An Introduction to Proportional-Integral-Derivative (PID) Controllers

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Motivation

- Growing gap between “real world” control problems and the theory for analysis and design of linear control systems
- Design techniques based on linear system theory have difficulties with accommodating nonlinear effects and modeling uncertainties
- Increasing complexity of industrial process as well as household appliances

Effective control strategies are required to achieve high performance for uncertain dynamic systems

Usefulness of PID Controls

- Most useful when a mathematical model of the plant is not available
- Many different PID tuning rules available
- Our sources
  - K. Ogata, *Modern Control Engineering*, Fifth Edition, Prentice Hall, 2010, Chapter 8
  - *IEEE Control Systems Magazine,* Feb. 2006, Special issue on PID control

☞ **Proportional-integral-derivative (PID)** control framework is a method to control uncertain systems

Type A PID Control

- Transfer function of the type A PID controller

$$G_{PID}(s)=\frac{U(s)}{E(s)}=K_p\left(1+\frac{1}{T_i s}+T_d s\right)$$

- The three term control signal,

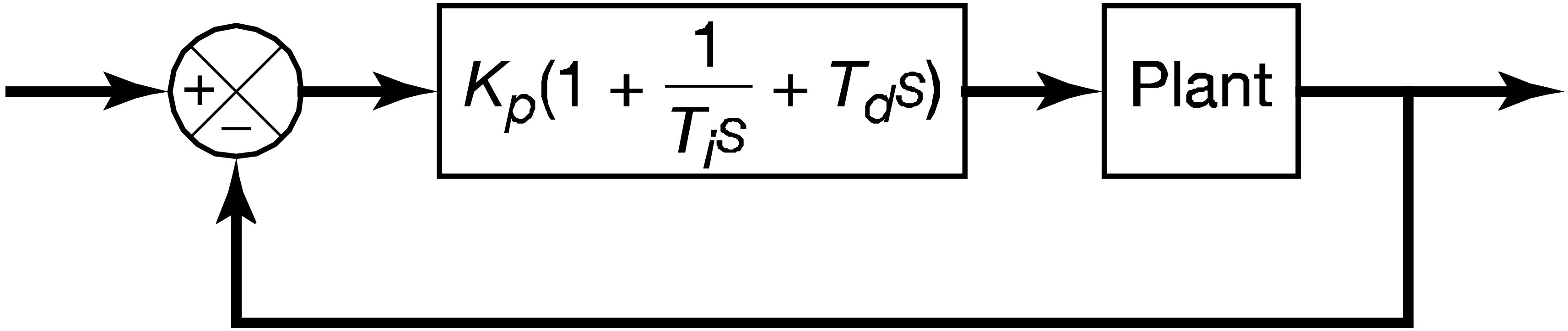
$$U(s)=K_p E(s)+K_i\frac{1}{s}E(s)+K_d sE(s)$$

- In the time domain,

$$u(t)=K_p e(t)+\frac{K_p}{T_i}\int_0^t e(\tau)d\tau+K_p T_d\frac{de(t)}{dt}$$

PID-Controlled System

PID controller in forward path

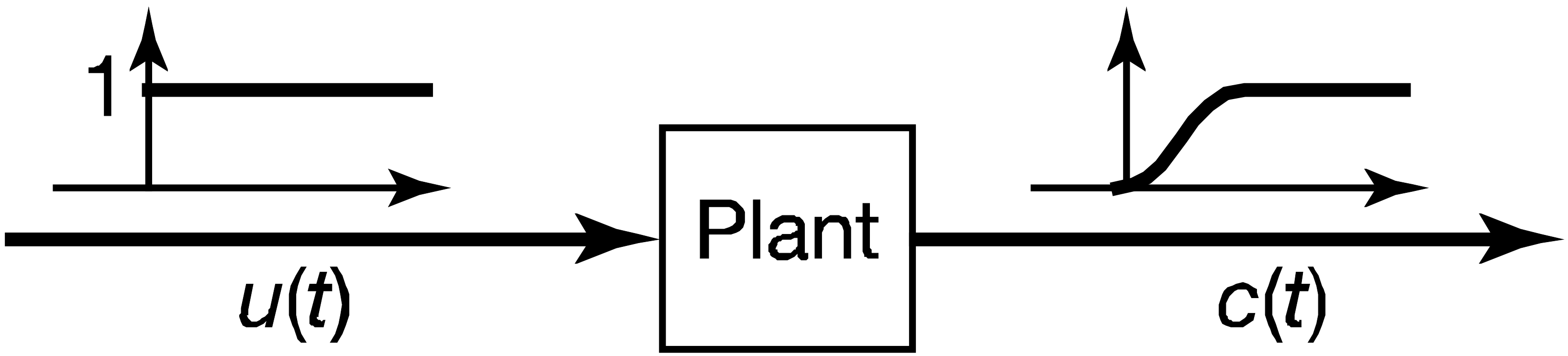


PID Tuning

- Controller tuning---the process of selecting the controller parameters to meet given performance specifications
- PID tuning rules---selecting controller parameter values based on experimental step responses of the controlled plant
- The first PID tuning rules proposed by Ziegler and Nichols in 1942
- The Ziegler-Nichols tuning rules provide a starting point for fine tuning
- Our exposition is based on K. Ogata, *Modern Control Engineering*, Prentice Hall, Fifth Edition, 2010, Chapter 8

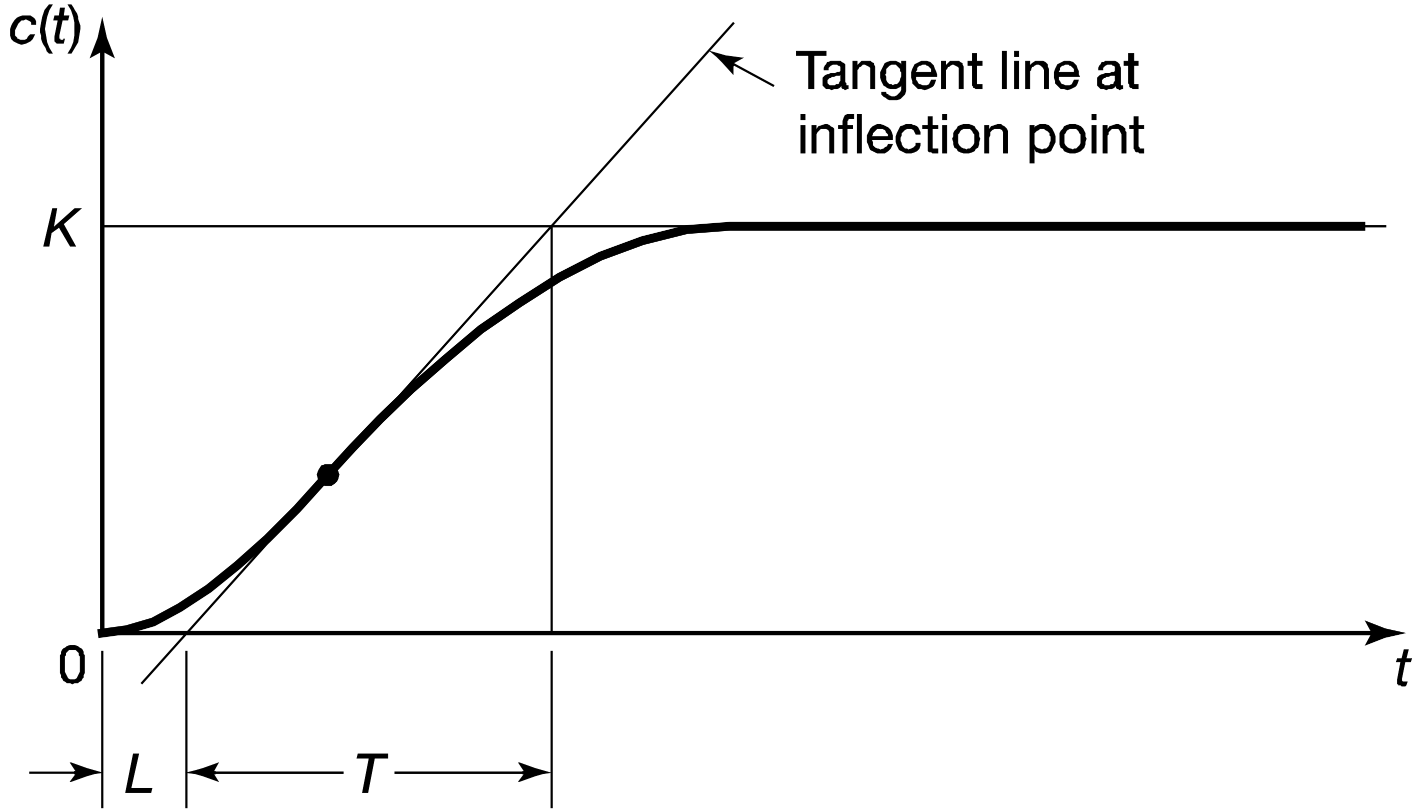
PID Tuning---First method (open-loop method)

Start with obtaining the step response



The S-shaped Step Response

Parameters of the S-shaped step response



Transfer Function of System With S-Shaped Step Response

- The S-shaped curve may be characterized by two parameters: lag (delay) time *L*, and time constant *T*
- The transfer function of such a plant may be approximated by a first-order system with a transport delay

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts+1}$$

PID Tuning---First method (open-loop method)

Table 10–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

Transfer Function of PID Controller Tuned Using the First Method

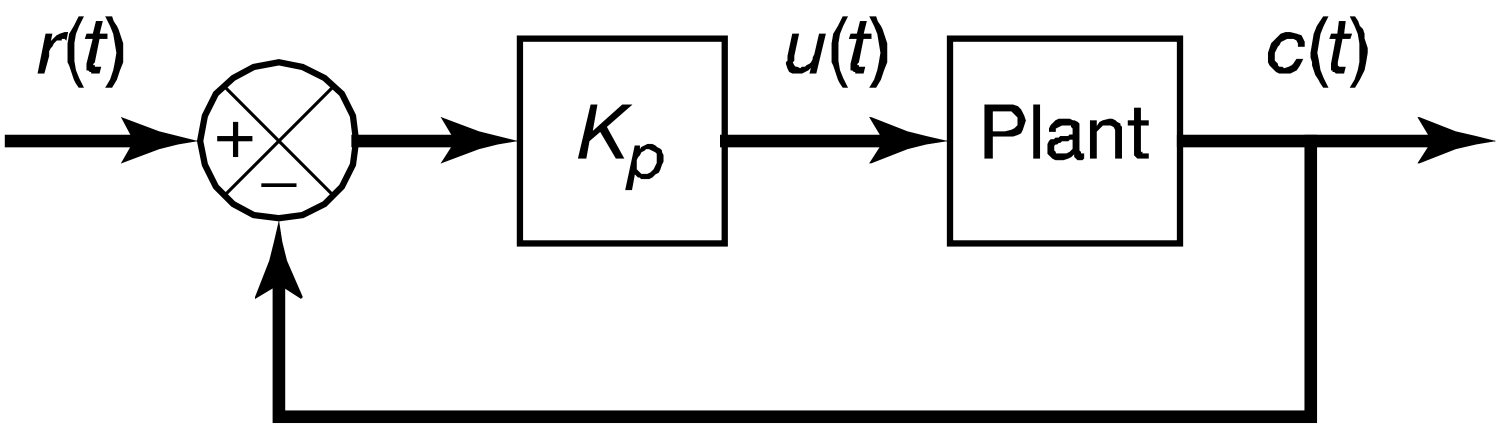
$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right)$$

$$= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s}$$

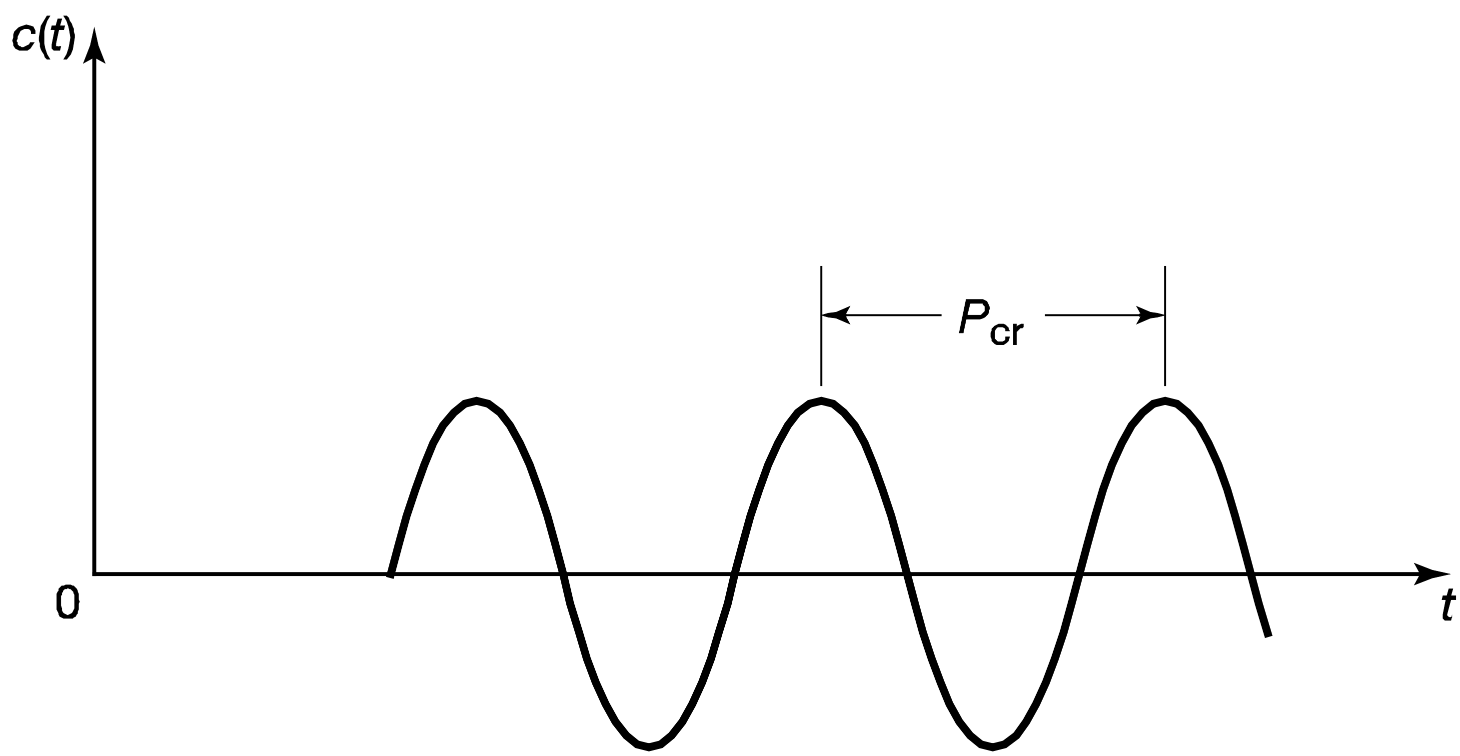
Ziegler-Nichols PID Tuning---Second method (closed-loop method)

Use the proportional controller to force sustained oscillations



PID Tuning---Second method (closed-loop method)

Measure the period of sustained oscillation



PID Tuning Rules---Second method (closed-loop method)

Table 10–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Transfer Function of PID Controller Tuned Using the Second Method

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

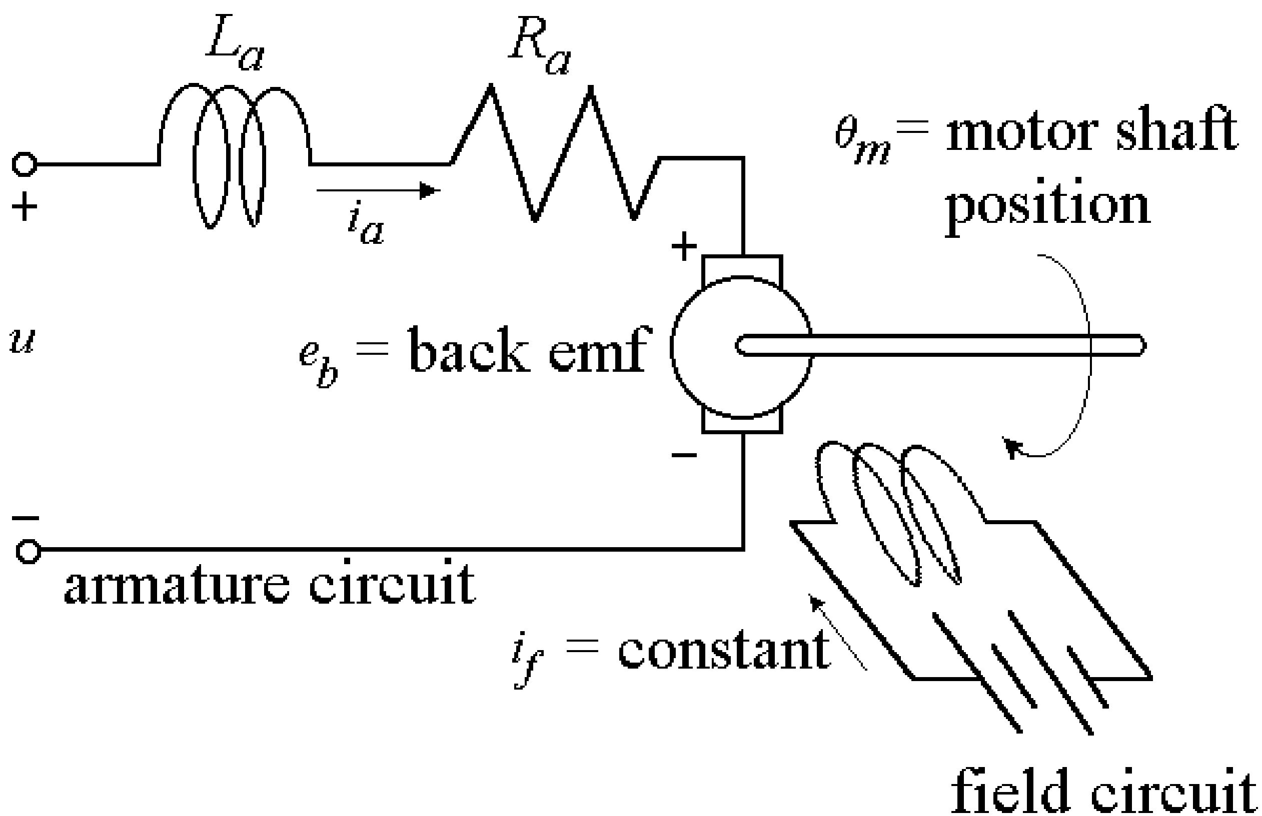
$$= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right)$$

$$= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}$$

Example 1---PID Controller for DC Motor

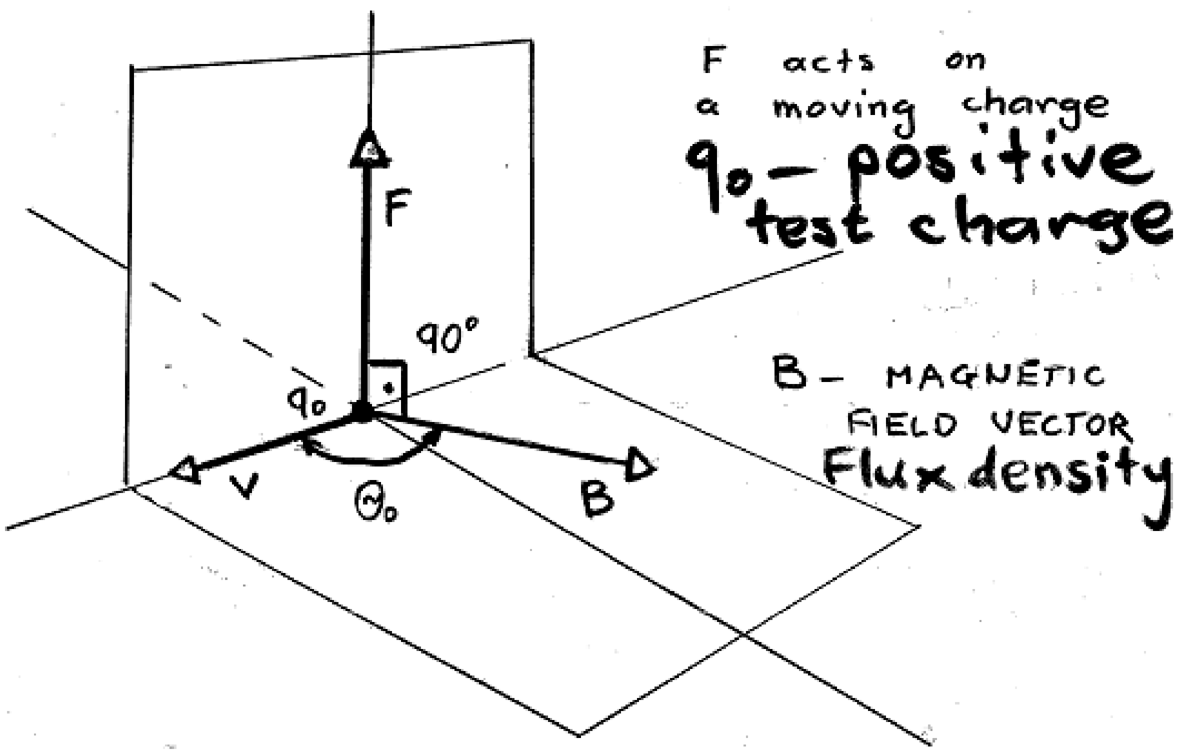
- Plant---Armature-controlled DC motor; MOTOMATIC system produced by Electro-Craft Corporation
- Design a Type A PID controller and simulate the behavior of the closed-loop system; plot the closed-loop system step response
- Fine tune the controller parameters so that the max overshoot is 25% or less

Armature-Controlled DC Motor Modeling



Physics---The Magnetic Field

Oersted (1820): A current in a wire can produce magnetic effects; it can change the orientation of a compass needle



Force Acting on a Moving Charge in a Magnetic Field

- Force

$$\vec{F} = q_0 \vec{v} \times \vec{B}$$

- Magnitude

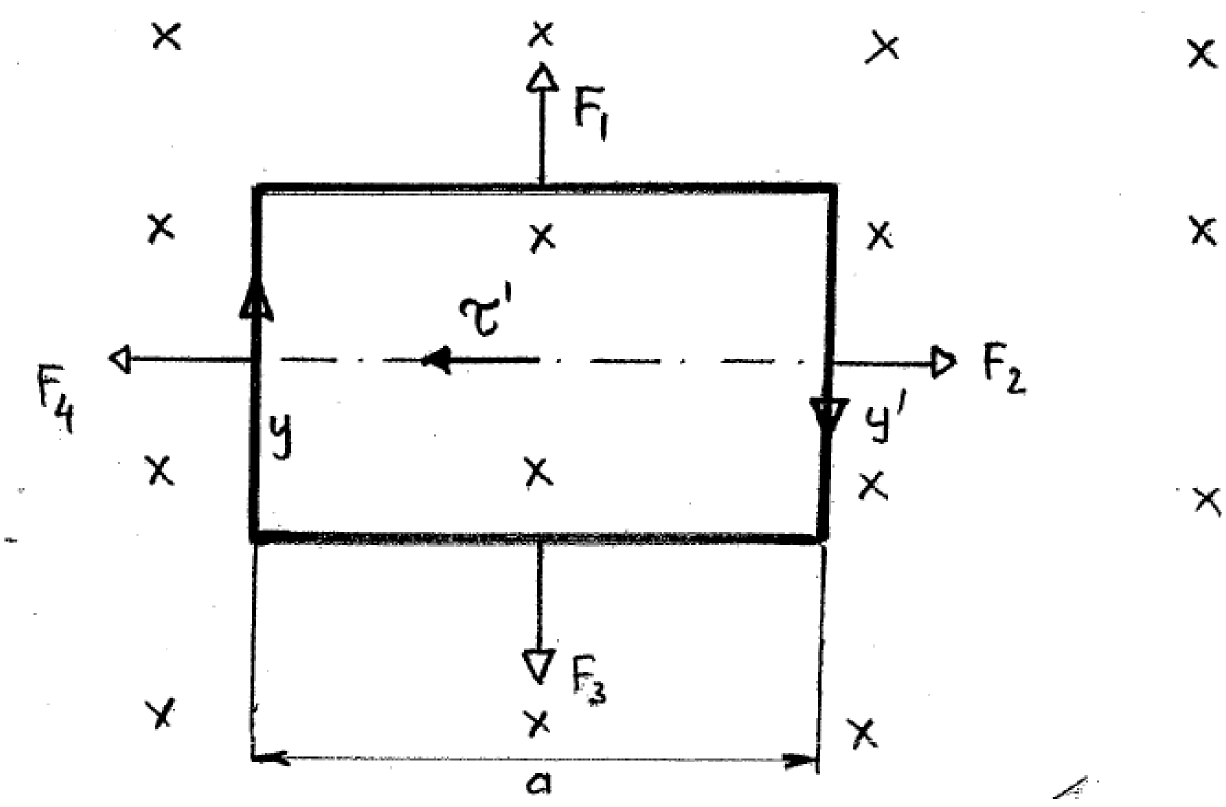
$$F = q_0 v B \sin\theta$$

- The unit of B (flux density)---1Tesla, where

$$1\,\text{Tesla} = \frac{1\,\text{Weber}}{1\,\text{m}^2} = 10^4\,\text{Gauss}$$

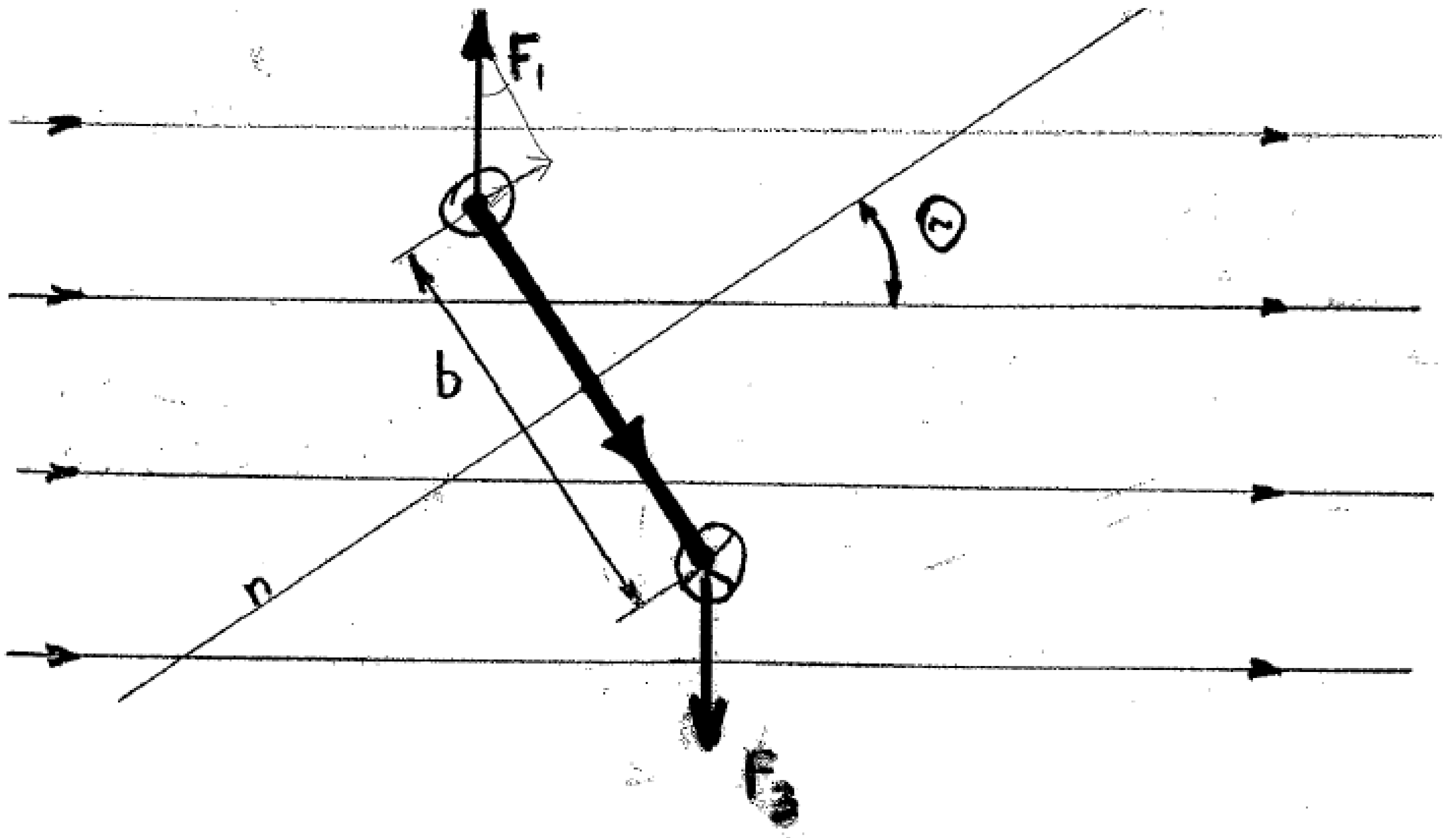
Torque on a Current Loop

The force F_4 has the same magnitude as F_2 but points in the opposite direction

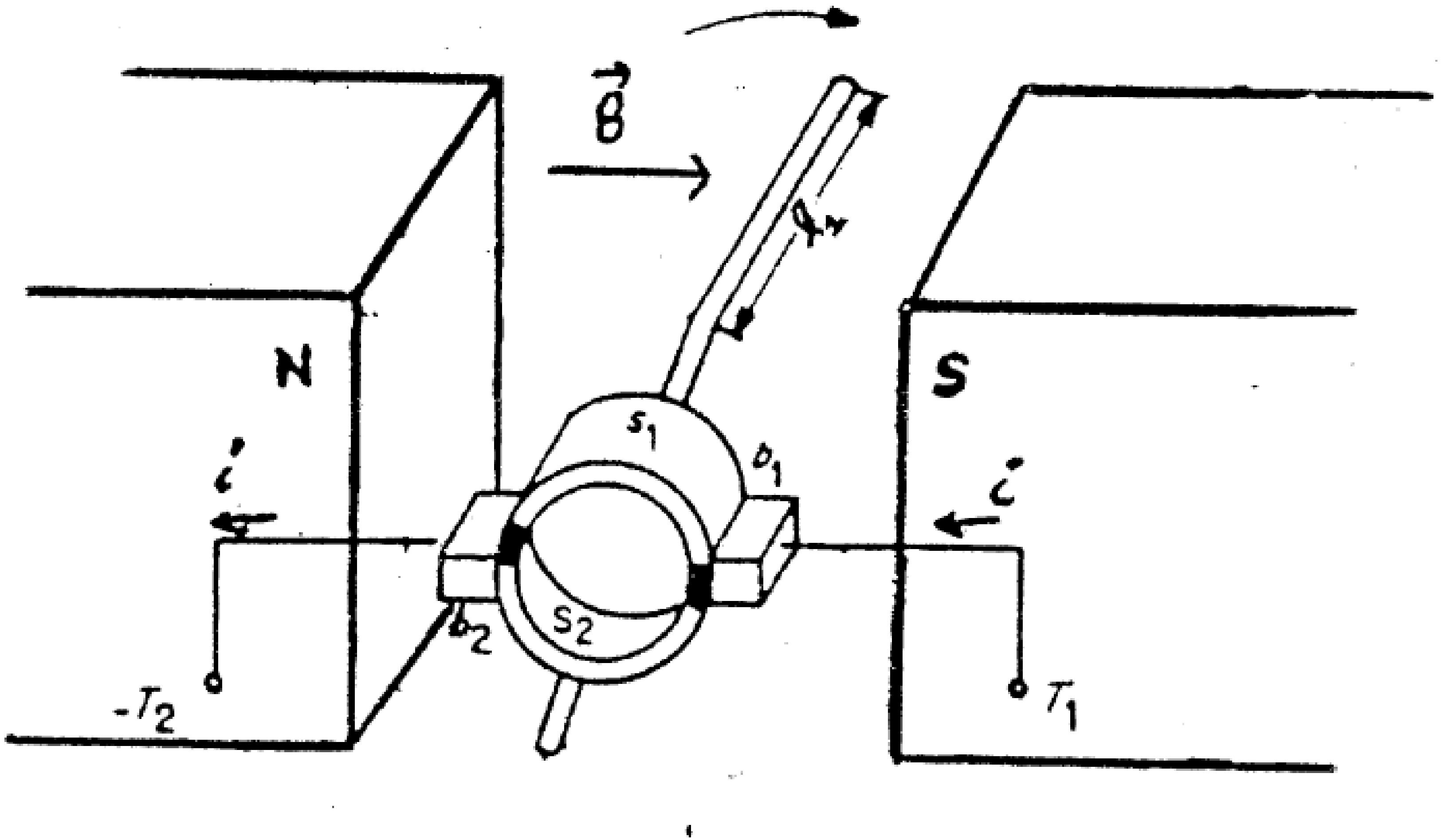


An End View of the Current Loop

The common magnitude of F_1 and F_3 is iaB



Building a Motor From a Current Loop



DC Motor Construction

- To keep the torque in the same direction as the loop rotates, change the direction of the current in the loop---do this using slip rings at 0 and π (pi) or - π
- The brushes are fixed and the slip rings are connected to the current loop with electrical contact made by the loop's slip rings sliding against the brushes

Modeling Equations

- Kirchhoff's Voltage Law to the armature circuit

$$U(s) = (L_a s + R_a) I_a(s) + E_b(s)$$

- Back-emf (equivalent to an ”electrical friction”)

$$E_b(s) = K_b \Omega_m(s)$$

- Torque developed by the motor

$$T_m(s) = (J_m s^2 + B_m s)\Theta_m(s)$$

- Electromechanical coupling

$$T_m(s) = K_t I_a(s)$$

Relationship between K_t and K_b

- Mechanical power developed in the motor armature (in watts)

$$p = e_b(t) i_a(t)$$

- Mechanical power can also be expressed as

$$p = T_m(t) \omega_m(t)$$

- Combine

$$p = T_m \omega_m = e_b i_a = K_b \omega_m \frac{T_m}{K_t}$$

In SI Units $K_t = K_b$

- The back-emf and the motor torque constants are equal in the SI unit system

$$K_t \left(\frac{\mathrm{V}}{\mathrm{rad/sec}} \right) = K_b \left(\mathrm{N \cdot m / A} \right)$$

Transfer Function of the DC Motor System

- Transfer function of the DC motor

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{0.1464}{7.89\times10^{-7}s^3 + 8.25\times10^{-4}s^2 + 0.00172s}$$

where $Y(s)$ is the angular displacement of the motor shaft and $U(s)$ is the armature voltage

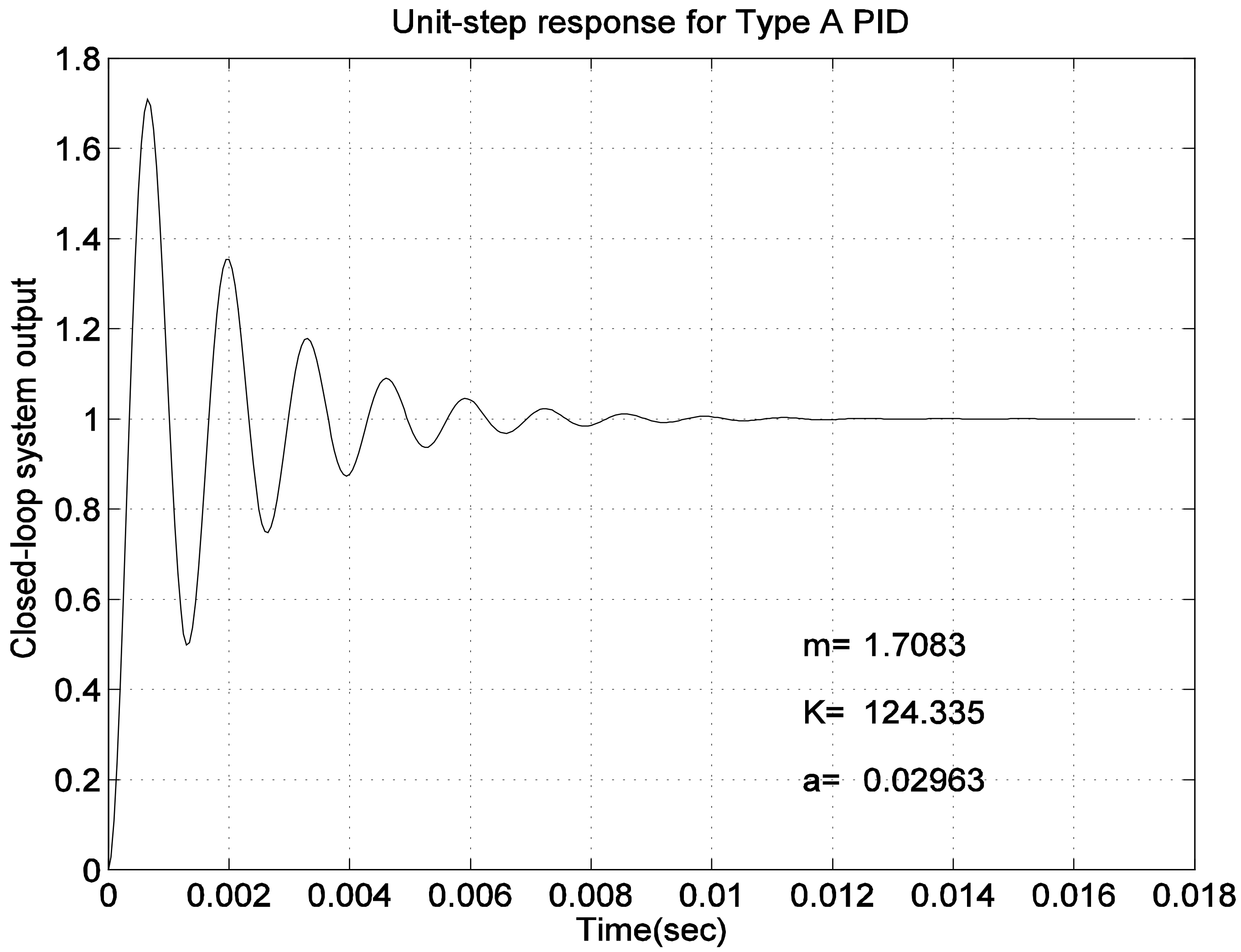
Tuning the Controller Using the Second Method of Ziegler and Nichols

- Use the Routh-Hurwitz stability test; see page 212 of the Text
- Determine K_{cr}
- Determine P_{cr}
- Compute the controller parameters

Generating the Step Response

```
t=0:0.00005:.017;
K_cr=12.28; P_cr=135;
K=0.075*K_cr*P_cr; a=4/P_cr;
num1=K*[1 2*a a^2]; den1=[0 1 0];
tf1=tf(num1,den1);
num2=[0 0 0 0.1464];
den2=[7.89e-007 8.25e-004 0.00172 0];
tf2=tf(num2,den2);
tf3=tf1*tf2;
sys=feedback(tf3,1);
y=step(sys,t); m=max(y);
```

Closed-Loop System Performance



Example 2 (Based on Ex. 10-3 in Ogata, 2002)

- Use a computational approach to generate an optimal set of the DC motor PID controller's parameters

$$G_c(s) = K\frac{(s+a)^2}{s}$$

- Generate the step response of the closed-loop system

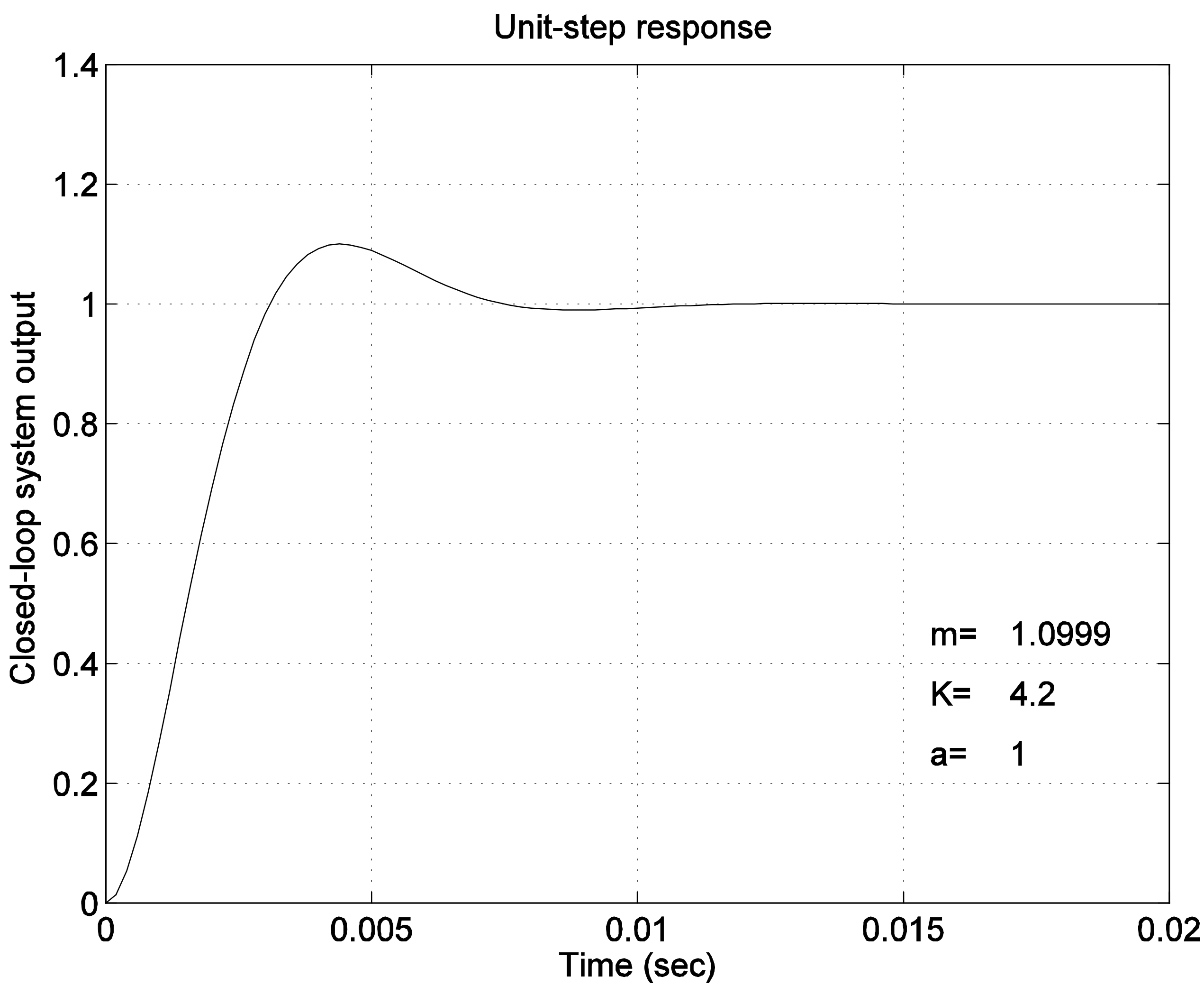
Optimizing PID Parameters

```
t=0:0.0002:0.02;
font=14;
for K=5:-0.2:2%Outer loop to vary the values of
   %the gain K
    for a=1:-0.01:0.01;%Outer loop to vary the
   %values of the parameter a
        num1=K*[1 2*a a^2]; den1=[0 1 0];
        tf1=tf(num1,den1);
        num2=[0 0 0 0.1464];
        den2=[7.89e-007 8.25e-004 0.00172 0];
        tf2=tf(num2,den2);
        tf3=tf1*tf2;
        sys=feedback(tf3,1);
        y=step(sys,t); m=max(y);
```

Finishing the Optimizing Program

```
if m<1.1 & m>1.05;
            plot(t,y);grid;set(gca,'Fontsize',font)
sol=[K;a;m]
            break % Breaks the inner loop
        end
    end
    if m<1.1 & m>1.05;
        break; %Breaks the outer loop
    end
end
```

Closed-Loop System Performance



Modified PID Control Schemes

- If the reference input is a step, then because of the presence of the derivative term, the controller output will involve an impulse function
- The derivative term also amplifies higher frequency sensor noise
- Replace the pure derivative term with a derivative filter---PIDF controller
- Set-Point Kick---for step reference the PIDF output will involve a sharp pulse function rather than an impulse function

The Derivative Term

- Derivative action is useful for providing a phase lead, to offset phase lag caused by integration term
- Differentiation increases the high-frequency gain
- Pure differentiator is not proper or causal
- 80% of PID controllers in use have the derivative part switched off
- Proper use of the derivative action can increase stability and help maximize the integral gain for better performance

Remedies for Derivative Action---PIDF Controller

- Pure differentiator approximation

$$\frac{T_d s}{\gamma T_d s + 1}$$

where γ is a small parameter, for example, 0.1

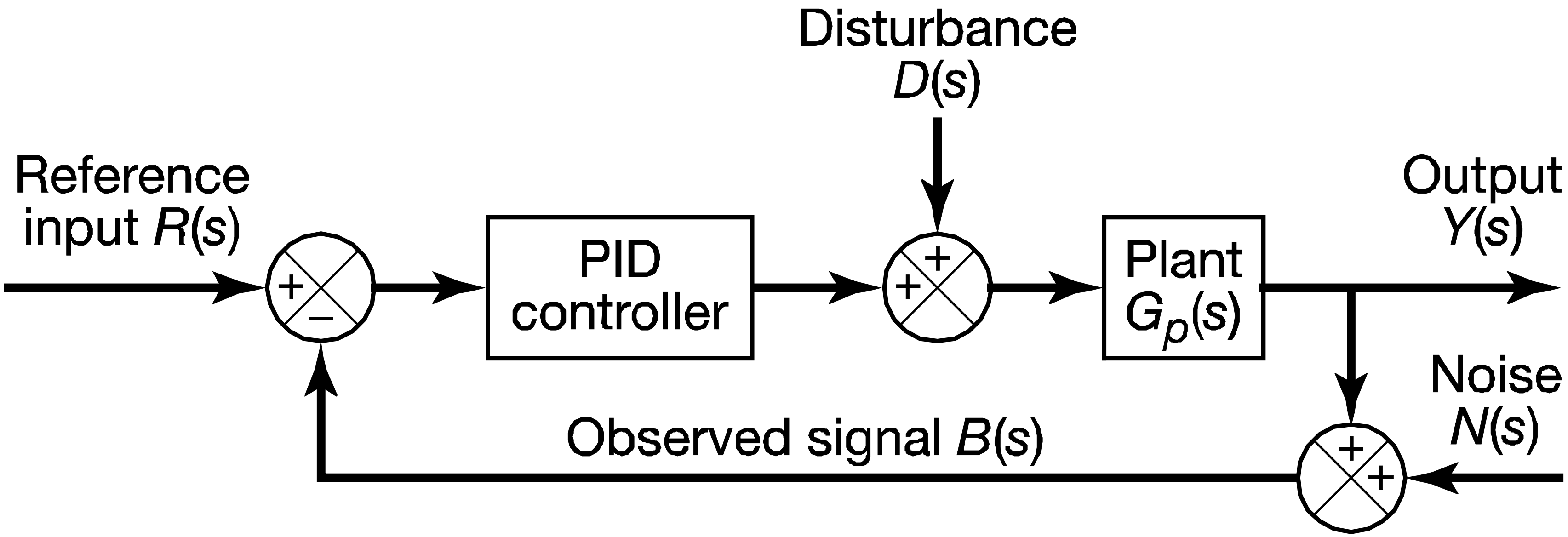
- Pure differentiator cascaded with a first-order low-pass filter

The Set-Point Kick Phenomenon

- If the reference input is a step function, the derivative term will produce an impulse (delta) function in the controller action
- Possible remedy---operate the derivative action only in the feedback path; thus differentiation occurs only on the feedback signal and not on the reference signal

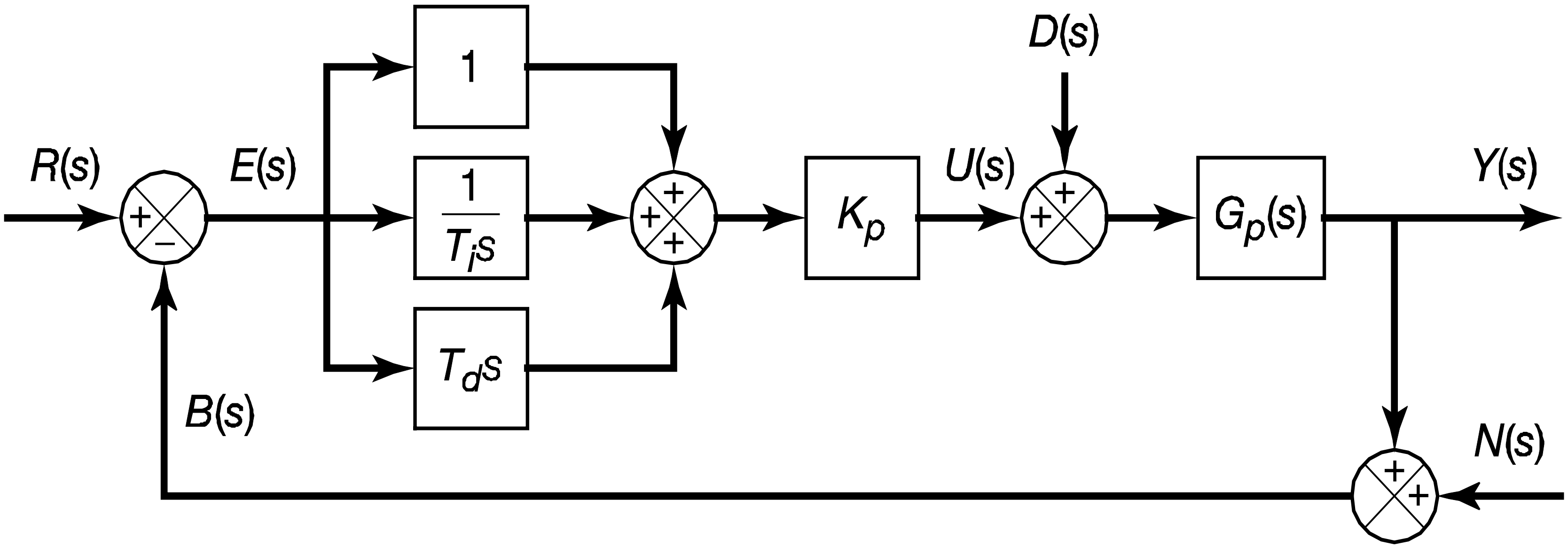
Eliminating the Set-Point Kick

PID controller revisited



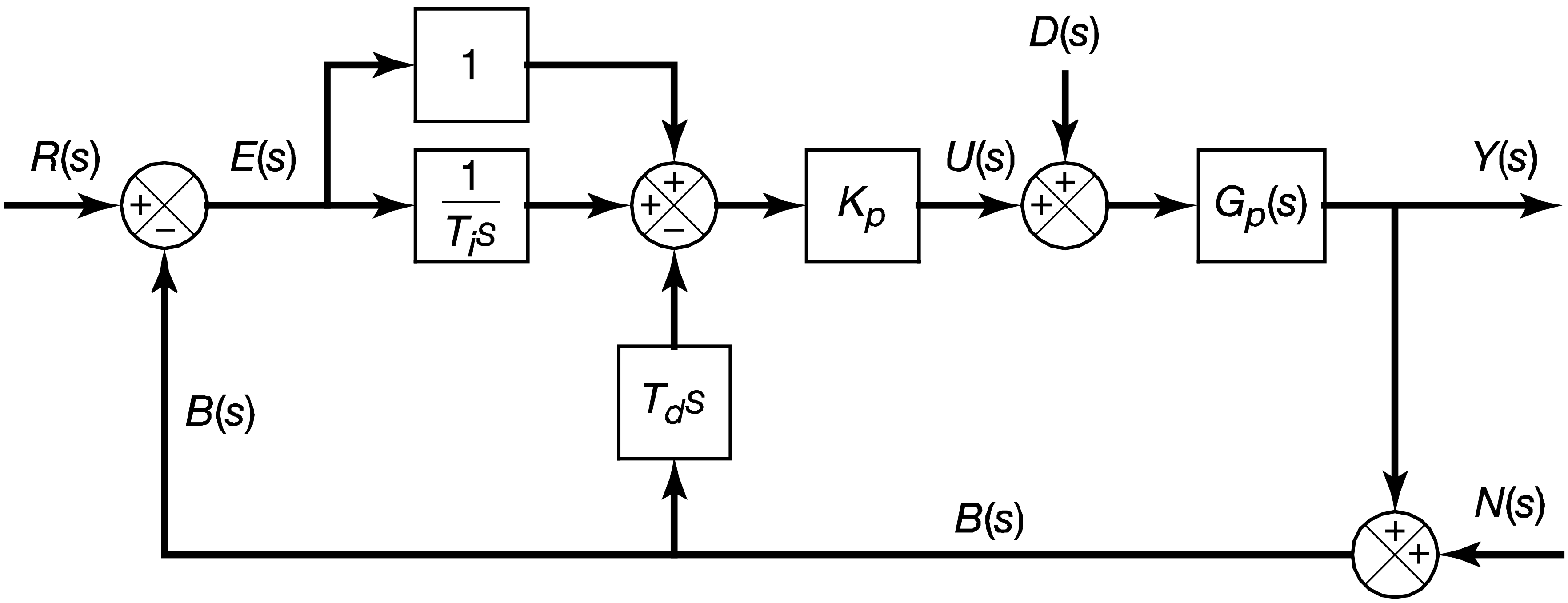
Eliminating the Set-Point Kick---Finding the source of trouble

More detailed view of the PID controller



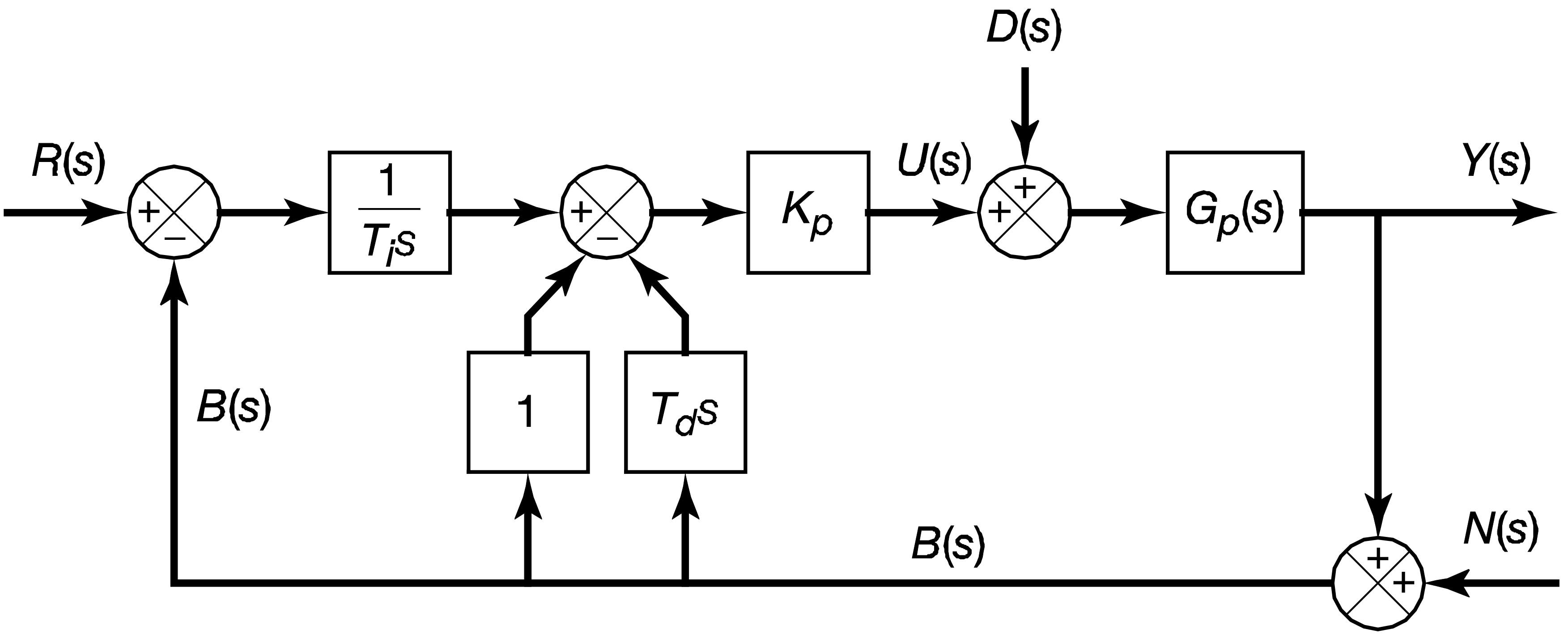
Eliminating the Set-Point Kick---PI-D Control or Type B PID

Operate derivative action only in the feedback



I-PD---Moving Proportional and Derivative Action to the Feedback

I-PD control or Type C PID



I-PD Equivalent to PID With Input Filter (No Noise)

Closed-loop transfer function $Y(s)/R(s)$ of the I-PD-controlled system

$$\frac{Y(s)}{R(s)} = \frac{\frac{K_p}{T_i s} G_p(s)}{1 + K_p\left(1 + \frac{1}{T_i s} + T_d s\right) G_p(s)}$$

PID-Controlled System

- Closed-loop transfer function $Y(s)/R(s)$ of the PID-controlled system with input filter

$$\frac{Y(s)}{R(s)} = \frac{1}{1+T_i s+T_i T_d s^2} \frac{K_p\left(1+\frac{1}{T_i s}+T_d s\right)G_p(s)}{1+K_p\left(1+\frac{1}{T_i s}+T_d s\right)G_p(s)}$$

- After manipulations it is the same as the transfer function of the I-PD-controlled closed-loop system

PID, PI-D and I-PD Closed-Loop Transfer Function---No Ref or Noise

In the absence of the reference input and noise signals, the closed-loop transfer function between the disturbance input and the system output is the same for the three types of PID control

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s)\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

The Three Terms of Proportional-Integral-Derivative (PID) Control

- Proportional term responds immediately to the current tracking error; it cannot achieve the desired setpoint accuracy without an unacceptably large gain. Needs the other terms
- Derivative action reduces transient errors
- Integral term yields zero steady-state error in tracking a constant setpoint. It also rejects constant disturbances

☞ **Proportional-Integral-Derivative (PID)** control provides an efficient solution to many real-world control problems

Summary

- PID control---most widely used control strategy today
- Over 90% of control loops employ PID control, often the derivative gain set to zero (PI control)
- The three terms are intuitive---a non-specialist can grasp the essentials of the PID controller's action. It does not require the operator to be familiar with advanced math to use PID controllers
- Engineers prefer PID controls over untested solutions