ANALYSIS OF 2D TRUSSES BY STIFFNESS METHOD

Procedure for Truss Analysis

- **Step 1: Notation**
- Establish the x, y global coordinate system. You may take any joint as an origin
- Identify each joint and element numerically and specify near and far ends of each member.
- Specify the two code numbers at each joint, using the lowest numbers to identify degree of freedoms, followed by the highest degree of freedom to identify constrains.
- From the problem establish D_k and Q_k.

- **Step 2: Structure Stiffness Matrix**

- For each member of the truss determine λ_x and λ_y and the member stiffness matrix using the following general matrix

$$\mathbf{k} = \frac{AE}{L}\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccc} N_x & N_y & F_x & F_y \\ \left[\begin{array}{c} \lambda_x^2 \\ \lambda_x\lambda_y \\ -\lambda_x^2 \\ -\lambda_x\lambda_y \end{array}\right. & \begin{array}{c} \lambda_x\lambda_y \\ \lambda_y^2 \\ -\lambda_x\lambda_y \\ -\lambda_y^2 \end{array} & \begin{array}{c} -\lambda_x^2 \\ -\lambda_x\lambda_y \\ \lambda_x^2 \\ \lambda_x\lambda_y \end{array} & \left.\begin{array}{c} -\lambda_x\lambda_y \\ -\lambda_y^2 \\ \lambda_x\lambda_y \\ \lambda_y^2 \end{array}\right] \end{array}\begin{array}{c} \\ N_x \\ N_y \\ F_x \\ F_y \end{array}$$

- Assemble these matrices to form the stiffness matrix for the entire truss (as explained earlier on board).
- Note: The member and structure stiffness matrices should be symmetric

$$\lambda_x = \frac{x_F - x_N}{L}$$

$$\lambda_y = \frac{y_F - y_N}{L}$$

- **Step 3: Displacement and Loads**

- Partition the structure stiffness matrix for easier calculations

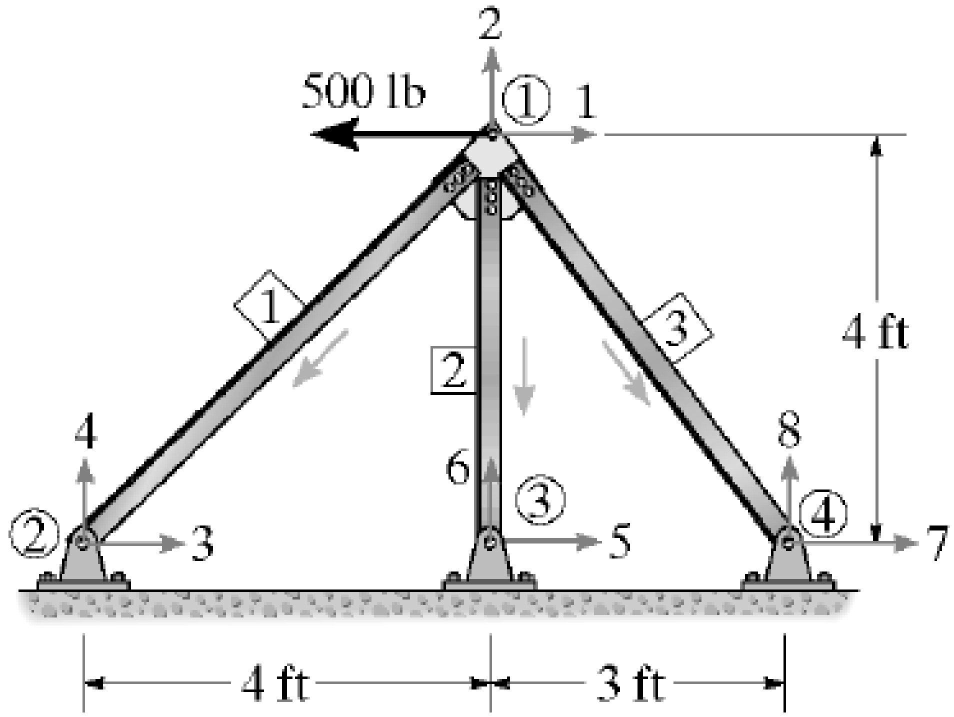
- Determine the unknown joint displacement D_x, the support reactions Q_x.

- Using all the unknowns the member forces in each truss element using basic rules of truss analysis

Solved Problem 1:

Determine the horizontal displacement of joint ① and the force in member $\boxed{2}$.

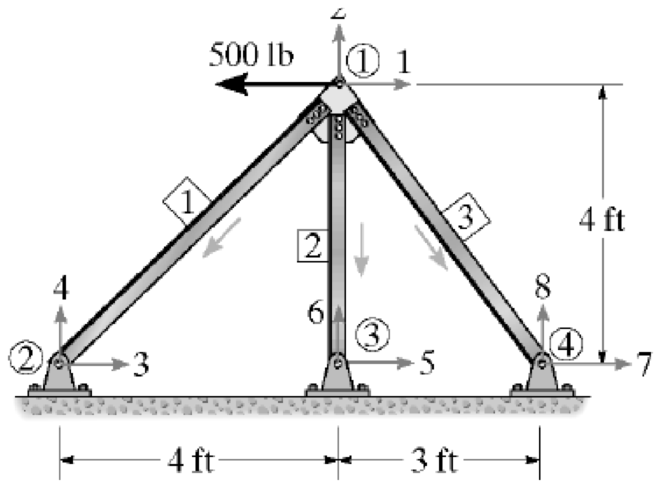
Take $A = 0.75\ in^2$ and $E = 29x10^3$ ksi.



General stiffness matrix

$$\mathbf{k} = \frac{AE}{L}\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} N_x & N_y & F_x & F_y \end{array} \\ \begin{bmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x^2 & -\lambda_x\lambda_y \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_x\lambda_y & -\lambda_y^2 \\ -\lambda_x^2 & -\lambda_x\lambda_y & \lambda_x^2 & \lambda_x\lambda_y \\ \lambda_x\lambda_y & -\lambda_y^2 & \lambda_x\lambda_y & \lambda_y^2 \end{bmatrix} \end{array}\begin{array}{c} \\ N_x \\ N_y \\ F_x \\ F_y \end{array}$$

- Member 1



$$\lambda_x = \frac{0-4}{\sqrt{32}} = -0.7071$$

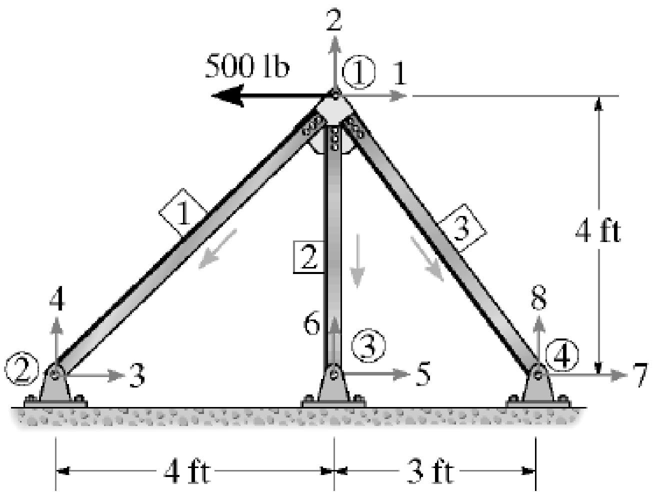
$$\lambda_y = \frac{0-4}{\sqrt{32}} = -0.7071$$

$$\mathbf{k}_1 = AE\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.08839 & 0.08839 & -0.08839 & -0.08839 \\ 0.08839 & 0.08839 & -0.08839 & -0.08839 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 \end{bmatrix} \end{array}$$

- Member 2

$$\lambda_x = \frac{4-4}{4} = 0$$

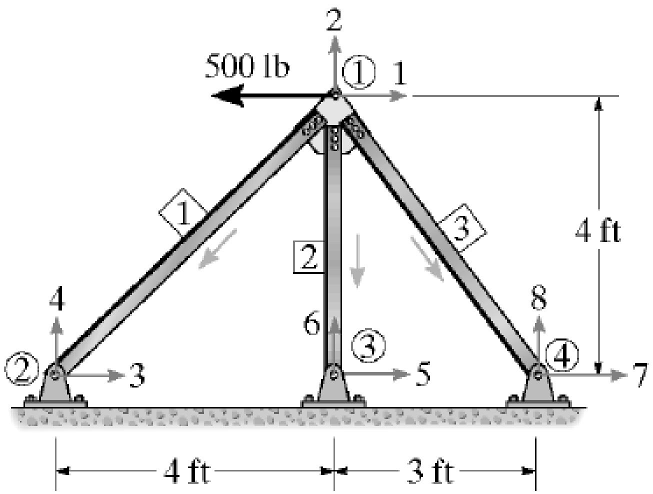
$$\lambda_y = \frac{0-4}{4} = -1$$



$$\begin{array}{c} \quad\quad\;\; \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \mathbf{k}_2 = AE\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & -0.25 \\ 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0.25 \end{bmatrix} \end{array}$$

$$\mathbf{k} = \frac{AE}{L}\begin{array}{c} \begin{array}{cccc} N_x & N_y & F_x & F_y \end{array} \\ \begin{bmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x^2 & -\lambda_x\lambda_y \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_x\lambda_y & -\lambda_y^2 \\ -\lambda_x^2 & -\lambda_x\lambda_y & \lambda_x^2 & \lambda_x\lambda_y \\ \lambda_x\lambda_y & -\lambda_y^2 & \lambda_x\lambda_y & \lambda_y^2 \end{bmatrix} \end{array} \begin{array}{c} N_x \\ N_y \\ F_x \\ F_y \end{array}$$

- Member 3



$\lambda_x = 0.6$

$\lambda_y = -0.8$

$$\mathbf{k}_3 = AE\begin{bmatrix} 0.072 & -0.096 & -0.072 & 0.096 \\ -0.096 & 0.128 & 0.096 & -0.128 \\ -0.072 & 0.096 & 0.072 & -0.096 \\ 0.096 & -0.128 & -0.096 & 0.128 \end{bmatrix}$$

- Structural Stiffness Matrix:

$$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$$

$$\mathbf{K} = AE \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0.16039 & -0.00761 & -0.08839 & -0.08839 & 0 & 0 & -0.072 & 0.096 \\ -0.00761 & 0.46639 & -0.08839 & -0.08839 & 0 & -0.25 & 0.096 & -0.128 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ -0.072 & 0.096 & 0 & 0 & 0 & 0 & 0.072 & -0.096 \\ 0.096 & -0.128 & 0 & 0 & 0 & 0 & -0.096 & 0.128 \end{bmatrix} \end{array}$$

- The Equation: **Q = K D**

$$\begin{bmatrix} -500 \\ 0 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = AE \begin{bmatrix} 0.16039 & -0.00761 & -0.08839 & -0.08839 & 0 & 0 & -0.072 & 0.096 \\ -0.00761 & 0.46639 & -0.08839 & -0.08839 & 0 & -0.25 & 0.096 & -0.128 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ -0.072 & 0.096 & 0 & 0 & 0 & 0 & 0.072 & -0.096 \\ 0.096 & -0.128 & 0 & 0 & 0 & 0 & -0.096 & 0.128 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

partition matrix

$$\begin{bmatrix} -500 \\ 0 \end{bmatrix} = AE \begin{bmatrix} 0.16039 & -0.00761 \\ -0.00761 & 0.46639 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$-500 = AE(0.16039D_1 - 0.00761D_2) \qquad (1)$$

$$0 = AE(-0.00761D_1 + 0.46639D_2) \qquad (2)$$

Solving Eq. (1) and (2) yields :

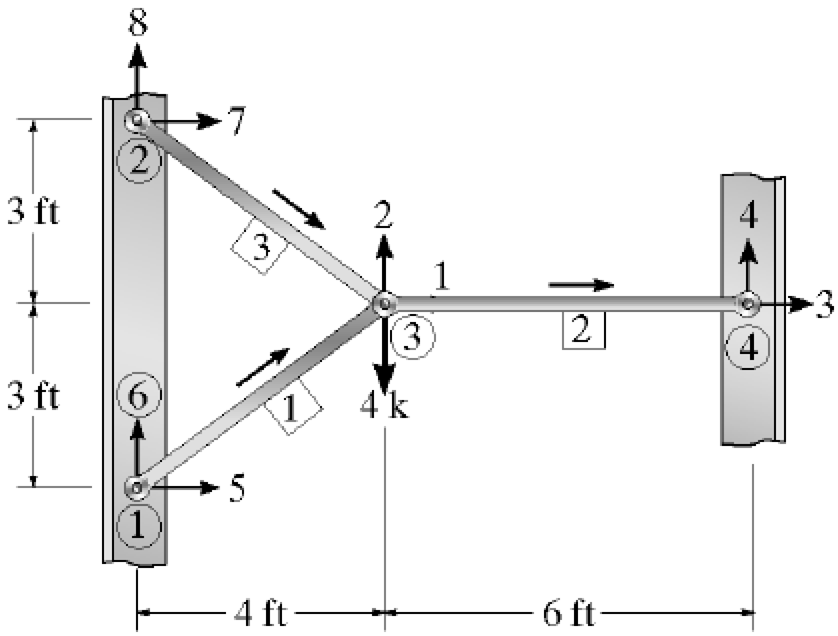
$$D_1 = \frac{-3119.82}{AE} = \frac{-3119.85(12 \text{ in./ft})}{0.75 \text{ in}^2 (29)(10^6) \text{ lb/in}^2} = -0.00172 \text{ in.}$$

$$D_2 = \frac{-50.917}{AE}$$

- Find out the values of Q_3, Q_4, Q_5, Q_6, Q_7 and Q_8.
- Then find out forces in member no. 2 using method of joints.

Practice Problems No. 1

- Determine the unknown support reactions in the following truss. Take $A = 0.5\ in^2$ and $E = 29x10^3$ ksi for each member. All supports are hinge supports

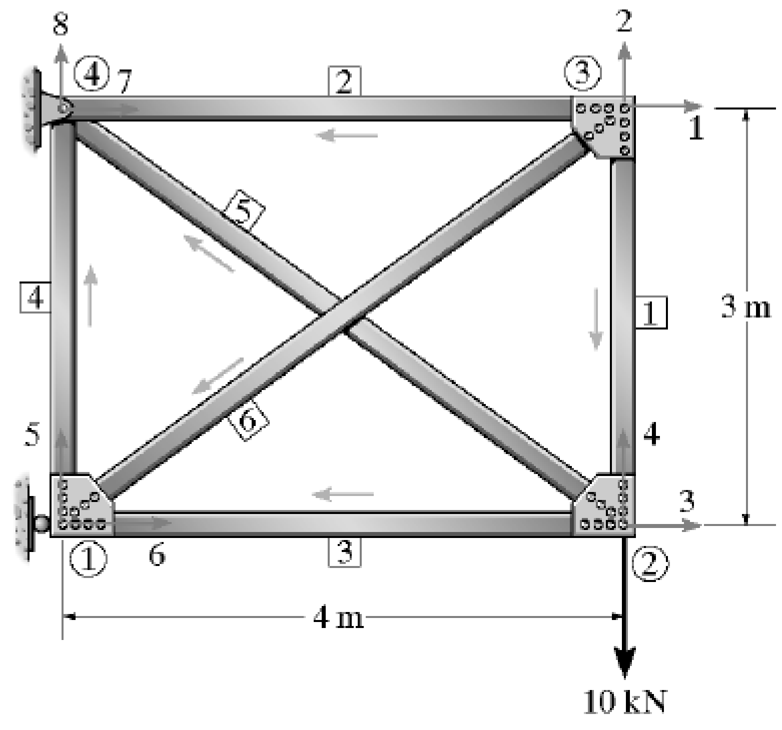


SOLUTION HINT

$$\begin{bmatrix} 0 \\ -4 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \begin{bmatrix} 510.72 & 0 & -201.39 & 0 & -154.67 & -116 & -154.67 & 116 \\ 0 & 174 & 0 & 0 & -116 & -87.0 & 116 & -87.0 \\ -201.39 & 0 & 201.39 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -154.67 & -116 & 0 & 0 & 154.67 & 116 & 0 & 0 \\ -116 & -87.0 & 0 & 0 & 116 & 87.0 & 0 & 0 \\ -154.67 & 116 & 0 & 0 & 0 & 0 & 154.67 & -116 \\ 116 & -87.0 & 0 & 0 & 0 & 0 & -116 & 87.0 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Practice Problems No. 2

- Determine the unknown support reactions in the following truss.
- Take $A = 0.005\ m^2$ and $E = 29\ Gpa$ for each member.

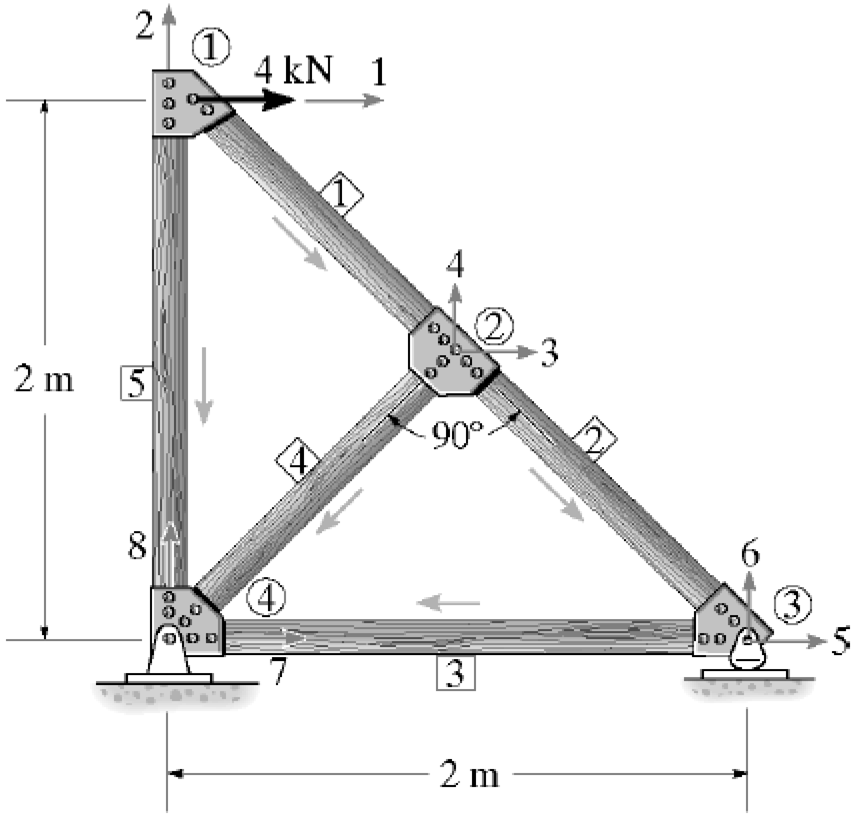


SOLUTION HINT

$$
\begin{bmatrix} 0 \\ 0 \\ 0 \\ -10 \\ 0 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = AE
\begin{bmatrix}
0.378 & 0.096 & 0 & 0 & -0.096 & -0.128 & -0.25 & 0 \\
0.096 & 0.4053 & 0 & -0.3333 & -0.072 & -0.096 & 0 & 0 \\
0 & 0 & 0.378 & -0.096 & 0 & -0.25 & -0.128 & 0.096 \\
0 & -0.3333 & -0.096 & 0.4053 & 0 & 0 & 0.096 & -0.072 \\
-0.096 & -0.072 & 0 & 0 & 0.4053 & 0.096 & 0 & -0.3333 \\
-0.128 & -0.096 & -0.25 & 0 & 0.096 & 0.378 & 0 & 0 \\
-0.25 & 0 & -0.128 & 0.096 & 0 & 0 & 0.378 & -0.096 \\
0 & 0 & 0.096 & -0.072 & -0.3333 & 0 & -0.096 & 0.4053
\end{bmatrix}
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Practice Problems No. 3

- Determine the unknown support reactions in the following truss.
- Take *AE as constant*

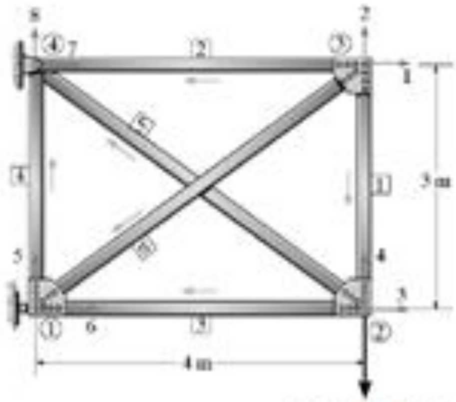


SOLUTION HINT

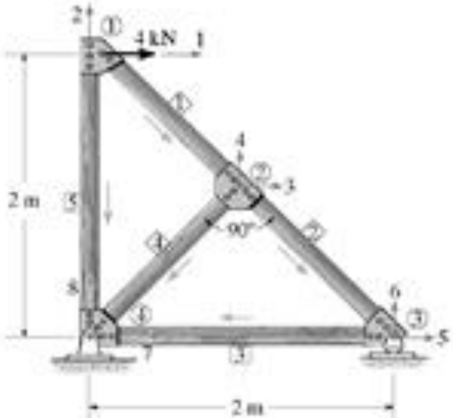
$$
\begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix}
= AE
\begin{bmatrix}
0.3536 & -0.3536 & -0.3536 & 0.3536 & 0 & 0 & 0 & 0 \\
-0.3536 & 0.8536 & 0.3536 & -0.3536 & 0 & 0 & 0 & -0.5 \\
-0.3536 & 0.3536 & 1.0607 & -0.3536 & -0.3536 & 0.3536 & -0.3536 & -0.3536 \\
0.3536 & -0.3536 & -0.3536 & 1.0607 & 0.3536 & -0.3536 & -0.3536 & -0.3536 \\
0 & 0 & -0.3536 & 0.3536 & 0.8536 & -0.3536 & -0.5 & 0 \\
0 & 0 & 0.3536 & -0.3536 & -0.3536 & 0.3536 & 0 & 0 \\
0 & 0 & -0.3536 & -0.3536 & -0.5 & 0 & 0.8536 & 0.3536 \\
0 & -0.5 & -0.3536 & -0.3536 & 0 & 0 & 0.3536 & 0.8536
\end{bmatrix}
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

ASSIGNMENT NO. 2

Q1) Determine the unknown support reactions in the following truss. Take A= 0.005 m^2 and E = 29 Gpa for each member.



Q1) Determine the unknown support reactions in the following truss. Take AE as constant.



Note: Submission date is 15-04-2015 (Wednesday)

HOW TO SOLVE MATRICES QUICKLY?

- In the matrices equation Q=KD, first the D matrix has to be determined. If there are 2 or three unknowns in D matrix then it can easily be determined by using equation method.

- But, if the no. of unknown increases it is recommended that you should use matrix inverse method for determining unknown D matrix.
- Since, Q=KD, so $D=K^{-1}Q$

- For determining inverse of a matrix, the Guass Jordan method can be used.

Gauss-Jordan Method for inverse Matrix?

Use a book, “Basic Structural Analysis by CS Reddy”, Appendix A.5.2, Page No. 790 for this method (scanned copies in next slide)

Noe:

For checking the results. an excel sheet can be used to determine inverse of a matrix. The excel sheet is provided alongwith.

A.5.2 Inversion of Matrix by Gauss-Jordan Method

When the size of matrix is larger than 4×4, inversion by Eq. A.62 becomes very cumbersome. Many additional methods have been developed for inverting large matrices. One method which is most commonly used is the Gauss-Jordan or complete elimination method. This is by far the quickest method for the inversion of a matrix on a computer.

As an example, consider the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 4 \\ -1 & 3 & 6 \end{bmatrix} \tag{A. 63}$$

As a first step, a rectangular matrix is formed by augmenting the given matrix with an identity matrix as shown

$$\begin{bmatrix} 2 & 4 & 3 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ -1 & 3 & 6 & 0 & 0 & 1 \end{bmatrix} \tag{A.64}$$

The Gauss-Jordan elimination process is applied to the rectangular matrix reducing the left part of the matrix to an identity matrix with the right part attaining the elements denoted by b_{ij}. The resulting matrix is of the form

$$\begin{bmatrix} 1 & 0 & 0 & b_{11} & b_{12} & b_{13} \\ 0 & 1 & 0 & b_{21} & b_{22} & b_{23} \\ 0 & 0 & 1 & b_{31} & b_{32} & b_{33} \end{bmatrix} \tag{A.65}$$

The inversion of $\mathbf{A}$ is

$$\mathbf{A}^{-1} = \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \tag{A.66}$$

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 4 \\ -1 & 3 & 6 \end{bmatrix}$$

Make an augmented matrix first

$$\begin{bmatrix} 2 & 4 & 3 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ -1 & 3 & 6 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 4 \\ -1 & 3 & 6 \end{bmatrix}$$

Apply the row operations to solve the matrix

$$\begin{bmatrix} 1 & 2 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

$R_1 / 2$

$R_2 - 1 \times R_1$

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 4 \\ -1 & 3 & 6 \end{bmatrix}$$

Apply the row operations to solve the matrix

$$\begin{bmatrix} 1 & 2 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \end{bmatrix}$$

$R_1 / 2$

$R_2 – 1 \times R_1$

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 4 \\ -1 & 3 & 6 \end{bmatrix}$$

Apply the row operations to solve the matrix

$$\begin{bmatrix} 1 & 2 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \end{bmatrix}$$

$R_1 / 2$

$R_2 – 1 \times R_1$

$R_3 + 1 \times R_1$

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 4 \\ -1 & 3 & 6 \end{bmatrix}$$

Apply the row operations to solve the matrix

$$\begin{bmatrix} 1 & 2 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & 5 & \frac{15}{2} & \frac{1}{2} & 0 & 1 \end{bmatrix}$$

$R_1 / 2$

$R_2 – 1 \times R_1$

$R_3 + 1 \times R_1$

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 4 \\ -1 & 3 & 6 \end{bmatrix}$$

Now don’t touch first row and the first column

$$\begin{bmatrix} 1 & 2 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & 5 & \frac{15}{2} & \frac{1}{2} & 0 & 1 \end{bmatrix}$$

$R_2 \,/\, (a_{22} = 1)$

$$\begin{bmatrix} 1 & 0 & -\frac{7}{2} & \frac{3}{2} & -2 & 0 \\ 0 & 1 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 & -\frac{3}{5} & 1 & -\frac{1}{5} \end{bmatrix}$$

$R_1 - 2\,R_2$

$R_3 - 5\,R_2$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{5} & \frac{3}{2} & -\frac{7}{10} \\ 0 & 1 & 0 & 1 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{3}{5} & 1 & -\frac{1}{5} \end{bmatrix}$$

$R_1 + 7/2\,R_3$

$R_2 - 5/2\,R_3$

Hence the inverse matrix is

$$\mathbf{A}^{-1} = \begin{bmatrix} -\dfrac{3}{5} & \dfrac{3}{2} & -\dfrac{7}{10} \\ 1 & -\dfrac{3}{2} & \dfrac{1}{2} \\ -\dfrac{3}{5} & 1 & -\dfrac{1}{5} \end{bmatrix}$$

The result can be verified by the relationship

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$\begin{bmatrix} -\dfrac{3}{5} & \dfrac{3}{2} & -\dfrac{7}{10} \\ 1 & -\dfrac{3}{2} & \dfrac{1}{2} \\ -\dfrac{3}{5} & 1 & -\dfrac{1}{5} \end{bmatrix} \begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 4 \\ -1 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

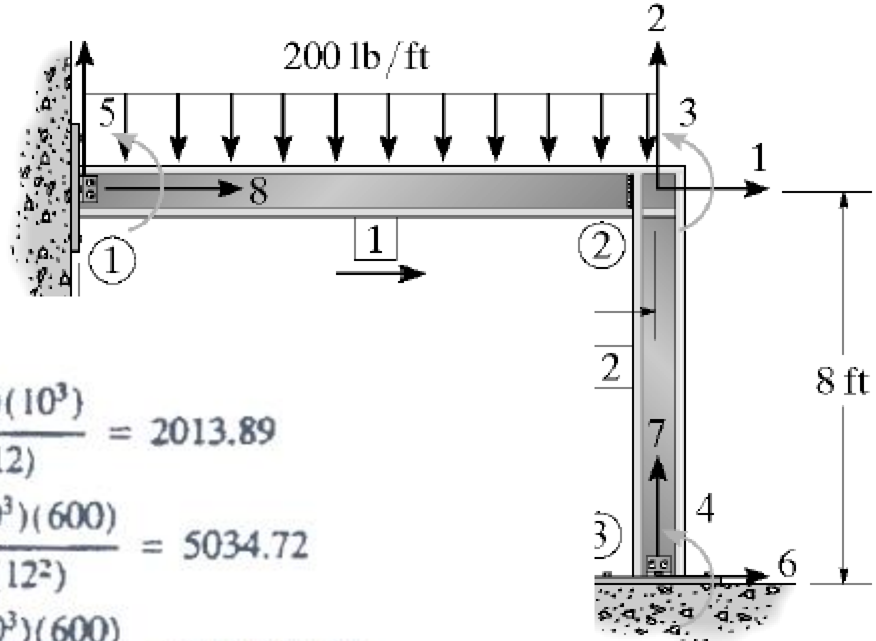
ANALYSIS OF FRAMES BY STIFFNESS METHOD

General Stiffness Matrix

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \quad N_x \quad & \quad N_y \quad & \quad N_z \quad & \quad F_x \quad & \quad F_y \quad & \quad F_z \quad \end{array} \\
\mathbf{k} = & \left[\begin{array}{cccccc}
\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\frac{6EI}{L^2}\lambda_y & -\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\frac{6EI}{L^2}\lambda_y \\
\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & \frac{6EI}{L^2}\lambda_x & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & \frac{6EI}{L^2}\lambda_x \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{4EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} \\
-\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y & \left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y \\
-\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & -\frac{6EI}{L^2}\lambda_x & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_x^2\right) & -\frac{6EI}{L^2}\lambda_x \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{4EI}{L}
\end{array}\right]
\begin{array}{c} N_x \\ N_y \\ N_z \\ F_x \\ F_y \\ F_z \end{array}
\end{array}
$$

Exercise 16-1

Determine the structure stiffness matrix **K** for the frame. Assume ① and ③ are pins. Take $E = 29(10^3)$ ksi, $I = 600\ \text{in}^4$, $A = 10\ \text{in}^2$ for each member.



Member 1

$$\lambda_x = \frac{12-0}{12} = 1 \qquad \lambda_y = 0 \qquad \frac{AE}{L} = \frac{(10)(29)(10^3)}{(12)(12)} = 2013.89$$

$$\frac{12EI}{L^3} = \frac{12(29)(10^3)(600)}{(12^3)(12^3)} = 69.93 \qquad \frac{6EI}{L^2} = \frac{6(29)(10^3)(600)}{(12^2)(12^2)} = 5034.72$$

$$\frac{4EI}{L} = \frac{4(29)(10^3)(600)}{(12)(12)} = 483\,333.33 \qquad \frac{2EI}{L} = \frac{2(29)(10^3)(600)}{(12)(12)} = 241\,666.67$$

$$\mathbf{k}_1 = \begin{bmatrix} 2013.89 & 0 & 0 & -2013.89 & 0 & 0 \\ 0 & 69.93 & 5034.72 & 0 & -69.93 & 5034.72 \\ 0 & 5034.72 & 483\,333.33 & 0 & -5034.72 & 241\,666.67 \\ -2013.89 & 0 & 0 & 2013.89 & 0 & 0 \\ 0 & -69.93 & -5034.72 & 0 & 69.93 & -5034.72 \\ 0 & 5034.72 & 241\,666.67 & 0 & -5034.72 & 483\,333.33 \end{bmatrix}$$

$$
\mathbf{k} = \begin{array}{c} \\ \end{array}
\begin{array}{cccccc|c}
N_x & N_y & N_z & F_x & F_y & F_z & \\
\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\frac{6EI}{L^2}\lambda_y & -\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\frac{6EI}{L^2}\lambda_y & N_x \\
\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & \frac{6EI}{L^2}\lambda_x & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & \frac{6EI}{L^2}\lambda_x & N_y \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{4EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} & N_z \\
-\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y & \left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y & F_x \\
-\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & -\frac{6EI}{L^2}\lambda_x & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & -\frac{6EI}{L^2}\lambda_x & F_y \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{4EI}{L} & F_z
\end{array}
$$

Member 2

$$\lambda_x = 0 \qquad \lambda_y = \frac{-8-0}{8} = -1 \qquad \frac{AE}{L} = \frac{10(29)(10^3)}{8(12)} = 3020.83$$

$$\frac{12EI}{L^3} = \frac{12(29)(10^3)(600)}{(8^3)(12^3)} = 236.00 \qquad \frac{6EI}{L^2} = \frac{6(29)(10^3)(600)}{(8^2)(12)^2} = 11\,328.13$$

$$\frac{4EI}{L} = \frac{4(29)(10^3)(600)}{8(12)} = 725\,000 \qquad \frac{2EI}{L} = \frac{2(29)(10^3)(600)}{8(12)} = 362\,500$$

$$
\mathbf{k}_2 = \begin{bmatrix}
236.00 & 0 & 11\,328.13 & -236.00 & 0 & 11\,328.13 \\
0 & 3020.83 & 0 & 0 & -3020.83 & 0 \\
11\,328.13 & 0 & 725\,000 & -11\,328.13 & 0 & 362\,500 \\
-236.00 & 0 & -11\,328.13 & 236.00 & 0 & -11\,328.13 \\
0 & -3020.83 & 0 & 0 & 3020.83 & 0 \\
11\,328.13 & 0 & 362\,500 & -11\,328.13 & 0 & 725\,000
\end{bmatrix}
$$

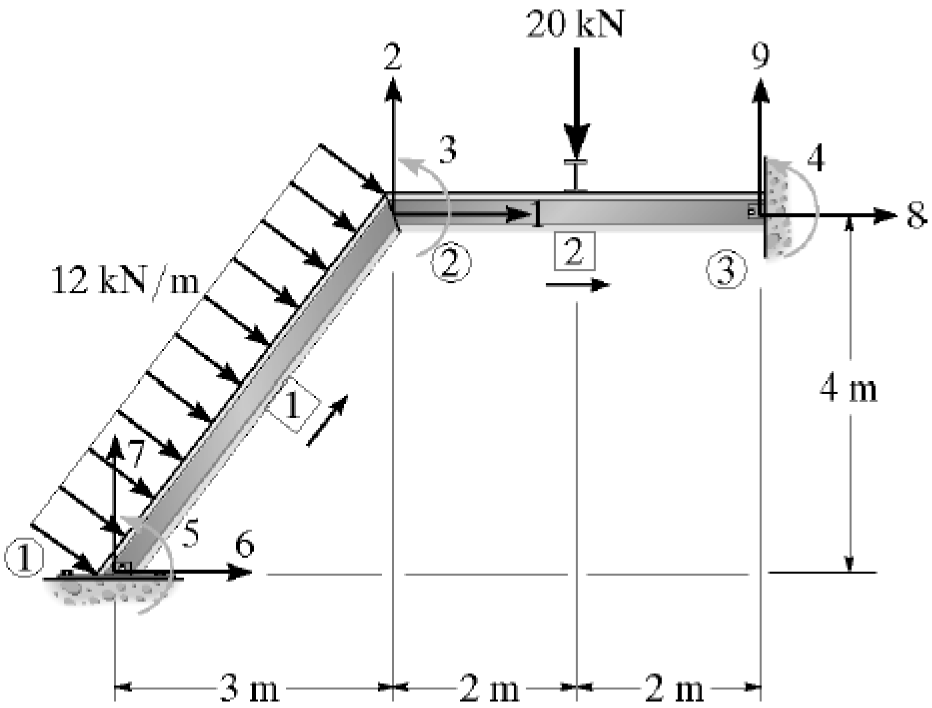
General Stiffness Matrix

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \quad N_x \quad & \quad N_y \quad & \quad N_z \quad & \quad F_x \quad & \quad F_y \quad & \quad F_z \quad \end{array} \\
\mathbf{k} = & \left[\begin{array}{cccccc}
\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\frac{6EI}{L^2}\lambda_y & -\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\frac{6EI}{L^2}\lambda_y \\
\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & \frac{6EI}{L^2}\lambda_x & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & \frac{6EI}{L^2}\lambda_x \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{4EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} \\
-\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y & \left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y \\
-\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & -\frac{6EI}{L^2}\lambda_x & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & -\frac{6EI}{L^2}\lambda_x \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{4EI}{L}
\end{array}\right]
\begin{array}{c} N_x \\ N_y \\ N_z \\ F_x \\ F_y \\ F_z \end{array}
\end{array}
$$

Structure stiffness matrix

$$
\begin{array}{c}
\begin{array}{ccccccccc} \quad 1 & \quad\quad 2 & \quad\quad 3 & \quad\quad 4 & \quad\quad 5 & \quad\quad 6 & \quad\quad 7 & \quad\quad 8 & \quad\quad 9 \end{array} \\
\mathbf{K} = \begin{bmatrix}
2249.89 & 0 & 11328.13 & 11328.13 & 0 & -236.00 & 0 & -2013.89 & 0 \\
0 & 3090.76 & -5034.72 & 0 & -5034.72 & 0 & -3020.83 & 0 & -69.93 \\
11328.13 & -5034.72 & 1208333.33 & 362500 & 241666.67 & -11328.13 & 0 & 0 & 5034.72 \\
11328.13 & 0 & 362500 & 725000 & 0 & -11328.13 & 0 & 0 & 0 \\
0 & -5034.72 & 241666.67 & 0 & 483\,333.33 & 0 & 0 & 0 & 5034.72 \\
-236.00 & 0 & -11328.13 & -11328.13 & 0 & 236.00 & 0 & 0 & 0 \\
0 & -3020.83 & 0 & 0 & 0 & 0 & 3020.83 & 0 & 0 \\
-2013.89 & 0 & 0 & 0 & 0 & 0 & 0 & 2013.89 & 0 \\
0 & -69.93 & 5034.72 & 0 & 5034.72 & 0 & 0 & 0 & 69.93
\end{bmatrix} \quad \text{Ans}
\end{array}
$$

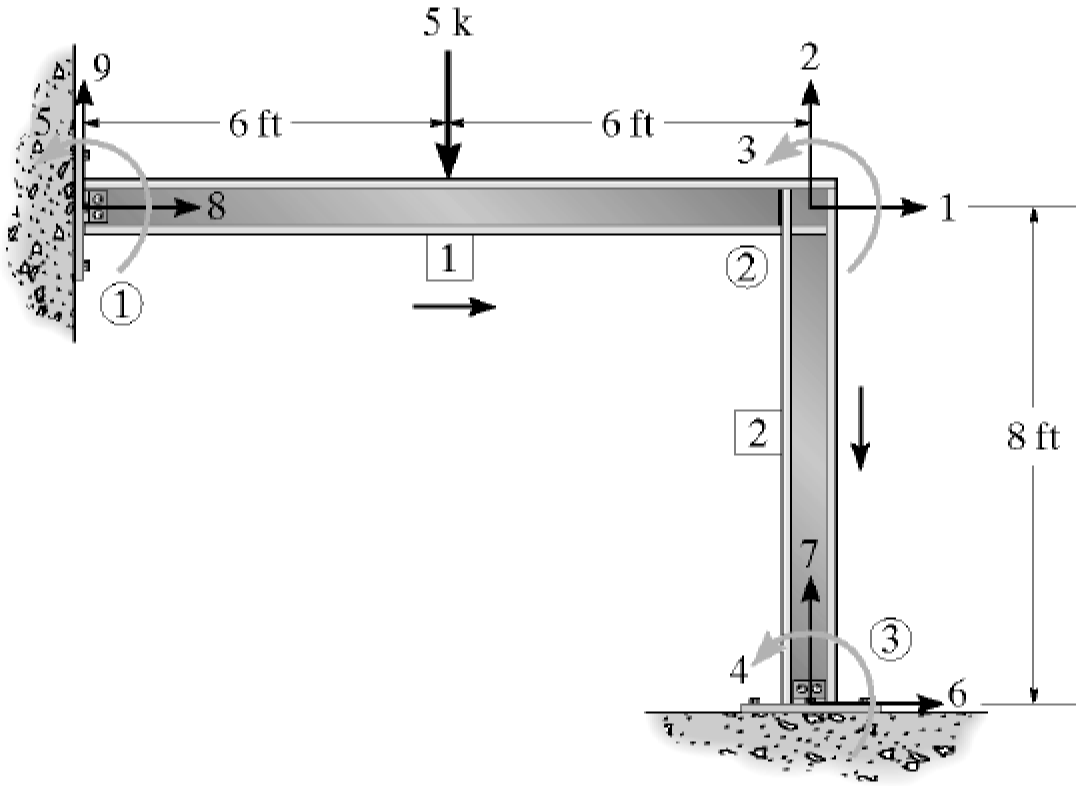
Exercise 16-14



For solution refer to class lectures

Exercise 16-10

Determine the structure stiffness matrix **K** for the frame. Take $E = 29(10^3)$ ksi, $I = 600$ in^4, $A = 10$ in^2 for each member. Assume joints ① and ③ are pinned; joint ② is fixed.



Member 1: $\lambda_x = \dfrac{12-0}{12} = 1$; $\lambda_y = \dfrac{0-0}{8} = 0$

$$\mathbf{k}_1 = \begin{bmatrix} 2013.89 & 0 & 0 & -2013.89 & 0 & 0 \\ 0 & 69.927 & 5034.722 & 0 & -69.927 & 5034.722 \\ 0 & 5034.722 & 483.33(10^3) & 0 & -5034.722 & 241.667(10^3) \\ -2013.89 & 0 & 0 & 2013.89 & 0 & 0 \\ 0 & -69.927 & -5034.722 & 0 & 69.927 & -5034.722 \\ 0 & 5034.722 & 241.667(10^3) & 0 & -5034.722 & 483.33(10^3) \end{bmatrix}$$

Member 2: $\lambda_x = \dfrac{12-12}{12} = 0$; $\lambda_y = \dfrac{-8-0}{8} = -1$

$$\mathbf{k}_2 = \begin{bmatrix} 236.003 & 0 & 11328.125 & -236.003 & 0 & 11328.125 \\ 0 & 3020.833 & 0 & 0 & -3020.833 & 0 \\ 11328.125 & 0 & 725000 & -11328.125 & 0 & 362500 \\ -236.003 & 0 & -11328.125 & 236.003 & 0 & -11328.125 \\ 0 & -3020.833 & 0 & 0 & 3020.833 & 0 \\ 11328.125 & 0 & 362500 & -11328.125 & 0 & 725000 \end{bmatrix}$$

$$
\mathbf{K} = \begin{bmatrix}
2249.892 & 0 & 11328.125 & 11328.125 & 0 & -236 & 0 & -2013.89 & 0 \\
0 & 3090.76 & -5034.722 & 0 & -5034.722 & 0 & -3020.833 & 0 & -69.927 \\
11328.125 & -5034.722 & 1208.33(10^{3}) & 362500 & 241666.67 & -11328.125 & 0 & 0 & 5034.722 \\
11328.125 & 0 & 362500 & 725000 & 0 & -11328.125 & 0 & 0 & 0 \\
0 & -5034.722 & 241666.67 & 0 & 483333.33 & 0 & 0 & 0 & 5034.722 \\
-236 & 0 & -11328.125 & -11328.125 & 0 & 236 & 0 & 0 & 0 \\
0 & -3020.833 & 0 & 0 & 0 & 0 & 3020.833 & 0 & 0 \\
-2013.89 & 0 & 0 & 0 & 0 & 0 & 0 & 2013.89 & 0 \\
0 & -69.927 & 5034.722 & 0 & 5034.722 & 0 & 0 & 0 & 69.927
\end{bmatrix}
$$

CONCEPT OF

TEMPERATURE EFFECTS,
LACK OF FIT
AND
SUPPORT SETTLEMENT

INTRODUCTION TO FINITE ELEMENT METHOD (FEM)

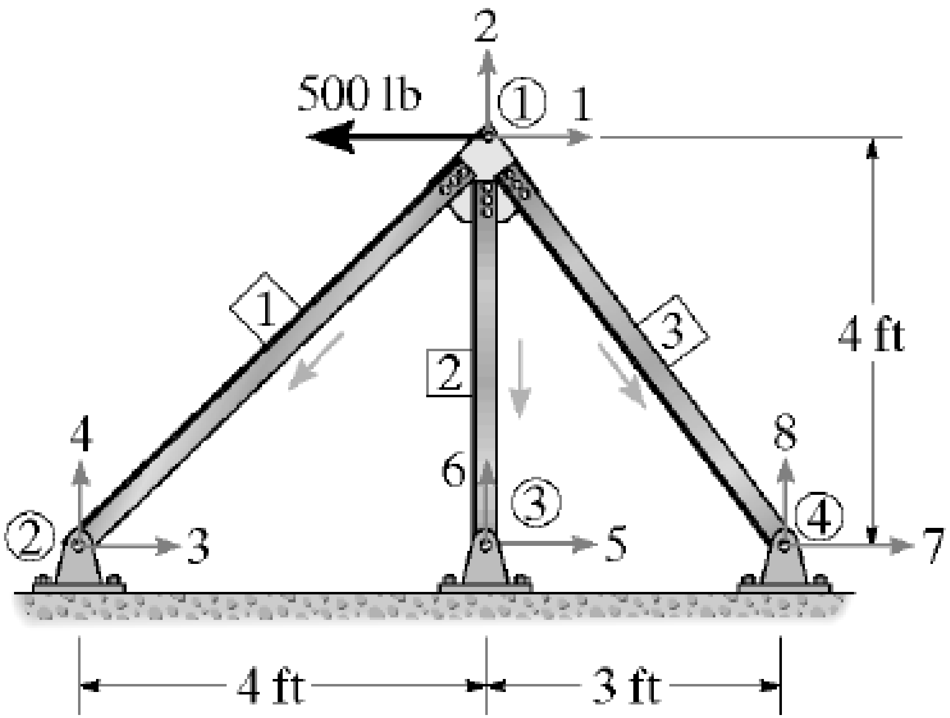
Topics to be covered

- Introduction
  - How it works
  - Concepts of Nodes
  - Use of stiffness matrices
  - Stress Analysis
  - Application of FEM

- Types of FEM
  - 1-D (Line element)
  - 2-D (Plane element)
  - 3-D (Solid element)

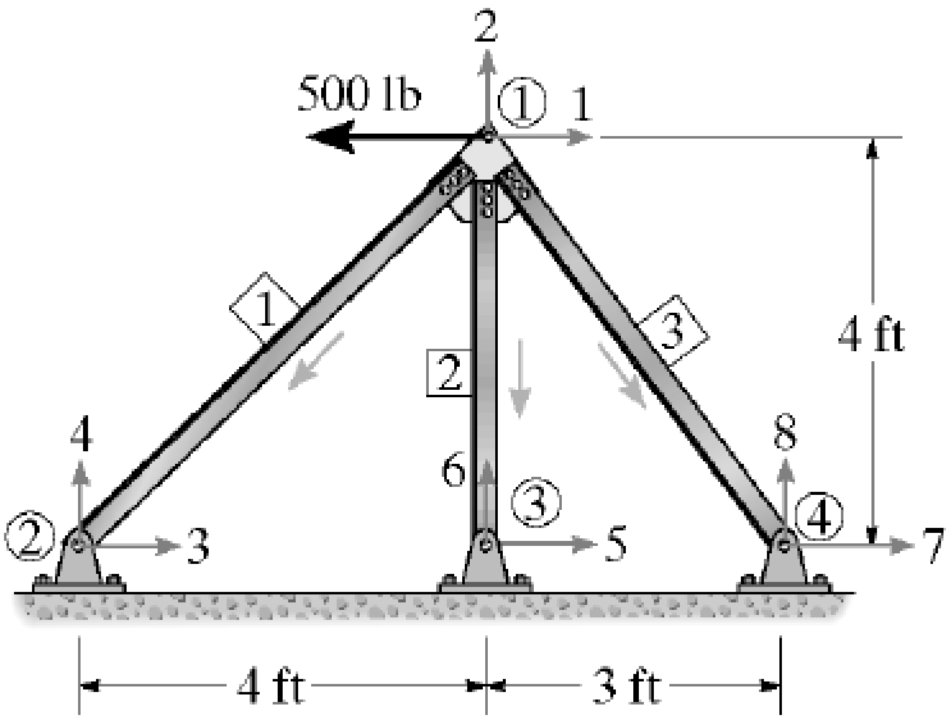
Structure having Support Settlement

- Redo the solved problem 1 with support settlement of 0.1ft downwards at Joint 1



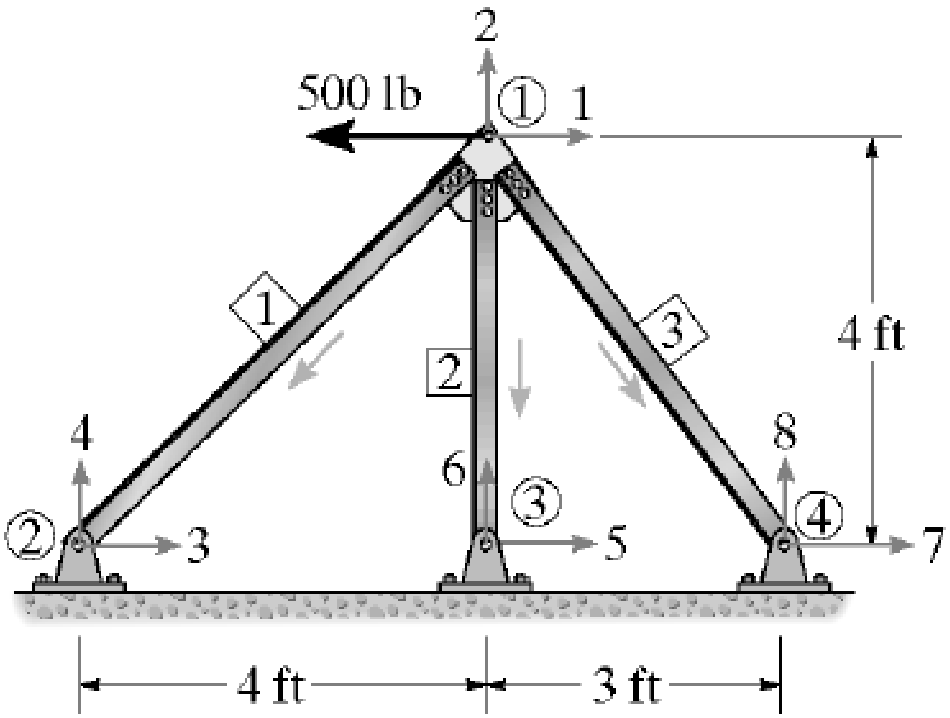
Lack of fit

- Redo the solved problem 1 if the member 1 is made 0;01ft too short before it was fitted into place.



Problem with temperature effect

- Redo the solved problem 1 when member 1 of the truss is subjected to an increase in temperature of 83 ^{o}C, tale temperature coefficient = 11.7 x 10-6/^{o}C.



FOR REFERENCE

- Solve examples in chapter 14 of RC Hibbeler, (6th or 7th Edition)

- Preferred examples are:
- Example 14-3
- Example 14-5 (problem with support settlement)
- Example 14-7 (problem for lack of fit)
- Example 14-8 (problem with temperature effect)

PRACTICE PROBLEMS

Following material related to this course has been uploaded on the site so far:

Lecture Schedule

Structural Analysis Book

Excel sheet for inverse of matrices

Lectures

Practice Problems

Book: Structural Analysis by RC Hibbeler, 7th Edition

Beams:

- Eg 15-1
- Exercise 15-2
- Exercise 15-3
- Exercise 15-4
- Exercise 15-9

Truss:

- Eg 14-3
- Eg 14-5 (problem with support settlement)
- Exercise 14-1, 14-2, 14-3
- Exercise 14-8, 14-9
- Exercise 14-11, 14-12
- Exercise 14-13, 14-14

Book: Structural Analysis by RC Hibbeler, 7th Edition

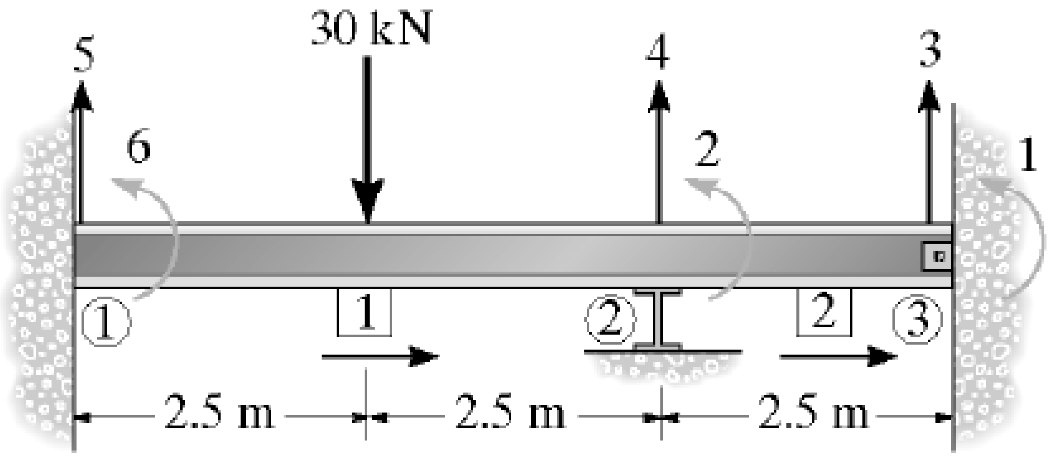
Frames:

- Exercise 16-1
- Exercise 16-10
- Exercise 16-14

BEAMS
EXERCISE SOLUTIONS

Exercise 15-2

Assume 1 as fixed, 2 as roller and 3 as pin.

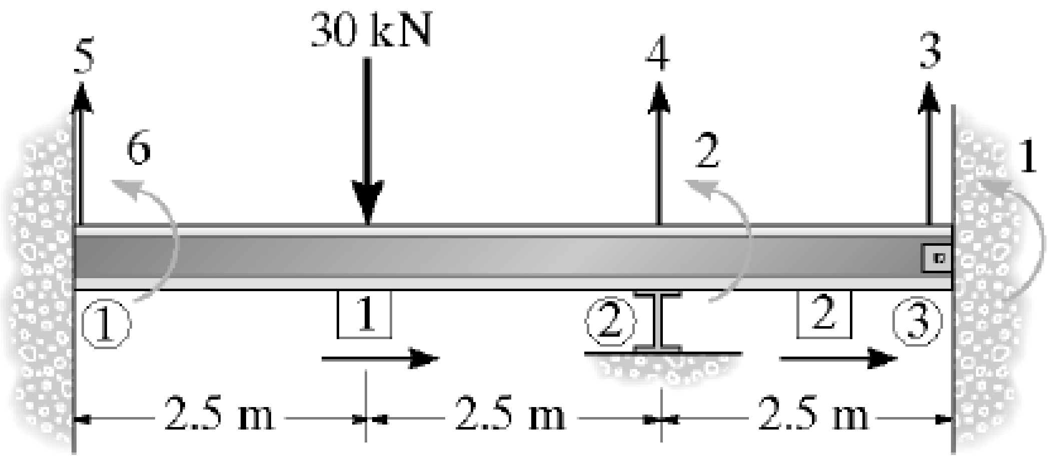


Member 1

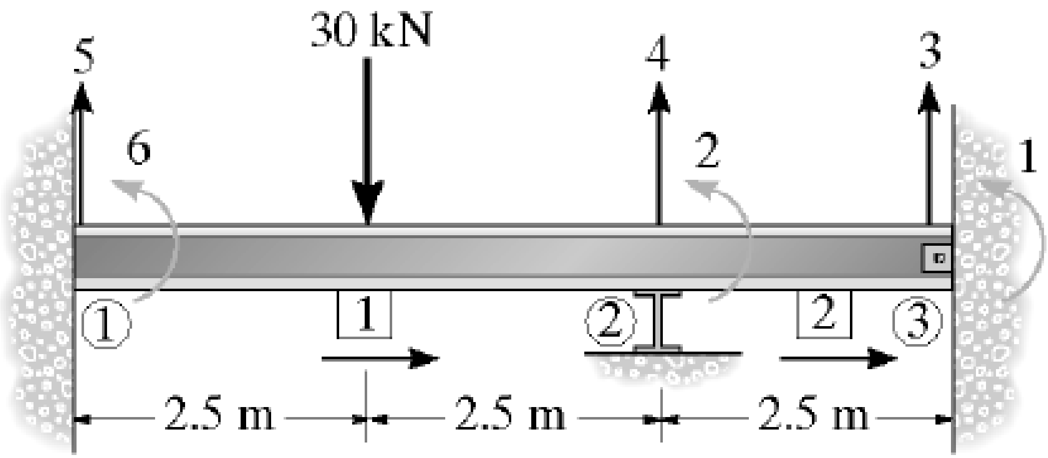
$$\mathbf{k}_1 = EI\begin{bmatrix} 0.096 & 0.24 & -0.096 \\ 0.24 & 0.80 & -0.24 \\ -0.096 & -0.24 & 0.096 \\ 0.24 & 0.40 & -0.24 \end{bmatrix}$$

Member 2

$$\mathbf{k}_2 = EI\begin{bmatrix} 0.768 & 0.96 & -0.768 & 0.96 \\ 0.96 & 1.60 & -0.96 & 0.80 \\ -0.768 & -0.96 & 0.768 & -0.96 \\ 0.96 & 0.80 & -0.96 & 1.60 \end{bmatrix}$$



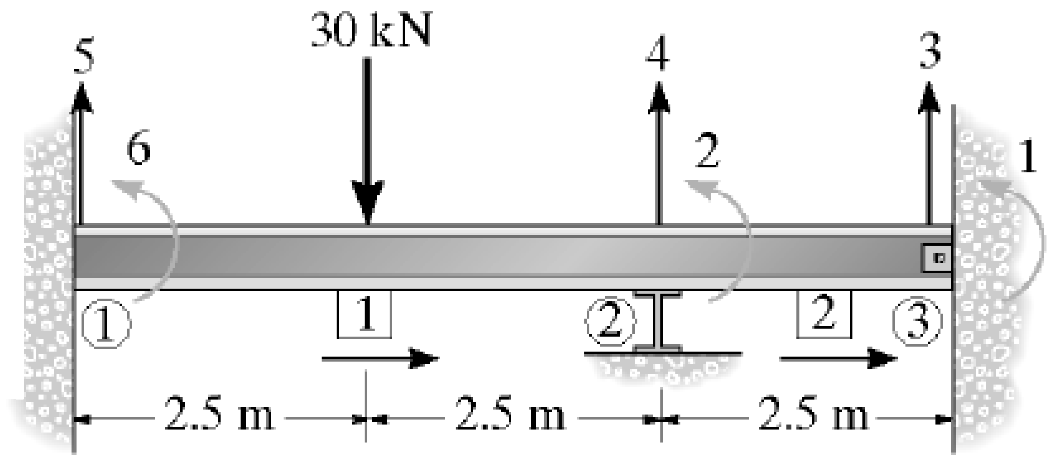
$$\mathbf{K} = EI\begin{bmatrix} 1.60 & 0.80 & -0.96 & 0.960 & 0 & 0 \\ 0.80 & 2.40 & -0.96 & 0.72 & 0.24 & 0.40 \\ -0.96 & -0.96 & 0.768 & -0.768 & 0 & 0 \\ 0.96 & 0.72 & -0.768 & 0.864 & -0.096 & -0.24 \\ 0 & 0.24 & 0 & -0.096 & 0.096 & 0.24 \\ 0 & 0.40 & 0 & -0.240 & 0.24 & 0.80 \end{bmatrix}$$



$$\begin{bmatrix} 0 \\ 18.75 \\ Q_3 \\ Q_4 - 15.0 \\ Q_5 - 15.0 \\ Q_6 - 18.75 \end{bmatrix} = EI \begin{bmatrix} 1.60 & 0.80 & -0.96 & 0.960 & 0 & 0 \\ 0.80 & 2.40 & -0.960 & 0.720 & 0.240 & 0.40 \\ -0.96 & -0.96 & 0.768 & -0.768 & 0 & 0 \\ 0.96 & 0.72 & -0.768 & 0.864 & -0.096 & -0.24 \\ 0 & 0.24 & 0 & -0.096 & 0.096 & 0.24 \\ 0 & 0.40 & 0 & -0.240 & 0.24 & 0.80 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By partition matrix

$$\begin{bmatrix} 0 \\ 18.75 \end{bmatrix} = EI\begin{bmatrix} 1.60 & 0.80 \\ 0.80 & 2.40 \end{bmatrix}\begin{bmatrix} D_1 \\ D \end{bmatrix}$$



$$D_1 = \frac{-4.6875}{EI}$$

$$D_2 = \frac{9.375}{EI}$$

$$\begin{bmatrix} q_5 \\ q_6 \\ q_4 \\ q_2 \end{bmatrix} = EI \begin{bmatrix} 0.096 & 0.24 & -0.096 & 0.24 \\ 0.24 & 0.80 & -0.24 & 0.40 \\ -096 & -0.24 & 0.096 & -0.24 \\ 0.24 & 0.40 & -0.24 & 0.80 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 9.375 \end{bmatrix} \frac{1}{EI} + \begin{bmatrix} 15 \\ 18.75 \\ 15 \\ -18.75 \end{bmatrix}$$

$$q_6 = 0.40EI\left(\frac{9.375}{EI}\right) + 18.75 = 22.5 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

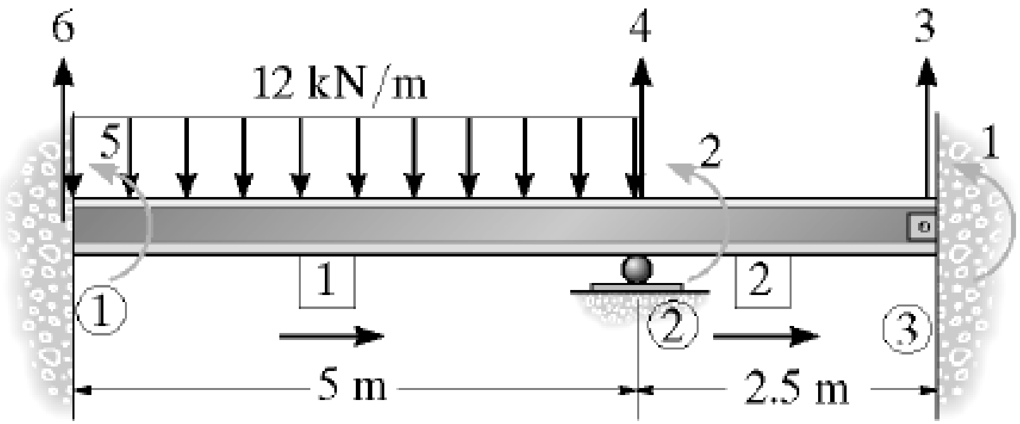
$$q_2 = 0.80EI\left(\frac{9.375}{EI}\right) - 18.75 = -11.25 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$q_5 = 0.24EI\left(\frac{9.375}{EI}\right) + 15.0 = 17.25 \text{ kN}$$

$$q_4 \;\; -0.24EI\left(\frac{9.375}{EI}\right) + 15.0 = 12.75 \text{ kN}$$

Exercise 15-3

Assume 1 as fixed, 2 as roller and 3 as pin.

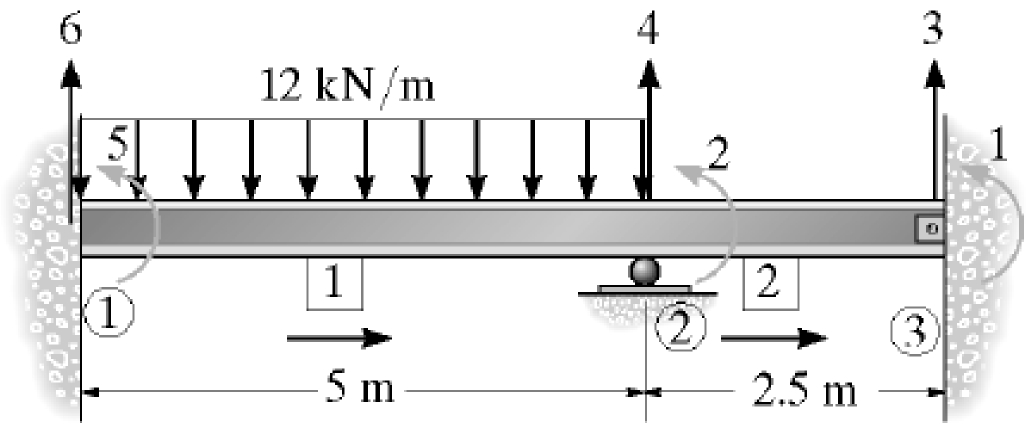


Member 1

$$\mathbf{k}_1 = EI\begin{bmatrix} 0.096 & 0.24 & -0.096 & 0.24 \\ 0.24 & 0.8 & -0.24 & 0.4 \\ -0.096 & -0.24 & 0.096 & -0.24 \\ 0.24 & 0.4 & -0.24 & 0.8 \end{bmatrix}$$

Member 2

$$\mathbf{k}_2 = EI\begin{bmatrix} 0.768 & 0.96 & -0.768 & 0.96 \\ 0.96 & 1.6 & -0.96 & 0.8 \\ -0.768 & -0.96 & 0.768 & -0.96 \\ 0.96 & 0.8 & -0.96 & 1.6 \end{bmatrix}$$



$$\begin{bmatrix} 0 \\ 25.0 \\ Q_3 \\ Q_4 - 30.0 \\ Q_5 - 25.0 \\ Q_6 - 30.0 \end{bmatrix} = EI \begin{bmatrix} 1.6 & 0.8 & -0.96 & 0.96 & 0 & 0 \\ 0.8 & 2.4 & -0.96 & 0.72 & 0.4 & 0.24 \\ -0.96 & -0.96 & 0.768 & -0.768 & 0 & 0 \\ 0.96 & 0.72 & -0.768 & 0.864 & -0.24 & -0.096 \\ 0 & 0.4 & 0 & -0.24 & 0.8 & 0.24 \\ 0 & 0.24 & 0 & -0.096 & 0.24 & 0.096 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Q_3 = -0.96EI\left(\frac{-6.25}{EI}\right) - 0.96EI\left(\frac{12.5}{EI}\right) = -6.00 \text{ kN} \qquad \textbf{Ans}$$

$$Q_4 - 30.0 = 0.96EI\left(\frac{-6.25}{EI}\right) + 0.72\left(\frac{12.5}{EI}\right)$$

$$Q_4 = 33 \text{ kN} \qquad \textbf{Ans}$$

$$D_1 = \frac{-6.25}{EI}$$

$$D_2 = \frac{12.5}{EI}$$

$$Q_5 - 25.0 = 0 + 0.4EI\left(\frac{12.5}{EI}\right)$$

$$Q_5 = 30 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$Q_6 - 30.0 = 0 + 0.24EI\left(\frac{12.5}{EI}\right)$$

$$Q_6 = 33.0 \text{ kN} \qquad \textbf{Ans}$$

$$\begin{bmatrix} q_5 \\ q_6 \\ q_4 \\ q_2 \end{bmatrix} = EI \begin{bmatrix} 0.096 & 0.24 & -0.096 & 0.24 \\ 0.24 & 0.80 & -0.24 & 0.40 \\ -096 & -0.24 & 0.096 & -0.24 \\ 0.24 & 0.40 & -0.24 & 0.80 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 9.375 \end{bmatrix} \frac{1}{EI} + \begin{bmatrix} 15 \\ 18.75 \\ 15 \\ -18.75 \end{bmatrix}$$

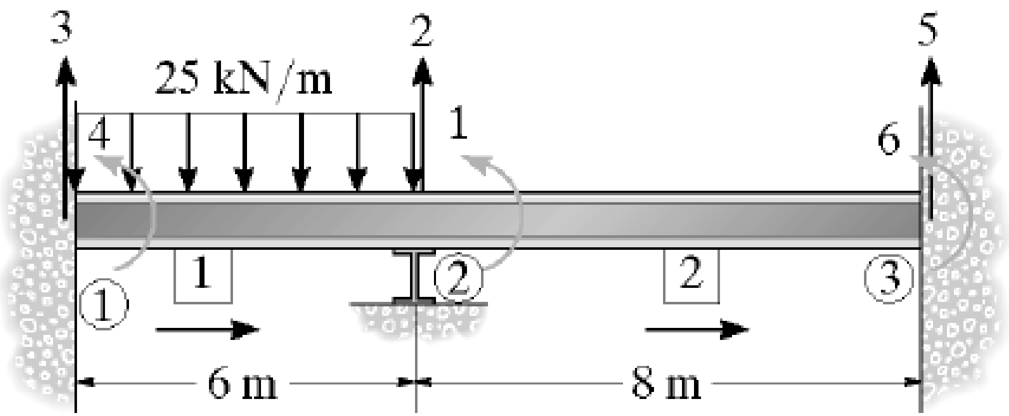
$$q_6 = 0.40EI\left(\frac{9.375}{EI}\right) + 18.75 = 22.5 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$q_2 = 0.80EI\left(\frac{9.375}{EI}\right) - 18.75 = -11.25 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$q_5 = 0.24EI\left(\frac{9.375}{EI}\right) + 15.0 = 17.25 \text{ kN}$$

$$q_4 \quad -0.24EI\left(\frac{9.375}{EI}\right) + 15.0 = 12.75 \text{ kN}$$

Exercise 15-4



$$\begin{bmatrix} 75 \\ Q_2 - 75.0 \\ Q_3 - 75.0 \\ Q_4 - 75.0 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{bmatrix} 1.1667 & -0.07292 & 0.16667 & 0.33333 & -0.09375 & 0.25 \\ -0.07292 & 0.07899 & -0.05556 & -0.16667 & -0.02344 & 0.09375 \\ 0.16667 & -0.05556 & 0.05556 & 0.16667 & 0 & 0 \\ 0.33333 & -0.16667 & 0.16667 & 0.66667 & 0 & 0 \\ -0.09375 & -0.02344 & 0 & 0 & 0.02344 & -0.09375 \\ 0.25 & 0.09375 & 0 & 0 & -0.09375 & 0.5 \end{bmatrix} \begin{bmatrix} D_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Partition matrix

$$75.0 = EI(1.16667)D_1$$

$$D_1 = \frac{64.286}{EI}$$

$$Q_2 - 75.0 = -0.07292EI\left(\frac{64.286}{EI}\right)$$

$$Q_2 = 70.31 \text{ kN}$$

$$Q_3 - 75.0 = 0.16667EI\left(\frac{64.286}{EI}\right)$$

$$Q_3 = 85.71 \text{ kN}$$

$$Q_4 - 75.0 = 0.33333EI\left(\frac{64.286}{EI}\right)$$

$$Q_4 = M_1 = 96.43 \text{ kN}\cdot\text{m} = 96.4 \text{ kN}\cdot\text{m} \qquad \text{Ans}$$

$$Q_5 = -0.09375EI\left(\frac{64.286}{EI}\right) = -6.03 \text{ kN} = 6.03 \text{ kN}$$

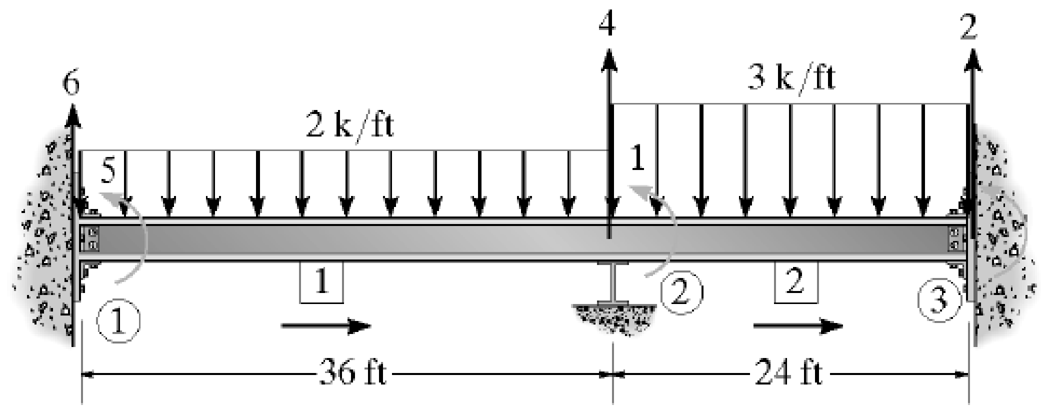
$$Q_6 = M_3 = 0.25EI\left(\frac{64.286}{EI}\right) = 16.07 \text{ kN}\cdot\text{m} = 16.1 \text{ kN}\cdot\text{m} \qquad \text{Ans}$$

Exercise 15-9

Support 2 settles 0.1ft

Member 1

$$\mathbf{k}_1 = EI\begin{bmatrix} \frac{12}{(36)^3} & \frac{6}{(36)^2} & \frac{-12}{(36)^3} & \frac{6}{(36)^2} \\ \frac{6}{(36)^2} & \frac{4}{36} & \frac{-6}{(36)^2} & \frac{2}{36} \\ \frac{-12}{(36)^3} & \frac{-6}{(36)^2} & \frac{12}{(36)^3} & \frac{-6}{(36)^2} \\ \frac{6}{(36)^2} & \frac{2}{36} & \frac{-6}{(36)^2} & \frac{4}{36} \end{bmatrix}$$



Member 1

$$\mathbf{k}_2 = EI\begin{bmatrix} \frac{12}{(24)^3} & \frac{6}{(24)^2} & \frac{-12}{(24)^3} & \frac{6}{(24)^2} \\ \frac{6}{(24)^2} & \frac{4}{24} & \frac{-6}{(24)^2} & \frac{2}{24} \\ \frac{-12}{(24)^3} & \frac{-6}{(24)^2} & \frac{12}{(24)^3} & \frac{-6}{(24)^2} \\ \frac{6}{(24)^2} & \frac{2}{24} & \frac{-6}{(24)^2} & \frac{4}{24} \end{bmatrix}$$

$$\begin{bmatrix} 72.0 \\ Q_2 - 36.0 \\ Q_3 + 144 \\ Q_4 - 72.0 \\ Q_5 - 216 \\ Q_6 - 36.0 \end{bmatrix} = EI \begin{bmatrix} \frac{5}{18} & \frac{-6}{(24)^2} & \frac{2}{24} & \frac{5}{864} & \frac{2}{36} & \frac{6}{(36)^2} \\ \frac{-6}{(24)^2} & \frac{12}{(24)^3} & \frac{-6}{(24)^2} & \frac{-12}{(24)^3} & 0 & 0 \\ \frac{2}{24} & \frac{-6}{(24)^2} & \frac{4}{24} & \frac{6}{(24)^2} & 0 & 0 \\ \frac{5}{864} & \frac{-12}{(24)^3} & \frac{6}{(24)^2} & \frac{35}{31\,104} & \frac{-6}{(36)^2} & \frac{-12}{(36)^3} \\ \frac{2}{36} & 0 & 0 & \frac{-6}{(36)^2} & \frac{4}{36} & \frac{6}{(36)^2} \\ \frac{6}{(36)^2} & 0 & 0 & \frac{-12}{(36)^3} & \frac{6}{(36)^2} & \frac{12}{(36)^3} \end{bmatrix} \begin{bmatrix} D_1 \\ 0 \\ 0 \\ -0.1 \\ 0 \\ 0 \end{bmatrix}$$

$$72.0 = 9500\left[\frac{5}{18}D_1 + \frac{5}{864}(-0.1)\right]$$

$$D_1 = 0.029368 \text{ rad}$$

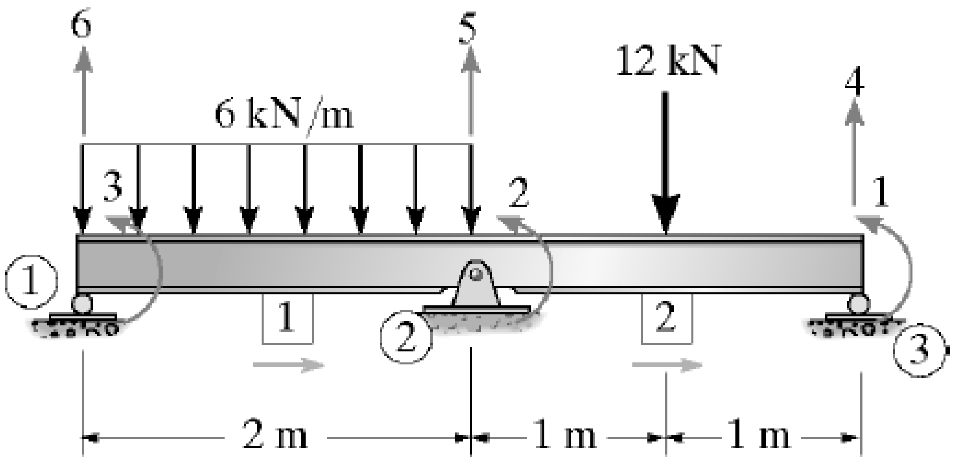
$$Q_3 + 144 = 9500\left[\frac{2}{24}(0.029368) + \frac{6}{(24)^2}(-0.1)\right]$$

$$Q_3 = -130.65 \text{ k}\cdot\text{ft} = 131 \text{ k}\cdot\text{ft} \qquad \textbf{Ans}$$

$$Q_5 + 216 = 9500\left[\frac{2}{36}(0.029368) + \frac{6}{(36)^2}(-0.1)\right]$$

$$Q_5 = 235.90 \text{ k}\cdot\text{ft} = 236 \text{ k}\cdot\text{ft} \qquad \textbf{Ans}$$

Exercise 15-9



Member 1

$$\mathbf{k}_1 = EI\begin{bmatrix} 1.5 & 1.5 & -1.5 & 1.5 \\ 1.5 & 2 & -1.5 & 1 \\ -1.5 & -1.5 & 1.5 & -1.5 \\ 1.5 & 1 & -1.5 & 2 \end{bmatrix}$$

Member 1

$$\mathbf{k}_2 = EI\begin{bmatrix} 1.5 & 1.5 & -1.5 & 1.5 \\ 1.5 & 2 & -1.5 & 1 \\ -1.5 & -1.5 & 1.5 & -1.5 \\ 1.5 & 1 & -1.5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -1 \\ -2 \\ Q_4 - 6 \\ Q_5 - 12 \\ Q_6 - 6 \end{bmatrix} = EI \begin{bmatrix} 2 & 1 & 0 & -1.5 & 1.5 & 0 \\ 1 & 4 & 1 & -1.5 & 0 & 1.5 \\ 0 & 1 & 2 & 0 & -1.5 & 1.5 \\ -1.5 & -1.5 & 0 & 1.5 & -1.5 & 0 \\ 1.5 & 0 & -1.5 & -1.5 & 3 & -1.5 \\ 0 & 1.5 & 1.5 & 0 & -1.5 & 1.5 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{3}{EI} = 2D_1 + 1D_2$$

$$\frac{-1}{EI} = 1D_1 + 4D_2 + 1D_3$$

$$\frac{-2}{EI} = 1D_2 + 2D_3$$

Solving these equations yields

$$D_1 = \frac{1.75}{EI}$$

$$D_2 = \frac{-0.50}{EI}$$

$$D_3 = \frac{-0.75}{EI}$$

$$Q_4 - 6.0 = -1.5EI\left(\frac{1.75}{EI}\right) - 1.5EI\left(\frac{-0.50}{EI}\right) + 0$$

$Q_4 = 4.125$ kN **Ans**

$$Q_5 - 12.0 = 1.5EI\left(\frac{1.75}{EI}\right) + 0 - 1.5EI\left(\frac{-0.75}{EI}\right)$$

$Q_4 = 15.75$ kN **Ans**

$$Q_6 - 6.0 = 0 + 1.5EI\left(\frac{-0.50}{EI}\right) + 1.5EI\left(\frac{-0.75}{EI}\right)$$

$Q_6 = 4.125$ kN **Ans**

Check for equilibrium

$\circlearrowleft + \Sigma M_2 = 0; \quad 4.125(2) + 12(1) - 4.125(2) - 12(1) = 0$ (Check)

$+\uparrow \Sigma F_y = 0; \quad 4.125 + 15.75 + 4.125 - 12 - 12 = 0$ (Check)