

Analysis On The Dynamic Effect of Shear Force On An H-Section Block

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Abstract

The Paper presents an analysis on the dynamic effect of shear force on an H-section block free to move and thereafter the concept is incorporated on a cast iron block to study whether the theoretically calculated results corresponds to the experimental results. A static analysis has also been done to identify the validity of the experiment performed. The paper also presents the conditions for failure when the shear force is applied on the extreme end of the H-section block. Although the experiment has been carried out for a cast iron block, the main goal of this experiment is to analyze the effect of shear force on large structures, beams and truss members. With further optimization this process can also be utilized to calculate the young's modulus of a material.

Keywords: Shear Force, Static Analysis, ANSYS, Fracture, Stress Intensity Factor.

I. INTRODUCTION

The mathematical treatment on the equilibrium of cantilever beams or any other beams subjected to load does not involve a great difficulty [1-4]. Nevertheless, unless small deflections are considered, an analytical solution does not exist, since for large deflections a differential equation with a non-linear term must be solved. The equation is said to involve geometrical non-linearity [5, 6]. An excellent treatment of the problem of deflection of beam, built-in at one end and loaded at the other with a vertical concentrated force, can be found in "The Feynman Lectures on Physics" [2], as well as in other university textbooks on physics, mechanics and elementary strength of materials. The analysis of large deflections of cantilever beams of elastic material can be found in Landau's book on elasticity [5], and the solution in terms of elliptic integrals was obtained by Bisshopp and Ducker [7]. The dynamic effect of shear force on a rectangular block has already been analyzed by Samanta and Bose [8].

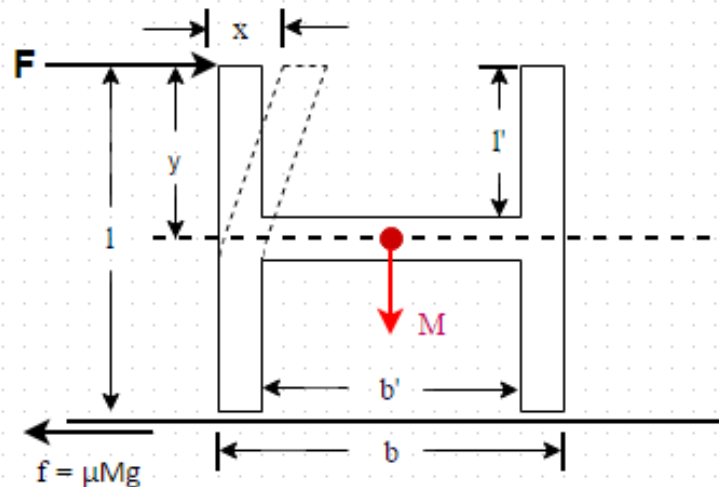


Fig. 1: Effect of Shear Force On A Rectangular Block

Moreover, with some modifications on the mathematical treatment for bending of beams, we can derive the mathematical expression for bending of a body subjected only to friction. From our general concept of physics and elementary strength of materials, we know that when the body is subjected to a shear force above the centroidal axis, it first undergoes a deformation and then comes into motion as shown in Figure 1. However, for such deformation, the force must be a shear force acting above the centroidal axis. In such case, the body can be considered as an H-section beam before the limiting friction is reached and with the help of certain assumptions and modification on the previously derived mathematical model for a rectangular block [8], the mathematical modeling for the H-section block can be carried out and the additional force can be calculated.

II. METHODOLOGY

A. Mathematical modeling

Let us consider the following nomenclature for deriving the mathematical expression for the additional force.

- E – Young's modulus of the material
- I – Moment of inertia of the material
- F – Shear force
- F_f – Frictional force
- F_t – Total force
- M – Mass of the body
- x – Linear deformation
- y – Depth from the top of the block
- l – Height of the block
- b – Width of the block
- μ – Coefficient of friction between the block and the surface
- g – Acceleration due to gravity

The H-section block is assumed to be homogeneous and isotropic. It is also assumed to have a uniform cross-sectional area. Thereafter, considering that the centre of gravity to exactly lie on the centroidal axis, we have

$$EI \frac{d^2x}{dy^2} = Fy - \mu Mgy$$

$$\Rightarrow EI \frac{dx}{dy} = \frac{Fy^2}{2} - \frac{\mu Mgy^2}{2} + C_1$$

At A, $\frac{dx}{dy} = 0$ and putting $y = l$

$$\Rightarrow C_1 = \frac{\mu Mgl^2 - Fl^2}{2}$$

$$\Rightarrow EI \frac{dx}{dy} = \frac{Fy^2}{2} - \frac{\mu Mgy^2}{2} + \frac{\mu Mgl^2 - Fl^2}{2}$$

$$\Rightarrow Elx = \frac{Fy^3}{6} - \frac{\mu Mgy^3}{6} + \frac{\mu Mgl^2y - Fl^2y}{2} + C_2$$

At A, $x = 0$ and putting $y = l$

$$\Rightarrow C_2 = \frac{\mu Mgl^3 - Fl^3}{6} - \frac{\mu Mgl^3 - Fl^3}{2} = \frac{Fl^3 - \mu Mgl^3}{3}$$

Maximum value of x will occur at point B due to maximum deformation due to bending.

\therefore At B, $x = x_{max}$ and $y = 0$

$$\Rightarrow Elx_{max} = \frac{Fy^3}{6} - \frac{\mu Mgy^3}{6} + \frac{\mu Mgl^2y - Fl^2y}{2} + \frac{Fl^3 - \mu Mgl^3}{3} = \frac{Fl^3 - \mu Mgl^3}{3}$$

$$\Rightarrow x_{max} = \frac{Fl^3 - \mu Mgl^3}{3EI}$$

Again, $Fl^3 = \mu Mgl^3 + 3Elx_{max}$

$$\Rightarrow F = \mu Mg + \frac{3Elx_{max}}{l^3}$$

For a H – section block, we have $I = \frac{1}{12} [bl^3 - 2b'l'^3]$

$$\therefore F = \mu Mg + \frac{E x_{max}}{4} [b - 2b'\gamma^3] \quad \text{where } \gamma = \frac{l'}{l}$$

$$\therefore \text{Additional force to move the body, } F_{ad} = F - \mu Mg = \frac{E b x_{max}}{4} [b - 2b'\gamma^3]$$

When $l' \ll l$, $F_{ad} = \frac{E b x_{max}}{4}$ which is same as the additional force required for a rectangular balock.

Fatigue toughness is an indication of the amount of stress required to propagate a pre-existing flow. It is a very important material property since the occurrence of flow is not completely available in the processing fabrication, or service of a material.

A parameter called stress-intensity factor (k) is used to determine the fracture toughness of most materials. It is used in fracture mechanics to predict the stress state near the tip of a crack caused by a remote load or residual stresses [9]. The stress intensity function is a function of loading crack size and structural geometry.

The stress intensity factor may be represented by the following equation:

$$k = \sigma_s \sqrt{\pi c}, \text{ where } c \text{ is the crack length [10]}$$

For an H section block, shear stress developed for the equivalent amount of force is

$$\sigma_s = E \cdot \left\{ \frac{2x_{max}}{b - b'} \right\}^p, \text{ where } p \text{ is the strain index that depends upon the plasticity of the body.}$$

Hence, the H section block will just fail when

$$\begin{aligned} k &= \lim_{c \rightarrow 0} \sigma_s \sqrt{\pi c} \\ \Rightarrow \sigma_s &= \lim_{c \rightarrow 0} \frac{k}{\sqrt{\pi c}} \\ \Rightarrow E \cdot \left\{ \frac{2x_{max}}{b - b'} \right\}^p &= \lim_{c \rightarrow 0} \frac{k}{\sqrt{\pi c}} \end{aligned}$$

B. Static Analysis

A static analysis calculates the effects of steady loading conditions on a structure, while ignoring inertia, damping effects and other reaction forces. In the present investigation, the static analysis has been done using finite element software called ANSYS. For the analysis, the H-section is considered as a simple solid structure fixed to the base and a shear force of 1 kN is applied on the side of the structure at a distance of 1 mm from the top. The purpose of the analysis is to obtain the appropriate deformation of the H-section when subjected to a shear force. The static analysis helps us to identify the validity of the experiment performed and whether it actually corresponds to the mathematical model derived. Figure 2 shows the level of deformation at various positions of the H-section when subjected to shear force as mentioned above whereas Figure 3 demonstrates the equivalent shear strain at the different positions.

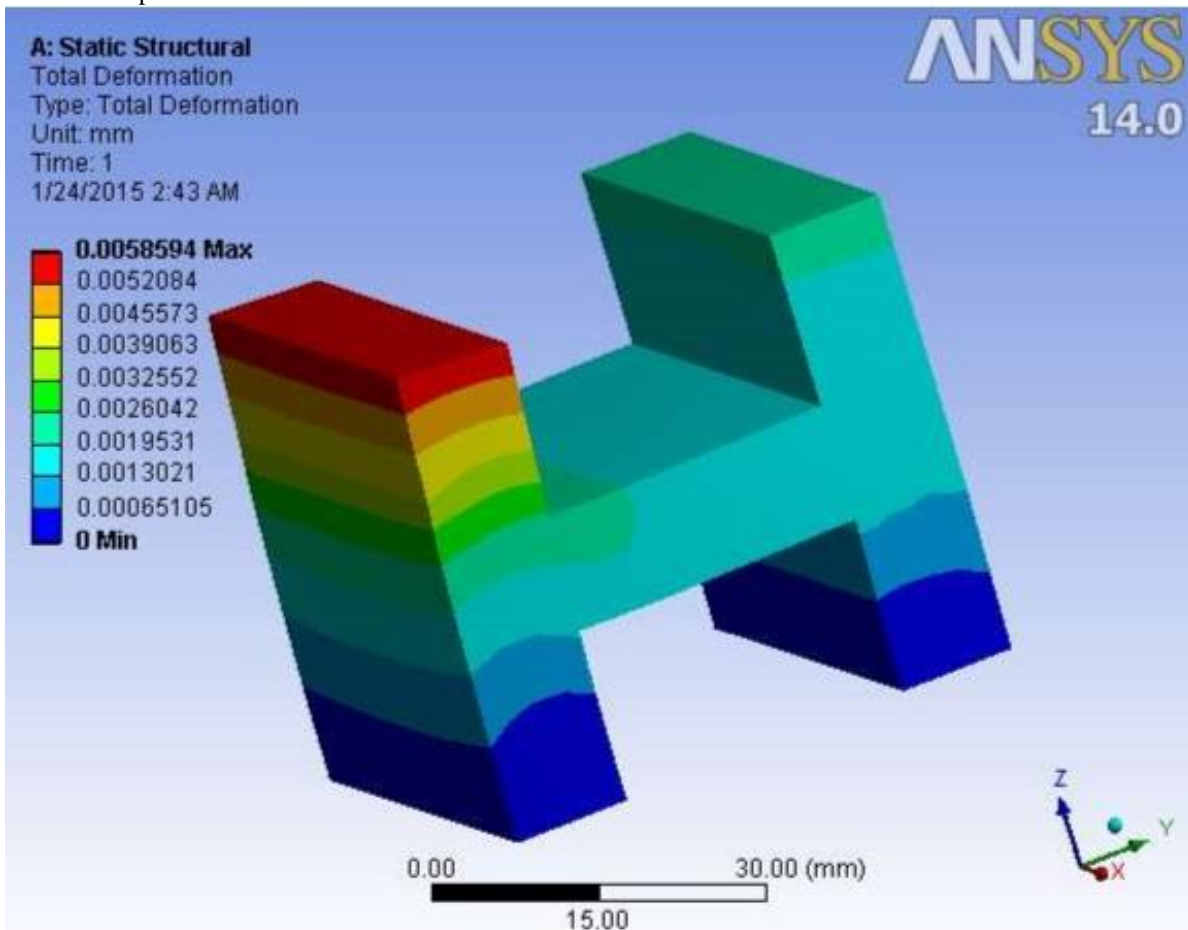


Fig. 2: Deformation At Various Positions of The H-Section

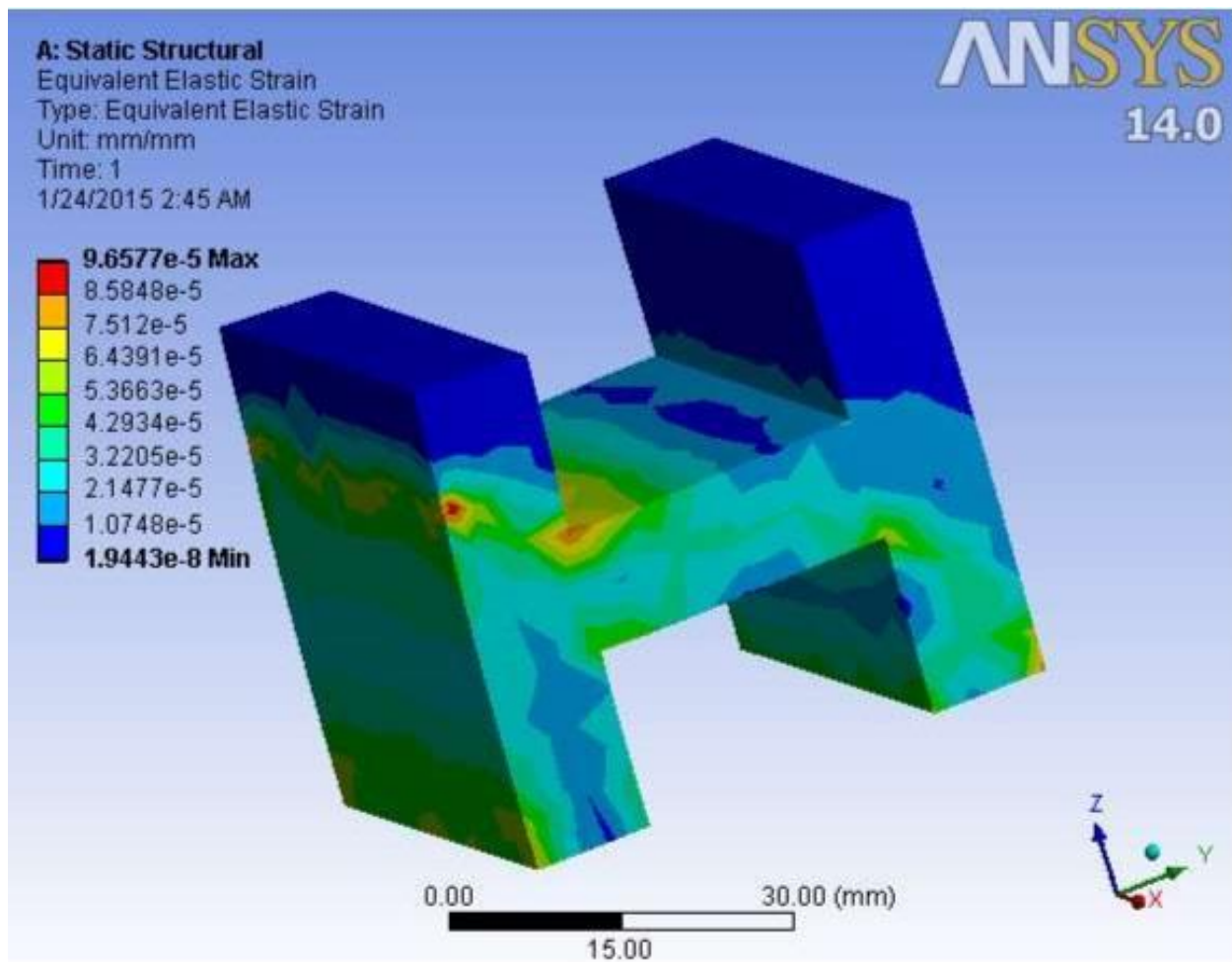


Fig. 3: Equivalent Shear Strain At The Different Positions

C. Experimental Details

Although this research finds its application in the analysis of large structures, a small cast iron block is chosen as a specimen to investigate the validity of the mathematical modeling. The purpose behind selecting a cast iron for the experiment is that cast iron is cheap and easily available. The block is then shaped into an H-section as per the design made for the experiment. An experimental setup was made with the combination of a dial indicator and a dynamometer. The tool attached to the dynamometer is slowly fed into the metal H-section block and the dial indicator measures the deflection on the cast iron block until the limiting friction is reached and the body just tends to slip on the surface. A proximity sensor is used to detect the slightest movement of the block on the surface. The net equivalent force is simultaneously measured and displayed on the digital screen of the dynamometer. The frictional force between the specimen and the surface is subsequently subtracted from the total force measured by the dynamometer to obtain the value of the additional force. The deformation obtained at various positions of the H-section is tallied with the static analysis made for the job. If the two results are observed to correspond to one another, the experimental and the theoretical values of the additional forces are tallied to investigate the validity of the mathematical model.

III. RESULTS AND DISCUSSION

In this investigation, a cast iron specimen is selected as the work piece. The specification of the test specimen is represented in Table 1 whereas Table 2 shows the results obtained from the experiment conducted on the basis of the aforementioned methodology.

Table – 1
Specification of the Cast Iron Block

Mass (M)	Young's Modulus (E)	Height (l)	Width (b)	l'	b'
471.2 gm	110 GPa	47.36 mm	49.64 mm	16.30 mm	27.80 mm

Let $b^* = b - 2b'\gamma^3$ where $\gamma = \frac{l'}{l} = 0.344$

Table - 2
Experimental Result on a Cast Iron Block

SL No.	y (mm)	x (μ m)	F_f (N)	F_t (N)	Additional Force, F_{ad} (N)	
					Experimental $F_{ad} = F_t - F_f$	Theoretical $F_{ad} = \frac{Eb^3x}{4}$
1	20	0.012	1.62	15.60	13.98	14.88
2	15	0.017	1.62	21.70	20.08	21.08
3	10	0.023	1.62	28.84	27.22	28.52
4	5	Topples	1.62	-	-	-
5	1	Topples	1.62	-	-	-

It can be clearly noticed from the result that the deflection is in the micro order and the entire experiment has to be carried out with highly sensitive mechanical instruments. However, the minor difference in the results of the experimentally obtained additional force and the theoretical obtained additional force can be explained in a number of ways. The instruments required for measuring the force and the deformation have to be very sensitive since the deformations are obtained in the micro order and the additional force is also quite small. A little error in the reading will lead the results to vary to a large extent. Moreover, the specimen is not absolutely homogenous and isotropic mainly due to casting defects and impurities. The toppling is also justified because for the selected geometry, the centre of gravity easily tends to shift downwards when the point of application of force moves further upwards from the base. However, both the result obtained closely corresponds to one another and is also in accordance with the results obtained from the static analysis. Hence the mathematical model is valid.

IV. CONCLUSION

From the above investigation, the following conclusions can be drawn with the help of the various mathematical models derived.

- (1) For an H-section block the additional force is F_{ad} is proportional to the elasticity of the material and the value of $[b - 2b'\gamma^3]$ and for a very small value of l' the additional force, F_{ad} gets reduced to the same as that obtained for a rectangular block.
- (2) The width of the work piece and the surface of contact play a very important role during toppling and fracture.
- (3) The mathematical model will be very helpful in civil and mechanical engineering during the analysis of bridges, machines, truss members and other large structures. The model can also be used to determine the elasticity and other properties of a material.

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