{\text{mixed}}}{C_{\text{unmixed}}} = \frac{C_a}{C_g} = 0.55$$

The number of transfer units is

$$NTU = \frac{U_o A_o}{C_{\min}} = \frac{U_o N \pi D_o L}{C_{\min}} = \frac{(73\,\text{W/m}^2)(40)\pi(2.5\times10^{-2}\,\text{m})(0.3\,\text{m})}{405\,\text{W/K}} = 0.17$$

From Figure 8.21, e ≈ 0.18

The rate of heat transfer is given by Equation (8.22)

$$q = \mathrm{E}\, C_{\min}\,(T_{g,\text{in}} - T_{a,\text{in}}) = 0.18\ (405\ \text{J/K})\ (870 - 40) = 60.5\ \text{kW}$$

PROBLEM 8.50

An oil having a specific heat of 2100 J/(kg K) enters an oil cooler at 82°C at the rate of 2.5 kg/s. The cooler is a counterflow unit with water as the coolant, the transfer area being 28 m^2 and the overall heat transfer coefficient being 570 W/(m^2 K). The water enters the exchanger at 27°C. Determine the water rate required if the oil is to leave the cooler at 38°C.

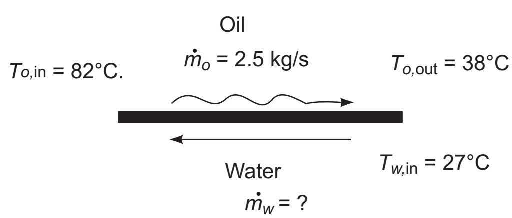
GIVEN

- Counterflow heat exchanger - water cools oil
- Oil specific heat (c_{po}) = 2100 J/(kg K)
- Oil temperatures
  - $T_{o,\text{in}}$ = 82°C
  - $T_{o,\text{out}}$ = 38°C
- Oil flow rate ($\dot{m}_o$) = 2.5 kg/s
- Transfer area (A) = 28 m^2
- The overall heat transfer coefficient (U) = 570 W/(m^2 K)
- Water inlet temperature ($T_{w,\text{in}}$) = 27°C

FIND

- The water flow rate ($\dot{m}_w$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water (c_{pw}) = 4175 J/(kg K) at 40°C

SOLUTION

The heat rate of the oil is

$$C_o = \dot{m}_o\ c_{po} = (2.5\ \text{kg/s})\ (2100\ \text{J/(kg K)}) = 5250\ \text{W/K}$$

Assuming $C_o = C_{\min}$, the effectiveness, from Equation (8.21a) is

$$\mathrm{E} = \frac{T_{o,\text{in}} - T_{o,\text{out}}}{T_{o,\text{in}} - T_{w,\text{in}}} = \frac{82 - 38}{82 - 27} = 0.80$$

Combining Equations (8.22) and (8.15)

$$q = e\ C_{\min}\,(T_{o,\text{in}} - T_{w,\text{in}}) = U\,A\,(LMTD)$$

Solving for the log mean temperature difference

$$LMTD = \frac{EC_{\min}}{U\,A}(T_{o,\text{in}} - T_{w,\text{in}}) = \frac{0.8(5250\text{ W/K})}{(570\text{ W/(m}^2\text{ K)})(28\text{ m}^2)}(80°\text{C} - 27°\text{C}) = 14.5°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{(T_{o,\text{in}} - T_{w,\text{out}}) - (T_{w,\text{out}} - T_{w,\text{in}})}{\ln\left(\frac{T_{o,\text{in}} - T_{w,\text{out}}}{T_{o,\text{out}} - T_{w,\text{in}}}\right)}$$

$$14.5°\text{C} = \frac{(82°\text{C} - T_{w,\text{out}}) - (38°\text{C} - 27°\text{C})}{\ln\left(\frac{82 - T_{w,\text{out}}}{38 - 27}\right)} = \frac{71°\text{C} - T_{w,\text{out}}}{\ln\left(\frac{82 - T_{w,\text{out}}}{11}\right)}$$

By trial and error $T_{w,\text{ou}}$t = 63°C

The flow rate of water can be calculated from an energy balance

$$\dot{m}_o\ c_{po}\,(C_{o,\text{in}} - T_{o,\text{out}}) = \dot{m}_w\ c_{pw}\,(T_{w,\text{out}} - T_{w,\text{in}})$$

$$\dot{m}_w = \dot{m}_o\left(\frac{c_{po}}{c_{pw}}\right)\left(\frac{T_{o,\text{in}} - T_{w,\text{out}}}{T_{o,\text{out}} - T_{w,\text{in}}}\right) = (2.5\text{ kg/s})\left(\frac{2100}{4175}\right)\left(\frac{82-38}{63-27}\right) = 1.54\text{ kg/s}$$

The heat capacity rate of the water is

$$C_w = \dot{m}_w\ c_{pw} = 1.54\text{ kg/s}\ (4175\text{ J/(kg K)}) = 6417\text{ W/K}$$

Therefore, the assumption that $C_o = C_{\min}$ is valid.

PROBLEM 8.51

Dry air is cooled from 65 to 38°C, while flowing at the rate of 1.25 kg/s in a simple counterflow heat exchanger, by means of cold air which enters at 15°C and flows at a rate of 1.6 kg/s. It is planned to lengthen the heat exchanger so that 1.25 kg/s. of air can be cooled from 65 to 26°C with a counterflow current of air at 1.6 kg/s entering at 15°C. Assuming that the specific heat of the air is constant, calculate the ratio of the length of the new heat exchanger to the length of the original.

GIVEN

A simple adiabatic air-to-air counter flow heat exchanger

Case 1

- Warm air temperatures
  - $T_{h,\text{in}}$ = 65°C
  - $T_{h,\text{out}}$ = 38°C
- Air flow rates
  - $\dot{m}_h$ = 1.25 kg/s
  - $\dot{m}_c$ = 1.6 kg/s
- Cold air inlet temperature ($T_{c,\text{in}}$) = 15°C

After lengthening heat exchanger

Case 2

- Warm air temperatures
  - $T_{h,\text{in}}$ = 65°C
  - $T_{h,\text{out}}$ = 26°C
- Air flow rates
  - $\dot{m}_h$ = 1.25 kg/s
  - $\dot{m}_c$ = 1.6 kg/s
- Cold air inlet temperature ($T_{c,\text{in}}$) = 15°C

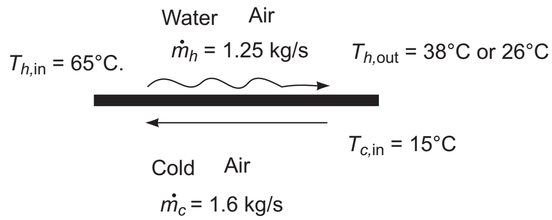
FIND

- The ratio of the length of the new heat exchanger to the length of the original

ASSUMPTIONS

- The specific heat of air is constant
- The overall heat transfer coefficient (U) is the same in both cases

SKETCH



SOLUTION

For both cases

$$\frac{C_{\min}}{C_{\max}} = \frac{C_h}{C_c} = \frac{\dot{m}_h c_p}{\dot{m}_c c_p} = \frac{\dot{m}_h}{\dot{m}_c} = \frac{1.25}{1.6} = 0.78$$

The effectiveness, from Equation (8.21a), is

$$\mathrm{E} = \frac{C_h}{C_{\min}} \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}}$$

Case 1

$$e_1 = \frac{65-38}{65-15} = 0.54$$

Case 2

$$e_2 = \frac{65-26}{65-15} = 0.78$$

From Figure 8.18 $\quad NTU_1 = 1.1 \quad\quad NTU_2 = 2.5$

$$\frac{NTU_1}{NTU_2} = \frac{\dfrac{U_2 A_2}{C_{\min 2}}}{\dfrac{U_1 A_1}{C_{\min 1}}} \quad \text{But } U_1 = U_2 \quad \text{and} \quad C_{\min 1} = C_{\min 2}$$

$$\therefore \quad \frac{A_2}{A_1} = \frac{NTU_2}{NTU_1} = \frac{2.5}{1.1} = 2.3$$

Since the area is directly proportional to the length.

$$\frac{L_2}{L_1} = 2.3$$

PROBLEM 8.52

Saturated steam at 1.35 atm condenses on the outside of a 2.6 m length of copper tubing heating 5m kg/hr of water flowing in the tube. The water temperatures, measured at 10 equally spaced stations along the tube length are

Station		**1**		**2**	**3**	**4**	**5**		**6**	**7**
8	**9**	**10**	**11**							
Temp °C		**18**		**43**	**57**	**67**	**73**	**78**		**82**
85	**88**	**90**	**92**							

Calculate (a) average overall heat transfer coefficient U_o based on the outside tube area; (b) average water-side heat transfer coefficient *hw* (assume steamside coefficient at h_s = 11,000 W/(m^2 K)), (c) local overall coefficient U_x based on the outside tube area for each of the 10 sections between temperature stations, and (d) local waterside coefficients h_{wx} for each of the 10 sections. Plot all items vs. tube length. Tube dimensions: *ID* = 2 cm, *OD* = 2.5 c. Temperature station 1 is at tube entrance and station 11 is at tube exit.

GIVEN

- Saturated steam condensing on copper tubing with water flowing within
- Steam pressure = 1.35 atm = 136,755 N/m^2
- Tube length (L) = 2.6 m
- Water flow rate $(\dot{m}_w)$ = 5 kg/h = 0.00139 kg/s
- Water temperatures given above as a function of distance along pipe
- Tube diameters
  - D_i = 2 cm = 0.02 m
  - D_o = 2.5 cm = 0.025 m

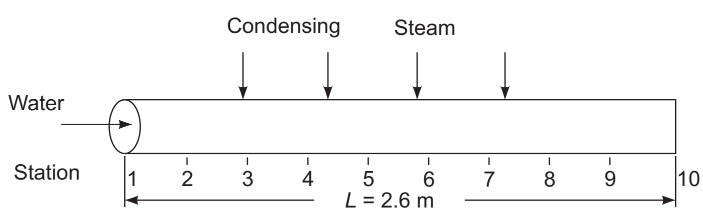
FIND

(a) Average overall heat transfer coefficient based on the outside tube area (U_o)
(b) Average water-side transfer coefficient $(\bar{h}_w)$
(c) Local overall coefficient (U_x) for each of the 10 sections
(d) Local water-side coefficient h_{wx} for each of the 10 sections
Plot all items vs. tube length

ASSUMPTIONS

- The steam-side heat transfer coefficient $\bar{h}_s$ = 11,000 W/(m^2 K)
- No scaling resistance
- Variation of the specific heat of water is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated steam at 136,755 N/m^2: T_s = 107°C
For water at the average temperature of 55°C, the specific heat (c_{pw}) = 4180 J/(kg K)
From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) at 127°C

SOLUTION

(a) The heat capacity rate of the water is

$$C_w = \dot{m}_w c_{pw} = (0.00139\ \text{kg/s}(4180\ \text{J/(kg K)}) = 5.81\ \text{W/K}$$

Since the heat capacity rate of the steam is essentially infinite, $C_{\min}/C_{\max}$ = 0.
The effectiveness of the heat exchanger is given by Equation (8.21b) ($C_c = C_{\min}$).

$$\text{E} = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}} = \frac{92-18}{107-18} = 0.83$$

For $C_{\min}/C_{\max} = 0$, the effectiveness for parallel or counterflow are the same and Equation (8.25) reduces to

$$e = 1 - e^{-NTU} \Rightarrow NTU = -\ln(1 - \text{e}) = -\ln(1 - 0.83) = 1.77$$

$$NTU = \frac{U_o A_o}{C_{\min}} \Rightarrow U_o = \frac{NTU\, C_{\min}}{A_o} = \frac{NTU\, C_{\min}}{\pi D_o L} = \frac{1.77(5.81\text{ W/K})}{\pi(0.025\text{ m})(2.6\text{ m})} = 50.4\text{ W/(m}^2\text{K)}$$

(b) From Equation (8.2)

$$\frac{1}{U_o} = \frac{A_o}{A_i \bar{h}_i} + \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{\bar{h}_o} = \frac{D_o}{D_i \bar{h}_w} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{1}{\bar{h}_s}$$

$$\therefore \quad \frac{1}{\bar{h}_w} = \frac{D_i}{D_o}\left(\frac{1}{U} - \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} - \frac{1}{\bar{h}_s}\right)$$

$$\frac{1}{\bar{h}_w} = \left(\frac{1}{\left(50.4\text{ W/(m}^2\text{K)}\right)} - \frac{(0.025\text{ m})\ln\left(\frac{2.5}{2}\right)}{2(392\text{ W/(m K)})} - \frac{1}{\left(11{,}000\text{ W/(m}^2\text{ K)}\right)}\right)$$

(c) Treating the first section as a separate heat exchanger and following the procedure of part (a) $C_{\max}/C_{\min} = 0$, $e = (43 - 18)/(107 - 18) = 0.28$, $NTU = -\ln(1 - 0.28) = 0.33$

$$\therefore \quad U_x = \frac{NTU\, C_{\min}}{A_o} = \frac{NTU\, C_{\min}}{\pi D_o L} = \frac{0.33(5.81\text{ W/K})}{\pi(0.025\text{ m})(0.26\text{ m})} = 93.9\text{ W/(m}^2\text{K)}$$

This procedure must be repeated for each section. The results are tabulated below section (d).

(d) Following the procedure of section (b), the only value that changes is the overall heat transfer coefficient

$$\frac{1}{h_{w1}} = \frac{2}{2.5}\left(\frac{1}{\left(93.9\text{ W/(m}^2\text{K)}\right)} - 0.000098\text{ (m}^2\text{ K)/W}\right)$$

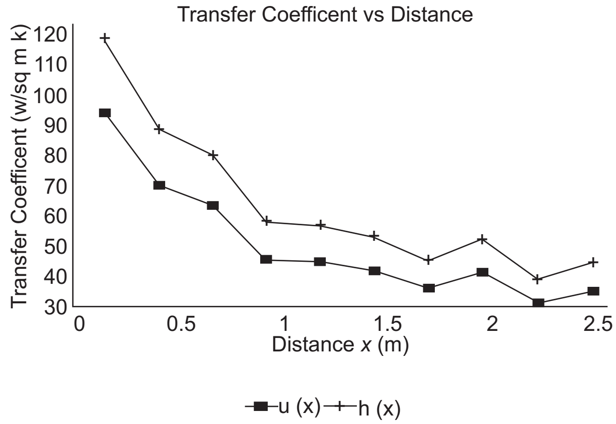
$$h_{w1} = 118.5\text{ W/(m}^2\text{ K)}$$

Repeating parts (c) and (d) for each section yields

Section	1	2	3	4	5	6	7	8	9	10
x (m)	0.13	0.39	0.65	0.91	1.17	1.43	1.69	1.95	2.21	2.47
U_x (W/(m^2 K))	93.9	70.2	63.5	46.2	45.3	42.2	36.4	41.7	31.6	35.6
h_{wx} (W/(m^2 K))	118.5	88.4	79.9	58.0	56.9	53.0	45.7	52.3	39.6	44.7

where x = distance from inlet to midpoint of section.

Plotting this data on a single graph



PROBLEM 8.53

Calculate the water side heat transfer coefficient and the coolant pressure drop per unit length of tube for the core of a compact air-to-water intercooler for a 3.7 MW gas turbine plant. The water flows inside of a flattened aluminum tube having the cross-section shown below

The inside diameter of the tube before it was flattened was 1.23 cm with a wall thickness (*t*) of 0.025 cm. The water enters the tube at 15.6°C and leaves at 26.7°C at a velocity of 1.34 m/s.

GIVEN

- Water flow in a flattened tube as shown above
- Inlet temperature (T_i) = 15.6°C
- Outlet temperature (T_o) = 26.7°C
- Water velocity (V) = 1.34 m/s

FIND

(a) The heat transfer coefficient on the inside of the tubes (h_c)
(b) The pressure drop per unit length ($\Delta p/L$)

ASSUMPTIONS

- Steady state
- Fully developed flow

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the average temperature of 21.1°C

Thermal conductivity (k) = 0.599 W/(m K)

Kinematic viscosity (ν) = 0.979×10^{-6} m^2/s

Density (ρ) = 998.0 kg/m^3

Prandtl number (Pr) = 6.80

SOLUTION

The hydraulic diameter of the flattened tube is

$$D_h = \frac{4A}{P} = \frac{4[(1.6\,\text{cm} - 0.2\,\text{cm})(0.2\,\text{cm}) + \pi(0.1\,\text{cm})^2]}{\pi(1.23\,\text{cm})} = 0.322\text{ cm} = 0.00322\text{ m}$$

The Reynolds number based on the hydraulic diameter is

$$Re_{D_h} = \frac{VD_h}{\nu} = \frac{(1.34\text{ m/s})(0.00322\,\text{m})}{\left(0.979\times10^{-6}\text{ m}^2\text{/s}\right)} = 4407 \text{ (turbulent)}$$

(a) The Nusselt number for turbulent flow is given by Equation (6.63)

$$\overline{Nu} = 0.023\ Re_{D_h}^{0.8}\ Pr^{0.4} = 0.023\ (4407)^{0.8}\ (608)^{0.4} = 40.7$$

$$\bar{h}_c = \overline{Nu}\ \frac{k}{D_h} = 40.7\frac{(0.599\text{ W/(m K)})}{0.00322\,\text{m}} = 7580\text{ W/(m}^2\text{K)}$$

(b) The friction factor for turbulent flow in smooth tubes is given by Equation (6.59)

$$f = 0.184\ Re_{D_h}^{-0.2} = 0.184\ (4407)^{-0.2} = 0.0344$$

The pressure drop is given by Equation (6.13)

$$\frac{\Delta P}{L} = \frac{f}{D_h}\frac{\rho V^2}{2g_c} = \frac{0.0344}{0.0322\text{m}}\frac{\left(998.0\text{ kg/m}^3\right)(1.34\text{ m/s})^2}{2\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(\text{N}/(\text{m}^2\text{Pa})\right)} = 9560\text{ Pa/m}$$

PROBLEM 8.54

An air-to-water compact heat exchanger is to be designed to serve as an intercooler for a 3.7 MW gas turbine plant. The exchanger is to meet the following heat transfer and pressure drop performance specifications

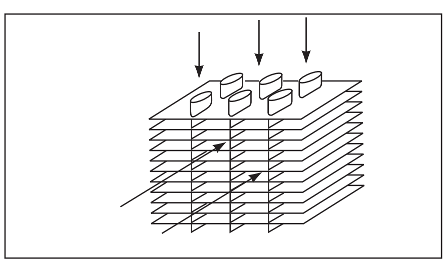
Air-side Operating Conditions

Flow rate	**25.2 kg/s**
Inlet Temperature	**400 K**
Outlet Temperature	**300 K**
Inlet Pressure(p_1)	**2.05×10^5 N/m^2**
Pressure Drop Ratio ($\Delta p/p_1$)	

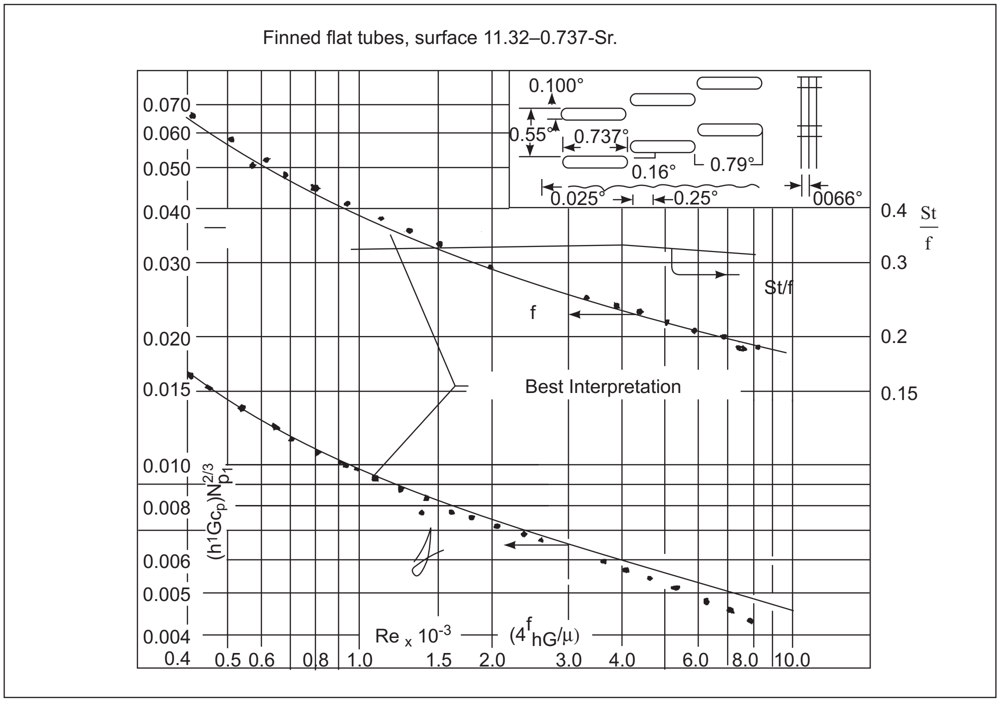
Water-side Operating Conditions

Flow rate	**50.4 kg/s**
Inlet Temperature	**289 K**

The exchanger is to have a cross-flow configuration with both fluids unmixed. The heat exchanger surface proposed for the exchanger consists of flattened tubes with continuous aluminum fins specified as a 11.32– 0.737 – SR surface in Ref. 10. The heat exchanger is shown schematically below.



The measured heat transfer and friction characteristic for this exchanger surface are shown in the graph below



Geometrical details for the proposed surface are

Air-side — **Flow passage hydraulic radius (r_h) = (0.0878 cm)**

Total transfer area/total volume (a_{air}) = (886 m²/m³)

Free flow area/frontal area (s) = 0.780

Fin area/total area (A_f/A) = 0.845

Fin metal thickness (t) = 0.00033 ft (0.0001 m)

Fin length (1/2 distance between tubes, L_f) = 0.225 in (0.00572 m)

Water side — **Tubes as given in Problem 8.53**

Water-side transfer area/total volume (a_{H2O}) = 42.1 ft²/ft³

The design should specify the core size, the air flow frontal area, and the flow length. The water velocity inside the tubes is 4.4 ft/s (1.34 m/s). See problem 8.53 for the calculation of the water side heat transfer coefficient.

Note: (i) the free-flow area is defined such that the mass velocity, G, is the air mass flow rate per unit free flow area, (ii) the core pressure drop is given by $\Delta p = fG^2L/2\rho r_h$ where L is the length of the core in the air flow direction, (iii) the fin length, L_f, is defined such that $L_f = 2A/P$ where A is the fin cross-sectional area for heat conduction and P is the effective fin perimeter.

GIVEN

- Air-to-Water Intercooler with the geometry and requirements specified above
- From Problem 8.53: Water side convective heat transfer coefficient ($h_{c,H2O}$) = 7580 W/(m² K)

FIND

(a) The air flow frontal area (A_{air})
(b) The flow length (L)
(c) The core size

ASSUMPTIONS

- Steady state
- Entrance effects are negligible
- Flow acceleration effects are negligible
- Negligible fouling resistance
- Negligible variation in thermal resistance
- The thermal resistance of the tube wall is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for air at the mean temperature of 77°C

Specific heat (c) = 1019 J/(kg K)

Density (ρ) = 0.977 kg/m^3

Prandtl number (Pr) = 0.71

Absolute viscosity (μ) = 20.6 × 10^6 (N s)/m^2

From Appendix 2, Table 13, for water at 20°C, c = 4182 J/(kg K)

From Appendix 2, Table 12, the thermal conductivity of aluminum at 320 K (k_a) = 238 W/(m K)

SOLUTION

The outlet water temperature is given by the conservation of energy

$$\dot{m}_{H2O}\ c_{H2O}\ (T_{in} - T_{out})_{H2O} = \dot{m}_{air}\ c_{air}\ (T_{in} - T_{out})_{air}$$

$$T_{out,H2O} = T_{in,H2O} + \frac{(\dot{m}c)_{air}}{(\dot{m}c)_{H2O}}\ (T_{in} - T_{out})_{air} = 289\text{ K} + \frac{25.2(1019)}{50.4(4182)}(400\text{ K} - 300\text{ K}) = 301\text{ K}$$

The effectiveness required for the specified performance is given by Equation (8.21a)

$$\text{E} = \frac{C_h\left(T_{h,in} - T_{h,out}\right)}{C_{min}\left(T_{h,in} - T_{c,in}\right)}$$

Since $C_h = C_{min}$

$$\text{E} = \frac{\left(T_{h,in} - T_{h,out}\right)}{\left(T_{h,in} - T_{c,in}\right)} = \frac{400\text{ K} - 300\text{ K}}{400\text{ K} - 289\text{ K}} = 0.90$$

The heat capacity rate ratio is

$$\frac{C_{min}}{C_{max}} = \frac{C_{air}}{C_{H2O}} = \frac{(\dot{m}c)_{air}}{(\dot{m}c)_{H2O}} = \frac{25.2(1019)}{50.4(4182)} = 0.122$$

From Figure 8.20 for cross-flow heat exchangers with e = 0.9 and C_{min}/C_{max} = 0.122, NTU_{max} = 2.75 = $U_{air}A_{air}/C_{min}$.

The solution will require iteration. For the first iteration, let $Re_D = 10^4 = 4\ r_h\ G/\mu$

Solving for the mass velocity

$$G = \frac{Re_d\,\mu}{4r_h} = \frac{(10^4)(20.6\times10^{-6}\text{ kg/m s})}{4(0.000878\text{ m})} = 58.7\text{ kg/(m}^2\text{s)}$$

From the graphical data at $Re = 10^4$

$$\frac{\bar{h}}{Gc_p}Pr^{\frac{2}{3}} \approx 0.0045 \qquad f \approx 0.018$$

$$\therefore\ \bar{h} = 0.0045\ Gc_p\ Pr^{-\frac{2}{3}} = (0.0045)\left(58.7\text{ kg}/(\text{m}^2\text{s})\right)\ (1019\ (\text{W s})/(\text{kg K}))\ (0.71)^{-\frac{2}{3}} = 337\text{ W}/(\text{m}^2\text{K})$$

Since $G = \dfrac{\dot{m}_a}{A_{\text{free flow.}}}$

$$A_{\text{free flow}} = \frac{\dot{m}_a}{G} = \frac{25.2\text{ kg/s}}{58.7\text{ kg}/(\text{m}^2\text{s})} = 0.429\text{ m}^2$$

$$A_{\text{frontal}} = \frac{A_{\text{free flow}}}{\sigma} = \frac{0.429\text{ m}^2}{0.78} = 0.55\text{ m}^2$$

We must calculate the fin efficiency per Chapter 2

$$m = \sqrt{\frac{h_a P}{kA}} = \sqrt{\frac{2h_a}{k_f L_f}} = \sqrt{\frac{2\left(337\text{ W/(m}^2\text{K)}\right)}{(283\text{ W/(m K)})(0.0001\text{ m})}} = 154\text{ m}^{-1}$$

$$m\,L_f = 154\text{ m}^{-1}\ (0.0572\text{ m}) = 0.883$$

The fin efficiency from Equation (2.65) is

$$\eta_f = \frac{\tanh\left(mL_f\right)}{mL_f} = 0.80$$

The total fin efficiency can be calculated from Equation (2.68)

$$\eta_f = 1 - \frac{A_f}{A}\ (1 - \eta_f) = 1 - 0.845\ (1 - 0.80) = 0.83$$

and the overall heat transfer coefficient from Equation (2.69) is

$$U_{\text{air}} = \left(\frac{1}{\eta_t h_a} + \frac{1}{\left(\dfrac{\alpha_{\text{H2O}}}{\alpha_{\text{air}}}\right) h_{\text{H2O}}}\right)^{-1} = \left(\frac{1}{0.83\left(337\text{ W/(m}^2\text{K)}\right)} + \frac{1}{\left(\dfrac{42.1}{270}\right)\left(7580\text{ W/(m}^2\text{K)}\right)}\right)^{-1}$$

$$A_{\text{air}} = \frac{NTU\ \dot{m}_a c_{pa}}{U_{air}} = \frac{(2.75)(25.2\text{ kg/s})(1019\text{ W s/(kg K)})}{\left(227\text{ W/(m}^2\text{K)}\right)} = 311\text{ m}^2$$

$$\text{Heat exchanger volume } V = \frac{A_{\text{air}}}{\alpha_{\text{air}}} = \frac{311\text{ m}^2}{886\text{ m}^2/\text{m}^3} = 0.35\text{ m}^3$$

$$\text{Core length } L = \frac{V}{A_{\text{frontal}}} = \frac{0.35\text{ m}^3}{0.55\text{ m}^2} = 0.64\text{ m}$$

The core pressure drop is

$$\Delta p = f\frac{G^2}{2\rho}\frac{L}{r_h} = \frac{(0.018)\left(58.5\text{ kg/(m}^2\text{s)}\right)^2}{2\left(0.977\text{ kg/m}^3\right)}\ \frac{0.64\text{ m}}{0.000878\text{ m}} = 22{,}980\ \text{ N/m}^2$$

$$\frac{\Delta p}{p_1} = \frac{22,980}{205,000} = 11\% > 7.6\% \text{ (Too High)}$$

Repeating this procedure until $\Delta p/p_1 = 7.6\%$ yields the following results

$Re = 8400$	$A_{\text{free}} = 0.513 \text{ m}^2$	$A_{\text{air}} = 342 \text{ m}^2$
$h\ Pr^{\frac{2}{3}}/G\ c_p = 0.0047$	$A_{\text{frontal}} = 0.657 \text{ m}^2$	$V = 0.386 \text{ m}^3$
$f = 0.019$	$\eta_f = 0.821$	$L = 0.587$ m
$G = 49.14 \text{ kg/(m}^2 \text{ s)}$	$\eta_{oa} = 0.849$	$\Delta p = 15,685 \text{ N/m}^2$
$h = 295 \text{ W/(m}^2 \text{ K)}$	$U_{\text{air}} = 207 \text{ W/(m}^2 \text{ K)}$	$\frac{\Delta p}{\Delta p_1} = 0.077$ as required

PROBLEM 8.55

Microchannel compact heat exchangers can be used to cool high heat flux micro-electronic devices. The sketch below shows a schematic view of a typical microchannel heat sink. Micro-fabrication techniques can be used to mass produce aluminum channels and fins with the following dimensions:

$W_c = W_w = 50$ **micrometers**
$b = 200$ **micrometers**
$L = 1.0$ **cm**
$t = 100$ **micrometers**

Assuming there are a total of 100 fins and that water at 30°C is used as the cooling medium at a Reynolds number of 2000 estimate

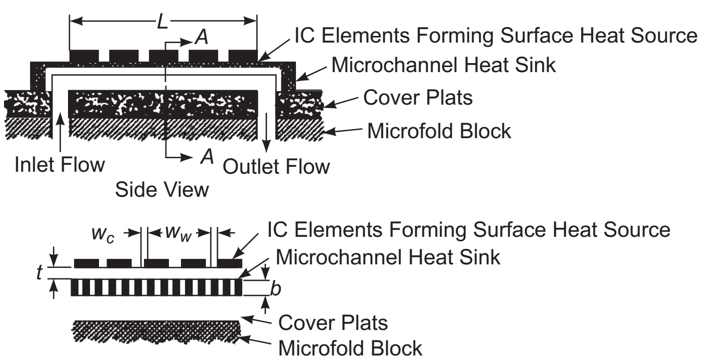
(a) The water flow rate through all the channels

(b) The Nusselt number

(c) The heat transfer coefficient

(d) The effective thermal resistance between the *IC* elements forming the heat source and the cooling water

(e) The rate of heat dissipation allowable if the temperature difference between source and water is not to exceed 100 K



Section A–A

GIVEN

- An aluminum microchannel heat exchanger as shown above
- $w_c = w_w = 50$ micrometers
- $b = 200$ micrometers
- $L = 1.0$ cm

- t = 100 micrometers
- Total number of fins (N) = 100
- Cooling water temperature (T_w) = 30°C
- Reynolds number (Re) = 2000

FIND

(a) The water flow rate $(\dot{m})$ through all the channels

(b) The Nusselt number $(\overline{Nu})$

(c) The heat transfer coefficient $(\bar{h}_c)$

(d) The effective thermal resistance (R_{eff}) between the IC elements forming the heat source and the cooling water

(e) The rate of heat dissipation (Q_{heatsink}) allowable if the temperature difference between source and water ($T_{\text{IC}} - T_{\text{fluid}}$) is not to exceed 100 K

ASSUMPTIONS

- Steady state
- Uniform and constant heat generation
- The heat generation chip is the same size as the heat exchanger
- A conducting paste has been applied between the heat sink and the IC to eliminate contact resistance
- The cover plate is an insulator

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 30°C

Absolute viscosity (μ) = 792.4×10^{-6} (N s)/m^2

Thermal conductivity (k) = 0.615 W/(m K)

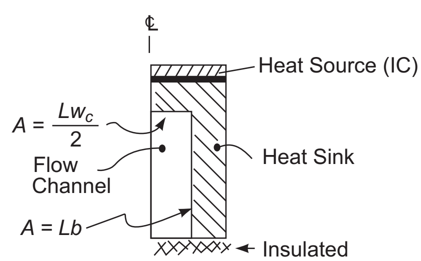
Specific heat (c_p) = 4176 J/(kg K)

From Appendix 2, Table 12

The thermal conductivity of aluminum at 30°C (k_w) = 238 W/(m K)

SOLUTION

A cross-section of the flow channel is shown schematically below



Since we assume there is no contact resistance between the IC and the heat sink, the top of the heat sink is at the IC temperature, T_{IC}. Heat is transferred by conduction through the heat sink directly from the IC to the area on top of the flow channel (area $Lw_c/2$) to the coolant and also along the tall portion of the heat sink to area Lb to the coolant. This latter part of the heat sink acts as a fin because the temperature of this part of the heat sink will decrease as we move down from the IC to the cover plate. As described in Chapter 2, this temperature decrease can be accounted for by the fin efficiency. Given the average heat transfer coefficient, $\bar{h}_c$ in the flow channel and the temperature of the heat sink at the top of the flow channel, T_{TOP}, we can write the rate of heat transfer to the coolant for the half flow channel shown in the sketch above as

$$q = \bar{h}_c L \left(\frac{w_c}{2} + b\eta_f\right)(T_{\text{TOP}} - T_{\text{fluid}})$$

where η_f is the fin efficiency of the heat sink.
The temperature drop from the IC to the top of the flow channel can be estimated by

$$\frac{k_w L\left(\dfrac{w_w + w_w}{2}\right)(T_{\text{IC}} - T_{\text{TOP}})}{t} = q$$

solving for T_{TOP}

$$T_{\text{TOP}} = T_{\text{IC}} - \frac{2tq}{k_w\,(w_w + w_c)\,L}$$

We can now eliminate T_{TOP} from the equation for q

$$q = \frac{\bar{h}_c L\left(\dfrac{w_c}{2 + b\eta_f}\right)(T_{\text{IC}} - T_{\text{fluid}})}{1 + \dfrac{\bar{h}_c L\left(\dfrac{w_c}{2 + b\eta_f}\right)2t}{k_w\,(w_w + w_c)\,L}}$$

so the effective thermal resistance is given by

$$R_{\text{eff}}\ \frac{T_{\text{IC}} - T_{\text{fluid}}}{q} = \frac{1 + \dfrac{\bar{h}_c\left(\dfrac{w_c}{2 + b\eta_f}\right)2t}{k_w\,(w_w + w_c)}}{L\bar{h}_c\left(\dfrac{w_c}{2 + b\eta_f}\right)}$$

We find the average heat transfer coefficient as follows. The hydraulic diameter of the channel is

$$D_h = \frac{4A_c}{P} = \frac{4bw_c}{2(b + w_c)} = \frac{4(2\times10^{-4})(5\times10^{-5})}{2(2\times10^{-4} + 5\times10^{-5})} = 8 \times 10^{-5}\ \text{m}$$

(a) The total mass flow rate can be calculated from the definition of the Reynolds number

$$Re_{D_h} = \frac{D_h\rho}{\mu}U_\infty = \frac{D_h\rho}{\mu}\left(\frac{\dot{m}}{Nbw_c\rho}\right) = \frac{D_h\dot{m}}{Nbw_c\mu}$$

$$m = \frac{ReNbw_c\mu}{D_h} = \frac{2000(100)(2\times10^{-4}\ \text{m})(5\times10^{-5}\ \text{m})(792.4\times10^{-6}\ (\text{N s})/\text{m}_2)((\text{kg m})/(\text{N s}^2))}{8\times10^{-5}\ \text{m}} = 0.020\ \text{kg/s}$$

(b) The aspect ratio of the channels is b/w_c = 200/50 = 4. The length-to-hydraulic diameter ratio is L/D_h = (0.01 m)/(8 × 10^{-5} m) = 125, therefore, the flow in the channels should be fully developed and we find the Nusselt number, from Table 6.1. If we assume that the fin efficiency will be high, then it is safe to assume that the flow chanenl is isothermal at any cross section. (In this argument, we are neglecting the fact that the channel is insulated from below). Therefore, we need Nu_{H1} which is 5.33.

(c)
$$h_c = \frac{k}{D_h}\ Nu = \frac{(0.615\ \text{W/(m K)})}{(8\times10^{-5})}(5.33) = 40{,}974\ \ \text{W/(m}^2\text{K)}$$

The fin efficiency can be determined from Equation (2.71)

$$\eta_f = \frac{\tanh\sqrt{\dfrac{\bar{h}_c P L^2{}_f}{kA}}}{\sqrt{\dfrac{\bar{h}_c P L^2{}_f}{kA}}}$$

For this fin we have

$$\bar{h}_c = 40{,}974 \ \text{W/(m}^2\text{K)}$$

$$P = 2L$$

$$L_f = 200 \times 10^{-6} \text{ m}$$

$$A = w_w L$$

First, calculate

$$\sqrt{\frac{\bar{h}_c P L^2{}_f}{kA}} = \sqrt{\frac{(40{,}974 \text{ W/(m}^2\text{K)})\, 2L\,(200\times 10^{-6} \text{ m})^2}{(238 \text{ W/(m K)})(50\times 10^{-6} \text{ m})\, L}} = 0.524$$

and then

$$\eta_f = \frac{\tanh(0.524)}{0.524} = 0.92$$

Our assumption that the flow channel all is isothermal is fairly good.

(d) We can now quantify the effective thermal resistance. First calculate the quantity

$$\bar{h}_c\left(\frac{w_c}{2 + b\eta_f}\right) = 40{,}974 \ \left(\text{W/(m}^2\text{ K)}\right) (25 \times 10^{-6} \text{ m} + (200 \times 10^{-6} \text{ m})\ (0.92)) = 8.56 \ \text{W/(m K)}$$

so

$$R_{\text{eff}} = \frac{1 + \dfrac{(8.56 \text{ W/(m K)})(2)(100\times 10^{-6} \text{ m})}{(238 \text{ W/(m K)})(100\times 10^{-6} \text{ m})}}{(8.56 \text{ W/(m K)})(0.01 \text{m})} = 12.5 \text{ K/W}$$

(e) The heat transfer for the half channel is therefore

$$q = \frac{(100 \text{ K})}{(12.5 \text{ K/W})} = 7.98 \text{ W}$$

and for the entire heat sink

$$Q_{\text{heatsink}} = 7.98 \times 2 \times 100 = 1597 \text{ W}$$

Chapter 9

PROBLEM 9.1

For an ideal radiator (hohlraum) with a 10 cm diameter opening, located in black surroundings at 16°C, (a) calculate the net radiant heat transfer rate for hohlraum temperatures of 100°C and 560°C, (b) the wavelength at which the emission is a maximum, (c) the monochromatic emission at λ_{max}, and (d) the wavelengths at which the monochromatic emission is 1 per cent of the maximum value.

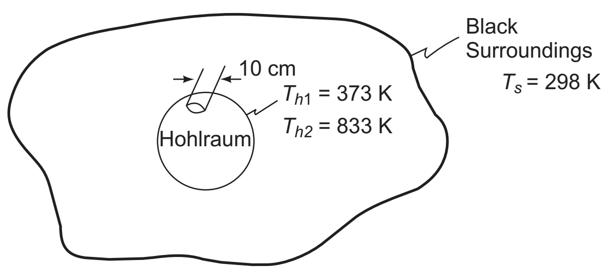
GIVEN

- An ideal radiator (hohlraum) in black surroundings
- Radiator opening diameter (D) = 10 cm = 0.1 m
- Surrounding temperature (T_s) = 16°C = 289 K
- Hohlraum temperatures
  - T_{h1} = 100°C = 373 K
  - T_{h2} = 560°C = 833 K

FIND

(a) The net radiant heat transfer rate (q_r)
(b) The wavelength at which the emission is maximum (λ_{max})
(c) The monochromatic emission at λ_{max} ($E_{\lambda max}$)
(d) The wavelengths at which the monochromatic emission is 1% $E\lambda_{max}$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

All parts of the problem will first be solved for T_h = 373 K

(a) The net radiative transfer between any two black surfaces is given by Equation (9.47)

$$q_{12} = A_1 F_{12} (E_{b1} - E_{b2})$$

where $A_1 = (\pi/4)D^2$

F_{12} = 1, since surface 1 is surrounded by surface 2.

From Equation (9.3) $E_{b7} = \sigma T_h^4$ and $E_{b2} = \sigma T_s^4$

$$q_{12} = \frac{\pi}{4} D^2 \sigma (T_h^4 - T_s^4) = \frac{\pi}{4} (0.1 \text{ m})^2 \left(5.67\times10^{-8} \text{ W/(m}^2 \text{K}^4)\right) [(373 \text{ K})^4 - (289 \text{ K})^4] = 5.51 \text{ W}$$

(b) The wavelength at which the maximum emission occurs for a black body is given by Equation (9.2)

$$\lambda_{max} T_h = 2.898 \times 10^{-3} \text{ m K}$$

$$\lambda_{max} = \frac{2.898\times10^{-3} \text{m K}}{T_h} = \frac{2.898\times10^{-3} \text{m K}}{373 \text{K}} = 7.77 \times 10^{-6} \text{ m} = 7.77 \text{ μ m}$$

(c) The monochromatic emission is given by Equation (9.1)

$$E_{b\lambda} = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)}$$

where $C_1 = 3.7415 \times 10^{-16}$ W m^2

$C_2 = 1.4388 \times 10^{-2}$ m K

$$E_{b\lambda\max} = \frac{3.7415 \times 10^{-16}\,\text{Wm}^2}{(7.77 \times 10^{-6}\,\text{m})^5 \left[\exp\left(\dfrac{1.4388 \times 10^{-2}\,\text{m K}}{(7.77 \times 10^{-6}\,\text{m})(373\,\text{K})}\right) - 1\right]} = 9.29 \times 10^7 \text{ W/m}^3$$

(d) 1% $E_{b\lambda\max} = E_{b\lambda} = 9.29 \times 10^5$ W/m^3

$$\frac{C_1}{\lambda^5 \left(e^{\frac{C_2}{\lambda T}} - 1\right)} = 9.29 \times 10^5 \text{ W/m}^3$$

$$\frac{3.7415 \times 10^{-16}\,\text{Wm}^2}{\lambda^5 \left[\exp\left(\dfrac{1.4388 \times 10^{-2}\,\text{m K}}{(\lambda)(373\,\text{K})}\right) - 1\right]} - 9.29 \times 10^5 \text{ W/m}^3 = 0$$

(There will be one solution below λ_{max} and one above λ_{max})

By trial and error $\lambda_1 = 2.55$ μ m

$\lambda_2 = 51.4$ μ m

Repeating the above proceduur for $T_h = 833$ K (a) 211 W (c) 5.15×10^9 W/m^2

(b) 3.48 μ m (d) $\lambda_1 = 1.14$ μ m

$\lambda_2 = 23.05$ μ m

PROBLEM 9.2

A tungsten filament is heated to 2700 K. At what wavelength is the maximum amount of radiation emitted? What fraction of the total energy is in the visible range (0.4 to 0.75 μ m)? Assume that the filament radiates as a gray body.

GIVEN

- Heated tungsten filament
- Filament temperature (T_f) = 2700 K

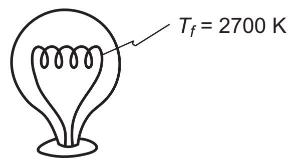
FIND

(a) Wavelength at which the maximum radiation is emitted (λ_{max})
(b) Percentage of radiation in the visible ranger (0.4 to 0.75 μ m)

ASSUMPTIONS

- The filament radiates as a gray body

SKETCH



SOLUTION

(a) Since the tungsten is a gray body, the maximum emission occurs at the same wavelength as a black body. From Equation (9.2)

$$\lambda_{max} = \frac{2.898 \times 10^{-3}\,\text{m K}}{T} = \frac{2.898 \times 10^{-3}\,\text{m K}}{2700\,\text{K}} = 1.07 \times 10^{-6} = 1.07\ \mu\text{ m}$$

(b) Gray body radiation is proportional to the black body radiation at all wavelengths. Therefore, the percentage of the gray body radiation in the visible spectrum is the same as the percentage of black body radiation in the visible spectrum.

For the limits of the visible range $\lambda_1\, T = (0.4 \times 10^{-6}\text{ m})\,(2700\text{ K}) = 1.08 \times 10^{-3}\text{ m K}$

$$\lambda_2\, T = (0.75 \times 10^{-6}\text{ m})\,(2700\text{ K}) = 2.025 \times 10^{-3}\text{ m K}$$

From Table 9.1

$$\frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} = 0.0011$$

$$\frac{E_b(0 \to \lambda_2 T)}{\sigma T^4} = 0.071$$

The percent within the visible spectrum is

$$\frac{E_b(\lambda_1 T \to \lambda_2 T)}{\sigma T^4} = 0.071 - 0.0011 = 0.07 = 7.0\%$$

PROBLEM 9.3

Determine the total average hemispherical emissivity and the emissive power of a surface that has a spectral hemispherical emissivity of 0.8 at wavelengths less than 1.5 μm. 0.6 from 1.5 to 2.5 μm, and 0.4 at wavelengths longer than 2.5 μm. The surface temperature is 7777 K.

GIVEN

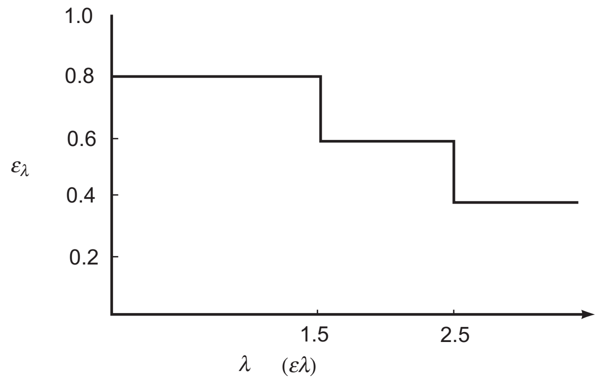
- A surface at temperature (T) = 1111 K
- Spectral hemispherical emittance (ε_λ) = 0.8 for $\lambda < 1.5$ μm (ε_1)
  = 0.6 for 1.5 μm < λ < 2.5 μ m (ε_2)
  = 0.4 for $\lambda > 2.5$ μ m (ε_3)

FIND

(a) The total average hemispherical emittance, $\varepsilon\,(T)$
(b) The emissive power, $E(T)$

SKETCH

The hemispherical emittance is shown graphically below



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) The total average hemispherical emittance is given by Equation (9.31)

$$\varepsilon\,(\mathrm{T}) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda)\,E_{b\lambda}(\lambda,T)\,d\,\lambda}{\int_0^\infty E_{b\lambda}(\lambda,T)\,d\,\lambda} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda)\,E_{b\lambda}(\lambda,T)\,d\,\lambda}{\sigma T^4}$$

For $\lambda_1 = 1.5$ μ m $\qquad \lambda_1\,T = (1.5 \times 10^{-6}\text{ m})\,(1111\text{ K}) = 1.67 \times 10^{-3}$ m K

For $\lambda_2 = 2.5$ μ m $\qquad \lambda_2\,T = (2.5 \times 10^{-6}\text{ m})\,(1111\text{ K}) = 2.78 \times 10^{-3}$ m K

Interpolating from Table 9.1

$$\frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} = 0.0266$$

$$\frac{E_b(0 \to \lambda_2 T)}{\sigma T^4} = 0.2234$$

$$\therefore \quad \frac{E_b(\lambda_1 T \to \lambda_2 T)}{\sigma T^4} = 0.2234 - 0.0266 = 0.1968$$

$$\therefore \quad \frac{E_b(\lambda_2 T \to \infty)}{\sigma T^4} = 1 - \frac{E_b(0 \to \lambda_2 T)}{\sigma T^4} = 1 - 0.2234 = 0.7766$$

Therefore

$$\varepsilon\,(T) = \varepsilon_1 \frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} + \varepsilon_2 \frac{E_b(\lambda_1 T \to \lambda_2 T)}{\sigma T^4} + \varepsilon_3 \frac{E_b(\lambda_2 T \to \infty)}{\sigma T^4}$$

$$\varepsilon\,(T) = 0.8\,(0.0266) + 0.6\,(0.1968) + 0.4\,(0.7766) = 0.45$$

(b) From Equation (9.31)

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{E(T)}{\sigma T^4}$$

$$E(T) = \varepsilon(T)\,\sigma T^4 = 0.45\,\left(5.67\times 10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\,(1111\ \text{K})^4 = 3.89\times 10^4\ \text{W/m}^2$$

PROBLEM 9.4

Shown that (a) $E_{b\lambda}/T^5 = f(\lambda T)$. Also, for λT = 5000 μm K, calculate $E_{b\lambda}/T^5$.

GIVEN

- λT = 5000 μ m K = 0.005 m K

FIND

Shown that

(a) $E_{b\lambda}/T^6 = f(\lambda T)$

(b) Calculate $E_{b\lambda}/T^5$ for the λT given

SOLUTION

(a) Starting with Equation (9.1)

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}$$

Where $C_1 = 3.7415\times 10^{-16}$ W m^2

$C_2 = 1.4388\times 10^{-2}$ m K

$$\frac{E_{b\lambda}}{T^5} = \frac{C_1}{(\lambda T)^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)} = f(\lambda T)$$

(b) At λT = 0.005 m K

$$\frac{E_{b\lambda}}{T^5} = \frac{3.7415\times 10^{-16}\,\text{Wm}^2}{(0.005\,\text{m K})^5\left[\exp\left(\frac{1.4388\times 10^{-2}\,\text{m K}}{(0.005\,\text{m K})}\right) - 1\right]} = 7.14\times 10^{-6}\ \text{W/(m}^3\ \text{K}^5)$$

PROBLEM 9.5

Compute the average emittance of anodized aluminum at 100°C and 650°C from the spectral curve in Fig. 9.16. Assume ε_λ = 0.8 for λ > 9 μm

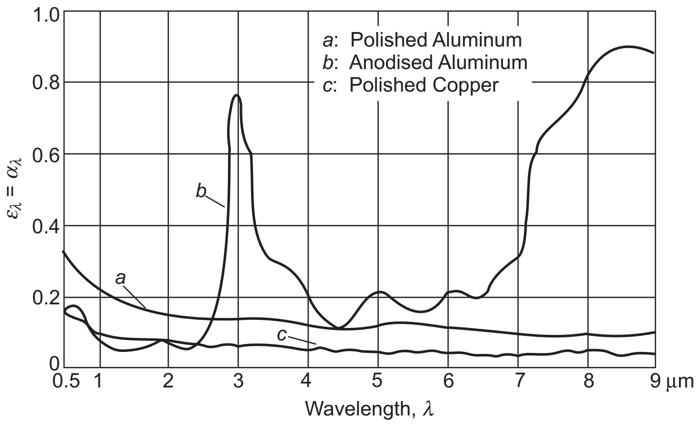
GIVEN

The spectral curve of Figure 9.16 for anodized aluminum

FIND

The average emittance (ε) at (a) 100°C = 373 K, and, (b) 650°C = 923 K

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 10^{-8} W/(m^2 K^4)

SOLUTION

The average emittance is given by Equation (9.31)

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{b\lambda}(\lambda,T)\,d\lambda}{\int_0^\infty E_{b\lambda}(\lambda,T)\,d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{b\lambda}(\lambda,T)\,d\lambda}{\sigma T^4} = \frac{\int_0^{\lambda_1} \varepsilon_\lambda(\lambda) E_{b\lambda}(\lambda,T)\,d\lambda}{\sigma T^4} + \varepsilon_\lambda(\lambda)\frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4}$$

The second term can be calculated from the following expression

$$\frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4} = 1 - \frac{E_b(0 \to \lambda_1 T)}{\sigma T^4}$$

At T = 923 K $\qquad \lambda_1 T = (9 \times 10^{-6}\text{ m})\,(923\text{ K}) = 8.31 \times 10^{-3}$ m K

From Table 9.1

$$\frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} = 0.8677$$

$$\therefore\ \varepsilon_\lambda(\lambda)\,\frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4} = 0.8\,(1 - 0.8677) = 0.1058$$

The first term for the average emittance can be approximated by divided Figure 9.16 into 12 segments.

$$\frac{\int_0^{\lambda_1} \varepsilon_\lambda(\lambda) E_{b\lambda}(\lambda,T)\,d\lambda}{\int_0^\infty E_{b\lambda}(\lambda,T)\,d\lambda} \approx \sum_{n=0}^{12} \varepsilon_{\lambda n}\, E_{b\lambda n}\, \Delta\lambda_n$$

where $\varepsilon_{\lambda n}$ = The average ε for the n^{th} segment

$$E_{b\lambda n} = \frac{C_1}{\lambda_n^5\left(e^{\frac{C_2}{\lambda a t}} - 1\right)}$$

$\lambda_n = \lambda$ at the center of segment n

$C_1 = 3.7415 \times 10^{-16}$ W/m^2

$C_2 = 1.4388 \times 10^{-16}$ m K

$\Delta\lambda_n$ = The width of segment n

The values of these quantities are tabulated below for T = 923 K (λ = wavelength at the end of the segment).

n	λ(μm)	λn(μm)	$\varepsilon_{\lambda n}$	$\varepsilon_{\lambda n}$ $E_{\lambda}n$ $\Delta\lambda_n$ (W/m²)
	0.5			
1	1	0.75	0.12	0.2362
2	2	1.5	0.06	90.673
3	2.5	2.25	0.07	222.73
4	3	2.75	0.41	1689.9
5	3.3	3.15	0.51	1318.6
6	4	3.65	0.27	1546.6
7	5	4.5	0.17	1113.8
8	6	5.5	0.18	835.44
9	7	6.5	0.22	709.18
10	8	7.5	0.58	1307.8
11	8.5	8.25	0.86	749.58
12	9	8.75	0.88	649.88
			Sum	10234.4 W/m²

Therefore, for T = 923 K

$$\varepsilon = \frac{10234\,\text{W/m}^2}{\sigma T^4} + 0.1058 = \frac{10234\,\text{W/m}^2}{\left(5.67\times10^{-8}\,\text{W/(m}^2\,\text{K}^4)\right)(923\,\text{K})^4} + 0.1058 = 0.35$$

Repeating this procedure for T = 100°C = 373 K yields, ε = 0.677

PROBLEM 9.6

A large body of nonluminous gas at a temperature of 1100°C has emission bands between 2.5 and 3.5 μm and between 5 and 8 μm. At 1100°C, the effective emittance in the first band is 0.8 and in the second 0.6. Determine the emissive power of this gas in W/m².

GIVEN

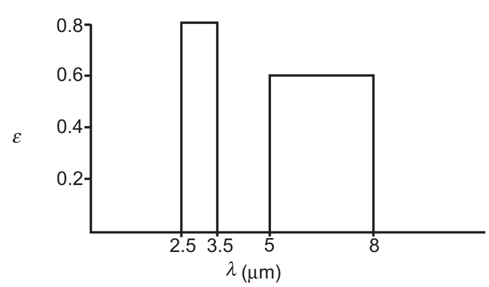
- A large body of nonluminous gas
- Gas temperature (T) = 1100°C = 1373 K
- Emission bands: 2.5 μ m < λ_1 < 3.5 μ m   5 μ m < λ_2 < 8 μ m
- Effective emittances: ε_1 = 0.8; ε_2 = 0.6

FIND

- The emissive power (E) in W/m²

SKETCH

The effective emittance can be represented graphically as shown below



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The total emittance is given by Equation (9.31)

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda)\, E_{b\lambda}(\lambda,T)\, d\lambda}{\int_0^\infty E_{b\lambda}(\lambda,T)\, d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda\, E_{b\lambda}\, d\lambda}{\sigma T^4} = \varepsilon_1\ \frac{E_b(\lambda_{11}T \to \lambda_{12}T)}{\sigma T^4} * + \varepsilon_2\ \frac{E_b(\lambda_{21}T \to \lambda_{22}T)}{\sigma T^4}$$

where $\lambda_{11}\ T = (2.5 \times 10^{-6}\ \text{m})\ (1373\ \text{K}) = 3.4 \times 10^{-3}\ \text{m K}$

$\lambda_{12}\ T = (3.5 \times 10^{-6}\ \text{m})\ (1373\ \text{K}) = 4.8 \times 10^{-3}\ \text{m K}$

$\lambda_{21}\ T = (5 \times 10^{-6}\ \text{m})\ (1373\ \text{K}) = 6.9 \times 10^{-3}\ \text{m K}$

$\lambda_{22}\ T = (8 \times 10^{-6}\ \text{m})\ (1373\ \text{K}) = 11.0 \times 10^{-3}\ \text{m K}$

From Table 9.1

$$\frac{E_b(0 \to \lambda_{11}T)}{\sigma T^4} = 0.3618$$

$$\frac{E_b(0 \to \lambda_{12}T)}{\sigma T^4} = 0.6076$$

$$\frac{E_b(0 \to \lambda_{21}T)}{\sigma T^4} = 0.8022$$

$$\frac{E_b(0 \to \lambda_{22}T)}{\sigma T^4} = 0.9320$$

$$\frac{E_b(\lambda_{11}T \to \lambda_{12}T)}{\sigma T^4} = 0.6076 - 0.3618 = 0.2458$$

$$\frac{E_b(\lambda_{21}T \to \lambda_{22}T)}{\sigma T^4} = 0.9320 - 0.8022 = 0.1298$$

$$\varepsilon = 0.8(0.2458) + 0.6(0.1298) = 0.2745$$

$$\therefore E = \varepsilon E_b = \varepsilon\, \sigma\, T^4 = 0.2745\left(5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)\right)(1373)^4 = 5.53 \times 10^4\ \text{W/m}^2$$

PROBLEM 9.7

A flat plate is in a solar orbit 150,000,000 km from the sun. It is always oriented normal to the rays of the sun and both sides of the plate have a finish which has a spectral absorptance of 0.95 at wavelengths shorter than 3 μ m and a spectral absorptance of 0.06 at wavelengths longer than 3 μm. Assuming that the sun is a 5550 K blackbody source with a diameter of 1,400,000 km, determine the equilibrium temperature of the plate.

GIVEN

- A flat plate in solar orbit oriented normal to the rays of the sun
- Distance from the sun (R) = 150,000,000 km = 1.5×10^{11} m
- Spectral absorplatace (ε_λ) of both sides $\varepsilon_{\lambda 1} = 0.95$ for $\lambda_1 < 3$ μ m
  $\varepsilon_{\lambda 2} = 0.06$ for $\lambda_1 > 3$ μ m

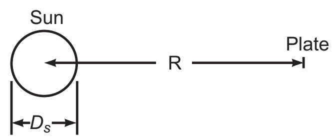
FIND

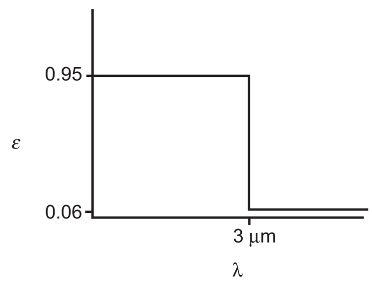
- The equilibrium temperature of the plate (T_p)

ASSUMPTIONS

- The sun is a black body at T_s = 5550 K
- Sun diameter (D_s) = 1,400,000 km = 1.4×10^9 m

SKETCH





SOLUTION

From Equation (9.31)

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda \, E_{b\lambda} \, d\lambda \int_0}{\int_0^\infty E_{b\lambda} \, d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) \, E_{b\lambda}(\lambda, T) \, d\lambda}{\sigma T^4}$$

$$\varepsilon = \varepsilon_{\lambda 1} \frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} + \varepsilon_{\lambda 2} \frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4}$$

For the sun λ_1 $T = (3 \times 10^{-6}$ m) (5550 K) = 16.7×10^{-3} m K

From Table 9.1

$$\frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} = 0.9764$$

$$\frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4} = 1 - 0.9764 = 0.0236$$

The absorptance of the plate is given by Equation (9.33)

$$\alpha_p = \frac{\int_0^\infty \sigma_\lambda \, G_\lambda \, d\lambda}{\int_0^\infty G_b \, d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda \, E_{b\lambda} \, d\lambda}{\sigma T^4} \quad (\text{Since } \alpha_\lambda = \varepsilon_\lambda)$$

$$\alpha_p = \varepsilon_{\lambda 1} \frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} + \varepsilon_{\lambda 2} \frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4} = 0.95\,(0.9764) + (0.06)\,(0.0236) = 0.9290$$

Let q_p = the flux incident on the plate

Energy leaving the sun surface = energy crossing sphere of radius R

$$\sigma T_s^4 A_S = q_p A_R$$

$$\sigma T_s^4 \pi D_s^2 = q_p \, 4 \pi R^2$$

$$q_p - \sigma T_s^4 \frac{D_s^2}{4 R^2}$$

Performing the energy balance on the plate

Energy absorbed from sun = energy emitted form both sides of the plate

$$\alpha q_P = 2\,\sigma T_P^4\,\varepsilon_P$$

$$\alpha\,\sigma T_s^4 \frac{D_s^2}{4R^2} = 2\,\sigma T_P^4 \left(\varepsilon_{\lambda 1} \frac{E_b(0 \to \lambda_1 T_p)}{\sigma T_P^4} + \varepsilon_{\lambda 2} \frac{E_b(\lambda_1 T_P \to \infty)}{\sigma T_P^4} \right)$$

$$T_P = \left[\frac{\sigma T_s^4 D_s^2}{8R\left(\varepsilon_{\lambda 1} \frac{E_b(0 \to \lambda_1 T_p)}{\sigma T_P^4} + \varepsilon_{\lambda 2} \frac{E_b(\lambda_1 T_P \to \infty)}{\sigma T_P^4} \right)} \right]^{\frac{1}{4}}$$

$$\frac{\alpha T_S^4}{8} = \frac{D_s^2}{R^2}\;\frac{0.9290(5550\,\text{K})^4}{8}\left(\frac{1.4\times 10^9}{1.5\times 10^{11}}\right)^2 = 9.60\times 10^9\ \text{K}^4$$

$$\therefore\ T_P = \left[\frac{9.60\times 10^9\,\text{K}^4}{(0.95)\left(\frac{E_b(0 \to \lambda_1 T_p)}{\sigma T_P^4} + (0.06)\frac{E_b(\lambda_1 T_P \to \infty)}{\sigma T_P^4} \right)} \right]^{\frac{1}{4}}$$

This can be solved iteratively. For a first guess, let T_P = 500 K.

$$\lambda_1 T_p = (3\times 10^{-6}\ \text{m})\,(500\ \text{K}) = 1.5\times 10^{-3}\ \text{m K}$$

$$\frac{E_b(0 \to \lambda_1 T_P)}{\sigma T_P^4} = 0.01376$$

$$\frac{E_b(\lambda_1 T_P \to \infty)}{\sigma T_P^4} = 1 - 0.01376 = 0.98624$$

$$T_p = \left[\frac{9.60\times 10^9\,K^4}{0.95(0.01376) + 0.06(0.98624)} \right]^{\frac{1}{4}} = 604\ \text{K}$$

Repeating this procedure

T_p (K)	$\lambda\,T_p$ (m K)	$\frac{E_b(0 \to \lambda_1 T_P)}{\sigma T_P^4}$	Tp (K)
604	0.00181	0.0407	562
562	0.00169	0.0286	579
579	0.00173	0.0325	573
573	0.00172	0.0315	575

$$T_p = 575\ \text{K} = 302°\text{C}$$

COMMENTS

Note that the total absorptance (α) = 0.929 but the total emittance (ε) = 0.088.

PROBLEM 9.8

By substituting Equation 9.1 for $E_{b\lambda}$ (T) in Equation 9.4 and performing the integration over the entire spectrum, derive a relationship between σ and the constants C_1 and C_2 in Equation 9.1.

GIVEN

- Equation 9.1

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}$$

where $C_1 = 3.7415 \times 10^{-16}$ W m^2 $C_2 = 1.4388 \times 10^{-2}$ m K

- Equation 9.4

$$\int_0^\infty E_{b\lambda}\, d\lambda = \sigma T^4 = E_b$$

FIND

- A relationship between σ and C_1 and C_2

SOLUTION

$$\int_0^\infty E_{b\lambda}\, d\lambda = \int_0^\infty \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}\, d\lambda = \sigma T^4$$

This can be solved using the transformation of variables.

$$\text{Let } \zeta = \frac{C_2}{\lambda T} \quad \text{and} \quad d\lambda = \left(-\frac{T\lambda^2}{C_2}\right) d\left(\frac{C_2}{\lambda T}\right) = \left(\frac{T\lambda}{C_2}\right)^2 \left(-\frac{C_2}{\lambda}\right) d\left(\frac{C_2}{\lambda T}\right)$$

$$\sigma T^4 = \int_0^\infty \frac{C_1}{\lambda^5\left(\frac{e_2}{\lambda T} - 1\right)}\, d\lambda = \int_0^\infty \left(\frac{C_2}{\lambda T}\right)^5 \left(\frac{T}{C_2}\right)^5 \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)} \left(\frac{T\lambda}{C_2}\right)^2 \left(-\frac{C_2}{\lambda}\right) \left(\frac{C_2}{\lambda T}\right)$$

$$\sigma T^4 = -\frac{C_1}{C_2^4}\, T^4 \int_0^\infty \frac{\zeta^3}{e^\zeta - 1}\, d\zeta$$

From a table of integrals

$$\int_0^\infty \frac{\zeta^3}{e^\zeta - 1}\, d\zeta = -\frac{\pi^4}{15}$$

$$\therefore \sigma T^4 = \frac{C_1 \pi^4}{15 C_2^4}\, T^4$$

$$\sigma = \frac{C_1 \pi^4}{15 C_2^4} = \frac{(3.7415 \times 10^{-16}\ \text{Wm}^2)\pi^4}{15(1.4388 \times 10^{-2}\ \text{mK})^4} = 5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)$$

PROBLEM 9.9

Determine the ratio of the total hemispherical emissivity to the normal emissivity for a nondiffuse surface if the intensity of emission varies as the cosine of the angle measured from the normal.

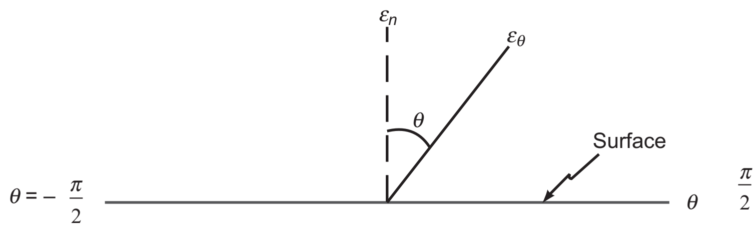
GIVEN

- A nondiffuse surface
- Intensity of emission varies as the cosine of the angle measured from normal

FIND

- The ratio of the total hemispherical emissivity (ε) to the normal emissivity (ε_n)

SKETCH



SOLUTION

From Equation (9.35), the intensity of emission varies with cosθ, then the emissivity must be proportional to cosθ

$$\varepsilon_\theta = \varepsilon_n \cos\theta$$

The average hemispherical value is

$$\varepsilon = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \varepsilon_n \cos\theta \, d\theta = \frac{\varepsilon_n}{\pi}\sin\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{2}{\pi}\,\varepsilon_n \sin\frac{\pi}{2} = \frac{2}{\pi}\,\varepsilon_n$$

The ratio $\varepsilon/\varepsilon_n$ is given by

$$\frac{\varepsilon}{\varepsilon_n} = \frac{2}{\pi} = 0.637$$

PROBLEM 9.10

Derive an expression for the geometric shape factor $F_{1\text{–}2}$ for a rectangular surface A_1, 1 by 20 m placed parallel to and centered 5 m above a 20-m-square surface A_2.

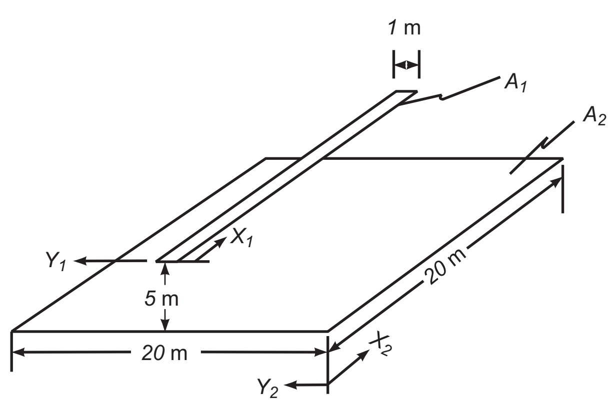
GIVEN

- Rectangular surface A_1 and square surface A_2
- A_1 is parallel to and centered 5 m above A_2
- Dimensions of A_1 = 1 m × 20 m
- A_2 is 20 m square

FIND

- The shape factor F_{12}

SKETCH



SOLUTION

From Equation (9.53)

$$A_1 F_{12} = \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, dA_2\, dA_1$$

Given a differential element dA_1, at $x,y,z = (x_1, y_1, -5\text{ m})$

Given a differential element dA_2, at $x,y,z = (x_2, y_2\ 0)$

The distance between elements (r) is: $r^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (5\text{ m} - 0)^2$

Since the surfaces are parallel: $\cos\theta_1 = \cos\theta_2 = (5\text{ m})/r$

The double integral can be expanded into the following quadruple integral:

$$A_1 F_{12} = \int_{x_1=0}^{20\text{m}}\int_{y_1=0}^{1\text{m}}\int_{x_2=0}^{20\text{m}}\int_{y_2=0}^{20\text{m}} \frac{\left(\frac{5\text{m}}{r}\right)\left(\frac{5\text{m}}{r}\right)}{\pi r^2}\, dy_2\, dx_2\, dy_1\, dx_1$$

$$A_1 F_{12} = \frac{25}{\pi}\int_{x_1=0}^{20\text{m}}\int_{y_1=0}^{1\text{m}}\int_{x_2=0}^{20\text{m}}\int_{y_2=0}^{20\text{m}} \frac{dy_2\ dx_2\ dy_1\ dx_1}{[(x_1-x_2)^2+(y_1-y_2)^2+25\text{m}^2]^2}$$

This can be simplified somewhat by trigonometric substitutions, however, it is fairly simple to solve numerically as it is. Let all the dx terms equal Δx, where $20/\Delta x$ and $1/\Delta x$ are both integers. The integral can then be approximated by

$$\frac{25\,\Delta x^4}{\pi}\sum_{ix_1=1}^{20/\Delta x}\ \sum_{iy_1=1}^{1/\Delta x}\ \sum_{ix_2=1}^{20/\Delta x}\ \sum_{iy_2=1}^{20/\Delta x} [(x_1-x_2)^2+(y_1-y_2)^2+25\text{ m}^2]^{-2}$$

where $x_1 = (ix_1)(\Delta x) - \Delta x/2 \qquad x_2 = (ix_2)(\Delta x) - \Delta x/2$

$y_1 = (iy_1)(\Delta x) - \Delta x/2 \qquad y_2 = (iy_2)(\Delta x) - \Delta x/2$

This is implemented in the Pascal program shown below

```
var     dx,dx2,sum,r4,.real;
        ix1,ix2,iy1,iy2,nx1,nx2,ny1,ny2.integer;
        x1,x2,y1,y2:real;
begin
        dx = 1.00;
        dx2 = dx/2;
        nx1 = trunc(20/dx);
        nx2 = trunc(20/dx);
        ny1 = trunc(1/dx);
        ny2 = trunc(20/dx);
        sum = 0.00;
for ix1 = 1 to nx1 do
        begin
          x1 = ix1*dx-dx2;
          writeln(x 1:8.3);
          for iy1 = 1 to ny 1 do
          begin
            y1 = iy1*dx-dx2;
            for ix2 = 1 to nx2 do
            begin
              x2 = ix2*dx-dx2;
              for iy2 = 1 to ny2 do
              begin
                y2 = iy2*dx-dx2;
                r4 = (x1-x2)*(x1-x2) + (y1 - y2)* (y1 - y2) + 25;
```

```
                r4 = 1/r4/r4;
                sum = sum + r4;
            end;
          end;
        end;
      end;
      sum = sum* 25/3.14159265/20*dx*dx*dx*dx;
  writen(*F12 = Sum:8.4);
  end.
```

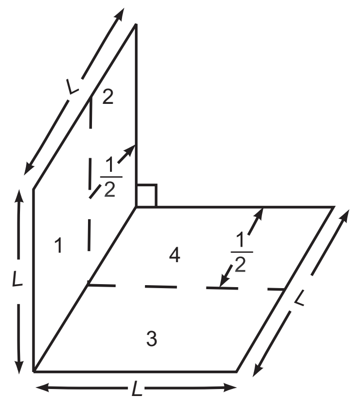
Running this program yields the following result

$$F_{12} = 0.427$$

Comment: If the geometry is approximated as two-dimensional so that the view factor can be calculated using the crossed string method, we get $F_{12} = 0.97$, a significant error.

PROBLEM 9.11

Determine the shape factor $F_{1–4}$ for the geometrical configuration shown below.



GIVEN

- Geometrical configurations shown above
- The shape factor $F_{1–4}$

SOLUTION

Let $A_5 = A_1 + A_2$ and $A_6 = A_3 + A_4$

Applying Equation (9.55)

$$A_{12}\, F_{12–34} = A_{12}\, F_{12–3} + A_{12}\, F_{12–4} = A_3\, F_{3–12} + A_4\, F_{4–12}$$

$$A_3\, F_{3–12} = A_3\, F_{3–1} + A_3\, F_{3–2}$$

$$A_4\, F_{4–12} = A_4\, F_{4–1} + A_4\, F_{4–2}$$

Combining these equations $A_{12}\, F_{12–34} = A_3\, F_{3–1} + A_3\, F_{3–2} + A_4\, F_{4–1} + A_4\, F_{4–2}$

By symmetry $F_{1–4} = F_{2–3} = F_{4–1} = F_{3–2}$ and $F_{3–1} = F_{1–3} = F_{2–4} = F_{4–2}$

Also $A_1 = A_2 = A_3 = A_4$ and $A_{12} = 2A_1$

Therefore $A_{12}\, F_{12–34} = 2A_1\,(F_{1–3} + F_{1–4})$

Solving for $F_{1–4}$ $F_{1–4} = F_{12–34} - F_{1–3}$

The shape factors $F_{1–3}$ and $F_{12–34}$ can be determined from Figure 9.27.

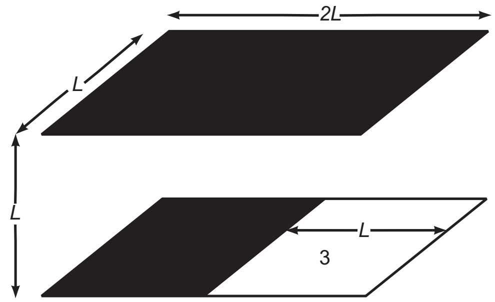
For $F_{1–3}$ $Y = Z = \dfrac{L}{\left(\dfrac{L}{2}\right)} = 2 \quad \rightarrow \quad F_{1–3} = 0.15$

For $F_{12–34}$ $Y = Z = \dfrac{L}{L} = 1 \quad \rightarrow \quad F_{12–34} = 0.20$

$F_{1-4} = 0.20 - 0.15 = 0.05$

PROBLEM 9.12

Determine this shape factor F_{1-2} for the geometrical configuration shown



GIVEN

- The geometrical configuration shown above

FIND

- The shape factor F_{1-2}

SOLUTION

From Equation (9.55) $F_{1-23} = F_{1-2} + F_{1-3}$

By symmetry $F_{1-2} = F_{1-3}$

Therefore, $F_{1-23} = 2\,F_{1-2} \rightarrow \quad F_{1-2} = 0.5\,F_{1-23}$

The shape factor (F_{1-23} is given in Figure 9.28

$$\frac{y}{D} = \frac{2L}{L} = 2 \text{ and } \frac{x}{D} = \frac{L}{L} = 1$$

From Figure 9.28 $F_{1-23} = 0.30$

$$F_{1-2} = 0.5(0.30) = 0.15$$

PROBLEM 9.13

Using basic shape-factor definitions, estimate the equilibrium temperature of the planet Mars which has a diameter of 6600 kms and revolves around the sun at a distance of 225×10^6 kms. The diameter of the sun is 1.384×10^6 kms. Assume that both the planet Mars and the sun act as blackbodies with the sun having an equivalent blackbody temperature of 5600 K. Then repeat your calculations assuming that the albedo of Mars (the fraction of the incoming radiation returned to space) is 0.15.

GIVEN

- The planet Mars revolving around the sun
- Diameter of Mars (D_m) = 6600 kms
- Distance from the sun (R_{ms}) = 2.25×10^8 kms
- Diameter of the sun (D_s) = 1.384×10^6 kms

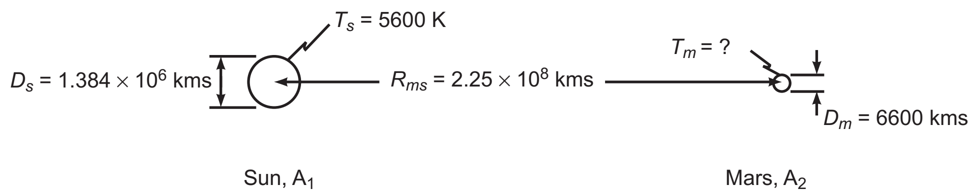
FIND

(a) The equilibrium temperature of Mars (T_m)
(b) T_m assuming that the albedo of Mars = 0.15

ASSUMPTIONS

- Both Mars and the sun act as blackbodies
- The sun has an equivalent blackbody temperature (T_s) of 5600 K

SKETCH



PROPERTIES AND CONSTANTS

The Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The energy from the sun that is incident on Mars can be calculated by integrating Equation (9.50)

$$q_{1-2} = E_{b1} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, dA_2\, dA_1$$

where θ_1 and θ_2 are the angles shown in Figure 9.24. For this case, since $D_m << R_{ms}$ and $D_s << R_{ms}$, the following approximations can be used cos θ_1 and cos θ_2 = 1, $r = R_{ms}$.

$$dA_1 = \frac{\pi}{4} D_s^2 \quad dA_2 = \frac{\pi}{4} Dm^2$$

Also from Equation (9.3) $E_{b1} = \sigma T_s^4$

$$\therefore\ q_{1-2} = \sigma T_s^4 \left(\frac{D_s D_m}{R_{ms}}\right)^2 \frac{\pi}{16}$$

$$q_{1-2} = 5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)\ (5600\ \text{K})^4 \left[\frac{(1.384\times10^9\ \text{m})(6.6\times10^6\ \text{m})}{2.25\times10^{11}\ \text{m}}\right]^2 \frac{\pi}{16}$$

$$q_{1-2} = 1.8 \times 10^{16}\ \text{W}$$

Case (a)

If Mars behaves as a blackbody, it will absorb all the sun's energy incident on it. For equilibrium, the energy radiated by Mars must equal the incident solar energy

$$q_m = A_2\, E_{b2} = \pi D_m^2\ \sigma T_m^4 = q_{1-2}$$

Solving for the temperature of Mars

$$T_m = \left(\frac{q_{1-2}}{\pi D_m^2\, \sigma}\right)^{\frac{1}{4}} = \left[\frac{1.8\times10^{16}\ \text{W}}{\pi(6.6\times10^6\ \text{m})^2\ (5.67\times10^{-8}\ \text{W/(m}^2\text{K))}}\right]^{\frac{1}{4}} = 219\ \text{K}$$

$$T_m = -\ 54°\text{C}$$

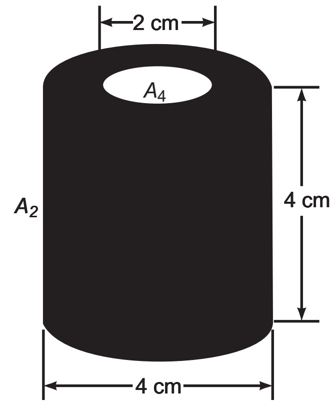
Case (b)

For an albedo of 0.15, Mars absorbs only 85% of the incident solar radiation, therefore

$$T_m = \left(\frac{0.85\, q_{1-2}}{\pi D_m^2 \sigma}\right)^{0.25} = 219\,(0.85)^{\frac{1}{4}} = 210.3 \text{ K} = -62.7°\text{C}$$

PROBLEM 9.14

A 4-cm-diameter cylindrical enclosure of black surfaces, as shown in the accompanying sketch, has a 2-cm hole in the top cover. Assuming the walls of the enclosure are at the same temperature, determine the percentage of the total radiation emitted from the walls which will escape through the hole in the cover.



GIVEN

- Cylindrical enclosure of black surfaces shown above
- Cylinder diameter (D) = 4 cm = 0.04 m
- Diameter of hole in top (D_h) = 2 cm = 0.02 m

FIND

- The percentage of the total radiation emitted from the wall which will escape through the hole in the cover (F_{e4}).

ASSUMPTIONS

- The walls of the enclosure are at the same temperature

SOLUTION

The total area of the interior of the enclosure (A_e) is

$$A_e = A_1 + A_2 + A_3 = \frac{\pi}{4} D^2 + \pi D\, L + \frac{\pi}{4}\,(D^2 - D_h^2)$$

$$A_e = \pi\left[\left(\frac{0.04\,\text{m}}{4}\right)^2 + (0.04\,\text{m})\,(0.04\text{m}) + \frac{1}{4}[(0.04\,\text{m})^2 - (0.02\,\text{m})^2]\right] = 0.0023\ \pi\text{m}^2$$

The shape factor between A_4 and the enclosure is unity $F_{4e} = 1$. From Equation (9.46)

$$A_e\, F_{e4} = A_4\, F_{4\text{e}} = A_4$$

$$\therefore\ F_{e4} = \frac{A_4}{A_e} = \frac{\pi D_h^2}{4\, A_e} = \frac{\pi (0.02\,\text{m})^2}{4(0.0023\pi \text{m}^2)} = 0.044$$

PROBLEM 9.15

Show that the temperature of the re-radiating surface T_r in Figure 9.37 is

$$\boldsymbol{T_R} = \left(\frac{A_1 F_{1R} T_1^4 + A_2 F_{2R} T_2^4}{AF_{1R} + A_2 F_{2R}}\right)^{\frac{1}{4}}$$

GIVEN

- Figure 9.37 shown below

FIND

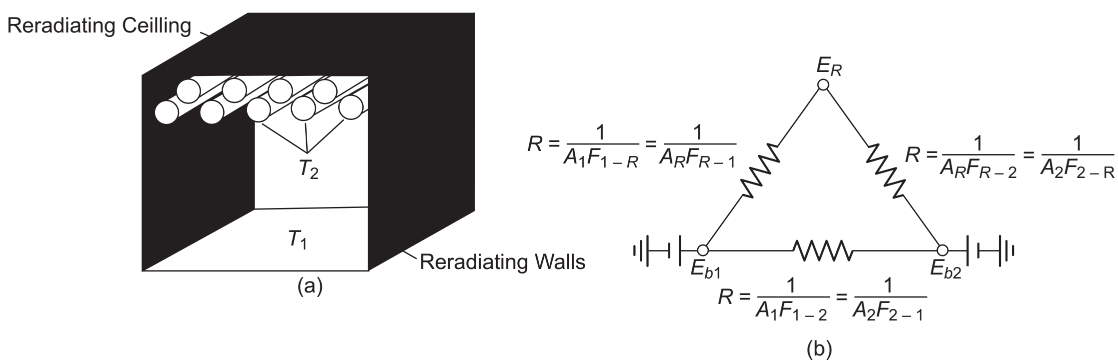
- Show that the temperature of the re-radiating surface T_R, is

$$T_R = \left(\frac{A_1 F_{1R} T_1^4 + A_2 F_{2R} T_2^4}{AF_{1R} + A_2 F_{2R}}\right)^{\frac{1}{4}}$$

ASSUMPTIONS

- Steady state
- The re-radiating surface temperature is uniform

SKETCH



SOLUTION

The following equation can be written from the thermal circuit

$$q_{R-2} = A_2\, F_{2R}\,(E_R - E_{b2}) = \sigma A_2\, F_{2R}\,(T_R^4 - T_2^4)$$
$$q_{r-1} = A_1\, F_{1R}\,(E_R - E_{b1}) = \sigma A_1\, F_{1R}\,(T_R^4 - T_1^4)$$

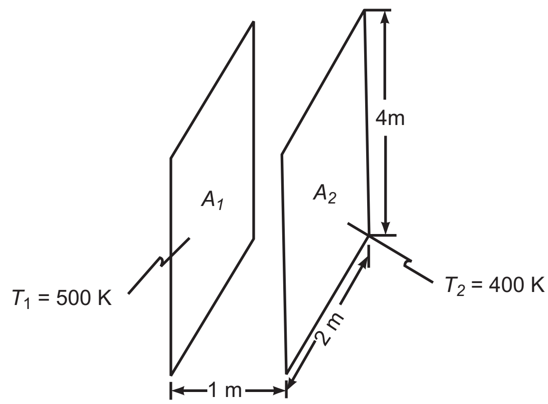
For steady state, heat flows must sum to zero

$$q_{R-2} + q_{R-1} = 0$$
$$\sigma(A_2\, F_{2R}\, T_R^4 - A_2\, F_2^R\, T_2^4 + A_1\, F_{1R}\, T_R^4 - A_1\, F_{1R}\, T_1^4) = 0$$
$$T_R^4\,(A_1\, F_{1R} + A_2\, F_{2R}) = A_1\, F_{1R}\, T_1^4 + A_2\, F_{2R}\, T_2^4$$

$$T_R = \left(\frac{A_1 F_{1R} T_1^4 + A_2 F_{2R} T_2^4}{AF_{1R} + A_2 F_{2R}}\right)^{\frac{1}{4}}$$

PROBLEM 9.16

In the construction of a space platform, two equally sized structural members with surfaces that may be considered black are placed relative to each other as shown schematically below. Assuming that the left member attached to the platform is at 500 K while the other is at 400 K and that the surroundings may be treated as though black at 0 K, calculate (a) the rate at which the warmer surface must be heated to maintain its temperature, (b) the rate of heat loss from the cooler surface to the surroundings, (c) the net rate of heat loss to the surroundings for both members.



GIVEN

- Two black surfaces as shown above on a space platform

FIND

(a) The rate of heating of the warmer surface (q_1)
(b) Net rate of heat loss to the surroundings (q_s)
(c) The rate of heat loss from the cooler surface to the surroundings (q_{2s})

ASSUMPTIONS

- Steady state
- The surroundings behave as a blackbody enclosure at T_s = 0 K
- The plate also lose heat from their back surface

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The shape factor F can be read off Figure 9.29 line 3

$$\text{Ratio} = (2\text{ m})/(1\text{ m}) = 2 \;\rightarrow\; F_{12} \approx 0.51$$

By symmetry, $F_{21} = F_{12}$.

Since neither A1 nor A2 can see itself $F_{11} = F_{22} = 0$.

The shape factors for any given surface must sum to unity

$$F_{11} + F_{12} + F_{1s} = 1 \quad \rightarrow \quad F_{1s} = 1 - F_{12} = 0.49$$

$$F_{21} + F_{12} + F_{2s} = 1 \quad \rightarrow \quad F_{2s} = 1 - F_{21} = 1 - F_{12} = 0.49$$

(a) The rate of heating of A_1 must equal the net rate of heat transfer from A_1 which is the sum of the net heat transfer rate to A_2 and the heat transfer to the surroundings

$$q_1 = q_{12} + q_{1s} = \sigma A_1 \, [F_{12} \, (T_1^4 - T_2^4) + F_{1s} \, T_1^4 + T_1^4] = \sigma A_1 \, [(F_{12} + F_{1s} + 1) \, T_1^4 - F_{12} \, T_2^4]$$

$$q_1 = \sigma A_1 \, (2T_1^4 - F_{12} \, T_2^4) = \left(5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4)\right) (2 \text{ m}) \, (4 \text{ m})$$

$$[2(500 \text{ K})^4 - 0.51(400 \text{ K})^4] = 5.08 \times 10^4 \text{ W}$$

(b) The rate of heat loss of A_2 to its surroundings is given by Equation (9.47)

$$q_{2s} = A_2 \, F_{2s} \, (E_{b2} - E_{bs}) + A_2 \, E_{b2} = A_2 \, (F_{2s} + 1) \, \sigma T_2^4$$

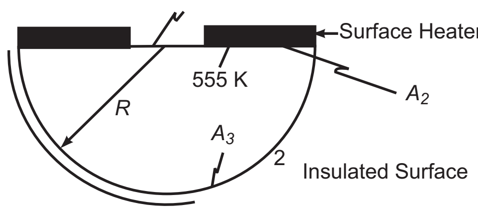
$$q_{2s} = (2 \text{ m}) \, (4 \text{ m}) \, (1.49)\left(5.67 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4)\right) (400 \text{ K})^4 = 17300 \text{ W}$$

(c) The net rate of heat loss to the surroundings equals the total heat loss from both members. Less q_{12}

$$q_{\text{net}} = 50800 + 17300 - \sigma A_1 \, F_{12} \, (T_1^4 - T_2^4) = 59543 \text{ W}$$

PROBLEM 9.17

A radiation source is to be built, as shown in the diagram, for an experimental study of radiation. The base of the hemisphere is to be covered by a circular plate having a centered hole of radius *R*/2. The underside of the plate is to be held at 555 K by heaters embedded in its surface. The heater surface is back. The hemispherical surface is well-insulated on the outside. Assume gray diffuse processes and uniform distribution of radiation. (a) Find the ratio of the radiant intensity at the opening to the intensity of emission at the surface of the heated plate. (b) Find the radiant energy loss through the opening in watts for *R* = 0.3 m. (c) Find the temperature of the hemispherical surface.



GIVEN

- A radiation source as shown above
- Radius of hole = $R/2$
- Temperature of underside of plate (T_2) = 555 K
- Underside of plate is black
- Hemispherical surface is well insulated on the outside

FIND

(a) The ratio of the radiant intensity at the opening to the intensity at the surface of the heated plate (G_1/ E_{b2})
(b) The radiant energy loss through the opening (q_1) in watts for R = 0.3 m
(c) The temperature of the hemispherical surface (T_3)

ASSUMPTIONS

- Gray diffuse processes
- Uniform distribution of radiation
- Radiation entering A_1 is negligible, i.e., A_1 as a black body at 0 K
  Heat loss through insulation is negligible

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The problem consists of radiative exchang between two black surfaces and a gray surface. It can be solved by simplifying Equation (9.69) which applies to gray surfaces

$$A_1\, G_1 = J_1\, A_1\, F_{11} + J_2\, A_2\, F_{21} + J_3\, A_3\, F_{31}$$

$$A_2\, G_2 = J_1\, A_1\, F_{12} + J_2\, A_2\, F_{22} + J_3\, A_3\, F_{32}$$

$$A_3\, G_3 = J_1\, A_1\, F_{13} + J_2\, A_2\, F_{23} + J_3\, A_3\, F_{33}$$

The radiosities, from Equation (9.66) are

$$J_1 = \rho_1\, G_1 + \varepsilon_1\, E_{b1}$$

$$J_2 = \rho_2\, G_2 + \varepsilon_2\, E_{b2}$$

$$J_3 = \rho_3\, G_3 + e_3\, E_{b3}$$

Let the opening the surface 1, the heater surface be surface 2, and the hemisphere be surface 3.

Since A_1 and A_2 cannot see themselves or each other: $F_{11} = F_{22} = F_{12} = F_{21} = 0$

Since A_1 and A_2 are black $\varepsilon_1 = \varepsilon_2 = 1$ and $\rho_1 = \rho_2 = 0$

Neglecting radiation entering A_1 $E_{b1} = 0 \rightarrow J_1 = 0$

In steady state, surface A_3 has no net heat gain or loss $q_3 = 0$. Applying Equation (9.67)

$$0 = A_3\,(J_3 - G_3) \rightarrow J_3 = G_3$$

Incorporating these simplifications into the above equations

(1) $A_1\, G_1 = G_3\, 0 A_3\, F_{31}$

(2) $A_2\, G_2 = G_3\, A_3\, F_{32}$

(3) $A_3\, G_3 = E_{b2}\, A_2\, F_{23} + G_3\, A_3\, F_{33} \quad \rightarrow \quad A_3\, G_3 = (E_{b2}\, A_2\, F_{23})/(1 - F_{33})$

(a) Combining Equations (1) and (3)

$$\frac{G_1}{E_{b2}} = \frac{A_2}{A_1}\,\frac{F_{23}\,F_{31}}{1 - F_{33}}$$

The shape factors must sum to unity: $F_{31} + F_{32} + F_{33} = 1$

$$\therefore\ \frac{G_1}{E_{b2}} = \frac{A_2}{A_1}\,\frac{F_{23}\,F_{31}}{F_{31} + F_{32}}$$

From examination of the geometry, it is clear that $F_{13} = 1$ and $F_{23} = 1$

And from Equation 9.46 $\quad A_1\, F_{13} = A_3\, F_{31} \rightarrow F_{31} = \dfrac{A_1}{A_3}$

$$A_2\, F_{23} = A_3\, F_{32} \rightarrow F_{32} = \frac{A_1}{A_3}$$

$$\frac{G_1}{E_{b2}} = \frac{A_2}{A_1}\,\frac{\left(\frac{A_1}{A_3}\right)}{\left(\left(\frac{A_1}{A_3}\right) + \left(\frac{A_2}{A_3}\right)\right)} = \frac{A_2}{A_1 + A_2} = \frac{\pi\left(R^2 - \left(\frac{R}{2}\right)^2\right)}{\pi R^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

(b) The radiation energy loss at the opening is given by the irradiance of surface 1

$$q_1 = G_1\, A_1 = \left(\frac{G_1}{E_{b2}}\right) E_{b2}\, A_1 = \left(\frac{G_1}{E_{b2}}\right) \sigma T_2^{\,4}\, \frac{\pi}{4} R^2$$

$$q_1 = \left(\frac{3}{4}\right)\left(5.67 \times 10^{-8}\ \mathrm{W/(m^2K^4)}\right)(555\ \mathrm{K})^4\, \frac{\pi}{4}\,(0.3\mathrm{m})^2 = 285\ \mathrm{W}$$

(c) From Equation (1)

$$G_3 = \frac{G_1 A_1}{F_{31} A_3} = \left(\frac{G_1}{E_{b2}}\right) E_{b2} \frac{A_1}{A_3} \left(\frac{A_3}{A_1}\right) = \frac{3}{4} \sigma T_2^4$$

Since $J_3 = G_3$, Equation 9.66 yields

$$G_3 = \rho_3 G_3 + \varepsilon_3 E_{b3} \rightarrow G_3 = \frac{\varepsilon_3}{1-\rho_3} E_{b3}$$

But A_3 is opaque, so $\theta_3 = 0$ and from Equations (9.23) and (9.30)

$$(1 - \rho_3) = \varepsilon_3$$

$$\therefore G_3 = E_{b3} = \sigma T_3^4$$

Combining these two equations

$$\sigma T_3^4 = \frac{3}{4} \sigma T_2^4$$

$$T_3 = \left(\frac{3}{4} T_2^4\right)^{0.25} = \left(\frac{3}{4} (555\,\text{K})^4\right)^{0.25} = 516\ \text{K}$$

PROBLEM 9.18

A large slab of steel 0.1 m thick has in it a 0.1 m-diam hole, with axis normal to the surface. Considering the sides of the hole to be black, specify the rate of radiative heat loss from the hole in W. The plate is at 811 K, the surroundings are at 300 K.

GIVEN

- A large slab of steel with a hole whose axis is normal to the surface
- Slab thickness (S) = 0.1 m
- Hole diameter (D) = 0.1 m
- Plate temperature (T_1) = 811 K
- Temperature of surrounding (T_∞) = 300 K

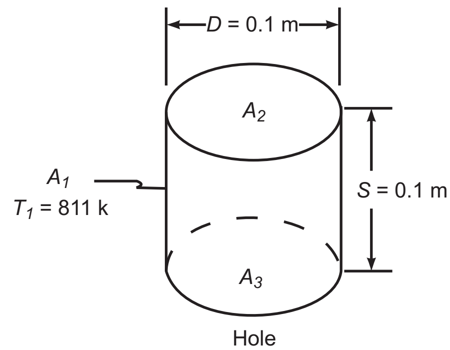
FIND

- The rate of radiative heat loss from the hole (q_r)

ASSUMPTIONS

- The sides of the hole are black

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

From Equation (9.69), for A_2 $\quad A_2\,G_2 = J_1\,A_1\,F_{12} + J_2\,A_2\,F_{22} + J_3\,A_3\,F_{32}$

From Equation (9.66) $\quad J_1 = \rho_1\,G_1 + \varepsilon_1\,E_{b1}$

$$J_2 = \rho_2\,G_2 + \varepsilon_2\,E_{b2}$$

$$J_3 = \rho_3\,G_3 + \varepsilon_3\,E_{b3}$$

Since all surfaces behave as blackbodies $\rho_1 = \rho_2 = \rho_3 = 0$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$

Therefore

$$J_1 = E_{b1} \quad J_2 = E_{b2} \quad \text{and} \quad J_3 = E_{b3} = E_{b2} \text{ (by symmetry)}$$

Substituting these into the above equation yields

$$A_2\,G_2 = E_{b1}\,A_1\,F_{12} + E_{b2}\,(A_2\,F_{22} + A_3\,F_{32})$$

Since A_2 cannot see itself $F_{22} = 0$

$$G_2 = Eb_1\,\frac{A_1}{A_2}\,F_{12} + E_{b2}\,\frac{A_3}{A_2}\,F_{32}$$

By symmetry $F_{12} = F_{13}$

The sum of the shape factors from one surface must be unity

$$F_{11} + F_{12} + F_{13} = 1 \quad \rightarrow \quad F_{11} + 2\,F_{12} = 1$$

$$F_{21} + F_{22} + F_{23} = 1 \quad \rightarrow \quad F_{21} = 1 - F_{23}$$

From Equation (9.46)

$$A_1\,F_{12} = A_2\,F_{21} \rightarrow F_{12} = \frac{A_2}{A_1}\,F_{21} = \frac{A_2}{A_1}\,(1 - F_{23})$$

and $\quad A_2\,F_{23} = A_3\,F_{32} \rightarrow F_{32} = \dfrac{A_2}{A_3}\,F_{23}$

$$G_2 = E_{b1}\frac{A_1}{A_2}\left(\frac{A_2}{A_1}(1 - F_{23})\right) + E_{b2}\,\frac{A_3}{A_2}\left(\frac{A_2}{A_3}F_{23}\right) = E_{b1}\,(1 - F_{23}) + E_{b2}\,F_{23}$$

$$= \sigma[T_1^{\,4}(1 - F_{23}) + T_\infty^{\,4}\,F_{23}]$$

The shape factor F_{23} can be determined from Figure (9.29) for D/S = 0.1 m/0.1 m = 1.0, and for disks with direct radiation, curve 1 applies and the shape factor $F_{23} \approx 0.19$.

$$G_2 = 5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)\ [(811\ \text{K})^4\ (1 - 0.19) + (300\ \text{K})^4\ (0.19)] = 1.996 \times 10^4\ \text{W/m}^2$$

The rate of heat transfer through A_2 is given by Equation (9.67)

$$q_2 = A_2\,(J_2 - G_2) = \frac{\pi}{4} D^2\,(E_{b2} - G_2) = \frac{\pi}{4} D^2\,(\sigma T_2^{\,4} - G_2)$$

$$q_2 = \frac{\pi}{4}(0.1\ \text{m})^2 \left[\left(5.67\times10^{-8}\,\text{W/(m}^2\ \text{K}^4)\right)(300\,\text{K})^4 - 1.996\times10^4\,\text{W/m}^2\right] = -151\ \text{W}$$

The negative sign indicates net heat loss through A_2. By symmetry, the total energy leaving the hole is

$$q_{\text{total}} = q_2 + q_3 = 2\,q_2 = 2(151\ \text{W}) = 302\ \text{W}$$

PROBLEM 9.19

A 15 cm black disk is placed halfway between two black 3 m diameter disks which are 7 m apart with all disk surfaces parallel to each other. If the surroundings are 0 K, determine the temperature of the two larger disks required to maintain the smaller disk at 540°C.

GIVEN

- A black disk (A_1) halfway between two other black disks (A_2 & A_3)
- Diameter of A_1 (D_1) = 15 cm = 0.15 m
- Diameter of A_2 and A_3: ($D_2 = D_3$) = 3 m
- Distance between A_2 and A_3 ($2L$) = 7 m
- Surrounding temperature (T_∞) = 0 K
- Temperature of A_1 (T_1) = 540°C = 813 K

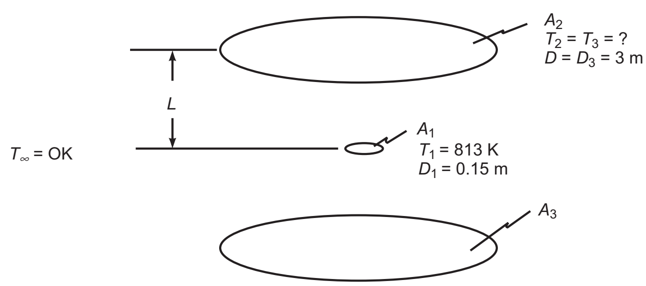
FIND

- The temperature A_2 and A_3 required

ASSUMPTIONS

- A_2 and A_3 are at the same temperature ($T_2 = T_3$)
- Steady state conditions

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The shape factor for the geometry is given in Table 9.3 as

$$F_{12} = \frac{1}{2a^2}\left[L^2 + a^2 + b^2 - \sqrt{(L^2 + a^2 + b^2)^2 - 4a^2 b^2}\right]$$

where $a = D_1/2$ and 0.075 m $b = D_2/2 = 1.5$ m $L = 3.5$ m

By symmetry $F_{13} = F_{12} = 0.155$.

The sum of the shape factors, including the shape factor with the surroundings must be unity

$F_{12} + F_{13} + F_{1\infty} = 1 \quad \rightarrow \quad F_{1\infty} = 1 - 2\,(F_{12}) = 1 - 2\,(0.155) = 0.690$

The net rate of heat transfer from A_1 to A_2 is given by Equation (9.47)

$$q_{1-2} = A_1\,F_{12}\,(E_{b1} - E_{b2}) = \sigma A_1\,F_{12}\,(T_1^4 - T_2^4)$$

Similarly

$$q_{1-3} = \sigma A_1\,F_{13}\,(T_1^4 - T_3^4) = \sigma A_1\,F_{12}\,(T_1^4 - T_2^4)$$

$$q_{1-\infty} = \sigma A_7\,F_{1\infty}\,(T_1^4 - T_\infty^4) = \sigma A_1\,F_{1\infty}\,T_1^4$$

For steady state, these rates of heat transfer must sum to zero

$$q_{1-2} + q_{1-3} + q_{1-\infty} = 0$$

$$\sigma A_1\,[2\,F_{12}\,(T_1^4 - T_2^4) + F_{1\infty}\,T_1^4] = 0$$

Solving for T_2

The net rate of heat transfer from A_1 to A_2 is given by Equation (9.47)

$$q_{1-2} = A_1\,F_{12}(E_{b1} - E_{b2}) = \sigma A_1\,F_{12}\,(T_1^4 - T_2^4)$$

PROBLEM 9.20

Show that the effective conductance. $A_1\,\overline{F}_{1-2}$ for two black parallel planes of equal area connected by re-radiating walls at a constant temperature is

$$\boldsymbol{A_1\,\overline{F}_{1-2} = A_1\left(\frac{1 + F_{1-2}}{2}\right)}$$

GIVEN

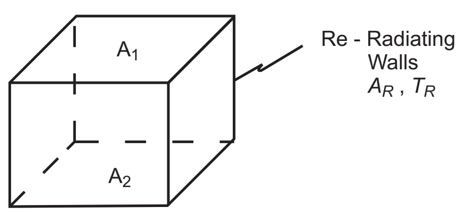
- Two black parallel planes of equal area connected by re-radiating walls at a constant temperature

FIND

- Show that

$$A_1\,F_{1-2} = A_1\,\left(\frac{1 + F_{1-2}}{2}\right)$$

SKETCH



SOLUTION

From Equation (9.79)

$$A_1\,F_{1-2} = A_1\left[F_{1-2} + \frac{1}{\dfrac{1}{F_{1-R}} + \dfrac{A_1}{A_2\,F_{2-R}}}\right]$$

Since A_1 and A_2 cannot see themselves, $F_{1-1} = F_{2-2} = 0$.

The shape factors from a single surface must sum to unity

$F_{1-1} + F_{1-2} + F_{1-R} = 1 \quad \rightarrow \quad F_{1-R} = 1 - F_{1-2}$

$F_{2-1} + F_{2-2} + F_{2-R} = 1 \quad \rightarrow \quad F_{2-R} = 1 - F_{2-1}$

From equation (9.46) $A_1 F_{1-2} = A_2 F_{2-1} \rightarrow F_{1-2} = F_{2-1} \rightarrow F_{2-R} = F_{1-R} = 1 - F_{1-2}$

Substituting this and $A_1 = A_2$ into the expression for $A_1 F_{12}$

$$A_1 F_{1-2} = A_1\left[F_{1-2} + \frac{1}{\dfrac{1}{1-F_{1-2}} + \dfrac{1}{1-F_{1-2}}}\right] = A_1\left[F_{1-2} + \frac{1}{\left(\dfrac{2}{1-F_{1-2}}\right)}\right] = A_1\left[F_{1-2} + \frac{1-F_{1-2}}{2}\right]$$

$$= A_1\left(\frac{1+F_{1-2}}{2}\right)$$

PROBLEM 9.21

Calculate the net radiant-heat-transfer rate if the two surfaces in Problem 9.10 are black and connected by a refractory surface of 500-sq-m area. A_1 is at 555 K and A_2 is at 278 K. What is the refractory surface temperature?

From Problem 9.10: Derive an expression for the geometric shape factor F_{1-2} for a rectangular surface A_1. 1 by 20 m placed parallel to and centered 5 m above a 20-m-square surface A_2.

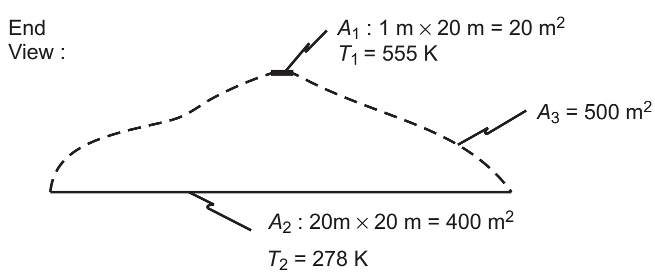
GIVEN

- Rectangular surfaces A_1 and A_2 connected by a refractory surface A_3
- A_1 is parallel to and centered 5 m above A_2
- Dimensions of A_1 = 1 m × 20 m
- A_2 is 20 m^2
- A_3 is 500 m^2
- Temperature of A_1 (T_1) = 555 K
- Temperature of A_2 (T_2) = 278 K
- A_1 and A_2 are black

FIND

(a) The net radiating heat transfer rate (q_1)
(b) The refractory surface temperature (T_3)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

From the solution to Problem 9.10

$$F_{12} = 0.427$$

For the black surfaces A_1 and A_2

$$\rho_1 = \rho_2 = 0 \quad \text{and} \quad \varepsilon_1 = \varepsilon_2 = 1$$

From Equation (9.66)

$$J_1 = E_{b1} \quad \text{and} \quad J_2 = E_{b2}$$

Since $q_3 = 0$, from Equation (9.67)

$$J_3 = G_3$$

From Equation (9.66)

$$G_3 = \frac{\varepsilon_3}{1-\rho_3} E_{b3} = E_{b3}$$

Since neither A_1 nor A_2 can see themselves, $F_{11} = F_{22} = 0$

With these simplifications, Equation (9.69) reduces to

[1] $A_1\, G_1 = Eb_2\, A_2\, F_{21} + E_{b3}\, A_3\, F_{31}$

[2] $A_2\, G_2 = E_{b1}\, A_1\, F_{12} + E_{b3}\, A_3\, F_{32}$

[3] $A_3\, G_3 = E_{b1}\, A_1\, F_{13} + E_{b2}\, A_2\, F_{23} + E_{b3}\, A_3\, F_{33}$

From Equation [3] $\quad E_{b3} = \dfrac{R_{b1}\, A_1 F_{13} + F_{b2}\, A_2 F_{23}}{A_3\,(1 - F_{33})}$

The shape factors are calculated below

$$A_2\, F_{21} = A_1\, F_{12} \rightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20\,\text{m}^2}{400\,\text{m}^2}\ (0.427) = 0.0214$$

$$F_{11} + F_{12} + F_{13} = 1 \rightarrow F_{13} = 1 - F_{12} = 1 - 0.427 = 0.573$$

$$A_3\, F_{31} = A_1\, F_{13} \rightarrow F_{31} = \frac{A_1}{A_3} F_{13} = \frac{20\,\text{m}^2}{500\,\text{m}^2}\ (0.573) = 0.0229$$

$$F_{21} + F_{22} + F_{31} = 1 \rightarrow F_{23} = 1 - F_{21} = 1 - 0.0214 = 0.9786$$

$$A_3\, F_{32} = A_2\, F_{23} \rightarrow F_{32} = \frac{A_2}{A_3} F_{23} = \frac{400\,\text{m}^2}{500\,\text{m}^2}\ (0.9786) = 0.7829$$

$$F_{31} + F_{32} + F_{33} = 1 \rightarrow F_{33} = 1 - F_{31} - F_{32} = 1 - 0.0229 - 0.7829 = 0.1942$$

Solving part (b) first

$$E_{b3} = \sigma T_3^{\,4} = \frac{\sigma T_1^4\, A_1\, F_{13} + \sigma T_2^4\, A_2\, F_{23}}{A_3\,(1 - F_{33})}$$

$$T_3 = \left(\frac{T_1^4\, A_1\, F_{13} + T_2^4\, A_2\, F_{23}}{A_3\,(1 - F_{33})}\right)^{0.25}$$

$$T_3 = \left[\frac{(555\,\text{K})^4 (20\,\text{m}^2)(0.573) + (278\,\text{K})^4 (400\text{m}^2)(0.9786)}{500\,\text{m}^2 (1 - 0.1942)}\right]^{0.25} = 304\ \text{K}$$

(a) The rate of heat loss from A1 is given by Equation (9.67)

$$q_1 = A_1\,(J_1 - G_1) = A_1\,(E_{b1} - G_1)$$

From Equation [1]

$$G_1 = \frac{\sigma(T_2^4 A_2 F_{21} + T_3^4 A_3 F_{31}}{A_1}$$

$$\therefore q_1 = \sigma(T_1^4 A_1 - T_2^4 A_2 F_{21} - T_3^4 A_3 F_{31})$$

$$q_1 = \left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right) [(555\ \text{K})^4 (20\ \text{m}^2) - (278\ \text{K})^4 (400\ \text{m}^2)$$

$$(0.0214) - (304\ \text{K})^4 (500\ \text{m}^2) (0.0229)]$$

$$q_1 = 99{,}150\ \text{W (loss)}$$

As a check

$$q_2 = A_2 (E_{b2} - G_2) = \sigma[T_2^4 A_2 - T_1^4 A_1 F_{12} - T_3^4 A_3 F_{32}] = -100{,}040\ \text{W (gain)}$$

PROBLEM 9.22

A black sphere (2.5 cm diam) is placed in a large infrared heating oven whose walls are maintained at 370°C. The temperature of the air in the oven is 90°C and the heat-transfer coefficient for convection between the surface of the sphere and the air is 30 W/(m^2 K). Estimate the net rate of heat flow to the sphere when its surface temperature is 35°C.

GIVEN

- A black sphere in a large infrared heating oven
- Sphere diameter (D) = 2.5 cm = 0.025 m
- Oven wall temperature (T_2) = 370°C = 643 K
- Oven air temperature (T_∞) = 90°C = 363 K
- Convective heat transfer coefficient (h_c) = 30 W/(m^2 K)
- Sphere surface temperature (T_1) = 35°C = 308 K

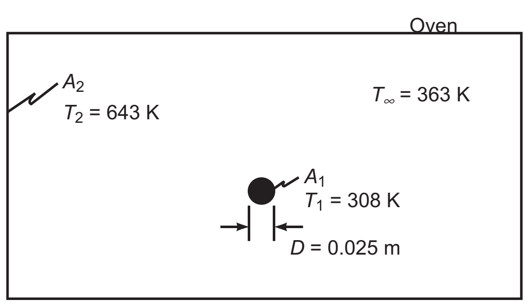
FIND

- The net rate of heat flow to the sphere (q_{total})

ASSUMPTIONS

- The oven walls are diffuse

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = $5.67 \times 10^{-8}\ \dfrac{\text{W}}{\text{m}^2\ \text{K}^4}$

SOLUTION

Since the sphere is black: $\rho_1 = 0$; $\varepsilon_1 = 1$

From Equation (9.66) $\quad J_1 = E_{b1}$ and $J_2 = \rho_2\, G_2 + \varepsilon_2\, E_{b2}$

From Equation (9.69) $\quad$ [1] $\quad A_1\, G_1 = J_1\, A_1\, F_{11} + J_2\, A_2\, F_{21}$

[2] $\quad A_2\, G_2 = J_1\, A_1\, F_{12} + J_2\, A_2\, F_{22}$

Since A_1 cannot see itself, $F_{11} = 0$

Also $\quad F_{12} = 1 \quad \to \quad A_1\, F_{12} = A_2\, F_{21} \quad \to \quad F_{21} = \dfrac{A_1}{A_2}$

Solving for J_2

$$J_2 = \rho_2 \left(E_{b1} \left(\frac{A_1}{A_2} \right) F_{12} + J_2\, F_{22} \right) + \varepsilon_2\, E_{b2}$$

$$J_2 = \frac{E_{b1} \left(\frac{A_1}{A_2} \right) \rho_2 + \varepsilon_2\, E_{b2}}{1 - \rho_2\, F_{22}}$$

$$\therefore\ A_1\, G_1 = \frac{A_2 F_{21} \left(E_{b1} \left(\frac{A_1}{A_2} \right) \rho_2 + \varepsilon_2\, E_{b2} \right)}{1 - \rho_2\, F_{22}}$$

From Equation (9.67)

$$q_1 = A_1\, (J_1 - G_1) = A_1\, E_{b1} - \frac{A_2 \left(\frac{A_1}{A_2} \right) \left(E_{b1} \left(\frac{A_1}{A_2} \right) \rho_2 + \varepsilon_2\, E_{b2} \right)}{1 - \rho_2\, F_{22}}$$

since $\varepsilon_2 = (1 - \rho_2)$, this simplifies to

$$q_1 = \frac{A_1 (1 - \rho_2)(E_{b1} - E_{b2})}{1 - \rho_2\, F_{22}}$$

Since A_2 is very large compared to A_1: $F_{22} \approx 1.0$ and $q_1 = A_1(E_{b1} - E_{b2}) = \sigma A_1(T_1^4 - T_2^4)$

The total rate of heat transfer is the sum of the convective and radiative heat transfer

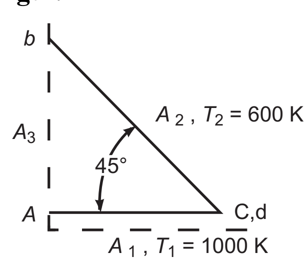
$$q_{\text{total}} = q_c + q_1 = A_1\, [h_c\, (T_\infty - T_1) + \sigma (T_2^4 - T_1^4)]$$

$$q_{\text{total}} = p\ (0.025\ \text{m})^2\ \left[\left(30\ \text{W/(m}^2\text{K)}\right)(363\,\text{K} - 308\,\text{K}) + \left(5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\right) [(643\,\text{K})^4 - (308\,\text{K})^4] \right]$$

$$q_{\text{total}} = 21\ \text{W}$$

PROBLEM 9.23

The wedge-shaped cavity shown in the accompanying sketch consists of two long strips joined along one edge. Surface 1 is 1 m wide and has an emittance of 0.4 and a temperature of 1000 K. The other wall has a temperature of 600 K. Assuming gray diffuse processes and uniform flux distribution, calculate the rate of energy loss from surface 1 and 2 per meter length.



GIVEN

- The wedge shaped cavity shown above
- Width of A_1 (W_1) = 1 m
- Emittance of A_1 (ε_1) = 0.4
- Temperature of A_1 (T_1) = 1000 K
- Temperature of A_2 (T_2) = 600 K
- A_2 is black

FIND

- The rate of energy loss from A_1 and A_2 per meter length (q_1/L and q_2/L)

ASSUMPTIONS

- Enclosure temperature (T_e) = 0 K
- Gray diffuse processes
- Uniform flux distribution

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

Width of A_3 (W_3) = 1 m

Width of A_2 (W_2) = $\sqrt{(1\text{m})^2 + (1\text{m})^2} = \sqrt{2\,\text{m}}$

The crossed-string methods can be used to calculate F_{12}. From Equation (9.54)

$$F_{12} = \frac{(ab+cb)-(ab+cd)}{2\,W_1} = \frac{(1\,\text{m}+\sqrt{2}\,\text{m})-(1\,\text{m}-0)}{2\,(1\,\text{m})} = \frac{\sqrt{2}}{2}$$

From Equation (9.46)

$$A_1 F_{12} = A_2 F_{21} \quad \rightarrow \quad F_{21} = \left(\frac{A_1}{A_2}\right) F_{12}$$

Since neither A_1 nor A_2 can see itself, $F_{11} = F_{22} = 0$

Since A_2 is black: $\rho_2 = 0$ and $\varepsilon_2 = 1$

From Equation (9.66) $\quad J_1 = \rho_1 G_1 + \varepsilon_1 E_{b1} = \rho_1 G_1 + \varepsilon_1 \sigma T_1^4$

$$J_2 = E_{b2} = \sigma T_2^4$$

From Equation (9.69) for A_1

$$A_1 G_1 = J_1 A_1 F_{11} + J_2 A_2 F_{21} = E_{b2} A_2 F_{21} = \sigma T_2^4 A_1 F_{21}$$

Solving for G_1

$$G_1 = \sigma T_2^4 \frac{A_1}{A_2} F_{12} = \left(5.67\times 10^{-8}\ \text{W/(m}^2\text{K}^4)\right)(600\text{K}^4)\frac{(1\,\text{m})\,L}{(\sqrt{2}\,\text{m})\,L}\left(\frac{\sqrt{2}}{2}\right) = 3674\,\text{W/m}^2$$

From Equation (9.67)

$$q_1 = A_1 (J_1 - G_1) = A_1 [(\rho_1 G_1 + \varepsilon_1 \sigma T_1^4) - G_1]$$

Since $\rho_1 = 1 - \varepsilon_1$

$$\frac{q_1}{L} = W_1 \varepsilon_1 (G_1 + \sigma T_1^4) = (1\text{m})\,(0.4)\left[\left(3674\,\text{W/(m}^2\,\text{K)}\right) + \left(5.67\times 10^{-8}\ \text{W/(m}^2\text{K}^4)\right)(1000\,\text{K})^4\right]$$

$$\frac{q_1}{L} = 24{,}150\,\frac{\text{W}}{\text{m}} \ \text{(loss)}$$

From Equation (9.69) for A_2

$$A_2\, G_2 = J_1\, A_1\, F_{12} + J_2\, A_2\, F_{22} = (\rho_1\, G_1 + \varepsilon_1\, \sigma T_1^4)\, A_1\, F_{12}$$

$$G_2 = (\rho_1\, G_1 + \varepsilon_1\, \sigma T_1^4) \left(\frac{A_1}{A_2}\right) F_{12}$$

From Equation (9.67)

$$q_2 = A_2\,(J_2 - G_2) = W_2\, L \left[E_{b2} - (\rho_1\, G_1 + \varepsilon_1\, \sigma T_1^4) \frac{A_1}{A_2} F_{12} \right]$$

Since $\rho_1 = 1 - \varepsilon_1$

$$\frac{q_2}{L} = W_2 \left[\sigma T_2^4 - \left(\frac{A_1}{A_2}\right) F_{12} [(1-\varepsilon_1)\, G_1 + \varepsilon_1\, \sigma T_1^4] \right]$$

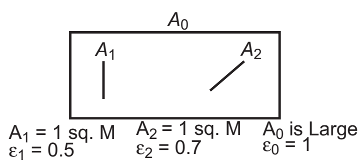
$$\frac{q_1}{L} = (\sqrt{2}\,\text{m})$$

$$\left[\left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(600\,\text{K})^4 - \left(\frac{1}{\sqrt{2}}\right)\frac{\sqrt{2}}{2}[(1-0.4)\left(3674\,\text{W/(m}^3\text{K)}\right) + 0.4\left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(1000\,\text{K})^4] \right]$$

$$\frac{q_2}{L} = -7204 \text{ W/m (gain)}$$

PROBLEM 9.24

Derive an equation for the net rate of radiant heat transfer from surface 1 in the system shown in the accompanying sketch. Assume that each surface is at a uniform temperature and that the geometrical shape factor $F_{1\text{–}2}$ is 0.1.



GIVEN

- The system shown above

FIND

- An expression for the net rate of radiant heat transfer from surface 1 (q_1)

ASSUMPTIONS

- Steady state
- A_1 and A_2 are gray, A_0 is black
- Each surface is at a uniform temperature
- The shape factor $F_{12} = 0.1$

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

All of the shape factors for the problem can be expressed in terms of F_{12} using Equation (9.46) and the fact that all shape factors from a given surface must sum to unity.

Also

$$A_1 = A_2 \text{ and } \frac{A_1}{A_0} = \frac{A_2}{A_0} \approx 0$$

$$A_1\, F_{12} = A_2\, F_{21} \rightarrow F_{21} = F_{12} = 0.1$$

$$F_{10} + F_{11} + F_{12} = 1 \text{ and } F_{11} = 0 \rightarrow F_{10} = 1 - F_{12} = 0.9$$
$$F_{20} + F_{21} + F_{22} = 1 \text{ and } F_{22} = 0 \rightarrow F_{20} = 1 - F_{21} = 0.9$$
$$A_1 F_{10} = A_0 F_{01} \rightarrow F_{01} = \left(\frac{A_1}{A_0}\right) F_{10} \approx 0$$
$$A_2 F_{20} = A_0 F_{02} \rightarrow F_{02} = \left(\frac{A_2}{A_0}\right) F_{20} \approx 0$$
$$F_{00} + F_{01} + F_{02} = 1 \rightarrow F_{00} = 1$$

The net rate of heat transfer from surface 1 is given by Equation (9.67)

$$q_1 = A_1 (J_1 - G_1)$$

Where the radiosity (J_1) and the irradiation (G_1) can be calculated using Equations (9.69) and (9.66)

$$[1]\ A_1 G_1 = J_0 A_0 F_{01} + J_1 A_1 F_{11} + J_2 A_2 F_{21} = J_2 A_2 F_{21}$$
$$[2]\ A_2 G_2 = J_0 A_0 F_{02} + J_1 A_1 F_{12} + J_2 A_2 F_{22} = J_1 A_1 F_{12}$$
$$[3]\ J_1 = \rho_1 G_1 + \varepsilon_1 E_{b1}$$
$$[4]\ J_2 = \rho_2 G_2 + \varepsilon_2 E_{b2}$$

Substituting [4] into [1]

$$A_1 G_1 = (\rho_2 G_2 + \varepsilon_2 E_{b2}) A_2 F_{21}$$

Substituting [2] and [3] into this Equation

$$A_1 G_1 = [\rho_2(\rho_1 G_1 + \varepsilon_1 E_{b1}) \left(\frac{A_1}{A_2}\right) F_{12} + \varepsilon_2 E_{b2}] A_2 F_{21}$$
$$G_1 = \frac{\rho_2 \varepsilon_1 E_{b1} A_1 F_{12} F_{21} + \varepsilon_2 E_{b2} A_2 F_{21}}{A_1 - \rho_2 \rho_1 A_1 F_{21} F_{12}}$$
$$q_1 = A_1 (J_1 - G_1) = A_1[(\rho_1 G_1 + e_1 E_{b1}) - G_1] = A_1[\varepsilon_1 E_{b1} + G_1 (\rho_1 - 1)]\ A_1(\varepsilon_1 E_{b1} - G_1\varepsilon_1)$$
$$q_1 = A_1 e_1 (E_{b1} - G_1) = A_1 \varepsilon_1 \left[E_{b1} - \left(\frac{\rho_2 \varepsilon_1 E_{b1} A_1 F_{12} F_{21} + \varepsilon_2 E_{b2} A_2 F_{21}}{A_1 - \rho_2 \rho_1 A_1 F_{21} F_{12}}\right)\right]$$

where $\rho 1 = 1 - \varepsilon_1 = 0.5$ $\rho_2 = 1 - \varepsilon_2 = 0.3$ and $E_{bi} = \sigma T_i^4$

$$\therefore q_1 = \varepsilon_1 \sigma \left[A_1 T_1^4 - \left(\frac{\rho_2 \varepsilon_1 A_1 F_{12} F_{21} T_1^4 + \varepsilon_2 A_2 F_{21} T_2^4}{1 - \rho_2 \rho_1 F_{21} F_{12}}\right)\right]$$
$$q_1 = (0.5)\ \left(5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\right)$$
$$\left[(1\text{m}^2) T_1^4 - \left(\frac{(0.3)(0.5)(1\text{m}^2)(0.1)(0.1) T_1^4 + (0.7)(1\text{m}^2)(0.1) T_2^4}{1 - (0.5)(0.3)(0.1)(0.1)}\right)\right]$$
$$q_1 = \left(2.83 \times 10^{-8}\ \text{W/K}^4\right) T_1^4 - \left(1.98 \times 10^{-9}\ \text{W/K}^4\right) T_2^4$$

PROBLEM 9.25

Two 1.5 m-square and parallel flat plates are 0.3 m apart. Plate A_1 is maintained at a temperature of 1100 K and A_2 at 500 K. The emittances of the plates are 0.5 and 0.8, respectively. Considering the surroundings black at 0 K and including multiple inter-reflections, determine (a) the net radiant exchange between the plates and (b) the heat input required by surface A_1 to maintain its temperature. The outer-facing surfaces of the plates are adiabatic.

GIVEN

- Two square parallel flat plates, 0.3 m
- Temperature of A_1 (T_1) = 1100 K
- Temperature of A_2 (T_2) = 500 K
- Emittances
  - $\varepsilon_1 = 0.5$
  - $\varepsilon_2 = 0.8$
- Surroundings are black at (T_3) = 0 K

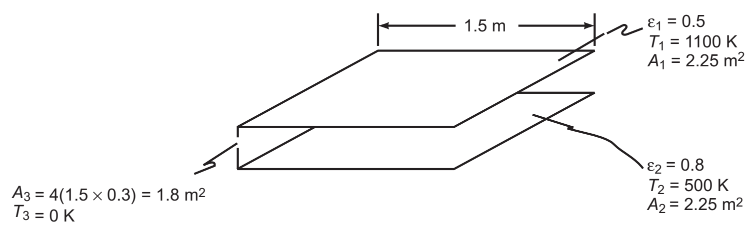
FIND

Including multiple inter-reflections, determine

(a) The net radiant exchange (q_{1-2})

(b) The heat input at surface A_1 (q_1) required to maintain its temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5*,* the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The gap between the plates can be considered to be a blackbody square cylindrical surface as shown in the sketch.

$$\rho_3 = 0 \quad \text{and} \quad \varepsilon_3 = 1 \rightarrow \text{From Equation (9.66) } J_3 = E_{b3} = \sigma T_3^4 = 0$$

Also, since neither A_1 nor A_2 can see itself, $F_{11} = F_{22} = 0$.

From Equation (9.46) $A_1 F_{12} = A_2 F_{21} \rightarrow F_{12} = F_{21}$ (This is apparent from the symmetry of the problem).

The radiosities J_1 and J_2 are needed to calculate the rate of radiant heat transfer and can be determined using Equations (9.69) and (9.66)

From Equation (9.69)

$$[1]\ A_1 G_1 = J_1 A_1 F_{11} + J_2 A_2 F_{21} + J_3 A_3 F_{31} = J_2 A_2 F_{21}$$

$$[2]\ A_2 G_2 = J_1 A_1 F_{12} + J_2 A_2 F_{22} + J_3 A_3 F_{32} = J_1 A_1 F_{12}$$

$$[3]\ A_3 G_3 = J_1 A_1 F_{13} + J_2 A_2 F_{23} + J_3 A_3 F_{33} = J_1 A_1 F_{13} + J_2 A_2 F_{23}$$

From Equation (9.66)

$$[4]\ J_1 = \rho_1 G_1 + \varepsilon_1 E_{b1}$$

$$[5]\ J_2 = \rho_2 G_2 + \varepsilon_2 E_{b2}$$

$$[6]\ J_3 = 0$$

Substituting Equations [4] and [5] into [1] and [2]

$$A_1 G_1 = (\rho_2 G_2 + \varepsilon_2 E_{b2}) A_2 F_{21}$$

$$A_2 G_2 = (\rho_1 G_1 + \varepsilon_1 E_{b1}) A_1 F_{12}$$

Substituting G$_1$ from the first equation into the second and using $F_{21} = F_{12}$ yields

$$A_2 G_2 = \left(\rho_1 \frac{A_2}{A_1} F_{12} (\rho_2 G_2 + \varepsilon_2 E_{b2}) + \varepsilon_1 E_{b1} \right) A_1 F_{12}$$

since $A_1 = A_2$ and $E_{bi} = \sigma T_i^4$

$$G_2 = \frac{\sigma F_{12}(\varepsilon_2 T_2^4 F_{12} \rho_1 + \varepsilon_1 T_1^4)}{1-(F_{12})^2 \rho_2 \rho_1}$$

The shape factor F_{12} can be determined from Figure 9.28: for $x/D = y/D = 5 \rightarrow F_{12} = 0.71$

Also

$$\rho_1 = 1 - \varepsilon_1 = 0.5 \text{ and } \rho_2 = 1 - \varepsilon_2 = 0.2$$

$$\therefore \quad G_2 = (5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4))(0.71)\left[\frac{0.8(500\text{K})^4(0.71)(0.5)+(0.5)(1100\text{K})^4}{1-(0.71)^2(0.5)(0.2)}\right]$$

$$\Rightarrow \quad G_2 = \frac{3.0185 \times 10^4}{0.9496} \text{ W/m}^2 = 31,787 \text{ W/m}^2$$

From Equation [5]

$$J_2 = \rho_2 G_2 + \varepsilon_2 E_{b2} = \rho_2 G_2 + \varepsilon_2 \sigma T_2^4$$

$$\Rightarrow \quad J_2 = 0.2 \text{ (31787 W/m}^2) + 0.8 \text{ (5.67} \times 10^{-8} \text{ W/(m}^2\text{ K)) (500 K)}^4$$

$$\Rightarrow \quad J_2 = 9192 \text{ W/m}^2$$

From Equation [1]

$$G_1 = F_{21} J_2 = F_{12} J_2 = (0.71) \text{ (9192 W/m}^2)$$

$$\Rightarrow \quad G_1 = 6526 \text{ W/m}^2$$

and from Equation [4]

$$J_1 = \rho G_1 + \varepsilon_1 \sigma T_1^4 = 0.5(6526 \text{ W/m}^2) + 0.5(5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4))(1100 \text{ K})^4$$

$$\Rightarrow \quad J_1 = 44770 \text{ W/m}^2$$

(a) The net radiant exchange is given by Equation (9.73)

$$q_{1-2} = (J_1 - J_2) A_1 F_{12} = (44770 - 9192) \text{ W/m}^2 \text{ (2.25 m}^2\text{) (0.71)}$$

$$\Rightarrow \quad q_{1-2} = 56836 \text{ W}$$

(b) The required input to surface A_1 is equal to the rate of radiative loss from surface A_1 which is given by Equation (9.67)

$$q_1 = A_1 (J_1 - G_1) = 2.25 \text{ m}^2 \text{ (44770} - 9192) \text{ W/m}^2$$

$$\Rightarrow \quad q_1 = 80050 \text{ W}$$

PROBLEM 9.26

Two concentric spheres 0.2 m and 0.3 m in diameter, with the space between them evacuated, are to be used to store liquid air (133 K). If the surfaces of the spheres have been flashed with aluminum and the liquid air has a talent heat of vaporization of 209 kJ/kg, determine the number of kilograms of liquid air evaporated per hour.

GIVEN

- Two concentric spheres with the space between them evacuated and liquid air in the inner sphere
- Diameters
  - $D_1 = 0.2$ m
  - $D_2 = 0.3$ m
- Liquid air temperature (T_a) = 133 K
- Room temperature (T_∞) = 293 K
- Surfaces of the spheres have been flashed with aluminum
- Heat of vaporization of liquid air (h_{fg}) = 209 kJ/kg = 209,000 J/kg

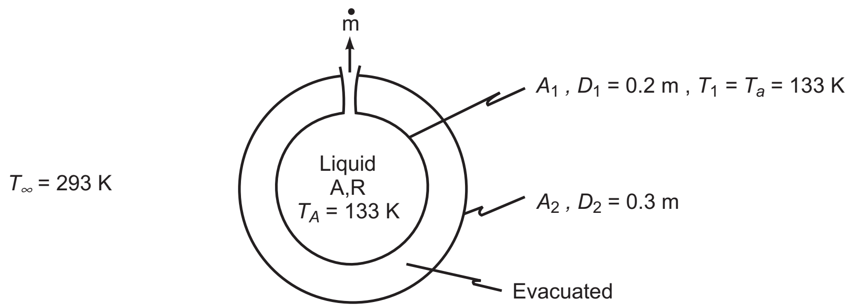
FIND

- The number of kilograms of liquid air evaporated per hour $(\dot{m})$

ASSUMPTIONS

- Steady state
- Convective thermal resistance between the liquid air and interior sphere is negligible
- Thermal resistance of the sphere walls is negligible
- Natural convection on the exterior is negligible
- The room behaves as a blackbody enclosure
- The thickness of the sphere walls in negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5*,* the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Table 9.2*,* the hemispherical emissivity of the spheres will be approximated by that for oxidized aluminum at 310 K: $\varepsilon = \varepsilon_1\ \varepsilon_2 = 0.11$

SOLUTION

Since A_2 completely surrounds A_1 and A_1 cannot see itself, $F_{12} = 1.0$ and $F_{11} = 0$
From Equation (9.46)

$$A_2\, F_{21} = A_1\, F_{12} \rightarrow\ F_{21} = \frac{A_1}{A_2} = \frac{\pi D_1^2}{\pi D_2^2} = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{0.2}{0.3}\right)^2 = 0.444$$

The shape factors from a given surface must sum to unity

$$F_{21} + F_{22} = 1 \rightarrow\ \ F_{22} = 1 - F_{21} = 0.556$$

Also

$$\rho = \rho_1 = \rho_2 = 1 - \varepsilon = 0.89$$

The net rate of heat transfer from A_1 to A_2 must equal the rate of heat transfer from the exterior sphere to the surroundings

$$q = q_{12} = \sigma\, \varepsilon\, A_2\, (T_2^{\,4} - T_\infty^{\,4}) \qquad [1]$$

The rate of heat transfer between the spheres is given by Equation (9.75)

$$q_{12} = A_1\, \mathcal{F}_{12}\, (E_{b1} - E_{b2}) = A_1\, \mathcal{F}_{21}\, \sigma (T_1^{\,4} - T_2^{\,4}) \qquad [2]$$

where f_{12} is given in Equation (9.76) for concentric spheres.

$$\mathcal{F}_{12} = \frac{1}{\left(1 - \dfrac{\varepsilon_1}{\varepsilon_1}\right) + 1 + \left(\dfrac{A_1}{A_2}\right)\left(1 - \dfrac{\varepsilon_1}{\varepsilon_2}\right)}$$

$$\frac{A_1}{A_2} = \frac{\pi D_1^2}{\pi D_2^2} = \left(\frac{D_1}{D_2}\right)^2$$

$$F_{12} = \frac{1}{\left(\left(\frac{1-0.11}{0.11}\right)+1+\left(\frac{0.2}{0.3}\right)^2\left(\frac{1-0.11}{0.11}\right)\right)} = 0.0788$$

$$A_1 \, F_{12} \, \sigma(T_1^4 - T_2^4) = \sigma\,\varepsilon\,A_2\,(T_2^4 - T_\infty^4)$$

Solving Equations [1] and [2] for T_2

$$T_2 = \left[\frac{\left(\frac{A_1}{A_2}\right)\left(\frac{F_{12}}{\varepsilon}\right)T_1^4 + T_\infty^4}{1+\left(\frac{A_1}{A_2}\right)\left(\frac{F_{12}}{\varepsilon}\right)}\right]^{0.25}$$

where $\frac{A_1}{A_2}\frac{F_{12}}{\varepsilon} = \left(\frac{0.2}{0.3}\right)^2 \frac{0.0788}{0.11} = 0.3185$

$$T_2 = \left(\frac{0.3185\,(133\text{K})^4 + (293\text{K})^4}{1+0.3185}\right)^{0.25} = 274\ \text{K}$$

The rate of evaporation of the liquid air is

$$\dot{m} = \frac{q}{h_{fg}} = \frac{\sigma\,\varepsilon\,A_2(T_\infty^4 - T_2^4)}{h_{fg}} = \frac{\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\pi(0.11)(0.3\,\text{m})^2\,[(293\text{K})^4 - (274\text{K})^4]}{(209{,}000\,\text{J/kg})((\text{Ws})/\text{J})(\text{h}/3600\,\text{s})}$$

$$\dot{m} = 0.053\ \text{kg/h}$$

COMMENT

The rate of evaporation would be reduced to 0.024 kg/h if the two aluminum surfaces in the evacuated space could remain polished so that $\varepsilon_1 = \varepsilon_2 = 0.04$.

PROBLEM 9.27

Determine the steady-state temperatures of two radiation shields placed in the evacuated space between two infinite planes at temperatures of 555 K and 278 K. The emissivity of all surfaces is 0.8.

GIVEN

- Two radiant shields placed in the evacuated space between two infinite planes
- Temperature of the planes
  - $T_1 = 555$ K
  - $T_4 = 278$ K
- Emissivity of all surface (ε) = 0.8

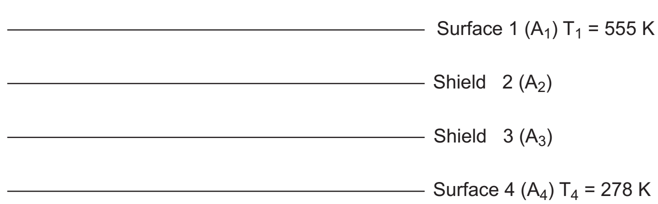
FIND

- The steady state temperatures of the shields (T_2, T_3)

ASSUMPTIONS

- All surfaces are gray and diffuse

SKETCH



SOLUTION

Since the space is evacuated, convection and conduction are negligible. Since the surfaces are simply infinite planes, equivalent conductance $A_1 f_{12}$ can be used.

The net rate of heat transfer from A_1 to A_2 is given by Equation (9.75)

$$q_{12} = A_1 \mathcal{F}_{12} (E_{b1} - E_{b2})$$

For steady state $q_{12} = q_{23} = q_{34}$

Also, because of all the emittances and areas are identical

$$\mathcal{F}_{12} = \mathcal{F}_{23} = \mathcal{F}_{34}$$

Therefore $\quad E_{b1} - E_{b2} = E_{b2} - E_{b3} = E_{b3} - E_{b4}$

$$E_{b2} = 0.5\,(E_{b1} + E_{b3}) \quad \text{and} \quad E_{b3} = 0.5(E_{b2} + E_{b4})$$

Solving for E_{b2}

$$E_{b2} = \frac{1}{2}\left[E_{b1} + \frac{1}{2}(E_{b2} + E_{b4})\right] \rightarrow \quad E_{b2} = \frac{4}{3}\left[\frac{1}{2}E_{b1} + \frac{1}{4}E_{b4}\right] = \sigma T_2^4$$

$$T_2 = \left[\frac{1}{3\sigma}(2\sigma T_1^4 + \sigma T_4^4)\right]^{0.25} = \left[\frac{1}{3}(2T_1^4 + T_4^4)\right]^{0.25} = \left[\frac{1}{3}[2(555\,\text{K})^4 + (278\,\text{K})^4]\right]^{0.25} = 505\ \text{K}$$

Similarly $\quad T_3 = \left[\frac{1}{3}(2T_4^4 + T_1^4)\right]^{0.25} = \left[\frac{1}{3}[2(278\,\text{K})^4 + (555\,\text{K})^4]\right]^{0.25} = 434\ \text{K}$

PROBLEM 9.28

Three thin sheets of polished aluminum are placed parallel to each other so that the distance between them is very small compared to the size of the sheets. If one of the outer sheets is at 280°C, and the other outer sheet is at 60°C, calculate the temperature of the intermediate sheet and the net rate of heat flow by radiation. Convection may be ignored.

GIVEN

- Three thin sheets of polished aluminum parallel to each other
- The distance between the sheets is small compared to the size of the sheets
- Outer sheet temperatures
  - T_1 = 280°C = 553 K
  - T_2 = 60°C = 333 K
- Convection may be ignored

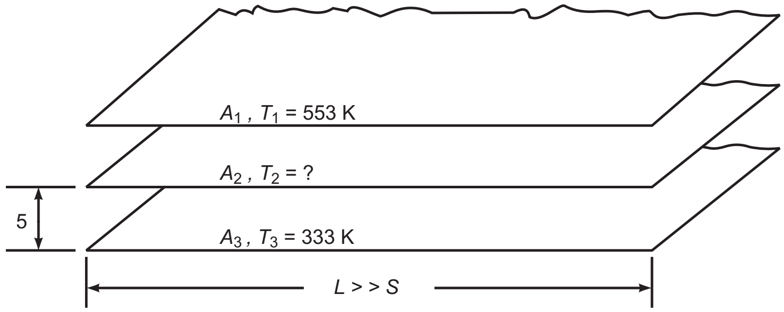
FIND

(a) The temperature of the intermediate sheet
(b) The net rate of heat flow by radiation

ASSUMPTIONS

- Steady state
- All surfaces are gray

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Table 9.2, the emissivity of polished aluminum at the average temperature of 443 K ($\varepsilon = \varepsilon_1 = \varepsilon_2 = \varepsilon_3$) = 0.05

SOLUTION

The plates may be approximated by infinite parallel planes, therefore, the shape factors are, $F_{12} = F_{21} = F_{23} = F_{32} = 1.0$

(a) For steady state, the net heat flow from surface 2, from Equation (9.75) must be zero

$$q_2 = q_{21} + q_{23} = A_2\, \mathrm{F}_{21}\,(E_{b2} - E_{b1}) + A_2\, \mathrm{F}_{23}\,(E_{b2} - E_{b3}) = 0$$

By symmetry

$$f_{21} = f_{23}$$

Therefore $\quad 2\,E_{b2} - E_{b1} - E_{b3} = 0 \;\rightarrow\; 2\,T_7^4 = T_7^4 + T_7^4$

Solving for T_2

$$T_2 = \left[\frac{T_1^4 + T_3^4}{2}\right]^{0.25} = \left[\frac{(553\,\mathrm{K})^4 + (333\,\mathrm{K})^4}{2}\right]^{0.25} = 480\ \mathrm{K}$$

(b) From Equation (9.78) for infinitely large parallel plates

$$\mathrm{F}_{12} = \frac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{1}{\dfrac{2}{\varepsilon} - 1} = \frac{1}{\left(\dfrac{2}{0.05}\right) - 1} = 0.0256$$

The rate of heat transfer is

$$q = q_{12} = A_1\, \mathrm{F}_{12}\,(E_{b1} - E_{b2})$$

$$\frac{q}{A} = \mathrm{F}_{12}\,\sigma(T_1^4 - T_2^4) = 0.0256\,\left(5.67\times 10^{-8}\ \mathrm{W/(m^2K^4)}\right)[(553\ \mathrm{K})^4 - (480\ \mathrm{K})^4] = 59\ \mathrm{W/m^2}$$

PROBLEM 9.29

Determine the rate of heat transfer between two 1 by 1 m parallel flat plates placed 0.2 m apart and connected by re-radiating walls. Assume that plate 1 is maintained at 1500 K and plate 2 at 500 K. (a) Plate 1 has an emissivity of 0.9 over the entire spectrum and plate 2 has an emissivity of 0.1. (b) Plate 1 has an emissivity of 0.1 between 0 and 2.5 μ m and an emissivity of 0.9 at wavelengths longer than 2.5 μ m, while plate 2 has an emissivity of 0.1 over the entire spectrum. (c) The emissivity of plate 1 is the same as in part (b), and plate 2 has an emissivity of 0.1 between 0 and 4.0 μ m and an emissivity of 0.9 at wavelengths larger than 4.0 μ m.

GIVEN

- Two parallel flat plates connected by re-radiating walls
- Plates dimensions: 1 m × 1 m
- Distance between plates (s) = 0.2 m
- Plate temperatures
  - T_1 = 1500 K
  - T_2 = 500 K

FIND

The rate of heat transfer between plates if

(a) The emissivity of plates 1 (ε_1)= 0.9 and the emissivity of plates 2 (ε_2) = 0.1

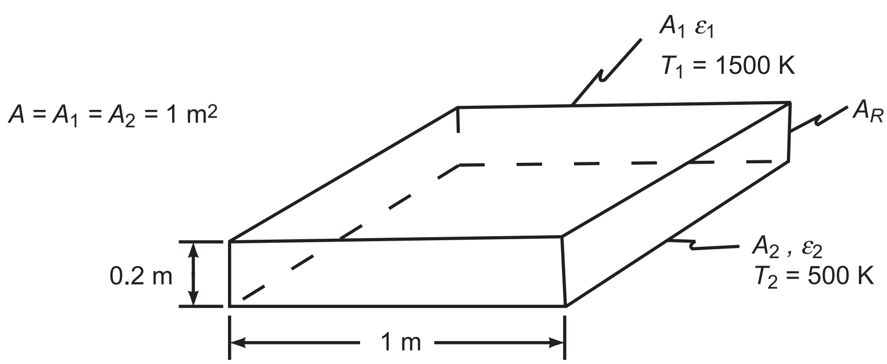
(b) ε_1 = 0.1 for 0 < λ < 2.5 μ m; ε_1 = 0.9 for λ > 2.5 μ m; ε_2 = 0.1

(c) ε_1 is same as (b); ε_2 = 0.1 for 0 < λ < 4.0 μ m, and ε_2 = 0.9 for λ > 4.0 μ m

ASSUMPTIONS

- Convective heat transfer is negligible
- The re-radiating surface is gray
- Steady state conditions

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67× 10^{-8} W/(m^2 K^4)

SOLUTION

The shape factor F_{12} is given by Figure 9.28 $x/D = y/D$ = (1 m)/(0.2 m) = 5 → $F_{12} \approx 0.71$

From Equation (9.46) $A_1 F_{12} = A_2 F_{21}$ → $F_{21} = F_{21} = 0.71$

Since neither A_1 nor A_2 can see itself, $A_{11} = A_{22} = 0$

The shape factors from a given surface must sum to zero

$$F_{11} + F_{12} + F_{1R} = 1 \rightarrow F_{1R} = 1 - F_{12} = 0.29$$

$$F_{21} + F_{22} + F_{2R} = 1 \rightarrow F_{2R} = 1 - F_{21} = 0.29$$

The rate of heat transfer is given by Equation (9.80)

$$q_{12} = A_1 \mathcal{F}_{12}\, \sigma (T_1^4 - T_2^4)$$

where $A_1 F_{12}$ is given by Equation (9.79)

$$A_1\mathcal{F}_{12} = \frac{1}{\frac{1}{A_1}\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) + \frac{1}{A_1\bar{F}_{12}}}$$

$$\text{where } A_1\,\overline{F}_{12} = A_1\left(F_{12} + \frac{1}{\frac{1}{F_{1R}} + \frac{A_1}{A_2F_{2R}}}\right) = 1\text{ m}^2\left(0.71 + \frac{1}{\frac{1}{0.29}+\frac{1}{0.29}}\right) = 0.855\text{ m}^2$$

(a) For $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.9$

$$A_1\,\mathcal{F}_{12} = \left[\frac{1}{1\text{m}^2}\left(\frac{1}{0.1}-1\right)+\frac{1}{1\text{m}^2}\left(\frac{1}{0.9}-1\right)\frac{1}{0.855\text{m}^2}\right]^{-1} = 0.0973\text{ m}^2$$

$$q_{12} = (0.0973\text{ m}^2)\left(5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\right)[(1500\text{ K})^4 - (500\text{ K})^4] = 2.76\times10^4\text{ W}$$

(b) For $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$

$$A_1\,\mathcal{F}_{12} = \left[\frac{1}{1\text{m}^2}\left(\frac{1}{0.1}-1\right)+\frac{1}{0.855\text{m}^2}\right]^{-1} = 0.052\text{ m}^2$$

Following the procedure demonstrated in Section 9.7.2

For band 1: $0 < \lambda < 2.5\ \mu$ m, $A_1 f_{12} = 0.052$ m^2

$$\lambda T_1 = (2.5\times10^{-6}\text{ m})\,(1500\text{ K}) = 3.8\times10^{-3}\text{ m K}$$

From Table 9.1 $\dfrac{E_b(0\to\lambda T)}{\sigma T^4} = 0.4434$

$$\lambda T_2 = (2.5\times10^{-6}\text{ m})\,(500\text{ K}) = 1.3\times10^{-3}\text{ m K}$$

From Table 9.1 $\dfrac{E_b(0\to\lambda T)}{\sigma T^4} = 0.004963$

$$q_{12}\Big|_0^{2.5\text{m}} = A_1\,\mathcal{F}_{12}\,(\varepsilon_1 = 0.1,\ \varepsilon_2 = 0.1)\left[\frac{E_b(0\to\lambda T_1)}{\sigma T_1^4}\sigma T_1^4 - \frac{E_b(0\to\lambda T_2)}{\sigma T_2^4}\sigma T_2^4\right]$$

$$q_{12}\Big|_\infty^{2.5\text{m}} = 0.052\text{ m}^2\left(5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\right)[0.4434\,(1500\text{ K})^4 - 0.004963\,(500\text{ K})^4] = 6618\text{ W}$$

For band 2: $2.5\ \mu < \lambda$

$$q_{12}\Big|_{2.5\text{m}}^{\infty} = A_1\,\mathcal{F}_{12}\,(\varepsilon_1 = 0.9,\ \varepsilon_2 = 0.1)\left[\frac{E_b(\lambda T_1\to\infty)}{\sigma T_1^4}\sigma T_1^4 - \frac{E_b(\lambda T_2\to\infty)}{\sigma T_2^4}\sigma T_2^4\right]$$

$$q_{12}\Big|_t^{\infty} = 0.0973\text{ m}^2\left(5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\right)[(1-0.4434)(1500\text{ K})^4 - (1-0.004963)\,(500\text{ K})^4]$$

$$q_{12}\Big|_{2.5\text{m}}^{\infty} = 15{,}202\text{ W}$$

The total rate of heat transfer is the sum of the rate of heat transfer in the two bands

$$q_{12,\text{total}} = q_{12}\Big|_0^{2.5\text{m}} + q_{12}\Big|_{2.5\text{m}}^{\infty} = 6618\text{ W} + 15{,}202\text{ W} = 2.18\times10^4\text{ W}$$

(c) For this case, the spectrum must be broken into three bands

$0 < \lambda < 2.5\ \mu$ m	$\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$, $A_1 f_{12} = 0.052$ m^2
$2.5\ \mu$ m $> \lambda$, $4\ \mu$ m	$\varepsilon_1 = 0.9$, $\varepsilon_2 = 0.1$, $A_1 f_{12} = 0.0973$ m^2
$4\ \mu$ m < 1	ε_1 0.9, $\varepsilon_2 = 0.9$

For $\lambda > 4\ \mu$ m

$$A_1\,\mathcal{F}_{12} = \left[\frac{2}{1\text{m}^2}\left(\frac{1}{0.9}-1\right)+\frac{1}{0.855\text{m}^2}\right]^{-1} = 0.719\text{ m}^2$$

At $\lambda = 4$ μ m

$$\lambda\, T_1 = (4 \times 10^{-6}\text{ m})\,(1500\text{ K}) = 6 \times 10^{-3}\text{ m K}$$

From Table 9.1: $\dfrac{E_b(0 \to \lambda T_1)}{\sigma T_1^4} = 0.7379$

$$\lambda\, T_2 = (4 \times 10^{-6}\text{ m})\,(500\text{ K}) = 2 \times 10^{-3}\text{ m K}$$

From Table 9.1: $\dfrac{E_b(0 \to \lambda T_2)}{\sigma T_2^4} = 0.0667$

Band 1

Same as part (b) $q_{12}\Big|_0^{2.5\text{m}} = 6618$ W

Band 2

2.5 μ m < l < 4 μ m

$$q_{12}\Big|_{2.5}^{4} = A_1\, \mathrm{F}_{12}\,(\varepsilon_1 = 0.9, \varepsilon_2 = 0.1)$$

$$\left[\left(\frac{E_b(0 \to 4T_1)}{\sigma T_1^4} - \frac{E_b(0 \to 2.5T_1)}{\sigma T_1^4}\right)\sigma T_1^4 \left(\frac{E_b(0 \to 2.5T_2)}{\sigma T_2^4} - \frac{E_b(0 \to 2.5T_2)}{\sigma T_2^4}\right)\sigma T_2^4\right]$$

$$q_{12}\Big|_0^{2.5\text{m}} = 0.0973\text{ m}^2\ \left(5.67 \times 10^{-8}\text{ W/(m}^2\text{K}^4)\right)\ [(0.7379 - 0.4434)\,(1500\text{ K})^4 - (0.0667 - 0.004963)\,(500\text{ K})^4]$$

$$q_{12}\Big|_0^{2.5\text{m}} = 27908\text{ W}$$

Band 3

$\lambda > 4\ \mu$ m

$$q_{12}\Big|_4^{\infty} = A_1\, \mathrm{F}_{12}\,(\varepsilon_1 = 0.9, \varepsilon_2 = 0.9)\left[\left(1 - \frac{E_b(0 \to 4T_1)}{\sigma T_1^4}\right)\sigma T_1^4 - \left(1 - \frac{E_b(0 \to 4T_2)}{\sigma T_2^4}\right) s\, T_2^4\right]$$

$$q_{12}\Big|_4^{\infty} = 0.719\text{ m}^2\ \left(5.67 \times 10^{-8}\text{ W/(m}^2\text{K}^4)\right) [(1 - 0.7379)\,(1500\text{ K})^4 - (1 - 0.0667\,(500\text{ K})^4]$$

$$q_{12}\Big|_0^{2.5\text{m}} = 204{,}006\text{ W}$$

$$q_{12,\text{total}} = (6618 + 27908 + 204{,}006)\text{ W} = 2.39 \times 10^5\text{ W}$$

PROBLEM 9.30

A small sphere (2.5 cm diam) is placed in a heating oven whose cavity is a 0.3 m cube filled with air at 101 kPa (abs), contains 3 per cent water vapor at 810 K, and whose walls are at 1370 K. The emissivity of the sphere is equal to 0.44 – 0.00018 *T*, where *T* is the surface temperature in K. When the surface temperature of the sphere is 810 K, determine (a) the total irradiation received by the walls of the oven from the sphere, (b) the net heat transfer by radiation between the sphere and the walls of the oven, and (c) the radiant heat transfer coefficient.

GIVEN

- A small sphere in a 0.3 cm cubic heat oven filled with air
- Sphere diameter (D) = 2.5 cm
- Air pressure = 1 atm = 101 kPa
- Air contains 3% water vapor
- Air temperature (T_m) = 810 K
- Oven wall temperature (T_2) = 1370 K
- Sphere emissivity (ε_1) = 0.44 – 0.00018 T (T in K)
- Sphere surface temperature (T_1) = 810 K

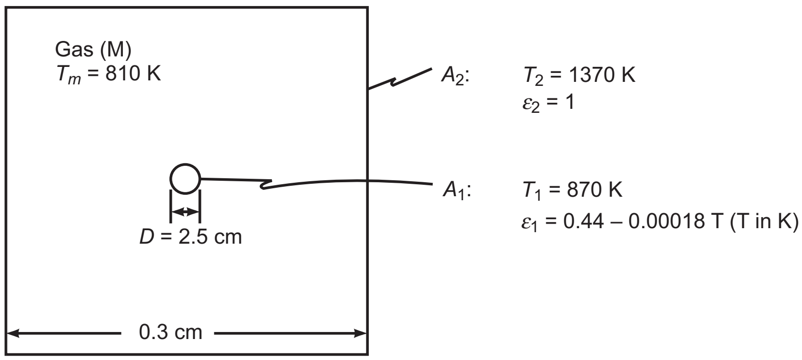
FIND

(a) Total irradiation received by the walls from the sphere (q_2)
(b) The net heat transfer by radiation between the sphere and the walls (q_{12})
(c) The radiant heat transfer coefficient (h_r)

ASSUMPTIONS

- The gas is a gray body
  The oven walls are black ($\varepsilon_2 = 1$)
- The sphere is near the center of the oven

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) At T_1 = 810 K ε_1 = 0.44 – 0.00018 (810) = 0.294
The surface area of the sphere is

$$A_1 = \pi(2.5 \times 10^{-2}\text{ m})^2 = 1.96 \times 10^{-3}\text{ m}^2$$

Since the air and the oven completely enclose the sphere, F_{12} = 1.0 and F_{1g} = 1.0
From Section 9.7.3, the portion of the total radiation leaving the sphere that is received by the walls (q_{1R2}) = $J_1 A_1 F_{12} \tau_m$ where τ_m is the transmissivity of the air and the radio sity of the sphere, J_1, is given by Equation 9.72

$$J_1 = E_{b1} - q_1 \frac{1-\varepsilon_1}{A_1 \varepsilon_1}$$

Since the air temperature is the same as the sphere temperature, there will be no net heat transfer between the air and the sphere and the heat transfer from the sphere (q_1) will be the same as the net heat transfer between the sphere and the walls (q_{12}). Simplifying Equation (9.109) with the shape factors above and
$A_1 << A_2$, $\varepsilon_2 = 1$ yields

$$q_{1-2} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1\left(\tau_m + \frac{1}{\varepsilon_m}\right)}}$$

The beam length to calculate τ_m and ε_m can be found in Table 9.7. Since the sphere is near the center of the cube, one half the beam length for a cube will be used

$$L_{eff} = \frac{L_{cube}}{2} = \frac{\left(\frac{2}{3}\right)(\text{edge length})}{2} = 0.1\text{ m}$$

The partial pressure of the water vapor is

$P_{H2O} = 3\%\, p_{air} = 0.03(1 \text{ atm}) = 0.03 \text{ atm}$

The emissivity of the water vapor (ε_m) can be calculated from Figure 9.46 where $T_m = 811$ K

$$P_{H2O}\, L = (0.03 \text{ atm})\,(0.1 \text{ m}) = 0.0030 \text{ atm m} \Rightarrow \varepsilon_m = 0.012$$

By Kirchoff's radiation law

$$\alpha_m = \varepsilon_m = 0.012$$

Also $\tau_m = 1 - \varepsilon_m = 1 - 0.012 = 0.988$

$$q_{1-2} = \frac{\left(5.67\times10^{-8}\ \text{W/(m}^2\,\text{K}^4)\right)[(810\,\text{K})^4 - (1370\,\text{K})^4]}{\dfrac{1-0.294}{0.294(1.96\times10^{-3}\ \text{m}^2)} + \dfrac{1}{1.96\times10^{-3}\ \text{m}^2\left(0.988 + \dfrac{1}{0.012}\right)}} = -\,143 \text{ W}$$

(b) The net radiation between the sphere and walls is 143 W from the walls to the sphere.

(a) $$J_1 = (5.67\times10^{-8}\ \text{W/(m}^2\,\text{K}^4))\,(810\text{ K})^4 - (-\,143\text{ W})\,\frac{1-0.294}{(1.96\times10^{-3}\ \text{m}^2)\,0.294}$$

$\Rightarrow$ $$J_1 = 199.6\ \text{kW/m}^2$$

$$q_{1R2} = J_1\, A_1\, F_{12}\, \tau_m = 199.6\times10^3\ \text{W/m}^2\,(1.96\times10^{-3}\ \text{m}^2)\,(1.0)\,(0.988)$$

$$= 386.5 \text{ W}$$

(c) The radiative heat transfer coefficient must satisfy the following equation

$$q_{12} = \bar{h}_r\, A_1\, \Delta_T = \bar{h}_r\, A_1\,(T_2 - T_1)$$

$$\bar{h}_r = \frac{q_{12}}{A_1(T_2 - T_1)} = \frac{143\,\text{W}}{(1.96\times10^{-3}\ \text{m}^2)(1370\,\text{K} - 810\,\text{K})} = 130.3\ \text{W/(m}^2\,\text{K)}$$

PROBLEM 9.31

A 0.61 m radius hemisphere (811 K surface temperature) is filled with a gas mixture at 533 K and 2-atm pressure containing 6.67 percent CO_2 and water vapor at 0.5 percent relative humidity. Determine the emissivity and absorptivity of the gas, and the net rate of radiant heat flow to the gas.

GIVEN

- A hemisphere filled with a gas mixture
- Hemisphere radius (r) = 0.61 m
- Hemisphere surface temperature (T_1) = 811 K
- Gas temperature (T_m) = 533 K
- Gas pressure (p_T) = 2 atm
- Gas mixture: 6.67% of CO_2 and water vapor at R.H. = 0.5%

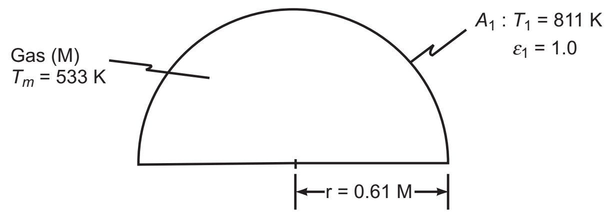
FIND

(a) The emissivity (ε_m) and absorptivity (τ_m) of the gas
(b) The net rate of radiant heat flow to the gas (q_{1-m})

ASSUMPTIONS

- The hemisphere surface is black ($\varepsilon_1 = 1$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Appendix 2, Table 13, the saturation pressure of water at 533 K ($P_{sat,H2O}$) = 4.694×10^6 N/m^2

SOLUTION

(a) The path length for the hemisphere is

$$L = 3.4\ \frac{\text{volume}}{\text{area}} = 3.4\ \frac{\frac{1}{2}\left(\frac{4}{3\pi r^3}\right)}{2\pi r^2 + \pi r^2} = 3.4\ (2/9)\ r = 0.461\text{ m}$$

For a relative humidity of 0.5% , the partial pressure of the water vapor is

$$p_{H2O} = \text{(R.H.)}\ (p_{sat,H2O}) = (0.005)\ \left(46.940\times 10^5\text{ N/m}^2\right)\left(\frac{1\,\text{atm}}{101{,}330\,\text{N/m}^2}\right) = 0.232\text{ atm}$$

$$p_{H2O}\,L = (0.232\text{ atm})\ (0.461\text{ m}) = 0.107\text{ atm m}$$

From Figure 9.46, for 1 atm pressure $(\varepsilon_{H2O})_{pT=1}$ = 0.18
This must be corrected for the total pressure

$$\frac{p_{H2O} + p_t}{2} = \frac{0.232\,\text{atm} + 2\,\text{atm}}{2} = 1.116\text{ atm}$$

In Figure 9.48, $C_{H2O} \approx 1.5$
Repeating this procedure for the CO_2
Partial pressure

$$p_{CO2} = p_T\,(6.67\%) = 2\text{ atm }(0.0667\text{ m}) = 0.733\text{ atm}$$

$$p_{CO2}\,L = (0.133\text{ atm})(0.461\text{ m}) = 0.061\text{ atm m}$$

From Figure 9.47, $(\varepsilon_{CO2})_{pT=1} \approx 0.085$
From Figure 9.49, $C_{CO2} \approx 1.25$
The emissivity of the mixture (ε_m) is given by Equation (9.114)

$$\varepsilon_m = C_{H2O}\ (\varepsilon_{H2O})_{pT=1} + C_{CO2}\ (\varepsilon_{CO2})_{pT=1} - \Delta\varepsilon$$

where Δe is determined by interpolating between values from Figure 9.50

$$\frac{p_{H2O}}{p_{CO2} + p_{H2O}} = \frac{0.232}{0.1334+0.232} = 0.635$$

$$p_{CO2}\,L = p_{H2O}\,L = (0.061 + 0.107\text{ atm m} = 0.168\text{ atm m}$$

At T_m = 400 K $\Delta\varepsilon \approx 0.006$ and at T_m = 811 K: $\Delta\ \varepsilon \approx 0.007$

Therefore

At T_m = 811 K: $\Delta\varepsilon \approx 0.0063$

$$\varepsilon_m = 1.5(0.18) + (1.25)(0.085) - 0.0063 = 0.37$$

$$\tau_m = 1 - \varepsilon_m = 1 - 0.37 = 0.63$$

(b) To evaluate the rate of heat transfer, the emissivity and absorptivity must be evaluated at the surface temperature

$$\text{At } T_s = 811\text{ K: } p_{H2O} L = (0.107 \text{ atm m}) \frac{T_s}{T_{H2O}} = 0.107 \text{ atm m}) \left(\frac{811}{533}\right) = 0.163 \text{ atm m}$$

From Figure (9.46), $\varepsilon'_{H2O} \approx 0.19$

From Equation (9.115)

$$a_{H2O} = C_{H2O}\, \varepsilon'_{H2O} \left(\frac{T_{H2O}}{T_s}\right)^{0.45} = 1.5(0.19) \left(\frac{533}{811}\right)^{0.45} = 0.236$$

At T_s =811 K: $P_{CO2}\, L = (0.061 \text{ atm m}) \left(\frac{811}{533}\right) = 0.093$

From Figure (9.47), $\varepsilon'_{CO2} \approx 0.11$

$$\alpha_{CO2} = 1.25(0.11) \left(\frac{533}{811}\right)^{0.65} = 0.105$$

The total absorptivity is the sum of the H_2O and CO_2 absorptivities

$$\alpha_G = 0.236 + 0.105 = 0.341$$

The rate of heat transfer is given by Equation (9.117)

$$q_r = \sigma A_G (\varepsilon_G T_G^4 - \alpha_G T_s^4) = \sigma (2\pi r^2 + \pi r^2) (\varepsilon_m T_m^4 - \alpha_G T_7^4)$$

$$q_r = \left(5.67\times10^{-8} \text{ W/(m}^2\text{K}^4)\right) [3\pi(0.61 \text{ m})^2][0.37(533 \text{ K})^4 - 0.34(811 \text{ K})^4]$$

$$q_r = -2.33 \times 10^4 \text{ W (from the surface to the gas)}$$

PROBLEM 9.32

Two infinitely large black plane surfaces are 0.3 m apart and the space between them is filled by an isothermal gas mixture at 811 K and atmospheric pressure consisting of 25% CO_2, 25% H_2O, and 50% N_2 by volume. If one of the surfaces is maintained at 278 K and the other at 1390 K respectively, calculate

(a) the effective emissivity of the gas at its temperature

(b) the effective absorptivity of the gas to radiation from the 1390 K surface

(c) the effective absorptivity of the gas to radiation from the 278 K surface

(d) the net rate of heat transfer to the gas per square meter of surface area

GIVEN

- Two infinitely large black plane surfaces with an isothermal gas mixture between them
- Distance between surfaces (s) = 0.3 m
- Gas mixture temperature (T_m) = 811 K
- Gas mixture pressure = 1 atm
- Gas contents: 25% CO_2, 25% H_2O, 50 % N_2 by volume
- Surface temperatures
  - T_1 = 278 K
  - T_2 = 1390 K

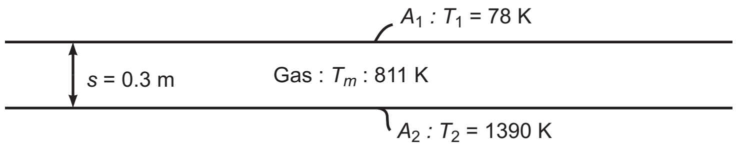
FIND

(a) The effective emissivity of the gas at its temperature (ε_{mix})
(b) The effective absorptivity of the gas to radiation from A_1
(c) The effective absorptivity of the gas to radiation from A_2
(d) The net rate of heat transfer to the gas per square meter of surface are (q_m/A)

ASSUMPTIONS

- Steady state
- Convection is negligible

SKETCH



SOLUTION

(a) The partial pressures of the CO_2 and H_2O are both 0.25 atm. The equivalent mean hemi- spherical beam length, L, is from Table 9.7

$$L = 2s = 0.6\text{m}$$

$$p_{CO2}\,L = p_{H2O}\,L = 0.25 \text{ atm } (0.6\text{m}) = 0.15 \text{ atm m}$$

From Figure 9.46 $(\varepsilon_{H2O})_{pT} \approx 0.18$ From Figure 9.47 $(\varepsilon_{CO2})_{pT} \approx 0.11$

$$p_{CO2}\,L + p_{H2O}\,L = 2(0.15 \text{ atm m}) = 0.30 \text{ atm m and } \frac{p_{H2O}}{p_{H2O}+p_{CO2}} = \frac{0.25}{0.50} = 0.5$$

From Figure 9.50 $\Delta\varepsilon \approx 0.014$
From Equations (9.114), (9.113a), and (9.113b)

$$\varepsilon_{mix} = C_{H2O}(\varepsilon_{H2O})_{p_T} + C_{CO2}(\varepsilon_{CO2})_{p_T} - \Delta\varepsilon = (1)\,0.18 + (1)\,0.11 - 0.014 = 0.276$$

(b) To find the absorptivity to radiation from A_1, the emittances of the H_2O and CO_2 must first be evaluated at $T_1 = 278$ K.

Using the procedure shown above with: $p_{H2O}\,L\left(\frac{T_s}{T_{H2O}}\right) = 0.15\left(\frac{278\text{K}}{811\text{K}}\right) = 0.051$

From Figure 9.46, $\varepsilon'_{H2O} \approx 0.11$ From Figure 9.47, $\varepsilon'_{CO2} \approx 0.085$
Applying Equation (9.115)

$$\alpha_{H2O} = C_{H2O}\,\varepsilon'_{H2O}\left(\frac{T_{H2O}}{T_s}\right)^{0.45} = (1)\,(0.11)\left(\frac{811}{278}\right)^{0.45} = 0.178$$

Applying equation (9.116)

$$\alpha_{CO2} = C_{CO2}\,\varepsilon'_{CO2}\left(\frac{T_{CO2}}{T_s}\right)^{0.65} = (1)\,(0.085)\left(\frac{811}{278}\right)^{0.65} = 0.171$$

$$\alpha_1 = \alpha_{H2O} + \alpha_{CO2} = 0.178 + 0.171 = 0.349$$

(c) Repeating this procedure for $T_2 = 1390$ K and $p_{H2O}\,L\,(T_s/T_{H2O}) = 0.257$

From Figure 9.46, $\varepsilon'_{H2O} \approx 0.17$ From Figure 9.47, $\varepsilon'_{CO2} \approx 0.14$

$$\alpha_{H2O} = (0.17)\left(\frac{811}{1390}\right)^{0.45} = 0.133 \quad \text{and} \quad \alpha_{CO2} = (0.14)\left(\frac{811}{1390}\right)^{0.65} = 0.099$$

$$\alpha_2 = 0.133 + 0.099 = 0.232$$

(d) The rate of heat flow from the gas to A_1 is given by Equation (9.117)

$$\frac{q_{r1}}{A}\ \sigma(\varepsilon_{\text{mix}}\, T_m^{\ 4} - \alpha_1\, T_1^{\ 4}) = \left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)[0.276\ (811\ \text{K})^4 - 0.349\ (278\ \text{K})^4] = 6651\ \text{W/m}^2$$

The rate of heat flow from the gas to A_2 is

$$\frac{q_{r2}}{A}\ \sigma(\varepsilon_{\text{mix}}\, T_m^{\ 4} - \alpha_2\, T_2^{\ 4}) = \left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)[0.276(811\ \text{K})^4 - 0.232(1390\ \text{K})^4] = 42{,}336\ \text{W/m}^2$$

The net rate of heat transfer from the gas is

$$\frac{q_m}{A} = \frac{q_{r1}}{A} + \frac{q_{r2}}{A} = (6651 - 42{,}336)\ \text{W} = -35684\ \text{W (gain to gas)}$$

PROBLEM 9.33

A manned spacecraft capsule has a shape of a cylinder 2.5 m in diameter and 9 m long. The air inside the capsule is maintained at 20°C and the convection-heat-transfer coefficient on the interior surface is 17 W/(m^2 K). Between the outer skin and the inner surface is a 15 cm layer of glass-wool insulation having a thermal conductivity of 0.017 W/(m K). If the emissivity of the skin is 0.05 and there is no aerodynamic heating or irradiation from astronomical bodies, calculate the total heat transfer rate into space at 0 K.

GIVEN

- A glass-wool insulated cylinder in space filled with air
- Diameter (D) = 2.5 cm
- Length (L) = 9 m
- Air temperature (T_a) = 20°C = 293 K
- Interior convective heat transfer coefficient (h_c) = 17 W/(m^2 K)
- Insulation thickness (t) = 15 cm = 0.15 m
- Thermal conductivity of insulation (k) = 0.017 W/(m K)
- Emissivity of the skin (ε) = 0.05
- No aerodynamic heating or irradiation from astronomical bodies

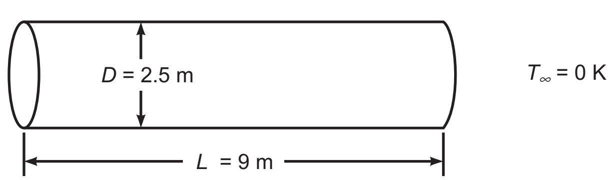
FIND

- The total rate of heat transfer into space at T_∞ = 0 K

ASSUMPTIONS

- Steady state
- Thermal resistance of the capsule walls is negligible compared to that of the insulation

SKETCH



PROPERTIES AND CONSTANTS

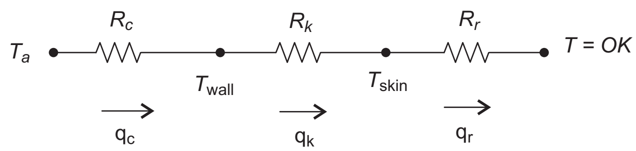
From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

Since $D >> t$ the effect of the cylinder's curvature can be neglected. The total surface area is

$$A = \pi D\, L + 2\,\frac{\pi}{4} D^2 = \pi D \left(L + \frac{D}{2} \right) = \pi\ (2.5\text{m})(9\text{m} + 1.25\text{m}) = 80.5\ \text{m}^2$$

The thermal circuit for the problem is shown below



where R_c = convective thermal resistance

R_k = conductive thermal resistance of the insulation

R_r = radiative thermal resistance

q_c = convective heat transfer rate to the interior wall = $h_c A\ (T_a - T_{\text{wall}})$

q_k = conductive heat transfer rate to the insulation = $(k/t)A(T_{\text{wall}} - T_{\text{skin}})$

q_r = radiative heat transfer from the skin = $\sigma\,\varepsilon\,T_{\text{skin}}^4$

For steady state, all three rates of heat transfer must be equal

$$h_c(T_a - T_{\text{wall}}) = \frac{k}{t}\,(T_{\text{wall}} - T_{\text{skin}}) = \sigma\,\varepsilon\,T_{\text{skin}}^4$$

solving for the wall temperature

$$T_{\text{wall}} = \frac{T_a + \dfrac{k}{t\bar{h}_c} T_{\text{skin}}}{1 + \dfrac{k}{t\bar{h}_c}} \qquad \text{Let } B = \frac{k}{t\bar{h}_c} = \frac{0.017\,\text{W/(m K)}}{0.15\,\text{m}\left(17\,\text{W/(m}^2\text{K)}\right)} = 0.00667$$

$$\therefore \quad \bar{h}_c \left(T_a - \frac{T_a + BT_{\text{skin}}}{1+B} \right) = \sigma\,\varepsilon\,T_{\text{skin}}^4$$

$$\sigma\,\varepsilon\,T_{\text{skin}}^4 - \bar{h}_c \left(T_a - \frac{T_a + BT_{\text{skin}}}{1+B} \right) = 0$$

$$\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\ (0.05)T_{\text{skin}}^4 - \left(17\,\text{W/(m}^2\text{K)}\right) \left(293\,\text{K} - \frac{293\,\text{K} + 0.00667\left(T_{\text{skin}}\right)}{1 + 0.00667} \right) = 0$$

Checking the units, then eliminating them for clarity

$$2.835 \times 10^{-9}\ T_{\text{skin}}^4 + 0.1126\ T_{\text{skin}} - 33.0 = 0$$

By trial and error

$$T_{\text{skin}} = 227\ \text{K}$$

$$q = \sigma\,\varepsilon\,A\ T_{\text{skin}}^4 = \left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(0.05)\ (80.5\text{m}^2)\ (227\ \text{K})^4 = 610\ \text{W}$$

PROBLEM 9.34

A 1 m × 1 m square solar collector is placed on the roof of a house. The collector receives a solar radiation flux of 800 W/m^2. Assuming that the surroundings act as a blackbody at an effective sky temperature of 30°C, calculate the equilibrium temperature of the collector (a) assuming its surface is black and the conduction and convection are negligible, and (b) assuming that the collector is horizontal and loses heat by natural convection.

GIVEN

- A square solar collector on the roof of a house
- Collector dimensions = 1 m × 1 m
- Solar flux on collector (q_s) = 800 W/m^2

FIND

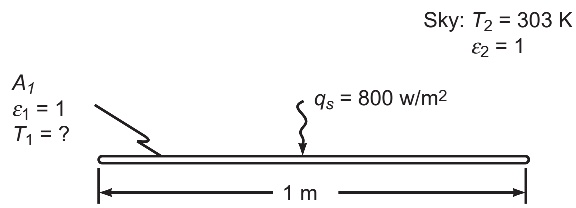
The equilibrium temperature of the collector (T_1) assuming

(a) The collector surface is black (ε_1 = 1) and conduction and convection are negligible
(b) The collector is horizontal and loses heat by natural convection

ASSUMPTIONS

- Steady state conditions
- The surroundings act as a blackbody at an effective sky temperature (T_2) = 30°C = 303 K
- The surrounding air temperature (T_∞) = T_2 = 303 K

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table *5*, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

(a) For equilibrium, the heat loss flux by radiation to the sky must equal the incident solar flux

$$q_s = \sigma\,\varepsilon_1\,(T_1^4 - T_2^4)$$

Solving for the collector temperature

$$T_1 = \left(\frac{q_s}{\sigma\varepsilon_1} + T_2^4\right)^{0.25} = \left[\frac{(800\,\text{W/m}^2)}{(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4))(1)} + (303\,\text{K})^4\right]^{0.25} = 387\text{ K} = 114\text{°C}$$

(b) The heat loss flux by radiation and convection must equal the incident solar flux

$$q_s = \sigma\,\varepsilon_1\,(T_1^4 - T_2^4) + h_c\,(T_1 - T_\infty)$$

The natural convection heat transfer coefficient depends on the collector temperature, T_1. Therefore, an iterative solution is required. Natural convection will tend to lower the collector temperature calculated in part (a). For the first iteration, let T_1 = 363 K.

From Appendix 2, Table 27, for dry air at the film temperature of (363 K + 303 K) = 333 K

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Raleigh number is

$$Ra_L = Gr_L\, Pr = \frac{g\beta(\Delta T)\, L^3 Pr}{\nu^2} = \frac{(9.8\,\text{m/s})(0.0031/\text{K})(363\,\text{K}-303\,\text{K})(1\,\text{m})^3\,(0.71)}{\left(19.4\times10^{-6}\,\text{m}^2/\text{s}\right)} = 3.33 \times 10^9$$

The Nusselt number for a horizontal plate with upper surface heated in this Raleigh number range is given by Equation (5.16)

$$\overline{Nu}_L = 0.15\ Ra_L^{\frac{1}{3}} = 0.15\ (3.33\times10^9)^{\frac{1}{3}} = 224$$

$$\bar{h}_c = \overline{Nu}_L \frac{k}{L} = 224\ \frac{(0.0279\,\text{W/(m K)})}{1\,\text{m}} = 6.25\ \text{W/(m}^2\ \text{K)}$$

Using this in the energy balance

$$800\ \text{W/m}^2 = \left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)(1)\ [T_1^{\,4} - (303\ \text{K})^4] + \left(6.25\,\text{W/(m}^2\text{K)}\right)(\text{T1} - 303\ \text{K})$$

$$5.67 \times 10^{-8}\ T_1^{\,4} + 6.25\ T_1 - 3171.7 = 0$$

By trial and error T_1 = 358 K

The properties of air will not change enough to justify another iteration.

PROBLEM 9.35

A thin layer of water is placed in a pan 1 m in diameter in the desert. The upper surface is exposed to 300 K air and the convection heat transfer coefficient between the upper surface of the water and the air is estimated to be 10 W/(m^2 K). The effective sky temperature depends on atmospheric conditions and is often assumed to be 0 K for a clear night and 200 K for a cloudy night. Calculate the equilibrium temperature of the water on a clear night and a cloudy night.

GIVEN

- A thin layer of water in a circular pan in the desert
- Pan diameter (D) = 1 m
- Air temperature (T_∞) = 300 K
- Convective heat transfer coefficient (h_c) = 10 W/(m^2 K)
- Effective sky temperature (T_2) = 0 K for a clear night, 200 K for a cloudy night

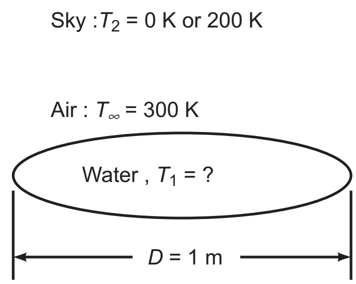
FIND

- The equilibrium temperature of the water (T_1) (a) on a clear night and (b) on a cloudy night

ASSUMPTIONS

- Steady state conditions
- The effect of the sides of the pan is negligible
- Heat transfer to the ground is negligible
- Edge losses are negligible
- Neglect losses due to evaporation

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Table 9.2, the emissivity of water (ε) ≈ 0.96

SOLUTION

For equilibrium, the heat gain by convection to the water must equal the heat loss by radiation

$$\bar{h}_c (T_\infty - T_1) = \varepsilon\, \sigma (T_1^4 - T_2^4)$$

(a) For T_2 = 0 K

$$\varepsilon\, \sigma T_1^4 - \bar{h}_c (T_\infty - T_1) = 0$$

$$(0.96)\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\ T_1^4 - \left(10\ \text{W/(m}^2\text{K)}\right)\ (300\ \text{K} - T_1) = 0$$

Checking units, then eliminating for clarity

$$5.443 \times 10^{-8}\ T_1^4 + 10\ T_1 - 3000 = 0$$

By trial and error T_1 = 271 K = –2°C (water will freeze)
(b) For T_2 = 200 K

$$5.443 \times 10^{-8}\ T_1^4 + 10\ T_1 - 3087.1 = 0$$

$$T_1 = 277\ \text{K} = 4°\text{C}$$

PROBLEM 9.36

Liquid nitrogen is stored in a dewar made of two concentric spheres with the space between them evacuated. The inner sphere has an outside diameter of 1 m and the space between the two spheres is 0.1 m. The surfaces of both spheres are gray with an emissivity of 0.2. If the saturation temperature for nitrogen at atmospheric pressure is 78 K and its latent heat of vaporization is 2 × 10^5 J/kg, estimate its boil-off rate under the following conditions

(a) The outer sphere is at 300 K.

(b) The outer surface of the surrounding sphere is black and loses heat by radiation to surroundings at 300 K. Assume convection is negligible.

(c) Repeat item (b) but include the effect of heat loss by natural convection.

GIVEN

- Liquid nitrogen in two concentric spheres with the space between them evacuated
- Inner sphere diameter (D_i) = 1 m
- Space between spheres (s) = 0.1 m
- Both surfaces are gray with equal emissivites ($\varepsilon_1 = \varepsilon_2$) = 0.2

- Saturation temperature of nitrogen (T_n) = 78 K
- Latent heat of vaporization of nitrogen (h_{fg}) = 2×10^5 J/kg

FIND

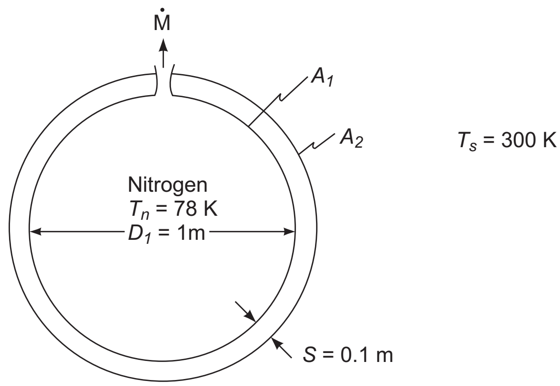
The boil-off rate $(\dot{m})$ under the following conditions

(a) Outer sphere temperature (T_2) = 300 K
(b) Outer surface of outer sphere is black (ε_o = 1) and loses heat by radiation to surroundings at (T_s) = 300 K, convection is negligible, and
(c) Repeat part (b) but include natural convection

ASSUMPTIONS

- Steady state
- Thermal resistance of the sphere walls is negligible
- Thermal resistance between the nitrogen and the inner sphere is negligible ($T_1 = T_n$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

$D_2 = D_1 + 0.2 = 1.2$ m

(a) The heat transfer is given by Equation (9.75)

$$q_{12} = A_1 \,\mathrm{F}_{12}\, (E_{b1} - E_{b2}) = \pi D_i^2 \,\mathrm{F}_{12}\, \sigma (T_1^4 - T_2^4)$$

where f_{12} for concentric spheres is given by Equation (9.76)

$$\mathrm{F}_{12} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2}} = \frac{1}{\frac{1-0.2}{0.2} + 1 + \left(\frac{\pi(1\,\mathrm{m})^2}{\pi(1.2\,\mathrm{m})^2}\right)\frac{1-0.2}{0.2}} = 0.129$$

$$q_{12} = \pi (1\text{ m})^2\, (0.129)\, \left(5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}\right)\, [(78\text{ K})^4 - (300\text{ K})^4]$$

$$q_{12} = -\,185.3\text{ W (heat gained by nitrogen)}$$

The boil-off rate of nitrogen is given by

$$\dot{m} = \frac{q_{12}}{h_{fg}} = \frac{185.3\,\mathrm{W(J/(W\,s))(3600\,s/h)}}{2\times10^5\ \mathrm{J/kg}} = 3.3\text{ kg/h}$$

(b) A heat balance on the outer sphere yields

$$q_{12} = q_{2s}$$

$$A_1 \, F_{12} \, \sigma (T_1^4 - T_2^4) = A_2 \, \sigma (T_2^4 - T_s^4)$$

$$T_2 = \left[\frac{T_s^4 + \frac{A_1}{A_2} F_{12} T_1^4}{1 + \frac{A_1}{A_2} F_{12}} \right]^{0.25} = \left[\frac{(300\,\text{K})^4 + \frac{\pi(1\,\text{m})^2}{\pi(1.2\,\text{m})^2}(0.129)(78\,\text{K})^4}{1 + \frac{\pi(1\,\text{m})^2}{\pi(1.2\,\text{m})^2}(0.129)} \right]^{0.25} = 294 \text{ K}$$

$$\therefore \quad q_{12} = \pi(1\text{m})^2(0.129) \, \left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right) \, [(78 \text{ K})^4 - (294 \text{ K})^4] = -170.8\text{W}$$

$$\dot{m} = \frac{q_{12}}{h_{fg}} = \frac{170\,\text{W}(\text{J/(W s)})(3600\,\text{s/h})}{2\times10^5\,\text{J/kg}} = 3.1 \text{ kg/h}$$

(c) A heat balance on the sphere yields: $q_{12} = q_{2s} + q_c$

$$A_1 \, F_{12} \, \sigma (T_1^4 - T_2^4) = A_2 \, \sigma (T_2^4 - T_s^4) + h_c \, A_2 \, (T_2 - T_s)$$

The natural convection heat transfer coefficient, h_c, depends on the temperature T_2, therefore, an iterative solution is required. For the first iteration, let T_2 = 296 K.

From Appendix 2, Table 27, for dry air at the film temperature of (296 K + 300 K)/2 = 295 K = 25°C

Thermal expansion coefficient (β) = 0.00336 1/K
Thermal conductivity (k) = 0.0255 W/(m K)
Kinematic viscosity (ν) = 16.2×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

The Nusselt number for 3-D bodies is given by Equation (5.25)

$$Nu^+ = 5.75 + 0.7511 \left[\frac{Ra^+}{F(Pr)} \right] 0.252$$

$$\text{where} \quad F(Pr) = \left[1 + \left(\frac{0.49}{Pr} \right)^{\frac{9}{16}} \right]^{\frac{16}{9}} = 2.876$$

$$L^+ = \frac{A}{\left(\frac{4\,A_{\text{horz}}}{\pi} \right)^{0.5}} = \frac{\pi D_2^2}{\left[\frac{4}{\pi} \left(\frac{\pi}{4} D_2^2 \right) \right]^{0.5}} = \pi D = \pi(1.2\text{m}) = 3.77 \text{ m}$$

The Rayleigh number is

$$Ra^+ = Gr^+ Pr = \frac{g\beta(\Delta T)\left(L^+\right)^3 Pr}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\left(0.00336\frac{1}{\text{K}}\right)(4\,\text{K})(3.77\,\text{m})^3(0.71)}{(16.2\times10^{-6}\,\text{m}^2/\text{s})^2} = 1.91 \times 10^{10}$$

Although this is outside of the Rayleigh number range for the above correlation, the correlation will be used to estimate the Nusselt number for lack of a better method

$$Nu^+ = 5.75 + 0.75 \left(\frac{1.91\times10^{10}}{2.876} \right)^{0.252} = 229$$

$$h_c = Nu^+ \frac{\text{K}}{L^+} = 229 \, \frac{(0.0255\,\text{W/(m K)})}{3.77\,\text{m}} = 1.55 \text{ W/(m}^2\text{K)}$$

Using this value in the heat balance

$$\pi(1\text{m})^2(0.129)\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)[(78\ \text{K})^4 - T_2^4] = \pi(1.2\text{m})^2$$

$$\left[\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)[T_2^4-(300\,\text{K})^4]+\left(1.55\,\text{W/(m}^2\text{K)}\right)(T_2-300\,\text{K})\right]$$

$$2.795\times10^{-7}\ T_2^4 + 7.01\ T_2 - 4182 = 0$$

By trial and error

$$T_2 = 295\ \text{K}$$

The effect of natural convection is negligible

$$\dot{m} = 3.1\ \text{kg/h}$$

PROBLEM 9.37

A Package of electronic equipment is enclosed in a sheet-metal box which has a 0.3 m square base and is 0.15 m high. The equipment uses 1200 W of electrical power and is placed on the floor of a large room. The emissivity of the walls of the box is 0.80 and the room air and the surrounding temperature is 21°C. Assuming that the average temperature of the container wall is uniform, estimate that temperature.

GIVEN

- A sheet metal box of electronics in a large room
- Box dimensions: 0.3 m × 0.3 m × 0.15 m high
- Power dissipation of electronics $(\dot{q}_G)$ = 1200 W
- Emissivity of the walls of the box (ε) = 0.80
- Room air and surrounding temperature (T_∞) = 21°C = 294 K

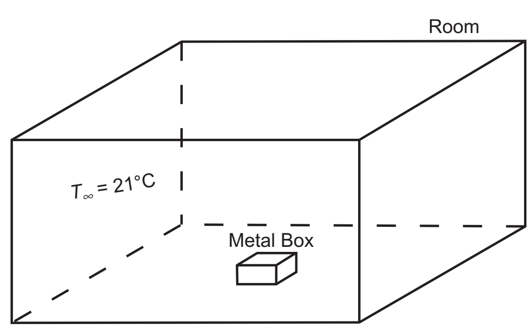
FIND

- The average temperature of the container walls (T_b)

ASSUMPTIONS

- The average temperature of the container walls is uniform
- Steady state
- The room behaves as a blackbody enclosure
- Heat loss from the bottom of the box is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The heat loss by natural convection and radiation from the box must equal the rate of electrical power dissipation.

$$q_{c,T} + q_{c,\text{sides}} + q_r = \dot{q}_G$$

$$(A_{\text{top}}\, h_{c,\text{top}} + A_{\text{sides}}\, h_{c,\text{sides}})\,(T_b - T_\infty) + \sigma\,\varepsilon\,(A_{\text{top}} + A_{\text{sides}})\,(T_b^{\,4} - T_\infty^{\,4})$$

The natural convection heat transfer coefficients are dependent on Tb, therefore, an iterative solution must be used. The initial guess for the box temperature will be based on the box temperature neglecting convection

$$q_r = \dot{q}_G$$

$$\sigma\,\varepsilon\,(A_{\text{top}} + A_{\text{sides}})\,(T_b^{\,4} - T_\infty^{\,4}) = \dot{q}_G$$

$$T_b = \left(\frac{\dot{q}_G}{\sigma\varepsilon\left(A_{\text{top}} + A_{\text{sides}}\right)} + T_\infty^{\,4}\right)^{0.25}$$

$$T_b = \left(\frac{1200\,\text{W}}{\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)0.80\left(0.09\,\text{m}^2 + 0.18\,\text{m}^2\right)} + (294\,\text{K})^4\right)^{0.25} = 570\text{ K}$$

Natural convection will cause the box temperature to be lower than this value. For a first guess, let T_b = 500 K

From Appendix 2, Table 27, for dry air at the film temperature of 397 K (124°C)

Thermal expansion coefficient (β) = 0.00254 1/K

Thermal conductivity (k) = 0.0322 W/(m K)

Kinematic viscosity (ν) = 26.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Grashof number, based on the length of a side of the top of the box is

$$Gr_L = \frac{g\beta(T_b - T_\infty)\,L^3}{\nu^2} = \frac{\left(9.8\,\text{m/s}^2\right)\left(0.00254\,\frac{1}{\text{K}}\right)(500\,\text{K} - 294\,\text{K})(0.3\,\text{m})^3}{\left(26.4\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1.98 \times 10^8$$

The Nusselt number for the top of the box is given by Equation (5.16)

$$\overline{Nu_L} = 0.15\,(Gr_L\,Pr)^{\frac{1}{3}} = 0.15\,[(1.98\times10^8)\,(0.71)]^{\frac{1}{3}} = 78.0$$

$$\bar{h}_{c,\text{top}} = \overline{Nu_L}\,\frac{k}{L} = 78.0\,\frac{(0.0322\,\text{W/(m K)})}{0.3\,\text{m}} = 8.37\text{ W/(m}^2\text{K)}$$

The Grashof number for the sides of the box is

$$Gr_H = \frac{g\beta(T_b - T_\infty)\,H^3}{\nu^2} = \frac{\left(9.8\,\text{m/s}^2\right)\left(0.00254\,\frac{1}{\text{K}}\right)(500\,\text{K} - 294\,\text{K})(0.15\,\text{m})^3}{\left(26.4\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 2.48 \times 10^7$$

The Nusselt number is given by Equation (5.12b)

$$\overline{Nu}_H = 0.68\ Pr^{\frac{1}{2}}\ \frac{\mathrm{Gr_H}^{\frac{1}{4}}}{\left(0.952+\mathrm{Pr}^{\frac{1}{4}}\right)} = 0.68\,(0.71)^{\frac{1}{2}}\ \frac{\left(2.48\times10^{7}\right)^{\frac{1}{4}}}{(0.952+0.71)^{\frac{1}{4}}} = 35.62$$

$$\bar{h}_{c,\mathrm{sides}} = \overline{Nu}_H\ \frac{k}{H} = 35.62\,\frac{(0.0322\ \mathrm{W/(m\,K)})}{0.15\,\mathrm{m}} = 7.65\ \mathrm{W/(m^2\ K)}$$

Substituting these into the energy balance

$$\left[(0.09\,\mathrm{m}^2)\left(8.37\,\mathrm{W/(m^2K)}\right)+(0.18\,\mathrm{m}^2)\left(7.65\,\mathrm{W/(m^2K)}\right)\right]\ (T_b-294\ \mathrm{K})$$

$$+\ \left(5.67\times10^{-8}\ \mathrm{W/(m^2K^4)}\right)\ (0.8)(0.27\ \mathrm{m}^2)\ [T_b^{\,4}-(294\ \mathrm{K})^4] = 1200\ \mathrm{W}$$

Checking the units, then eliminating for clarity

$$1.225\times10^{-8}\ T_b^{\,4} + 2.130\ T_b - 1918 = 0$$

By trial and error: T_b = 510 K

Performing another iteration yields the following results

Film temperature = 402 K

k = 0.0325 W/(m K)

β = 0.00251 1/K

$\nu = 27.1\times10^{-6}\ \mathrm{m^2/s}$

Pr = 0.71

$h_{c,\mathrm{top}} = 8.41\ \mathrm{W/(m^2\ K)}$

$h_{c,\mathrm{sides}} = 7.69\ \mathrm{W/(m^2\ K)}$

T_b = 510 K = 237°C

COMMENTS

Note that neglecting natural convection leads to an error of 60 K.

PROBLEM 9.38

An 0.2 m *OD* oxidized steel pipe at a surface temperature of 756 K passes through a large room in which the air and the walls are at 38°C. If the heat transfer coefficient by convection from the surface of the pipe to the air in the room is 28 W/(m^2 K), estimate the total heat loss per meter length of pipe.

GIVEN

- An Oxidized steel pipe passes through a large room
- Pipe outside diameter (D) = 0.2 m
- Pipe surface temperature (T_s) = 756 K
- Air and wall temperature (T_∞) = 38°C = 311 K
- Convective heat transfer coefficient (h_c) = 28 W/(m^2 K)

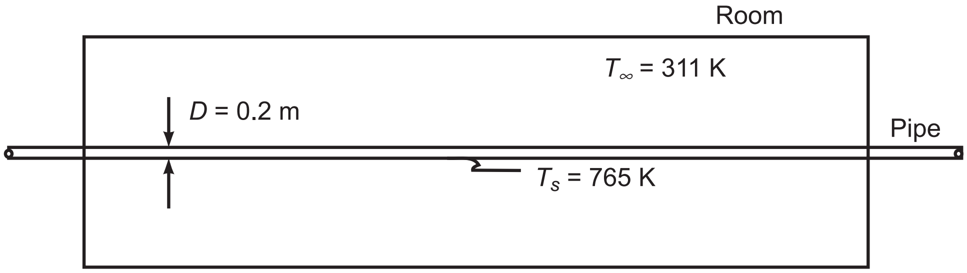
FIND

- The total heat loss per meter of pipe (q/L)

ASSUMPTIONS

- Steady state
- The walls of the room are black (ε_w = 1.0)
- The H_2O and CO_2 in the room air are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Table 9.2, the emissivity of oxidized steel (ε_s) ≈ 0.80

SOLUTION

The total rate of heat transfer is the sum of the convective and radiative rates

$$q = h_c A_t (T_s - T_\infty) + \sigma \varepsilon_s A (T_s^4 - T_\infty^4)$$

where $A = \pi D L$

$$\frac{q}{L} = \pi D [h_c(T_s - T_\infty) + \sigma \varepsilon_s (T_s^4 - T_\infty^4)]$$

$$\frac{q}{L} = \pi (0.2\text{m}) \left[\left(28\,\text{W/(m}^2\text{K)}\right)(756\,\text{K} - 311\,\text{K}) + \left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(0.80)\left[(756\,\text{K})^4 - (311\,\text{K})^4\right]\right]$$

$$q/L = 1.68 \times 10^4 \text{ W/m}$$

PROBLEM 9.39

A 6 mm thick sheet of polished 304 stainless steel is suspended in a comparatively large vacuum-drying oven with black walls. The dimensions of the sheet are 30 cm × 30cm, and its specific heat is 565 J/(kg K). If the walls of the oven are uniformly at 150°C and the metal is to be heated from 10 to 120°C, estimate how long the sheet should be left in the oven if (a) heat transfer by convection may be neglected and (b) the heat transfer coefficient is 3 W/(m^2 K).

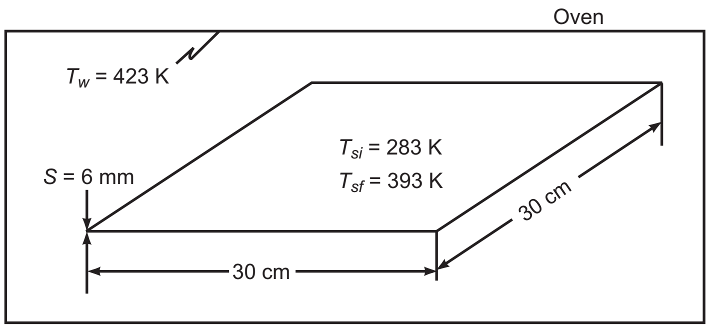
GIVEN

- A sheet of polished stainless steel in a large vacuum drying oven with black walls
- Sheet thickness (s) = 6 mm = 0.006 m
- Sheet dimensions = 30 cm × 30 cm = 0.3 m × 0.3 m
- Specific heat of the sheet (c) = 565 J/(kg K)
- Oven wall temperature (T_w) = 150°C = 423 K
- Sheet temperatures
  - Initial (T_{si}) = 10°C = 283 K
  - Final (T_{sf}) = 120°C = 393 K

FIND

- How long the sheet should be left in the oven if (a) convection may be neglected and, (b) the convective heat transfer coefficient (h_c) = 3 W/(m^2 K)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)
From Appendix 2, Table 10, the thermal conductivity of type 304 stainless steel (k_s) = 14.4 W/(m K) and its density (ρ_s) = 7817 kg/m^3
From Table 9.2, the emissivity of polished stainless steel at the average temperature of 65°C (338 K) (ε_g) = 0.15

SOLUTION

(a) Neglecting convection, the rate of heat transfer is given by

$$q_r = \sigma \varepsilon A\,(T_w^4 - T_s^4)$$

The radiative heat transfer coefficient is given by Equation (9.118)

$$h_r = \frac{q_r}{A(T_w - T_s)} = \sigma \varepsilon \left[\frac{T_w^4 - T_s^4}{T_w - T_s}\right]$$

For the final conditions

$$h_{rf} = \left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)(0.15)\left[\frac{(423)^4-(393\,\text{K})^4}{(423\,\text{K}-393\,\text{K})}\right] = 2.31\ \text{W/(m}^2\text{K)}$$

For the initial conditions

$$h_{rf} = 1.56\ \text{W/(m}^2\text{K)}$$

The Biot number based on half of the sheet thickness is

$$Bi_{\max} = \frac{\overline{h_{r,\max}}\, s}{2K_s} = \frac{\left(2.31\,\text{W/(m}^2\text{K)}\right)(0.006\,\text{m})}{2(14.4\,\text{W/(m K)})} = 0.0005 << 0.1$$

Therefore, the internal thermal resistance of the steel sheet may be neglected. The temperature change of the sheet over a small time step is given by

$$\Delta T = \frac{q\Delta T}{m\,c} = \frac{\sigma\,\varepsilon\,A\left(T_w^{\,4} - T_s^{\,4}\right)\Delta T}{\rho(\text{volume})\,c} = \frac{2\,\sigma\,\varepsilon\left(T_w^{\,4} - T_s^{\,4}\right)\Delta T}{\rho\,s\,c}$$

$$\Delta T = \frac{2\left(5.67\times10^{-8}\ \text{W}/(\text{m}^2\text{K}^4)\right)(0.15)(\text{J}/(\text{Ws}))}{\left(7817\ \text{kg/m}^3\right)(0.006\ \text{m})(565\ \text{J}/(\text{kg K}))}\,[(423\ \text{K})^4 - T_s^4]\,\Delta t = (0.0206 - 6.42\times10^{-13}\,T_s^4)\,\Delta t$$

As the plate heats up, the rate of heat transfer will diminish. Therefore, the following numerical solution will be followed until $T_s = T_{sf}$:

1. Let Δt = 20 min = 1200 s
2. Calculate ΔT using T_{si}
3. Update T_s: $T_s = T_{si} + \Delta T$
4. Use the new T_s to calculate a new ΔT and repeat the procedure

t (min)	ΔT (K)	T_s (K)
0	---	282
20	19.8	301.8
40	18.3	320.1
60	16.6	336.6
80	14.8	351.4
100	12.9	364.3
120	11.1	375.4
140	9.4	384.4
160	7.8	392.5
162	0.6	393.1

The time required = 162 min = 2.7 hours.

(b) The rate of heat transfer by radiation and convection is

$$q = q_c + q_r = h_c\,A\,(T_w - T_s) + \sigma\,\varepsilon\,A\,(T_w^4 - T_s^4)$$

$$\Delta T = \frac{(q_c + q_r)\,\Delta T}{m\,c} = \frac{2\,h_c\,(T_w - T_s) + 2\,\sigma\,\varepsilon\,(T_w^{\,4} - T_s^{\,4})}{\rho\,s\,c}\,\Delta t$$

$$\Delta T = \frac{2\left(3\ \text{W}/(\text{m}^2\text{K})\right)(423\,\text{K} - T_s) + 2\left(5.67\times10^{-8}\ \text{W}/(\text{m}^2\text{K}^4)\right)(0.15)\left[(423\,\text{K})^4 - T_s^{\,4}\right]}{\left(7817\ \text{kg/m}^3\right)(0.006\ \text{m})(565\ \text{J}/(\text{kg K}))((\text{Ws})/\text{J})}\,\Delta t$$

$$\Delta T = (-6.419\times10^{-13}\,T_s^4 - 0.000226\,T_s + 0.1163)\ (\text{K/s})\,\Delta t$$

Following the procedure of part (a), Let Δt = 10 min initially

t (min)	ΔT (K)	T_s (K)
0	---	282
10	29.1	311.1
20	24.0	335.1
30	19.5	354.6
40	15.6	370.2
50	12.3	382.5
60	9.7	392.1
61	0.88	393.1

Time required = 61 min = 1.02 hour.

PROBLEM 9.40

> **Calculate the equilibrium temperature of a thermocouple in a large air duct if the air temperature is 1367 K, the duct-wall temperature 533 K, the emissivity of the thermocouple 0.5, and the convective heat transfer coefficient, h_c, is 114 W/(m^2 K).**

GIVEN

- A thermocouple in a large air duct
- Air temperature (T_a) = 1367 K
- Duct wall temperature (T_d) = 533 K
- The emissivity of the thermocouple (ε_{tc}) = 0.5
- The convective heat transfer coefficient (h_c) = 114 W/(m^2 K)

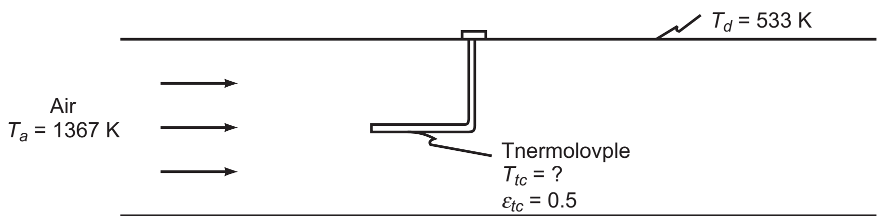
FIND

- The equilibrium temperature of the thermocouple (T_{tc})

ASSUMPTIONS

- Conduction along the thermocouple is negligible
- The walls of the duct are black
- The CO_2 and H_2O in the air are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

In steady state, the heat gain by convection must equal the heat loss by radiation

$$h_c\, A\, (T_a - T_{tc}) = \sigma\, \varepsilon_{tc}\, A\, (T_{tc}^4 - T_d^4)$$

$$\left(114\,\text{W/(m}^2\text{K)}\right)\ (1367\text{K} - T_{tc}) = \left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\ (0.5)\ [T_{tc}^4 - (533\ \text{K})^4]$$

Checking in units, then eliminating them for clarity

$$2.835 \times 10^{-8}\ T_{tc}^4 + 114\ T_{tc} - 158{,}126 = 0$$

By trial and error

T_{tc} = 1066 K.

COMMENTS

Assuming the purpose of the thermocouple is to measure the temperature of the air flowing in the duct, we have an error of 301 K. This so-called thermocouple radiation error can be reduced by increasing the convective heat transfer coefficient via higher air velocity, by reducing the thermocouple emissivity, or by the addition of a radiation shield, see problem 9.41.

PROBLEM 9.41

Repeat Problem 9.40 with the addition of a radiation shield with emissivity ε_s = 0.1.

From Problem 9.40: Calculate the equilibrium temperature of a thermocouple in a large air duct if the air temperature is 1367 K, the duct-wall temperature 533 K, the emittance of the couple 0.5, and the convective heat transfer coefficient, h_c, is 114 W/(m^2 K).

GIVEN

- A thermocouple surrounded by a radiation shield in a large air duct
- Air temperature (T_s) = 1367 K
- Duct wall temperature (T_d) = 533 K
- The emissivity of the thermocouple (ε_{tc}) = 0.5
- The convective heat transfer coefficient (h_c) = 114 W/(m^2 K)
- Shield emissivity (ε_s) = 0.1

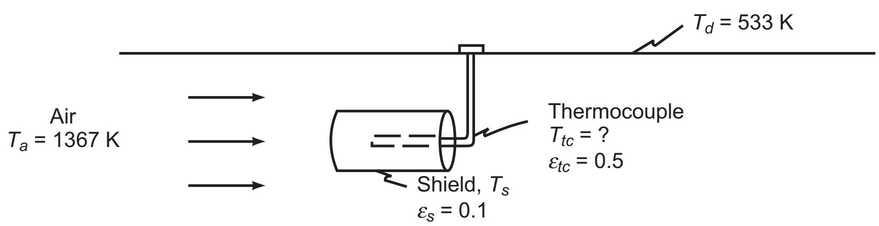
FIND

- The equilibrium temperature of the thermocouple (T_{tc})

ASSUMPTIONS

- Conduction along the thermocouple is negligible
- The walls of the duct are black
- The CO_2 and H_2O in the air are negligible
- The heat transfer coefficient on the inside and outside of the shield is h_c
- The conductive thermal resistance of the shield is negligible
- The view factor between the shield and the thermocouple ≈ 1
- The surface area of the shield is large compared to that of the thermocouple
  Shield and thermocouple are gray
- The thermocouple and shield can be approximated by infinitely long concentric cylinders
- Convective heat transfer between the shield and thermocouples is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/ (m^2 K^4)

SOLUTION

Let A_s = the inside area of the shield ≈ the outside area of the shield.
A heat balance on the radiation shield yields

$$q_s + \overline{h_{cs}}\, 2\, A_s\, (T_a - T_s) = \sigma\, \varepsilon_s\, A_s\, (T_s^4 - T_d^4)$$

Where q_s is the radiative heat transfer to the shield from the thermocouple which is given by Equation (9.74) for long concentric cylinders

$$q_s = \frac{A_{tc}\left(E_{btc} - E_{bs}\right)}{\frac{1}{\varepsilon_{tc}} + \frac{A_{tc}}{A_s}\left(\frac{1-\varepsilon_s}{\varepsilon_s}\right)} \qquad \text{but } \frac{A_{tc}}{A_s} << 1 \qquad \therefore\ q_s = A_{tc}\, \varepsilon_{tc}\, \sigma\, (T_{tc}^4 - T_s^4)$$

Substituting this into the energy balance and dividing by A_s

$$\frac{A_{tc}}{A_s}\, \varepsilon_{tc}\, \sigma (T_{tc}^4 - T_s^4) + 2\,\overline{h_{cs}}\,(T_a - T_s) = \sigma\, \varepsilon_s\, (T_s^4 - T_d^4)$$

$$\therefore \qquad 2\, h_{cs}\,(T_a - T_s) \approx \sigma\, \varepsilon_s\,(T_s^4 - T_d^4)$$

This shown that since the thermocouple is small compared to the shield, the effect of the thermocouple wire on the shield temperature can be neglected

$$2\left(114\,\text{W/(m}^2\text{K)}\right)(1367\text{K} - T_s) = \left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(0.1)\,[T_s^4 - (533\text{ K})^4]$$

$$5.67 \times 10^{-7}\, T_s^4 + 228\, T_{tc} - 312{,}134 = 0$$

By trial and error

$$T_s = 1298\text{ K.}$$

Performing a heat balance on the thermocouple

$$\overline{h_c}\, A_{tc}\,(T_s - T_{tc}) = \sigma\, \varepsilon_{tc}\, A_{tc}\,(T_{tc}^4 - T_s^4)$$

$$\left(114\,\text{W/(m}^2\text{K)}\right)(1367\text{K} - T_{tc}) = \left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(0.5)\,[T_{tc}^4 - (1298\text{ K})^4]$$

$$2.835 \times 10^{-8}\, T_{tc}^4 + 114\, T_{tc} - 236{,}311 = 0$$

By trial and error

$$T_{tc} = 1319\text{ K.}$$

COMMENTS

The thermocouple error has been reduced from 301 K to 48 K by use of the radiation shield.

PROBLEM 9.42

A thermocouple is used to measure the temperature of a flame in a combustion chamber. If the thermocouple temperature is 1033 K and the walls of the chamber are at 700 K, what is the error in the thermocouple reading due to radiation to the walls? Assume all surfaces are black and the convection coefficient is 568 W/(m^2 K) on the thermocouple.

GIVEN

- A thermocouple in a combustion chamber flame
- Thermocouple temperature (T_{tc}) = 1033 K
- Chamber wall temperature (T_w) = 700 K
- Convection coefficient (h_c) = 568 W/(m^2 °C)
- All surfaces are black (ε = 1.0)

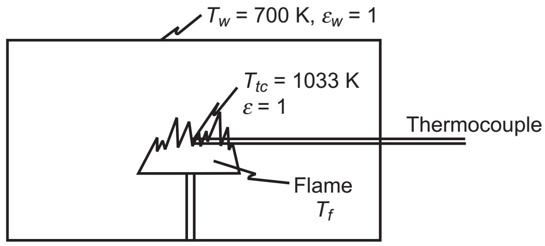
FIND

- The error in the thermocouple reading due to radiation to the walls

ASSUMPTIONS

- Conduction along the thermocouple is negligible
- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

In steady state, the rate of heat gain by convection from the flame must equal the heat loss by radiation to the walls:

$$h_c\, A\, (T_f - T_{tc}) = \sigma\, \varepsilon\, A\, (T_{tc}^4 - T_w^4) \qquad [\varepsilon = 1]$$

Solving for the flame temperature

$$T_f = T_{tc} + \frac{\sigma}{h_c}(T_{tc}^4 - T_w^4) = 1033\text{ K} + \frac{\left(5.678\times10^{-8}\text{ W/(m}^2\text{K}^4)\right)}{\left(568\text{ W/(m}^2\text{K)}\right)}[(1033\text{K})^4 - (700\text{K})^4] = 1123\text{ K}$$

$$\text{Error} = T_f - T_{tc} = 1123 - 1033 = 90\text{ K}$$

PROBLEM 9.43

A metal plate is placed in the sunlight. The incident radiant energy G is 780 W/m^2. The air and the surroundings are at 10°C. The heat transfer coefficient by natural convection from the upper surface of the plate is 17 W/(m^2 K). The plate has an average emissivity of 0.9 at solar wavelengths and 0.1 at long wavelengths. Neglecting conduction losses on the lower surface, determine the equilibrium temperature of the plate.

GIVEN

- A metal plate is sunlight
- Incident radiant energy (G) = 780 W/m^2
- Temperature of air and surroundings (T_∞) = 10°C = 283 K
- Natural convection heat transfer coefficient (h_c) = 17 W/(m^2 K)
- Plate emissivity (ε) = 0.9 at solar wavelengths, 0.1 at long wavelengths

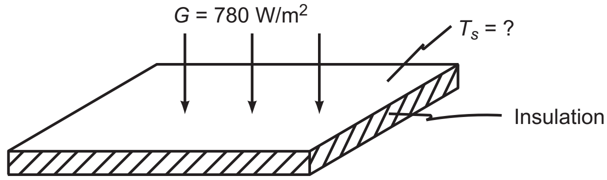
FIND

- The equilibrium temperature of the plate (T_p)

ASSUMPTIONS

- Steady state
- Conduction losses on the lower surface of the plate are negligible
- The surroundings behave as a blackbody enclosure

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The plate will absorb the solar radiation with the absorptivity (α) = ε = 0.9 according to Kirchoff's Law. However, it will radiate to its surroundings at longer infrared wavelengths with ε = 0.1. The heat gain from solar radiation must equal the heat flux loss by radiation and convection at steady state

$$\alpha G = h_c (T_p - T_\infty) + \sigma \varepsilon (T_p^4 - T_\infty^4)$$

$$(0.9)\left(780\,\text{W/m}^2\right) = \left(17\,\text{W/(m}^2\text{K)}\right)(T_p - 283\text{K}) + \left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(0.1)\,[T_p^4 - (283\text{ K})^4]$$

$$5.67 \times 10^{-9}\, T_p^4 + 17\, T_p - 5549.37 = 0$$

By trial and error

$$T_p = 323\text{ K} = 50°\text{C}$$

PROBLEM 9.44

> **A 0.6-m-square section of panel heater is installed in the corner of the ceiling of a room having a 2.7 m × 3.6 m floor area with an 2.4 m ceiling. If the surface of the heater, made from oxidized iron, is at 147°C and the walls and the air of the room are at 20°C in the steady state, determine (a) the rate of heat transfer to the room by radiation, (b) the rate of heat transfer to the room by convection (h_c = 11 W/(m^2 K), (c) the cost of heating the room per day if the cost of electricity is Rs.3.50 per kWh.**

GIVEN

- A 0.6 m square panel heater in the corner of the ceiling of a room
- Room dimensions: 2.7 m × 3.6 m × 2.4 m
- Heater has oxidized iron surface
- Surface temperature (T_s) = 147°C = 420 K
- Room air and walls (T_∞) = 20°C = 293 K
- Convective heat transfer coefficient = 11 W/(m^2 K)

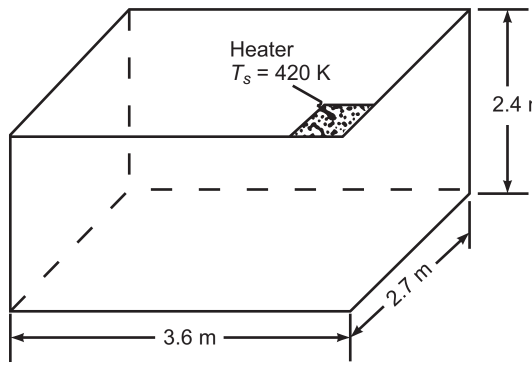
FIND

(a) Rate of radiative heat transfer to the room (q_r)
(b) Rate of convective heat transfer to the room (q_c)
(c) Cost of heating the room at Rs.3.50/(kW/h)

ASSUMPTIONS

- The walls of the room are black (ε_w = 1)
- Steady state conditions
- Effect of H_2O and CO_2 in the air is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Table 9.2, the emissivity of cast oxidized iron at 420 K (ε_s) = 0.64

SOLUTION

(a) Since the view factor of the heater to the room is unity, the rate of heat transfer by radiation is

$$q_r = \sigma \varepsilon_s A (T_s^4 - T_\infty^4)$$

where A = area of heater = 0.36 m^2

$$\Rightarrow \quad q_r = (5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4)) (0.64) [(420 \text{ K})^4 - (293 \text{ K})^4] (0.36 \text{ m}^2) = 310 \text{ W}$$

(b) The rate of heat transfer by convection is

$$q_c = h_c A (T_s - T_{\text{air}}) = (11 \text{ W/(m}^2\text{ K)}) (0.36 \text{ m})^2 (420 - 293)\text{K} = 503 \text{ W}$$

(c)

$$\text{Cost} = (q_r + q_c) \text{ (energy cost)}$$

$$\text{Cost} = \frac{(310 \text{ W} + 503 \text{ W})}{1000 \text{ W/kW}} \times (24 \text{ hr/day}) (\text{Rs.}3.50/\text{kW hr})$$

$$= \text{Rs.}68.3/\text{day}$$

PROBLEM 9.45

In a manufacturing process, a fluid is transported through a cellar maintained at a temperature of 300 K. The fluid is contained in a pipe having an external diameter of 0.4 m and whose surface has an emissivity of 0.5. To reduce heat losses, the pipe is surrounded by a thin shielding pipe having an *ID* of 0.5 m and an emissivity of 0.3. The space between the two pipes is effectively evacuated to minimize heat losses and the inside pipe is at a temperature of 550 K. (a) Estimate the heat loss from the liquid per meter length, (b) If the fluid inside the pipe is an oil flowing at a velocity of 1 m/s, calculate the length of pipe for a temperature drop of 1 K.

GIVEN

- Fluid in concentric pipes, with the space between the pipes evacuated, running through a cellar space
- Cellar temperature (T_∞) = 300 K
- External diameter of inner pipe (D_1) = 0.4 m
- Emissivity of outer pipe surface (ε_1) = 0.5

- Inside diameter of outer pipe (D_2) = 0.5 m
- Emissivity of inner pipe (ε_2) = 0.3
- Inside pipe temperature (T_1) = 550 K

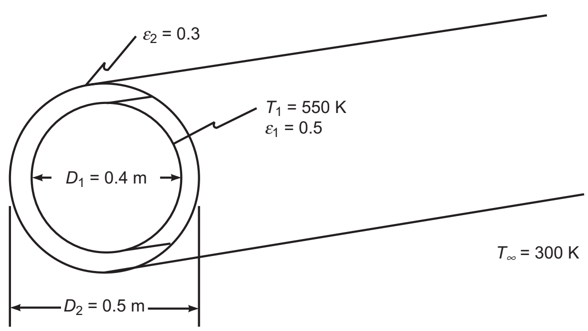
FIND

(a) The heat loss from the liquid per meter length (q/L)
(b) The length of pipe for a temperature drop of 1 K if the fluid is oil flowing at a velocity (V) = 1 m/s

ASSUMPTIONS

- Steady state
- Convection between the pipes is negligible
- The thermal resistance of the pipe walls is negligible
- The thickness of the outer pipe wall is negligible (Inside surface area ≈ Outside surface area)
- Area of the cellar is large compared to the pipe so that cellar behaves as a blackbody enclosure at T_∞
- Oil has the thermal properties of unused engine oil
- The temperature of the inner pipe is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
Extrapolating Appendix 2, Table 16, for unused engine oil at 550 K

Density (ρ) = 742 kg/m^3

Specific heat (c) 2998 J/(kg K)

SOLUTION

The rate of heat transfer between the pipes is given by Equation (9.75)

$$q_{12} = A_1 \mathrm{F}_{12} (E_{b1} - E_{b2}) = A_1 \mathrm{F}_{12}\, \sigma (T_1^{\,4} - T_2^{\,4})$$

where F_{12} is given for infinite concentric cylinders by Equation (9.76)

$$\mathrm{F}_{12} = \frac{1}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + 1 + \dfrac{A_1}{A_2}\dfrac{1-\varepsilon_2}{\varepsilon_2}} = \frac{1}{\dfrac{1-0.5}{0.5} + 1 + \left(\dfrac{\frac{\pi}{4}(0.4\,\mathrm{m})L}{\frac{\pi}{4}(0.5\,\mathrm{m})L}\right)\dfrac{1-0.3}{0.3}} = 0.259$$

The rate of heat transfer from the outer pipe to the surroundings is the sum of the rates of convective and radiative heat transfer

$$q_{2\infty} = \overline{h_c}\, A_2\, (T_2 - T_\infty) = \sigma\, \varepsilon_2\, A_2\, (T_2^{\,4} - T_\infty^{\,4})$$

An energy balance on the outer pipe yields

$$q_{12} = q_{2\infty}$$

$$\mathrm{F}_{12}\, \sigma (T_1^{\,4} - T_2^{\,4}) = \overline{h_c}\, \frac{A_2}{A_1} (T_2 - T_\infty) + \sigma\, \varepsilon_2 \frac{A_2}{A_1} (T_2^{\,4} - T_\infty^{\,4})$$

$$0.259\left(5.67\times10^{-8}\ \mathrm{W/(m^2K^4)}\right)\ [(550\ \mathrm{K})^4 - T_2^{\,4}]$$

$$= \overline{h_c}\left(\frac{0.5}{0.4}\right)\ (T_2 - 300\mathrm{K}) + \left(5.67\times10^{-8}\ \mathrm{W/(m^2K^4)}\right) (0.3)\ \left(\frac{0.5}{0.4}\right) [T_2^{\,4} - (300\ \mathrm{K})^4]$$

Checking the units, then eliminating them for clarity

$$3.595 \times 10^{-8}\ T_2^{\,4} + 1.25\ h_c\ T_2 - 375\ h_c - 375\ h_c - 1516 = 0$$

Since the value of the natural convection heat transfer coefficient, h_c, depends on T_2, an iterative solution must be used. For the first iteration, let $T_2 = 400$ K.

The Grashof number is

$$Gr_D = \frac{g\beta(\Delta T)\, D^3}{\nu_a^{\,2}} = \frac{\left(9.8\ \mathrm{m/s^2}\right)(0.00283\ 1/\mathrm{K})(100\ \mathrm{K})(0.5\mathrm{m})^3}{\left(21.5\times10^{-6}\ \mathrm{m^2/s}\right)^2} = 7.5 \times 10^8$$

$$Gr_D\, Pr = 7.5 \times 10^8\ (0.71) = 5.32 \times 10^8$$

The Nusselt number for this geometry is given by Equation (5.20)

$$\overline{Nu_D} = 0.53\ (Gr_D\ Pr)^{\frac{1}{4}} = 0.53\ (5.32\times10^8)^{\frac{1}{4}} = 80.5$$

$$\overline{h_c} = \overline{Nu_D}\, \frac{k}{D} = 80.5 \frac{(0.0293\ \mathrm{W/(m\,K)})}{0.5\,\mathrm{m}} = 4.72\ \mathrm{W/(m^2K)}$$

Substituting this value into the energy balance yields

$$3.595 \times 10^{-8}\ T_2^{\,4} + 5.896\ T_2 - 3285 = 0$$

By trial and error

$$T_2 = 400\ \mathrm{K}$$

(a) The rate of heat transfer from the liquid is

$$\frac{q}{L} = \pi D_1\ \mathrm{F}_{12}\ \sigma (T_1^{\,4} - T_2^{\,4}) = \pi (0.4\ \mathrm{m})\ (0.259)\ \left(5.67\times10^{-8}\ \mathrm{W/(m^2K^4)}\right)\ [(550\ \mathrm{K})^4 - (400\ \mathrm{K})^4]$$

$$\frac{q}{L} = 1216\ \mathrm{W/m}$$

(b) The length of pipe for a temperature drop (ΔT) of 1 K can be determined from the following

$$\frac{q}{L} = \frac{\dot{m}\, c\, \Delta T}{L} = \frac{\rho V\, A_c\, c\Delta T}{L} = \frac{\rho V \left(\frac{\pi}{4}\right) D_1^{\,2}\, c\Delta T}{L}$$

Solving for L

$$L = \frac{\pi \rho V D_1^2 \, c\Delta T}{4\left(\dfrac{q}{L}\right)} = \frac{\pi\left(742\ \text{kg/m}^3\right)(1\,\text{m/s})(0.4\,\text{m})^2\,(2998\ \text{J/(kg K)})(1\,\text{K})}{(4)(1216\,\text{W/m})(\text{J/(Ws)})} = 230\ \text{m}$$

PROBLEM 9.46

45 kgs of carbon dioxide is stored in a high-pressure cylinder 25 cm in diameter (*OD*), 1.2 m long and 1.2 cm thick. The cylinder is fitted with a safety rupture diaphragm designed to fail at 140 bar (gauge) (with the specified charge, this pressure will be reached when the temperature increases to 50°C). During a fire, the cylinder is completely exposed to the irradiation from flames at 1097°C (ε = 1.0). For the specified conditions, *c* = 2.5 kJ/(kg K) for CO_2. Neglecting the convective heat transfer, determine the time the cylinder may be exposed to this irradiation before the diaphragm will fail if the initial temperature is 21°C and (a) the cylinder is bare oxidized steel (ε = 0.79), (b) the cylinder is painted with aluminum paint (ε = 0.30).

GIVEN

- CO_2 in a high pressure cylinder exposed to flames
- Mass of CO_2 = 45 kg
- Cylinder dimensions
  - Outside Diameter (D) = 25 cm
  - Length (L) = 1.2 m
  - Thickness (s) = 1.2 cm
- Rupture diaphragm fails at 140 bar (gauge) (T_{gf} = 323 K)
- Temperature of flames (T_f) = 1097°C = 1370 K (ε_f = 1.0)
- Specific heat of CO_2 (c_v) = 2.5 kJ/(kg K)
- Initial temperature (T_{gf}) = 21°C = 294 K

FIND

The time for the diaphragm to fail if the cylinder is

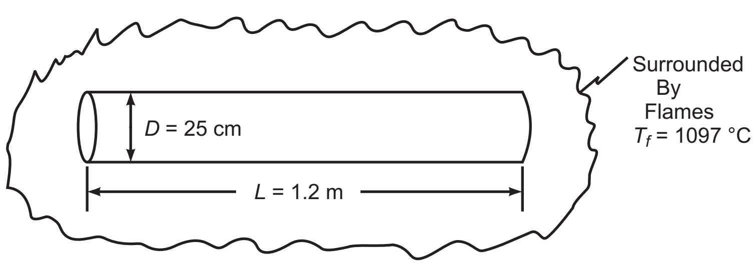
(a) bare oxidized steel (ε_s = 0.79) or

(b) painted with aluminum paint (ε_s = 0.30)

ASSUMPTIONS

- Convective heat transfer is negligible
- Cylinder is 1% carbon steel
- Irradiation is constant and uniform over the entire cylinder
- Quasi-steady state
- Thermal resistance between the gas and the cylinder is negligible ($T_s = T_g$)
- Variation of specific heat of gas and cylinder with temperature is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Appendix 2, Table 10, for 1% carbon steel at 20°C

Density (ρ_s) = 8020 kg/m^3

Specific heat (c_s) = 473 J/(kg K)

SOLUTION

The exterior surface area of the cylinder is

$$A_s = \pi D L + 2 \frac{\pi}{4} D_2 = \frac{\pi}{2} D (2 L + D) = \frac{\pi}{2} (0.25 \text{ m}) [2(1.2 \text{ m}) + 0.25 \text{ m}] = 1.04 \text{ m}^2$$

The mass of steel in the cylinder is

$$m_s = \rho_s (\text{volume}) = \rho_s A_s s = (8020 \text{ kg/m}^3) (1.04 \text{ m}^2) (1.2 \times 10^{-2} \text{ m}) = 100 \text{ kg}$$

Performing an energy balance on the gas

heat input = rate of increase in enthalpy

$$\sigma \varepsilon_s A_s (T_f^4 - T_g^4) = (m_g c_v + m_s c_s) \frac{dT_g}{dt} \qquad [T_g = T_s]$$

Solving for the rate of change of the gas temperature

$$\frac{dT_g}{dt} = \frac{\sigma \varepsilon_s A_s}{m_g c_v + m_s c_a} (T_f^4 - T_g^4)$$

$$\Rightarrow \quad \frac{dT_g}{dt} = \frac{(5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4)) \varepsilon_s (1.04 \text{ m}^2)}{(45 \text{ kg})(2500 \text{ J/(kg K)}) + (100 \text{ kg})(473 \text{ J/(kg K)})} [(1370 \text{ K})^4 - T_g^4]$$

$$\Rightarrow \quad \frac{dT_g}{dt} = 3.69 \times 10^{-13} \varepsilon_s (3.523 \times 10^{12} - T_g^4) \text{ K/s}$$

Case (a)
Initially

$$\frac{dT_g}{dt} = 3.69 \times 10^{-13} (0.79)[3.523 \times 10^{12} - (294)^4] \text{ K/s} = 1.03 \text{ K/s}$$

Finally

$$\frac{dT_g}{dt} = 3.69 \times 10^{-13} (0.79)[3.523 \times 10^{12} - (323 \text{ K})^4] \text{ K/s} = 1.024 \text{ K/s}$$

The rate of change of the gas temperature is essentially constant, and hence the time required for the gas to reach 50°C is

$$t = \frac{50°\text{C} - 21°\text{C}}{1.024 °\text{C/s}} = 28.3 \text{ seconds}$$

Case (b)

$$\frac{dT_g}{dt} = 3.69 \times 10^{-13} (0.30)[3.523 \times 10^{12} - (294 \text{ K})^4] \text{ K/s} = 0.39 \text{ K/s or °C/s}$$

$$\therefore \quad t = \frac{50°\text{C} - 21°\text{C}}{0.39 °\text{C/s}} = 74 \text{ seconds}$$

PROBLEM 9.47

A hydrogen bomb may be approximated by a fireball at a temperature of 7200 K according to a report published in 1950 by the Atomic Energy Commission. (a) Calculate the total rate of radiant-energy emission in watts, assuming that the gas radiates as a blackbody and has a diameter of 1.5 km, (b) If the surrounding atmosphere absorbs radiation below 0.3 μ m, determine the per cent of the total radiation emitted by the bomb which is absorbed by the atmosphere, (c) Calculate the rate of irradiation on a 1 m² area of the wall of a house 40 km from the center of the blast if the blast occurs at an altitude of 16 km and the wall faces in the direction of the blast, (d) Estimate the total amount of radiation absorbed assuming that the blast lasts approximately 10 sec and that the wall is covered by a coat of red paint, (e) If the wall were made of oak whose flammability limit is estimated to be 650 K and that had a thickness of 1 cm, determine whether or not the wood would catch on fire. Justify your answer by an engineering analysis stating carefully all assumptions.

GIVEN

- A hydrogen bomb fireball
- Fireball temperature (T_1) = 7200 K
- Surrounding atmosphere absorbs radiation below 0.3 μm
- The blast occurs at an altitude (H) of 16 km = 16,000 m

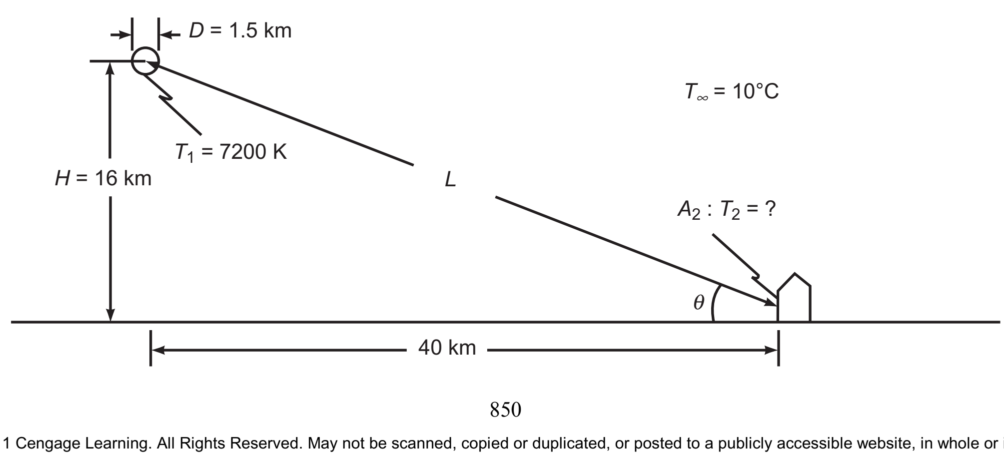
FIND

(a) The total rate of radiant-energy emission in watts (q_r)
(b) The percent of the total radiation absorbed by te atmosphere
(c) The rate of irradiation on a 1 m² area of the wall of a house 40 km (40,000 m) from the center of the blast and facing the blast (G_2)
(d) Total amount of radiation absorbed if the blast lasts 10 seconds and the wall is covered with red paint
(e) If the walls are oak with a flammability limit of 650 K and a thickness (s) of 1 cm, will the wood catch fire?

ASSUMPTIONS

- The gas radiates as a blackbody
- Diameter of the fireball (D) = 1.5 km
- The air and surrounding temperature (T_∞) = 10°C
- The surroundings behave as a blackbody enclosure
- The heat transfer from the oak walls to its surroundings during the 10 seconds of irradiation can be neglected
- The house wall is initially at the surroundings temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

From Table 9.2 the emissivity of red paint at short wavelengths (ε_{2s}) = 0.74

the emissivity of red paint at long wavelengths (ε_{21}) = 0.97

From Appendix 2, Table 11, for oak Specific heat (c) = 2390 J/(kg K)

Thermal conductivity (k_s) = 0.19 W/(m K)

Density (ρ) ≈ 700 kg/m^3

Thermal diffusivity (α_{th}) ≈ 0.011×10^{-5} m^2/s

SOLUTION

(a) The total rate of radiation emission is the blackbody emissive power, from Equation (9.3), times the area

$$q_1 = E_{b1}\, A = \sigma T_1^4\, \pi D^2 = \left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(7200\text{ K})^4\, \pi (1500\text{ m})^2 = 1.08 \times 10^{15}\text{ W}$$

(b) For $\lambda = 0.3\ \mu$m, $T\lambda$ = (7200 K)(0.3×10^{-6} m) = 2.16×10^{-3} m K

The fraction of energy absorbed is the fraction, e of the total radiation below 0.3 μm which can be read directly from Table 9.1

$$\text{\% absorbed by atmosphere} = \frac{E_{b1}(0 \to \lambda T)}{\sigma T^4} \times 100 = (0.09406)(100) = 9.4\%$$

(c) The distance between the house and the fireball center (L) is

$$L = \sqrt{(16\,\text{km})^2 + (40\,\text{km})^2} = 43\text{ km}$$

The energy calculated in part (a) will spread evenly in all directions from the fireball. Therefore, the flux at the distance $L = q_1/A_{sL}$ where A_{sL} is the surface area of a sphere of radius L

$$\frac{q_1}{4\pi L^2} = \frac{1.08\times10^{15}\,\text{W}}{4\pi(43{,}000\,\text{m})^2} = 46{,}480\text{ W/m}^2$$

However, the atmosphere will absorb 9.4% of this energy.

$$\text{Energy flux at wall} = 46{,}480\left(46{,}480\,\text{W/m}^2\right)(1 - 0.094) = 42{,}110\text{ W/m}^2$$

The angle between this flux and the (normal to the) wall surface, θ, is given by

$$\tan\theta = \frac{16\,\text{km}}{40\,\text{km}} \quad \Rightarrow \quad \theta = 21.8°$$

Therefore, the irradiation on the wall is

$$G_2 = (42{,}110\ \text{W/m}^2)\cos\theta = 39{,}100\text{ W/m}^2$$

(d) By Kirchoff's law, the absorptivity (α_2) = ε_2

$$\text{Energy absorbed} = G_2\, \varepsilon_2\, t = 39{,}100\left(39{,}100\,\text{W/m}^2\right)\left(\text{J/(Ws)}\right)(0.74)(10\text{s})\left(\text{kJ/(1000 J)}\right) = 289\ \text{kJ/m}^2$$

(e) The radiative heat transfer coefficient (hr) is given by

$$G_2 = \overline{h_r}\,(T_f - T_s) \quad \rightarrow \quad \overline{h_r} = \frac{G_2}{T_f - T_s}$$

Since $T_s << T_f$, the heat transfer coefficient will not vary much as the T_f changes. To estimate hr, let $T_f = 500$ K

$$\therefore \quad \overline{h_r} = \frac{\left(39{,}100\ \text{W/m}^2\right)}{7200\,\text{K} - 500\,\text{K}} = 5.84\ \text{W/(m}^2\text{K)}$$

The Biot number for the wall is

$$Bi = \frac{\overline{h_r}\, s}{2k} = \frac{\left(5.84\ \text{W/(m}^2\text{K)}\right)(0.01\,\text{m})}{2\left(0.19\,\text{W/(m\,K)}\right)} = 0.154 > 0.1$$

Therefore, the internal thermal resistance of the oak is significant and the chart solution for Figure 2.37 will be used to estimate the surface temperature of the oak after 10 seconds: ($L = s/2 = (0.01\text{ m})/2 = 0.005\text{ m}$)

The Fourier number is

$$Fo = \frac{\alpha_{th}\, t}{L^2} = \frac{\left(0.011\times 10^{-5}\ \text{m}^2/\text{s}\right)(10\,\text{s})}{(0.005\,\text{m})^2} = 0.044$$

$$\frac{1}{Bi} = 6.5$$

From Figure 2.37

$$\frac{T(0,t) - T_t}{T_o - T_t} \approx 1.0$$

(The center of the oak is still at the initial temperature after 10 s).

From Figure 2.32(b) for $x/L = 1.0$

$$\frac{T(L,t) - T_f}{T(0,t) - T_f} = \frac{T(L,t) - T_f}{T_o - T_f} = 0.92$$

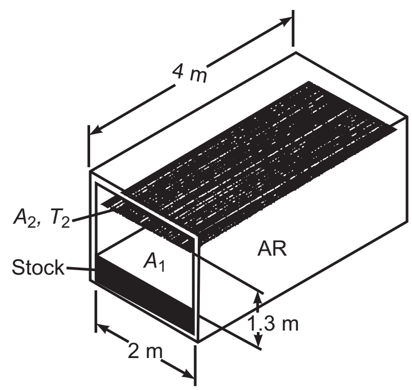
where $T(L,t)$ = the surface temperature of the wall after 10 seconds of exposure to the radiation.

$$\therefore \quad T(L,t) = T_f + 0.92\,(T_o - T_f) = 7200\text{K} + 0.92\,(283\text{K} - 7200\text{K}) = 836\text{K}$$

Therefore, the walls will catch on fire.

PROBLEM 9.48

An electric furnace is to be used for batch heating a certain material (specific heat of 670 J/(kg K)) from 20 to 760°C. The material is placed on the furnace floor which is 2m × 4m in area as shown in the accompanying sketch. The side walls of the furnace are made of a refractory material. Parallel to the plane of the roof, but several inches below it, a grid of round resistor rods is installed. The resistors are 13 mm in diameter and are spaced 5 cm center to center. The resistor temperature is to be maintained at 1100°C, under which conditions the emissivity of the resistor surface is 0.6. If the top surface of the stock may be assumed to have an emissivity of 0.9, estimate the time required for heating a 6 metric ton batch. External heat losses from the furnace may be neglected, the temperature gradient through the stock may be considered negligibly small, and steady-state conditions may be assumed.



GIVEN

- Batch heating of material in the furnace shown above
- Specific heat of material (c) = 670 Jkg K
- Material temperatures
  - Initial (T_{1i}) = 20°C = 293 K
  - Final (T_{hf}) = 760°C = 1033 K
- Furnace dimensions: 2 m × 4 m × 1.3 m high
- Side walls are refractory material
- Resistor rod diameter (D_r) = 13 mm = 0.013 m
- Resistor center to center distance (s) = 5 cm = 0.05 m
- Resistor temperature (T_2) = 1100°C = 1373 K
- Emissivity of the resistor surface (ε_2) = 0.6
- Emissivity of the material surface (ε_1) = 0.9
- Mass of material (m) = 6 metric tons = 6000 kg

FIND

- The time required (t) for heating the 6 metric ton batch

ASSUMPTIONS

- Quasi-steady state conditions
- External heat losses are negligible
- Temperature gradient through the material is negligible (negligible internal thermal resistance)
- Material is gray
- Convective heat transfer is negligible

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The shape factor F_{21} can be read off Figure 9.30: For s/D = 50/13 = 3.85 and one row: $F_{21} \approx 0.60$. Note that $A_1 = A_2$, therefore, $F_{21} = F_{12}$.

The sum of the shape factors from a given surface must sum to unity

$$F_{11} + F_{12} + F_{1R} = 1 \quad \rightarrow \quad F_{1R} = 1 - F_{12}$$

$$F_{21} + F_{12} + F_{2R} = 1 \quad \rightarrow \quad F_{2R} = 1 - F_{21} = 1 - F_{12}$$

The rate of radiative heat transfer, between two gray surfaces connected by re-radiating surfaces is given by Equation (9.80)

$$q_{12} = A_1\, \mathcal{F}_{21}\, \sigma (T_2^4 - T_1^4)$$

where $A_2 f_{21}$ is given by Equation (9.79), note that $A_1 = A_2 = A$

$$A_1\, \mathcal{F}_{21} = \frac{1}{\frac{1}{A}\left(\frac{1}{\varepsilon_2} - 1\right) + \frac{1}{A}\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{A\overline{F_{21}}}}$$

$$\text{where} \quad A\,\overline{F_{21}} = A\left(F_{21} + \frac{1}{\frac{1}{F_{2R}} + \frac{1}{AF_{1R}}}\right) = A\left[F_{12} + \frac{1 - F_{12}}{2}\right]$$

$$A\,\mathcal{F}_{21} = \frac{1}{\left(\frac{1}{\varepsilon_1} - 1\right) + \left(\frac{1}{\varepsilon_2} - 1\right) + \left[F_{12} + \frac{1 - F_{12}}{2}\right]} = \frac{(4\text{m})(2\text{m})}{\left(\frac{1}{0.9} - 1\right) + \left(\frac{1}{0.6} - 1\right) + \left[0.6 + \frac{1 - 0.6}{2}\right]}$$

$$A\,\mathcal{F}_{21} = (8\text{m}^2)\,(0.634) = 5.07\ \text{m}^2$$

The temperature changes in the material is given by

$$\Delta T_1 = \frac{q_{21}\Delta t}{mc} = \frac{A\mathcal{F}_{21}\sigma\left(T_2^4 - T_1^4\right)\Delta t}{mc}$$

$$\Delta T_1 = \frac{\left(5.07\,\text{m}^2\right)\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\left[(373\,\text{K})^4 - T_1^4\right]\Delta t}{(6000\,\text{kg})(670\ \text{J/(kg K)})((\text{Ws})/\text{J})}$$

$$\Delta T_1 = \left[(0.2541(\text{K/s})) - \left(7.15\times10^{-14}\ 1/(\text{K}^3\text{s})\right)T_1^4\right]\Delta t$$

As T_1 increases, the rate of heat transfer will decrease. Therefore, the equation above will be solved for a chosen time increment and the temperature T_1 will then be updated. This procedure will be repeated until T_1 = 760°C = 1033 K.

Let Δt = 5 min = 300 s initially

	Time (min)	ΔT_1 (K)	T_1 (K)
	0	----	293
	5	76.1	369.1
	10	75.8	444.9
	15	75.4	520.3
	20	74.7	595.0
	25	73.5	668.5
	30	72.0	740.5
	35	69.8	810.3
Let Δt = 1 min	40	67.0	877.3
	45	63.5	940.8
	50	59.4	1000.2
	51	11.0	1011.2
	52	10.8	1021.9
	53	10.6	1032.5

The time required ≈ 53 min.

PROBLEM 9.49

A rectangular flat water tank is placed on the roof of a house with its lower portion perfectly insulated. A sheet of glass whose transmission characteristics are tabulated below is placed 1 cm above the water surface. Assuming that the average incident solar radiation is 630 W/m^2, calculate the equilibrium water temperature for a water depth of 12 cm if the heat transfer coefficient at the top of the glass is 8.5 W/(m^2 K) and the surrounding air temperature of 20°C. Disregard intereflections.

τ_λ of glass	**= 0 for wavelength from 0 to0.35 μm**
	= 0.92 for wavelength from 0.35 to 2.7 μm
	= 0 for wavelength larger than 2.7 μm
ρ_λ of glass	**= 0.08 for all wavelengths**

GIVEN

- A glass covered water tank on the roof of a house
- Lower portion of tank is perfectly insulated
- Distance between glass cover and water surface (δ) = 1 cm = 0.01 m
- Average incident solar radiation (I_s) = 630 W/m^2
- Water depth = 12 cm = 0.12 m
- Heat transfer coefficient on the top of the glass (h_{co}) = 8.5 W/(m^2 K)
- Surrounding air temperature (T_∞) = 20°C = 293 K
- Transmissivity of glass (λ_λ) = 0 for $0 < \lambda < 0.35$ μ m
  = 0.92 for $0.35 < \lambda < 2.7$ μ m
  = 0 for $\lambda > 2.7$ m m
- Reflectivity of Glass (ρ_λ) = 0.08

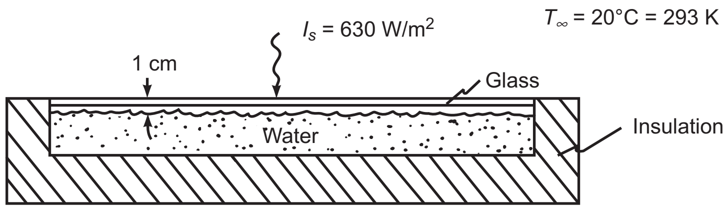
FIND

- The equilibrium temperature of the water (T_w)

ASSUMPTIONS

- The effect of inter-reflections is negligible
- The water temperature is uniform (internal resistance of the water is negligible)
- Steady state conditions
- I_s value given is normal to the glass surface
- The water absorbs all the radiation reaching it
- Water behaves as a blackbody
- The conductive thermal resistance of the glass is negligible
- The sky behaves as a blackbody enclosure at T_{sky} = 0 K
- The sun is blackbody at 6000 K (see Table 9.2)
- The shape factor between the surface and the glass can be taken to be unity
- The air properties are the same as dry air properties
- The glass acts as a black surface for the reradiated energy

SKETCH

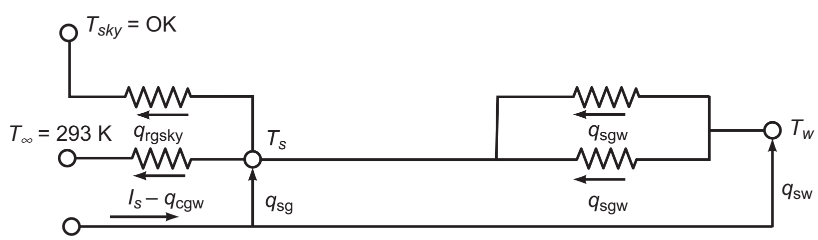


PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The thermal circuit for the problem is shown below



where q_r = radiative heat transfer flux
q_c = convective heat transfer flux
q_s = solar radiation

The radiative heat transfer from the water through the glass to the sky ($q_{rw,sky}$) ≈ 0 because the majority of the radiation from the water will be at long wavelengths for which the transmissivity of the glass is zero.

An expression can be written for each of the heat fluxes

$$q_{rgsky} = \sigma \varepsilon_g (T_g^4 - T_{sky}^4) = \sigma T_g^4$$

$$q_{rwg} = \sigma \varepsilon_w (T_w^4 - T_g^4) = \sigma (T_w^4 - T_g^4)$$

$$q_{eg\infty} = h_{co} (T_g - T_\infty)$$

$$q_{cwg} = h_\infty (T_w - T_g)$$

$$q_{cg} = (1 - \rho_\lambda)(1 - \tau_\lambda)\, I_s =$$

$$(1-\rho)\left[\left(1-\tau_{(0-0.35)}\right)\frac{E_{b(0-0.35)}}{\sigma T^4}I_s+\left(1-\tau_{(0.35-2.7)}\right)\frac{E_{b(0.35-2.7)}}{\sigma T^4}I_s+\left(1-\tau_{(2.7\to 0)}\right)\frac{E_{b(2.7\to 0)}}{\sigma T^4}I_s\right]$$

$$q_{sw} = (1-\rho_\lambda)\,\tau_\lambda\, I_{s\lambda} = (1-\rho_\lambda)\,\tau_{(0.35\,\to\,2.7)}\,\frac{E_{b(0.35-2.7)}}{\sigma T^4}\,I_s$$

Only the last two expressions are frequency dependent. From Table 9.1

For $\lambda T = (0.35 \times 10^{-6}\text{ m})(6000\text{ K}) = 2.1 \times 10^{-3}\text{ m K}$

$$\frac{E_b(0\to 0.35T)}{\sigma T^4} = 0.08382$$

For $\lambda T = (2.7 \times 10^{-6}\text{ m})(6000\text{ K}) = 16.2 \times 10^{-3}\text{ m K}$

$$\frac{E_b(0\to 2.7T)}{\sigma T^4} = 0.9746$$

$$\frac{E_b(0.35T\to 2.7T)}{\sigma T^4} = 0.9746 - 0.08382 = 0.8908$$

$$\therefore \qquad q_{sw} = (1-0.08)(0.92)(0.8908)\left(630\text{ W/m}^2\right) = 475\text{ W/m}^2$$

$$q_{sg} = (1-0.08)[(1)(0.08382)(630) + (1-0.92)(0.8908)(630) + (1)(1-0.9746)(630)] = 105\text{ W/m}^2$$

The natural convection Nusselt number between the glass and water is given by Equation (5.30)

$$\overline{Nu_\delta} = 1 + 1.44\left[1-\frac{1708}{Ra_\delta}\right]^{\bullet} + \left[\left(\frac{Ra_\delta}{5830}\right)^{\frac{1}{3}}-1\right]^{\bullet} \qquad 1700 < Ra_\delta < 10^8$$

where the notation [ ]· indicates that if the quantity inside the brackets is negative, the quantity is to be taken as zero. The Rayleigh number is given by

$$Ra_\delta = Gr_\delta Pr = \frac{g\beta\left(T_w - T_\delta\right)\delta^3 Pr}{{\nu_a}^2}$$

Since both T_w and T_g are unknown, an iterative solution must be used. For the first iteration, let $T_w = 80°\text{C}$ and $T_g = 40°\text{C}$.

From Appendix 2, Table 27, for dry air at the average temperature of $(T_w + T_g)/2 = 60°\text{C}$

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

$$Ra_\delta = \frac{\left(9.8\,\text{m/s}^2\right)\left(0.003\frac{1}{\text{K}}\right)(40°\text{C})(0.01\,\text{m})^3(0.71)}{\left(19.4\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 2219$$

$$\overline{Nu_\delta} = 1 + 1.44\left[1-\frac{1708}{2219}\right]^{\bullet} + \left[\left(\frac{2219}{5830}\right)^{\frac{1}{3}}-1\right]^{\bullet} = 1 + 0.3316 + 0 = 1.33$$

$$\overline{h_{c\delta}} = \overline{Nu_\delta}\,\frac{k}{\delta} = 1.33\,\frac{\left(0.0279\ \text{W/(m K)}\right)}{0.01\,\text{m}} = 3.72\ \text{W/(m}^2\text{K)}$$

An energy balance on the glass plate yields

$$Q_{sg} + q_{cwg} + q_{rwg} = q_{rg\text{sky}} + q_{cg\infty}$$

$$Q_{sg} + \overline{h_{c\delta}}\,(T_w - T_g) + \sigma(T_w^{\ 4} - T_g^{\ 4}) = \sigma T_g^{\ 4} + h_{co}\,(T_g - T_\infty)$$

Rearranging

$$\left(\overline{h_\infty} + \overline{h_{c\delta}}\right)\ T_g = q_{sg} + \overline{h_{co}}\ T_\infty + \overline{h_{c\delta}}\ T_w + \sigma(T_w^{\ 4} - 2T_g^{\ 4})$$

$$[1] \quad T_g = K_1 + K_2\,T_w + K_3\,(T_w^{\ 4} - 2\,T_g^{\ 4})$$

where

$$K_1 = \frac{q_{sg} + \overline{h_{co}}\,T_\infty}{\overline{h_{co}} + \overline{h_{c\delta}}} = \frac{\left(105\ \text{W/m}^2\right) + \left(8.5\ \text{W/(m}^2\text{K)}\right)(293\,\text{K})}{(8.5 + 3.72)\ \text{W/(m}^2\text{K)}} = 212.4\ \text{K}$$

$$K_2 = \frac{\overline{h_{c\delta}}}{\overline{h_{co}} + \overline{h_{c\delta}}} = \frac{3.72}{8.5 + 3.72} = 0.3044$$

$$K_3 = \frac{\sigma}{\overline{h_\infty} + \overline{h_{c\delta}}} = \frac{\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)}{\left((8.5 + 3.72)\ \text{W/(m}^2\text{K)}\right)} = 4.64 \times 10^{-9}\ 1/\text{K}^3$$

An energy balance on the water yields

$$q_{sw} = q_{rwg} + q_{cwg} = \sigma(T_w^{\ 4} - T_g^{\ 4}) + \bar{h}_{c\delta}\ (T_w - T_g)$$

Rearranging

$$[2] \quad T_w = K_4 - K_5\,(T_w^{\ 4} - T_g^{\ 4}) + T_g$$

where

$$K_4 = \frac{q_{sw}}{\bar{h}_{c\delta}} = \frac{\left(475\ \text{W/m}^2\right)}{\left(3.72\ \text{W/(m}^2\ \text{K)}\right)} = 127.7\ \text{K}$$

$$K_5 = \frac{\sigma}{\bar{h}_{c\delta}} = \frac{\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)}{\left(3.72\ \text{W/(m}^2\text{K)}\right)} = 1.524 \times 10^{-8}\,1/\text{K}^3$$

These two simultaneous 4th order equations may by solved iteratively as follows:

1. Guess values of T_w and T_g
2. Iterative equation [2] to generate a new value of T_w
3. Using this value of T_w, iterate equation [1] to generate a new rate for T_g.
4. Repeat the procedure until the difference between the values of T_w and T_g for successive iterations is below a chosen tolerance.

This procedure is implemented in the Pascal program shown below

```
var
   Tw1,Tw,Tg1,Tg,Diff_g,Diff_w:real;
Const
   K1 = 212.4;
   K2 = 0.3044;
   K3 = 4.64E-9;
   K4 = 127.7;
   K5 = 1.524E-8;
   gain = 0.4;
```

```
Begin
   {Let Tw1 = 353 K and Tg1 = 313 K be the initial guesses}
   Tw:=0.0;
   Tg:=0.0;
   Tw1:=353.0;
   Tg1:=313.0;
   Repeat
      Repeat
         {Iterate equation [2] to calculate a new Tw}
         Tw:=K4 – K5*(Tw1*Tw1* Tw1* Tw1 – Tg1* Tg1* Tg1* Tg1)+ Tg1;
         Diff_w:=Tw-Tw1;
         Tw1:=Tw1+gain*Diff_w;
      Until abs(Diff_w) < 0.1;
      Repeat
         {Iterate equation [1] to calculate a new Tg}
         Tg:=K1 + K2*Tw + K3*(Tw* Tw* Tw* Tw – 2.0*Tg1* Tg1* Tg1 Tg1);
         Diff_g:=Tg-Tg1;
         Tg1:=Tg1+gain*Diff_g;
      Until abs(Diff_g) < 0.1;
      Tw:=K4 – K5*(Tw1* Tw1* Tw1* Tw1* – Tg1* Tg1* Tg1* Tg1*) + Tg1;
      Diff_w:=abs(Tw-Tw1);
      Tw1:=Tw;
   Until Diff_w < 0.1;
   Writeln(‘ Tw = ’,Tw:6.1, K  Tg = ’,Tg:6.1,’ K’);
end.
```

(Note: the gain factor slows down the convergence but is often necessary for non-linear problems)

The output from the first run of this program is

$$T_w = 345.4 \text{ K} \quad T_g = 304.2 \text{ K}$$

These values are different enough from the initial guesses that another iteration will be performed:

Mean temperature = 324.8 K

$\beta = 0.00308$ 1/K

$k = 0.0273$ W/(m K)

$\nu = 18.66 \times 10^{-6}$ m²/s

$Pr = 0.71$

$Ra_\delta = 2536$

$\bar{h}_{c\delta} = 4.01$ W/(m² K)

$K_1 = 207.42$ K

$K_2 = 0.3205$

$K_3 = 4.532 \times 10^{-9}$ 1/K³

$K_4 = 118.45$ K

$K_5 = 1.414 \times 10^{-8}$ 1/K³

Running the program again, the new constants and the above temperatures as the initial guesses yields:

$$T_w = 344.5 \text{ K} \quad T_g = 304.1 \text{ K}$$

The water temperature is approximately 71.5°C.

COMMENTS

Consistent with our assumption, he glass temperature is low enough so that all radiation emitted from the glass will be beyond 2.7 μm so that the glass can be considered black.

PROBLEM 9.50

Mercury is to be evaporated at 317°C in a furnace. The mercury flows through a 2.5 cm BWG No. 18 gauge 304 stainless-steel tube, which is placed in the center of the furnace whose cross section, perpendicular to the tube axis, is a square 20 cm × 20 cm. The furnace is made of brick having an emissivity of 0.85, with the walls maintained uniformly at 977°C. If the convective heat transfer coefficient on the inside of the tube is 2.8 kW/(m^2 K) and the emittance of the outer surface of the tube is 0.60, calculate the rate of heat transfer per foot of tube, neglecting convection within the furnace.

GIVEN

- Mercury flow through a tube in the center of a furnace
- Mercury temperature (T_m) = 317°C = 590 K
- Tube specification: 1 in BWG no 18 gauge stainless steel
- Furnace cross section is 20 cm × 20 cm
- Furnace emissivity (ε_2) = 0.85
- Furnace wall temperature (T_2) = 977°C = 1250 K
- Tube interior heat transfer coefficient (h_{ci}) = 2800 W/(m^2 K)
- Tube exterior emissivity (ε_1) = 0.60

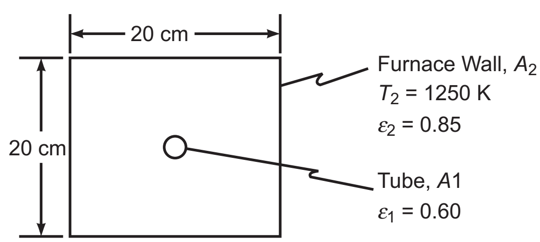
FIND

- The rate of heat transfer per foot of tube

ASSUMPTIONS

- Steady state
- Convection within the furnace is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

From Appendix 2, Table 42, for 1 in 18 BWG tubes

Inside diameter (D_i) = 2.29 cm

Outside diameter (D_o) = 2.54 cm

From Appendix 2*,* Table 10, for type 304stainless steel,the thermal conductivity (k_s) = 14.4 W/(m K)

SOLUTION

The tube and furnace can be thought of as two infinitely long concentric gray cylinders. The rate of radiative heat transfer is given by Equation (9.75)

$$q_{12} = A_1 \, \mathtt{F}_{12} (E_{b1} - E_{b2}) = \sigma A_2 \, \mathtt{F}_{12} (T_1^4 - T_2^4)$$

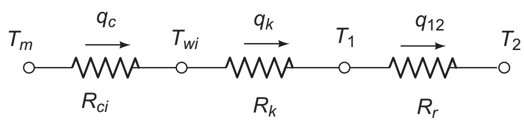
From Equation (9.76)

$$\text{F}_{12} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2}}$$

where $$\frac{A_1}{A_2} = \frac{\pi(2.54\,\text{cm})}{4(20\,\text{cm})} = 0.1$$

$$\text{F}_{12} = \frac{1}{\frac{1-0.6}{0.6} + 1 + 0.1\,\frac{1-0.85}{0.85}} = 0.594$$

The thermal circuit for this problem is shown below



where

R_{ci} = Convective thermal resistance inside the tube
R_k = Conductive thermal resistance of the tube wall
R_r = Radiative thermal resistance
q_c = Convective heat transfer to the tube wall interior = $h_{ci}\, A_i\, (T_m - T_{wi})$
q_k = Conductive heat transfer through the tube wall = $\dfrac{2\pi k_s L}{\ln\left(\frac{D_o}{D_i}\right)}(T_{wi} - T_1)$
q_{1-2} = Radiative heat transfer = $\sigma A_1 f_{12}\,(T_1^4 - T_2^4)$

For steady state, these three heat transfer rates must be equal

$$h_{ci}\, A_\text{i}\,(T_m - T_{wi}) = \frac{2\pi k_s L}{\ln\left(\frac{D_o}{D_i}\right)}(T_{wi} - T_1) = \sigma A_1\, \text{F}_{12}\,(T_1^4 - T_2^4)$$

Solving for T_{w1} from the first part of the equation

$$T_{wi} = \frac{\bar{h}_{ci}\, A_i\, T_m + \frac{2\pi k_s L}{\ln\left(\frac{D_o}{D_i}\right)}T_1}{\bar{h}_{ci}\, A_i + \frac{2\pi k_s}{\ln\left(\frac{D_o}{D_i}\right)}}$$

Substituting this into the second part of the equation to solve for T_1 yields

$$\bar{h}_{ci}\, A_\text{i}\left[T_m - \frac{\bar{h}_{ci}\, A_i\, T_m + \frac{2\pi k_s L}{\ln\left(\frac{D_o}{D_i}\right)}T_1}{\bar{h}_{ci}\, A_i + \frac{2\pi k_s}{\ln\left(\frac{D_o}{D_i}\right)}}\right] - \sigma A_1\, \text{F}_{12}\,(T_1^4 - T_2^4) = 0$$

Here $$\frac{2\pi k_s L}{\ln\left(\frac{D_o}{D_i}\right)} = \frac{2\pi(14.4\ \text{W/(m K)})\,L}{\ln\left(\frac{2.54\,\text{cm}}{2.29\,\text{cm}}\right)} = 873\, L\ \text{W/K}$$

and $$\bar{h}_{ci} A_i = \bar{h}_{ci} \pi D_i L = 2800 \text{ W/(m}^2\text{ K)} \pi (2.29 \times 10^{-2} \text{ m}) L = 200 L \text{ W/K}$$

$$200 L \text{ W/K} \left[590\text{K} - \frac{(200L)(590) + (893L)(T_1)}{(200L + 873L)} \right] - 5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4)$$

$$[\pi (2.54 \times 10^{-2} \text{ m})L](0.595)\ [T_1^4 - (1250 \text{ K})^4] = 0$$

Checking the units and then eliminating them for clarity

$$\Rightarrow \qquad -2.69 \times 10^{-9}\ T_1^4 - 162.72\ T_1 + 102575 = 0$$

By trial and error

$$T_1 \simeq 630 \text{ K}$$

$$\frac{q}{L} = \sigma \frac{A_1}{L} \mathrm{F}_{12} (T_1^4 - T_2^4) = 5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4)\ [\pi (2.5 \times 10^{-2} \text{ m})]\ (0.595)[(630 \text{ K})^4 - (1250 \text{ K})^4]$$

$$\Rightarrow \qquad \frac{q}{L} = -6048 \text{ W/m}$$

COMMENTS

Negative sign in the answer indicates heat is transferred to the mercury. Note that $f_{12} \approx \varepsilon_1$, because $A_2 >> A_1$. For $A_1/A_2 = 0$ Equation (9.76) reduces to $f_{12} = \varepsilon_1$.

PROBLEM 9.51

A 2.5 cm diameter cylindrical refractory crucible for melting lead is to be built for thermocouple calibration. An electrical heater immersed in the metal is shut off at some temperature above the melting point. The fusion-cooling curve is obtained by observing the thermocouple emf as a function of time. Neglecting heat losses through the wall of the crucible, estimate the cooling rate (W) for the molten lead surface (melting point 327.3°C, surface emissivity 0.8) if the crucible depth above the lead surface is (a) 2.5 cm, (b) 17 cm. Assume that the emissivity of the refractory surface is unity and the surroundings are at 21°C. (c) Noting that the crucible would hold about 0.09 kg of lead for which the heat of fusion is 23,260 J/kg, comment on the suitability of the crucible for the purpose intended.

GIVEN

- A cylindrical refractory crucible filled with molten lead
- Cylinder diameter (D) = 2.5 cm
- Melting point of lead (T_1) = 327.2°C = 600.3 K
- Surface emissivity of lead (ε_1) = 0.8
- Mass of lead in crucible (m) ≈ 0.09
- Heat of fusion of lead (h_{fg}) = 23,260 J/kg

FIND

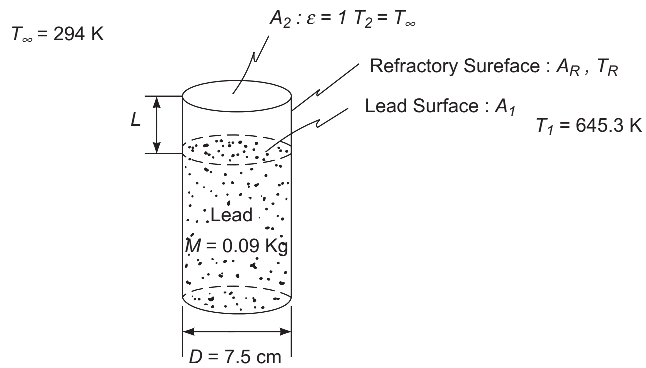
The cooling rate (q) if the crucible depth above the lead surface (L) is

(a) 2.5 cm = 0.025 m
(b) 17 cm = 0.17 m
(c) Comment on the suitability of the crucible for thermocouple calibration

ASSUMPTIONS

- Heat loss through the wall of the crucible is negligible
- The emissivity of the refractory surface (crucible wall above the lead) is unity ($\varepsilon_2 = 1$)
- The surroundings behave as a blackbody enclosure
- The temperature of the refractory surface is uniform at T_R

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

From Appendix 2, Table 27, for air at the film temperature of $(T_1 + T_\infty)/2$ = 447.2 K = 174.2 C

Thermal expansion coefficient (β) = 0.00226 1/K

Thermal conductivity (k) = 0.0354 W/(m K)

Kinematic viscosity (ν) = 32.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The total cooling rate is the sum of natural convection and radiation

$$q = h_c A_1 (T_1 - T_\infty) + q_{12}$$

where q_{12} is the radiative heat transfer between the two surfaces connected by a refractory wall and is given by Equation (9.80)

$$q_{12} = A_1 f_{12}\, \sigma (T_1^4 - T_2^4)$$

where $A_1 f_{12}$ is given by Equation (9.79) (Note that $\varepsilon_2 = 1.0$ and $A_2 = A_1$)

$$A_1 \mathrm{F}_{12} = \frac{1}{\dfrac{1}{A_1}\left(\dfrac{1}{\varepsilon_1} - 1\right) + \dfrac{1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right) + \dfrac{1}{A_1 \overline{\overline{F_{12}}}}} = \frac{1}{\dfrac{1}{A_1}\left(\dfrac{1}{\varepsilon_1} - 1\right) + \dfrac{1}{A_1 \overline{\overline{F_{12}}}}}$$

where $$A_1 \overline{F_{12}} = A_1 \left(F_{12} + \frac{1}{\dfrac{1}{F_{1R}} + \dfrac{A_1}{A_2 F_{2R}}} \right) = A_1 \left(F_{12} \frac{1}{\dfrac{1}{F_{1R}} + \dfrac{1}{F_{2R}}} \right)$$

The shape factor is given in Table 9.3 #6 by letting $a = b = D/s$ and

$$F_{12} = \frac{2}{D^2}\left(L^2 + \frac{D^2}{2} - \sqrt{\left(L^2 + \frac{D^2}{2}\right)^2 - \frac{D^4}{4}}\right)$$

For Case (a) $$F_{12} = \frac{2}{(2.5)^2}\left((2.5)^2 + \frac{(2.5)^2}{2} - \sqrt{\left((2.5)^2 + \frac{(2.5)^2}{2}\right)^2 - \frac{(2.5)^4}{4}}\right) = 0.17$$

For Case (b) $$F_{12} = \frac{2}{(2.5)^2}\left((17)^2 + \frac{(2.5)^2}{2} - \sqrt{\left((17)^2 + \frac{(2.5)^2}{2}\right)^2 - \frac{(2.5)^4}{4}}\right) = 0.0053$$

By symmetry

$$F_{21} = F_{12}$$

The shape factors from a given surface must sum to unity

$$F_{11} + F_{12} + F_{1R} = 1 \quad \rightarrow \quad F_{1R} = 1 - F_{12}$$

$$F_{21} + F_{12} + F_{2R} = 1 \quad \rightarrow \quad F_{2R} = 1 - F_{21} = 1 - F_{12}$$

$$\therefore A_1\, \overline{F_{12}} = A_1\left(F_{12} + \frac{1-F_{12}}{2}\right)$$

$$A_1\, \mathrm{F}_{12} = \frac{A_1}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{\left(F_{12} + \frac{1-F_{12}}{2}\right)}}$$

$$q = A_1\left[\bar{h}_c\,(T_1 - T_\infty) + \frac{\sigma}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{F_{12} + \frac{1-F_{12}}{2}}}\left(T_1^4 - T_2^4\right)\right]$$

The heat transfer coefficient, h_c, can be calculated from Equation (5.15) or (5.16)

$$Ra_D = Gr_D\, Pr = \frac{g\beta(T_1 - T_\infty)\, D^3 Pr}{\nu^2}$$

$$Ra_D = \frac{\left(9.8\ \mathrm{m/s^2}\right)(0.00226\ 1/\mathrm{K})(600.3\,\mathrm{K} - 294\,\mathrm{K})(0.025\,\mathrm{m})^3\,(0.71)}{\left(32.4 \times 10^{-6}\ \mathrm{m^2/s}\right)^2} = 7.17 \times 10^4$$

Although this is slightly below its lower Rayleigh number range, Equation (5.15) will be used to estimate the Nusselt number

$$\overline{Nu}_D = 0.54\ Ra_D^{\frac{1}{4}} = 0.54\,(7.17 \times 10^4)^{\frac{1}{4}} = 8.84$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 8.84\,\frac{(0.0354\ \mathrm{W/(m\,K)})}{0.025\,\mathrm{m}} = 12.5\ \mathrm{W/(m^2K)}$$

(a)

$$q = \frac{\pi}{4}(0.025\text{ m})^2 \left[\left(12.5\,\text{W/(m}^2\text{K)}\right)\left(600.3\,\text{K} - 294\,\text{K}\right) + \frac{\left(5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\right)}{\left(\frac{1}{0.8}-1\right)+\frac{1}{0.17+\frac{1-0.17}{2}}}[(600.3\,\text{K})^4 - (294\,\text{K})^4] \right]$$

$$q = 3.26\text{ W}$$

(b) $F_{12} = 0.0053 \quad \rightarrow \quad q = 3.4$ W

(c) The time required for the lead to solidify at the cooling rate (q) of 3.62 W is given by

$$t = \frac{m\,h_{fg}}{q} = \frac{(0.09\,\text{kg})(23{,}260\,\text{J/kg})}{3.62\,\text{W}(\text{J/(W s)})} = 579\text{ s} = 9.6\text{min}$$

The technician would have about 9.6 minutes to do the calibration. This should be enough time to accomplish the task.

PROBLEM 9.52

A spherical satellite circling the sun is to be maintained at a temperature of 25°C. The satellite rotates continuously and is covered partly with solar cells having a gray surface with an absorptivity of 0.1. The rest of the sphere is to be covered by a special coating which has an absorptivity of 0.8 for solar radiation and an emissivity of 0.2 for the emitted radiation. Estimate the portion of the surface of the sphere which can be covered by solar cells. The solar irradiation may be assumed to be 1,420 W/m^2 of surface perpendicular to the rays of the sun.

GIVEN

- A spherical satellite partially covered with solar cells is orbiting the sun
- Satellite temperature (T_s) = 25°C = 298 K
- Solar cell absorptivity (α_c) = 0.1
- Absorptivity of the rest of the satellite (α_{2s}) = 0.8
- Emissivity of the rest of the satellite (ε_s) = 0.2

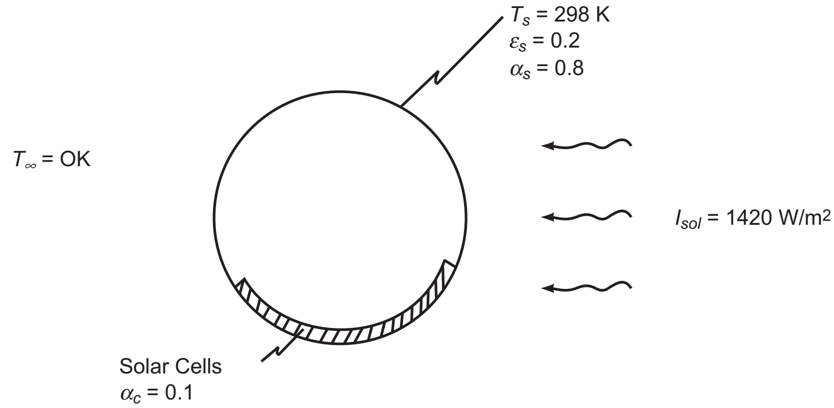
FIND

- The portion of the surface which can be covered by solar cells

ASSUMPTIONS

- Ambient temperature (T_∞) = 0 K
- Quasi steady state
- Solar irradiation (I_{sol}) = 1420 W/m^2 of the surface perpendicular to the rays of the sun
- Satellite and cell surfaces are gray
- Surface temperature of the satellite is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

Let x be the fraction of the total satellite surface area (A_T) which is covered by solar cells. By definition, the absorptivity is the fraction of the total irradiation absorbed by a body

$$\alpha = \frac{\frac{q_{\text{in}}}{A}}{I_{\text{sol}}}$$

$$\therefore \quad q_{\text{in}} = \alpha I_{\text{sol}} A = I_{\text{sol}} (\alpha_s A_s + \alpha_c A_c) = I_{\text{sol}} (\alpha_c \times A_T + \alpha_s (1 - x) A_T)$$

$$q_{\text{in}} = I_{\text{sol}} A_T [\alpha_s + (\alpha_c - \alpha_s) x]$$

The rate of radiation from he satellite is the emissive power of the satellite from Equation (9.34) multiplied by its area

$$q_{\text{out}} = E A = \varepsilon \sigma A T_s^4 = \sigma T_s^4 (\varepsilon_c \times A_T + \varepsilon_s (1 - x) A_T) = \sigma A_T T_s^4 [\varepsilon_s + (\varepsilon_c - \varepsilon_s) x]$$

For steady state $\quad q_{\text{in}} = q_{\text{out}}$

$$I_{\text{sol}} A_T [\alpha_s + (\alpha_c - \alpha_s) x] = \sigma A_T T_s^4 [\varepsilon_s + (\varepsilon_c - \varepsilon_s) x]$$

Solving for the fraction covered by solar cells

$$x = \frac{\left(\frac{I_{\text{sol}}}{\sigma T_s^4}\right)(\alpha_s - \varepsilon_s)}{(\varepsilon_c - \varepsilon_s) - \left(\frac{I_{\text{sol}}}{\sigma T_s^4}\right)(\alpha_c - \alpha_s)}$$

$$\frac{I_{\text{sol}}}{\sigma T_s^4} = \frac{(1420\,\text{W/m}^2)}{(5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4))(298\,\text{K})^4} = 3.176$$

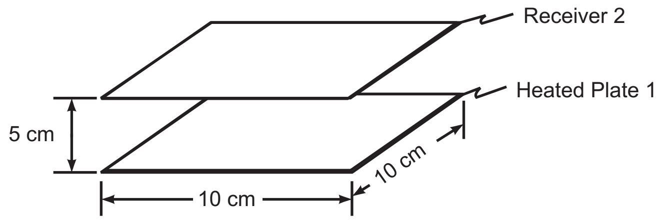
$$\varepsilon_c = 1 - \alpha_c = 1 - 0.1 = 0.9$$

$$x = \frac{3.176(0.8) - 0.2}{(0.9 - 0.2) - 3.176(0.1 - 0.8)} = 0.801$$

80.1% of the satellite can be covered by solar cells.

PROBLEM 9.53

An electrically heated plate, 10 cm square, is placed in a horizontal position 5 cm below a second plate of the same size as shown schematically below.



The heating surface is gray (emissivity = 0.8) while the receiver has a black surface. The lower plate is heated uniformly over its surface with a power input of 300 W. Assuming that heat losses from the backs of the radiating surface and the receiver are negligible and that the surroundings are at a temperature of 27°C, calculate the following

(a) The temperature of the receiver

(b) The temperature of the heated plate.

(c) The net radiation heat transfer between the two surfaces.

(d) The net radiation loss to the surroundings.

(e) Estimate the effect of natural convection between the two surfaces on the rate of heat transfer.

GIVEN

- A square heated plate below a second plate of equal size as shown above
- Plate size = 10 cm × 10 cm = 0.1 m × 0.1 m
- Distance between plates (L) = 5cm = 0.05 m
- Heated surface (A_1) is gray with an emissivity (ε_1) = 0.8
- Receiver (A_2) is black (ε_2 = 1.0)
- Heater power input $\left(\dot{q}_G\right)$ = 300 W

FIND

(a) The temperature of the receiver (T_2)
(b) The temperature of the heated transfer (T_1)
(c) The net radiation heat transfer (q_{12})
(d) The net radiation loss to the surroundings (q_s)
(e) Estimate the effect of natural convection

ASSUMPTIONS

- Steady state
- Heat losses from the back of each plate are negligible
- Temperature of surroundings (T_3) = 27°C = 300 K
- The surroundings behave as a blackbody enclosure

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

In steady state, the net rate of heat transfer from the heater must be equal to the power input

$$q_1 = \dot{q}_G = 300 \text{ W}$$

The net rate of heat transfer to the receiver in steady state must be zero: $q_2 = 0$

Also, since the receiver and the surroundings are black

$$J_2 = E_{b2} = \sigma T_2^4 \quad \text{and} \quad J_3 = E_{b3} = \sigma T_3^4$$

The shape factor F_{12} can be read from Figure 9.28

From Figure 9.28 $x/D = y/D = 10/5 = 2 \quad \rightarrow \quad F_{12} = 0.43$

Since $A_1 = A_2$, from Equation (9.46) $F_{21} = F_{12}$ (This is also clear from the symmetry of the problem).

Since neither A_1 nor A_2 can see itself, $F_{11} = F_{22} = 0$

The shape factors from any given surface must sum to unity

$$F_{11} + F_{12} + F_{13} = 1 \quad \rightarrow \quad F_{13} = 1 - F_{12} = 0.57$$

$$F_{21} + F_{12} + F_{23} = 1 \quad \rightarrow \quad F_{23} = 1 - F_{21} = 1 - F_{12} = 0.57$$

From Equation (9.46)

$$F_{31} = \frac{A_1}{A_3} F_{13} = \frac{A_1}{A_3} (1 - F_{12}) = F_{32}$$

The net rate transfer from a gray surface is given be Equation (9.67)

[1] $q_1 = A_1 (J_1 - G_1)$

[2] $q_2 = A_2 (J_2 - G_2) = A_2 (\sigma T_2^4 - G_2) = 0$

From Equation (9.69)

[3] $A_1 G_1 = J_1 A_1 F_{11} + J_2 A_2 F_{21} + J_3 A_3 F_{31} = A_2 F_{12} \sigma T_2^4 + A_1(1 - F_{12}) \sigma T_3^4$

[4] $A_2 G_2 = J_1 A_1 F_{12} + J_2 A_2 F_{22} + J_3 A_3 F_{32} = J_1 A_1 F_{12} + A_1(1 - F_{12}) \sigma T_3^4$

Combining Equations [2] and [4]

$$\sigma T_2^4 = G_2 = J_1 F_{12} + (1 - F_{12}) \sigma T_3^4$$

$$[5] \qquad J_1 = \left(\frac{\sigma}{F_{12}}\right) [T_2^4 - (1 - F_{12}) T_3^4]$$

Substituting Equation [5] and Equation [3] into [1]

$$q_1 = A_1 \left[\frac{\sigma}{F_{12}} \left[T_2^4 - (1 - F_{12}) T_3^4 \right] - \left[F_{12}\, \sigma T_2^4 + (1 - F_{12})\, \sigma T_3^4 \right] \right]$$

Solving for T_2

$$T_2 = \left[\frac{(1 - F_{12})\left(\left(\frac{1}{F_{12}}\right) + 1\right) T_3^4 + \left(\frac{q_1}{(\sigma A_1)}\right)}{\frac{1}{F_{12}} - F_{12}} \right]^{0.25}$$

$$T_2 = \left[\frac{(1 - 0.43)\left(\left(\frac{1}{0.43}\right) + 1\right) (300\,\text{K})^4 + \frac{300\,\text{W}}{\left(5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(0.1\,\text{m})(0.1\,\text{m})}}{\left(\left(\frac{1}{0.43}\right) - 0.43\right)} \right]^{0.25}$$

$$T_2 = 732\ \text{K} = 459°\text{C}$$

(b) From Equation [3]

$$G_1 = \sigma[F_{12} T_2^4 + (1 - F_{12}) T_3^4] = 5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\ [0.43\ (732\ \text{K})^4 + (1 - 0.43)(300\ \text{K})^4] = 7261.7\ \text{W/m}^2$$

From Equation [5]

$$J_1 = \frac{5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)}{0.43}\ [(732\ \text{K})^4 - (1 - 0.43)\ (300\ \text{K})^4] = 37{,}249\ \text{W/m}^2$$

Applying Equation (9.66) $J_1 = \rho_1\, G_1 + \varepsilon_1\, E_{b1} = (1 - \varepsilon_1)\, G_1 + \varepsilon_1\, \sigma T_1^{\,4}$

$$T_1 = \left[\frac{1}{\varepsilon_1\sigma}[J_1 - (1-\varepsilon_1)\, G_1]\right]^{0.25}$$

$$T_1 = \left[\frac{1}{0.43\left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)}\left((37{,}249\,\text{W/m}^2) - (1-0.43)(7261.7\,\text{W/m}^2)\right)\right]^{0.25}$$

$$T_1 = 1080\ \text{K} = 807°\text{C}$$

(c) The net rate of heat transfer between A_1 and A_2 is given by Equation (9.73)

$$q_{12} = (J_1 - J_2)\, A_1\, F_{12} = (J_1 - \sigma T_2^{\,4})\, A_1\, F_{12}$$

$$q_{12} = \left[37{,}249\ \text{W/m}^2 - \left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)(732\,\text{K})^4\right](0.1\text{m})\ (0.1\text{m})\ (0.43) = 90.2\ \text{W}$$

(d) Applying the conservation of energy of both plates and the surroundings yields

$$q_1 + q_2 + q_3 = 0 \quad \rightarrow \quad q_3 = -\,q_1 - q_2 = -300\ \text{W} - 0 = -300\ \text{W}$$

The surroundings gain 300 watts from the plates.

(e) An estimate of the natural convection heat transfer will be made by treating the heater and receiver as a horizontal enclosed space, heated from below with the surface temperature calculated above.

From Appendix 2, Table 27, for dry air at the average temperature of (459°C + 807°C)/2 = 633°C

Thermal expansion coefficient (β) = 0.00116 1/K

Thermal conductivity (k) = 0.0599 W/(m K)

Kinematic viscosity (ν) = 108 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.73

The Rayleigh number is

$$Ra_\delta = Gr_\delta Pr = \frac{g\beta(\Delta T)\,\delta^3 Pr}{\nu_a^{\,2}} = \frac{\left(9.8\,\text{m/s}^2\right)(0.00116\,1/\text{K})(807°\text{C} - 459°\text{C})(0.05\,\text{m})^3(0.73)}{\left(108\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 30{,}949$$

Applying Equation (5.30a)

$$\overline{Nu_\delta} = 1 + 1.44\left[1 - \frac{1708}{Ra_\delta}\right]^{\bullet} + \left[\left(\frac{Ra_\delta}{5830}\right)^{\frac{1}{3}} - 1\right]^{\bullet}$$

where $[\ ]^{\bullet}$ indicates that the quantity in the brackets should be taken to be zero if it is negative.

$$\overline{Nu}_{\delta} = 1 + 1.44\left[1 - \frac{1708}{30,949}\right]^{\bullet} + \left[\left(\frac{30,949}{5830}\right)^{\frac{1}{3}} - 1\right]^{\bullet} = 3.11$$

$$\bar{h}_c = \overline{Nu}_{\delta}\,\frac{k}{\delta} = 3.11\,\frac{0.0599\,\text{W/(m K)}}{0.05\,\text{m}} = 3.72\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer by convection is given by

$$q = \bar{h}_c\,A\,(\Delta T) = \left(3.72\,\text{W/(m}^2\text{K)}\right)(0.1\text{m})(0.1\text{m})(807°\text{C} - 459°\text{C}) = 13\ \text{W}$$

COMMENTS

The natural convection heat transfer rate is only about 4% of the total heat transfer rate. Therefore, the estimate of natural convection is probably adequate.

Chapter 10

PROBLEM 10.1

> **Water at atmospheric pressure is boiling in a pot with a flat copper bottom on an electric range which maintains the surface temperature at 115°C. Calculate the boiling heat transfer coefficient.**

GIVEN

- Water at atmospheric pressure boiling in a copper bottom pot
- Surface temperature of the pot bottom (T_s) = 115°C

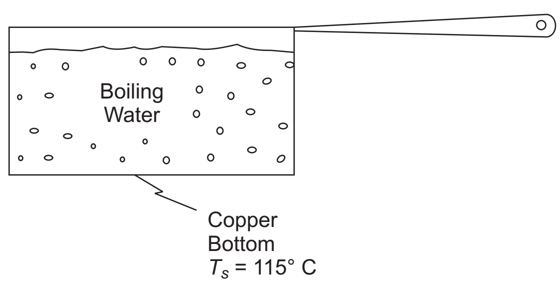
FIND

- The boiling heat transfer coefficient (h_b)

ASSUMPTIONS

- Temperature of the pan bottom is uniform
- The copper is polished

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 1 atm (T_{sat} = 100°C)

Density (ρ_l) = 958.4 kg/m^3

Specific heat (c_l) = 4211 J/(kg K)

Absolute viscosity (μ_l) = 277.5 × 10^{-6} (Ns)/m^2

Prandtl number (Pr_l) = 1.75

Heat of vaporization (h_{fg}) = 2257 kJ/kg = 2.257 × 10^6 J/kg

From Appendix 2, Table 34, the density of steam at 100° (ρ_v) = 0.5977 kg/m^3

From Table 10.2, Surface tension at 100°C (σ) = 0.0589 N/m

From Table 10.1, The coefficient, C_{sf}, for water on emery polished copper = 0.0128

SOLUTION

Assuming the boiling is nucleate boiling, the heat flux, q'', is given by Equation (10.2)

$$\frac{c_l\,\Delta T_x}{h_{fg}\,Pr_l^{\,n}} = C_{sf}\left[\frac{q''}{\mu_l\,h_{fg}}\sqrt{\frac{g_c\,\sigma}{g(\rho_l-\rho_v)}}\right]^{0.33}$$

where $\Delta T_x = T_s - T_{sat} = 115°\text{C} - 100°\text{C} = 15°\text{C}$

$g_c = 1.0$ (in the SI system)

$n = 1.0$ for water

$g = 9.8\ \text{m/s}^2$

Rearranging

$$q'' = \left(\frac{c_l\,\Delta T_x}{h_{fg} Pr_l^n\, C_{sf}}\right)^3 \frac{\mu_l\, h_{fg}}{\sqrt{\dfrac{g_c\,\sigma}{g\,(\rho_l - \rho_v)}}}$$

$$q'' = \left(\frac{(4211\ \text{J/(kg K)})\ (15°\text{C})}{(2.257 \times 10^6\ \text{J/kg})\ (1.75)\ (0.0128)}\right)^3 \frac{(277.5 \times 10^{-6}\ (\text{N s})/\text{m}^2)((\text{kg m})/(\text{s}^2\text{N}))(2.257 \times 10^6\ \text{J/kg})((\text{W s})/\text{J})}{\sqrt{\dfrac{(0.0589\ \text{N/m})((\text{kg m})/(\text{s}^2\text{N}))}{(9.8\ \text{m/s}^2)\ (958.4 - 0.5977)\ \text{kg/m}^3}}}$$

$$= 4.87 \times 10^5\ \text{W/m}^2$$

The critical heat flux for nucleate boiling is given by Equation (10.4)

$$q''_c = \frac{\pi}{24}\ \rho_v^{0.5}\ h_{fg}\ [\sigma\, g\,(\rho_l - \rho_v)\, g_c]^{\frac{1}{4}}$$

$$q''_c = \frac{\pi}{24}\ (0.5977\ \text{kg/m}^3)^{0.5}\ (2.257 \times 10^6\ \text{J/kg})\ ((\text{Ws})/\text{J})$$

$$[(0.0589\ \text{J/kg})\ ((\text{kg m})/(\text{s}^2\text{N}))\ (9.8\ \text{m/s}^2)\ (958.4 - 0.5977)\text{kg/m}^3]^{\frac{1}{4}}$$

$$q''_c = 1.11 \times 10^6\ \text{W/m}^2$$

Since $q'' < q''_c$, the nucleate boiling assumption is valid.

By definition

$$h_b = \frac{q''}{\Delta T_x} = \frac{(4.87 \times 10^5\ \text{W/m}^2)}{15\ \text{K}} = 3.25 \times 10^4\ \text{W/(m}^2\text{K)}$$

PROBLEM 10.2

> **Predict the nucleate-boiling heat transfer coefficient for water boiling at atmospheric pressure on the outside surface of a 1.5 cm *OD* vertical copper tube 1.5 m long. Assume the tube-surface temperature is constant at 10 K above the saturation temperature.**

GIVEN

- Water boiling at atmospheric pressure on the outside surface of vertical copper tube
- Tube outside diameter (D) = 1.5 cm = 0.015 m
- Tube length (L) = 1.5 m
- Tube surface temperature above saturation temperature (ΔT_x) = 10 K

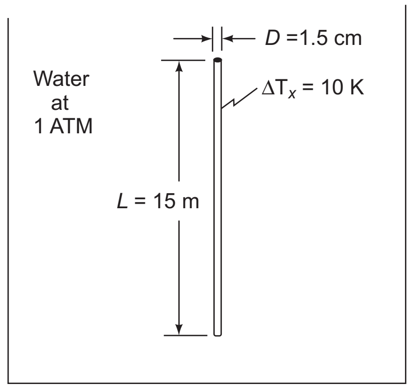
FIND

- The nucleate-boiling heat transfer coefficient (h_b)

ASSUMPTIONS

- The water is at saturation temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 1 atm (T_{sat} = 100°C)

Density (ρ_l) = 958.4 kg/m^3

Specific heat (c_l) = 2411 J/(kg K)

Absolute viscosity (μ_l) = 277.5 × 10^{-6} (Ns)/m^2

Prandtl number (Pr_l) = 1.75

Heat of vaporization (h_{fg}) = 2257 kJ/kg = 2.257 × 10^6 J/kg

From Appendix 2, Table 34, the density of steam at 100°C (ρ_v) = 0.5977 kg/m^3

From Table 10.2, Surface tension at 100°C (σ) = 0.0589 N/m

From Table 10.1, The coefficient, C_{sf}, for water on copper = 0.0130

SOLUTION

As stated near the end of Section 10.2.2, 'the geometric shape of the heating surface has no appreciable effect on the nucleate boiling mechanism'.

Assuming the boiling is nucleate boiling, the heat flux q'', is given by Equation (10.2)

$$\frac{c_l \, \Delta T_x}{h_{fg} Pr_l^{\,n}} = C_{sf} \left[\frac{q''}{\mu_l \, h_{fg}} \sqrt{\frac{g_c \, \sigma}{g(\rho_l - \rho_v)}} \right]^{0.33}$$

where $\Delta T_x = T_s - T_{sat}$ = 10 K

g_c = 1.0 (in the SI system)

n = 1.0 for water

g = 9.8 m/s^2

Rearranging

$$q'' = \left(\frac{c_l \, \Delta T_x}{h_{fg} Pr_l^{\,n} C_{sf}} \right)^3 \frac{\mu_l \, h_{fg}}{\sqrt{\dfrac{g_c \, \sigma}{g\,(\rho_l - \rho_v)}}}$$

$$q'' = \left(\frac{(4211\,\text{J/(kg K)})\,(10°\text{C})}{\left(2.257 \times 10^6\ \text{J/kg}\right)(1.75)\,(0.0130)} \right)^3$$

$$\times \frac{\left(277.5\times10^{-6}\,(\text{N s})/\text{m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(2.257\times10^6\ \text{J/kg}\right)\left((\text{W s})/\text{J}\right)}{\sqrt{\dfrac{0.0589\ \text{N/m}\left((\text{kg m})/(\text{s}^2\text{N})\right)}{\left(9.8\ \text{m/s}^2\right)(958.4-0.5977)\ \text{kg/m}^3}}}$$

$$= 1.37\times10^5\ \text{W/m}^2$$

The critical heat flux for nucleate boiling is given by Equation (10.4)

$$q''_c = \left(\frac{\pi}{24}\right)\rho_v^{0.5}\,h_{fg}\,[\sigma\,g\,(\rho_l-\rho_v)\,g_c]^{\frac{1}{4}}$$

$$q''_c = \left(\frac{\pi}{24}\right)\left(0.5977\ \text{kg/m}^3\right)^{0.5}\left(2.257\times10^6\ \text{J/kg}\right)\left((\text{Ws})/\text{J}\right)$$

$$\left[(0.0589\ \text{J/kg})\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(9.8\ \text{m/s}^2\right)(958.4-0.5977)\ \text{kg/m}^3\right]^{\frac{1}{4}}$$

$$q''_c = 1.11\times10^6\ \text{W/m}^2$$

Since $q'' < q''_c$, the nucleate boiling assumption is valid.
By definition

$$h_b = \frac{q''}{\Delta T_x} = \frac{\left(1.37\times10^5\ \text{W/m}^2\right)}{10\ \text{K}} = 1.37\times10^4\ \text{W/(m}^2\text{K)}$$

PROBLEM 10.3

Estimate the maximum heat flux obtainable with nucleate pool boiling on a clean surface for (a) water at 1 atm on brass, (b) water at 10 atm on brass.

GIVEN

- Nucleate pool boiling on a clean surface

FIND

- The maximum heat flux obtainable for (a) water at 1 atm on brass; and (b) water at 10 atm on brass

ASSUMPTIONS

- Water is at saturation temperature

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water from Table 10.2 for surface tension

Pressure (atm)	1	10
Saturation Temperature (°C)	100	180.4
Liquid density, ρ_l (kg/m^3)	958.4	886.1
Heat of Vaporization, h_{fg} (J/kg)	2.257×10^6	2.013×10^6
Vapor density, $\rho_v = 1/v_g$ (kg/m^3)	0.5977	5.22
Surface tension, σ (N/m)	0.0589	0.0422

SOLUTION

The maximum heat flux for nucleate boiling is given by Equation 10.4

$$q''_c = \left(\frac{\pi}{24}\right)\rho_v^{0.5}\,h_{fg}\,[\sigma\,g\,(\rho_l-\rho_v)\,g_c]^{\frac{1}{4}}$$

Case (a)

$$q''_c = \left(\frac{\pi}{24}\right)\left(0.5977\,\text{kg/m}^3\right)^{0.5}\left(2.257\times10^6\,\text{J/kg}\right)\,((\text{Ws})/\text{J})$$

$$\left[(0.0589\,\text{J/kg})\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(9.8\,\text{m/s}^2\right)(958.4-0.5977)\text{kg/m}^3\right]^{\frac{1}{4}}$$

$$q''_c = 1.11\times10^6\ \text{W/m}^2$$

Case (b)

$$q''_c = \left(\frac{\pi}{24}\right)\left(5.22\,\text{kg/m}^3\right)^{0.5}\left(2.013\times10^6\,\text{J/kg}\right)\,((\text{Ws})/\text{J})$$

$$\left[(0.0422\,\text{J/kg})\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(9.8\,\text{m/s}^2\right)(886.1-5.22)\,\text{kg/m}^3\right]^{\frac{1}{4}}$$

$$q''_c = 2.63\times10^6\ \text{W/m}^2$$

PROBLEM 10.4

Determine the excess temperature at one-half of the maximum heat flux for the fluid-surface combinations in Problem 10.3.

From Problem 10.3: Estimate the maximum heat flux obtainable with nucleate pool boiling on a clean surface for (a) water at 1 atm on brass, (b) water at 10 atm on brass.

GIVEN

- Nucleate pool boiling on a clean surface

FIND

The excess temperature (ΔT_x) at one half of the maximum heat flux for
(a) water at 1 atm on brass
(b) water at 10 atm on brass

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water Table 10.2 for surface tension

Pressure (atm)	1	10
Saturation Temperature (°C)	100	180.4
Liquid density, ρ_l (kg/m^3)	958.4	886.1
Heat of Vaporization, h_{fg} (J/kg)	2.257×10^6	2.013×10^6
Vapor density, $\rho_v = 1/V_g$ (kg/m^3)	0.5977	5.22
Surface tension, σ (N/m)	0.0589	0.0422
Specific heat, c_l (J/kg K)	4211	4398
Absolute viscosity, μ_l (Ns/m^2)	277.5×10^{-6}	151.7×10^{-6}
Prandtl number, Pr	1.75	1.01

From Table 10.1, the coefficient, C_{sf}, for water on brass = 0.0060

SOLUTION

Solving Equation (10.2) for the excess temperature

$$\Delta T_x = \frac{C_{sf}\,h_{fg}\,Pr_l^{\,n}}{c_l}\left[\frac{q''}{\mu_l\,h_{fg}}\sqrt{\frac{g_c\,\sigma}{g\,(\rho_l-\rho_v)}}\right]^{0.33}$$

where n = 10 for water.

(a) From the solution to Problem 110.3, $q''_{max} = 1.11 \times 10^6$ W/m^2.
For $q'' = q''_{max}/2 = 5.55 \times 10^5$ W/m^2, and water at 1 atm on brass

$$\Delta T_x = \frac{(0.0060)\left(2.257\times10^6 \text{ J/kg}\right)(1.75)}{(4211 \text{J/(kg K)})}$$

$$\left[\frac{5.55\times10^5 \text{ W/m}^2 \text{ (J/(W s))}}{\left(277.5\times10^{-6} \text{ (N s)/m}^2\right)\left(2.257\times10^6 \text{ J/(kg)}\right)\left(\text{(kg m)/(s}^2\text{N)}\right)}\sqrt{\frac{0.0589 \text{ N/m}\left(\text{(kg m)/(s}^2\text{N)}\right)}{\left(9.8 \text{ m/s}^2\right)(958.4-0.5977)\text{kg/m}^3}}\right]^{0.33}$$

$$\Delta T_x = 7.3 \text{ K}$$

(b) From the solution to Problem 10.3, $q''_{max} = 2.63 \times 10^6$ W/m^2.
For $q'' = q''_{max}/2 = 1.315 \times 10^6$ W/m^2; and water at 10 atm on brass

$$\Delta T_x = \frac{(0.0060)\left(2.013\times10^6 \text{ J/kg}\right)(1.01)}{(4398 \text{ J/(kg K)})}$$

$$\left[\frac{1.315\times10^6 \text{ W/m}^2 \text{ (J/(W s))}}{\left(151.7\times10^{-6} \text{ (N s)/m}^2\right)\left(2.013\times10^6 \text{ J/kg}\right)\left(\text{(kg m)/(s}^2\text{N)}\right)}\sqrt{\frac{0.0422 \text{ N/m}\left(\text{(kg m)/(s}^2\text{N)}\right)}{\left(9.8 \text{ m/s}^2\right)(886.1-5.22)\text{kg/m}^3}}\right]^{0.33}$$

$$\Delta T_x = 5.8 \text{ K}$$

PROBLEM 10.5

In a pool boiling experiment with water boiling on a large horizontal surface at atmospheric pressure, a heat flux of 4 × 10^5 W/m^2 was measured at an excess temperature of 14.5 K. What was the boiling surface material?

GIVEN

- Water at atmospheric pressure boiling on an unknown surface
- Heat flux and excess temperature

FIND

(a) The boiling surface

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the properties of water at 100°C are

Density (ρ_l) = 958.4 kg/m^3

Specific heat (c_l) = 4211 J/(kg K)

Absolute viscosity (μ_l) = 277.5 × 10^{-6} kg/ms

Prandtl number (Pr_l) = 1.75

Heat of vaporization (h_{fg}) = 2.257 × 10^6 J/kg

Vapor density (ρ_v) = 0.598 kg/m^3

Table 10.2 gives

Surface tension (σ) = 58.9 × 10^{-3} N/m

SOLUTION

With the given data, we can find the coefficient C_{sf} for the surface and then by using Table 10.1, determine what surface was used. Solving Equation (10.2) for C_{sf}:

$$C_{sf} = \frac{c_l\,\Delta T_x}{h_{fg}\,Pr_l^{\,n}} \left[\frac{q''}{\mu_l\,h_{fg}} \sqrt{\frac{g_c\,\sigma}{g\,(\rho_l - \rho_v)}} \right]^{-0.33}$$

For water, $n = 1$ in this equation.
So

$$C_{sf} = \frac{(4211\,\text{J/(kg K)})(14.5\,\text{K})}{\left(2.257\times10^{6}\,\text{J/kg}\right)(1.75)}$$

$$\left[\frac{\left(4\times10^{5}\,\text{W/m}^2\right)}{\left(277.5\times10^{-6}\,\text{kg/(ms)}\right)\left(2.257\times10^{6}\,\text{J/kg}\right)} \sqrt{\frac{(0.0589\,\text{N/m})}{\left(9.81\,\text{m/s}^2\right)(958.4-0.598)\,\text{kg/m}^3}} \right]^{-0.33} = 0.013$$

From Table 10.1, mechanically-polished stainless steel was most likely the boiling surface used in the experiment.

PROBLEM 10.6

Compare the critical heat flux for a flat horizontal surface with that for a submerged horizontal wire of 3 mm diameter in water at saturation temperature and pressure.

GIVEN

- Flat, horizontal surface and a submerged, horizontal wire

FIND

(a) Critical heat flux for both geometries

ASSUMPTIONS

- The water is at atmospheric pressure

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13

Liquid density (ρ_l) = 958.4 kg/m^3

Vapor density (ρ_v) = 0.598 kg/m^3

and Table 10.2 gives

Surface tension (σ) = 0.0589 N/m

SOLUTION

From Table 10.3, entry #5, the ratio of critical heat fluxes for two geometries is

$$\frac{q''_{\max,\text{wire}}}{q''_{\max,\text{Z}}} = 0.94 \left(\frac{R}{L_b}\right)^{-\frac{1}{4}}$$

The bubble length scale, L_b is calculated from

$$L_b = \sqrt{\frac{\sigma}{g(\rho_l - \rho_v)}} = \sqrt{\frac{(0.0589\,\text{N/m})\left((\text{kg m})/(\text{s}^2\text{N})\right)}{(9.81\,\text{m/s}^2)(958.4 - 0.598)\left(\text{kg/m}^3\right)}} = 0.0025 \text{ m} = 2.5 \text{ mm}$$

Since $\frac{R}{L_b} = \frac{1.5}{2.5} = 0.6$, we have

$$\frac{q''_{\max,\text{wire}}}{q''_{\max,Z}} = 0.94\ (0.6)^{\frac{1}{4}} = 1.068$$

The wire has about 7% higher critical heat flux than the horizontal plate.

PROBLEM 10.7

For saturated pool boiling of water on a horizontal plate, calculate the peak heat flux at pressures of 10, 20, 40, 60, and 80 percent of the critical pressure p_c and plot your rsesults as q''_{max}/p_c versus p/p_c. The surface tension of water may be taken as σ = 0.0743 (1 – 0.0026 T), where σ is in newtons per meter and T in degrees Celsius. The critical pressure of water is 22.09 MPa.

GIVEN

- Saturated pool boiling of water on a horizontal plate
- Surface tension of water (σ) = 0.0743 (1 – 0.0026 T) (Where σ is in N/m and T is in °C)
- The critical pressure (p_c) = 22.09 MPa = 2.209×10^4 kPa

FIND

- The peak heat flux, q''_{max} for pressures of 10, 20, 40, 60, and 80 percent of the critical pressure, p_c

ASSUMPTIONS

- Steady state
- The plate is clean

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water

Percent of, p_c	10	20	40	60	80
Pressure, p (kPa)	2209	4418	8836[(a)]	13,254[(a)]	17,672[(a)]
Saturation Temperature T_{sat} (°C)	217	256	302	332	355
Liquid density, ρ_l (kg/m^3)	844.6	789.7	709.7	634.0	550.9
Vapor density, ρ_v (kg/m^3)	11.1	22.3	47.9	80.7	128.5
Heat of vaporization, $h_{fg} \times 10^{-6}$ (J/kg)	1.8635	1.6768	1.3844	1.1093	0.803

(a) data from steam tables

SOLUTION

The peak heat flux is given by Equation 10.4

$$q''_c = \left(\frac{\pi}{24}\right) \rho_v^{0.5} h_{fg} \left[\sigma g(\rho_l - \rho_v) g_c\right]^{\frac{1}{4}}$$

For $p = 0.1\ p_c$:

$$q''_c = \left(\frac{\pi}{24}\right) \left(11.1\ \text{kg/m}^3\right)^{0.5} \left(1.8635 \times 10^6\ \text{J/kg}\right) \left((\text{Ws})/\text{J}\right)$$

$$\left[(0.0743[1 - 0.0026(217)]\text{N/m}\right) \left((\text{kg m})/(\text{s}^2\text{N})\right) \left(9.8\ \text{m/s}^2\right) (844.6 - 11.1\text{kg/m}^3\Big]^{\frac{1}{4}}$$

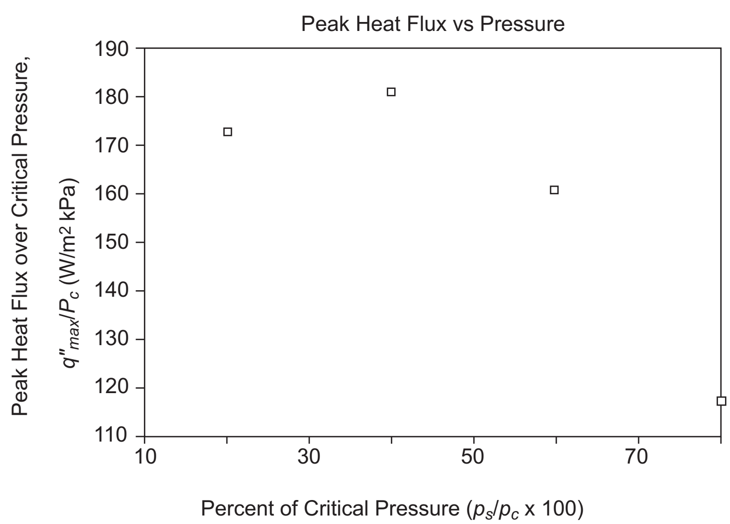
$$q''_c = 3.28 \times 10^6\ \text{W/m}^2$$

$$\frac{q''_{max}}{P_c} = \frac{\left(3.28 \times 10^6\ \text{W/m}^2\right)}{2.209 \times 10^4\ \text{kPa}} = 148\ \frac{\text{W/m}^2}{\text{kPa}}$$

Repeating this procedure for the rest of the cases

Percent of p_c	10	20	40	60	80
$q''_{max} \times 10^{-6}$ (W/m^2)	3.28	3.83	4.00	3.56	2.63
q''_{max}/p_c (W/m^2 kPa)	148	173	181	161	119

These results are plotted below



COMMENTS

Note the similarity of the graph to Figure 10.9.

PROBLEM 10.8

A 0.6-cm-thick flat plate of stainless steel, 7.5 cm wide and 0.3 m long, is immersed horizontally at an initial temperature of 980°C in a large water bath at 100°C and at atmospheric pressure. Determine how long it will take this plate to cool to 540°C.

GIVEN

- A flat stainless steel plate is immersed horizontally in water
- Plate thickness (s) = 0.6 cm = 0.006 m
- Plate width (w) = 7.5 cm = 0.075 m
- Plate length (L) = 0.3 m
- Initial plate temperature (T_{pi}) = 980°C = 1253 K
- Pressure = 1 atm
- Water bath temperature (T_b) = 100°C (saturation) = 373 K

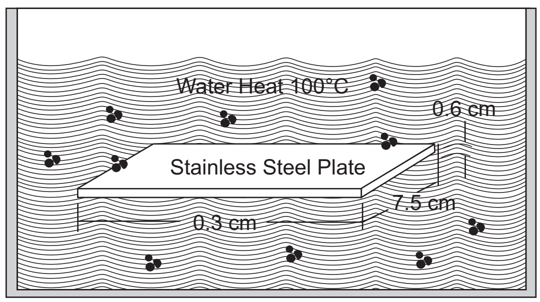
FIND

- Time for plate to cool to T_{pf} = 540°C = 813 K

ASSUMPTIONS

- The heat fluxes from the bottom and top of the plate are equal
- The plate is polished

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the pool temperature of 100°C

Density (ρ) = 958.4 kg/m^3

Thermal conductivity (k) = 0.682 W/(m K)

Absolute viscosity (μ) = 277 × 10^{-6} (Ns)/m^2

Prandtl number (Pr) = 1.75

Specific heat (c) = 4211 J/(kg K)

Heat of vaporization (h_{fg}) = 2.257 × 10^6 J/kg

From Appendix 2, Table 34, for steam at 100°C

k_v = 0.0249 W/(m K) c_v = 2034 J/(kg K) μ_v = 12.10 × 10^{-6} (Ns)/m^2

ρ = 0.5977 kg/m^3

From Table 10.2, The surface tension, σ, for water @ 100°C = 0.0589 N/m

From Table 10.1, The coefficient, C_{sf}, for water on mechanically polished stainless steel = 0.0132

From Table 9.2, the emissivity of polished stainless steel at the average temperature of (980°C + 540°C)/2 = 760°C (ε) = 0.22

From Appendix 1, Table 5, the Stephen-Botzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

From Appendix 2, Table 10, for type 304 stainless steel

(k_{steel}) = 14.4 W/(m K) ρ_{steel} = 7817 kg/m^3 c_{steel} = 461 J/(kg K).

SOLUTION

As the plate cools, the heat flux from the plate will diminish. Therefore, the heat flux will be assumed to be constant over a small time step, then the plate temperature will be updated. This procedure will be repeated until the plate temperature drops to 540°C.

The initial excess temperatures, $\Delta T_x = T_{\text{pi}} - T_{\text{sat}}$ = 980°C – 100°C = 880°C.

Assuming nucleate boiling, the heat flux is given by Equation (10.2)

$$\frac{c_l\,\Delta T_x}{h_{fg}\,Pr_l^n} = C_{sf}\left[\frac{q''}{\mu_l\,h_{fg}}\sqrt{\frac{g_c\,\sigma}{g\,(\rho_l-\rho_v)}}\right]^{0.33}$$

Solving for the heat flux

$$q'' = \left(\frac{c_l\,\Delta T_x}{h_{fg}Pr_l^n\,C_{sf}}\right)^3 \frac{\mu_l\,h_{fg}}{\sqrt{\dfrac{g_c\,\sigma}{g\,(\rho_l-\rho_v)}}}$$

$$q'' = \left(\frac{(4211\,\text{J/(kg K)})\,(880\,°\text{C})}{\left(2.257\times10^6\ \text{J/kg}\right)(1.75)\,(0.0132)}\right)^3 \frac{\left(277.5\times10^{-6}\,(\text{N s})/\text{m}^2\right)(\text{kg m})/(\text{s}^2\text{N})\left(2.257\times10^6\ \text{J/kg}\right)((\text{W s})/\text{J})}{\sqrt{\dfrac{0.0589\,\text{N/m}\left((\text{kg m})/(\text{s}^2\text{N})\right)}{\left(9.8\ \text{m/s}^2\right)(958.4-0.5977)\text{kg/m}^3}}}$$

$$= 8.96\times10^{10}\ \text{W/m}^2$$

The critical heat flux for nucleate boiling is given be Equation (10.4)

$$q''_c = \frac{\pi}{24}\rho_v^{0.5}\,h_{fg}\,[\sigma\,g\,(\rho_l-\rho_v)\,g_c]^{\frac{1}{4}}$$

$$q''_c = \left(\frac{\pi}{24}\right)\left(0.5977\,\text{kg/m}^3\right)^{0.5}\left(2.257\times10^6\ \text{J/kg}\right)\ ((\text{Ws})/\text{J})$$

$$\left[(0.0589\ \text{J/kg})\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(9.8\,\text{m/s}^2\right)(958.4-0.5977)\ \text{kg/m}^3\right]^{\frac{1}{4}}$$

Since $q'' > q''_c$, The nucleate boiling assumption is invalid and film boiling will occur. Note that at the final plate temperature, $T_{b,f}$ = 540°C, ΔT_x = 440°C, and $q'' = 1.12\times10^{10} > q''_c$. Therefore, film boiling will occur during the entire cooling period.

For film boiling on flat horizontal surfaces, the conduction heat transfer coefficient is given by Equation (10.7)

$$\bar{h}_c = \left(0.59+0.69\frac{\lambda}{D}\right)\left\{\frac{g\,(\rho_l-\rho_v)\rho_v\,k_v^3\,[h_{fg}+0.68c_v\Delta T_x]}{\lambda\,\mu_v\,\Delta T_x}\right\}^{\frac{1}{4}}$$

$$\text{where } \lambda = 2\pi\left[\frac{g_c\,\sigma}{g\,(\rho_l-\rho_c)}\right]^{\frac{1}{2}} = 2\pi\left[\frac{\left((\text{kg m})/(\text{s}^2\text{N})\right)(0.0589\,\text{N/m})}{\left(9.8\ \text{m/s}^2\right)(958.4-0.5977)\ \text{kg/m}^3}\right]^{\frac{1}{2}} = 0.01574\ \text{m}$$

For a flat plate, $D \to \infty$, therefore $\lambda/D \to 0$

$$\bar{h}_c = \left[\frac{(9.8\ \text{m/s}^2)(958.4 - 0.5977)\ \text{kg/m}^3 (0.5977\ \text{kg/m}^3)(0.0249\ \text{W/(m K)})^3}{0.01574\ \text{m}(12.1 \times 10^{-6}\ (\text{N s})/\text{m}^2)\Delta T_x} \right.$$

$$\left. \left[2.257 \times 10^6\ \text{J/kg} + 0.68(2034\ \text{J/(kg K)})\Delta T_x\right] \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 15.32 \left[\frac{2.257 \times 10^6 + 1383.1 \Delta T_x}{\Delta T_x} \right]^{\frac{1}{4}}\ \text{W/(m}^2\text{K)}\ \ (\Delta T_x \text{ in °C})$$

Initially

$$\bar{h}_c = 15.32 \left[\frac{2.257 \times 10^6 + 1383.1(880°\text{C})}{(880°\text{C})} \right]^{\frac{1}{4}} = 121.4\ \text{W/(m}^2\text{K)}$$

The radiation heat transfer coefficient is given by Equation (10.9)

$$\bar{h}_r = \sigma \varepsilon \left(\frac{T_p^4 - T_{\text{sat}}^4}{T_p - T_{\text{sat}}} \right) = (5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4))\ (0.22) \left(\frac{T_p^4 - (373\text{K})^4}{T_p - (373\text{K})} \right)$$

Initially

$$\bar{h}_r = (1.247 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)) \left(\frac{(1253\text{K})^4 - (373\text{K})^4}{(1253\text{K}) - (373\text{K})} \right) = 34.7\ \text{W/(m}^2\text{K)}$$

The total heat transfer coefficient is given by Equation (10.8)

$$h_{\text{total}} = \bar{h}_c + 0.75\ \bar{h}_r = [121.4 + (0.75)\ (34.7)]\ \text{W/(m}^2\text{K)} = 147.4\ \text{W/(m}^2\text{K)}$$

The initial rate of heat transfer is

$$q = \bar{h}_{\text{total}}\ A\ (T_p - T_b) = \bar{h}_{\text{total}}\ 2H\,L\ (T_p - T_b)$$

$$q = (147.4\ \text{W/(m}^2\text{K)})\ 2\ (0.075\ \text{m})(0.3\ \text{m})\ (980°\text{C} - 100°\text{C}) = 5838\ \text{W}$$

The initial Biot number is

$$Bi = \frac{h_{\text{total}}\, s}{2\,k_{\text{steel}}} = \frac{(147.4\ \text{W/(m}^2\text{K)})(0.006\ \text{m})}{2(14.4\ \text{W/(m K)})} = 0.03 < 0.1$$

Therefore, the internal resistance of the steel is negligible. The change in the plate temperature is given by

$$\Delta T = \frac{q\,\Delta t}{m_{\text{steel}}\, c_{\text{steel}}} = \frac{q\,\Delta t}{\rho_{\text{steel}}\ (\text{volume})\, c_{\text{steel}}} = \frac{q\,\Delta t}{H\ L\ s\ \rho_{\text{steel}}\ c_{\text{steel}}}$$

For $\Delta t = 5$ s

$$\Delta T = \frac{5838\ \text{W}(5\ \text{s})}{(0.075\text{m})(0.3\text{m})(0.006\text{m})(7817\ \text{kg/m}^3)(461\ \text{J/(kg K)})((\text{W s})/\text{J})} = 60\ \text{K}$$

Therefore, after 5 s

$$T_p = T_{pi} - \Delta T = 980°C - 60°C = 920°C$$

Repeating this procedure: for Δt = 5 s

Time (s)	T_p (°C)	h_{total} (W/m^2 K)	q (W)	ΔT (K)
0	980	147.4	5838	60
5	920	145.7	5378	55.3
10	864.7	144.6	4858	49.9
15	814.8	143.8	4626	47.5
20	767.2	143.4	4307	44.3
25	723.0	143.4	4019	41.3
30	681.7	143.5	3757	38.6
35	643.1	143.9	3518	36.2
40	606.9	144.6	3299	33.9
45	573.0	145.4	3096	31.8
50	540			

The plate will cool to 540°C in approximately 50 seconds.

PROBLEM 10.9

Calculate the maximum heat flux attainable in nucleate boiling with saturated water at 2 atm pressure in a gravitational field equivalent to one-tenth that of the earth.

GIVEN

- Nucleate boiling with saturated water
- Pressure = 2 atm
- Gravitational field = 1/10 that of earth

FIND

- The maximum heat flux (q''_{max})

ASSUMPTIONS

- Steady state conditions
- Nucleate pool boiling

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 2 atm (2.0264×10^5 N/m^2)

Saturation temperature (T_s): 120.5°C

Water density, (ρ_l): 943.5 kg/m^3

Heat of vaporization, (h_{fg}): 2201 J/kg

Vapor density, ($\rho_v = 1/v_g$): 1.13 kg/m^3

From Table 10.2, the surface tension (σ) = 0.0547 N/m

Acceleration due to gravity on earth (g_e) = 9.8 m/s^2

SOLUTION

The maximum heat flux is given by Equation (10.4)

$$q''_{max} = \frac{\pi}{24}\,\rho_v^{0.5}\,h_{fg}\,[\sigma\, g\,(\rho_l - \rho_v)g_c]^{\frac{1}{4}}$$

where
$$g = \left(\frac{1}{10}\right) g_{earth} = 0.98\ \text{m/s}^2$$

$$q''_{max} = \left(\frac{\pi}{24}\right)\left(1.13\ \text{kg/m}^3\right)^{0.5}\left(2.201\times10^6\ \text{J/kg}\right)\ ((\text{Ws})/\text{J})$$

$$\left[(0.0547\ \text{J/kg})\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(0.98\ \text{m/s}^2\right)(943.5-1.13)\text{kg/m}^3\right]^{\frac{1}{4}}$$

$$q''_{\max} = 8.17 \times 10^5\ \text{W/m}^2$$

PROBLEM 10.10

Prepare a graph showing the effect of subcooling between 0 and 50°C on the maximum heat flux calculated in Problem 10.9.
From Problem 10.9: Calculate the maximum heat flux attainable in nucleate boiling with saturated water at 2 atm pressure in a gravitational field equivalent to one-tenth that of the earth.

GIVEN

- Nucleate boiling with saturated water
- Pressure = 2 atm
- Gravitational field = 1/10 that of earth
- From Problem 10.9, the maximum heat flux ($q''_{\max,sat}$) = 8.17×10^5 W/m^2

FIND

- Prepare a graph showing the effect of sub cooling ($T_{sat} - T_{liquid}$) between 0 and 50°C on $q''_{\max}$

ASSUMPTIONS

- Steady state conditions
- Nucleate pool boiling

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 2 atm (2.0264×10^5 N/m^2)

Saturation temperature (T_s): 120.5°C
Water density, (ρ_l): 943.5 kg/m^3
Heat of vaporization, (h_{fg}): 2201 J/kg
Vapor density, ($\rho_v = 1/v_g$): 1.13 kg/m^3
Thermal conductivity (k_l) = 0.685 W/(m K)
Thermal diffusivity (α_l) = 0.171×10^{-6} m^2/s

From Table 10.2, the surface tension (σ) = 0.0547 N/m
Acceleration due to gravity on earth (g_e) = 9.8 m/s^2

SOLUTION

For subcooling, the effect on the maximum heat flux is given by Equation (10.5)

$$q''_{\max} = q''_{\max,sat}\left(1+\left[\frac{2k_l(T_{sat}-T_{liquid})}{\sqrt{\pi\alpha_l\,\tau}}\right]\frac{24}{\pi h_{fg}\rho_v}\left[\frac{\rho_v^2}{g_c\,\sigma\,g(\rho_l-\rho_v)}\right]^{\frac{1}{4}}\right)$$

$$\text{where } \tau = \left(\frac{\pi}{3}\right)\sqrt{2\pi}\left[\frac{g_c\sigma}{g(\rho_l-\rho_v)}\right]^{\frac{1}{2}}\left[\frac{\rho_v^2}{g_c\,\sigma\,g(\rho_l-\rho_v)}\right]^{\frac{1}{4}}$$

$$\tau = \left(\frac{\pi}{3}\right)\sqrt{2\pi}\left[\frac{\left((\text{kg m})/(\text{s}^2\text{N})\right)0.0547\ \text{N/m}}{\left(0.98\ \text{m/s}^2\right)(943.5-1.13)\,\text{kg/m}^3}\right]^{\frac{1}{2}}$$

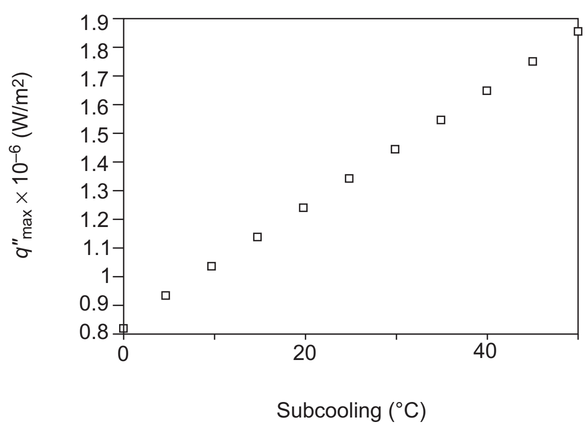
$$\left[\frac{\left(1.13\ \text{kg/m}^3\right)^2}{\left((\text{kg m})/(\text{s}^2\text{N})\right)(0.0547\ \text{N/m})\left(0.98\ \text{m/s}^2\right)(943.5-1.13)\ \text{kg/m}^3}\right]^{\frac{1}{4}} = 0.008055\ \text{s}$$

$$q''_{max} = \left(8.17\times10^5\ \text{W/m}^2\right)\left[1+\left[\frac{2(0.685\ \text{W/(m K)})(T_{sat}-T_{liquid})}{\sqrt{\pi(0.171\times10^{-6}\ \text{m}^2/\text{s})(0.008055\,\text{s})}}\right]\frac{24}{\pi\left(2.201\times10^6\ \text{J/kg}\right)\left(1.13\ \text{kg/m}^3\right)((\text{W s})/\text{J})}\left[\frac{\left(1.13\ \text{kg/m}^3\right)^2}{(\text{kg m})/(\text{s}^2\text{N})(0.0547\,\text{N/m})\left(0.98\ \text{m/s}^2\right)(943.5-1.13)\text{kg/m}^3}\right]\right]$$

$$q''_{max} = \left(8.17\times10^5\ \text{W/m}^2\right)\ [1 + 0.02551\ (T_{sat} - T_{liquid})]$$

This is tabulated and graphed fro different values of ($T_{sat} - T_{liquid}$) below

$T_{sat} - T_{liquid}$ (°C)	$q''_{max} \times 10^{-6}$ (W/m^2)
0	0.817
5	0.921
10	1.03
15	1.13
20	1.23
25	1.34
30	1.44
35	1.55
40	1.65
45	1.75
50	1.86



PROBLEM 10.11

A thin-walled horizontal copper tube of 0.5 cm *OD* is placed in a pool of water at atmospheric pressure and 100°C. Inside the tube, an organic vapor is condensing and the outside surface temperature of the tube is uniform at 232°C. Calculated the average heat transfer coefficient on the outside of the tube.

GIVEN

- A horizontal copper tube in a pool of water at atmospheric pressure
- Tube outside diameter (D) = 0.5 cm = 0.005 m
- Water temperature (T_w) = 100°C = 373 K
- Tube outside surface temperature (T_t) = 232°C = 505 K (uniform)

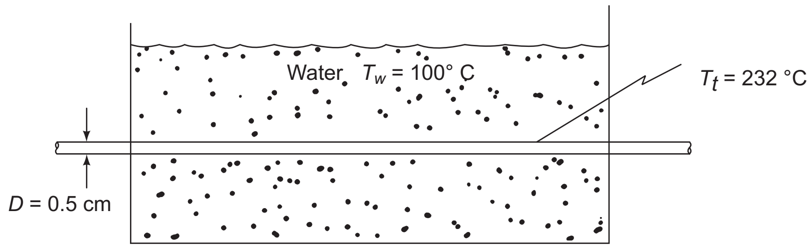
FIND

- The average heat transfer coefficient (h_{total})

ASSUMPTIONS

- Steady state
- The copper tube is polished

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 1 atm, 100°C (saturation temperature)

Density (ρ_l) = 958.4 kg/m^3

Specific heat (c_l) = 4211 J/(kg K)

Absolute viscosity (μ_l) = 277.5 × 10^{-6} (Ns)/m^2

Prandtl number (Pr) = 1.75

Heat of vaporization (h_{fg}) = 2257 kJ/kg = 2.257 × 10^6 J/kg

From Appendix 2, Table 34, for steam at 100°C

Density ρ_v = 0.5977 kg/m^3

Specific heat (c_v) = 2034 J/(kg K)

Thermal conductivity (k_v) = 0.0249 W/(m K)

Absolute viscosity (μ_v) = 12.10 × 10^{-6} (Ns)/m^2

From Table 10.2, the surface tension (σ) = 0.0589 N/m

From Table 10.1, for water on polished copper, the constant C_{sf} = 0.0128

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

From Table 9.2, the emissivity of polished copper (ε) ≈ 0.04.

SOLUTION

The excess temperature $\Delta T_x = T_t = T_{\text{sat}}$ = 232°C – 100°C = 132°C. This high an excess temperature will probably lead to film boiling. This can be checked by calculating the nucleate boiling heat flux and comparing it to the critical flux. The nucleate boiling heat flux is given by Equation (10.2)

$$\frac{c_l \Delta T_x}{h_{fg} Pr_l^n} = C_{sf}\left[\frac{q''}{\mu_l\, h_{fg}}\sqrt{\frac{g_c\,\sigma}{g(\rho_l - \rho_v)}}\right]^{0.33}$$

Rearranging

$$q'' = \left(\frac{c_l\,\Delta T_x}{h_{fg} Pr_l^n\, C_{sf}}\right)^{\left(\frac{1}{0.33}\right)} \frac{\mu_l\, h_{fg}}{\sqrt{\dfrac{g_c\,\sigma}{g(\rho_l - \rho_v)}}}$$

$$q'' = \left(\frac{(4211\ \text{J/(kg K)})(132\,°\text{C})}{(2.257 \times 10^6\ \text{J/kg})(1.75)(0.0128)} \right)^{\left(\frac{1}{0.33}\right)}$$

$$\frac{(277.5 \times 10^{-6}\ (\text{N s})/\text{m}^2)\ (\text{kg m})/(\text{s}^2\text{N})(2.257 \times 10^6\ \text{J/kg})((\text{W s})/\text{J})}{\sqrt{\dfrac{0.0589\ \text{N/m}\ ((\text{kg m})/(\text{s}^2\text{N}))}{(9.8\ \text{m/s}^2)(958.4 - 0.5977)\text{kg/m}^3}}}\ 3.57 \times 10^8\ \text{W/m}^2$$

The critical heat flux for nucleate boiling is given by Equation (10.4)

$$q''_c = \left(\frac{\pi}{24}\right)\ \rho_v^{0.5}\, h_{fg}\ [\sigma\, g\, (\rho_l - \rho_v)\, g_c]^{\frac{1}{4}}$$

$$q''_c = \left(\frac{\pi}{24}\right)\ (0.5977\ \text{kg/m}^3)^{0.5}\ (2.257 \times 10^6\ \text{J/kg})\ ((\text{Ws})/\text{J})$$

$$[(0.0589\ \text{J/kg})\ ((\text{kg m})/(\text{s}^2\text{N}))\ (9.8\ \text{m/s}^2)\ (958.4 - 0.5977)\text{kg/m}^3]^{\frac{1}{4}}$$

$$q''c = 1.11 \times 10^6\ \text{W/m}^2$$

Since $q'' > q''_{\text{max}}$, film boiling will exist. The conductive heat transfer coefficient for film boiling on tubes is given by Equation (10.6)

$$\bar{h}_c = 0.62 \left(\frac{g\,(\rho_l - \rho_v)\,\rho_v\, k_v^3\, [h_{fg} + 0.68\, c_{pv}\, \Delta t_x]}{D\,\mu_v \Delta t_x} \right)^{\frac{1}{4}}$$

$$\bar{h}_c = 0.62 \left[\frac{(9.8\ \text{m/s}^2)(958.4 - 0.5977)\text{kg/m}^3\ (0.5977\ \text{kg/m}^3)(0.0249\ \text{W/(m K)})^3}{(0.005\,\text{m})(12.10 \times 10^{-6}\ (\text{N s})/\text{m}^2)((\text{kg m})/(\text{s}^2\text{N}))(132°\text{C})} \right.$$

$$\left. \frac{[2.257 \times 10^6\ \text{J/kg} + 0.68(2034\ \text{J/(kg K)})(132\,°\text{C})((\text{W s})/\text{J})]}{} \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 250\ \text{W/(m}^2\text{K)}$$

The radiative heat transfer coefficient is given by Equation (10.9)

$$\bar{h}_r = \sigma_r\, \varepsilon \left(\frac{T_t^4 - T_w^4}{T_t - T_w} \right) = (5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4))$$

$$(0.04) \left(\frac{(505\,\text{K})^4 - (373\,\text{K})^4}{(505\,\text{K}) - (373\,\text{K})} \right) = 0.78\ \text{W/(m}^2\text{K)}$$

The total heat transfer coefficient is given by Equation (10.8)

$$h_{\text{total}} = \bar{h}_c + 0.75\ \bar{h}_r = [250 + (0.75)\ (0.78)]\ \text{W/(m}^2\text{K)} = 251\ \text{W/(m}^2\text{K)}$$

COMMENTS

Note that the contribution of radiation is very small due to the low emissivity of the polished copper surface.

PROBLEM 10.12

In boiling (and condensation) heat transfer, the convection coefficient, h, is expected to depend on the difference between surface and saturation temperature $\Delta T = T_{surface} - T_{saturation}$, the body force arising from the density difference between liquid and vapor, $g(\rho_l - \rho_v)$, the latent heat, h_{fg}, the surface tension, σ, a characteristic length of the system, L, and the thermophysical properties of the liquid or vapor: ρ, c, k, μ. Thus we can write

$h = h(\Delta T, g(\rho_l - \rho_v), h_{fg}, \sigma, L, \rho, c, k, \mu)$

Determine (a) the number of dimensionless groups necessary to correlate experimental data, and (b) appropriate dimensionless groups that should include the Prandtl number, the Jakob number, and the Bond number ($g\Delta\rho L^2/\sigma$).

GIVEN

- Heat transfer coefficient as a function of several dimensional quantities

FIND

(a) The number of dimensionless groups necessary to correlate experimental data
(b) The appropriate dimensionless groups including *Pr*, *Ja*, and *Bo*

SOLUTION

(a) We have 10 physical quantities and 4 dimensions (Mass, Length, Time, and Temperature) therefore, there must be 10 – 4 = 6 dimensionless groups necessary to correlate experimental data for boiling or condensation.
(b) We are given four of these dimensionless groups

$$\pi_1 = Nu = \frac{hL}{k}$$

$$\pi_2 = Pr = \frac{\mu c}{k}$$

$$\pi_3 = Ja = \frac{c\Delta T}{h_{fg}}$$

$$\pi_4 = Bo = \frac{g\Delta\rho L^2}{\sigma}$$

so there must be two other dimensionless groups.
For either of these dimensionless groups, we can write

$$\pi = \Delta T^a \, (g\Delta\rho)^b \, h_{fg}{}^c \, \sigma^d \, L^e \, \rho^f \, c^g \, k^h \, \mu^i$$

or in terms of the dimension of each of the physical quantities we have

$$[\pi] = [T]^a \left[\frac{M}{L^2 t^2}\right]^b \left[\frac{L^2}{t^2}\right]^c \left[\frac{M}{t^2}\right]^d [L]^e \left[\frac{M}{L^3}\right]^f \left[\frac{L^2}{T t^2}\right]^g \left[\frac{ML}{T t^3}\right]^h \left[\frac{M}{L t}\right]^i$$

In order for either of the two new groups to be dimensionless, the following four equations in the powers (a, b, …) must be satisfied

$[T]^0 \quad \Rightarrow \quad a - g - h = 0 \qquad (1)$

$[L]^0 \quad \Rightarrow \quad -2b + 2c + e - 3f + 2g + h - i = 0 \qquad (2)$

$[t]^0 \quad \Rightarrow \quad -2b - 2c - 2d - 2g - 3h - i = 0 \qquad (3)$

$[M]^0 \quad \Rightarrow \quad b + d + f + h + i = 0 \qquad (4)$

Since there are 4 equations in 9 unknowns, we are free to select 5 of these powers. The following table will help determine which powers to select

	π_2	π_3	π_4	π_5	π_6
	Pr	*Ja*	*Bo*		
a	0	1	0		
b	0	0	1		
c	0	–1	0		
d	0	0	–1		
e	0	0	2		
f	0	0	0		
g	1	1	0		
h	–1	0	0		
i	1	0	0		

For π_5, let's set $a = c = d = g = 0$ and $f = 1$ to ensure that our solution vector is not proportional to those for the previous π,*s*. Then equation (1) gives $h = 0$, equation (2) gives $-2b + e - 3 - i = 0$, equation (3) gives $-2b - i = 0$ and equation (4) gives $b + 1 + i = 0$. These can be solved to find $b = 1$, $i = -2$, and $e = 3$. So we have

$$\pi_5 = \frac{g\Delta\rho L^3\rho}{\mu^2}$$

For π_6, let's $a = d = i = 1$, and $c = g = 0$. This will ensure that this vector is not proportional to the others. The four equations can then be solved to give h = 1, $b = -3$, $f = 0$, and $e = -6$, giving

$$\pi_6 = \frac{\Delta T\sigma k\mu}{(g\Delta\rho)^3 L^6}$$

PROBLEM 10.13

Environmental concerns have recently motivated the search for replacements for chlorofluorocarbon refrigerants. An experiment has been devised to determine the feasibility of such a replacement. A silicon chip is bonded to the bottom of a thin copper plate as shown in the sketch below. The chip is 0.2 cm thick and has a thermal conductivity of 125 W/(m K). The copper plate is 0.1 cm thick and there is no contact resistance between the chip and the copper plate. This assembly is to be cooled by boiling a saturated liquid refrigerant on the copper surface. The electronic circuit on the bottom of the chip generates heat uniformly at a flux of $q'' = 5 \times 10^4$ W/m². Assume that the sides and the bottom of the chip are insulated. Calculate the steady state temperature at the copper surface and the bottom of the chip, as well as the maximum heat flux in pool boiling, assuming that the boiling coefficient, C_{sf}, is the same as for *n*-pentane on lapped copper. The physical properties of this new coolant are: T_{sat} = 60°C, c_p = 1100 J/(kg K), $h_{fg} = 8.4 \times 10^4$ J/kg, ρ_l = 1620 kg/m³, ρ_v = 13.4 kg/m³, σ = 0.081 N/m, $\mu_l = 4.4 \times 10^{-4}$ kg/(ms) and Pr_l = 9.0.

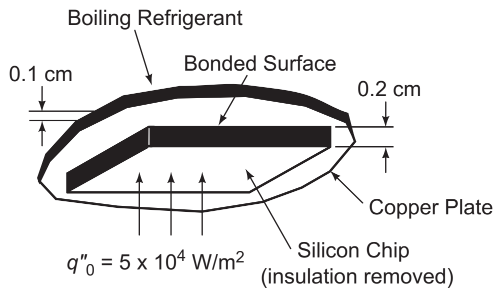
GIVEN

- A new refrigerant boiling on top of a copper plate, cooling a silicon chip
- The refrigerant is a saturated liquid
- Properties of the refrigerant and C_{sf}
- No contact resistance between the copper plate and the chip
- Uniform heat flux produced by the chip is 5×10^4 W/m²

FIND

(a) Steady state temperature at the copper surface
(b) Steady state temperature at the bottom of the chip
(c) Maximum heat flux in pool boiling

SKETCH



PROPERTIES AND CONSTANTS

From Table 10.1, the boiling coefficient, C_{sf}, for *n*-pentane boiling on lapped copper is $C_{sf} = 0.0049$

SOLUTION

The right side of the sketch above shows the thermal circuit. The heat generated at the bottom of the chip is transferred by conduction up through the chip and through the copper plate and is then transferred to the boiling refrigerant on top of the chip.

Assuming that the heat flux does not exceed the critical heat flux, we can determine the temperature drop as the excess temperature. $\Delta Tx = T_{\text{copper}} - T_{\text{sat}}$, from Equation (10.2)

$$\frac{c_l\,\Delta T_x}{h_{fg} Pr_l^{\,n}} = C_{fs}\left[\frac{q''}{\mu_l\, h_{fg}}\sqrt{\frac{g_e\,\sigma}{g\,(\rho_l - \rho_v)}}\right]^{0.33}$$

Since the refrigerant is not water, $n = 1.7$, so the right side of the above equation is

$$(0.0049)\left[\frac{\left(5\times10^4\ \text{W/m}^2\right)}{\left(4.4\times10^{-4}\ \text{kg/(ms)}\right)\left(8.4\times10^4\ \text{J/kg}\right)}\sqrt{\frac{\left(1\,(\text{kg m})/(\text{N s}^2)\right)(0.081\ \text{N/m})}{\left(9.81\ \text{m/s}^2\right)(1620-13.4)\ \text{kg/m}^3}}\right]^{0.33} = 0.007093$$

and

$$\Delta T_x = \frac{(0.007093)\left(8.4\times10^4\ \text{J/kg}\right)(9^{1.7})}{\left(1100\ \text{J/(kg K)}\right)} = 22.7\ \text{K}$$

(a) Therefore, $T_{\text{copper}} = T_{\text{sat}} + \Delta T_x = 60°\text{C} + 22.7 = 82.7°\text{C}$.

Now, the thermal conductivity of copper at ~90°C is, from Figure 1.6, $k_{\text{copper}} = 400$ W/(m K), so the thermal resistance of the copper is

$$R_{\text{copper}} = \left(\frac{t}{k}\right)_{\text{copper}} = \frac{10^{-3}\,\text{m}}{\left(400\ \text{W/(m K)}\right)} = 2.5\times10^{-6}\ (\text{m}^2\text{K})/\text{W}$$

The temperature drop across the copper plate is

$$\Delta T_{\text{copper}} = q_o''\,R_{\text{copper}} = \left(5\times10^4\ \text{W/m}^2\right)\ \left(2.5\times10^{-6}\ (\text{m}^2\text{K})/\text{W}\right) = 0.125\text{K}$$

The thermal resistance of the chip is

$$R_{\text{chip}} = \left(\frac{t}{k}\right)_{\text{chip}} = \frac{(2\times10^{-3}\,\text{m})}{(125\ \text{W/m})} = 1.6\times10^{-5}\ (\text{m}^2\text{K})/\text{W}$$

So the temperature drop across the chip is

$$\Delta T_{\text{chip}} = q_o'' \, R_{\text{chip}} = \left(5 \times 10^4 \ \text{W/m}^2\right) \ \left(1.6 \times 10^{-5} \ (\text{m}^2\text{K})/\text{W}\right) = 0.8 \ \text{K}$$

(b) and the temperature of the bottom of the chip is therefore $T_{\text{chip}} =$ 82.7+ 0.125+ 0.8 = 83.65°C.
The maximum heat flux can be calculated from Equation (10.4)

$$q''_{\max} = \left(\frac{\pi}{24}\right) \rho_v^{\frac{1}{2}} \ h_{fg} \ [\sigma g g_c \ (\rho_l - \rho_v)]^{\frac{1}{4}}$$

or
(c)

$$q''_{\max} = \left(\frac{\pi}{24}\right) \left(13.4 \ \text{kg/m}^3\right)^{\frac{1}{2}} \left(8.4 \times 10^4 \ \text{J/kg}\right)$$

$$\left[(0.081 \ \text{N/m}) \left(9.81 \ \text{m/s}^2\right) \left(1 \ (\text{kg m})/(\text{N s}^2)\right) (1620 - 13.4) \left(\text{kg/m}^3\right)\right]^{\frac{1}{4}} = 240{,}590 \ \text{W/m}^2$$

(Using Lienhard's recommendation, this critical heat flux would be about 11% larger.)
Since ΔT_x is proportional to (heat flux)$^{0.33}$, we can recalculate the chip temperature as follows

$$T'_{\text{chip}} = 60 + 22.7 \left(\frac{240{,}590}{50{,}000}\right)^{0.33} + 0.125 + 0.8 = 99.0°\text{C}$$

This temperature could be compared to the maximum permissible chip operating temperature to determine the maximum permissible chip power dissipation.

PROBLEM 10.14

It has recently been proposed by Andraka el al. of Sandia National Laboratories, Albuquerque, in Sodium Reflux Pool-Boiler Solar Receiver On-Sun Test Results (SAND89-2773, June 1992), that the heat flux from a parabolic dish solar concentrator could be delivered effectively to a Stirling engine by a liquid-metal pool boiler. The sketch below shows a cross-section of the pool boiler receiver assembly. Solar flux is absorbed on the concave side of a hemispherical absorber dome, boiling molten sodium metal on the convex side of the dome. The sodium vapor condenses on the engine heater tube as shown near the top of the figure. Condensing sodium transfers its latent heat to the engine working fluid which circulates inside the tube. Calculations indicate that a maximum heat flux of 75 W/cm^2 delivered by the solar concentrator to the absorber dome is to be expected.

After the receiver had been tested for about 50 hours, a small spot on the absorber dome suddenly melted and the receiver failed. Is it possible that the critical flux for the boiling sodium was exceeded? Use the following properties for the sodium: $\tilde{\rho}$ = 0.056 kg/m^3, ρ_l = 779 kg/m^3, h_{fg} = 4.039 × 10^6 J/kg, σ_l = 0.138 N/m, μ_l = 1.8 × 10^{-4} kg/ms.

GIVEN

- Sodium pool-boiler solar receiver
- Failure after about 50 hours operation
- Expected peak heat flux was 75 W/cm^2 = 750,000 W/m^2

FIND

(a) Whether the critical heat flux could have been exceeded

ASSUMPTIONS

- To first order, the absorber dome can be treated as flat, horizontal surface

PROPERTIES AND CONSTANTS

As given in the problem statement, the pertinent properties of the sodium vapor and liquid are

Heat of vaporization (h_{fg}) = 4.039×10^6 J/kg

Vapor density (ρ_v) = 0.056 kg/m^3

Liquid density (ρ_l) = 779 kg/m^3

Surface tension (σ) = 0.138 N/m

SOLUTION

The critical heat flux can be calculated from Equation (10.4)

$$q''_{\max,Z} = \left(\frac{\pi}{24}\right) \rho_v^{\frac{1}{2}} h_{fg} \left[\sigma g (\rho_l - \rho_v) g_c\right]^{\frac{1}{4}}$$

For the property values listed above we have

$$q''_{\max,Z} = \left(\frac{\pi}{24}\right) \left(0.056 \text{ kg/m}^3\right)^{\frac{1}{2}} \left(4.039 \times 10^6 \text{ J/kg}\right)$$

$$\left[(0.138 \text{ N/m})\left(9.81 \text{ m/s}^2\right)(779 - 0.056)\left(\text{kg/m}^3\right)\left((\text{kg m})/(\text{Ns}^2)\right)\right]^{\frac{1}{4}} = 712{,}970 \text{ W/m}^2$$

If we use Lienhard and Dhir's recommendation that the factor $\pi/24$ be replaced by 0.149, the critical heat flux would be

$$q''_{\max,Z} = 712{,}970 \left(\frac{0.149}{\left(\frac{\pi}{24}\right)}\right) = 811{,}550 \text{ W/m}^2$$

which exceeds the expected maximum flux. Therefore, exceeding the critical heat flux is a possible factor in the failure of the receiver.

COMMENTS

The correlation for critical heat flux used above has not necessarily been tested with liquid metals so the result should be used with some caution.

PROBLEM 10.15

Calculate the peak heat flux for nucleate pool boiling of water at 3 atm and 110°C on clean copper.

GIVEN

- Nucleate pool boiling of water on clean copper
- Pressure = 3 atm
- Water temperature (T_w) = 390°C

Find

- The peak heat flux ($q''_{\max}$)

ASSUMPTIONS

- Steady state

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 3 atm (3.04×10^5 Pa) pressure

Saturation temperature (T_{sat}) = 133°C

Liquid density (ρ_l) = 932.3 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 1.55 kg/m^3
Thermal conductivity (k_l) = 0.684 W/(m K)
Heat of vaporization (h_{fg}) = 2164 kJ/kg = 2.164 × 10^6 J/kg
Thermal diffusivity (α_l) = 0.172 × 10^{-6} m^2/s
Absolute viscosity (μ_l) = 213.0 × 10^{-6} (Ns)/m^2
Prandtl number (pr_l) = 1.30

From Table 10.2, the surface tension at 133°C (σ) = 0.0522 N/m

SOLUTION

The maximum heat flux for water at saturation temperature is given by Equation (10.4)

$$q''_{\text{max,sat}} = \left(\frac{\pi}{24}\right) \rho_v^{0.5} h_{fg} \left[\sigma g(\rho_l - \rho_v) g_c\right]^{\frac{1}{4}}$$

$$q''_{\text{max,sat}} = \left(\frac{\pi}{24}\right) \left(1.55\ \text{kg/m}^3\right)^{0.5} \left(2.164\times10^6\ \text{J/kg}\right) \left((\text{Ws})/\text{J}\right)$$

$$\left[(0.0522\ \text{J/kg})\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(9.8\ \text{m/s}^2\right)(932.3-1.55)\text{kg/m}^3\right]^{\frac{1}{4}}$$

$$q''_{\text{max,sat}} = 1.65\times10^6\ \text{W/m}^2$$

The maximum heat flux for a subcooled liquid is given by Equation (10.5)

$$q''_{\text{max}} = q''_{\text{max,sat}} \left(1+\left[\frac{2k_l(T_{\text{sate}} - T_{\text{liquid}})}{\sqrt{\pi\alpha_l\tau}}\right]\frac{24}{\pi h_{fg}\rho_v}\left[\frac{\rho_v^2}{g_c\sigma g(\rho_l-\rho_v)}\right]^{\frac{1}{4}}\right)$$

$$\text{where } \tau = \left(\frac{\pi}{3}\right) \sqrt{2\pi} \left[\frac{g_c\sigma}{g(\rho_l-\rho_v)}\right]^{\frac{1}{2}} \left[\frac{\rho_v^2}{g_c\sigma g(\rho_l-\rho_v)}\right]^{\frac{1}{4}}$$

$$t = \left(\frac{\pi}{3}\right) \sqrt{2\pi} \left[\frac{\left((\text{kg m})/(\text{s}^2\text{N})\right)(0.0522\ \text{N/m})}{\left(9.8\ \text{m/s}^2\right)(932.3-1.55)\text{kg/m}^3}\right]^{\frac{1}{2}}$$

$$\left[\frac{\left(1.55\ \text{kg/m}^3\right)^2}{\left((\text{kg m})/(\text{s}^2\text{N})\right)(0.0522\ \text{N/m})\left(9.8\,\text{m/s}^2\right)(932.3-1.55)\ \text{kg/m}^3}\right]^{\frac{1}{4}} = 0.00167\ \text{s}$$

$$q''_{\text{max}} = \left(1.65\times10^6\ \text{W/m}^2\right)$$

$$\left[1+\left[\frac{2(0.684\ \text{W/(m K)})(133°\text{C}-110°\text{C})}{\sqrt{\pi\left(0.172\times10^{-6}\ \text{m}^2/\text{s}\right)(0.00167\text{s})}}\right]\frac{24}{\pi\left(2.164\times10^6\ \text{J/kg}\right)\left(1.55\ \text{kg/m}^3\right)\left((\text{Ws})/\text{J}\right)}\right.$$

$$\left.\left[\frac{\left(1.55\ \text{kg/m}^3\right)^2}{\left((\text{kg m})/(\text{s}^2\text{N})\right)(0.0522\ \text{N/m})\left(9.8\ \text{m/s}^2\right)(932.3-1.55)\ \text{kg/m}^2}\right]^{\frac{1}{4}}\right]$$

$$q''_{\text{max}} = 2.69\times10^6\ \text{W/m}^2$$

PROBLEM 10.16

Calculate the heat transfer coefficient for film boiling of water on a 1.3 cm horizontal tube if the tube temperature is 550°C and the system is placed under pressure of 1/2 atm.

GIVEN

- Film boiling of water on a horizontal tube
- Tube outside diameter (D) = 1.3 cm = 0.013 m
- Tube temperature (T_t) = 550°C
- Pressure = 1/2 atm = 50,660 N/m^2

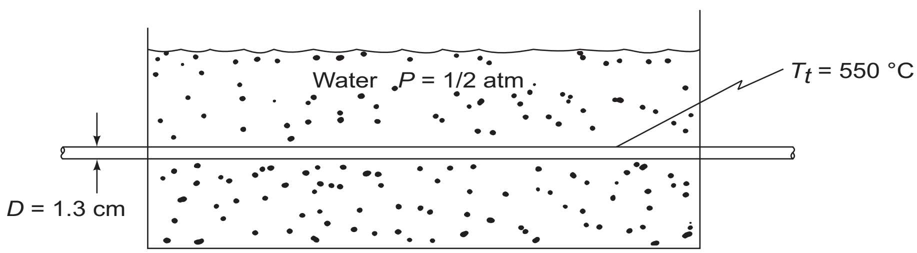
FIND

- The heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- Tube temperature is uniform and constant
- The viscosity, specific heat, and thermal conductivity of the vapor can be approximated by those of steam at 1 atm pressure
- Radiation across the vapor film is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 50,660 N/m^2

Saturation temperature (T_{sat}): 81.5°C

Water density, (ρ_l): 970.6 kg/m^3

Heat of vaporization, (h_{fg}): 2304 kJ/kg = 2.304×10^6 J/kg

Vapor density, ($\rho_v = 1/v_g$): 0.3070 kg/m^3

Extrapolating from Appendix 2, Table 34, for steam at 1 atm and 81.5°C

Absolute viscosity (μ_v) = 10.49×10^{-6} (Ns)/m^2

Thermal conductivity (k_v) = 0.0257 W/(m K)

Specific heat (c_v) = 1965 J/(kg K)

SOLUTION

The heat transfer coefficient for film boiling on tubes is given by Equation (10.6)

$$\bar{h}_c = 0.62 \left(\frac{g(\rho_l - \rho_v)\rho_v k_v^3 [h_{fg} + 0.68 c_{pv} \Delta T_x]}{D \mu_v \Delta T_x} \right)^{\frac{1}{4}}$$

$$\bar{h}_c = 0.62 \left[\frac{(9.8\ \text{m/s}^2)(970.6 - 0.3070)\ \text{kg/m}^3 (0.3070\ \text{kg/m}^3)(0.0257\ \text{W/(m K)})^3}{(0.013\ \text{m})(10.49 \times 10^{-6}\ (\text{N s})/\text{m}^2)((\text{kg m})/(\text{s}^2\text{N}))(550°\text{C} - 81.5°\text{C})} \right.$$

$$\left.\frac{\left[2.304\times10^{6}\ \text{J/kg}+0.68(1965\ \text{J/(kg K)})(550\,°\text{C}-81.5\,°\text{C})\right]((\text{W s})/\text{J})}{}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 135\ \text{W/(m}^2\text{K)}$$

PROBLEM 10.17

A metal-clad electrical heating element of cylindrical shape, as shown in the sketch below, is immersed in water at atmospheric pressure. The element has a 5 cm diameter and heat generation produces a surface temperature of 300°C. Estimate the heat flux under steady state conditions and the rate of heat generation per unit length.

GIVEN

- Cylindrical, electrical heating element
- Heat generation produces a 300°C surface temperature

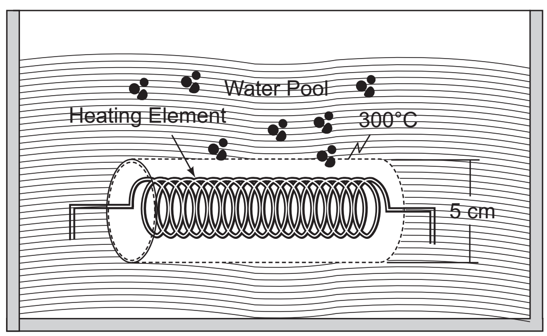
FIND

(a) Heat flux
(b) Rate of heat generation

ASSUMPTIONS

- The system operates at atmospheric pressure
- The heater surface is black

SKETCH



PROPERTIES AND CONSTANTS

Inspection of the boiling curve in Figure (10.1) indicates that at 300°C surface temperature, the heating system must operate in the film boiling regime. Hence, a vapor layer covers the surface of the heater and it is appropriate to evaluate the physical properties at the mean film temperature of (100 + 300)/2 = 200°C.

From Appendix 2, Table 34

Vapor density (ρ_v) = 0.4673 kg/m^3

Specific heat (c_{pv}) = 1982 J/(kg K)

Absolute viscosity (μ_v) = 1.61 × 10^{-5} kg/(ms)

Thermal conductivity (k_v) = 0.032 W/(m K)

From Appendix 2, Table 13

Heat of vaporization (h_{fg}) = 2.257 × 10^6 J/kg

Liquid density (ρ_l) = 958.4 kg/m^3

SOLUTION

We find the convection heat transfer coefficient from Equation (10.6)

$$\bar{h}_c = 0.62 \left\{ \frac{g(\rho_l - \rho_v)\rho_v k_v^3 [h_{fg} + 0.68\, c_{pv}\, \Delta T_x]}{D\mu_v\, \Delta T_x} \right\}^{\frac{1}{4}}$$

so

$$\bar{h}_c = 0.62 \left\{ \frac{(9.81\,\text{m/s}^2)(958.4 - 0.4673)(\text{kg/m}^3)(0.4673\,\text{kg/m}^3)(0.032\,\text{W/(m K)})^3}{(0.05\,\text{m})(1.61\times 10^{-5}\,\text{kg/(ms)})(200\,\text{K})} \right.$$

$$\left. \frac{[(2.257\times 10^6\ \text{J/kg}) + 0.68(1982\ \text{J/(kg K)})(200\,\text{K})]}{} \right\}^{\frac{1}{4}}$$

$$= 135\ \ \text{W/(m}^2\text{K)}$$

The radiation heat transfer coefficient can be determined from Equation (10.9)

$$\bar{h}_r = \sigma\varepsilon_s \left(\frac{T_s^4 - T_{\text{sat}}^4}{T_s - T_{\text{sat}}} \right) = (5.67\times 10^{-8}\ \text{W/(K}^4\text{m}^2))\,(1)\left(\frac{(573^4 - 373^4)(\text{K}^4)}{(573 - 373)(\text{K})} \right) = 25.1\ \ \text{W/(m}^2\text{K)}$$

The total heat transfer coefficient is from Equation (10.8)

$$\bar{h}_{\text{total}} = \bar{h}_c + 0.75\ \bar{h}_r = (135.0\ \text{W/(m}^2\text{K)}) + 0.75\ (25.1\,\text{W/(m}^2\text{K)}) = 154\ \ \text{W/(m}^2\text{K)}$$

(a) The heat flux is then

$$q'' = \bar{h}_{\text{total}}\ \Delta T = (154\ \text{W/(m}^2\text{K)})\ (200\ \text{K}) = 30{,}761\ \text{W/m}^2$$

(b) The rate of heat generation per unit length is

$$q_L = \bar{h}_{\text{total}}\ \Delta T\pi D = (154\ \text{W/(m}^2\text{K)})\ (200\ \text{K})\ (\pi)\ (0.05\ \text{m}) = 4838\ \ \text{W/m}$$

PROBLEM 10.18

> **Calculate the maximum safe heat flux in the nucleate-boiling regime for water flowing at a velocity of 15 m/s through a 1.2-cm-*ID* copper tube 0.31 m long if the water enters at 1 atm pressure and 100°C saturated liquid.**

GIVEN

- Nucleate boiling of water flowing through a tube
- Water velocity (V) = 15 m/s
- Tube inside diameter (D) = 1.2 cm = 0.012 m
- Tube length (L) = 0.31 m
- Water pressure (p) = 1 atm
- Water temperature (T_w) = 100°C

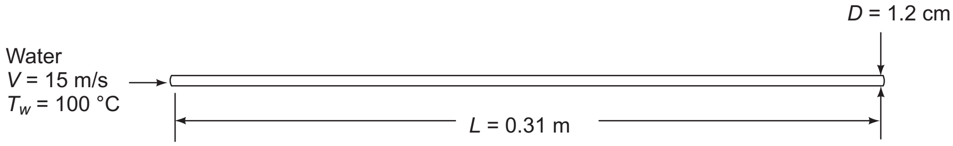
FIND

- The maximum safe heat flux in the nucleate boiling regime (q''_{max})

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 100°C, 1 atm

Density (ρ_l) = 958.4 kg/m^3

Thermal conductivity (k_l) = 0.682 W/(m K)

Absolute viscosity (μ_l) = 277.5 × 10^{-6} (Ns)/m^2

Prandtl number (Pr_l) = 1.75

Specific heat (c_l) = 4211 J/(kg K)

Kinematic viscosity (ν_l) = 0.294 × 10^{-6} m^2/s

Heat of vaporization (h_{fg}) = 2.275 × 10^6 J/kg

Enthalpy of saturated vapor (h_g) = 2.676 × 10^6 J/kg

Enthalpy of saturated liquid (h_b) = 0.419 × 10^6 J/kg

From Appendix 2, Table 34, for steam at 100°C: ρ_v = 0.5977 kg/m^3

From Table 10.2: Surface tension at 100°C (s) = 0.0589 N/m

From Table 10.1: For water on copper, C_{sf} = 0.0130

SOLUTION

Assuming that by 'safe' we mean that the critical heat flux is not exceeded, we can use the Griffith correlation, Figure 10.17. The critical pressure is P_c = 218.3 atm, P/P_c = 1/218.3 = 0.0046. From the figure, the ordinate is therefore 6000

$$6000 = \frac{41.5\, q_{\max}''}{(h_g - h_b)\rho_v \left[\left(\frac{\rho_l - \rho_v}{\mu_l}\right) g \left(\frac{k_l}{\rho_l\, c_l}\right)^2\right]^{\frac{1}{3}} F}$$

Since $T_s = T_b$, the parameter F simplifies to

$$F = 1 + 10^{-6}\left(\frac{UD}{\nu_l}\right) = 1 + 10^{-6}\left(\frac{(15\text{ m/s})(0.012\text{ m})}{0.249\times 10^{-6}\text{ m}^2/\text{s}}\right) = 1.612$$

The maximum heat flux is then given by

$$q''_{\max} = \frac{6000}{41.5}\,(h_g - h_b)\,\rho_v \left[\left(\frac{\rho_l - \rho_v}{\mu_l}\right) g \left(\frac{k_l}{\rho_l\, c_l}\right)^2\right]^{\frac{1}{3}} F$$

$$= \left(\frac{6000}{41.5}\right)(2.676 - 0.419)\left(10^6\text{ J/kg}\right)\left(0.5977\text{ kg/m}^3\right)$$

$$\left[\frac{(958.4 - 0.5977)\text{ kg/m}^3}{\left(277.5\times 10^{-6}(\text{N s})/\text{m}^2\right)}\left(9.8\text{ m/s}^2\right)\left(\frac{(0.682\text{ W/(m K)})}{\left(958.4\text{ kg/m}^3\right)(4211\text{ J/(kg K)})}\right)^2\right]^{\frac{1}{3}} 1$$

$$q''_{\max} = 3.108\times 10^6\text{ W/m}^2$$

PROBLEM 10.19

During the 1980s, solar thermal electric technology was commercialized with the installation of 350 MW of electrical power capacity in the California desert. The technology involved heating a heat transfer oil in receiver tubes placed at the focus of line-focus, parabolic trough solar concentrators. The heat transfer oil was then used to generate steam which, in turn, powered a steam turbine/electrical generator. Since the transfer of heat from the oil to the steam creates a temperature drop and a resulting loss in thermal efficiency, alternatives have been considered. In one alternative, steam would be generated directly inside the receiver tubes. Consider an example in which a heat flux of 50,000 W/m^2 is absorbed on the outside surface of a 12.7 mm i.d., stainless steel 316 tube with a wall thickness of 1.245 mm. Inside the tube, saturated liquid water at 300°C is flowing at a rate of 100 kg/hr. Determine the maximum tube wall temperature if the steam quality is to be increased to 0.5. Assume $\mu_v = 2.0 \times 10^{-5}$ kg/(ms). Neglect any heat losses from the outside of the receiver tube.

GIVEN

- 12.7 mm i.d. tube with flowing, boiling water inside
- 50,000 W/m^2 heat flux at tube o.d.
- 100 kg/hr water enters the tube at saturated liquid conditions, 300°C
- Absolute viscosity of the steam is $\mu_v = 2 \times 10^{-5}$ kg/(ms)
- Tube heat losses are negligible

FIND

(a) Tube wall temperature if steam quality is to be 0.5 at tube exit

ASSUMPTIONS

- The method of Chen is applicable

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for T_{sat} = 300°C we have

Heat of vaporization (h_{fg}) = 1.403×10^6 J/kg

Saturation pressure (P_{sat}) = 8.592×10^6 N/m^2

Vapor density (ρ_v) = 46.3 kg/m^3

Liquid density (ρ_l) = 712.5 kg/m^3

Liquid absolute viscosity (μ_l) = 92.2×10^{-6} kg/(ms)

Liquid Prandtl number (Pr_l) = 0.98

Liquid thermal conductivity (k_l) = 0.564 W/(m K)

Liquid specific heat (c_l) = 5694 J/(kg K)

and from Table 10.2

Surface tension (σ) = 0.0143 N/m

SOLUTION

We will follow the method of Chen described in Section 10.3.2. The tube flow area is $A_f = \pi D_i^2/4$ = 0.000127 m^2 and the tube outside radius is r_o = 12.7/2 mm + 1.245 = 7.6 mm. Then

$$G = \frac{\dot{m}}{A_f} = (100\ \text{kg/h})\ \frac{1}{(0.000127\,\text{m}^2)}(\text{h}/3600\,\text{s}) = 218.7\ \text{kg/(m}^2\text{s)}$$

The heat flux at the inner tube wall is

$$q''_i = q''_o \frac{r_o}{r_i} = 50{,}000\,\frac{7.6}{\frac{12.7}{2}} = 59{,}842\ \text{W/m}^2$$

The convective component of the heat transfer coefficient is

$$h_c = 0.023\left[\frac{G(1-x)D}{\mu_l}\right]^{0.8} Pr_l^{0.4}\,\frac{k_l}{D}\,F$$

We are interested in conditions at the end of the tube where the quality, x, is 0.5.

$$h_c = 0.023\left[\frac{\left(218.7\ \text{kg/(sm}^2)\right)(1-0.5)(0.0127\,\text{m})}{\left(92.2\times10^{-6}\ \text{kg/(ms)}\right)}\right]^{0.8} 0.98^{0.4}\,\frac{(0.564\ \text{W/(m K)})}{(0.0127\,\text{m})}\,F = 2229F\ \text{W/(m}^2\text{K)}$$

To find F, we must first find X from

$$\frac{1}{X_{tt}} = \left(\frac{x}{1-x}\right)^{0.9}\left(\frac{\rho_l}{\rho_v}\right)^{0.5}\left(\frac{\mu_v}{\mu_l}\right)^{0.1}$$

$$\frac{1}{X_{tt}} = \left(\frac{0.5}{1-0.5}\right)^{0.9}\left(\frac{712.5}{46.3}\right)^{0.5}\left(\frac{2\times10^{-5}}{92.2\times10^{-6}}\right)^{0.1} = 3.37 \text{ or } X_{tt} = 0.297$$

then F is

$$F = 2.35\left(\frac{1}{X_{tt}} + 0.213\right)^{0.736} = 6.01$$

and the convective heat transfer coefficient is

$$h_c = 2229\,F = \left(2229\ \text{W/(m}^2\text{K)}\right)(6.01) = 13{,}391\ \text{W/(m}^2\text{K)}$$

Now, the boiling heat transfer coefficient is given by Equation (10.12)

$$h_b = 0.00122\left(\frac{(0.564)^{0.79}(5694)^{0.45}(712.5)^{0.49}(1)^{0.25}}{(0.0143)^{0.5}(92.2\times10^{-6})^{0.29}(1.403\times10^{6})^{0.24}(46.3)^{0.24}}\right)\Delta T_x^{0.24}\,\Delta p_{\text{sat}}^{0.75}\,\text{S}$$

In this equation, we check for SI units and ΔP_{sat} is in N/m^2. We have

$$h_b = 1.567\,\Delta T_x^{0.24}\,\Delta p_{\text{sat}}^{0.75}\,\text{S}$$

To find S, we need Re_{TP}

$$Re_{TP} = \frac{G(1-x)D}{\mu_l}\,F^{1.25}\times10^{-4}$$

so

$$Re_{TP} = \frac{\left(218.7\ \text{kg/(sm}^2)\right)(1-0.5)(0.0127\,\text{m})}{\left(92.2\times10^{-6}\ \text{kg/(ms)}\right)}(6.01)^{1.25}\times10^{-4} = 14.17$$

and we find S from

$$S = (1 + 0.12\,Re_{TP}^{1.14})^{-1} = 0.2886$$

So the expression for the boiling heat transfer coefficient is

$$h_b = (1.567)\,(0.288)\Delta T_x^{0.24}\,\Delta p_{\text{sat}}^{0.75} = 0.4522\Delta T_x^{0.24}\,\Delta p_{\text{sat}}^{0.75}$$

From Table 13, we can approximate the relationship between saturation pressure and temperature

$$\frac{\Delta p_{\text{sat}}}{\Delta T_{\text{sat}}} = \frac{(85.917 - 64.191)\times 10^5 \left(\text{N/m}^2\right)}{20\,\text{K}} = 108{,}630\ (\text{N/m}^2)/\text{K}$$

So $\Delta p_{\text{sat}} \approx 108{,}630\ \Delta T_{\text{sat}}$

when ΔT_{sat} is expressed in K and ΔP_{sat} is expressed in N/m^2. Assuming that the excess temperature, ΔT_x is small, the above expression can be used to find Δp_{sat} by substituting ΔT_x for ΔT_{sat} in the above expression. Given this, we can further simplify the expression for the boiling heat transfer coefficient

$$h_b = 0.4522\Delta T_x^{0.24}\,(108{,}630\ \Delta T_x)^{0.75} = 2706\Delta T_x^{0.99} \approx 2706\Delta T_x$$

According to Chen, the two heat transfer coefficients can be added

$$h = h_c + h_b = 13{,}319 + 2706\Delta T_x$$

Since the heat flux can be written as

$$q'' = h\Delta T_x$$

we have the following relationship between the heat flux and the excess temperature

$$q'' = 13391\ \Delta T_x + 2706\ \Delta T_x^2 = 59{,}842$$

Solving this quadratic equation for the excess temperature we find

$$\Delta T_x = 2.84\ \text{K}$$

So we were justified in assuming that the excess temperature is small.

Finally, we need to calculate the temperature drop across the tube wall. From Equation (2.39)

$$L2\pi r_o q'' = (\Delta T_{\text{wall}})/\left[\ln\left(\frac{r_o}{r_i}\right)/(2\pi kL)\right]$$

From Figure 1.6, the thermal conductivity of the 316 stainless steel is k_{ss} = 17 W/(mK).

Solving for the wall temperature drop

$$\Delta T_{\text{wall}} = \frac{r_o\, q''_o \ln\left(\frac{r_o}{r_i}\right)}{k_{ss}} = \frac{(0.0076\,\text{m})\left(50{,}000\ \text{W/m}^2\right)\ln(7.5/6.35)}{(17\ \text{W/(m K)})} = 3.72\ \text{K}$$

The total temperature drop from the tube wall outer surface to the boiling water is

$$\Delta T_{\text{total}} = 2.8 + 3.7 = 6.5\ \text{K}$$

and the tube wall outer surface temperature is therefore

$$T_{\text{tube,outer}} = 300 + 6.5 = 306.5°\text{C, say } 307°\text{C}$$

PROBLEM 10.20

> **Calculate the average heat transfer coefficient for film-type condensation of water at pressures of 10 kPa and 101 kPa for (a) a vertical surface 1.5 m high (b) the outside surface of a 1.5-cm-OD vertical tube 1.5 m long (c) the outside surface of a 1.6-cm-*OD* horizontal tube 1.5 m long and (d) a 10-tube vertical bank of 1.6-cm-*OD* horizontal tubes 1.5 m long. In all cases, assume that the vapor velocity is negligible and that the surface temperatures are constant at 11°C below saturation temperature.**

GIVEN

- Film condensation of water

FIND

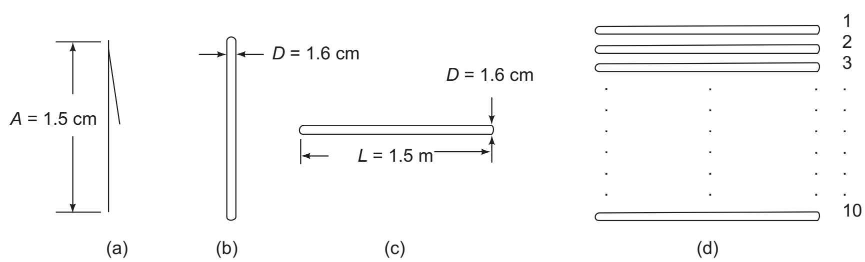
The average heat transfer coefficient at pressure of 10 kPa and 101 kPa for

(a)	A vertical surface of height (H) = 1.5 m	
(b)	The outside surface of a vertical tube	Outside diameter (D) = 1.6 cm = 0.016 m Height (H) = 1.5 m
(c)	The outside surface of a horizontal tube	Outside diameter (D) = 1.6 cm = 0.016m Length (L) = 1.5 m
(d)	A 10 tube vertical bank of horizontal tubes	Outside diameter (D) = 1.6 cm Length (L) = 1.5 m

ASSUMPTIONS

- Steady state
- Vapor velocity is negligible
- Surface temperatures (T_s) are constant at 11°C below saturation temperature
- Film thickness is much smaller than the pipe diameter
- Laminar condensate flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the saturation temperatures for water at

101 kPa (T_{sv1}) = 100°C, therefore $T_s = T_{sv} - 11°C = 89°C$

10 kPa (T_{sv2}) = 45.3°C, therefore, T_s = 34.3°C

The film temperatures, as given in Section 10.4.1 are

$T_{film1} = T_s + 0.25\,(T_{sv} - T_s) = 89°C + 0.25\,(11°C) = 91.8°C$

$T_{film2} = 34.3°C + 0.25\,(11°C) = 37.1°C$

From Appendix 2, Table 13, for water at the film temperatures

Film Temperature, °C	91.8	37.1
Density, ρ_l (kg/m^3)	963.8	993.3
Thermal conductivity, k (W/(m K))	0.678	0.628
Absolute viscosity, $\mu_l \times 10^6$ (Ns/m^2)	310.0	693.8
Vapor density, $\rho_v = 1/v_g$ (kg/m^3)	0.4468	0.0427
Heat of vaporization, $h_{fg} \times 10^{-6}$ (J/kg)	2.278	2.413
Specific heat, c_{pl} (J/(kg K))	4204	4175

SOLUTION

The solution will first be worked for p = 101 kPa

(a) The average heat transfer coefficient on a vertical plate is given by Equation (10.21)

$$\bar{h}_c = 0.943 \left[\frac{\rho_l(\rho_l - \rho_v) g\, h'_{fg}\, k^3}{\mu_l L (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + \frac{3}{8}\, c_{pl}(T_{sv} - T_s) = \left(2.278 \times 10^6 \text{ J/(kg)}\right) + \frac{3}{8}\,(4204 \text{ J/(kg K)})\,(11°\text{C})$

$$= 2.295 \times 10^6 \text{ J/kg}$$

Rohsenow's analysis showed that h'_{fg} should be replaced by h_{fg} + 0.68 c_{pl} (T_{sv} – T_s) if c_{pl} ($T_{sv} - T_s$)/h_{fg} < 1

$$\frac{c_{pl}(T_{sv} - T_s)}{h'_{fg}} = \frac{(4204 \text{ J/(kg K)})(11°\text{C})}{\left(2.295 \times 10^6 \text{ J/kg}\right)} = 0.0201 < 1$$

Therefore, the Rohsenow results will be used

$$h''_{fg} = h_{fg} + 0.68 c_{p1}\,(T_{sv} - T_s) = \left(2.278 \times 10^6 \text{ J/(kg)}\right)^{+}\ 0.68\,(4204 \text{ J/(kg K)})\,(11°\text{C}) = 2.309 \times 106 \text{ J/kg}$$

$\bar{h}_c = 0.943$

$$\left[\frac{\left(963.8 \text{ kg/m}^3\right)(963.8 - 0.4463) \text{ kg/m}^3 \left(9.8 \text{ m/s}^2\right)\left(2.309 \times 10^6 \text{ J/kg}\right)\left((\text{W s})/\text{J}\right)(0.678 \text{ W/(m K)})^3}{\left(310.0 \times 10^{-6} \text{ (N s)/m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)(1.5 \text{ m})(11°\text{C})} \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 5641 \text{ W/(m}^2\text{K)}$$

(b) For vertical tubes large in diameter compared to the film thickness, the heat transfer coefficient is the same as a vertical flat plate. Therefore, h_c = 5651 W/(m^2 K).

(c) The heat transfer coefficient for horizontal tubes is given by Equation (10.23)

$$\bar{h}_c = 0.725 \left[\frac{\rho_l(\rho_l - \rho_v) g\, h'_{fg}\, k^3}{\mu_l D (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

$\bar{h}_c = 0.725$

$$\left[\frac{\left(963.8 \text{ kg/m}^3\right)(963.8 - 0.4463)\text{kg/m}^3 \left(9.8 \text{ m/s}^2\right)\left(2.309 \times 10^6 \text{J/kg}\right)\left((\text{Ws})/\text{J}\right)(0.678 \text{W/(m K)})^3}{\left(310.0 \times 10^{-6} \text{ (N s)/m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)(0.016 \text{m})(11°\text{C})} \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 13{,}495 \text{ W/(m}^2\text{K)}$$

(d) The heat transfer coefficient on the tube bank is given by Equation (10.24)

$$\bar{h}_c = 0.778 \left[1 + 0.2 \frac{c_p(T_{sv} - T_s)}{h_{fg}} (N-1) \right] \left[\frac{\rho_l(\rho_l - \rho_v) g\, h'_{fg}\, k^3}{\mu_l\, N D (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

$$\text{Provided } \frac{(N-1)\,c_p\,(T_{sv}-T_s)}{h_{fg}} < 2$$

$$\frac{(10-1)\left(4204\ \text{J/(kg K)}\right)(11°\text{C})}{2.278\times10^6\ \text{J/kg}} = 0.183 < 2$$

$$\bar{h}_c = 0.728\ [1 + 0.2\ (0.183)]$$

$$\left[\frac{\left(963.8\ \text{kg/m}^3\right)(963.8-0.4463)\ \text{kg/m}^3\left(9.8\ \text{m/s}^2\right)\left(2.309\times10^6\ \text{J/kg}\right)\left((\text{Ws})/\text{J}\right)\left(0.678\ \text{W/(m K)}\right)^3}{\left(310.0\times10^{-6}\ (\text{N s})/\text{m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)10\,(0.016\,\text{m})\,(11°\text{C})}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 7899\ \text{W/(m}^2\text{K)}$$

Repeating this procedure for p = 10 kPa and tabulating all of the heat transfer coefficients in W/(m^2 K)

Pressure (kPa)	101	10
Case (a)	5641	4484
Case (b)	5641	4484
Case (c)	13,495	10,728
Case (d)	7899	6265

PROBLEM 10.21

The inside surface of a 1 m long vertical 5 cm-*ID* tube is maintained at 120°C. For saturated steam at 350 kPa condensing inside, estimate the average heat transfer coefficient and the condensation rate, assuming the steam velocity is small.

GIVEN

- Steam condensing inside a vertical tube
- Tube length (L) = 1 m
- Tube inside diameter (D) = 5 cm = 0.05 m
- Tube surface temperature (T_s) = 120°C
- Steam pressure (p) = 350 kPa

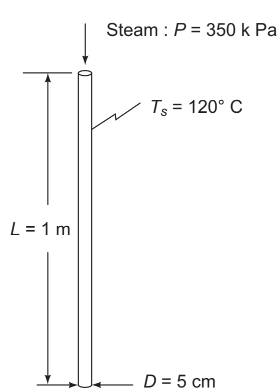
FIND

- The average heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- The steam velocity is small
- Film condensation occurs

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 350 kPa

Saturation temperature (T_{sv}) = 138.6°C

Liquid density (ρ_l) = 927.5 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 1.87 kg/m^3

Thermal conductivity (k_l) = 0.684 W/(m K)

Heat of vaporization (h_{fg}) = 2148 kJ/kg = 2.148 × 10^6 J/kg

Absolute viscosity (μ_l) = 203.4 × 10^{-6} (Ns)/m^2

Specific heat (c_{pl}) = 4255 J/(kg K)

SOLUTION

The condensate layer thickness (δ) at the bottom of the tube ($x = L$) can be estimated using Equation (10.17)

$$\delta = \left[\frac{4\mu_l\, k \times (T_{sv} - T_s)}{g\,\rho_l\,(\rho_l - \rho_v) h'_{fg}}\right]^{\frac{1}{4}}$$

$$\text{where } h'_{fg} = h_{fg} + \frac{3}{8} c_{pl}\,(T_{sv} - T_s)$$

$$h'_{fg} = \left(2.148\times10^6 \text{ J/kg}\right) + \frac{3}{8}\,(4255 \text{ J/(kg K)})\,(138.6°\text{C} - 120°\text{C}) = 2.178 \times 10^6 \text{ J/kg}$$

$$\delta = \left[\frac{4\left(203.4\times10^{-6}\text{ (N s)/m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)\left(0.684\text{ W/(m K)}\right)\left(\text{J/(W s)}\right)(1\text{ m})(138.6°\text{C} - 120°\text{C})}{\left(9.8\text{ m/s}^2\right)\left(927.5\text{ kg/m}^3\right)(927.5 - 1.87)\text{ kg/m}^3\left(2.178\times10^6\text{ J/kg}\right)}\right]^{\frac{1}{4}}$$

$$= 1.5 \times 10^{-4}\text{ m}$$

Since the condensate layer is much smaller than the tube diameter, Equation (10.21) can be used to estimate the average heat transfer coefficient.

$$\bar{h}_c = 0.943\left[\frac{\rho_l(\rho_l - \rho_v) g\, h'_{fg} k^3}{\mu_l\, L(T_{sv} - T_s)}\right]^{\frac{1}{4}}$$

Rohsenow's analysis showed that h'_{fg} should be replaced by $h'_{fg} + 0.68\ c_{pl}\ (T_{sv} - T_s)$ if $c_{pl}\ (T_{sv} - T_s)/h'_{fg} < 1$

$$\frac{c_{pl}(T_{sv} - T_s)}{h'_{fg}} = \frac{(4255\text{ J/(kg K)})(138.6°\text{C} - 120°\text{C})}{\left(2.178\times10^6\text{ J/kg}\right)} = 0.0360 < 1$$

$$h'_{fg} = h_{fg} + 0.68\ c_{pl}\,(T_{sv} - T_s) = 2.148 \times 10^6\text{ J/kg}$$

$$+ 0.68\,(4255\text{ J/(kg K)})\,(138.6°\text{C} - 120°\text{C}) = 2.202 \times 10^6\text{ J/kg}$$

$$\bar{h}_c = 0.943$$

$$\left[\frac{\left(927.5\text{ kg/m}^3\right)(927.5 - 1.87)\text{ kg/m}^3\left(9.8\text{ m/s}^2\right)\left(2.202\times10^6\text{ J/kg}\right)\left((\text{W s})/\text{J}\right)\left(0.684\text{ W/(m K)}\right)^3}{\left(203.4\times10^{-6}\text{ (N s)/m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)(1\text{ m})(138.6°\text{C} - 120°\text{C})}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 5933\ \text{W/(m}^2\text{K)}$$

The above analysis assumes the condensate layer is laminar. This assumption can be checked by checking the Reynolds number at the bottom of the tube. With the aid of Equation (10.14), the Reynolds number can be written as

$$Re_\delta = \frac{4\Gamma_c}{\mu_l} = \frac{4\rho_l^2 g \delta^3}{3\mu_l^2}$$

Substituting Equation (10.17) for δ yields

$$Re_\delta = \frac{4\rho_l^2 g}{3\mu_l^2}\left[\frac{4\mu_l\, k_l\, L\,(T_{sv} - T_s)}{g\,\rho_l^2\, h'_{fg}}\right]^{\frac{3}{4}} = 985 < 2000$$

Therefore, the laminar assumption is valid.

PROBLEM 10.22

> **A horizontal 2.5 cm-*OD* tube is maintained at a temperature of 27°C on its outer surface. Calculate the average heat transfer coefficient if saturated steam at 12 kPa is condensing on this tube.**

GIVEN

- Saturated steam condensing on a horizontal tube
- Tube outside diameter (D) = 2.5 cm = 0.025 m
- Tube outer surface temperature (T_s) = 27°C
- Steam pressure (p) = 12 kPa

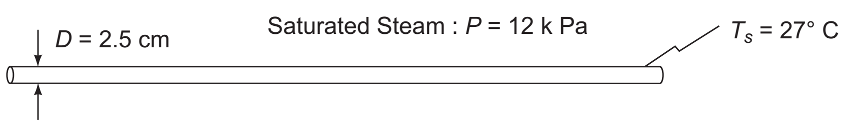
FIND

- The average heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- Film condensation occurs

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 12 kPa

Saturation temperature (T_s) = 49.3°C

Liquid density (ρ_l) = 988.4 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 0.0797 kg/m^3

Thermal conductivity (k_l) = 0.646 W/(m K)

Heat of vaporization (h_{fg}) = 2384 kJ/kg = 2.384×10^6 J/kg

Absolute viscosity (μ_l) = 562.1×10^{-6} (Ns)/m^2

Specific heat (c_l) = 4178 J/(kg K)

SOLUTION

The heat transfer coefficient for this geometry is given by Equation (10.23)

$$\bar{h}_c = 0.725 \left[\frac{\rho_l(\rho_l - \rho_v)\, g h'_{fg}\, k^3}{\mu_l D (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + 3/8\, c_{pl}\,(T_{sv} - T_s) = (2.384 \times \text{J/kg} + (3/8))\ (4178\ \text{J/(kg K)})\ (49.3°\text{C} - 27°\text{C})$

$$= 2.419 \times 10^6\ \text{J/kg}$$

$\bar{h}_c = 0.725$

$$\left[\frac{(988.4\ \text{kg/m}^3)(988.4 - 0.0797)\ \text{kg/m}^3 (9.8\ \text{m/s}^2)(2.419 \times 10^6\ \text{J/kg})((\text{Ws})/\text{J})(0.646\ \text{W/(m K)})^3}{(562.1 \times 10^{-6}\ (\text{N s})/\text{m}^2)((\text{kg m})/(\text{s}^2\text{N}))\ (0.025\,\text{m})(49.3\,°\text{C} - 27\,°\text{C})} \right]^{\frac{1}{4}}$$

$$= 8613\ \text{W/(m}^2\text{K)}$$

PROBLEM 10.23

Repeat Problem 10.22 for a tier of six horizontal 2.5 cm *OD* tubes under similar thermal conditions.

From Problem 10.22: Horizontal tubes are maintained at a temperature of 27°C on its outer surface. Calculate the average heat transfer coefficient if saturated steam at 12 kPa is condensing on this tube.

GIVEN

- Saturated steam condensing on horizontal tubes
- Tube outside diameter (D) = 2.5 cm = 0.025 m
- Tube outer surface temperature (T_s) = 27°C
- Steam pressure (p) = 12 kPa
- Number of tubes (n) = 6

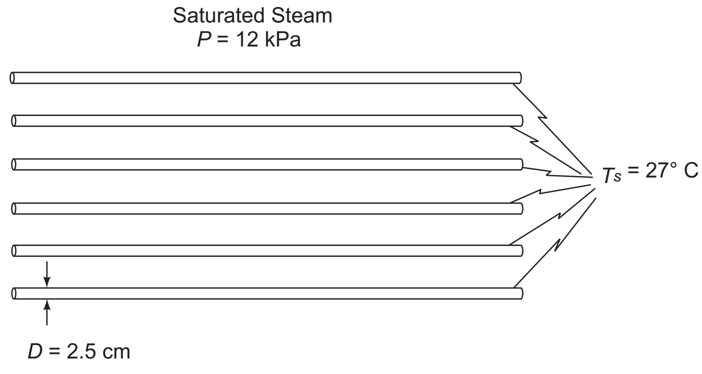
FIND

- The average heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- Film condensation occurs

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 12 kPa

Saturation temperature (T_s) = 49.3°C

Liquid density (ρ_l) = 988.4 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 0.0797 kg/m^3

Thermal conductivity (k_l) = 0.646 W/(m K)

Heat of vaporization (h_{fg}) = 2384 kJ/kg = 2.384 × 10^6 J/kg

Absolute viscosity (μ_l) = 562.1 × 10^{-6} (Ns)/m^2

Specific heat (c_l) = 4178 J/(kg K)

SOLUTION

The average heat transfer coefficient for the tube bank is given by Equation (10.24)

$$\bar{h}_c = 0.728\left[1+0.2\frac{c_p(T_{sv}-T_s)}{h_{fg}}(\mathrm{N}-1)\right]\left[\frac{\rho_l(\rho_l-\rho_v)\,g h'_{fg} k^3}{\mu_l N D(T_{sv}-T_s)}\right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + \frac{3}{8} c_{pl}(T_{sv} - T_s) = \left(2.384\times10^6\ \mathrm{J/kg}\right) + (4178\ \mathrm{J/(kg\,K)})\,(49.3°\mathrm{C} - 27°\mathrm{C})$

$$= 2.419\times10^6\ \mathrm{J/kg}$$

$$\text{Provided}\ \frac{(N-1)c_p(T_{sv}-T_s)}{h_{fg}} < 2$$

$$\frac{(6-1)(4178\ \mathrm{J/(kg\,K)})(49.3°\mathrm{C}-27°\mathrm{C})}{\left(2.384\times10^6\ \mathrm{J/kg}\right)} = 0.1954 < .2$$

$$\bar{h}_c = 0.728\ [1 + 0.2\ (0.1954)]$$

$$\left[\frac{(988.4\ \mathrm{kg/m^3})(988.4-0.0797)\mathrm{kg/m^3}(9.8\ \mathrm{m/s^2})(2.419\times10^6\ \mathrm{J/kg})((\mathrm{Ws})/\mathrm{J})(0.646\ \mathrm{W/(m\,K)})^3}{(562.1\times10^{-6}\ (\mathrm{N\,s})/\mathrm{m^2})((\mathrm{kg\,m})/(\mathrm{s^2N}))\,6(0.025\,\mathrm{m})(49.3°\mathrm{C}-27°\mathrm{C})}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 5742\ \mathrm{W/(m^2K)}$$

PROBLEM 10.24

Saturated steam at 34 kPa condenses on a 1 m tall vertical plate whose surface temperature is uniform at 60°C. Compute the average heat transfer coefficient and the value of the coefficient 1/3, 2/3, and 1 m from the top. Also, find the maximum plate height for which the condensate film will remain laminar.

GIVEN

- Saturated steam condensing on a vertical plate
- Steam pressure (p) = 34 kPa
- Plate height (L) = 1 m
- Plate surface temperature (T_s) = 60°C (uniform)

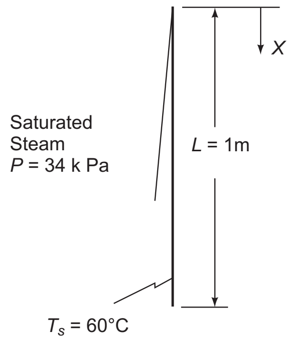
FIND

(a) The average heat transfer coefficient (h_c)
(b) The local heat transfer coefficient (h_α) at x = 1/3 L, 2/3 L, and L
(c) The maximum height for which the condensate film will remain laminar

ASSUMPTIONS

- Steady state
- Film condensation occurs

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 34 kPa

Saturation temperature (T_s) = 71.8°C

Liquid density (ρ_l) = 976.6 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 0.21 kg/m^3

Thermal conductivity (k_l) = 0.668 W/(m K)

Specific heat (c_l) = 4188 J/(kg K)

Heat of vaporization (h_{fg}) = 2329 kJ/kg = 2.329 × 10^6 J/kg

Absolute viscosity (μ_l) = 3.998 × 10^{-4} (N s)/m^2

SOLUTION

(a) The average heat transfer coefficient is given by Equation (10.21)

$$\bar{h}_c = 0.943 \left[\frac{\rho_l(\rho_l - \rho_v) g h'_{fg} k^3}{\mu_l L (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + \frac{3}{8} c_{pl}(T_{sv} - T_s)$

$$h'_{fg} = (2.329 \times 10^6 \text{ J / Kg}) + (4188 \text{ J/(kg K)}) (71.8°\text{C} - 60°\text{C}) = 2.348 \times 10^6 \text{ J/(kg K)}$$

For Equation (10.21), Rohsenow recommends h_{fg} be replaced by h_{fg} + 0.68 c_{p1} ($T_{sv} - T_s$) if c_{pl} ($T_{sv} - T_s$)/h'_{fg} < 1

$$\frac{C_{pl}(T_{sv} - T_s)}{h'_{fg}} = \frac{(4188 \text{ J/kg K})(11.8°\text{C})}{2.348 \times 10^6 \text{ J/kg}} = 0.021 < 1$$

$$h'_{fg} = h_{fg} + 0.68\, c_{pl}\, (T_{sv} - T_s) = (2.329 \times 10^6 \text{ J/kg}) + 0.68 (4188 \text{ J/(kg K)}) (11.8°\text{C})$$

$$= 2.363 \times 10^6 \text{ J/kg}$$

$$\bar{h}_c = 0.943$$

$$\left[\frac{(976.6 \text{ kg/m}^3)(976.6 - 0.21) \text{ kg/m}^3 (9.8 \text{ m/s}^2)(2.363 \times 10^6 \text{ J/kg})((\text{Ws})/\text{J})(0.668 \text{ W/(m K)})^3}{(3.998 \times 10^{-4} \text{ (N s)/m}^2)((\text{kg m})/(\text{s}^2\text{N}))\,(1 \text{ m})\,(11.8°\text{C})} \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 5763 \text{ W/(m}^2\text{K)}$$

(b) The local heat transfer coefficient is given by Equation (10.18)

$$h_x = \left[\frac{\rho_l(\rho_l-\rho_v)gh'_{fg}k^3}{4\mu_1 x(T_{sv}-T_s)}\right]^{\frac{1}{4}} = \bar{h}_c \left(\frac{L}{4x}\right)^{\frac{1}{4}} \left(\frac{1}{0.943}\right)$$

At $x = \frac{1}{3}L$

$$h_x = \left(5763\ \text{W/(m}^2\text{K)}\right) \left(\frac{3}{4}\right)^{\frac{1}{4}} \left(\frac{1}{0.943}\right) = 5687\ \text{W/(m}^2\text{K)}$$

At $x = \frac{2}{3}L$

$$h_x = \left(5763\ \text{W/(m}^2\text{K)}\right) \left(\frac{3}{8}\right)^{\frac{1}{4}} \left(\frac{1}{0.943}\right) = 4782\ \text{W/(m}^2\text{K)}$$

At $x = L$

$$h_x = \left(5763\ \text{W/(m}^2\text{K)}\right) \left(\frac{1}{4}\right)^{\frac{1}{4}} \left(\frac{1}{0.943}\right) = 4321\ \text{W/(m}^2\text{K)}$$

(c) Turbulence occurs when $Re_\delta = \frac{4\Gamma_c}{\mu_l} = 2000$

Combining the definition of the Reynolds number with Equation (10.14)

$$Re_\delta = \frac{4\rho_l^2 g\delta^3}{3\mu_l^2}$$

Solving this for the critical film thickness

$$\delta_c = \left[\frac{3Re_{\delta c}\mu_l^2}{4\rho_l^2 g}\right]^{\frac{1}{3}} = \left[\frac{3(2000)\left(3.998\times10^{-4}\ (\text{N s})/\text{m}^2\right)^2\left((\text{kg m})/(\text{s}^2\text{N})^2\right)}{4\left(976.6\ \text{kg/ m}^2\right)\left(9.8\ \text{m/s}^2\right)}\right]^{\frac{1}{3}} = 0.000295\ \text{m}$$

Solving Equation (10.17) for the distance x down a flat plate at which the film thickness is δ:

$$x = \frac{\delta^4 g\rho_l(\rho_l-\rho_v)h'_{fg}}{4\mu_l k(T_{sv}-T_s)}$$

$$x = \frac{(0.000295\,\text{m})\left(9.8\ \text{m/s}^2\right)\left(976.6\ \text{kg/m}^3\right)\left(976.6-0.21\ \text{kg/m}^3\right)\left(2.363\times10^6\ \text{J/kg}\right)}{4\left(3.998\times10^{-4}\ (\text{N s})/\text{m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)(0.668\ \text{W/(m K)})(\text{J/(W s)})\ (11.8°\text{C})} = 13.2\ \text{m}$$

PROBLEM 10.25

At a pressure of 490 kPa, the saturation temperature of sulfur dioxide (SO_2) is 32°C, the density is 1350 kg/m^3, the heat of vaporization is 343 kJ/kg, the absolute viscosity is 3.2 × 10^{-4} (Ns)/m^2, the specific heat is 1445 J/(kg K) and the thermal conductivity is 0.192 W/(m K). If the SO_2 is to be condensed at 490 kPa on a 20–cm flat surface, inclined at an angle at 45°, whose temperature is maintained uniformly at 24°C, calculate (a) the thickness of the condensate film 1.3 cm from the bottom, (b) the average heat transfer coefficient for the entire plate, and (c) the rate of condensation in kilograms per hour.

GIVEN

- SO_2 condensing on a flat surface inclined 45°
- Pressure (p) = 490 kPa

- SO_2 properties
  - Saturation temperature (T_{sv}) = 32°C
  - Liquid density (ρ_l) = 1350 kg/m^3
  - Heat of vaporization (h_{fg}) = 343 kJ/kg = 343, 000 J/kg
  - Absolute viscosity (μ_l) = 3.2×10^{-4} (Ns)/m^2
  - Specific heat (c_{pl} = 1445 J/(kg K))
  - Thermal conductivity (k) = 0.192 W/(m K)
- Surface temperature (T_s) = 24°C (uniform)
- Length of inclined edge of surface (L) = 20 cm = 0.2 m

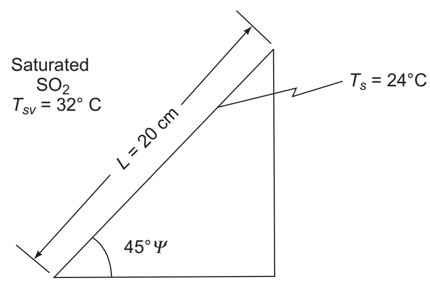
FIND

(a) Condensate film thickness (δ) at 1.3 cm from the bottom ($x = L - 1.3$ cm = 0.187 m)
(b) The average heat transfer coefficient (h_c), and
(c) The rate of condensation (m) in kg/h

ASSUMPTIONS

- Steady state
- Laminar condensate flow
- Vapor density is negligible compared to the liquid density
- Interfacial shear and momentum effects are negligible

SKETCH



SOLUTION

(a) Assuming the condensate element shown in Figure 10.17 is on an inclined plane at an angle ψ with the horizontal, the force balance on the element becomes

$$(\delta - y)\,(\rho_l - \rho_v)\,g \sin\psi = \mu_l \frac{du}{dy}$$

The constant sin ψ can be carried through the derivation shown in Section 10.4.1 to yield the following version of Equation (10.17)

$$\delta = \left[\frac{4\,\mu_l\,k\,x\,(T_{sv} - T_s)}{g \sin\psi\,\rho_l(\rho_l - \rho_v)h'_{fg}}\right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + \dfrac{3}{8}\,c_{pl}\,(T_{sv} - T_s)$

$$h'_{fg} = 343{,}000 \text{ J/kg} + \frac{3}{8}\,(1445 \text{ J/(kg K)})\,(32°\text{C} - 24°\text{C}) = 347{,}335 \text{ J/kg}$$

Neglecting the vapor density, the condensate film thickness at x = 0.187 m is

$$\delta = \left[\frac{4\left(3.2\times10^{-4}(\text{N s})/\text{m}^2\right)\left((\text{kg m})/(\text{s}^2\text{N})\right)(0.192\text{ W/(m K)})(\text{J/(Ws)})(0.187\text{ m})(8°\text{C})}{\left(9.8\text{ m/s}^2\right)(\sin 45°)\left(1350\text{ kg/m}^3\right)^2(347{,}335\text{ J/kg})}\right]^{\frac{1}{4}} = 9.57\times10^{-5}\text{ m}$$

(b) The average heat transfer coefficient is given by Equation (10.22a)

$$\bar{h}_c = 0.943 \left[\frac{\rho_l (\rho_l - \rho_v) g h'_{fg} k^3 \sin \psi}{\mu_l L (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 0.943 \left[\frac{\left(1350 \text{ kg/m}^3\right)^2 \left(9.8 \text{ m/s}^2\right)(347{,}335 \text{ J/kg})((\text{Ws})/\text{J})(0.0192 \text{ W/(m K)})^3}{\left(3.2 \times 10^{-4} \text{ (N s)/m}^2\right) \text{(kg m)/(s}^2\text{N)} (0.2 \text{ m}) (8°\text{C})} \right]^{\frac{1}{4}} = 2631 \text{ W/(m}^2\text{K)}$$

(c) An energy balance yields

$$\dot{m}\, h_{fg} = \bar{h}_c\ A\ (T_{sv} - T_s) = h_c\, L\, w\, (T_{sv} - T_s)$$

$$\frac{\dot{m}}{w} = \frac{\bar{h}_c\, L (T_{sv} - T_s)}{h_{fg}} = \frac{\left(2631 \text{ W/(m}^2\text{K)}\right)(\text{J/(W s)})(3600 \text{ s/h})(0.2 \text{ m})(8°\text{C})}{(343{,}000 \text{ J/kg})}$$

$$\frac{\dot{m}}{w} = 44.2 \text{ kg/h per meter width}$$

PROBLEM 10.26

Repeat Problem 10.25 part (b) and (c) but assume that condensation occurs on a 5 cm cm-*OD* horizontal tube.

From Problem 10.25: At a pressure of 490 kPa, the saturation temperature of sulfur dioxide (SO_2) is 32°C, the density is 1350 kg/m^3, the heat of vaporization is 343 kJ/kg, the absolute viscosity is 3.2×10^{-4} (Ns)/m^2, the specific heat is 1445 J/(kg K) and the thermal conductivity is 0.192 W/(m K). If the SO_2 is to be condensed at 490 kPa on a 20-cm flat surface, inclined at an angle of 45°, whose temperature is maintained uniformly at 24°C, calculate (a) the thickness of the condensate film 1.3 cm from the bottom, (b) the average heat transfer coefficient, and (c) the rate of condensation in kilograms per hour.

GIVEN

- SO_2 condensing on a horizontal tube
- Tube outside diameter (D) = 5 cm = 0.05 m
- Pressure (p) 490 kPa

- SO_2 properties
  - Saturation temperature (T_{sv}) = 32°C
  - Liquid density (ρ_l) = 1350 kg/m^3
  - Heat of vaporization (h_{fg}) = 343 kJ/kg = 343,000 J/kg
  - Absolute viscosity (μ_l) = 3.2×10^{-4} (N s)/m^2
  - Specific heat (c_{pl}) = 1445 J/(kg K)
  - Thermal conductivity (k) = 0.192 W/(m K)
- Surface temperature (T_s) = 24°C (uniform)

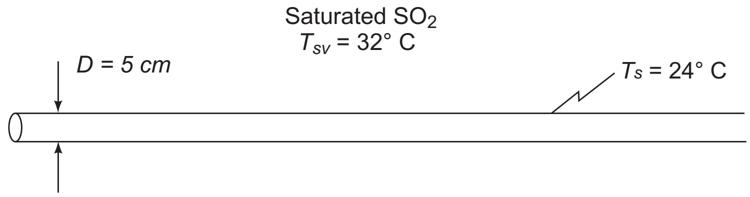
FIND

(a) Condensate film thickness (δ) at 1.3 cm from the bottom ($x = L$ – 1.3 cm = 0.187 m)
(b) The average heat transfer coefficient (h_c)
(c) The rate of condensation ($\dot{m}$) in kg/h

ASSUMPTIONS

- Steady state
- Laminar condensate flow
- Vapor density is negligible compared to the liquid density

SKETCH



SOLUTION

(a) (See solution for (a) in Problem 10.25.)
(b) The average heat transfer coefficient is given by Equation (10.23)

$$\bar{h}_c = 0.725 \left[\frac{\rho_l (\rho_l - \rho_v) g h'_{fg} k^3}{\mu_l D (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + \frac{3}{8} c_{pl} (T_{sv} - T_s)$

$$h'_{fg} = (343{,}000 \text{ J/kg}) + 3/8\ (1445 \text{ J/(kg K)})\ (32°\text{C} - 24°\text{C}) = 347{,}000 \text{ J/kg}$$

Neglecting the vapor density compared to the liquid density

$$\bar{h}_c = 0.725 \left[\frac{(1350 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(347{,}335 \text{ J/kg})((\text{W s})/\text{J})(0.192 \text{ W/(m K)})^3}{(3.2 \times 10^{-4} \text{ (N s)/m}^2)((\text{kg m})/(\text{s}^2\text{N}))(0.05 \text{ m})(8°\text{C})} \right]^{\frac{1}{4}} = 3120 \text{ W/(m}^2\text{K)}$$

(c) An energy balance yields

$$\dot{m}\, h_{fg} = \bar{h}_c\, A\, (T_{sv} - T_s) = \bar{h}_c\, \pi D\, L\, (T_{sv} - T_s)$$

$$\frac{\dot{m}}{L} = \frac{\bar{h}_c \pi D (T_{sv} - T_s)}{h_{fg}} = \frac{(3131 \text{ W/(m}^2\text{K)})(\text{J/(W s)})(3600 \text{ s/h})\,\pi(0.05 \text{ m})(8°\text{C})}{(343{,}000 \text{ J/kg})}$$

$$\frac{\dot{m}}{w} = 41.2 \text{ kg/h per meter length}$$

PROBLEM 10.27

In Problem 10.12, it was indicated that the Nusselt number for condensation depends on the Prandtl number and four other dimensionless groups including the Jacob number, the Bond number, and a nameless group resembling the Grashof number, $\rho g (\rho_l - \rho_v) L^3/\mu^2$. Give a physical explanation of each of these 3 groups and explain when you expect *Bo* and *Ja* to exert a significant influence and when their respective influence is negligible.

GIVEN

- Three of the dimensionless groups upon which the condensation Nusselt number depends

FIND

(a) Physical explanation for the three groups
(b) When *Ja* and *Bo* are important

SOLUTION

The Jacob number is

$$Ja = \frac{c_p \Delta T}{h_{fg}}$$

and it scales the maximum sensible heat that the liquid can absorb to the latent heat absorbed by the liquid during boiling. For most liquids, *Ja* is small. For large values of the excess temperature, ΔT_x, *Ja* could become significant.

The Bond number is

$$Bo = \frac{g \Delta \rho L^2}{\sigma}$$

and it scales the gravitational force to the surface tension force. For water at atmospheric pressure, as an example,

$$Bo \sim \frac{(9.81\ \text{m/s}^2)(1000\ \text{kg/m}^3)L^2}{(100\,\text{N/m})} = 98\ L^2$$

when *L* is given in meters. So, if the length scale for a given problem is the order of 0.1 m = 10 cm, the Bond number is order 1 and these forces are comparable. Clearly, for problems involving large length scales, the Bond number will be >> 1.

The nameless dimensionless group is

$$N_{??} = \frac{g \Delta \rho L^3 \rho}{\mu^2}$$

and, like the Grashof number, it scales the buoyant force to the viscous force. For water at atmospheric pressure, we find that $N_{??}$ >> 1 if $L \sim 0.1$ m, so for typical cases, the buoyant forces will be much larger than the viscous forces.

PROBLEM 10.28

> **Saturated methyl chloride at 4.3 bar (abs) condenses on a horizontal bank of tubes, ten-by-ten, 5 cm *OD*, equally spaced, 10 cm apart center-to-center on rows and columns. If the surface temperature of the tubes is maintained at 7°C by water pumped through them, calculate the rate of condensation of methyl chloride in kg/ms.**
>
> **The properties of saturated methyl chloride at 4.3 bar are shown below**

Saturation temperature = 16°C
Heat of vaporization = 390 kJ/ kg
Liquid density = 936 kg/m^3
Liquid specific heat = 1.6 kJ/(kg K)
Liquid absolute viscosity = 2×10^{-4} kg/ms
Liquid thermal conductivity = 0.17 W/(m K)

GIVEN

- Saturated methyl chloride condensing on a ten-by-ten bank of horizontal tubes
- Pressure = 4.3 bar
- Tube outside diameter (*D*) = 5 cm = 0.05 m
- Tube center-to-center spacing (*s*) = 10 cm = 0.10 m

- Methyl chloride properties given above
- Tube surface temperature (T_s) = 7°C

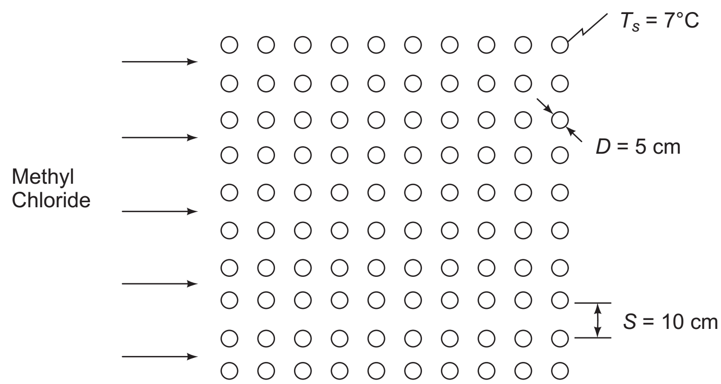
FIND

- The rate of condensation ($\dot{m}/L$) in kg/ms

ASSUMPTIONS

- Steady state
- Laminar flow condensation
- Interfacial shear is negligible
- Tube surface temperature is uniform and constant
- Vapor density is negligible compared to the liquid density

SKETCH



SOLUTION

The average heat transfer coefficient for a vertical row of tubes including liquid subcooling is given by Equation (10.24)

$$\bar{h}_c = 0.728\left[1+0.2\frac{c_p(T_{sv}-T_s)}{h_{fg}}(N-1)\right]\left[\frac{\rho_l(\rho_l-\rho_v)gh'_{fg}k^3}{\mu_l N D(T_{sv}-T_s)}\right]^{\frac{1}{4}}$$

Where N = the number of tubes in a vertical row = 10

$$\text{where } h'_{fg} = h_{fg} + \frac{3}{8}c_{pl}(T_{sv}-T_s)$$

$$h'_{fg} = 390\times10^3\ \text{J/kg} + \frac{3}{8}(1.6\times10^3\ \text{J/kg})\ (16°\text{C}-7°\text{C}) = 395.4\ \text{kJ/kg}$$

Assuming the vapor density is negligible compared to the liquid density

$$\bar{h}_c = 0.728\left[1+0.2\frac{(1.6\,\text{kJ/(kg K)})(16°\text{C}-9°\text{C})}{(390\ \text{kJ/kg})}(10-1)\right]$$

$$\left[\frac{9.81\times(936)^2\times395.4\times10^3\times(0.17)^3}{2\times10^{-4}\times10\times(5\times10^{-2})\times(16-7)}\right]^{\frac{1}{2}} = 1611\ \text{W/(m}^2\ \text{K)}$$

$$\bar{h}_c = 1611\ \text{W/(m}^2\ \text{K)}$$

The rate of heat transfer is

$$q = \bar{h}_c\ A_t\,(T_{sv}-T_s) = \bar{h}_c\ N_{\text{total}}\,\pi D\,T\,(T_{sv}-T_s)$$

The rate of condensate flow is given by

$$\dot{m} = \frac{q}{h_{fg}}$$

$$\frac{\dot{m}}{L} = \frac{\bar{h}_c}{h_{fg}} N_{\text{total}} \pi D (T_{sv} - T_s) = \frac{1611\,\text{W/(m}^2\text{K)}(\pi)(100)}{390 \times 10^3\ \text{J/kg}} (5 \times 10^{-2}\ \text{m})(16°\text{C} - 9°\text{C}) = 0.584\ \text{kg/m}$$

$$\Rightarrow \qquad \frac{\dot{m}}{L} = 0.584\ \text{kg/ms}$$

PROBLEM 10.29

A vertical rectangular water duct 1 m high and 0.1 m deep is placed in an environment of saturated steam at atmospheric pressure. If the outer surface of the duct is about 50°C, estimate the rate of steam condensation per unit length.

GIVEN

- Water-cooled rectangular duct, 1 m high, 0.1 m deep
- Duct surface is 50°C
- Steam environment

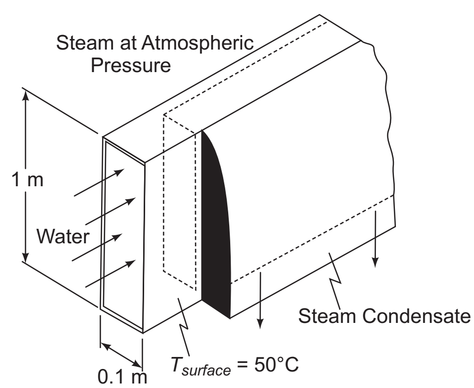
FIND

(a) Rate of steam condensation per unit length of the duct

ASSUMPTIONS

- The steam is at 1 atmosphere pressure
- Condensation from the horizontal duct surfaces can be neglected
- Laminar film condensation on the vertical surfaces (must be checked)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the properties of saturated liquid at the mean temperature of 75°C are

Density (ρ_l) = = 974.9 kg/m^3

Specific heat (c_l) = 4190 J/(kg K)

Thermal conductivity (k_l) = 0.671 W/(m K)

Absolute viscosity (μ_l) = 3.77×10^{-4} kg/ms

Heat of vaporization (h_{fg}) = 2.257×10^6 J/kg

From Appendix 2, Table 34, the properties of the saturated vapor at the saturation temperature of 100°C are

Density (ρ_v) = 0.597 kg/m^3

SOLUTION

The mean flow of condensate per unit duct length is from Equation (10.14)

$$\Gamma_c = \frac{\rho_l(\rho_l - \rho_v)\delta^3 g}{3\mu_l}$$

and the film thickness can be found from Equation (10.17)

$$\delta = \left[\frac{4\mu_l k_l \times (T_{sv} - T_s)}{g\,\rho_l(\rho_l - \rho_v)h'_{fg}}\right]^{\frac{1}{4}}$$

where

$$h'_{fg} = h_{fg} + 0.68\, c_l \Delta T = \left(2.257 \times 10^6 \text{ J/kg}\right) + (0.68)(4190 \text{ J/(kg K)})(100 - 50)(\text{K}) = 2.4 \times 10^6 \text{ J/kg}$$

Calculating the condensate film thickness at the bottom edge of the duct, we have

$$\delta = \left[\frac{(4)\left(3.77 \times 10^{-4}\text{ kg/(ms)}\right)(0.671\text{ W/(mK)})(1\text{ m})(50\text{ K})}{\left(9.81\text{m/s}^2\right)\left(974.9\text{ kg/m}^3\right)(974.9 - 0.597)(\text{kg/m}^3)(2.4 \times 10^6\text{ J/kg})}\right]^{\frac{1}{4}} = 2.18 \times 10^{-4}\text{ m}$$

Now, we can calculate the film flow per unit duct length

$$\Gamma = \frac{(974.9\text{ kg/m}^3)(974.9 - 0.597)(\text{kg/m}^3)(2.18 \times 10^{-4}\text{ m})^3(9.81\text{ m/s}^2)}{\left((3)(3.77 \times 10^{-4}\text{ kg/(ms)}\right)} = 0.085\text{ kg/ms}$$

Doubling this to account for both sides of the duct, we have 0.171 kg/m s for the rate of condensate flow, per unit duct length.

To confirm that the Reynolds number for the condensate film flow is laminar

$$Re = \frac{4\Gamma_c}{\mu_l} = \frac{(4)(0.085\text{ kg/(ms)})}{\left(3.77 \times 10^{-4}\text{ kg/(ms)}\right)} = 902$$

PROBLEM 10.30

A 1 m long tube-within-a-tube heat exchanger, as shown in the sketch, is used to condense steam at 2 atmospheres in the annulus, and water flows in the inner tube, entering at 90°C. The inner tube is made of copper with an *OD* of 1.27 cm and an *ID* of 1.0 cm. (a) Estimate the water flow rate required to keep its outlet temperature below 100°C. (b) Estimate the pressure drop and the pumping power for the water in the heat exchanger, neglecting inlet and outlet losses.

GIVEN

- Tube-within-a-tube condenser
- Cooling water flowing in the inner tube
- Steam at 2 atm condensing inside the annulus

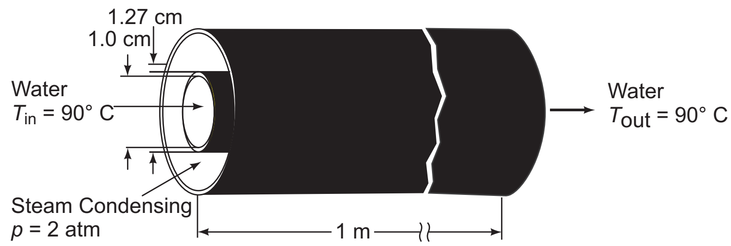
FIND

(a) Coolant water flow rate to maintain coolant outlet temperature below 100°C
(b) Coolant pressure drop and pumping power

ASSUMPTIONS

- Steady conditions
- The heat exchanger is horizontal

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the properties of water at 2 atm are

Saturation temperature, T_{sv} = 121°C

Liquid density, ρ_l = 944 kg/m^3

Liquid specific heat, c_{pl} = 4232 J/(kg K)

Liquid thermal conductivity, k_l = 0.685 W/(mK)

Liquid viscosity, $\mu_l = 2.35 \times 10^{-4}$ kg/(ms)

Heat of vaporization, $h_{fg} = 2.202 \times 10^6$ J/kg

Vapor density, ρ_v = 1.12 kg/m^3

SOLUTION

(a) We can use Equation (10.23) to calculate the average condensing heat transfer coefficient for a horizontal tube

$$\bar{h}_c = 0.725 \left[\frac{\rho_l (\rho_l - \rho_v) g h'_{fg} k^3}{D \mu_l (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where

$$h'_{fg} = h_{fg} + 0.68\, c_{pl}\, (T_{sv} - T_s)$$

Assuming that the average coolant temperature is 95°C and neglecting temperature drop across the copper tube, we have T_s = 95°C. Then

$$h'_{fg} = \left(2.202 \times 10^6 \text{ J/kg}\right) + (0.68)\left(4232 \text{ J/(kg K)}\right)(121 - 95)(\text{K}) = 2.277 \times 10^6 \text{ J/kg}$$

The average condensing heat transfer coefficient is then

$$\bar{h}_c = 0.725 \left[\frac{\left(944 \text{ kg/m}^3\right)(944 - 1.12) \text{ kg/m}^3 \left(9.81 \text{ m/s}^2\right)\left(2.277 \times 10^6 \text{ J/kg}\right)\left(0.685 \text{ W/(m K)}\right)^3}{(0.0127 \text{ m})\left(2.35 \times 10^{-4} \text{ kg/(ms)}\right)(121 - 95)(\text{K})} \right]^{\frac{1}{4}}$$

$$= 12{,}282 \text{ W/(m}^2\text{K)}$$

Performing a heat balance on the cooling water

$$\dot{m}\, c_{pl}(T_{\text{water,out}} - T_{\text{water,in}}) = \bar{h}_c\, \pi D L (T_{sv} - T_s)$$

we can solve for the coolant mass flow

$$\dot{m} = \frac{\left(12{,}282 \text{ W/(m}^2\text{K)}\right)(\pi)(0.0127 \text{ m})(1 \text{ m})(121 - 95)(\text{K})}{\left(4232 \text{ J/(kg K)}\right)(100 - 90)(\text{K})} = 0.30 \text{ kg/s}$$

(b) To determine the pressure drop and pumping power, we need to determine the Reynolds number for the coolant flow

$$Re_w = \frac{4\dot{m}}{\pi \mu_l D}$$

At the average bulk coolant temperature of 95°C, Table 13 gives

$\mu_l = 2.97 \times 10^{-4}$ kg/(ms)

$\rho_l = 961$ kg/m^3

$$Re_w = \frac{(4)(0.30\text{ kg/s})}{(\pi)\left(2.97\times 10^{-4}\text{ kg/(ms)}\right)(0.01\text{ m})} = 128{,}610$$

Assuming the tube is smooth, the friction from Figure 6.18 is

$$F = 0.0165$$

and Equation (6.13) gives the pressure drop

$$\Delta p = f\,\frac{L\rho U^2}{D\,2g_c}$$

The mean flow velocity for the coolant is

$$U = \frac{\dot{m}}{\frac{\rho\pi D^2}{4}} = \frac{(0.3\text{ kg/s})}{\left(961\text{ kg/m}^3\right)\left(\pi(0.01\text{ m})^2/4\right)} = 3.97\text{ m/s}$$

The pressure drop is then

$$\Delta p = (0.0165)\ (1\text{ m}/(0.01\text{ m}))\ \frac{\left(961\text{ kg/m}^3\right)(3.97\text{ m/s})^2}{(2)\left((\text{kg m})/(\text{s}^2\text{N})\right)} = 12{,}500\text{ N/m}^2$$

The pumping power can be determined from Equation (6.19)

$$P_{\text{pumping}} = \Delta p\ \frac{\dot{m}}{\rho} = 12{,}500\text{ N/m}^2\ \frac{(0.30\text{ kg/s})}{\left(961\text{ kg/m}^3\right)} = 3.9\ (\text{N m})/\text{s} = 3.9\text{ W}$$

PROBLEM 10.31

A one-pass condenser-heat exchanger, shown in the sketch, has 64 tubes arranged in a square array with 8 tubes per line. The tubes are 1.22 m long, made of copper with an outside diameter of 1.27 cm, in a shell at atmospheric pressure. Water flows inside the tubes whose outside wall temperature is 98°C. Calculate (a) the rate of steam condensation, (b) the temperature rise of the water if the flow rate per tube is 0.0454 kg/s. Express your answer in SI units.

GIVEN

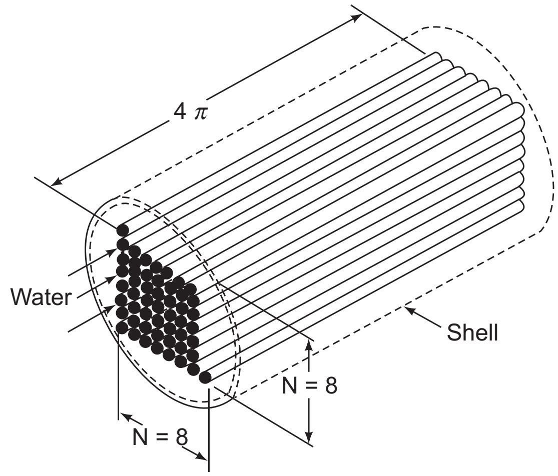
- One-pass condenser heat exchanger with 64 tubes in a square array

FIND

(a) Rate of steam condensation

(b) Water temperature rise

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the properties of saturated water at the film temperature of 99°C are

Density (ρ_l) = 958 kg/m^3

Absolute viscosity (μ_l) = 2.78×10^{-4} kg/(ms)

Thermal conductivity (k_l) = 0.682 W/(mK)

Specific heat (c_l) = 4211 J/(kg K)

Heat of vaporization (h_{fg}) = 2.257×10^6 J/kg

SOLUTION

Converting the remaining problem parameters we have

Wall temperature: 98°C

Saturation temperature: 100°C

Tube o.d.: 0.0127 m

Tube length: 1.22 m

Water flow rate: 0.0454 kg/s

From Equation (10.24), we can obtain the average heat transfer coefficient

$$\bar{h}_c = 0.728[1 + 0.2(\mathrm{N} - 1)Ja]\left[\frac{g\rho_l(\rho_l - \rho_v)k^3h'_{fg}}{N\,D\mu_l(T_{sv} - T_s)}\right]^{\frac{1}{4}}$$

where

$$Ja = \frac{cp\,(T_{sv} - T_s)}{h_{fg}} = \frac{(4211\,\mathrm{J/(kg\,K)})(2\,\mathrm{K})}{(2.257\times 10^6\,\mathrm{J/kg})} = 0.00373$$

and

$$h'_{fg} = h_{fg} + c_{pl}\,(T_{sv} - T_s) = 2.257 \times 10^6\ \mathrm{J/\,kg} + (4211\,\mathrm{J/(kg\ K)})\,(2\mathrm{K}) = 2.265 \times 10^6\ \mathrm{J/kg}$$

Neglecting the vapor density compared to the liquid density, the quantity in the right bracket in the equation for the heat transfer coefficient is

$$\left[\frac{(9.81\,\text{m/s}^2)(958\,\text{kg/m}^3)^2(0.68\,\text{W/(mK)})^3(2.265\times10^6\ \text{J/kg})}{(8)(0.0127\,\text{m})\left(2.78\times10^{-4}\,\text{kg/(ms)}\right)(2\,\text{K})}\right]^{\frac{4}{4}} = 18{,}355\ \ \text{W/(m}^2\text{K)}$$

so

$$\overline{h}_c = (0.728)[1 + (0.2)(7)(0.00373)][18{,}355] = 13{,}432\ \ \text{W/m}^2$$

The tube surface area is

$$N\pi DL = (64)(\pi)(0.0127\text{ m})(1.22\text{ m}) = 3.12\text{m}^2$$

The rate of heat transfer is therefore

$$q = \overline{h}_c\ A\ (T_{sv} - T_s) = (13{,}432\text{ W/m}^2\text{K})(3.12\text{m}^2)(2\text{K}) = 83{,}688\text{ W}$$

(a) The flow rate of condensate is then

$$\dot{m}_c = \frac{q}{h'_{fg}} = \frac{83{,}688\,\text{W}}{2.265\times10^6\,\text{(Ws)/kg}} = 0.03695\ \ \text{kg/s}$$

The heat transfer per tube is 83,688/64 W = 1308 W and this must equate to the increase in sensible heat in the cooling water, giving for the water temperature rise
(b)

$$\Delta T_w = \frac{1308\,\text{W}}{(0.0454\,\text{kg/s})(4211\,\text{J/(kgK)})} = 6.8\text{K}$$

PROBLEM 10.32

Show that the dimensionless equation for ice formation at the outside of a tube of radius r_o is

$$T^* = \frac{r^{*2}}{2}\ \ln r^* + \left(\frac{1}{2R^*}-\frac{1}{4}\right)(r^{*2} - 1)$$

where

$$r^* = \frac{\varepsilon + r_o}{r_o}\quad R^* = \frac{h_o r_o}{k}\quad t^* = \frac{(T_f - T_\infty)kt}{pLr_o^2}$$

Assume that the water is initially at the freezing temperature T_f, that the cooling medium inside the tube surface is below the freezing temperature at a uniform temperature T_∞, and that h_o is the total heat transfer coefficient between the cooling medium and the pipe-ice interface. Also show the thermal circuit.

GIVEN

- Water freezing on the outside of a tube
- Tube radius = r_o
- Cooling medium temperature = T_∞ (uniform)
- Heat transfer coefficient between the cooling medium and the pipe-ice interface = h_o

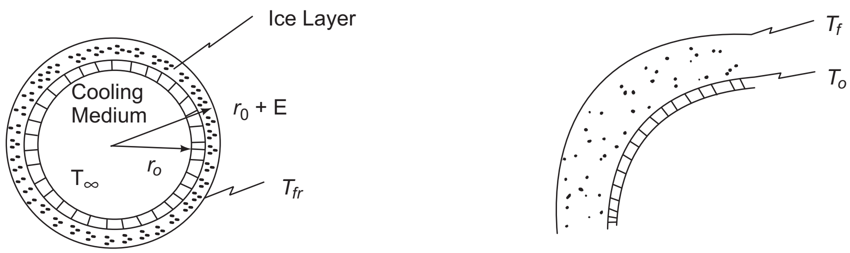
FIND

- Draw the thermal circuit and show that the dimensionless equation for ice formation is as shown above

ASSUMPTIONS

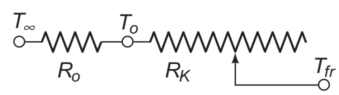
- Steady state
- The thermal capacitance of the ice layer is negligible
- T_∞ is constant
- The water is at the freezing temperature, T_f
- The properties of the ice are uniform

SKETCH



SOLUTION

The thermal circuit is shown below



where

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o 2\pi r_o L}$$

$$R_k = \frac{\ln\left(\dfrac{r_o + \varepsilon}{r_o}\right)}{2\pi L k}$$

The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{fr} - T_\infty}{R_o + R_k} = 2\,\pi L\, r_o \,\frac{T_{fr} - T_\infty}{\dfrac{1}{h_o} + \dfrac{r_o}{k}\ln\left(\dfrac{r_o + \varepsilon}{r_o}\right)}$$

This is the heat flow rate which removes the latent heat of fusion necessary for freezing the ice as shown by Equation (10.34)

$$q = A\,\rho\, L_f \,\frac{d\varepsilon}{dt} = 2\,\pi (r_o + \varepsilon)\, L\, \rho\, L_f \,\frac{d\varepsilon}{dt}$$

where ρ_{Lf} is the latent heat.

Combining these two equations

$$r_o \,\frac{T_{fr} - T_\infty}{\dfrac{1}{h_o} + \dfrac{r_o}{k}\ln\left(\dfrac{r_o + \varepsilon}{r_o}\right)} = (r_o + \varepsilon)\, \rho L_f \,\frac{d\varepsilon}{dt}$$

Rearranging and using $d\varepsilon = r_o\, d\left(\dfrac{r_o + \varepsilon}{r_o}\right)$

$$\frac{k\,(T_{fr} - T_\infty)}{r_o^2 \rho L_f}\, dt = \left[\frac{k}{r_o h_o} + \ln\left(\frac{r_o + \varepsilon}{r_o}\right)\right] \left(\frac{r_o + \varepsilon}{r_o}\right) d\left(\frac{r_o + \varepsilon}{r_o}\right)$$

$$\text{Let } t^* = \frac{(T_f - T_\infty)\,kt}{\rho L r_o^2} \quad \rightarrow \quad dt^* = \frac{(T_{fr} - T_\infty)\,k}{\rho L^2\, r_o^2}\, dt$$

$$r^* = \frac{\varepsilon + r_o}{r_o}$$

$$R^* = \frac{h_o\, r_o}{k}$$

Expressing the above equation in terms of these dimensionless parameters:

$$dt^* = \left(\frac{1}{R^*} + \ln r^*\right) r^*\, dr^*$$

Integrating

$$\int_0^{t^*} dt^* = \int_1^{r^*}\left(\frac{1}{R^*} + \ln r^*\right) r^*\, dr^*$$

$$t^* = \frac{r^{*2}}{2R^*}\,\frac{1}{2R^*} + r^{*2}\left(\frac{\ln r^*}{2} - \frac{1}{4}\right) + \frac{1}{4}$$

$$t^* = \frac{r^{*2}}{2}\,\ln r^* + \left(\frac{1}{2R^*} - \frac{1}{4}\right)(r^{*2} - 1)$$

PROBLEM 10.33

In the manufacture of can ice, cans having inside dimensions of 27.5 cm × 55 cm × 125 cm with 2.5 cm inside taper are filled with water and immersed in brine at a temperature of –12°C. [For details of the process see (81).] For the purpose of a preliminary analysis, the actual ice can be considered as an equivalent cylinder having the same cross-sectional area as the can, and end effects may be neglected. The overall conductance between the brine and the inner surface of the can is 225 W/(m^2 K). Determine the time required to freeze the water and compare with the time necessary if the brine circulation rate were increased to reduce the thermal resistance of the surface to one-tenth of the value specified above. The latent heat of fusion of ice is 334 kJ/kg, its density is 912.5 kg/m^3, and its thermal conductivity is 2.2 W/(m K).

GIVEN

- Ice formation within a can immersed in a brine solution
- Can dimensions: 27.5 cm × 55 cm × 125 cm (with 2.5 cm taper)
- Brine temperature (T_∞) = – 12°C
- Overall heat transfer coefficient between the brine and the outer surface of the can (h_o) = 225 W/ (m^2 K)
- Ice properties
  - Latent heat of fusion (L_f) = 334 kJ/kg
  - Density (ρ) = 912.5 kg/m^3
  - Thermal conductivity (k) = 2.2 W/(m K)

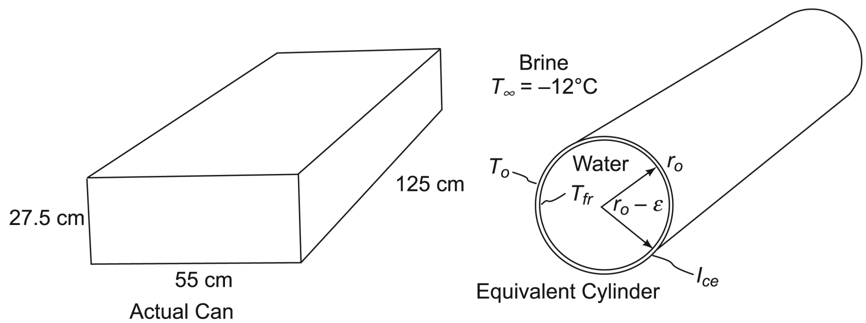
FIND

The time required to freeze the water in the can for
(a) the given h_o, and (b) for one-tenth the resistance

ASSUMPTIONS

- Steady state
- The capacitance of the layer can be neglected
- The brine temperature is constant and uniform
- The can can be treated as a cylinder having the same cross-sectional area
- End effects are negligible

SKETCH

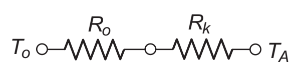


SOLUTION

The effective radius of the equivalent cylinder is

$$r_o = \frac{1}{2}\sqrt{\frac{4A}{\pi}} = \frac{1}{2}\sqrt{\frac{4(27.5\times10^{-2}\ \text{m})(55\times10^{-2}\ \text{m})}{\pi}} = 0.22\ \text{m}$$

The thermal circuit for the problem is shown below



where

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o\, 2\pi r_o L}$$

$$R_k = \frac{\ln\left(\dfrac{r_o+\varepsilon}{r_o}\right)}{2\pi L k}$$

The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{fr}-T_\infty}{R_o+R_k} = 2\,\pi L\, r_o\,\frac{T_{fr}-T_\infty}{\dfrac{1}{h_o}+\dfrac{r_o}{k}\ln\left(\dfrac{r_o+\varepsilon}{r_o}\right)}$$

This is the heat flow rate which removes the latent heat of fusion necessary for freezing, as shown by Equation (10.34)

$$q = A\,\rho\,L_f\,\frac{d\varepsilon}{dt} = -2\,\pi(r_o+\varepsilon)\,L\,\rho\,L_f\,\frac{d\varepsilon}{dt}$$

where ρL_f is the latent heat.

Combining these equations

$$\frac{k\,(T_{fr}-T_\infty)}{r_o^2\,\rho L_f}\;dt = -\left[\frac{k}{r_o h_o}+\ln\left(\frac{r_o+\varepsilon}{r_o}\right)\right]\left(\frac{r_o+\varepsilon}{r_o}\right)d\left(\frac{r_o+\varepsilon}{r_o}\right)$$

Let

$$t^* = \frac{(T_f-T_\infty)kt}{\rho L r_o^2} \to dt^* = \frac{(T_{fr}-T_\infty)k}{\rho L^2\, r_o^2}\,dt \qquad r^* = \frac{\varepsilon + r_o}{r_o} \qquad R^* = \frac{h_o\, r_o}{k}$$

$$dt^* = -\left(\frac{1}{R^*}+\ln r^*\right)\; r^*\, dr^*$$

Integrating

$$\int_0^{t^*} dt^* = \int_1^{r^*}\left(\frac{1}{R^*}+\ln r^*\right)\; r^*\, dr^*$$

$$t^* = \frac{r^{*2}}{2R^*} - \frac{1}{2R^*} + r^{*2}\left(\frac{\ln r^*}{2}-\frac{1}{4}\right)+\left(\frac{1}{4}\right)$$

$$t^* = \frac{r^{*2}}{2}\ln r^* + \left(\frac{1}{2R^*}-\frac{1}{4}\right)(r^{*2}-1)$$

All the ice is frozen when $\varepsilon = r_o \to r^* = 0$

At $\quad r^* = 0$

$$t^* = \frac{1}{2R^*} + \frac{1}{4}$$

$$\frac{k\,(T_{fr}-T_\infty)}{r_o^2\rho L_f}\;t = \frac{1}{2\left(\frac{h_o\, r_o}{k}\right)} + \left(\frac{1}{4}\right)$$

$$t = \frac{r_o^2\rho L_f}{k\,(T_{fr}-T_\infty)}\left(\frac{k}{2\,h_o\, r_o}+\frac{1}{4}\right)$$

(a) For h_o = 225 W/(m^2 K)

$$t = \frac{(0.22\,\text{m})^2\left(912.5\,\text{kg/m}^3\right)\left(334\times10^3\ \text{J/kg}\right)}{\left(2.2\ \text{W/(m K)}\right)(0-(-12°\text{C}))}\left[\frac{2.2\ \text{W/(m K)}}{2\left(225\ \text{W/(m}^2\,\text{K)}\right)(0.22\,\text{m})}+\frac{1}{4}\right] = 152\times10^3\ \text{s}$$

$$t = 42.25 \text{ hours}$$

(b) If the thermal resistance is one tenth of part (a), that is the same as saying the heat transfer coefficient is increased ten-fold: h_o = 2250 W/(m^2 K), we get

$$t = 39.15 \text{ hours}$$

PROBLEM 10.34

Estimate the time required to freeze vegetables in thin, tin cylindrical containers 15 cm in diameter. Air at –12°C is blowing at 4 m/s over the cans, which are stacked to form one long cylinder. The physical properties of the vegetables before and after freezing may be taken as those of water and ice, respectively.

GIVEN

- The freezing of vegetables in thin tin cylindrical cans with air flowing over the cans
- Container diameter (D) = 15 cm = 0.15 m
- Air temperature (T_∞) = –12°C

- Air velocity (U_∞) = 4 m/s
- Vegetables have the same properties as water and ice

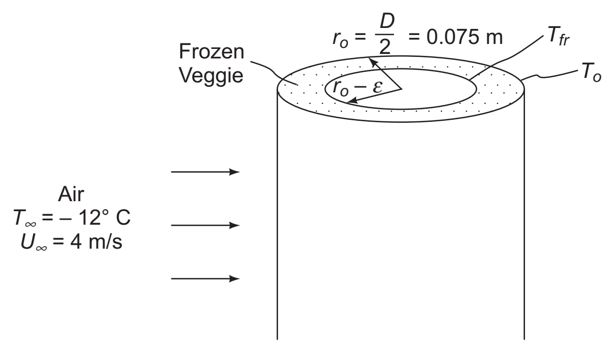
FIND

- The time (t) to freeze the vegetables

ASSUMPTIONS

- Air temperature is constant
- Thermal resistance of the tin can is negligible
- Thickness of the tin can is negligible
- Thermal capacitance of the frozen vegetable layer is negligible

SKETCH



PROPERTIES AND CONSTANTS

Converting the ice property values given in the problem statement of Problem 10.33 to SI units

Latent heat of fusion (L_f) = 333.780 J/kg

Density (ρ) = 918 kg/m^3

Thermal conductivity (k) = 2.22 W/(m K)

Extrapolating for Appendix 2, Table 27, for dry air at –12°C

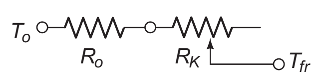
Thermal conductivity (k_a) = 0.0229 W/(m K)

Kinematic viscosity (ν_a) = 12.8 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The thermal circuit for the problem is shown below



where

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o\, 2\pi r_o\, L}$$

$$R_k = \frac{\ln\left(\dfrac{r_o + \varepsilon}{r_o}\right)}{2\pi L k}$$

The Reynolds number of the air flow is

$$Re_D = \frac{U_\infty D}{\nu_a} = \frac{(4\ \text{m/s})(0.15\ \text{m})}{\left(12.8 \times 10^{-6}\ \text{m}^2/\text{s}\right)} = 4.69 \times 10^4$$

The Nusselt number for the geometry is given by Equation (7.3)

$$\overline{Nu}_D = \frac{\bar{h}_o D}{k_a} = C\, Re_D^{\ m}\, Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where, for $Re_D = 4.69\ 10^4$

$C = 0.26$, $m = 0.6$, and $n = 0.37$

Since the surface temperature is between T_∞ and T_{fr}, the Prandtl number evaluated at the surface temperature (Pr_s) will be 0.71 and $Pr/Pr_s = 1.0$

$$\overline{Nu}_D = 0.26\,(4.69 \times 10^4)^{0.6}\,(0.71)^{0.37} = 145.9$$

$$\bar{h}_o = \overline{Nu}_D\, \frac{k_a}{D} = 145.9\, \frac{(0.0229\ \text{W/(m K)})}{0.15\ \text{m}} = 22.3\ \ \text{W/(m}^2\text{K)}$$

The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{fr} - T_\infty}{R_\text{o} + R_k} = 2\,\pi L\, r_o\, \frac{T_{fr} - T_\infty}{\dfrac{1}{h_o} + \dfrac{r_o}{k} \ln\left(\dfrac{r_o + \varepsilon}{r_o}\right)}$$

This is the heat flow rate which removes the latent heat of fusion necessary for freezing, as shown by Equation (10.34)

$$q = A\,\rho L_f\, \frac{d\varepsilon}{dt} = -\,2\,\pi (r_o + \varepsilon)\, L\, \rho L_f\, \frac{d\varepsilon}{dt}$$

where ρL_f is the latent heat.

Combining these Equations

$$\frac{k\,(T_{fr} - T_\infty)}{r_o^2 \rho L_f}\, dt = -\left[\frac{k}{r_o h_o} + \ln\left(\frac{r_o + \varepsilon}{r_o}\right)\right]\left(\frac{r_o + \varepsilon}{r_o}\right) d\left(\frac{r_o + \varepsilon}{r_o}\right)$$

$$\text{Let} \quad t^* = \frac{(T_f - T_\infty) kt}{\rho L r_o^2} \rightarrow dt^* = \frac{(T_{fr} - T_\infty) k}{\rho L^2\, r_o^2}\, dt \qquad r^* = \frac{\varepsilon + r_o}{r_o} \qquad R^* = \frac{h_o\, r_o}{k}$$

$$dt^* = -\left(\frac{1}{R^*} + \ln r^*\right) r^*\, dr^*$$

Integrating

$$\int_0^{t^*} dt^* = \int_1^{r^*} \left(\frac{1}{R^*} + \ln r^*\right) r^*\, dr^*$$

$$t^* = \frac{r^{*2}}{2R^*} - \frac{1}{2R^*} + r^{*2}\left(\frac{\ln r^*}{2} - \frac{1}{4}\right) + \frac{1}{4}$$

$$t^* = \frac{r^{2*}}{2} \ln r^* + \left(\frac{1}{2\,R^*} - \frac{1}{4}\right)(r^{*2} - 1)$$

All the ice is frozen when $\varepsilon = r_o \rightarrow r^* = 0$

At $\quad r^* = 0$

$$t^* = \frac{1}{2R^*} + \left(\frac{1}{4}\right)$$

$$\frac{k\,(T_{fr} - T_\infty)}{r_o^2 \rho L_f}\, t = \frac{1}{2\left(\frac{h_o\, r_o}{k}\right)} + \left(\frac{1}{4}\right)$$

$$t = \frac{r_o^2\, \rho L_f}{k\,(T_{fr} - T_\infty)} \left(\frac{k}{2 h_o\, r_o} + \frac{1}{4}\right)$$

$$t = 64{,}698 \text{ s } (0.6636 + 0.25) = 59{,}113 \text{ s} = 16.4 \text{ h}$$

PROBLEM 10.35

Estimate the time required to freeze a 3 cm thickness of water due to nocturnal radiation with ambient air and initial water temperature at 4°C. Neglect evaporation effects.

GIVEN

- Water exposed to nocturnal radiation and air
- Initial water temperature (T_{wi}) and ambient air temperature (T_∞) = 4°C

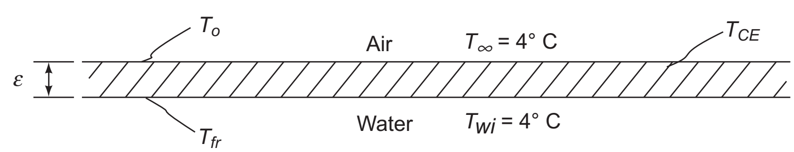
FIND

- The time (t_f) required to freeze an ice layer of thickness (ε_f) = 3 cm = 0.03 m

ASSUMPTIONS

- The thermal capacitance of the ice is negligible
- The energy required to lower the temperature of the water to the freezing point is negligible compared to the latent heat of fusion
- Natural convection from the upper and lower surfaces of the ice layer is negligible
- Effective sky temperature (T_s) = 0 K
- Ice surface behaves as a blackbody (ε_r = 1.0)

SKETCH



PROPERTIES AND CONSTANTS

Converting the ice property values given in the problem statement of Problem 10.33 to SI units:

Latent heat of fusion (L_f) = 333,780 J/kg

Density (ρ) = 918 kg/m^3

Thermal conductivity (k) = 2.22 W/(m K)

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The analysis of Section 10.6 can be applied to this problem by substituting the radiative heat transfer coefficient, h_r, for h_o.

However, h_r is a function of the ice surface temperature, T_o

$$h_r = \frac{\sigma(T_o^4 - R_s^4)}{T_o - T_s} = \sigma T_o^3$$

As ice first begins to form, the surface temperature is the same as the freezing temperature ($T_o - T_{fr}$) and the radiative heat transfer coefficient is

$$h_{ri} = \sigma T_{fi}^3 = \left(5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)\right)\ (273\ \text{K})^3 = 1.154\ \text{W/(m}^2\text{K)}$$

The final surface temperature can be calculated by equating the rate of conduction through the ice layer with the rate of radiation from the surface of the ice

$$\frac{k}{\varepsilon} = (T_{fr} - T_o) = \sigma T_o^4$$

$$\frac{2.22\,\text{W/(m K)}}{0.03\,\text{m}}\ (273\ \text{K} - T_o) = \left(5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\ T_o^4$$

By trial and error, T_o = 269 K

Therefore, the final value of h_r is

$$h_{rf} = \sigma T_o^3 = \left(5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\ (269\ \text{K})^3 = 1.104\ \text{W/(m}^2\text{K)}$$

Since the variation of hr is only about 5%. h_r will be considered constant at the average value of 1.13 W/(m^2 K). The time required is given by Equation (10.39)

$$\varepsilon^+ = -1 + \sqrt{1 + 2t^+}$$

where

$$\varepsilon^+ = \frac{h_r \varepsilon}{k} = \frac{\left(1.13\ \text{W/(m}^2\text{K)}\right)(0.03\,\text{m})}{\left(2.22\ \text{W/(m K)}\right)} = 0.0153$$

$$t^+ = t\, h_r^2\, \frac{T_{fr} - T_s}{\rho L_f\, k}$$

Solving for t

$$t = \frac{\rho L_f k}{2 \text{h}_r^2 (T_{fr} - T_s)} [(\varepsilon^+ + 1)^2 - 1]$$

$$= \frac{\left(918\ \text{kg/m}^3\right)(333{,}780\ \text{J/kg})\left((\text{W s})/\text{J}\right)(2.22\ \text{W/(m K)})}{2\left(1.13\ \text{W/(m}^2\text{K)}\right)^2 (273\,\text{K} - 0\ \text{K})}\ [0.0153 + 1)^2 - 1]$$

$$t = 30{,}084\ \text{s} = 8.4\ \text{h}$$

PROBLEM 10.36

The temperature of a round cooling pond, 100 m in diameter, is 7°C on a winter day. If the air temperature suddenly drops to –7°C, calculate the thickness of ice formed after three hours.

GIVEN

- A round cooling pond on a winter day
- Initial temperature (T_l) = 7°C
- Air temperature (T_∞) drops to –7°C
- Pond diameter (D) = 100 m

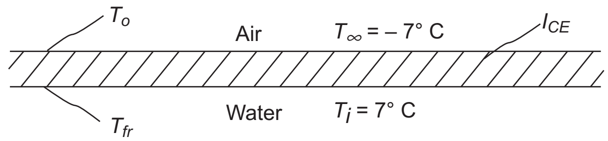
FIND

- The thickness of ice (ε) formed after 3 hours

ASSUMPTIONS

- The thermal capacitance of the ice layer is negligible
- Radiative heat transfer is negligible
- Bulk water temperature remains constant at 7°C
- Air temperature is constant at –7°C
- The air and water are still

SKETCH



PROPERTIES AND CONSTANTS

Converting the ice property values given in the problem statement of Problem 10.33 to SI units

Latent heat of fusion (L_f) = 333,780 J/kg

Density (ρ) = 918 kg/m^3

Thermal conductivity (k) = 2.22 W/(m K)

Extrapolating from Appendix 2, Table 27, for dry air at the estimated film temperature of –3.5°C

Density (ρ_a) = 1.267 kg/m^3

Thermal expansion coefficient (β_a) = 0.00370 1/K

Thermal conductivity (k_a) = 0.0235 W/(m K)

Kinematic viscosity (ν_a) = 13.6 × 10^{-6} m^2/s

Prandtl number (Pr_a) = 0.71

From Appendix 2, Table 13, for water at the estimated film temperature of 3.5°C

Density (ρ_w) = 1000 kg/m^3

Thermal expansion coefficient (β_w) = –0.12 × 10^{-4} 1/K

Thermal conductivity (k_w) = 0.565 W/(m K)

Kinematic viscosity (ν_w) = 1.611 × 10^{-6} m^2/s

Prandtl number (Pr_w) = 12.1

SOLUTION

The convective heat transfer coefficient on the air side (h_o) and the water side (h_ε) must be calculated before the analysis of Section 10.6 can be applied.

Water side

The characteristic length for the pond is

$$L = \frac{A_s}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4} = 25 \text{ m}$$

The Rayleigh number based on this length is

$$Ra_L = Gr_L\, Pr = \frac{g\,\beta_w (T_l - T_{fr}) L^3 Pr}{\nu_w^2}$$

$$Ra_L = \frac{(9.8 \text{ m/s}^2)(-0.12\times 10^{-4} \text{ 1/K})(7°\text{C} - 0°\text{C})(25 \text{m})^3 (12.1)}{(1.611\times 10^{-6} \text{ m}^2/\text{s})^2} = -6.12 \times 10^{13}$$

The minus sign indicates that even though we are cooling the water from above, the buoyancy effect is like that for the heating from above case.

Although this is out of its Rayleigh number range, Equation (5.17) will be used, for lack of a more appropriate correlation, to estimate the Nusselt number

$$\overline{Nu}_L = 0.27\, Ra_L^{\frac{1}{4}} = 0.27\,(6.12 \times 10^{13})^{\frac{1}{4}} = 755$$

$$\bar{h}_c = \overline{Nu}_L\, \frac{k}{L} = 755\, \frac{(0.565\ \text{W/(m K)})}{25\,\text{m}} = 17.1\ \text{W/(m}^2\text{K)}$$

Air side

The heat transfer coefficient on the air side (h_o) will depend on the ice surface temperature (T_o) which changes as the ice thickens. The transfer coefficient will be approximated as constant with the surface temperature equal to the freezing temperature. With these simplifications, the Rayleigh number is

$$Ra_L = \frac{\left(9.8\ \text{m/s}^2\right)(0.00370\ 1/\text{K})(7°\text{C})(25\,\text{m})^3(0.71)}{\left(13.6\times10^{-6}\text{m}^2/\text{s}\right)^2} = 1.52 \times 10^{13}$$

Once again, the correlation of Equation (5.16) will be extended to estimate the Nusselt number

$$\overline{Nu}_L = 0.15\, Ra_L^{\frac{1}{3}} = 0.15\,(1.52 \times 10^{13})^{\frac{1}{3}} = 3718$$

$$\bar{h}_o = \overline{Nu}_L\, \frac{k}{L} = 3718\, \frac{(0.0235\,\text{W/(m K)})}{25\,\text{m}} = 3.5\ \text{W/(m}^2\,\text{K)}$$

Applying Equation (10.14)

$$t^+ = -\frac{1}{(R^+T^+)^2}\ \ln\left(1+\frac{R^+T^+\varepsilon^+}{1+R^+T^+}\right) + \frac{\varepsilon^+}{R^+T^+}$$

where

$$\varepsilon^+ = \frac{\bar{h}_o \varepsilon}{k}$$

$$R^+ = \frac{\bar{h}_c}{\bar{h}_o} = \frac{17.1}{3.5} = 4.89$$

$$T^+ = \frac{T_l - T_{fr}}{T_{fr} - T_\infty} = \frac{7-0}{0+7} = 1.0$$

$$t^+ = t\, h_o^2\, \frac{T_{fr} - T_\infty}{\rho L_f k} = (3\text{hr})(3600\ \text{s/h})\ \left(3.5\ \text{W/(m}^2\text{K)}\right)^2$$

$$\frac{(0°\text{C} + 7°\text{C})}{\left(918\ \text{kg/m}^3\right)(333{,}780\ \text{J/kg})((\text{W s})/\text{J})(2.22\ \text{W/(m K)})} = 0.00136$$

$$0.00136 = -\frac{1}{(4.89)^2}\ \ln\left[1+\frac{4.89\,\varepsilon^+}{1+4.89}\right] + \frac{\varepsilon^+}{4.89}$$

By trial and error

$$\varepsilon^+ = 0.008$$

$$\therefore\ \varepsilon = \varepsilon^+\, \frac{k}{\bar{h}_o} = 0.008\ \frac{(2.22\ \text{W/(m K)})}{3.5\ \text{W/(m}^2\text{K)}} = 0.0051\ \text{m} = 5.1\ \text{mm}$$

PROBLEM 10.37

On a rainy Monday afternoon, Sherlock Holmes gets a call from a wealthy banker to arrange a breakfast appointment for the following day to discuss the collection of a loan from farmer Joe. When Holmes arrives at the home of the banker at 9 a.m. Tuesday, he finds the body of the banker in his kitchen. The farmer's house is located on the other side of a lake, approximately 10 km from the banker's home. Since there is no convenient road between the home of the farmer and that of the banker, Holmes phones the police to question the farmer. The police arrive at the farmer's home within the hour and interrogate him about the death of the banker. The farmer claims to have been home all night. The tires on his truck were dry and he explains that his boots were moist and soiled because he had been fishing at the lake early in the morning. The police then phone Holmes to eliminate farmer Joe as a murder suspect because he could not have been at the banker's home since Holmes spoke to him. Holmes then calls the local weather bureau and learns that, although the temperature had been between 2°C and 5°C for weeks, it had dropped to –30°C quite suddenly on Monday night. Remembering that a 3 cm layer of ice can support a man, Holmes takes out his slide rule and heat transfer text, lights his pipe, makes a few calculations, and then phones the police to arrest farmer Joe. Why?

GIVEN

- Murder suspect with flakey alibi

FIND

(a) How Sherlock Holmes was able to disprove the alibi

SOLUTION

Holmes has evidently surmised that with the cold snap, the lake could have frozen to a sufficient thickness of 3 cm. to support the farmer on his way to murder the banker. In addition, the frozen lake would have precluded the farmer from fishing that morning. Basically, we need to determine if a 10 km lake, originally at 5°C or cooler, could have frozen to a thickness of 3 cm overnight after exposure to an air temperature of –30°C.

We can use Equation (10.63) to determine the ice thickness as a function of time. The following parameters are given

Liquid temperature, T_1 = 5°C (this is conservative since the range was 2°C to 5°C)

Freezing temperature, T_{fr} = 0°C

Air temperature, T_∞ = –30°C

The properties of ice from Appendix 2, Table 11 are

Heat of fusion, $L_f = 3.3 \times 10^5$ J/kg

Thermal conductivity, k_{ice} = 2.2 W/(mK)

Density, ρ_{ice} = 913 kg/m^3

In addition to the above parameters, we need to calculate the heat transfer coefficient at the air-ice interface, $\bar{h}_o$ and at the ice-water interface $\bar{h}_\varepsilon$. For the air-ice interface, we assume conservatively that there is no wind. Also note that the ice is warmer than the air and faces up, so we can use Equation (5.15) or (5.16) to determine $\bar{h}_o$.

At the mean temperature of (0 + –30)/2 = –15°C, we have for the air properties from Appendix 2, Table 27 (via extrapolation from data at 0°C and 20°C to –15°C)

$g\beta/\nu^2 = 2.22 \times 10^8\ (\text{K m}^3)^{-1}$

Prandtl number, $Pr = 0.71$

thermal conductivity, $k_{\text{air}} = 0.023$ W/(mK)

Further, let us assume that the lake is approximately circular, 10 km in diameter. The length scale required in Equation (5.15) or (5.16) is

$$L = \frac{\text{area of lake}}{\text{perimeter of lake}} = \frac{\dfrac{\pi(10^4\,\text{m})^2}{4}}{\pi 10^4\ \text{m}} = 2500\ \text{m}$$

So, the Rayleigh number is

$$Ra_L = \frac{g\,\beta \Delta T\,L^3 Pr}{\nu^2} = (2.22 \times 10^8\ (\text{m}^3\ \text{K})^{-1})\ (30\text{K})(2500\text{m})^3(0.71) = 7.38 \times 10^{19}$$

which is well beyond the restriction on Equation (5.16). In lieu of a correlation equation for such a large Rayleigh number, we will use Equation (5.16) and take note of the assumption.

Then the mean Nusselt number is

$$\overline{Nu}_L = 0.15\,Ra_L^{\frac{1}{3}} = 6.28 \times 10^5$$

and the heat transfer coefficient is

$$\bar{h}_o = \frac{k_{\text{air}}}{L}\,\overline{Nu}_L = \frac{(0.023\,\text{W/(m K)})(6.28\times 10^5)}{(2500\,\text{m})} = 5.8\ \text{W/(m}^2\text{K)}$$

Equation (5.16) shows that the heat transfer coefficient is independent of L as long as we are in the turbulent regime. That is, $\bar{h}_o$ = 5.8 W/(m^{2}K) regardless of the Rayleigh number in the turbulent regime and we expect that this value of $\bar{h}_o$ would be a reasonable estimate for the 10 km lake surface.

At the bottom surface of the ice, we have a cooled surface facing down, therefore, the same equations apply. For water at the mean temperature of(0 + 5)/2 = 2.5°C, we have from Appendix 2, Table 13

$g\beta/\nu^2 = 0.551 \times 10^9\ (\text{m}^3\ \text{K})^{-1}$ (from 10°C data)

Prandtl number, $Pr = 12.6$

thermal conductivity, $k_{\text{water}} = 0.563$ W/(mK)

so

$$Ra_L = \frac{g\,\beta \Delta T\,L^3 Pr}{\nu^2} = (0.551 \times 10^9 (\text{m}^3\ \text{K})^{-1})\ (5\text{K})(2500\text{m})^3(12.6) = 5.42 \times 10^{20}$$

The Nusselt number is

$$Nu_L = 0.15\ Ra_L^{\frac{1}{3}} = 1.22 \times 10^6$$

and the heat transfer coefficient is

$$\bar{h}_c = \frac{k_{\text{water}}}{L}\,\overline{Nu}_L = \frac{(0.563\,\text{W/(m K)})(1.22\times 10^6)}{(2500\,\text{m})} = 275\ \text{W/(m}^2\text{K)}$$

The same comments on $\bar{h}_o$ apply to $\bar{h}_\varepsilon$.

Now, to use Equation (10.63) we need the following parameters

$$R^+ = \frac{\bar{h}_c}{\bar{h}_o} = \frac{275}{2.8} = 47.4$$

$$T^+ = \frac{T_l - T_{fr}}{T_{fr} - T_\infty} = \frac{5-0}{0-(-30)} = 0.166$$

$$R^+T^+ = (47.4)(0.166) = 7.89$$

To calculate how long is needed to freeze the 3 cm layer, we have $\varepsilon = 0.03$ m, so

$$\varepsilon^+ = \frac{\bar{h}_o \varepsilon}{k_{\text{ice}}} = \frac{\left(5.8\ \text{W/(m}^2\text{K)}\right)(0.03\,\text{m})}{(2.2\ \text{W/(m K)})} = 0.079$$

Equation (10.63) then given for the generalized time

$$t^+ = -\frac{1}{7.89^2}\ \ln\left(1+\frac{(7.89)(0.079)}{1+7.89}\right) + \frac{0.079}{7.89} = 0.0089$$

The definition of the generalized time is

$$t^+ = t\bar{h}_o^2\,\frac{T_{fr} - T_\infty}{\rho_{\text{ice}} L_f\ k_{\text{ice}}} = 0.0089$$

Solving for the dimensional time we have
or

$$t = \frac{(0.0089)\left(913\ \text{kg/m}^3\right)\left(3.3\times 10^5\right)((\text{W s})/\text{kg})(2.2\ \text{W/(m K)})}{\left(5.8\ \text{W/(m}^2\text{K)}\right)^2(0-(-30))(\text{K})} = 5861\ \text{s} = 1.63\ \text{h}$$

Therefore, less than 2 hours would be required for the lake to freeze and this is the information that led Sherlock Holmes to ask for the arrest of Farmer Joe.

PROBLEM 10.38

Estimate the cross-sectional area required for a 30 cm long methanol-nickel heat pipe to transport 30 W at atmospheric pressure.

GIVEN

- Methanol-nickel heat pipe
- 30 cm long
- Atmospheric pressure

FIND

(a) Cross-sectional area required to transport 30 W

ASSUMPTIONS

- The type of wick to be used is a threaded artery wick
- 100°C operation

SOLUTION

Table 10.6 gives

$$q''_{\text{axial}} = 0.45\ \text{W/cm}^2 = 0.45\times 10^4\ \text{W/m}^2$$

Since the total heat transported by the heat pipe is

$$q = q''_{\text{axial}} A$$

where A is the desired cross-sectional area, we have

$$A = \frac{q}{q''_{\text{axial}}}$$

$$\Rightarrow \qquad A = \frac{30\,\text{W}}{0.45 \times 10^4\ \text{W/m}^2} = 66.7 \times 10^{-4}\ \text{m}^2 = 66.7\ \text{cm}^2$$

The heat pipe diameter is

$$D_o = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 66.7\,\text{cm}^2}{\pi}} = 9.21\ \text{cm}$$

$$\Rightarrow \qquad D_o = 9.21\ \text{cm}$$

PROBLEM 10.39

> **Design a heat pipe cooling system for a spherical satellite that dissipates 5000 W/m^3, has a surface area of 5 m^2, and cannot exceed a temperature of 120°C. All the heat must be dissipated by radiation into space. State all your assumptions.**

GIVEN

- Spherical satellite, 5 m^2 surface area
- Dissipates 5000 W/m^3
- Maximum temperature of 102°C
- All heat rejection is by radiation to space

FIND

(a) A design for a heat pipe cooling system

ASSUMPTIONS

- Temperature drop between the satellite interior and the heat pipe evaporator is < 20°C
- Neglect vapor pressure drop in the heat pipe

SOLUTION

Since the satellite has a 5 m^2 surface area, the radius of the spherical satellite is

$$4\pi r_s^2 = 5\text{m}^2 \Rightarrow r_s = 0.631\text{m}^3$$

from which the satellite volume is

$$V_{\text{satellite}} = \frac{4}{3}\pi r_s^3 = 1.05\text{m}^3$$

and the total power dissipated by the satellite, and therefore by the heat pipe cooling system, is

$$q = \left(5000\ \text{W/m}^3\right)(1.05\text{m}^3) = 5260\ \text{W}$$

Since we have assumed that the temperature drop between the satellite interior and the heat pipe evaporator is less than 20°C, we can safely operate the heat pipe at 100°C.

From Figure 10.23, water has the highest figure of merit, M, at the desired temperature of 373 K and it should operate satisfactorily at the desired temperature. From that figure we find

$$M = \frac{\sigma_l\,\rho_l\,h_{fg}}{\mu_l} = 4 \times 10^4\ \text{kW/cm}^2$$

Since we have neglected vapor neglected vapor pressure drop, and since there is no gravitational head, Equation (10.40) simplifies to

$$\frac{2\sigma_l \cos\theta}{r_c} = \frac{\mu_l L_{\text{eff}}\, q}{\rho_l K_w A_w h_{fg}}$$

Let's try a 200 mesh nickel wick. Table 38 in Appendix 2 gives the pore size, r_c = 0.004 cm and the wick permeability, $K_w = 0.62 \times 10^{-10}$ m^2. Let us also assume perfect wetting of the wick by the water, giving $\theta = 0$.

Rearranging the previous equation to solve for the geometry of the heat pipe

$$\frac{L_{\text{eff}}}{A_w} = \frac{\sigma_l \rho_l h_{fg}}{\mu_l} \frac{2}{r_c} \frac{K_w}{q} = M \frac{2}{r_c} \frac{K_w}{q}$$

$$\frac{L_{\text{eff}}}{A_w} = \left(4 \times 10^4 \times 10^3 \text{ W/cm}^2\right) \left(\frac{2}{0.004 \text{cm}}\right) \frac{(0.62 \times 10^{-10} \text{m}^2)}{5260 \text{ W}} \frac{10^4 \text{cm}^2}{\text{m}^2} = 2.4 \text{ cm}^{-1}$$

For a reasonably sized heat pipe, let L_{eff} = 30 cm. Then the total wick cross-sectional area required is A_w = 30 cm/ 2.4 cm^{-1} = 12.5 cm^2. Assuming that we need N pipe of diameter D and thickness t, we have

$$12.5 \text{ cm}^2 = \pi D t N$$

If the exterior surface of the heat pipe condenser section is black, the following equation gives the required surface area of the condenser section

$$q = A_{\text{cond}}\, \sigma (T^4_{\text{cond}} - T^4_{\text{space}})$$

This equation assumes that the heat pipes are separated sufficiently that they all have a near-unity shape factor with respect to space.

Using T_{cond} = 100°C = 373 K and assuming T_{space} = 0 K and we can solve for the condenser surface area as follows

$$A_{\text{cond}} = \frac{5260 \text{ W}}{\left(5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4)\right)(373 \text{ K})^4} = 4.79 \text{ m}^2$$

If the condenser length is L_{cond}, then

$$N\pi D L_{\text{cond}} = 4.79 \text{ m}^2$$

Let L_{cond} = 10 cm giving $N\pi D$ = 47.9 m = 4790 cm. Since we found previously that πDtN = 12.5 cm^2, we can solve for the wick thickness t

$$t = \frac{12.5 \text{cm}^2}{4790 \text{cm}} = 0.0026 \text{ cm}$$

Choose a pipe diameter of 3 cm, then

$$N = \frac{47.9 \text{ m}}{\pi (0.03 \text{ m})} = 508 \text{ pipes required}$$

Since $L_{\text{eff}} = L + (L_{\text{cond}} + L_{\text{evap}})/2$, letting the evaporator and condenser lengths be the same, 10 cm, we find L = 20 cm and the overall length is 20 + 10 + 10 = 40 cm. We summarize the cooling system below

Pipe diameter = 3 cm

Pipe length = 40 cm

Condenser length = 10 cm

Evaporator length = 10 cm
Adiabatic section length = 20 cm
Wick: 0.0026 cm thickness of 200 mesh nickel powder
Working fluid: water
Pressure: atmospheric
Number of pipes required: 508

PROBLEM 10.40

Compare the axial heat flux achievable by a heat pipe using water as the working fluid with that of a silver rod. Assume that both are 20 cm long, that the temperature difference for the rod from end to end is 100°C and that the heat pipe operates at atmospheric pressure. State your other assumptions.

GIVEN

- Silver rod and a heat pipe, both 20 cm long
- End-to-end temperature difference of 100°C
- Heat pipe operates at atmospheric pressure

FIND

(a) Axial heat flux for both rods

ASSUMPTIONS

- Neglect heat pipe vapor pressure drop
- Horizontal operation for the heat pipe
- Perfect wetting of the heat pipe fluid, $\theta = 0$

PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the conductivity for silver at 63°C is k_{silver} = 424 W/(mK)

SOLUTION

The axial heat flux for the silver rod is

$$q'' = \frac{k\Delta T}{L} = \frac{(424\text{ W/(m K)})(100\text{ K})}{0.2\text{ m}} = 212{,}000 \text{ W/m}^2$$

With the above assumptions, Equation (5.40) simplifies to

$$\frac{2\sigma_l}{r_c} = \frac{\mu_l L_{\text{eff}}\, q}{\rho_l K_w A_w h_{fg}}$$

At the operating conditions, water is a satisfactory working fluid. From Figure 10.23, for an operating temperature near 100°C, we have for the figure of merit, M

$$M = \frac{\rho_l \sigma_l h_{fg}}{\mu_l} = 4 \times 10^4 \text{ kW/cm}^2$$

Rearranging Equation (5.40) to find the heat transport

$$q = M\left(\frac{2K_w A_w}{r_c L_{\text{eff}}}\right)$$

We have to make some assumptions about the wick material and thickness. Let's try 120 mesh nickel, with thickness t = 0.01 cm. Table 38 gives for the wick pore radius r_c = 0.019 cm and for the wick permeability, $K_w = 3.5 \times 10^{-10}$ m^2. We must also assume a length for the condenser and evaporator sections. Since the total length is 20 cm, a reasonable length for the condenser and evaporator is 8 cm. Then L = 20 – 8 – 8 = 4 cm and

$$L_{\text{eff}} = L + (L_{\text{cond}} + L_{\text{evap}})/2 = 12 \text{ cm}$$

Unlike the silver rod, we must select a diameter for the heat pipe, Let's try D = 2 cm giving a total corss-sectional area of $A_{\text{pipe}} = \pi 2^2/4 = 3.14$ cm^2. The wick cross-sectional area is

$$A_w = \pi D t = (\pi)\,(2\text{cm})\,(0.01\text{ cm}) = 0.0628\text{ cm}^2$$

We can now calculate the heat transport

$$q = \left(4\times10^7\text{ W/cm}^2\right)\frac{(2)(3.5\times10^{-10}\text{m}^2)(0.0628\,\text{cm}^2)}{(0.019\,\text{cm})(12\,\text{cm})}\left(\frac{10^4\text{cm}^2}{\text{m}^2}\right) = 77\text{ W}$$

The heat flux for the heat pipe is therefore

$$q''_{\text{pipe}} = \frac{77\,\text{W}}{3.14\,\text{cm}^2} = 245{,}000\ \text{W/m}^2$$

This is slightly more than the heat flux for the silver rod so the performance of the two methods of heat transport seems similar. However, the heat pipe provides two advantages: (i) it will undoubtedly cost much less, and (ii) it is isothermal, that is, the temperature difference from one end to the other will be very small. This means that we can afford some temperature drop at either end of the heat pipe to get the heat from the heat source into the evaporator section and out of the condenser section to the heat sink.