Chapter 10

Analysis of Statically Indeterminate Structures

Force Method of Analysis

Methods of Analysis

Two different Methods are available

- Force method
  - known as consistent deformation, unit load method, flexibility method, or the superposition equations method.
  - The primary unknowns in this way of analysis are forces
- Displacement method
  - Known as stiffness matrix method
  - The primary unknowns are displacements

	Unknowns	Equations Used for Solution	Coefficients of the Unknowns
Force Method	Forces	Compatibility and Force Displacement	Flexibility Coefficients
Displacement Method	Displacements	Equilibrium and Force Displacement	Stiffness Coefficients

Methods of Analysis

Force method of analysis

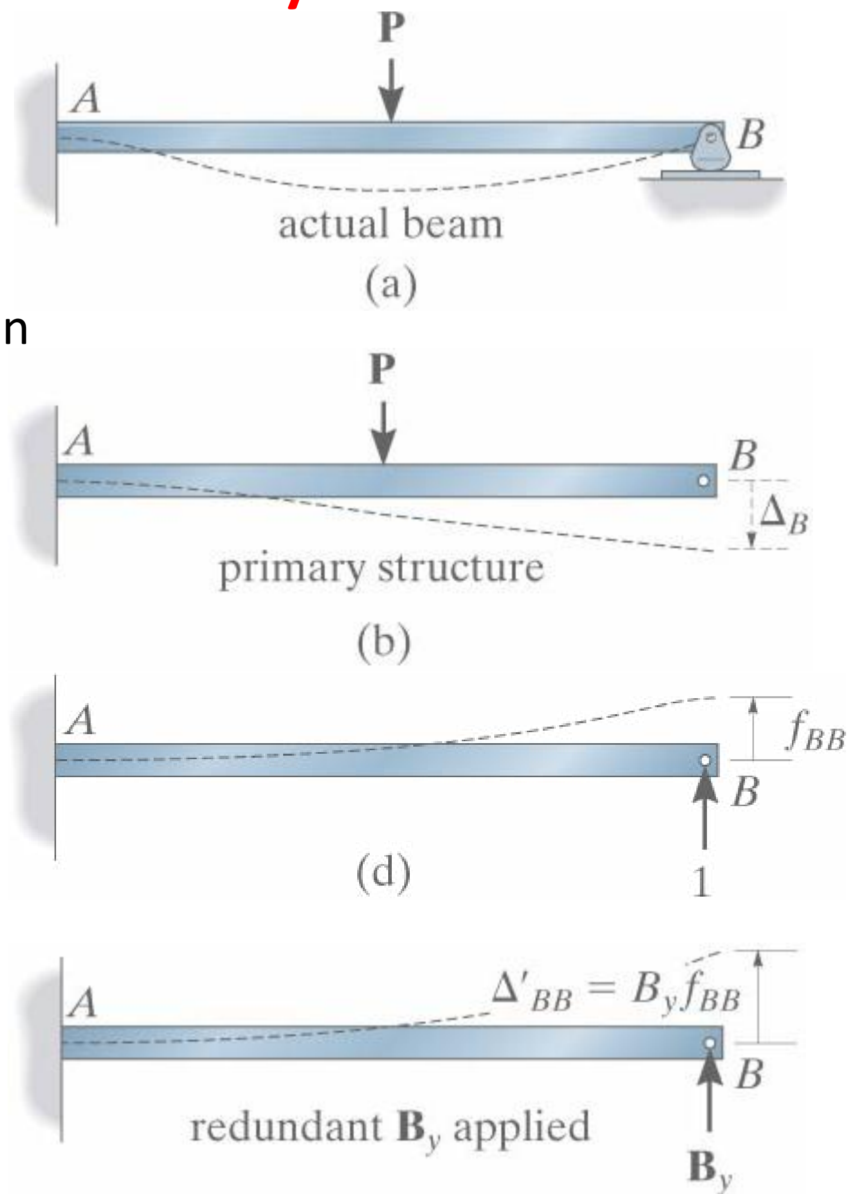
The deflection or slope at any point on a structure as a result of a number of forces, including the reactions, is equal to the algebraic sum of the deflections or slopes at this particular point as a result of these loads acting individually

Force Method of Analysis

- General Procedure
  - Indeterminate to the first degree
  - 1 Compatibility equation is needed
  - Choosing one of the support reaction as a redundant
  - The structure become statically determinate & stable
  - Downward displacement Δ_B at B calculated (load action)
  - f_{BB} upward deflection per unit force at B
  - Compatibility equation

$$0 = \Delta_B + B_y f_{BB}$$

  - Reaction B_y known
  - Now the structure is statically determinate

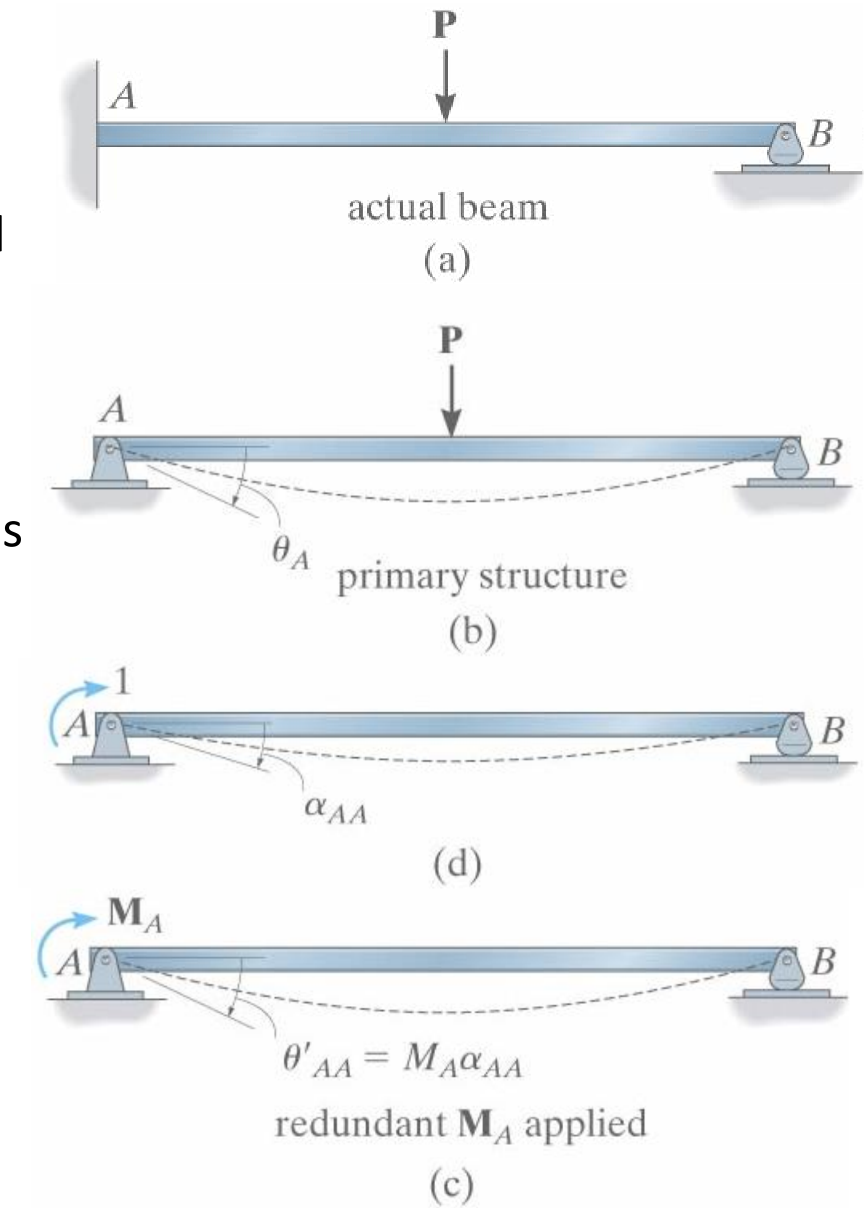


Force Method of Analysis

- General Procedure
  - Indeterminate to the first degree
  - 1 Compatibility equation is needed
  - Choosing M_A at A as a redundant
  - The structure become statically determinate & stable
  - Rotation θ_A at A caused by load P is determined
  - α_{AA} rotation caused by unit couple moment applied at A
  - Compatibility equation

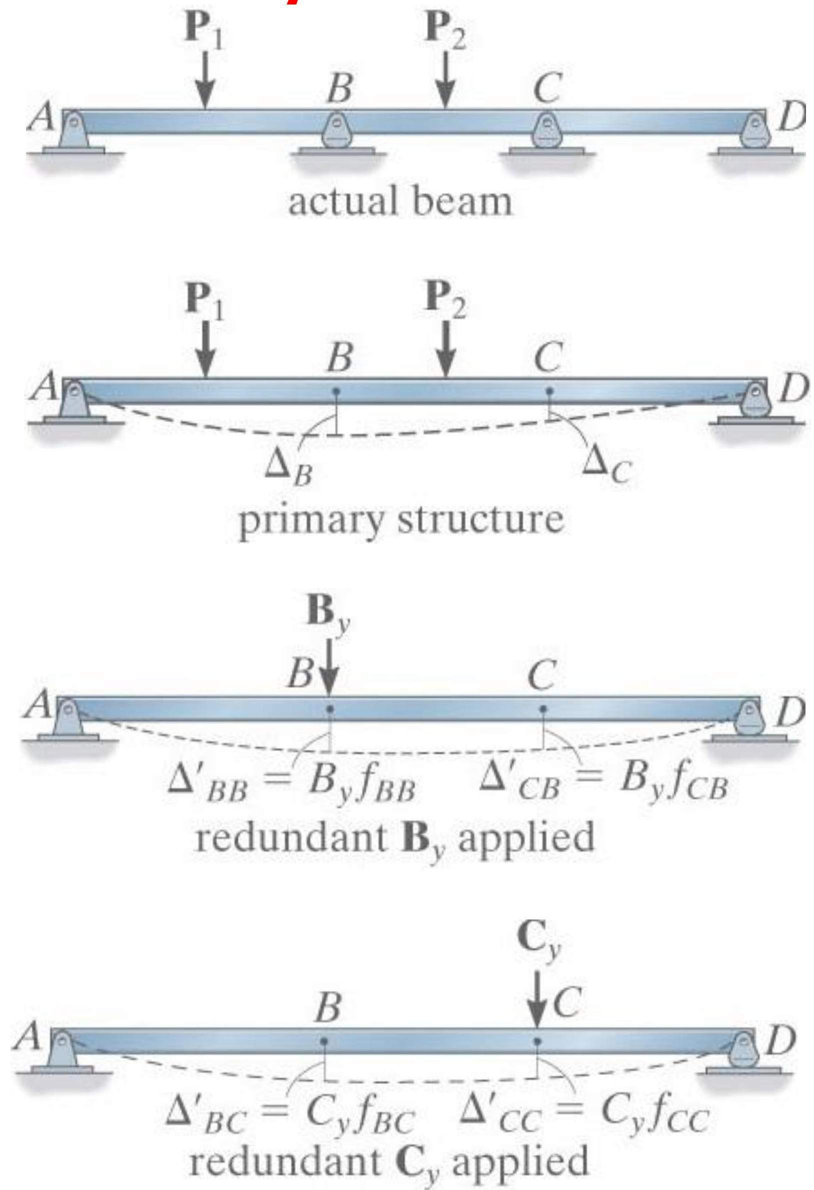
$$0 = \theta_A + M_A \alpha_{AA}$$

  - Moment M_A known
  - Now the structure is statically determinate



Force Method of Analysis

- General Procedure
  - Indeterminate to the 2nd degree
  - 2 Compatibility equations needed
  - Redundant reaction B & C
  - Displacement Δ_B & Δ_C caused by load P_1 & P_2 are determined



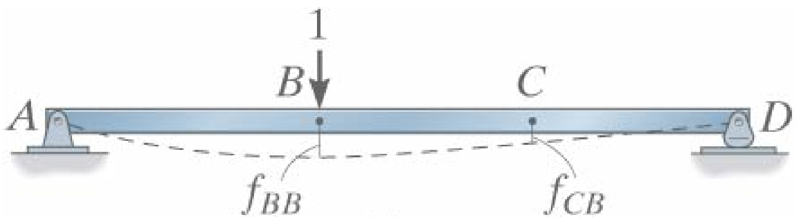
Force Method of Analysis

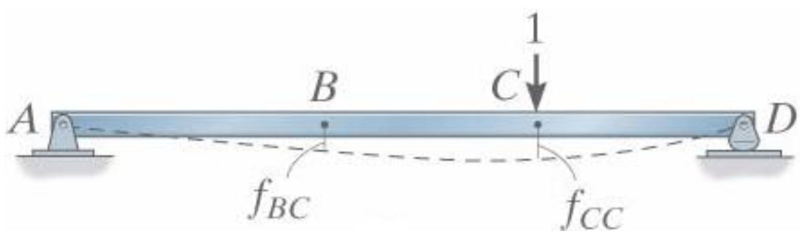
- General Procedure
  - f_{BB} & f_{BC} Deflection per unit force at B are determined
  - f_{CC} & f_{CB} Deflection per unit force at C are determined
  - Compatibility equations

$$0 = \Delta_B + B_y f_{BB} + C_y f_{BC}$$

$$0 = \Delta_C + B_y f_{CB} + C_y f_{CC}$$

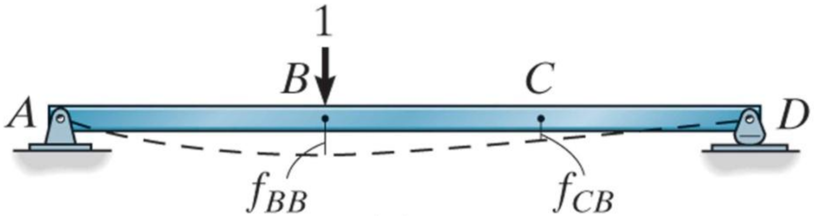
  - Reactions at B & C are known
  - Statically determinate structure





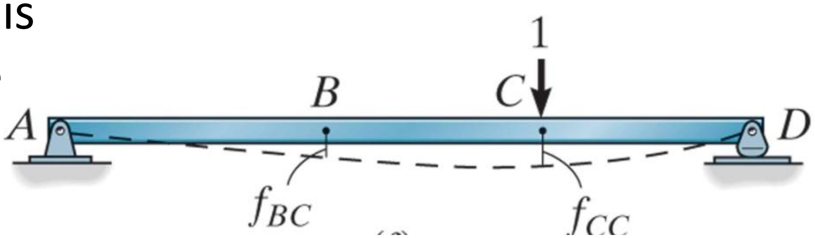
Maxwell's Theorem

- The displacement of a point B on a structure due to a unit load acting at a point A is equal to the displacement of point A when the unit load is acting at point B the is



$$f_{BC}=f_{CB}$$

- The rotation of a point B on a structure due to a unit moment acting at a point A is equal to the rotation of point A when the unit moment is acting at point B the is

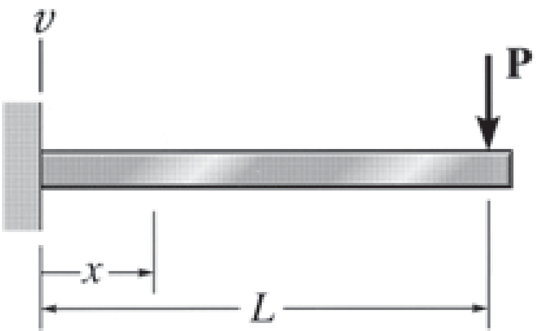


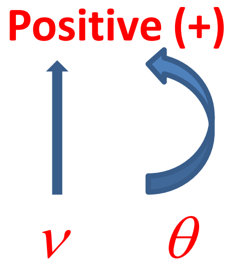
$$\alpha_{BC}=\alpha_{CB}$$

Force Method of Analysis

- Procedure for Analysis
  - Determine the degree of statically indeterminacy
  - Identify the redundants, whether it's a force or a moment, that would be treated as unknown in order to form the structure statically determinate & stable
  - Calculate the displacements of the determinate structure at the points where the redundants have been removed
  - Calculate the displacements at these same points in the determinate structure due to the unit force or moment of each redundants individually
  - Workout the compatibility equation at each point where there is a redundant & solve for the unknown redundants
  - Knowing the value of the redundants, use equilibrium to determine the remaining reactions

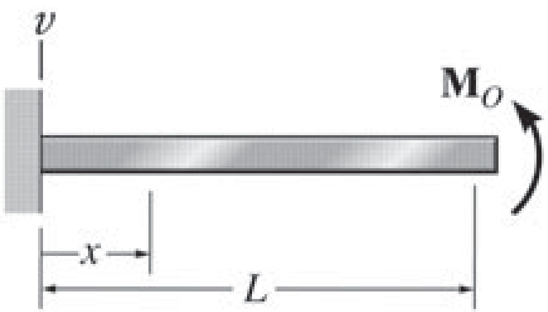
Beam Deflections and Slopes





$$v = \frac{P}{6EI}\left(x^3 - 3Lx^2\right)$$

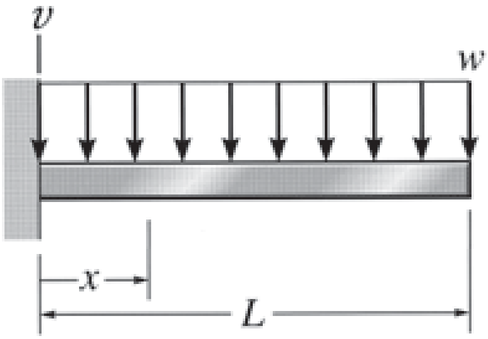
$$at\,x = L \qquad v_{\max} = -\frac{PL^3}{3EI} \qquad \theta_{\max} = -\frac{PL^2}{2EI}$$

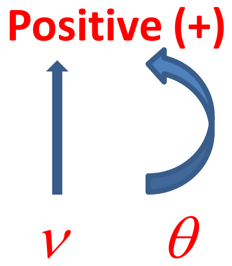


$$v = \frac{M_0}{2EI}x^2$$

$$at\,x = L \qquad v_{\max} = \frac{M_0L^2}{2EI} \qquad \theta_{\max} = \frac{M_0L}{EI}$$

Beam Deflections and Slopes





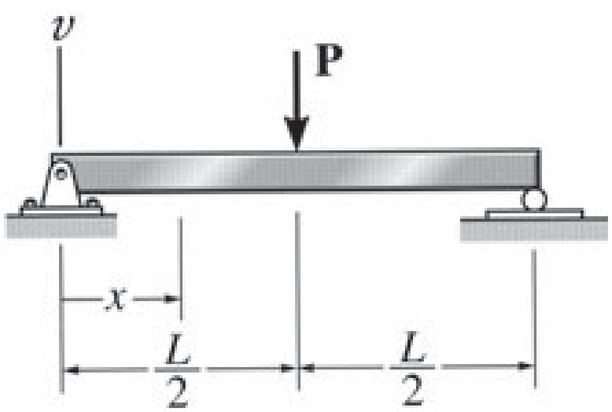
$$v = -\frac{w}{24EI}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$

$$at\,x = L \qquad v_{max} = -\frac{wL^4}{8EI} \qquad \theta_{max} = -\frac{wL^3}{6EI}$$

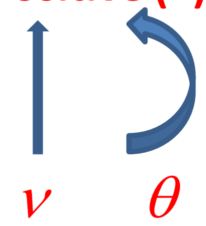
$$v = \frac{P}{48EI}\left(4x^3 - 3L^2x\right), \;\; 0 \le x \le \frac{L}{2}$$

$$at\,x = \frac{L}{2} \qquad v_{max} = -\frac{PL^3}{48EI}$$

$$at\,x = 0 \text{ or } \frac{L}{2} \qquad \theta_{max} = \pm\frac{PL^2}{16EI}$$

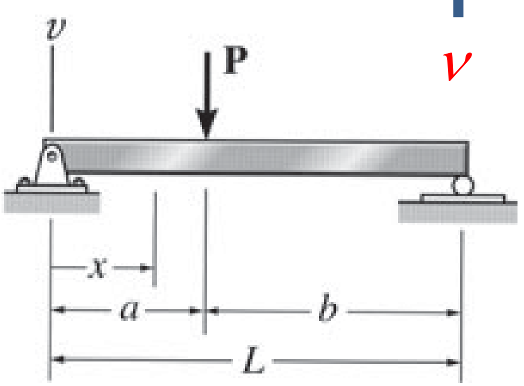


Beam Deflections and Slopes



$$v = -\frac{Pbx}{6LEI}\left(L^2 - b^2 - x^2\right), \qquad 0 \le x \le a$$

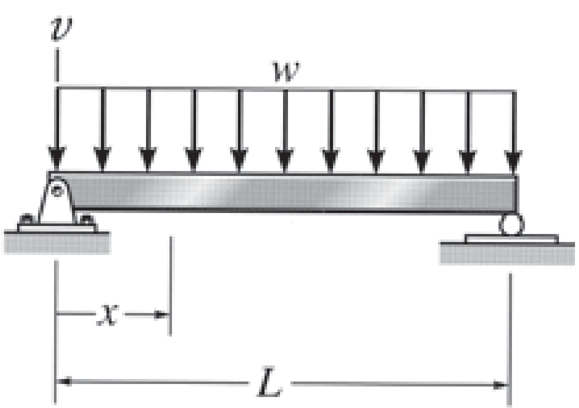
$$\theta_L = -\frac{Pab\left(L+b\right)}{6LEI} \qquad \theta_R = \frac{Pab\left(L+a\right)}{6LEI}$$



$$v = -\frac{wx}{24EI}\left(x^3 - 2Lx^2 + L^3\right)$$

$$at\, x = \frac{L}{2} \qquad v_{\max} = -\frac{5wL^4}{384EI}$$

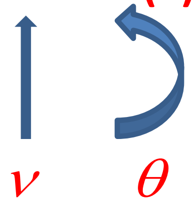
$$\theta_{\max} = \pm\frac{wL^3}{24EI}$$



Beam Deflections and Slopes

$$v = -\frac{wx}{384EI}\left(9L^3 - 24Lx^2 + 16x^3\right) \qquad 0 \le x \le L/2$$

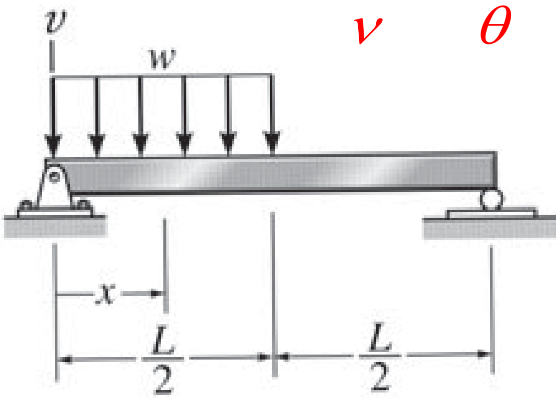
Positive (+)



$$v = -\frac{wL}{384EI}\left(8x^3 - 24Lx^2 + 17L^2x - L^3\right)$$

$$L/2 \le x \le L$$

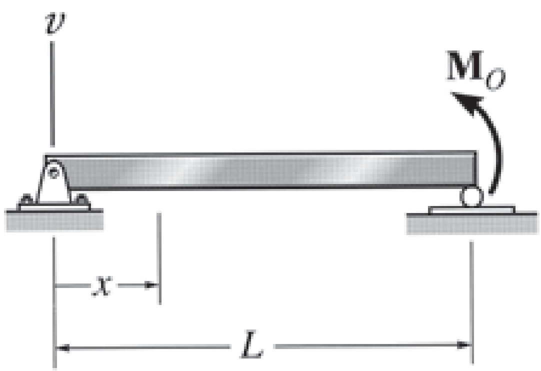
$$\theta_L = -\frac{3wL^3}{128EI} \qquad \theta_R = \frac{7wL^3}{384EI}$$



$$v = \frac{M_0 x}{6EIL}\left(x^2 - 3Lx + 2L^2\right)$$

$$v_{\max} = -\frac{M_0 L^2}{9\sqrt{2}EI}$$

$$\theta_L = -\frac{M_0 L}{6EI} \qquad \theta_R = \frac{M_0 L}{3EI}$$



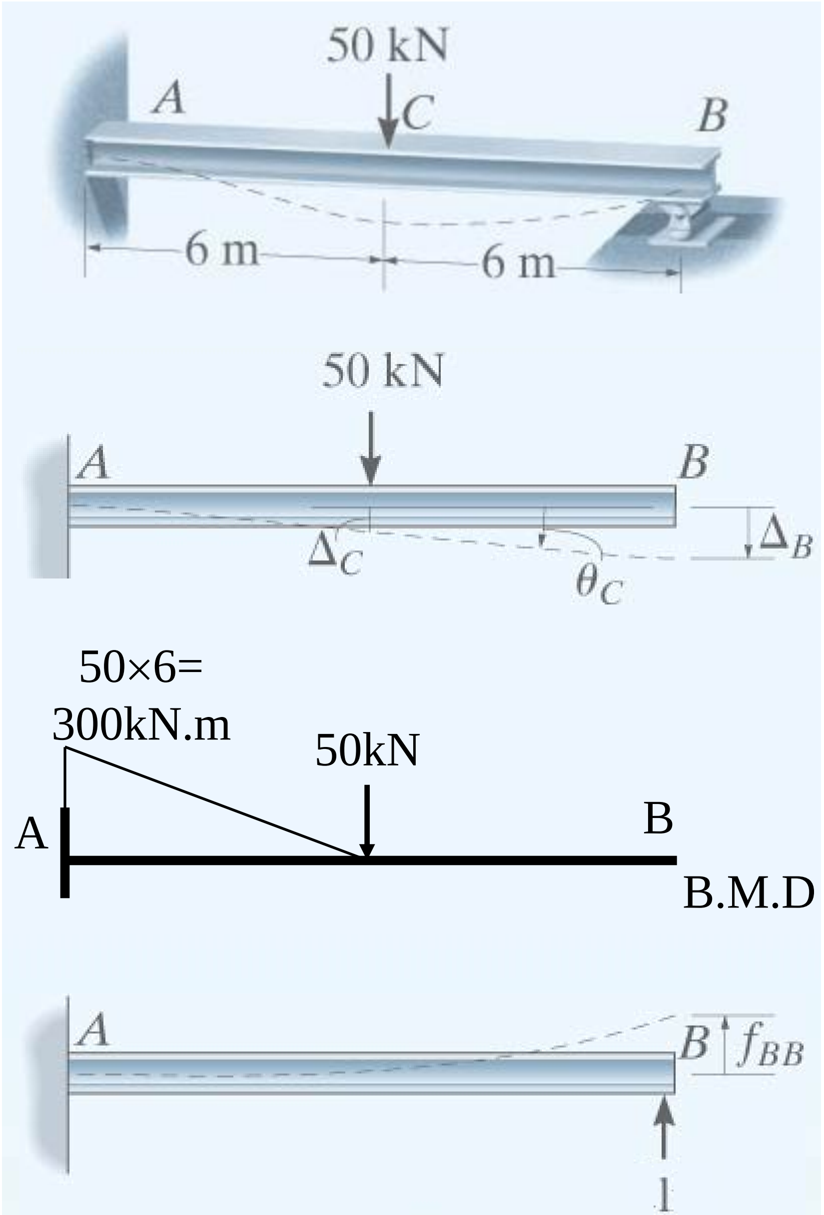
Example 1

- Determine the reaction at B
  - Indeterminate to the 1st degree thus one additional equation needed
  - Lets take B as a redundant
  - Determine the deflection at point B in the absence of support B. Using the moment-area method

$$\Delta_B = \frac{1}{EI}\left[300 \times \frac{6}{2} . \left(\frac{2}{3}6 + 6\right)\right]$$

$$\Delta_B = \frac{9000kN.m^3}{EI} \downarrow$$

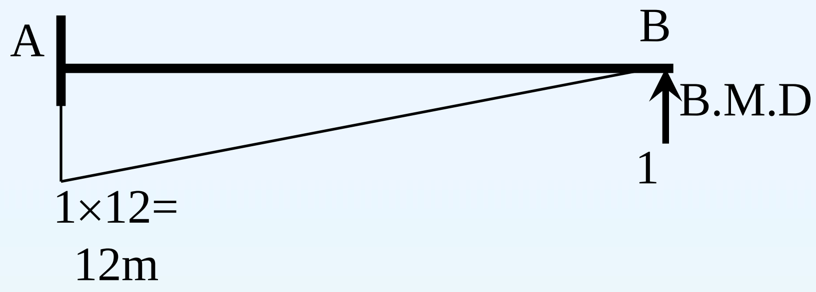
  - Determine the deflection caused by the unit load at point B



Example 1

$$f_{BB} = \frac{1}{EI}\left(12\times\frac{12}{2}\times\frac{2}{3}\times 12\right)$$

$$f_{BB} = \frac{576m^3}{EI}\uparrow$$



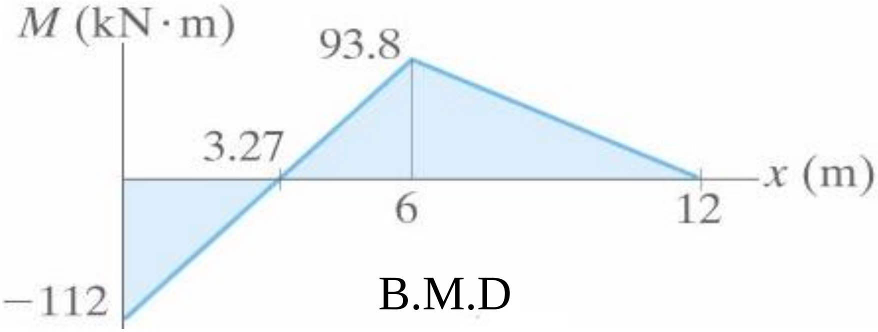
– Compatibility equation

$0 = \Delta_B + B_y f_{BB}$

$$0 = -\frac{9000}{EI} + B_y\frac{576}{EI}$$

$B_y = 15.6kN$

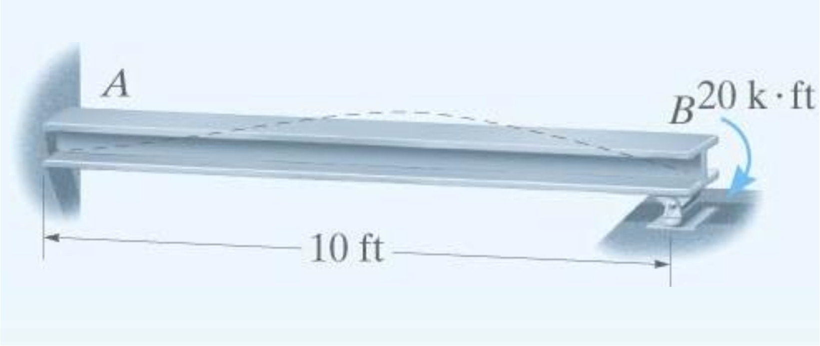
– The reaction at B is known now so the structure is statically determinate & equilibrium equations can be applied to get the rest of the unknowns

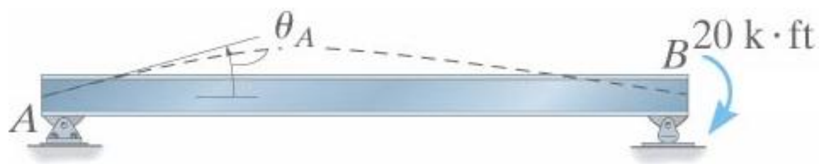


B.M.D

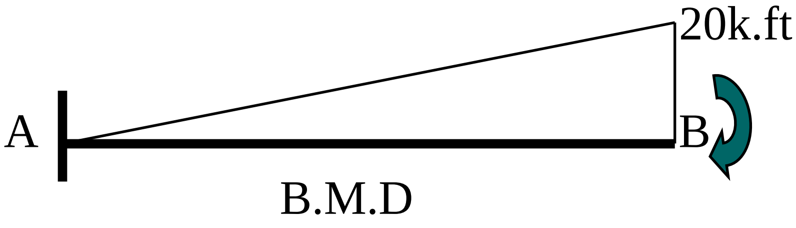
Example 2

- Determine the moment at A
  - Indeterminate to the 1st degree thus one additional equation needed
  - Lets take M_A as a redundant
  - Determine the slope θ_A at point A ignoring the fixation at A. Using the moment-area method



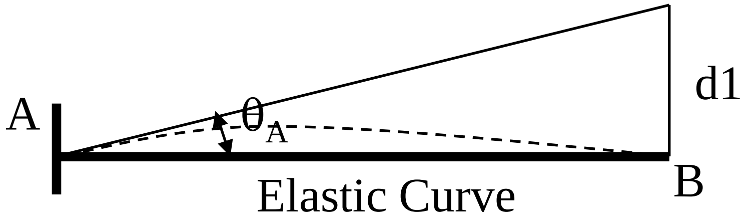


$$d_1 = \frac{1}{EI}\left(20 \times \frac{10}{2} \times \frac{10}{3}\right) = \frac{333.33}{EI}$$



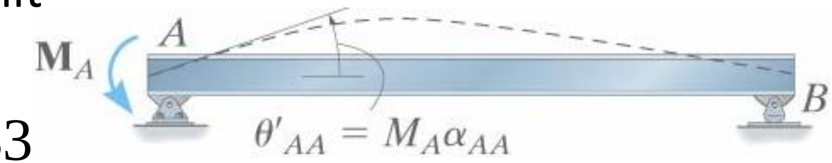
$$\theta_A = \frac{d_1}{L} = \frac{1}{EI} . \frac{333.33}{10}$$

$$\theta_A = \frac{33.33k.ft^2}{EI}$$



Example 2

- Determine rotation caused by the unit moment applied at A



$$d_2 = \frac{1}{EI}\left(1 \times \frac{10}{2} \times \frac{2 \times 10}{3}\right) = \frac{33.33}{EI}$$

$$\alpha_{AA} = \frac{d_2}{L} = \frac{1}{EI} . \frac{33.33}{10} = \frac{3.33ft}{EI}$$

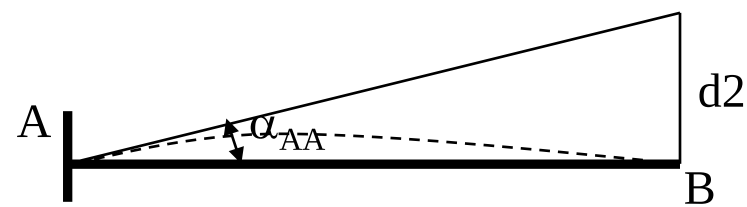
B.M.D

- Compatibility equation

$0 = \theta_A + M_A \alpha_{AA}$

$$0 = \frac{33.33}{EI} + M_A \frac{3.33}{EI}$$

M_A = -10k.ft

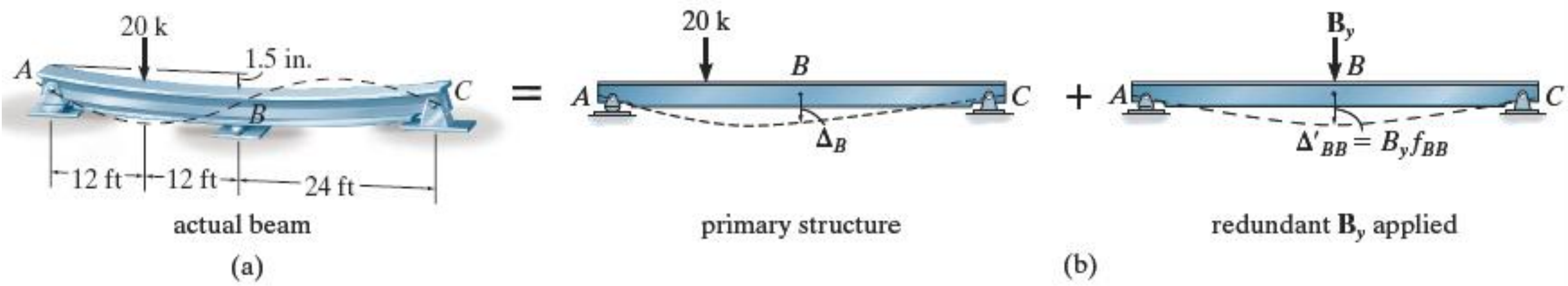


Elastic Curve

- The moment at A is known now so the structure is statically determinate

Example 3

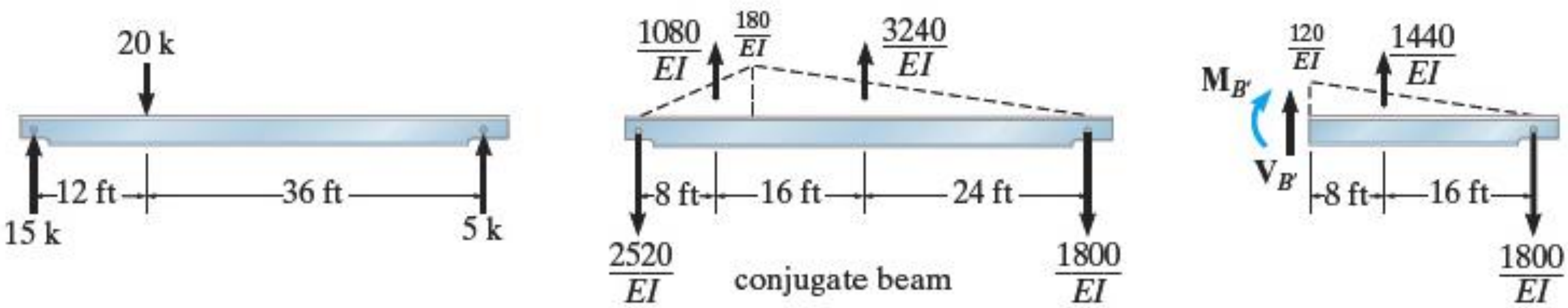
Draw the shear and moment diagrams for the beam shown in Fig. 10–10*a*. The support at *B* settles 1.5 in. Take $E = 29(10^3)$ ksi, $I = 750 \text{ in}^4$.



Example 3

Compatibility Equation. With reference to point B in Fig. 10–10b, using units of ft, we require

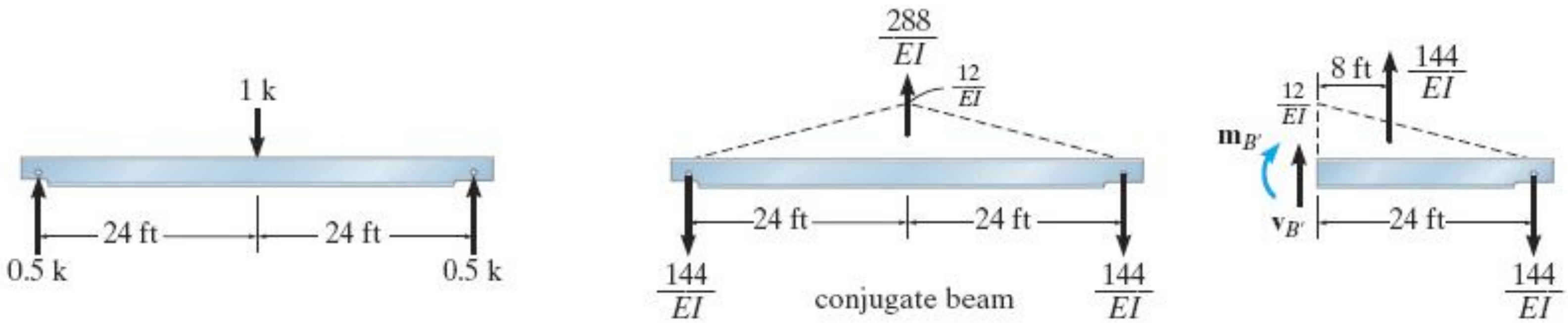
$$(+\downarrow) \qquad \frac{1.5}{12} = \Delta_B + B_y f_{BB} \qquad (1)$$



$$\circlearrowright + \Sigma M_{B'} = 0; \qquad -M_{B'} + \frac{1440}{EI}(8) - \frac{1800}{EI}(24) = 0$$

$$M_{B'} = -\frac{31\,680}{EI} = \frac{31\,680}{EI} \circlearrowleft$$

Example 3

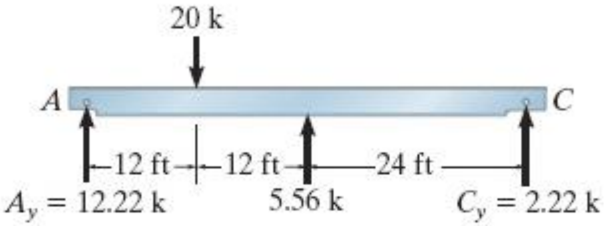


(d)

Verify the calculations in Fig. 10–10*d* for calculating f_{BB}. Note that

$$\circlearrowright + \Sigma M_{B'} = 0; \qquad -m_{B'} + \frac{144}{EI}(8) - \frac{144}{EI}(24) = 0$$

$$m_{B'} = -\frac{2304}{EI} = \frac{2304}{EI} \circlearrowleft$$



(e)

Example 3

Substituting these results into Eq. (1), we have

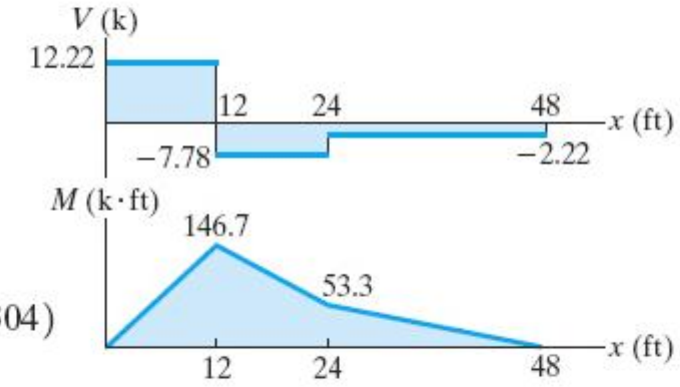
$$\frac{1.5}{12} = \frac{31\,680}{EI} + B_y\left(\frac{2304}{EI}\right)$$

Expressing the units of E and I in terms of k and ft, we have

$$\left(\frac{1.5}{12}\text{ft}\right)[29(10^3)\text{ k/in}^2((12)^2\text{ in}^2/\text{ft}^2)][750\text{ in}^4(\text{ft}^4/(12)^4\text{ in}^4)]$$

$$= 31\,680 + B_y(2304)$$

$$B_y = -5.56\text{ k}$$



(f)

Equilibrium Equations. The negative sign indicates that $\mathbf{B}_y$ acts *upward* on the beam. From the free-body diagram shown in Fig. 10–10*e* we have

$$\circlearrowleft + \Sigma M_A = 0; \qquad -20(12) + 5.56(24) + C_y(48) = 0$$

$$C_y = 2.22\text{ k}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 20 + 5.56 + 2.22 = 0$$

$$A_y = 12.22\text{ k}$$

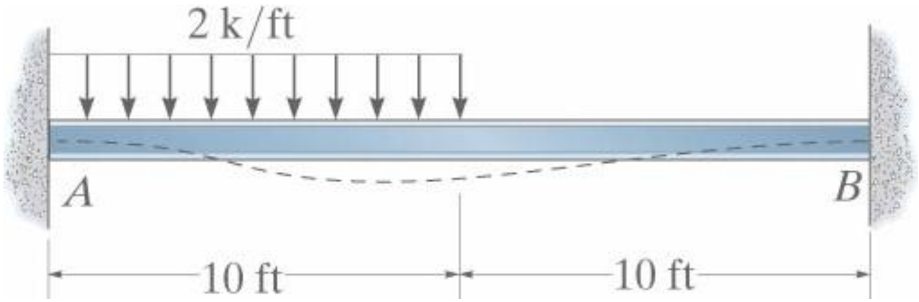
Using these results, verify the shear and moment diagrams shown in Fig. 10–10*f*.

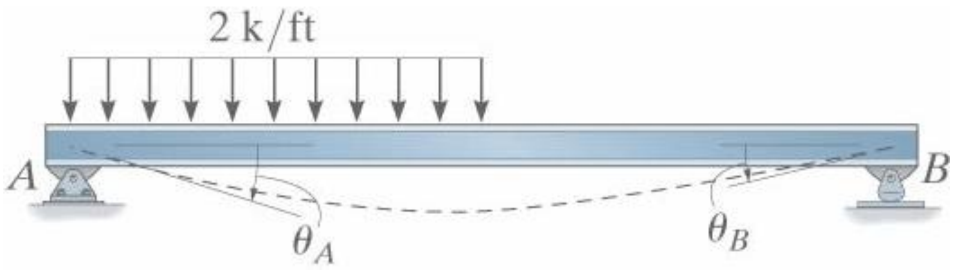
Example 4

- Neglect the axial load
- The end moments at A&B will be considered as redundants
- From Table inside the front cover

$$\theta_A = \frac{1}{EI}(375)$$

$$\theta_B = \frac{1}{EI}(291.7)$$





Example 4

-

$$\alpha_{AA} = \frac{6.67}{EI}$$

$$\alpha_{BA} = \frac{3.33}{EI}$$

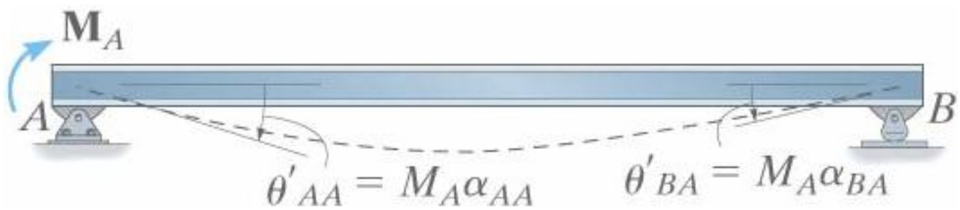
$$\alpha_{BB} = \frac{6.67}{EI}$$

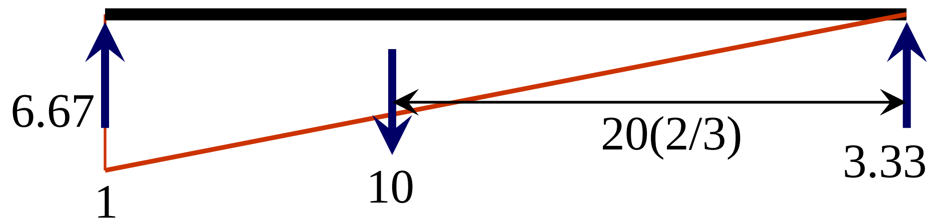
$$\alpha_{AB} = \frac{3.33}{EI}$$

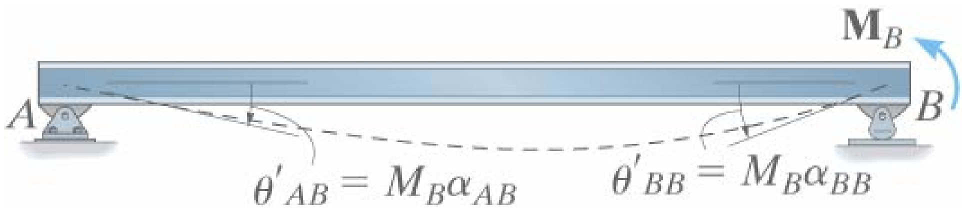
$$0 = \frac{375}{EI} + M_A\left(\frac{6.67}{EI}\right) + M_B\left(\frac{3.33}{EI}\right)$$

$$0 = \frac{291.7}{EI} + M_A\left(\frac{3.33}{EI}\right) + M_B\left(\frac{6.67}{EI}\right)$$

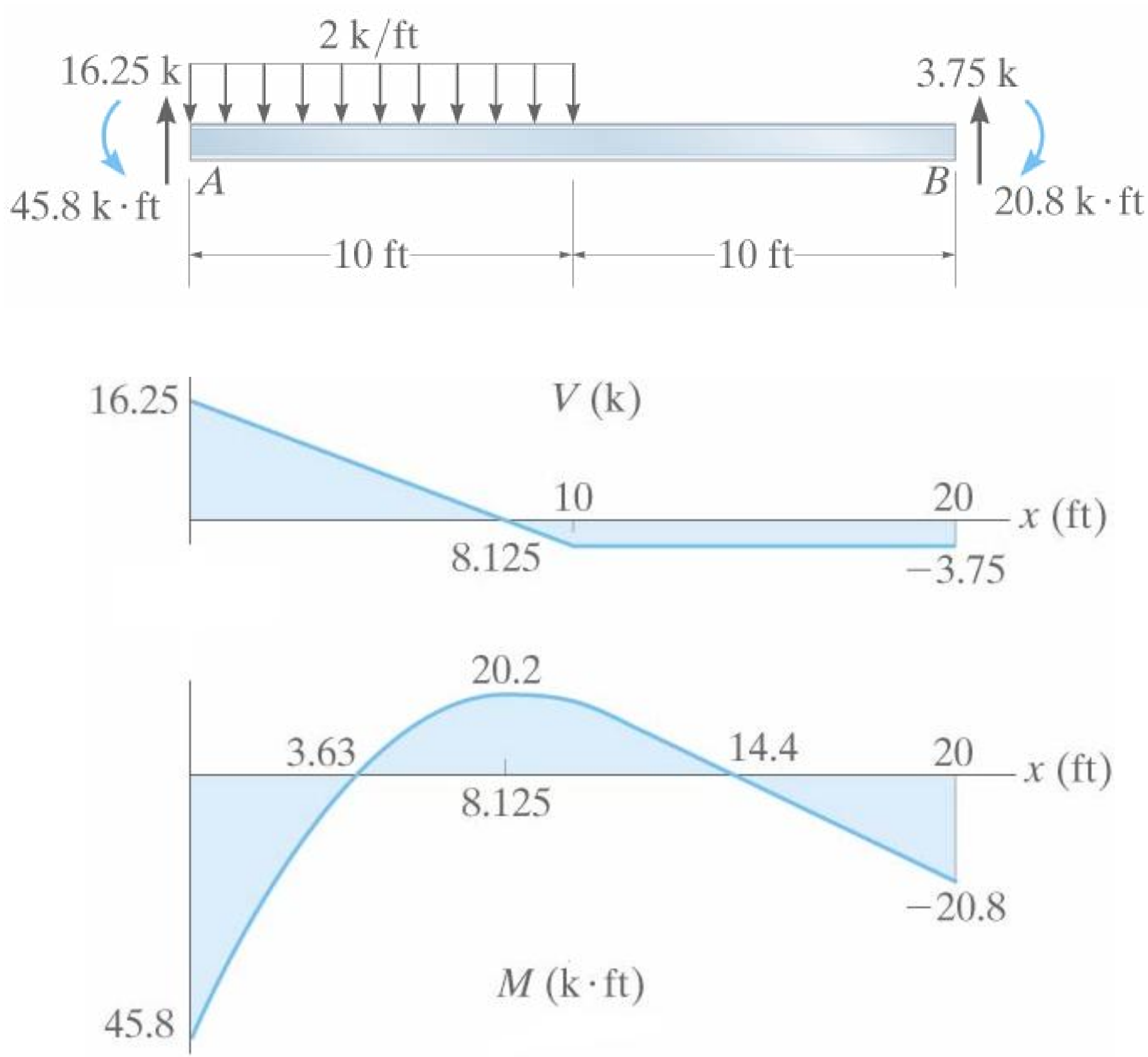
$$M_A = -45.8k.ft \qquad M_B = -20.8k.ft$$





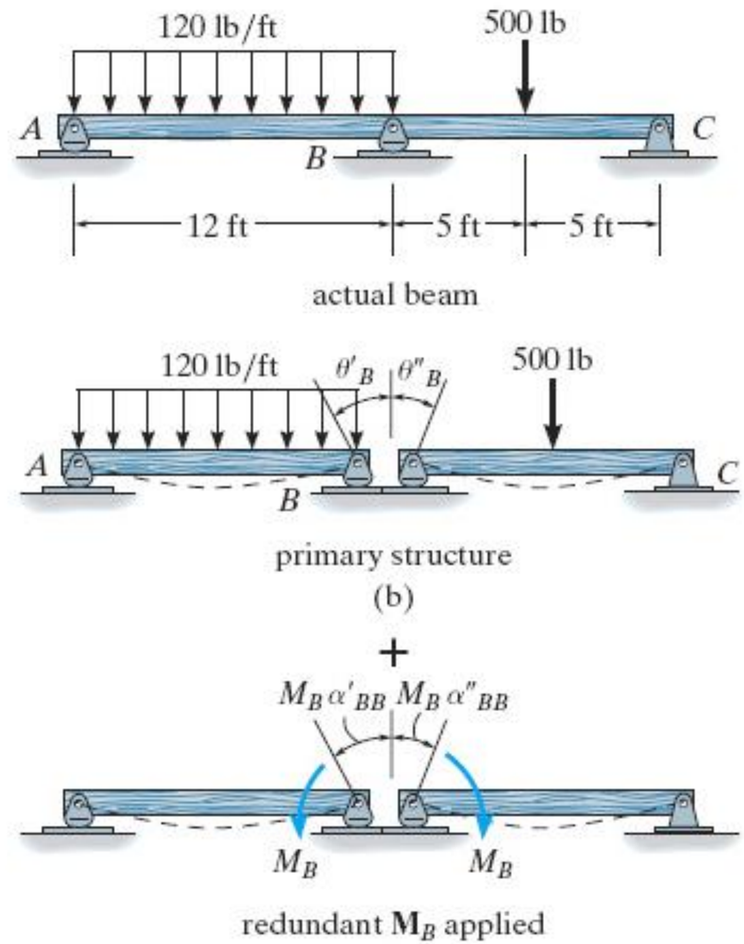


Example 4


2 k/ft
16.25 k
3.75 k
45.8 k·ft
20.8 k·ft
A
B
10 ft
10 ft
16.25
V (k)
10
20
x (ft)
8.125
−3.75
20.2
3.63
14.4
20
x (ft)
8.125
−20.8
M (k·ft)
45.8

Example 5

Determine the reaction at the support for the beam shown, EI is Constant.
Choose the internal moment at Internal support as the redundant



Compatibility Equations

$(\curvearrowleft +)$ $$\theta_B + M_B\alpha_{BB} = 0$$

where

$$\theta_B = \theta'_B + \theta''_B$$

and

$$\alpha_{BB} = \alpha'_{BB} + \alpha''_{BB}$$

Example 5

The slopes and angular flexibility coefficients can be determined from the table on the inside front cover, that is,

$$\theta'_B = \frac{wL^3}{24EI} = \frac{120(12)^3}{24EI} = \frac{8640 \text{ lb} \cdot \text{ft}^2}{EI}$$

$$\theta''_B = \frac{PL^2}{16EI} = \frac{500(10)^2}{16EI} = \frac{3125 \text{ lb} \cdot \text{ft}^2}{EI}$$

$$\alpha'_{BB} = \frac{ML}{3EI} = \frac{1(12)}{3EI} = \frac{4 \text{ ft}}{EI}$$

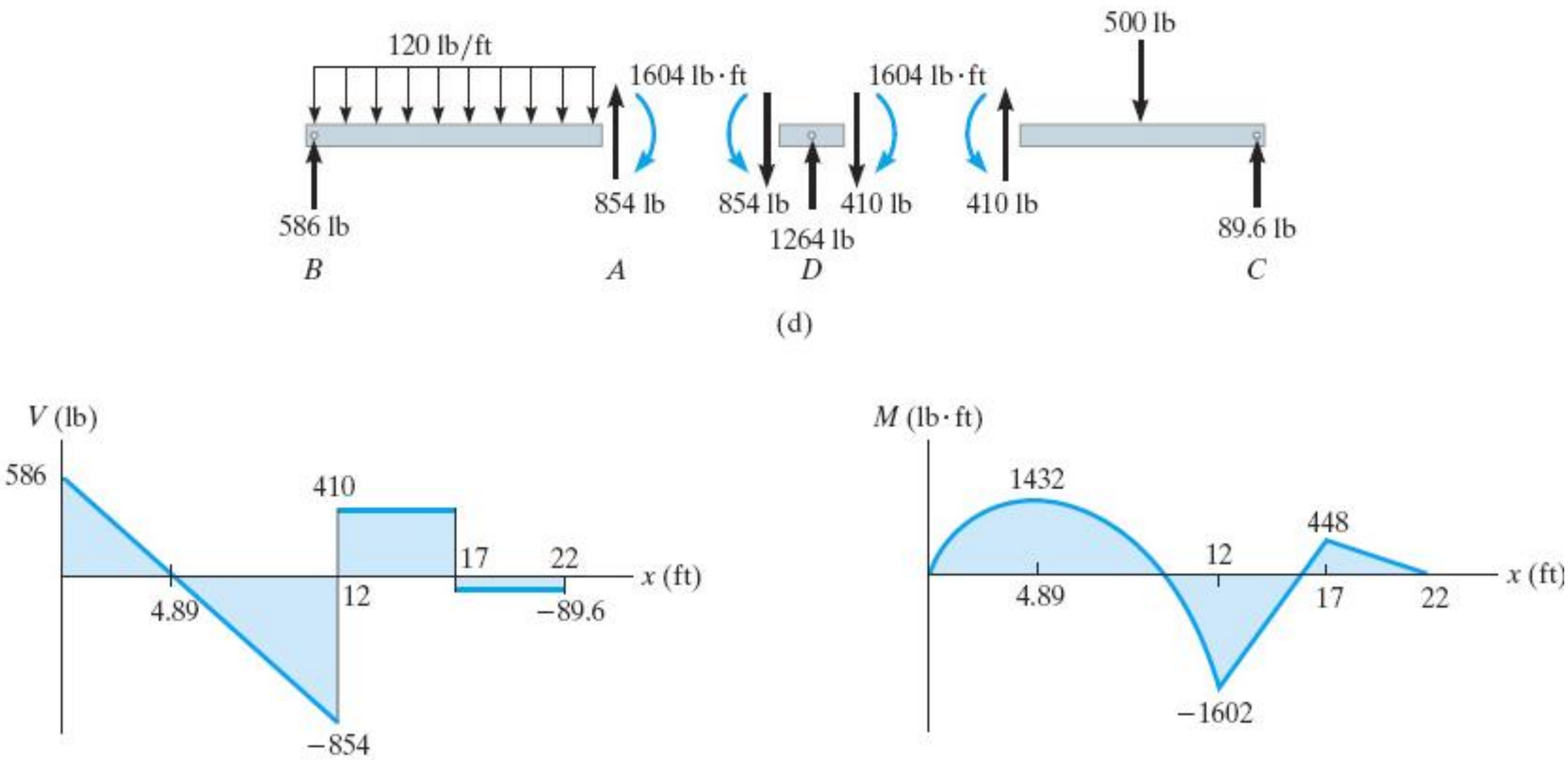
$$\alpha''_{BB} = \frac{ML}{3EI} = \frac{1(10)}{3EI} = \frac{3.33 \text{ ft}}{EI}$$

Thus

$$\frac{8640 \text{ lb} \cdot \text{ft}^2}{EI} + \frac{3125 \text{ lb} \cdot \text{ft}^2}{EI} + M_B\left(\frac{4 \text{ ft}}{EI} + \frac{3.33 \text{ ft}}{EI}\right) = 0$$

$$M_B = -1604 \text{ lb} \cdot \text{ft}$$

Example 5



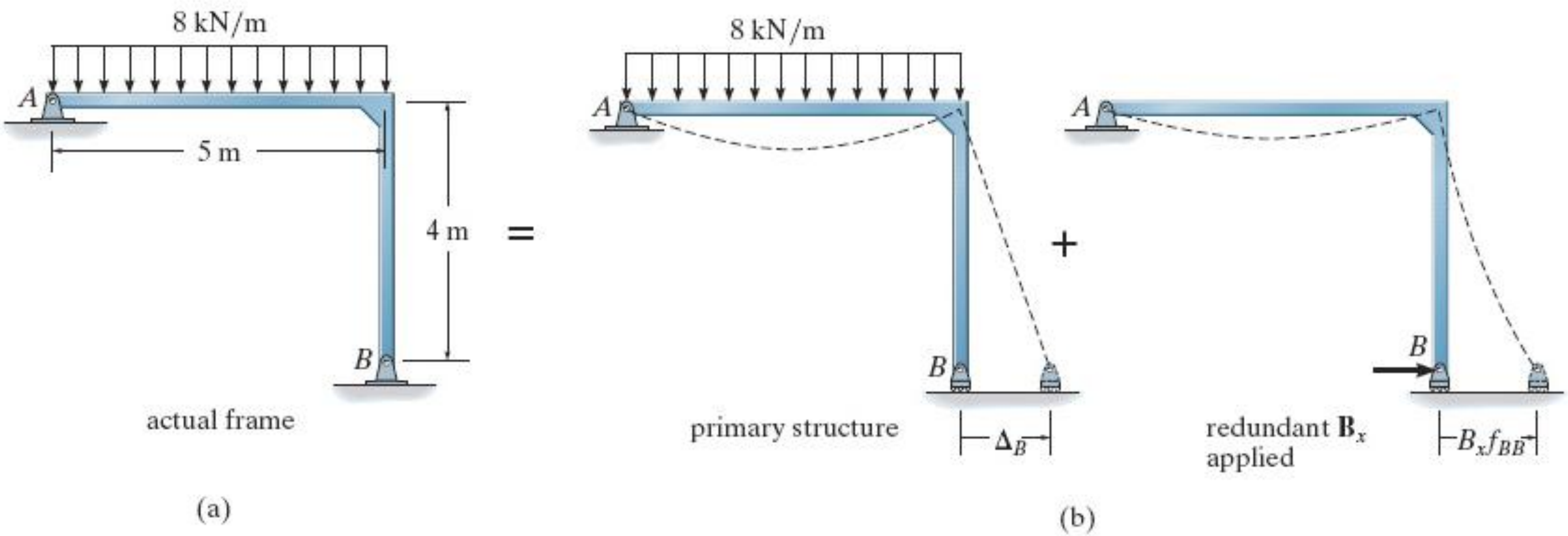
120 lb/ft
500 lb
1604 lb · ft
1604 lb · ft
586 lb
854 lb
854 lb
410 lb
410 lb
1264 lb
89.6 lb
B
A
D
C
(d)
V (lb)
586
410
17
22
x (ft)
4.89
12
−89.6
−854
M (lb · ft)
1432
448
12
x (ft)
4.89
17
22
−1602

Example 6

Determine the support reactions on the frame shown EI is Constant.

Compatibility Equation. Reference to point B in Fig. 10–13b requires

$$(\overset{+}{\rightarrow}) \qquad 0 = \Delta_B + B_x f_{BB} \qquad (1)$$



Example 6

$$\Delta_B = \int_0^L \frac{Mm}{EI}dx = \int_0^5 \frac{(20x_1 - 4x_1^2)(0.8x_1)\,dx_1}{EI} + \int_0^4 \frac{0(1x_2)\,dx_2}{EI}$$

$$= \frac{166.7}{EI} + 0 = \frac{166.7}{EI}$$

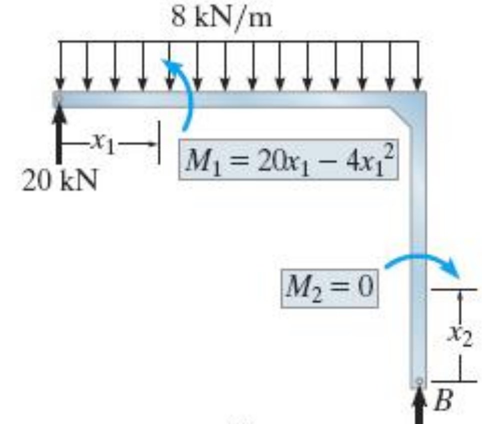
For f_{BB} we require application of a real unit load and a virtual unit load acting at B, Fig. 10–13d. Thus,

$$f_{BB} = \int_0^L \frac{mm}{EI}dx = \int_0^5 \frac{(0.8x_1)^2\,dx_1}{EI} + \int_0^4 \frac{(1x_2)^2\,dx_2}{EI}$$

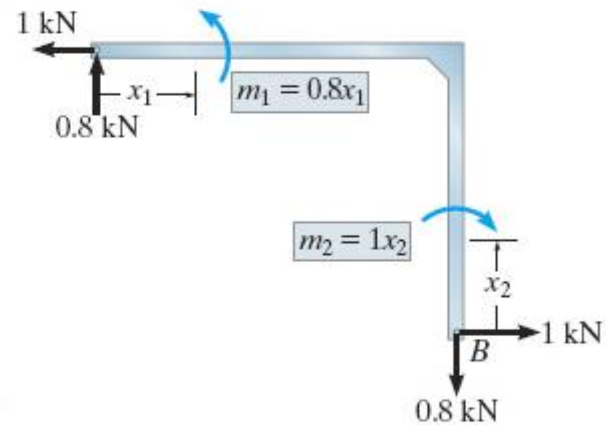
$$= \frac{26.7}{EI} + \frac{21.3}{EI} = \frac{48.0}{EI}$$

Substituting the data into Eq. (1) and solving yields

$$0 = \frac{166.7}{EI} + B_x\left(\frac{48.0}{EI}\right) \qquad B_x = -3.47\text{ kN}$$ ***Ans.***



(c)



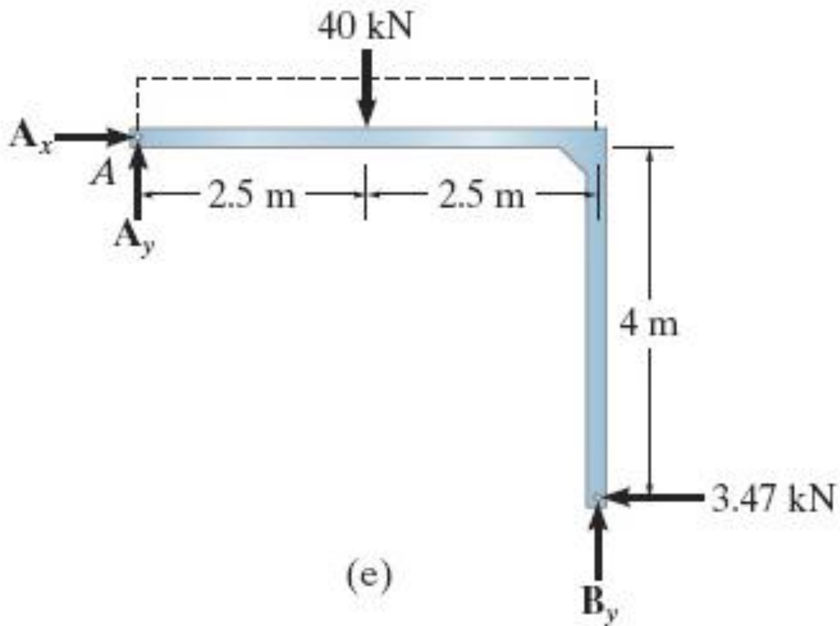
(d)

Example 6

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 3.47 = 0 \quad A_x = 3.47 \text{ kN}$ *Ans.*

$\circlearrowright + \Sigma M_A = 0; \quad -40(2.5) + B_y(5) - 3.47(4) = 0 \quad B_y = 22.8 \text{ kN}$ *Ans.*

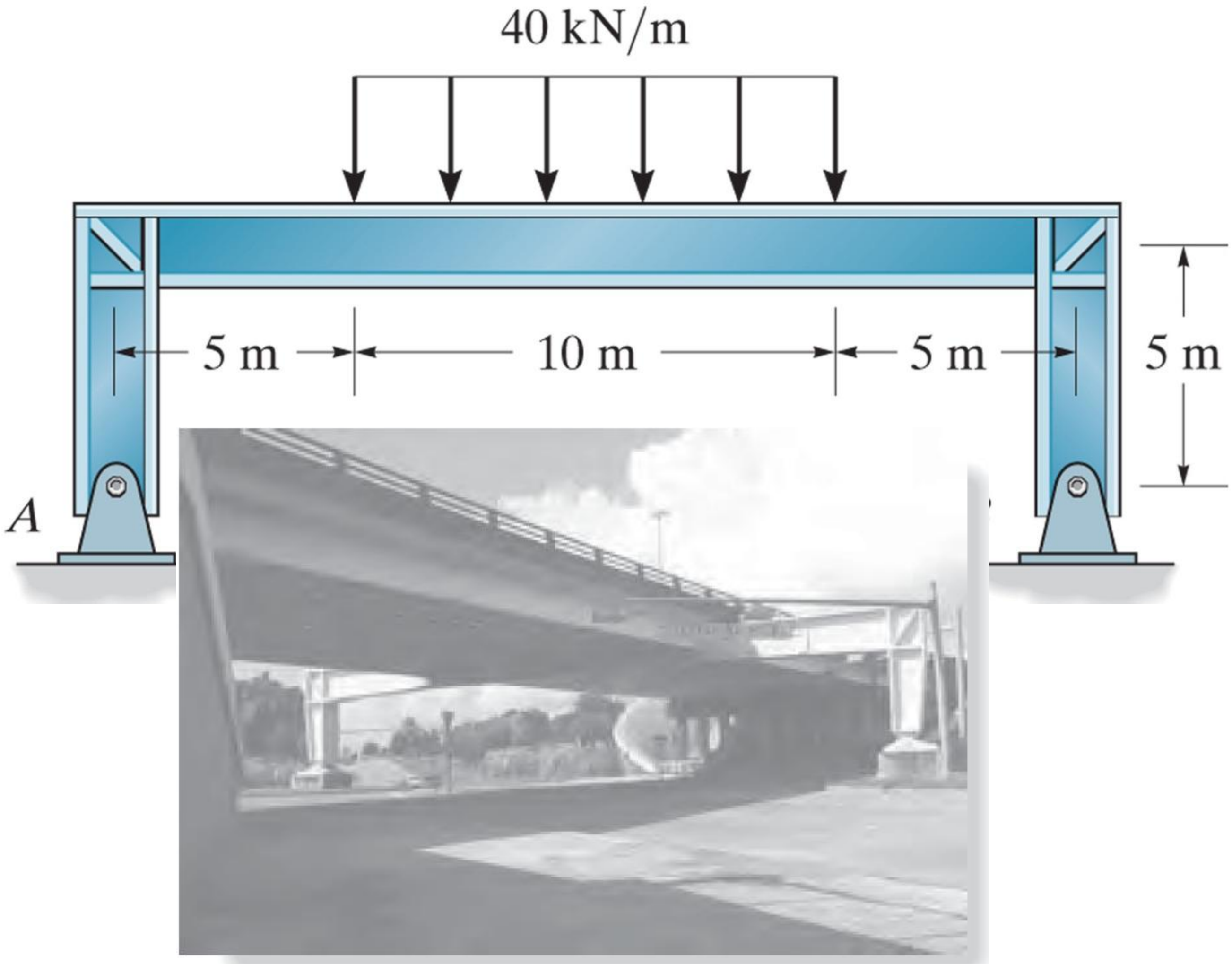
$+\uparrow \Sigma F_y = 0; \quad A_y - 40 + 22.8 = 0 \quad A_y = 17.2 \text{ kN}$ *Ans.*

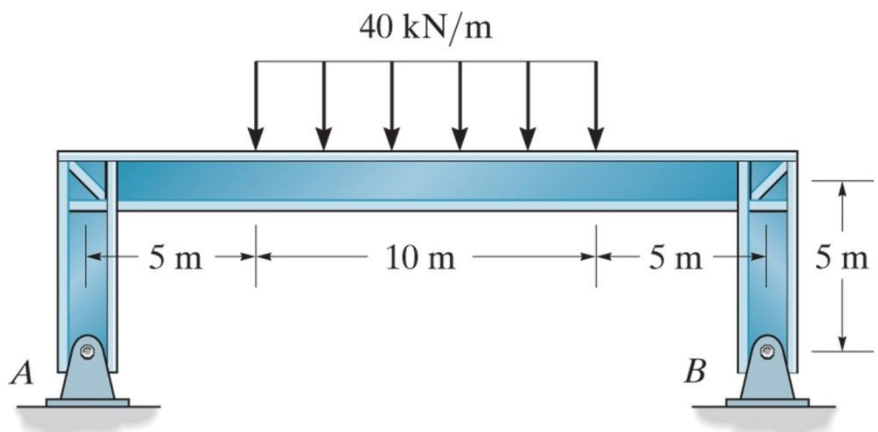


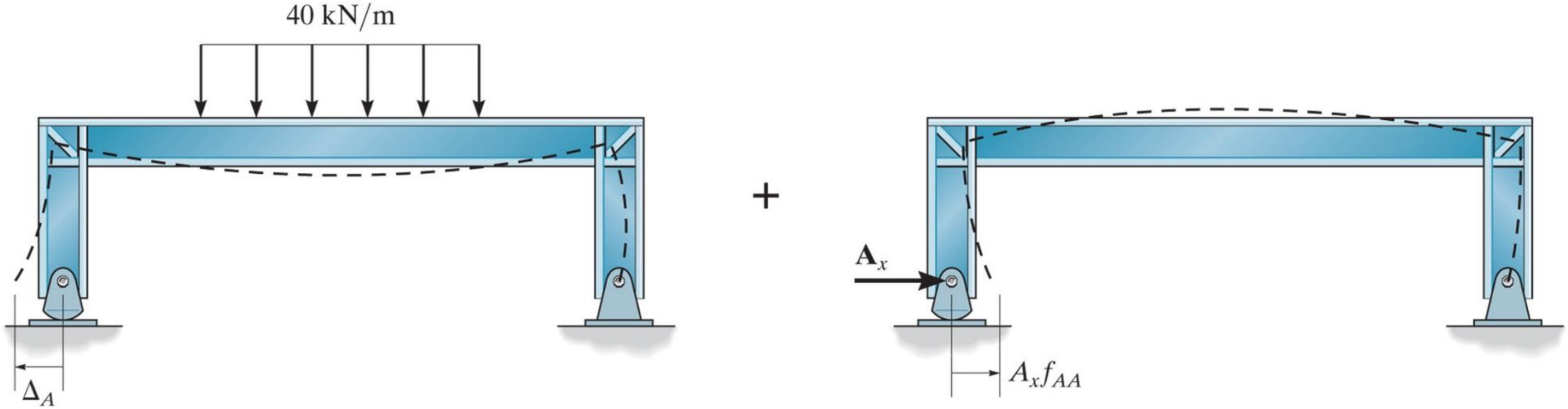
(e)

Example (Additional)

The frame, shown in the photo is used to support the bridge deck. Assuming EI is constant, a drawing of it along with the dimensions and loading is shown. Determine the support reactions.

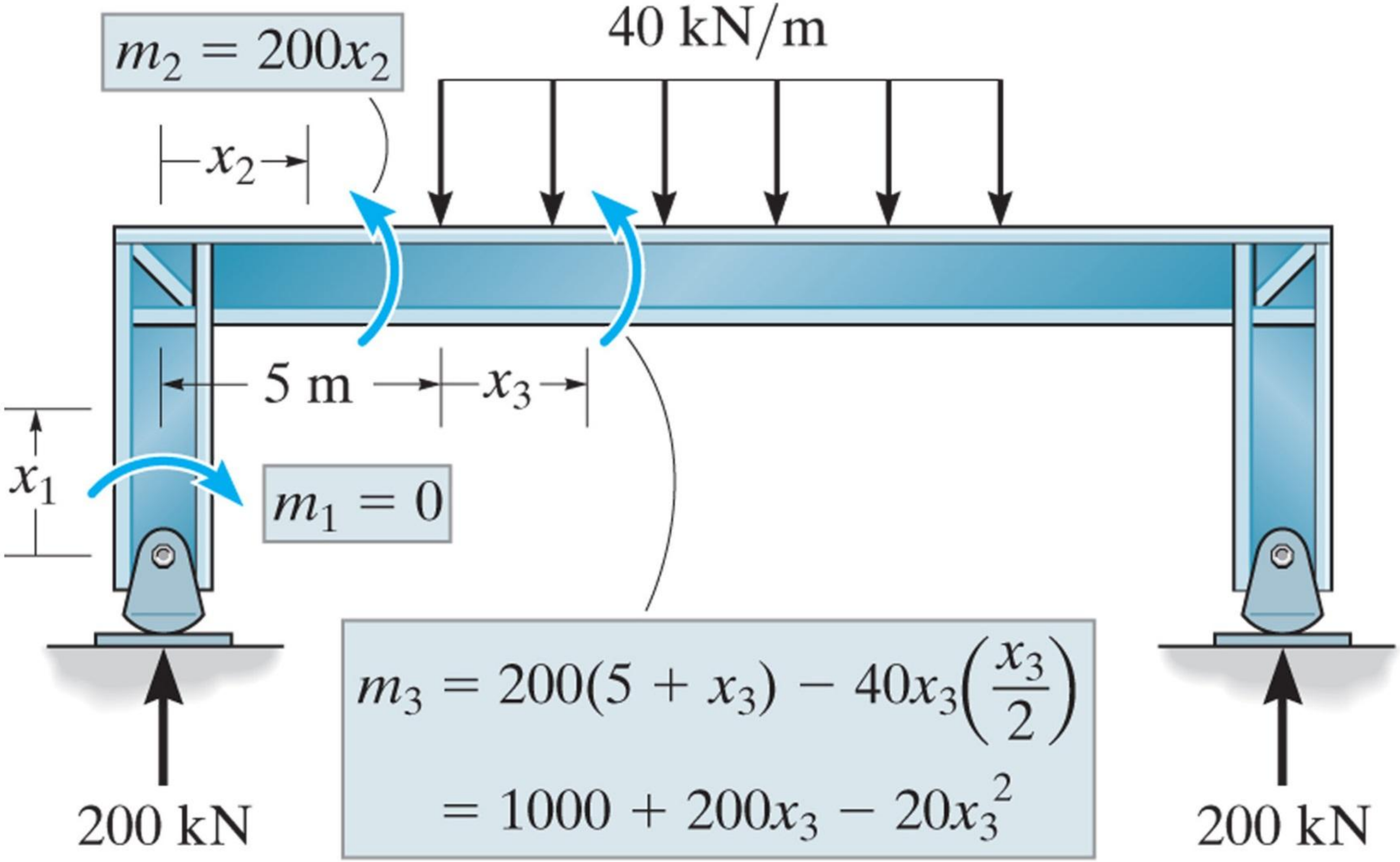




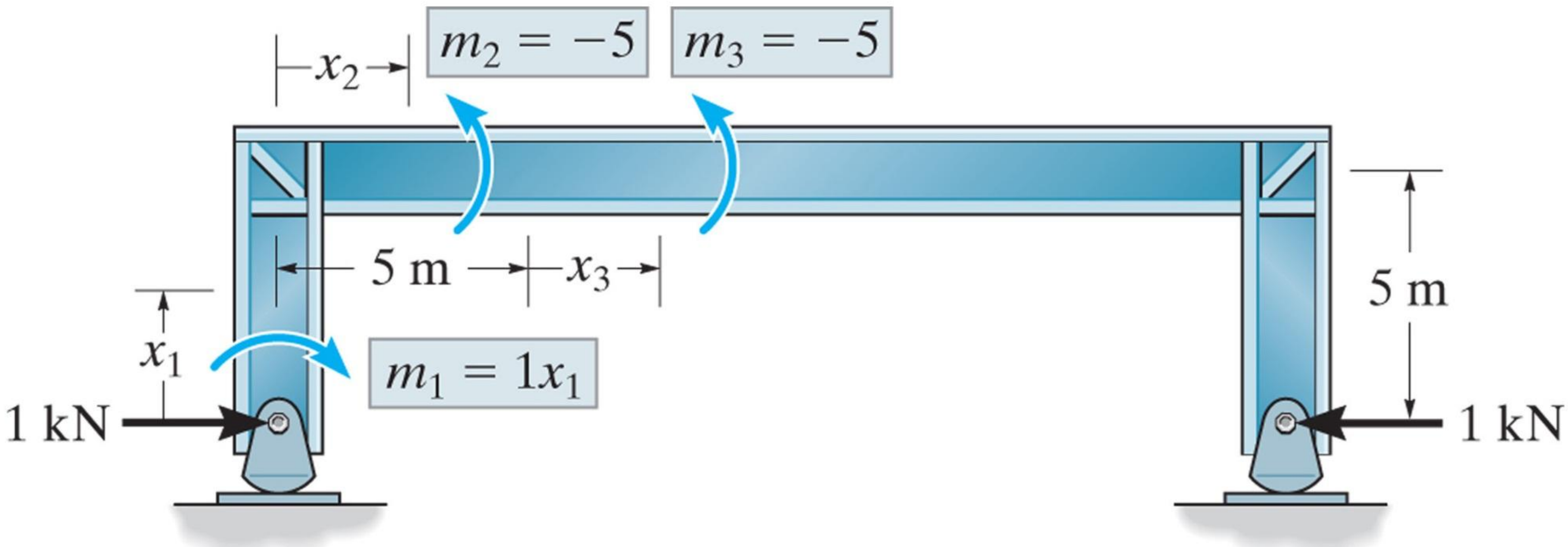


Compatibility Equation. Reference to point A in Fig. 10–12b requires

$$(\xrightarrow{+}) \qquad 0 = \Delta_A + A_x f_{AA} \qquad (1)$$



40 kN/m
$m_2 = 200x_2$
x_2
5 m
x_3
x_1
$m_1 = 0$
$m_3 = 200(5 + x_3) - 40x_3\left(\frac{x_3}{2}\right)$
$= 1000 + 200x_3 - 20x_3^2$
200 kN
200 kN



$$\Delta_A = \int_0^L \frac{Mm}{EI}dx = 2\int_0^5 \frac{(0)(1x_1)dx_1}{EI} + 2\int_0^5 \frac{(200x_2)(-5)dx_2}{EI}$$

$$+ 2\int_0^5 \frac{(1000 + 200x_3 - 20x_3^2)(-5)dx_3}{EI}$$

$$= 0 - \frac{25\,000}{EI} - \frac{66\,666.7}{EI} = -\frac{91\,666.7}{EI}$$

$$f_{AA} = \int_0^L \frac{mm}{EI}dx = 2\int_0^5 \frac{(1x_1)^2 dx_1}{EI} + 2\int_0^5 (5)^2 dx_2 + 2\int_0^5 (5)^2 dx_3$$

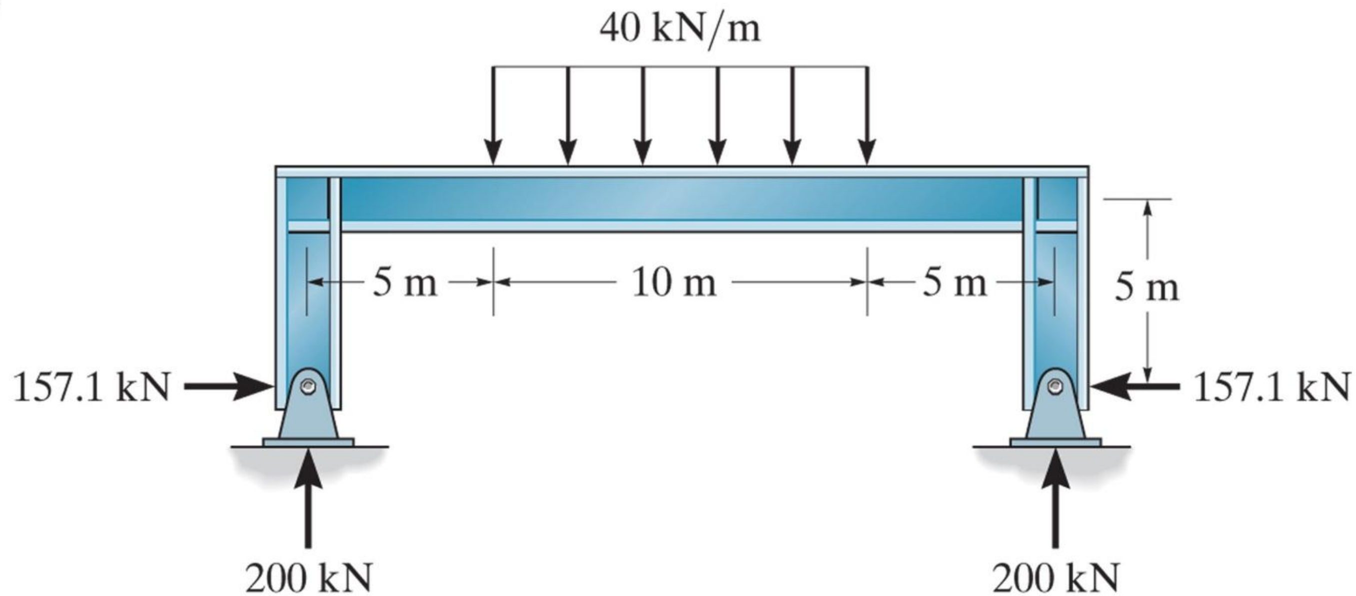
$$= \frac{583.33}{EI}$$

Substituting the results into Eq. (1) and solving yields

$$0 = \frac{-91\,666.7}{EI} + A_x\left(\frac{583.33}{EI}\right)$$

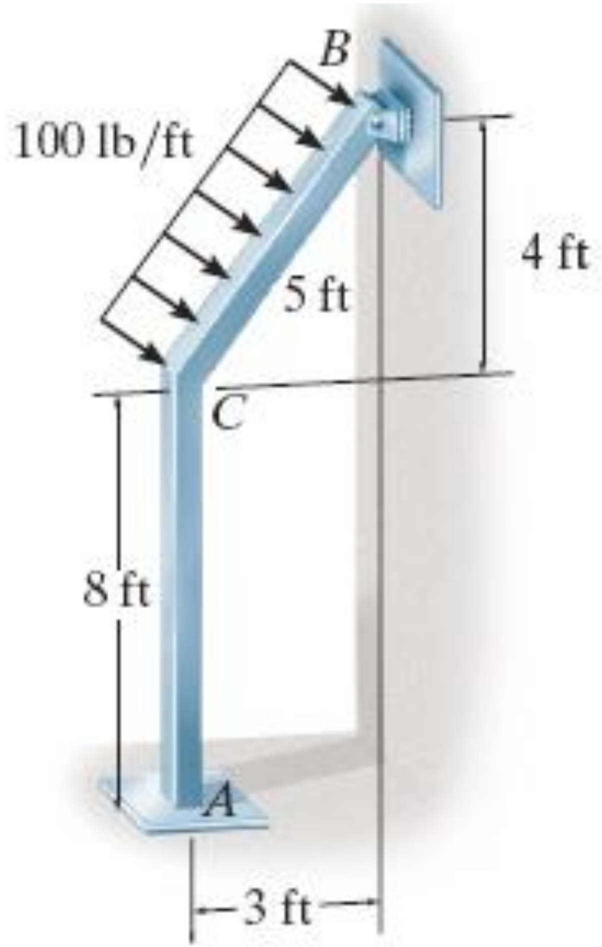
$A_x = 157$ kN

Ans



Example 7

Determine the moment at the fixed support A for the frame shown EI is Constant.

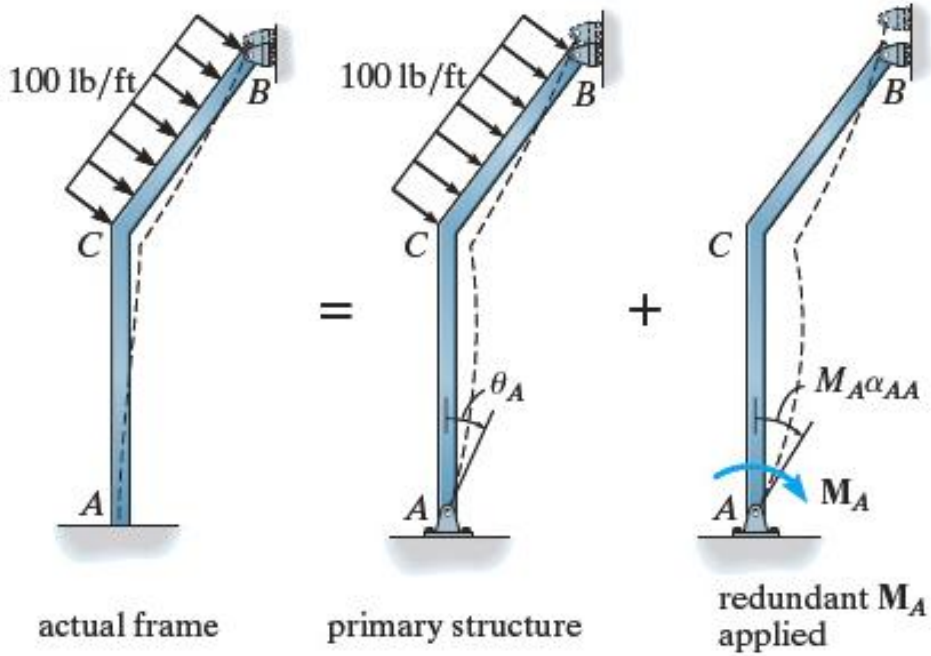


Example 7

Compatibility Equation. Reference to point A in Fig. 10–14b requires

$$(\curvearrowleft +) \qquad 0 = \theta_A + M_A \alpha_{AA} \qquad (1)$$

As in the preceding example, θ_A and α_{AA} will be computed using the method of virtual work. The frame's x coordinates and internal moments are shown in Fig. 10–14c and 10–14d.



Example 7

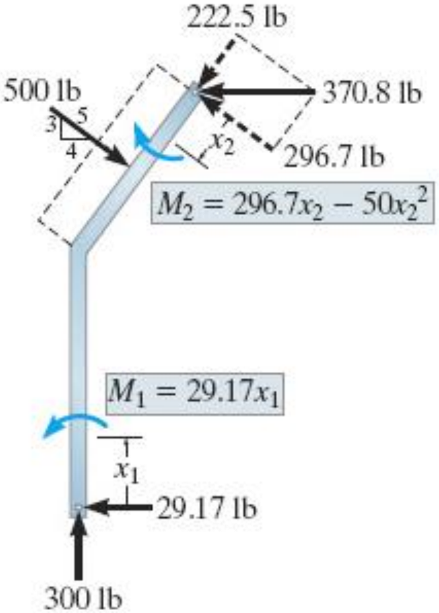
For θ_A we require application of the real loads, Fig. 10–14c, and a virtual unit couple moment at A, Fig. 10–14d. Thus,

$$\theta_A = \sum \int_0^L \frac{M m_\theta \, dx}{EI}$$

$$= \int_0^8 \frac{(29.17x_1)(1 - 0.0833x_1)\, dx_1}{EI}$$

$$+ \int_0^5 \frac{\left(296.7x_2 - 50x_2^2\right)(0.0667x_2)\, dx_2}{EI}$$

$$= \frac{518.5}{EI} + \frac{303.2}{EI} = \frac{821.8}{EI}$$



(c)

Example 7

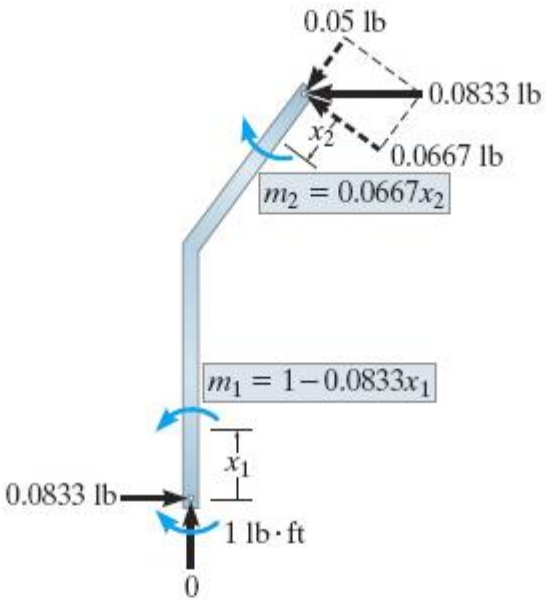
$$\alpha_{AA} = \sum \int_0^L \frac{m_\theta m_\theta}{EI} dx$$

$$= \int_0^8 \frac{(1 - 0.0833x_1)^2 \, dx_1}{EI} + \int_0^5 \frac{(0.0667x_2)^2 \, dx_2}{EI}$$

$$= \frac{3.85}{EI} + \frac{0.185}{EI} = \frac{4.04}{EI}$$

Substituting these results into Eq. (1) and solving yields

$$0 = \frac{821.8}{EI} + M_A\left(\frac{4.04}{EI}\right) \qquad M_A = -204 \text{ lb} \cdot \text{ft}$$ ***Ans.***



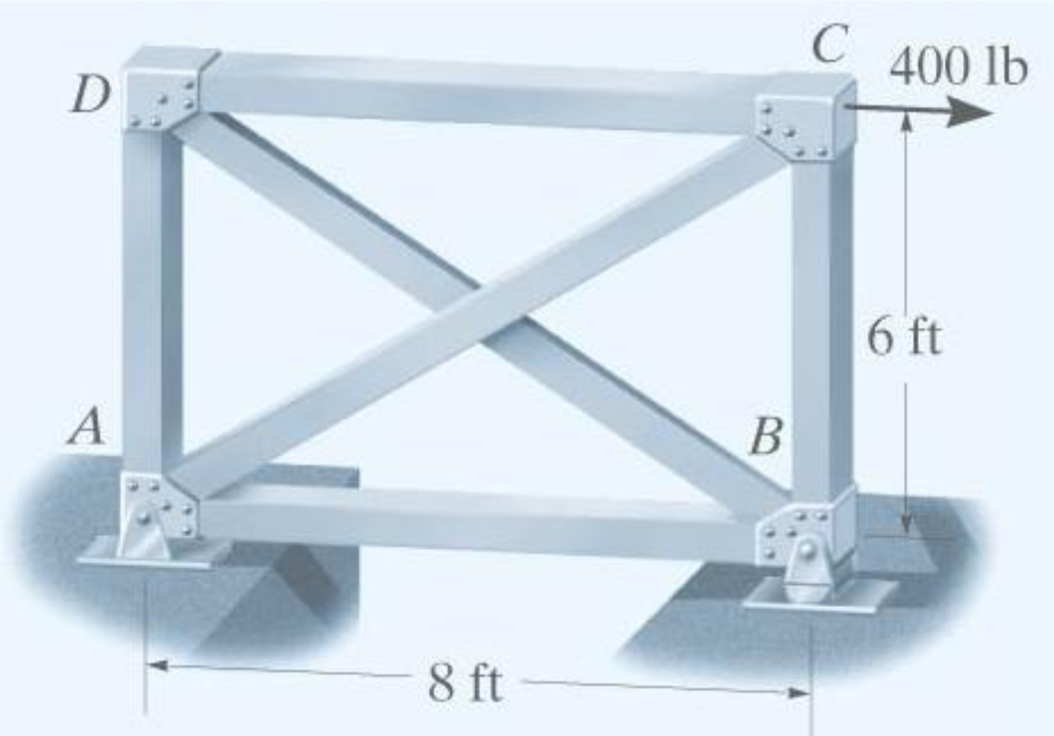
(d)

Force Method of Analysis: Truss

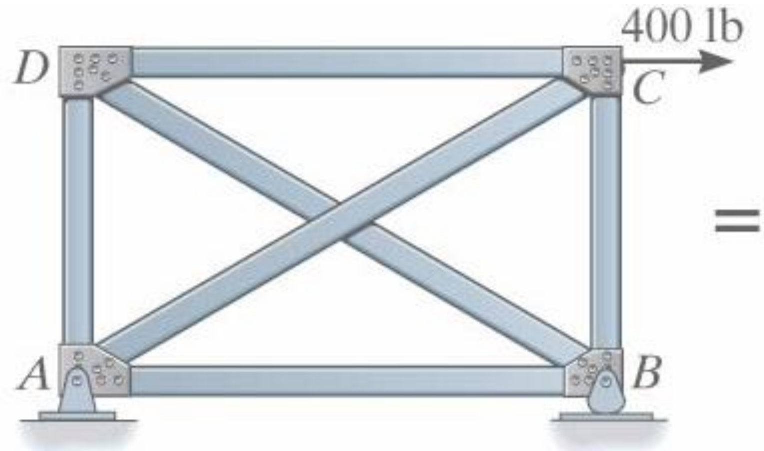
Example8

Determine the force in member AC.

Assume EA is the same for all the members

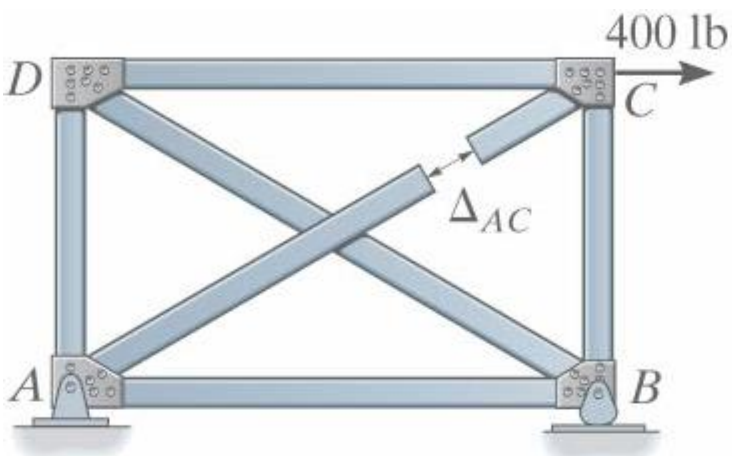


Example 8

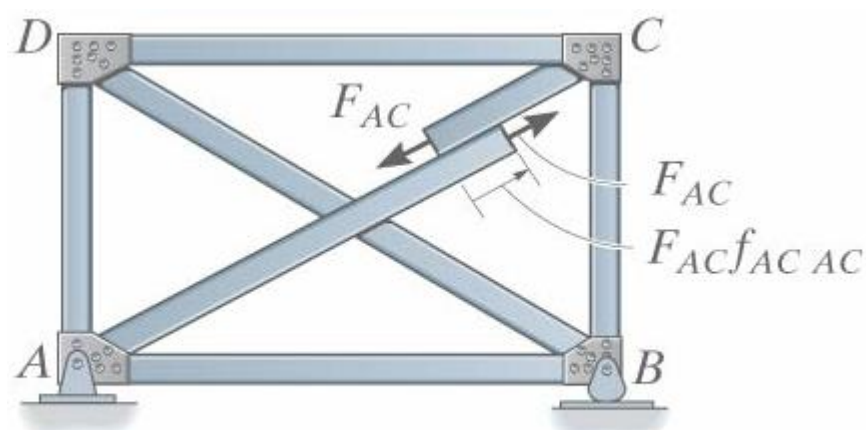


actual truss

$$0 = \Delta_{AC} + F_{AC} f_{AC\,AC}$$



primary structure



redundant $\mathbf{F}_{AC}$ applied

Force Method of Analysis: Truss

$$\Delta_{AC} = \sum \frac{nNL}{AE}$$

$$= 2\left[\frac{(-0.8)(400)(8)}{AE}\right] + \frac{(-0.6)(0)(6)}{AE} + \frac{(-0.6)(300)(6}{AE}$$

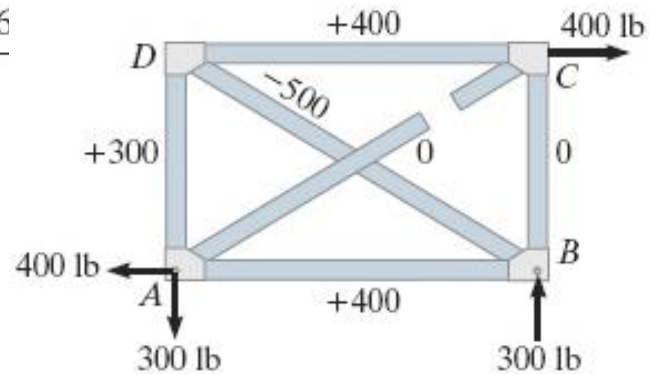
$$+ \frac{(1)(-500)(10)}{AE} + \frac{(1)(0)(10)}{AE}$$

$$= -\frac{11\,200}{AE}$$

$$f_{AC\,AC} = \sum \frac{n^2L}{AE}$$

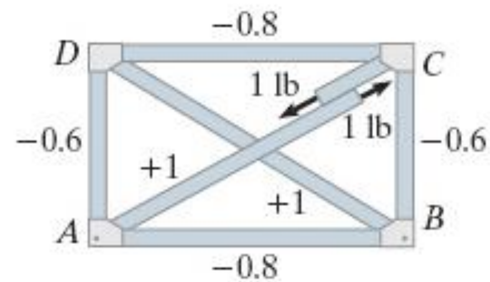
$$= 2\left[\frac{(-0.8)^2(8)}{AE}\right] + 2\left[\frac{(-0.6)^2(6)}{AE}\right] + 2\left[\frac{(1)^2 10}{AE}\right.$$

$$= \frac{34.56}{AE}$$



(c)

+



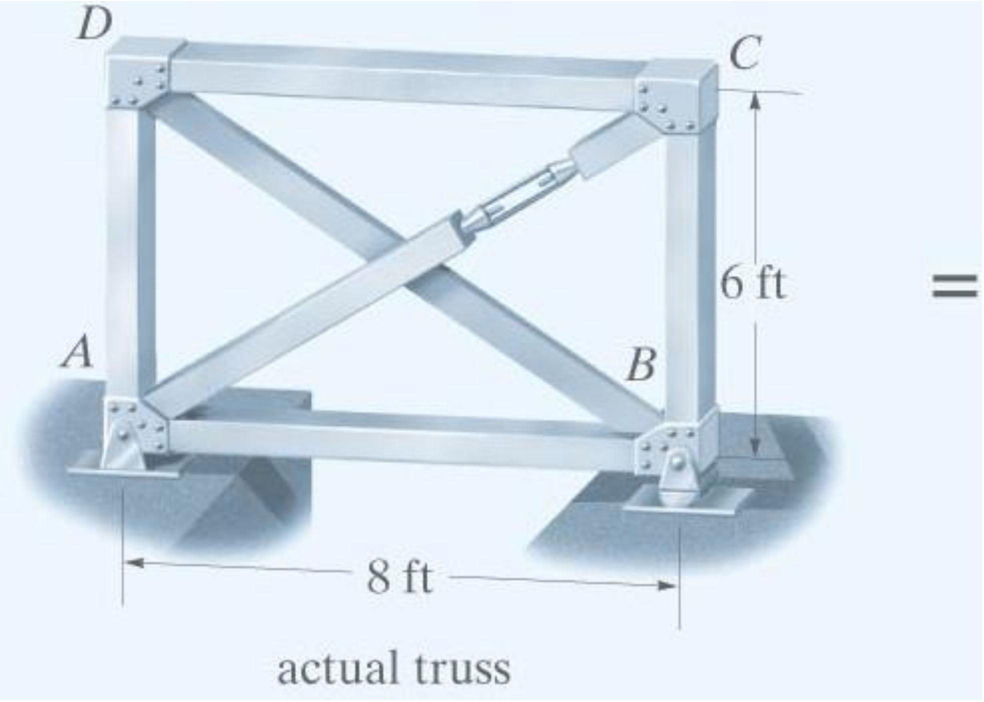
Substituting the data into Eq. (1) and solving yields

$$0 = -\frac{11\,200}{AE} + \frac{34.56}{AE} F_{AC}$$

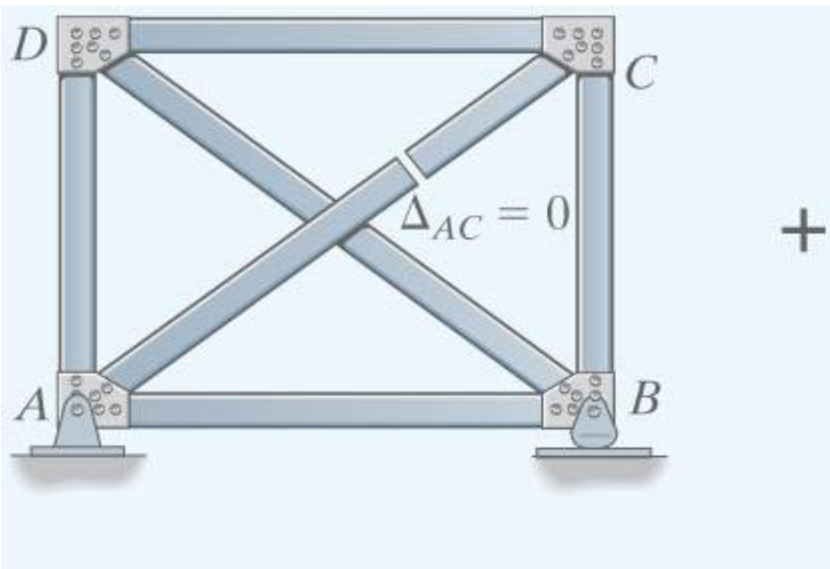
$$F_{AC} = 324 \text{ lb (T)}$$

Example 9

- Determine the force in each member if the turnbuckle on member AC is used to shorten the member by 0.5in. Each member has a cross-section area of 0.2 in^2 & E=29(10^6)psi

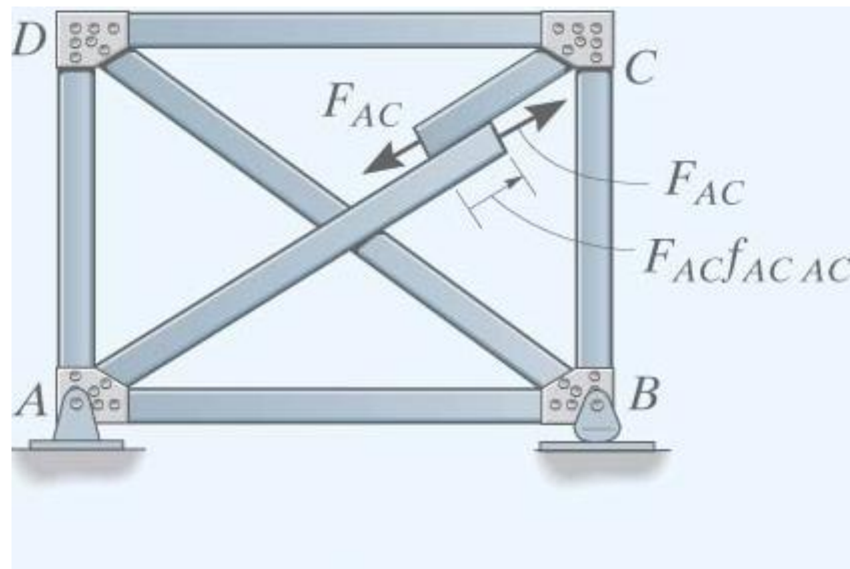


Example 9



primary structure

+



redundant $\mathbf{F}_{AC}$ applied

Example 9

$$f_{AC\,AC} = \frac{34.56}{AE}$$

Assuming the amount by which the bar is shortened is positive, the compatibility equation for the bar is therefore

$$0.5 \text{ in.} = 0 + \frac{34.56}{AE} F_{AC}$$

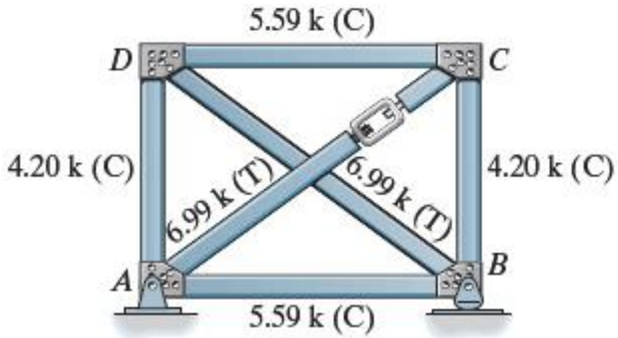
Realizing that $f_{AC\,AC}$ is a measure of displacement per unit force, we have

$$0.5 \text{ in.} = 0 + \frac{34.56 \text{ ft}(12 \text{ in./ft})}{(0.2 \text{ in}^2)[29(10^6) \text{ lb/in}^2]} F_{AC}$$

Thus,

$$F_{AC} = 6993 \text{ lb} = 6.99 \text{ k (T)} \qquad \textit{Ans.}$$

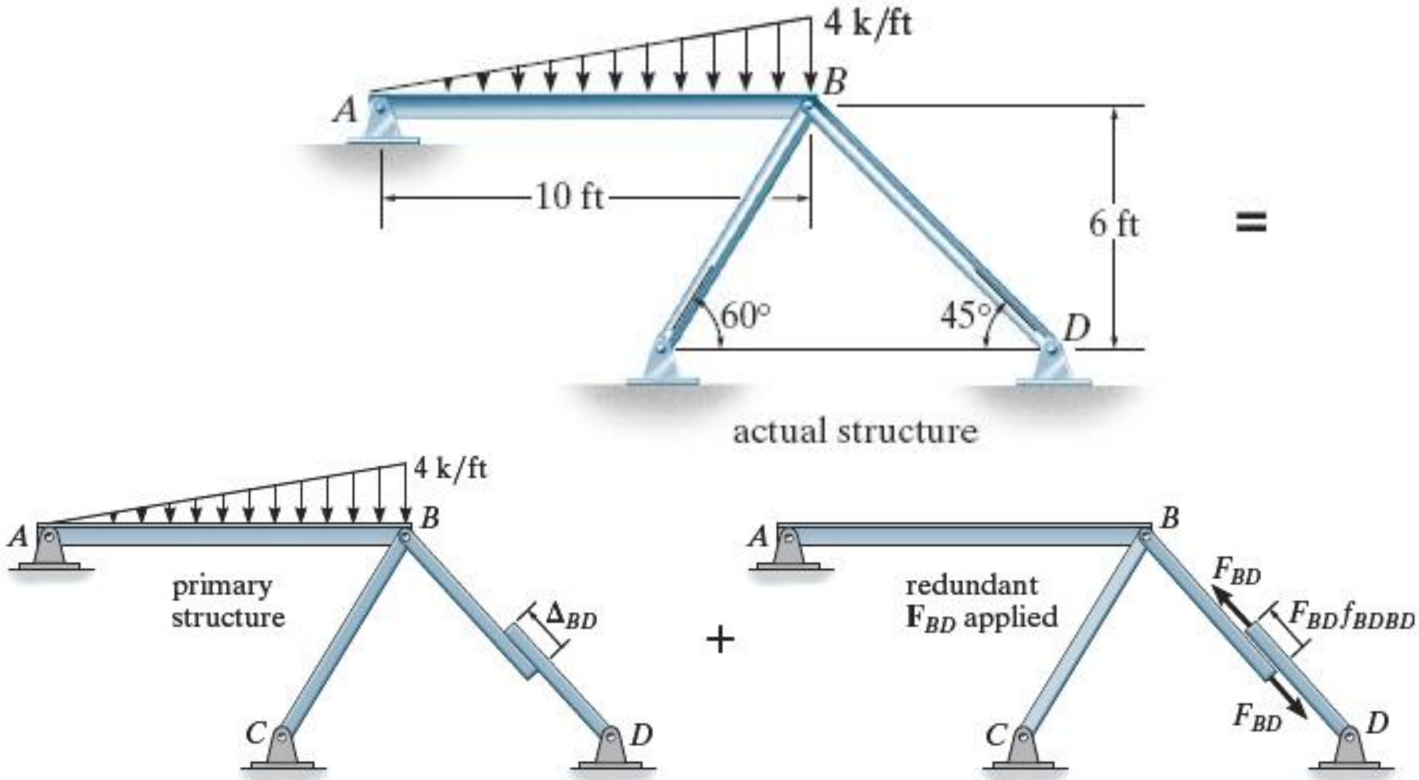
Since no external forces act on the truss, the external reactions are zero. Therefore, using F_{AC} and analyzing the truss by the method of joints yields the results shown in Fig. 10–16*c*.



(c)

Example 10

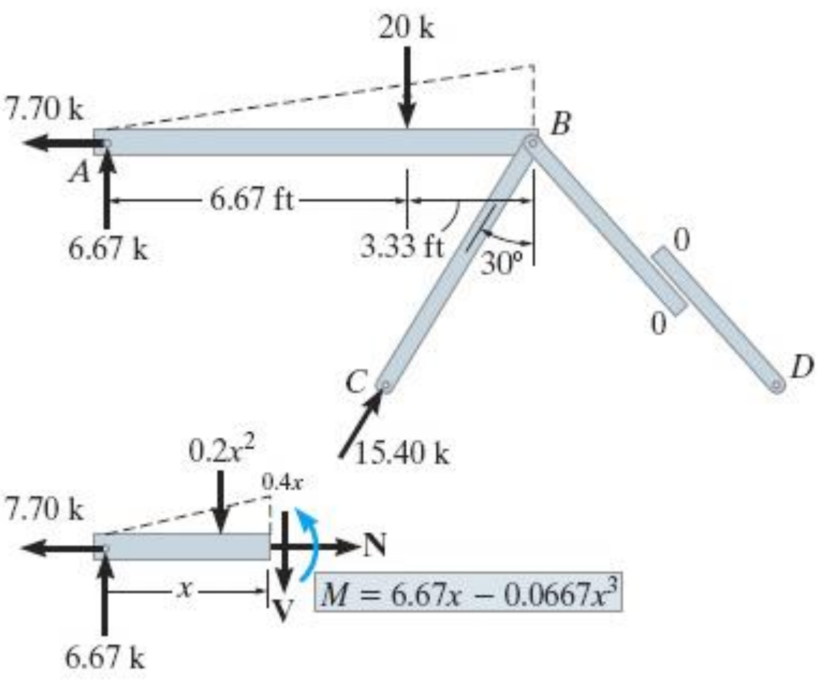
The beam shown is supported by a pin at A and two pin-connected bar at B. Determine the force in member BD. Take $E=29(10^3)$, $I=800$ in^4 for the beam and $A=3$ in^2 for each bar.



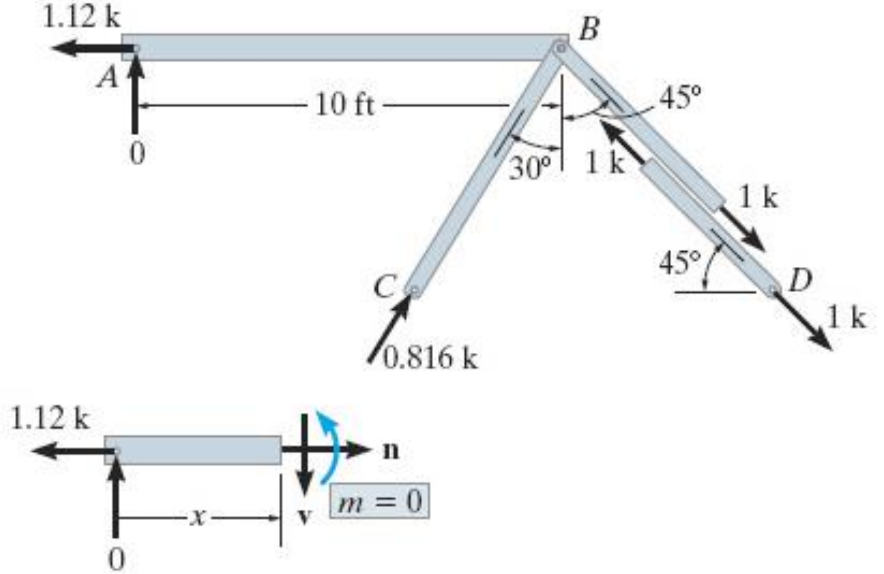
(b)

$$0 = \Delta_{BD} + F_{BD} f_{BD\,BD}$$

Example 10



(c)



(d)

Example 10

$$\Delta_{BD} = \int_0^L \frac{Mm}{EI}dx + \sum \frac{nNL}{AE} = \int_0^{10} \frac{(6.67x - 0.0667x^3)(0)\,dx}{EI} + \frac{(-15.40)(-0.816)(6/\cos 30°)(12)}{AE} + \frac{(0)(1)(6/\cos 45°)(12)}{AE}$$

$$= 0 + \frac{1045.1}{3[29(10^3)]} + 0 = 0.0120 \text{ in.}$$

For $f_{BD\,BD}$ we require application of a real unit load and a virtual unit load at the cut ends of member BD, Fig. 10–17d. Thus,

$$f_{BD\,BD} = \int_0^L \frac{m^2\,dx}{EI} + \sum \frac{n^2L}{AE} = \int_0^{10} \frac{(0)^2\,dx}{EI} + \frac{(-0.816)^2(6/\cos 30°)(12)}{AE} + \frac{(1)^2(6/\cos 45°)(12)}{AE} = \frac{157.2}{3[29(10^3)]} = 0.001807$$

Substituting the data into Eq. (1) yields

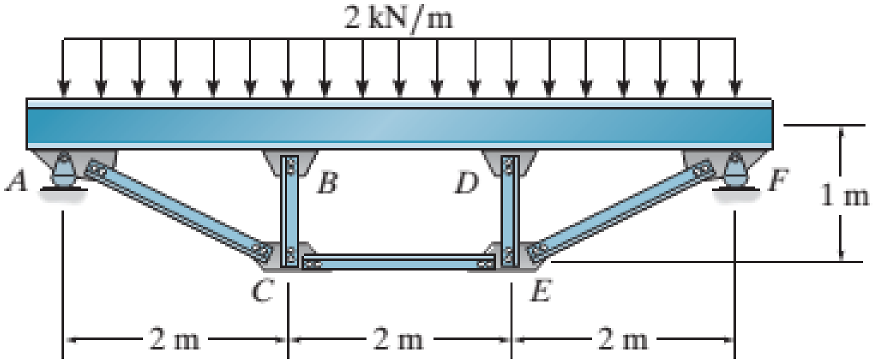
$$0 = \Delta_{BD} + F_{BD}f_{BD\,BD}$$

$$0 = 0.0120 + F_{BD}(0.001807)$$

$$F_{BD} = -6.65 \text{ k} = 6.65 \text{ k (C)} \qquad \textit{Ans.}$$

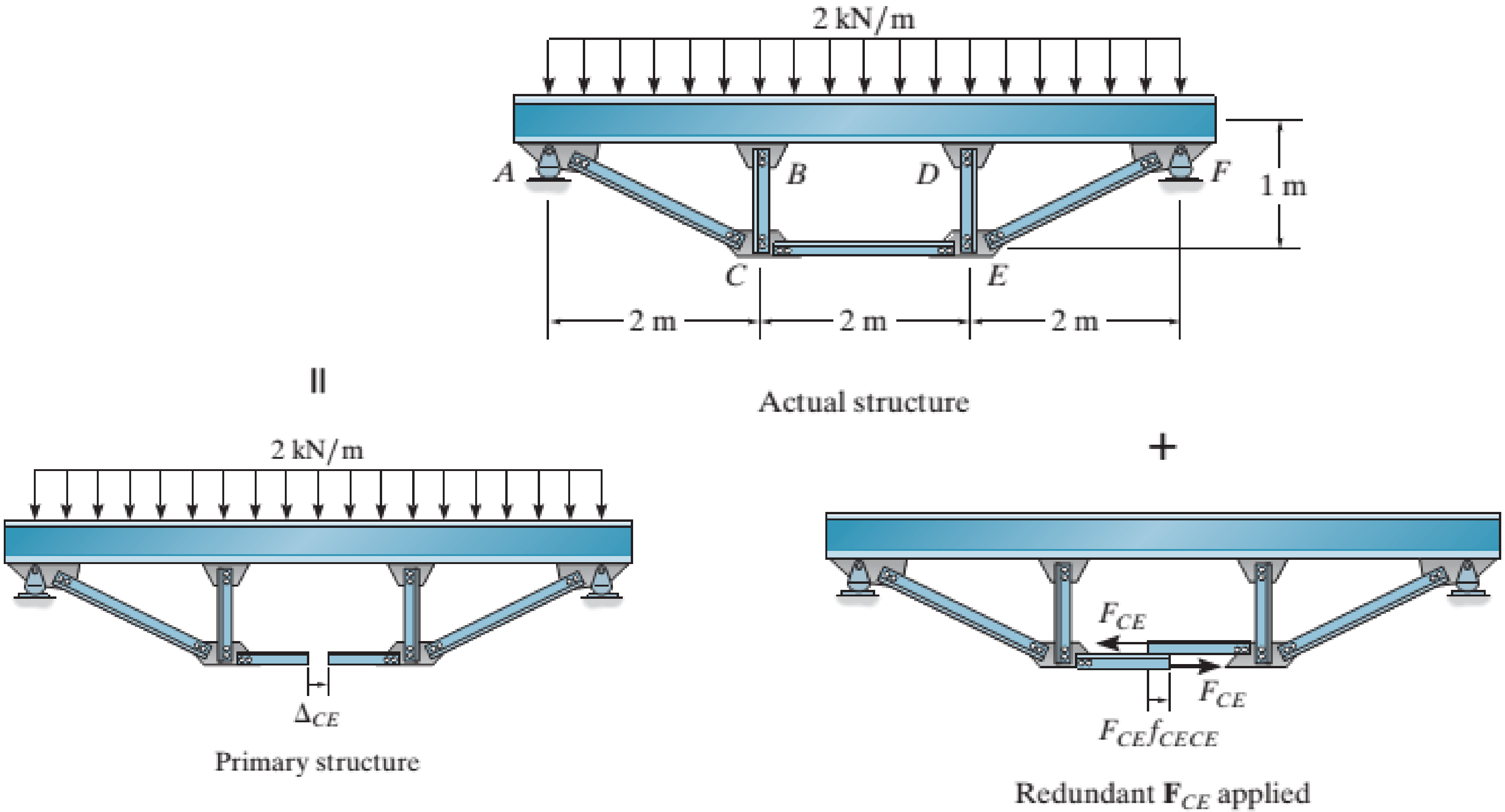
Example 11

The simply supported beam shown in the photo is to be designed to support a uniform load of 2 kN/m. Determine the force developed in member CE. Neglect the thickness of the beam and assume the truss members are pin connected to the beam. Also, neglect the effect of axial compression and shear in the beam. The cross-sectional area of each strut is 400 mm^2, and for the beam $I=20(10^6)mm^4$. Take E=200 GPa



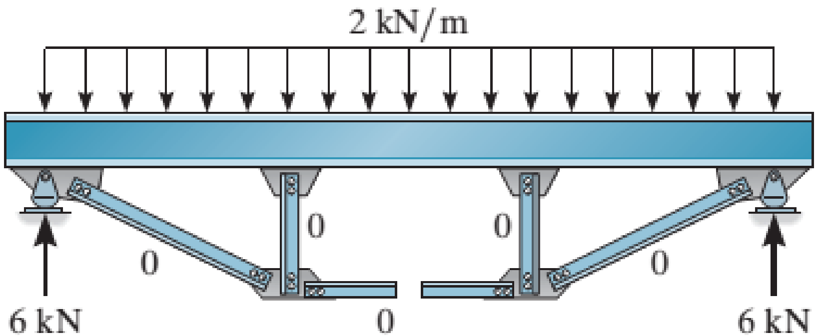
Actual structure

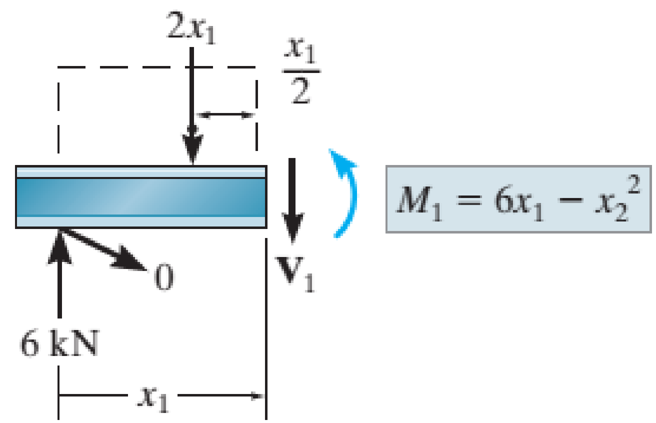
Example 11

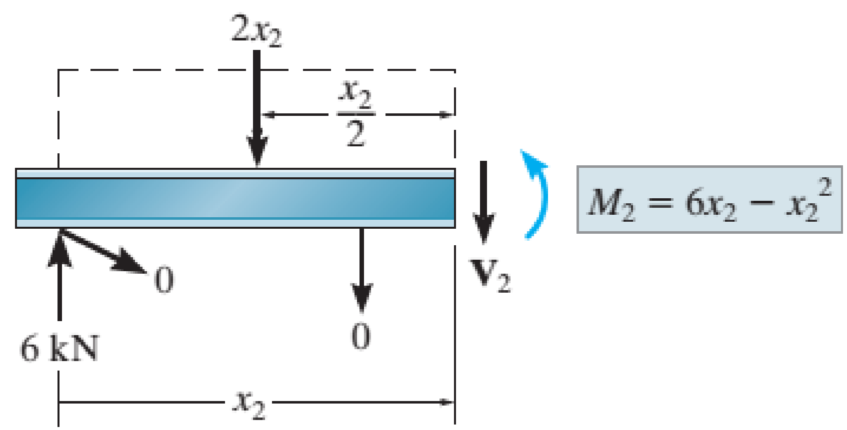


$$0 = \Delta_{CE} + F_{CE} f_{CECE}$$

Example 11

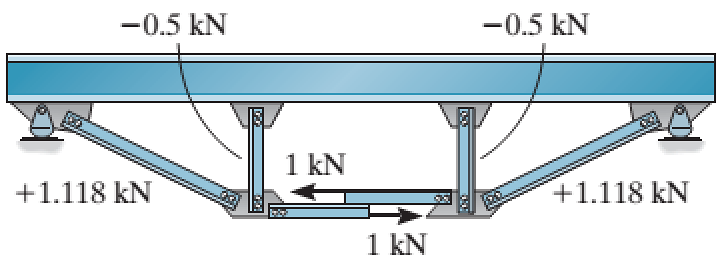


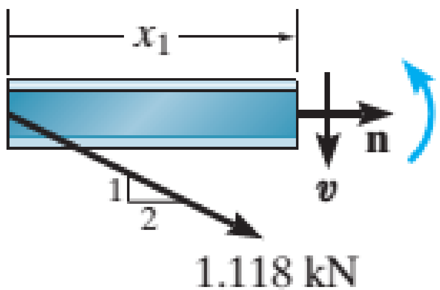




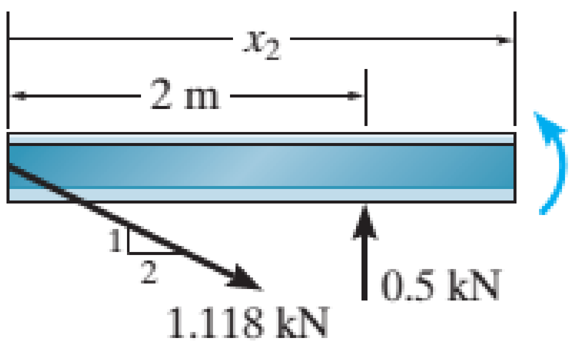
$$0 = \Delta_{CE} + F_{CE} f_{CECE}$$

Example 11





$$m_1 = -0.5x_1$$



$$\begin{aligned} m_2 &= -0.5x_2 + 0.5(x_2 - 2) \\ &= -1 \end{aligned}$$

$$0 = \Delta_{CE} + F_{CE} f_{CE\,CE}$$

Example 11

$$0 = \Delta_{CE} + F_{CE} f_{CE\,CE}$$

$$\Delta_{CE} = \int_0^L \frac{Mm}{EI} dx + \sum \frac{nNL}{AE} = 2\int_0^2 \frac{(6x_1 - x_1^2)(-0.5x_1)dx_1}{EI}$$

$$+ 2\int_2^3 \frac{(6x_2 - x_2^2)(-1)dx_2}{EI} + 2\left(\frac{(1.118)(0)(\sqrt{5})}{AE}\right)$$

$$+ 2\left(\frac{(-0.5)(0)(1)}{AE}\right) + \left(\frac{1(0)2}{AE}\right)$$

$$= -\frac{12}{EI} - \frac{17.33}{EI} + 0 + 0 + 0$$

$$= \frac{-29.33(10^3)}{200(10^9)(20)(10^{-6})} = -7.333(10^{-3})\text{ m}$$

Example 11

$$f_{CE\,CE} = \int_0^L \frac{m^2dx}{EI} + \sum \frac{n^2L}{AE} = 2\int_0^2 \frac{(-0.5x_1)^2dx_1}{EI} + 2\int_2^3 \frac{(-1)^2dx_2}{EI}$$

$$+ 2\left(\frac{(1.118)^2(\sqrt{5})}{AE}\right) + 2\left(\frac{(-0.5)^2(1)}{AE}\right) + \left(\frac{(1)^2(2)}{AE}\right)$$

$$= \frac{1.3333}{EI} + \frac{2}{EI} + \frac{5.590}{AE} + \frac{0.5}{AE} + \frac{2}{AE}$$

$$= \frac{3.333(10^3)}{200(10^9)(20)(10^{-6})} + \frac{8.090(10^3)}{400(10^{-6})(200(10^9))}$$

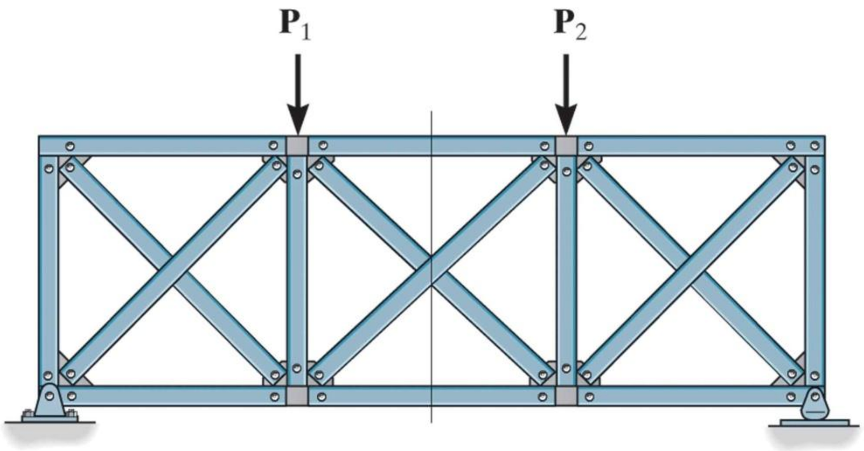
$$= 0.9345(10^{-3})\ \text{m/kN}$$

$$0 = \Delta_{CE} + F_{CE}f_{CE\,CE}$$

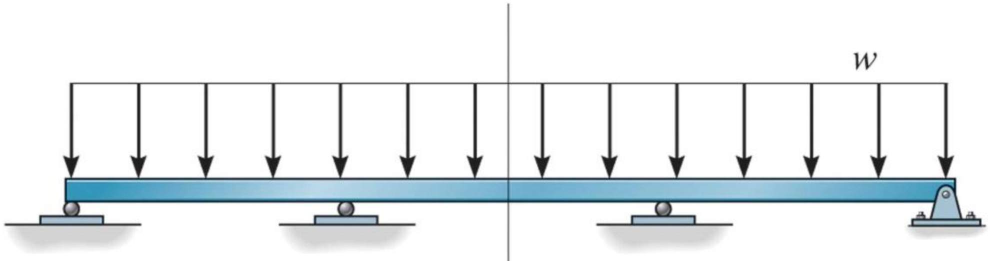
$$0 = -7.333(10^{-3})\ \text{m} + F_{CE}(0.9345(10^{-3})\ \text{m/kN})$$

$$F_{CE} = 7.85\ \text{kN}$$

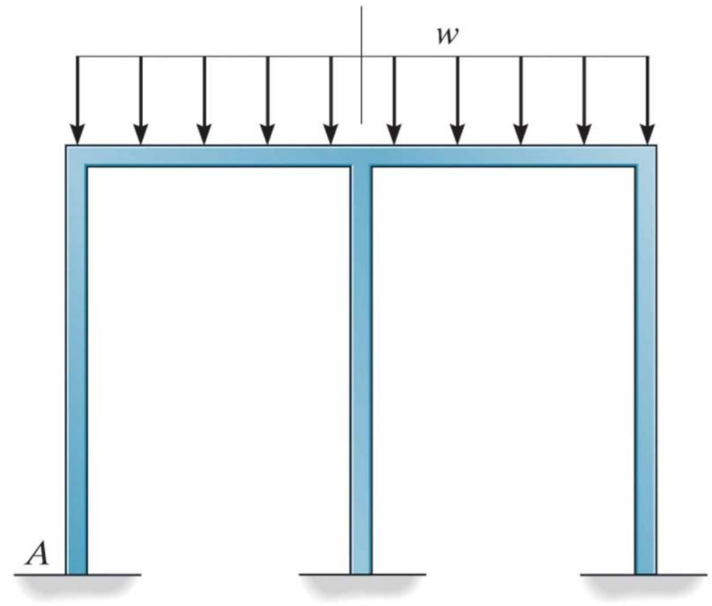
Symmetric Structures



axis of symmetry

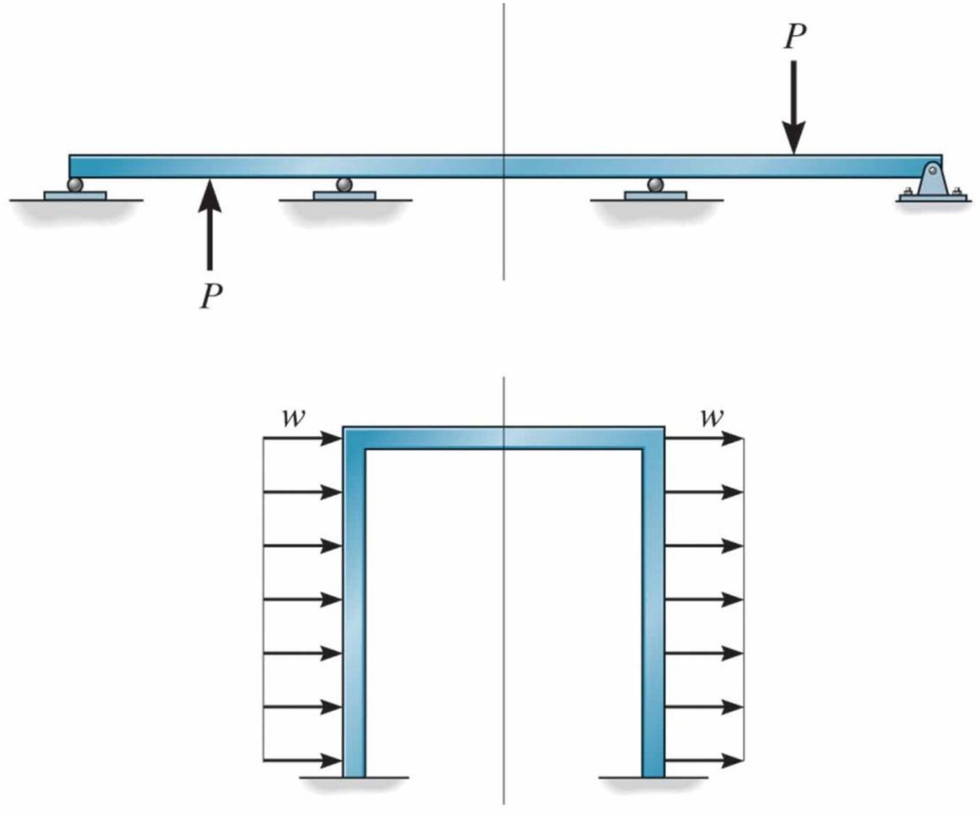


axis of symmetry



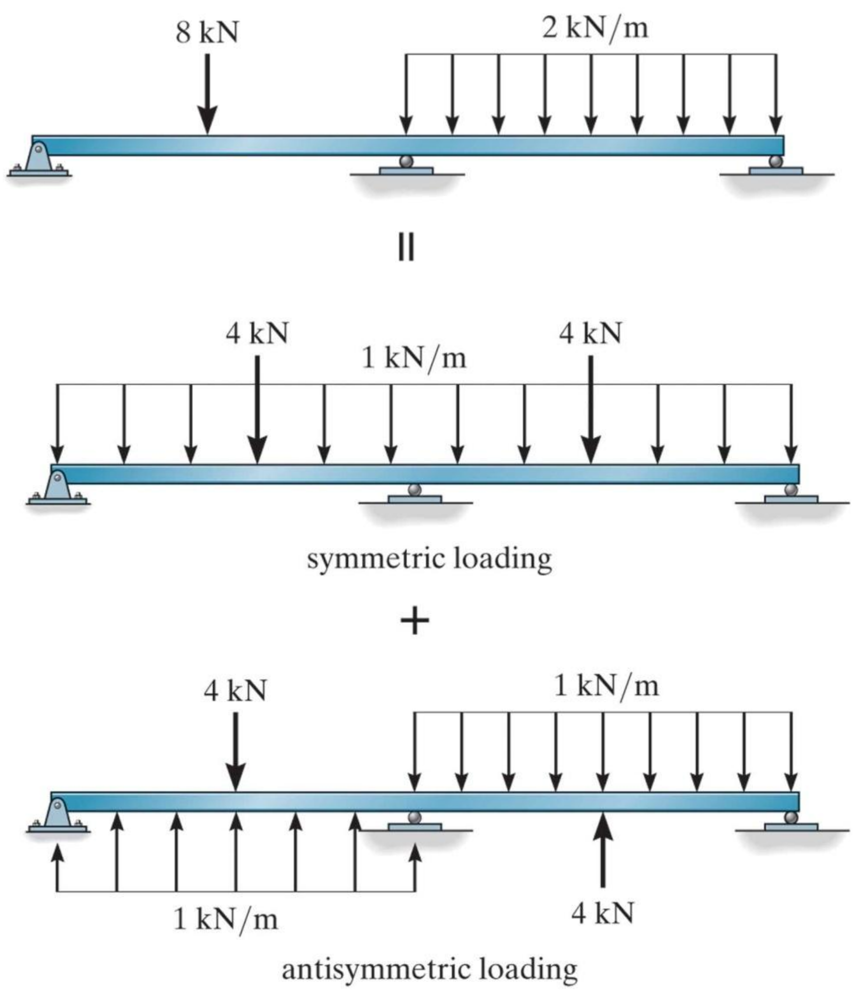
axis of symmetry

Antisymmetric Structures



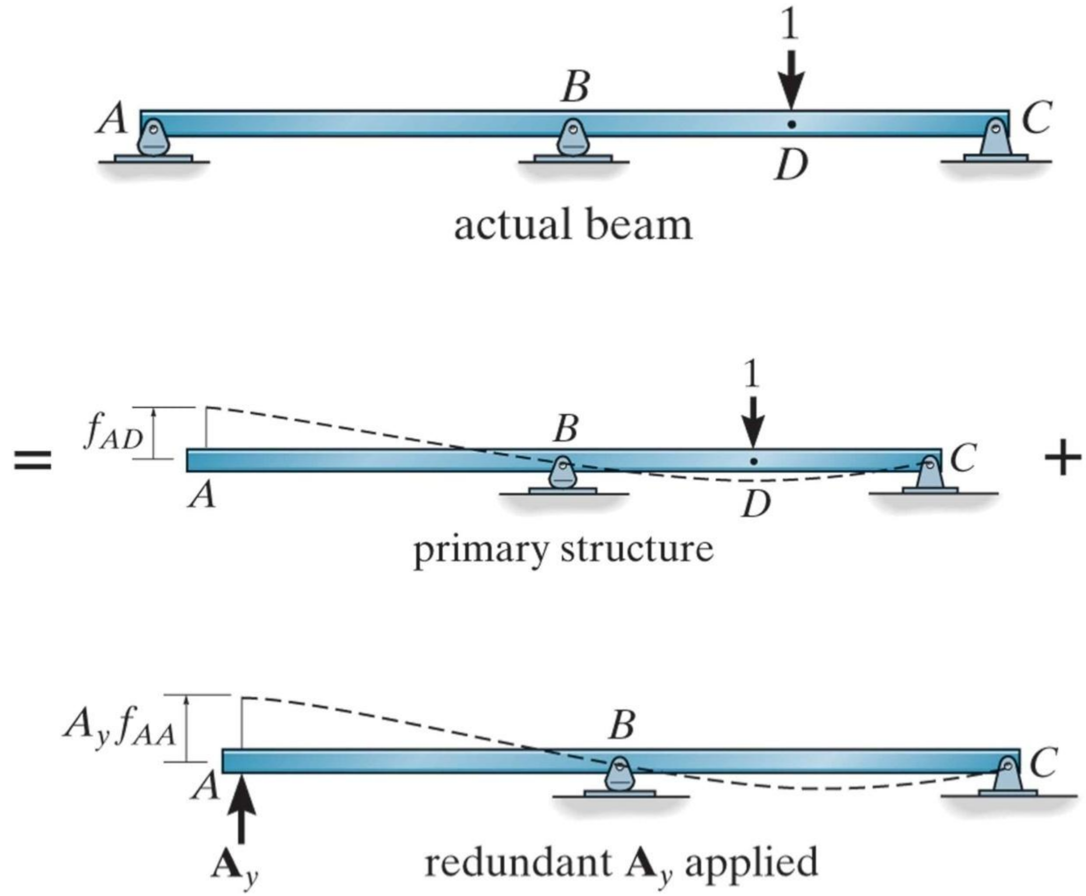
Antisymmetric loading

Transformation of Loading



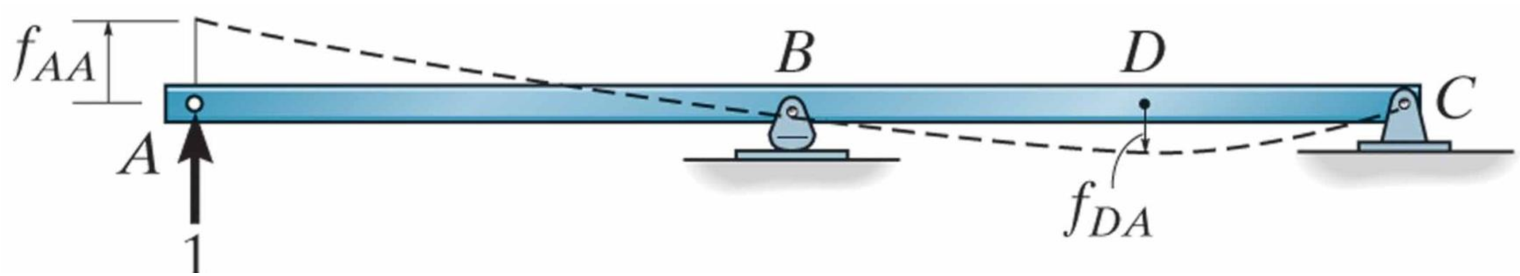
Influence Lines for Statically Indeterminate Beams

Reaction at A



Influence Lines for Statically Indeterminate Beams

Reaction at A

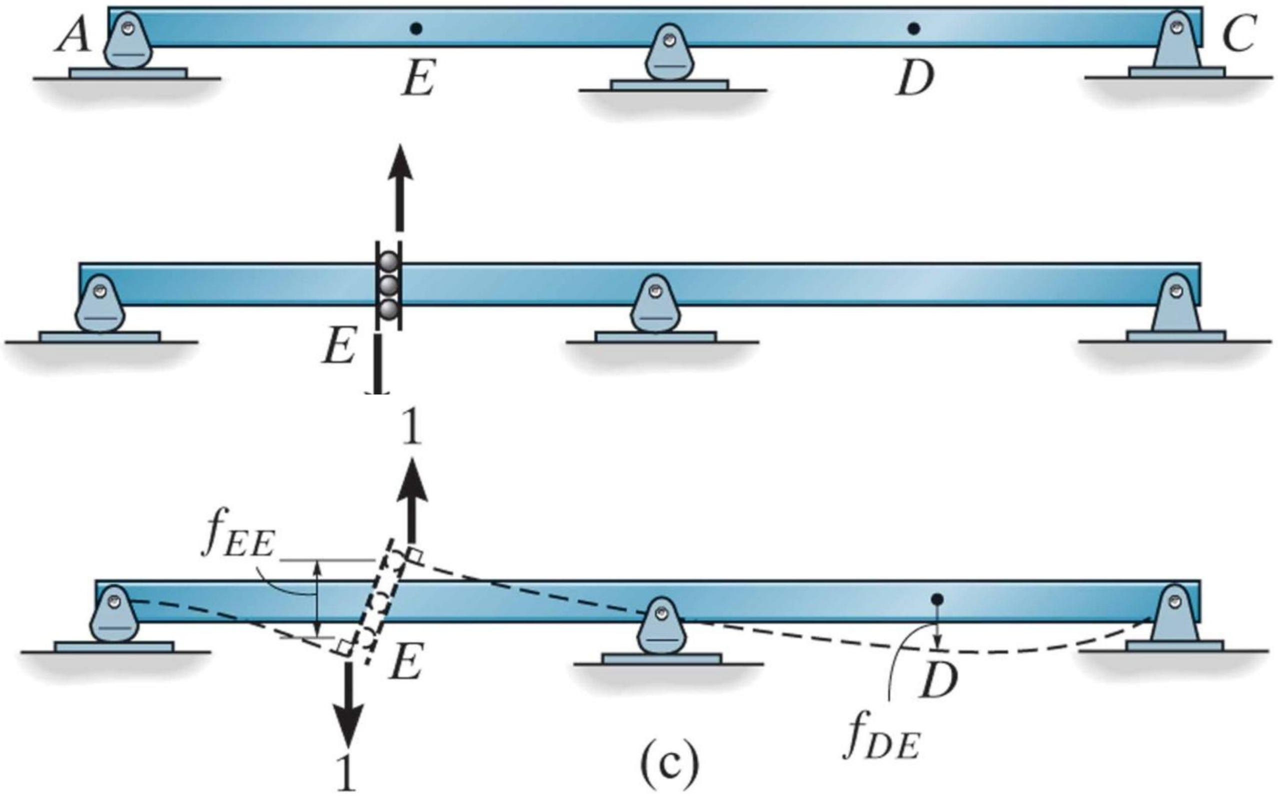


$$A_y = \left(\frac{1}{f_{AA}}\right) f_{DA}$$

Scale Factor $1/f_{AA}$

Influence Lines for Statically Indeterminate Beams

Shear at E



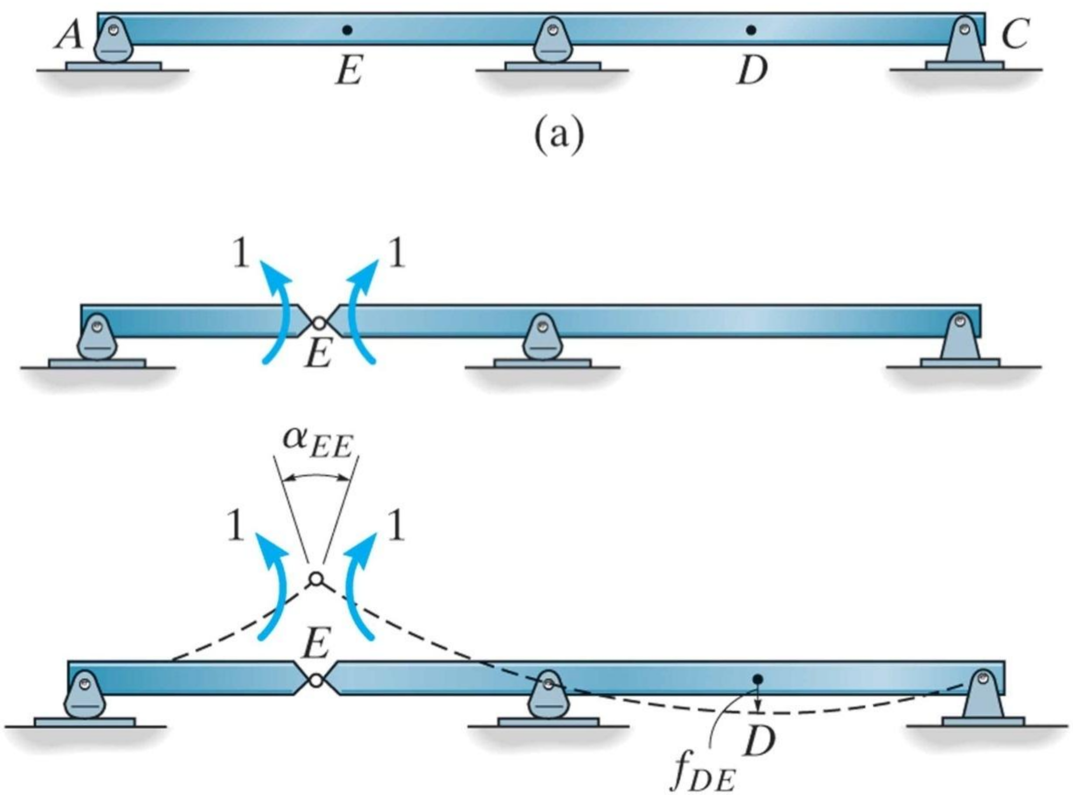
(c)

$$V_E = \left(\frac{1}{f_{EE}}\right) f_{DE}$$

Scale Factor $1/f_{EE}$

Influence Lines for Statically Indeterminate Beams

Moment at E

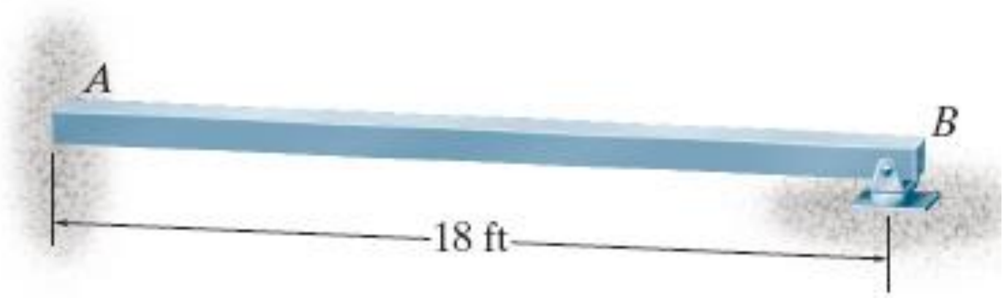


$$M_E = \left(\frac{1}{\alpha_{EE}}\right) f_{DE}$$

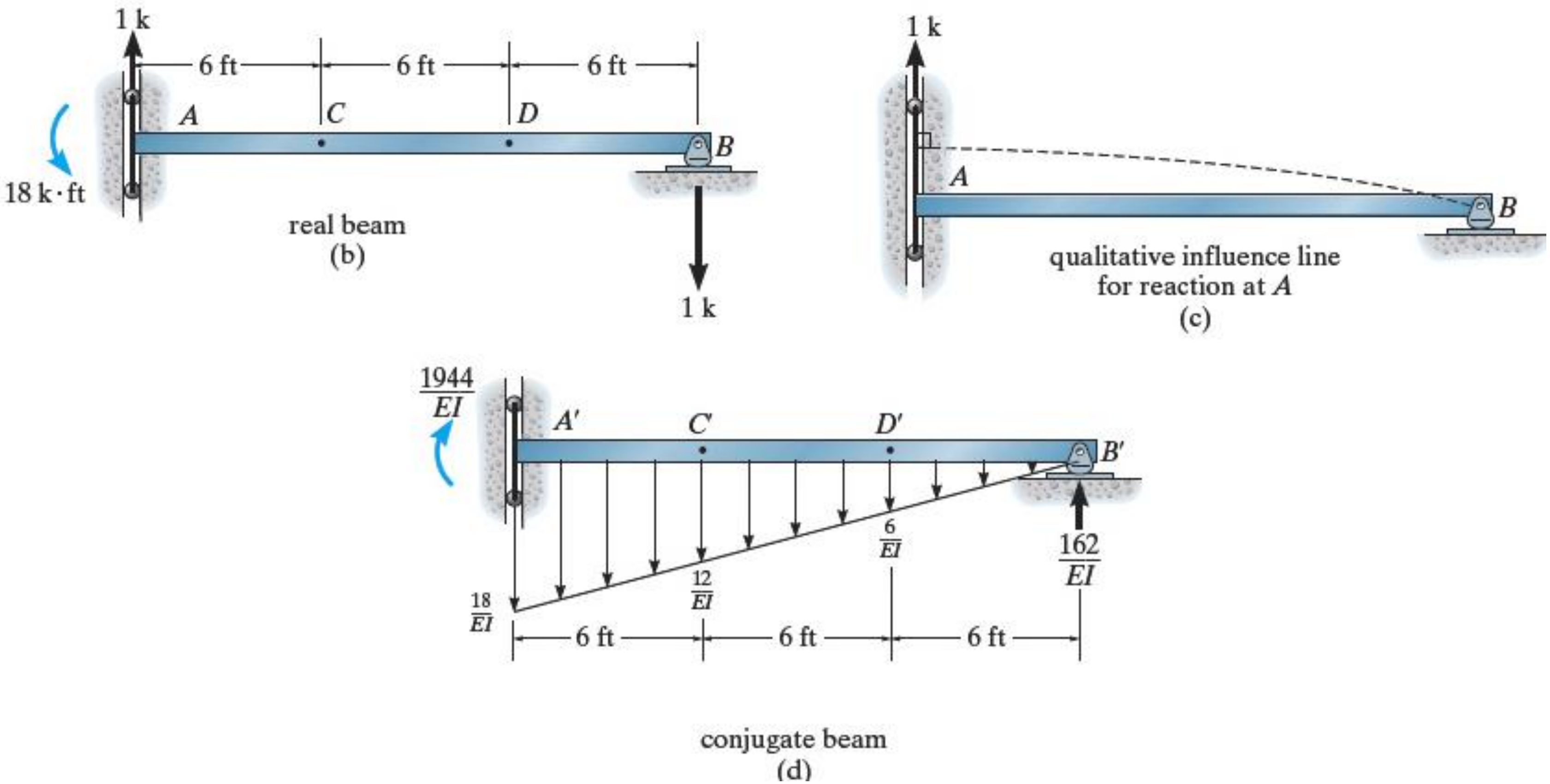
Scale Factor $1/\alpha_{EE}$

Example 11

Draw the influence line for the vertical reaction at A for the beam in Fig. 10–25*a*. EI is constant. Plot numerical values every 6 ft.



Example 11


1 k
6 ft
6 ft
6 ft
A
C
D
B
18 k·ft
real beam
(b)
1 k
1 k
A
B
qualitative influence line
for reaction at A
(c)
1944/EI
A′
C′
D′
B′
6/EI
162/EI
12/EI
18/EI
6 ft
6 ft
6 ft
conjugate beam
(d)

Example 11

For B', since no moment exists on the conjugate beam at B', Fig. 10–25*d*, then

$$\Delta_B = M_{B'} = 0$$

For D', Fig. 10–25*e*:

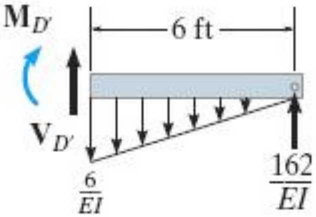
$$\Sigma M_{D'} = 0; \qquad \Delta_D = M_{D'} = \frac{162}{EI}(6) - \frac{1}{2}\left(\frac{6}{EI}\right)(6)(2) = \frac{936}{EI}$$

For C', Fig. 10–25*f*:

$$\Sigma M_{C'} = 0; \qquad \Delta_C = M_{C'} = \frac{162}{EI}(12) - \frac{1}{2}\left(\frac{12}{EI}\right)(12)(4) = \frac{1656}{EI}$$

For A', Fig. 10–27*d*:

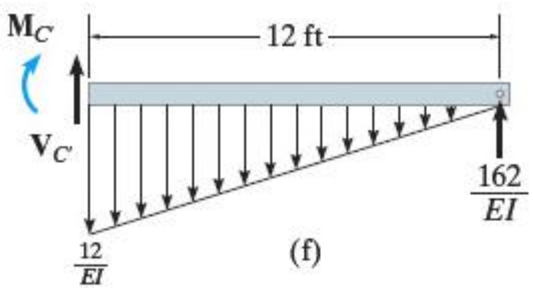
$$\Delta_A = M_{A'} = \frac{1944}{EI}$$



(e)

Example 11

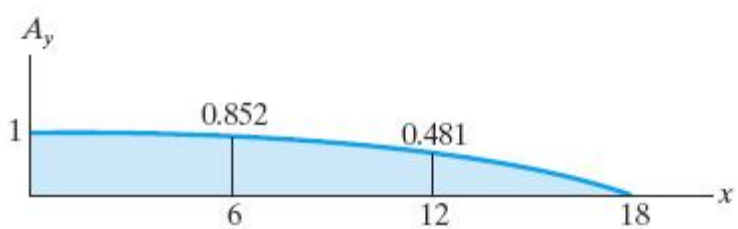
Since a vertical 1-k load acting at A on the beam in Fig. 10–25*a* will cause a vertical reaction at A of 1 k, the displacement at A, $\Delta_A = 1944/EI$, should correspond to a numerical value of 1 for the influence-line ordinate at A. Thus, dividing the other computed displacements by this factor, we obtain



(f)

x	A_y
A	1
C	0.852
D	0.481
B	0

A plot of these values yields the influence line shown in Fig. 10–25*g*.

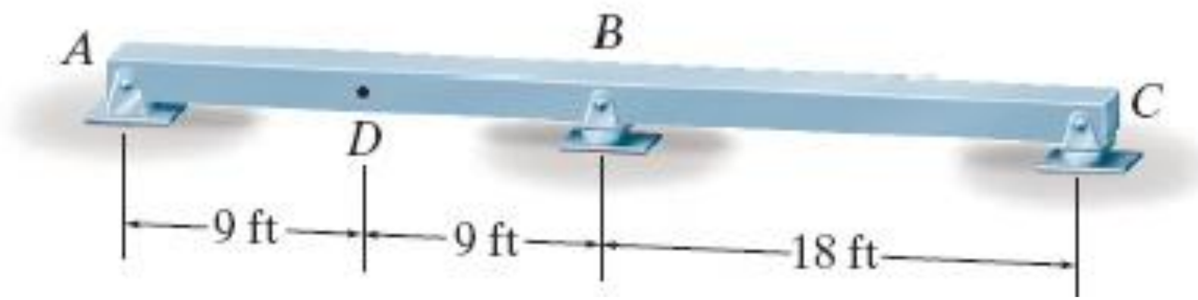


quantitative influence line for reaction at A

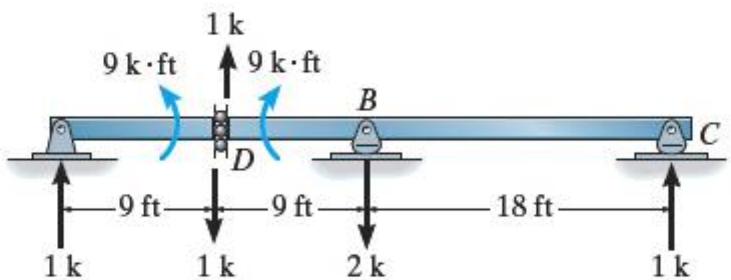
(g)

Example 12

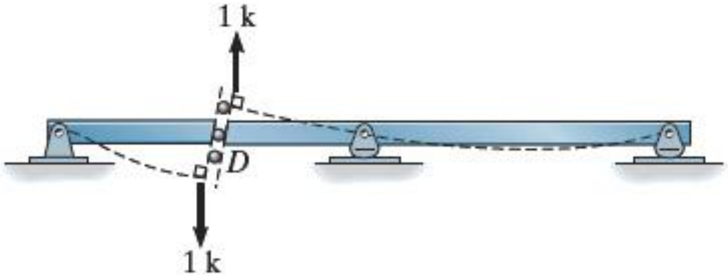
Draw the influence line for the shear at D for the beam in Fig. 10–26a. EI is constant. Plot numerical values every 9 ft.



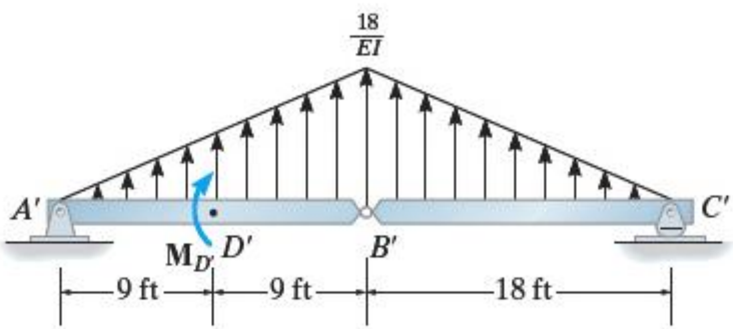
Example 12



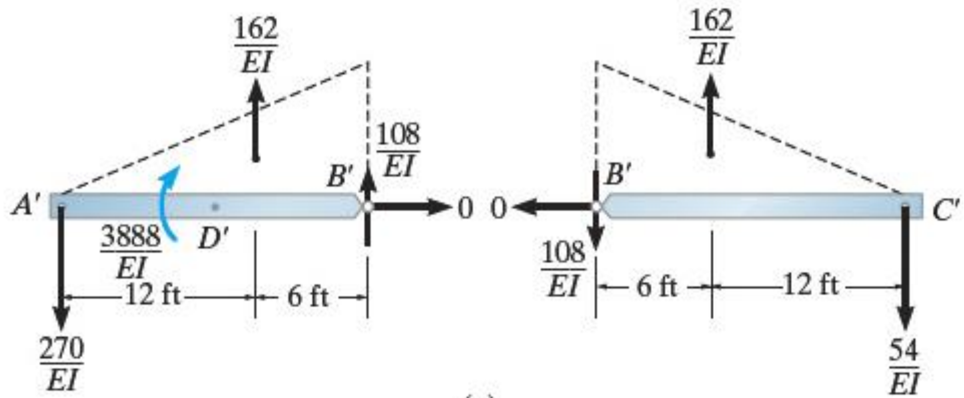
real beam
(b)



qualitative influence line
for shear at D
(c)



conjugate beam
(d)

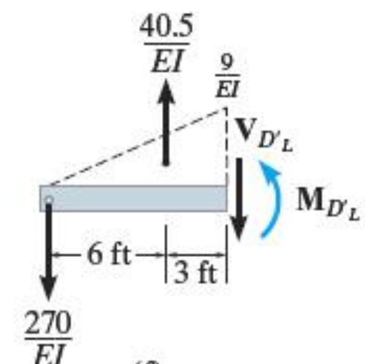


(e)

Example 12

Since there is a *discontinuity* of moment at D', the internal moment just to the left and right of D' will be computed. Just to the left of D', Fig. 10–26*f*, we have

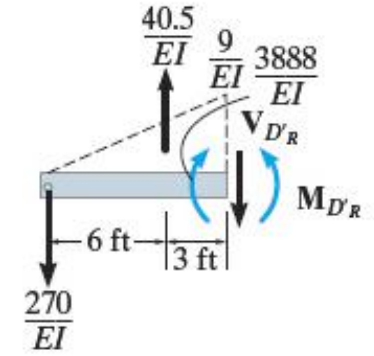
$$\Sigma M_{D'_L} = 0; \qquad \Delta_{D_L} = M_{D'_L} = \frac{40.5}{EI}(3) - \frac{270}{EI}(9) = -\frac{2308.5}{EI}$$



(f)

Just to the right of D', Fig. 10–26*g*, we have

$$\Sigma M_{D'_R} = 0; \qquad \Delta_{D_R} = M_{D'_R} = \frac{40.5}{EI}(3) - \frac{270}{EI}(9) + \frac{3888}{EI} = \frac{1579.5}{EI}$$



(g)

From Fig. 10–26*e*,

$$\Delta_A = M_{A'} = 0 \qquad \Delta_B = M_{B'} = 0 \qquad \Delta_C = M_{C'} = 0$$

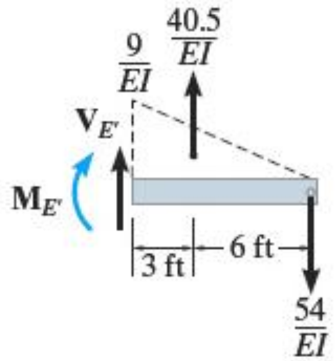
For point E, Fig. 10–26*b*, using the method of sections at the corresponding point E' on the conjugate beam, Fig. 10–26*h*, we have

$$\Sigma M_{E'} = 0; \qquad \Delta_E = M_{E'} = \frac{40.5}{EI}(3) - \frac{54}{EI}(9) = -\frac{364.5}{EI}$$

Example 12

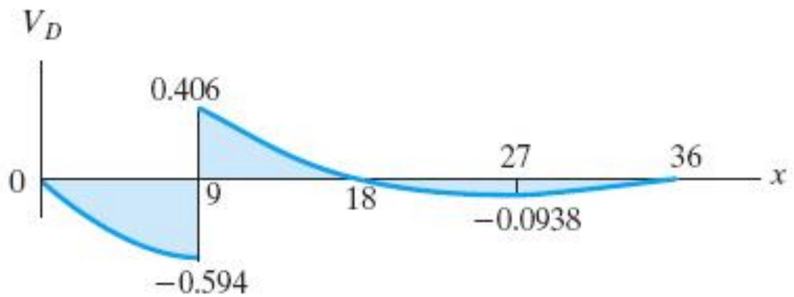
The ordinates of the influence line are obtained by dividing each of the above values by the scale factor $M_{D'} = 3888/EI$. We have

x	V_D
A	0
D_L	−0.594
D_R	0.406
B	0
E	−0.0938
C	0



(h)

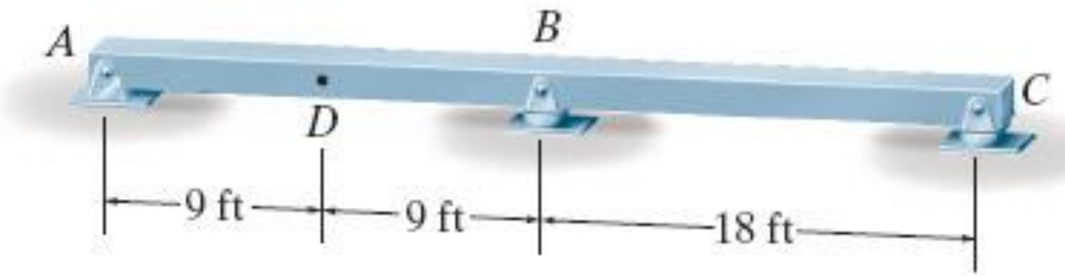
A plot of these values yields the influence line shown in Fig. 10–26*i*.



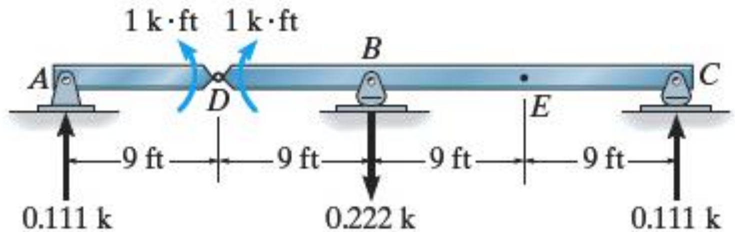
quantitative influence line
for shear at D
(i)

Example 13

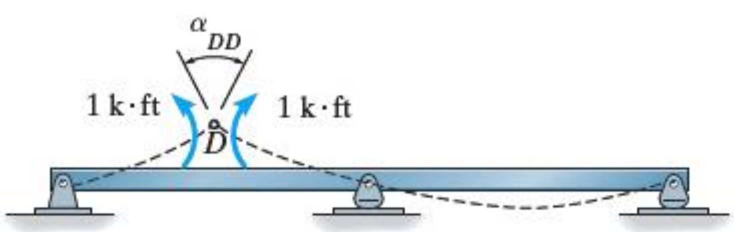
Draw the influence line for the moment at D for the beam in Fig. 10–27*a*. EI is constant. Plot numerical values every 9 ft.



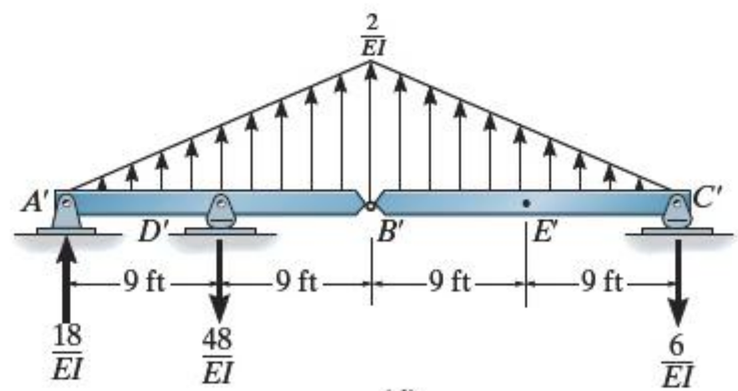
Example 13



real beam
(b)

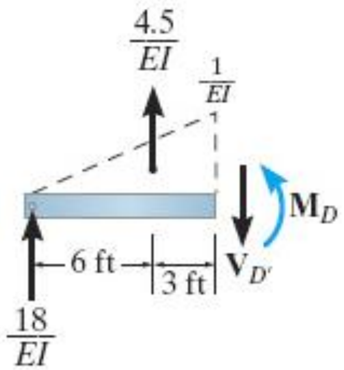


qualitative influence line for moment at D
(c)

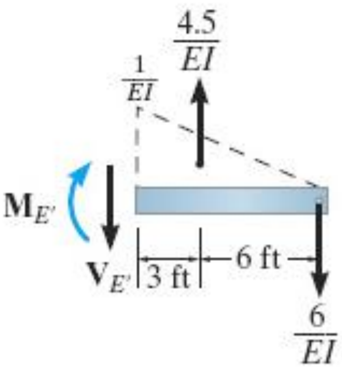


(d)

Example 13



(e)



(f)

For point D', Fig. 10–27*e*:

$$\Sigma M_{D'} = 0; \qquad \Delta_D = M_{D'} = \frac{4.5}{EI}(3) + \frac{18}{EI}(9) = \frac{175.5}{EI}$$

For point E', Fig. 10–27*f*:

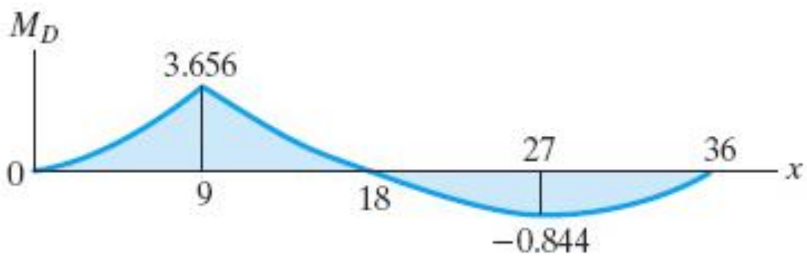
$$\Sigma M_{E'} = 0; \qquad \Delta_E = M_{E'} = \frac{4.5}{EI}(3) - \frac{6}{EI}(9) = -\frac{40.5}{EI}$$

Example 13

The angular displacement α_{DD} at D of the "real beam" in Fig. 10–27c is defined by the reaction at D' on the conjugate beam. This factor, $D'_y = 48/EI$, is divided into the above values to give the ordinates of the influence line, that is,

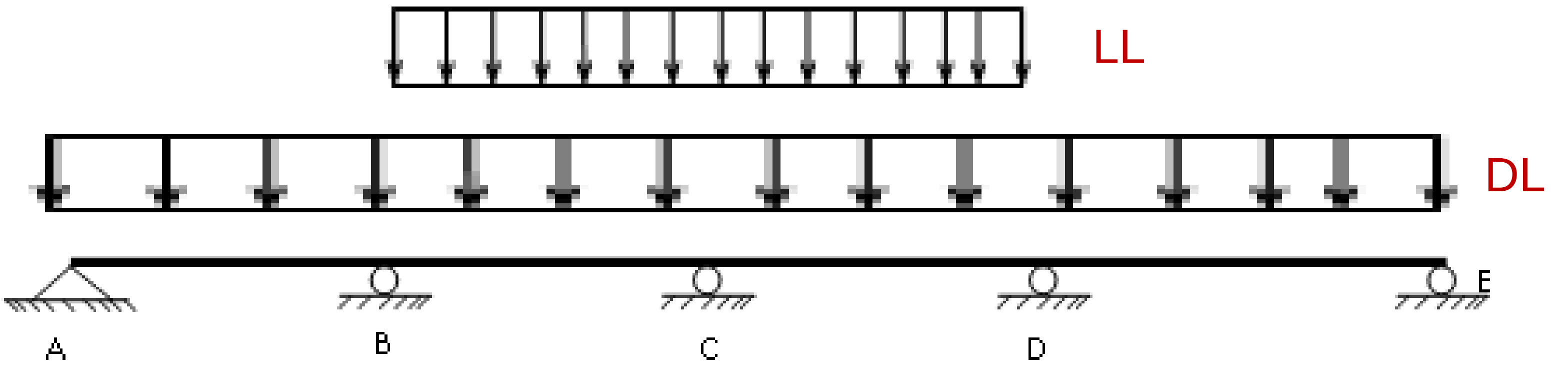
x	M_D
A	0
D	3.656
B	0
E	−0.844
C	0

A plot of these values yields the influence line shown in Fig. 10–27g.

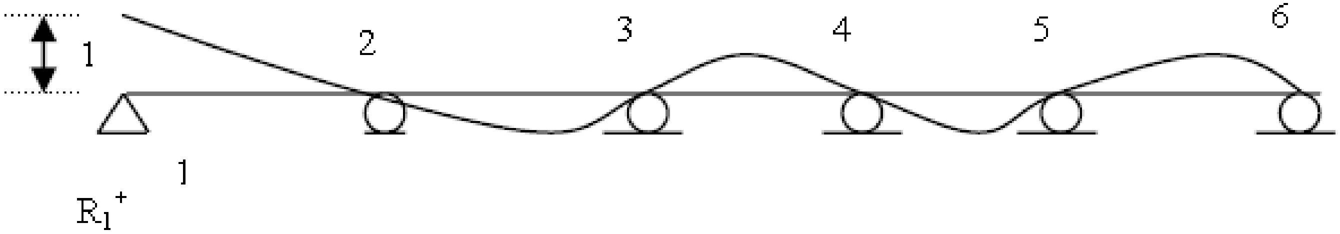


quantitative influence line
for moment at D
(g)

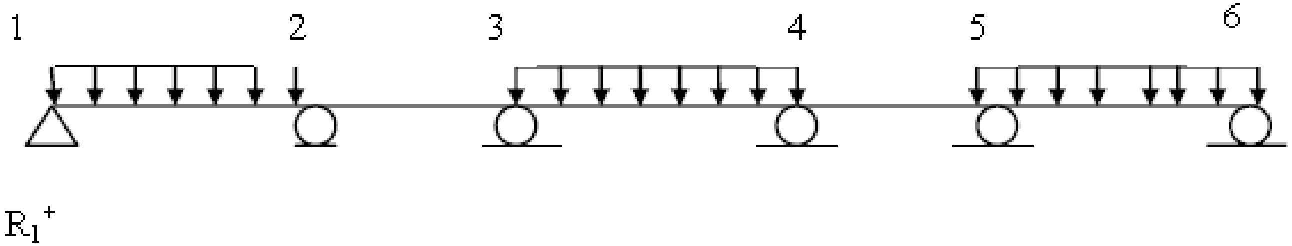
Live Load Pattern in Continuous Beams



Support Reactions

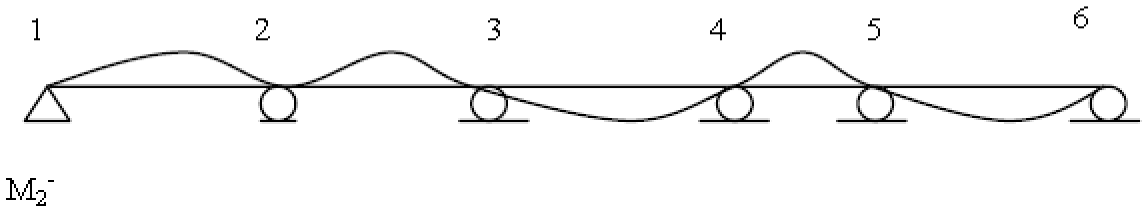


Influence Line for positive reaction at support 1

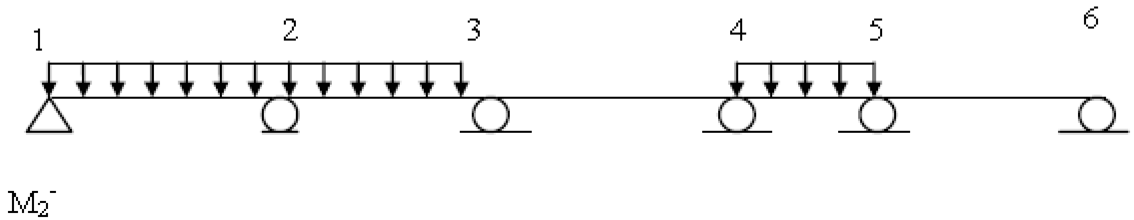


Load pattern for maximum positive reaction at support 1

Support Negative Moment

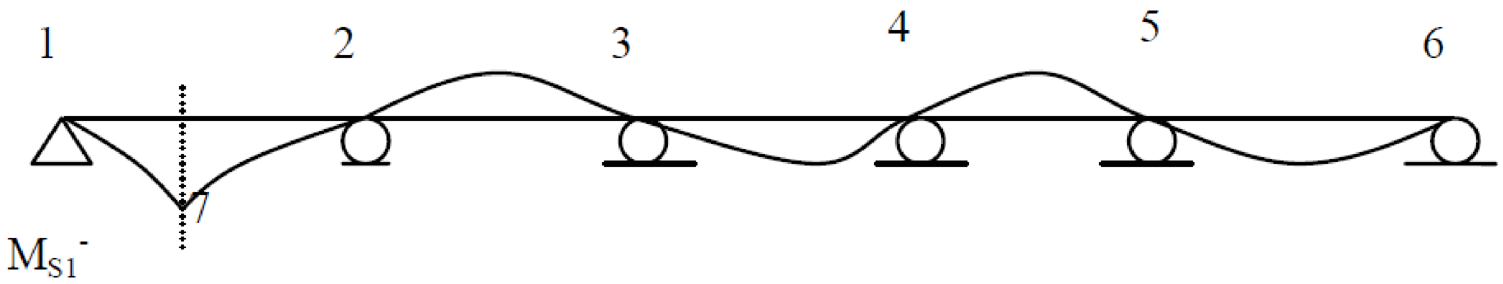


Influence line for negative moment at support 2

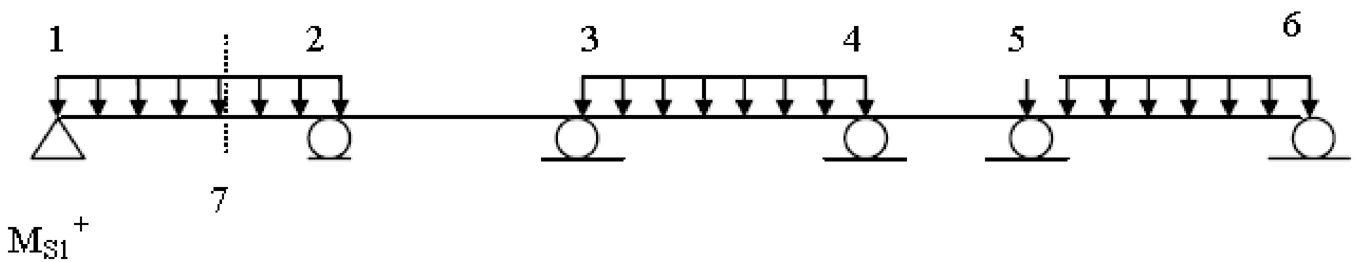


Load pattern for maximum negative moment at support 2

Span Positive Moment

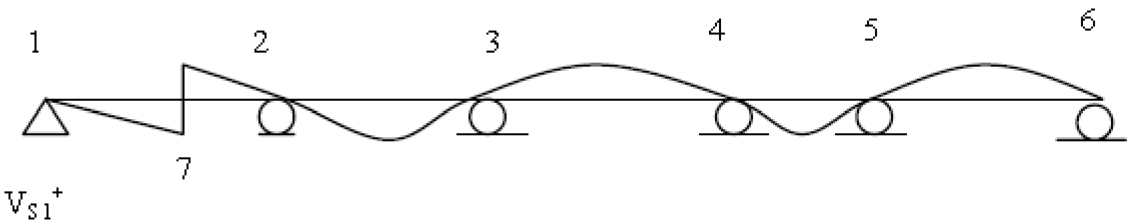


Influence line for positive moment at 7

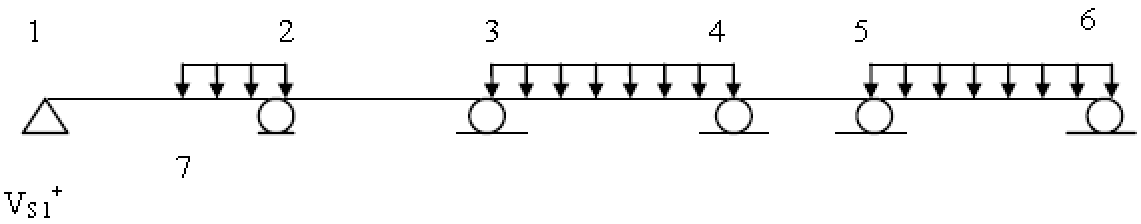


Load pattern for maximum positive moment at 7

Internal Shear



Influence line for positive shear at 7



Load pattern for maximum positive shear at 7

Live Load Pattern for Three Span Beam