Text Book

- Elementary Structural Analysis, by Norris, Wilbur and utku.
- Statically Indeterminate Structures by Chu-kia Wang.

References

- Analysis of Structural system by Jobn F. Fleming.
- Elementary Theory of Structures by Yuan Yu Hsieh.
- Structural Analysis by Hibbeler.
- Indeterminate Structural Analysis by Kinney.

Syllabus

- Introduction.
- Stability and determinacy of structures.
- Axial force, Shear force and bending moment diagram of frames and arches.
- Trusses
- Influence line and moving load.
- Elastic deformation of structures.
- Method of consistence deformation.
- Slope deflection method.
- Moment distribution method.
- Approximate analysis of indeterminate structures.
- Stiffness matrix analysis of structures.

Introduction.

1- Sign convention:

For the analysis procedure, the active and reactive load acting on the structure and the displacement of the joints will be expressed as components in a right hand orthogonal " ***Global Coordinate System***" with the three axes being designed as ***X, Y, and Z***, Fig,(1).

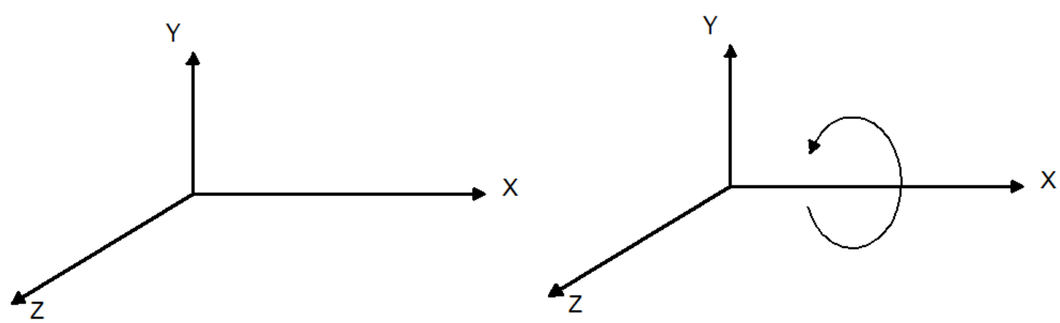


Figure (1): Sign convention

2- Structural loads:

Dead Load: it's usually consist of:

- Own weight of the structure.
- Immovable loads that are constant in magnitude and permanently attached to the structure (for celling, ducts …. ext).

The weight of several structural materials are shown in Table (1).

Table 1: Weight of Structural material.

Material	Weight (SI) kN/m^3
Concrete	23-25
Steel	76.97
Wood	6.28
Aluminum	25.92

Live load

a- Vertical live load

- Movable load.
- Moving load.
- Snow load

b- Horizontal live load

- Wind load.
- Earthquake load
- Soil pressure.
- Hydrostatic load.
- Thermal force.

3- Equilibrium and reactions

a- Equilibrium

A rigid body is in equilibrium if it is either;

- At rest (velocity = 0).
- State of constant motion (acceleration=0).

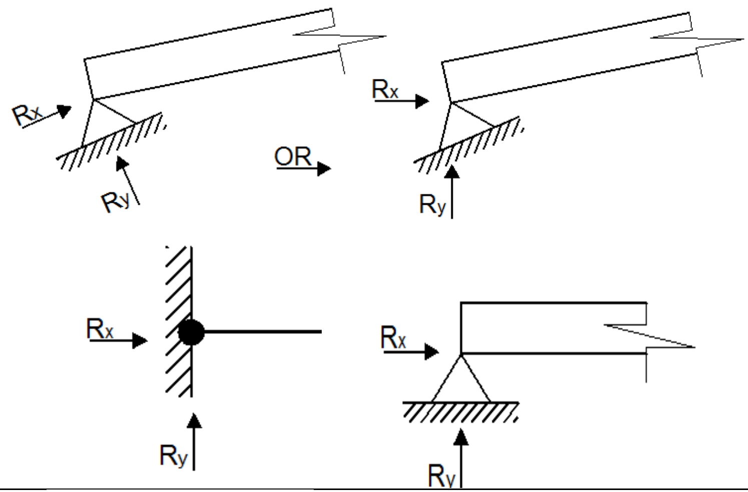
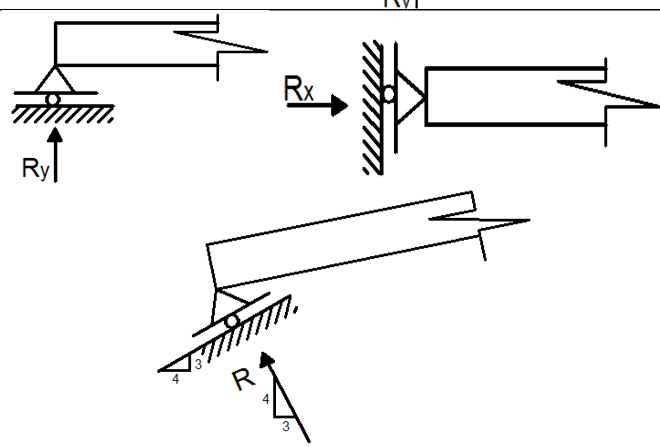
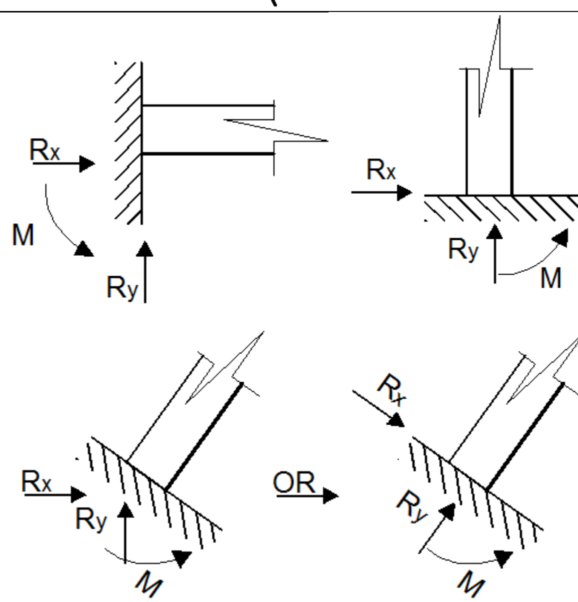
This require that:

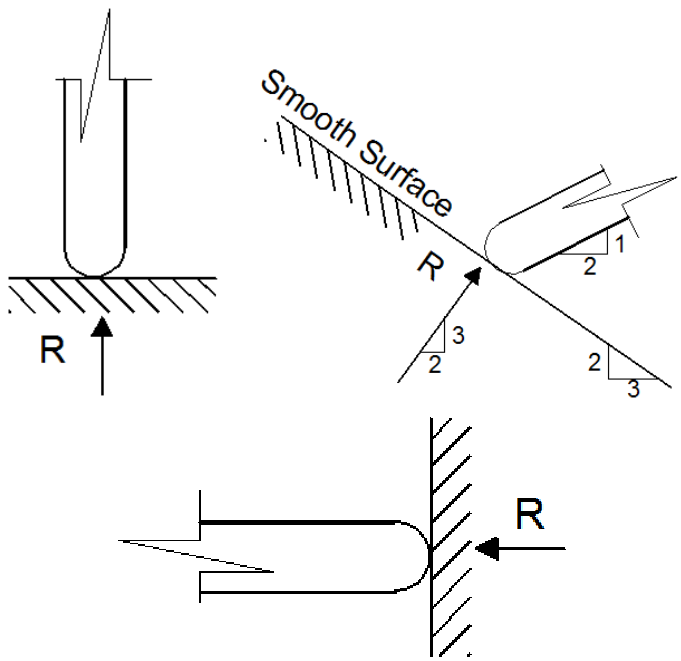
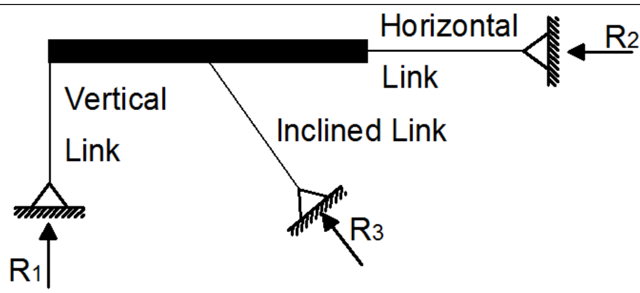
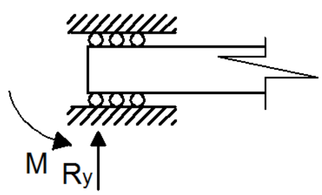
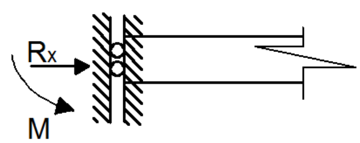
- The resultant force on the body must be zero to prevent linear acceleration $\sum F = 0$.
- The resultant moment on the body must be zero to prevent angular acceleration $\sum M = 0$.

For plane truss or frame (two dimensions), these two conditions are usually designed by the three equilibrium equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M = 0$$

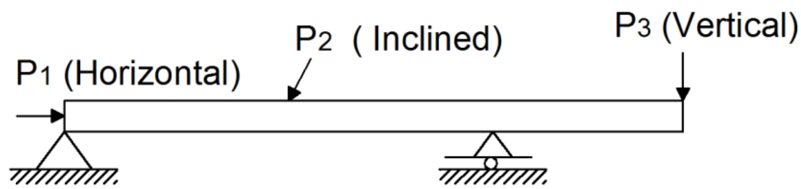
b- Types of Supports

Type of Support	Sample	Reaction
Hinge		Two reactions perpendicular to the supported surface and parallel to it, R_x and R_y .
Roller		One reaction perpendicular to the moving surface , R.
Fixed		Three reactions, two force and one moment. R_x , R_y and M.

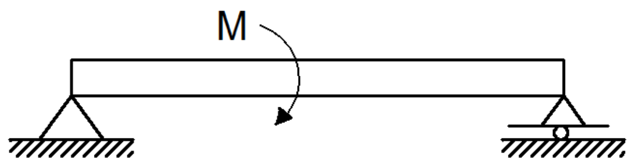
Smooth surface		One reaction perpendicular to the smooth surface , R
Link or strut	 Note : Link is a straight element of a pin end support and no external acting force a long its length.	One reaction in the direction of the link, R
Guide support	 Allow movement in X direction  Allow movement in Y direction	Two reactions moment and force, R and M.

c- Types of applied loads

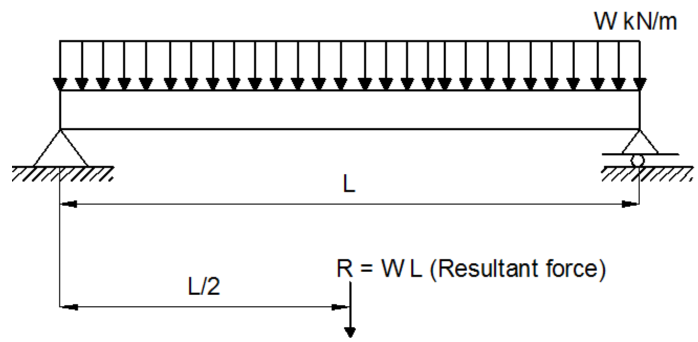
- Concentrated load.



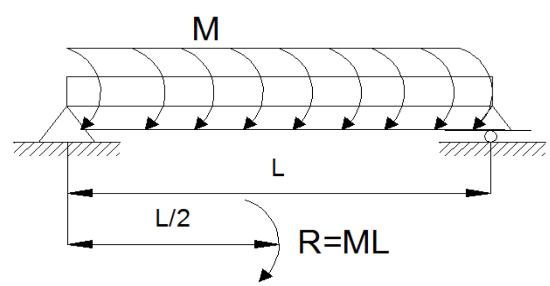
- Concentrated moment



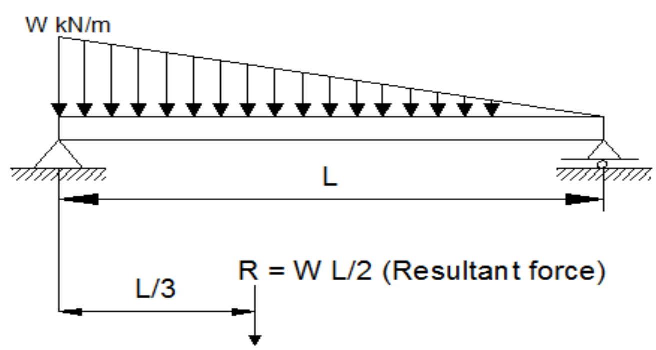
- Uniform distributed load.



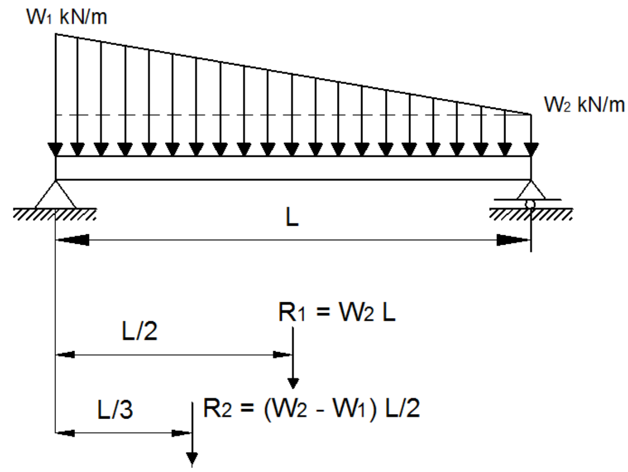
Uniform distributed Moment



- Triangular load

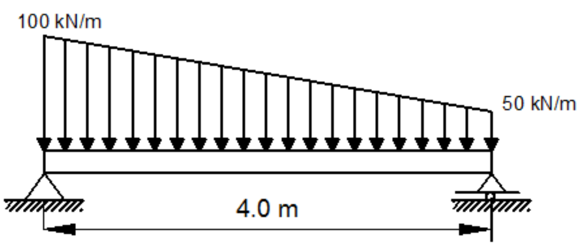


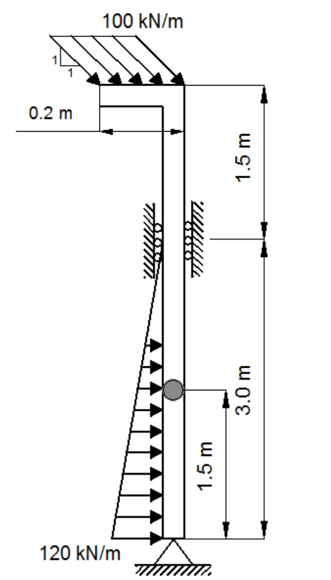
- Trapezoidal load.

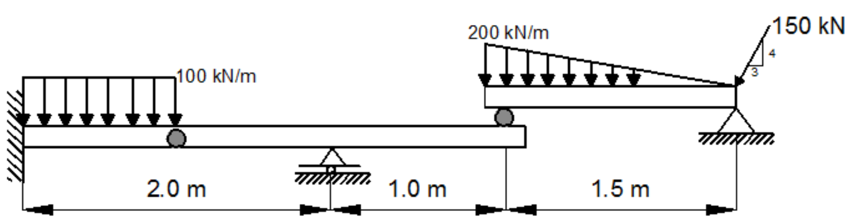


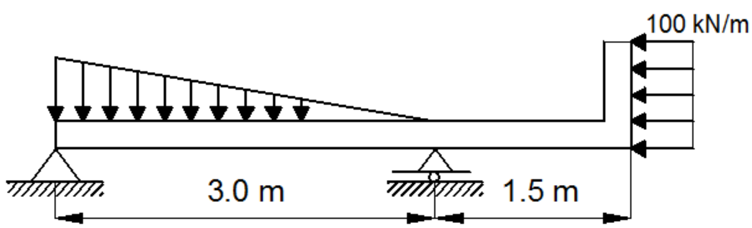
Notes:

- The applied loads may be a case of combination from the above mentioned load cases.
- Cases of distributed load can be partially distributed along the affected span.







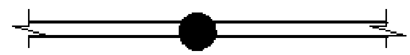


d- Determinate and indeterminate structure

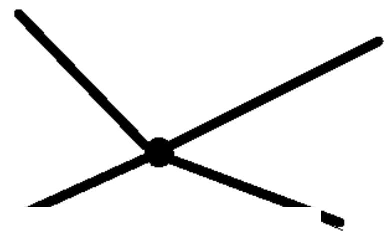
- The structure is said to be determinate if:
  (Number of unknown = Total number of equilibrium equations)
- The structure is said to be indeterminate if:
- ***(Number of unknown > Total number of equilibrium equation)***

Conditional equations (**C**)

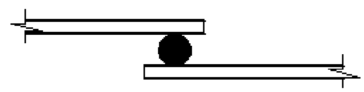
- Interior hinge "pin" that's connecting two members.
  C=1



- Interior hinge "pin" that's connecting **m** members.
  C=m-1



Interior roller
C=2
(M=0 and F_x =0)



Stability and determinacy of structures.

The structure is said to be unstable if:

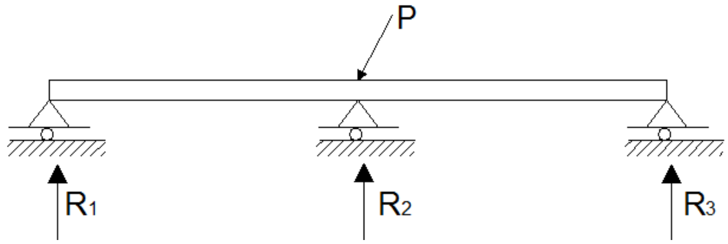
1- Numbers of Unknown (reactions, NR) < Total numbers of equilibrium equations (NEE)

$$NR < NEE$$

2- All reactions are parallel

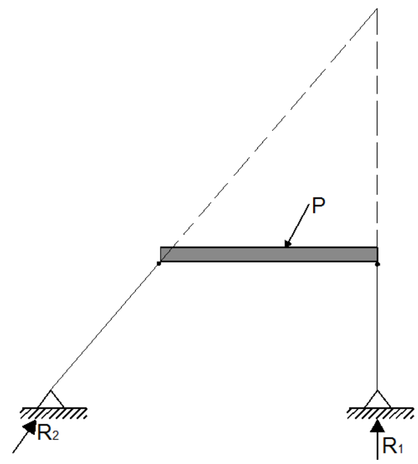
$R_1 \; // \; R_2 \; // \; R_3$

$\sum M \neq 0$

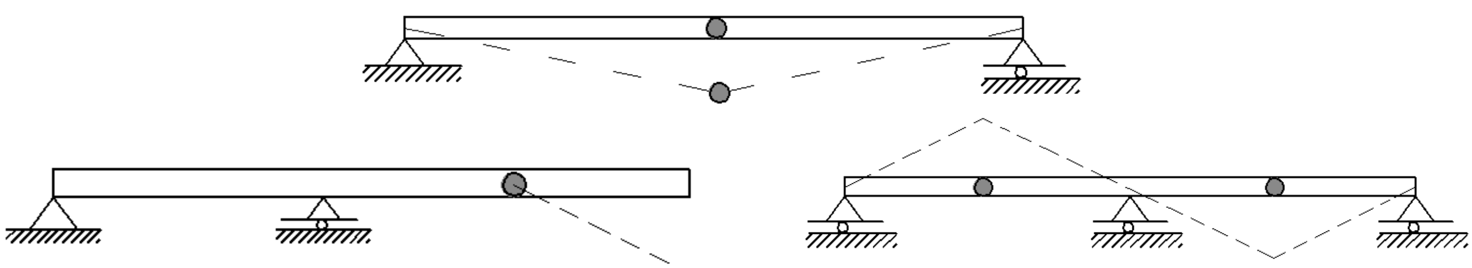


3- All reactions are concurrent (meat at one point).

$\sum M \neq 0$



4- When the structure is geometrically unstable.



a- Stability and determinacy of beams

Let r = No. of reactions (Unknowns).

And No. of equilibrium equations in a case of plane structure = 3 ($\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$)

∴Total No. of equilibrium equations = 3 + C

Where (C) is the number of the conditional equations.

<u>Therefore:</u>

If	r < (3 + C)	(The beam is unstable).
	r = (3 + C)	(The beam is determinate, if stable).
	r > (3 + C)	(The beam is indeterminate, if stable).

Example 1

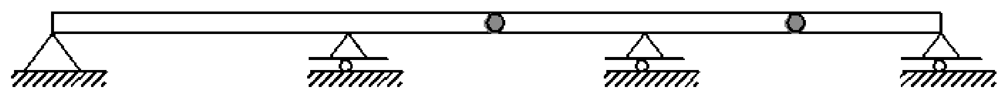
r = 5

C = 2

No parallel reactions

No concurrent reactions

r = (3 + C) = 5 (The beam is determinate and stable).



Example 2

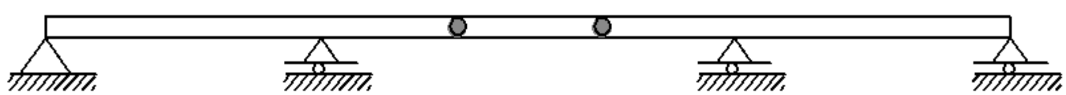
r = 5

C = 2

No parallel reactions

No con reactions

r = (3 + C) = 5 (The beam is determinate and stable)



Example 3

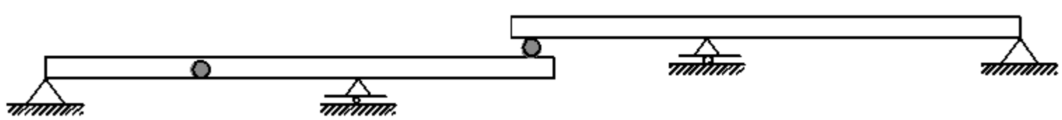
r = 6

C = 3

No parallel reactions

No concurrent reactions

r = (3 + C) = 6 (The beam is determinate and stable)



Example 4

r = 6

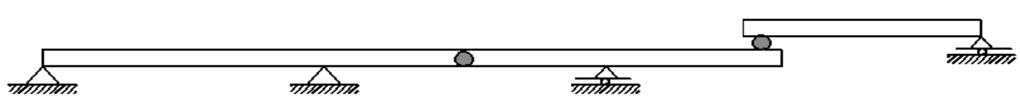
C = 3

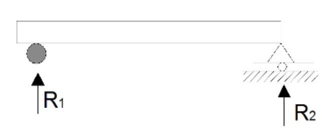
No parallel reactions

No concurrent reactions

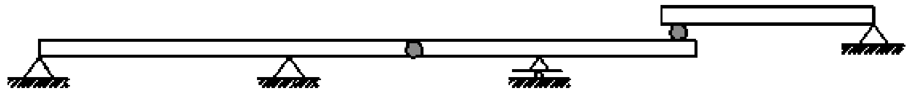
r = (3 + C) = 9

The beam is unstable because the reaction (R_1& R_2) are parallel



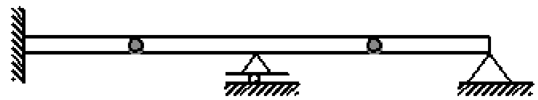


Example 5



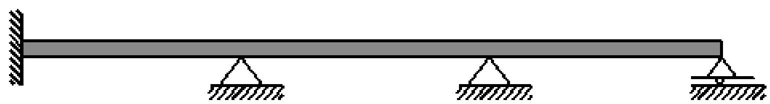
r = 7
C = 3
No parallel reactions
No concurrent reactions
r > (3 + C) = 6 (The beam is Indeterminate and stable)

Example 6



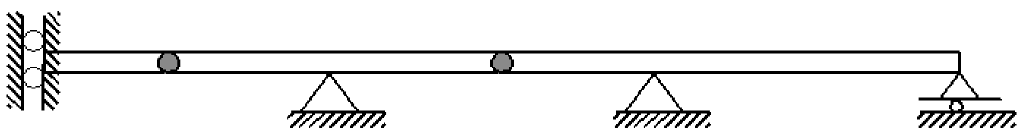
r = 6
C = 2
No parallel reactions
No concurrent reactions
r > (3 + C) = 5 (The beam is Indeterminate and stable)

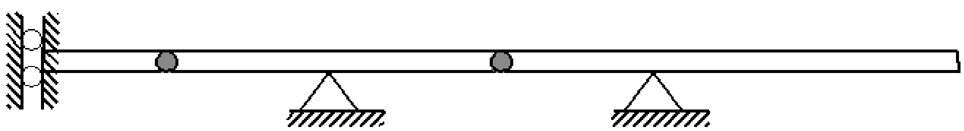
Example 7

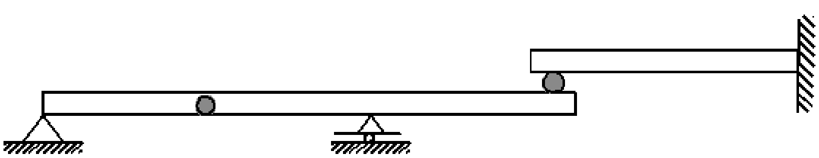


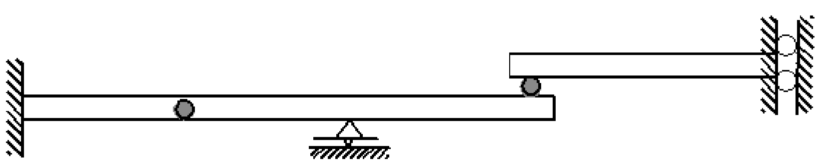
r = 8
C = 0
No parallel reactions
No concurrent reactions
r > (3 + C) = 3 (The beam is determinate and stable)

H.W.









b- Stability and determinacy of truss

Let : r= No. of reactions
b= No. of bars
j = No. of joints

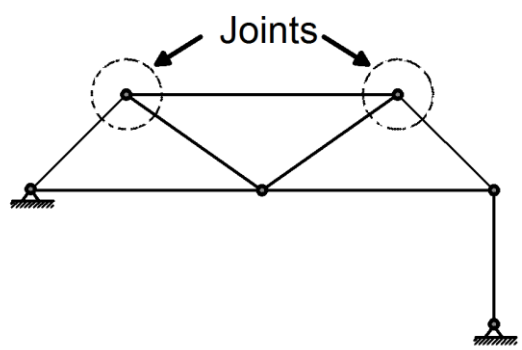
No. of unknown = b + r

No. of equilibrium Eqs. = 2j (Two equilibrium Eqs. can be written at each joint $\sum F_x = 0$, $\sum F_y = 0$)

<u>Therefore:</u>

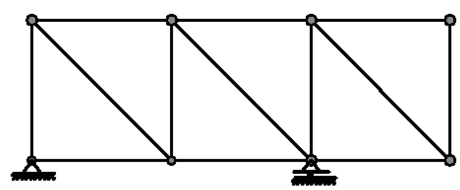
If r+b < 2j (The truss is unstable).
r +b= 2j (The truss is determinate, if stable).
r+b > 2j (The truss is indeterminate, if stable).

Example 1



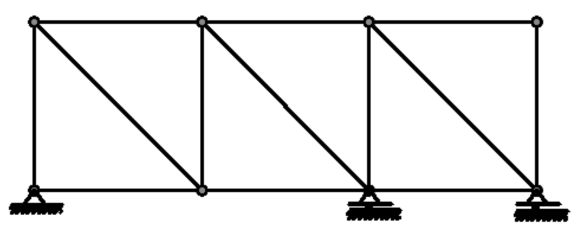
r = 4
b = 8
j = 6 → 2j = 12
r +b = 12
r +b= 2j (The truss is stable and determinate).

Example 2



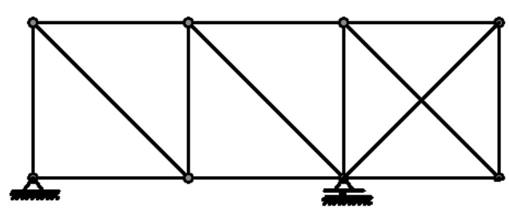
r = 3
b = 13
j = 8 → 2j = 16
r +b = 16
r +b= 2j (The truss is stable and determinate).

Example 3



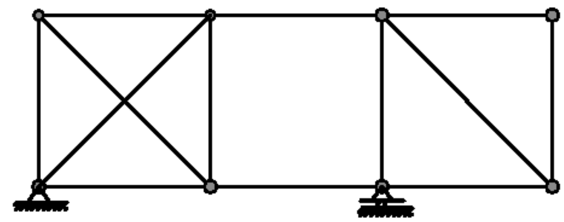
r = 4
b = 13
j = 8 → 2j = 16
r +b = 17
r +b> 2j (The truss is stable and indeterminate to the 1 st degree).

Example 4



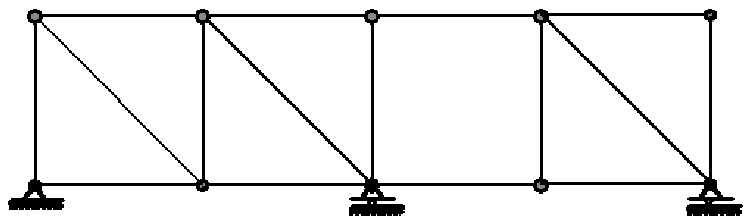
r = 3
b = 14
j = 8 → 2j = 16
r +b = 17
r +b> 2j (The truss is stable and determinate to the 1 st degree).

Example 5



r = 3
b = 13
j = 8 → 2j = 16
r +b = 16
r +b= 2j (un stable).

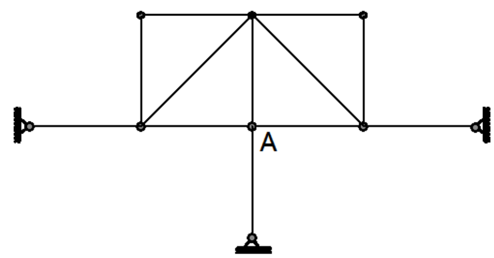
Example 6



r = 4
b = 16
j = 10 → 2j = 20
r +b = 20
r +b= 2j (The truss is stable and determinate).

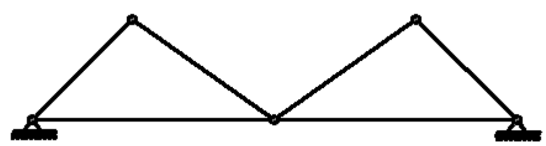
Example 7

All the reactions intersect at point A (Un stable)



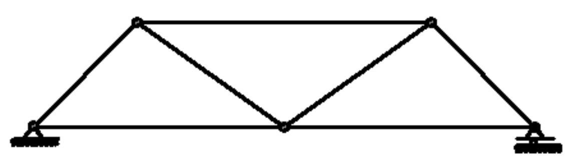
Example 8

(Un stable)



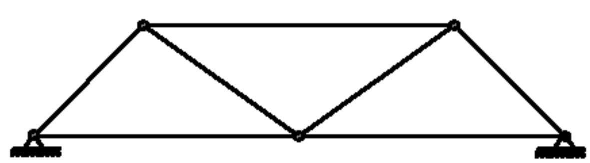
Example 9

r = 3
b = 7
j = 5 → 2j = 10
r +b = 10
r +b= 2j (The truss is stable and determinate).



Example 10

r = 4
b = 7
j = 5 → 2j = 10
r +b = 11
r +b> 2j (The truss is stable and indeterminate to the 1st degree).



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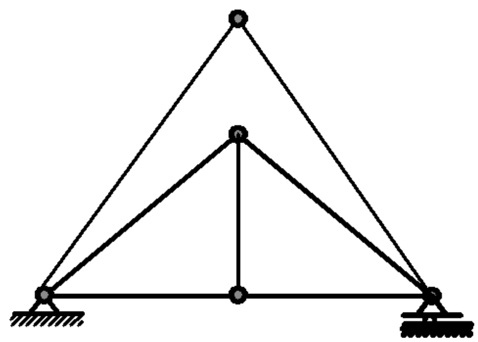
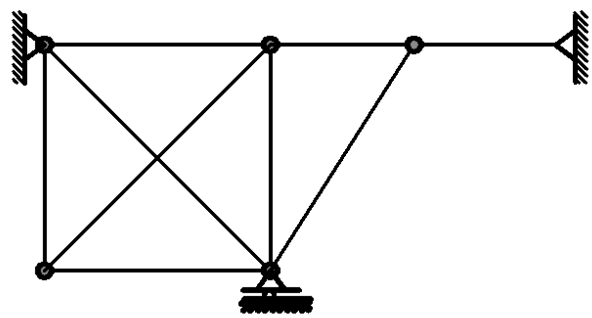
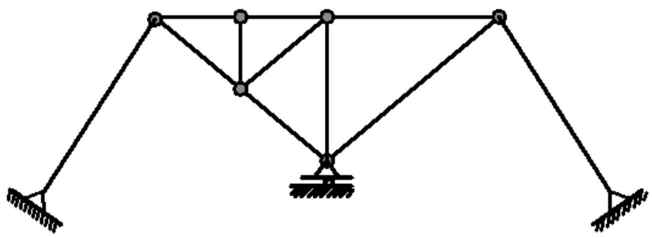
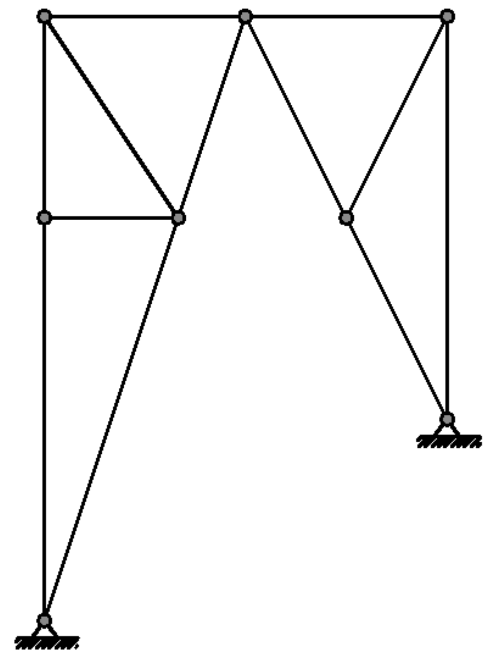








c- Stability and determinacy of frames and arches

1- Open frames and arches.

Determinacy of open frames and arches can be estimated in a way similar to that adopted in a case of beams determinacy, i e by applying the relation [$r \underset{<}{\overset{>}{=}} (3 + C)$].

Example 1

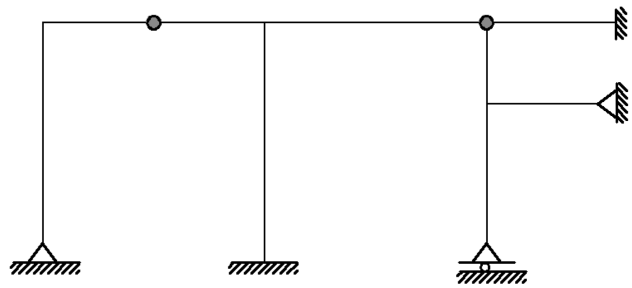
r = 11

C = 3

No parallel reactions

No concurrent reactions

r > (3 + C) = 6 (The frame is Indeterminate to the 5 th degree and stable).



Example 2

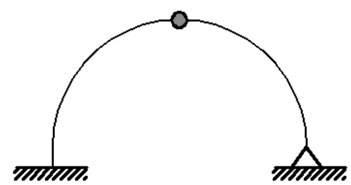
r = 5

C = 1

No parallel reactions

No concurrent reactions

r > (3 + C) = 4 (The frame is Indeterminate to the 1 th degree and stable)



Example 3

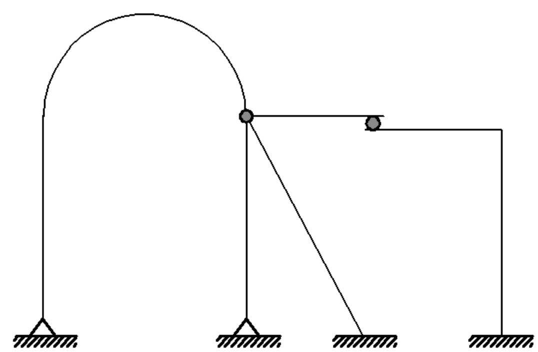
r = 10

C = 5

No parallel reactions

No concurrent reactions

r > (3 + C) = 8 (The frame is Indeterminate to the 2 nd degree and stable)



Example 4

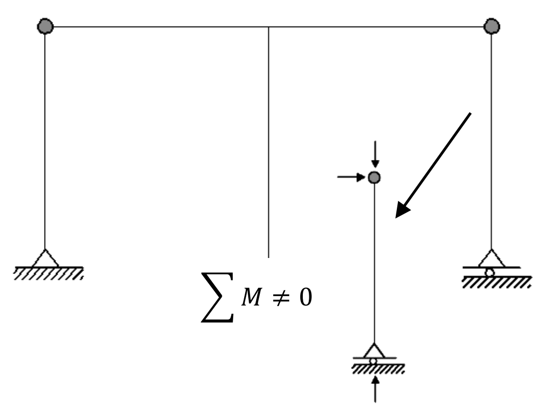
r = 6

C = 2

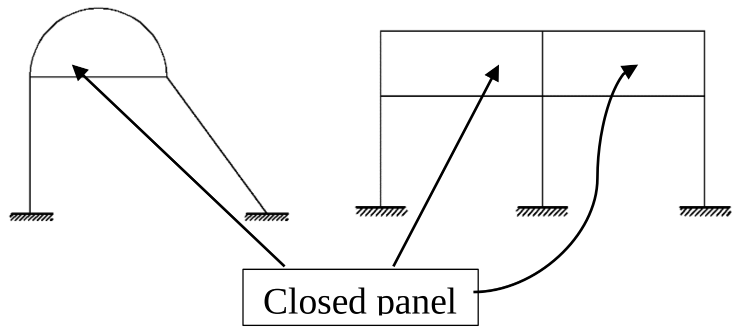
No parallel reactions

Concurrent reactions for the right column

r > (3 + C) = 5 (The frame is Indeterminate to the 1 st degree and unstable)



2- Closed frames and arches.
Frames are classified as closed frames if they contain at least one closed panel.



Let r = No. of reactions
b = No. of members.
j = No. of joints.

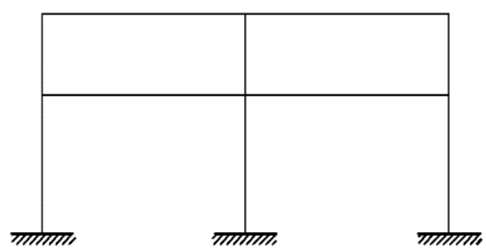
No. of unknowns = 3b + r ("3" denotes to the three interior unknown force "N, V and M" that is generated at each members).
∴Total No. of equilibrium equations = 3j + C (No. of equilibrium equations at each joints = 3)
($\sum F_x = 0 , \sum F_y = 0 , \sum M = 0$))

Therefore:

If 3b+r < (3j + C) (The frame is unstable).
3b+r = (3j + C) (The frame is determinate, if stable).
3b+r > (3j + C) (The frame is indeterminate, if stable).

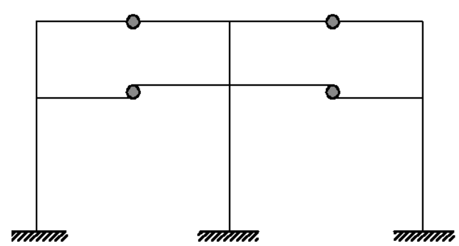
Example 1

r = 9 b = 10 C=0 j=9
3b+r = 3*10+9 = 39
3j + C = 3*9+ 0 = 27
3b+r > (3j + C) (The frame is Indeterminate to the 12 th degree and stable)



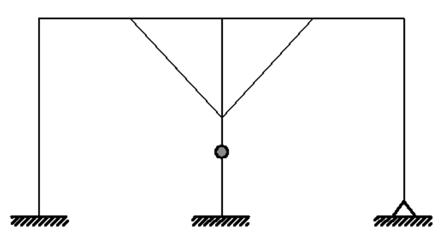
Example 2

r = 9 b = 14 C=6 j=13
3b+r = 3*14+9 = 51
3j + C = 3*13+ 6 = 45
3b+r > (3j + C) (The frame is Indeterminate to the 6 th degree and stable)



Example 3

r = 8 b = 11 C=1 j=10
3b+r = 3*11+8 = 41
3j + C = 3*10+ 1 = 31
3b+r > (3j + C) (The frame is Indeterminate to the 10 th degree and stable)



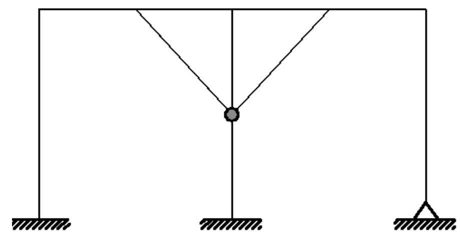
Example 4

r = 8 b = 10 C=3 j=9

3b+r = 3*10+8 = 38

3j + C = 3*9+ 3 = 30

3b+r > (3j + C) (The frame is Indeterminate to the 8 th degree and stable)



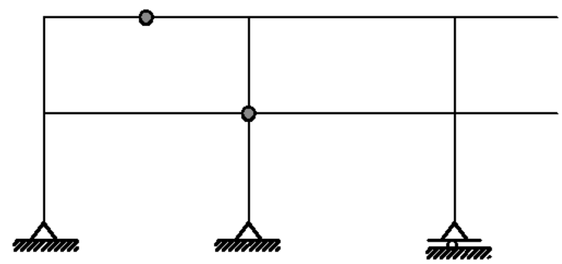
Example 5

r = 5 b = 11 C=4 j=10

3b+r = 3*11+5 = 38

3j + C = 3*10+ 4 = 34

3b+r > (3j + C) (The frame is Indeterminate to the 4 th degree and stable)



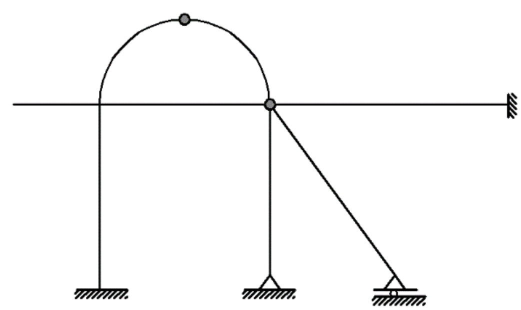
Example 6

r = 9 b = 7 C=5 j=7

3b+r = 3*7+9 = 30

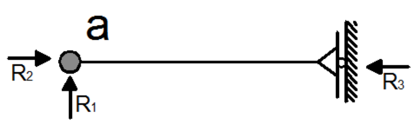
3j + C = 3*7+ 5 = 26

3b+r > (3j + C) (The frame is Indeterminate to the 4 th degree and stable)

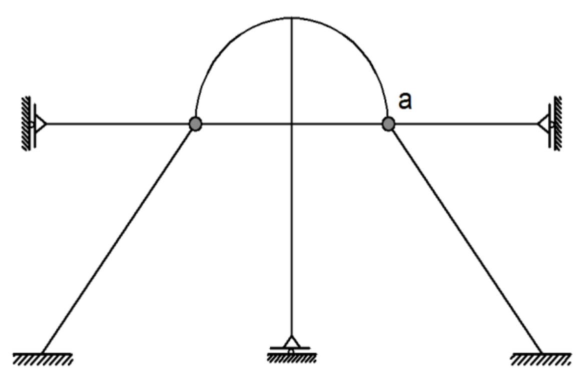


Example 7

Un stable



$$\sum M \neq 0$$

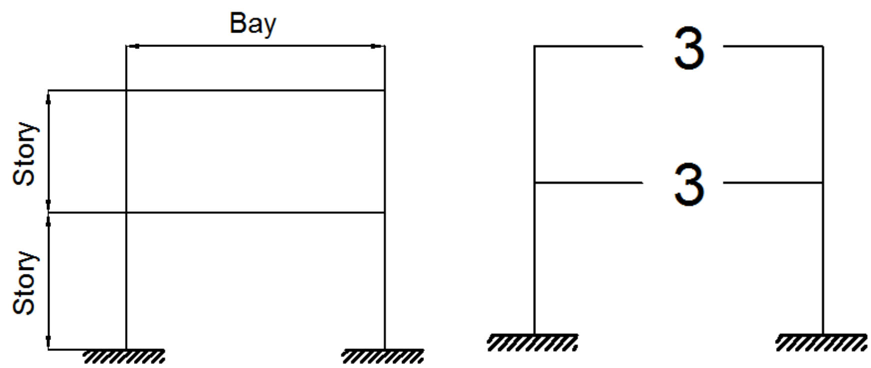


3- Multi- story or multi-bay buildings

An easier way to estimate the degree of indeterminacy for multi = story of multi bay frames of fixed base is as follow :

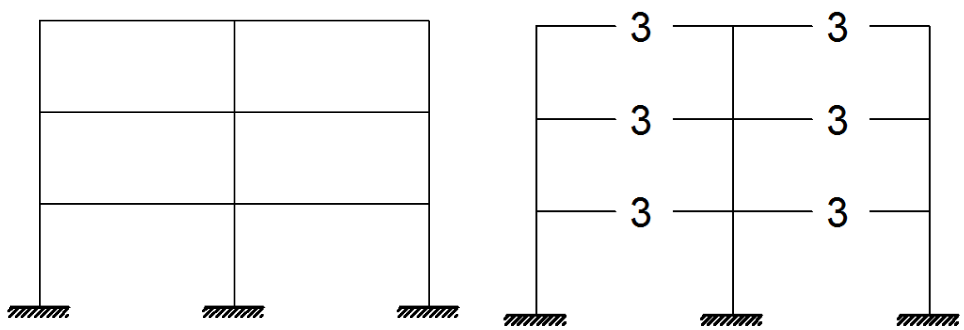
Example 1 one – bay , two – story frame

$3b + r = 3*6+3 = 24$
$3j + C = 3* 6 + 0 = 18$
$3b + r > 3j + C$
Stable and Indeterminate to the 6 th degree



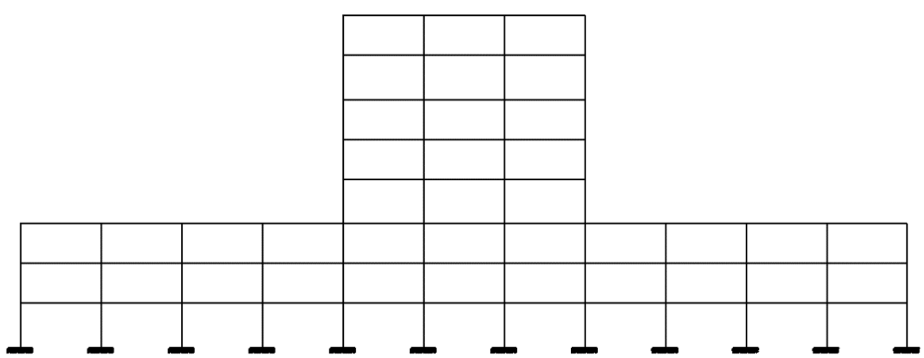
Example 2 two – bay , three – story frame

$3b + r = 3*15+9 = 54$
$3j + C = 3* 12 + 0 = 36$
$3b + r > 3j + C$
Stable and Indeterminate to the 18 th degree



Example 3

$3b + r = 3*[3*12+3*11+4*5+3*5]+3*12 = 348$
$3j + C = 3* (12*4+4*5) + 0 = 228$
$3b + r > 3j + C$
Stable and Indeterminate to the 120 th degree



4- Stability and determinacy of composite structures

A composite structure can be defined as a combination of flexural and axial members where the flexural members can resist bending moment (like beams, frames and arches) while the axial member can only resist axial force (like cable, strut, link and truss).

The solution process will be based on:

- **Each (F. B. D.) gives three equilibrium equations.**
- **Each pine connection involved two unknowns.**

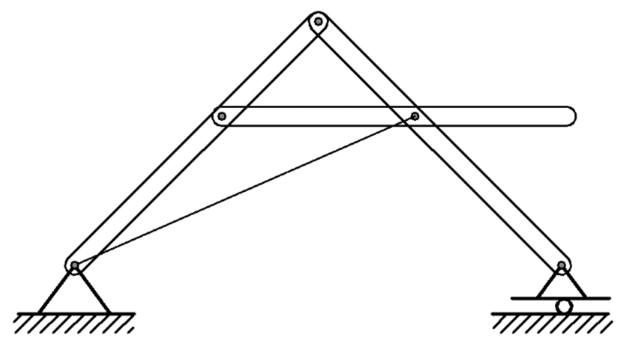
Example 1:

No. of (F. B. D.) = 3

No. of equilibrium equations = 3*3 = 9

No. of unknown = 10

Stable & Indeterminate to the 1st degree



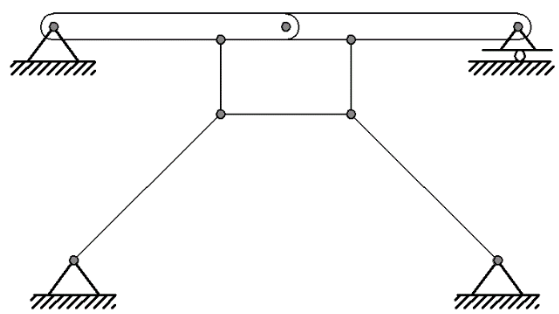
Example 2:

No. of (F. B. D.) = 3

No. of equilibrium equations = 3*3 = 9

No. of unknown = 9

Stable & determinate



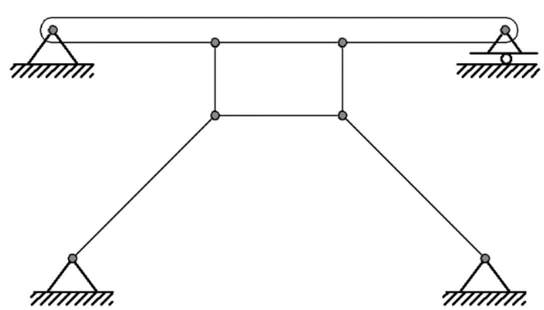
Example 3:

No. of (F. B. D.) = 2

No. of equilibrium equations = 2*3 = 6

No. of unknown = 7

Stable & Indeterminate to the 1st degree



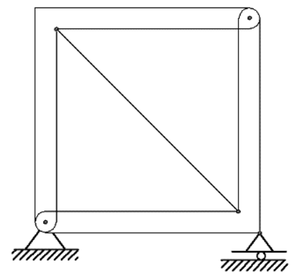
Example 4:

No. of (F. B. D.) = 2

No. of equilibrium equations = 2*3 = 6

No. of unknown = 8

Stable & Indeterminate to the 2nd degree



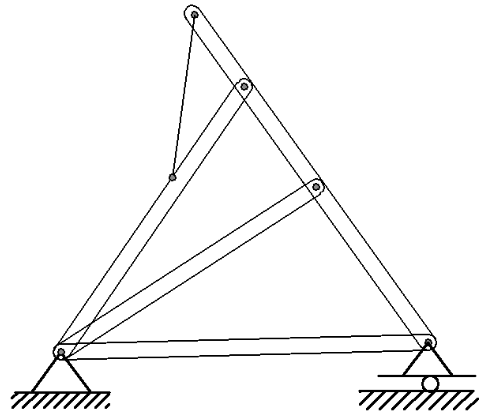
Example 5:

No. of (F. B. D.) = 4

No. of equilibrium equations = 4*3 = 12

No. of unknown = 14

Stable & Indeterminate to the 2nd degree



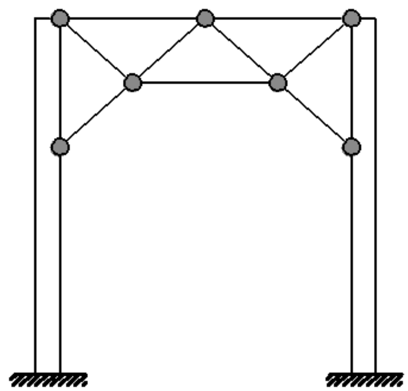
Example 6:

No. of (F. B. D.) = 2

No. of equilibrium equations = 3*3 = 9

No. of unknown = 12

Stable & Indeterminate to the 3rd degree



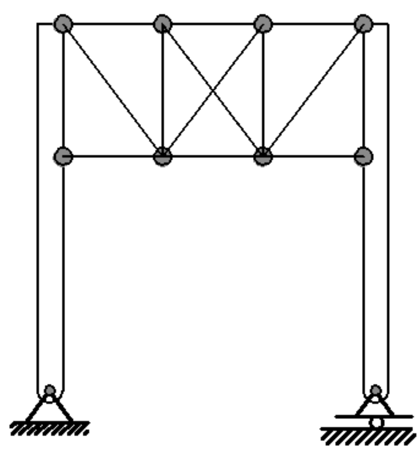
Example 7:

No. of (F. B. D.) = 3

No. of equilibrium equations = 3*3 = 9

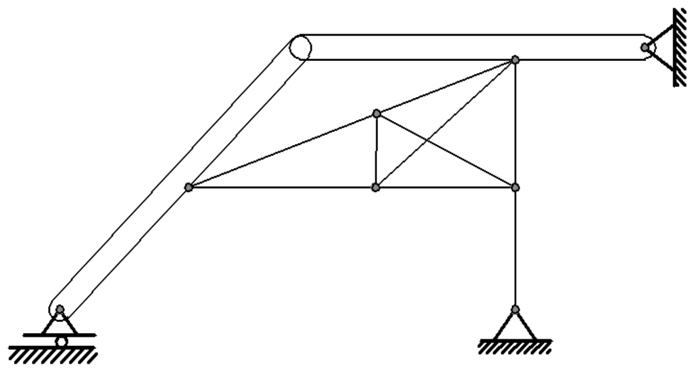
No. of unknown = 10

Stable & Indeterminate to the 1st degree



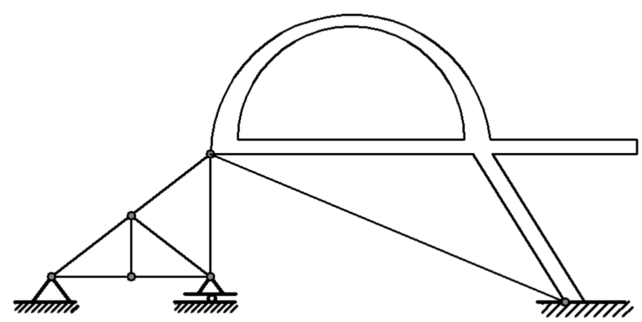
Example 8:

No. of (F. B. D.) = 3
No. of equilibrium equations = 3*3 = 9
No. of unknown = 11
Stable & Indeterminate to the 2nd degree



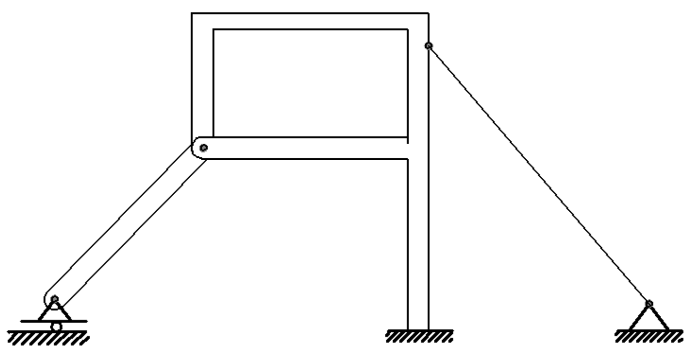
Example 9:

No. of (F. B. D.) = 2
No. of equilibrium equations = 2*3 = 6
No. of unknown = 9+3
Stable & Indeterminate to the 6th degree

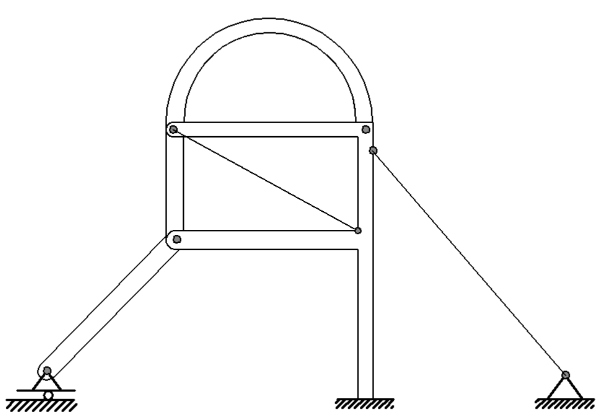


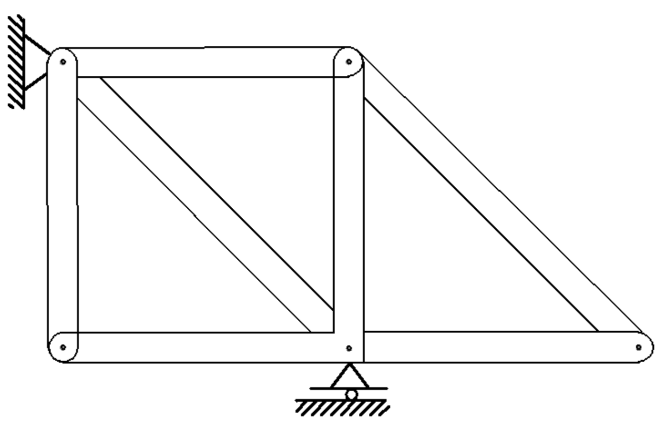
Example 10:

No. of (F. B. D.) = 2
No. of equilibrium equations = 2*3 = 6
No. of unknown = 7+3 = 10
Stable & Indeterminate to the 4th degree



H.W.





Axial, Shear & Bending Moment Diagram.

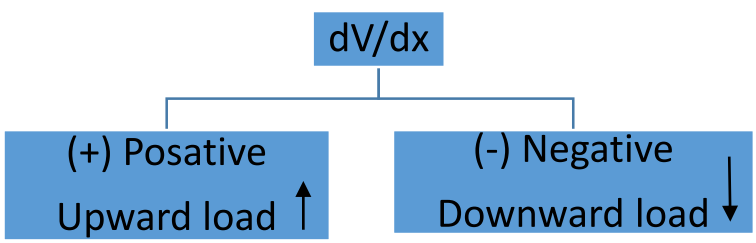
a- Left section partInterior force : three interior forces are generated if any beam element was cutting, these are:
- N= Axial Force
- V= Shear Force
- M= Bending Moment.

b- Sign convention:
- + N Tension Force (directed outward from the section)
- + V Downward direction on the left section part (clock wise rotation)
- + M Counter clock wise effect on the left section part.
- + W Upward ↑ and left side ⟵ reaction load.
- + R Upward ↑ and left side ⟵ reaction.
- Exterior applied moment is consider as positive moment if its clock wise affected.

c- Relationships between (applied vertical load and generated interior forces V & M).

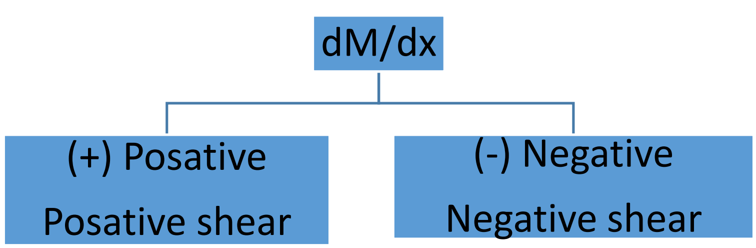
$V = \int W\,dx$ (Shear force at any section is equal to the summation of the applied vertical load from x=0 to the considered location)

or $\frac{dV}{dx} = w$ (Slop of the shear force diagram at any location equal to the applied vertical load at that location).

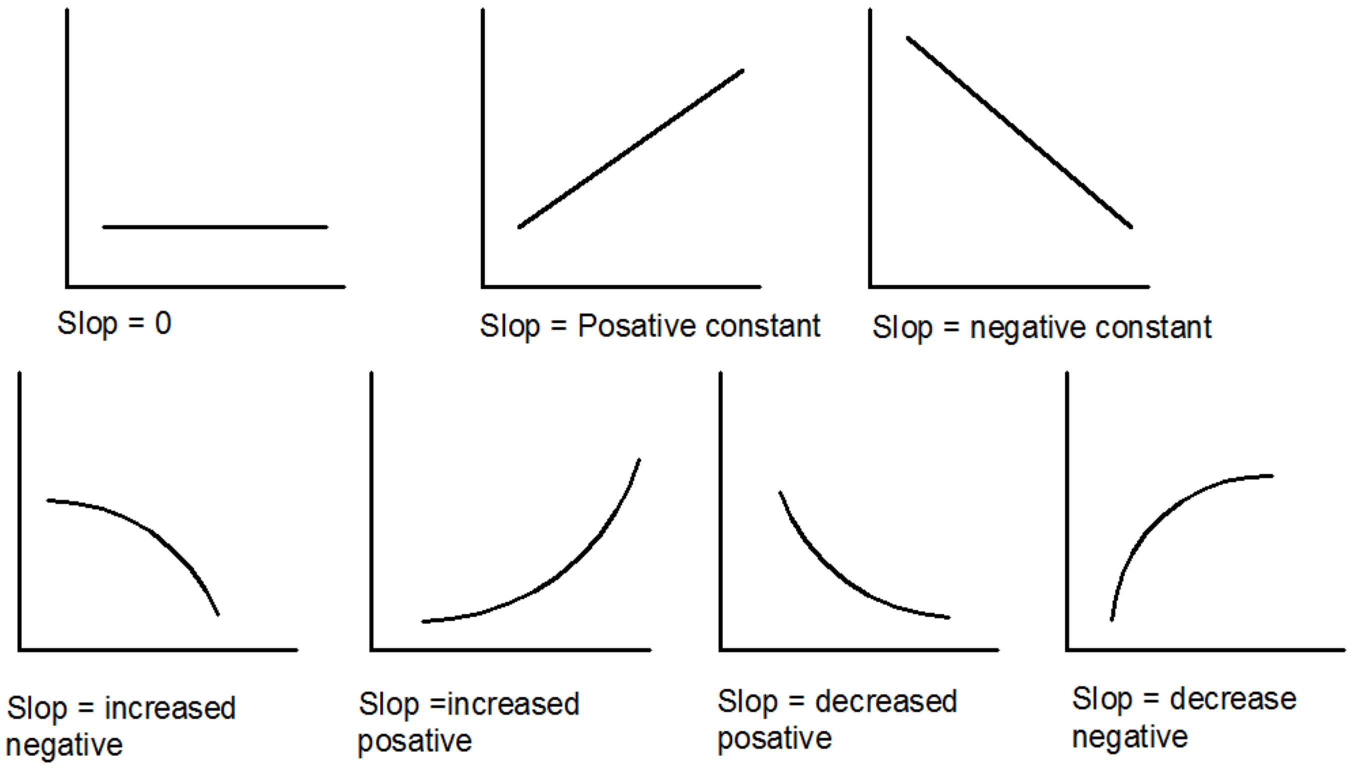


$M = \int V\,dx$ (The moment at any location is equal to the summation of the area under the shear force diagram from x=0 to the considered location)

or $\frac{dM}{dx} = V$ (Slop of the moment diagram at any location equal to the generated shear at that location).



d- Types of curves and lines slope.



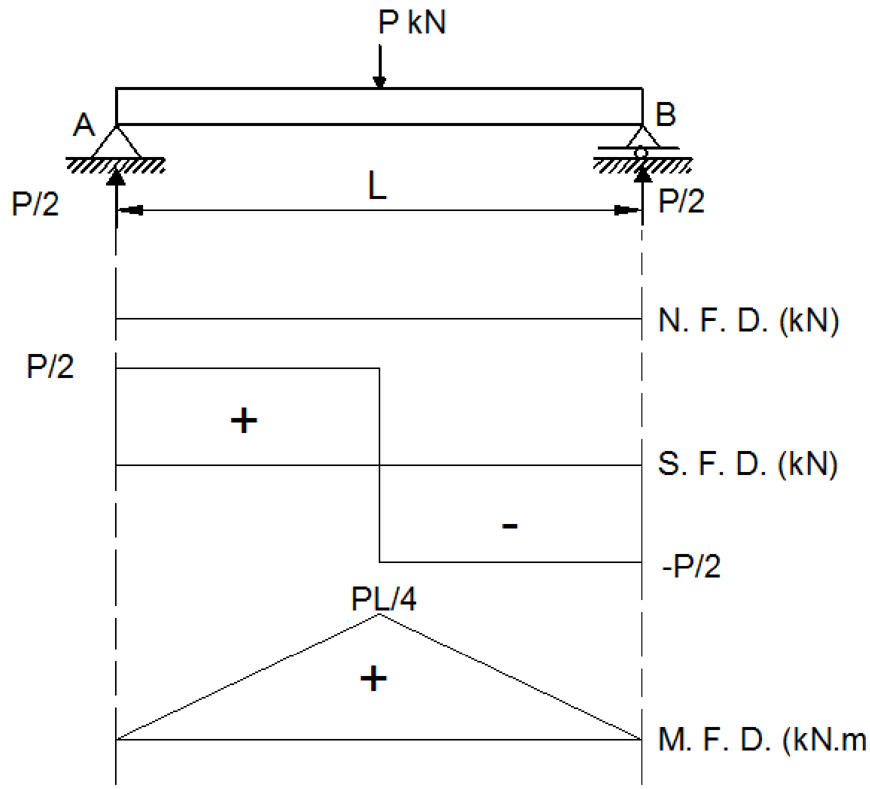
Notes:

- **Axial, shear and bending moment diagrams are usually started from the left side.**
- **Any concentrated load (force or moment) causes a jump in the corresponding diagram.**
- **The value of the interior force (axial, shear or moment) at any location is equal to the cumulative area under the corresponding force diagram from (x=0) to the considered location.**

Example 1: Draw axial, shear and bending moment diagram.

Solution:

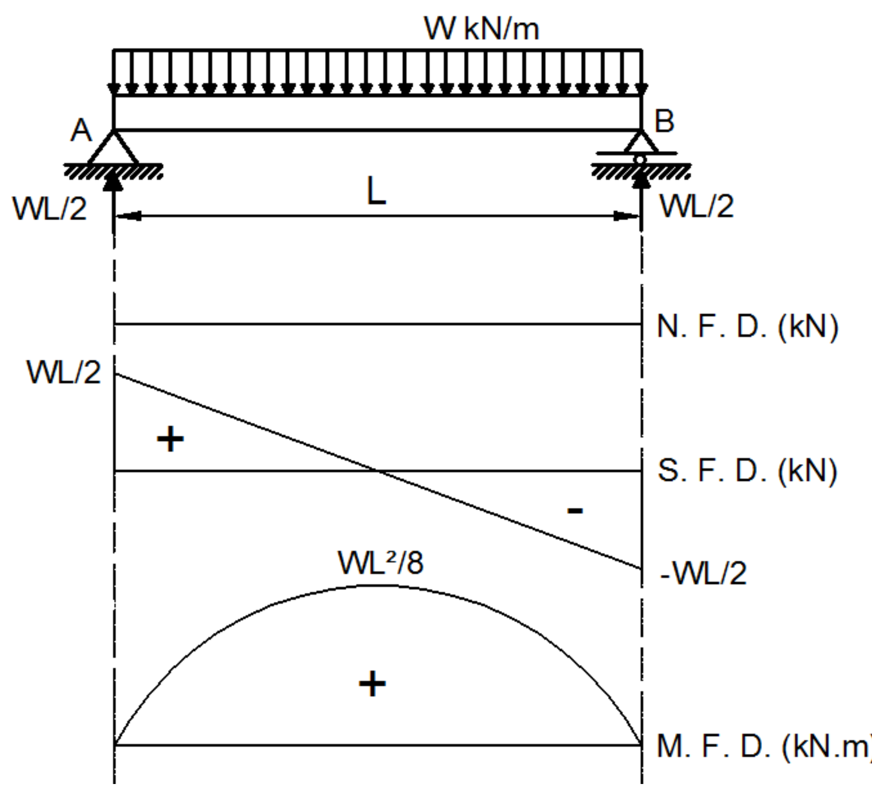
From symmetry the vertical reaction at (A & B) is equal ($A_y = B_y = P/2$)



Example 2: Draw axial, shear and bending moment diagram.

Solution:

From symmetry the vertical reaction at (A & B) is equal ($A_y = B_y = WL/2$)

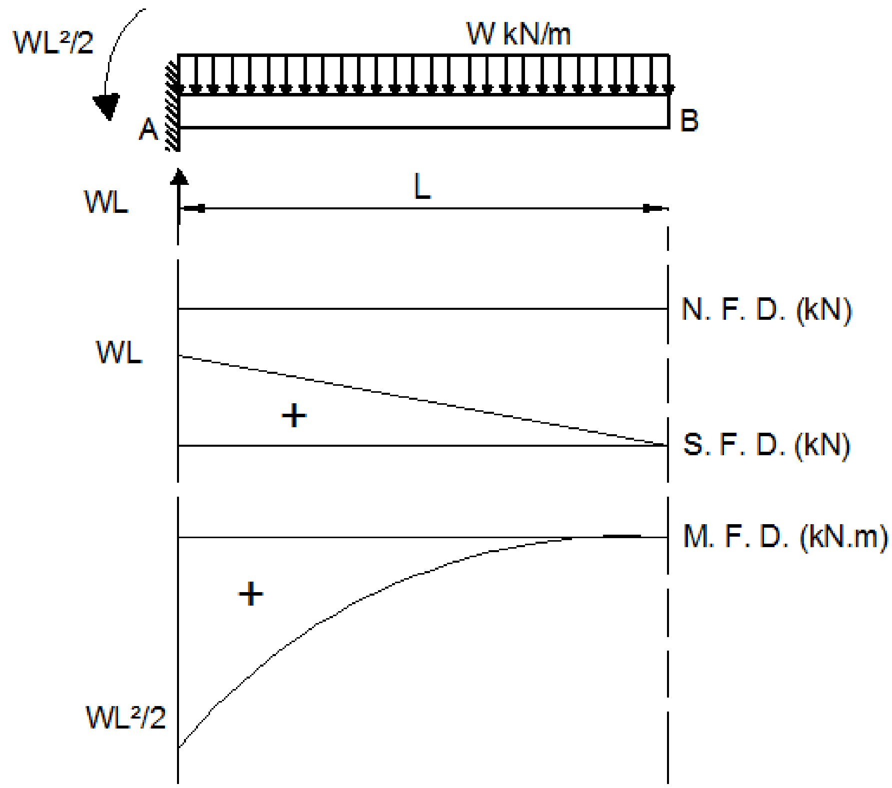


Example 3: Draw axial, shear and bending moment diagram.

Solution:

$$\sum F_y = 0 \rightarrow A_y = WL$$

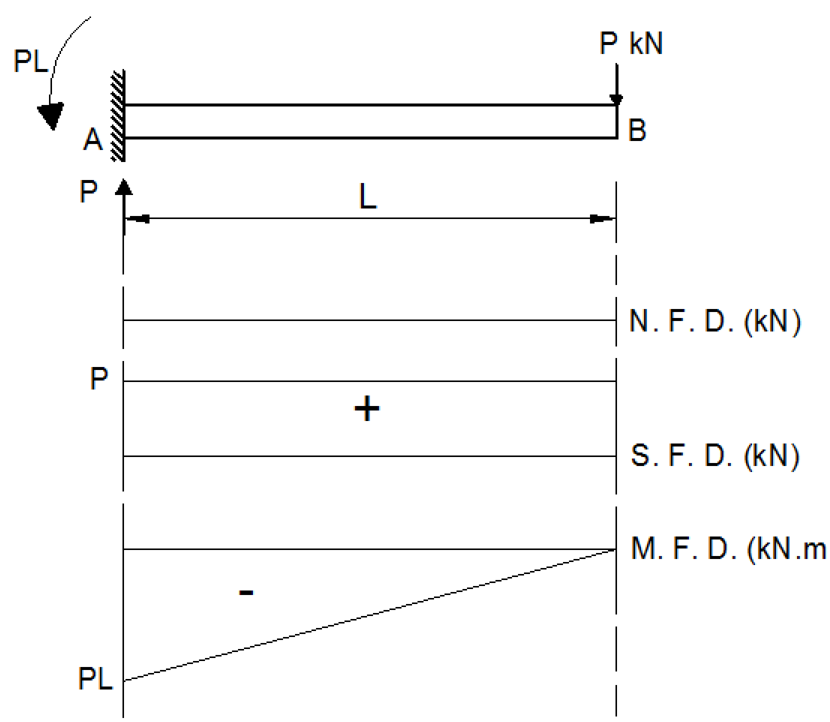
$$\sum M_{@A} = 0 \rightarrow M_A = \frac{WL^2}{2}$$



Example 4: Draw axial, shear and bending moment diagram.
Solution:

$$\sum F_y = 0 \rightarrow A_y = P$$

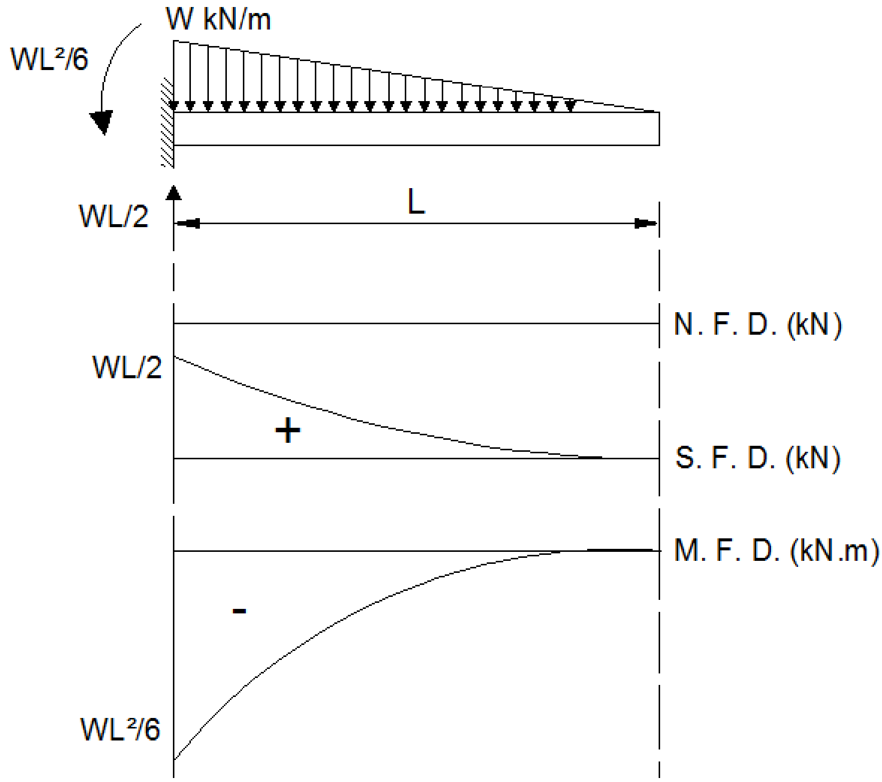
$$\sum M_{@A} = 0 \rightarrow M_A = PL$$



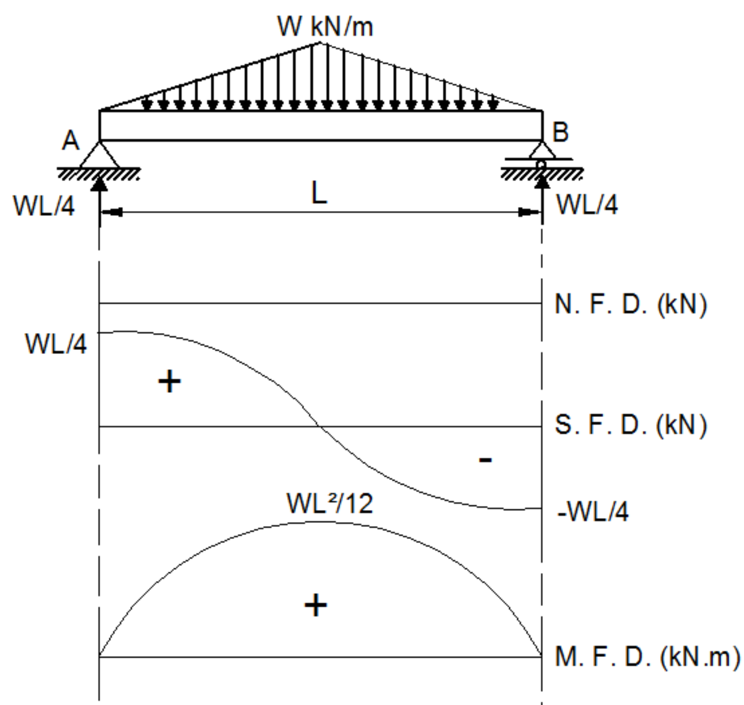
Example 5: Draw axial, shear and bending moment diagram.
Solution:

$$\sum F_y = 0 \rightarrow A_y = \frac{WL}{2}$$

$$\sum M_{@A} = 0 \rightarrow M_A = \frac{WL^2}{6}$$



Example 6: Draw axial, shear and bending moment diagram.
Solution:
From symmetry the vertical reaction at (A & B) is equal ($A_y = B_y = WL/4$)

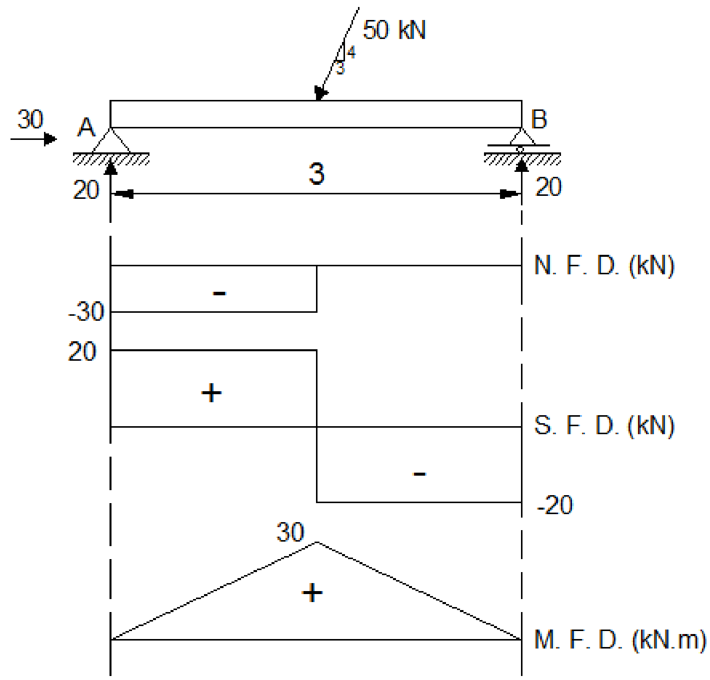


Example 7: Draw axial, shear and bending moment diagram.
Solution:

$\sum F_x = 0 \rightarrow A_x = 50 * \frac{3}{5} = 30\ kN$

$\sum M_{@A} = 0 \rightarrow B_y = 20$ kN

$\sum F_y = 0 \rightarrow A_y = 20\ kN$

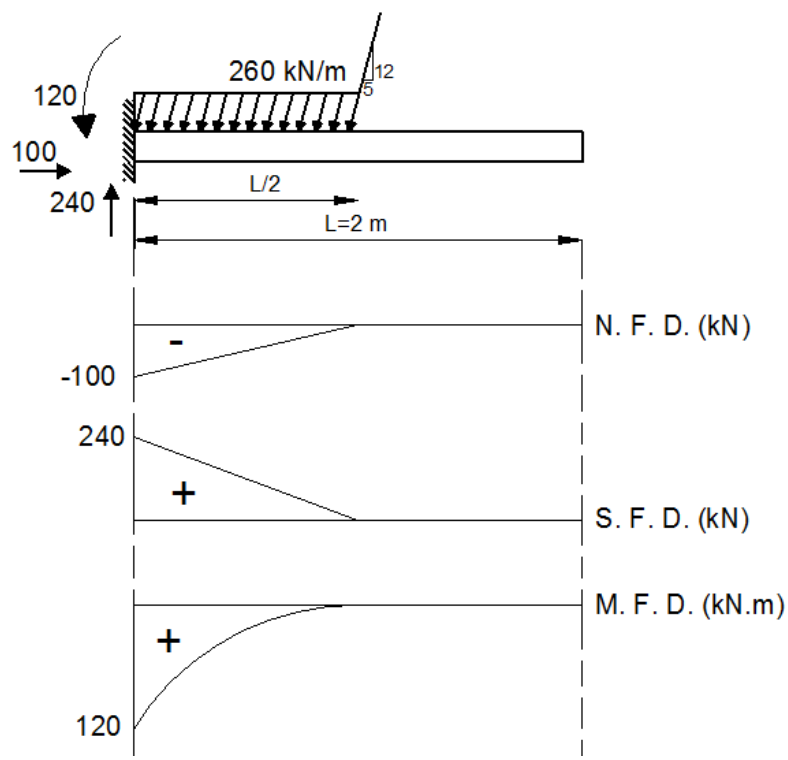


Example 8: Draw axial, shear and bending moment diagram.
Solution:

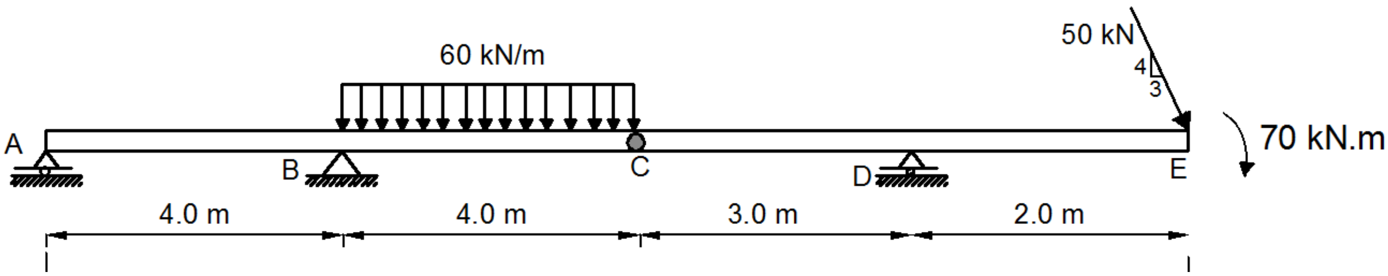
$\sum F_x = 0 \rightarrow A_x = 260 * \frac{5}{13} * 1 = 100\ kN$

$\sum F_y = 0 \rightarrow A_y = 260 * \frac{12}{13} * 1 = 240\ kN$

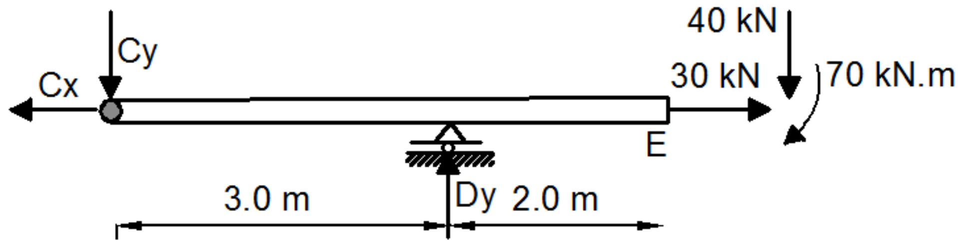
$\sum M_{@A} = 0 \rightarrow M_A = 240 *$
$0.5 = 120\ kN\ .m$



Example 9: Draw axial, shear and bending moment diagram.

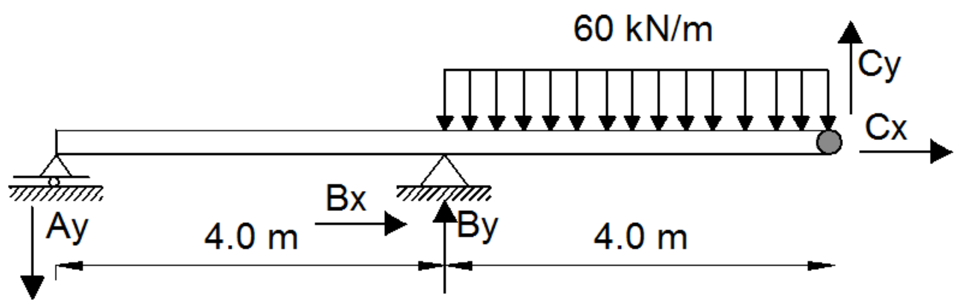


Solution:



$$\sum M_{@C} = 0 \rightarrow D_y = \frac{40 * 5 + 70}{3} = 90\ kN$$

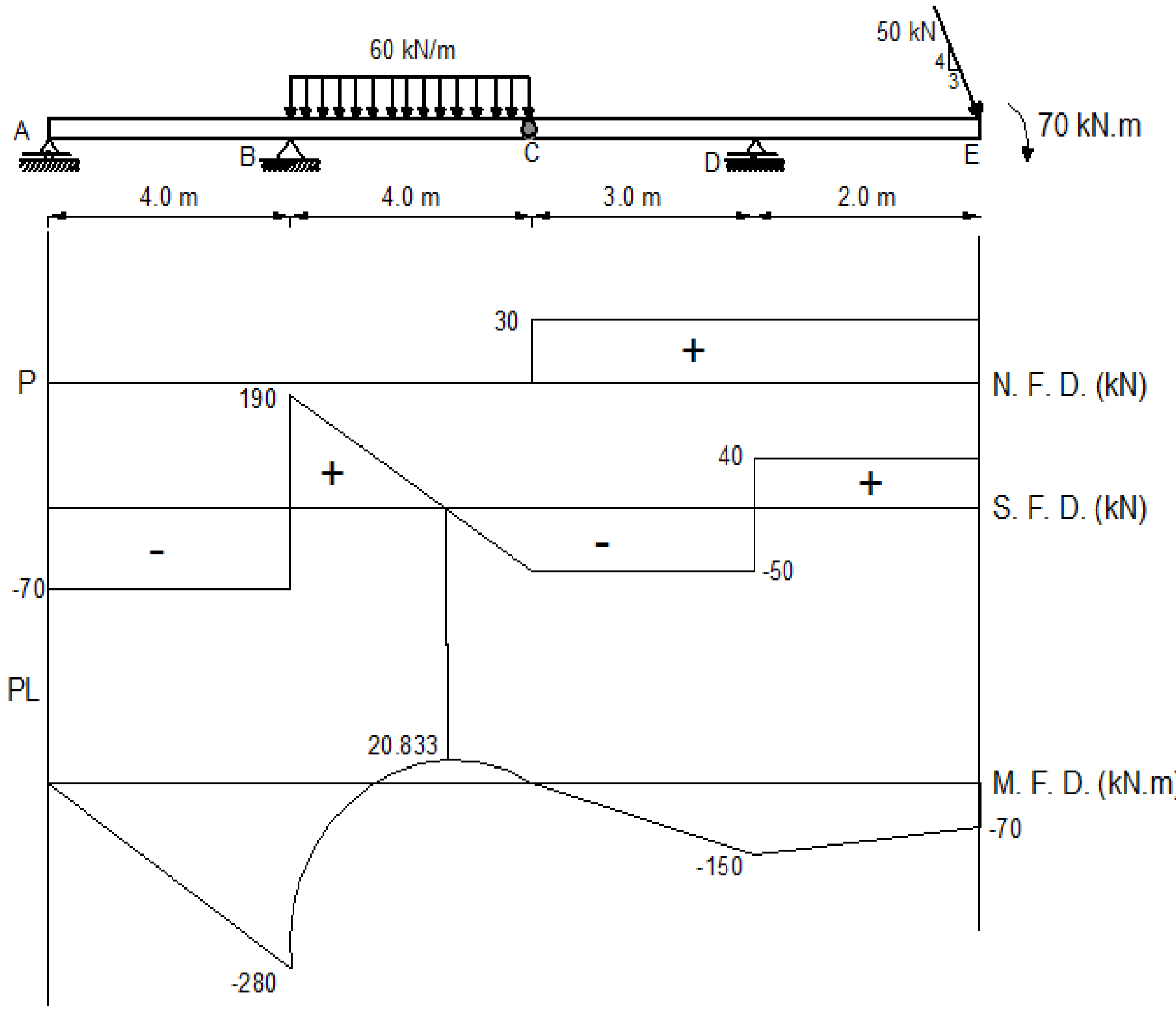
$$\sum F_y = 0 \rightarrow C_y = 90 - 40 = 50\ kN$$



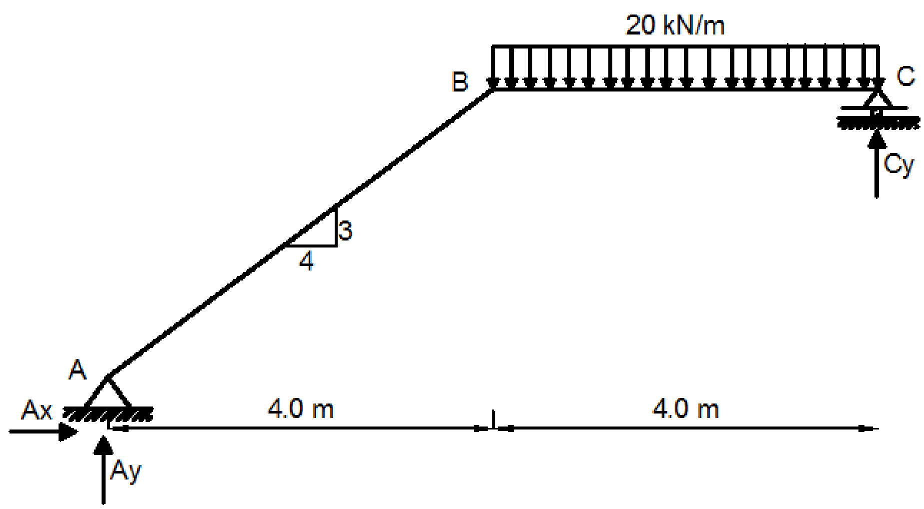
$$\sum M_{@A} = 0$$

$$B_y = \frac{-50 * 8 + 240 * 6}{4} = 260kN$$

$$\sum F_y = 0 \rightarrow A_y = 260 - 240 + 50 = 70\ kN$$


60 kN/m
50 kN
4
3
70 kN.m
A
B
C
D
E
4.0 m
4.0 m
3.0 m
2.0 m
30
+
P
N. F. D. (kN)
190
+
40
+
S. F. D. (kN)
-
-
-70
-50
PL
20.833
M. F. D. (kN.m)
-70
-150
-280

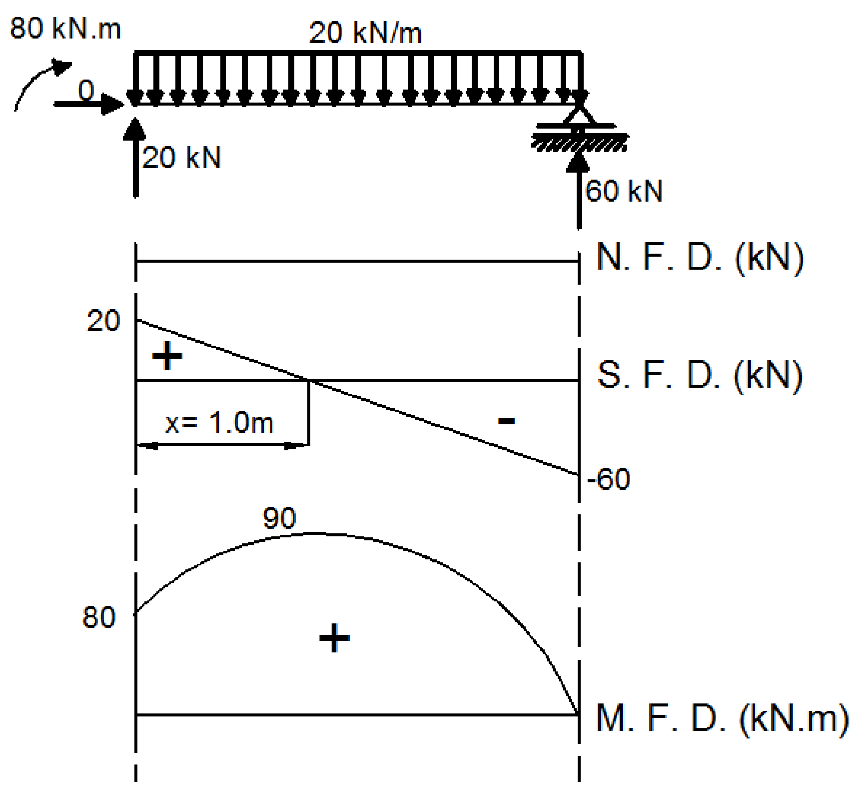
Example 10: Draw axial, shear and bending moment diagram

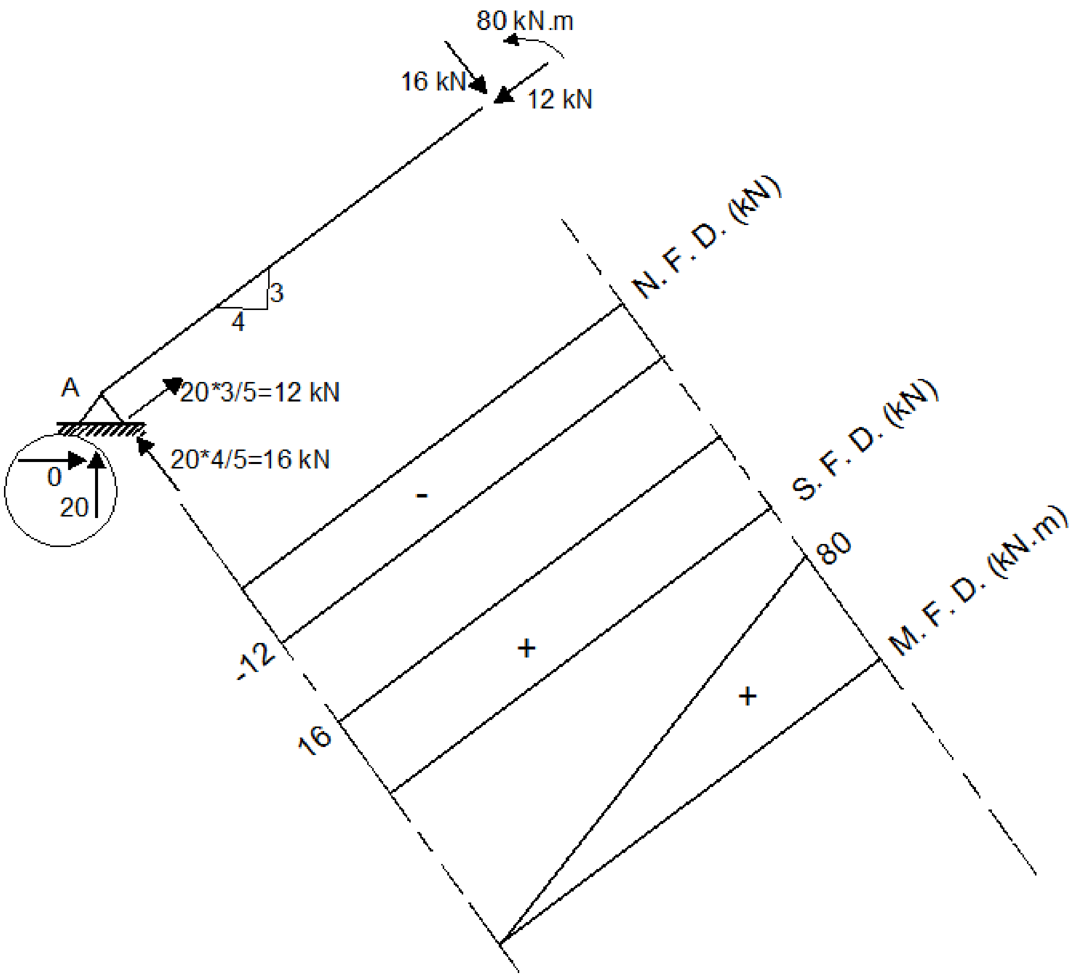


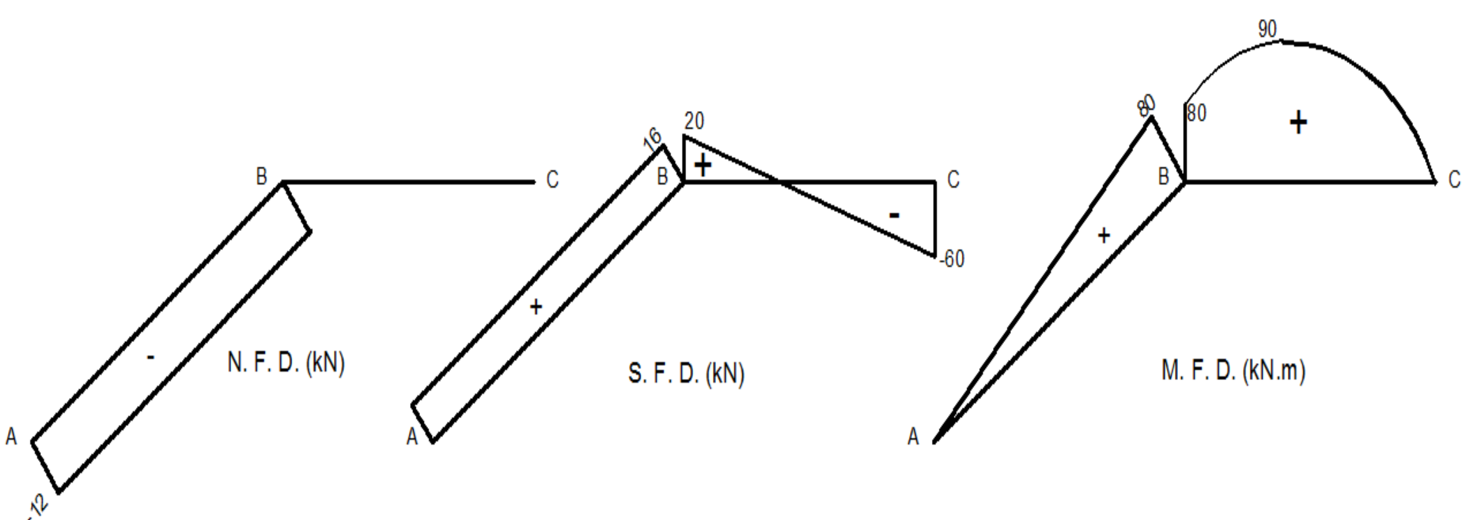
Solution:

$$\sum M_{@A} = 0 \ \rightarrow \ C_y = \frac{20 * 4 * 6}{8} = 60 \ kN$$

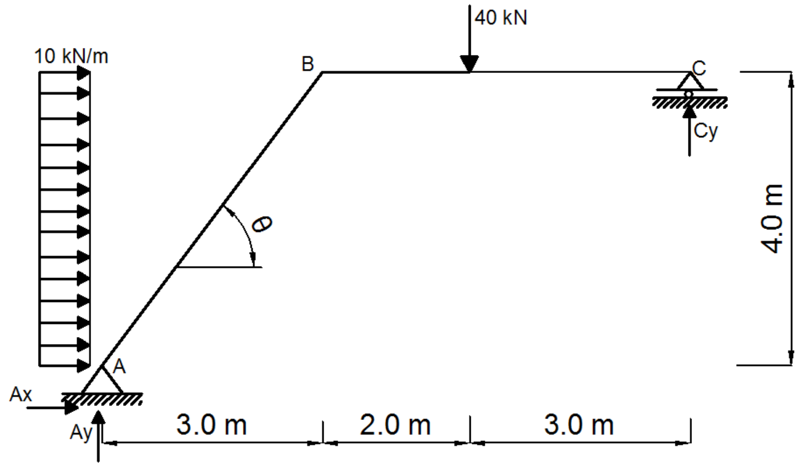
$$\sum F_y = 0 \ \rightarrow \ A_y = -60 + 20 * 4 = 20 \ kN$$




80 kN.m
16 kN
12 kN
3
4
A
20*3/5=12 kN
20*4/5=16 kN
0
20
N. F. D. (kN)
-
-12
S. F. D. (kN)
+
16
80
M. F. D. (kN.m)
+


B
C
A
-
-12
N. F. D. (kN)
16
20
B
+
C
-
-60
+
A
S. F. D. (kN)
90
80
80
B
+
C
+
A
M. F. D. (kN.m)

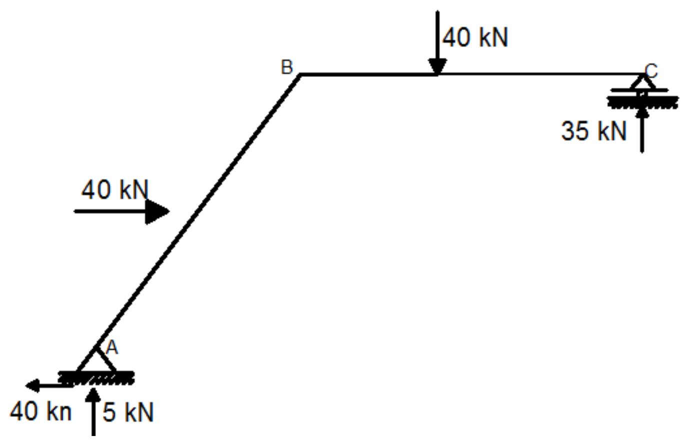
Example 11: Draw axial, shear and bending moment diagram

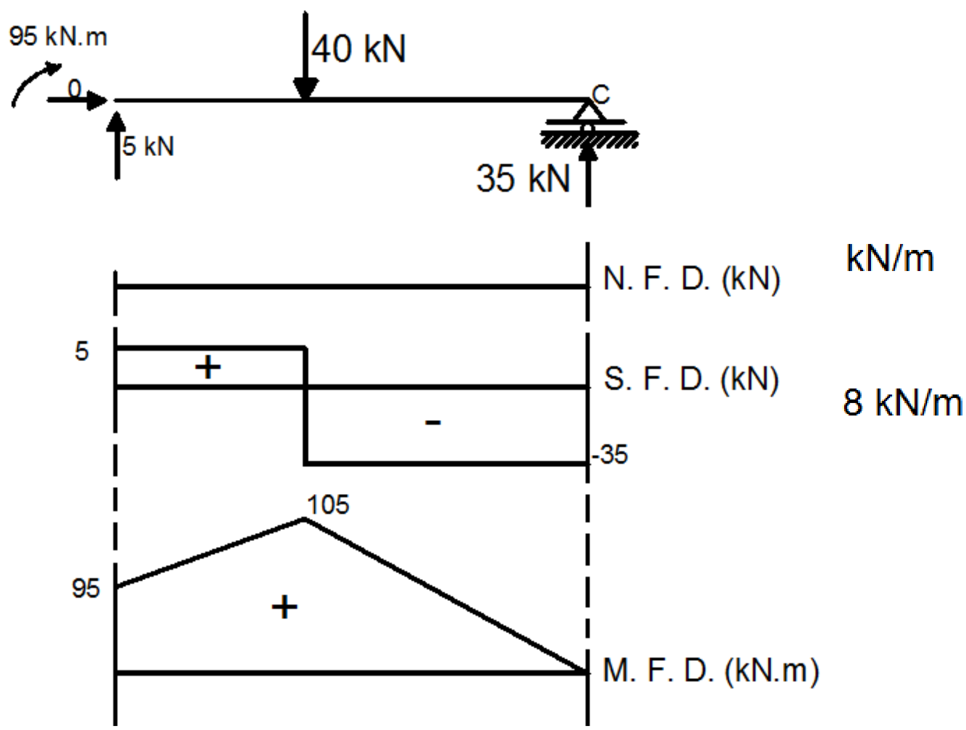


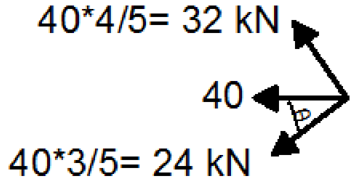
Solution:

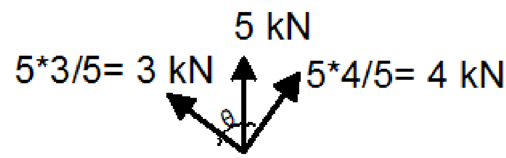
$$\sum M_{@A} = 0 \rightarrow C_y = \frac{40 * 5 + 40 * 2}{8} = 35\ kN$$

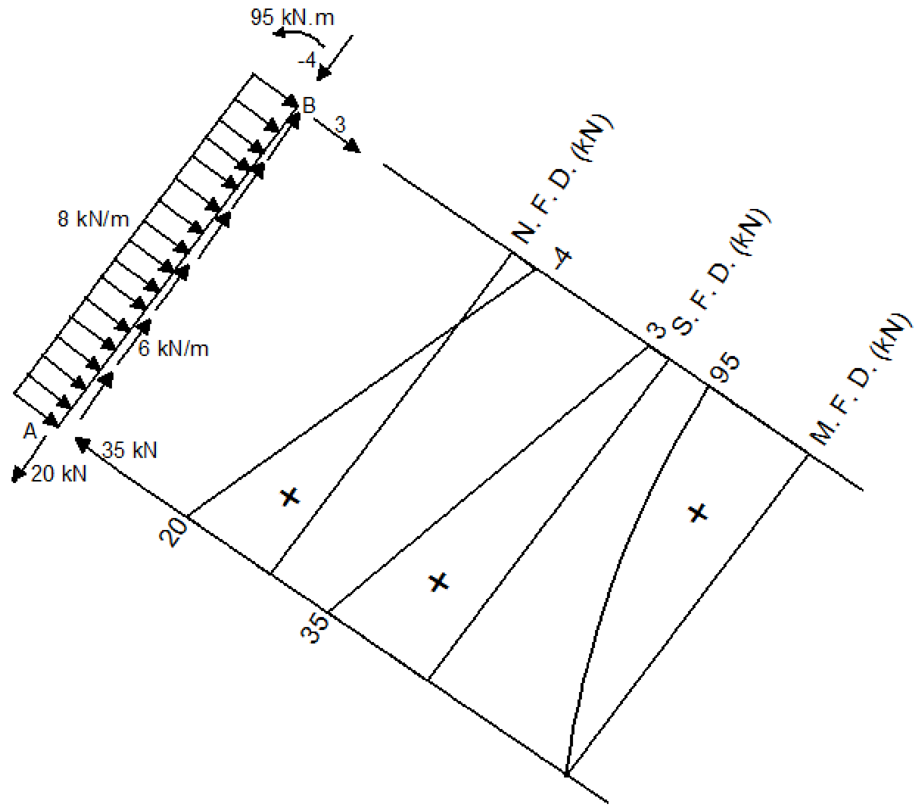
$$\sum F_y = 0 \rightarrow A_y = 40 - 35 = 20\ kN$$
$$\sum F_x = 0 \rightarrow A_x = 40\ kN$$

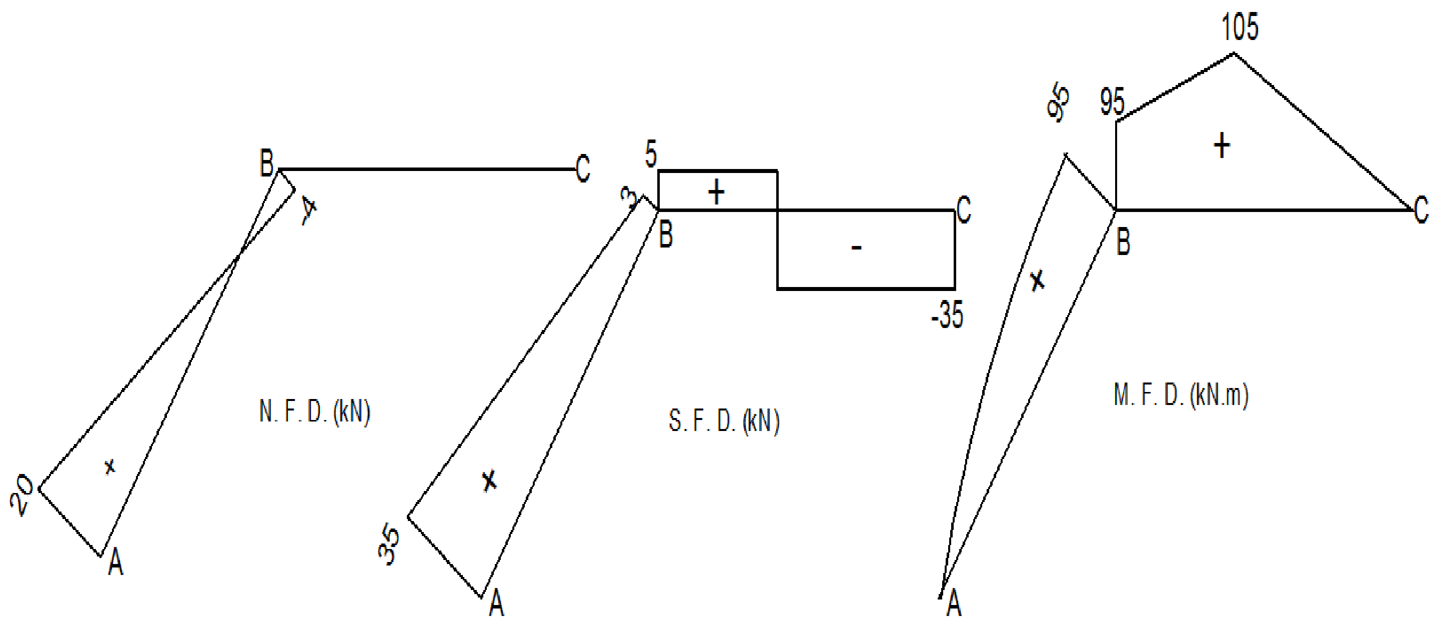




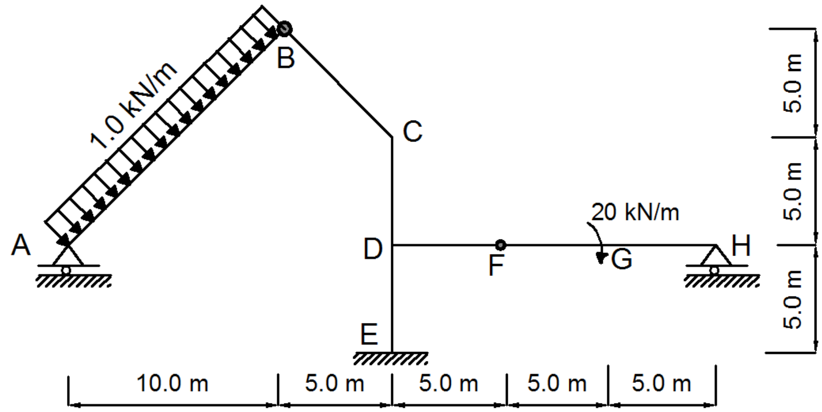

40*4/5= 32 kN
40
40*3/5= 24 kN


5 kN
5*3/5= 3 kN
5*4/5= 4 kN


95 kN.m
-4
B
3
8 kN/m
6 kN/m
A
35 kN
20 kN
N. F. D. (kN)
-4
20
3
S. F. D. (kN)
35
95
M. F. D. (kN)
+
+
+

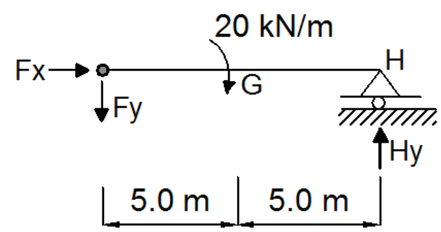

B
C
-4
20
A
N. F. D. (kN)
5
3
+
B
C
-
-35
35
A
S. F. D. (kN)
105
95
95
+
B
C
A
M. F. D. (kN.m)

Example 12: Draw axial, shear and bending moment diagram



Solution:

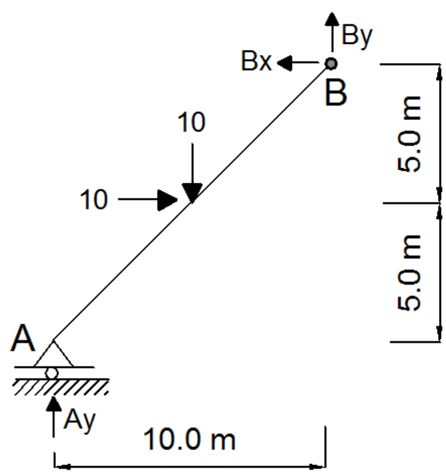
FGH as F.B.D,



$$\sum M_{@F} = 0 \rightarrow H_y = \frac{20}{10} = 2\ kN$$

$$\sum F_y = 0 \rightarrow F_y = 2\ kN$$
$$\sum F_x = 0 \rightarrow F_x = 0$$

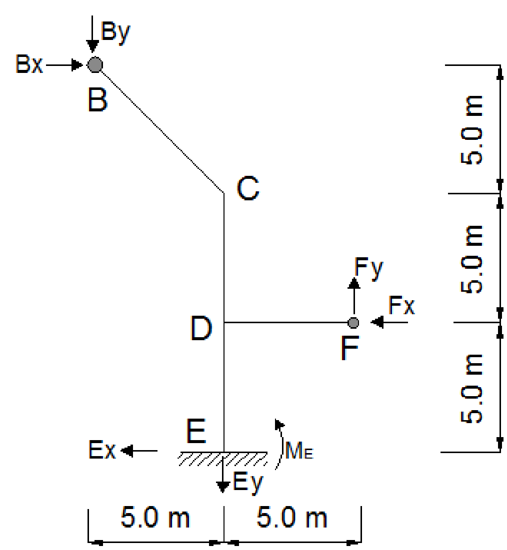
AB as F.B.D,



$$\sum M_{@B} = 0 \rightarrow A_y = \frac{10 * 5 + 10 * 5}{10} = 10\ kN$$

$$\sum F_y = 0 \rightarrow B_y = 0\ kN$$
$$\sum F_x = 0 \rightarrow B_x = 10\ kN$$

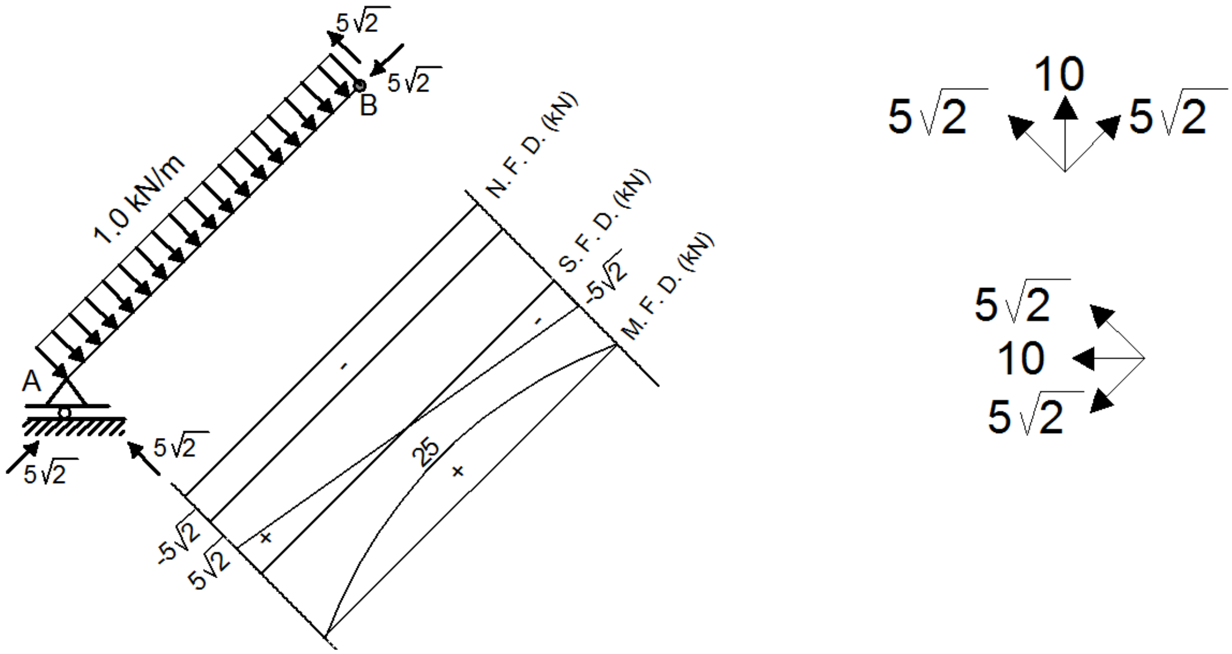
BCDEF as F.B.D,



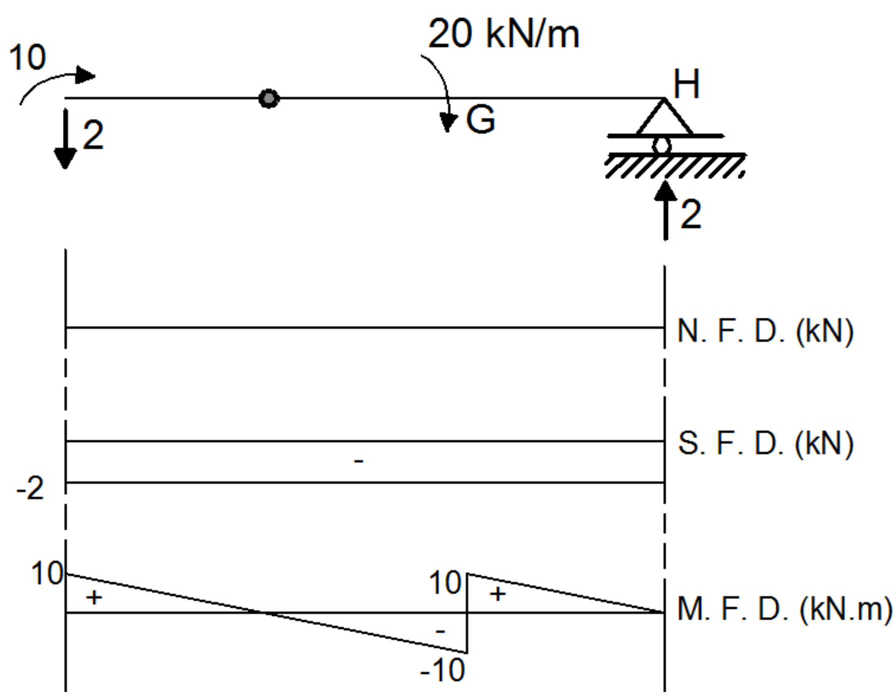
$$\sum M_{@E} = 0 \rightarrow M_E = 10 * 15 - 2 * 5 = 140\ kN$$

$$\sum F_y = 0 \rightarrow E_y = 2\ kN$$
$$\sum F_x = 0 \rightarrow E_x = 10\ kN$$

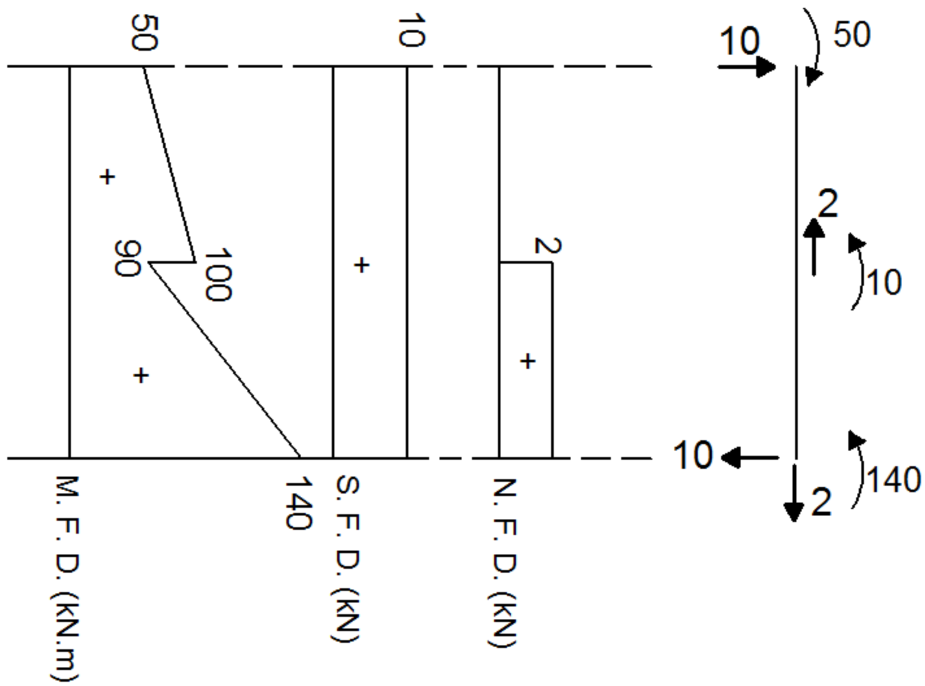
AB axial, shear and bending moment diagram



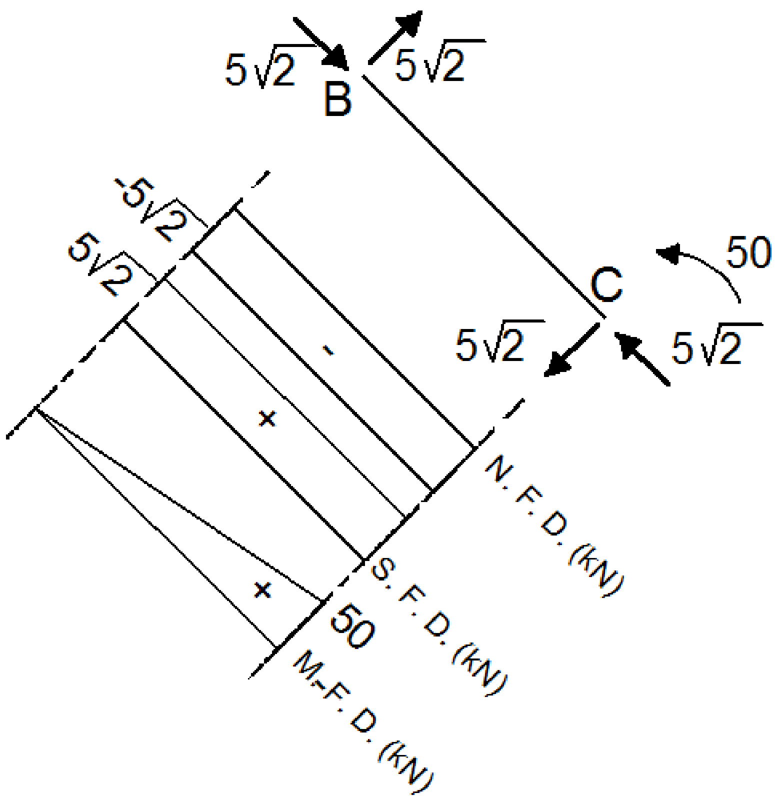
FGH axial, shear and bending moment diagram



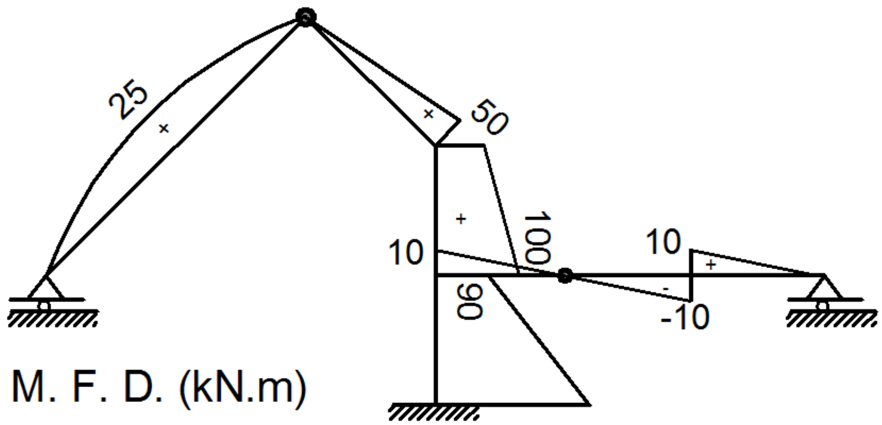
CDE axial, shear and bending moment diagram



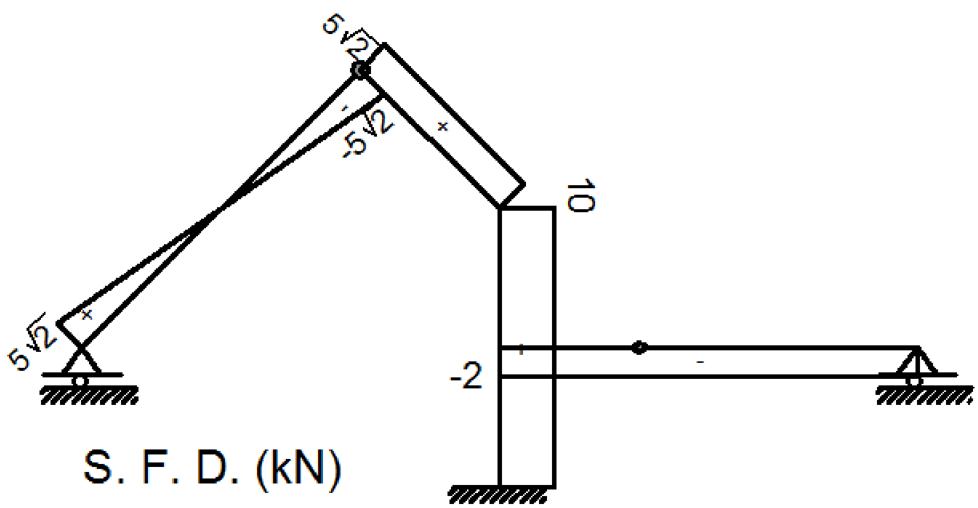
BC axial, shear and bending moment diagram



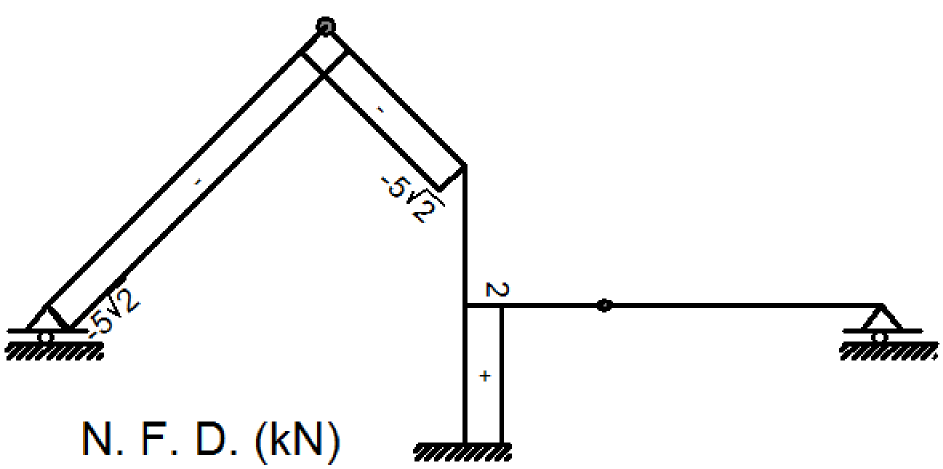
Whole frame axial, shear and bending moment diagram



M. F. D. (kN.m)

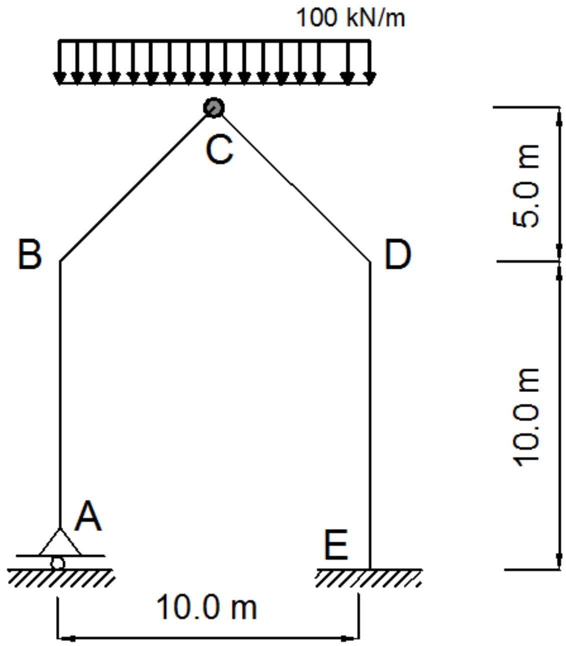


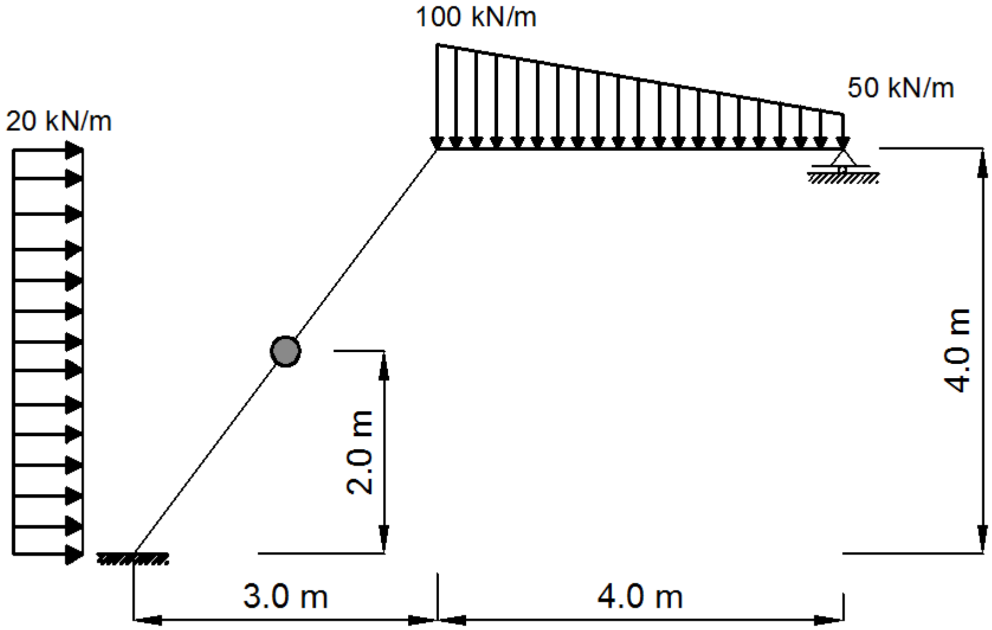
S. F. D. (kN)



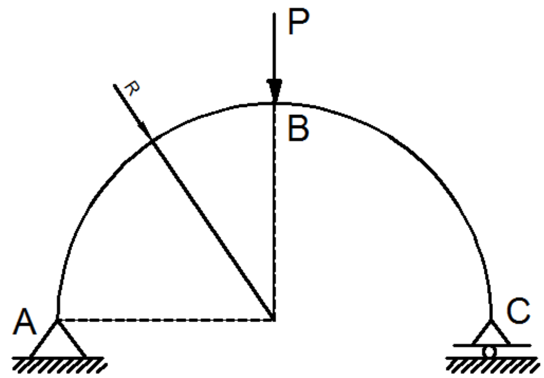
N. F. D. (kN)

H.W.: Draw axial, shear and bending moment diagram





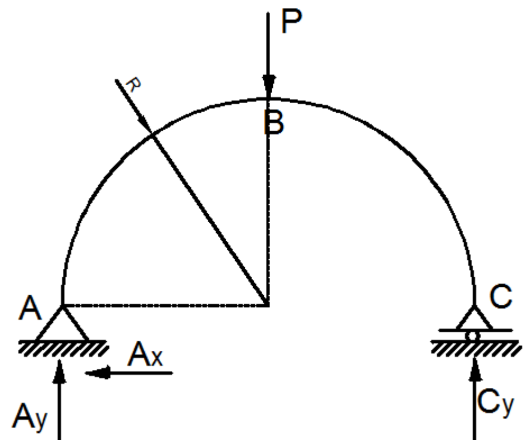
Example 13: Draw axial, shear and bending moment diagram



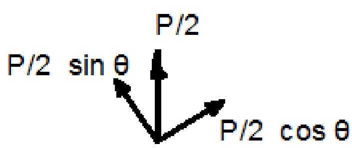
Solution:

From symmetry, the vertical reactions at (A & C) are equal ($A_y = C_y = P/2$)

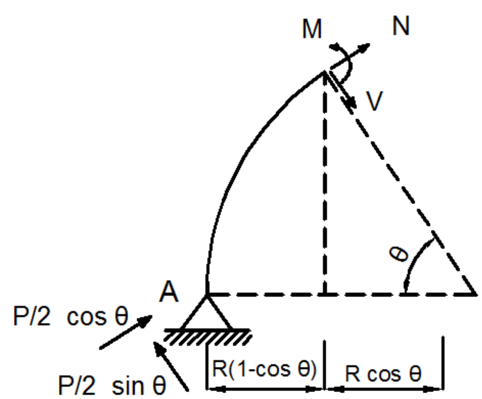
$\sum F_x = 0 \;\rightarrow\; A_x = 0$



Only one section is required (at angle (θ) from point A or C) due to the symmetrical status.



Taking a section at angle (θ) from point A. Three interior forces will be appear, these are (N, V and M) that's will be calculated as a function of (R and θ) by applying the three equilibrium equations.

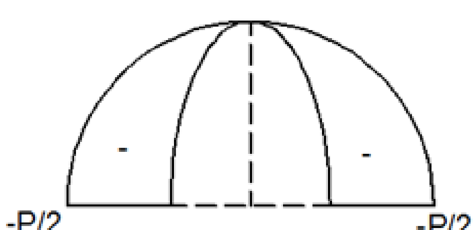


$$N = -\frac{P}{2}\cos\theta$$

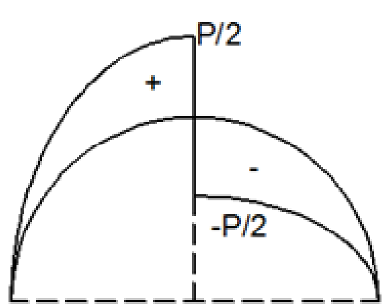
$$V = \frac{p}{2}\sin\theta$$

$$M = \frac{RP}{2}(1-\cos\theta)$$

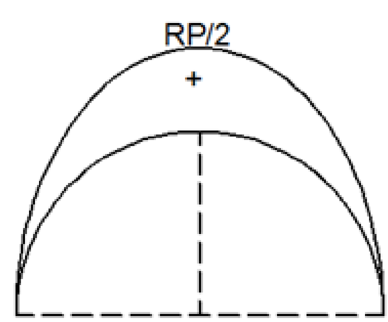
θ	N	V	M
0	$-\frac{P}{2}$	0	0
90	0	$\frac{P}{2}$	$\frac{RP}{2}$



N. F. D. (kN)

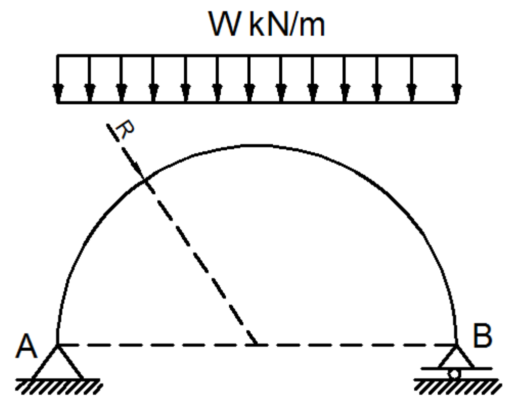


S. F. D. (kN)



M. F. D. (kN.m)

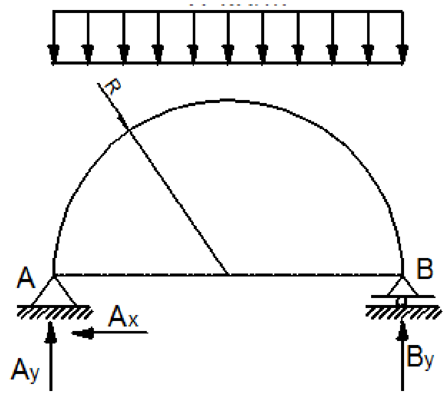
Example 14: Draw axial, shear and bending moment diagram



Solution:

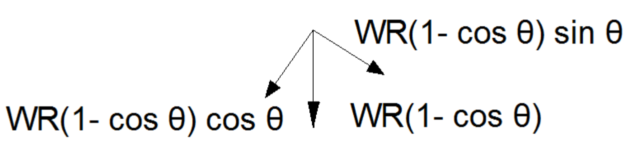
From symmetry, the vertical reactions at (A & B) are equal ($A_y = B_y = WR$)

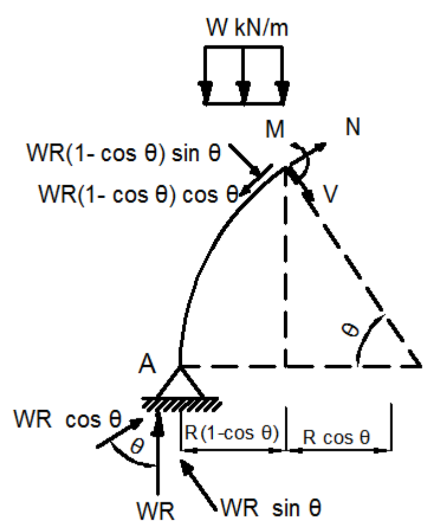
$\sum F_x = 0 \rightarrow A_x = 0$



Only one section is required (at angle (θ) from point A or B) due to the symmetrical status.

Taking a section at an angle (θ) from point A. Three interior forces will be appear, these are (N, V and M) that's will be calculated as a function of (R and θ) by applying the three equilibrium equations.





$N = WR(1-\cos\theta)\cos\theta - WR\cos\theta = -WR\cos^2\theta$

$V = WR\sin\theta - WR(1-\cos\theta)\sin\theta =$

$\quad WR\sin\theta\cos\theta = \frac{WR}{2}\sin 2\theta$

$M = WR^2(1-\cos\theta) - WR^2\frac{(1-\cos\theta)^2}{2}$

$M = WR^2\left(1-\cos\theta - \frac{1-2\cos\theta+\cos^2\theta}{2}\right)$

$M = WR^2\left(1-\frac{1}{2}-\frac{\cos^2\theta}{2}\right) = \frac{WR^2}{2}(1-\cos^2\theta)$

$M = \frac{WR^2}{2}\sin^2\theta$

The drawing will be set at three values of the angle (θ), these are (zero, 90° and angle of critical section)

a- Angle of axial force critical section: the section will be found by setting the slope of the axial force to zero ($\frac{dN}{d\theta} = 0$).

$$\frac{dN}{d\theta} = -2WR\cos\theta\,(-\sin\theta) = WR\ \sin 2\theta$$

$$\frac{dN}{d\theta} = 0 \ \rightarrow\ WR\ \sin 2\theta = 0 \ \rightarrow\ \sin 2\theta = 0 \ \rightarrow\ 2\theta = 0^o or\ 180^o \ \rightarrow\ \theta = 0^o\ or\ 90^o$$

b- Angle of shear force critical section: the section will be found by setting the slope of the shear force to zero ($\frac{dV}{d\theta} = 0$).

$$\frac{dV}{d\theta} = WR\ \cos 2\theta$$

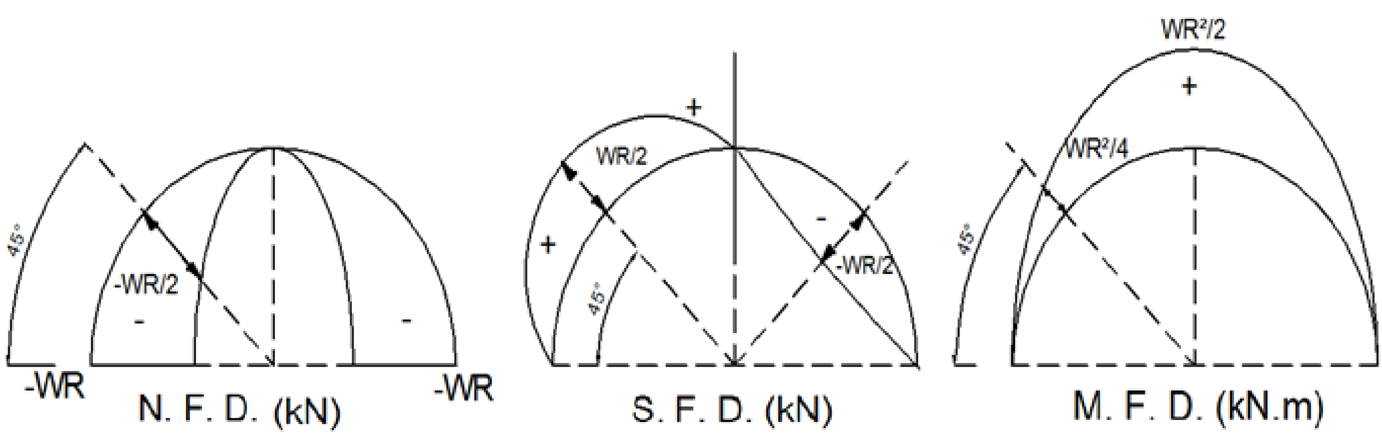
$$\frac{dV}{d\theta} = 0 \ \rightarrow WR\ \cos 2\theta = 0 \ \rightarrow\ \cos 2\theta = 0 \ \rightarrow\ 2\theta = 90^o \ \rightarrow\ \theta = 45^o$$

c- Angle of moment critical section: the section will be found by setting the slope of the moment to zero ($\frac{dM}{d\theta} = 0$).

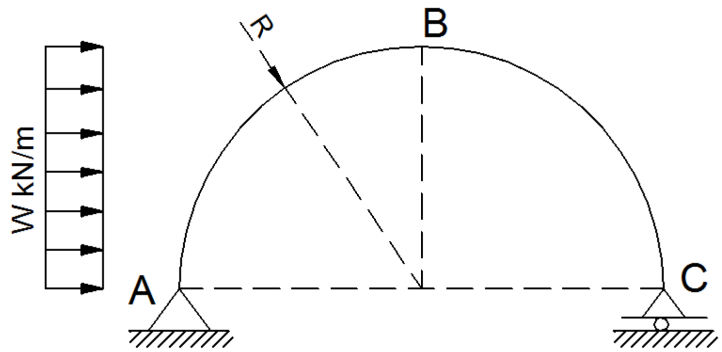
$$\frac{dM}{d\theta} = WR^2 \sin\theta\cos\theta = \frac{WR^2}{2}\ sin2\,\theta$$

$$\frac{dM}{d\theta} = 0 \ \rightarrow \frac{WR^2}{2}\ sin2\,\theta = 0 \ \rightarrow\ sin2\,\theta = 0 \ \rightarrow\ 2\theta = 0^o or\ 180^o \ \rightarrow\ \theta = 0^o\ or\ 90^o$$

θ	N	V	M
0°	$-WR$	0	0
45°	$-\frac{WR}{2}$	$\frac{WR}{2}$	$\frac{WR^2}{4}$
90°	0	0	$\frac{WR^2}{2}$



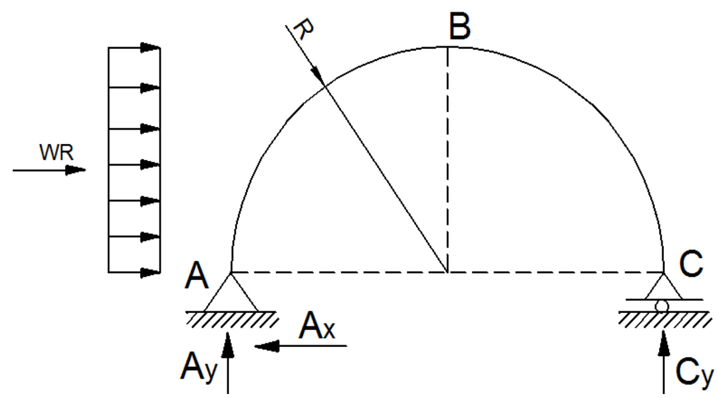
Example 15: Draw axial, shear and bending moment diagram



Solution:
For the whole structure

$$\sum M_{@A} = 0 \ \rightarrow \ C_y = \frac{WR * R/2}{2R} = \frac{WR}{4} kN$$

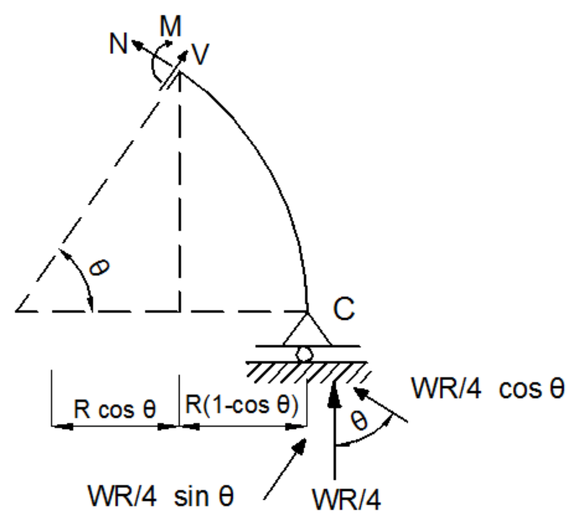
$$\sum F_y = 0 \ \rightarrow \ A_y = -\frac{WR}{4} \ kN$$
$$\sum F_x = 0 \ \rightarrow \ A_x = WR \ kN$$



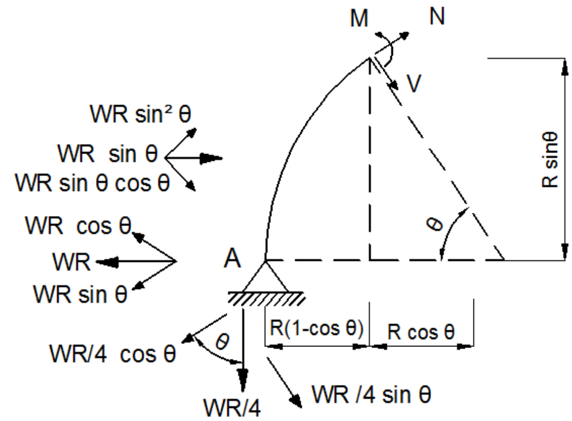
Since the structure is not symmetry, two sections will be considered. The first at an angle (θ) from point A and the second at an angle (θ) from point C. Three interior forces will be appear, these are (N, V and M) that's will be calculated as a function of (R and θ) by applying the three equilibrium equations.

1- First section (Portion BC):

$$N = -\frac{WR}{4} \cos\theta$$
$$V = -\frac{WR}{4} \sin\theta$$
$$M = \frac{WR^2}{4} \ (1 - \cos\theta)$$



2- Second section (Portion AB):



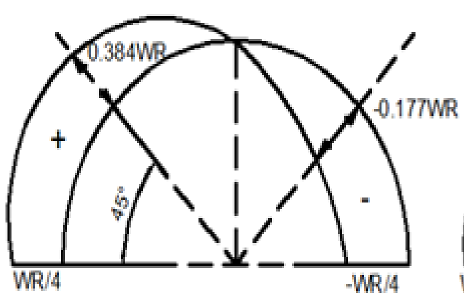
$$N = \frac{WR}{4}\cos\theta + WR\sin\theta - WR\sin^2\theta = WR\left(\frac{\cos\theta}{4} + \sin\theta - \sin^2\theta\right)$$

$$V = WR\cos\theta - WR\sin\theta\ \cos\theta - \frac{WR}{4}\sin\theta = WR\left(\cos\theta - \frac{\sin 2\theta}{2} - \frac{\sin\theta}{4}\right)$$

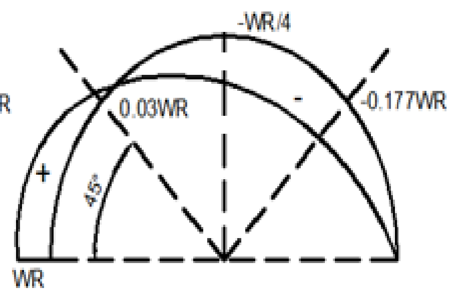
$$M = WR^2\sin\theta - \frac{WR^2}{4}(1-\cos\theta) - \frac{WR^2}{2}\sin^2\theta = WR^2\left(\sin\theta - \frac{1-\cos\theta}{4} - \frac{\sin^2\theta}{2}\right)$$

Portion AB			
θ	N	V	M
0°	$\frac{WR}{4}$	WR	0
45°	$0.384WR$	$0.03Wr$	$0.384\,WR^2$
90°	0	$-\frac{WR}{4}$	$\frac{WR^2}{4}$

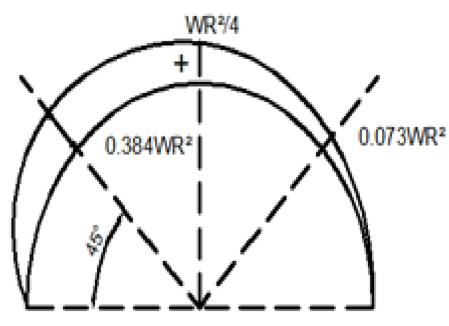
Portion BC			
θ	N	V	M
0°	$-\frac{WR}{4}$	0	0
45°	$-\frac{WR}{4\sqrt{2}}$	$-\frac{WR}{4\sqrt{2}}$	$\frac{0.414WR^2}{4\sqrt{2}}$
90°	0	$-\frac{WR}{4}$	$\frac{WR^2}{4}$



N. F. D. (kN)

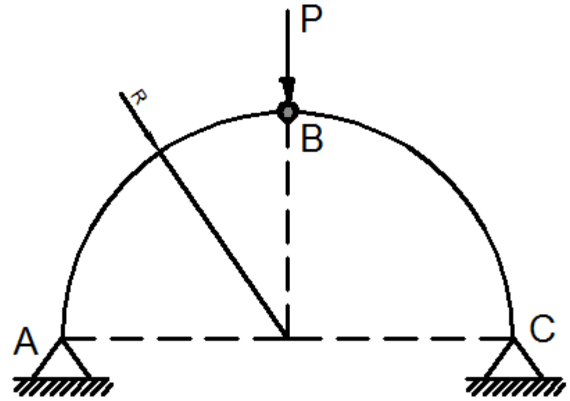


S. F. D. (kN)



M. F. D. (kN.m)

Example 16: Draw axial, shear and bending moment diagram



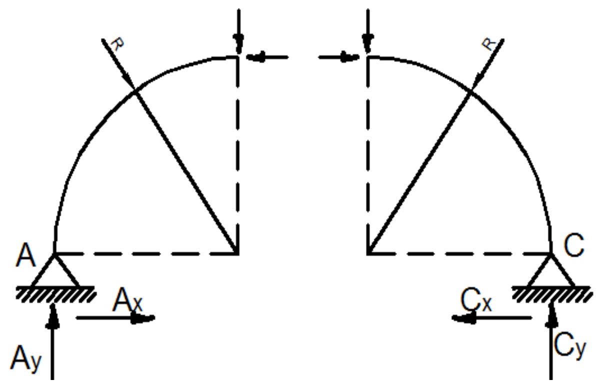
Solution:
From symmetry, the vertical reactions at (A & C) are equal ($A_y = C_y = P/2$)

The horizontal reactions at point (A and C) will be found by applying the equilibrium equation ($\sum M = 0$) at part (AB or BC)

$$\sum M_{@B} = 0 \;\rightarrow\; A_x * R = A_y * R \rightarrow A_x = \frac{P}{2} kN$$

For the whole structure

$$\sum F_x = 0 \;\rightarrow\; C_x = \frac{P}{2} kN$$

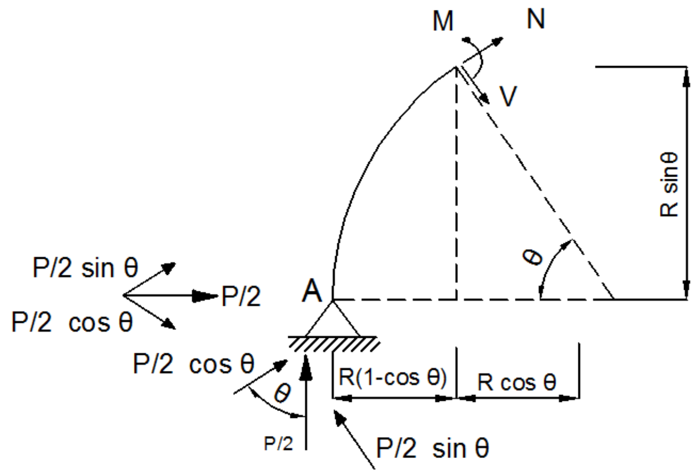


Taking a section at an angle (θ) from point A. Three interior forces will be appear, these are (N, V and M) that's will be calculated as a function of (R and θ) by applying the three equilibrium equations.

$$N = -\frac{P}{2}(\cos\theta + \sin\theta)$$

$$V = \frac{P}{2}(\sin\theta - \cos\theta)$$

$$M = \frac{PR}{2}(1 - \cos\theta - \sin\theta)$$



The drawing will be set at three values of the angle (θ), these are (zero, 90° and angle of critical section)

a- Angle of axial force critical section: the section will be found by setting the slope of the axial force to zero ($\frac{dN}{d\theta} = 0$).

$$\frac{dN}{d\theta} = -\frac{P}{2}(\cos\theta - \sin\theta)$$

$$\frac{dN}{d\theta} = 0 \;\rightarrow\; \cos\theta - \sin\theta = 0 \;\rightarrow\; \tan\theta = 1 \;\rightarrow\; \theta = 45^o$$

b- Angle of shear force critical section: the section will be found by setting the slope of the shear force to zero ($\frac{dV}{d\theta} = 0$).

$$\frac{dV}{d\theta} = \ (\cos\theta + \sin\theta \ )$$

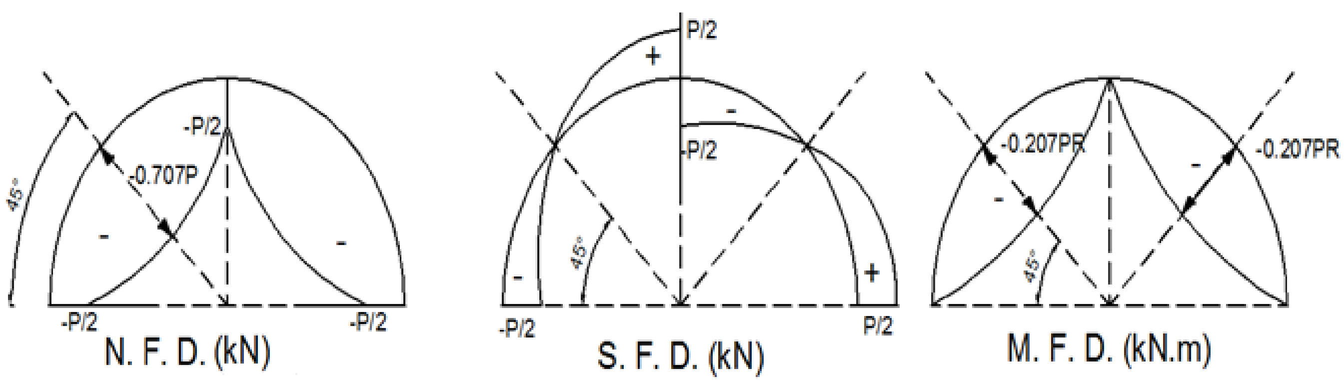
$$\frac{dN}{d\theta} = 0 \ \rightarrow \cos\theta + \sin\theta \ = 0 \ \rightarrow \ \tan\theta = -1 \ \rightarrow \ \theta = -45^o \ or \ 270^o \ (Trivial \ sol.)$$

c- Angle of moment critical section: the section will be found by setting the slope of the moment to zero ($\frac{dM}{d\theta} = 0$).

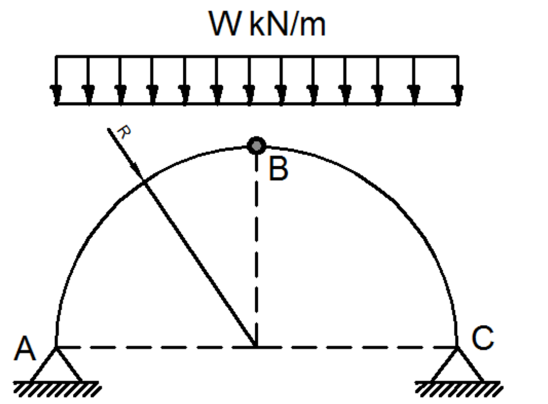
$$\frac{dM}{d\theta} = \frac{PR}{2} \ (\sin\theta - \cos\theta \ )$$

$$\frac{dM}{d\theta} = 0 \ \rightarrow (\sin\theta - \cos\theta \ ) = 0 \ \rightarrow \ \tan\theta = 1 \ \rightarrow \ \theta = 45^o = 0^o$$

θ	N	V	M
0°	$-\frac{P}{2}$	$-\frac{P}{2}$	0
45°	$-\frac{P}{\sqrt{2}}$	0	-0.207PR
90°	$-\frac{P}{2}$	$-\frac{P}{2}$	0



Example 17: Draw axial, shear and bending moment diagram



Solution:

From symmetry, the vertical reactions at (A & C) are equal ($A_y = C_y = WR$)

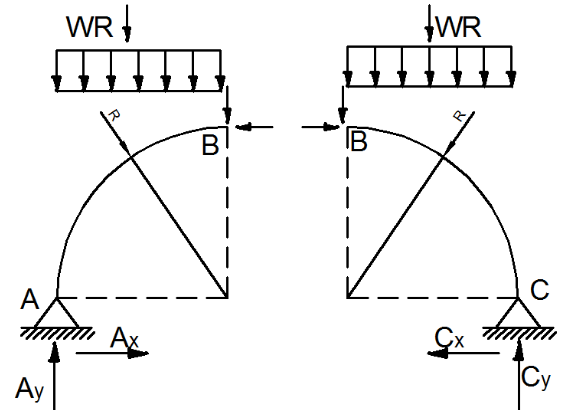
The horizontal reactions at point (A and C) will be found by applying the equilibrium equation ($\sum M = 0$) at part (AB or BC)

$$\sum M_{@B} = 0 \;\rightarrow\; A_x * R = A_y * R - WR * \frac{R}{2}$$

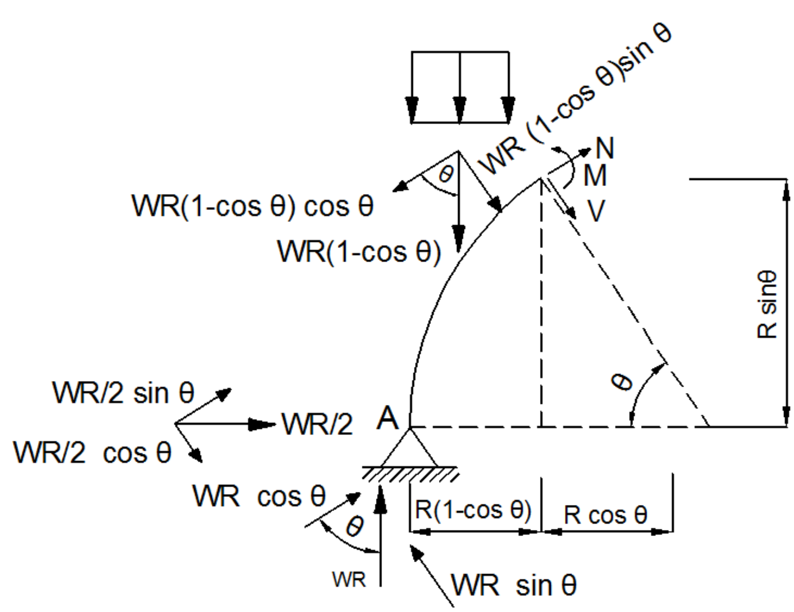
$$\rightarrow A_x = \frac{WR}{2} kN$$

For the whole structure

$$\sum F_x = 0 \;\rightarrow\; C_x = \frac{WR}{2}\, kN$$



Taking a section at an angle (θ) from point A. Three interior forces will be appear, these are (N, V and M) that's will be calculated as a function of (R and θ) by applying the three equilibrium equations.



$$N = -WR\,\cos\theta - \frac{WR}{2}\sin\theta + WR(1-\cos\theta)\,cos\theta$$

$$N = -WR\left(\frac{\sin\theta}{2} + cos^2\,\theta\right)$$

$$V = WR\left(\sin\theta - \frac{\cos\theta}{2} - \sin\theta + sin\,\theta\, cos\theta\right)$$

$$V = WR\left(sin\,\theta\, cos\theta - \frac{\cos\theta}{2}\right)$$

$$M = WR^2(1 - \cos\theta - \frac{\sin\theta}{2} - \frac{(1-\cos\theta)^2}{2}$$
$$M = WR^2(1 - \cos\theta - \frac{\sin\theta}{2} - \frac{1-2\cos\theta + \cos\theta^2}{2})$$
$$M = \frac{WR^2}{2}(1 - \sin\theta - \cos\theta^2) = \frac{WR^2}{2}(\sin\theta^2 - \sin\theta)$$

The drawing will be set at three values of the angle (θ), these are (zero, 90° and angle of critical section)

a- Angle of axial force critical section: the section will be found by setting the slope of the axial force to zero ($\frac{dN}{d\theta} = 0$).

$$\frac{dN}{d\theta} = -WR\,(\frac{\cos\theta}{2} - 2\cos\theta\sin\theta)$$
$$\frac{dN}{d\theta} = 0 \rightarrow \frac{\cos\theta}{2} - 2\cos\theta\sin\theta = 0 \rightarrow \cos\theta\,(\frac{1}{2} - 2\sin\theta) = 0$$
$$\text{either } \cos\theta = 0 \rightarrow \theta = 90^o \quad \rightarrow N = \frac{-WR}{2}$$
$$or\ (\frac{1}{2} - 2\sin\theta) = 0 \rightarrow \sin\theta = \frac{1}{4} \rightarrow \theta = sin^{-1}(0.25) = 14.48^o \rightarrow N = -1.09WR$$

b- Angle of shear force critical section: the section will be found by setting the slope of the shear force to zero ($\frac{dV}{d\theta} = 0$).

$$\frac{dV}{d\theta} = WR\,(-sin^2\theta + cos^2\theta + \sin\theta) = WR\left(1 - 2sin^2\theta + \frac{1}{2}\sin\theta\right)$$
$$\frac{dN}{d\theta} = 0 \rightarrow 1 - 2sin^2\theta + \frac{1}{2}\sin\theta = 0 \rightarrow sin^2\theta - \frac{1}{4}\sin\theta - \frac{1}{2} = 0$$
$$\sin\theta = \frac{0.25 \pm \sqrt{0.25^2 + 2}}{2}$$
$$= 0.843(neglect\ the\ negative\ value\ since\ it\ gives\ a\ negative\ angle)$$
$$\rightarrow \theta = sin^{-1}(0.843) = 57.46^o \rightarrow V = -0.185WR$$

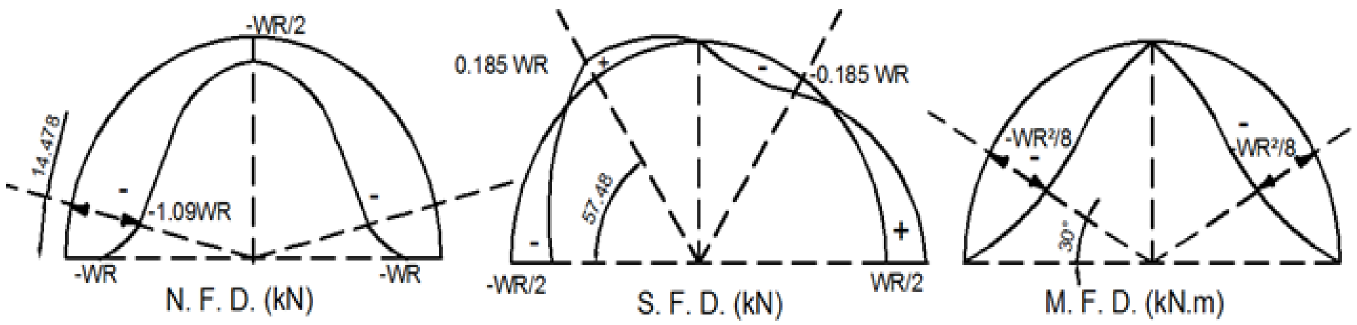
c- Angle of moment critical section: the section will be found by setting the slope of the moment to zero ($\frac{dM}{d\theta} = 0$).

$$\frac{dM}{d\theta} = \frac{WR^2}{2}(2\sin\theta\cos\theta - \cos\theta)$$
$$\frac{dM}{d\theta} = 0 \rightarrow \cos\theta\,(2\sin\theta - 1) = 0$$
$$\text{either } \cos\theta = 0 \rightarrow \theta = 90^o \quad \rightarrow M = 0$$
$$or\ (2\sin\theta - 1) = 0 \rightarrow \sin\theta = \frac{1}{2} \rightarrow \theta = 30^o \quad \rightarrow M = \frac{WR^2}{8}$$

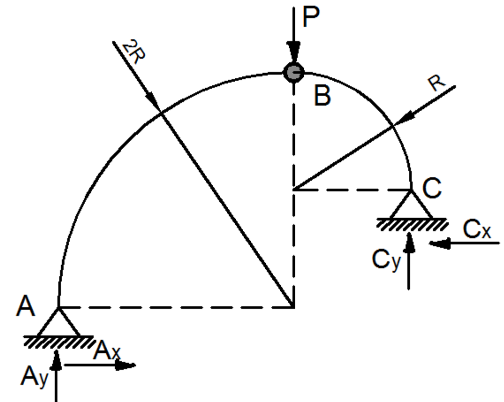
θ	N
0^o	$-WR$
14.478^o	$-1.09WR$
90^o	$-\frac{WR}{2}$

θ	V
0^o	$-\frac{WR}{2}$
57.47^o	$-0.185\,WR$
90^o	0

θ	M
0^o	0
30^o	$-\frac{WR^2}{8}$
90^o	0



Example 18: Draw axial, shear and bending moment diagram



Solution:

For the whole body:

$\sum F_x = 0 \rightarrow A_x = C_x$

For part AB:

$\sum M_{@B} = 0 \rightarrow A_x * 2R = A_y * 2R$

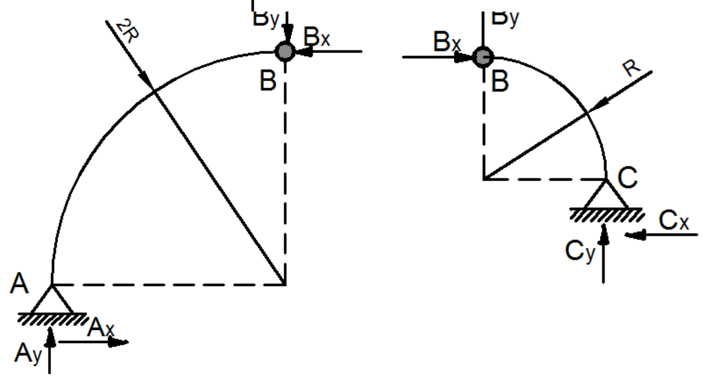
$\rightarrow A_x = A_y$

For part CB:

$\sum M_{@B} = 0 \rightarrow C_x * R = C_y * R$

$\rightarrow C_x = C_y$

But $(A_x = C_x) \rightarrow (A_x = C_x = C_y = A_y)$



For the whole body:

$\sum F_y = 0 \rightarrow A_y + C_y = P \rightarrow A_y = C_y = \frac{P}{2}$

The two parts (AB & BC) will have the same equations of (N, V & M) since they are identical in their parameters $(A_x = C_x = C_y = A_y)$ that's included in the calculations steps.

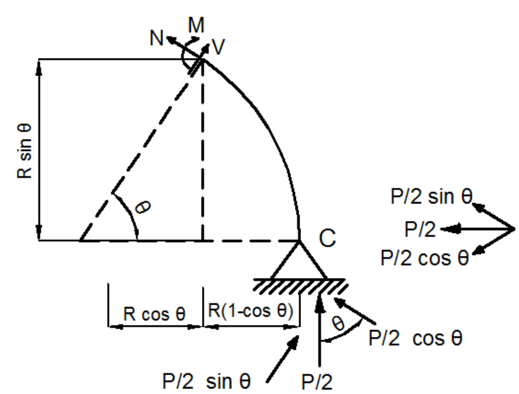
$N = -\frac{P}{2}\cos\theta - \frac{P}{2}\sin\theta$

$N = -\frac{P}{2}(\cos\theta + \sin\theta)$

$V = \frac{P}{2}\cos\theta - \frac{P}{2}\sin\theta$

$V = \frac{P}{2}(\cos\theta - \sin\theta)$

$M = \frac{PR}{2}(1 - \cos\theta - \sin\theta)$



The drawing will be set at three values of the angle (θ), these are (zero, 90° and angle of critical section)

a- Angle of axial force critical section: the section will be found by setting the slope of the axial force to zero ($\frac{dN}{d\theta} = 0$).

$$\frac{dN}{d\theta} = -\frac{P}{2}(-\sin\theta + \cos\theta)$$

$$\frac{dN}{d\theta} = 0 \rightarrow -\sin\theta + \cos\theta = 0 \rightarrow \cos\theta = \sin\theta \rightarrow \theta = 45^o$$

b- Angle of shear force critical section: the section will be found by setting the slope of the shear force to zero ($\frac{dV}{d\theta} = 0$).

$$\frac{dV}{d\theta} = \frac{P}{2}(-\sin\theta + \cos\theta)$$

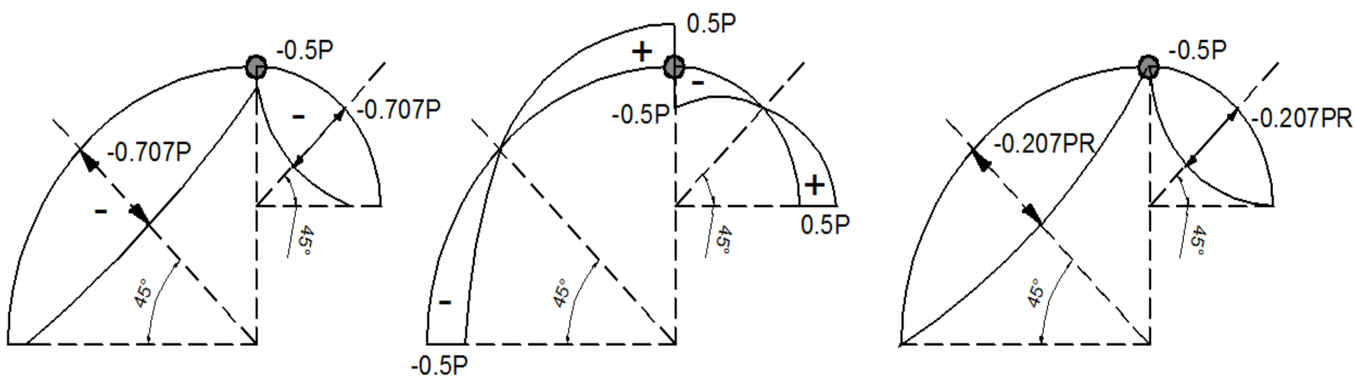
$$\frac{dN}{d\theta} = 0 \rightarrow -\sin\theta + \cos\theta = 0 \rightarrow \cos\theta = \sin\theta \rightarrow \theta = 45^o$$

c- Angle of moment critical section: the section will be found by setting the slope of the moment to zero ($\frac{dM}{d\theta} = 0$).

$$\frac{dM}{d\theta} = \frac{PR}{2}(\sin\theta - \cos\theta)$$

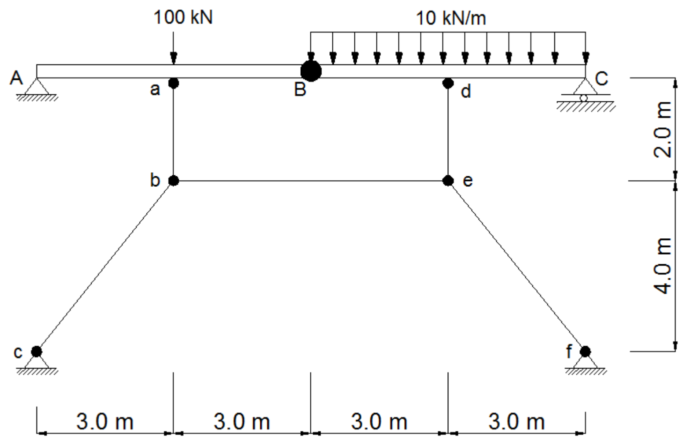
$$\frac{dM}{d\theta} = 0 \rightarrow (\sin\theta - \cos\theta) = 0 \rightarrow \cos\theta = \sin\theta \rightarrow \theta = 45^o$$

θ	N	V	M
0°	$-\frac{P}{2}$	$\frac{P}{2}$	0
45°	$-\frac{P}{\sqrt{2}}$	0	-0.207PR
90°	$-\frac{P}{2}$	$-\frac{P}{2}$	0



N. F. D. (kN) S. F. D. (kN) M. F. D. (kN.m)

Example 18: For the composite structure shown below, draw axial, shear and bending moment diagram for beam ABC.



Solution:

For Joint b

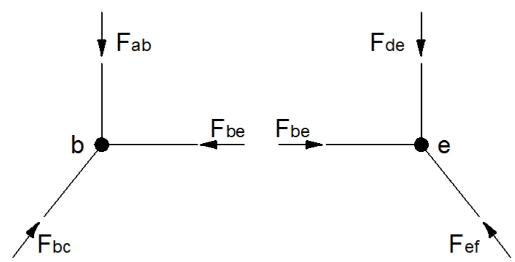
$\sum F_x = 0 \rightarrow F_{be} = \frac{3}{5} F_{bc}$ ……… 1

$\sum F_y = 0 \rightarrow F_{ab} = \frac{4}{5} F_{bc}$ ……… 2

For Joint e

$\sum F_x = 0 \rightarrow F_{be} = \frac{3}{5} F_{ef}$ ……… 3

$\sum F_y = 0 \rightarrow F_{de} = \frac{4}{5} F_{ef}$ ……… 4

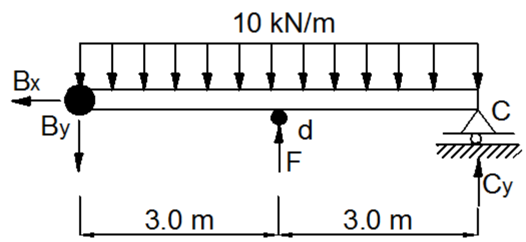


From Eqs. (1&3) → $F_{bc} = F_{ef}$ ……… 5

From Eqs. (5,2&4) → $F_{ab} = F_{de} = F$

BC as F.B.D. :

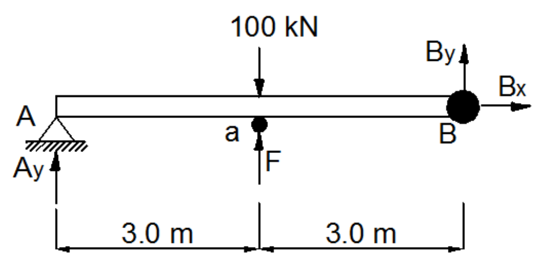
$\sum M_{@C} = 0 \rightarrow B_y = \frac{F * 3 - 60 * 3}{6} \rightarrow B_y = \frac{F}{2} - 30$ ……… 6



AB as F.B.D. :

$\sum M_{@A} = 0 \rightarrow B_y = \frac{100 * 3 - F * 3}{6}$

$\rightarrow B_y = 50 - \frac{F}{2}$ ……… 7



From Eqs. (6&7) → $\frac{F}{2} - 30 = 50 - \frac{F}{2} \rightarrow F = 80kN$ & $B_y = 10\ kN$

BC as F.B.D. :

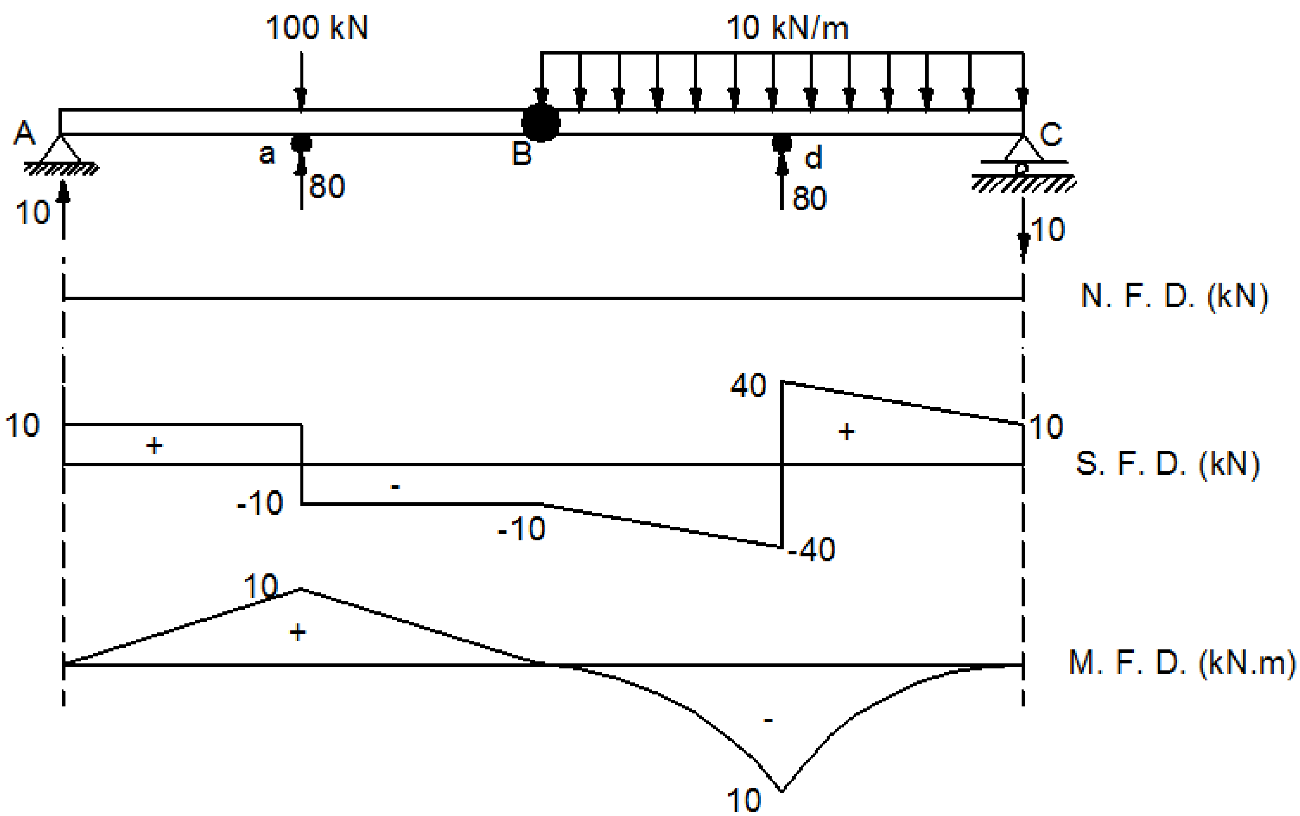
$\sum F_y = 0 \rightarrow C_y = 60 + 10 - 80 = -10\ kN$

AB as F.B.D. :

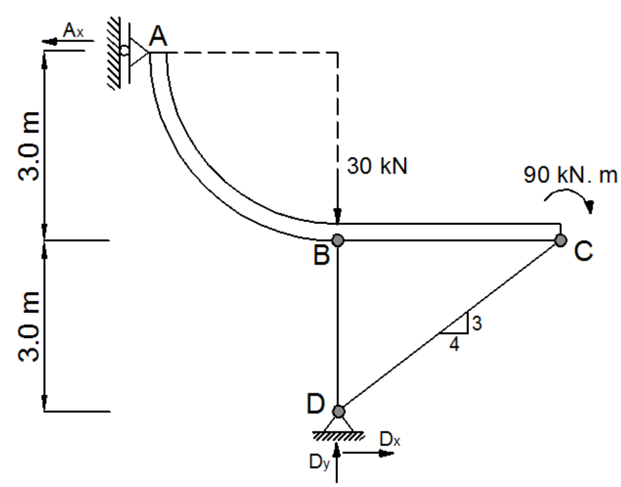
$\sum F_y = 0 \rightarrow A_y = 100 - 80 - 10 = 10\ kN$

Whole body as F.B.D:

$$\sum F_x = 0 \rightarrow A_x = 0\ kN$$



Example 19: For the composite structure shown below, draw axial, shear and bending moment diagram.



Solution:

Whole body as F.B.D:

$$\sum M_{@D} = 0 \rightarrow A_y = \frac{90}{6} = 15\ kN$$

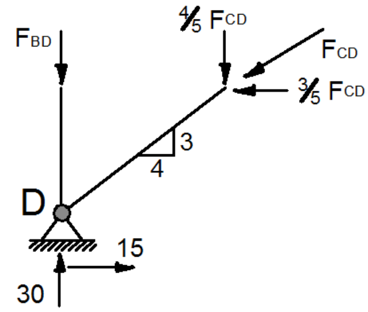
$$\sum F_x = 0 \rightarrow D_x = 15\ kN$$

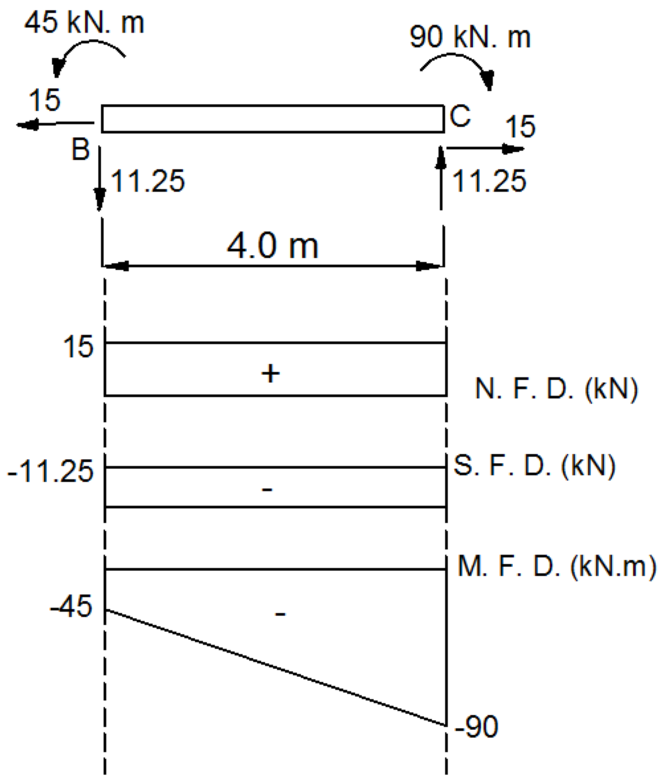
$$\sum F_y = 0 \rightarrow D_y = 30\ kN$$

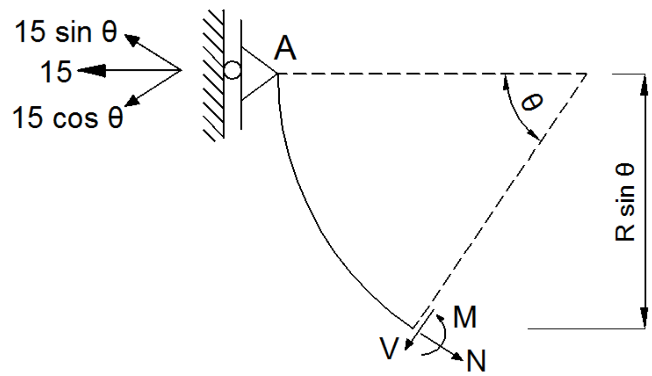
<u>For Joint D</u>

$$\sum F_x = 0 \rightarrow F_{DC} * \frac{4}{5} = 15 \rightarrow F_{DC} = 18.75\ kN$$

$$\sum F_y = 0 \rightarrow F_{DC} * \frac{3}{5} + F_{DB} = 30 \rightarrow F_{DB} = 11.25\ kN$$

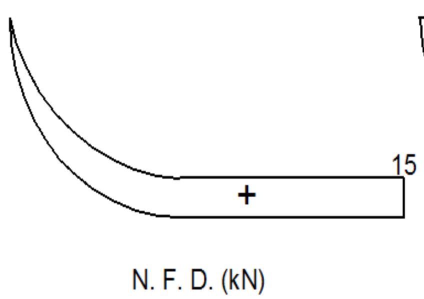


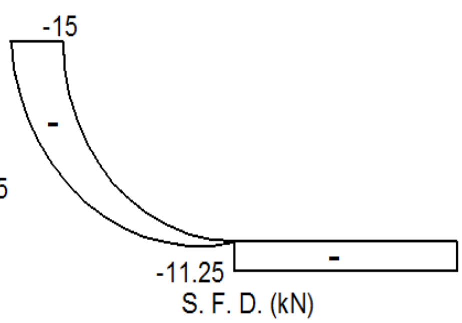


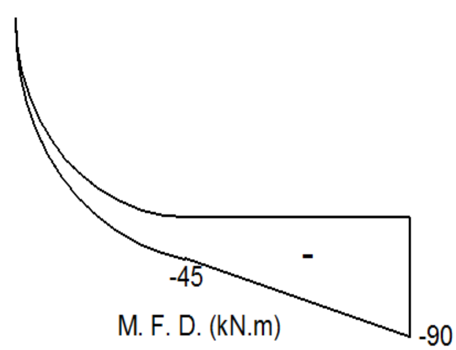


$N = 15\ \sin\theta$
$V = -15\ \cos\theta$
$M = -45\ \sin\theta$

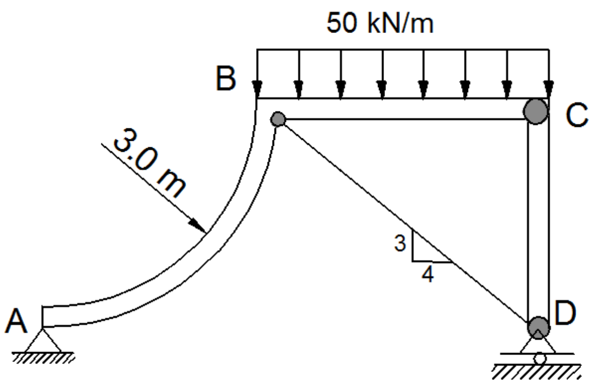
θ	N	V	M
0^o	0	-15	0
90^o	15	0	-45

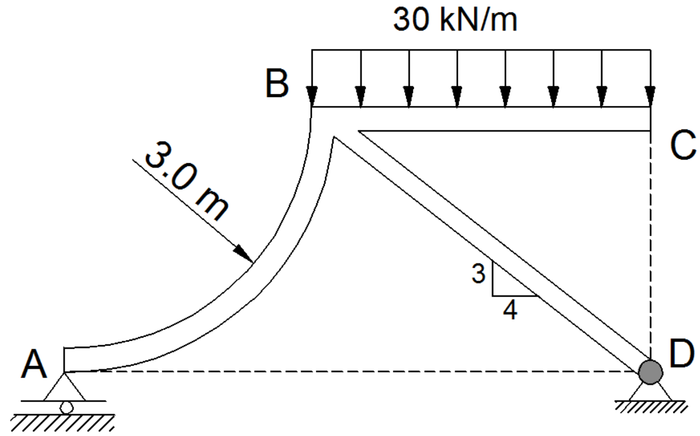






H.W.: Draw axial, shear and bending moment diagram





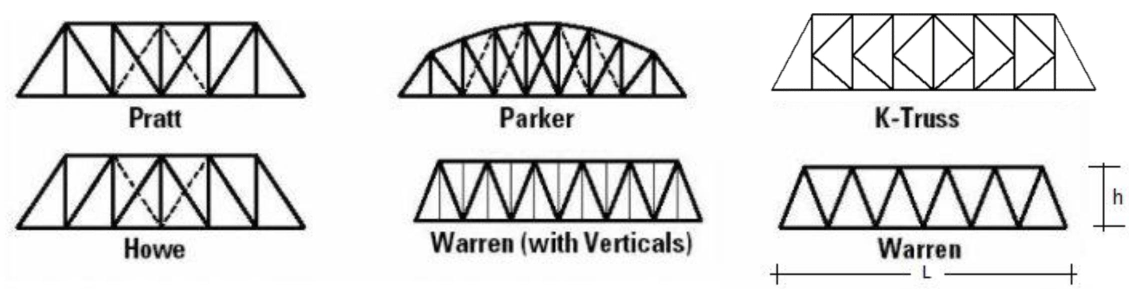
Trusses

Truss is usually defined as a structure that's composed of slender members (two-force members) joined together at their end points. Trusses are physically stronger than other ways of arranging structural elements, because nearly every material can resist a much larger load in tension or compression than in shear, bending, torsion, or other kinds of force. It is usually used for large- span panels since it will be more economic and strength.

Some common trusses are named according to their "web configuration"

<u>Common types of bridges truss</u> (h = $\frac{L}{10}$ to $\frac{L}{15}$)

- Warren truss.
- Pratt truss.
- Howe Truss.
- K- Truss.
- Parker.

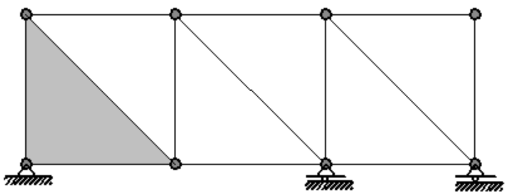


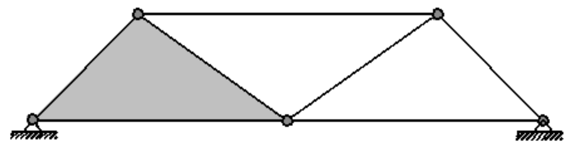
<u>Common types of roof truss</u> (h = $\frac{L}{4}$)

- Fink.
- Pratt truss.
- Howe Truss.
- Fan.

Types of statically determined trusses (b+r= 2j)

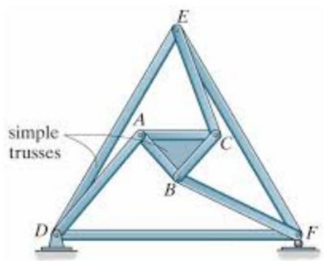
a- <u>Simple truss</u>: this type of trusses are consisted of a base triangle (three members connecting at three joints) and any additional two members connected at an additional joint.

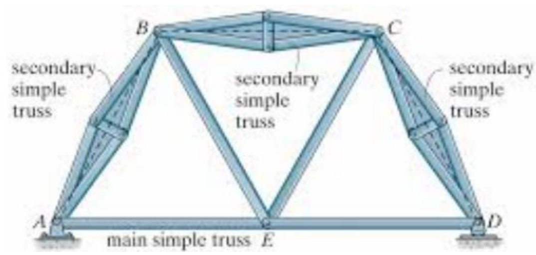




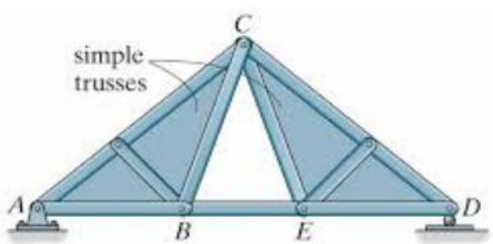
b- <u>Compound truss:</u> can be defined as two or more simple trusses compound together using one of the following processes:

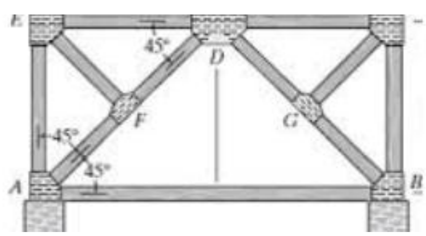
1- Three links that are neither parallel nor concurrent.



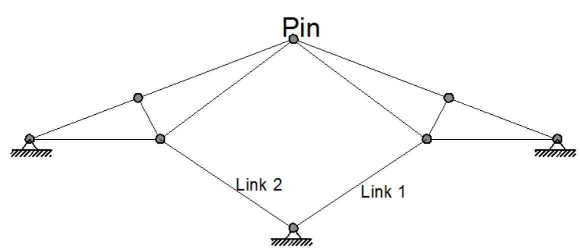


2- A pin and a link.

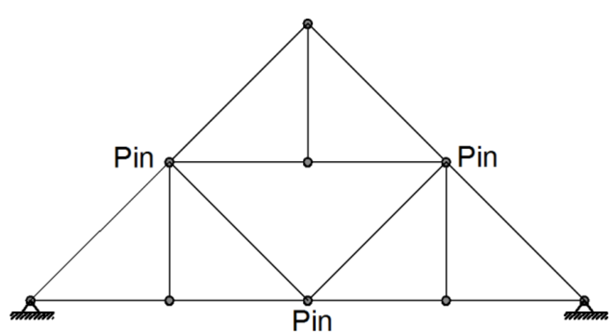




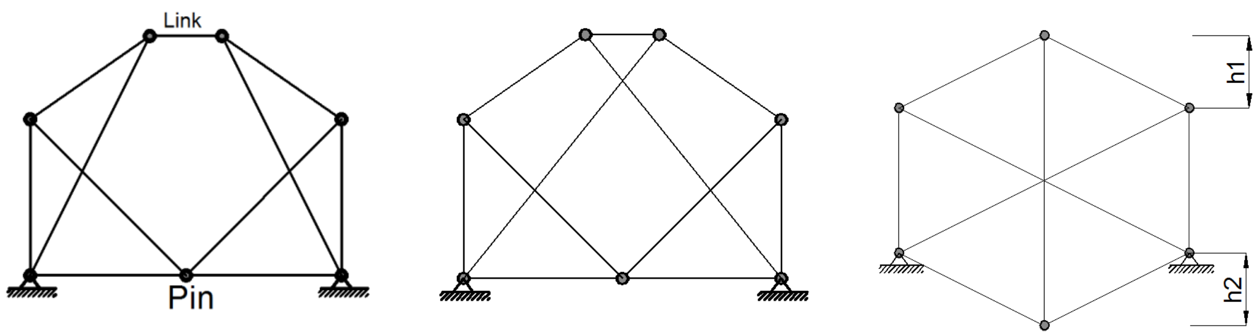
3- A pin and two links intersecting at a support.



4- Three pins.



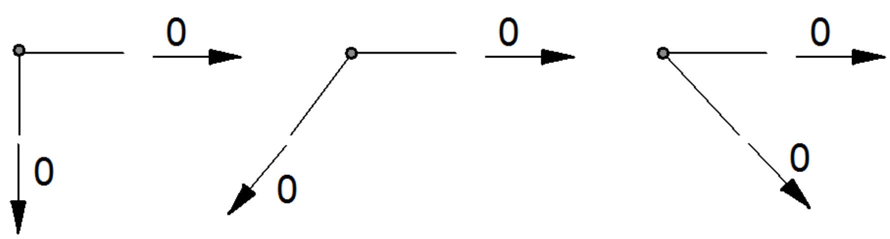
c- <u>Complex trusses:</u> In this type of trusses, three or more members are connected in each joints. It cannot be solved easy since many members will be intersect at any adopted section.



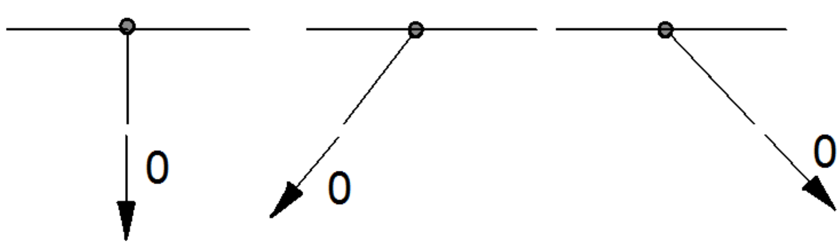
If h1 = h2 (unstable)

Notes:

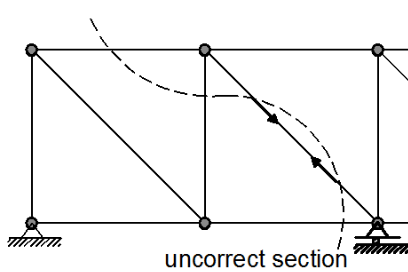
1- If not more than two members are joint together (not straight) and there was no applied force on it then; the axial force in these members are equal to zero.



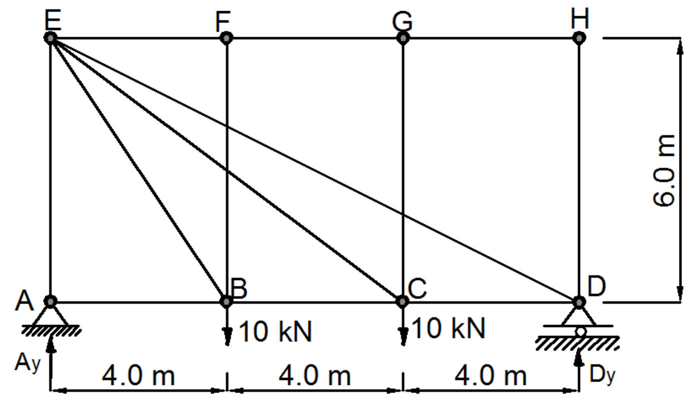
2- If three members are joint together (two of them configurator a straight line) and there was no applied load on the joint, the axial force in the third member will be zero.



3- The path of any section cannot pass through the same members more than one time. This will cause canceling the axial force in this member.



Example 1: Find the axial force in the members of the truss ABCDEFGH

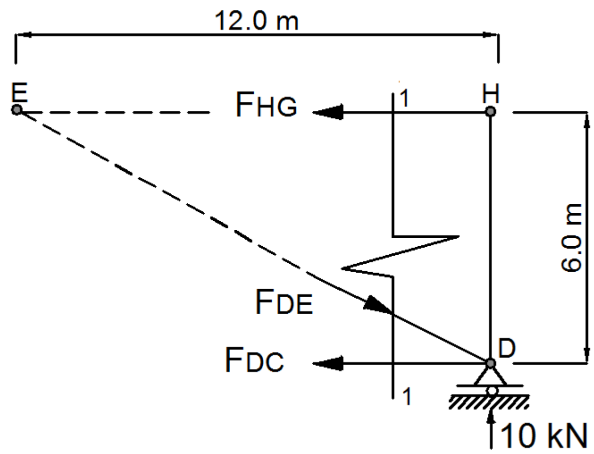


Solution:

It is a simple truss. Du to symmetry

$A_y = D_y = 10\ kN$

$\sum F_x = 0 \rightarrow A_x = 0$



Joint H as F. B. D.

$\sum F_x = 0 \rightarrow F_{HD} = 0$

$\sum F_y = 0 \rightarrow F_{HG} = 0$

Joint G as F. B. D.

$\sum F_x = 0 \rightarrow F_{GF} = 0$

$\sum F_y = 0 \rightarrow F_{GC} = 0$

Joint F as F. B. D.

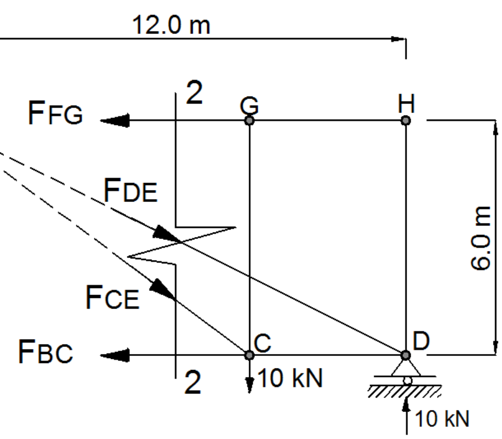
$\sum F_x = 0 \rightarrow F_{FE} = 0$

$\sum F_y = 0 \rightarrow F_{FB} = 0$

Joint A as F. B. D.

$\sum F_x = 0 \rightarrow F_{AB} = 0$

$\sum F_y = 0 \rightarrow F_{AE} = 10\ kN\ Comp$



From Sec 1-1

$$\sum M_{@E} = 0 \rightarrow F_{DC} = \frac{10 * 12}{6} = 20\ kN\ Ten$$

$$\sum F_y = 0 \rightarrow F_{DE} * \frac{1}{\sqrt{5}} = 10 \rightarrow F_{DE} = 10\sqrt{5}\ kN\ Comp$$

From Sec 2-2

$$\sum M_{@E} = 0 \rightarrow F_{BC} = \frac{10 * 12 - 10 * 8}{6} = 6.67\ kN\ Ten$$

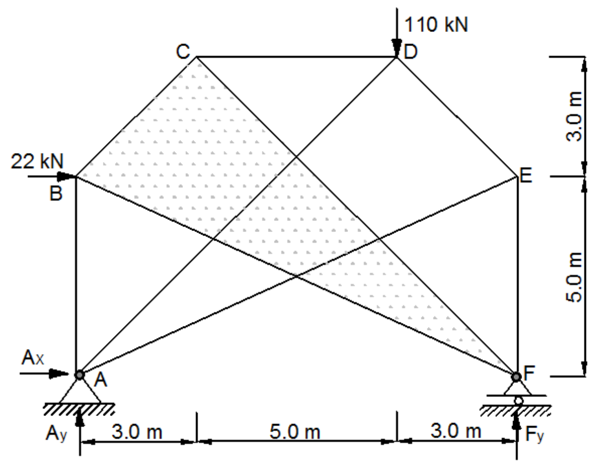
$$\sum F_y = 0 \rightarrow F_{CE} * \frac{3}{5} = 10 \rightarrow F_{CE} = 16.67\ kN\ Comp$$

Joint B as F. B. D.

$$\sum F_y = 0 \rightarrow F_{BE} * \frac{3}{\sqrt{13}} = 10 \rightarrow F_{BE} = 12.02\ kN\ Ten$$

Member	Force (kN)	Type	Member	Force (kN)	Type
AB	0		BF	0	
BC	6.67	T	CG	0	
CD	20	T	DH	0	
EF	0		DE	$10\sqrt{5}$	C
FG	0		CE	16.67	C
GH	0		BE	12.02	T
AE	10	C			

Example 2: Find the axial force in members AB, CD & EF

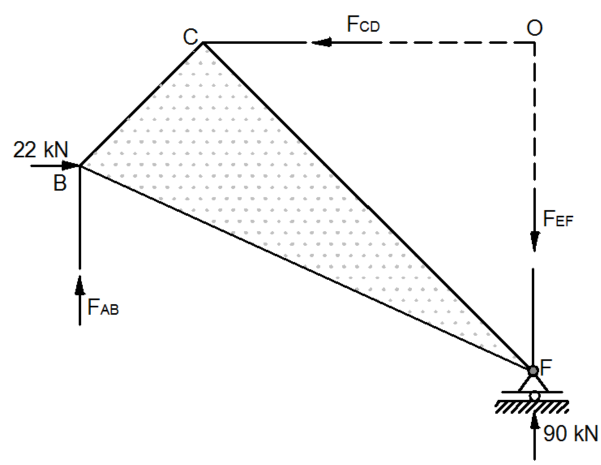


Solution:
It is a combined truss.

$$\sum M_{@A} = 0 \rightarrow B_y = \frac{110 * 8 + 22 * 5}{11} = 90\ kN$$

$$\sum F_y = 0 \rightarrow A_y = 110 - 90 \rightarrow A_y = 20\ kN$$

$$\sum F_x = 0 \rightarrow A_x = 22\ kN$$

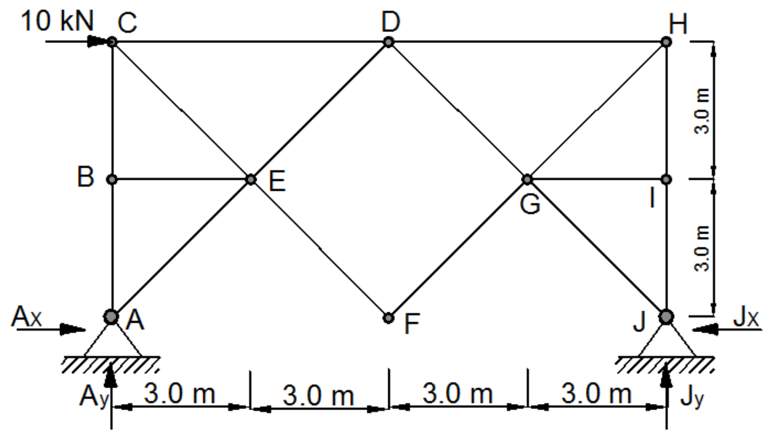


$$\sum F_x = 0 \rightarrow F_{CD} = 22\ kN\ Comp$$

$$\sum M_{@O} = 0 \rightarrow F_{AB} = \frac{22 * 3}{11} = 6\ kN\ Comp$$

$$\sum F_y = 0 \rightarrow F_{EF} = 90 + 6 \rightarrow F_{EF} = 96\ kN\ \ Comp$$

Example 3: Find the axial force in all members of the truss shown below:



Solution:

$$\sum M_{@j} = 0 \rightarrow A_y = \frac{10 * 6}{12} = 5\ kN$$

$$\sum F_y = 0 \rightarrow J_y = 5\ kN$$

Joint F as F. B. D.

$F_{FG} = F_{FE} = 0$

Joint B & I as F. B. D.

$F_{BE} = F_{GI} = 0$

Joint E & G as F. B. D.

$F_{CE} = F_{GH} = 0$

Joint H as F. B. D.

$F_{HD} = F_{HI} = 0$

Joint C as F. B. D.

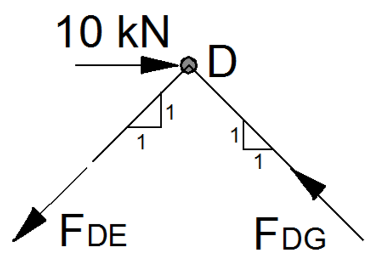
$F_{CD} = 10$ kN Comp

$F_{CB} = 0$

Joint B as F. B. D.

$F_{AB} = 0$

Joint A as F. B. D.



$$\sum F_y = 0 \rightarrow \frac{1}{\sqrt{2}} F_{AE} = 5 \rightarrow F_{AE} = 5\sqrt{2}\ kN \text{ Comp}$$

$$\sum F_x = 0 \rightarrow A_x = \frac{1}{\sqrt{2}} F_{AE} = 5\ kN$$

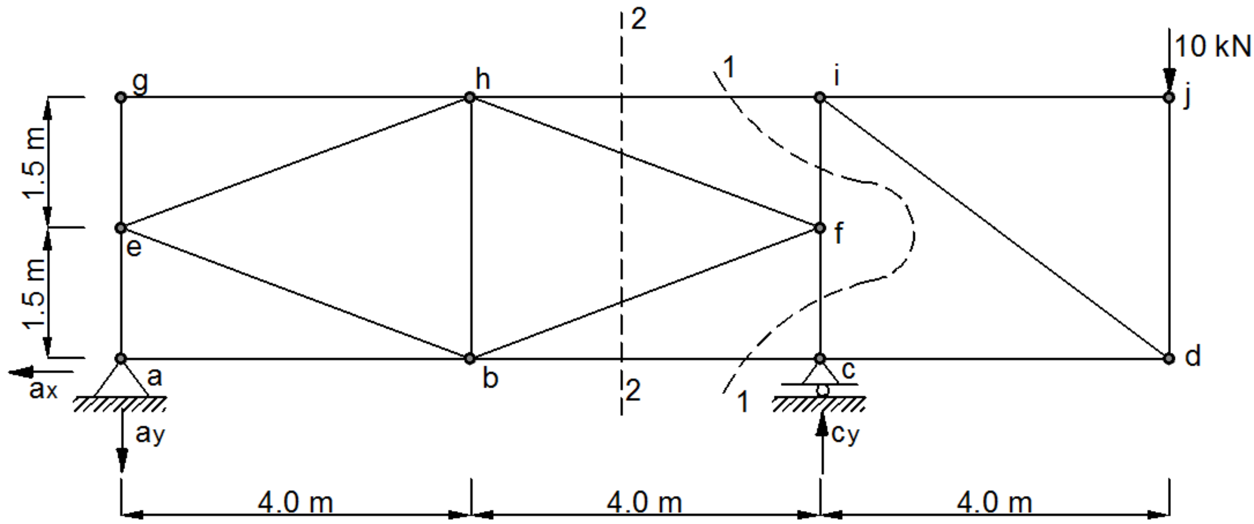
Joint D as F. B. D.

$$\sum F_y = 0 \rightarrow \frac{1}{\sqrt{2}} F_{DE} = \frac{1}{\sqrt{2}} F_{DG} \rightarrow F_{DE} = F_{DG}$$

$$\sum F_x = 0 \rightarrow \frac{1}{\sqrt{2}} F_{DE} + \frac{1}{\sqrt{2}} F_{DG} - 10 \rightarrow \frac{2}{\sqrt{2}} F_{DE} = 10 \rightarrow F_{DE} = 5\sqrt{2}\ kN\ Ten$$

$$F_{DG} = 5\sqrt{2}\ kN\ Comp$$

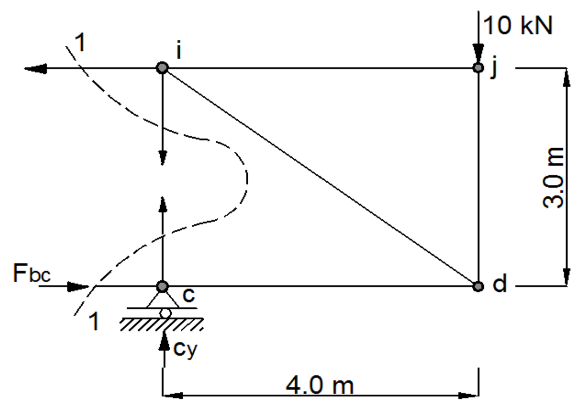
Example 4: Find the axial force in members (bc, bf & id) of the truss shown below:



Solution:

$$\sum F_x = 0 \rightarrow a_x = 0$$

$$\sum M_{@a} = 0 \rightarrow c_y = \frac{10 * 12}{8} = 15\ kN$$

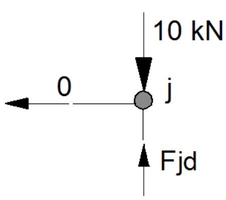


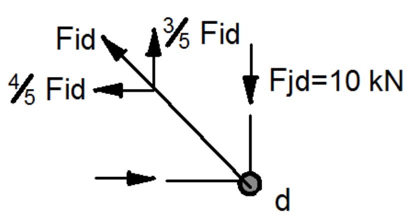
From Sec 1-1

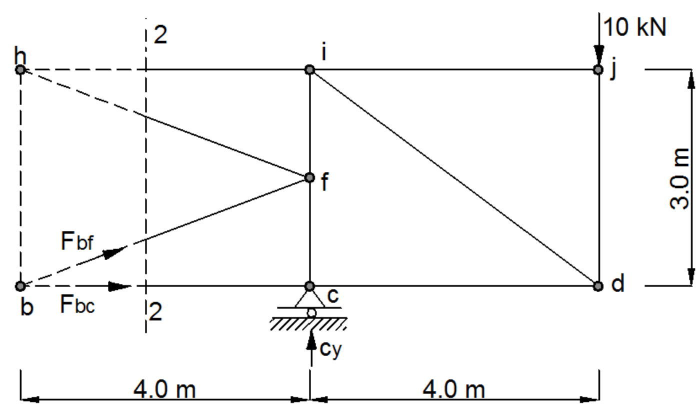
$$\sum M_{@i} = 0 \rightarrow F_{bc} = \frac{10 * 4}{3} = \frac{40}{3}\ kN\ Comp$$

From Sec 2-2

$$\sum M_{@h} = 0 \rightarrow \frac{4}{\sqrt{18.25}} F_{bc} * 3 + 15 * 4 - 10 * 8 + \frac{40}{3} * 3 = 0 \rightarrow F_{bc} = 7.12\ kN\ Comp$$







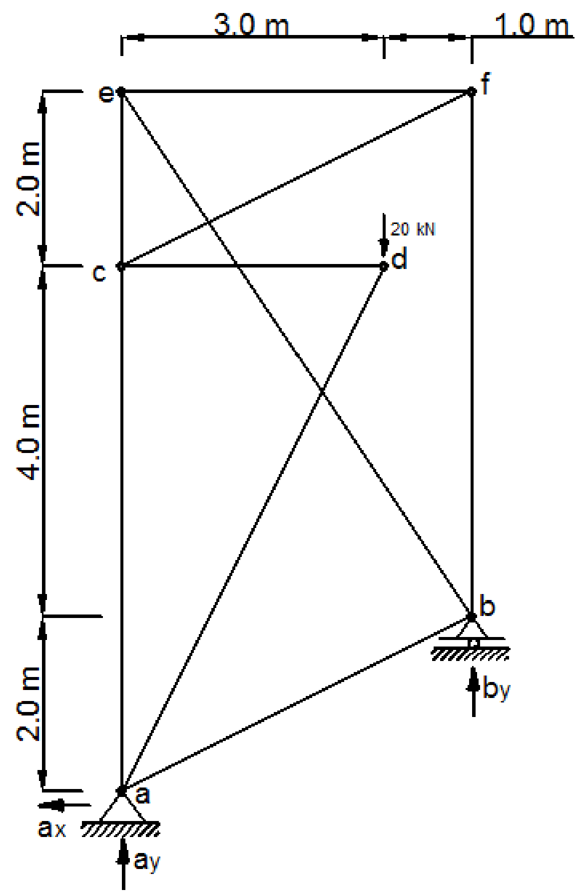
Joint j as F. B. D.

$$\sum F_y = 0 \rightarrow F_{jd} = 10\ kN\ \ Comp$$

Joint d as F. B. D.

$$\sum F_y = 0 \rightarrow \frac{3}{5} F_{di} = 10 \rightarrow F_{di} = 16.67\ \ kN\ \ Ten$$

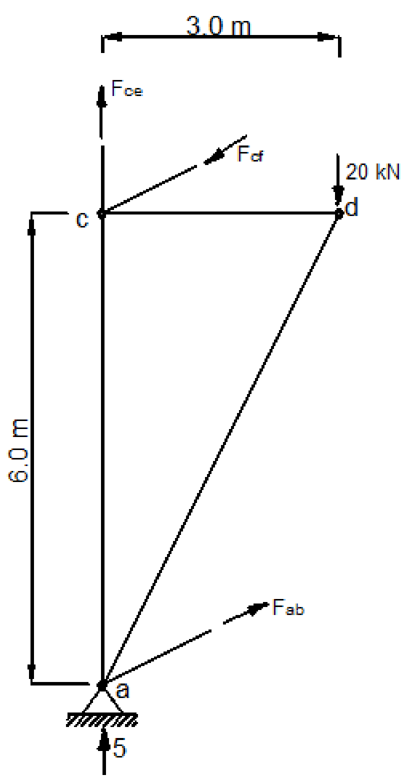
Example 5: Find the axial force in members (cf, ce & ab) of the truss shown below:



Solution:

$$\sum F_x = 0 \rightarrow a_x = 0$$

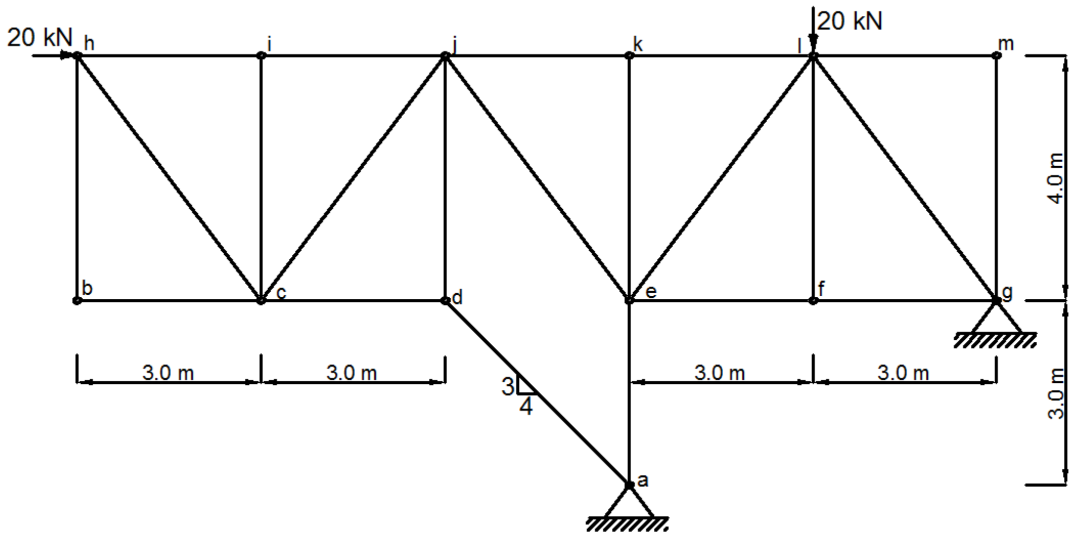
$$\sum M_{@a} = 0 \rightarrow b_y * 4 = 20 * 3 \rightarrow b_y = 15 = kN$$



$$\sum M_{@c} = 0 \rightarrow \frac{2}{\sqrt{5}} F_{ab} * 6 = 20 * 3 \rightarrow F_{ab} = 5\sqrt{5}\, kN\ Ten$$

$$\sum F_x = 0 \rightarrow \frac{2}{\sqrt{5}} F_{ab} = \frac{2}{\sqrt{5}} F_{cf} \rightarrow F_{cf} = 5\sqrt{5}\, kN\ Comp$$

$$\sum F_y = 0 \rightarrow F_{ce} = 20 - 5 \rightarrow F_{cf} = 15\, kN\ Ten$$


20 kN
h
i
j
k
l
20 kN
m
4.0 m
b
c
d
e
f
g
3.0 m
3.0 m
3
4
3.0 m
3.0 m
3.0 m
a

Influence Line

Influence line can be defined as a graph of a response function for a structure relating to the position of a downward unit load moving across the structure (Response Function may be support reaction, axial force, shear force, or bending moment)

Coals of the Influence line: Once an influence line is constructed, the following aims can be obtained

- Determining where to place the live load on a structure to maximize the considered response function.
- Evaluate the maximum magnitude of the response function.

Notes:

1- The influence line of a statically determinate structure is usually characterized by connecting lines while it's represented for a statically indeterminate structure by connecting curves.

2- The path of the influence line can be divided to some parts depending on the geometry of the structure (No. of F,B.D. = No. of parts in the I.L.)

3- No movement is allowed for all supporting points along the path of the influence line for any response function except the one that represents the considered response function.

4- The rotation of the interior pin is allowed if the next connected part is free to rotate.

5- Points of shear influence line are separated from each other vertically. The summation of the absolute vertical distance between them is equal to one unit.

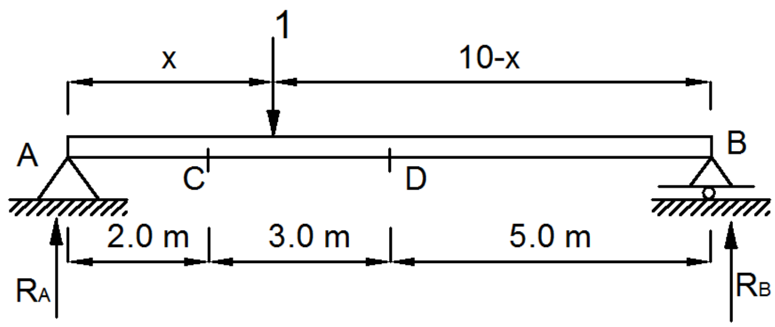
6- Points of moment influence line are connected to each other (either at zero elevation or at the same elevation value). The summation of the absolute rotated angles of them is equal to one unit

7- No discontinuity (bent or broken) is allowed for any part of the influence line path.

Method of Drawing

1- **Equilibrium Method:** In this method the corresponding response function will be calculated in term of (x) for a unite load that placed at distance (x).

Example 1: Draw I.L. for (R_A, R_B, V_C, M_C, V_D & M_D).

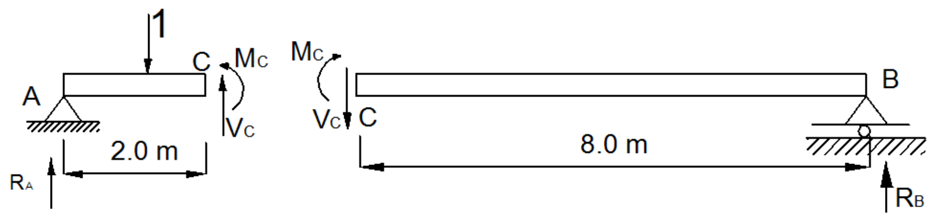


Solution:

Allocate a unit weight at distance (x) from the support (A). The reactions in both end-supports will be found in term of (x) as shown below:

$$\sum M_{@B} = 0 \rightarrow R_A = \frac{10-x}{10} = 1 - \frac{x}{10}$$

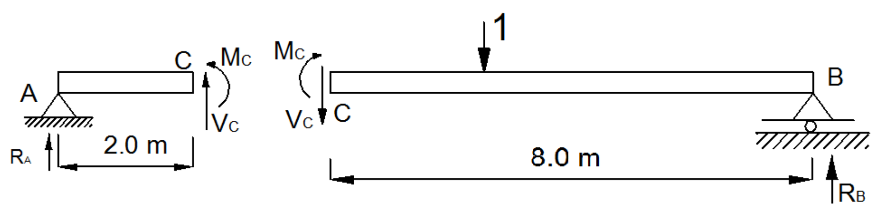
$$\sum F_y = 0 \rightarrow R_B = \frac{x}{10}$$



$0 \leq x \leq 2$

$$\sum F_y = 0 \rightarrow V_C = \frac{x}{10}$$

$$\sum M_{@c} = 0 \rightarrow M_C = 2\left(1 - \frac{x}{10}\right) - (2-x) = \frac{4x}{5}$$



$2 \leq x \leq 10$

$$\sum F_y = 0 \rightarrow V_C = 1 - \frac{x}{10}$$

$$\sum M_{@c} = 0 \rightarrow M_C = 8\left(\frac{x}{10}\right) - (x-2) = 2 - \frac{x}{5}$$

$0 \leq x \leq 5$

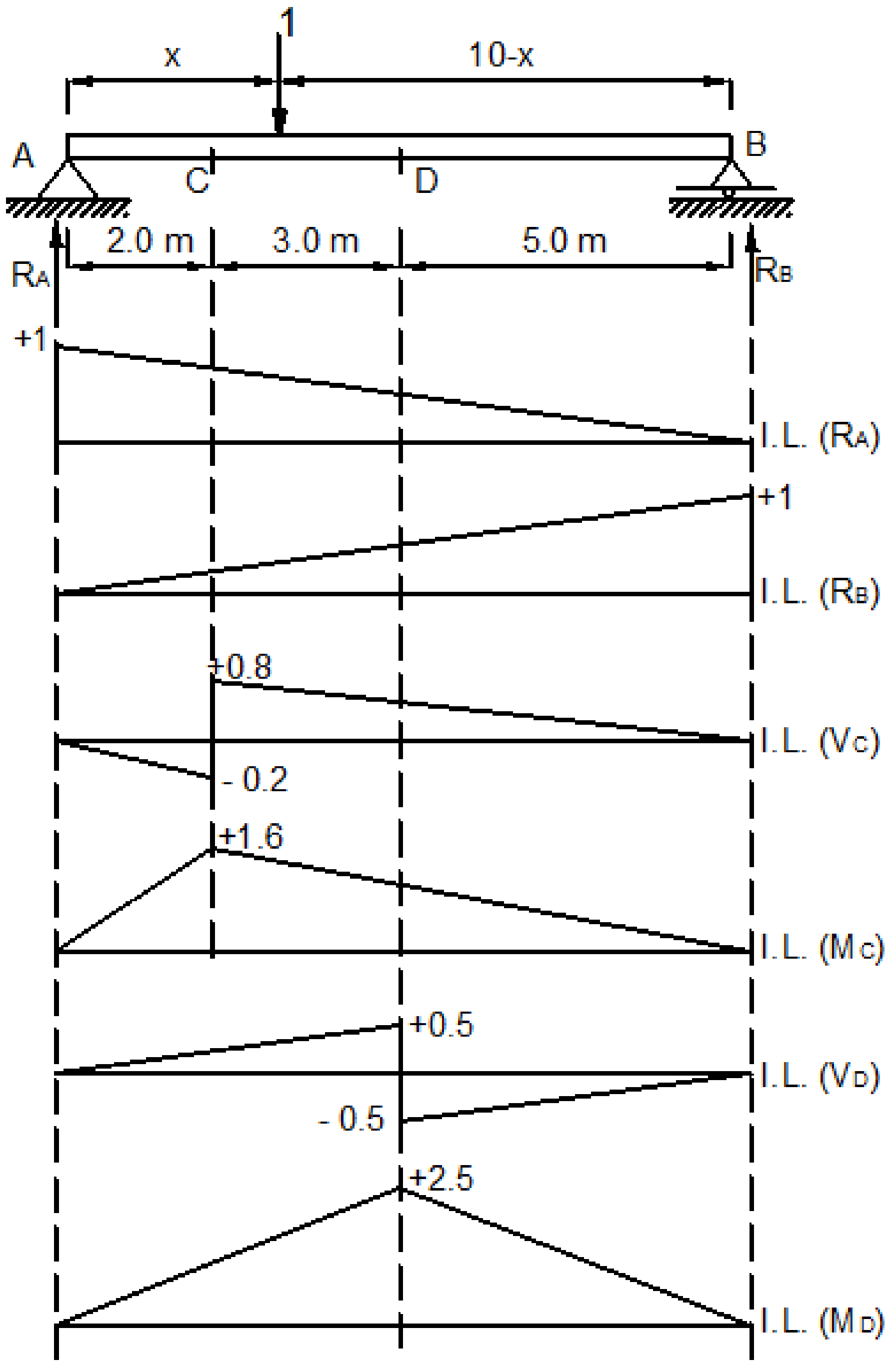
$$\sum F_y = 0 \rightarrow V_D = \frac{x}{10}$$

$$\sum M_{@D} = 0 \rightarrow M_D = 5\left(1 - \frac{x}{10}\right) - (5-x) = \frac{x}{2}$$

$5 \leq x \leq 10$

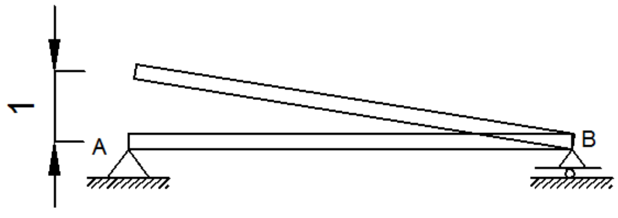
$$\sum F_y = 0 \rightarrow V_D = 1 - \frac{x}{10}$$

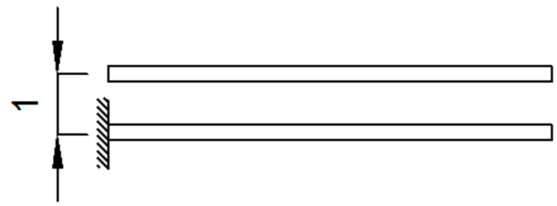
$$\sum M_{@D} = 0 \rightarrow M_D = 5\left(\frac{x}{10}\right) - (x-5) = 5 - \frac{x}{2}$$



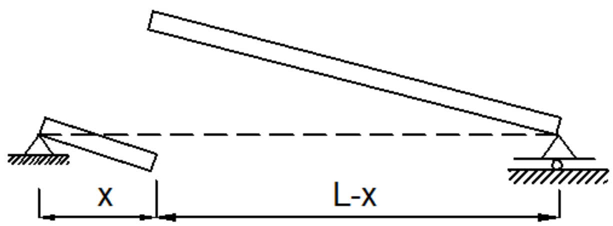
2- **Muller-Breslau Principle**: In this method the virtual work is applied to obtain the influence line for a response function by removing the displacement constraint corresponding to the response function of interest from the original structure.

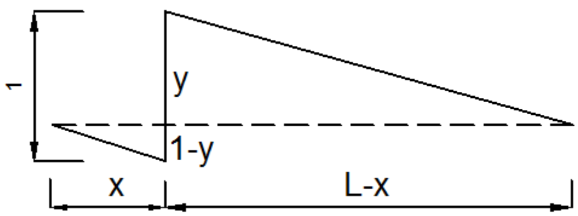
a- Vertical reaction: Since the reaction sign convention is (+) for upward direction, the corresponding support will be lift up for one unit to get the I.L. graph for the support reaction.



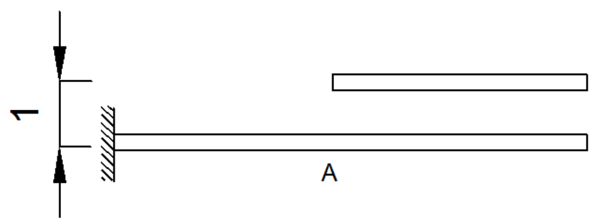


b- Shear: The positive assumed direction of the interior shear at left and right side of the beam section () will be adopted to draw the I.L. of the shear force. The corresponding point of the lift side will be lift upward and the right will be lift to the down .The total absolute summation for the vertical elevation should be equal to one unite.

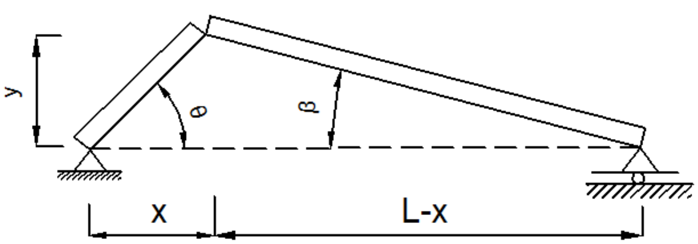


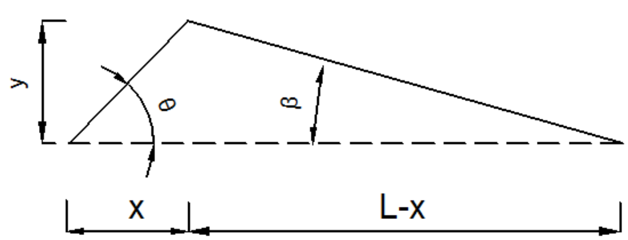


$$\frac{1-y}{x} = \frac{y}{L-x} \rightarrow y = 1 - \frac{x}{L}$$



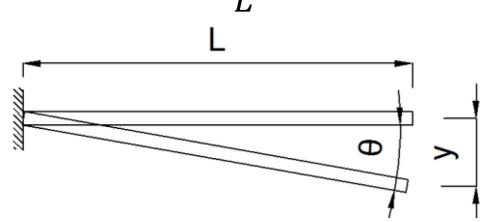
c- Moment: The positive assumed direction of the interior moment at left and right side of the beam section () will be adopted to draw the I.L. of the interior moment. The corresponding point of the lift side will be rotate counter clockwise and the right will be rotate clockwise. The total absolute summation for the rotated angles should be equal to one unite.



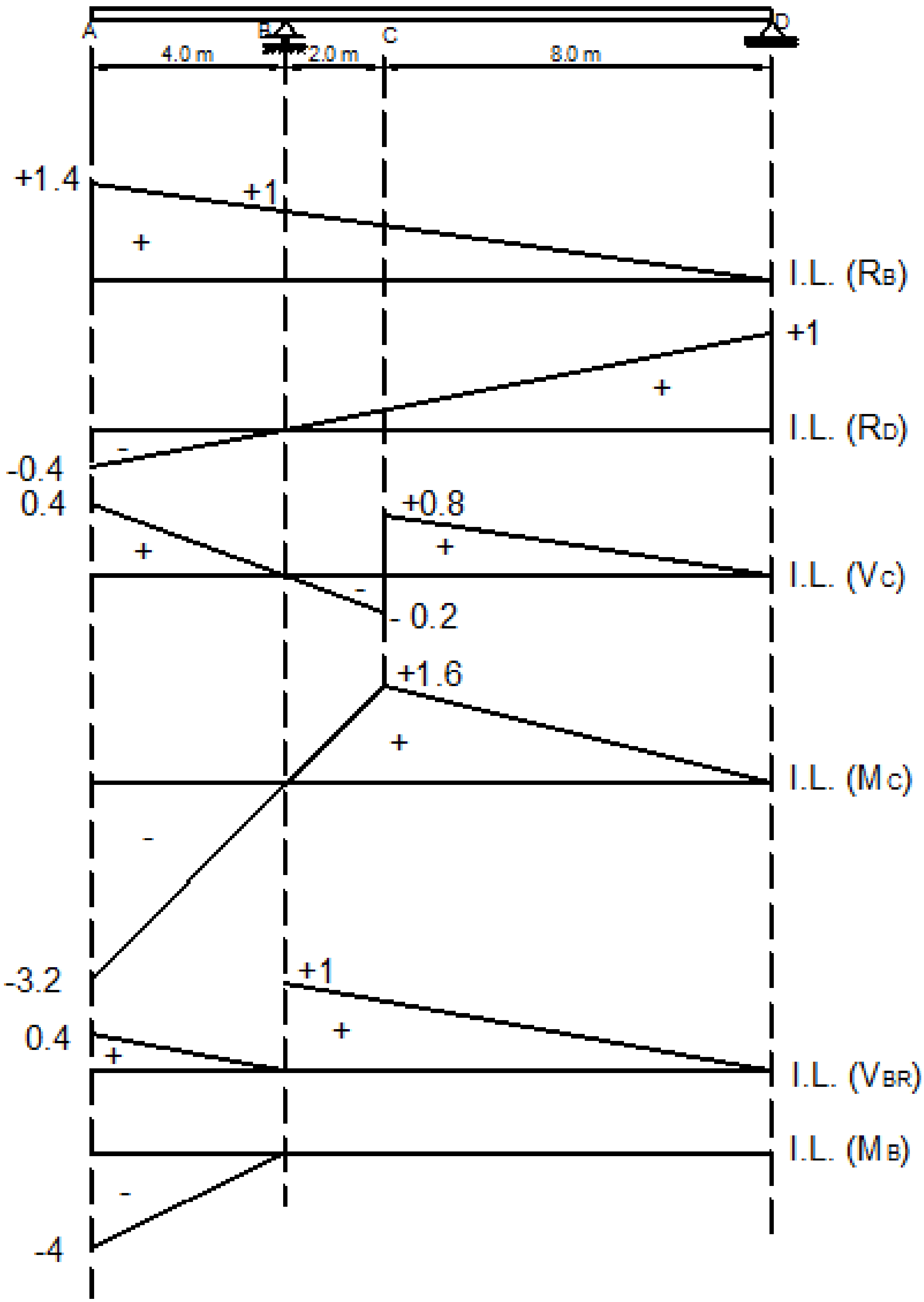


$$\theta + \beta = 1 \rightarrow \tan\theta + \tan\beta = 1 \rightarrow \frac{y}{x} + \frac{y}{L-x} = 1 \rightarrow y = \frac{x(L-x)}{L}$$

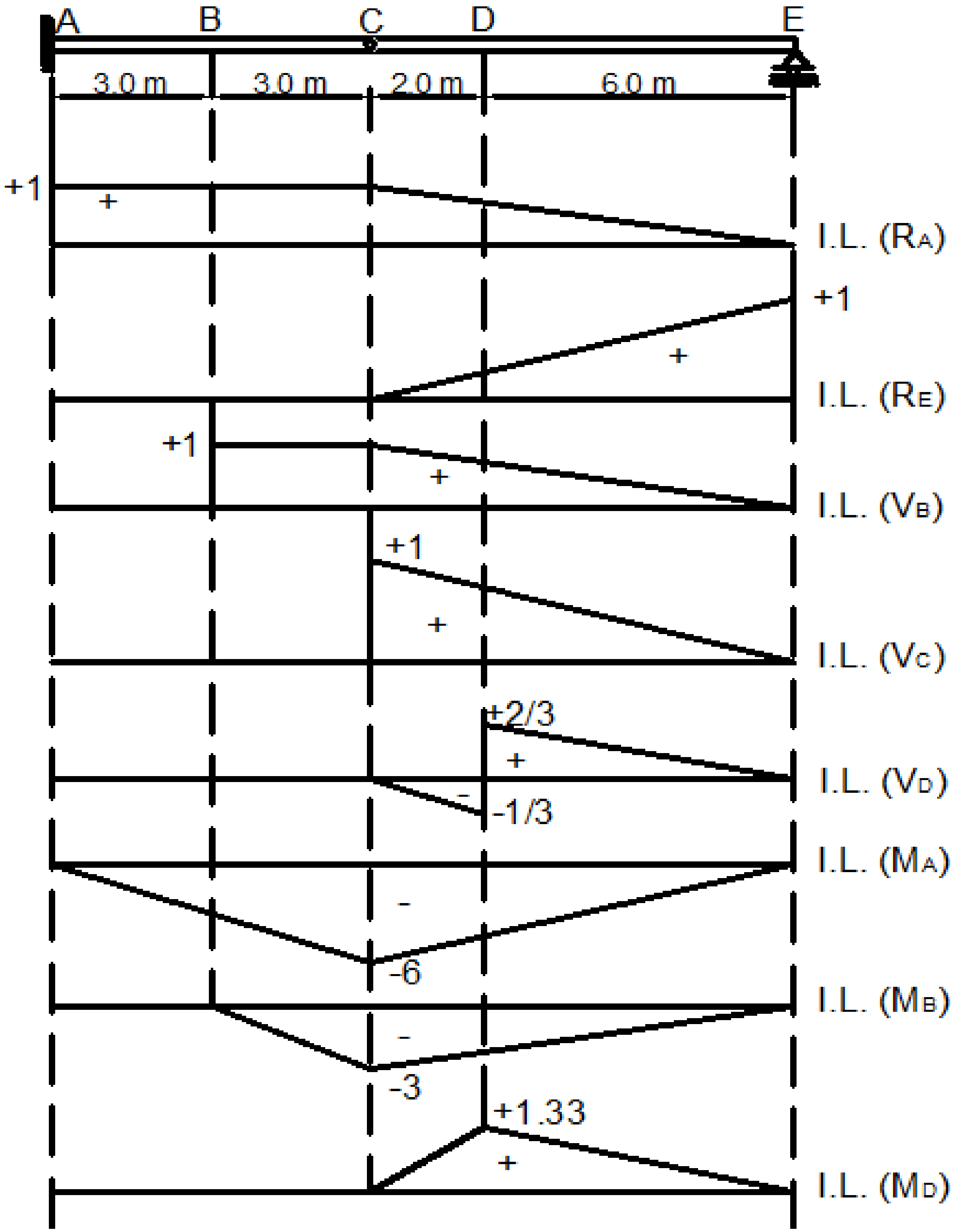
$$\theta = 1 \rightarrow \tan\theta = 1 \rightarrow \frac{y}{L} = 1 \rightarrow y = L$$



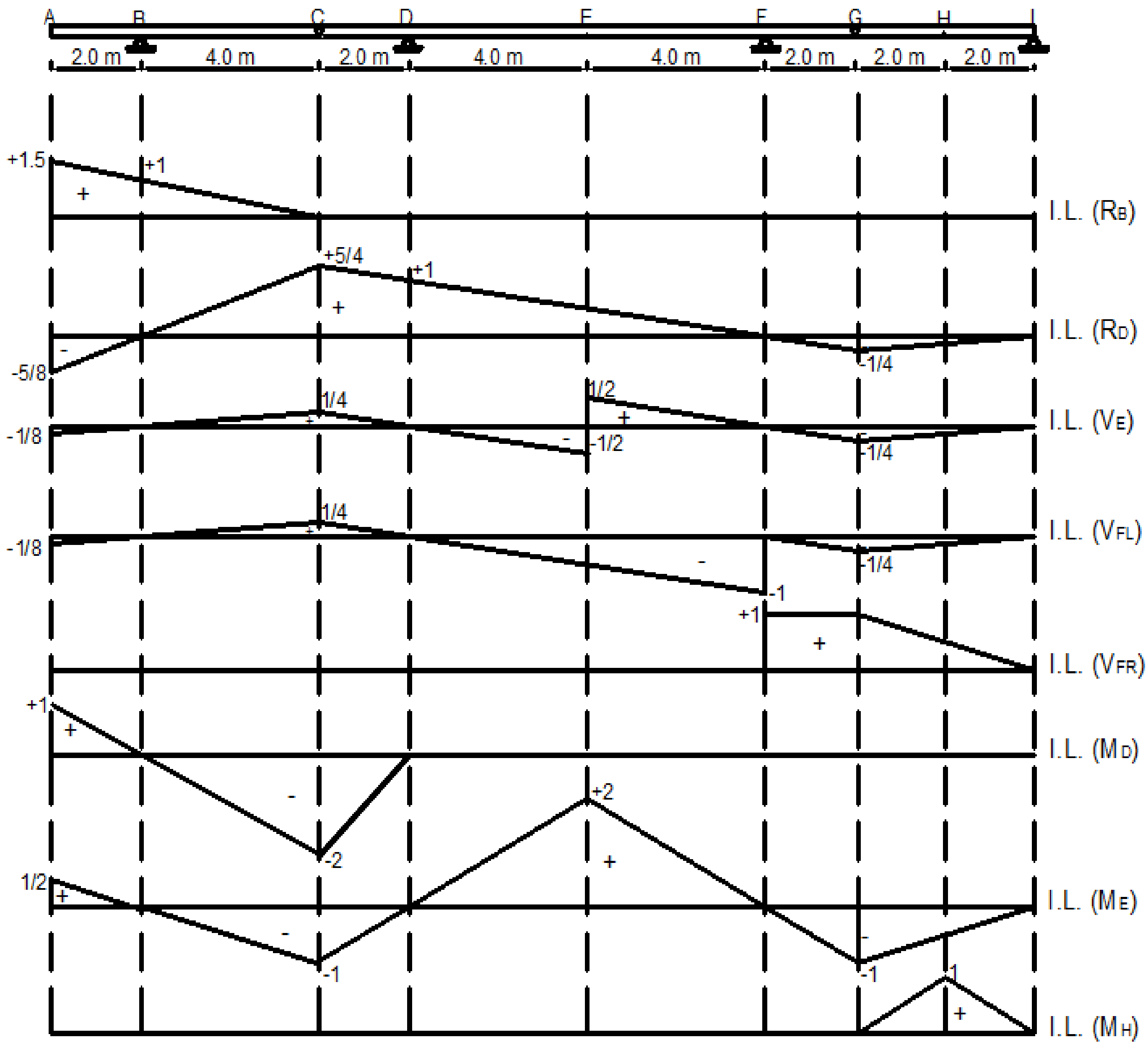
Example 2: Draw I.L. for (R_B, R_D, V_C, M_C, V_{BR} & M_B)



Example 3: Draw I.L. for (R_A, R_E, V_B, V_C, V_D , M_A, M_B & M_D)



Example 4: Draw I.L. for (R_B, R_D, V_E, V_{GL}, V_{GR} , M_D, M_E & M_H)



Maximum value of a response function

The maximum value of a response function (Moment, Shear or Reaction) can be produced by multiplying the considered load case by the corresponding I.L. under curve area. Since the applied load can be either concentrated (moving load) or uniformly distributed (dead and live load) the maximum value of the response function can be calculated using one of the forms shown below:

a- $f^-_{max} = P * max\ negative\ ordinate\ of\ the\ I.L$ (For a case of concentrated load)

b- $f^+_{max} = P * max\ posative\ ordinate\ of\ the\ I.L$ (For a case of concentrated load)

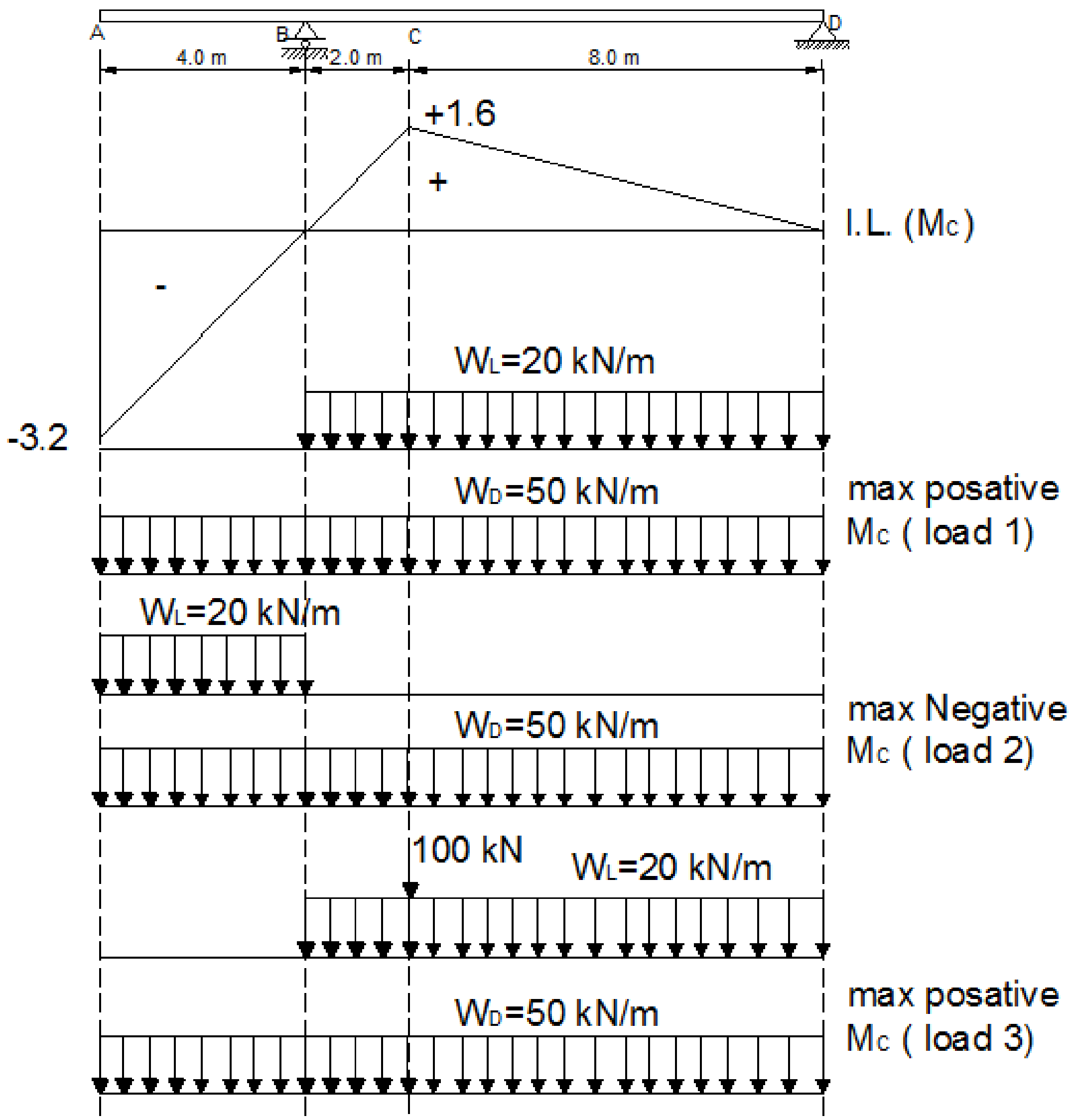
c- $f^-_{max} = W * \sum negative\ area\ of\ the\ I.L$ (For a case of uniform distributed load)

d- $f^+_{max} = W * \sum posative\ area\ of\ the\ I.L$ (For a case of uniform distributed load)

Example 1: Find the maximum moment at point C due to the following load combination:

1- D.L. = 50 kN/m + L.L= 20 kN/m (max positive moment).
2- D.L. = 50 kN/m + L.L= 20 kN/m (max negative moment).
3- D.L. = 50 kN/m + L.L= 20 kN/m + P= 100 kN (max positive moment)

Solution:



Load case 1

$$M_c^+ = (20+50)*\left(\frac{1.6*10}{2}\right) - 50*\left(\frac{3.2*4}{2}\right) = 240\ kN.m$$

Load case 2

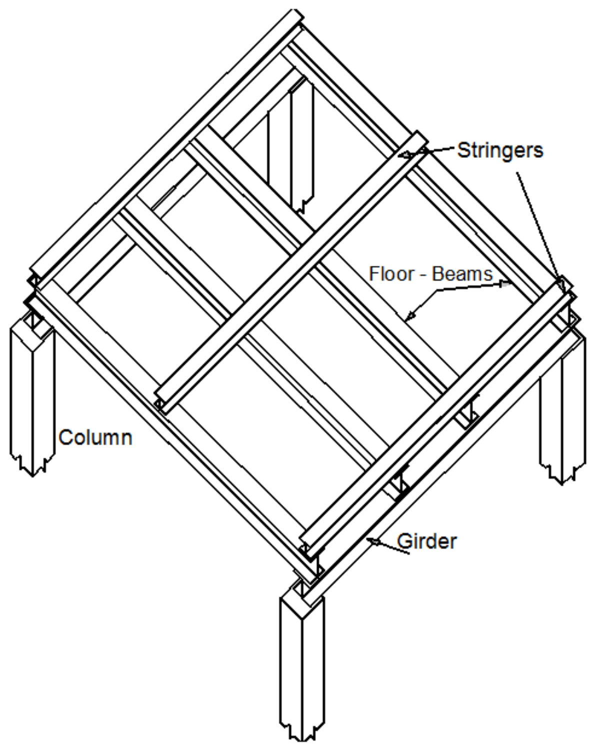
$$M_c^- = 50*\left(\frac{1.6*10}{2}\right) - (50+20)*\left(\frac{3.2*4}{2}\right) = -48\ kN.m$$

Load case 3

$$M_c^+ = (20+50)*\left(\frac{1.6*10}{2}\right) - 50*\left(\frac{3.2*4}{2}\right) + 100*1.6 = 400\ kN.m$$

Girder – Floor Beam – Stringer System

Some of structure types like bridges and stories have a large spans that's required a large slab thickness if it's constructed as a girder–slab system. Hence; these types are recommended to be constructed by girder–floor beam – stringer system.



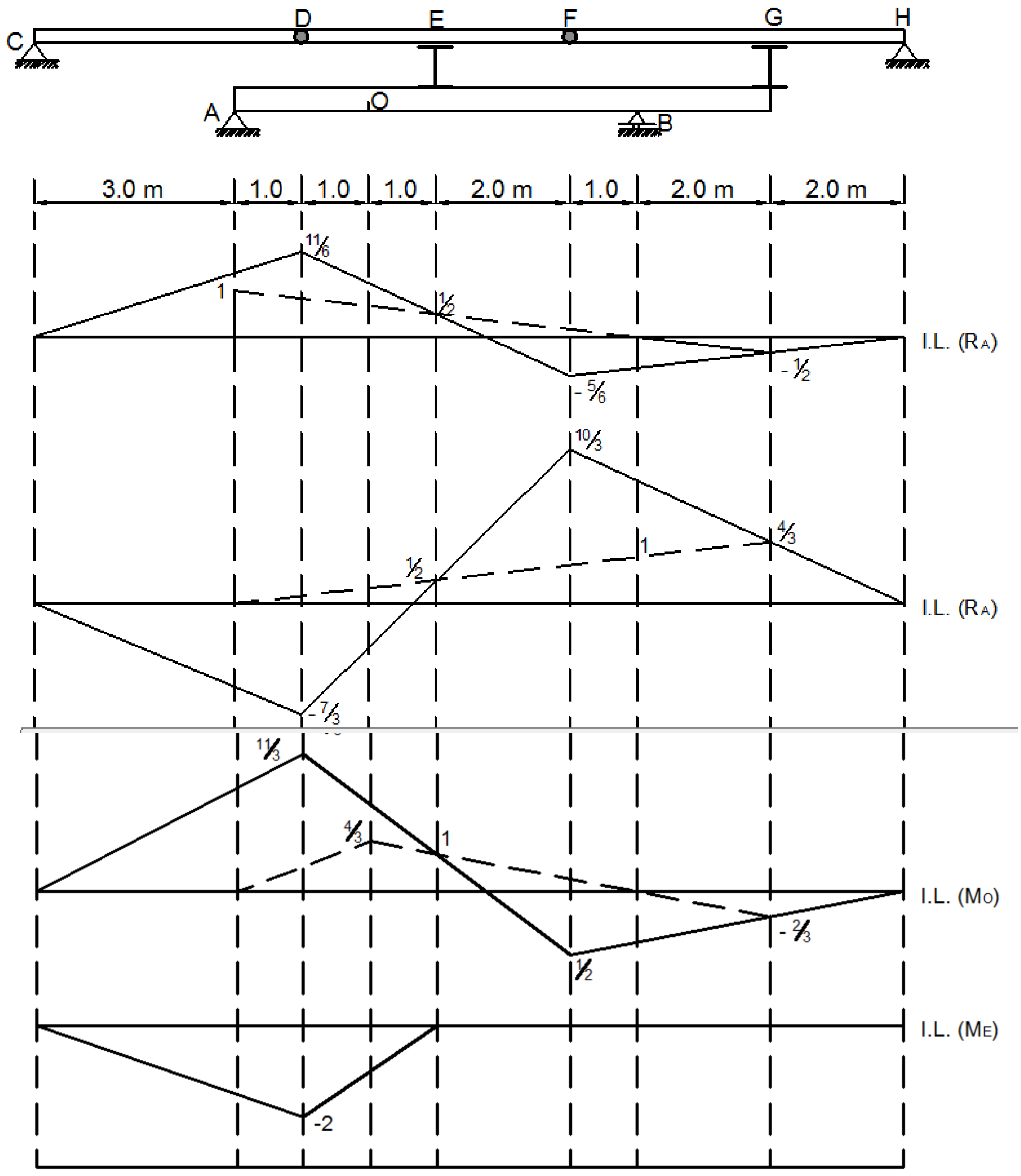
Advantage of the girder–floor beam – stringer system:

- Minimize the required thickness of the slab.
- Reduce the overall weight of the structure.
- Increase the strength of the constructed building.

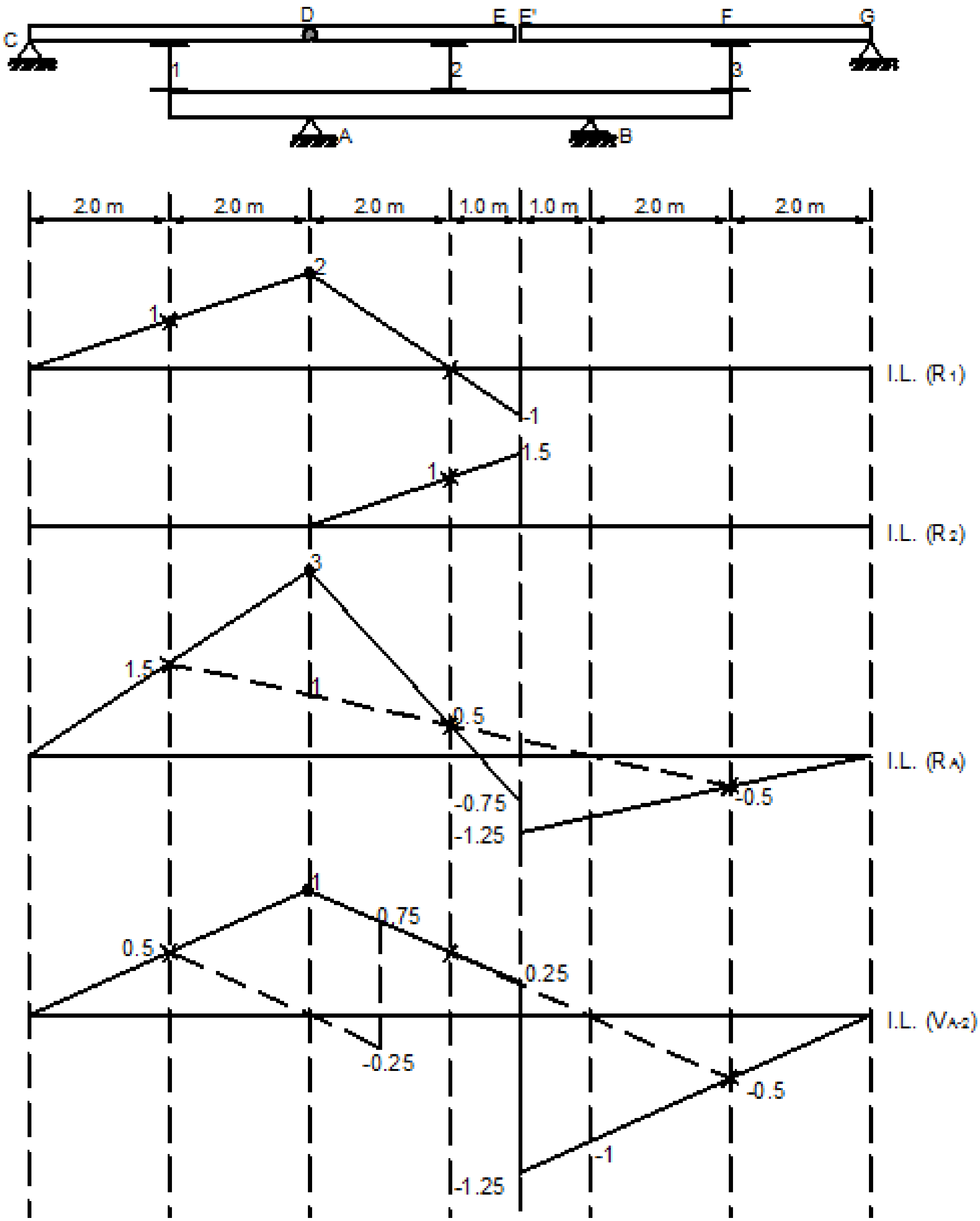
Steps of I. L. drawing in a case of Girder – Floor Beam - Stringer system

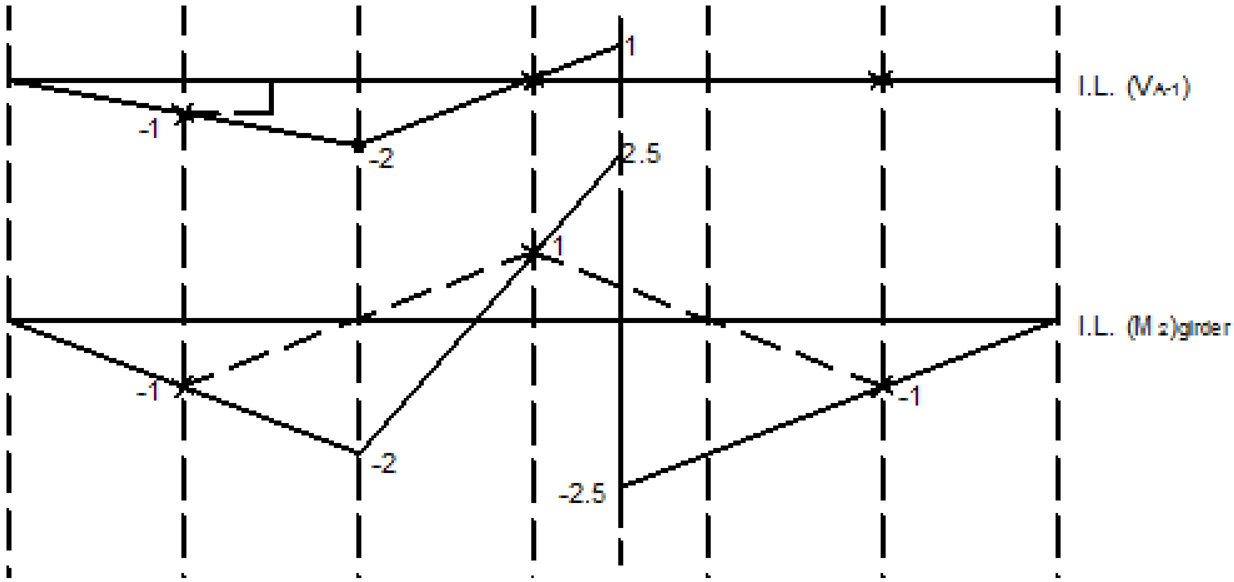
1- All the floor and stringer beams should be canceled.
2- Drawing the I. L of the corresponding response function for the girder.
3- Allocate all the floor beams on theirs corresponding locations along the path of the I.L.
4- Connecting the points of the floor beams locations to get the final shape of the I.L. (all the supports of the stringers are considered as zero elevated points on the path of the I.L.
5- The I.L. of any response function that's belong to the stringer is drawn traditionally.

Example 1: For the girder – floor beam – stringer system shown in below Draw I.L. for (R_A, R_B, , M_E & M_O)

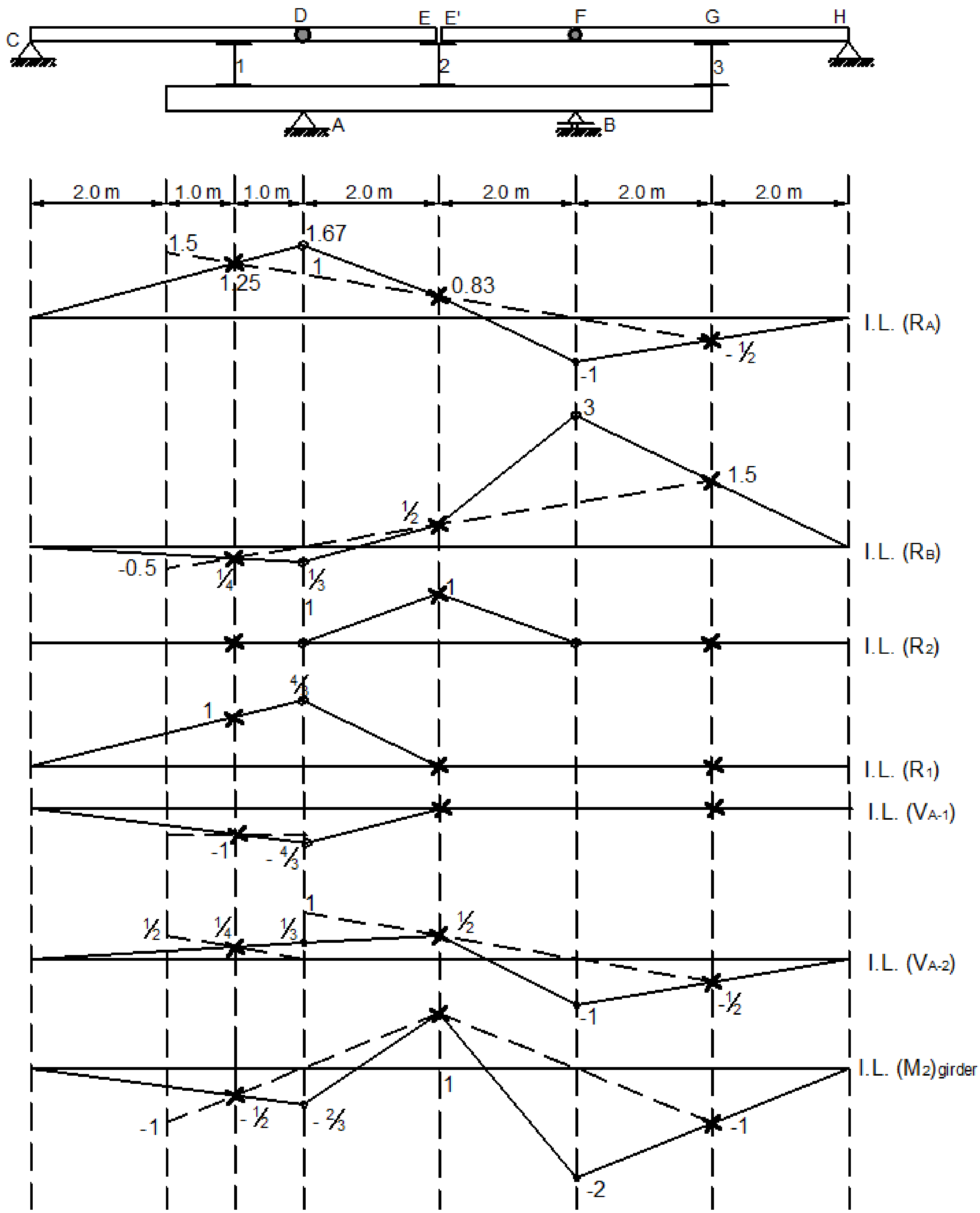


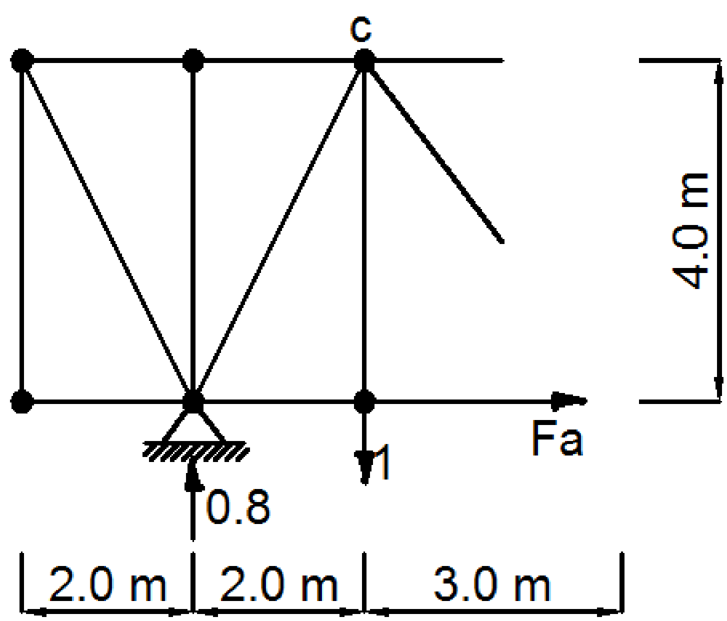
Example 2: For the girder – floor beam – stringer system shown in below Draw I.L. for (R_1, $R_{2,}$ R_A, R_B, V_{A-2}, $V_{A-1,}$ $M_{1\text{-girder}}$)

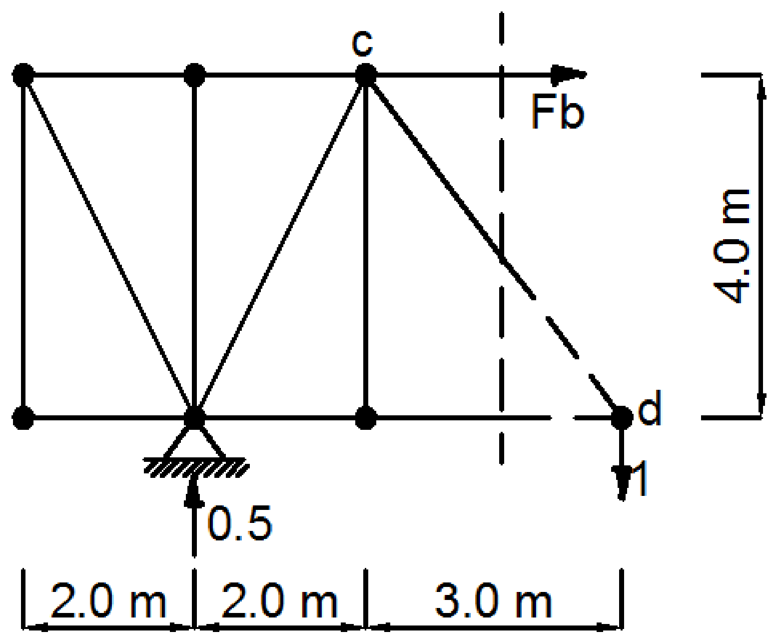


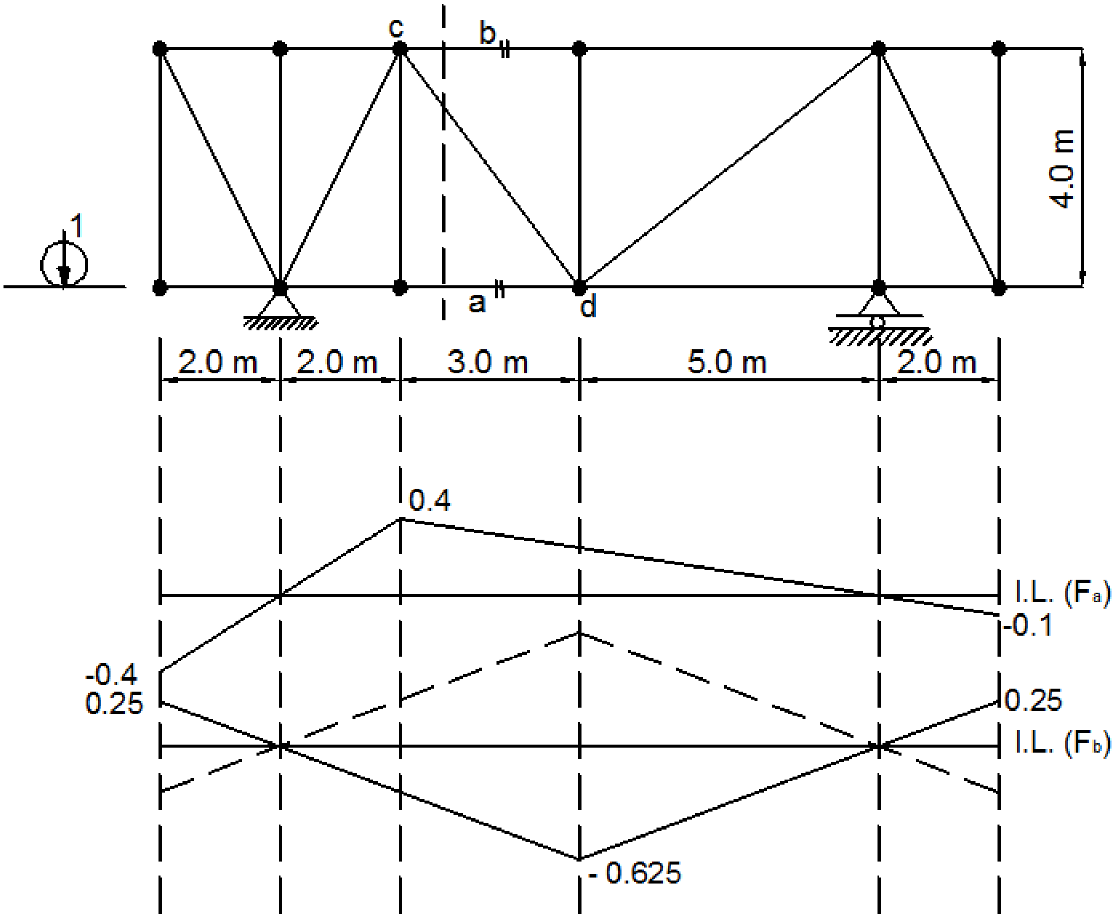

1
-1
-2
I.L. (VA-1)
2.5
1
I.L. (M 2)girder
-1
-2
-2.5
-1

Example 3: For the girder – floor beam – stringer system shown below Draw I.L. for (R_A, R_B, R_1, R_2, V_{A-2}, V_{A-1}, $M_{2\text{-girder}}$)




c
4.0 m
Fa
1
0.8
2.0 m
2.0 m
3.0 m


c
Fb
4.0 m
d
1
0.5
2.0 m
2.0 m
3.0 m


c
b
1
4.0 m
a
d
2.0 m
2.0 m
3.0 m
5.0 m
2.0 m
0.4
I.L. (Fa)
-0.1
-0.4
0.25
0.25
I.L. (Fb)
- 0.625