An Introduction to Orthogonal Frequency Division Multiplexing

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Abstract: Orthogonal Frequency Division Multiplexing (OFDM) is one of the latest modulation techniques used in order to combat the frequency-selectivity of the transmission channels, achieving high data rate without inter–symbol interference. The basic principle of OFDM is gaining a wide spread popularity within the wireless transmission community. Furthermore, OFDM is one of the main techniques proposed to be employed in 4^{th} Generation Wireless Systems. Therefore, it is crucial to understand the concepts behind OFDM. In this paper it is given an overview of the basic principles on which this modulation scheme is based.

1. Introduction

Due to the spectacular growth of the wireless services and demands during the last years, the need of a modulation technique that could transmit high data rates at high bandwidth efficiency strongly imposed. The problem of the inter–symbol interference (ISI) introduced by the frequency selectivity of the channel became even more imperative once the desired transmission rates dramatically grew up.

Using adaptive equalization techniques at the receiver in order to combat the ISI effects could be the solution, but there are practical difficulties in operating this equalization in real-time conditions at several Mb/s with compact, low-cost hardware. OFDM is a promising candidate that eliminates the need of very complex equalization.

In a conventional serial data system, the symbols are transmitted sequentially, one by one, with the frequency spectrum of each data symbol allowed to occupy the entire available bandwidth. A high rate data transmission supposes a very short symbol duration, conducing at a large spectrum of the modulation symbol. There are good chances that the frequency selective channel response affects in a very distinctive manner the different spectral components of the data symbol, hence introducing the ISI [1]. The same phenomenon, regarded in the time domain consists in smearing and spreading of information symbols such, the energy from one symbol interfering with the energy of the next ones, in such a way that the received signal has a high probability of being incorrectly interpreted.

Intuitively, one can assume that the frequency selectivity of the channel can be mitigated if, instead of transmitting a single high rate data stream, we transmit the data simultaneously, on several narrow-band subchannels (with a different carrier corresponding to each subchannel), on which the frequency response of the channel looks "flat" (see fig. 1). Hence, for a given overall data rate, increasing the number of carriers reduces the data rate that each individual carrier must convey, therefore lengthening the symbol duration on each subcarrier. Slow data rate (and long symbol

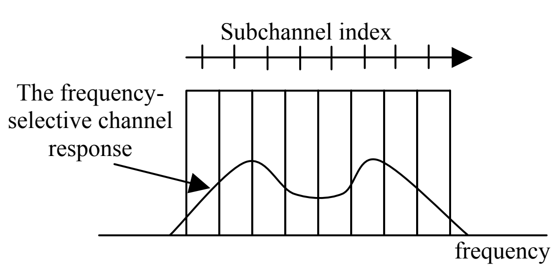

Fig. 1: The frequency-selective channel response and the relatively flat response on each subchannel

duration) on each subchannel merely means that the effects of ISI are severely reduced.

This is in fact the basic idea that lies behind OFDM. Transmitting the data among a large number of closely spaced subcarriers accounts for the "frequency division multiplexing" part of the name. Unlike the classical frequency division multiplexing technique, OFDM will provide much higher bandwidth efficiency. This is due to the fact that in OFDM the spectra of individual subcarriers are allowed to overlap. In fact, the carriers are carefully chosen to be orthogonal one another. As it is well known, the orthogonal signals do not interfere, and they can be separated at the receiver by correlation techniques. The orthogonality of the subcarriers accounts for the first part of the OFDM name.

2. The block diagram of an OFDM system

In figure 2, a classical OFDM transmission scheme using FFT (Fast Fourier Transform) is presented:

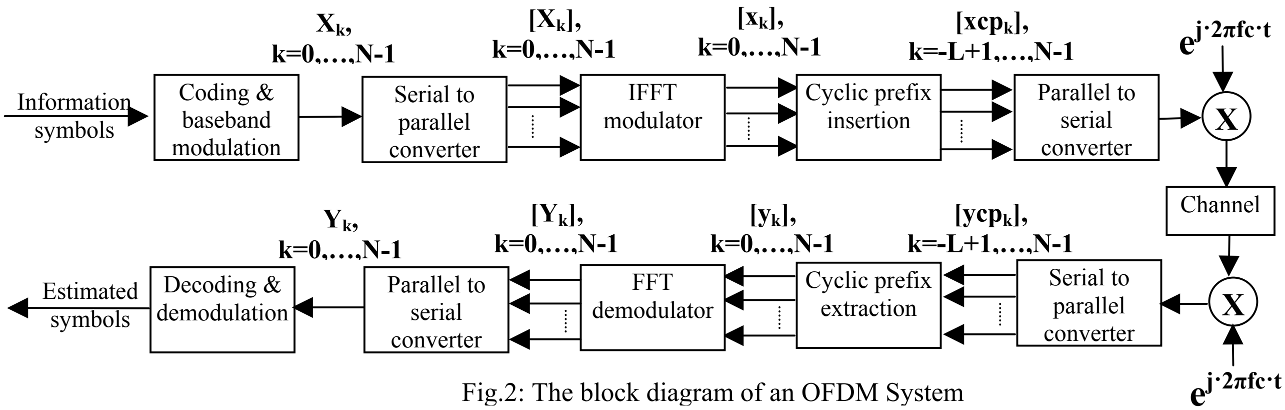

Fig.2: The block diagram of an OFDM System

The input data sequence is baseband modulated, using a digital modulation scheme. Various modulation schemes could be employed such as BPSK, QPSK (also with their differential form) and QAM with several different signal constellations. There are also forms of OFDM where a distinct modulation on each subchannel is performed (e.g. transmitting more bits using an adequate modulation method on the carriers that are more „confident", like in ADSL systems). Also, data can be encoded „in frame" (the baseband signal modulation is performed on the serial data, that is inside of what we name a „DFT frame"), or „inter frame" (the modulation is performed on each parallel substream, that is on the symbols belonging to adjacent DFT frames). The data symbols are parallelized in N different substreams. Each substream will modulate a separate carrier through the IFFT modulation block, which is in fact the key element of an OFDM scheme, as we will see later. A cyclic prefix is inserted in order to eliminate the inter-symbol and inter-block interference (IBI). This cyclic prefix of length L is a circular extension of the IFFT-modulated symbol, obtained by copying the last L samples of the symbol in front of it. The data are back-serial converted, forming an OFDM symbol that will modulate a high-frequency carrier before its transmission through the channel. The radio channel is generally referred as a linear time-variant system.To the receiver, the inverse operations are performed: the data are down-converted to the baseband and the cyclic prefix is removed. The coherent FFT demodulator will ideally retrieve the exact form of transmitted symbols. The data are serial converted and the appropriated demodulation scheme will be used to estimate the transmitted symbols.

3. OFDM Principles

In this section, the key points of OFDM are presented: the principles of a multicarrier (parallel) transmission, the usage of FFT and the cyclic prefix „trick". We will also discuss the main challenges that this technique must deal with.

3.1. The concept of multicarrier (parallel) transmission

In a mobile radio environment, the signal is carried by a large number of paths with different strength and delays. Such multipath dispersion of the signal is commonly referred as „channel-induced ISI"and yields the same kind of ISI distortion caused by an electronic filter [2]. In fact, the multipath dispresion leads to an upper limitation of the transmission rate in order to avoid the frequency selectivity of the channel or the need of a complex adaptive equalization in the receiver. In order to mitigate the time-dispersive nature of the channel, the finding of the multicarrier technique was to replace a single-carrier serial transmission at a high data rate with a number of slower parallel data streams. Each parallel stream will be then used to sequentially modulate a different carrier. By creating N parallel substreams, we will be able to decrease the bandwidth of the modulation symbol by the factor of N, or, in other words, the duration of a modulation symbol is increased by the same factor. The summation of all of the individual subchannel data rates will result in total desired symbol rate, with the drastic reduction of the ISI distortion. The price to pay is of course very important, since the multicarrier transmission seems to act as a frequency multiplexation, which will generate problems in terms of bandwidth efficiency usage. The things go however better than seemed, because in OFDM the carriers are orthogonal to each-other and they are separated by a frequency interval of $\Delta f=1/T$. The frequency spectrum of the adjacent subchannels will overlap one another, but the carriers orthogonality will eliminate in principle the interchannel interference that we feared of.

3.2 The Discrete Fourier Transform (DFT) and the orthogonality of the carriers

As already noted, OFDM transmits a large number of narrowband subchannels. The frequency range between carriers is carefully chosen in order to make them orthogonal one another. In fact, the carriers are separated by an interval of 1/T, where T represents the duration of an OFDM symbol (or, equivalently, the duration of a parallel modulation symbol, as indicated previously). The frequency spectrum of an OFDM transmission is illustrated in figure 3. Each sinc of the frequency spectrum below corresponds to a sinusoidal carrier modulated by a rectangular waveform representing the information symbol. One could easily notice that the frequency spectrum of one carrier exhibits zero-crossing at central frequencies corresponding to all other carriers. At these frequencies, the intercarrier interference is eliminated, although the individual spectra of subcarriers overlap. As it is well known, orthogonal signals can be separated at the receiver by correlation techniques. The receiver

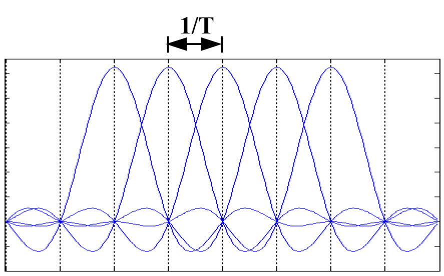

Fig. 3: OFDM transmission spectrum

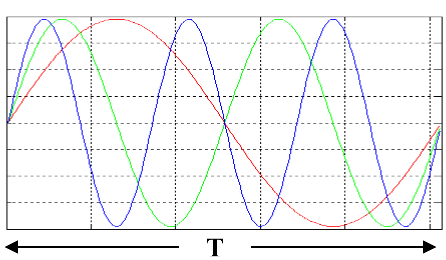

Fig.4: The waveform of the carriers in an OFDM transmission

acts as a bank of demodulators, translating each carrier down to baseband, the resulting signal then being integrated over a symbol period to recover the data. If the other carriers all beat down to frequencies which, in the time domain means an integer number of cycles per symbol period (T), then the integration process results in a zero contribution from all these carriers. The waveform of some carriers in a OFDM transmission is illustrated in figure 4.

Introduced in the 1950, the multicarrier technique was regarded with circumspection. The main reason that hindered the OFDM expansion for a very long time was practical. Indeed, it has seemed difficult to generate such a signal, and even harder to receive and appropriately demodulate him. This technique required a very large array of sinusoidal generators and also a large array of coherent demodulators to make it work. Therefore, the hardware solution proved impractical. At this point, the alternative came as a consequence of the explosive development of digital signal processors (DSP), which can be used for generating and demodulating an OFDM signal, overcoming the complexity issue. The magic idea was to use Fast Fourier Transform (FFT), a modern DSP technique. FFT merely represents a rapid mathematical method for computer applications of Discrete Fourier Transform (DFT). The ability to generate and to demodulate the signal using a software implementation of FFT algorithm is the key of OFDM current popularity. In fact, the signal is generated using the Inverse Fast Fourier Transform (IFFT), the fast implementation of Inverse Discrete Fourier Transform (IDFT). But let's take a closer look to IDFT in order to find the mysterious connection between this transform and the concept of multicarrier modulation. According to its mathematical distribution, IDFT summarizes all sine and cosine waves of amplitudes stored in X[k] array, forming a time domain signal:

$$x[n] = \sum_{k=0}^{N-1} X[k] \cdot e^{j \cdot k \cdot \frac{2\pi}{N} \cdot n} = \sum_{k=0}^{N-1} X[k](\cos(k\frac{2\pi}{N}n) + j \cdot \sin(k\frac{2\pi}{N}n)), \quad n = 0,1,\ldots,N-1 \tag{1}$$

Carefully studying the relation (1), we can simply observe that IDFT takes a series of complex exponential carriers, modulate each of them with a different symbol from the information array X[k], and multiplexes all this to generate N samples of a time domain signal (fig. 5). And what is really important, the complex exponential carriers are orthogonal to each other, as we know from the Fourier decomposition. These carriers are frequency spaced with $\Delta\Omega = 2\pi/N$. If we consider that the N data symbols X[k] come from sampling an analog information with a frequency of f_s, an easy to make discrete to analog frequency conversion indicates a $\Delta f = 1/T$ spacing between the subcarriers of the transmitted signal. The schema presented in figure 6, relies on a classical signal synthesis algorithm. The N samples of the time domain signal are synthesized from sinusoids and cosinusoids of frequencies $k \cdot 2\pi/N$. The „weight" with which each complex exponential contributes to the time domain signal waveform is given by the modulation symbol X[k]. Therefore, the information X[k] to be transmitted could be regarded as being defined in the frequency domain. In its most simplest form, when X[k] stores a binary information („0" and „1"), each symbol to the IDFT entry will simply indicate the presence (an „1") or the absence (a „0") of a certain carrier in the composition of the time domain signal.

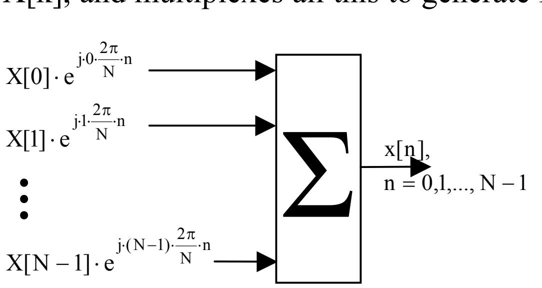

Fig.5: The multicarrier modulation using IDFT

To the receiver, the inverse process is realized: the time domain signal constitutes the input to a DFT „signal analyser”, implemented of course using the FFT algorithm. The FFT demodulator takes the N time domain transmitted samples and determines the amplitudes and phases of sine and cosine waves forming the received signal, according to the equation below:

$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\cdot e^{-j\cdot n\cdot\frac{2\pi}{N}\cdot k} = \frac{1}{N}\sum_{n=0}^{N-1} x[n](\cos(k\frac{2\pi}{N}n) - j\cdot\sin(k\frac{2\pi}{N}n)), \quad k = 0,1,\ldots,N-1 \tag{2}$$

3.3 The cyclic prefix

Since OFDM transmits data in blocks (usually a block is referred to as an „OFDM symbol”) any type of a non-ideal transmission channel (such as a multipath channel, in mobile communications system, or a classical dispersive channel as in wired transmissions) will „spread” the OFDM symbol, causing the blocks of signal to interfere one another. This type of interference is called Inter-Block Interference. IBI will eventually lead to ISI, since two adjacent blocks will overlap, causing the distortion of the symbol affected by overlapping (fig. 6).

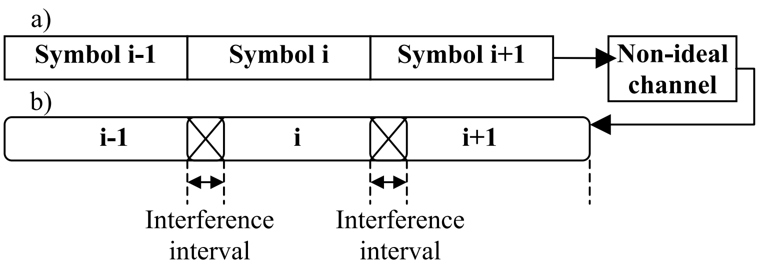

Fig.6: The transmitted symbols frame structure (a) and the received symbols are overlapping during the “interference interval”(b)

In order to combat this interference, one of the possible approaches was to introduce „a silence period” between the transmitted frames. Known as „zero prefix”, this silence period consists in a number of zero samples, added to the front of each symbol. The residual effect of the previous transmitted frame will affect only the „zero padding” portion (if the channel is considered to be linear). These altered samples are discarded to the receiver, and useful samples, unaffected by the IBI are used in order to demodulate the signal. However, the zero-padding doesn’t seem the ideal solution, because the zero prefix will destroy the periodicity of the carriers. The demodulation process that uses FFT will be facilitated by keeping this periodicity, as we will see next.

Instead of this „quiet period” we could use a cyclic prefix (CP) at the beginning of each symbol. The cyclic prefix consists of the last L samples of the OFDM symbol that are copied in front of the data block [3]. If CP duration spans more than the channel impulse response or than the multipath delay, the residual contribution from the previous block is entirely absorbed by the cyclic prefix samples, that are thrown up to the receiver [3][4][5]. The same thing happened if a zero prefix is used, but the CP facilitates the receiver carrier synchronization, since some signals instead of a long silence period are transmitted. Furthermore, using a circular extension maintains the carriers periodicity, which is important in order to simplify the proper reconstruction of the signal using DFT. The beneficent effect of the CP is illustrated in the figure 7. The transmitted OFDM symbols are two cosinusoides, the second being phase shifted with 180^0. If we consider a radio channel, multipath attenuated replica of the first symbol will arrive to the receiver with a certain delay. In figure 7a) the delayed replica of the symbol i-1 will affect the next received symbol for a duration marked with the „interference” label. If a cyclic extension of duration T_g=T/4 is inserted in front of the useful data, the delayed replica of the first symbol, will affect only the CP portion of the second symbol, that is actually discarded to the receiver. If the guard interval is longer than the multipath delay (or, equivalently than the channel impulse response duration), the useful content is not distorted by the delayed replica (fig. 7b). Of course, the price to pay is a decrease of the transmission efficiency with a factor of $T/(T+T_g)$, representing a ratio between the useful and the total transmission time for an OFDM symbol. Speaking in a more „mathematical" language, using the CP, the convolution between the data block and the channel impulse response is transformed into a circular convolution [3,6]. This way, the OFDM symbols preserve their temporal support, and can independently be processed by the receiver. In addition, the channel effect on the transmitted signal can be totally eliminated by a simple „one tap" frequency domain equalizer.

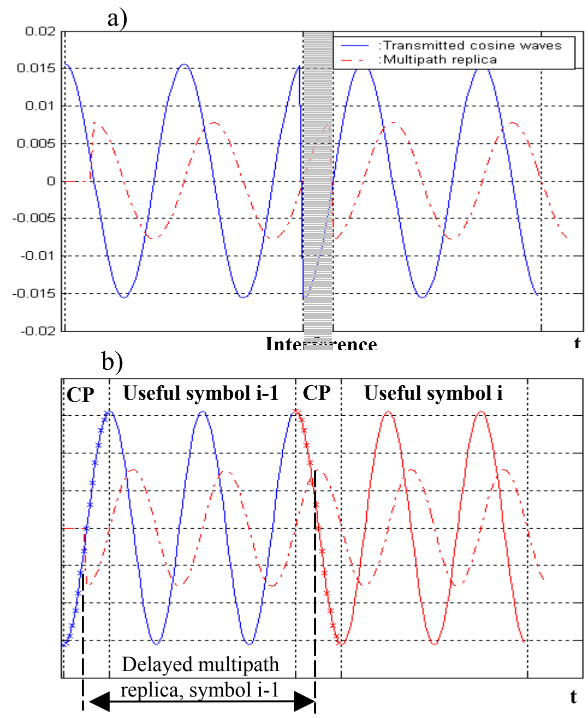

Fig. 7: IBI-two adjacent OFDM symbols interfere(a), the cyclic prefix eliminates this interference (b)

3.4 OFDM Challenges

A pertinent conclusion regarding OFDM performances and capabilities cannot be given without a brief review of the main drawbacks and challenges that this technique must deal with. There are, for example, major practical difficulties to achieve real time synchronization for OFDM frames. On the other hand, the technique is extremely sensitive to the frequency offsets that could cause inter-carrier interference. Also, spectral nulls in the useful transmission band will conduce to severe performance degradation on the affected sub-carriers. OFDM symbols have a high peak-to-average power ratio (PAPR) that makes them unsuitable for RF amplifiers, which could "clip" the signal peaks, hence causing distortion. Finally, but not last, full capabilities of OFDM can be achieved only if the channel impulse response is known, assumption that is not always met; complex channel estimation techniques must be used in order to achieve this need. Present studies in OFDM range focus on these drawbacks and in the finding of the means to overcome them.

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