Analysis on the use of a Strain Gauge, Accelerometer, and Gyroscope

For calculating the displacement at the end of a cantilever beam.

By – Tyler Cone

For – Dr. David Turcic

ME – 410

Due 5/15/2014

Portland State University

Mechanical and Materials Engineering Department

Contents

1 – INTRODUCTION

The purpose of this experiment was to analyze the accuracy and effectiveness of using a strain gauge, accelerometer, and gyroscope for measuring the displacement at the end of a cantilever beam. This was done by applying a step input to the end of the beam, and letting the beam oscillate freely. The natural frequency and damping ratio was also calculated for the beam.

2 – THEORY

Each of the instruments used operate on a different theory.

2.1 – Strain Gauge

A strain gauge consists of a length of wire with several loops that have been mounted on a piece of flexible backing. The backing is then mounted to a beam and will deform with the beam and will exert a compressive or tensile force on the wire. As the wire is put under tensile and compressive stress, its resistance will change accordingly. This relationship is expressed in Eq. 1

$$\frac{\Delta R}{R} = (GF)\epsilon \qquad \text{Eq. 1}$$

Where ϵ is the strain in the gauge, R is the nominal resistance of the gauge, ΔR is the change in resistance, and GF is the Gauge Factor that is provided with the strain gauge.

Due to the usual small values for strain, ΔR is usually very small. Thus, the best way to measure ΔR is to use a wheatstone bridge which will output a measureable voltage with a minimal change in resistance. Figure 1 is a schematic of a wheatstone bridge where R_{SG} is the strain gauge, $R_2, R_3, \& \, R_4$ are resistors, V_E is the excitation voltage, and V_m is the measured voltage. Equation 2 gives the

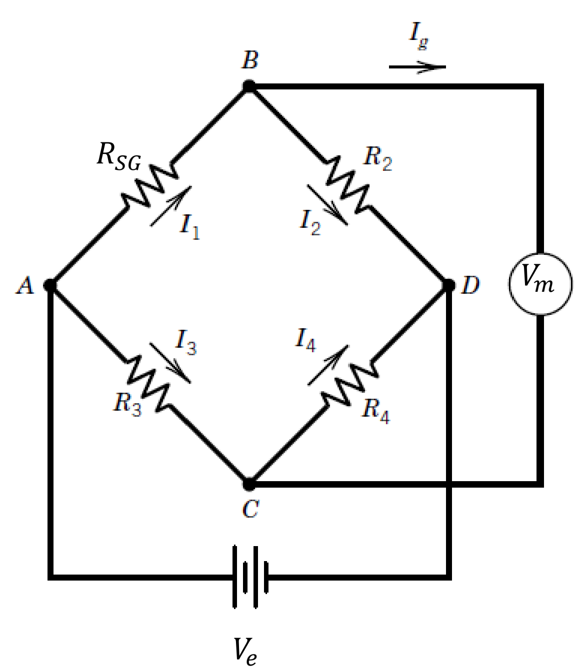

Figure 1- Wheatstone bridge

relationship for the measured voltage in terms of the excitation voltage

$$V_m = \left[\frac{R_2R_4 - R_1R_3}{(R_1 + R_2)(R_3 + R_4)}\right] V_e \qquad \text{Eq. 2}$$

It can be seen, that if $R_1 = R_2 = R_3 = R_4$, then no voltage will pass through to the voltmeter at V_m. At this point, the bridge is considered to be "balanced". If one of the resistances were to change, then there would be a measured voltage. It was then passed through a Differential Amplifier with a gain of G where G is defined as

$$G = 1 + \frac{50{,}000\ \Omega}{R_g} \qquad \text{Eq. 3}$$

And R_g is a resistor that can be changed to give the desired amplification.

According to Appendix A, in Lab 5: Position Estimator using a Strain Gauge, an Accelerometer, and a Gyroscope, the equations for strain and voltage can be rewritten as

$$\delta_{max} = -\frac{4}{3}\left[\frac{V_m L^3}{((G)(GF)(L-x)(V_e)(c)]}\right] \qquad \text{Eq. 4}$$

Where L is the length of the cantilever beam, x is the distance from the Strain Gauge to the support, and c is the distance from the neutral axis to the outermost fiber in the cantilever beam.

2.2 – Accelerometer

An accelerometer outputs a voltage proportional to the acceleration. The voltage is then converted to acceleration using Eq. 5

$$a = S_{acc}V_{acc} \qquad \text{Eq. 5}$$

Where S_{acc} is the sensitivity in mV/g as found in the Data Sheet and V_{acc} is the voltage from the accelerometer.

Since displacement is the double integral of acceleration, the displacement can be found.

$$\delta = \iint a\ dt \qquad \text{Eq. 6}$$

2.3 – Gyroscope

A gyroscope works by converting angular velocity to a voltage. Like Eq. 5, the value for the angular velocity can be calculated as

$$\theta = S_{gyr}V_{gyr} \qquad \text{Eq. 7}$$

Where S_{gyr} is the sensitivity in $mV/cycle$. The value for the angular velocity was then converted to rad/sec. Then, the angular displacement is the integral of the angular velocity.

$$\theta = \int \omega \, dt \qquad \text{Eq. 8}$$

Where θ is the angular displacement and ω is the angular velocity. Then, using Appendix B from Lab 5: Position Estimator using a Strain Gauge, an Accelerometer, and a Gyroscope, the displacement can be written as

$$\delta_{max} = \frac{2}{3}\theta L \qquad \text{Eq. 9}$$

2.4 – Natural Frequency And Damping Ratio

To calculate the ringing frequency, the period of the wave was inverted and than multiplied by 2π such as in Eq. 9

$$\omega_d = \frac{1}{T} * 2\pi \quad (rad/s) \qquad \text{Eq. 10}$$

Where T is the period of the oscillation.

To calculate the damping ratio, the logarithmic decrement was used.

$$\delta' = \frac{1}{n-1}\ln\left(\frac{y_1}{y_n}\right) \qquad \text{Eq. 11}$$

Where y_1 is the height of the first peak, y_{1+n} is the height of another peak n peaks away. The damping ratio could then be found

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta'}\right)^2}} \qquad \text{Eq. 12}$$

The natural frequency is

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} \quad \text{Eq. 13}$$

3 – EXPERIMENTAL SETUP

An aluminum beam was obtained and mounted to a table using a clamp and the dimensions of the beam were recorded. The strain gauge, accelerometer, and gyroscope were mounted. An experimental schematic can be found in Figure 2 and the dimensions are located in Table 1

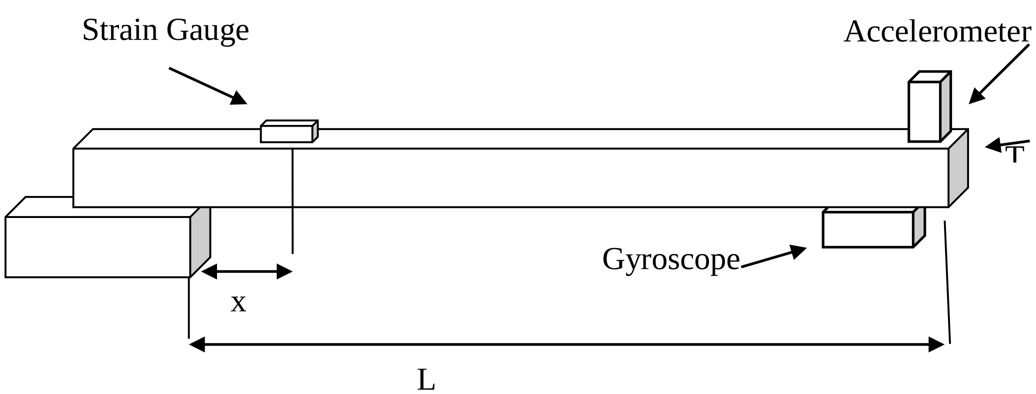

Figure 2 - Experimental Setup Schematic

Table 1 - Dimensional Qualities for a cantilever beam.

Item	Variable	Value
Length	L	9.825 inches
Distance from the table to the strain gauge	X	1.5 inches
Thickness	T	0.0661 inches

3.1 – Strain Gauge

The Wheatstone bridge was constructed using three 120Ω resistors, the strain gauge, and a $10\ k\Omega$ potentiometer to balance the bridge. The output of the Wheatstone bridge was then passed through a INA105E differential amplifier to remove the noise. The wheat stone bridge was powered by a constant 8V and the differential amplifier was powered by +8V and -8V. The idea output of the amplifier was a maximum of 2V, so a resistor of $180\ \Omega$ was chosen to be the gain

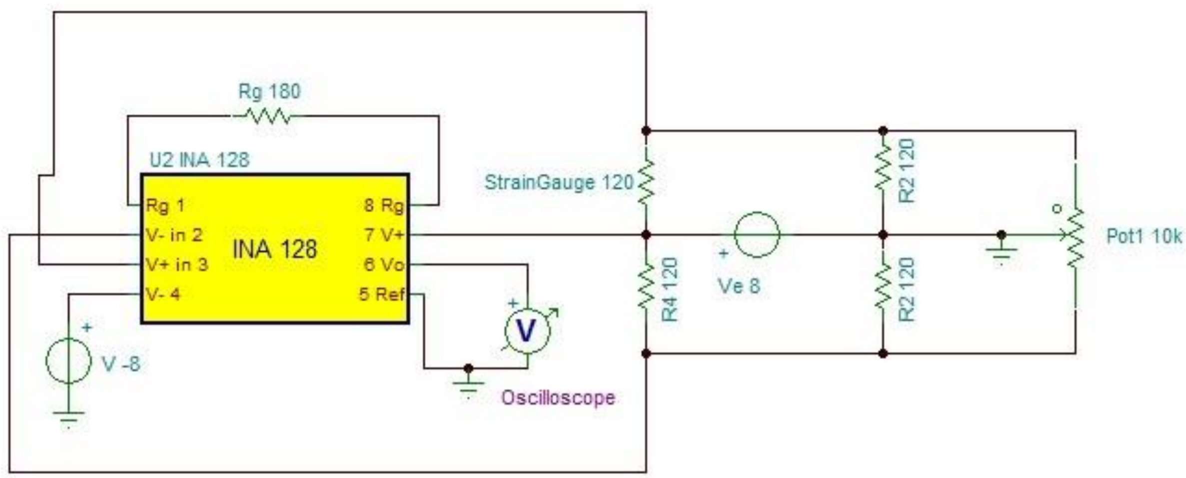

Figure 3 - Wheatstone bridge with the differential amplifier.

resistor which gave an amplification of 278 according to Eq. 3. An oscilloscope was then used to measure the output of the differential amplifier and referenced to ground. Figure 3 is a schematic showing the setup.

Once the Wheatstone bridge and differential amplifier were built, the power was turned on and after approximately 5 minutes, the bridge was balanced using the potentiometer. The five minutes was allowed to elapse to allow the resistors to heat up and reach a constant resistance.

3.2 – Accelerometer & Gyroscope

The accelerometer was attached to the end of the cantilever beam along the x-axis. It was then wired using the schematic in Figure 4. A power supply of +8V was passed through a 3.3V Voltage Regulator and the output was run to the VCC pin of the accelerometer. The GND pin was wired to ground and the oscilloscope was attached to the X pin.

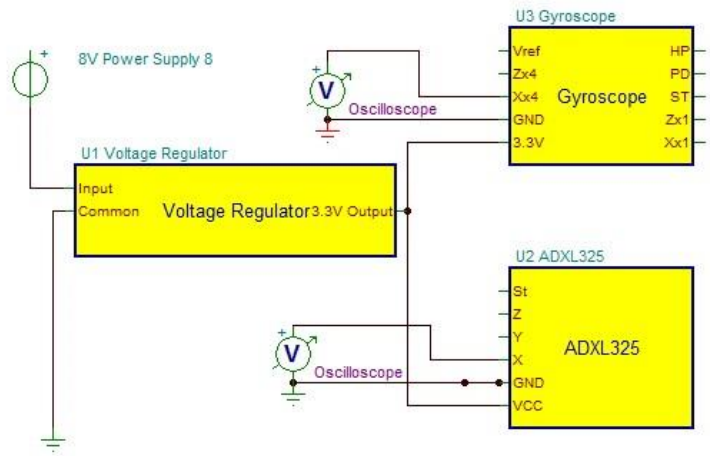

Figure 4 - Schematic of the accelerometer & gyroscope wiring

The gyroscope was attached to the end of the cantilever beam in the X direction. The 3.3V was wired to the output of the Voltage Regulator, and GND was attached to ground, and the Xx4 was attached to the Oscilloscope. Later, it was realized that the Oscilloscope had been hooked up to the four times amplification and the Eq. 8 was changed to

$$\delta_{max} = \frac{\frac{2}{3}\theta L}{4} \qquad \text{Eq. 14}$$

to compensate for the amplification.

3.3 – Experimental Procedure

After the experiment was setup, the beam was depressed by approximately 1 inch and allowed to vibrate. The oscilloscope was set up to capture first ten cycles of the beam. The data was then saved to a thumb drive in Comma Separated Values (.csv) format. Three runs were conducted for the purposes of this experiment.

4 – RESULTS & DISCUISSION

Two sets of runs were conducted. The first set was conducted in the middle of the table and the second set was conducted on the side by a supporting member. Figure 5 is a graph of the displacement as recorded by the strain gauge and the accelerometer. It can be seen that the strain gauge adequately records the displacement. The accelerometer, however gives a peculiar output. When the cantilever beam was depressed, then suddenly released, part of the force was transmitted to the table causing it to oscillate to. The vibrations caused by this were then transmitted to the accelerometer located at the end of the beam. That is why the accelerometer data is peculiar looking while the strain gauge looks normal. The problem was then rectified by mounting the beam close to a table leg which. This decreased the oscillations in the table. Figure 6 is a plot of the displacement measured by the strain gauge, accelerometer, and the gyroscope. It can be seen that the strain gauge has a very consistent decrease in amplitude. The accelerometer and the gyroscope though, appear to lag behind the response of the strain gauge.

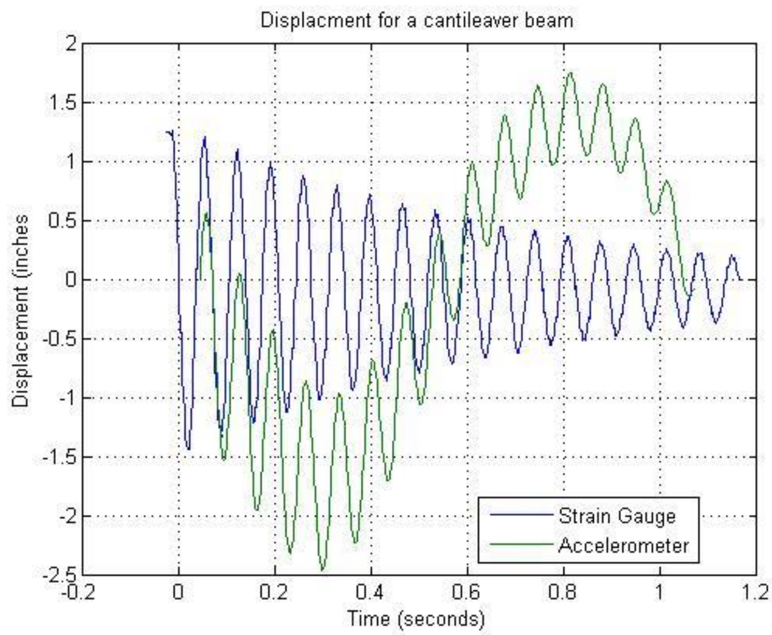

Figure 5 - The displacement for the Strain Gauge and Accelerometer for a cantilever beam mounted in the middle of the table.

For each test, the beam was displaced approximately one inch. This was accomplished by holding a ruler up next to the beam. Thus, it would appear that the strain gauge gives the most accurate, initial displacement. For this purpose, it can be argued that the strain gauge is the best measurement of the displacement of the beam.

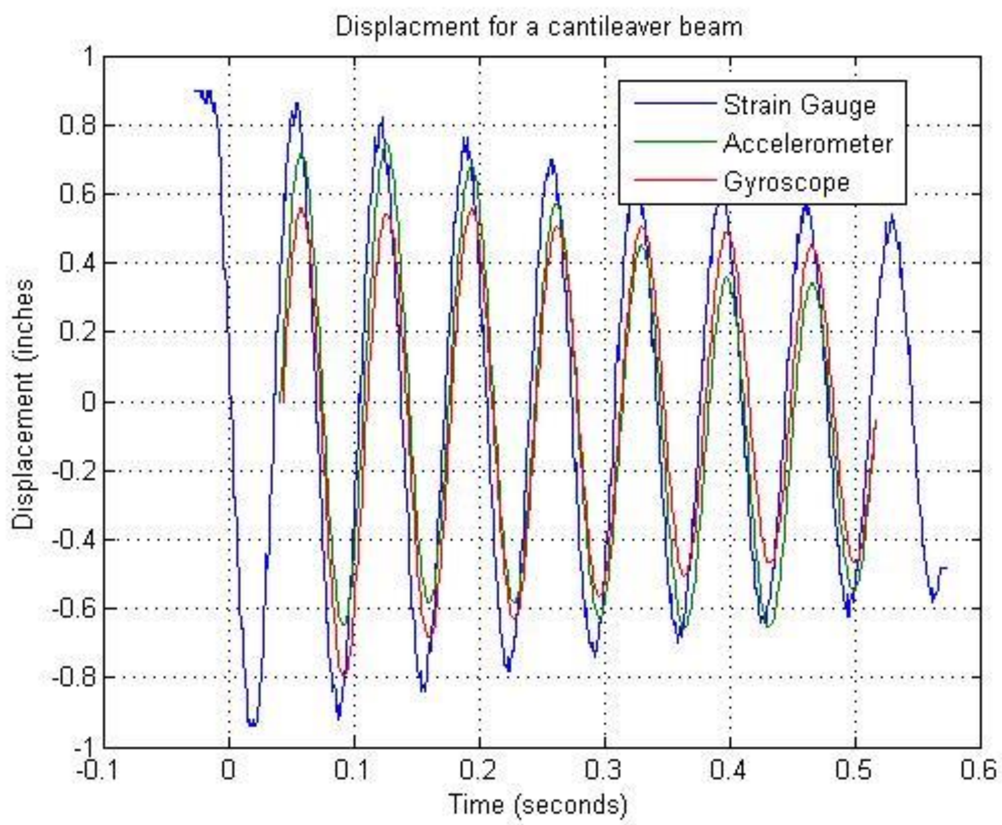

Figure 6 - Displacement of a cantilever beam according to a Strain Gauge, Accelerometer, and Gyroscope.

4.1 – Natural Frequency and Damping Ratio

The natural frequency and damping ratio was calculated for all three sensors. They were calculated using plots generated by the m-code and equations 9-12. Sample calculations can be found in Appendix C. Table 2 lists the results for each sensor as an average between two runs

Table 2 - Damping Ratio and Natural Frequency for each sensor.

	Strain Gauge	Accelerometer	Gyroscope
Damping Ratio	.0114	.0229	.00353
Natural Frequency (rad/s)	92.21	92.54	92.62

It can be seen, that there is a significant discrepancy between the damping ratio for the sensors. This is due to the nature of the data recorded by the accelerometer and the gyroscope. Appendix

D contains graphs of the displacement for the strain gauge, accelerometer, and gyroscope. It can be seen that the strain gauge has a smooth decrease of the amplitude, while the accelerometer and the gyroscope does not. This is why there is such a discrepancy for the damping ratio for the sensors. It is to be noted, that with the accelerometer and gyroscope mounted at the end of the beam could have changed the natural frequency and the damping ratio. The natural frequency though, is reasonable for each sensor.

5 – CONCLUSION

The purpose of the experiment was to analyze the displacement recorded for a cantilever beam using a strain gauge, accelerometer, and gyroscope. It was found that the strain gauge produced a nice, reasonable plot similar to what was expected. The accelerometer and the gyroscope though, produced graphs that did not show the characteristic oscillation expected of underdamped oscillations. Therefore, the conclusion is that the strain gauge is the optimal sensor for this purpose.

APPENDIX A – MATLAB CODE

The following is the code used for the purposes of this report. For the code behind the "`zerodata`" function, please see Appendix B

```
clear
close all

%Load the data
filename1='\\khensu\Home07\tcone\My Documents\ME-410 Mechatronics\Lab 5 -
Cantileaver Beam\NewFile1.csv';
fid=fopen(filename1,'r');

%Define Variables
A1=load(filename1);
Time=A1(:,1);
Strain=A1(:,2);
RawAccel=A1(:,3);
RawGyro=A1(:,4);


%Define Strain Constants
Rg=180;          %Gain Resistor
G=1+50000/Rg;    %Gain Factor
GF=2.060;        %Gauge Factor
L=9.825;         %Total Length of the Beam (in)
x=1.5;           %Distance the strain gauge is from the mounting edge (in)
T=.0661;         %Thickeness of the beam (inches)
c=T/2;           %Distance to the neutral axis
Ve=8;            %Excitation Voltage (V)

%Calculate the Strain
deltamax=-4/3*(Strain.*L^3)/(G*GF*(L-x)*Ve*c);


%To find Displacmenet from the Accelerometer

%Convert the voltage to gravity and then to inches per second squared.
AccSens=64e-3;                        %Sensitivity factor
Accel=RawAccel*(1/AccSens)*386.4;     %Converts to in/s^2

%Determine Integration Start/End points for Acceleration
Acc_start=49;
Acc_end=595;

%Subtact the constant value of integration
AccZeroed = zerodata(Time(Acc_start:Acc_end), Accel(Acc_start:Acc_end));

%Integrate between Peek Values for Acceleration
Vel=cumtrapz(Time(Acc_start:Acc_end),AccZeroed);
```

APPENDIX A – CONTINUED

```
%Determine Integration Start/End points for Velocity
Vel_start=20;
Vel_end=495;

%Subtract the constant value of velocity.
%Note that the time vector must be shifted by both the original
%Acceleration Shift plus the velocity shift
VelZeroed=zerodata(Time((Acc_start+Vel_start):(Acc_start+Vel_end)),
Vel(Vel_start:Vel_end));

%Integrate Vetween Peak Values for Velocity
DispAcc =
cumtrapz(Time((Acc_start+Vel_start):(Acc_start+Vel_end)),VelZeroed);

%To find the displacmenet from the gyroscope.

%Convert the voltage to angular velocity
GyroSens=0.167e-3;
OmegaGyro=RawGyro*(1/GyroSens)*(pi/180);    %Convert to radians/sec


%Define Integration Points
OmegaGyro_Start=71;
OmegaGyro_End=544;

% Remove the trace of the gyroscope
GyroZeroed=zerodata(Time(OmegaGyro_Start:OmegaGyro_End),
OmegaGyro(OmegaGyro_Start:OmegaGyro_End));

% Differentiate angular velocity with respect to time
theta=cumtrapz(Time(OmegaGyro_Start:OmegaGyro_End), GyroZeroed);

% Covert theta to displacement.
dispGyro=(2/3)*L*theta;


%Plot all on one graph
figure
plot(Time,deltamax,Time((Acc_start+Vel_start):(Acc_start+Vel_end)),DispAcc,Ti
me(OmegaGyro_Start:OmegaGyro_End),dispGyro)
grid on
xlabel('Time (seconds)')
ylabel('Displacement (inches')
title('Displacment for a cantileaver beam')
legend('Strain Gauge', 'Accelerometer', 'Gyroscope','Location','Best')
```

APPENDIX B – ZERODATA CODE

The following is the code for the "`zerodata` " function as provided by Dr. David Turcic.

```
function Yout = zerodata(Xin, Yin)

p = polyfit(Xin, Yin, 1);
Yout = Yin-polyval(p, Xin);

%figure
%plot(Xin,Yin,Xin,polyval(p, Xin))

end
```

APPENDIX C – SAMPLE CALCULATIONS

For the purposes of this sample calculation of the natural frequency and damping ratio, the values for the strain gauge will be used. Figure C1 is a plot of the displacement according to the strain gauge. It can be seen that the time and displacement values have been found for the maximum points at the start and the end of the data.

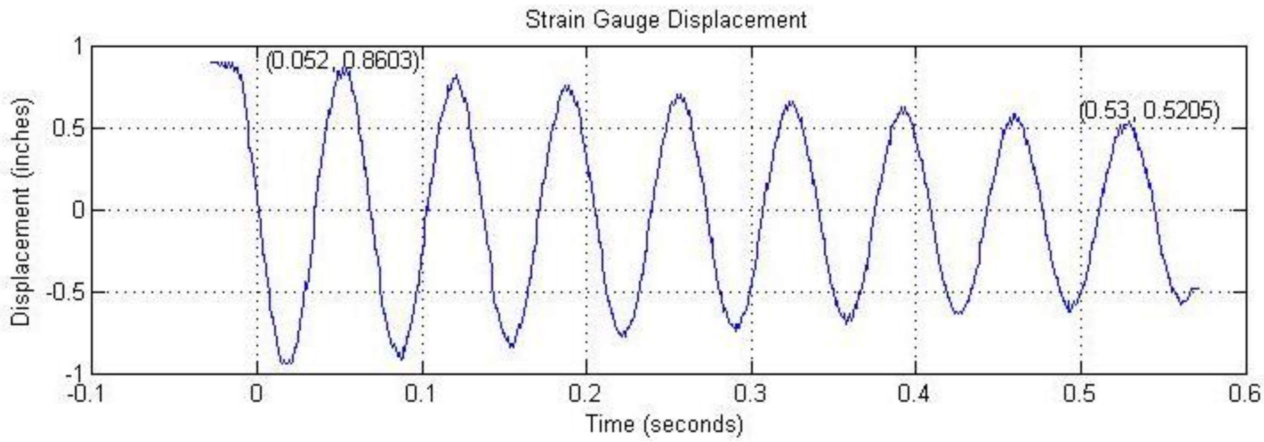

Figure 7 - Displacement according to the Strain Gauge

Thus, define $y_1 = 0.8603\ in,\ y_8 = 0.5205\ in,\ t_1 = 0.052\ s,$ and $t_8 = 0.53\ in.$ Then, using Eq. 10, where the period is defined as

$$T = \frac{t_n - t_1}{n - 1} \qquad \text{Eq. C1}$$

Such that

$$T = \frac{0.53\ s - 0.052\ s}{8 - 1} \rightarrow T = 0.06829\ s$$

Thus, the damped frequency is

$$\omega_d = \frac{1}{T} * 2\pi \rightarrow \omega_d = 92.01\ \frac{rad}{s} \qquad \text{Eq. C2}$$

The logarithmic decrement can then be calculated using Eq. 11

APPENDIX C – CONTINUED

$$\delta' = \frac{1}{n-1}\ln\left(\frac{y_1}{y_n}\right) \rightarrow \delta' = \frac{1}{8-1}\ln\left(\frac{.8603\ in}{.5205\ in}\right) \rightarrow \delta' = 0.07178 \qquad \text{Eq. C3}$$

Then using equation 12

$$\zeta = \frac{1}{\sqrt{1+\left(\frac{2\pi}{\delta'}\right)^2}} \rightarrow \zeta = \frac{1}{\sqrt{1+\left(\frac{2\pi}{0.07178}\right)^2}} \rightarrow \zeta = 0.011 \qquad \text{Eq. C4}$$

And now the natural frequency can be calculated from equation 13.

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} \rightarrow \omega_n = \frac{92.01\frac{rad}{s}}{\sqrt{1-0.011^2}} \rightarrow \omega_n = 92.02\frac{rad}{s} \qquad \text{Eq. C5}$$

APPENDIX D – DISPLACEMENT GRAPHS FOR EACH SENSOR

Figures D1 – D3 are graphs for the displacement as recorded by each sensor.

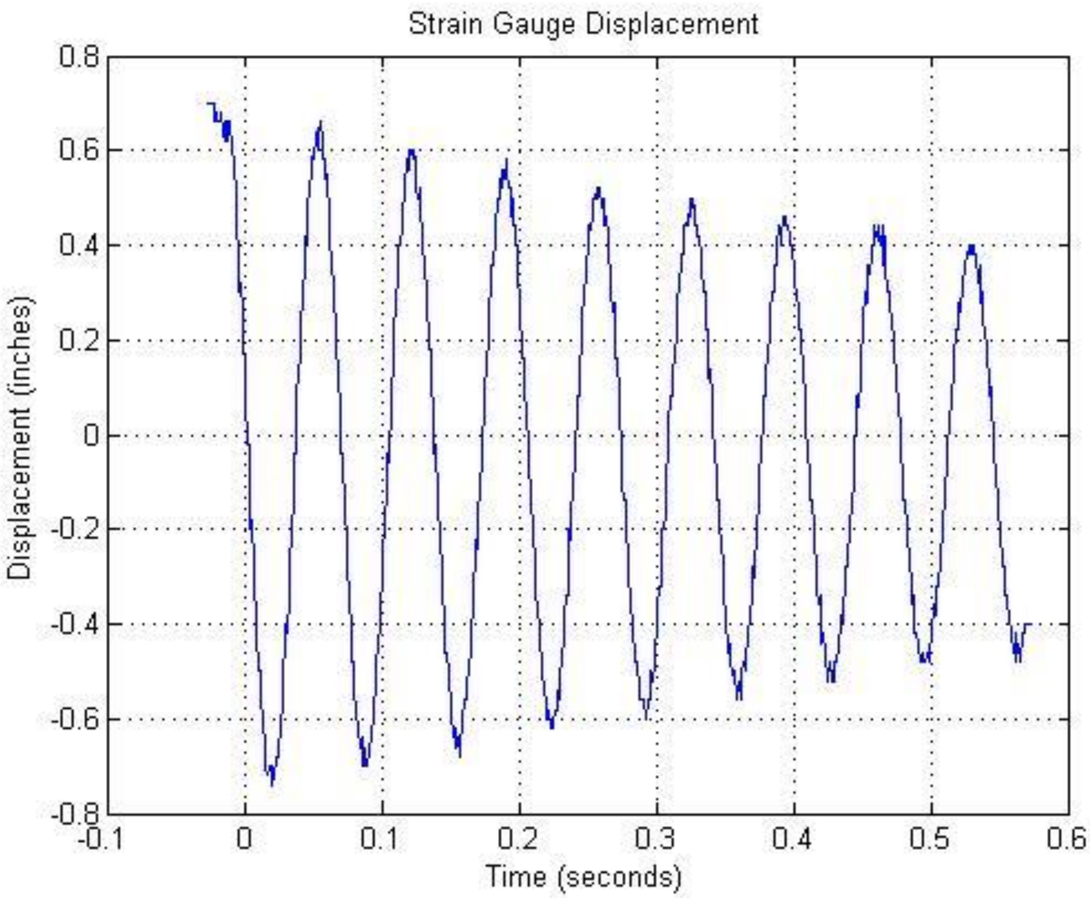

Figure D1 - Displacement for the strain gauge.

APPEDNX D – CONTINUED

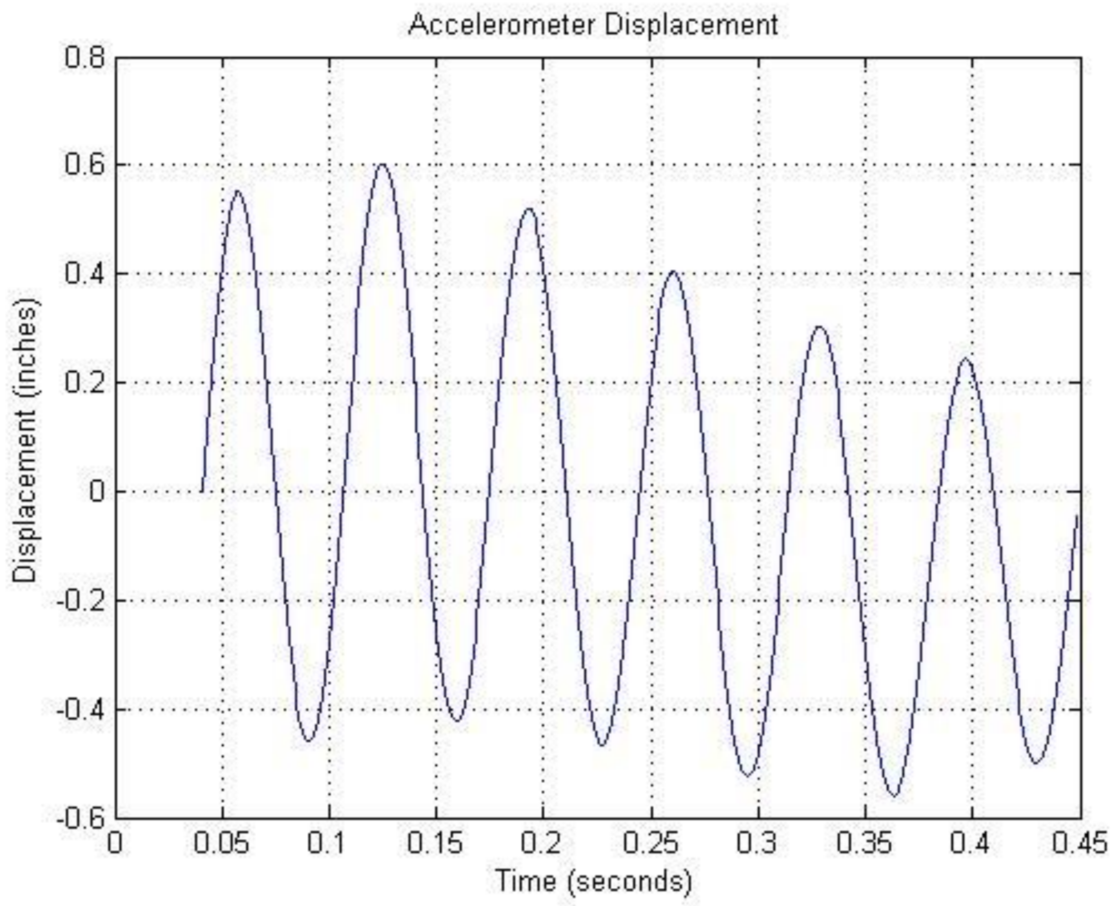

Figure 8 - Displacement for the accelerometer

APPEDNX D – CONTINUED

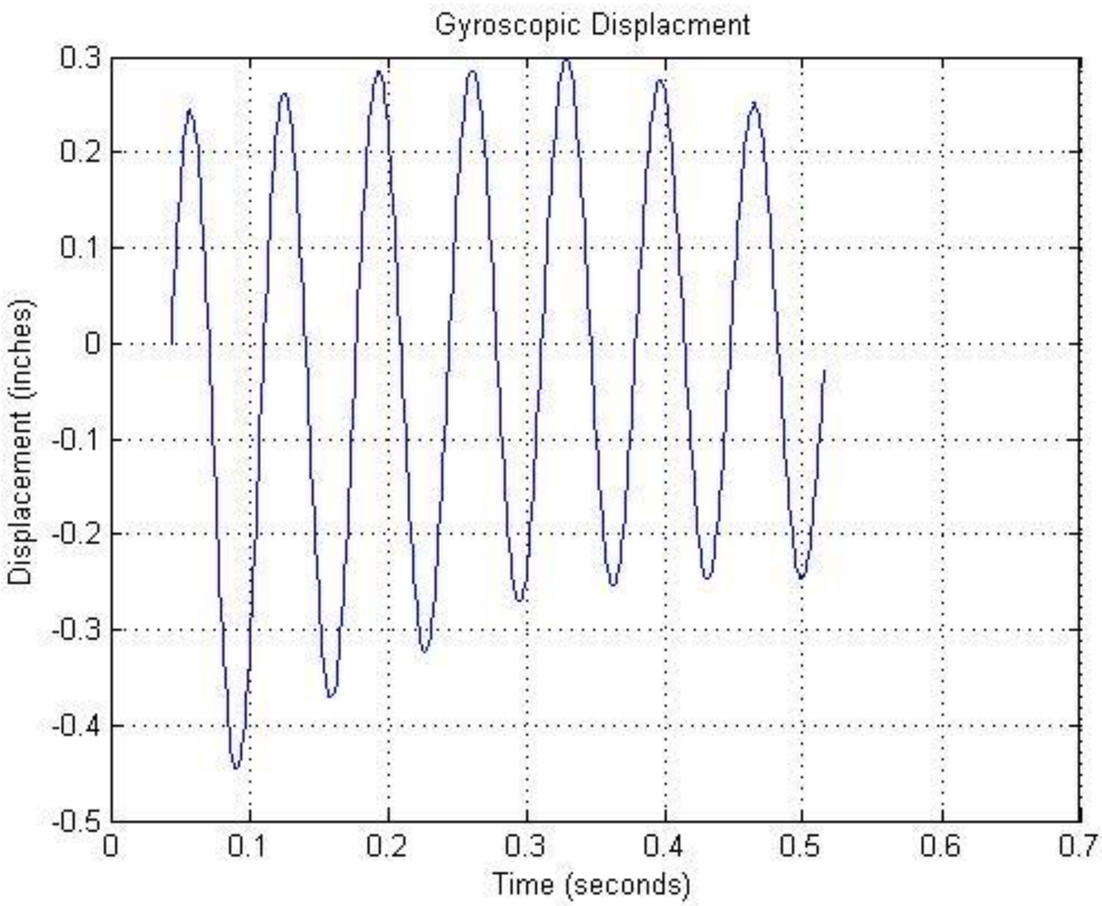

Figure 9 - Displacement for the gyroscope