Analysis and Design of Structural Glass Systems

José Sanches

Instituto Superior Técnico, Technical University of Lisbon

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jose.sanches@ist.utl.pt

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Introduction

Glass is one of the most popular construction materials due to a combination of transparency, strength and durability. Since it offers the possibility of natural light transmission, soon it earned a major influence in window glazing systems. Glass resistance is highly dependent on surface imperfections, which reduce the tensile strength.

Great developments in the quality of glass were made in last decades, not only because of the float process invention but also because new chemical and thermal treatments (tempering process) were also invented. Thermal treatments consist on heating the glass until 1500°C (approximately) followed by a controlled cooling stage until room temperature. Residual compressive stresses keep permanent on surface, which improves the bending resistance (Fig. 1). Depending on the cooling rate, there are two types of glass, heat-strengthened (HSG) and toughened (TOGH) glass. The residual stresses are higher in the latter, due to a faster cooling stage. Therefore, toughened glass is able to resist to higher tensile stresses. When the limit of tensile stress is reached, the result is an ”explosion” that splits glass into hundreds of small fragments, reducing the injury risk.

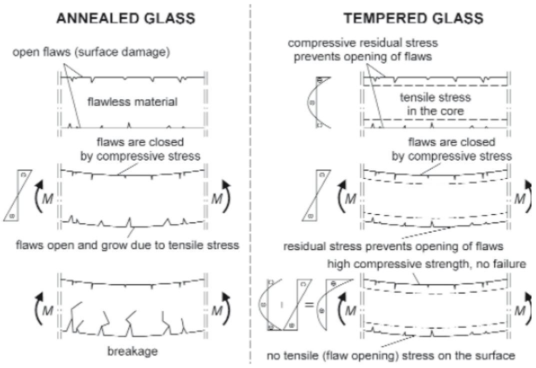

Fig. 1 - Tempering effect (2).

However, glass still has a brittle behavior and it breaks without warning. In order to overcome this lack of plasticity, laminated glass was invented. Multiple panes of glass are bonded with polymer interlayers. When broken, the fragments stick to the plastic interlayer, giving a post-breakage behavior and avoiding an instantaneous collapse. The residual resistance is fragment’s size dependent and this is the reason why TOGH is not used in all the cases.

With such developments, glass is no longer stuck to simple windows and there are several outstanding examples of structural use of glass in glass façades (Fig. 2) or stairs (Fig. 3) all over the World. Even though, most countries do not have appropriate standards for the use of glass as a structural material, thus the design of this type of structures is still in its early days.

Fig. 2 – Oslo Opera House (3).

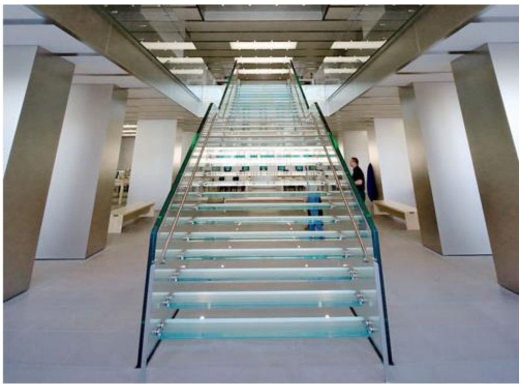

Fig. 3 - Apple Store, Hamburg (4).

This paper presents three typical cases of structural glass and explains its design method, while considering the most important issues in each case.

Case study 1

Definition

In this example, a glass façade supported by 15 meters length glass fins was designed (Fig. 4). The infill panels are 1,2 meters height and simply supported in fins (Fig. 5), which are simply supported as well. The vertical reaction was released on bottom support for fins, preventing them against instability problems caused by self-weight and thermal movements. FE models were made for fins, using *SAP2000 v.14.2 advanced*.

In the following analysis, the wind pressure acting on infill panels surface is assumed as 1,5 kPa.

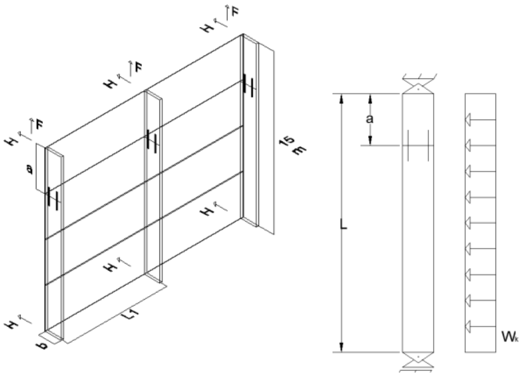

Fig. 4 - Case study 1 - Glass façade and glass fin

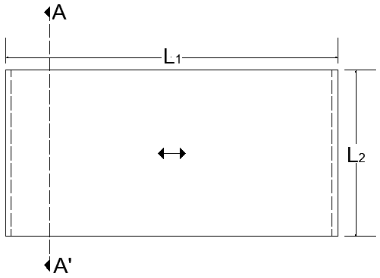

Fig. 5 - Infill panel, structural system.

Material selection

Before starting the structural analysis, an in depth study about materials selection and their influence on global analysis was considered.

Laminated safety glass is used for the infill panels, which are insulated due to comfort requirements (Fig. 6). The monolithic sheet is a toughened glass and the laminated glass consists of a HSG and another TOGH.

Fig. 6 - Infill panel, cross section.

When a insulated glass is loaded (wind, snow, etc.), as the gas inside is completely sealed, the deflection of both sheets will be approximately equal (Fig. 7).

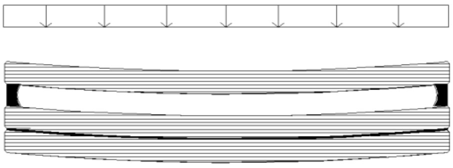

Fig. 7 - Infill panel, typical lateral deflection.

Usually, PVB is used as an intermediate material to join different glass sheets. Its ability to transfer shear load has a major effect in the global performance of laminated glass. Two borderline cases may be considered depending on interlayer stiffness. If interlayer shear modulus is assumed as infinite, the laminated glass behaves as a monolithic piece. On the other hand, shear modulus is assumed to be nil, the sheets are free-sliding and an equivalent thickness may be considered for structural calculations (deflection) :

$$e_{eq} = \sqrt[3]{\sum_i t_i^3} \qquad \textbf{Eq. 1}$$

PVB interlayer is quite flexible and some applications, like long fins and columns, may have buckling problems. In these cases it is recommended to use stiffer interlayers such as SentryGlasPlus® (SGP).

As fins have a structural function, some ductility is required, therefore laminated glass of four panes (15 mm each), with two toughened and two heat-strengthened bonded by SGP, was selected to get a balance between strength and post-breakage behavior.

Due to the impossibility to produce glass pieces over 11 meters, an intermediate connection was designed, by means of four bolted steel plates (Fig. 8).

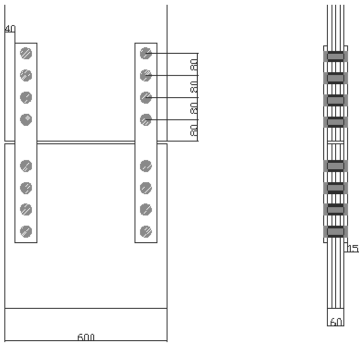

Fig. 8 – Intermediate connection

With 40 mm holes diameter and 30 mm bolts, there is a clearance that shall be fulfilled for some appropriate material to avoid the direct contact between glass and bolts and the interaction of different materials must be correctly modeled to get realistic results. Two different behaviors were simulated depending on the bushing material used.

When bending forces are acting on beam, elastic tensile and compressive stresses are installed (Fig. 9). The two glass segments tend to separate below the neutral surface and to get closer above it, inducing shear forces on bolts.

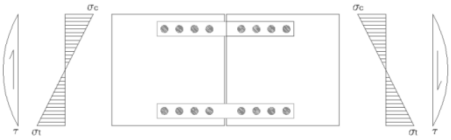

Fig. 9 - Stress transmition.

Traditionally the contact between the bolts and glass were avoided through a thin neoprene interlayer which is not able to redistribute uniformly the bolt stresses around the hole, therefore high stresses were installed in glass. Recently, some developments were made in the direction of reducing the stresses around the hole and there are already some materials (such as silicone) which achieve an uniformly stress field around the hole (Fig. 10).

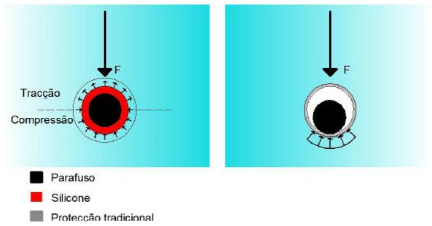

Fig. 10 - Stress transmition between bolt and glass. Bushing material effect.

A FE analysis was made to understand how dangerous a wrong choice on the bushing material can become. The traditional solution was modeled considering that only part of the bolt was in contact with glass and silicone solution was considered acting all around the hole. The results revealed a 10 MPa difference. The following assumptions were made to run the model:

- 16 bolts in intermediate connection
- L1= 3 m (Fig. 4)
- a=4 m (Fig. 4)

Number of bolts

To optimize the number of bolts in the intermediate connection, several solutions with 8, 12, 16, 20, and 24 bolts were tested. Additionally, silicone was selected as the bushing material. The chart in Fig. 11 shows the evolution of the maximum tensile stress in glass per number of bolts introduced in half of steel plate. The most efficient solution is with 4 bolts (a total of 16) because additional bolts will not reduce the maximum tensile stress and besides that, it would represent a higher cost during the conception.

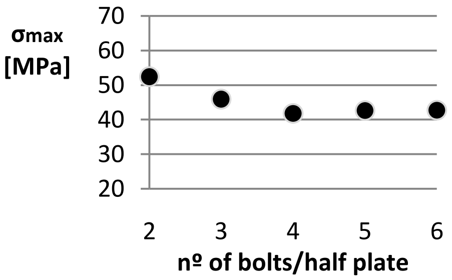

Fig. 11 - Maximum glass tensile stress per number of bolts

In further analysis, only solutions with 16 bolts were considered.

Connection position

The connection shall be made as far possible from the mid span, since it is the maximum value of the bending moment diagram. Notwithstanding, other few solutions were tested to understand the evolution of the maximum tensile stress in glass. As it was expected, a maximum stress grows when the relation a/L grows and it reaches the maximum value when the connection is positioned exactly at the mid span (Fig. 12). For this analysis, a 1,5kPa wind pressure and a 3 meters distance between fins were assumed.

Distance between glass fins – Optimization

The distance between fins, l(m), were tested for three cross sections with different widths (A - 0,6m; B - 0,7m and C - 0,8m). The optimization was done by controlling the tensile stress according to NF DTU 39 and ASTM E 1300 limits (Table 1) .

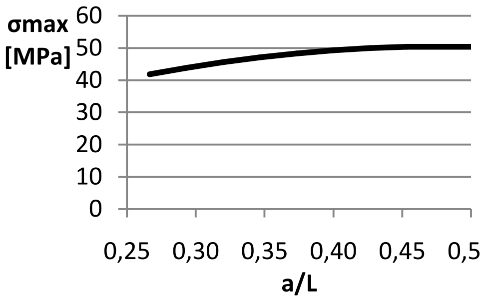

Fig. 12 - Maximum tensile stress by connection position.

Table 1 - Allowable tensile stress by international standards.

	Maximum tensile stress [MPa]		
	ANN	**HSG**	**TOGH**
NF DTU39*	20(16)	35(28)	50(40)
ASTM E 2751**	18,3(5,7)	36,5(20,3)	73(49,4)

( ) – for permanent loads
*For permanent loads, the maximum tensile stress shall be multiplied by a specific coefficient depending on support conditions.
** For seamed edges

. Optimal solutions are, according to Fig. 13:

- Solution A : l=2,5m
- Solution B: l=3,0m

Solution C has capacity to handle longer distances, but that will lead to large deflections of the infill panel, so the selected solution is:

- Solution C: l=3,5m

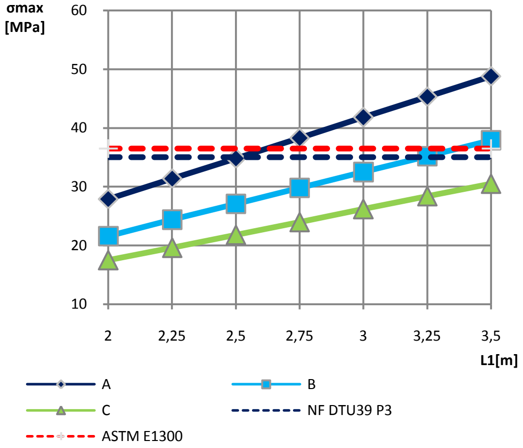

Fig. 13 - Distance between fins, optimization.

Deformation

The limits for deflection are, for this case, L/175 and L/150 by American and French standards respectively. Results show that deflection criteria can hardly be broken for such long glass fins.

- $\delta_A = 35{,}1\ mm$
- $\delta_B = 26{,}6\ mm$
- $\delta_C = 20{,}9\ mm$

Torsional Buckling analysis

Torsional Buckling analysis was conducted by a FE software and the first modes were calculated for the fins. Results are summarized in Table 2 and deformed shapes are illustrated in Fig. 14. The safety factor is calculated by:

$$|F_s| = \frac{critical\ load\ 1st\ mode}{applied\ load} > 1 \qquad \textbf{Eq. 2}$$

Table 2 - Torsional buckling safety factors

Solution	F_s
A	-10,9
B	-10,3
C	-9,4

The negative value means that the load which conducts the beam to the 1st buckling mode is acting on the opposite direction (suction instead of wind pressure). This happens because when wind pressure is acting, the façade provides additional stability to the fins (on compression side of cross section).

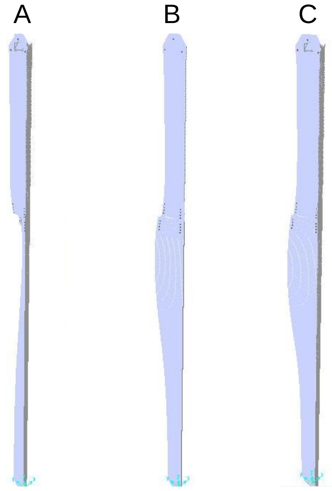

Fig. 14 - 1st buckling mode.

Infill panels

American standard (ASTM E 1300-09) refers that load can be shared for both panes proportionally to their stiffness, in other words, proportionally to the third power of thickness.

$$P_i = \frac{t_i^3}{\sum_i t_i^3} * P \qquad \textbf{Eq. 3}$$

As one of the glass panes is laminated, an equivalent thickness must be calculated according to the following equations:

$$h_{ef,w} = \sqrt[3]{h_1^3 + h_2^3 + 12.\Gamma.I_s} \qquad \textbf{Eq. 4}$$

$$h_{1;ef;\sigma} = \sqrt[2]{\frac{h^3_{ef;w}}{h_1 + 2.\Gamma.h_{s;2}}}$$ **Eq. 5**

$$h_{2;ef;\sigma} = \sqrt[2]{\frac{h^3_{ef;w}}{h_2 + 2.\Gamma.h_{s;1}}}$$ **Eq. 6**

$$h_{s;1} = \frac{h_s h_1}{h_1 + h_2}$$ **Eq. 7**

$$h_{s;2} = \frac{h_s h_2}{h_1 + h_2}$$ **Eq. 8**

$$h_s = 0{,}5 * (h_1 + h_2) + h_v$$ **Eq. 9**

$$\Gamma = \frac{1}{1 + 9{,}6 * \frac{E I_s h_v}{G h_s^2 a^2}}$$ **Eq. 10**

$$I_s = h_1 h_{s;2}^2 + h_2 h_{s;1}^2$$ **Eq. 11**

where,

$h_{ef,w}$ – effective thickness for deformation

$h_{i;ef;\sigma}$ – effective thickness for stress calculation of i panel.

Γ – Shear transfer coefficient

E – Glass Elasticity Modulus

G – Interlayer shear modulus

Several solutions were analyzed with analytical equations and results are summarized in **Erro! A origem da referência não foi encontrada.**, **Erro! A origem da referência não foi encontrada.** and **Erro! A origem da referência não foi encontrada.**. Considering 35 MPa as the limit, optimal solutions are:

- Solution A: $t_2 = t_3 = 8\,mm,\ t_1 = 6\,mm$
- Solution B: $t_2 = t_3 = 10\,mm,\ t_1 = 8\,mm$
- Solution C: $t_2 = t_3 = 12\,mm,\ t_1 = 10\,mm$

The results were compared with SJ MEPLA 3.5, which is a software based on FE theory where the interaction between different layers can be correctly modeled (PVB shear modulus was considered equal to 0.5MPa, corresponding to a load duration of 3sec and a 50°C temperature).

The charts presented from Fig. to Fig. 17 show the evolution of the tensile stress for one sheet of the laminated pane as a function of its thickness for each solution.

SJ MEPLA led to the same thicknesses for A and B cases. For C, MEPLA led to a thinner solution.

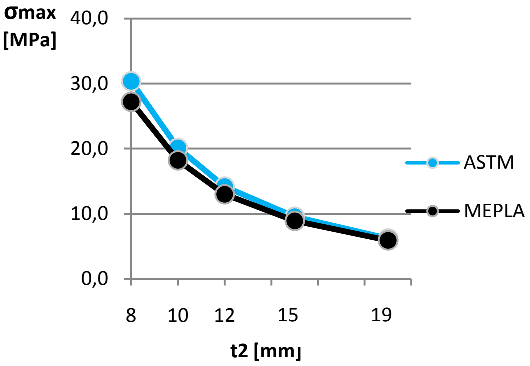

Fig. 15 – Solution A - Laminated pane. Maximum tensile stress by one sheet thickness.

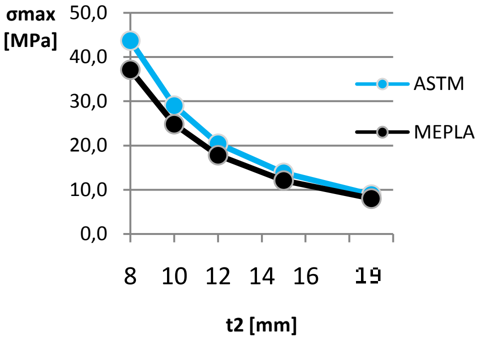

Fig. 16 - Solution B - Laminated pane. Maximum tensile stress by one sheet thickness.

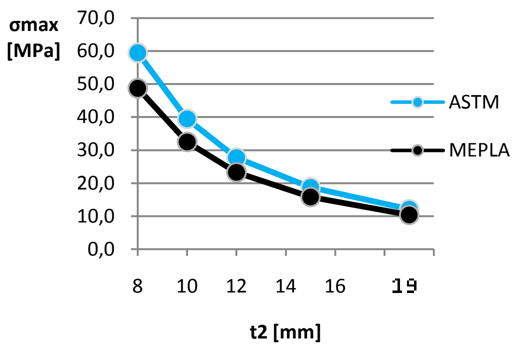

Fig. 15 - Solution C - Laminated pane. Maximum tensile stress by one sheet thickness.

French standard describes an alternative method for glass thickness. For an insulated glass unit, the total thickness of glass sheets shall not be less than:

$$e_t = \frac{t_2 + t_3}{\varepsilon_2} + t_1 \geq e_1 * \varepsilon_1 \qquad \textbf{Eq. 12}$$

and for a simply supported panel on two edges:

$$e_1 = \frac{L * \sqrt{W}}{4{,}9} \qquad \textbf{Eq. 13}$$

where,

L – Span length in meters

W – Wind pressure in Pascal

ε_1 – equivalent factor for insulated glass (1,5 for double units)

ε_2 – equivalent factor for laminated glass (1,3 for two sheets)

Moreover, the deformation shall be less than L/150 and calculated according to:

$$f = \alpha . \frac{P}{1{,}2} . \frac{c^4}{e_2^3} \qquad \textbf{Eq. 14}$$

and,

$$e_2 = \frac{\frac{t_2 + t_3}{\varepsilon_2} + t_1}{\varepsilon_1} \qquad \textbf{Eq. 15}$$

where,

α – equal to 2,1143 for panels supported only on two edges.

c – unsupported edge length in meters.

For solution A,

$$e_1 = \frac{L * \sqrt{W}}{4{,}9} = \frac{2{,}5\sqrt{1500}}{4{,}9} = 19{,}76\ mm$$

$$e_t = \frac{t_2 + t_3}{\varepsilon_2} + t_1 \geq e_1 * \varepsilon_1 = 19{,}76 * 1{,}5$$
$$= 29{,}64\ mm$$

If the monolithic glass is 10 mm thick,

$$e_t = \frac{2 * (t_2 - 0{,}2)}{1{,}3} + (10 - 0{,}2) \geq 29{,}64\ mm$$

$$=> \ t_2 \geq 13{,}10\ mm$$

Usually, sheets with different thicknesses are not bonded together so,

- $t_2 = t_3 = 15\ mm\ e\ t_1 = 12\ mm$

Deformation control

$$e_2 = \frac{\frac{t_2 + t_3}{\varepsilon_2} + t_1}{\varepsilon_1} = \frac{\frac{14{,}8 + 14{,}8}{1{,}3} + 11{,}8}{1{,}5}$$
$$= 23{,}05\ mm$$

$$f = 2{,}1143 * \frac{1500}{1{,}2} * \frac{2{,}5^4}{23{,}05^3} = 8{,}43\ mm$$
$$< 16{,}67\ mm\ (L/150)$$

The same method was applied for solution B and C, and the results are:

- B: $t_2 = t_3 = 19\ mm\ e\ t_1 = 15\ mm$
- C: $t_2 = t_3 = 19\ mm\ e\ t_1 = 15\ mm$

Case study 2

Definition

An example of glass floor design is now presented, according to ASTM E 2751-11. Fig. 18 and Fig. 19 show a 3750x6000 mm glass floor, supported by HEA steel beams on four edges.

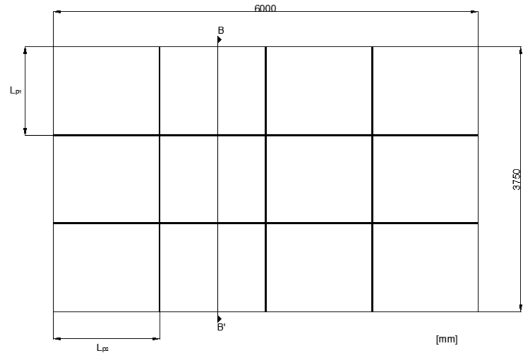

Fig. 16 - Case study 2 - Glass floor.

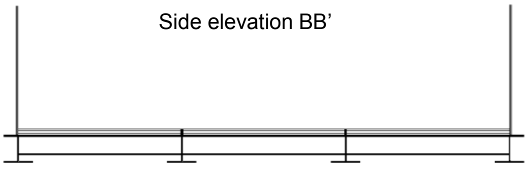

Fig. 17 - Glass floor - Side elevation

Material selection

Once glass is broken, the remaining structure shall be stiff and resistant enough to keep its integrity, therefore more than one sheet shall be bonded.

The type of glass that shall be used is not unanimous among specialists. Some say that, although heat-strengthened having a smaller fragment in case of rupture HSG has also some imperfections, resulting from the factory process, which turns the bonding process between two HSG complicated and even some bubbles may appear. However, the heat treated glasses are much more resistant and some of the most popular glass floor around the World apply HSG (1).

Design

The chart of Fig. 20, shows tensile stress evolution for both software as a function of floor's thickness.

Floors are subject to permanent loads, and as a result, the maximum tensile stress admissible is quite smaller (Table 1), due to the stress corrosion phenomenon (2).

The load cases considered were:

- Uniform distributed load, $qk = 7kN/m^2$
- Concentrated load, $Qk = 1340\ N$

Once American standard makes no reference to calculation of the effective thickness for more than two panes in a laminated glass, on safety side, the shear coefficient may be consider equal to zero .In this case the following equations are valid:

$$h_{ef,w} = \sqrt[3]{h_1^3 + h_2^3 + h_3^3} \qquad \textbf{Eq. 16}$$

$$h_{1;ef;\sigma} = \sqrt[2]{\frac{h_{ef;w}^3}{h_1}} \qquad \textbf{Eq. 17}$$

For a 3 pane laminated glass the results are summarized in Table 3 and Table 4 for HSG and ANN glass respectively.

Table 3 - Floor solutions. Heat-strengthened glass.

		HSG	
L_2/L_1 [mm]	1500	2000	2500
750	3x8	3x8	3x8
1250	3x10	3x12	3x12

Table 4 - Floor solutions - Annealed glass.

		ANN	
L_2/L_1 [mm]	1500	2000	2500
750	3x15	3x15	3x15
1250	n.a	n.a	n.a

n.a – not available.

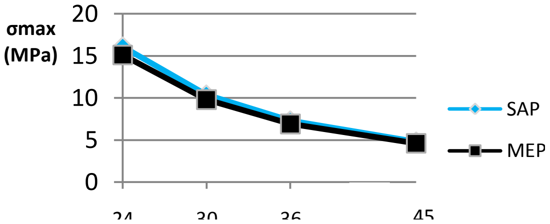

Fig. 18 - Maximum tensile stress per thickness floor.

More sheets may be added to give a better post-breakage performance. Bennison in (1) says that a 8 mm thick toughened sheet was bonded to ensure additional safety and impact resistance.

Case Study 3

Definition

Fig. 21 and Fig. 22 illustrate a real glass balustrade used in a balcony. A laminated glass panel is bolted to a steel structure, which is fixed to a concrete slab.

Forty millimeters diameter holes are introduced to prevent high tensile stresses in glass. Between glass and steel plate is introduced a polymeric bushing material (Teflon, neoprene, silicone, etc).

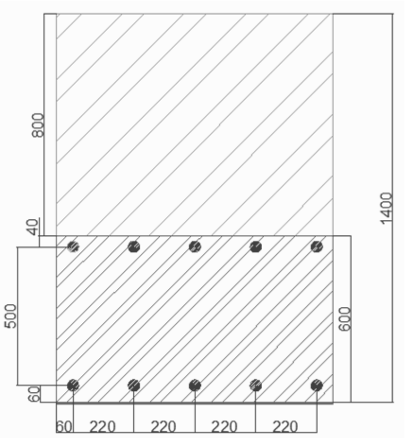

Fig. 19 – Case study 3 - glass balustrade.

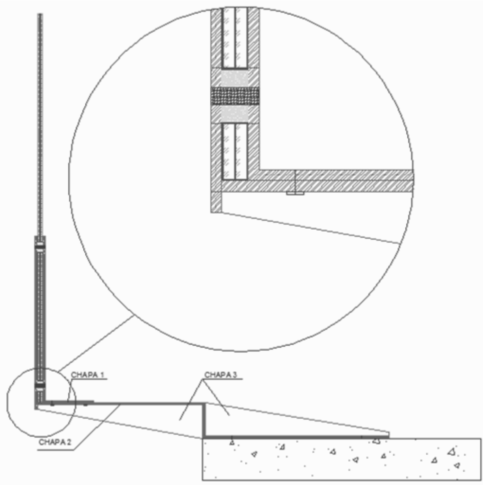

Fig. 20 – Case study 3 - glass balustrade, side elevation.

Material selection

The laminated glass is composed by two 10 mm sheets bonded with 2 PVB. For security reasons, one sheet is a toughened glass and the other is a heat-strengthened glass. The steel structure is composed by two bolted steel plates (S355JR) with 8 mm thick and welded to three longitudinal reinforcements. The bolts are 5.6 type with 12 mm diameter.

Design

French standard is used to limit the tensile stress in glass. For a heat-strenghtened glass, the maximum tensile stress is 35 MPa.

The load cases considered are:

- Self-weight
- Uniform linear load (1kN/m), acting on top edge.
- Wind (1,2 kN/m²).

Load Combination:

Ultimate Limit State – Main load – wind/uniform linear load – Steel structure and bolts design

$$S_d = 1.35 \times S_{G,k} + 1.5 \times \left(S_{q,K} ou S_{W,k}\right)$$

Service Limit State – Main load – wind/uniform linear load – Glass panel analysis.

$$S_{Raro} = S_{G,k} + \left(S_{q,K} ou S_{W,k}\right)$$

Results

A finite element model (shell elements) was created in order to analyze the maximum stress in the glass panel and in the steel structure. The results were obtained with SAP2000 software (Fig. 23 to Fig. 27).

For the glass panel was assumed that each sheet supports half of the load and a model with one sheet thickness was run.

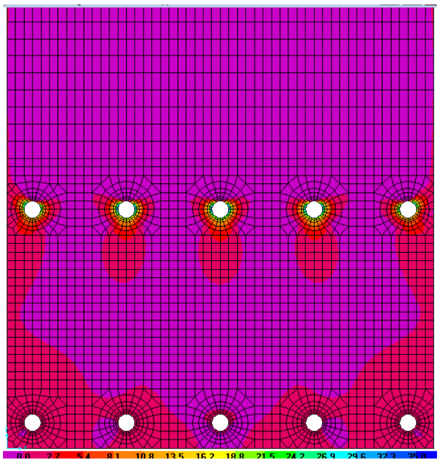

Fig. 21 – Maximum tensile stress in glass $\sigma_{max} = 31,9\ MPa$.

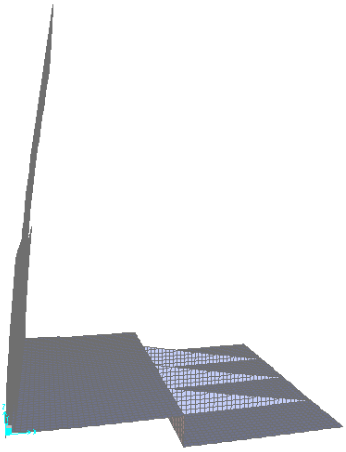

Fig. 22 – Deformed shape $\delta_{max} = 21,8\ mm$.

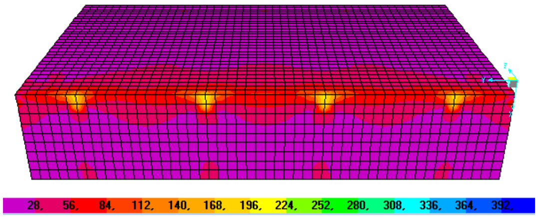

Fig. 23 – Maximum combination stress, L plate $\sigma_{comb} = 214\ MPa$

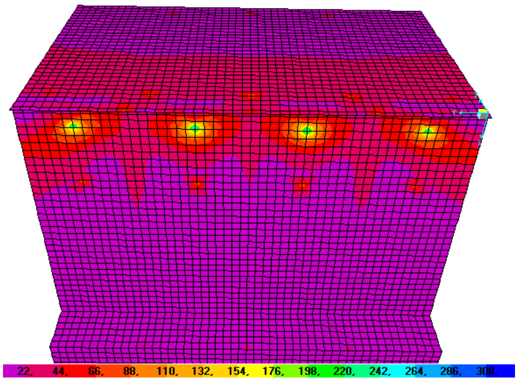

Fig. 24 – Maximum combination stress, main plate. $\sigma_{comb} = 333\ MPa$

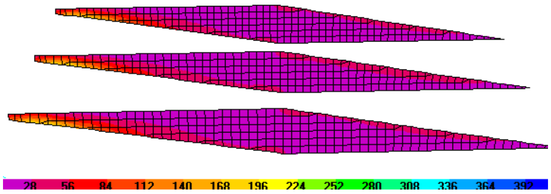

Fig. 25 – Maximum combination stress, reinforcement plates $\sigma_{max} = 175\ MPa$.

4 – Conclusions

Even with the recent development in glass industry and with such improvements in quality, the structural glass design process is far from being a trivial task. The engineer has to be aware of several distinct problems, such as material's behavior and its interaction with others, safety requirements, FE analysis etc. Additionally, international standards are still having some difficulties to set accurate design methods based in rules of thumb, which turns the engineer’s work more complicated.

This paper showed how to design two examples where frequently glass is used as a structural material. In the façade case, it was demonstrated how the connection geometry and chosen material may influence the maximum stress in glass fins.

The design of infill panels according to two different methods showed that American Standard is more permissive than French standard. MEPLA results were compared with ASTM and it was seen that the maximum relative error was about 22% and in general, it decreases with thickness and grows for longer spans. Even though, same solutions were selected for case A and B. For the laminated pane SAP conducted to higher stresses in all cases.

For the floor case, FE simplified models by SAP were used in accordance to ASTM requirements and then compared with MEPLA. The results are more encouraging for simple glazing systems against double glazing, with a maximum relative error of 7%. It was verified that for a shortest span of 1250 mm it is necessary to bond more sheets.

For the third case, the model is quite conservative because it only considers that stress transmission occurs through bolts which is not true, due to the direct contact that exists all over the surface.

References

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