# Analysis of Pile Foundation Subjected to Lateral and Vertical Loads 

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#### Abstract

Pile foundations are common foundations for bridge abutment, piers and buildings resting on soft soil strata. The pile is subjected to both vertical and horizontal forces. The objective of the current study is Lateral\& Vertical loaded analysis of pile by using various methods. Vertical load analysis of pile is done by P-Y curves and Vesic's methods in cohesive\& cohesion less soils with different soil parameters. The piles are modelled as linear elements. The effect of soil structure interaction is taken into account by assuming it as vertical and horizontal soil spring (winkler soil spring). Lateral subgrade modulus and vertical subgrade modulus of soil ( $K_{H}$ and $K_{v}$ ) is calculated as per is code 2911. The lateral load analysis is carried out in FEM (Finite Element Method) Staad pro soft ware, L Pile software \& by empirical equations (Brom's method and Vesic's method). The above problem solved as per the Brom's method mentioned in IS 2911 and comparative results are also presented.


Keywords - Laterally and Vertically Loaded Pile Foundation, P-Y curves, Softwares: staad pro, ENSoft L-pile, empirical equations (Brom's method and Vesic's method), IS 2911 Part I SECTION I, SECTION II.

## I. INTRODUCTION

When a soil of low bearing capacity extends to a considerable depth, piles are generally used to transmit vertical and lateral loads to the surrounding soil media. Piles that are used under tall chimneys, television towers, high rise buildings, high retaining walls, offshore structures, etc. are normally subjected to high lateral loads. These piles or pile groups should resist not only vertical movements but also lateral movements. The requirements for a satisfactory foundation are,

1. The vertical settlement or the horizontal movement should not exceed an acceptable Maximum value,
2. There must not be failure by yield of the surrounding soil or the pile material. Vertical piles are used in foundations to take normally vertical loads and small lateral loads. In the case of foundations of bridges, transmission towers, offshore structures and for other type of huge structures, piles are also subjected to lateral loads. Extensive theoretical and experimental investigation
has been conducted on single vertical piles subjected to lateral loads by many investigators. Generalized solutions for laterally loaded vertical piles are given by Matlock and Reese (1960). The effect of vertical loads in addition to lateral loads has been evaluated by Davisson (1960) in terms of non-dimensional parameters. Brom's (1964a, 1964b) and Poulos and Davis (1980) have given different approaches for solving laterally loaded pile problems. Brom's method is ingenious and is based primarily on the use of limiting values of soil resistance. The method of Poulos and Davis is based on the theory of elasticity. The finite difference method of solving the differential equation for a laterally loaded pile is very much in use where computer facilities are available. Matlock (1970) have developed the concept of ( $p-y$ ) curves for solving laterally loaded pile problems.
Many numerical techniques such as Finite Difference Method, Variational Method Boundary Element Method Finite Element Method are being used for the engineering analysis of Piles. Finite Element Method itself as a powerful numerical technique, especially for Geotechnical Engineering problems complicated geometrical behaviour and boundary conditions. Using Finite element these problems can be solved easily.
II. GENERAL SOLUTIONS FOR VERTICAL PILES
1) Differential Equations of Elastic Curves for Vertical Piles Subjected To Lateral Loads
The standard differential equations for slope, moment, shear and soil reaction for a beam on an elastic foundation are equally applicable to laterally loaded piles.

(a) Laterally loaded pile

The relationships between $y$, slope, moment, shear and soil reaction at any point on the Pile is
deflection of the pile $=y$

| slope of the deflected pile $S$ | $=\frac{d y}{d x}$ |
| :--- | :--- | eqn 3.1

where El is the flexural rigidity of the pile material. The soil reaction $p$ at any point at a distance $x$ along the axis of the pile may be expressed as

$$
p=-E_{s} y \quad \text { eqn } 3.5
$$

where " $y$ " is the deflection at point $x$, and $E_{s}$ is the soil modulus. Eqs (3.4) and (3.5) when combined gives

$$
E I \frac{d^{4} y}{d x^{4}}+E_{s} y=0 \quad \text { eqn } 3.6
$$

which is called the differential equation for the elastic curve with zero axial load.

$$
E_{s}=-\frac{p}{y} \text { eqn } 3.7
$$

## 2) Non-Dimensional Solutions for Vertical Piles Subjected To Lateral Loads

Matlock and Reese (1960) have given equations for the determination of $y, S, M, V$, and $p$ at any point $x$ along the pile based on dimensional analysis. The equations are

| deflection, | $y=\left[\frac{P_{t} T^{3}}{E I}\right] A_{y}+\left[\frac{M_{t} T^{2}}{E I}\right] B_{y}$ | eqn 3.9 |
| :--- | :--- | :--- |
| slope, | $S=\left[\frac{P_{t} T^{2}}{E I}\right] A_{s}+\left[\frac{M_{t} T}{E I}\right] B_{s}$ | eqn 3.10 |
| moment, | $M=\left[P_{t} T\right] A_{m}+\left[M_{t}\right] B_{m}$ | eqn 3.11 |
| shear, | $V=\left[P_{t}\right] A_{v}+\left[\frac{M_{t}}{T}\right] B_{v}$ | eqn 3.12 |
| soil reaction, | $p=\frac{P_{t}}{T} A_{p}+\frac{M_{t}}{T^{2}} B_{p}$ | eqn 3.13 |

where $T$ is the relative stiffness factor expressed as

$$
\begin{aligned}
& \qquad T=\left[\frac{E I}{n_{h}}\right]^{\frac{1}{5}} \\
& \text { for } \quad E_{s}=n_{h} x \\
& \text { For a general case } T=\left[\frac{E I}{n_{h}}\right]^{\frac{1}{n+4}} \\
& \qquad \mathrm{Z}=\mathrm{xl} \mathrm{~T} \quad \text { eqn (3.14) } \\
& \text { III. VERTICAL LOAD ANALYSIS OF } \\
& \text { 1) Vertical Load Analysis Of Piles By P-Y } \\
& \text { Curves }
\end{aligned}
$$

## Load Transfer Mechanism

When the ultimate load applied on the top of the pile is Qu , a part of the load is transmitted to the soil along the length of the pile and the balance is transmitted to the pile base. The load transmitted to the soil along the length of the pile is called the ultimate friction load or skin load Qf and that transmitted to the base is called the base or point load $Q b$. The total ultimate load $Q u$ is expressed as the sum of these two, that is,

## Ultimate Load Bearing Capacity

$$
Q u=Q b+Q f=q_{b} A_{b}+f_{s} A_{S}
$$

Where $Q u=$ ultimate load applied on the top of the pile
$q b=$ ultimate unit bearing capacity of the pile at the base
$A b=$ bearing area of the base of the pile
As $=$ total surface area of pile embedded below ground surface
$f s$ - unit skin friction (ultimate)

## allnwable Load Bearing Capacity

ty factor of 2.5 is normally used.
ore we may write

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{a}}=\frac{\mathrm{Q}_{\mathrm{b}}+Q_{f}}{2.5} \tag{2}
\end{equation*}
$$

e where the values of Qb and Q . can be ed independently, the allowable load can be 1 as

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{a}}=\frac{Q_{b}}{3}+\frac{Q_{f}}{1.5} \tag{3}
\end{equation*}
$$

General Theory For Ultimate Bearing Capacity According To Vesic (1967)
ling to Vesic (1967)
The total failure load Qu may be written as follows
$\mathrm{Qu}=\mathrm{Qu}+\mathrm{Wp}=\mathrm{Qb}+\mathrm{Q} \mathrm{f}+\mathrm{Wp} \quad$ Eq (4)
Where $\mathrm{Qu}=$ load at failure applied to the pile
Qb = base resistance
$\mathrm{Qf}=$ shaft resistance $\quad \mathrm{Wp}=$ weight of the pile.


The general equation for the base resistance may be written as

$$
Q_{b}=c N_{c}+g_{e}^{\prime} N_{q}+\frac{1}{2} y d N_{y} A_{b}
$$

Eq (5)
where $\mathrm{d}=$ width or diameter of the shaft at base level
$\mathrm{q}^{\prime}{ }_{0}=$ effective overburden pressure at the base level of the pile
$\mathrm{A}_{\mathrm{b}}=$ base area of pile
$\mathrm{c}=$ cohesion of soil
$\gamma=$ effective unit weight of soil
$\mathrm{N}_{\mathrm{c}}, \mathrm{N}_{\mathrm{q}}, \mathrm{N} \gamma=$ bearing capacity factors which take into account the shape factor

## Cohesion less Soils

For cohesion less soils, $\mathrm{c}=0$ and the term 1I2ydNy becomes insignificant in comparison with the term $\mathrm{q}_{\mathrm{o}} \mathrm{N}_{\mathrm{q}}$ for deep foundations. Therefore Eq. (5) reduces to
$\mathrm{Q}_{\mathrm{b}}=\mathrm{q} \sim \mathrm{N}_{\mathrm{q}} \mathrm{A}_{\mathrm{b}}=\mathrm{q}_{\mathrm{b}} \mathrm{A}_{\mathrm{b}}$
Eq. (15.4) may now be written as
$\mathrm{Q}_{\mathrm{b}}=\mathrm{Q}_{\mathrm{u}}+\mathrm{W}_{\mathrm{p}}=\mathrm{q} \sim \mathrm{N}_{\mathrm{q}} \mathrm{A}_{\mathrm{b}}+\mathrm{W}_{\mathrm{p}}+\mathrm{Q}_{\mathrm{f}}$
The net ultimate load in excess of the overburden pressure load $\mathrm{q}_{0} \mathrm{~A}_{\mathrm{b}}$ is

$$
Q_{u}+W_{p}-q_{0} A_{b}=q_{0} N_{q} A_{b}+W_{p}-q_{0}^{\prime} A_{b}+Q_{f}
$$

Eq (8)
If we assume, for all practical purposes, $\mathrm{W}_{\mathrm{p}}$ and $\mathrm{q}_{\mathrm{o}} \mathrm{A}_{\mathrm{b}}$ are roughly equal for straight side or moderately tapered piles. Eq. (8) reduces to

$$
\begin{aligned}
& Q_{u}=q_{o}^{\prime} N_{q} A_{b}+Q_{f} \\
& Q_{u}=q_{o}^{\prime} N_{q} A_{b}+A_{s} \bar{q}_{o}^{\prime} \bar{K}_{s} \tan \delta
\end{aligned}
$$

Where $A_{s}=$ surface area of the embedded length of the pile
$q^{\prime}{ }_{o}=$ average effective overburden pressure over the embedded depth of the pile
$K_{s}=$ average lateral earth pressure coefficient $\gamma=$ angle of wall friction.

## Cohesive Soils

For cohesive soils such as saturated clays (normally consolidated), we have for </> = $0, N-1$ and $N=0$. The ultimate base load from Eq. (5) is

$$
\bar{Q}_{b}=\left(c_{b} N_{c}+q_{o}^{\prime}\right) A_{b}
$$

The net ultimate base load is

$$
\left(\bar{Q}_{b}-q_{o}^{\prime} A_{b}\right)=Q_{b}=c_{b} N_{c} A_{b}
$$

Therefore, the net ultimate load capacity of the pile, $Q_{u^{\prime}}$, is

$$
Q_{u}=c_{b} N_{c} A_{b}+Q_{f}
$$

$$
Q_{k}=c_{b} N_{c} A_{b}+A_{s} \alpha \bar{c}_{u}
$$

$\alpha=$ adhesion factor
$c_{u}=$ average undrained shear strength of clay along the shaft
$c_{b}=$ undrained shear strength of clay at the base level
$N_{C}=$ bearing capacity factor

## Vertical Load Bearing Capacity Of Pile Calculations:

From analysis of structure, it is found that maximum axial load in working condition is 2932 kN . Pile capacity is checked for above value of axial load required to be transmitted. Bearing capacity of piles is calculated as per procedure given in Appendix B IS: 2911-1979 part 1/section- II

Ultimate Skin Resistance
Qs $=\left(\alpha^{*} \mathrm{C}+\mathrm{K} * \mathrm{Pdi}{ }^{*} \tan \delta\right)^{*}$ Asi
Ultimate End Bearing Capacity
$\mathrm{Qb}=\left(\mathrm{Cp} * \mathrm{Nc}+\mathrm{Pd} \mathrm{N}^{*} \mathrm{Nq}+0.5^{*} \gamma^{*} \mathrm{~B}^{*} \mathrm{~N} \gamma\right)^{*} \mathrm{Ap}$
Ultimate Bearing Capacity of Soil
$\mathrm{Qu}=\mathrm{Qs}+\mathrm{Qb}-\mathrm{W}$
Where,
$\alpha=$ reduction factor,
$\mathrm{C}=$ average cohesion throughout layer,
$\mathrm{K}=$ coefficient of earth pressure,
$\mathrm{P}_{\mathrm{di}}=$ effective over burden pressure for ith layer,
$\delta=$ angle of wall friction between soil and pile,
$\mathrm{A}_{\mathrm{si}}=$ surface area of pile for ith layer,
$\mathrm{C}_{\mathrm{p}}=$ cohesion at the base of pile,
$B=$ diameter of pile,
$\mathrm{A}_{\mathrm{p}}=$ area of pile tip,
$\mathrm{W}=$ weight of pile,
$\gamma=$ effective unit weight of soil,
$\mathrm{N}_{\mathrm{c}}, \mathrm{N}_{\mathrm{q}}, \mathrm{N} \gamma=$ bearing capacity factors as per IS: 2911-1979 part 1/sec 2

## Soil data for input:

Design bed level $=(+) 1.15 \mathrm{~m}$

|  |  | Layer |  | Submerged |  |  | Angle of |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Layer | Depth | Thicknes <br> s | Density | density | N | Cohesio <br> n | friction |
| No. | below | $(\mathrm{m})$ | $(\mathrm{kN} / \mathrm{m} 3)$ | $(\mathrm{kN} / \mathrm{m} 3)$ | value | $(\mathrm{kN} / \mathrm{m} 2)$ | $(\mathrm{deg})$ |
| 1 | 6.53 | 6.53 | 18 | 7.75 | 38 | 150 | 0 |
| 2 | 15.53 | 9 | 18 | 7.75 | 26 | 80 | 0 |
| 3 | 23.03 | 7.5 | 18 | 7.75 | 18 | 160 | 35 |
| 4 | 26.03 | 3 | 18 | 7.75 | 50 | 0 | 35 |
| 5 | 29.15 | 3.12 | 18 | 7.75 | 80 | 0 | 0 |
| 6 | 31.15 | 2 | 20 | 7.75 | 80 | 300 | 0 |

## Calculation Of Skin frictional resistance:

## Layer 1:

Layer thickness $=6.53 \mathrm{~m}$
$\gamma$ sub $=7.75 \mathrm{kN} / \mathrm{m} 3, \mathrm{C}=150 \mathrm{kN} / \mathrm{m} 2$
Angle of internal friction $=0$ deg
SPT ' $N$ ' value $=38$
Level of water table $=(+) 5.10 \mathrm{~m}$
Length of pile above bed level $=11.245 \mathrm{~m}$
Critical depth $=20$ times dia.
Factor of safety $=2.5$
Surface area $=20.518 \mathrm{~m} 2$
Reduction factor $=0.3$
Wall friction between soil and pile $=0 \mathrm{deg}$
Co-efficient of earth pressure $=2$
Avg. over burden pressure $=50.6075 \mathrm{kN} / \mathrm{m} 2$
Design over burden pressure $=50.6075 \mathrm{kN} / \mathrm{m} 2$
Skin frictional resistance, Qsf1=923 kN
Layer 2:
Layer thickness $=9 \mathrm{~m}$
$\gamma=7.75 \mathrm{kN} / \mathrm{m} 3, \mathrm{C}=80 \mathrm{kN} / \mathrm{m} 2$
Angle of internal friction $=0$ deg
SPT ' $N$ ' value $=26$
Surface area $=28.274 \mathrm{~m} 2$
Reduction factor $=0.3$
Wall friction between soil and pile $=0 \mathrm{deg}$
Co-efficient of earth pressure $=1$
Avg. over burden pressure $=69.75 \mathrm{kN} / \mathrm{m} 2$
Design over burden pressure $=120.3575 \mathrm{kN} / \mathrm{m} 2$

## Skin frictional resistance, Qsf3 = $1131 \mathbf{k N}$

Layer 3:
Layer thickness $=3 \mathrm{~m}, \gamma=0 \mathrm{kN} / \mathrm{m} 2$
Angle of internal friction $=35$ deg
SPT ' N ' value $=50$
Surface area $=9.425 \mathrm{~m} 2$
Reduction factor $=0.3$
Wall friction between soil and pile $=35 \mathrm{deg}$
Co-efficient of earth pressure $=2$
Avg. over burden pressure $=23.25 \mathrm{kN} / \mathrm{m} 2$
Design over burden pressure $=155 \mathrm{kN} / \mathrm{m} 2$
Skin frictional resistance, Qsf4 = 2045.8 kN
Total Skin Frictional Resistance, Qsf = 4778.5 kN
End Bearing Resistance:

## Layer 4:

Angle of internal friction = 35 deg
Pile tip area $=0.7853 \mathrm{~m} 2$
$\mathrm{Nq}=50, \mathrm{~N} \gamma=48$
Design over burden pressure $=155 \mathrm{kN} / \mathrm{m} 2$
End bearing resistance at pile tip, $\mathrm{Qb}=4241 \mathrm{kN}$

## Weight of Pile:

Weight of pile above scour level Wp1 $=220.893 \mathrm{kN}$
Weight of pile below scour level Wp2 $=301.548 \mathrm{kN}$
Total ultimate resistance of pile
Qsf $+\mathrm{Qb}-\mathrm{Wp} 2=8717.452 \mathrm{kN}$
Allowable load (8717.452 / F.S.) - Wp1 = 3266 kN .
From above calculations,
Required depth $=26.03 \mathrm{~m}$ below design seabed level
E.G.L. $=(+) 1.15 \mathrm{~m} \mathrm{CD}$

## IV. LATERALLY LOADED ANALYSIS

 OF PILE1) Laterally Loaded Analysis By Using Subgrade Reaction Using Vesics Method:
A pile may be subjected to transverse force from a number of causes, such as wind, earthquake, water current, water waves, earth pressure, effect of moving vehicles or ships, plant and equipment, etc.
The lateral load carrying capacity of a single pile depends not only on the horizontal subgrade modulus of the surrounding soil but also on the structural strength of the pile shaft against bending consequent upon application of a lateral load. While considering lateral load on piles, effect of other coexistent loads including the axial load on the pile should be taken into consideration for checking the structural capacity of the shaft. There are various methods available for analysis of laterally loaded piles such as Equivalent Fixity Depth Approach As per IS: 2911-1979, Subgrade Modulus Approach (FEM or Matrix method), Closed Form Solution, Non dimensional Method, p-y Curve Method, Brom's Method, Vesic's Method etc.
A horizontal load on a vertical pile is transmitted to the subsoil primarily by horizontal subgrade reaction generated in the upper part of the shaft. A single pile is normally designed to carry load along its axis. Transverse load bearing capacity of a single pile depends on the soil reaction developed and the structural capacity of the shaft under bending.
In case the horizontal loads are of higher magnitude, it is essential to investigate the phenomena using principles of horizontal subsoil reaction adopting appropriate values for horizontal modulus of the soil. In this study, piles are analyzed using modulus of subgrade reaction and lateral resistance offered by soil is modeled by providing springs having stiffness derived using modulus of subgrade reaction. The modulus of subgrade reaction is seldom measured in lateral pile load test. Node values of "Ks" are required in FEM solution for lateral piles.
However in absence of test results, this value may be approximated as per procedure given below: As per Vesic (1961), modulus of subgrade reaction can be computed using stress-strain modulus Es based on as,

$$
k_{s}^{\prime}=0.65 \sqrt[12]{\frac{E_{s} B^{4}}{E_{f} I_{f}}} \frac{E_{s}}{1-\mu^{2}}
$$

Where Es, Ef $=$ modulus of soil and footing respectively, in consistent units B, If = footing width and its moment of inertia based on cross section in consistent units
One can obtain ks from ks’ as,

$$
k_{s}=\frac{k_{s}^{\prime}}{B}
$$

Since the twelfth root of any value multiplied by 0.65 will be close to 1 , for practical purposes the Vesic's equation reduces to,

$$
k_{s}=\frac{E_{s}}{B\left(1-\mu^{2}\right)}
$$

Now, we know that immediate (elastic) settlement,

$$
\Delta H=q_{0} B \frac{1-\mu^{2}}{E_{s}} I_{f}
$$

Where $q_{o}=$ foundation pressure
B = width of foundation
$\mu=$ Poisson's ratio
$\mathrm{I}_{\mathrm{f}}=$ influence factor
Put $E_{s}^{\prime}=\frac{\left(1-\mu^{2}\right)}{E_{s}}$ in above equation,
$\Delta H=q_{0} B E_{s}^{\prime} I_{f}$
But we know ks = ratio of soil pressure to deflection

$$
k_{s}=\frac{\Delta q}{\Delta H}=\frac{1}{B E_{s}^{\prime} I_{f}}
$$

But since one does not often have values of Es, other approximations are useful and quite satisfactory if the computed deflection is reasonable. It has been found that bending moments and computed soil pressure are not very sensitive to what is used for 'Ks' because the structural member stiffness is usually 10 or more as great as soil stiffness as defined by 'Ks'.
Vesic has suggested the following for approximating 'Ks' from the allowable bearing capacity qa based on geotechnical data:

$$
\mathrm{k}_{\mathrm{s}}=40(\mathrm{SF}) \mathrm{q}_{\mathrm{a}} \mathrm{kN} / \mathrm{m}^{3}
$$

Where, qa is in kPa . This equation is based on assumption that ultimate soil pressure occurs at a settlement of 0.0254 m . For other values of $\Delta \mathrm{H}=6$, $12,20 \mathrm{~mm}$ etc., the factor 40 can be adjusted to 160 , 83,50 etc. 40 is reasonably conservative but smaller assumed displacement can always be used. The most general form for either horizontal or lateral modulus of subgrade reaction is,
$k_{s}=A_{s}+B_{s} Z^{n}$
A $\mathrm{s}=$ constant for either horizontal or vertical members
Bs = coefficient based on depth variation
Z = depth of interest below ground
$\mathrm{n}=$ exponent to give ks the best fit.
We know that ultimate bearing capacity is given by,
$q_{u l t}=c N_{c} S_{c}+\gamma Z N_{q} S_{q}+0.5 \gamma B N_{\gamma} S_{\gamma}$
Observing that,

$$
A_{s}=C\left(c N_{c} S_{c}+0.5 \gamma B N_{\gamma} S_{\gamma}\right) \text { and } B_{s} Z^{n}=C\left(\gamma N_{q} S_{q}\right) Z^{n}
$$

The $C$ factor is 40 for SI units and 12 for FPS, using the same reasoning that $q_{u l t}$ occurs at a $0.0254-\mathrm{m}$ and $1-\mathrm{in}$. settlement but with no SF, since this equation directly gives $q_{u l t}$.

Table-A may be used to estimate a value of 'Ks' to determine the correct order of magnitude of the subgrade modulus obtained using one of the approximations given here. Obviously if a computed value is two or three times larger than the table range indicates, the computations should be rechecked for a possible gross error. Note, however, if you use a reduced value of displacement (say, 6 mm or 12 mm ) instead of 0.0254 m you may well exceed the table range other than this, if no computational error (or a poor assumption) is found then use judgment in what value to use.

Range of modulus of subgrade reaction ks.

| Soil | Ks ( kN/m3) |
| :--- | :--- |
| Loose sand | $4800-6000$ |
| Medium dense sand | $9600-80000$ |
| Dense sand | $64000-128000$ |
| Clayey medium <br> dense sand | $32000-80000$ |
| Slity medium <br> dense sand | $24000-48000$ |
| Clayey soil |  |
| qa $\leq 200 \mathrm{kPa}$ | $12000-24000$ |
| $200 \quad \mathrm{qa}$ <br> kPa | $24000-48000$ |
| qa $>800 \mathrm{kPa}$ | $>48000$ |


following formula for finding spring constants representing soil in the model.

$$
\begin{gathered}
K_{1}=\frac{B L}{24}\left(7 k_{S 1}+6 k_{S 2}-k_{S 3}\right) \\
K_{n}=\frac{B L}{24}\left(7 k_{S n}+6 k_{S(n-1)}-k_{S(n-2)}\right)
\end{gathered}
$$

## Any other K,

$$
K_{i}=\frac{B L}{12}\left(k_{S(i-1)}+10 k_{S i}-k_{S(i+1)}\right)
$$

## Calculation of Soil Spring Constant:

Input Data:
Design scour level $=(+) 1.15 \mathrm{~m}$
Depth of consideration $=26.00 \mathrm{~m}$
Diameter of pile $=1.00 \mathrm{~m}$
The horizontal modulus of subgrade reaction

$$
k_{s}=A_{s}+B_{s} Z^{n}
$$

Where $A_{s}=C m C\left(c N_{c}+0.5 \gamma B N_{\gamma}\right)$ and $B_{s} Z^{n}=C m C\left(\gamma N_{q}\right) Z^{n}$
Exponent $\mathrm{n}=0.5$
Size factor $\mathrm{Cm}=1.555824$
Factor depending on displacement of pile $\mathrm{C}=40$

## Soil data and calculation is as under:

## For layer 1,

Thickness of layer $=5 \mathrm{~m}$
Angle of internal friction $=0$
Cohesion of soil $=150 \mathrm{kN} / \mathrm{m} 2$
Submerged unit weight of soil $=7.75 \mathrm{kN} / \mathrm{m} 3$
Bearing capacity factor, $\mathrm{Nc}=5.14, \mathrm{Nq}=1, \mathrm{~N} \gamma=0$
As $=1.555 * 40 *(150 * 5.14+0.5 * 7.75 * 1 * 0)=$ $47981.6 \mathrm{kN} / \mathrm{m} 3$
$\mathrm{Bs}=1.555^{*} 40^{*}\left(7.75^{*} 1\right)=482.32 \mathrm{kN} / \mathrm{m} 3$
$\mathrm{Ks}=47981.6+\left(482.32 * \mathrm{Z}^{\wedge} 0.5\right) \mathrm{kN} / \mathrm{m} 3$
Similarly for other layers, Ks is found out and from that, value of spring constant is also found out for every 1 m interval as per equations given above. Values of spring constants throughout Similarly for other layers, 'Ks' is found out and from that, value of spring constant is also found out for every 1 m interval as per equations given above. Values of spring constants throughout the entire depth are calculated using spread sheet "Spring Constant".
Calculated values are shown below in Table-C and D

## Table C

| Layer | C | $\Phi$ | $\Gamma$ sub |  |  |  | As | Bs |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Thick, $(\mathrm{m})$ | $(\mathrm{kN} / \mathrm{m} 2)$ | $(\mathrm{Deg})$ | $(\mathrm{kN} / \mathrm{m} 3)$ | Nc | Nq | $\mathrm{N} \gamma$ | $(\mathrm{kN} / \mathrm{m} 2)$ | $(\mathrm{kN} / \mathrm{m} 3)$ |
| 5 | 150 | 0 | 7.75 | 5.14 | 1 | 0 | 47981.6 | 482.31 |
| 1.53 | 150 | 0 | 7.75 | 5.14 | 1 | 0 | 47981.6 | 482.31 |
| 5 | 80 | 0 | 7.75 | 5.14 | 1 | 0 | 25590.2 | 482.31 |
| 4 | 80 | 0 | 7.75 | 5.14 | 1 | 0 | 25590.2 | 482.31 |
| 5 | 160 | 0 | 7.75 | 5.14 | 1 | 0 | 51180.4 | 482.31 |
| 2.5 | 160 | 0 | 7.75 | 5.14 | 1 | 0 | 51180.4 | 482.31 |
| 3 | 0 | 35 | 7.75 | 46.12 | 33.3 | 48.03 | 11582.6 | 16061 |

## Table D

| Depth (m) | $\begin{gathered} \mathrm{As} \\ (\mathrm{kN} / \mathrm{m} 2) \end{gathered}$ | $\begin{gathered} \mathrm{Bs} \\ (\mathrm{kN} / \mathrm{m} 3) \end{gathered}$ | $\begin{gathered} \mathrm{Ks} \\ (\mathrm{kN} / \mathrm{m} 3) \end{gathered}$ | $\mathrm{K} \quad(\mathrm{kN} / \mathrm{m})$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 47981.61 | 482.3054 | 47981.61 | 24082.96 |
| 1 | 47981.61 | 482.3054 | 48463.91 | 48440.37 |
| 2 | 47981.61 | 482.3054 | 48663.69 | 48659.82 |
| 3 | 47981.61 | 482.3054 | 48816.98 | 48814.98 |
| 4 | 47981.61 | 482.3054 | 48946.22 | 48944.94 |
| 5 | 47981.61 | 482.3054 | 49060.07 | 49059.16 |
| 6 | 47981.61 | 482.3054 | 49163.01 | 47296.37 |
| 7 | 25590.19 | 482.3054 | 26866.25 | 28731.66 |
| 8 | 25590.19 | 482.3054 | 26954.36 | 26953.91 |
| 9 | 25590.19 | 482.3054 | 27037.11 | 27036.73 |
| 10 | 25590.19 | 482.3054 | 27115.37 | 27115.05 |
| 11 | 25590.19 | 482.3054 | 27189.82 | 27189.54 |
| 12 | 25590.19 | 482.3054 | 27260.94 | 27260.7 |
| 13 | 25590.19 | 482.3054 | 27329.17 | 27328.95 |
| 14 | 25590.19 | 482.3054 | 27394.81 | 27394.62 |
| 15 | 25590.19 | 482.3054 | 27458.15 | 29590.49 |
| 16 | 51180.38 | 482.3054 | 53109.6 | 50976.93 |
| 17 | 51180.38 | 482.3054 | 53168.98 | 53168.83 |
| 18 | 51180.38 | 482.3054 | 53226.63 | 53226.5 |
| 19 | 51180.38 | 482.3054 | 53282.7 | 53282.58 |
| 20 | 51180.38 | 482.3054 | 53337.32 | 53337.2 |
| 21 | 51180.38 | 482.3054 | 53390.58 | 53390.48 |
| 22 | 51180.38 | 482.3054 | 53442.59 | 53442.5 |
| 23 | 51180.38 | 482.3054 | 53493.44 | 56553.41 |
| 24 | 11582.56 | 16060.77 | 90263.94 | 87334.94 |
| 25 | 11582.56 | 16060.77 | 91886.41 | 91883.73 |
| 26 | 11582.56 | 16060.77 | 93476.74 | 46474.65 |

## 2) Lateral Load Analysis Using Broms Method

## Analysis of piles using Broms methods by IS 2911

Analysis of a single pile according to Broms is described in Broms, 1964. This method exclusively assumes a pile in the homogeneous soil. Thus the analysis method does not allow for layered subsoil.
The lateral soil resistance for granular soils and normally consolidated clays which have varying soil modulus is modelled according to the equation:

$$
\frac{p}{y}=\eta_{\mathrm{h}} z
$$

where
$p=$ lateral soil reaction per unit length of pile at depth $z$ below ground level;
$y=$ lateral pile deflection; and
$\phi h=$ modulus of subgrade reaction for which the recommended values are given in IS 2911

Modulus of Subgrade Reaction for Granular Soils, $\phi$ h, in kN/m3


The lateral soil resistance for preloaded clays with constant soil modulus is modelled according to the equation:

$$
\frac{p}{y}=K
$$

where

$$
K=\frac{k_{1}}{1.5} \times \frac{0.3}{B}
$$

## 3) Lateral Load Analysis Using Staad Pro

Laterally loaded single pipe pile in soft clay, representative of a test performed was analyzed in the StaadPro Parameters of the pile and soft clay are tabulated in Tables D and E. The lateral load was applied to the pile head at a distance 0.0635 m ( 2.5 in) above the ground line, and the water table was kept above the ground line.

Table D-Parameters for the Pile

| Outsi <br> de <br> Diame <br> ter | Pile <br> Leng <br> th | Momen <br> t <br> of <br> inert <br> ia | Elast <br> ic <br> modul <br> us, | Yield <br> ing <br> momen <br> t | Mome <br> nt <br> at <br> full <br> hing <br> e, |
| :--- | :--- | :--- | :--- | :--- | :--- |
| M | M | $\mathrm{M}^{4}$ | $\mathrm{kN} / \mathrm{m}^{2}$ | My1 <br> $((\mathrm{kN}-$ <br> $\mathrm{m})$ | My2 <br> $(\mathrm{kN}-$ <br> $\mathrm{m})$ |
| 1.2 | 12.8 | 1.44 x <br> $10^{-4}$ | $2.18 \times$ <br> $10^{8}$ | 231 | 304 |

Table E-Parameters for the Sand

| Effective unit <br> weight, <br> $\mathrm{kN} / \mathrm{m}^{3}$ | Friction angle, $\varphi$ | Strain at $50 \%$ of <br> the maximum <br> stress, $\varepsilon_{50}$ |
| :--- | :--- | :--- |
| 10 | 32.3 | 0.012 |

Step 1: Open STAAD Pro, create a model of the structure, and assign properties to the model, as shown in Figure. Once the properties are assigned, select the pile.
Step 2: Open the Staad pro. By clicking "SelectPiles"; the pile dimensions are shown in the table. Then, by clicking "Strata, Vertical Axis, and Loading Directions", select the directions associated with the deposit of the strata, global vertical axis in STAAD Pro, and lateral loading in global coordinate system in STAAD Pro



The Structure Model of Group of 9 piles after being assigned with
(a) Elastic Soil springs
(b) Multi linear Soil Springs


Comparison of Lateral Displacement of the Pile Group
(a)Interior Pile
(b)Exterior Pile; at Ft $=1500 \mathrm{kN}$ (337 kips) (the Displacements Are Amplified 100Time


## 4) Lateral Load Analysis Using Lpile Software

The LPile software uses a finite difference scheme to analyze individual laterally loaded piles. To determine pile displacements and stresses, the basic differential equation for a beam-column is solved using a finite difference approximation. The different subsoil layers and their properties were modelled along with the physical and elastic properties of the pile.





EN-SOFT L-PILE SOFTWARE


| S oil properties |  |
| :---: | :---: |
|  | k_s oil |
| Element | [kN/m ${ }^{2}$ ] |
| 1 | 1000 |
| 2 | 1500 |
| 3 | 2000 |
| 4 | 2500 |
| 5 | 3000 |
| 6 | 3500 |
| 7 | 4000 |
| 8 | 4500 |
| 9 | 5000 |
| 10 | 5500 |
| 11 | 6000 |
| 12 | 6500 |
| 13 | 7000 |
| 14 | 7500 |
| 15 | 8000 |
| 16 | 8500 |
| 17 | 9000 |
| 18 | 9500 |
| 19 | 10000 |
| 20 | 10500 |
| 21 | 11000 |
| 22 | 11500 |
| 23 | 12000 |
| 24 | 12500 |
| 25 | 13000 |
| 26 | 13500 |
| 27 | 14000 |
| 28 | 14500 |
| 29 | 15000 |
| 30 | 15500 |
| 31 | 16000 |
| 32 | 16500 |
| 33 | 17000 |
| 34 | 17500 |
| 35 | 18000 |
| 36 | 18500 |
| 37 | 19000 |
| 38 | 19500 |
| 39 | 20000 |
| 40 | 20500 |



| Sping forces |  |  |  | Pile forces |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F_son | z | F_son | z | V | M |  |
| Elernent | [kNrm] | [m] | [kN] | [m] | [kN] | [kNrn] |  |
| 1 | -19.97 | 0.00 | -6.3 | 0.00 | 100.0 | 0.0 |  |
| 2 | -29.89 | -0.50 | -13.7 | -0.50 | 93.7 | -46.8 |  |
| 3 | -33.58 | -1.00 | -16.2 | -1.00 | 80.0 | -86.9 |  |
| 4 | -34.57 | -1.50 | -17.3 | -1.50 | 63.8 | -118.8 |  |
| 5 | -33.30 | -2.00 | -17.2 | -2.00 | 46.5 | -1420 |  |
| 6 | -30.29 | -2.50 | -16.1 | -2.50 | 29.2 | -156.6 |  |
| 7 | -26.06 | -3.00 | -14.2 | -3.00 | 13.2 | -163.2 |  |
| 8 | -21.11 | -3.50 | -11.9 | -3.50 | -1.1 | -1627 |  |
| 9 | -15.86 | -4.00 | -9.3 | -4.00 | -12.9 | -156.2 |  |
| 10 | -10.71 | -4.50 | -6.6 | -4.50 | -22.2 | -145.1 |  |
| 11 | -5.93 | -5.00 | -4.1 | -5.00 | -28.8 | -130.7 |  |
| 12 | -1.75 | -5.50 | -1.8 | -5.50 | -32.9 | -114.3 |  |
| 13 | 1.71 | -6.00 | 0.1 | -6.00 | -34.7 | -96.9 |  |
| 14 | 4.39 | -6.50 | 1.6 | -6.50 | -34.6 | -79.6 |  |
| 15 | 6.29 | -7.00 | 2.8 | -7.00 | -33.0 | -63.1 |  |
| 16 | 7.47 | -7.50 | 3.5 | -7.50 | -30.2 | -48.0 |  |
| 17 | 8.01 | -8.00 | 3.9 | -8.00 | -26.7 | -34.6 |  |
| 18 | 8.02 | -8.50 | 4.1 | -8.50 | -22.8 | -23.3 |  |
| 19 | 7.61 | -9.00 | 3.9 | 9.00 | -18.7 | -13.9 |  |
| 20 | 6.90 | -9.50 | 3.7 | 9.50 | -14.8 | -6.5 |  |
| 21 | 6.00 | -10.00 | 3.2 | -10.00 | -11.1 | -1.0 |  |
| 22 | 5.01 | -10.50 | 2.8 | -10.50 | -7.9 | 2.9 |  |
| 23 | 4.01 | -11.00 | 2.3 | -11.00 | -5.1 | 5.5 |  |
| 24 | 3.05 | -11.50 | 1.8 | -11.50 | -2.8 | 6.9 |  |
| 25 | 2.18 | -12.00 | 1.3 | -12.00 | -1.1 | 7.4 |  |
| 26 | 1.43 | -12.50 | 0.9 | -12.50 | 0.2 | 7.3 |  |
| 27 | 0.81 | -13.00 | 0.5 | -13.00 | 1.1 | 6.8 |  |
| 28 | 0.32 | -13.50 | 0.3 | -13.50 | 1.6 | 6.0 |  |
| 29 | -0.05 | -14.00 | 0.1 | -14.00 | 1.9 | 5.0 |  |
| 30 | -0.30 | -14.50 | 0.1 | -14.50 | 2.0 | 4.0 |  |
| 31 | -0.46 | -15.00 | -0.2 | -15.00 | 1.9 | 3.1 |  |
| 32 | -0.54 | -15.50 | -0.3 | -15.50 | 1.7 | 2.3 |  |
| 33 | -0.56 | -16.00 | -0.3 | -16.00 | 1.4 | 1.6 |  |
| 34 | -0.53 | -16.50 | -0.3 | -16.50 | 1.1 | 1.0 |  |
| 35 | -0.47 | -17.00 | -0.3 | -17.00 | 0.8 | 0.6 |  |
| 36 | -0.39 | -17.50 | -0.2 | -17.50 | 0.6 | 0.3 |  |
| 37 | -0.30 | -18.00 | -0.2 | -18.00 | 0.4 | 0.1 |  |
| 38 | -0.20 | -18.50 | -0.1 | -18.50 | 0.2 | 0.0 |  |
| 39 | -0.09 | -19.00 | -0.1 | -19.00 | 0.1 | 0.0 |  |
| 40 | 0.00 | -19.50 | 0.0 | -19.50 | 0.0 | 0.0 |  |
|  |  | -20.00 | 0.0 | -20.00 | 0.0 | 0.0 |  |
|  |  |  |  |  |  |  |  |

Bending moment [ kNm ]


Shear Force [kN]



## V. COMPARISON AND CONCLUSION

## Comparison of Maximum Bending Moment of Pile

The configurations of the pile group and the pile cap have been assumed for this calculation. The parameters for the piles are tabulated, and the center-to-center spacing of the piles has been taken as three times the outside diameter of the pile ( $3 D$ ). The pile cap only serves to rigidly connect the piles together, and was assigned an elastic modulus of $2.17 \times 10^{7} \mathrm{kN} / \mathrm{m}^{2} \quad(2,147 \mathrm{ksi})$ and dimensions of $1 \mathrm{~m} \times 1.83 \mathrm{~m} \times 1.83 \mathrm{~m} \quad(3.28 \mathrm{ftx} 6 \mathrm{ftx} 6 \mathrm{ft}) \quad$ (thickness $\times$ length $\times$ width).
The piles were embedded 0.5 m (1.64ft) into thickness of the pile cap. The ground line was taken at the same elevation as the pile cap base.

## Parameters for the Pile

| Outside <br> diameter, <br> D | Inside <br> diameter, <br> d | Pile <br> length, <br> L | Moment of <br> inertia, <br> I | Elastic <br> modulus, <br> E | Yielding <br> moment, <br> $M_{y l}$ | Moment at <br> full hinge, <br> $M_{y 2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| M | m | M | $\mathrm{m}^{4}$ | $\mathrm{kN} / \mathrm{m}^{2}$ | $\mathrm{kN}-\mathrm{m}$ | $\mathrm{kN}-\mathrm{m}$ |
| 0.61 | 0.59 | 21 | $8.08 \times 10^{4}$ | $2.02 \times 10^{8}$ | 640 | 828 |
| $(\mathrm{ft})$ | $(\mathrm{ft})$ |  | $\left(\mathrm{ft}^{4}\right)$ | $(\mathrm{ksi})$ | $(\mathrm{k}-\mathrm{ft})$ | $(\mathrm{k}-\mathrm{ft})$ |
| $(2.00)$ | $(1.94)$ | $(68.9)$ | $\left(9.36 \times 10^{-2}\right)$ | $(29,297)$ | $(472)$ | $(611)$ |



Comparison of Lateral Displacement of Pile Head


Comparison of Maximum Bending Moment from Field Test,

Staad and

LPILE


## VI. CONCLUSION

1. The analytical methods like Broms presented almost half a century back still holds good in estimating the pile head deflections under the lateral loads. Even though these methods overestimate the deflection, they can still be adopted considering the soil as a complex and nonlinear material which is influenced by many variables, including its history, nature of loading, changes in the environment, method of pile installation etc.
2. Broms methods which gave fairly accurate results can be adopted for small scale projects and when softwares are not available for the analysis
3. The analysis using softwares such as LPile also gives conservative outputs, but has an advantage
over the manual analysis that other soil pile behaviour such as pile bending moment, soil reactions, p-y curves and other such soil-pile behaviour can be studied within short time with less effort.
4. The STAAD Pro analysis, though applicable for structural analysis and design, can be effectively used to evaluate pile head deflections with some multiplication factor due to its consistently lower deflection values. Its dependence on soil spring constants and ultimately on the accuracy in the estimation of soil elastic modulus based on soil properties and field data shall be well understood before its use.

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