ANALYTIC TRIGONOMETRY

Precalculus

Chapter 05

- This Slideshow was developed to accompany the textbook
  - *Precalculus*
  - *By Richard Wright*
  - *https://www.andrews.edu/~rwright/Precalculus-RLW/Text/TOC.html*
- Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

5-01 FUNDAMENTAL TRIGONOMETRIC IDENTITIES PART A

In this section, you will:

- Use fundamental identities to evaluate trigonometric expressions.
- Use fundamental identities to simplify trigonometric expressions.

5-01 Fundamental Trigonometric Identities Part A

- Uses for identities
  - Evaluate trig functions
  - Simplify trig expressions
  - Develop more identities
  - Solve trig equations

5-01 Fundamental Trigonometric Identities Part A

- Reciprocal Identities
- $\sin u = \frac{1}{\csc u}$ $\csc u = \frac{1}{\sin u}$
- $\cos u = \frac{1}{\sec u}$ $\sec u = \frac{1}{\cos u}$
- $\tan u = \frac{1}{\cot u}$ $\cot u = \frac{1}{\tan u}$

- Quotient Identities
- $\tan u = \frac{\sin u}{\cos u}$ $\cot u = \frac{\cos u}{\sin u}$

- Pythagorean Identites
- $\sin^2 u + \cos^2 u = 1$
- $\tan^2 u + 1 = \sec^2 u$
- $1 + \cot^2 u = \csc^2 u$

Colored ones should be memorized.

5-01 Fundamental Trigonometric Identities Part A

- Even/Odd Identities
- $\cos(-u) = \cos u$
- $\sec(-u) = \sec u$
- $\sin(-u) = -\sin u$
- $\tan(-u) = -\tan u$
- $\csc(-u) = -\csc u$
- $\cot(-u) = -\cot u$

- Cofunction Identities
- $\sin\left(\frac{\pi}{2} - u\right) = \cos u$
- $\cos\left(\frac{\pi}{2} - u\right) = \sin u$
- $\tan\left(\frac{\pi}{2} - u\right) = \cot u$
- $\cot\left(\frac{\pi}{2} - u\right) = \tan u$
- $\sec\left(\frac{\pi}{2} - u\right) = \csc u$
- $\csc\left(\frac{\pi}{2} - u\right) = \sec u$

5-01 Fundamental Trigonometric Identities Part A

- If $\sin\theta = -1$ and $\cot\theta = 0$, evaluate $\cos\theta$
- Evaluate $\tan\theta$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$0 = \frac{\cos\theta}{-1}$$
$$0 = \cos\theta$$

$$\tan\theta = \frac{1}{\cot\theta}$$
$$\tan\theta = \frac{1}{0} = undefined$$

5-01 Fundamental Trigonometric Identities Part A

- Simplify $\frac{\sec^2 x - 1}{\sin^2 x}$

$$\frac{\sec^2 x - 1}{\sin^2 x}$$
$$\frac{(1 + \tan^2 x) - 1}{\sin^2 x}$$
$$\frac{\tan^2 x}{\sin^2 x}$$
$$\left(\frac{\sin^2 x}{\cos^2 x}\right)\frac{1}{\sin^2 x}$$
$$\frac{1}{\cos^2 x}$$
$$\sec^2 x$$

5-01 Fundamental Trigonometric Identities Part A

- Simplify $\sin\varphi\,(\csc\varphi - \sin\varphi)$

$$\sin\varphi\,(\csc\varphi - \sin\varphi)$$
$$\sin\varphi\csc\varphi - \sin^2\varphi$$
$$\sin\varphi\left(\frac{1}{\sin\varphi}\right) - \sin^2\varphi$$
$$1 - \sin^2\varphi$$
$$1 - (1 - \cos^2\varphi)$$
$$\cos^2\varphi$$

5-01 Fundamental Trigonometric Identities Part A

- Simplify $\frac{1-\sin^2 x}{\csc^2 x-1}$

$$\frac{1-\sin^2 x}{\csc^2 x-1}$$

$$\frac{\cos^2 x}{(\cot^2 x+1)-1}$$

$$\frac{\cos^2 x}{\cot^2 x}$$

$$\frac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}}$$

$$\cos^2 x\left(\frac{\sin^2 x}{\cos^2 x}\right)$$

$$\sin^2 x$$

5-01 Fundamental Trigonometric Identities Part A

- Simplify $\cos\left(\frac{\pi}{2} - x\right)(\sec x)$

$$\cos\left(\frac{\pi}{2} - x\right)(\sec x)$$
$$\sin x \sec x$$
$$\sin x \frac{1}{\cos x}$$
$$\frac{\sin x}{\cos x}$$
$$\tan x$$

5-02 Fundamental Trigonometric Identities Part B

In this section, you will:

- Factor and multiply trigonometric expressions.
- Use trigonometric identities with rational expressions.
- Use trigonometric substitution.

5-02 Fundamental Trigonometric Identities Part B

- Factor and simplify $\sin^4 x - \cos^4 x$

$$\sin^4 x - \cos^4 x$$
$$(\sin^2 x)^2 - (\cos^2 x)^2$$
$$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$$
$$(\sin^2 x - \cos^2 x)(1)$$
$$\sin^2 x - (1 - \sin^2 x)$$
$$2\sin^2 x - 1$$

5-02 Fundamental Trigonometric Identities Part B

- Multiply and simplify $(2\csc x + 2)(2\csc x - 2)$

$$(2\csc x + 2)(2\csc x - 2)$$
$$4\csc^2 x - 4$$
$$4(\csc^2 x - 1)$$
$$4\left((\cot^2 x + 1) - 1\right)$$
$$4\cot^2 x$$

5-02 Fundamental Trigonometric Identities Part B

- Simplify $\frac{\cos x}{1+\sin x}+\frac{1+\sin x}{\cos x}$

$$\frac{\cos x}{1+\sin x}+\frac{1+\sin x}{\cos x}$$

$$\frac{\cos^2 x}{(1+\sin x)\cos x}+\frac{(1+\sin x)^2}{(1+\sin x)\cos x}$$

$$\frac{\cos^2 x+1+2\sin x+\sin^2 x}{(1+\sin x)\cos x}$$

$$\frac{(1-\sin^2 x)+1+2\sin x+\sin^2 x}{(1+\sin x)\cos x}$$

$$\frac{2+2\sin x}{(1+\sin x)\cos x}$$

$$\frac{2(1+\sin x)}{(1+\sin x)\cos x}$$

$$\frac{2}{\cos x}$$

$$2\sec x$$

5-02 Fundamental Trigonometric Identities Part B

- Rewrite not as a fraction: $\frac{3}{\sec x - \tan x}$

$$\frac{3}{\sec x - \tan x}$$
$$\frac{3(\sec x + \tan x)}{(\sec x - \tan x)(\sec x + \tan x)}$$
$$\frac{3(\sec x + \tan x)}{\sec^2 x - \tan^2 x}$$
$$3(\sec x + \tan x)$$

5-02 Fundamental Trigonometric Identities Part B

- Use trig substitution: $\sqrt{x^2 - 9}$ with $x = 3\sec\theta$

$$\sqrt{x^2 - 9}$$
$$\sqrt{(3\sec\theta)^2 - 9}$$
$$\sqrt{9\sec^2\theta - 9}$$
$$\sqrt{9(\sec^2\theta - 1)}$$
$$\sqrt{9\tan^2\theta}$$
$$3\tan\theta$$

5-03 VERIFY TRIGONOMETRIC IDENTITIES

In this section, you will:

- Verify trigonometric identities algebraically.
- Verify trigonometric identities graphically.

5-03 Verify Trigonometric Identities

- Show that trig identities are true by turning one side into the other side

- Guidelines

1. Work with 1 side at a time. Start with the more complicated side.
2. Try factor, add fractions, square a binomial, etc.
3. Use fundamental identities
4. If the above doesn't work, try rewriting in sines and cosines
5. Try something!

5-03 Verify Trigonometric Identities

- Verify $(1 + \sin\alpha)(1 - \sin\alpha) = \cos^2\alpha$

$$(1 + \sin\alpha)(1 - \sin\alpha)$$
$$1 - \sin\alpha + \sin\alpha - \sin^2\alpha$$
$$1 - \sin^2\alpha$$
$$1 - (1 - \cos^2\alpha)$$
$$\cos^2\alpha$$

5-03 Verify Trigonometric Identities

- Verify $\sin^2 \alpha - \sin^4 \alpha = \cos^2 \alpha - \cos^4 \alpha$

$$\sin^2 \alpha - \sin^4 \alpha$$
$$\sin^2 \alpha \,(1 - \sin^2 \alpha)$$
$$(1 - \cos^2 \alpha)(1 - (1 - \cos^2 \alpha)$$
$$(1 - \cos^2 \alpha) \cos^2 \alpha$$
$$\cos^2 \alpha - \cos^4 \alpha$$

5-03 Verify Trigonometric Identities

- Verify $\frac{\cot^2 t}{\csc t} = \csc t - \sin t$

$$\frac{\cot^2 t}{\csc t}$$
$$\frac{\csc^2 t - 1}{\csc t}$$
$$\frac{\csc^2 t}{\csc t} - \frac{1}{\csc t}$$
$$\csc t - \sin t$$

5-03 Verify Trigonometric Identities

- Verify $\frac{1}{\sec x \tan x} = \csc x - \sin x$

$$\frac{1}{\sec x \tan x}$$
$$\cos x \cot x$$
$$\left(\frac{\cos x}{1}\right)\left(\frac{\cos x}{\sin x}\right)$$
$$\frac{\cos^2 x}{\sin x}$$
$$\frac{1 - \sin^2 x}{\sin x}$$
$$\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$
$$\csc x - \sin x$$

5-03 Verify Trigonometric Identities

- Verify $\frac{\cos\theta\cot\theta}{1-\sin\theta} - 1 = \csc\theta$

$$\frac{\cos\theta\cot\theta}{1-\sin\theta} - 1$$

$$\frac{\left(\frac{\cos\theta}{1}\right)\left(\frac{\cos\theta}{\sin\theta}\right)}{1-\sin\theta} - 1$$

$$\frac{\left(\frac{\cos^2\theta}{\sin\theta}\right)}{1-\sin\theta} - 1$$

$$\frac{\cos^2\theta}{\sin\theta\,(1-\sin\theta)} - 1$$

$$\frac{1-\sin^2\theta}{\sin\theta\,(1-\sin\theta)} - 1$$

$$\frac{(1-\sin\theta)(1+\sin\theta)}{\sin\theta\,(1-\sin\theta)} - 1$$

$$\frac{1+\sin\theta}{\sin\theta} - 1$$

$$\frac{1}{\sin\theta} + \frac{\sin\theta}{\sin\theta} - 1$$

$$\csc\theta + 1 - 1$$

$\csc\theta$

5-04 Solve Trigonometric Equations

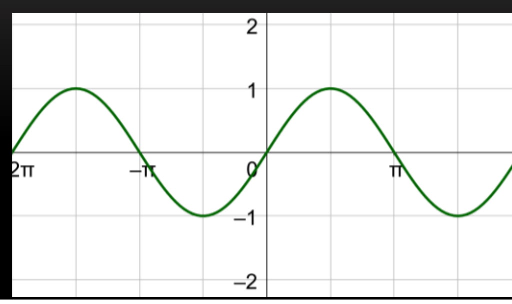
In this section, you will:

- Solve trigonometric simple equations.
- Solve trigonometric equations by factoring.
- Solve trigonometric equations using identities.
- Find all solutions and all solutions on the interval [0, 2π) to trigonometric equations.

5-04 Solve Trigonometric Equations

- Main goal – Isolate a trig expression
  - Try identities to simplify
  - Try solving by factoring

- Number of solutions
  - $\sin x = 0$
  - Infinite solutions so describe
  - $0 + n\pi = n\pi$



5-04 Solve Trigonometric Equations

- Solve $\sin x - \sqrt{2} = -\sin x$

$$-\sqrt{2} = -2\sin x$$

$$\frac{\sqrt{2}}{2} = \sin x$$

Use a unit circle to find solutions

$$x = \frac{\pi}{4} + 2\pi n, \frac{3\pi}{4} + 2\pi n$$

5-04 Solve Trigonometric Equations

- Solve $4\sin^2 x - 3 = 0$

$$4\sin^2 x = 3$$
$$\sin^2 x = \frac{3}{4}$$
$$\sin x = \pm\sqrt{\frac{3}{4}}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$

Use a unit circle to find the solutions

$$x = \frac{\pi}{3} + \pi n, \frac{2\pi}{3} + \pi n$$

It is +πn because solutions are directly opposite each other on the circle so that adding π moves to another solution.

5-04 Solve Trigonometric Equations

- Solve $\sin^2 x = 2\sin x$

$$\sin^2 x - 2\sin x = 0$$
$$\sin x\,(\sin x - 2) = 0$$
$$\sin x = 0 \qquad \sin x - 2 = 0$$

Use a unit circle for each equation to find the solutions

$$x = 0 + \pi n \qquad x = No\ Solution$$

Only solution is $x = \pi n$

5-04 Solve Trigonometric Equations

- Solve $3\sec^2 x - 2\tan^2 x - 4 = 0$

$$3(\tan^2 x + 1) - 2\tan^2 x - 4 = 0$$
$$3\tan^2 x + 3 - 2\tan^2 x - 4 = 0$$
$$\tan^2 x - 1 = 0$$
$$\tan^2 x = 1$$
$$\tan x = \pm\sqrt{1}$$
$$\tan x = \pm 1$$

Use a unit circle to find all the soutions

$$x = \frac{\pi}{4} + \pi n, \frac{3\pi}{4} + \pi n$$
$$x = \frac{\pi}{4} + \frac{\pi}{2} n$$

5-04 Solve Trigonometric Equations

- Solve in the interval [0, 2π)
- $\sin x + 1 = \cos x$

$$(\sin x + 1)^2 = \cos^2 x$$
$$\sin^2 x + 2\sin x + 1 = \cos^2 x$$
$$\sin^2 x + 2\sin x + 1 = 1 - \sin^2 x$$
$$2\sin^2 x + 2\sin x = 0$$
$$2\sin x(\sin x + 1) = 0$$
$$\sin x = 0 \qquad \sin x + 1 = 0$$
$$\sin x = -1$$

Use a unit circle for each equation to find all solutions between 0 and 2π

$$x = 0, \pi \qquad x = \frac{3\pi}{2}$$

5-04 Solve Trigonometric Equations

- Solve on the interval [0, 2π)
- $\sin 2x = \frac{\sqrt{3}}{2}$

$$\sin 2x = \frac{\sqrt{3}}{2}$$

Use a unit circle to find all the solutions. Because 2x, you need to go around circle twice

$$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}\ (+2\pi n)$$

$$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$$

5-04 Solve Trigonometric Equations

- Solve $4\tan^2 x + 5\tan x = 6$

Quadratic type

$$4\tan^2 x + 5\tan x - 6 = 0$$
$$(\tan x + 2)(4\tan x - 3) = 0$$
$$\tan x + 2 = 0 \qquad 4\tan x - 3 = 0$$
$$\tan x = -2 \qquad 4\tan x = 3$$
$$\tan x = \frac{3}{4}$$

Use a unit circle for each equation

$$x = \arctan(-2) + \pi n \qquad x = \arctan\left(\frac{3}{4}\right) + \pi n$$

5-05 Sum and Difference Formulas

In this section, you will:

- Apply the sum and difference formulas to evaluate trigonometric expressions.
- Apply the sum and difference formulas to simplify trigonometric expressions.
- Apply the sum and difference formulas to solve trigonometric equations.

5-05 Sum and Difference Formulas

- $\sin(u+v) = \sin u \cos v + \cos u \sin v$
- $\sin(u-v) = \sin u \cos v - \cos u \sin v$

- $\cos(u+v) = \cos u \cos v - \sin u \sin v$
- $\cos(u-v) = \cos u \cos v + \sin u \sin v$

- $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$
- $\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$

5-05 Sum and Difference Formulas

- Use a sum or difference formula to find the exact value of $\tan 255°$

$$\tan 255° = \tan(225° + 30°)$$

$$= \frac{\tan 225° + \tan 30°}{1 - \tan 225° \tan 30°}$$

$$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1\left(\frac{\sqrt{3}}{3}\right)}$$

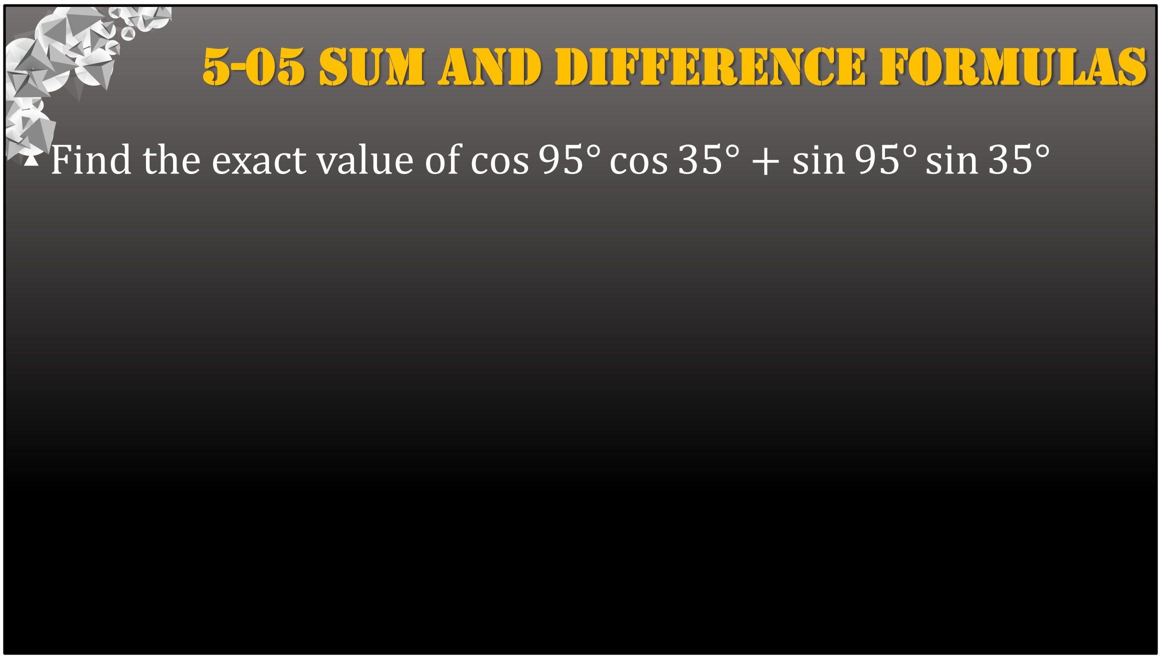
$$= \frac{\frac{3 + \sqrt{3}}{3}}{\frac{3 - \sqrt{3}}{3}}$$

$$= \frac{3 + \sqrt{3}}{3} \cdot \frac{3}{3 - \sqrt{3}}$$

$$= \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{(3 + \sqrt{3})(3 + \sqrt{3})}{(3 - \sqrt{3})(3 + \sqrt{3})}$$

$$= \frac{9 + 6\sqrt{3} + 3}{9 - 3}$$
$$= \frac{12 + 6\sqrt{3}}{6}$$
$$= 2 + \sqrt{3}$$



$$\cos u \cos v + \sin u \sin v = \cos(u - v)$$
$$\cos(95° - 35°)$$
$$\cos(60°)$$
$$\frac{1}{2}$$

5-05 Sum and Difference Formulas

- Derive a reduction formula for $\sin\left(t+\frac{\pi}{2}\right)$

$$\sin t\cos\frac{\pi}{2}+\cos t\sin\frac{\pi}{2}$$
$$(\sin t)(0)+(\cos t)(1)$$
$$\cos t$$

5-05 Sum and Difference Formulas

- Find all solutions in [0, 2π)
- $\cos\left(x - \frac{\pi}{3}\right) + \cos\left(x + \frac{\pi}{3}\right) = 1$

$$\cos\left(x - \frac{\pi}{3}\right) + \cos\left(x + \frac{\pi}{3}\right) = 1$$
$$\left(\cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3}\right) + \left(\cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3}\right) = 1$$
$$2\cos x \cos\frac{\pi}{3} = 1$$
$$2(\cos x)\frac{1}{2} = 1$$
$$\cos x = 1$$
$$x = 0$$

5-06 MULTIPLE ANGLE FORMULAS

In this section, you will:

- Use multiple angle formulas to evaluate trigonometric functions.
- Use multiple angle formulas to derive new trigonometric identities.
- Use multiple angle formulas to solve trigonometric equations.

5-06 Multiple Angle Formulas

- Double-Angle Formulas
- $\sin 2u = 2\sin u \cos u$
- $\cos 2u = \cos^2 u - \sin^2 u$
- $\qquad = 2\cos^2 u - 1$
- $\qquad = 1 - 2\sin^2 u$
- $\tan 2u = \frac{2\tan u}{1-\tan^2 u}$

These come from the sum formulas where u = v

5-06 Multiple Angle Formulas

- If $\sin u = \frac{3}{5}$ and $0 < u < \frac{\pi}{2}$,
- Find $\sin 2u$
- $\cos 2u$
- $\tan 2u$

Use a right triangle in the first quadrant with y = 3 and r = 5 to find x = 4

$$\sin 2u = 2 \sin u \cos u$$
$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

$$\cos 2u = 1 - 2\sin^2 u$$
$$= 1 - 2\left(\frac{3}{5}\right)^2$$
$$= 1 - 2\left(\frac{9}{25}\right)$$
$$= 1 - \frac{18}{25} = \frac{7}{25}$$

$$\tan 2u = \frac{2\left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{3}{2}}{1 - \frac{9}{16}}$$

$$= \frac{\frac{3}{2}}{\frac{7}{16}}$$

$$= \frac{3}{2} \cdot \frac{16}{7} = \frac{24}{7}$$

5-06 Multiple Angle Formulas

- Derive a triple angle formula for $\cos 3x$

$$\begin{aligned}
\cos 3x &= \cos(2x + x) \\
&= \cos 2x \cos x - \sin 2x \sin x \\
&= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x \\
&= 2\cos^3 x - \cos x - 2\sin^2 x \cos x \\
&= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x \\
&= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x \\
&= 4\cos^3 x - 3\cos x
\end{aligned}$$

5-06 Multiple Angle Formulas

- Power-Reducing Formulas
- $\sin^2 u = \frac{1-\cos 2u}{2}$
- $\cos^2 u = \frac{1+\cos 2u}{2}$
- $\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$

5-06 Multiple Angle Formulas

- Rewrite $\cos^4 x$ as a sum of 1st powers of cosines.

$$\cos^4 x$$
$$\cos^2 x \cos^2 x$$
$$\left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$$
$$\frac{1+2\cos 2x+\cos^2 2x}{4}$$
$$\frac{1+2\cos 2x+\frac{1+\cos 2(2x)}{2}}{4}$$
$$\frac{2+4\cos 2x+1+\cos 4x}{8}$$
$$\frac{3+4\cos 2x+\cos 4x}{8}$$

5-06 Multiple Angle Formulas

- Half-Angle Formulas
- $\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$
- $\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$
- $\tan\frac{u}{2} = \frac{1-\cos u}{\sin u}$
- $= \frac{\sin u}{1+\cos u}$

- Find the exact value of $\cos 105°$

$$\cos 105° = \cos\left(\frac{210°}{2}\right)$$
$$= \pm\sqrt{\frac{1+\cos 210°}{2}}$$
$$= \pm\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$
$$= \pm\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$
$$= \pm\sqrt{\frac{2-\sqrt{3}}{4}}$$

Pick the negative because 105° falls in quadrant II where cos is negative

$$= -\frac{\sqrt{2-\sqrt{3}}}{2}$$

5-07 PRODUCT-TO-SUM FORMULAS

In this section, you will:

- Use product-to-sum formulas to evaluate trigonometric functions.
- Use product-to-sum formulas to derive new trigonometric identities.
- Use product-to-sum formulas to solve trigonometric equations.

5-07 Product-to-Sum Formulas

- Product-to-Sum Formulas

- $\sin u \sin v = \frac{1}{2}(\cos(u - v) - \cos(u + v))$
- $\cos u \cos v = \frac{1}{2}(\cos(u - v) + \cos(u + v))$
- $\sin u \cos v = \frac{1}{2}(\sin(u + v) + \sin(u - v))$
- $\cos u \sin v = \frac{1}{2}(\sin(u + v) - \sin(u - v))$

5-07 Product-to-Sum Formulas

- Rewrite $\sin 5\theta \cos 3\theta$ as a sum or difference.

$$\frac{1}{2}(\sin(5\theta + 3\theta) + \sin(5\theta - 3\theta))$$

$$\frac{1}{2}(\sin 8\theta + \sin 2\theta)$$

5-07 Product-to-Sum Formulas

- Sum-to-Product Formulas

- $\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$
- $\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$
- $\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$
- $\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$

5-07 Product-to-Sum Formulas

- Find the exact value of $\sin 195° + \sin 105°$

$$2\sin\left(\frac{195° + 105°}{2}\right)\cos\left(\frac{195° - 105°}{2}\right)$$

$$2\sin 150° \cos 45°$$

$$2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{2}$$

5-07 Product-to-Sum Formulas

- Solve on the interval [0, 2π)
- $\sin 4x - \sin 2x = 0$

$$2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$$

$$2\cos 3x \sin x = 0$$

$$\cos 3x = 0 \qquad\qquad \sin x = 0$$

Use a unit circle for each equation. The cos 3x needs to be gone around 3 times

$$3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2} \qquad\qquad x = 0, \pi$$

$$x = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

5-07 Product-to-Sum Formulas

- Verify $\frac{\sin 6x+\sin 4x}{\cos 6x+\cos 4x} = \tan 5x$

$$\frac{\sin 6x + \sin 4x}{\cos 6x + \cos 4x}$$

$$\frac{2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)}{2\cos\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)}$$

$$\frac{2\sin 5x\cos x}{2\cos 5x\cos x}$$

$$\frac{\sin 5x}{\cos 5x}$$

$$\tan 5x$$