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ANALYSIS OF MUTUAL INDUCTANCE AND COUPLING FACTOR OF INDUCTIVELY COUPLED COILS FOR WIRELESS ELECTRICITY

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ABSTRACT

A basic analysis of inductive coils and its parameter calculations are presented. The simulations of mutual inductance, coupling factor calculations are demonstrated with graphical analysis. Three different lab-scale coil models such as square, circular and rectangular coils are wounded to evaluate the magnetic field by experiment, to validate the performance of Wireless Power System (WPT). In the open literature, circular coils are employed in most of the works, but few works have been reported in the parameter analysis. Further investigations on parameter exploration seems as a prerequisite for magnetic field measurement by estimating the parameters such as mutual inductance(M), coupling factor(k), magnetic flux(Φ) and magnetic field(B). It helps us to select the coils according to the applications. In this work, it is observed that circular performs well than other shaped coils in terms of parametrical analysis which are mentioned above. The simulation, and experimental results are tabulated as well as supported graphical plots are shown as proving circular coils performs well in the WPT scenario.

Keywords: coupling factor, mutual inductance, magnetic field, inductive coils, wireless power transfer.

INTRODUCTION

The development of WPT grown as a matured technology that can deliver the small amount of power into the remotely located devices [1]. It can be used for various applications such as electrical vehicles, sensor devices and biomedical implantable devices [2]. The concept behind that is to transfer the electricity from one point to other without cables. There are many ways to transfer power wirelessly, but standard method is electromagnetic induction. When the load coil is placed in the magnetic field of source coil, a voltage is induced in a load coil by inductive coupling and thereby power transfer occurs [2], [3]. The magnetic field generates around the region of source coil, will be picked up by load coil is the most efficient way. However, power transfer efficiency gets degraded by lowering mutual inductance [4], [5]. The analysis of magnetic attraction (coupling between coils) is very specific about mutual inductance calculation. This means indirectly that it is very useful to know the characteristics of mutual inductance associated with coupling factor of coils [6], [7]. Many scholars have been studied about mutual inductance calculation based on Neumann's formula, Biot-Savart law and the Maxwell's formula [8-10]. Few years ago, interesting method of mutual inductance calculation was presented using Bessel functions [11], [12]. In addition, many numerical and analytical methods such as Boundary Element Method (BEM) and Finite element Method (FEM) are used to obtain the parameters [13], [14].

Most of the simulated works does not shown their design and how to calculate the parameters through simulation as well as mapping with the expressions [15]. This is enabled us to conduct the further work on parameter calculations in the field of wireless electricity for a large scale applications which is mentioned earlier. Hence, the aim of this work is to design the wireless power transfer system with an optimum geometry to achieve the maximum power transfer.

The paper is organized as follows: Section II derives the analytical expressions for mutual inductance and coupling factor for parameter analysis. Then, simulation model for circular, square and rectangular are developed using FEM along with experimental models are presented in Section III. Finally, Section IV concludes the presented work.

MATHEMATICAL MODELING

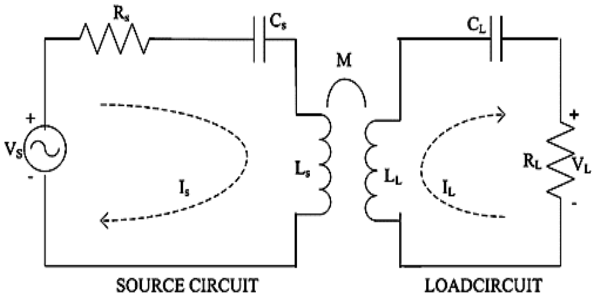

Figure-1. Equivalent circuit for WPT system.

The schematic of electric equivalent of WPT system is shown in Figure-1. A source coil (L_S) is excited by supply voltage with parasitic resistance (R_S) and capacitance (L_S). Power transmitted through inductive link to the load coil with inductance (L_L).The load (R_L) is connected via rectifier and parasitic capacitor (C_L) to the load coil. The input impedance Z_{in} related with a load R_L connected in the load coil can be written as,

$$Z_{in} = R_{SS} + j\omega L_S + \frac{\omega^2 M^2}{j\omega L_L R_{LS} + Z_L} \tag{1}$$

where R_{SS} denotes the combination of source resistance R_S and mutual inductance (M). It gives the relation of coupling factor

$$k = \frac{M}{\sqrt{L_S L_L}}. \tag{2}$$

From Eq. (1) and (2), the input impedance as well as coupling factor will vary according to the mutual inductance.

A. Mutual inductance calculation

Let us consider the two circular coils separated with the distance d and its radius is r_1 and r_2 respectively. The mutual inductance between the coils can be calculated using Neumann's equation [3]:

$$M = \frac{\mu}{4\pi} \iint \frac{\cos\varphi}{D} ds ds' \tag{3}$$

where D is total distance between coils.

$$D = \sqrt{r_1^2 + r_2^2 + d^2 - 2r_1 \cos(\phi - \phi')} \tag{4}$$

$$ds = r_1 d\phi \;\; ds^1 = r_2 d\phi \tag{5}$$

By substituting the above values,

$$M = \frac{\mu}{4\pi} \int\int_0^{2\pi} \frac{r_1 r_2 \cos(\phi - \phi^1)}{\sqrt{r_1^2 + r_2^2 + d^2 - 2r_1 \cos(\phi - \phi')}} d\phi d\phi' \tag{6}$$

The above integration can be solved using elliptic integrals,

$$M(m) = \frac{2\mu\sqrt{r_1 r_2}}{m} \left[\left(1 - \frac{m^2}{2}\right) K(m) - E(m) \right] \tag{7}$$

In the Eq. 7, $K(m)$ and $E(m)$ are the elliptic integrals of first and second kind, its values can be defined as,

$$K(m) = \frac{\pi}{2} + \frac{\pi}{8} \frac{m^2}{1 - m^2} \tag{8}$$

$$E(m) = \frac{\pi}{2} + \frac{\pi}{8} m^2. \tag{9}$$

The value of m is defined as

$$m = \sqrt{4r_1 r_2 \big/ (r_1 + r_2)^2 + d^2} \tag{10}$$

Eq. (8) and (9) will substitute in (7),

$$M(m) = \frac{\mu\pi\sqrt{r_1 r_2}}{8} \frac{m^3}{1 - m^2} \tag{11}$$

After that, substituting (11) in the mutual inductance Eq. (6),

$$M = \frac{\mu r_1^2 r_2^2}{\sqrt{(r_1 + r_2)^2 + d^2}\,[(r_1 + r_2)^2 + d^2]} \tag{12}$$

The Eq. (12) can be adjusted by choosing two coils with equal number of turns, n_1, n_2

$$M = \frac{\mu\pi n_1 n_2 r_1^2 r_2^2}{\sqrt{(r_1 + r_2)^2 + d^2}\,[(r_1 + r_2)^2 + d^2]} \tag{13}$$

The expression in (13) provides the relationship of mutual inductance related with coil turnsn_1, n_2, distance d, radius of coils r_1, $r_2$and permeability μ .

B. Coupling factor calculation

The electric equivalent two port model of inductive link is shown in Figure-2.

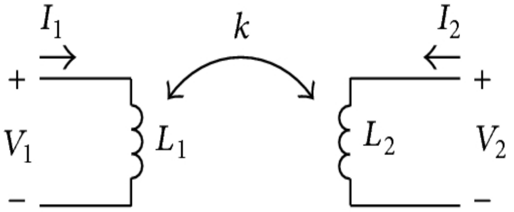

Figure-2. Two port model inductive link.

The relationship between input and output voltages can be written as

$$V_S = j\omega(I_S L_S + M I_L) \tag{14}$$

$$V_L = j\omega(I_L L_L + M I_S) \tag{15}$$

When load coil shorted, the current flow in the load coil can be expressed as

$$I_L = -\frac{M I_S}{L_L} \tag{16}$$

Substituting (16) in (14) yields,

$$V_S = j\omega I_S \left(L_S - \frac{M}{L_2} \right) \tag{17}$$

Assume, $L_{SS} = \left(L_S - \frac{M}{L_2}\right)$ and then mutual inductance is expressed as

$$M = L_L\left(L_S - L_{SS}\right) \tag{18}$$

Finally, the coupling factor in terms of source inductance and inductance measured with shorted load:

$$K = \sqrt{\left(1 - \frac{L_{SS}}{L_S}\right)}. \tag{19}$$

C. Magnetic field calculation

The voltage produced in the load coil can be increased by increasing the magnetic field. It is ensured that the higher magnetic field produced by a coil does the better performance in WPT system. By using Biot-Savart law, the following equations of magnetic field were derived for the circular, rectangular and square shapes respectively.

For circular, $$B = \frac{\mu_0 INr^2}{2(z^2 + r^2)^{\frac{3}{2}}} \tag{20}$$

where r is the radius, I is the current flowing through the coil, z is the distance from the centre on axis of symmetry of the coil to a point and N is the number of turns.
For rectangular,

$$B = \frac{2\mu_0 IN}{\pi}\left[\frac{a^2}{\left(4z^2 + a^2\right)\left(4z^2 + 2a^2\right)^{\frac{1}{2}}} + \frac{b^2}{\left(4z^2 + b^2\right)\left(4z^2 + 2b^2\right)^{\frac{1}{2}}}\right] \tag{21}$$

where a, b is length and breadth of rectangular coil.

For square, $$B = \frac{4\mu_0 INa^2}{\pi(4z^2 + a^2)(4z^2 + 2a^2)^{\frac{1}{2}}} \tag{22}$$

where 'a' is the length of one side the square coil.

From this analysis, the interesting fact is that the magnetic field, mutual inductance and coupling factors are the vital parameters but controversial each other. Mutual inductance is more reliable on the magnetic field and also coupling factor more reliable on distance of separation between coils. Hence the performance deciders of WPT system are magnetic field (output voltage in the load coil) and coupling factor (distance), both are varied mutually in opposite direction. This is where the researcher to find the optimised solution by bringing up the novel shape and coil dimensions.

The graph shown in Figure-1 indicates the variation of output voltage with respect to the separation distance between coils. In other words, the harvested voltage in the load coil reduces when increasing the distance of separation between coils in terms of coupling factor.

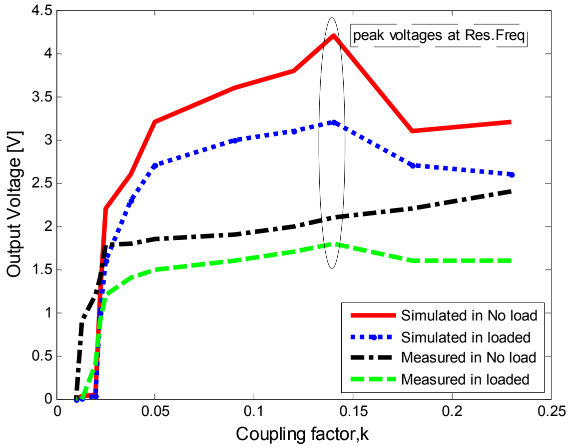

Figure-3. Analysis of output voltage with respect to separation distance (coupling factor).

RESULTS AND DISCUSSIONS

To evaluate the performance of coils, simulation and experimental models of different coils such as circular, square and rectangular are designed and verified to check the accuracy of equation (13). In addition, practical results are analysed to confirm which shape of coil produces good values and voltage efficiency. Different shapes of coils can give different results for power transfer in a WPT system. In simulation, a rectangular and a circular coil having 2 turns each will be simulated in Ansoft Maxwell 3D modelling software to determine the mutual inductance. Then, the power transferred from the source can be calculated using this mutual inductance value. The simulation designs of circular, rectangular, and square shaped coils and its corresponding measurements are shown in the below Figure-2 and Table-1.

The Table-1 depicts the coil parameters of circular, square and rectangular coils for WPT applications. To each shape, 16 cm length of copper wire with thickness 1cm was chosen and the distance between coils was set to be 2cm.

Table-1. Coil Parameters used for WPT Measurement.

Shape	**Self-inductance** (L)	**Mutual inductance** (M)	**Coupling factor** (k)
Circular	131.74 nH	9.54 nH	0.0724
Rectangular	107.35 nH	4.1nH	0.03806
Square	107.05 nH	3.4 nH	0.03106

As noticed from the results obtained, the rectangular and square coils has lower mutual inductance than the circular coil, hence proving that circular performs better than the rectangular shape. In addition, magnetic

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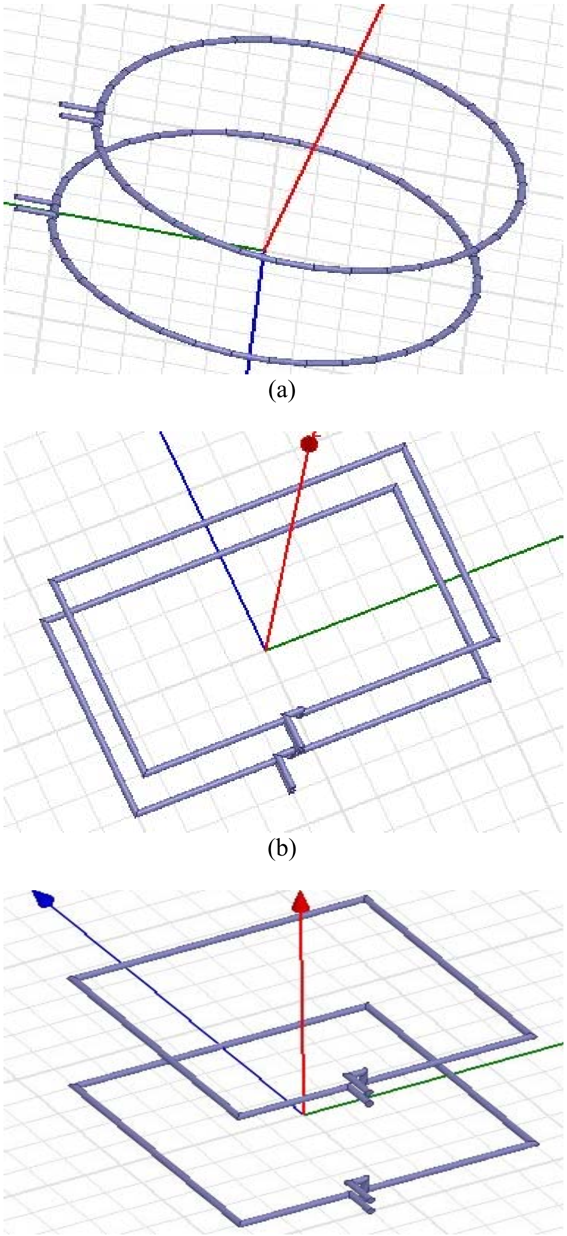
field analysis also conducted through experimental models shows in the Figure-3 by setting coil length of 16cm with 1cm thickness and the distance from the coils to the point was fixed to 3cm for all the shapes. The numbers of turns were varied for each shape and the respective values for the magnetic field were calculated using the Eq. (20), (21) and (22). The experiment was conducted in the lab and the values are recorded in the Table-2 in order to analyse the distinction of the coil shapes to be used in the inductively coupled wireless power transfer applications.



(a)

(b)

(c)

Figure-4. Simulation model of (a) circular, (b) rectangular and (c) square coils used to measure the mutual inductance values for WPT.

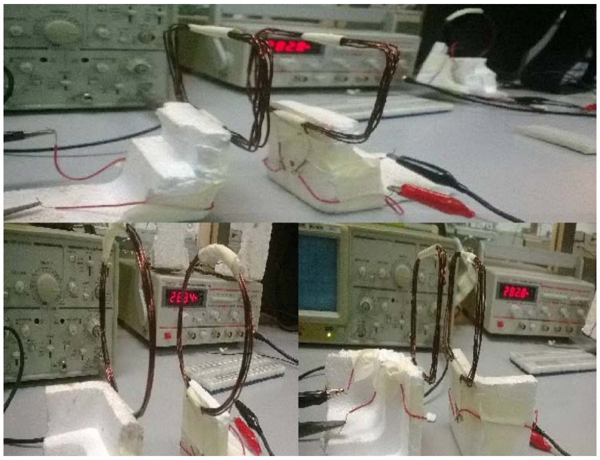


Figure-5. Prototype models of coils used to find the magnetic field.

The Figures5 and 6 shows the analysis of magnetic field of three coils with respect to number of turns and distance of separation. It clearly depicts that the circular coil produces higher magnetic field than the rectangular and followed with square shaped coils. Initially the difference in the magnetic field of the 3 shapes is negligible but as the turn increases, the difference gets larger. There is a linear increase in magnetic field for all the three shapes but the slope of the graph explains circular produces the best results than other two coils.

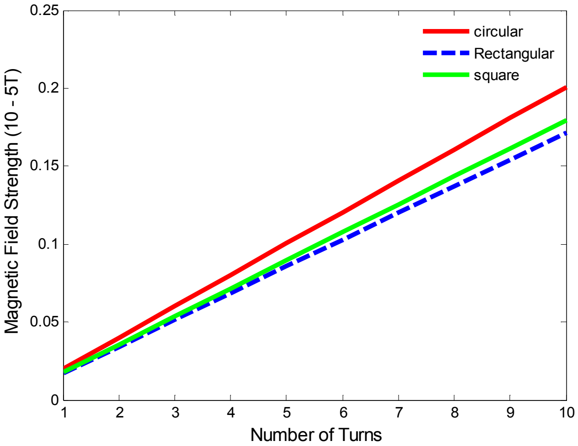


Figure-6. Plot for magnetic field versus number of turns.

As similarly to the coil parameter analysis shown previously, the coil analysis is enhanced further by analysing the magnetic field as a function of distance. The distance barrier is the significant matter in the wireless electricity. The coverage distance of two coils mainly depends on the magnetic flux and field produced by the transmitter coil and its own coil geometry for good flux linkage. Here, it is conducted the magnetic field test by varying the distance to explore the ability of the transmitter coil to power up the remotely located loads, which is shown in the Figure 7.

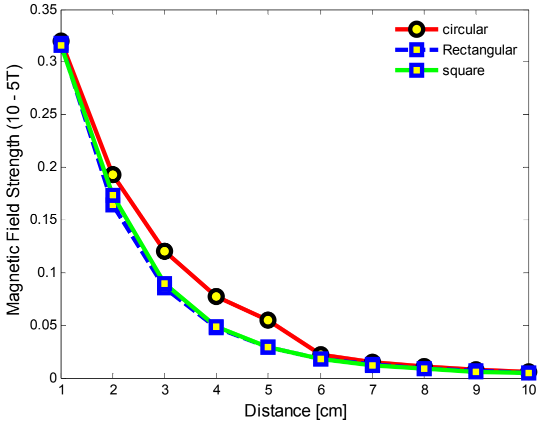


Figure-7. Plot for magnetic field versus distance.

As per the analysis shown in Figure-7, the magnetic field gradually reduces as the point gets away from the coil. Comparing the performance of the coils by increasing the distance, it is observed that the magnetic field linearly falls as the distance increases but at a given distance circular has the highest magnetic field.

CONCLUSION

In this paper, two distinct analysis of inductive coils are conducted to explore the significance of coil geometry in the wireless power transfer applications. First, mutual inductance and coupling factor of three different shaped coils such as circular, square and rectangular are analysed as a function of number of turns and distance separation between source and load coils. In addition, the equation for mutual inductance and magnetic field are derived using Neumann's equations associated with elliptical integrals. Secondly, the lab scale models are developed in order to conduct the data analysis for magnetic field measurements. The mutual inductance analysis was performed through Maxwell 3D simulation platform and thereafter magnetic field analysis was conducted through the experiment of developed prototype. The experimental as well as simulation results are demonstrated that circular coils offers the best performance in terms of coil parameters such as mutual inductance, coupling coefficient, magnetic flux and magnetic field.

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