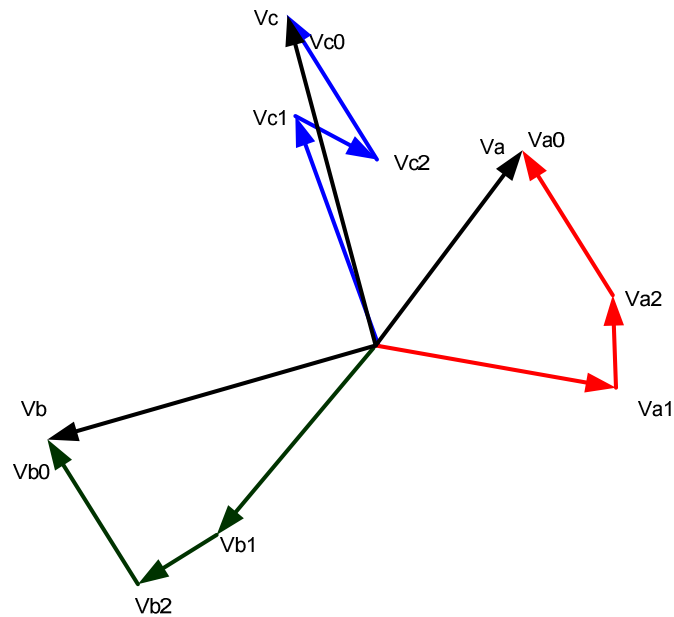# An Introduction to Symmetrical Components, System Modeling and Fault Calculation

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Introduction

The electrical power system normally operates in a balanced three-phase sinusoidal steady-state mode. However, there are certain situations that can cause unbalanced operations. The most severe of these would be a fault or short circuit. Examples may include a tree in contact with a conductor, a lightning strike, or downed power line.

In 1918, Dr. C. L. Fortescue wrote a paper entitled "Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks." In the paper Dr. Fortescue described how arbitrary unbalanced 3-phase voltages (or currents) could be transformed into 3 sets of balanced 3-phase components, Fig I.1. He called these components "symmetrical components." In the paper it is shown that unbalanced problems can be solved by the resolution of the currents and voltages into certain symmetrical relations.
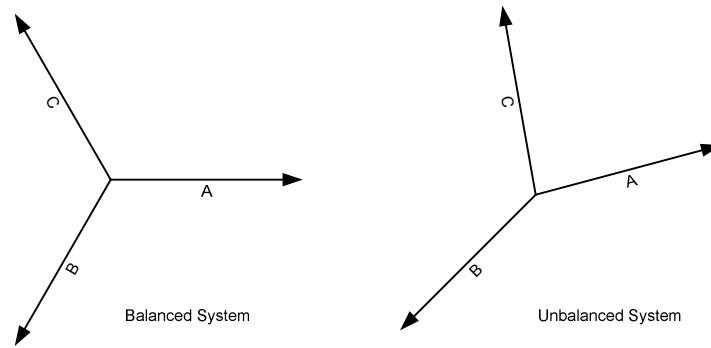

Fig I.1

By the method of symmetrical coordinates, a set of unbalanced voltages (or currents) may be resolved into systems of balanced voltages (or currents) equal in number to the number of phases involved. The symmetrical component method reduces the complexity in solving for electrical quantities during power system disturbances. These sequence components are known as positive, negative and zero-sequence components, Fig I.2
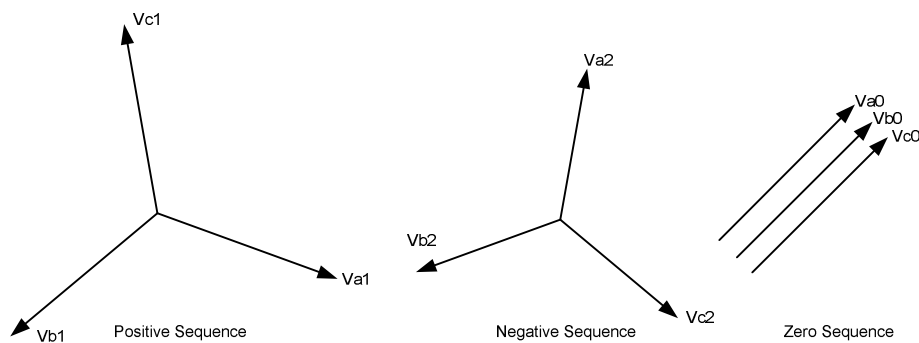

Fig I.2

The purpose of this paper is to explain symmetrical components and review complex algebra in order to manipulate the components. Knowledge of symmetrical components is important in performing mathematical calculations and understanding system faults. It is also valuable in analyzing faults and how they apply to relay operations.

1. Complex Numbers

The method of symmetrical components uses the commonly used mathematical solutions applied in ordinary alternating current problems. A working knowledge of the fundamentals of algebra of complex numbers is essential. Consequently this subject will be reviewed first.

Any complex number, such as $a + jb$, may be represented by a single point p, plotted on a Cartesian coordinates, in which a is the abscissa on the x axis of real quantities and b the ordinate on the y axis of imaginary quantities. This is illustrated in Fig. 1.1
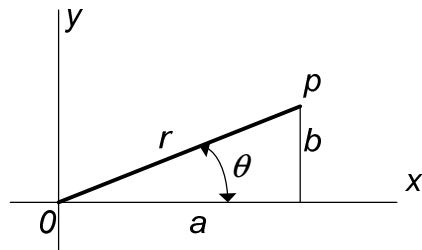

Fig. 1.1

Referring to Fig. 1.1, let r represent the length of the line connecting the point p to the origin and θ the angle measured from the x-axis to the line r. It can be observed that

$$a = r \cdot \cos \theta \qquad\qquad (1.1)$$
$$b = r \cdot \sin \theta \qquad\qquad (1.2)$$

2. Properties of Phasors

A vector is a mathematical quantity that has both a magnitude and direction. Many quantities in the power industry are vector quantities. The term phasor is used within the steady state alternating linear system. It is used to avoid confusion with spatial vectors: the angular position of the phasor represents position in time, not space. In this document, phasors will be used to document various ac voltages, currents and impedances.

A phasor quantity or phasor, provides information about not only the magnitude but also the direction or angle of the quantity. When using a compass and giving directions to a house, from a given location, a distance and direction must be provided. For example one could say that a house is 10 miles at an angle of 75 degrees (rotated in a clockwise direction from North) from where I am standing. Just as we don't say the other house is -10 miles away, the magnitude of

the phasor is always a positive, or rather the absolute value of the "length of the phasor." Therefore giving directions in the opposite direction, one could say that a second house is 10 miles at an angle of 255 degrees. The quantity could be a potential, current, watts, etc.

Phasors are written in polar form as

$$Y = |Y| \angle \theta \qquad (2.1)$$
$$= |Y|\cos\theta + j|Y|\sin\theta \qquad (2.2)$$

where Y is the phasor, $|Y|$ is the amplitude, magnitude or absolute value and θ is the phase angle or argument. Polar numbers are written with the magnitude followed by the $\angle$ symbol to indicate angle, followed by the phase angle expressed in degrees. For example $Z = 110 \angle 90^o$. This would be read as 110 at an angle of 90 degrees. The rectangular form is easily produced by applying Eq. (2.2)

The phasor can be represented graphically as we have demonstrated in Fig. 1.1, with the real components coinciding with the x axis.

When multiplying two phasors it is best to have the phasor written in the polar form. The magnitudes are multiplied together and the phase angles are added together. Division, which is the inverse of multiplication, can be accomplished in a similar manner. In division the magnitudes are divided and the phase angle in the denominator is subtracted from the phase angle in the numerator.

Example 2.1
    Multiply $A \cdot B$ where $A = 5 \angle 35^o$ and $B = 3 \angle 45^o$.
Solution
$$A \cdot B = 5 \angle 35^o \cdot 3 \angle 45^o = (5 \cdot 3) \angle (35^o + 45^o)$$
$$= 15 \angle 80^o$$

Example 2.2
    Solve $\dfrac{C}{D}$ where $C = 15 \angle 35^o$ and $D = 3 \angle 50^o$.
Solution
$$\frac{C}{D} = \frac{15 \angle 35^o}{3 \angle 50^o} = \left(\frac{15}{3}\right) \angle (35^o - 50^o)$$
$$= 5 \angle -15^o$$

3. The j and a operator

Recall the operator j. In polar form, $j = 1\angle 90^o$. Multiplying by j has the effect of rotating a phasor 90° without affecting the magnitude.

<u>Table 3.1 - Properties of the vector j</u>

$$1 = 1.0 + j0.0$$
$$j = 1\angle 90^o$$
$$j^2 = 1\angle 180^o = -1$$

$$j^3 = 1\angle 270^o = -j$$
$$-j = 1\angle -90^o$$
$$j = \sqrt{-1}$$

Example 3.1

Compute jR where $R = 10\angle 60^o$.

Solution

$$jR = 1\angle 90^o (10\angle 60^o)$$
$$= 10\angle 150^o$$

Notice that multiplication by the j operator rotated the Phasor $\overline{R}$ by 90°, but did not change the magnitude. Refer to Fig. 3.1

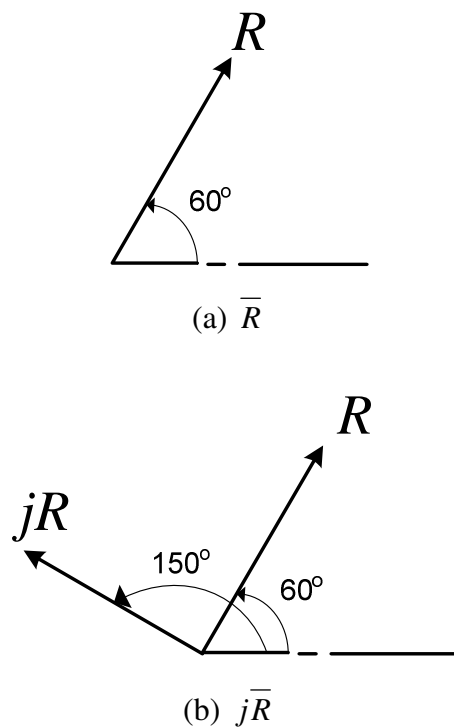(a) $\overline{R}$

(b) $j\overline{R}$

Fig. 3.1. j effects

In a similar manner the a operator is defined as unit vector at an angle of 120°, written as $a = 1\angle 120^o$. The operator a^2, is also a unit vector at an angle of 240°, written $a^2 = 1\angle 240^o$.

Example 3.2

Compute aR where $R = 10\angle 60^o$.

Solution

$$aR = 1\angle 120^o (10\angle 60^o)$$
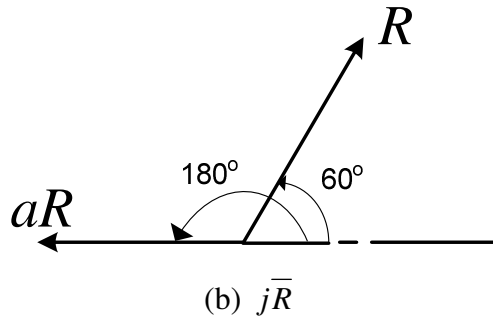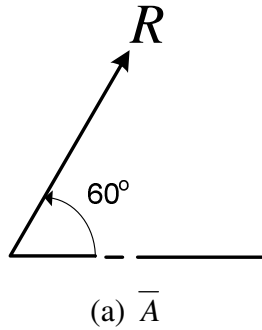$$= 10\angle 180^o$$

(a) $\overline{A}$

(b) $j\overline{R}$

Fig. 3.2. a effects

<u>Table 3.2 - Properties of the vector a</u>

$1 = 1.0 + j0.0$

$a = 1\angle 120^o$

$a^2 = 1\angle 240^o$

$a^3 = 1\angle 360^o = 1\angle 0^o$

$1 + a^2 + a = 0$

$a + a^2 = -1$

$1 + a = 1\angle 60^o$

$1 + a^2 = 1\angle - 60^o$

$a - a^2 = j\sqrt{3}$

$a^2 - a = -j\sqrt{3}$

$1 - a = \sqrt{3}\angle - 30^o$

$1 - a^2 = \sqrt{3}\angle 30^o$

4. The three-phase System and the relationship of the $\sqrt{3}$

In a Wye connected system the voltage measured from line to line equals the square root of three, $\sqrt{3}$, times the voltage from line to neutral. See Fig. 4.1 and Eq. (4.1). The line current equals the phase current, see Eq. (4.2)
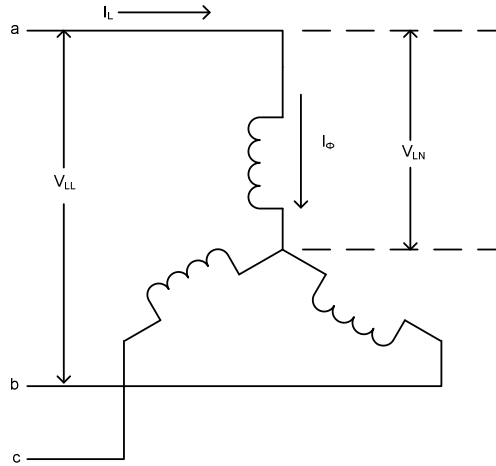

Fig. 4.1

$$V_{LL} = \sqrt{3}V_{LN} \tag{4.1}$$
$$I_L = I_\Phi \tag{4.2}$$

In a Delta connected system the voltage measured from line to line equals the phase voltage. See Fig. 4.2 and Eq. (4.3). The line current will equal the square root of three, $\sqrt{3}$, times the phase current, see Eq. (4.4)
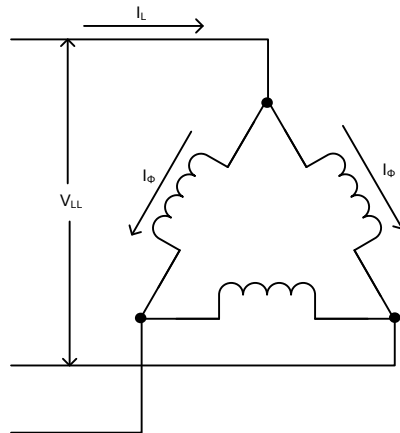

Fig. 4.2

$$V_{LL} = V_\Phi \tag{4.3}$$
$$I_L = \sqrt{3}I_\Phi \tag{4.4}$$

The power equation, for a three phase system, is

$$S = \sqrt{3}V_{LL}I_L \qquad\qquad (4.5a)$$
$$P = \sqrt{3}V_{LL}I_L \cos\psi \qquad\qquad (4.5b)$$
$$Q = \sqrt{3}V_{LL}I_L \sin\psi \qquad\qquad (4.5c)$$

where S is the apparent power or complex power in volt-amperes (VA). P is the real power in Watts (W, kW, MW). Q is the reactive power in VARS (Vars, kVars, MVars).

5. The per-unit System

5.1 Introduction

In many engineering situations it is useful to scale, or normalize, dimensioned quantities. This is commonly done in power system analysis. The standard method used is referred to as the *per-unit* system. Historically, this was done to simplify numerical calculations that were made by hand. Although this advantage is eliminated by the calculator, other advantages remain.

- Device parameters tend to fall into a relatively narrow range, making erroneous values conspicuous.

- Using this method all quantities are expressed as ratios of some base value or values.

- The *per-unit* equivalent impedance of any transformer is the same when referred to either the primary or the secondary side.

- The *per-unit* impedance of a transformer in a three-phase system is the same regardless of the type of winding connections (wye-delta, delta-wye, wye-wye, or delta-delta).

- The *per-unit* method is independent of voltage changes and phase shifts through transformers where the base voltages in the winding are proportional to the number of turns in the windings.

- Manufactures usually specify the impedance of equipment in per-unit or percent on the base of its nameplate rating of power (usually kVA) and voltage (V or kV).

The *per-unit* system is simply a scaling method. The basic *per-unit* scaling equation is

$$per - unit = \frac{actual\_value}{base\_value} \qquad\qquad (5.1)$$

The base value always has the same units as the actual value, forcing the *per-unit* value to be dimensionless. The base value is always a real number, whereas the actual value may be complex. The subscript *pu* will indicate a *per-unit* value. The subscript *base* will

indicate a base value, and no subscript will indicate an actual value such as Amperes, Ohms, or Volts.

Per-unit quantities are similar to percent quantities. The ratio in percent is 100 times the ratio in per-unit. For example, a voltage of 70kV on a base of 100kV would be 70% of the base voltage. This is equal to 100 times the per unit value of 0.7 derived above.

The first step in using *per-unit* is to select the base(s) for the system.

S_{base} = power base, in VA. Although in principle S_{base} may be selected arbitrarily, in practice it is typically chosen to be 100 MVA.

V_{base} = voltage base in V. Although in principle V_{base} is also arbitrary, in practice V_{base} is equal to the nominal line-to-line voltage. The term nominal means the value at which the system was designed to operate under normal balanced conditions.

From Eq. (4.5a) it follows that the base power equation for a three-phase system is:

$$S_{3\Phi base} = \sqrt{3} V_{base} I_{base} \tag{5.2}$$

Solving for current:

$$I_{base} = \frac{S_{3\Phi base}}{\sqrt{3} V_{base}}$$

Because $S_{3\Phi base}$ can be written as kVA or MVA and voltage is usually expressed in kilo-volts, or kV, current can be written as:

$$I_{base} = \frac{kVA_{base}}{\sqrt{3} kV_{base}} \, amperes \tag{5.3}$$

Solving for base impedance:

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}$$

$$Z_{base} = \frac{kV_{base}^2 x 1000}{kVA_{base}} ohms \tag{5.4a}$$

or

$$Z_{base} = \frac{kV_{base}^2}{MVA_{base}} ohms \tag{5.4b}$$

Given the base values, and the actual values: $V = IZ$, then dividing by the base we are able to calculate the *pu* values

$$\frac{V}{V_{base}} = \frac{IZ}{I_{base}Z_{base}} \Rightarrow V_{pu} = I_{pu}Z_{pu}$$

After the base values have been selected or calculated, then the *per-unit* impedance values for system components can be calculated using Eq. (5.4b)

$$Z_{pu} = \frac{Z(\Omega)}{Z_{base}} = \left(\frac{MVA_{base}}{kV_{base}^2}\right) \cdot Z(\Omega) \qquad (5.5a)$$

or

$$Z_{pu} = \left(\frac{kVA_{base}}{1000 \cdot kV_{base}^2}\right) \cdot Z(\Omega) \qquad (5.5b)$$

It is also a common practice to express *per-unit* values as percentages (i.e. 1 pu = 100%). (Transformer impedances are typically given in % at the transformer MVA rating.) The conversion is simple

$$per-unit = \frac{percent\_value}{100}$$

Then Eq. (5.5a) can be written as

$$\%Z = \frac{100MVA_{base} \cdot Z(\Omega)}{kV_{base}^2} = \frac{kVA_{base}Z(\Omega)}{10kV_{base}^2} \qquad (5.6)$$

It is frequently necessary, particularly for impedance values, to convert from one (old) base to another (new) base. The conversion is accomplished by two successive application of Eq. (5.1), producing:

$$Z_{pu}^{new} = Z_{pu}^{old}\left(\frac{Z_{base}^{old}}{Z_{base}^{new}}\right)$$

Substituting for Z_{base}^{old} and Z_{base}^{new} and re-arranging the new impedance in *per-unit* equals:

$$Z_{pu}^{new} = Z_{pu}^{old}\left(\frac{kVA_{base}^{new}}{kVA_{base}^{old}}\right)\left(\frac{kV_{base}^{old}}{kV_{base}^{new}}\right)^2 \qquad (5.7)$$

In most cases the turns ratio of the transformer is equivalent to the system voltages, and the equipment rated voltages are the same as the system voltages. This means that the voltage-squared ratio is unity. Then Eq. (5.7) reduces to

$$Z_{pu}^{new} = Z_{pu}^{old} \left(\frac{MVA_{base}^{new}}{MVA_{base}^{old}} \right)$$ (5.8)

We can quickly change from one impedance value in ohms, to another impedance value in ohms by dividing by the old base voltage and multiplying by the new base voltage in ohms. This is shown in Eq. (5.9)

$$Z_{ohm}^{new} = Z_{ohm}^{old} \cdot \left(\frac{kV_{base}^{new}}{kV_{base}^{old}} \right)^2$$ (5.9)

Example 5.1

A system has $S_{base} = 100$ MVA, calculate the base current for
    a) $V_{base} = 230$ kV
    b) $V_{base} = 525$ kV
Then using this value, calculate the actual line current and phase voltage where $I = 4.95_{pu}$, and $V = 0.5_{pu}$ at both 230 kV and 525 kV.

Solution

Using Eq. (5.3) $I_{base} = \dfrac{kVA_{base}}{\sqrt{3}kV_{base}} amperes$

a) $I_{base} = \dfrac{1000 \times 100}{\sqrt{3} \times 230} amperes = 251A$

b) $I_{base} = \dfrac{1000 \times 100}{\sqrt{3} \times 525} amperes = 110.0A$

From Eq. (5.1)

$$I_{actual} = I_{pu} \cdot I_{base}$$ (5.9)
$$V_{actual} = V_{pu} \cdot V_{base}$$ (5.10)

At 230 kV
c) $I_{actual} = (4.95) \cdot (251A) = 1242A$
d) $V_{actual} = (0.5) \cdot (230kV) = 115kV$

At 525 kV
e) $I_{actual} = (4.95) \cdot (110.0A) = 544A$
f) $V_{actual} = (0.5) \cdot (525kV) = 263kV$

Example 5.2

A 900 MVA 525/241.5 autotransformer has a nameplate impedance of 10.14%

a) Determine the impedance in ohms, referenced to the 525 kV side.

b) Determine the impedance in ohms, referenced to the 241.5 kV side

Solution

First convert from % to *pu*.

$$Zpu = \frac{Z\%}{100} = 0.1014$$

Arranging Eq. (5.5a) and solving for Z_{actual} gives

$$Z(\Omega) = Z_{pu}\frac{kV_{base}^2}{MVA_{base}} \; ; \text{therefore}$$

a) $Z_{525kV} = 0.1014 \times \dfrac{525^2}{900}$

$$= 31.05\Omega$$

b) $Z_{241.5kV} = 0.1014 \times \dfrac{241.5^2}{900}$

$$= 6.57\Omega$$

A check can be made to see if the high-side impedance to the low-side impedance equals the turns ratio squared.

$$\frac{31.05}{6.57} = 4.726 \qquad\qquad \left(\frac{525}{241.5}\right)^2 = 4.726$$

5.1 Application of per-unit

Appling this to relay settings, a practical example can be shown in calculation of the settings for a relay on a transmission line. For distance relays a common setting for zone 1 is 85% of the line impedance. Zone 2 should be set not less than 125% of the line, with care to not over reach the zone 1 of the next line section. If this does then zone 2 will need to be coordinated with the next line section zone 2.

Referring to Fig. 5.1 the line impedance for the 161 kV line is $Z = 59.3\angle 81^o$ ohms. Using the above criteria of 85% for zone 1 and 125 % for zone 2 the relays would be set at

For zone 1

$Z_1(\Omega) = 85\%(59.31\angle 81^o)$

$Z_1(\Omega) = 50.4\angle 81^o$

For zone 2

$$Z_2(\Omega) = 125\%(59.31\angle 81^o)$$
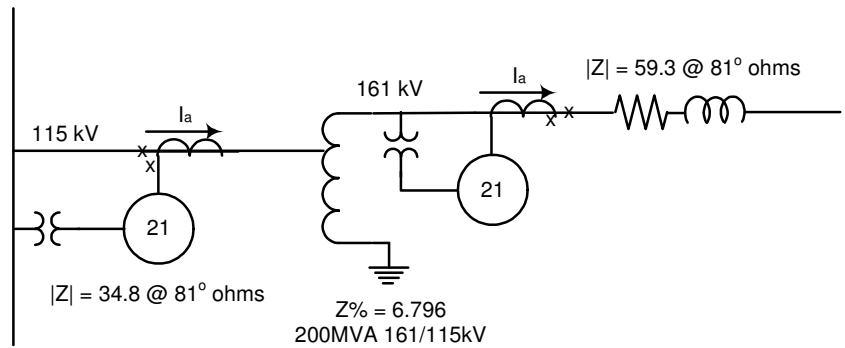
$$Z_2(\Omega) = 74.1\angle 81^o$$

Fig 5.1

For the relays on the 115 kV side of the transformer, the impedance of the transformer needs to be calculated. From example 5.2 we see that

$$Z_{115kV} = 0.06796 \times \frac{115^2}{200}$$

$$= 4.494\Omega$$

Next the line impedance needs referenced to the 115 kV side of the transformer. Using equation 5.9

$$Z_{ohm}^{new} = Z_{ohm}^{old} \cdot \left(\frac{kV_{base}^{new}}{kV_{base}^{old}} \right)^2 \tag{5.9}$$

Substituting, the line impedance equals

$$Z_{ohm}^{115kV} = 59.3 \cdot \left(\frac{115}{161} \right)^2 = 30.3 ohms$$

Adding this to the transformer, the impedance setting for the relays on the 115 kV side of the transformer is $Z = 34.8\angle 82^o$

Using the same criteria for zone 1 and zone 2 reach.

For zone 1

$$Z_1(\Omega) = 85\%(34.8\angle 82^o)$$

$$Z_1(\Omega) = 29.6\angle 81^o$$

For zone 2

$$Z_2(\Omega) = 125\%(34.8\angle 81^o)$$

$$Z_2(\Omega) = 43.5\angle 81^o$$

Given these values, one can easily see that by ignoring the base values of the voltages the relay settings would not be adequate. For example if the 161 kV settings were applied to the 115 kV relays, zone 1 would over reach the remote terminal. Conversely, if the 115

kV settings were applied to the 161 kV relays zone 2 would not reach past the remote terminal and would thus not protect the full line.
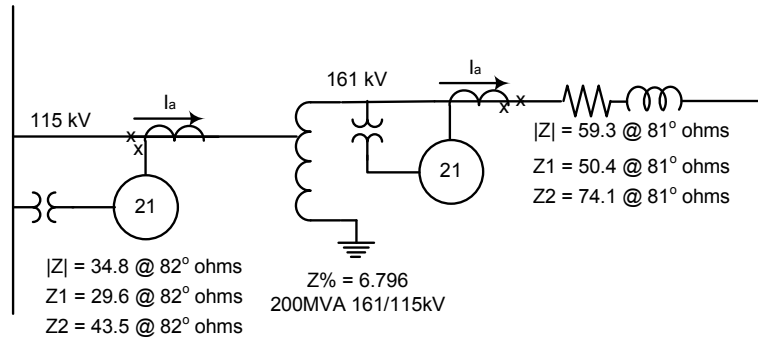

161 kV

115 kV

|Z| = 59.3 @ 81° ohms
Z1 = 50.4 @ 81° ohms
Z2 = 74.1 @ 81° ohms

|Z| = 34.8 @ 82° ohms
Z1 = 29.6 @ 82° ohms
Z2 = 43.5 @ 82° ohms

Z% = 6.796
200MVA 161/115kV

Fig. 5.2

5.2 Calculating actual values from per-unit

In the following sections we will discuss symmetrical faults. The analysis of the faults uses the per-unit. A impedance and voltage of the system is express in per-unit. Then the fault current and fault voltage is solved and that value will be given in per unit. Next we need to convert from per-unit to actual amps and volts by using the base values. Using the above equations it is easy to prove the following equations.

The MVA for a three phase fault is given as

$$MVA_{Fault} = \frac{MVA_{Base}}{Z_{Fault}PU} \tag{5.10}$$

Or

$$MVA_{Fault} = \frac{100}{Z_{Fault}PU} \text{ for a 100 MVA}_{Base} \tag{5.11 a}$$

$$I_{Fault\_Current} = \frac{I_{Base}}{Z_{Fault}PU} \tag{5.12}$$

Or

$$I_{Fault\_Current} = \frac{100,000}{(Z_{Fault}PU) \cdot \sqrt{3}(kV_{Base})} \tag{5.12 a}$$

5.3 Converting per-unit

Before using the per-unit impedance of a transformer from a manufacture nameplate you must first convert it to a per-unit value of your system. Typically the three-phase power base of 100MVA is used. This is done by first converting the per unit impedance to an actual impedance (in ohms) at 525kV and then converting the actual impedance to a per-unit impedance on the new base. Repeat, this time converting the per unit impedance to

an actual impedance (in ohms) at 241.5kV and then converting the actual impedance to a per-unit impedance on the new base.

In the problem 3 at the end of this document, the transformer nameplate data is for a ratio of 525/241.5kV or 2.174, whereas BPA's ASPEN model uses nominal voltages of 525kV and 230kV for a ratio of 2.283. Because BPA used a transformer ratio in ASPEN model that was different than the transformer nameplate values, we have a discrepancy in the per-unit impedance values that we obtained. The problem arises because when a transformer is applied to the BPA system the transformer tap used will often be different than the one used in the nameplate calculations.

What is the correct way to convert the per-unit impedance to the BPA base?

Because the actual impedance of the transformer will vary when different taps are used, the most accurate way to model the impedance would be to actually measure the impedance with the transformer on the tap that will normally be used on the BPA system. This impedance would then be converted to a per-unit value on the BPA model base. Since this isn't normally possible, a close approximation can be made by assuming that the per-unit impedance given on the nameplate will remain the same for the different tap positions of the transformer. Find the transformer tap position that most closely matches the ratio of the ASPEN model (2.283 for a 525/230kV transformer), then convert the nameplate per-unit impedance to an actual value based on either the high- or low-side voltage given for that tap position. This actual impedance is then converted to a per-unit value on the BPA model base, using the high-side BPA voltage base if the high-side voltage was used for the conversion to actual impedance, or using the low-side BPA voltage base if the low-side voltage was used for the conversion to actual impedance. See problem 4.

6. Sequence Networks

Refer to the basic three-phase system as shown in Fig. 6.1. There are four conductors to be considered: *a, b, c* and neutral *n*.
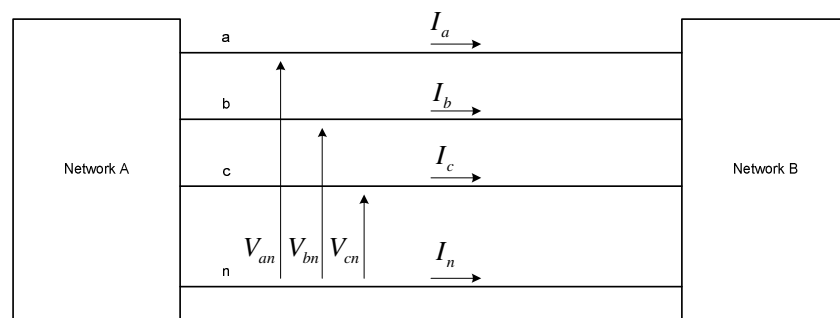

Fig. 6.1

The phase voltages, V_p, for the balanced 3Φ case with a phase sequence *abc* are

$$V_{an} = V_a = V_p \angle 0^o \tag{6.1a}$$

$$V_{bn} = V_b = V_p \angle -120^o \tag{6.1b}$$

$$V_{cn} = V_c = V_p \angle +120^0 = V_p \angle -240^o \tag{6.1c}$$

The phase-phase voltages, V_{LL}, are written as

$$V_{ab} = V_a - V_b = V_{LL} \angle 30^o \tag{6.2a}$$

$$V_{bc} = V_b - V_c = V_{LL} \angle -90^o \tag{6.2b}$$

$$V_{ca} = V_c - V_a = V_{LL} \angle 150^o \tag{6.2c}$$

Equation (6.1) and (6.2) can be shown in phasor form in Fig. 6.2.
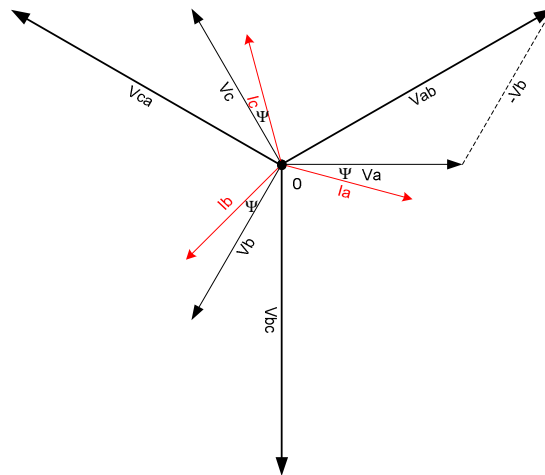
Fig. 6.2

There are two balanced configurations of impedance connections within a power system. For the wye case, as shown in Fig. 4.1, and with an impedance connection of $Z \angle \Psi$, the current can be calculated as

$$I_a = \frac{V}{Z_Y} = \frac{V_P}{Z_Y} \angle 0^o - \psi \tag{6.3}$$

Where Ψ is between -90° and $+90^o$. For Ψ greater than zero degrees the load would be inductive (I_a lags V_a). For ψ less than zero degrees the load would be capacitive (I_a leads V_a).

The phase currents in the balanced three-phase case are

$$I_a = I_p \angle 0^o - \psi \qquad\qquad (6.4a)$$

$$I_b = I_p \angle -120^o - \psi \qquad\qquad (6.4b)$$

$$I_c = I_p \angle -240^o - \psi \qquad\qquad (6.4c)$$

See Fig. 6.2. for the phasor representation of the currents.

7. Symmetrical Components Systems

The electrical power system operates in a balanced three-phase sinusoidal operation. When a tree contacts a line, a lightning bolt strikes a conductor or two conductors swing into each other we call this a fault, or a fault on the line. When this occurs the system goes from a balanced condition to an unbalanced condition. In order to properly set the protective relays, it is necessary to calculate currents and voltages in the system under such unbalanced operating conditions.

In Dr. C. L. Fortescue's paper he described how symmetrical components can transform an unbalanced condition into symmetrical components, compute the system response by straight forward circuit analysis on simple circuit models, and transform the results back into original phase variables. When a short circuit fault occurs the result can be a set of unbalanced voltages and currents. The theory of symmetrical components resolves any set of unbalanced voltages or currents into three sets of symmetrical balanced phasors. These are known as positive, negative and zero-sequence components. Fig. 7.1 shows balanced and unbalanced systems.
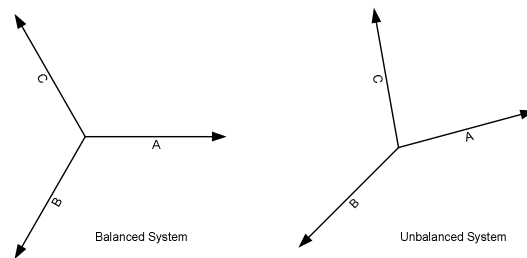
Fig. 7.1

Consider the symmetrical system of phasors in Fig. 7.2. Being balanced, the phasors have equal amplitudes and are displaced 120° relative to each other. By the definition of symmetrical components, $\overline{V}_{b1}$ *always lags* $\overline{V}_{a1}$ by a fixed angle of 120° and always has the same magnitude as $\overline{V}_{a1}$. Similarly $\overline{V}_{c1}$ leads $\overline{V}_{a1}$ by 120°. It follows then that

$$V_{a1} = V_{a1} \qquad\qquad (7.1a)$$

$$V_{b1} = (1 \angle 240^o) V_{a1} = a^2 V_{a1} \qquad\qquad (7.1b)$$

$$V_{c1} = (1 \angle 120^o) V_{a1} = a V_{a1} \qquad\qquad (7.1c)$$

Where the subscript (1) designates the positive-sequence component. The system of phasors is called positive-sequence because the order of the sequence of their maxima occur *abc*.

Similarly, in the negative and zero-sequence components, we deduce

$$V_{a2} = V_{a2} \qquad\qquad (7.2a)$$

$$V_{b2} = (1\angle 120^o)V_{a2} = aV_{a2} \qquad\qquad (7.2b)$$

$$V_{c2} = (1\angle 240^o)V_{a2} = a^2 V_{a2} \qquad\qquad (7.2c)$$

$$V_{a0} = V_{a0} \qquad\qquad (7.3a)$$

$$V_{b0} = V_{a0} \qquad\qquad (7.3b)$$

$$V_{c0} = V_{a0} \qquad\qquad (7.3c)$$

Where the subscript (2) designates the negative-sequence component and subscript (0) designates zero-sequence components. For the negative-sequence phasors the order of sequence of the maxima occur *cba*, which is opposite to that of the positive-sequence. The maxima of the instantaneous values for zero-sequence occur simultaneously.
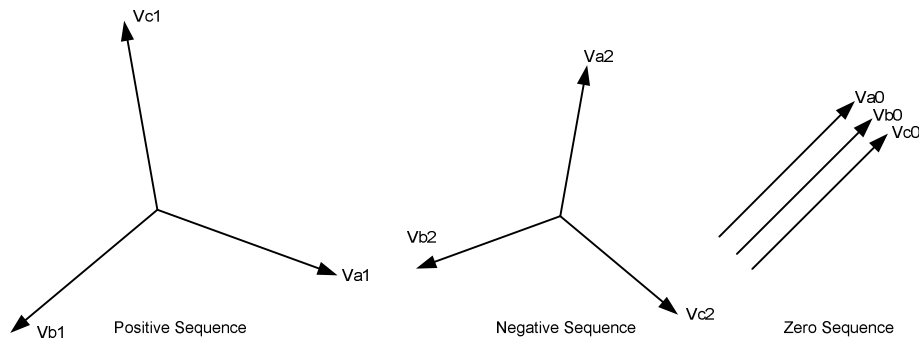
Fig.7.2

In all three systems of the symmetrical components, the subscripts denote the components in the different phases. The total voltage of any phase is then equal to the sum of the corresponding components of the different sequences in that phase. It is now possible to write our symmetrical components in terms of three, namely, those referred to the *a* phase (refer to section 3 for a refresher on the *a* operator).

$$V_a = V_{a0} + V_{a1} + V_{a2} \qquad\qquad (7.4a)$$

$$V_b = V_{b0} + V_{b1} + V_{b2} \qquad\qquad (7.4b)$$

$$V_c = V_{c0} + V_{c1} + V_{c2} \qquad\qquad (7.4c)$$

We may further simplify the notation as follows; define

$$V_0 = V_{a0} \tag{7.5a}$$
$$V_1 = V_{a1} \tag{7.5b}$$
$$V_2 = V_{a2} \tag{7.5c}$$

Substituting their equivalent values

$$V_a = V_0 + V_1 + V_2 \tag{7.6a}$$
$$V_b = V_0 + a^2 V_1 + a V_2 \tag{7.6b}$$
$$V_c = V_0 + a V_1 + a^2 V_2 \tag{7.6c}$$

These equations may be manipulated to solve for V_0, V_1, and V_2 in terms of V_a, V_b, and V_c.

$$V_0 = \frac{1}{3}\left(V_a + V_b + V_c\right) \tag{7.7a}$$

$$V_1 = \frac{1}{3}\left(V_a + a V_b + a^2 V_c\right) \tag{7.7b}$$

$$V_2 = \frac{1}{3}\left(V_a + a^2 V_b + a V_c\right) \tag{7.7c}$$

It follows then that the phase currents are

$$I_a = I_0 + I_1 + I_2 \tag{7.8a}$$
$$I_b = I_0 + a^2 I_1 + a I_2 \tag{7.8b}$$
$$I_c = I_0 + a I_1 + a^2 I_2 \tag{7.8c}$$

The sequence currents are given by

$$I_0 = \frac{1}{3}\left(I_a + I_b + I_c\right) \tag{7.9a}$$

$$I_1 = \frac{1}{3}\left(I_a + a I_b + a^2 I_c\right) \tag{7.9b}$$

$$I_2 = \frac{1}{3}\left(I_a + a^2 I_b + a I_c\right) \tag{7.9c}$$

The unbalanced system is therefore defined in terms of three balanced systems. Eq. (7.6) may be used to convert phase voltages (or currents) to symmetrical component voltages (or currents) and vice versa [Eq. (7.7)].

Example 7.1

Given $V_a = 5\angle 53^o$, $V_b = 7\angle -164^o$, $V_c = 7\angle 105^o$, find the symmetrical components. The phase components are shown in the phasor form in Fig. 7.3
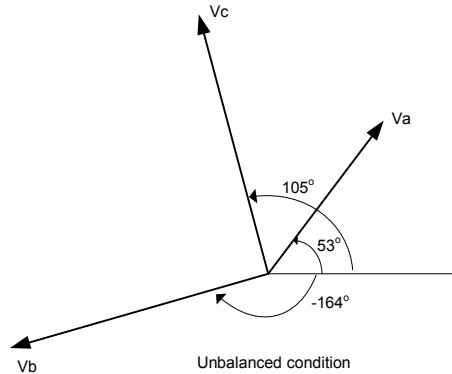
Fig. 7.3

Solution

Using Eq. (7.7a)

Solve for the zero-sequence component:

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$

$$= \frac{1}{3}(5\angle 53^o + 7\angle -164^o + 7\angle 105^o)$$

$$= 3.5\angle 122^o$$

From Eq. (7.3b) and (7.3c)

$$V_{b0} = 3.5\angle 122^o$$

$$V_{c0} = 3.5\angle 122^o$$

Solve for the positive-sequence component:

$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2 V_c)$$

$$= \frac{1}{3}(5\angle 53^o + (1\angle 120^o \cdot 7\angle -164^o) + (1\angle 240^o \cdot 7\angle 105^o))$$

$$= 5.0\angle -10^o$$

From Eq. (7.1b) and (7.1c)

$$V_{b1} = 5.0\angle -130^o$$

$$V_{c1} = 5.0\angle 110^o$$

Solve for the negative-sequence component:

$$V_{a2} = \frac{1}{3}(V_a + a^2 V_b + aV_c)$$

$$= \frac{1}{3}\left(5\angle 53^o + \left(1\angle 240^o \cdot 7\angle -164^o\right) + \left(1\angle 120^o \cdot 7\angle 105^o\right)\right)$$

$$= 1.9\angle 92^o$$

From Eq. (7.2b) and (7.2c)

$$V_{b2} = 1.9\angle -148^o$$

$$V_{c2} = 1.9\angle -28^o$$

The sequence components can be shown in phasor form in Fig. 7.4.
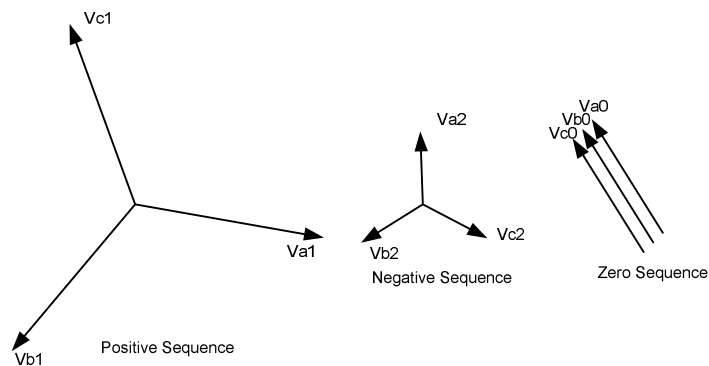
Fig. 7.4

Using Eq. (7.6) the phase voltages can be reconstructed from the sequence components.

Example 7.2

Given $V_0 = 3.5\angle 122^o$, $V_1 = 5.0\angle -10^o$, $V_2 = 1.9\angle 92^o$, find the phase sequence components. Shown in the phasor form in Fig. 7.4

Solution

Using Eq. (7.6)

Solve for the A-phase sequence component:

$$V_a = V_0 + V_1 + V_2$$
$$= 3.5\angle 122^o + 5.0\angle -10^o + 1.9\angle 92^o$$
$$= 5.0\angle 53^o$$

Solve for the B-phase sequence component:

$$V_b = V_0 + a^2 V_1 + a V_2$$
$$= 3.5\angle 122^o + 5.0\angle -130^o + 1.9\angle -148^o$$
$$= 7.0\angle -164^o$$

Solve for the C-phase sequence component:

$$V_c = V_0 + aV_1 + a^2V_2$$
$$= 3.5\angle 122^o + 5.0\angle 110^o + 1.9\angle -28^o$$
$$= 7.0\angle 105^o$$

This returns the original values given in Example 5.2.

This can be shown in phasor form in Fig. 7.5.
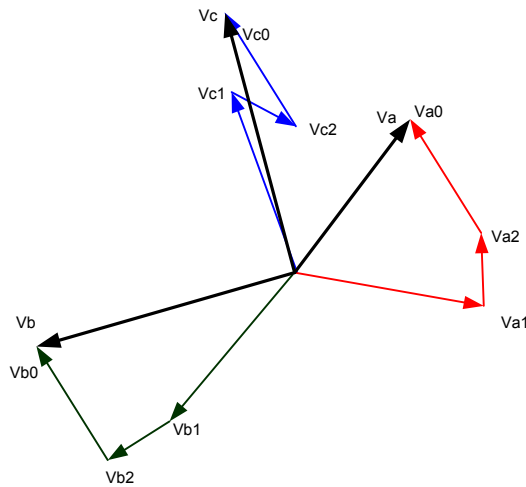
Fig. 7.5

Notice in Fig. 7.5 that by adding up the phasors from Fig. 7.4, that the original phase, Fig. 7.3 quantities are reconstructed.

8. Balanced and Unbalanced Fault analysis

Let's tie it together. Symmetrical components are used extensively for fault study calculations. In these calculations the positive, negative and zero-sequence impedance networks are either given by the manufacturer or are calculated by the user using base voltages and base power for their system. Each of the sequence networks are then connected together in various ways to calculate fault currents and voltages depending upon the type of fault.

Given a system, represented in Fig. 8.1, we can construct general sequence equivalent circuits for the system. Such circuits are indicated in Fig. 8.2.
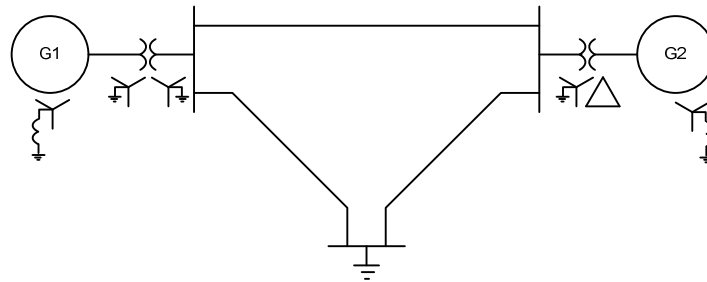
Fig. 8.1

The positive-sequence impedance system data for this example in per-unit is shown in Fig. 8.2.
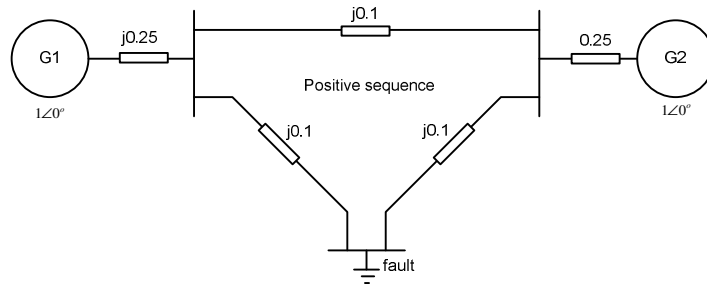
Fig. 8.2

Assuming the negative-sequence equals the positive-sequence, then the negative-sequence is shown in Fig 8.3
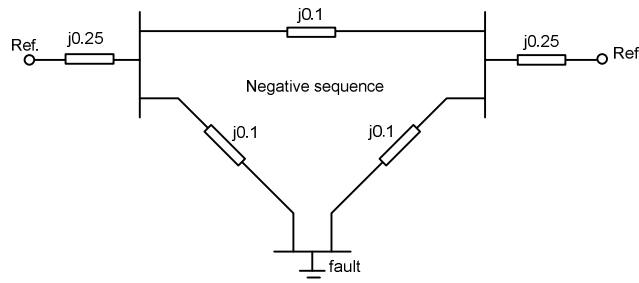
Fig. 8.3

The zero-sequence impedance is greater then the positive and for our purpose is assumed to be three times greater. Also because of the wye-delta transformer, zero-sequence from the generator will not pass through the transformer. This will be shown in section 10.2. Zero-sequence is shown in Fig 8.4
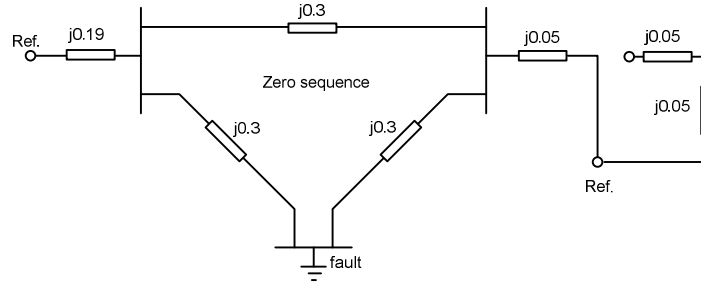
Fig. 8.4

The Thevenin equivalents for each circuit is reduced and shown in Fig. 8.5
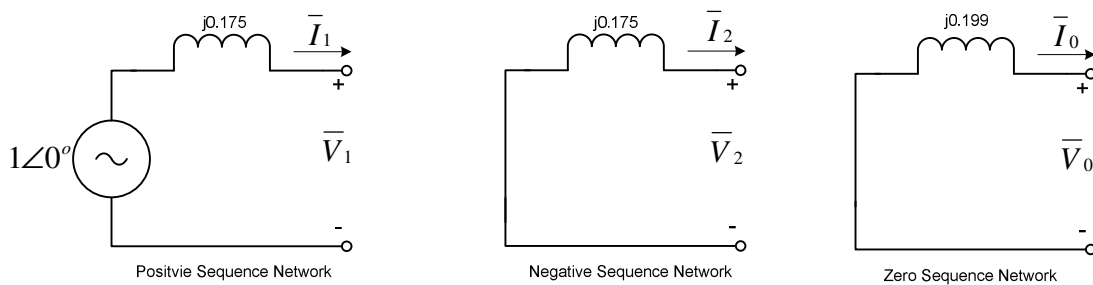
Fig. 8.5

Each of the individual sequence may be considered independently. Since each of the sequence networks involves symmetrical currents, voltages and impedances in the three phases, each of the sequence networks may be solved by the single-phase method. After converting the power system to the sequence networks, the next step is to determine the type of fault desired and the connection of the impedance sequence network for that fault. The network connections are listed in Table 8.1

<div align="center">Table 8.1 - Network Connection</div>

- Three-phase fault - The positive-sequence impedance network is only used in three-phase faults. Fig. 8.3
- Single Line-to-Ground fault - The positive, negative and zero-sequence impedance networks are connected in series. Fig. 8.5
- Line-to-line fault - The positive and negative-sequence impedance networks are connected in parallel. Fig. 8.7
- Double Line-to-Ground fault - All three impedance networks are connected in parallel. Fig. 8.9

The system shown in Fig. 8.1 and simplified to the sequence network in Fig. 8.5 and will be used throughout this section.

Example 8.1

Given $Z_0 = 0.199\angle 90^o \ pu$, $Z_1 = 0.175\angle 90^o \ pu$,

$Z_2 = 0.175\angle 90^o \ pu$, compute the fault current and voltages for a Three-phase fault. Note that the sequence impedances are in *per-unit*. This means that the solution for current and voltage will be in *per-unit*.

Solution

The sequence networks are interconnected, and shown

Note that for a three phase fault, there are no negative or zero-sequence voltages.

$$V_0 = V_2 = 0$$

$$I_0 = I_2 = 0$$

The current I_1 is the voltage drop across Z_1

$$I_1 = \frac{V_1}{Z_1}$$

$$I_1 = \frac{1\angle 0^o}{j0.175}$$

$$= -j5.71$$

The phase current is converted from the sequence value using Eq. (7.8).

$$I_a = 0 - j5.71 + 0 = 5.71\angle -90^o \ pu$$
$$I_b = 0 + a^2(-j5.71) + a(0) = 5.71\angle 150^o \ pu$$
$$I_c = 0 + a(-j5.71) + a^2(0) = 5.71\angle 30^o \ pu$$

Calculating the voltage drop, the sequence voltages are

$$V_0 = V_2 = 0$$
$$V_1 = 1\angle 0^o - Z_1 I_1$$
$$V_1 = 1 - j0.175(-j5.71) = 0.0$$
$$= 0.0 \ pu$$
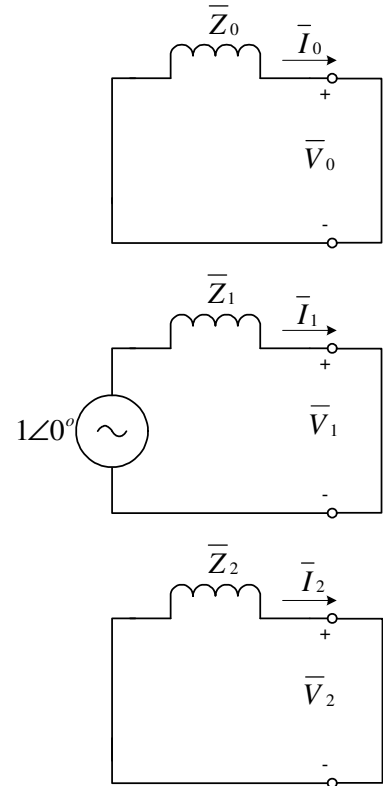
The phase voltages are converted from the sequence value using Eq. (7.6).

$$V_a = 0.0 + 0.0 + 0.0 = 0.0\,pu$$

$$V_b = 0.0 + a^2(0.0) + a(0.0) = 0.0\,pu$$

$$V_c = 0.0 + a(0.0) + a^2(0.0) = 0.0\,pu$$

The *per-unit* value for the current and voltage would now be converted to actual values using Eq. (5.9) and Eq. (5.10) and knowing the base power and voltage for the given system. See example 5.1 for a reference.

The currents and voltages can be shown in phasor form.
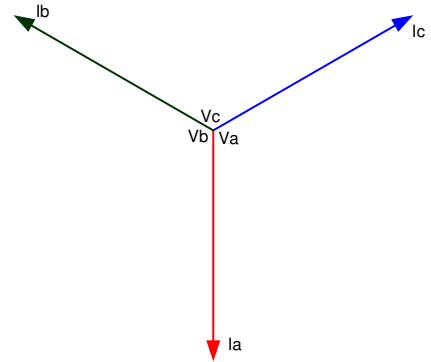
Example 8.2

Given $Z_0 = 0.199\angle 90^o\,pu$, $Z_1 = 0.175\angle 90^o\,pu$,

$Z_2 = 0.175\angle 90^o\,pu$, compute the fault current and voltages for a Single line-to-ground fault. Note that the sequence impedances are in *per-unit*. This means that the results for current and voltage will be in *per-unit*.

Solution

The sequence networks are interconnected in series, as shown.

Because the sequence currents are in series, and using ohms law.

$$I_0 = I_1 = I_2$$

$$I_0 = \frac{V_1}{(Z_0 + Z_1 + Z_2)}$$

$$I_0 = \frac{1\angle 0^o}{(j0.199 + j0.175 + j0.175)}$$

$$= -j1.82\,pu$$

The phase currents are converted from the sequence value using Eq. (7.8). Substituting $I_0 = I_1 = I_2$ into
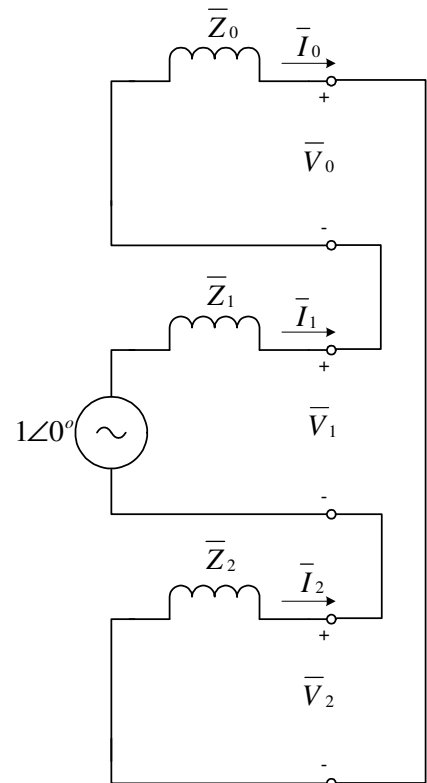
Eq. (7.8) gives

$$I_a = I_0 + I_0 + I_0 = 3I_0$$
$$I_b = I_0 + a^2I_0 + aI_0 = 0$$
$$I_c = I_0 + aI_0 + a^2I_0 = 0$$

Refer to Table 3.2: $\left(1 + a + a^2 = 0\right)$

Note that $I_a = 3I_0$. This is the quantity that the relay "see's" for a Single Line-to-Ground fault.

Substituting $I_0 = -j1.82\, pu$

$$I_a = 3I0 = 3(-j1.82)$$
$$= -j5.46\, pu$$

Calculating the voltage drop, the sequence voltages are

$$V_0 = -Z_0 I_0$$
$$V_1 = V - Z_1 I_1$$
$$V_2 = -Z_2 I_2$$

Substituting in the impedance and current from above

$$V_0 = -j0.199(-j1.82) = -0.362$$
$$V_1 = 1 - j0.175(-j1.82) = 0.681$$
$$V_2 = -j0.175(-j1.82) = -0.319$$

The phase voltages are converted from the sequence value using Eq. (7.6).

$$V_a = -0.362 + 0.681 - 0.319 = 0$$
$$V_b = -0.362 + a^2(0.681) + a(-0.319) = 1.022\angle 238^o\, pu$$
$$V_c = -0.362 + a(0.681) + a^2(-0.319) = 1.022\angle 122^o\, pu$$

The *per-unit* value for the current and voltage would now be converted to actual values using Eq. (5.9) and Eq. (5.10) and knowing the base power and voltage for the given system. See example 5.1 for a reference.

The currents and voltages can be shown in phasor form.
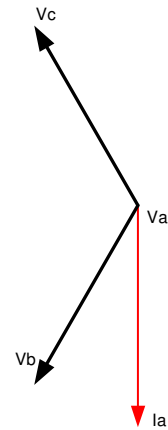
Example 8.3

Given $Z_0 = 0.199\angle 90^o\, pu$, $Z_1 = 0.175\angle 90^o\, pu$,

$Z_2 = 0.175\angle 90^o\, pu$, compute the fault current and voltages for a Line-to-Line fault. Note that the sequence impedances are in *per-unit*. This means that the solution for current and voltage will be in *per-unit*.

Solution

The sequence networks are interconnected, as shown.

Because the sequence currents sum to one node, it follows that

$$I_1 = -I_2$$

The current I_1 is the voltage drop across Z_1 in series with Z_2

$$I_1 = \frac{V_1}{Z_1 + Z_2}$$

$$I_1 = \frac{1\angle 0^o}{j0.175 + j0.175}$$

$$= -j2.86\, pu$$

$$I_2 = +j2.86\, pu$$
$$I_0 = 0$$

The phase current is converted from the sequence value using Eq. (7.8).

$$I_a = 0 - j2.86 + j2.86 = 0\, pu$$
$$I_b = 0 + a^2(-j2.86) + a(j2.86) = -4.95\, pu$$
$$I_c = 0 + a(-j2.86) + a^2(j2.86) = 4.95\, pu$$

Calculating the voltage drop, and referring to Fig. 8.7, the sequence voltages are

$$V_1 = V_2$$
$$V_2 = -Z_2 I_2$$
$$= -(j1.75)(j2.86)$$
$$= 0.5\, pu$$
$$V_0 = 0$$
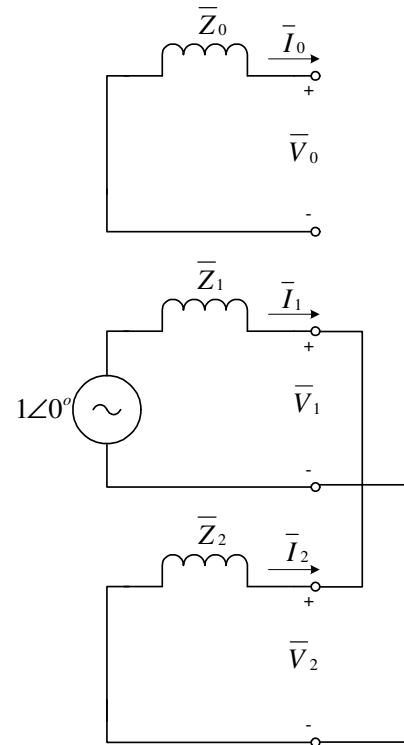
The phase voltages are converted from the sequence value using Eq. (7.6).

$$V_a = 0.0 + 0.5 + 0.5 = 1.0 \, pu$$
$$V_b = 0.0 + a^2(0.5) + a(0.5) = -0.5 \, pu$$
$$V_c = 0.0 + a(0.5) + a^2(0.5) = -0.5 \, pu$$

The *per-unit* value for the current and voltage would now be converted to actual values using Eq. (5.9) and Eq. (5.10) and knowing the base power and voltage for the given system. See example 5.1 for a reference.
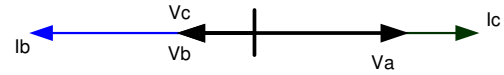
The currents and voltages can be shown in phasor form.

Example 8.4

Given $Z_0 = 0.199\angle 90^o \, pu$, $Z_1 = 0.175\angle 90^o \, pu$, $Z_2 = 0.175\angle 90^o \, pu$, compute the fault current and voltages for a Double Line-to-Ground fault. Note that the sequence impedances are in *per-unit*. This means that the solution for current and voltage will be in *per-unit*.

Solution

The sequence networks are interconnected, as shown in Fig. 8.9

Because the sequence currents sum to one node, it follows that

$$I_1 = -(I_0 + I_2)$$

The current I_1 is the voltage drop across Z_1 in series with the parallel combination of Z_0 and Z_2

$$I_1 = \frac{V_1}{Z_1 + \left(\dfrac{Z_0 Z_2}{Z_0 + Z_2} \right)}$$

Substituting in $V_1 = 1\angle 0^o$, and Z_0, Z_1, and Z_2, then solving for I_1
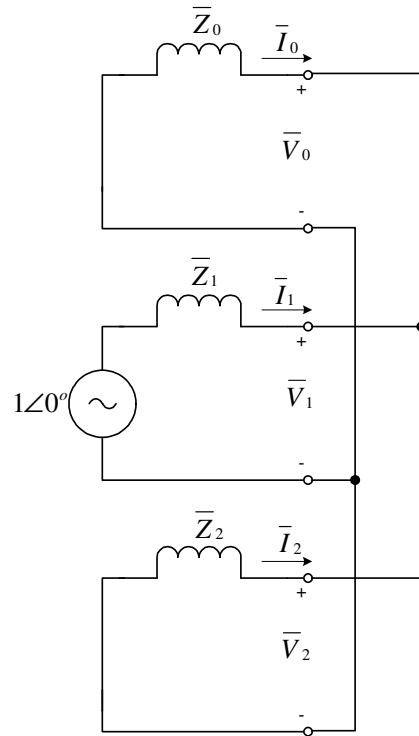
$$I_1 = -j3.73\,pu$$

$$I_0 = \frac{Z_2}{(Z_0 + Z_2)}I_1$$
$$= +j1.75$$

$$I_2 = \frac{Z_0}{(Z_0 + Z_2)}I_1$$
$$= +j1.99$$

The phase current is converted from the sequence value using Eq. (7.8).

$$I_a = j1.75 - j3.73 + j1.99 = 0\,pu$$
$$I_b = j1.75 + a^2(-j3.73) + a(j1.99) = 5.60\angle 152.1^o\,pu$$
$$I_c = j1.75 + a(-j3.73) + a^2(j1.99) = 5.60\angle 27.9^o\,pu$$

Calculating the voltage drop, and referring to Fig. 8.9, the sequence voltages are

$$V_0 = V_1 = V_2$$
$$V_0 = -Z_0 I_0$$
$$= -(j1.75)(j0.199)$$
$$= 0.348\,pu$$

The phase voltages are converted from the sequence value using Eq. (7.6).

$$V_a = 0.348 + 0.348 + 0.348 = 1.044\,pu$$
$$V_b = 0.348 + a^2(0.348) + a(0.348) = 0\,pu$$
$$V_c = 0.348 + a(0.348) + a^2(0.348) = 0\,pu$$
$$\text{Refer to Table 3.2: } \left(1 + a + a^2 = 0\right)$$

The *per-unit* value for the current and voltage would now be converted to actual values using Eq. (5.9) and Eq. (5.10) and knowing the base power and voltage for the given system. See example 5.1 for a reference.

The currents and voltages can be shown in phasor form.
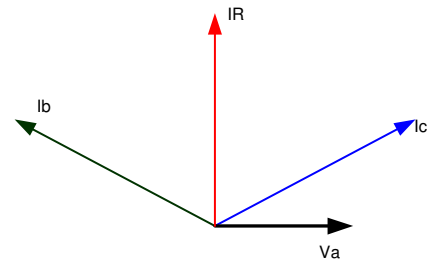
9. Oscillograms and Phasors

Attached are four faults that were inputted into a relay and then captured using the relay software.

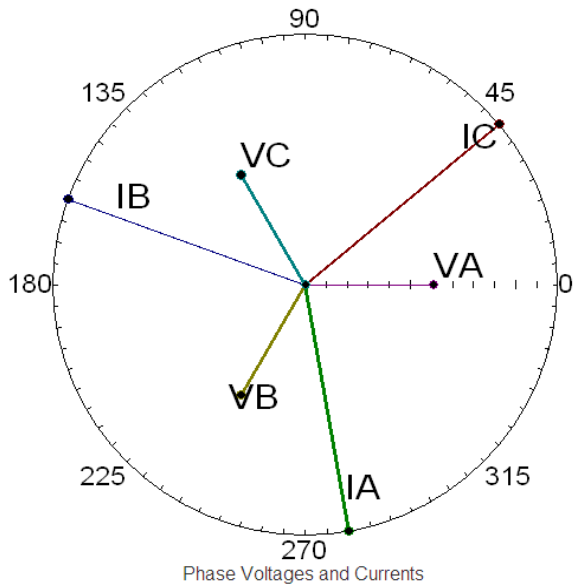Three-phase fault. Compare to example (8.1)

Fig 9.1a

Phase Voltages and Currents
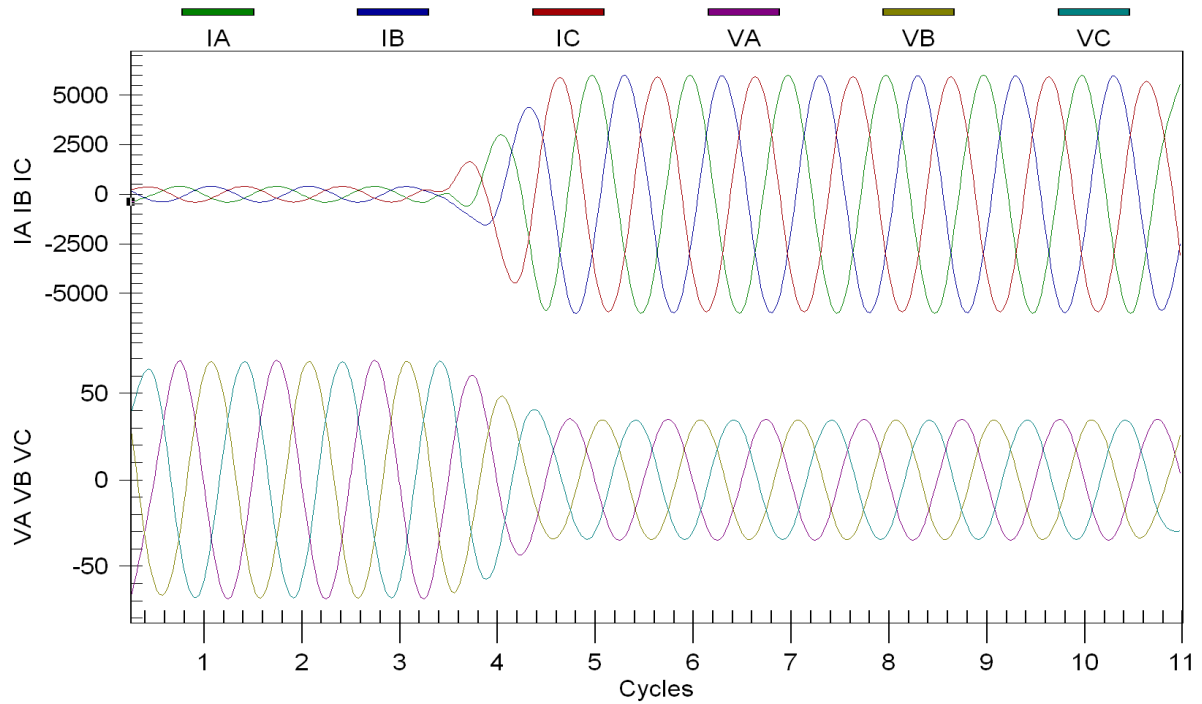
Fig 9.1b
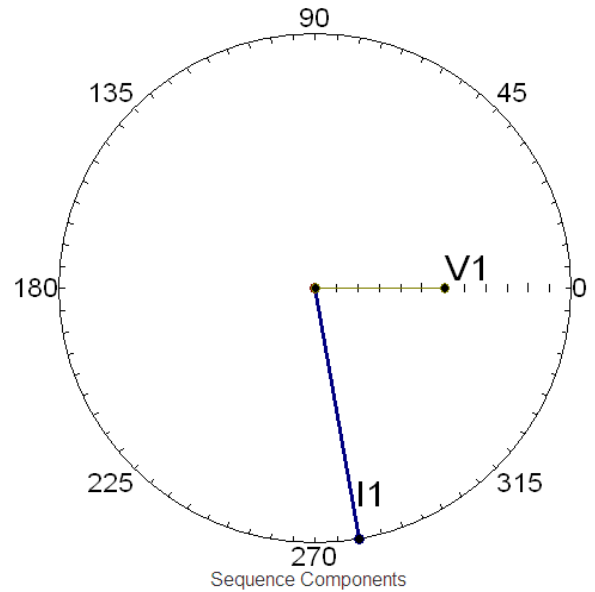
Sequence Components

Fig 9.1c

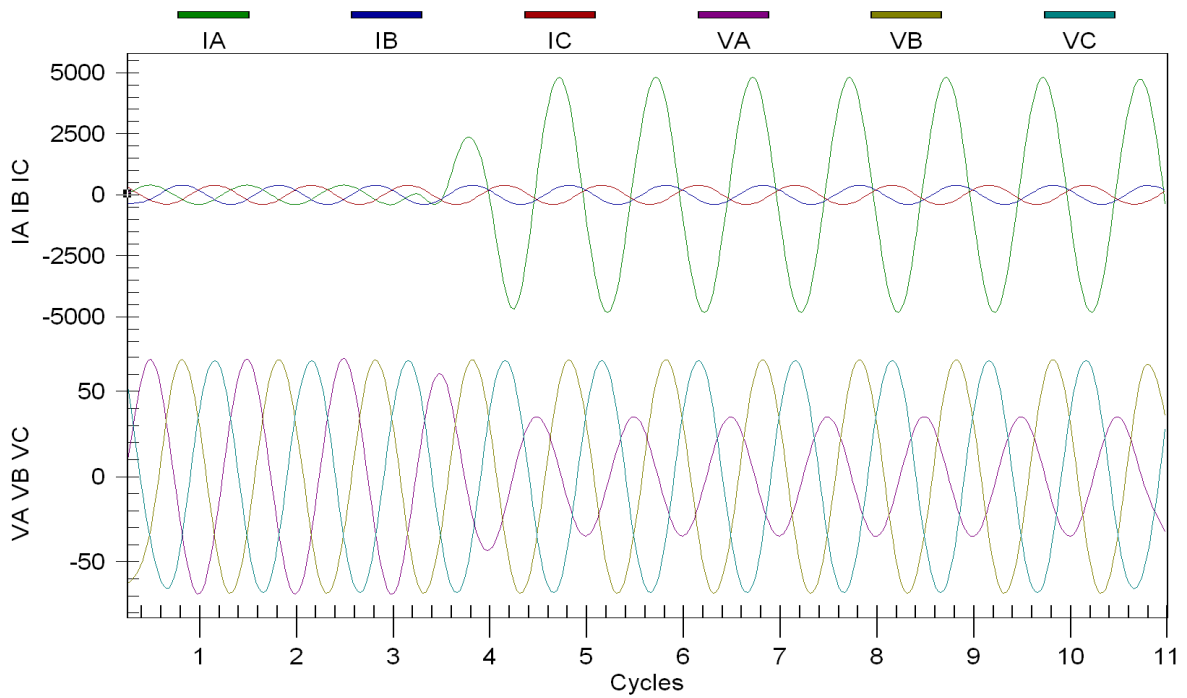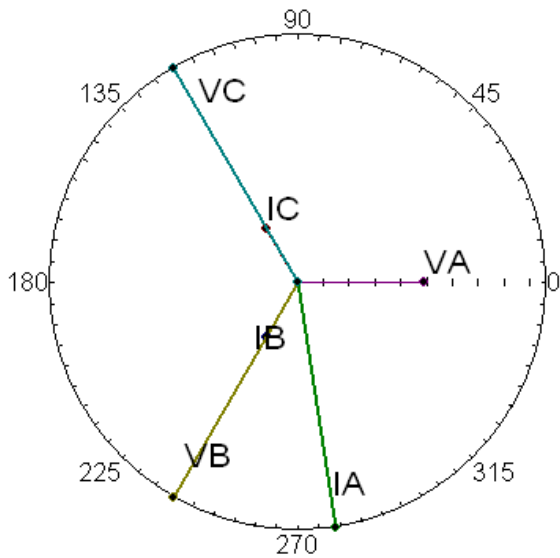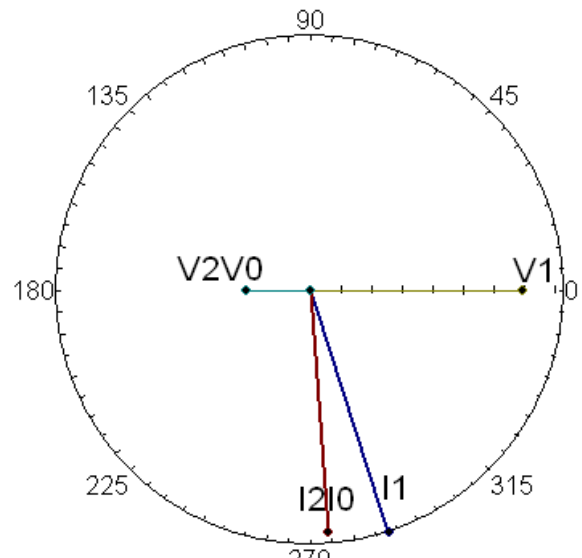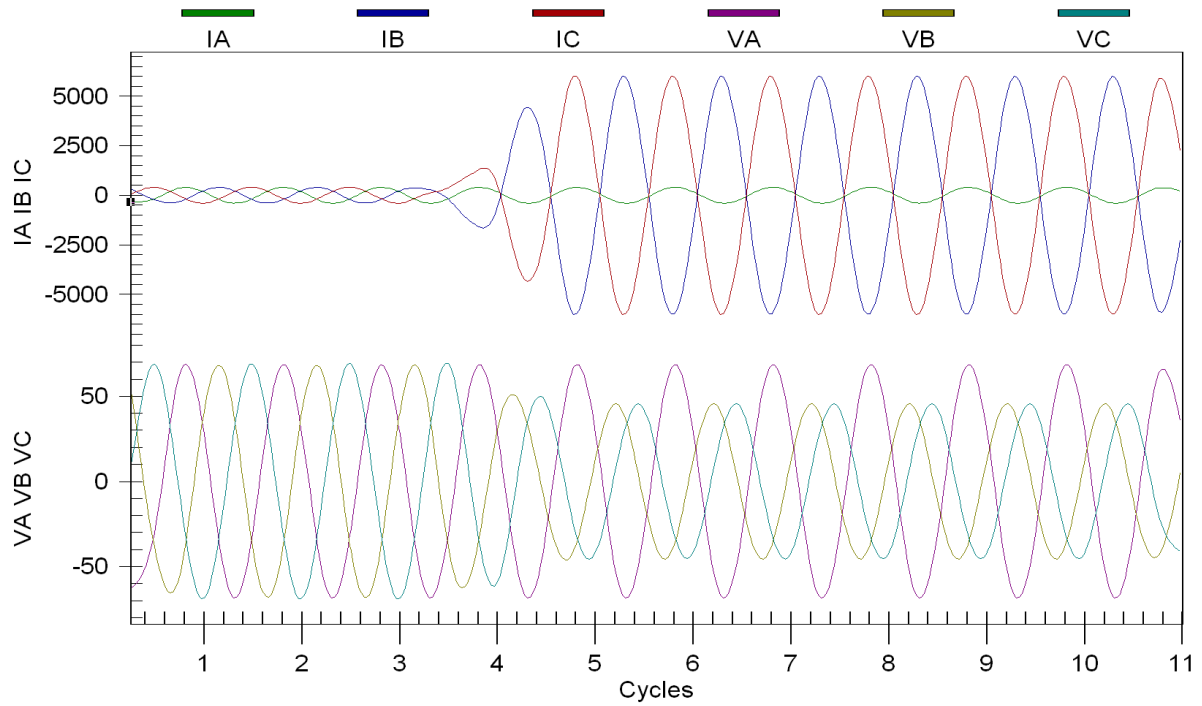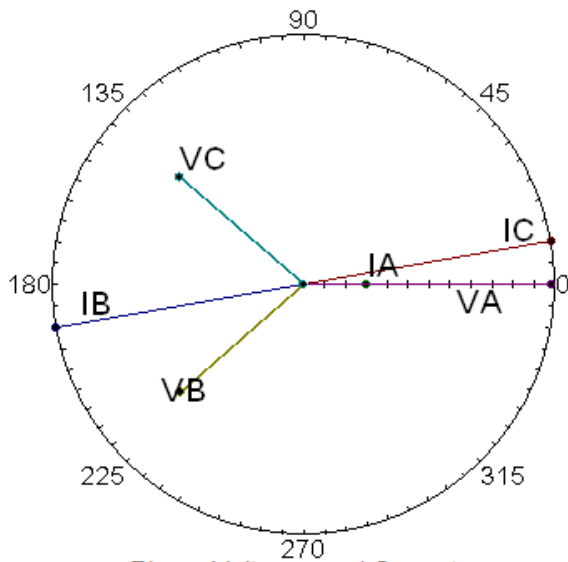Single Line-to-Ground fault. Compare to example (8.2)

Fig 9.2a

Phase Voltages and Currents
Fig 9.2b

Sequence Components
Fig 9.2c

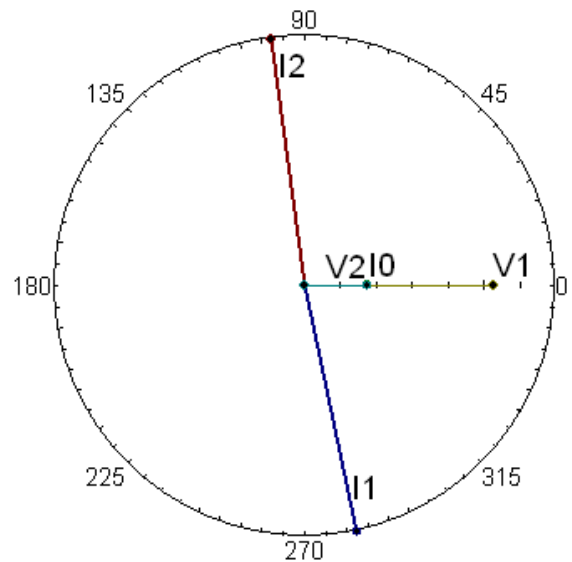Line-to-Line fault. Compare to example (8.3)

Fig 9.3a

Phase Voltages and Currents
Fig 9.3b

Sequence Components
Fig 9.3c

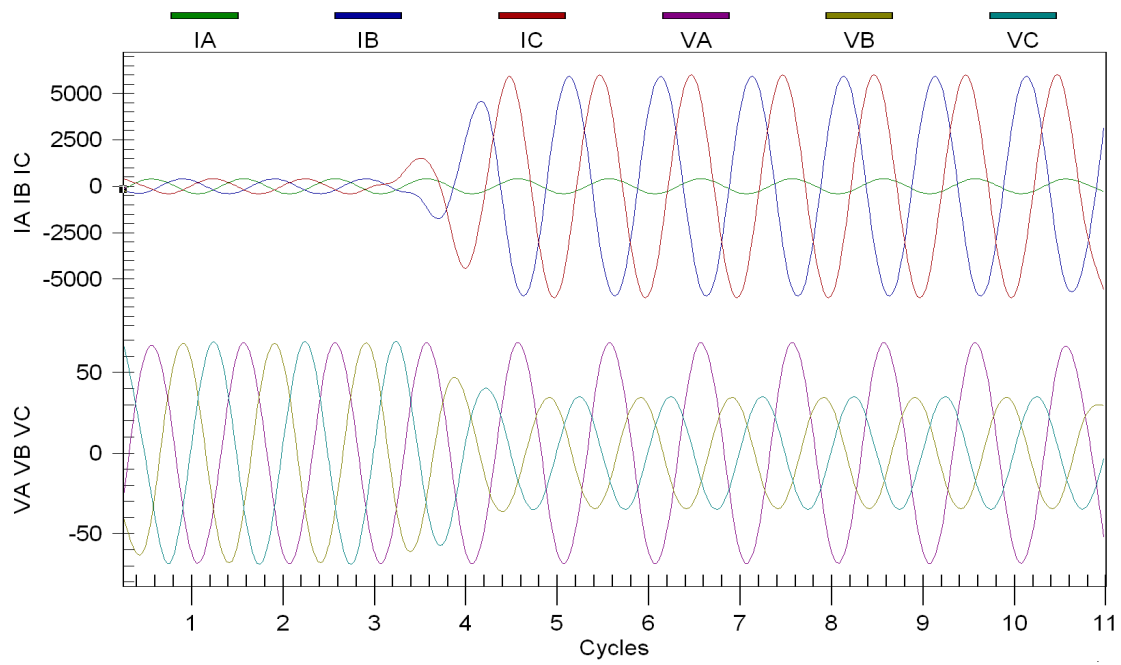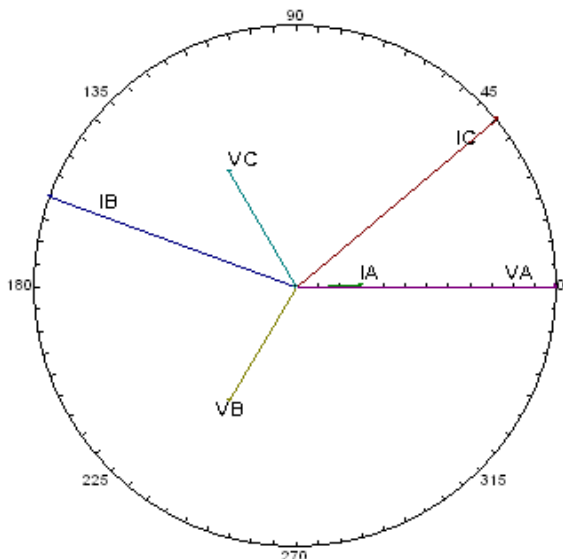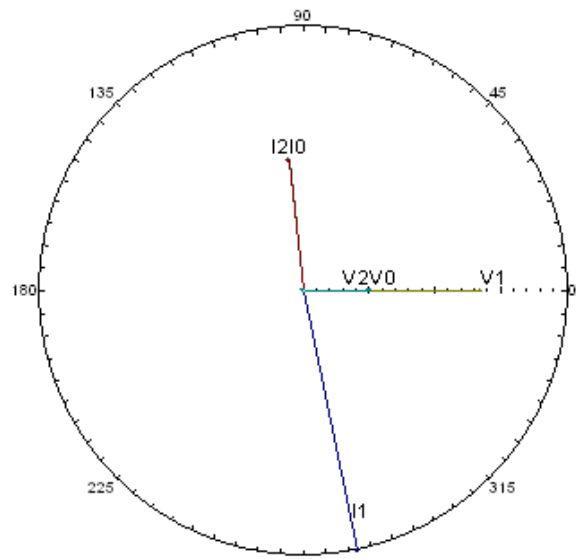Double Line-to-Ground fault. Compare to example (8.4)

Fig 9.4a

Phase Voltages and Currents
Fig 9.4b

Sequence Components
Fig 9.4c

10. Addition Symmetrical Components considerations

10.1 Symmetrical Components into a Relay

Using a directional ground distance relay it will be demonstrated how sequential components are used in the line protection. To determine the direction of a fault, a directional relay requires a reference against which the line current can be compared. This reference is known as the polarizing quantity. Zero-sequence line current can be referenced to either zero-sequence current or zero-sequence voltage, or both may be used. The zero-sequence line current is obtained by summing the three-phase currents. See Fig. 10.1
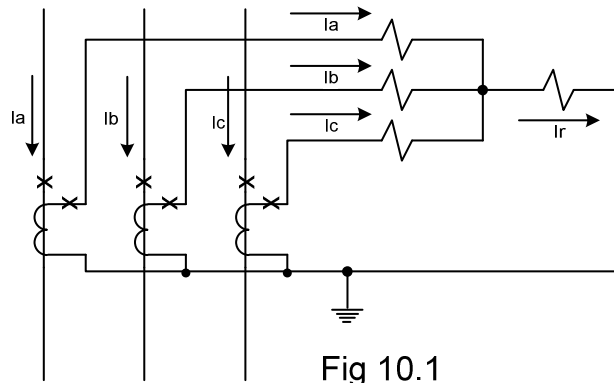
Fig 10.1

From Eq. (7.9)

$$\left(I_a + I_b + I_c\right) = 3I_0 = I_r \tag{10.1}$$

This is known as the residual current or simply $3I_0$.

The zero-sequence voltage at or near the bus can be used for directional polarization. The polarizing zero-sequence voltage is obtained by adding an auxiliary potential transformer to the secondary voltage. The auxiliary transformer is wired as a broken-delta and the secondary inputted to the relay. See Fig 10.2

Fig 10.2
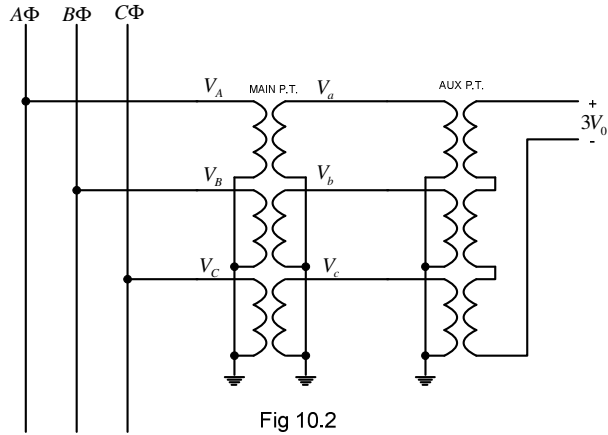
From Eq. (7.7a) the zero-sequence voltage equals

$$V_0 = \frac{1}{3}\left(V_a + V_b + V_c\right) \tag{10.2a}$$

$$3V_0 = \left(V_a + V_b + V_c\right) \tag{10.2a}$$

Example 10.1

Using the values obtained from example 8.2, calculate $3V_0$.

Solution

$$V_a = 0$$
$$V_b = 1.022\angle 238^o \; pu$$
$$V_c = 1.022\angle 122^o \; pu$$

$$3V_0 = 0 + 1.022\angle 238^o + 1.022\angle 122^o$$
$$= 1.08\angle 180^o \; pu$$

The zero-sequence voltage is $1.08\angle 180^o \; pu$. By connecting the value in the reverse gives $-3V_0$ which equals $1.08\angle 0^o \; pu$. Plotting this, we can show in phasor form what the relay see's, Ia lagging $-3V_0$ by the line angle. In this case resistance is neglected, therefore Ia lags by 90°. (see Fig 10.3).
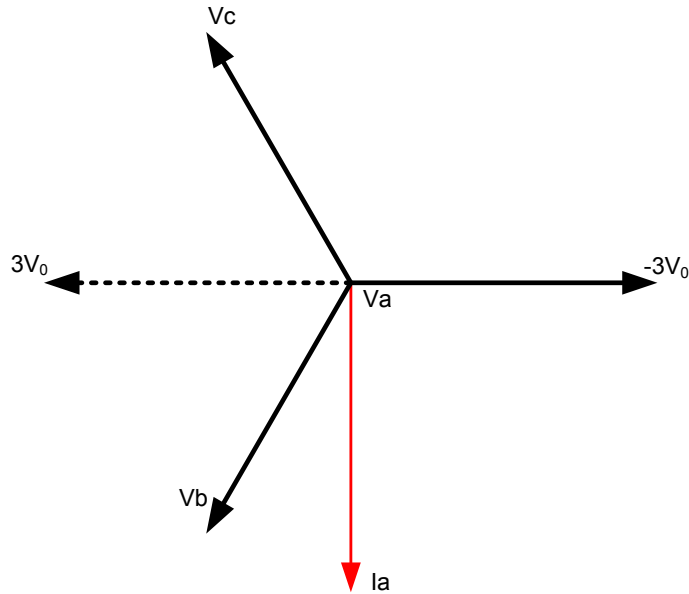
Fig 10.3

10.2 Symmetrical Components through a Transformer

This section will look at current flow through a wye-delta transformer bank. It will be shown in the next chapter that for faults that include ground that zero-sequence quantities will be generated. It can be shown using symmetrical components that zero-sequence components cannot pass through delta-wye transformer banks. If zero-sequence is flowing on the wye side, the currents will be reflected to the other side, but circulate within the delta. Fig 10.4 The current on the left side is
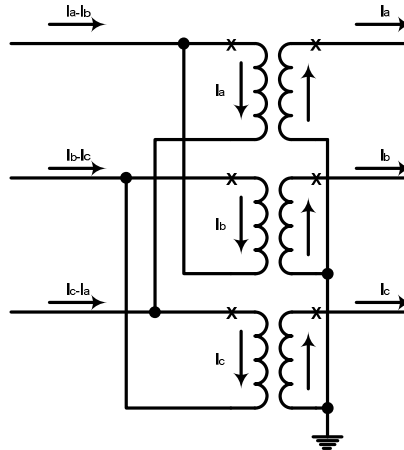
$$I_a = \frac{1}{n}\left(I_A - I_B\right)$$

Fig 10.4

From equation 7.2 we have

$$I_A = I_{A0} + I_{A1} + I_{A2} \qquad\qquad\qquad (10.3\ a)$$

$$I_B = I_{B0} + I_{B1} + I_{B2} \qquad\qquad\qquad (10.3\ b)$$

Substituting on the right side of the equation 8.1 gives

$$(I_A - I_B) = (I_{A0} - I_{B0}) + (I_{A1} - I_{B1}) + (I_{A2} - I_{B2}) \qquad\qquad (10.4)$$

The zero-sequence currents are in-phase, therefore equation 10.3 simplifies to

$$(I_A - I_B) = (I_{A1} - I_{B1}) + (I_{A2} - I_{B2}) \qquad\qquad\qquad (10.5)$$

Where $(I_{A1} - I_{B1}) = \sqrt{3} I_{A1} \angle 30^o$ and $(I_{A2} - I_{B2}) = \sqrt{3} I_{B2} \angle -30^o$

$$I_a = \frac{1}{n}(\sqrt{3} I_{A1} \angle 30^o) + (\sqrt{3} I_{B2} \angle -30^o)$$

$$I_a = \frac{\sqrt{3}}{n}(I_{A1} \angle 30^o + I_{B2} \angle -30^o) \qquad\qquad\qquad (10.6)$$

In a balanced system where there is no negative or zero-sequence current then equation 10.6 reduces to

$$I_a = \frac{\sqrt{3}}{n}(I_A \angle 30^o) \qquad\qquad\qquad (10.7)$$

As can be seen the current will shift by 30° when transferring through a transformer connected delta-wye. The same can be prove when looking at the voltages.

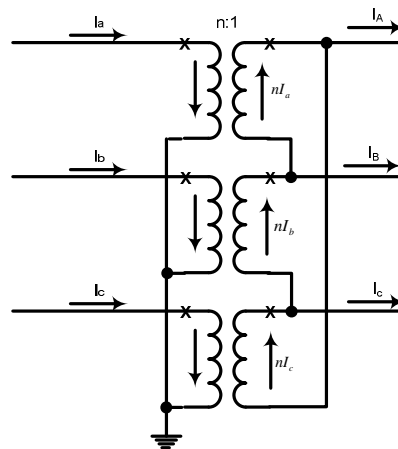Now consider the connection in Fig 10.5.

Fig 10.5

$$I_A = n(I_a - I_c)$$

Substituting equation 7.2 and reducing gives

$$(I_A - I_C) = (I_{A0} - I_{C0}) + (I_{A1} - I_{C1}) + (I_{A2} - I_{C2}) \qquad (10.8)$$

$$I_a = n(\sqrt{3}I_{A1}\angle - 30^o) + (\sqrt{3}I_{C2}\angle 30^o)$$

$$I_a = n\sqrt{3}(I_{A1}\angle - 30^o + I_{C2}\angle 30^o) \qquad (10.9)$$

As seen from the prior example equation 10.9 will reduce to

$$I_a = n\sqrt{3}(I_A \angle - 30^o)$$

if there is no negative or zero-sequence current, which is the case for a balanced system.

By inspection of the equations above for ANSI standard connected delta-wye transformer banks if the positive-sequence current on one side leads the positive current on the other side by 30°, the negative-sequence current correspondingly will lag by 30°. Similarly if the positive-sequence current lags in passing through the bank, the negative-sequence quantities will lead 30°.

The direction of the phase shifts between the delta-connected winding and the wye-connected winding depends on the winding connections of the transformer.

The winding configurations of a transformer will determine whether or not zero-sequence currents can be transformed between windings. Because zero-sequence currents do not add up to zero at a neutral point, they cannot flow in a neutral without a neutral conductor or a ground connection. If the neutral has a neutral conductor or if it is grounded, the zero-sequence currents from the phases will add together to equal 3I0 at the neutral point and then flow through the neutral conductor or ground to make a complete path.

Following are some different transformer winding configurations and their effect on zero-sequence currents

1. **Transformers with at least two grounded wye windings**

   When a transformer has at least two grounded-wye windings, zero-sequence current can be transformed between the grounded-wye windings. The I0 currents will add up to 3I0 in the neutral and return through ground or the neutral conductor. The I0 currents will be transformed into the secondary windings and flow in the secondary circuit. Any impedance between the transformer neutral points and ground must be represented in the zero-sequence network as three times its value to correctly account for the zero-sequence voltage drop across it.

   Below on the left is a three-phase diagram of a grounded-wye, grounded-wye transformer connection with its zero-sequence network model on the right. Notice the resistance in the neutral of the secondary winding is modeled by 3R in the zero-sequence network model.
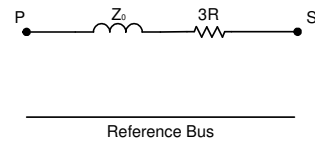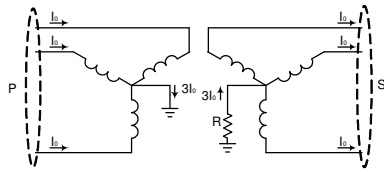
2. **Transformers with a grounded-wye winding and a delta winding**

When a transformer has a grounded-wye winding and a delta winding, zero-sequence currents will be able to flow through the grounded-wye winding of the transformer. The zero-sequence currents will be transformed into the delta winding where they will circulate in the delta without leaving the terminals of the transformer. Because the zero-sequence current in each phase of the delta winding is equal and in phase, current does not need to enter or exit the delta winding. Below on the left is a three-phase diagram of a grounded-wye-delta transformer connection with its zero-sequence network model on the right.
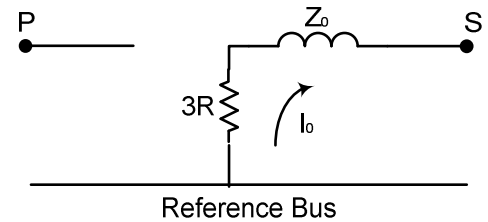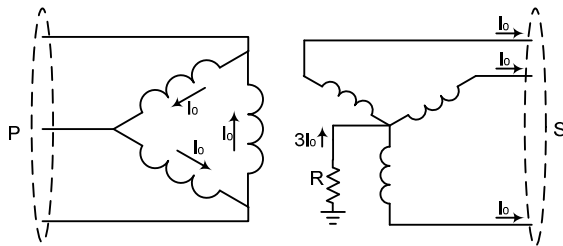
3. **Autotransformers with a grounded neutral**

Autotransformers can transform zero-sequence currents between the primary and secondary windings if the neutral is grounded. Zero-sequence current will flow through both windings and the neutral ground connection. Below on the left is a three-phase diagram of a grounded neutral autotransformer with its zero-sequence network model on the right.
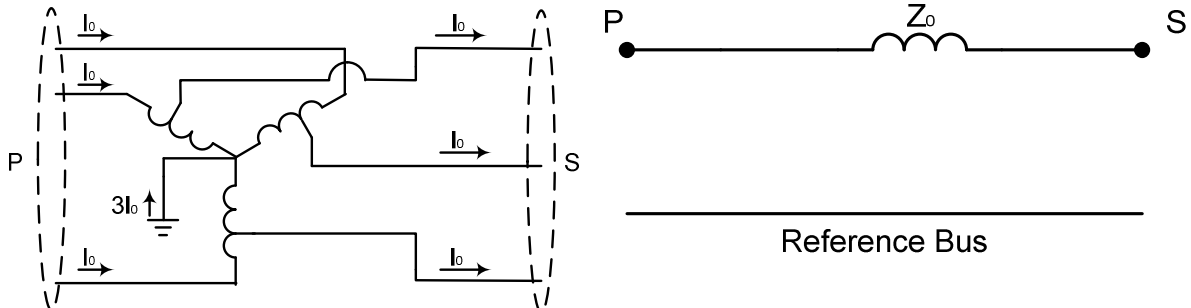
4. **Autotransformers with a delta tertiary**

If an autotransformer has a delta tertiary, zero-sequence current can flow through either the primary or secondary winding even if the other winding is open circuited in the same manner that zero-sequence current can flow in a grounded-wye-delta transformer. If the ground is removed from the neutral, zero-sequence current can still flow between the primary and secondary windings, although there will not be any transformation of currents between the primary and secondary windings—only between the partial winding between the primary and secondary terminals and the delta tertiary. This is not a normal condition though, so it will not be analyzed here.

Note that when modeling three-winding transformers the impedance needs to be broken into the impedance of the individual windings.
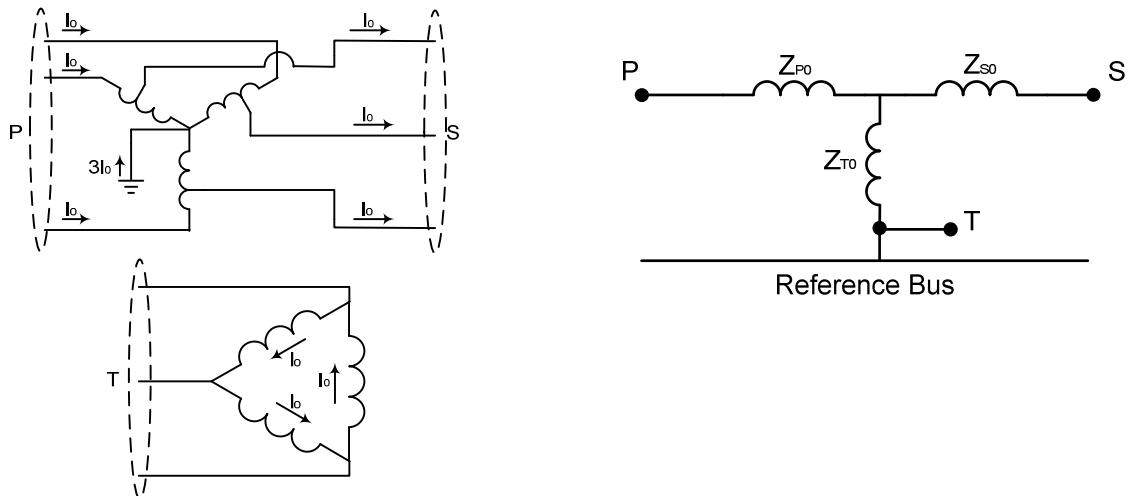
5. **Other transformers**

Other transformer configurations, such as ungrounded wye-ungrounded wye, grounded wye-ungrounded wye, ungrounded wye-delta, and delta-delta will not allow zero-sequence currents to flow and will have an open path in the zero-sequence network model. Some of these configurations are shown below with their zero-sequence network models.
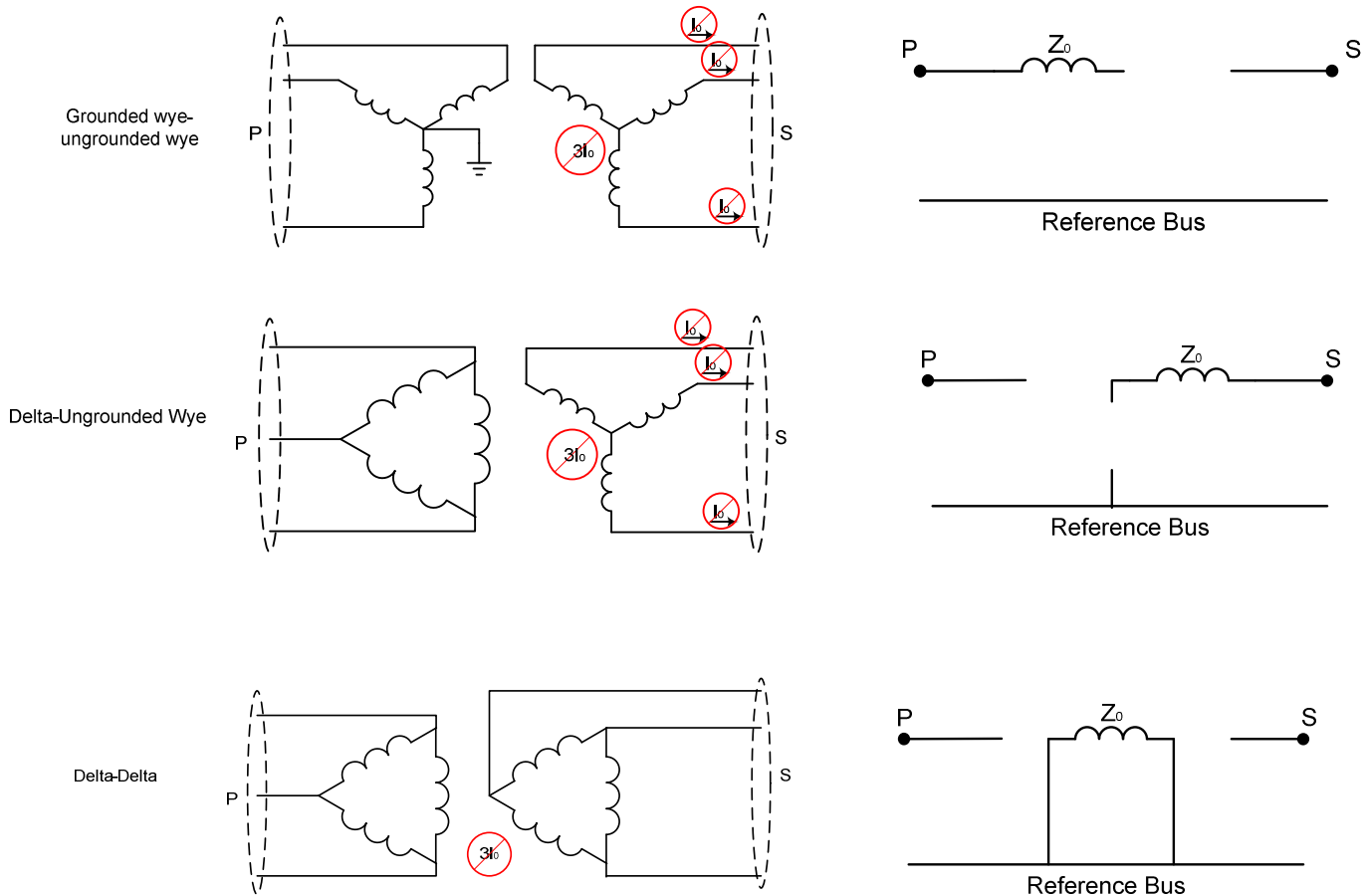
In the preceding transformer connection diagrams the values of I0 at the terminals of the primary and secondary windings will be equal on a per-unit basis. They will also have the same per-unit values within the wye and delta windings; however, the per-unit values of current within the windings of an autotransformer are somewhat more difficult to determine because part of the winding carries both primary and secondary currents. If the magnitude of current within the winding of an autotransformer needs to be known, it can be determined by equating the ampere turns of the primary winding to those of the secondary winding and solving. If a tertiary is involved, it will need to be included in the equation also.

Magnitude of transformer zero-sequence impedance

The zero-sequence impedance of a single-phase transformer is equal to the positive-sequence impedance. When three single-phase units are connected as a three-phase unit in a configuration that will transform zero-sequence currents (grounded wye-grounded wye, grounded wye-delta, etc.), the zero-sequence impedance of the three-phase unit will normally be equal to the positive-sequence impedance.

In transformers built as three-phase units, i.e. with a three-phase core, in a configuration capable of transforming zero-sequence currents, the zero-sequence impedance will be the same as the positive-sequence impedance if the transformer core is of the shell type. If the core is of the core type, the zero-sequence impedance will be different than the positive-sequence impedance. This is because the zero-sequence excitation flux does not sum to zero where the three legs of the core come together and is forced to travel outside of the iron core, through the oil or the transformer tank where the magnetic permeability is much less than the iron core. This results in a low impedance (high conductance) in the magnetizing branch of the transformer model. The larger zero-sequence magnetizing current results in a lower apparent zero-sequence impedance. Using a lower value of zero-sequence impedance in the transformer zero-sequence model is sufficient for most fault studies, but to obtain a highly accurate zero-sequence model of a three-phase core-form transformer, the magnetizing branch can not be neglected.

11. System Modeling

11.1 System Modeling: Transmission Lines

Transmission lines are represented on a one-line diagram as a simple line connecting busses or other circuit elements such as generators, transformers etc.

Transmission lines are also represented by a simple line on impedance diagrams, but the diagram will include the impedance of the line, in either ohm or per-unit values. Sometimes the resistive element of the impedance is omitted because it is small compared to the reactive element.

Here is an example of how a transmission line would be represented on an impedance diagram with impedances shown in ohms:
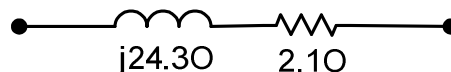
In a balanced three-phase system the impedance of the lines and loads are the same, and the source voltages are equal in magnitude. We can calculate the single-phase current, but must take into account the voltage drop across the mutual impedance caused by the other phase currents. From Fig 11.1, the voltage drop in A-phase is
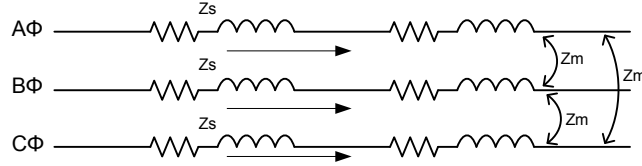
Fig 11.1

$$V_a = Z_S I_A + Z_m I_B + Z_m I_C \tag{11.1a}$$

For the case of a balanced three-phase current $(I_B + I_C) = -I_A$. Thefore:

$$V_a = (Z_S - Z_m)I_A \tag{11.1b}$$

Dividing by I_A shows the positive-sequence impedance of the line equals the self impedance minus the mutual impedance.

$$Z_{a1} = \frac{V_{A0}}{I_{A0}} = (Z_S - Z_m) \tag{11.2}$$

The negative-sequence current encounters a negative-sequence impedance which is equal to the positive-sequence impedance

$$Z_{a2} = \frac{V_A}{I_A} = (Z_S - Z_m) \tag{11.3}$$

For the zero-sequence impedance, because I_{a0}, I_{b0} and I_{c0} are in phase with each other,

$$I_{A0} = I_{B0} = I_{C0}$$

then zero-sequence voltage drop is given in equation 11.4

$$V_{a0} = Z_S I_{A0} + Z_m I_{B0} + Z_m I_{C0} = Z_S I_{A0} + (Z_m + Z_m)I_{A0} \tag{11.4a}$$
$$V_{a0} = (Z_S + 2Z_m)I_{A0} \tag{11.4b}$$

Dividing each side by I_{A0} give the zero-sequence impedance:

$$Z_{a0} = \frac{V_{A0}}{I_{A0}} = (Z_S - Z_m) \tag{11.5}$$

The result gives the zero-sequence impedance as function of the self and mutual impedance of the line. The zero-sequence impedance is always larger than the positive-sequence because we are adding two times the mutual impedance to the self impedance, instead of subtracting the mutual impedance from the self impedance.

11.2 System Modeling: Subtransient, Transient, and Synchronous Reactance of Synchronous Generators

A synchronous generator is modeled by an internal voltage source in series with an internal impedance.
Below is a typical one-line diagram symbol for a generator.

The circle represents the internal voltage source. The symbol to the left of the circle indicates that the three phases of the generator are wye-connected and grounded through a reactance. The symbol for a synchronous motor is the same as a synchronous generator.

A typical impedance diagram representation of a synchronous generator is shown in Fig. 11.2.
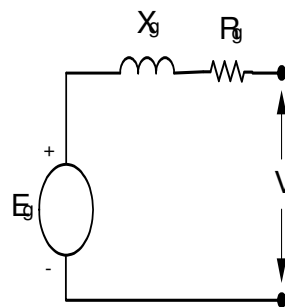

Fig. 11.2

When modeling the impedance of a synchronous generator (or motor), the resistive component is usually omitted because it is small compared to the reactive component.

When a fault is applied to a power system supplied by a synchronous generator, the initial current supplied by the generator will start at a larger value, and over a period of several cycles it will decrease from its initial value to a steady state value.

The initial value of current is called the subtransient current or the initial symmetrical rms current. Subtransient current decreases rapidly during the first few cycles after a fault is initiated, but its value is defined as the maximum value that occurs at fault inception.

After the first few cycles of subtransient current, the current will continue to decrease for several cycles, but at a slower rate. This current is called the transient current. Although, like the subtransient current, it is continually changing, the transient current is defined as its maximum value, which occurs after the first few cycles of subtransient current.

After several cycles of transient current, the current will reach a final steady state value. This is called the steady state current or the synchronous current.

Symmetrical Components

The reason why the current supplied by the synchronous generator is changing after a fault is because the increased current through the armature of the generator creates a flux that counteracts the flux produced by the rotor. This results in a reduced flux through the armature and therefore a reduced generated voltage. However, because the decrease in flux takes time, the generator voltage will be initially higher and decrease over time.

We account for the changing generator voltage in our model by using different values of reactance in series with the internal generator voltage.

We use three values of reactance to model the generator during the period after fault inception: the subtransient reactance (Xd'') is used during the initial few cycles; the transient reactance (Xd') is used for the period following the initial few cycles until a steady state value is reached; the synchronous reactance (Xd) is used for the steady state period.

The impedance diagrams for a synchronous generator (or motor) during the subtransient, transient, and synchronous periods are shown in Fig. 11.3.
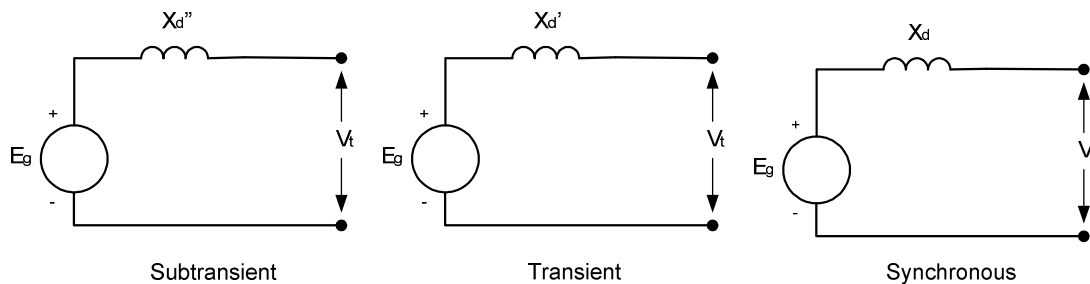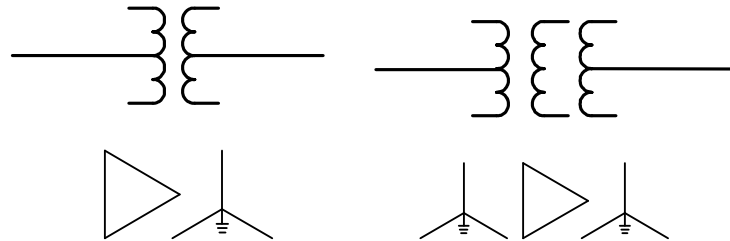

Fig. 11.3

The reactance of synchronous motors are the same as for synchronous generators. If the line to a synchronous motor develops a three-phase fault, the motor will no longer receive electrical energy from the system, but its field remains energized and the inertia of its rotor and connected load will keep the rotor turning for some time. The motor is then acting like a generator and contributes current to the fault

11.3 System Modeling: Transformers

Transformers are represented in one-line diagrams by several symbols. Below are some typical ones.

The first is a two-winding transformer connected delta- grounded wye, and the second is a three-winding transformer connected grounded wye-delta-grounded wye.

An impedance model of a practical two-winding transformer is shown in Fig. 11.4.
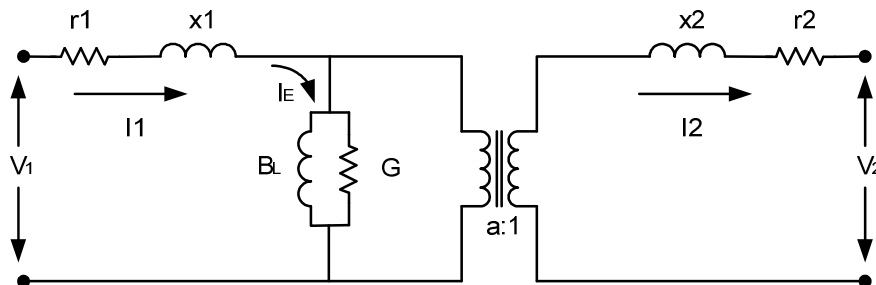

Fig. 11.4

In the model, a:1 represents the winding ratio of the ideal transformer shown by the two coupled coils, BL in parallel with G represents the magnetizing susceptance and conductance which make up the magnetizing branch, IE represents the excitation current, r1 and x1 represent the leakage impedance of winding 1,r2 and x2 represent the leakage impedance of winding 2, V1 and I1 represents the primary voltage and current respectively, and V2 and I2 represent the secondary voltage and current respectively.
Because normal fault and load currents are very much larger than the magnetizing current, IE, we can omit the magnetizing branch from our model. We can also omit the ideal transformer if we refer the leakage impedances to either the primary- or secondary-side of the transformer. The leakage impedance of one side of the transformer can be referred to the other side of the transformer by multiplying it by the square of the turns ratio. Below is the simplified impedance diagram with the magnetizing branch removed and the leakage impedance of the secondary winding referred to the primary side of the transformer.
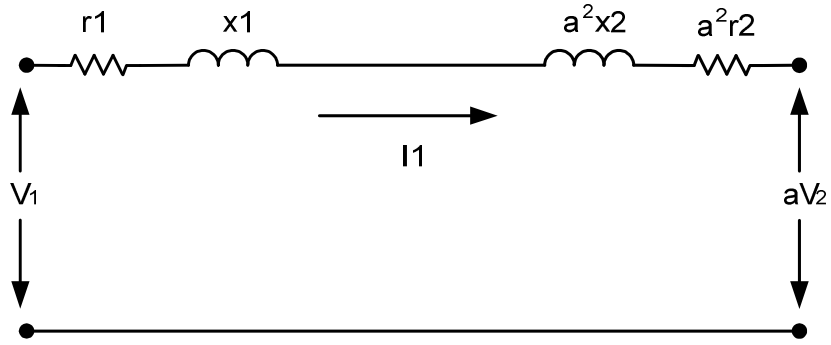
Our impedance model can be further simplified by letting
$$R1 = r1 + a^2 r^2$$
$$X1 = x1 + a^2 x^2$$
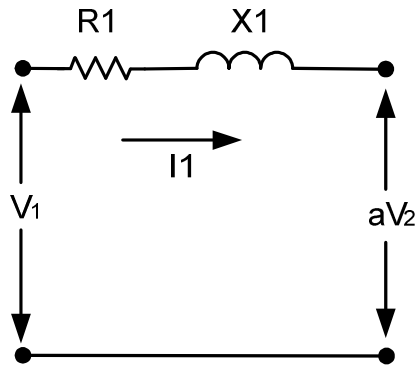
When using this simplified model, any impedances and voltages connected to the secondary side of the circuit must now be referred to the primary side.

As an example, the following transformer model will be converted to the simplified impedance model. The magnetizing branch and the leakage resistances have been omitted to simplify the problem.
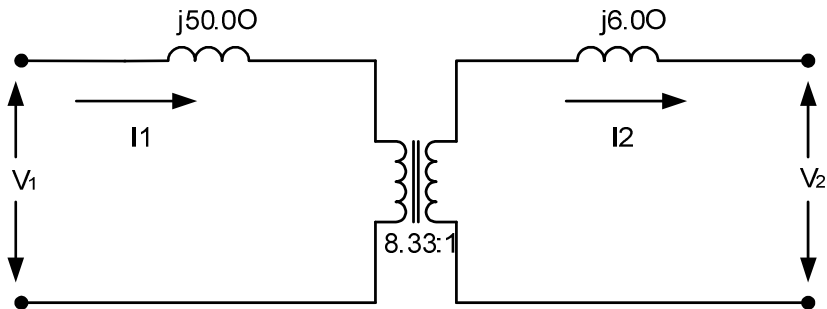
The secondary-side impedance is multiplied by the square of the turns ratio before being transferred to the primary side.
$$j6.0 * 8.33^2 = j416.3\Omega$$

This is added to the high side to get an impedance of $j50\Omega + j416.3\Omega = j466.3\Omega$
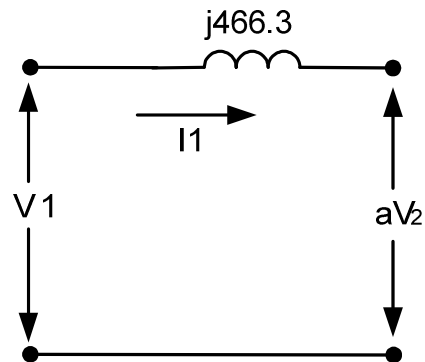
The simplified model is shown in Fig. 11.5

Fig. 11.5

11.4 Some additional points – DC Offset

In a transmission network, the sudden occurrence of a short circuit will result in a sinusoidal current that is initially larger and decreases due to the changing air gap flux in the synchronous generators. We've seen that this is modeled by subtransient, transient, and synchronous reactances in our generator model. In a circuit containing resistance and inductance (RL circuit), such as in a transmission network, the sudden occurrence of a short circuit will also result in DC offset in the current that occurs after a fault is applied. Consider the RL circuit below:
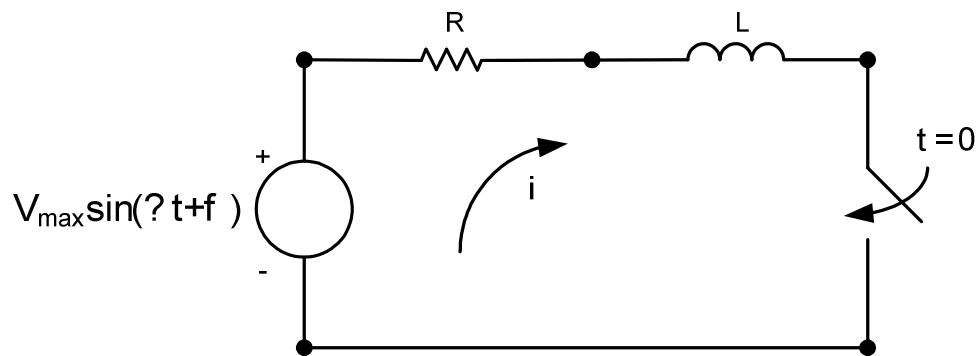
If the switch is closed at time t=0, the voltage around the circuit is
$Vmax\sin(\omega t+\varphi) = Ri + Ldi/dt$

Solving this differential equation for the instantaneous current, i, gives
$i = Vmax\ [\sin(\omega t+\varphi-\theta) - e-Rt/L\sin(\varphi-\theta)]\ /\ |Z|$

Where $|Z| = \sqrt{(R2 + (\omega L)2}$ and $\theta = \tan-1(\omega L/R)$

The important thing to note from the solution is that there is a sinusoidal component that represents the steady-state solution for the current (Vmax sin(ωt+φ-θ) / │Z│) and a exponentially decaying component (-Vmax e-Rt/Lsin(φ-θ) / │Z│).

Some points to note about the exponentially decaying—or DC offset—component:

The initial value of the DC offset is determined by what point in the cycle the voltage waveform is at when the fault occurs (the value of φ) and will range from 0 up to the value of the steady state component.

The dc component will decrease with a time constant of L/R. The larger the ratio of inductance to resistance in the circuit, the larger the time constant, and the slower the dc component will decay.

Three time constants after the switch is closed, the dc offset will have decayed to 5% of its initial value.

DC offset is an important consideration in sizing breakers.

Most modern microprocessor-based relays are immune to DC offset because after the analog signals are converted to digital signals, they can be mathematically filtered to remove the DC component. Therefore the DC component doesn't need to be considered in the relay settings.

Some electromechanical relays are immune to DC offset, and some aren't. Clapper and plunger type units are generally not immune, and DC offset will have to be allowed for in the relay settings (one guideline is to set pickup at 160% of the desired ac pickup current). Cylinder type units, used in distance relays, are immune to DC offset.

The different values of the AC fault current should be considered in the relay settings. The subtransient fault current should be used in setting instantaneous current elements, whereas the synchronous fault current should be used in current elements with long time delays.

Problems

Problem 1

BPA's system model uses a three-phase power base of 100MVA. The line-to-line voltage base is 525kV for the 500 system, 230kV for the 230 system, and 115kV for the 115 system.

a) An undervoltage relay on the 115 system is set to pick up at 0.85 pu (per unit) of the phase-to-ground voltage. What is the phase-to-ground voltage that the undervoltage relay will pick up at?

b) A three-phase fault on the 500 system results in a fault current of 2750A. What is the per unit value of this current?

c) What is the base impedance for the 500 system?

d) What is the base impedance for the 230 system?

e) What is the base impedance for the 115 system?

Problem 2

From our example 5.2, the percent impedance of a 525/241.5kV autotransformer is 10.14% based on its nameplate value of 900MVA. Suppose we need to model this transformer in BPA's ASPEN model which uses a 100MVA power base. What would the per-unit impedance be?
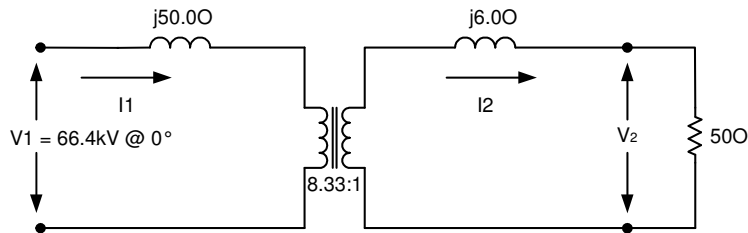
Problem 3

From our example in 5.2, convert the per-unit impedance to a per-unit value in a three-phase power base of 100MVA.
a) First convert the per unit impedance to an actual impedance (in ohms) at 525kV and then convert the actual impedance to a per-unit impedance on the new base.
b) Repeat, this time converting the per unit impedance to an actual impedance (in ohms) at 241.5kV and then converting the actual impedance to a per-unit impedance on the new base

Problem 4

Convert the per-unit impedance of the transformer in the example to a per-unit value in the BPA model with a three-phase power base of 100MVA by first converting the per unit impedance to an actual impedance (in ohms) at 230 kV and then converting the actual impedance to a per-unit impedance on the new base.


Problem 5

Using the transformer model convert from ohms to per-unit.
The voltage base for the primary side will be 115kV, and the voltage base for the
secondary side will be 13.8kV. The power base for both sides is 100MVA.

Problem 6

Below is a one line diagram of a partial power system.
The two generators are identical, each rated 13.8kV and 50MVA with a subtransient
reactance of $X_d" = 15\%$. The two generators are tied to a common bus which is
connected to a transmission line with a delta-grounded wye transformer rated at
150MVA, 13.8kV/115kV and an impedance of 9.7%. The transmission line is 30 miles
long and has an impedance of $5.43 + j22.5\Omega$. At the end of the transmission line is a
grounded wye-grounded wye transformer, rated 225MVA, 115kV/230kV with an
impedance of 7.4% that connects the line to a 230kV bus. The remaining power system
connected to the 230kV bus is not shown.
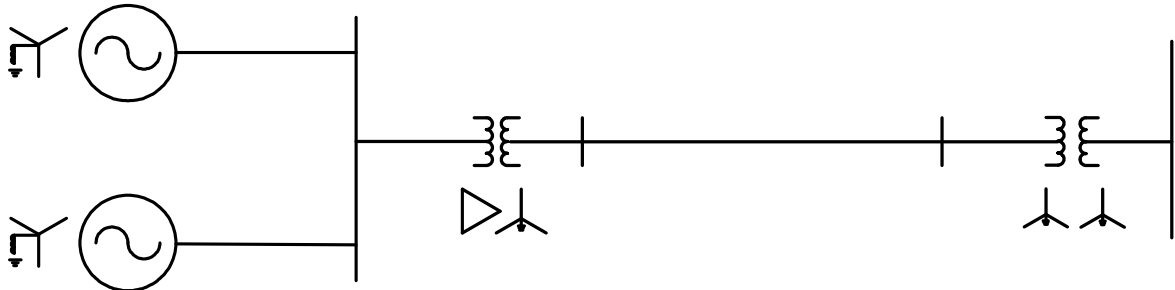
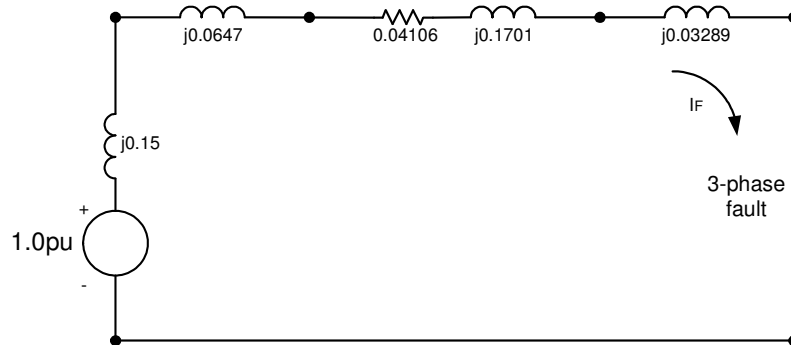
From the above information, draw the impedance diagram with impedances shown in
their per-unit values. Use voltage bases of 13.8kV, 115kV, and 230kV for the
corresponding parts of the system, and use a power base of 100MVA for the whole
system.

Problem 7

From the impedance diagram, determine the per-unit and ampere values of subtransient
current in each generator and at the fault for a three-phase fault applied on the 230kV bus
with both generators operating at 1.0pu voltage.

The generators can be combined into their Thevenin equivalent as shown below.

Problem 8

From the one line diagram of a partial power system that we used in problem 6.

From the above information, we drew the positive-sequence impedance diagram using subtransient impedances for the generators and with impedances shown in their per-unit values. Normally the positive-sequence network is drawn with the reference bus (which is the neutral point) shown at the top instead of the bottom.

The negative-sequence reactance of the generators is equal to their positive-sequence subtransient reactance. Draw the positive and negative-sequence networks for the power system with impedances shown in their per-unit values.

Problem 9

Each generator has a zero-sequence reactance of 5% and is grounded through a reactance of 2Ω. The transmission line has a zero-sequence impedance of 12.9 + j75.9Ω. The grounded wye-grounded wye transformer has a zero-sequence reactance of 4.8%.
Draw the zero-sequence impedance diagram.

Solutions

Problem 1

a) $V_{BL\text{-}G} = V_{BL\text{-}L} / \sqrt{3}$
   $V_{BL\text{-}G} = 115kV / \sqrt{3} = 66.4kV$

   $Z_{PU} = Z_A / Z_B$
   $Z_A = Z_{PU}*Z_B$
   $Z_A = 0.85*66.4kV$
   $Z_A = 56.4kV$

b) $I_B = P_{B3\Phi} / \sqrt{3}*V_{BL\text{-}L}$
   $I_B = 100\text{x}10^6 / \sqrt{3}*525\text{x}10^3$
   $I_B = 110.0 \ A$

   $I_{PU} = I_A / I_B$
   $I_{PU} = 2750 \ A / 110 \ A$
   $I_{PU} = 25.0 \ pu$

c) $Z_B = V_{BL\text{-}L2} / P_{B3\Phi}$
   $Z_B = (525\text{x}10^3)^2 / 100\text{x}10^6$
   $Z_B = 2756.25\Omega$

d) $Z_B = V_{BL\text{-}L2} / P_{B3\Phi}$
   $Z_B = (230\text{x}10^3)^2 / 100\text{x}10^6$
   $Z_B = 529.0\Omega$

e) $Z_B = V_{BL\text{-}L2} / P_{B3\Phi}$
   $Z_B = (115\text{x}10^3)^2 / 100\text{x}10^6$
   $Z_B = 132.25\Omega$

Problem 2

$Z_{pu\ new} = Z_{pu\ old} *(V_{BL\text{-}L\ old} / V_{BL\text{-}L\ new})^2 * (P_{B3\Phi\ new} / P_{B3\Phi\ old})$

$Z_{pu\ old} = 10.14 / 100 = 0.1014$
$V_{BL\text{-}L\ old} = 525kV, \quad P_{B3\Phi\ old} = 900MVA$
$V_{BL\text{-}L\ new} = 525kV, \quad P_{B3\Phi\ new} = 100MVA$

$Z_{pu\ new} = 0.1014 *(525kV / 525kV)^2 * (100MVA / 900MVA)$
$Z_{pu\ new} = 0.1014 *1* (100 / 900)$
$Z_{pu\ new} = 0.01127 \ pu$

Problem 3

$Z_{PU} = Z_A / Z_B$
$Z_A = Z_{PU} * Z_B$

$Z_B = V_{BL\text{-}L}{}^2 / P_{B3\Phi}$

a) Using the <u>high-side</u> voltage:
   $Z_{B\ old} = 525{,}000^2 / 900\text{x}10^6$
   $Z_{B\ old} = 306.25\Omega$

   $Z_A = 0.1014 * 306.25$
   $Z_A = 31.05\Omega$

   Converting to the 100MVA base:
   $Z_{B\ new} = V_{BL\text{-}L\ new}{}^2 / P_{B3\Phi\ new}$
   $Z_{B\ new} = 525{,}000^2 / 100\text{x}10^6$
   $Z_{B\ new} = 2756.25\Omega$

   $Z_{PU\ new} = Z_A / Z_{B\ new}$
   $Z_{PU\ new} = 31.05\Omega / 2756.25\Omega$
   $Z_{PU\ new} = 0.01127\ pu$

b) Using the <u>low-side</u> voltage:
   $Z_{B\ old} = \mathbf{241{,}500}^2 / 900\text{x}10^6$
   $Z_{B\ old} = 64.80\Omega$

   $Z_A = 0.1014 * 64.80$
   $Z_A = 6.57\Omega$

   Converting to the 100MVA base:
   $Z_{B\ new} = V_{BL\text{-}L\ new}{}^2 / P_{B3\Phi\ new}$
   $Z_{B\ new} = \mathbf{230{,}000}^2 / 100\text{x}10^6$
   $Z_{B\ new} = 529.0\Omega$

   $Z_{PU}\ new = Z_A / Z_{B\ new}$
   $Z_{PU}\ new = 6.57\Omega / 529.0\Omega$
   $Z_{PU}\ new = 0.01242\ pu$

Problem 4

Repeat problem 3 assuming the transformer has a tap with a ratio of 525 /230 kV and using the low side voltage.

Problem 5

<u>Answer:</u>

The base impedance of the secondary side is $Z_B = V_{BL-L2} / P_{B3\Phi}$

$Z_B = (13.8*10^3)2 / 100*10^6$
$Z_B = 1.904\Omega$

The per-unit impedance of the secondary leakage reactance is
$X2 = j6.0 / 1.094 = j3.151$ pu
The per-unit value of the load resistance is $R_L = 50 / 1.904 = 26.26$ pu

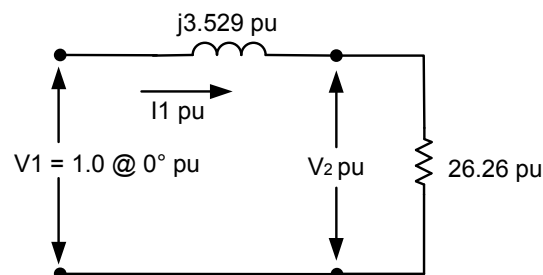The base impedance of the primary side is $Z_B = V_{BL-L2} / P_{B3\Phi}$

$Z_B = (115*10^3)2 / 100*10^6$
$Z_B = 132.25\Omega$

The per-unit impedance of the primary leakage reactance is
$X1 = j50.0 / 132.25 = j0.3781$ pu

The total per-unit impedance of our model can be obtained by simply adding together the per-unit values of the primary and secondary impedances.

$X = X1 + X2 = j0.3781 + j3.151 = j3.529$ pu

Problem 6

Answer:

Converting the impedances to per-unit on a 100MVA base using
$Z_{pu\ new} = Z_{pu\ old} * (V_{BL-L\ old} / V_{BL-L\ new})^2 * (P_{B3\Phi\ new} / P_{B3\Phi\ old})$

Each generator subtransient reactance is $X_d" = j0.15 * (13.8kV / 13.8kV)^2 * (100MVA / 50MVA)$
$X_d" = j0.30$ pu

The 13.8kV / 115kV transformer impedance is $X = 0.097 * (13.8kV / 13.8kV)^2 *$ (100MVA / 150MVA)
$X = j0.06467$ pu

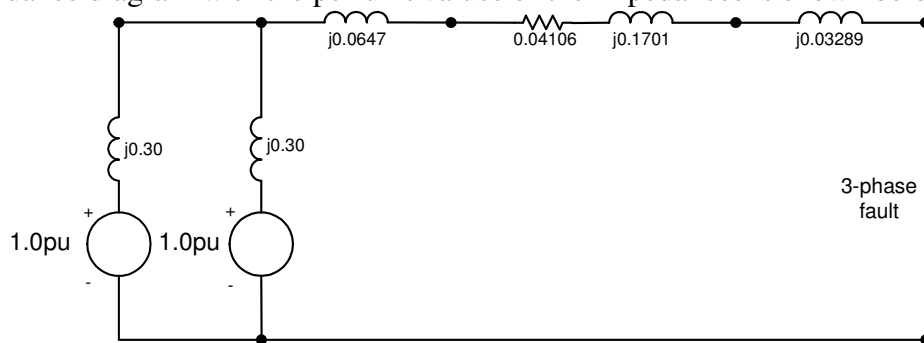The base impedance for the 115kV line is $Z_B = V_{BL-L2} / P_{B3\Phi}$
$Z_B = (115 \times 10^3)^2 / 100 \times 10^6 = 132.25\Omega$
The per-unit impedance of the 115kV transmission line is $(5.43+j22.5) / 132.25 =$ 0.04106+j0.1701 pu

The 115kV / 230kV transformer impedance is $X = 0.074 * (115kV / 115kV)^2 *$ (100MVA / 225MVA)
$X = j0.03289$ pu

The impedance diagram with the per-unit values of the impedances is shown below.

Problem 7

Answer:

The fault current is
$I_F = 1.0 / (0.04106 + j0.15 + j0.0647 + j0.1701 + j0.03289)$
$I_F = 1.0 / (0.04106 + j0.41769)$
$I_F = 2.382$ @ $-84.4°$ pu

At the generators, the total fault current is $I_{FGT} = 2.382 * I_B$
$I_B = P_{B3\Phi} / \sqrt{3}*V_{BL-L} = 100x10^6 / \sqrt{3}*13.8x10^3 = 4184$ A
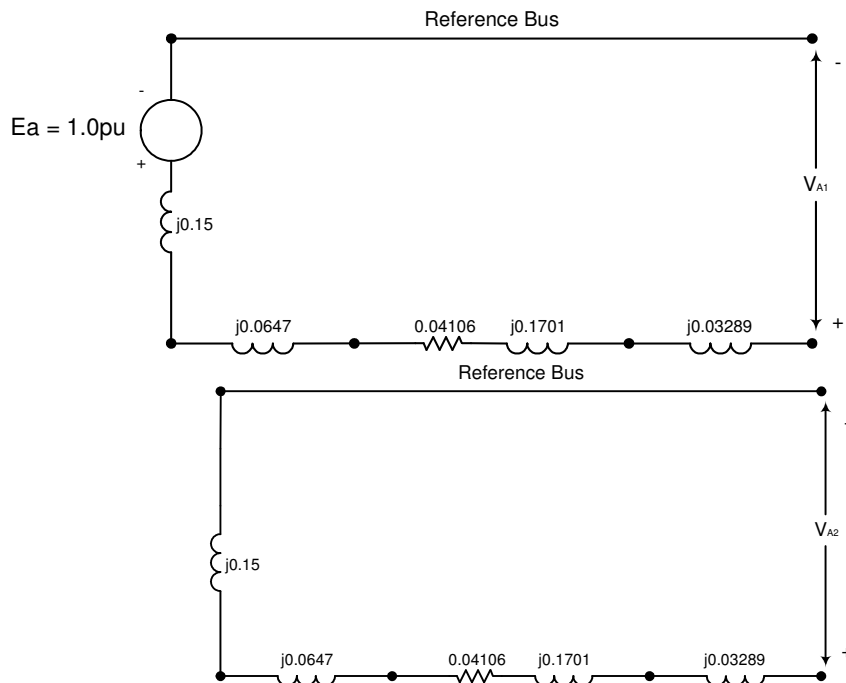$I_{FGT} = 2.382 * 4184 = 9966$ A
Each generator contributes half of this current
$I_{FG} = 9966 / 2 = 4983$ A

At the fault, the total fault current is $I_F = 2.382 * I_B$
$I_B = P_{B3\Phi} / \sqrt{3}*V_{BL-L} = 100x10^6 / \sqrt{3}*230x10^3 = 251.0$ A
$I_F = 2.382 * 251.0 = 597.9$ A

Problem 8

Problem 9

The zero-sequence reactance of each generator is 5%, or 0.05pu on a 13.8kV, 50MVA base. Converting this to a 100MVA base gives

$Z_{pu\ new}$ = j0.05 * (100 / 50) = j0.10 pu

Each generator is grounded through a reactance of 2Ω. The base impedance at 13.8kV, 100MVA is ZB = (13.8x103)2 / (100x106) = 1.9044Ω. The per-unit impedance of each grounding reactor is Zpu = j2.0 / 1.9044 = j1.05pu. The grounding reactances will need to be multiplied by three for the zero-sequence network, giving a value of 3 * j1.05 = j3.15pu.

Because a value is not given for the zero-sequence impedance of the delta-grounded wye transformer, it can be assumed that the zero-sequence impedance is the same as the positive-sequence impedance.

The zero-sequence impedance of the transmission line is 12.9 + j75.9Ω. The base impedance at 115kV, 100MVA is $Z_B = (115 \times 103)^2 / (100 \times 10^6) = 132.25Ω$. Converting the zero-sequence line impedance to a per-unit value gives Z_{L0} = (12.9 + j75.9) / 132.25 = 0.0975 + j0.574pu.

The zero-sequence impedance of the grounded wye-grounded wye transformer is 4.8%, or j0.048pu on a base of 115kV, 225MVA. Converting to a 115kV, 100MVA base gives

$Z_{pu\ new}$ = j0.048 * (100 / 225) = j0.0213 pu

The zero-sequence network is shown below. Notice the interruption in the path caused by the delta-wye transformer.
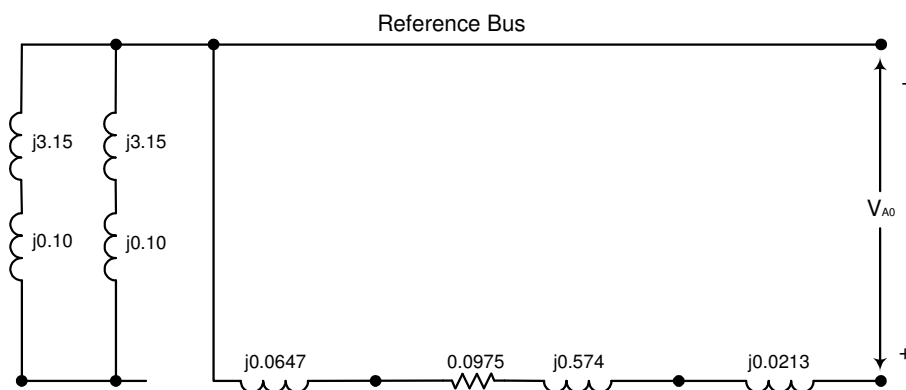
Here is a simplified version of the zero-sequence network with the two generator branches combined into an equivalent branch.
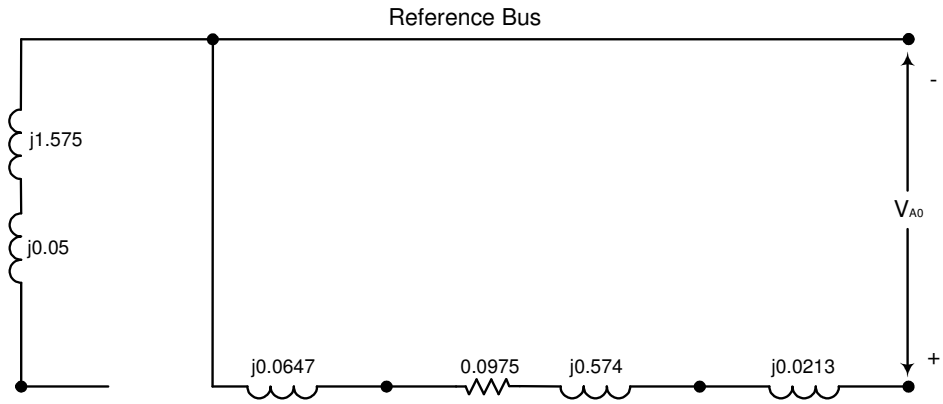
Appendix

Three Phase System

$S = \sqrt{3}V_{LL}I_L$, $P = \sqrt{3}V_{LL}I_L \cos\Theta$, $Q = \sqrt{3}V_{LL}I_L \sin\Theta$

Per-Unit

First step in using *per-unit* is to select the base(s) for the system.

    Sbase = Power base, in VA   Sbase = 100 MVA

    Vbase = voltage base in V   Vbase = Nominal voltage rated line-to-line

$$per-unit = \frac{actual\_value}{base\_value} \qquad\qquad per-unit = \frac{percent\_value}{100}$$

$$\frac{V}{V_{base}} = \frac{IZ}{I_{base}Z_{base}} \qquad\qquad V_{pu} = I_{pu}Z_{pu}$$

$$I_{base} = \frac{kVA_{base}}{\sqrt{3}kV_{base}} amperes \qquad\qquad \boxed{I_{base} = \frac{100kVA_{base}}{\sqrt{3}(230)V_{base}} amperes = 251A}$$

Ex: 230kV base, 100MVA base

$$MVA_{Fault} = \frac{MVA_{Base}}{Z_{Fault}PU} \qquad\qquad I_{Fault\_Current} = \frac{I_{Base}}{Z_{Fault}PU}$$

$$Z_{base} = \frac{kV_{base}^2 x1000}{kVA_{base}} ohms \text{ (in kVA)} \qquad\qquad Z_{base} = \frac{kV_{base}^2}{MVA_{base}} ohms \text{ (in MVA)}$$

$$Z_{base} = \frac{V_{base}^2}{100} \text{ (for a 100 MVA base)}$$

$$Z_{pu} = \frac{Z(\Omega)}{Z_{base}} \qquad\qquad Z_{pu} = \left(\frac{MVA_{base}}{kV_{base}^2}\right) \cdot Z(\Omega) \text{ (in MVA)}$$

$$\%Z = \frac{100MVA_{base} \cdot Z(\Omega)}{kV_{base}^2} \text{ (percent in MVA)}$$

$$Z_{ohm}^{new} = Z_{ohm}^{old} \cdot \left(\frac{kV_{base}^{new}}{kV_{base}^{old}}\right)^2 \text{ (new impedance reflective through a transformer)}$$

$$\boxed{Z_{ohm}^{new} = 7.2 \cdot \left(\frac{115}{230}\right)^2 = 1.8 ohms}$$

Ex: 115kV line impedance on the 115kV side of a 230/115kV transformer

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Symmetrical Components

a Operator

$$a = 1\angle 120^o$$
$$a^2 = 1\angle 240^o$$
$$a^3 = 1$$

$$V_a = V_0 + V_1 + V_2$$
$$V_b = V_0 + a^2V_1 + aV_2$$
$$V_c = V_0 + aV_1 + a^2V_2$$

$$V_0 = \frac{1}{3}\left(V_a + V_b + V_c\right)$$
$$V_1 = \frac{1}{3}\left(V_a + aV_b + a^2V_c\right)$$
$$V_2 = \frac{1}{3}\left(V_a + a^2V_b + aV_c\right)$$

$$I_a = I_0 + I_1 + I_2$$
$$I_b = I_0 + a^2I_1 + aI_2$$
$$I_c = I_0 + aI_1 + a^2I_2$$

$$I_0 = \frac{1}{3}\left(I_a + I_b + I_c\right)$$
$$I_1 = \frac{1}{3}\left(I_a + aI_b + a^2I_c\right)$$
$$I_2 = \frac{1}{3}\left(I_a + a^2I_b + aI_c\right)$$

$3I_0 = \left(I_a + I_b + I_c\right)$ (residual currents or sum of the three phase currents)

Three-Phase fault

$$MVA_{Fault} = \frac{MVA_{Base}}{Z_{Fault}\,pu}$$
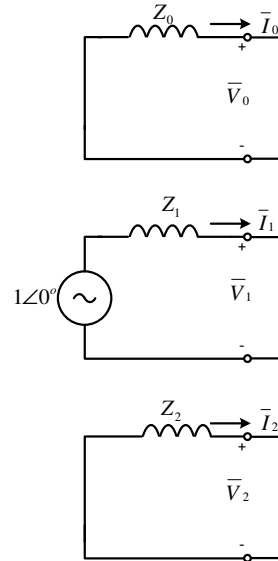

$$I_1 = \frac{E_a}{Z_1}$$

$$I_2 = I_0 = 0$$

$$I_A = I_1 = \left(\frac{1}{Z_1}\right)\left(\frac{100kVA}{\sqrt{3}\cdot kV}\right)$$

$$I_B = a^2 I_A$$

$$I_B = a I_A$$

$$E_1 = 1 - I_1 Z_1$$

$$E_2 = E_0 = 0$$

0ne-line to ground fault

$$MVA_{Fault} = \frac{3\cdot MVA_{Base}}{Z_1 + Z_2 + Z_0\,pu}$$

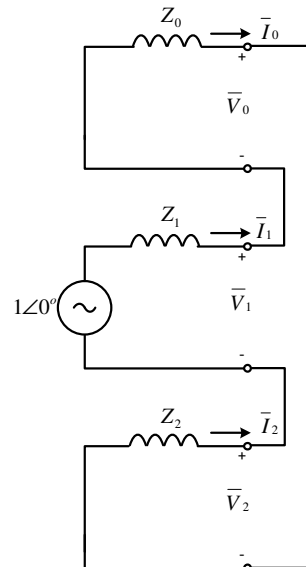$$I_0 = I_1 = I_2 = \frac{1}{Z_1 + Z_2 + Z_0}$$

$$I_A = I_0 + I_1 + I_2 = 3I_0$$

$$I_B = I_c = 0$$

$$E_1 = 1 - I_1 Z_1$$

$$E_2 = -I_2 Z_2$$

$$E_0 = -I_0 Z_0$$

Line-Line fault, or Phase-to-phase fault

$$I_1 = -I_2 = \frac{1}{Z_1 + Z_2} = \frac{1}{2Z_1}$$

$$I_0 = 0$$

$$I_A = 0$$

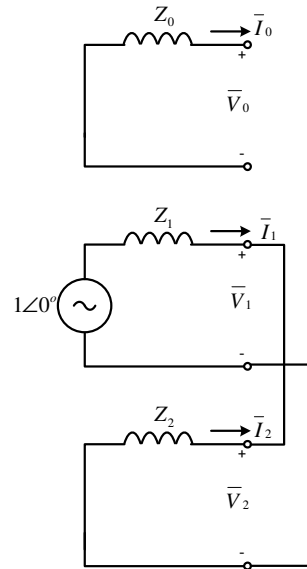$$I_B = I_0 + a^2 I_1 + a I_2 = a^2 I_1 + a I_2 = a^2 I_1 + a I_1$$

$$I_B = \frac{(a^2 - a)E}{Z_1 + Z_2}$$

$$I_C = -I_B \text{ when } Z_1 = Z_2$$

$$E_1 = 1 - I_1 Z_1$$

$$E_2 = -I_2 Z_2 = E_1$$

$$E_0 = 0$$

Double Line-Line fault, or Two phase to Ground fault

$$I_1 = \frac{1}{Z_1 + \left(\frac{Z_0 Z_2}{Z_0 + Z_2}\right)}$$
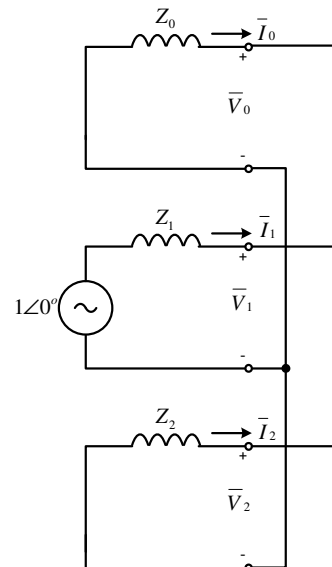
$$I_2 + I_0 = -I_1$$

$$I_2 = \frac{Z_0 I_1}{Z_0 + Z_2}$$

$$I_0 = \frac{Z_2 I_1}{Z_0 + Z_2}$$

$$E_1 = 1 - I_1 Z_1$$

$$E_2 = -I_2 Z_2 = E_1$$

$$E_0 - I_0 Z_0 = E_1$$

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