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An Introductory Course on Mathematical Game Theory

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Preface

This is a mathematical book on game theory and, thus, starts with a definition of game theory. This introduction also provides some preliminary considerations about this field, which mainly lies in the boundary between mathematics and economics. Remarkably, in recent years, the interest for game theory has grown in utterly disparate disciplines such as psychology, computer science, biology, and political science.

A definition of game theory. Game theory is the mathematical theory of interactive decision situations. These situations are characterized by the following elements: (a) there is a group of agents, (b) each agent has to make a decision, (c) an outcome results as a function of the decisions of all agents, and (d) each agent has his own preferences on the set of possible outcomes. Robert J. Aumann, one of the most active researchers in the field, said in an interview with Hart (2005): "game theory is optimal decision making in the presence of others with different objectives".

A particular collection of interactive decision situations are the so-called parlor games. Game theory borrows the terminology used in parlor games to designate the various elements that are involved in interactive decision situations: the situations themselves are called *games*, the agents are called *players*, their available decisions are called *strategies*, etc.

Classical game theory is an ideal normative theory, in the sense that it prescribes, for every particular game, how rational players should behave. By rational player we mean one who (a) knows what he wants, (b) has the only objective of getting what he wants, and (c) is able to identify the strategies that best fit his objective. More recently, a normative game theory for bounded-rational players and even a pure descriptive game theory have been developed. However, this book lies mostly inside the borders of classical game theory.

Noncooperative and cooperative models. Game theory is concerned with both noncooperative and cooperative models. The differences between these two types of models are explained, for instance, in van Damme and Furth (2002). They state that "noncooperative models assume that all the possibilities for cooperation have been included as formal moves in the game, while cooperative models are 'incomplete' and allow players to act outside of the detailed rules that have been specified". Another view comes from Serrano (2008): "One clear advantage of the (noncooperative) approach is that it is able to model how specific details of the interaction may impact the final outcome. One limitation, however, is that its predictions may be highly sensitive to those details. For this reason it is worth also analyzing more abstract approaches that attempt to obtain conclusions that are independent of such details. The cooperative approach is one such attempt". The necessity of cooperative models has been perceived by game theorists since the very beginning because in some situations the cooperation mechanisms are too complex as to be fully described by a mathematical model. Thus, cooperative game theory deals with coalitions and allocations, and considers groups of players willing to allocate the joint benefits derived from their cooperation (however it takes place). On the other hand, noncooperative game theory deals with strategies and payoffs, and considers players willing to use the strategies that maximize their individual payoffs.

Some stepping stones in the history of game theory. Most likely, the first important precursor of game theory was Antoine Augustin Cournot and his analysis of duopoly published in 1838. In Cournot (1838), the author studied a concept very close to Nash equilibrium for a duopoly model. At the beginning of the 20th century, some important mathematicians like Zermelo and Borel became interested in parlor games and in two-player zero-sum games.

However, the first important result of game theory was the minimax theorem proved by Hungarian mathematician John von Neumann in 1928. Some years later, von Neumann came back to interactive decision situations when he met the Austrian economist Oskar Morgenstern in Princeton. In 1944 they published their famous book, "The Theory of Games and Economic Behavior", which is considered to be the seminal work of game theory. Since then, game theory has advanced as a result of the cooperation between mathematicians and economists.

In 1950, John Nash published his first paper on the equilibrium concept. Nowadays, Nash equilibrium and, in general, equilibrium theory, which started from that concept, play a central role in the development of social sciences and, especially, of economic theory. In 1994, John Nash, John Harsanyi and Reinhard Selten were awarded the Nobel Prize in Economics “for their pioneering analysis of equilibria in the theory of noncooperative games”. More recently, other game theorists have also received this award. In 2005 the prize was awarded to Robert Aumann and Thomas Schelling “for having enhanced our understanding of conflict and cooperation through game-theory analysis”. In 2007 the prize went to L. Hurwicz, E. Maskin, and R. Myerson “for having laid the foundations of mechanism design theory” (through game theory). These awards illustrate the role of game theory as the main mathematical tool to analyze social interactions, where economics is the major field of application. This interaction between mathematics and social sciences continuously produces challenging problems for the scientific community.

In 1999, the Game Theory Society (GTS) was created to promote the research, teaching, and application of game theory. On its web site, the GTS provides resources related to game theory such as software tools, journals, and conferences. Periodically, the GTS organizes a world conference on game theory; the first one was held in Bilbao (Spain) in 2000.

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The authors maintain a web page that will be periodically updated to include the corrections of typos and other errors that we may be aware of. The web page is hosted by the AMS at: www.ams.org/bookpages/gsm-115/. In this respect, we appreciate any comments from our readers to improve this book. You may contact us via email at julio.gonzalez@usc.es.

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Notations

We do not pretend to provide a comprehensive list of all the notations used in this book. We just intend to provide a brief reference list containing those notations that might be unclear to some readers and also those conventions that are not completely standard.

$\mathbb{N}$	The set of natural numbers $\{1, 2, \ldots\}$
$B \subset A$	Set B is a subset of A (possibly equal)
$B \subsetneq A$	Set B is a subset of A and B is not equal to A
$\lvert A \rvert$	The number of elements of set A
$\operatorname{conv}(A)$	The convex hull of set A, *i.e.*, $\operatorname{conv}(A) := \{\sum_{i=1}^{k} \alpha_i x_i : k \in \mathbb{N},\ x_i \in A,\ \alpha_i \in \mathbb{R}, \alpha_i \geq 0,$ and $\sum_{i=1}^{k} \alpha_i = 1\}$
$\operatorname{ext}(A)$	The extreme points of set A, *i.e.*, $\operatorname{ext}(A) := \{x \in A :$ for each pair $y, z \in A$ and each $\alpha \in (0,1), x = \alpha y + (1-\alpha) z$ implies $x = y = z\}$
2^A	The set of all subsets of set A
B^A	Maps from the set A to the set B
ΔA	$\{x \in [0,1]^A : \lvert\{a \in A : x(a) > 0\}\rvert < \infty$ and $\sum_{a \in A} x(a) = 1\}$
$\lvert \mathcal{A} \rvert$	The determinant of matrix $\mathcal{A}$
$\mathcal{A}^t$	The transpose of matrix $\mathcal{A}$
I_m	The identity $m \times m$ matrix
$\mathbb{1}_m$	The vector $(1, \ldots, 1) \in \mathbb{R}^m$
$\mathbb{E}(X)$	The expectation of random variable X
$\mathbb{E}(X\,\lvert_Y)$	The expectation of variable X conditional on Y
$P(A)$	The probability of event A
$P(A\,\lvert_B)$	The probability of event A conditional on event B

$\{x^k\}$	Abbreviation for $\{x^k\}_{k\in\mathbb{N}}$
$\{x^k\} \to x$	Abbreviation for $\lim_{k\to\infty}\{x^k\} = x$
$\operatorname{argmin}_{x\in\Omega} f(x)$	The set $\{y \in \Omega : f(y) = \min_{x\in\Omega} f(x)\}$
Let $a, b \in \mathbb{R}^n$:	
$a \geq b$	For each $i \in \{1,\ldots,n\}$, $a_i \geq b_i$
$a > b$	For each $i \in \{1,\ldots,n\}$, $a_i > b_i$
(a_{-i}, b_i)	The vector $(a_1,\ldots,a_{i-1},b_i,a_{i+1},\ldots,a_n)$

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