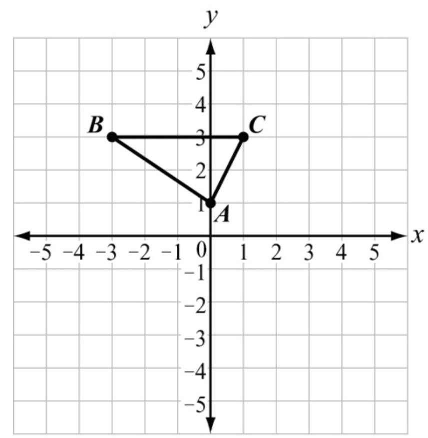
REVIEW EXAMPLES

1) Draw a triangle with vertices at $A(0, 1)$, $B(-3, 3)$, and $C(1, 3)$. Dilate the triangle using a scale factor of 1.5 and a center of (0, 0). Name the dilated triangle $A'B'C'$.

Solution:

Plot points $A(0, 1)$, $B(-3, 3)$, and $C(1, 3)$. Draw $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$.



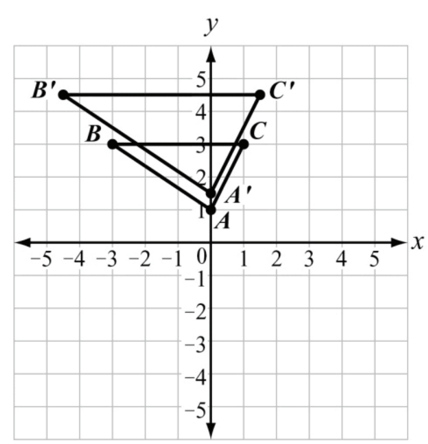
The center of dilation is the origin, so to find the coordinates of the image, multiply the coordinates of the pre-image by the scale factor 1.5.

Point A': $(1.5 \bullet 0, 1.5 \bullet 1) = (0, 1.5)$

Point B': $(1.5 \bullet (-3), 1.5 \bullet 3) = (-4.5, 4.5)$

Point C': $(1.5 \bullet 1, 1.5 \bullet 3) = (1.5, 4.5)$

Plot points $A'(0, 1.5)$, $B'(-4.5, 4.5)$, and $C'(1.5, 4.5)$. Draw $\overline{A'B'}$, $\overline{A'C'}$, and $\overline{B'C'}$.



Note: Since no part of the pre-image passes through the center of dilation, $\overline{BC} \parallel \overline{B'C'}$, $\overline{AB} \parallel \overline{A'B'}$, and $\overline{AC} \parallel \overline{A'C'}$.

2) Line segment *CD* is 5 inches long. If line segment *CD* is dilated to form line segment $C'D'$ with a scale factor of 0.6, what is the length of line segment $C'D'$?

Solution:

The ratio of the length of the image and the pre-image is equal to the scale factor.

$$\frac{C'D'}{CD} = 0.6$$

Substitute 5 for *CD*.

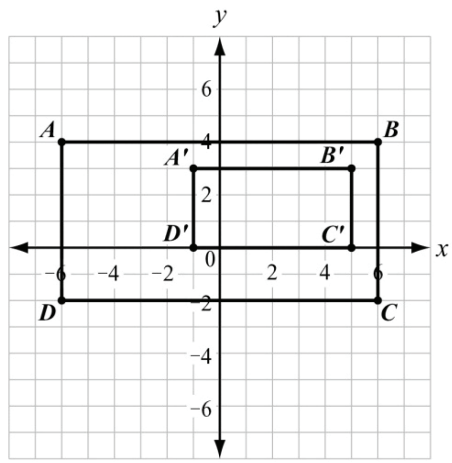
$$\frac{C'D'}{5} = 0.6$$

Solve for $C'D'$.

$$C'D' = 0.6 \bullet 5$$
$$C'D' = 3$$

The length of line segment $C'D'$ is 3 inches.

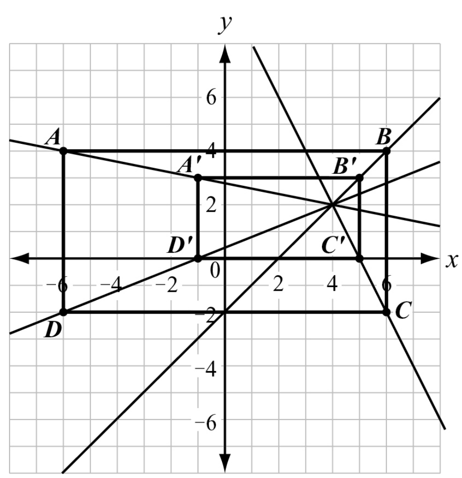
3) Figure $A'B'C'D'$ is a dilation of figure $ABCD$.



a. Determine the center of dilation.
b. Determine the scale factor of the dilation.
c. What is the relationship between the sides of the pre-image and corresponding sides of the image?

Solution:

a. To find the center of dilation, draw lines connecting each corresponding vertex from the pre-image to the image. The lines meet at the center of dilation.



The center of dilation is (4, 2).

b. Find the ratios of the lengths of the corresponding sides.

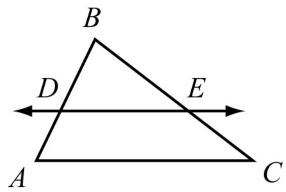
$$\frac{A'B'}{AB} = \frac{6}{12} = \frac{1}{2}$$
$$\frac{B'C'}{BC} = \frac{3}{6} = \frac{1}{2}$$
$$\frac{C'D'}{CD} = \frac{6}{12} = \frac{1}{2}$$
$$\frac{A'D'}{AD} = \frac{3}{6} = \frac{1}{2}$$

The ratio for each pair of corresponding sides is $\frac{1}{2}$, so the scale factor is $\frac{1}{2}$.

c. Each side of the image is parallel to the corresponding side of its pre-image and is $\frac{1}{2}$ the length.

REVIEW EXAMPLES

1) In the triangle shown, $\overline{AC} \parallel \overleftrightarrow{DE}$.

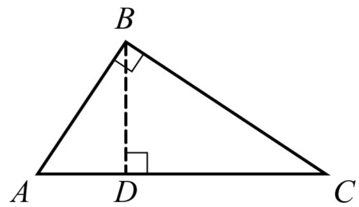


Prove that $\overleftrightarrow{DE}$ divides $\overline{AB}$ and $\overline{CB}$ proportionally.

Solution:

Step	Statement	Justification
1	$\overleftrightarrow{DE} \parallel \overline{AC}$	Given
2	$\angle BDE \cong \angle BAC$	If two parallel lines are cut by a transversal, then corresponding angles are congruent.
3	$\angle DBE \cong \angle ABC$	***Reflexive Property of Congruence*** because they are the same angle.
4	$\triangle DBE \sim \triangle ABC$	Angle-Angle (AA) Similarity
5	$\frac{BA}{BD} = \frac{BC}{BE}$	Corresponding sides of similar triangles are proportional.
6	$BD + DA = BA$ $BE + EC = BC$	Segment Addition Postulate
7	$\frac{BD+DA}{BD} = \frac{BE+EC}{BE}$	Substitution
8	$\frac{BD}{BD} + \frac{DA}{BD} = \frac{BE}{BE} + \frac{EC}{BE}$	Rewrite each fraction as sum of two fractions.
9	$1 + \frac{DA}{BD} = 1 + \frac{EC}{BE}$	Substitution
10	$\frac{DA}{BD} = \frac{EC}{BE}$	Subtraction Property of Equality
11	$\overleftrightarrow{DE}$ divides $\overline{AB}$ and $\overline{CB}$ proportionally	Definition of proportionality

2) Gale is trying to prove the Pythagorean Theorem using similar triangles. Part of her proof is shown below.



Step	Statement	Justification
1	$\angle ABC \cong \angle BDC$	All right angles are congruent.
2	$\angle ACB \cong \angle BCD$	Reflexive Property of Congruence
3	$\triangle ABC \sim \triangle BDC$	Angle-Angle (AA) Similarity
4	$\frac{BC}{DC} = \frac{AC}{BC}$	Corresponding sides of similar triangles are proportional.
5	$BC^2 = AC \bullet DC$	In a proportion, the product of the means equals the product of the extremes.
6	$\angle ABC \cong \angle ADB$	All right angles are congruent.
7	$\angle BAC \cong \angle DAB$	Reflexive Property of Congruence
8	$\triangle ABC \sim \triangle ADB$	Angle-Angle (AA) Similarity
9	$\frac{AB}{AD} = \frac{AC}{AB}$	Corresponding sides of similar triangles are proportional.
10	$AB^2 = AC \bullet AD$	In a proportion, the product of the means equals the product of the extremes.

What should Gale do to finish her proof?

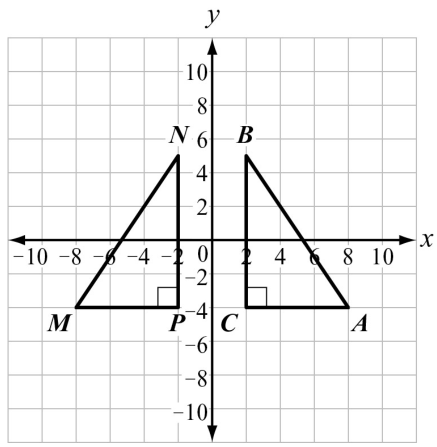
Solution:

Step	Statement	Justification
11	$AB^2 + BC^2 = AC \bullet AD + AC \bullet DC$	Addition
12	$AB^2 + BC^2 = AC(AD + DC)$	Distributive property
13	$AC = AD + DC$	Segment Addition Property
14	$AB^2 + BC^2 = AC \bullet AC$	Substitution
15	$AB^2 + BC^2 = AC^2$	Definition of exponent

$AB^2 + BC^2 = AC^2$ is a statement of the Pythagorean Theorem, so Gale's proof is complete.

REVIEW EXAMPLES

1) Is $\triangle ABC$ congruent to $\triangle MNP$? Explain.



(scale unit = 2)

Solution:

$\overline{AC}$ corresponds to $\overline{MP}$. Both segments are 6 units long. $\overline{BC}$ corresponds to $\overline{NP}$. Both segments are 9 units long. Angle C (the included angle of $\overline{AC}$ and $\overline{BC}$) corresponds to angle P (the included angle of $\overline{MP}$ and $\overline{NP}$). Both angles measure 90°. Because two sides and an included angle are congruent, the triangles are congruent by SAS.

Or, $\triangle ABC$ is a reflection of $\triangle MNP$ over the y-axis. This means that all of the corresponding sides and corresponding angles are congruent, so the triangles are congruent. (Reflections preserve angle measurement and lengths, therefore, corresponding angles and sides are congruent.)

2) Rectangle $WXYZ$ has coordinates $W(1, 2)$, $X(3, 2)$, $Y(3, –3)$, and $Z(1, –3)$.

   a. Graph the image of rectangle $WXYZ$ after a rotation of 90° clockwise about the origin. Label the image $W'X'Y'Z'$.

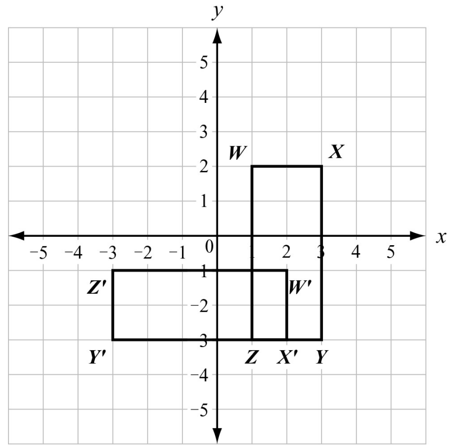
   b. Translate rectangle $W'X'Y'Z'$ 2 units left and 3 units up.

   c. Is rectangle $WXYZ$ congruent to rectangle $W''X''Y''Z''$? Explain.

Solution:

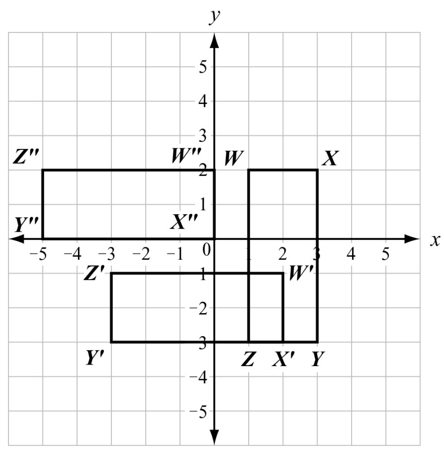
a. For a 90° clockwise rotation about the origin, use the rule $(x, y) \rightarrow (y, -x)$.

$W(1, 2) \rightarrow W'(2, -1)$
$X(3, 2) \rightarrow X'(2, -3)$
$Y(3, -3) \rightarrow Y'(-3, -3)$
$Z(1, -3) \rightarrow Z'(-3, -1)$



b. To translate rectangle $W'X'Y'Z'$ 2 units left and 3 units up, use the rule $(x, y) \rightarrow (x - 2, y + 3)$.

$W'(2, -1) \rightarrow W''(0, 2)$
$X'(2, -3) \rightarrow X''(0, 0)$
$Y'(-3, -3) \rightarrow Y''(-5, 0)$
$Z'(-3, -1) \rightarrow Z''(-5, 2)$



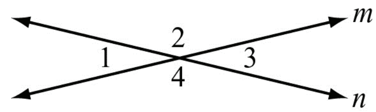
c. Rectangle $W''X''Y''Z''$ is the result of a rotation and a translation of rectangle $WXYZ$. These are both rigid transformations, so the shape and the size of the original figure are unchanged. All of the corresponding parts of $WXYZ$ and $W''X''Y''Z''$ are congruent, so $WXYZ$ and $W''X''Y''Z''$ are congruent.

8. Some important key ideas about parallelograms include:

- Opposite sides are congruent and opposite angles are congruent.
- The diagonals of a parallelogram bisect each other.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- A rectangle is a parallelogram with congruent diagonals.

REVIEW EXAMPLES

1) In this diagram, line *m* intersects line *n*.



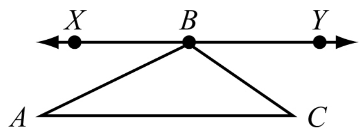
Write a two-column proof to show that vertical angles $\angle 1$ and $\angle 3$ are congruent.

Solution:

Construct a proof using intersecting lines.

Step	Statement	Justification
1	line *m* intersects line *n*	Given
2	$\angle 1$ and $\angle 2$ form a linear pair $\angle 2$ and $\angle 3$ form a linear pair	Definition of a linear pair
3	$m\angle 1 + m\angle 2 = 180°$ $m\angle 2 + m\angle 3 = 180°$	Angles that form a linear pair have measures that sum to 180°
4	$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Substitution
5	$m\angle 1 = m\angle 3$	Subtraction Property of Equality
6	$\angle 1 \cong \angle 3$	Definition of congruent angles

2) In this diagram, $\overleftrightarrow{XY}$ is parallel to $\overline{AC}$, and point B lies on $\overleftrightarrow{XY}$.

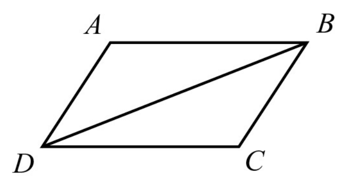


Write a paragraph to prove that the sum of the angles in a triangle is 180°.

Solution:

$\overline{AC}$ and $\overleftrightarrow{XY}$ are parallel, so $\overline{AB}$ is a transversal. The alternate interior angles formed by the transversal are congruent. So, $m\angle A = m\angle ABX$. Similarly, $\overline{BC}$ is a transversal, so $m\angle C = m\angle CBY$. The sum of the angle measures that make a straight line is 180°. So, $m\angle ABX + m\angle ABC + m\angle CBY = 180°$. Now, substitute $m\angle A$ for $m\angle ABX$ and $m\angle C$ for $m\angle CBY$ to get $m\angle A + m\angle ABC + m\angle C = 180°$.

3) In this diagram, $ABCD$ is a parallelogram and $\overline{BD}$ is a diagonal.



Write a two-column proof to show that $\overline{AB}$ and $\overline{CD}$ are congruent.

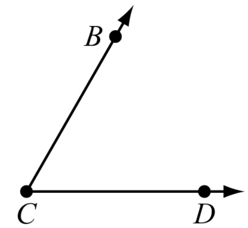
Solution:

Construct a proof using properties of the parallelogram and its diagonal.

Step	Statement	Justification
1	$ABCD$ is a parallelogram	Given
2	$\overline{BD}$ is a diagonal	Given
3	$\overline{AB}$ is parallel to $\overline{DC}$ $\overline{AD}$ is parallel to $\overline{BC}$	Definition of parallelogram
4	$\angle ABD \cong \angle CDB$ $\angle DBC \cong \angle BDA$	Alternate interior angles are congruent.
5	$\overline{BD} \cong \overline{BD}$	Reflexive Property of Congruence
6	$\triangle ADB \cong \triangle CBD$	ASA
7	$\overline{AB} \cong \overline{CD}$	CPCTC

REVIEW EXAMPLES

1) Allan drew angle *BCD*.

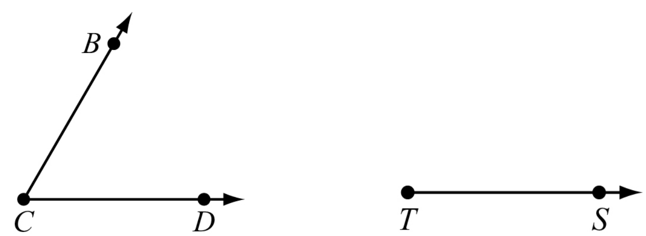


a. Copy angle *BCD*. List the steps you used to copy the angle. Label the copied angle *RTS*.

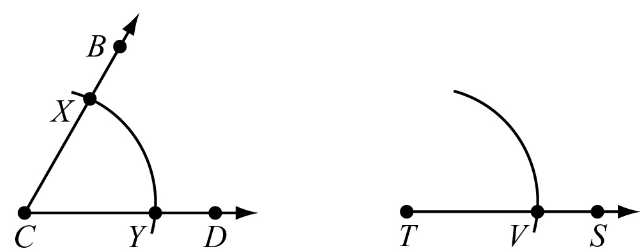
b. Without measuring the angles, how can you show they are congruent to one another?

Solution:

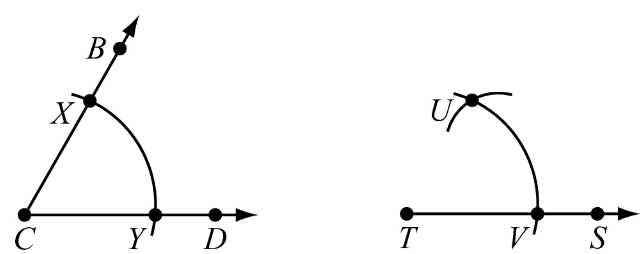
a. Draw point *T*. Draw $\overrightarrow{TS}$.



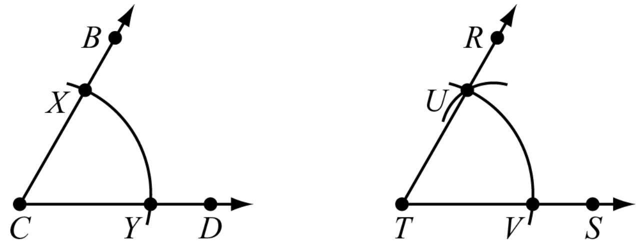
Place the point of a compass on point *C*. Draw an arc. Label the intersection points *X* and *Y*. Keep the compass width the same, and place the point of the compass on point *T*. Draw an arc and label the intersection point *V*.



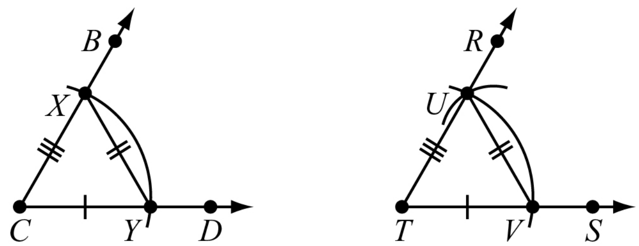
Place the point of the compass on point *Y* and adjust the width to point *X*. Then place the point of the compass on point *V* and draw an arc that intersects the first arc. Label the intersection point *U*.



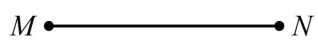
Draw $\overrightarrow{TU}$ and point R on $\overrightarrow{TU}$. Angle BCD has now been copied to form angle RTS.



b. Connect points X and Y and points U and V to form $\triangle XCY$ and $\triangle UTV$. $\overline{CY}$ and $\overline{TV}$, $\overline{XY}$ and $\overline{UV}$, and $\overline{CX}$ and $\overline{TU}$ are congruent because they were drawn with the same compass width. So, $\triangle XCY \cong \triangle UTV$ by SSS, and $\angle C \cong \angle T$ because congruent parts of congruent triangles are congruent.

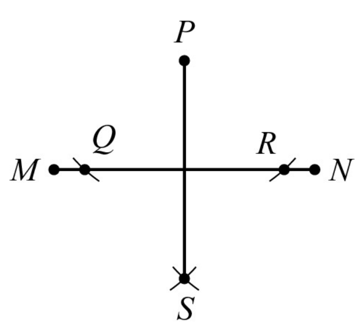


2) Construct a line segment perpendicular to $\overline{MN}$ from a point not on $\overline{MN}$. Explain the steps you used to make your construction.



Solution:

Draw a point P that is not on $\overline{MN}$. Place the compass point on point P. Draw an arc that intersects $\overline{MN}$ at two points. Label the intersections points Q and R. Without changing the width of the compass, place the compass on point Q and draw an arc under $\overline{MN}$. Place the compass on point R and draw another arc under $\overline{MN}$. Label the intersection point S. Draw $\overline{PS}$. Segment PS is perpendicular to and bisects $\overline{MN}$.



3) Construct equilateral $\triangle HIJ$ inscribed in circle K. Explain the steps you used to make your construction.

Solution:

(This is an alternate method from the method shown in Key Idea 7.) Draw circle K. Draw segment $\overline{FG}$ through the center of circle K. Label the intersection points I and P. Using the compass setting you used when drawing the circle, place a compass on point P and draw an arc passing through point K. Label the intersection points at either side of the circle points H and J. Draw $\overline{HJ}$, $\overline{IJ}$, and $\overline{HI}$. Triangle HIJ is an equilateral triangle inscribed in circle K.