Analog and Digital Signals, Time and Frequency Representation of Signals

Required reading:
Garcia 3.1, 3.2

CSE 3213, Fall 2010
Instructor: N. Vlajic

- **Data vs. Signal**
- **Analog vs. Digital**
- **Analog Signals**
  - **Simple Analog Signals**
  - **Composite Analog Signals**
- **Digital Signals**

Data vs. Signal

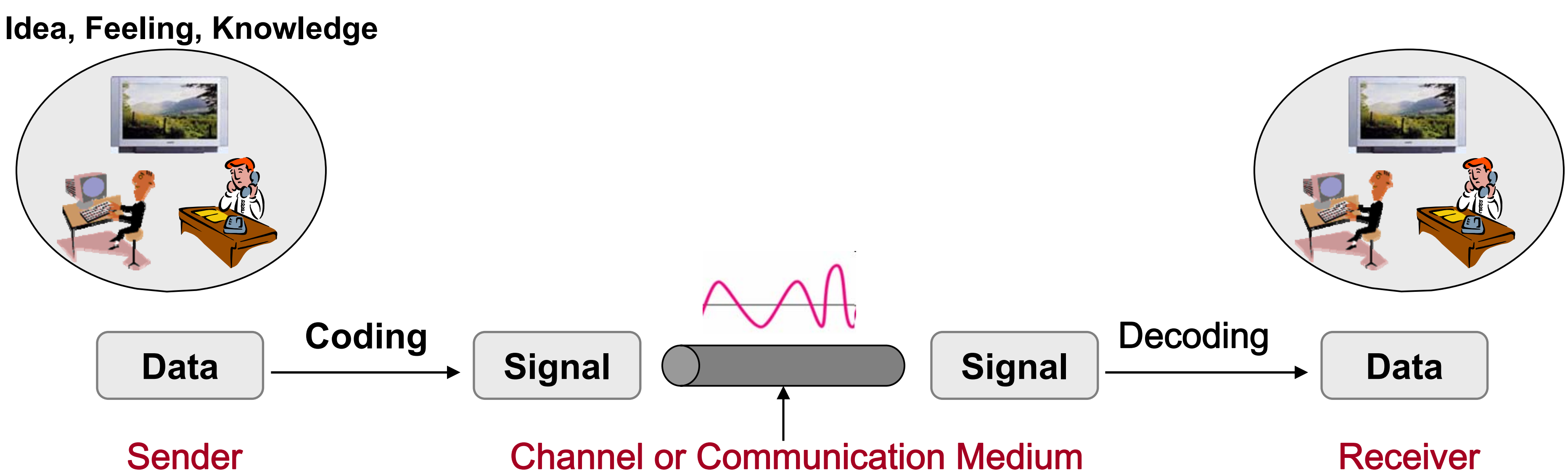
Data – information formatted in human/machine readable form

- examples: voice, music, image, file

Signal – electric or electromagnetic representation of data

- transmission media work by conducting energy along a physical path; thus, to be transmitted, data must be turned into energy in the form of electro-magnetic signals

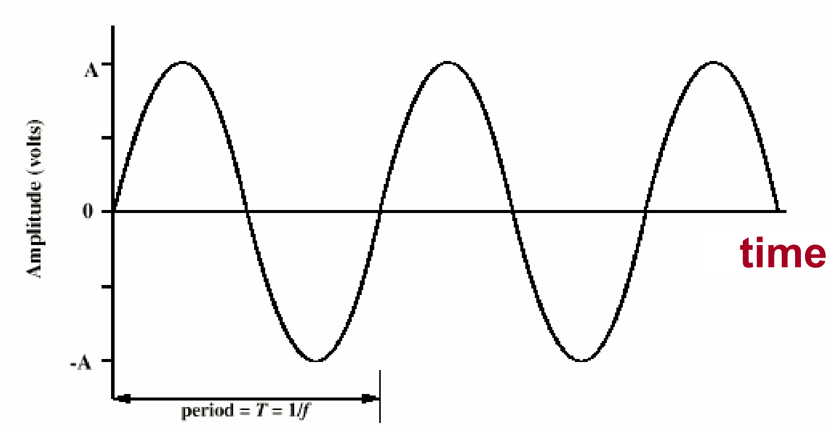
Transmission – communication of <u>data</u> through propagation and processing of <u>signals</u>

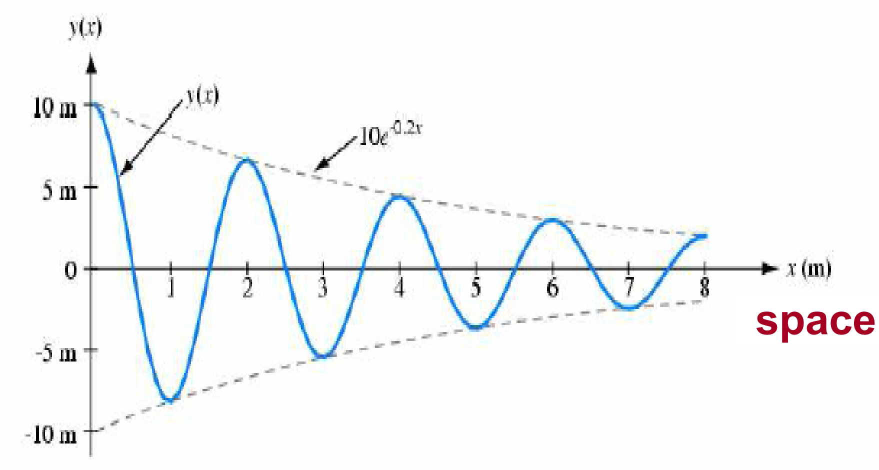


Signal Representation

Signal Representation – **typically in 2D space, as a function of time, space or frequency**

- **when horizontal axis is time, graph displays the value of a signal at one particular point in space as a function of time**
- **when horizontal axis is space, graph displays the value of a signal at one particular point in time as a function of space**

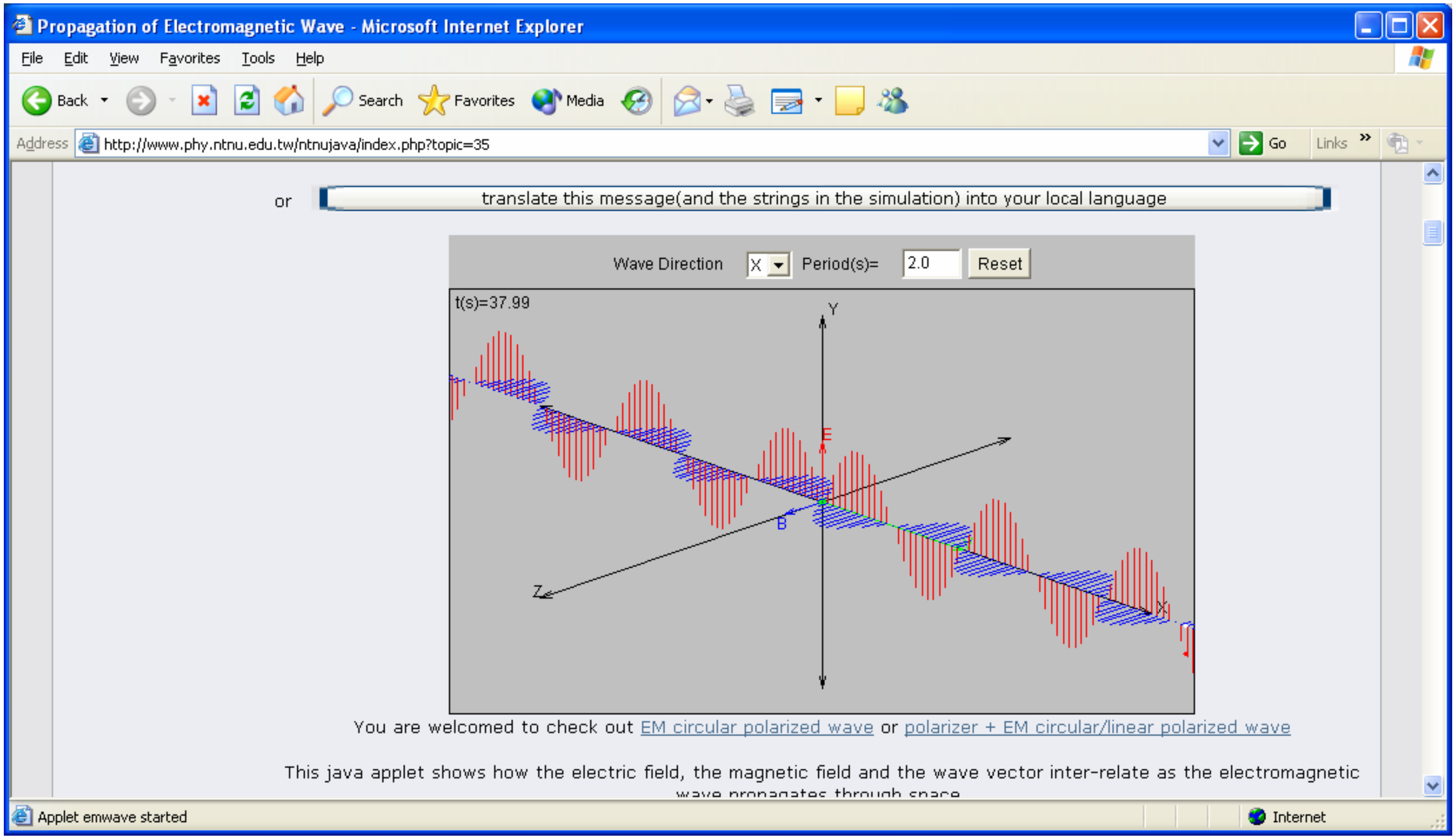




The time- and space- representation of a signal often resemble each other, though the signal envelope in the space-representation is different (signal attenuates over distance).

Signal Representation (cont.)

Example [signal in time and space]



http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=35

- Data vs. Signal
- **Analog vs. Digital**
- Analog Signals
  - Simple Analog Signals
  - Composite Analog Signals
- Digital Signals

Analog vs. Digital

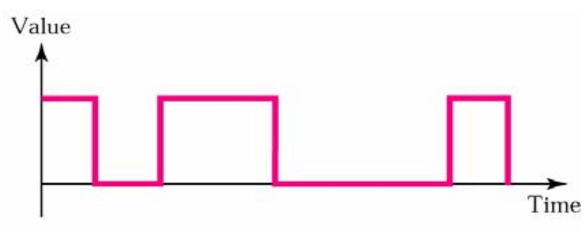
Analog vs. Digital Data

- **analog data** – representation variable takes on continuous values in some interval, e.g. voice, temperature, etc.
- **digital data** – representation variable takes on discrete (a finite & countable number of) values in a given interval, e.g. text, digitized images, etc.

Analog vs. Digital Signal

- **analog signal** – signal that is continuous in time and can assume an infinite number of values in a given range (continuous in time and value)
- **discrete (digital) signal** – signal that is continuous in time and assumes only a limited number of values (maintains a constant level and then changes to another constant level)

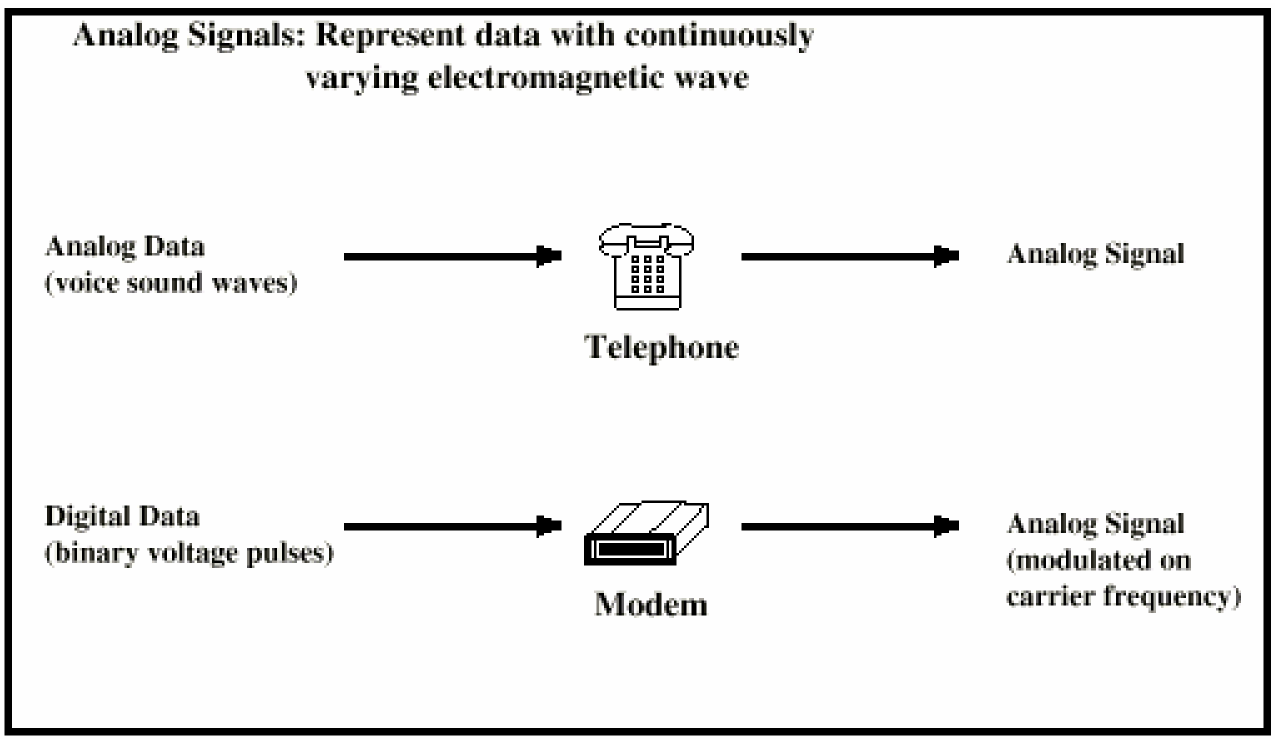
a. Analog signal

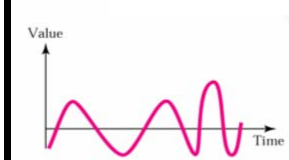


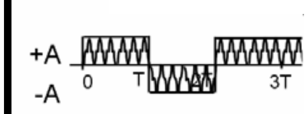
b. Digital signal

Analog vs. Digital (cont.)

Both analog and digital data can be transmitted using either analog or digital signals.







example: analog signaling of analog and digital data

… will talk more about this later …

Analog vs. Digital (cont.)

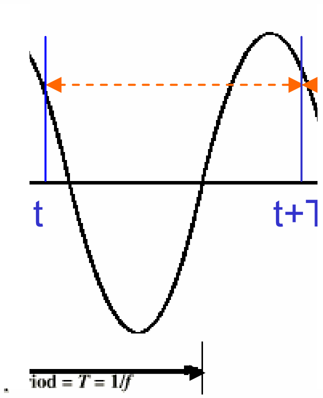
Periodic vs. Aperiodic Signals

- **periodic signal – completes a pattern within some measurable time frame, called a period (T), and then repeats that pattern over sub-sequent identical periods**

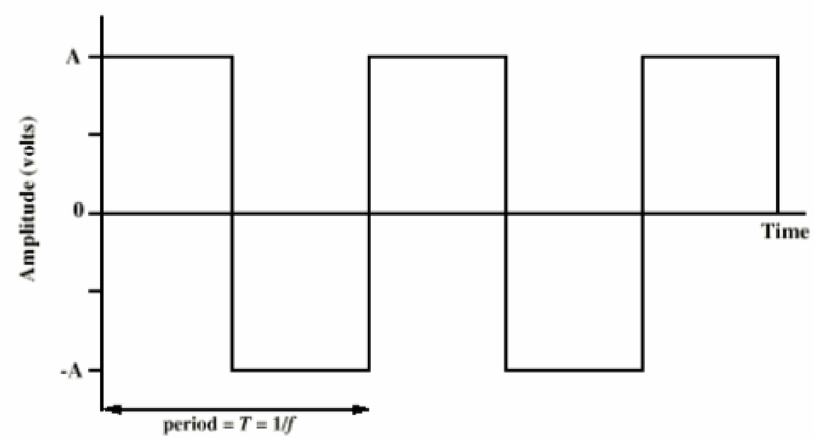
$$\exists T \in R \quad s.t. \quad s(t+T) = s(t), \ \forall t \in \langle -\infty, +\infty \rangle$$

  - **T - smallest value that satisfies the equation**
  - **T is (typically) expressed in seconds**

- **aperiodic signal – changes without exhibiting a pattern that repeats over time**



periodic analog signal



periodic digital signal

- **Data vs. Signal**
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- **Digital Signals**

Figure 2.6 *Two or more sinusoidal waves can produce a composite wave*

2.3.4 Simple Harmonic Motion (SHM)

Sinusoidal waves are not mathematical abstractions, they can be produced in the real world by a phenomenon called **Simple Harmonic Motion** (SHM). An SHM is a specific type of oscillation exhibited by the spring loaded weight in Fig. 2.7. Assume that point P is rotating along a circle with uniform speed. A spring loaded weight *W* is attached to the point P with a flexible string T. As P moves along the circle in the anti-clockwise direction, the weight *W* attached to P oscillates up and down in synchronism. If a pen is attached to the backside of *W* which marks on a paper moving slowly to the right to represent the flow of time, the pen would trace out a sinusoidal curve. If ω is the angular frequency of the point *P* in radian/sec, then the frequency of the generated wave is given by:

$$f = \omega/2\pi$$

where f is in cycles/sec. This is apparent from Fig. 2.7, because as a complete cycle is traced out by the weight *W*, the point P moves by 2π radians, i.e. a complete circle. In 1 second since f cycles are traced out, the points moves a total of $2\pi f$ radians, which by definition is the angular frequency ω.

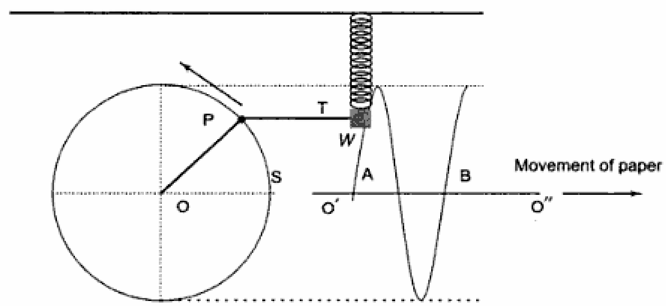


Figure 2.7 *Simple Harmonic Motion (SHM)*

Analog Signals

Classification of Analog Signals

(1) Simple Analog Signal – cannot be decomposed into simpler signals

- **sinewave – most fundamental form of periodic analog signal – mathematically described with 3 parameters**

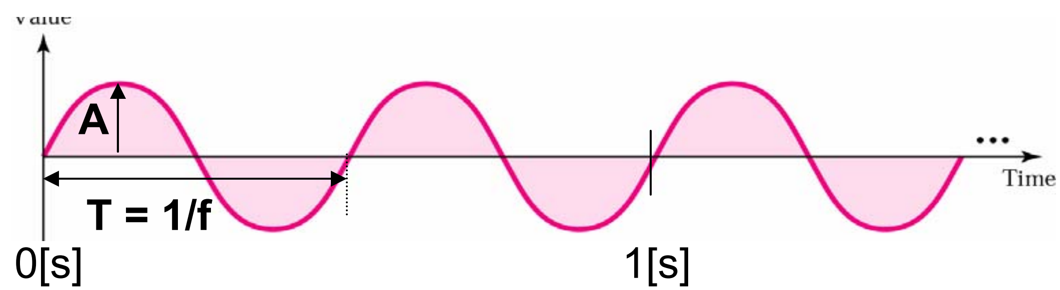
$$s(t) = A \cdot \sin(2\pi ft + \varphi)$$

(1.1) peak amplitude (A) – absolute value of signal's highest intensity – unit: volts [V]

(1.2) frequency (f) – number of periods in one second – unit: hertz [Hz] = [1/s] – inverse of period (T)!

(1.3) phase (φ) – absolute position of the waveform relative to an arbitrary origin – unit: degrees [º] or radians [rad]

The origin is usually taken as the last previous passage through zero from the negative to the positive direction.

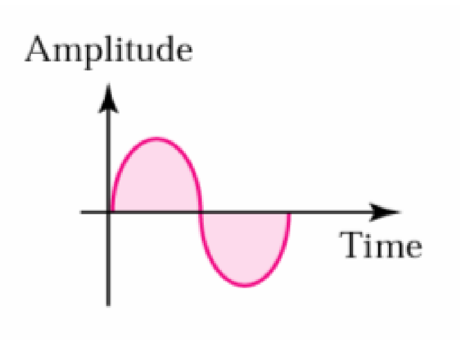


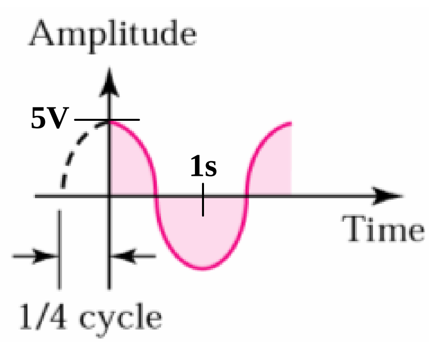
(2) Composite Analog Signal – composed of multiple sinewaves

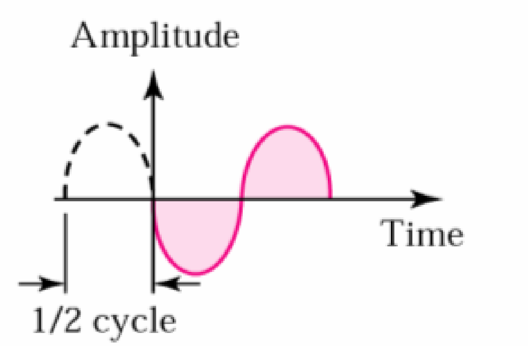
Simple Analog Signals

Phase in Simple Analog Signals – **measured in degrees or radians**

- **360º = 2π rad**
- **1º = 2π/360 rad**
- **1 rad = (360/2π)º** = 57.29578°
- **phase shift of 360º = shift of 1 complete period**
- **phase shift of 180º = shift of 1/2 period**
- **phase shift of 90º = shift of 1/4 period**







φ = 0º or 360º

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Analog Signals

Example [period and frequency]

Unit	Equivalent	Unit	Equivalent
seconds (s)	1 s	hertz (Hz)	1 Hz
milliseconds (ms)	10^{-3} s	kilohertz (KHz)	10^{3} Hz
microseconds (μs)	10^{-6} s	megahertz (MHz)	10^{6} Hz
nanoseconds (ns)	10^{-9} s	gigahertz (GHz)	10^{9} Hz
picoseconds (ps)	10^{-12} s	terahertz (THz)	10^{12} Hz

units of period and respective frequency

(a) Express a period of 100 ms in microseconds.

$100\ ms = 100 \times 10^{-3}\ s = 100 \times 10^{-3} \times 10^{6}\ \mu s = 10^{5}\ \mu s$

(b) Express the corresponding frequency in kilohertz.

$100\ ms = 100 \times 10^{-3}\ s = 10^{-1}\ s$

$f = 1/10^{-1}\ Hz = 10 \times 10^{-3}\ KHz = 10^{-2}\ KHz$

Simple Analog Signals

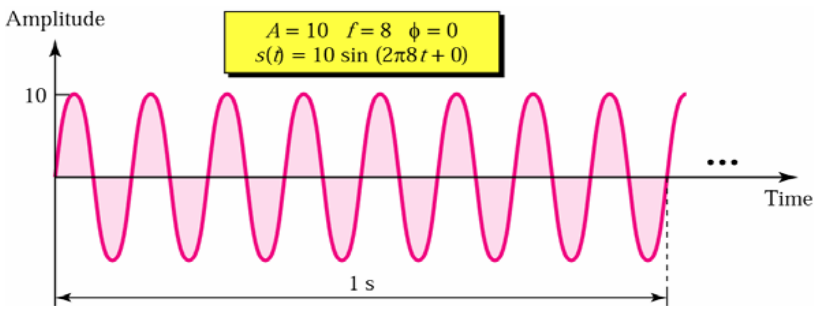
Frequency in Simple Analog Signals – rate of signal change with respect to time

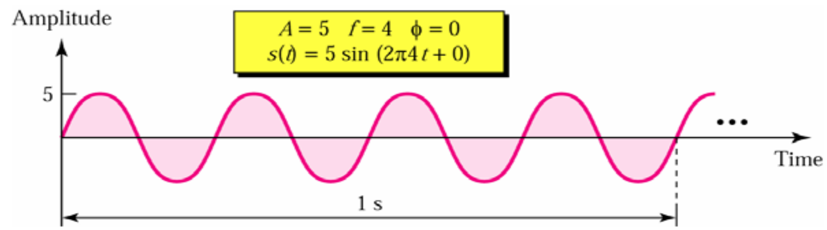
- change in a short span of time ⇒ high freq.
- change over a long span of time ⇒ low freq.
- signal does not change at all ⇒ zero freq.

??

- signal changes instantaneously ⇒ ∞ freq.

??





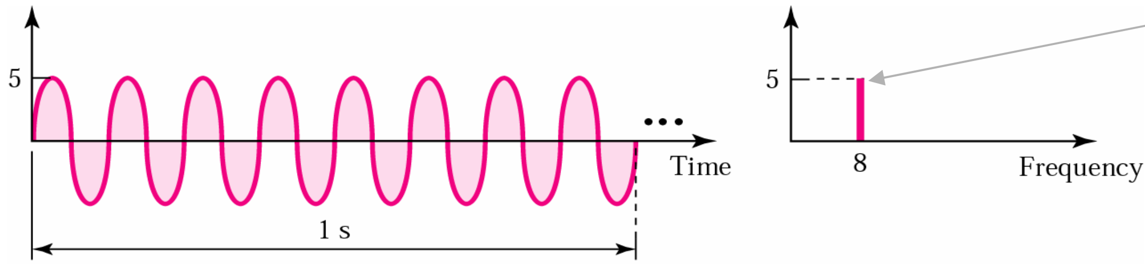
Time Domain Plot – specifies signal amplitude at each instant of time

- does NOT express explicitly signal's phase and frequency

Frequency Domain Plot – specifies peak amplitude with respect to freq.

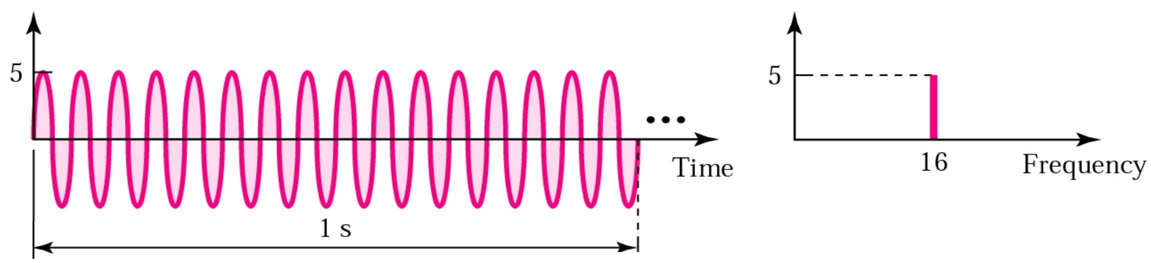
- phase CANNOT be shown in the frequency domain

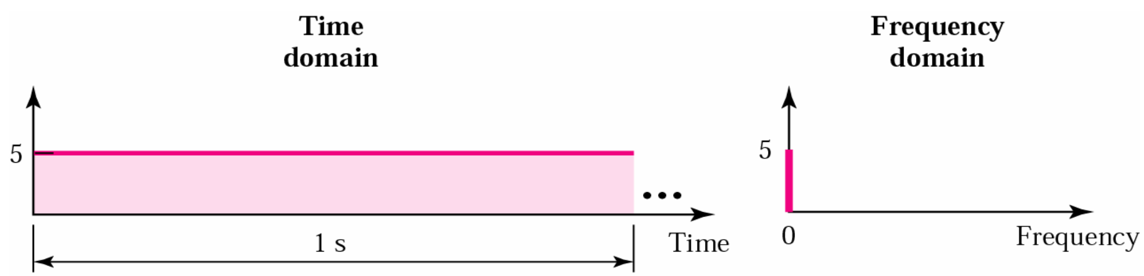
Simple Analog Signals



b. A signal with frequency 8

One 'spike' in frequency domain shows two characteristics of the signal:
spike position = signal frequency,
spike height = peak amplitude.

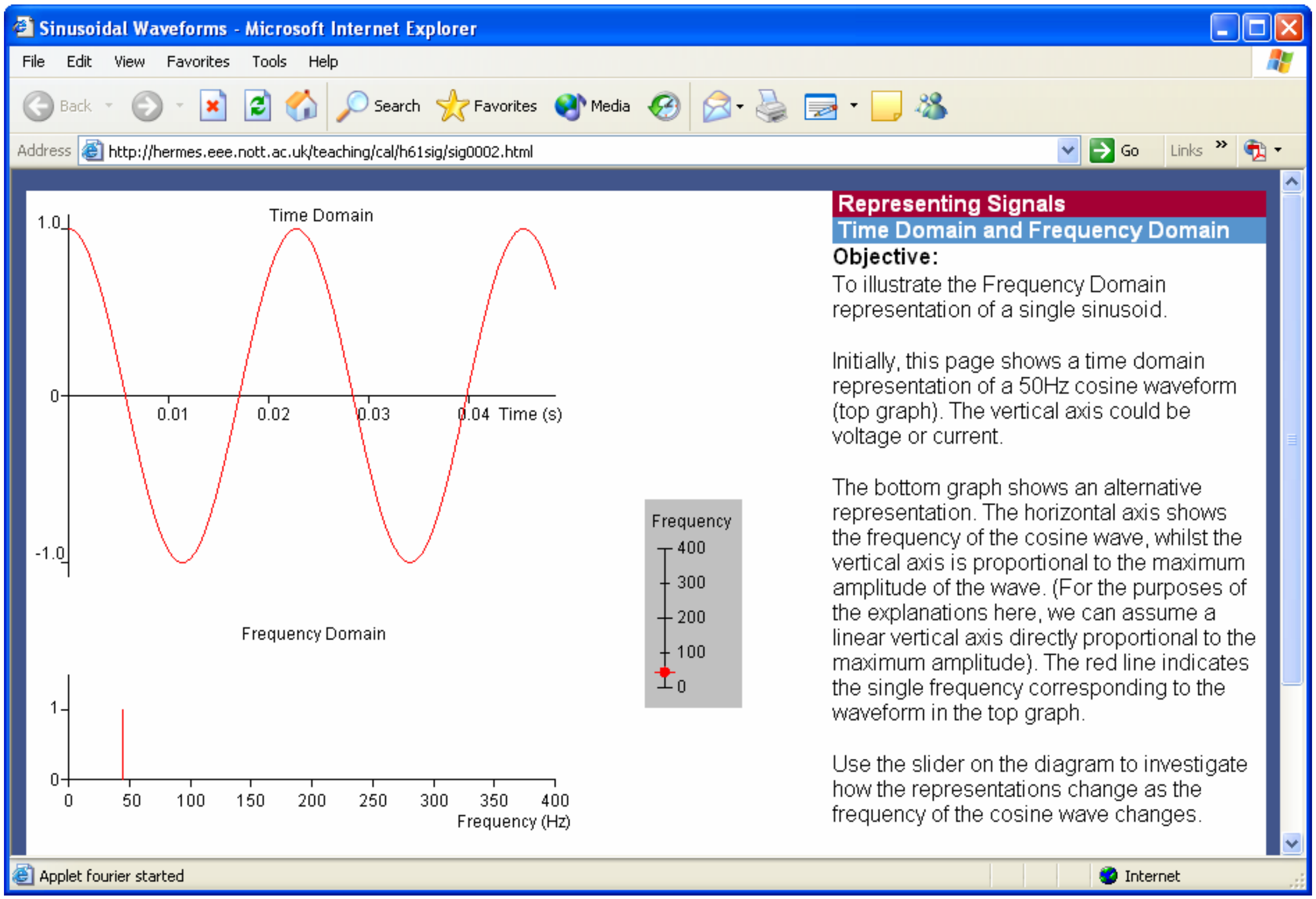




Analog signals are best represented in the frequency domain.

Simple Analog Signals

Example [time vs. frequency domain]



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Composite Analog Signals

Fourier Analysis – **any composite signal can be represented as a combination of simple sine waves with different frequencies, phases and amplitudes**

$$s(t) = A_1 \sin(2\pi f_1 t + \varphi_1) + A_2 \sin(2\pi f_2 t + \varphi_2) + \ldots$$

- **periodic composite signal (period=T, freq. = f_0=1/T) can be represented as a sum of simple sines and/or cosines known as *Fourier series*:**

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t) \right]$$

With the aid of good table of integrals, it is easy to determine the frequency-domain nature of many signals.

$$A_n = \frac{2}{T} \int_0^T s(t) \cos(2\pi n f_0 t) dt, \quad n = 0,1,2, \ldots$$

$$B_n = \frac{2}{T} \int_0^T s(t) \sin(2\pi n f_0 t) dt, \quad n = 1,2,3, \ldots$$

- **f_0 is referred to as '<u>fundamental frequency</u>'**
- **integer multiples of f_0 are referred to as <u>harmonics</u>**

Composite Analog Signals (cont.)

Angular Frequency – **aka radian frequency – number of 2π revolutions during a single period of a given signal**

$$\omega = \frac{2\pi}{T} = 2\pi \cdot f$$

- **simple multiple of ordinary frequency**

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t) \right]$$

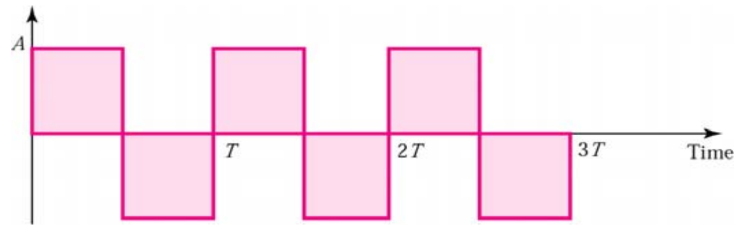
$$A_n = \frac{2}{T} \int_0^T s(t) \cos(n\omega_0 t) dt, \qquad n = 0,1,2,...$$

$$B_n = \frac{2}{T} \int_0^T s(t) \sin(n\omega_0 t) dt, \qquad n = 1,2,...$$

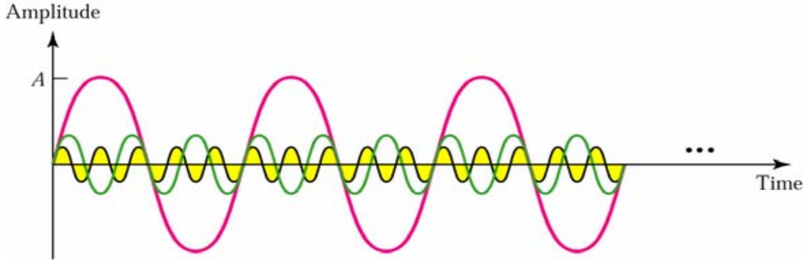
Composite Analog Signals (cont.)

Example [periodic square wave]

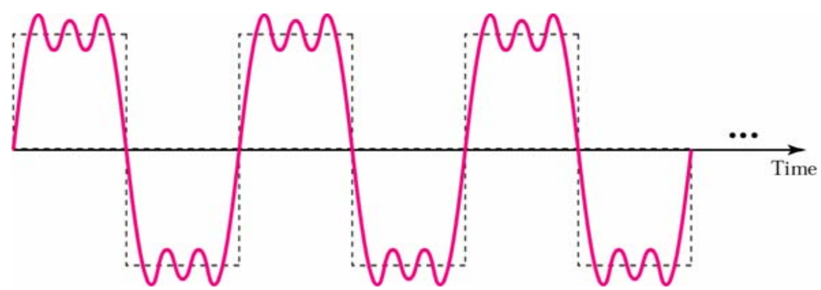
No DC component!!!



$$s(t) = \frac{4A}{\pi}\sin(2\pi ft) + \frac{4A}{3\pi}\sin(2\pi(3f)t) + \frac{4A}{5\pi}\sin(2\pi(5f)t) + \ldots$$



three harmonics



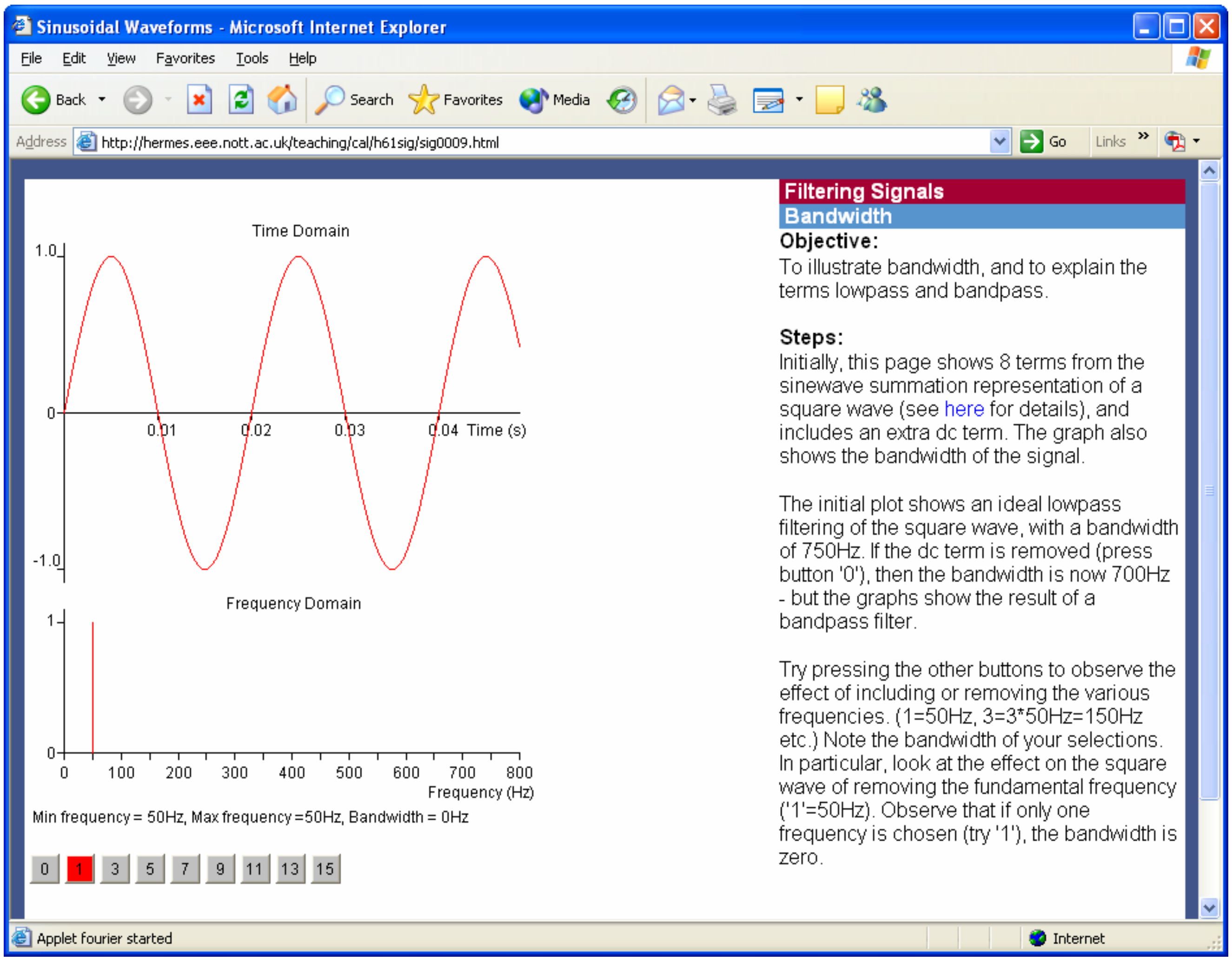
adding three harmonics

With three harmonics we get an approximation of a square wave.
To get the actual square, all harmonics up to ∞ should be added.

http://www.nst.ing.tu-bs.de/schaukasten/fourier/en_idx.html
http://www.phy.ntnu.edu.tw/java/sound/sound.html

Composite Analog Signals (cont.)

Example **[composite analog signal]**



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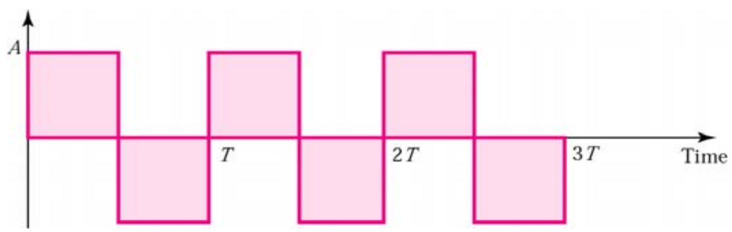
Composite Analog Signals (cont.)

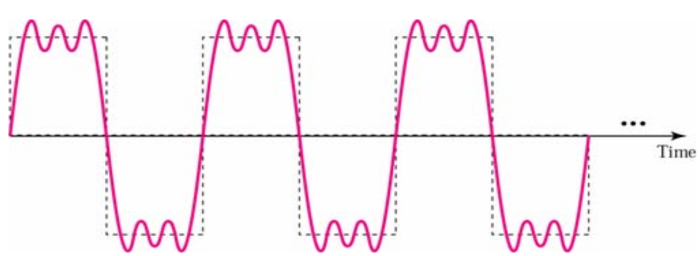
Frequency Spectrum of Analog Signal – range (set) of frequencies that signal contains

Absolute Bandwidth of Analog Signal – width of signal spectrum: $B = f_{highest} - f_{lowest}$

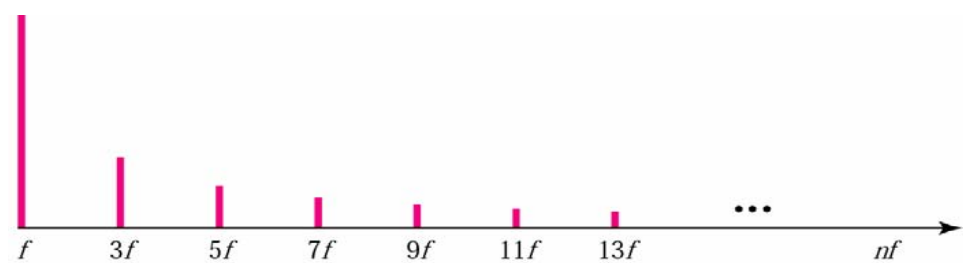
Effective Bandwidth of Analog Signal – range of frequencies where signal contains most of its power/energy



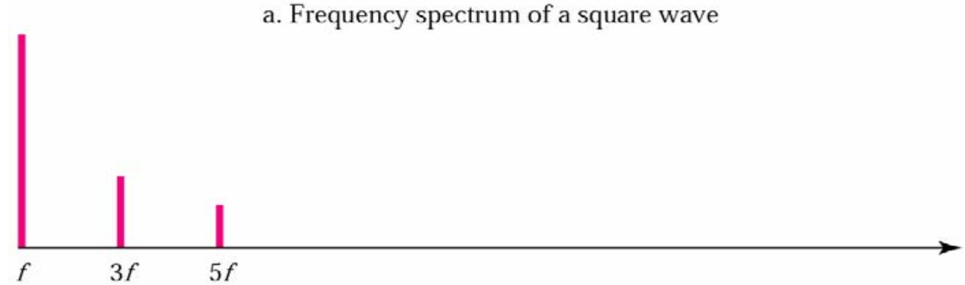
square wave



3-harmonic representation



a. Frequency spectrum of a square wave



b. Frequency spectrum of an approximation with only three harmonics

Composite Analog Signals (cont.)

Example **[frequency spectrum and bandwidth of analog signal]**

A periodic signal is composed of five sinewaves with frequencies of 100, 300, 500, 700 and 900 Hz.

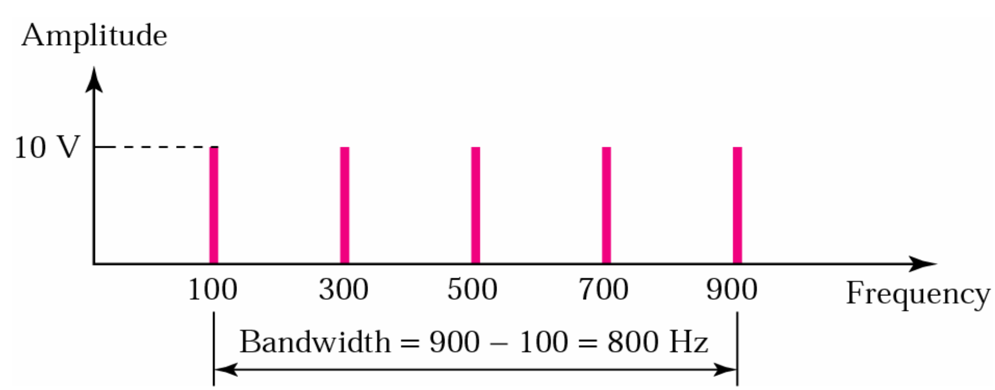
What is the bandwidth of this signal?

Draw the frequency spectrum, assuming all components have a max amplitude of 10V.

Solution:

$B = f_{highest} - f_{lowest} = 900 - 100 = 800$ Hz

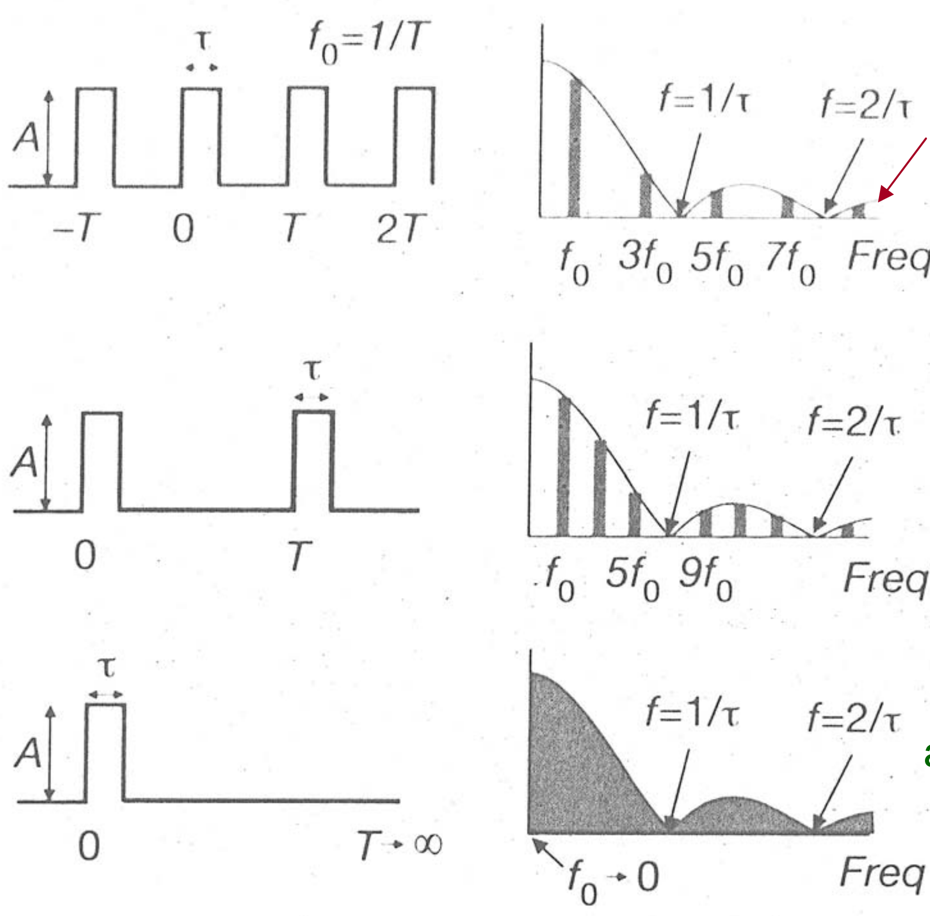
The spectrum has only five spikes, at 100, 300, 500, 700, and 900.



Composite Analog Signals (cont.)

Example **[frequency spectrum of a data pulse]**

$$\left|\frac{2A\tau}{T}\cdot\frac{\sin\left(\pi n\,\tau/T\right)}{\pi n\,\tau/T}\right| \approx \left|\frac{\sin(\pi\cdot f\cdot\tau)}{\pi\cdot f\cdot\tau}\right| \text{ or } \left|\frac{\sin(x)}{x}\right|$$



envelope of frequency spectrum

periodic signal ⇒ discrete freq. spectrum

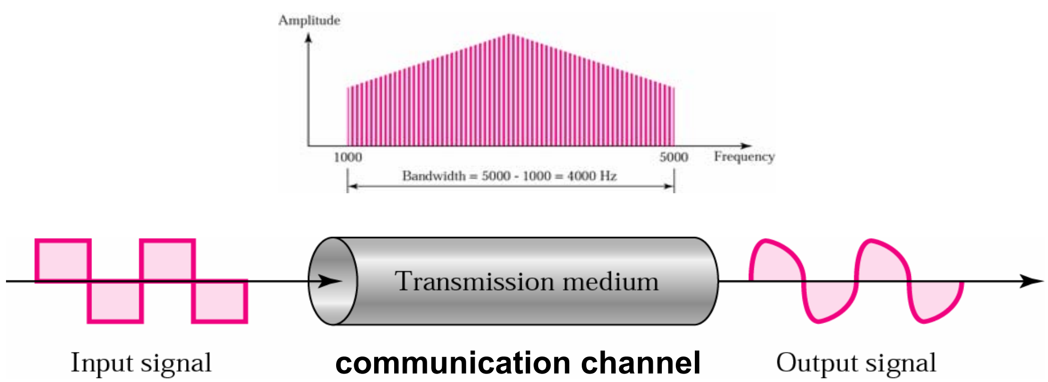
aperiodic signal ⇒ continuous freq. spectrum

What happens if $\tau \to 0$???

Composite Analog Signals (cont.)

Composite Signals and Transmission Medium – no transmission medium is perfect – each medium passes some frequencies and blocks or weakens others

- composite signal sent at one end of transmiss. medium (comm. channel), may not be received in the same form at the other end
- passing a square wave through any medium will always deform the signal !!!



Channel Bandwidth – range of frequencies passed by the channel – difference between highest and lowest frequency that channel can satisfactorily pass

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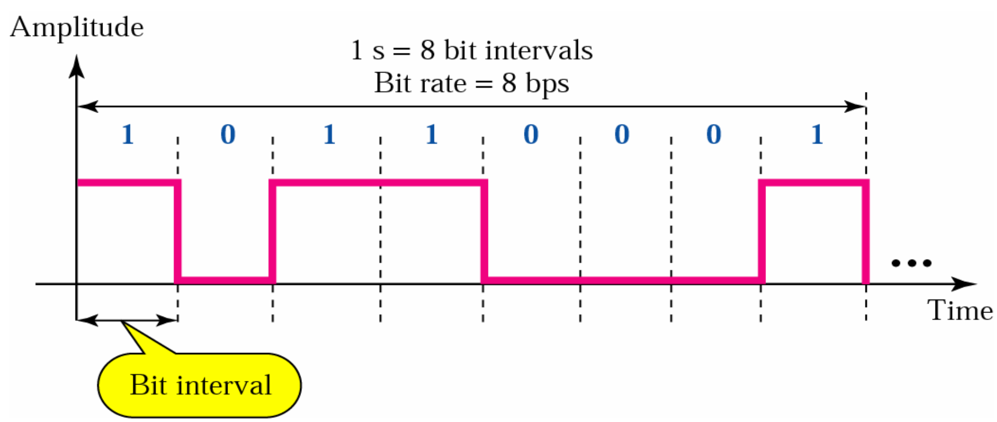
Digital Signals

Digital Signals – **sequence of voltage pulses (DC levels) – each pulse represents a *signal element***

- **binary data are transmitted using only 2 types of signal elements (1 = positive voltage, 0 = negative voltage)**
- **key digital-signals terms:**
  - **bit interval – time required to send a single bit, unit: [sec]**
  - **bit rate – number of bit intervals per second – unit: [bps]**



Most digital signals are aperiodic,
so it is not appropriate / correct to talk about their period.

Digital Signals (cont.)

Digital Signal as a Composite Analog Signal

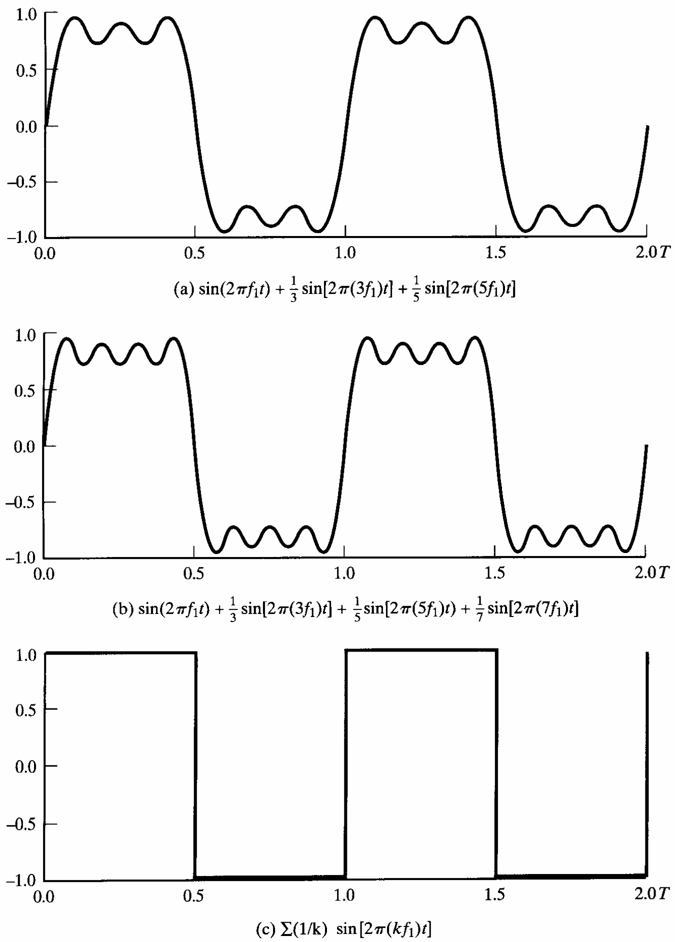


FIGURE 4.6 Frequency Components of a Square Wave ($T = 1/f_1$).

– digital signal, with all its sudden changes, is actually a composite signal having an infinite number of frequencies

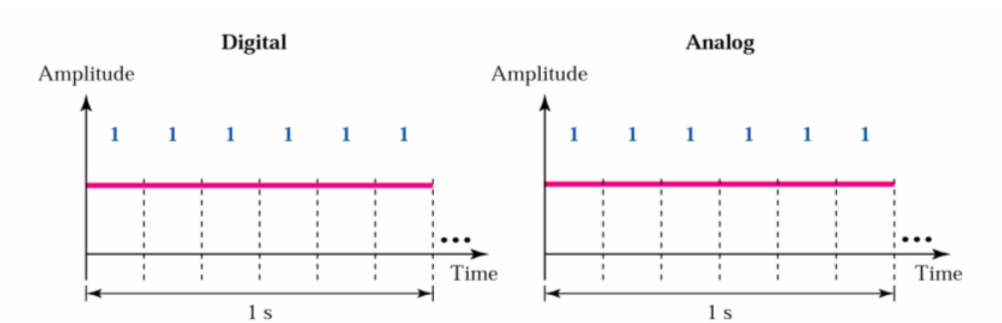
- a digital signal is a composite signal with an infinite bandwidth
- if a medium has a wide bandwidth, a digital signal can be sent through it
  - some frequencies will be weakened or blocked; still, enough frequencies will be passed to preserve a decent signal shape
- what is the minimum required bandwidth B [Hz] of a band-limited medium if we want to send n [bps]?

Digital Signals (cont.)

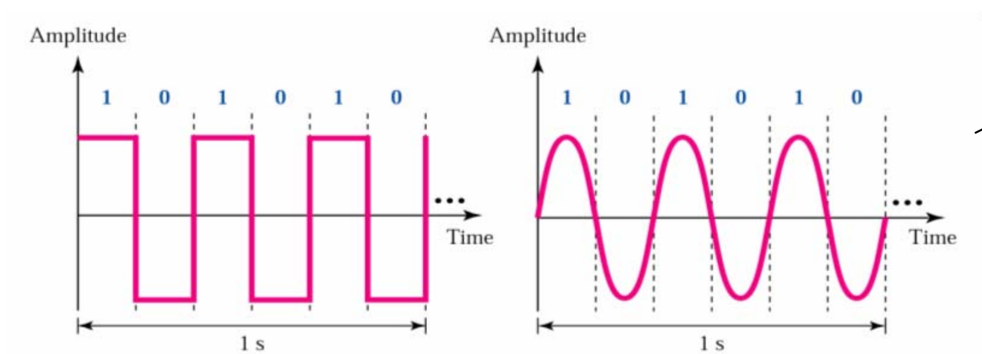
Example [approximation of digital signal's spectrum using 1st harmonic]

Assume our computer generates 6 bps.
Possibilities (periodic combinations) : 000000, 111111, 110011, 101010 etc.

1. **Best case: min # of changes ⇒ min freq. of substitute analog signal**



2. **Worst case – max # of changes ⇒ max freq. of substitute analog signal**



bit rate: n = 6 [bps]
frequency: B = 3 [Hz]

$$\mathbf{B} \approx \frac{\mathbf{n}}{\mathbf{2}}$$

Exercise

1. Before data can be transmitted, they must be transformed to ________________.
   (a) periodic signals
   (b) electromagnetic signals
   (c) aperiodic signals
   (d) low-frequency sinewaves
2. In a frequency-domain plot, the vertical axis measures the ________________.
   (a) peak amplitude
   (b) frequency
   (c) phase
   (d) slope
3. In a time-domain plot, the vertical axis measures the ________________.
   (a) peak amplitude
   (b) amplitude
   (c) frequency
   (d) time
4. If the bandwidth of a signal is 5 KHz and the lowest frequency is 52 KHz, what is the highest frequency ________________.
   (a) 5 KHz
   (b) 10 KHz
   (c) 47 KHz
   (d) 57 KHz

Exercise

5. If one of the components of a signal has a frequency of zero, the average amplitude of the signal _______________.
   (a) is greater than zero
   (b) is less than zero
   (c) is zero
   (d) (a) or (b)

6. Give two sinewaves A and B, if the frequency of A is twice that of B, then the period of B is ________________ that of A.
   (a) one-half
   (b) twice
   (c) the same as
   (d) indeterminate from

7. A device is sending out data at the rate of 1000 bps.
   (a) How long does it take to send out 10 bits?
   (b) How long does it take to send out a single character (8 bits)?
   (c) How long does it take to send a file of 100,000 characters?