An Introduction to Classical Real Analysis

Karl R. Stromberg

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*Sections marked with a single asterisk are not actually needed in conjunction with any of the unmarked sections. These sections may be safely omitted if time permits only a short course.

**Sections marked with a double asterisk are not actually needed in conjunction with any other section. They are included as interesting and useful applications of the theory.

PREFACE

This volume has evolved from lectures that I have given at the University of Oregon and at Kansas State University during the past twenty years. The subject is classical analysis. It is "real analysis" in the sense that none of the Cauchy theory of analytic functions is discussed. Complex numbers, however, do appear throughout. Infinite series and products are discussed in the setting of complex numbers. The elementary functions are defined as functions of a complex variable. I do depart from the classical theme in Chapter 3, where limits and continuity are presented in the contexts of abstract topological and metric spaces.

The approach here is to begin with the axioms for a complete ordered field as the definition of the real number system. Based only upon that, an uncompromisingly rigorous Definition-Theorem-Proof style is followed to completely justify all else that is said. For better or for worse, I have scrupulously avoided any presumption at all that the reader has any knowledge of mathematical concepts until they are formally presented here. Thus, for example, the number π is not mentioned until it has been precisely defined in Chapter 5.

I hope that this book will be found useful as a text for the sort of courses in analysis that are normally given nowadays in most American universities to advanced undergraduate and beginning graduate students. I have included every topic that I deem necessary as a preparation for learning complex and abstract analysis. I have also included a selection of optional topics. The table of contents is a brief guide to the topics included and to which ones may be safely omitted without disturbing the logical continuity of the presentation. I also hope that this book will be found useful as a reference tool for mature mathematicians and other scientific workers.

One significant way in which this book differs from other texts at this level is that the integral which we first mention is the Lebesgue integral on the real line. There are at least three good reasons for doing this. First, the F. Riesz approach (after which mine is modelled) is no more difficult to understand than is the traditional theory of the Riemann integral as it currently appears in nearly every calculus book. Second, I feel that students profit from acquiring a thorough understanding of Lebesgue integration on Euclidean spaces before they enter into a study of abstract measure theory. Third, this is the integral that is most useful to current applied mathematicians and theoretical scientists whether or not they ever study abstract mathematics. Of course, it is clearly shown in Chapter 6 how the Riemann integral is a special case of the Lebesgue integral. Stieltjes integration is presented in a graded sequence of exercises. The proofs of these exercises are easy, but any instructor who wishes to include them in his lectures is obviously free to do so.

I sincerely hope that the exercise sets will prove to be a particularly attractive feature of this book. I spent at least three times as much effort in preparing them as I did on the main text itself. Most of the exercises take the form of simple assertions. The exercise is to prove the assertion. A great many of the exercises are projects of many parts which, when completed in the order given, lead the student by easy stages to important and interesting results. Many of the exercises are supplied with copious hints. I feel that the only way to truly learn mathematics is by just plain hard work. It does not suffice to simply read through a book and agree with the author. I do encourage all serious students to work diligently through the exercises provided here. Thomas Edison's dictum that genius is ten percent inspiration and ninety percent perspiration has never been truer than it is here.

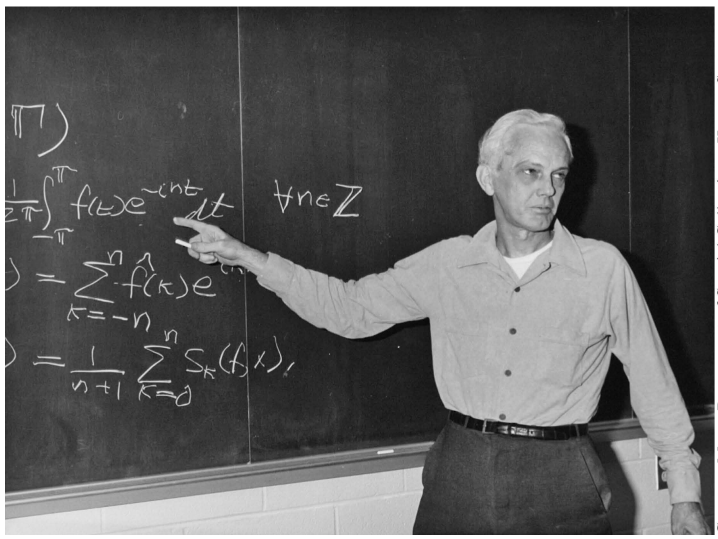
I have found that for a two semester (or three quarter) course, it is easy to cover all the sections in Chapters 1 through 7 that are not marked with asterisks in the Table of Contents. I also find time to include some of the optional sections or part of Chapter 8. In doing this, I make it a practice of assigning a lot of the easier textual material as reading for the students, while I work through many of the harder exercises in class. I see no point in copying the text onto the blackboard.

If it is only possible to spend one semester (about fifteen weeks) on classical real analysis, then one can proceed as follows. Assign all of Chapter 0 and much of Chapter 1 as reading. Omit all sections which bear asterisks in the Table of Contents. Spend only one week on each of Chapters 1, 5, and 7 and only three weeks on each of Chapters 2, 3, 4, and 6 by making the following additional omissions. In Chapter 3, proceed only through "Uniform Convergence," omitting "Baire Category." In Chapter 4 omit "Differentiability Almost Everywhere." In Chapter 6 stop with "The Riemann Integral," but be sure to work through many of the exercises at the end of that section. In Chapter 7, stop with "Some Theorems of Abel."

I take great pleasure in offering my warmest thanks to my good friends Bob Burckel and Louis Pigno who gave me such valuable assistance in preparing this book through their constant encouragement, their proofreading and their many stimulating conversations. I also thank the four women who valiantly typed the technically complicated manuscript. They are Twila Peck, Judy Bernhart, Marie Davis, and Marlyn Logan. Finally, it is a pleasure to thank my publishers and editors John Martindale, Arthur Weber, Paul Prindle, John Kimmel, and David Foss for their excellent help and for their patience and understanding through this seemingly interminable project.

Karl Stromberg
Manhattan, Kansas
July, 1980

About the Author



Courtesy Morse Department of Special Collections, Kansas State University Libraries, Photo Collection—People, "Stromberg, Karl".

Karl Stromberg (1 December 1931 — 3 July 1994) received his Ph.D. in mathematics from the University of Washington under the direction of Edwin Hewitt in 1958. After postdoctoral years at Yale University and the University of Chicago, he served on the faculty of the University of Oregon from 1960 until 1968. In 1966, he was given the Ersted Award as the outstanding teacher at the University of Oregon. From 1968 on he was Professor of Mathematics at Kansas State University where he received the William L. Stamey Award for exceptional teaching in 1990. He lectured at many universities in the United States and Europe; he spent 1966-7 at Uppsala University (Sweden) and 1974-5 at the University of York (England) as Visiting Professor. He also served on the National Research Council in the United States. In addition to his many research papers in mathematical analysis (see "Other Work by the Author" in the backmatter of this book), he wrote the well-known text *Real and Abstract Analysis* together with Edwin Hewitt. His last major work was a high-level text, *Probability for Analysts*, published in 1994.

The absence of figures and the few typographical imperfections in this present book should be attributed to the fact that for his whole professional life the author was virtually (and, indeed, legally) blind.

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