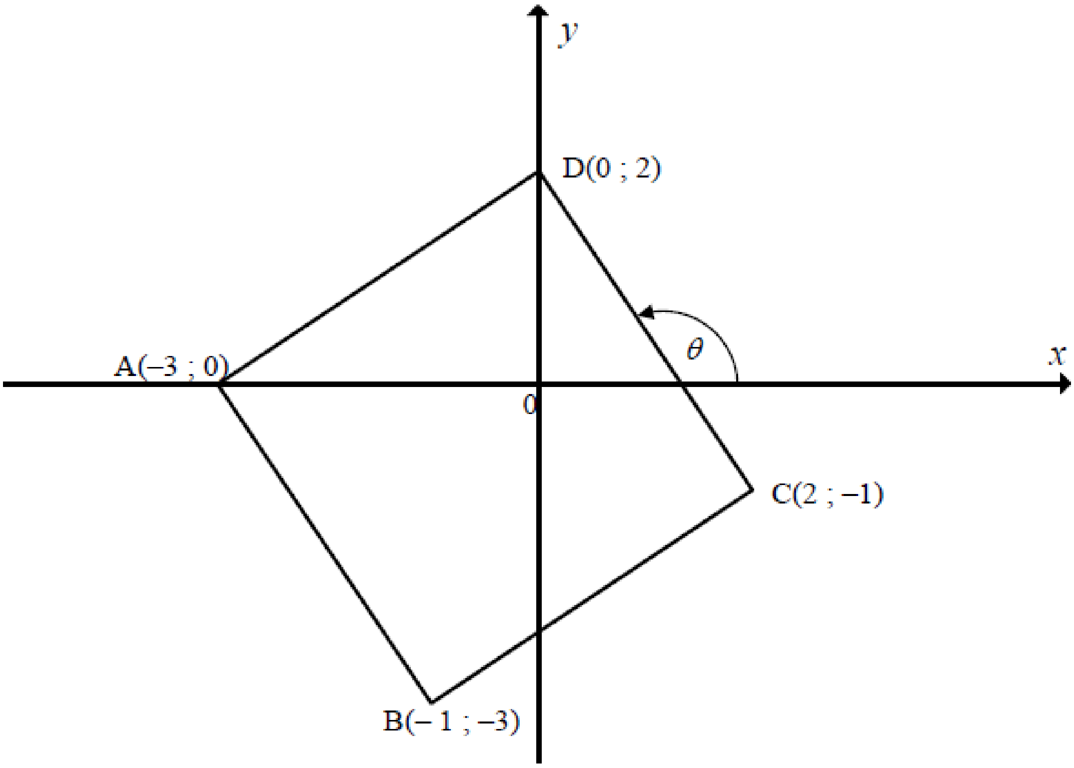
Analytical Geometry – Past Papers (Questions & Solutions)

November 2008

QUESTION 1

ABCD is a quadrilateral with vertices A(– 3 ; 0), B(– 1 ; – 3), C(2 ; –1) and D(0 ; 2).



1.1 Determine the coordinates of M, the midpoint of AC. (2)

1.2 Show that AC and BD bisect each other. (3)

1.3 Prove that $A\hat{D}C = 90°$. (4)

1.4 Show that ABCD is a square. (6)

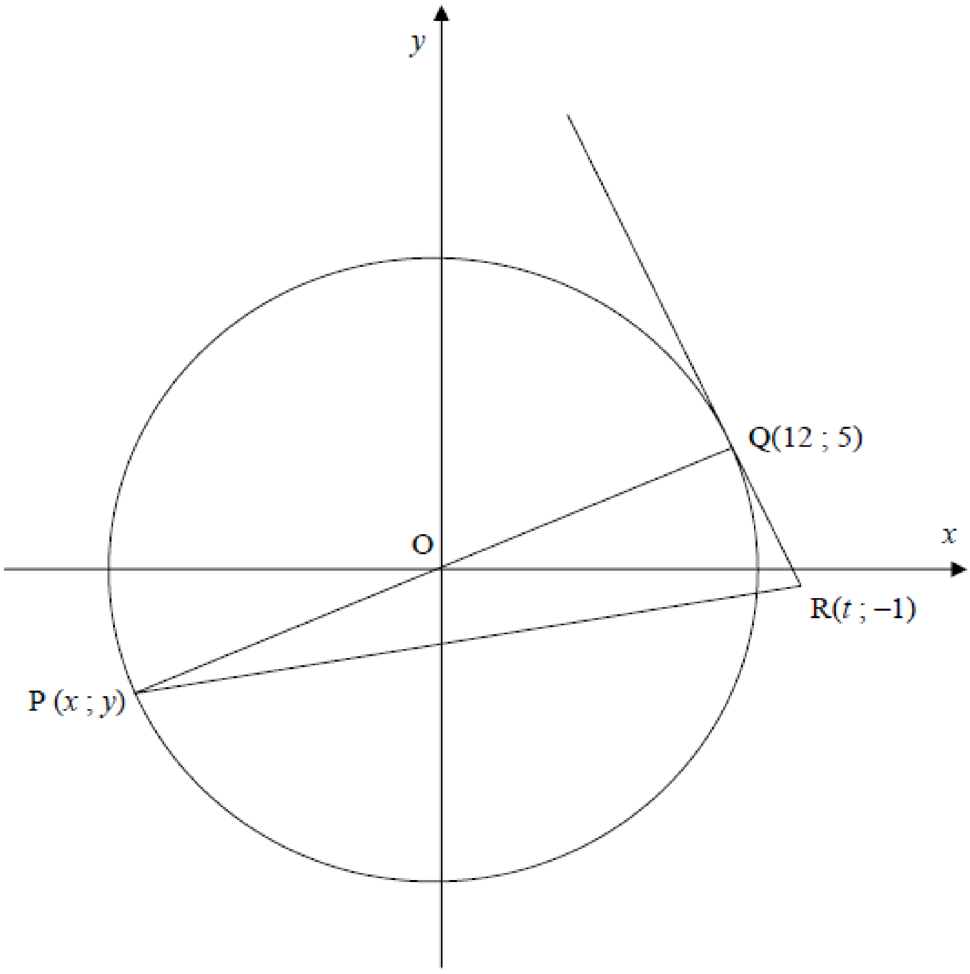
1.5 Determine the size of θ, the angle of inclination of DC, correct to ONE decimal place. (3)

1.6 Does C lie inside or outside the circle with centre (0 ; 0) and radius 2? Justify your answer. (2)

[20]

QUESTION 2

O is the centre of the circle in the figure below. P(x ; y) and Q (12 ; 5) are two points on the circle. POQ is a straight line. The point R(t ; – 1) lies on the tangent to the circle at Q.



2.1 Determine the equation of the circle. (3)

2.2 Determine the equation of the straight line through P and Q. (2)

2.3 Determine x and y, the coordinates of P. (2)

2.4 Show that the gradient of QR is $-\frac{12}{5}$. (2)

2.5 Determine the equation of the tangent QR in the form $y = \ldots$ (3)

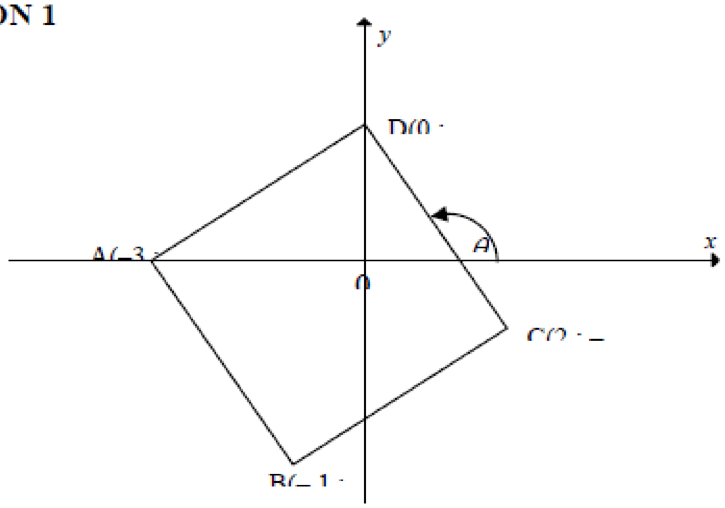
2.6 Calculate the value of t. (2)

2.7 Determine an equation of the circle with centre Q(12 ; 5) and passing through the origin. (3)

[17]

- **Continued accuracy applies as a rule in the memorandum.**
- **If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.**

QUESTION 1



1.1	M $\left(\frac{2-3}{2};\frac{-1+0}{2}\right)$ $=\left(-\frac{1}{2};-\frac{1}{2}\right)$	✓ substitution into midpoint formula ✓ answer for both coordinates (2) Answer only: 1 mark per coordinate Wrong formula: 0 / 2
1.2	Midpoint BD $=\left(\frac{-1+0}{2};\frac{-3+2}{2}\right)$ $=\left(-\frac{1}{2};-\frac{1}{2}\right)$ $\therefore$ Midpoint of AC and BD are the same point therefore AC and BD bisect each other **OR**	✓ substitution into formula ✓ answer ✓ conclusion (midpoints are the same) (3)

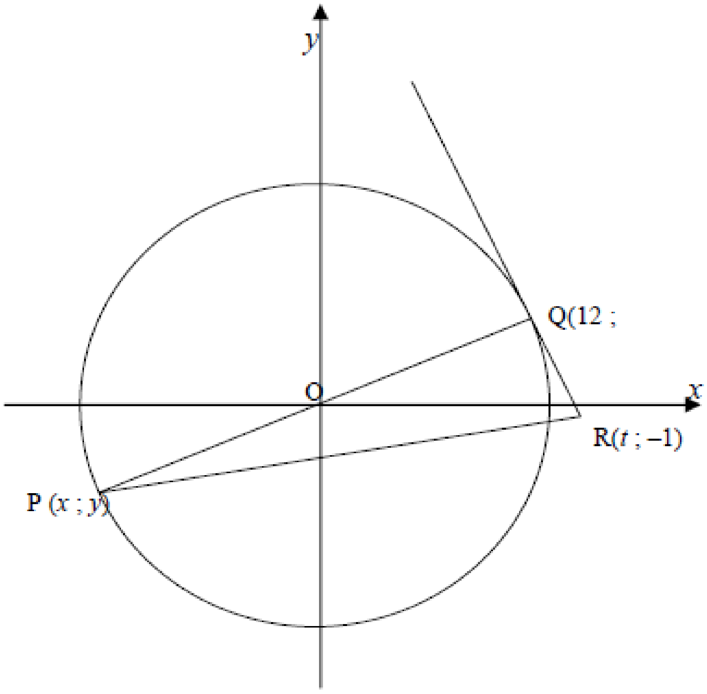
	$AM = \sqrt{\left(-3+\frac{1}{2}\right)^2 + \left(0+\frac{1}{2}\right)^2}$ $AM = \sqrt{6,5}$ $CM = \sqrt{\left(2+\frac{1}{2}\right)^2 + \left(-1+\frac{1}{2}\right)^2}$ $CM = \sqrt{6,5}$ $BM = \sqrt{\left(-1+\frac{1}{2}\right)^2 + \left(-3+\frac{1}{2}\right)^2}$ $BM = \sqrt{6,5}$ $DM = \sqrt{\left(0+\frac{1}{2}\right)^2 + \left(2+\frac{1}{2}\right)^2}$ $DM = \sqrt{6,5}$ AC and BD bisect each other	2 / 3 for answer on the left (because candidate did not show that M is on BD)
1.3	$m_{AD} = \frac{2-0}{0+3}$ $m_{AD} = \frac{2}{3}$ $m_{CD} = \frac{-1-2}{2-0}$ $m_{CD} = -\frac{3}{2}$ $m_{AD} \times m_{CD}$ $= \frac{2}{3} \times -\frac{3}{2}$ $= -1$ $\therefore AD \perp CD$ $\therefore \ A\hat{D}C = 90°$ Note: If do: $m_{AD} \times m_{CD} = -1$ $\frac{2}{3} \times -\frac{3}{2} = -1$ $-1 = -1$ then **3** / **4** if calculated the gradients correctly. If $m_{AD} \times m_{CD} = -1$ and conclude AD $\perp$ CD without any working, then **1** / **4**	✓ answer m_{AD} ✓ answer m_{CD} ✓ $m_{AD} \times m_{CD} = -1$ ✓ conclude $A\hat{D}C = 90°$ (4)
	OR	
	$\tan\theta = m_{CD}$ $\tan\theta = -\frac{3}{2}$ $\theta = 123,69°$ $\tan D\hat{A}C = \frac{2}{3}$ $D\hat{A}C = 33,69°$ $A\hat{D}C = 123,69° - 33,69°$ $A\hat{D}C = 90°$	✓ $\tan\theta = m_{CD}$ ✓ $\theta = 123,69°$ ✓ $D\hat{A}C = 33,69°$ ✓ $A\hat{D}C = 90°$ (4)

	OR $AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ $AC^2 = (0-(-1))^2 + (-3-2)^2$ $AC^2 = 26$ $AD^2 + DC^2$ $= 13 + 13$ $= 26$ $= AC^2$ $\therefore$ AD $\perp$ DC $\therefore\ A\hat{D}C = 90°$	✓ $AD^2 = 13$ ✓ $DC^2 = 13$ ✓ $AC^2 = 26$ ✓ conclusion (4)
1.4	$BD = \sqrt{(2+3)^2 + (0+1)^2}$ $= \sqrt{26}$ $AC = \sqrt{(-3-2)^2 + (0+1)^2}$ $= \sqrt{26}$ diagonals are equal diagonals bisect each other (Proved in 1.2) (i.e. ABCD is a rectangle) $m_{AC} . m_{BD}$ $= \frac{1}{-5} \times \frac{5}{1}$ $= -1$ AC $\perp$ BD **OR** $AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ The figure is a rectangle and one pair of adjacent sides are equal in length $\therefore$ it is a square. **OR**	✓ answer for BD ✓ answer for AC ✓diagonals are equal ✓ bisect each other ✓ $m_{AC} . m_{BD} = -1$ ✓. AC $\perp$ BD (6) ✓ substitution ✓ answer for AD ✓ substitution ✓ answer for DC ✓✓ conclusion (6)

	$AD^2=(2-0)^2+(0-(-3))^2$ $AD^2=13$ $DC^2=(2-(-1))^2+(0-2)^2$ $DC^2=13$ $AB^2=(-3-(-1))^2+(0-(-3))^2$ $AB^2=13$ $BC^2=(2-(-1))^2+(-1-(-3))^2$ $BC^2=13$ All four sides equal and one internal angle equal to 90°	✓ answer for AD ✓ answer for AB ✓ answer for DC ✓ answer for BC ✓ all four sides are equal ✓ one internal angle equal to 90° (6)
	OR	
	The diagonals bisect one another $A\hat{D}C=90°$ $AD^2=(2-0)^2+(0-(-3))^2$ $AD^2=13$ $DC^2=(2-(-1))^2+(0-2)^2$ $DC^2=13$ $\therefore$ adjacent sides equal in length $\therefore$ ABCD is a square	✓ diagonals bisect each other ✓ $A\hat{D}C=90°$ ✓ substitution into distance formula ✓ answer for AD ✓ answer for DC ✓ conclusion (6)
1.5	$\tan\theta=\frac{2+1}{0-2}$ $\tan\theta=-\frac{3}{2}$ $\theta=-56{,}30993247\ldots+180°$ $\theta=123{,}7°$	✓ gradient of CD ✓ $\tan\theta=-\frac{3}{2}$ ✓ answer (3)
	OR	
	$\tan D\hat{A}O=\frac{2}{3}$ $D\hat{A}O=33{,}7°$ $A\hat{D}C=90°$ $\theta=90°+33{,}7°$ $\theta=123{,}7°$ Penalty 1 for incorrect rounding	✓ $\theta=90°+D\hat{A}O$ ✓ $\tan D\hat{A}O=\frac{2}{3}$ ✓ answer (3)

1.6	$OC^2 = (2-0)^2 + (-1-0)^2$ $OC^2 = 5$ $OC = 2{,}236067977$ $OC > 2$ C lies outside the circle OR $OC^2 = (2-0)^2 + (-1-0)^2$ $OC^2 = 5$ $OC^2 > 4$ C lies outside the circle OR $x^2 + y^2 = 4$ $(2)^2 + (-1)^2 = 5 > 4$ C lies outside the circle	✓ OC^2 ✓ answer (2) Answer only: 0 / 2 **[20]**

QUESTION 2



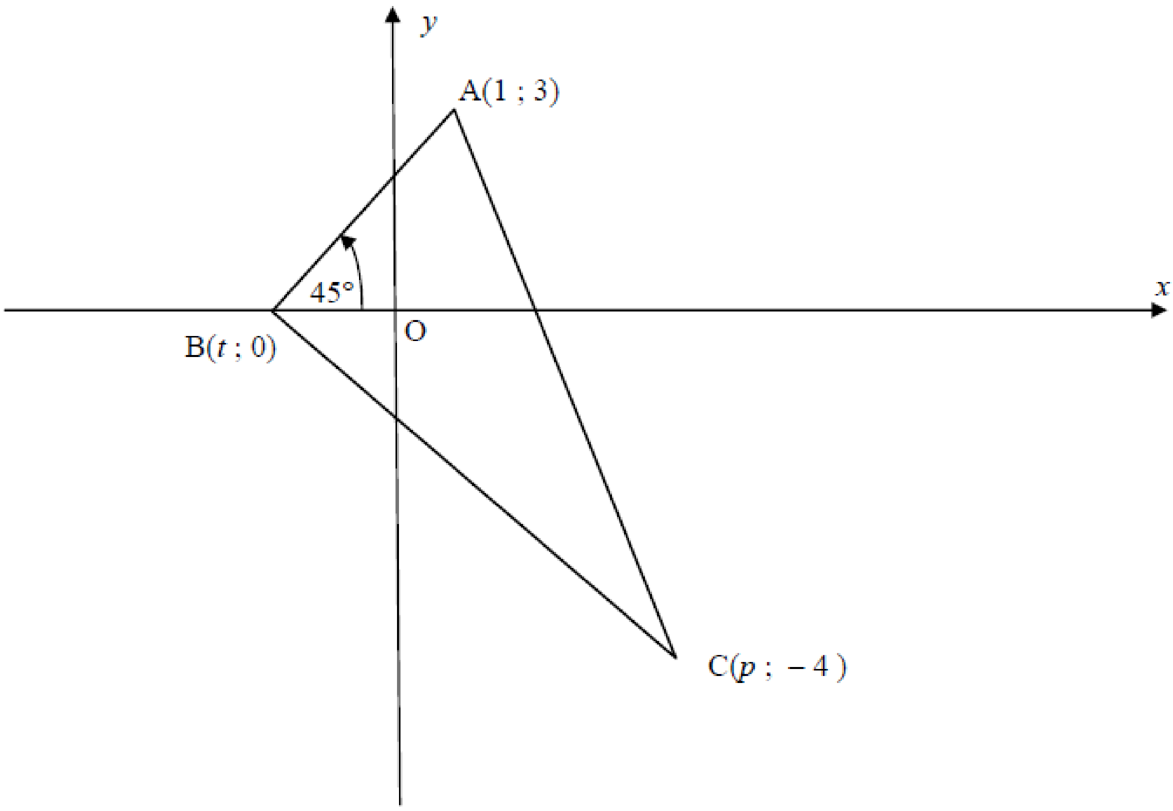
2.1	$r^2 = OQ^2$ $= (5)^2 + (12)^2$ $= 169$ $\therefore x^2 + y^2 = 169$ OR $x^2 + y^2 = (5)^2 + (12)^2 = 169$	✓ substituting (5 ; 12) into $x^2 + y^2$ ✓ 169 ✓ $x^2 + y^2 = 169$ (3) ✓ $x^2 + y^2 = r^2$ ✓ substitution coordinates ✓ 169 (3) Answer only: Full marks
2.2	$m_{PQ} = \frac{5-0}{12-0)}$ $m_{PQ} = \frac{5}{12}$ $\therefore y = \frac{5}{12}x$	✓ gradient ✓ $c = 0$ (2)

2.3	P(– 12; – 5) (By symmetry) **OR** $x^2 + y^2 = 169$ $x^2 + \left(\frac{5}{12}x\right)^2 = 169$ $144x^2 + 25x^2 = 169 \times 144 = 24336$ $169x^2 = 24336$ $x^2 = 144$ $x = \pm 12$ $x = -12$ $y = -5$	✓ $x = -12$ ✓ $y = -5$ (2)
2.4	tangent ⊥ diameter $m_{PQ} \times m_{QR} = -1$ $m_{PQ} = \frac{5}{12}$ $\therefore m_{QR} = -\frac{1}{\frac{5}{12}} = -\frac{12}{5}$ **OR** PQ ⊥ QR $m_{QR} = -\frac{12}{5}$	✓✓ $m_{PQ} \times m_{QR} = -1$ (2) ✓✓ PQ ⊥ QR (2)
2.5	$y = \frac{-12}{5}x + c$ $5 = \frac{-12}{5}(12) + c$ $c = \frac{169}{5}$ $y = -\frac{12}{5}x + \frac{169}{5}$ OR $y = -2,4x + 33,8$ **OR**	✓ $y = mx + c$ ✓substitution of gradient and (12 ; 5) ✓calculation of c. (3)

	$y-y_1=m(x-x_1)$ $y-5=-\frac{12}{5}(x-12)$ $5y-25=-12(x-12)$ $5y=-12x+144+25$ $5y=-12x+169$ $12x+5y-169=0$ $y=-\frac{12}{5}x+\frac{169}{5}$	✓ formula ✓ substitution of gradient and (12 ; 5) ✓equation in correct form (3)
2.6	$-1=\frac{-12}{5}(t)+\frac{169}{5}$ $12t=174$ $t=\frac{174}{12}$ $t=14{,}5$ **OR** $m_{QO}\times m_{QR}=-1$ $\frac{5}{12}\times\frac{-6}{t-12}=-1$ $t=14{,}5$ **OR** $PQ^2+QR^2=PR^2$ $576+100+(12-t)^2+36=(t+12)^2+16$ $712+144-24t+t^2=t^2+24t+144+16$ $-48t=-696$ $t=14{,}5$	✓substitution of $(t\,;-1)$ ✓answer (2) ✓ $\frac{5}{12}\times\frac{-6}{t-12}=-1$ ✓ answer (2) ✓ Pythagoras with substitution ✓ answer (2)
2.7	$(x-12)^2+(y-5)^2=OQ^2$ $OQ^2=(12-0)^2+(5-0)^2=169$ $(x-12)^2+(y-5)^2=169$ **OR** $(x)^2+(y)^2=169$ By translating 12 units right and 5 units up $(x-12)^2+(y-5)^2=169$	✓$(x-12)^2$ ✓$(y-5)^2$ ✓169 (3) If answer only: $(x-12)^2+(y-5)^2=169$: 3 / 3 **[17]**

QUESTION 4

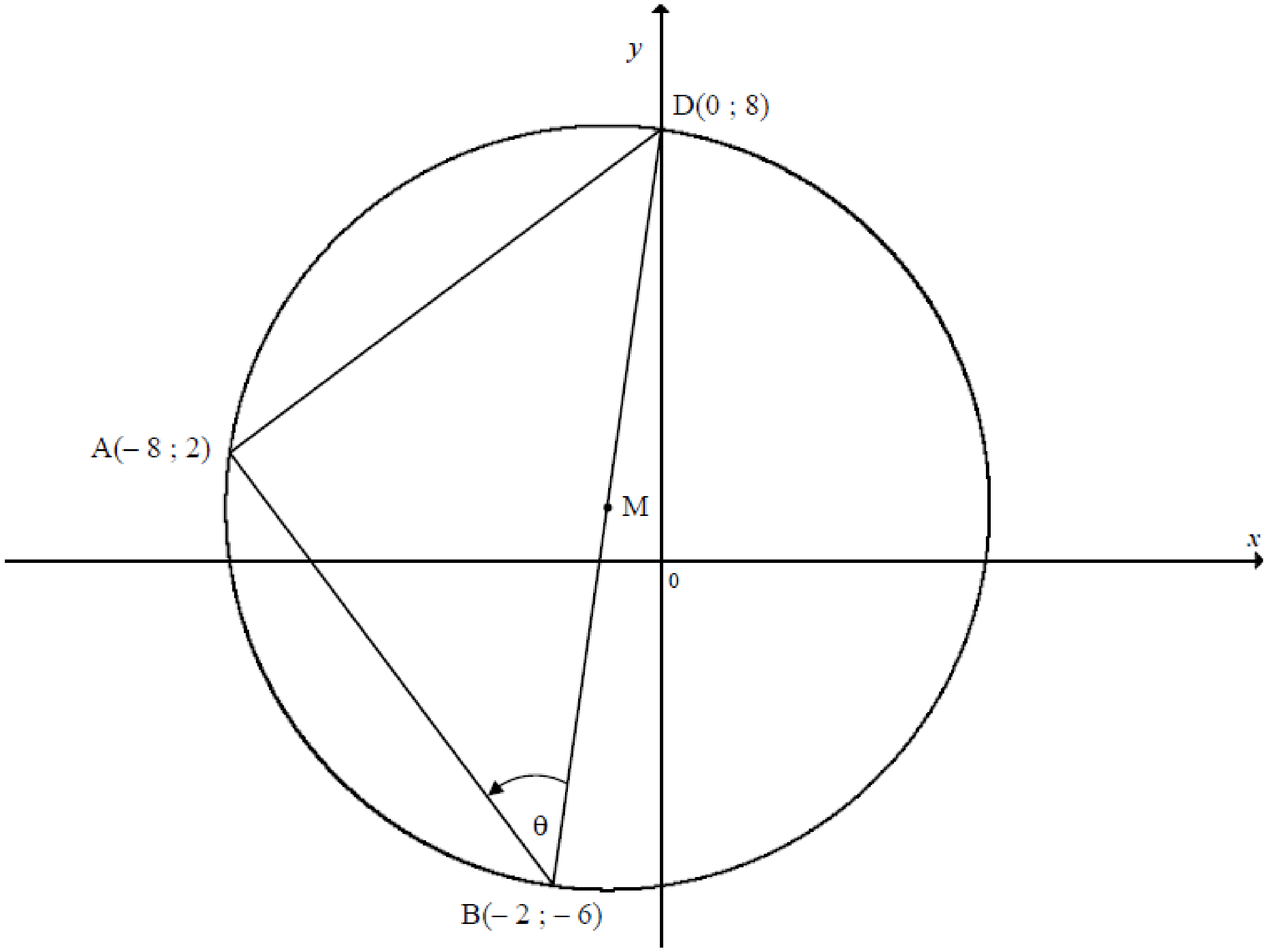
ABC is a triangle with vertices A(1 ; 3), B(t ; 0) and C(p ; −4), with $p > 0$, in a Cartesian plane.
AB makes an angle of 45° with the positive x-axis.   AC = $\sqrt{50}$.



4.1	Determine the gradient of AB.	(2)
4.2	Calculate the value of t.	(2)
4.3	Calculate p, the x-coordinate of point C.	(4)
4.4	Hence, determine the midpoint of BC.	(2)
4.5	Determine the equation of the line parallel to AB, passing through point C.	(3)
		[13]

QUESTION 5

A(−8 ; 2), B(−2 ; −6) and D(0 ; 8) are the vertices of a triangle that lies on the circumference of a circle with diameter BD and centre M, as shown in the figure below.



5.1 Calculate the coordinates of M. (2)

5.2 Show that (–8 ; 2) lies on the line $y = 7x + 58$. (1)

5.3 What is the relationship between the line $y = 7x + 58$ and the circle centred at M? Motivate your answer. (5)

5.4 Calculate the lengths of AD and AB. (4)

5.5 Prove $D\hat{A}B = 90°$. (3)

5.6 Write down the size of angle θ. (1)

5.7 A circle, centred at a point Z inside ΔABD, is drawn to touch sides AB, BD and DA at N, M and T respectively. Given that BMZN is a kite, calculate the radius of this circle. A diagram is provided on DIAGRAM SHEET 2. (6)

[22]

QUESTION 4

4.1	$\tan 45° = m_{AB}$ $= 1$ OR $m_{AB} = \frac{3-0}{1-t} = \frac{3}{1-t}$	✓ $\tan 45°$ ✓ answer (2) Answer only: full marks
4.2	$\frac{3-0}{1-t} = \tan 45° = 1$ $1-t=3$ $t=-2$ OR $y = mx + c$ $3 = (1)(1) + c$ $c = 2$ $y = x + 2$ $(t;0)$ in $y = mx + 2$ $0 = t + 2$ $t = -2$	✓equating ✓value (2) ✓c=2 ✓value (2) Answer only: full marks

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
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4.3	$\sqrt{(1-p)^2+(3+4)^2}=\sqrt{50}$ $(1-p)^2+(3+4)^2=50$ $1-2p+p^2+49=50$ $p^2-2p=0$ $p(p-2)=0$ $p\neq 0$ or $p=2$ OR $(1-p)^2+(3+4)^2=50$ $(1-p)^2=50-49$ $(1-p)^2=1$ $1-p=1$ or $1-p=-1$ $p\neq 0$ $\quad p=2$ OR $Let\ p=2$ $AC=\sqrt{(1-2)^2+(3+4)^2}$ $=\sqrt{1+49}$ $=\sqrt{50}$ which is true $\therefore p=2$	✓ substitution into distance formula ✓ expansion ✓factors ✓answer Note: If an answer was not chosen: 3/4 (4) ✓ substitution into distance formula ✓ expansion ✓factors ✓answer (4) If gradient of BC assumed as -1 and p calculated correctly: 0/4 Answer only: 1/4 ✓ substitution into distance formula ✓ $\sqrt{50}$ ✓which is true(justification) ✓answer (4) If equating to $\sqrt{50}$ from the start, then 3/4
4.4	midpoint of BC $=\left(\frac{-2+2}{2};\frac{0-4}{2}\right)$ midpoint of BC $=(0\ ;-2)$	✓ x-value ($x=\frac{t+p}{2}$) ✓y-value (2)
4.5	Gradient of line = $m_{AB}=1$ Equation of line is: $y+4=1(x-2)$ $y=x-6$ OR $y=mx+c$ $y=x-p-4$	✓ gradients are equal ✓substitution of (p;-4) ✓equation in any form (3) **[13]**

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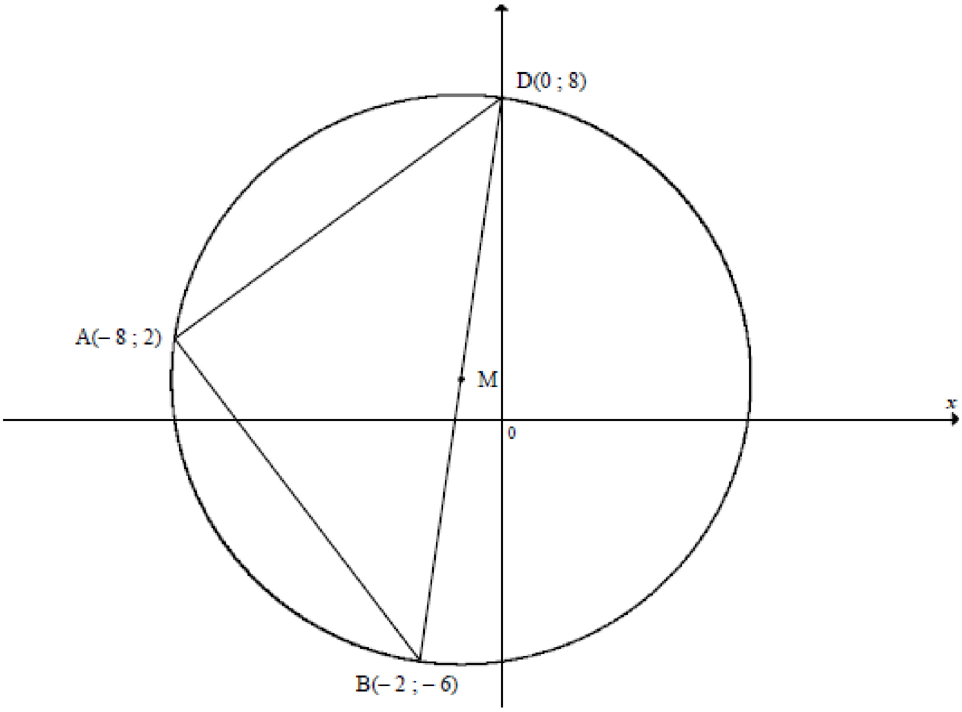
QUESTION 5



5.1	Midpoint BD $\left(\frac{0-2}{2};\frac{8-6}{2}\right)$ $=(-1;1)$	✓x-coordinate ✓y-coordinate (2)
5.2	$y=7(-8)+58$ $=2$ $\therefore$ A lies on the line.	✓substitution (1) Substitute both at the same time with justification (1)
5.3	The line $y = 7x + 58$ is a tangent to the circle at A. $m_{line}=7$ $m_{AM}=\frac{2-1}{-8-(-1)}=-\frac{1}{7}$ $m_{line}\times m_{AM}=7\times-\frac{1}{7}=-1$ $\therefore$ AM $\perp$ to the line OR	✓relationship ✓✓ $m_{AM}=\frac{2-1}{-8-(-1)}=-\frac{1}{7}$ ✓ $m_{line}=7$ ✓product (5)

NOTE:
$m_{line}=7$ and CA gradient of AM then no relationship: 4/5

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5.3 contd	**OR** $m_{BD} = 7$ $m_{line} = 7$ $\therefore$ line // diameter	✓✓ $m_{BD} = 7$ ✓ $m_{line} = 7$ ✓✓conclusion (5) Note: Only lines parallel 4/5
	OR $(x+1)^2 + (y-1)^2 = 50$ $x^2 + 2x + 1 + y^2 - 2y + 1 = 50$ $x^2 + 2x + 1 + (7x+58)^2 - 2(7x+58) + 1 = 50$ $x^2 + 2x + 1 + 49x^2 + 812x + 3364 - 14x - 116 + 1 = 50$ $50x^2 + 800x + 3200 = 0$ $x^2 + 16x + 64 = 0$ $(x+8)^2 = 0$ $x = -8$ $y = 2$ $y = 7x + 58$ is a tangent to the circle	✓ circle equation ✓substitution of $y = 7x + 58$ ✓standard form ✓ answer ✓ tangent (5)
5.4	$\text{AD} = \sqrt{(8-2)^2 + (0+8)^2}$ $= \sqrt{36+64}$ $= 10$ $\text{AB} = \sqrt{(2+6)^2 + (-8+2)^2}$ $= \sqrt{64+36}$ $= 10$	✓ substitution ✓ answer ✓ substitution ✓ answer (4) Note: Answers $\sqrt{10}$ then 3/4

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

5.5	$m_{AD} = \frac{8-(2)}{0-(-8)}$ $m_{AD} = \frac{3}{4}$ $m_{AB} = \frac{2-(-6)}{-8-(-2)}$ $= -\frac{4}{3}$ $m_{AB}.m_{AD} = -\frac{4}{3} \times \frac{3}{4}$ $= -1$ $D\hat{A}B = 90°$	✓ gradient of AD ✓ gradient of AB ✓ PRODUCT (3)
	OR $BD^2 = (8+6)^2 + (0+2)^2$ $= 200$ $= AD^2 + AB^2$ $\therefore D\hat{A}B = 90°$	✓ distance formula ✓ Pythagoras ✓ conclusion (3)
	OR $a^2 = b^2 + d^2 - 2(b)(d)\cos A$ $200 = 100 + 100 - 2(10)(10)\cos A$ $0 = -200\cos A$ $A = 90°$	✓ cos rule ✓ substitution ✓ conclusion (3)
	OR $(AD)^2 = 100$ $(AB)^2 = 100$ $BD^2 = (-2-0)^2 + (-6-8)^2$ $= 4 + 196$ $= 200$ $\therefore BD^2 = AD^2 + AB^2$ $\therefore D\hat{A}B = 90°$ (Pyth)	✓ $BD^2 = 200$ ✓ $BD^2 = AD^2 + AB^2$ ✓ conclusion (3)
	OR $\hat{A} = 90°$ (angles in semi - circle)	✓ ✓✓reason (3)
5.6	$\theta = 45°$	✓ answer (1)

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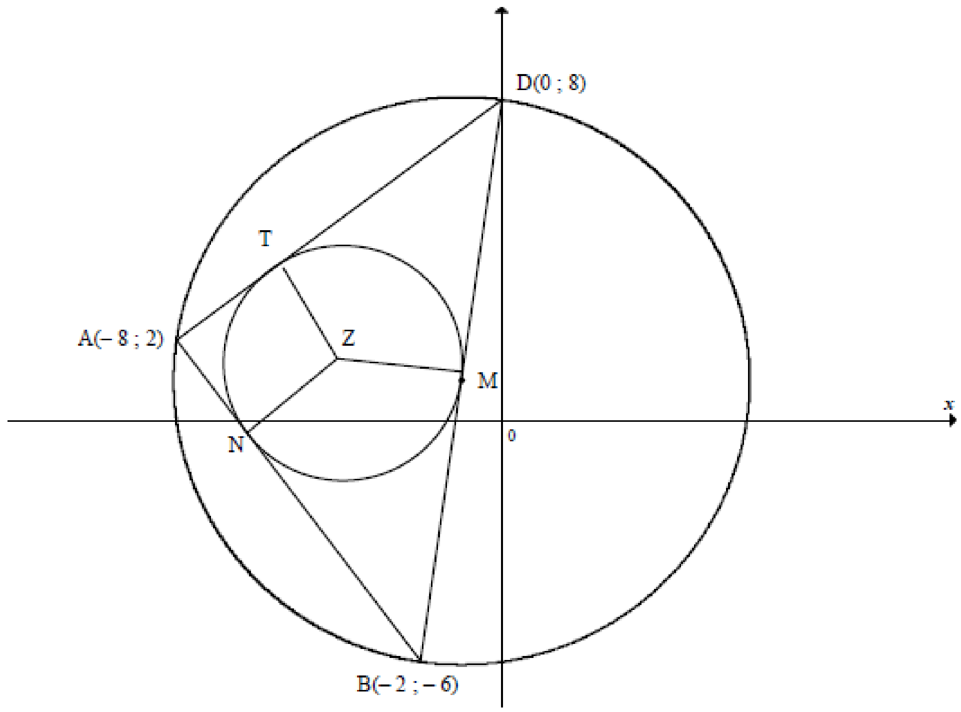

5.7	Let the radius of circle TNM be r NB = BM (properties of a kite) AN = TZ = r (TZNA is a square) NB = $10 - r$ BD = 2MB $\sqrt{(8-(-6))^2+(0-(-2))^2} = 2(10-r)$ $\sqrt{200} = 2(10-r)$ $10\sqrt{2} = 2(10-r)$ $r = 10 - 5\sqrt{2}$ $= 2{,}93$ **OR** $Z\hat{M}B = 90°$ $MB = \frac{1}{2}\sqrt{200}$ $= 7{,}07$ $\frac{ZM}{MB} = \tan 22{,}5°$ $ZM = 7{,}07 \tan 22{,}5°$ $= 2{,}93$ **OR**	✓ NB = BM ✓ AN = TZ = r ✓ NB = $10 - r$ ✓ BD = 2MB ✓BD = $\sqrt{200}$ ✓answer (6) ✓tan radius theorem ✓✓MB ✓✓tan 22,5° ✓answer (6)

- Consistent Accuracy will apply as a general rule.
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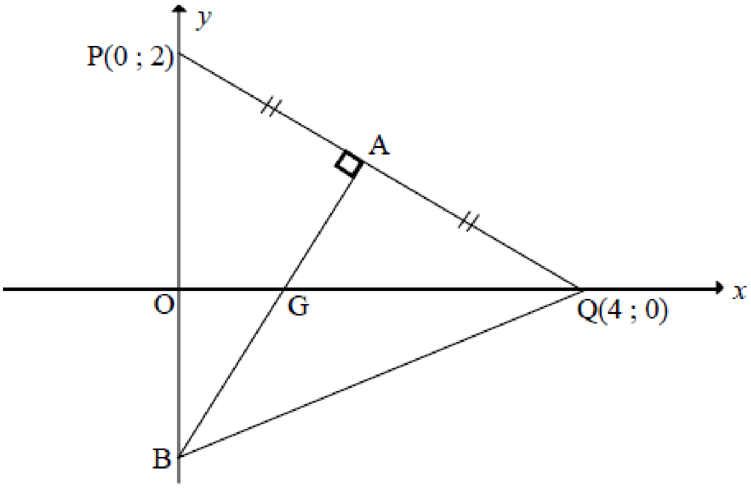
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

5.7 contd	$MB^2 = (-1+2)^2 + (1+6)^2$ $= 1 + 49$ $= 50$ $MB = \sqrt{50}$ $\frac{ZM}{MB} = \tan 22{,}5°$ $ZM = 7{,}07 \tan 22{,}5°$ $= 2{,}93$	✓✓MB ✓✓tan 22,5° ✓✓answer (6)
	OR By a well known formula Area $\Delta ABD = r \times$ (semi—perimeter) $\frac{1}{2} \times 10 \times 10 = r \times \frac{1}{2}(20 + \sqrt{200})$ $50 = r(10 + 5\sqrt{2})$ $r = 2{,}93$	✓✓ formula ✓ $\sqrt{200}$ ✓✓ answer (6)
	OR $MB = \sqrt{50}$ (radius of circle) $NB = \sqrt{50}$ (adjacent sides of kite) AB = 10 AN = $10 - \sqrt{50}$ $= 2{,}93$ But TANZ is a square $\therefore$ AN = ZN $\therefore$ radius = 2,93	✓MB ✓ NB ✓✓AN = 2,93 ✓ square ✓answer (6)

QUESTION 4

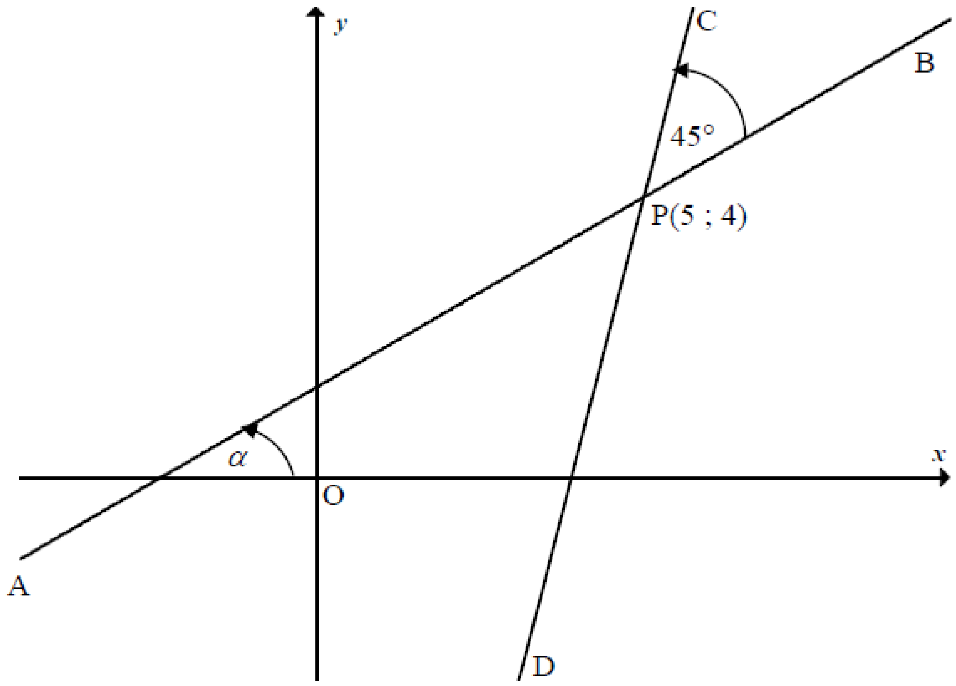
The diagram below shows the points P(0 ; 2) and Q(4 ; 0). Point A is the midpoint of PQ. The line AB is perpendicular to PQ and intersects the x-axis at G and the y-axis at B.



4.1	Show that the gradient of PQ is $-\frac{1}{2}$.	(1)
4.2	Determine the coordinates of A.	(2)
4.3	Determine the equation of the line AB.	(5)
4.4	Calculate the length of BQ.	(3)
4.5	Show that ΔBPQ is isosceles.	(2)
4.6	If PBQR is a rhombus, determine the coordinates of R.	(3)
		[16]

QUESTION 5

The straight line AB has the equation $5y-3x-5=0$. Another straight line CD is drawn to intersect AB at P(5 ; 4) such that the acute angle between AB and CD is 45°.



5.1 Determine the gradient of the line CD. (5)

5.2 Hence, or otherwise, determine the equation of the line CD. (2)
[7]

QUESTION 6

6.1 Determine the centre and radius of the circle with the equation $x^2+y^2+8x+4y-38=0$. (4)

6.2 A second circle has the equation $(x-4)^2+(y-6)^2=26$. Calculate the distance between the centres of the two circles. (2)

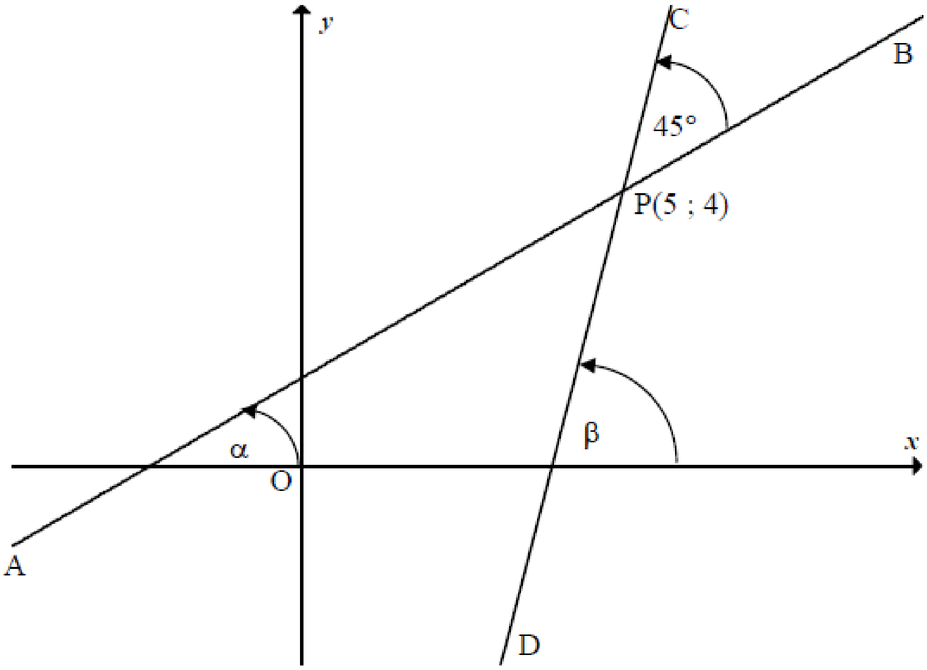
6.3 Hence, show that the circles described in QUESTION 6.1 and QUESTION 6.2 intersect each other. (3)

6.4 Show that the two circles intersect along the line $y=-x+4$. (4)
[13]

QUESTION 4

4.1	$m_{PQ} = \frac{2-0}{0-4} = -\frac{1}{2}$	✓ substitution (1)
4.2	A: $\left(\frac{0+4}{2};\frac{2+0}{2}\right)$ A (2 ; 1)	✓ x-coordinate ✓ y-coordinate (2)
4.3	$m_{AB}.m_{PQ} = -1$ $m_{AB}.(-1/2) = -1,\ \therefore m_{AB} = 2$ Equation of AB is $y = 2x + c$ $\therefore\ 1 = 2(2) + c$ $c = -3$ Equation of AB is $y = 2x - 3$. **OR** $m_{AB}.m_{PQ} = -1$ $m_{AB}.(-1/2) = -1,\ \therefore m_{AB} = 2$ $y - 1 = 2(x-2)$ $y - 1 = 2x - 4$ $y = 2x - 3$	✓ $m_{AB}.m_{PQ} = -1$ ✓ $m_{AB} = 2$ ✓ equation of AB ✓ $y = 2x - 3$ ✓ $c = -3$ (5) ✓ $m_{AB}.m_{PQ} = -1$ ✓ $m_{AB} = 2$ ✓ gradient of AB ✓ substitution into formula ✓ equation of AB (5)
4.4	B is the point (0 ; –3) BQ $= \sqrt{(0-4)^2 + (-3-0)^2}$ $= 5$	✓ coordinates of B ✓ substitution ✓ answer (3)
4.5	BP $= \sqrt{(0-0)^2 + (-3-2)^2}$ $= 5$ BP = BQ $\therefore \Delta$BPQ is isosceles. **OR** BP = 2 + 3 = 5 BP = BQ $\therefore \Delta$BPQ is isosceles	✓ BP = 5 ✓ BP = BQ (2) ✓ BP = 5 ✓ BP = BQ (2)
4.6	If PBQR is a rhombus then A is the midpoint of BR. Let the coordinates of R be $(x\ ;\ y)$ $\frac{x+0}{2} = 2$ and $\frac{y-3}{2} = 1$ $x = 4$ $y = 5$ $\therefore$R(4 ; 5) **OR** RQ || PB so $x_R = 4$ RQ = PB = 5, so $y_R = 5$ $\therefore$R(4 ; 5)	✓ A is the midpoint of BR ✓ x coordinate ✓ y coordinate (3) ✓ RQ || PB ✓ x coordinate ✓ y coordinate (3) **[16]**

QUESTION 5



5.1	AB is defined as $5y-3x-5=0$ which can be written as $y=\frac{3}{5}x+1$ $m_{AB}=\frac{3}{5}$ Let α be the inclination of AB. $\tan\alpha=\frac{3}{5}$ $\alpha=30.96°$. Let β be the inclination of CD β = 45° + 30,96° = 75,96° Gradient of CD = tan 75,96° = 4. **OR** $\tan\beta=\tan(\alpha+45°)$ $=\frac{\tan\alpha+\tan 45°}{1-\tan\alpha.\tan 45°}$ $=\frac{\frac{3}{5}+1}{1-\frac{3}{5}\times 1}$ $=4$ $m_{CD}=\tan\beta$ $m_{CD}=4$	✓ $m_{AB}=\frac{3}{5}$ ✓ $\tan\alpha=\frac{3}{5}$ ✓ $\alpha=30.96°$ ✓ β = 75,96° ✓ gradient of CD (5) ✓ expansion ✓ tan 45° = 1 ✓ tan α = $\frac{3}{5}$ ✓ substitution ✓ answer (5)

5.2	Equation of CD is $y = 4x + c$ $\therefore\ 4 = 4(5) + c$ $c = -16$ Equation of CD is $y = 4x - 16$. **OR** $y - 4 = 4(x - 5)$ $y - 4 = 4x - 20$ $y = 4x - 16$	✓ y- intercept ✓ equation of CD (2) ✓ substitution ✓ equation of CD (2) [7]

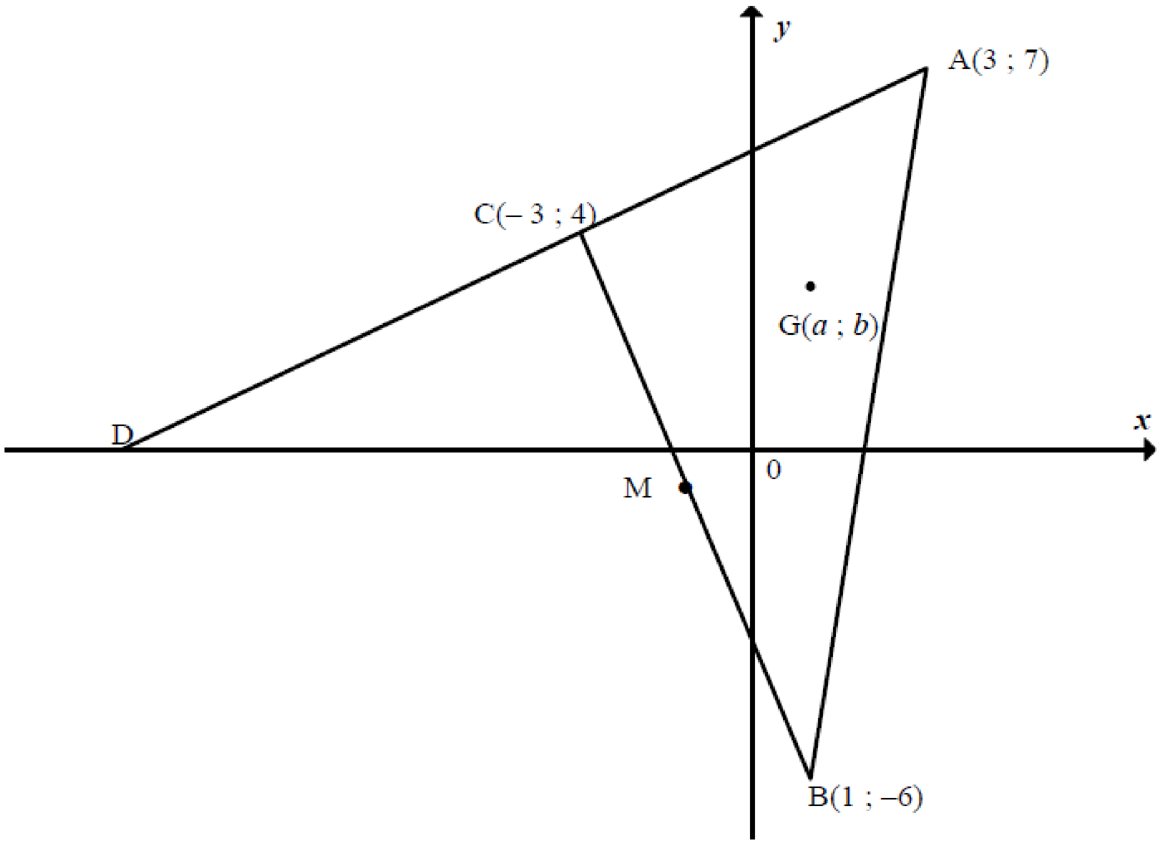
QUESTION 6

6.1	$x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + 8x + 16 + y^2 + 4y + 4 = 16 + 4 + 38$ $(x+4)^2 + (y+2)^2 = 58$ Centre is $(-4\ ;\ -2)$ and the radius is $\sqrt{58}$	✓completing the square (both or one) ✓ factor form ✓ centre ✓ radius (4)
6.2	Centre of second circle is $(4\ ;\ 6)$ Distance between centres is $\sqrt{(4+4)^2 + (6+2)} = \sqrt{128} = 11{,}31$	✓centre ✓distance (2)
6.3	Sum of radii = $\sqrt{58} + \sqrt{26} = 12{,}71$ Distance between centres is 11,31. sum of the radii > distance between the centres $\therefore$ the circles must overlap and hence the circles must intersect.	✓✓ sum of radii ✓ conclusion (3)
6.4	Equation of second circle: $(x-4)^2 + (y-6)^2 = 26$ $x^2 - 8x + 16 + y^2 - 12y + 36 = 26$ $x^2 - 8x + y^2 - 12y + 26 = 0$ Let $(x\ ;\ y)$ be either of the two points on intersection. Then $x^2 + y^2 + 8x + 4y - 38 = 0$ and $x^2 + y^2 - 8x - 12y + 26 = 0$ Subtract $16y + 16x - 64 = 0$ $y = -x + 4$ Both points of intersection lie on this line. $\therefore\ y = -x + 4$ is the equation of the common chord. **OR**	✓ equation of circle in form = 0 ✓ statement – two points of intersection ✓ subtracting ✓ simplification (4)

	Check that the line $y = -x + 4$ cuts the two circles at the same points: $(x-4)^2 + (-x-2)^2 = 26$ $x^2 - 8x + 16 + x^2 + 4x + 4 = 26$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ $x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$ $x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $x = 3$ or $x = -1$	✓ substitution ✓ answer ✓ substitution ✓ answer (4) **[13]**

QUESTION 5

In the diagram below, A, B and C are the vertices of a triangle. AC is extended to cut the x-axis at D.



5.1 Calculate the gradient of:

5.1.1 AD (2)

5.1.2 BC (1)

5.2 Calculate the size of $D\hat{C}B$. (3)

5.3 Write down an equation of the straight line AD. (2)

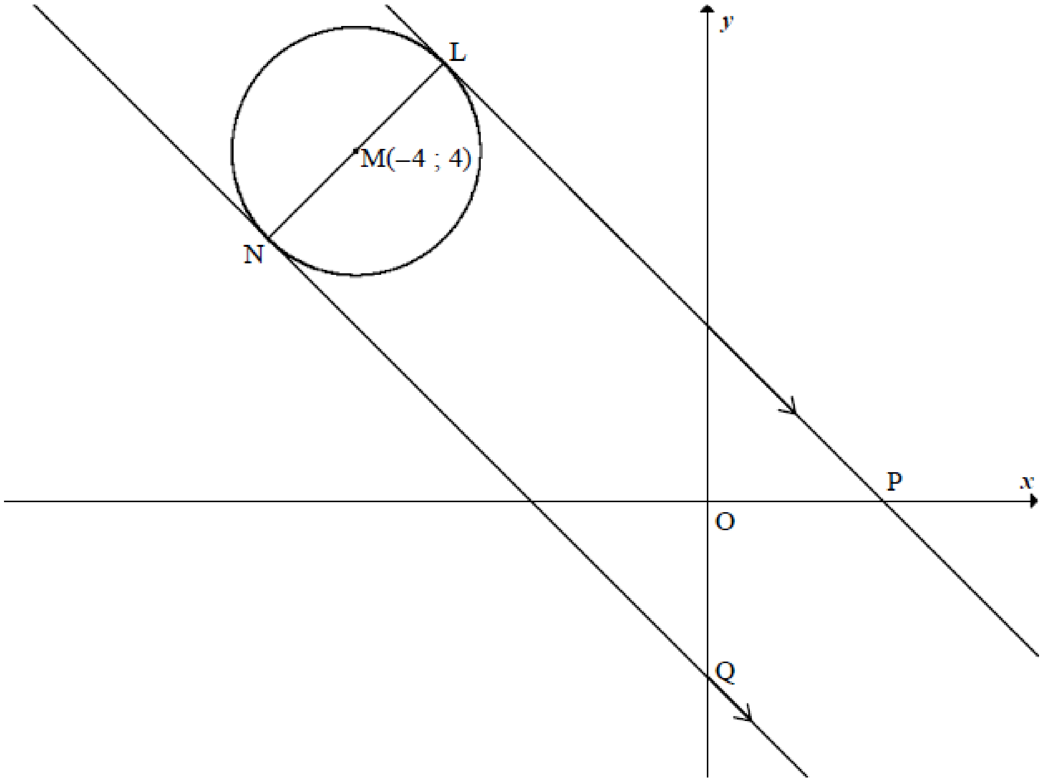
5.4 Determine the coordinates of M, the midpoint of BC. (2)

5.5 If G(a ; b) is a point such that A, G and M lie on the same straight line, show that $b = 2a + 1$. (4)

5.6 Hence calculate TWO possible values of b if $GC = \sqrt{17}$. (6)

[20]

QUESTION 6

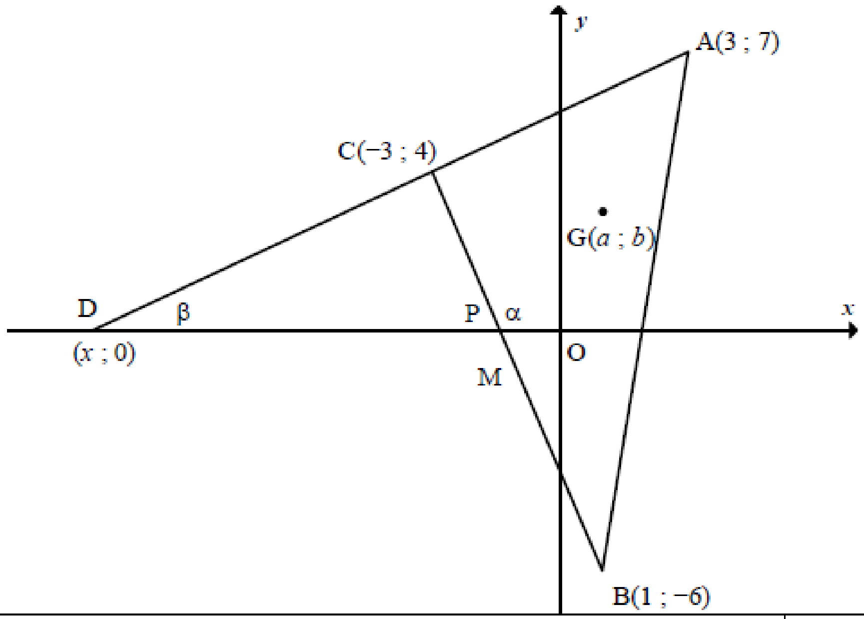


The line LP, with equation $y + x - 2 = 0$, is a tangent at L to the circle with centre M(– 4 ;4). LN is a diameter of the circle. Also LP || NQ, where P lies on the x-axis, and Q lies on the y-axis.

6.1	Determine the equation of the diameter LN.	(3)
6.2	Calculate the coordinates of L.	(2)
6.3	Determine the equation of the circle.	(3)
6.4	Write down the coordinates of N.	(3)
6.5	Write down the equation of NQ.	(3)
6.6	If the length of the diameter is doubled and the circle is translated horizontally 6 units to the right, write down the equation of the new circle.	(3)

[17]

QUESTION 5



5.1.1	$m_{AD} = m_{AC}$ $= \frac{7-4}{3-(-3)}$ $= \frac{3}{6}$ $= \frac{1}{2}$ **OR** $m_{AD} = m_{AC}$ $= \frac{4-7}{-3-(3)}$ $= \frac{-3}{-6}$ $= \frac{1}{2}$ **Note:** If candidate gives $m_{AD} = \frac{7}{3-x}$ then **1/2 marks**	✓ substitution of A and C into correct formula ✓ answer (2)
5.1.2	$m_{BC} = \frac{-6-4}{1-(-3)}$ $= \frac{-10}{4}$ $= \frac{-5}{2}$ **OR** $m_{BC} = \frac{4-(-6)}{-3-(1)}$ $= \frac{10}{-4}$ $= \frac{-5}{2}$	✓ answer (1)
5.2	$m_{AD} = \frac{1}{2} = \tan C\hat{D}O$ $C\hat{D}O = 26{,}56505....°$ $m_{BC} = \frac{-5}{2} = \tan \alpha$ $\alpha = 111{,}814\,095°$ $D\hat{C}B = 111{,}801\,409\,5....° - 26{,}56505....°$ $= 85{,}236359°$ $= 85{,}24°$ $\approx 85{,}2°$ **OR**	✓ 26,57° ✓ 111,80° ✓answer (3)

$$\tan C\hat{D}O = \frac{1}{2}$$

$$C\hat{D}O = 26{,}56505....^\circ$$ ✓ 26,57°

$$\tan(180^\circ - \alpha) = \frac{5}{2}$$

$$180^\circ - \alpha = 68{,}19859051...^\circ$$ ✓ 68,2°

$$D\hat{C}B = 180^\circ - (26{,}56505....^\circ + 68{,}19859051...^\circ)$$

$$= 85{,}236359^\circ$$

$$= 85{,}24^\circ$$ ✓answer

(3)

OR

$$D\hat{C}B = \alpha - C\hat{D}O$$

$$\tan D\hat{C}B = \frac{m_{CB} - m_{CD}}{1 + m_{CB}.m_{CD}}$$ ✓ $\tan D\hat{C}B = \frac{m_{CB} - m_{CD}}{1 + m_{CB}.m_{CD}}$

$$= \frac{-\frac{5}{2} - \frac{1}{2}}{1 + \left(-\frac{5}{2}\right)\left(\frac{1}{2}\right)}$$ ✓ substitution

$$= 12$$ ✓ answer

$$D\hat{C}B = 85{,}24^\circ$$

(3)

OR

$$AC = \sqrt{45} \quad BC = \sqrt{116} \quad AB = \sqrt{173}$$

$$\cos A\hat{C}B = \frac{AC^2 + BC^2 - AB^2}{2AC.BC}$$ ✓ cosine rule

$$= \frac{45 + 116 - 173}{2(\sqrt{45})(\sqrt{116})}$$ ✓ substitution into cosine rule

$$= -0{,}083045...$$

$$A\hat{C}B = 94{,}76...^\circ$$

$$D\hat{C}B = 180^\circ - 94{,}76...^\circ$$

$$= 85{,}24^\circ$$ ✓ answer

(3)

OR

D(– 11 ; 0)

$$DC = \sqrt{80} \quad BC = \sqrt{116} \quad DB = \sqrt{180}$$

$$\cos D\hat{C}B = \frac{DC^2 + BC^2 - DB^2}{2DC.BC}$$ ✓ cosine formula

$$= \frac{80 + 116 - 180}{2(\sqrt{80})(\sqrt{116})}$$ ✓ substitution into cosine rule

$$= 0{,}08304547985...$$

$$D\hat{C}B = 85{,}24^\circ$$ ✓ answer

(3)

OR

Equation AC: $2y = x + 11$

D(– 11 ; 0)

C(– 3 ; 4)

$$\begin{aligned} DC^2 &= (x_C - x_D)^2 + (y_C - y_D)^2 \\ &= (-3+11)^2 + (4-0)^2 \\ &= 80 \end{aligned}$$

Equation BC: $2y = -5x - 7$

$P\left(-\frac{7}{5};0\right)$

$$\begin{aligned} PC^2 &= (-3+\tfrac{7}{5})^2 + (4-0)^2 \\ &= \frac{464}{25} \end{aligned}$$

$$\begin{aligned} DP^2 &= (-\tfrac{7}{5}+11)^2 \\ &= \frac{2304}{25} \end{aligned}$$

In ΔDCP: $DP^2 = DC^2 + CP^2 - 2DC.CP.\cos D\hat{C}P$ — ✓ cosine formula

$$\frac{2304}{25} = \frac{2000}{25} + \frac{464}{25} - 2\left(\frac{\sqrt{2000}}{5}\right)\left(\frac{\sqrt{464}}{5}\right).\cos D\hat{C}P$$

✓ substitution into cosine rule

$D\hat{C}P = 85{,}23635...$

$D\hat{C}P = 85{,}24°$

✓ answer

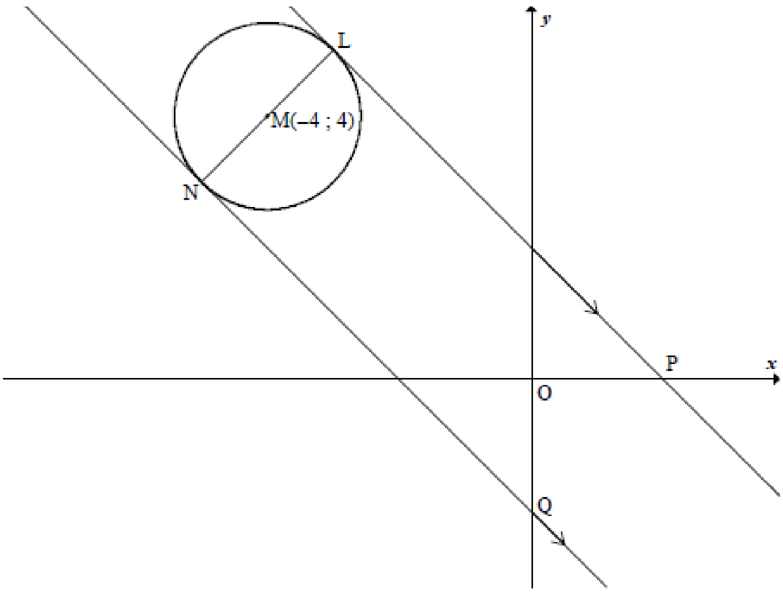
(3)

5.3	$y-7=\frac{1}{2}(x-3)$ $y=\frac{1}{2}x+\frac{11}{2}$ $x-2y+11=0$ **OR** $y-4=\frac{1}{2}(x+3)$ $y=\frac{1}{2}x+\frac{11}{2}$ $x-2y+11=0$ **OR** $y=\frac{1}{2}x+c$ $(7)=\frac{1}{2}(3)+c$ $c=\frac{11}{2}$ $y=\frac{1}{2}x+\frac{11}{2}$ $x-2y+11=0$ **Note:** If candidate leaves answer as $y-7=\frac{1}{2}(x-3)$ or $y-4=\frac{1}{2}(x+3)$: **max 1 / 3 marks**	✓ substitution of (3 ; 7) into $y-y_1=m(x-x_1)$ ✓ answer in any form (2) ✓ substitution of (– 3 ; 4) into $y-y_1=m(x-x_1)$ ✓ answer in any form (2) ✓ substitution of (3 ; 7) into $y=mx+c$ ✓ answer in any form (2)
5.4	$\text{M}(x;y)=\left(\frac{-3+1}{2};\frac{4-6}{2}\right)$ $\text{M}(x;y)=(-1;-1)$	✓substitution ✓answer (2)

5.5	$m_{AM}=\frac{7-(-1)}{3-(-1)}=2$ $y=2x+c$ $-1=2(-1)+c$ $\therefore c=1$ $y=2x+1$ G(a ; b) lies on the line $\therefore b=2a+1$ **Note:** If the candidate does not conclude $b = 2a + 1$ from $y = 2x + 1$: **max 3 / 4 marks**	✓gradient = 2 ✓substitution (– 1 ; – 1) ✓c = 1 ✓conclusion (4)
	OR $\frac{7-b}{3-a}=\frac{b+1}{a+1}$ $(7-b)(a+1)=(b+1)(3-a)$ $7a+7-ab-b=3b-ab+3-a$ $8a-4b=-4$ $2a-b=-1$ $b=2a+1$	✓$\frac{7-b}{3-a}$ ✓$\frac{b+1}{a+1}$ ✓equating ✓simplification leading to $2a-b=-1$ (4)
	OR Using the point (– 1 ; – 1) $\frac{b+1}{a+1}=\frac{8}{4}$ $\frac{b+1}{a+1}=2$ $b+1=2a+2$ $b=2a+1$	✓ substitution of (– 1 ; – 1) into gradient ✓ gradient = 2 ✓ equating ✓ simplification leading to $b+1=2a+2$ (4)
	OR Using the point (3 ; 7) $\frac{7-b}{3-a}=\frac{8}{4}$ $\frac{7-b}{3-a}=2$ $7-b=6-2a$ $b=2a+1$	✓ substitution of (3 ; 7) into gradient ✓ gradient = 2 ✓ equating ✓ simplification leading to $7-b=6-2a$ (4)

5.6	$GC = \sqrt{17}$ $GC^2 = 17$ $(a+3)^2 + (b-4)^2 = 17$ $(a+3)^2 + (2a+1-4)^2 = 17$ $a^2 + 6a + 9 + 4a^2 - 12a + 9 - 17 = 0$ $5a^2 - 6a + 1 = 0$ $(5a-1)(a-1) = 0$ $a = \frac{1}{5}$ or $a = 1$ $\therefore b = \frac{7}{5}$ or $b = 3$ **Note:** If candidate swops a and b around: **max 2 / 6 marks**	✓ distance formula in terms of a and b ✓substitution of $b = 2a + 1$ ✓standard form ✓factors or correct substitution into formula ✓values of a ✓values of b (6)
	OR $a = \frac{b-1}{2}$ $17 = (a+3)^2 + (b-4)^2$ $17 = \left(\left(\frac{b-1}{2}\right) + 3\right)^2 + (b-4)^2$ $17 = \left(\frac{b+5}{2}\right)^2 + (b-4)^2$ $17 = \frac{b^2 + 10b + 25 + 4b^2 - 32b + 64}{4}$ $68 = 5b^2 - 22b + 89$ $0 = 5b^2 - 22b + 21$ $0 = (5b-7)(b-3)$ $\therefore b = \frac{7}{5}$ or $b = 3$	✓ $a = \frac{b-1}{2}$ ✓ distance formula in terms of a and b ✓ substitution of $a = \frac{b-1}{2}$ ✓ standard form ✓ factors or correct substitution into formula ✓ values of b (6) **[20]**

QUESTION 6



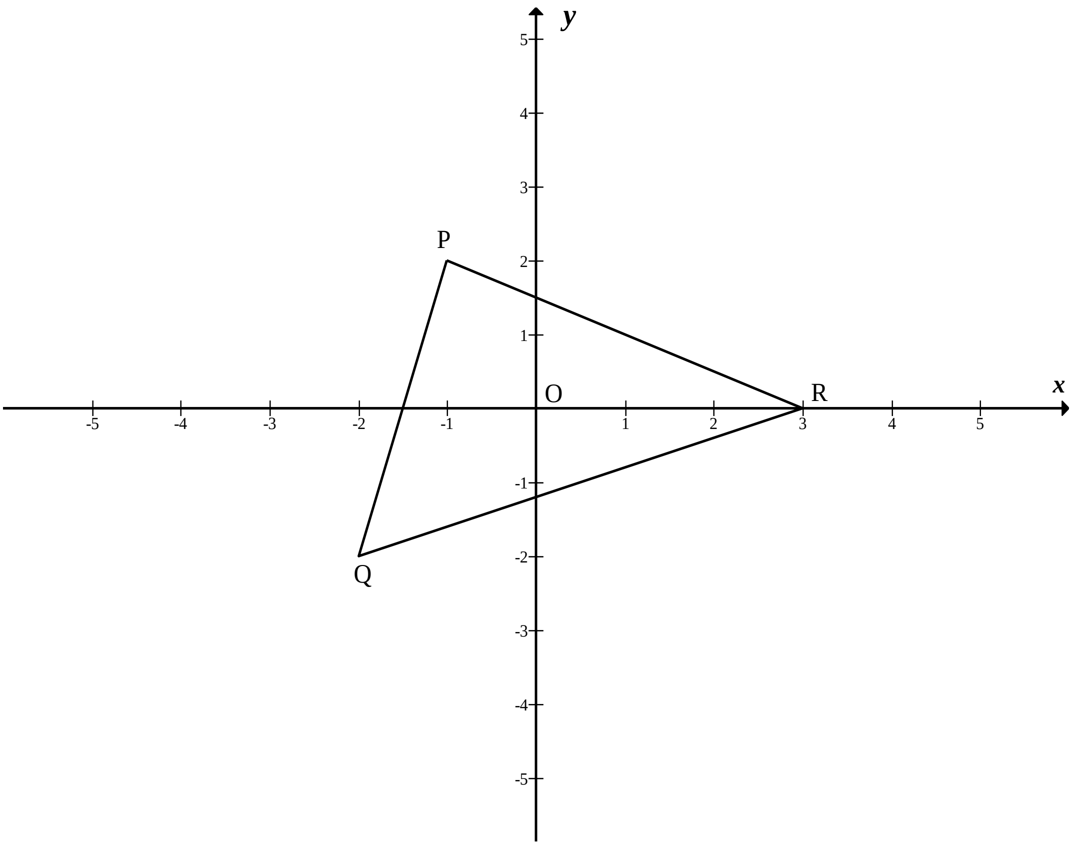
6.1	$y = -x + 2$ $m_{LP} = -1$ $\therefore m_{LN} = \frac{-1}{-1} = 1$ $y = x + c$ $4 = -4 + c$ $\therefore c = 8$ $y = x + 8$ **Note:** If candidate leaves it as $y - 4 = x + 4$ **max 2 / 3 marks** Answer only: **Full marks** **OR** $y - 4 = 1(x + 4)$ $y = x + 8$	✓ $m_{LP} = -1$ ✓ $m_{LN} = 1$ ✓equation (3) ✓ $m = 1$ ✓ substitution of $y - y_1 = m(x - x_1)$ ✓ answer (3)
6.2	$x + 8 = -x + 2$ $2x = -6$ $x = -3$ $y = -3 + 8$ $y = 5$ L$(-3; 5)$ **OR** $y + x = 2$..........(1) $y - x = 8$..........(2 $2y = 10$ $\therefore y = 5$ $\therefore x = -3$ L$(-3; 5)$ **Note:** No penalty for not leaving in coordinate form	✓x-value ✓y-value Equations leading to these values must be used (2)

6.3	$(x+4)^2+(y-4)^2=r^2$ $(-3+4)^2+(5-4)^2=r^2$ $\therefore r^2=2$ $(x+4)^2+(y-4)^2=2$ Equation can be left as: $x^2+8x+y^2-8y+30=0$ **Note:** If the candidate only uses the distance formula to determine the radius $(-3+4)^2+(5-4)^2=r^2$ $\therefore r^2=2$ then **2 / 3 marks**	✓ $(x+4)^2+(y-4)^2=r^2$ ✓substitution of (– 3 ; 5) ✓ $r^2=2$ (3)
6.4	Let N(x, y). Since M(– 4 ; 4) is the midpoint of LN and L(– 3 ; 5) $\frac{x-3}{2}=-4$; $\frac{y+5}{2}=4$ $\therefore x=-5$; $y=3$ **OR** $y=x+8$ $(x+4)^2+(y-6)^2=2$ $(x+4)^2+(x+8-4)^2-2=0$ $x^2+8x+16+x^2+8x+16-2=0$ $2x^2+16x+30=0$ $x^2+8x+15=0$ $(x+5)(x+3)=0$ $x=-3$ or $x=-5$ $y=5$ $\quad$ $y=3$ $\therefore$ N(– 5 ; 3) **Note:** Answer only: **Full marks**	✓ using the fact that M is the midpoint of LN ✓ $x=-5$ ✓ $y=3$ (3) ✓ $(x+4)^2+(x+8-4)^2-2=0$ ✓ $x=-5$ ✓ $y=3$ (3)
6.5	$m_{NQ}=-1$ $y=-x+c$ $3=-(-5)+c$ $c=-2$ $y=-x-2$ **Note:** Answer only: **Full marks** **OR** $m_{NQ}=-1$ $y-3=-(x+5)$ $y=-x-2$ **OR** Equation of LP is $x+y=2$ NQ || LP $\therefore$ equation of NQ is $x+y=k$ for some $k\in R$ But N(– 5 ; 3) lies on NQ $\therefore x+y=-5+3=-2$	✓gradient ✓substitution of (– 5 ; 3) into $y=mx+c$ ✓$c=-2$ (3) ✓gradient ✓substitution of (– 5 ; 3) into $y-y_1=m(x-x_1)$ ✓equation (3) ✓ $x+y=k$ ✓ substitution of (– 5 ; 3) ✓ equation (3)

	OR NQ is a reflection of LP ($y + x = 2$) in the line $y = x$ $\therefore$ equation of NQ is $x + y = -2$	
6.6	Let new radius of circle be R and centre be M′. $M'(-4+6;4)$ $=(2;4)$ $R = 2r$ $R^2 = 4r^2$ $=4(2)$ $=8$ $\therefore (x-2)^2 + (y-4)^2 = (2\sqrt{2})^2$ $\therefore (x-2)^2 + (y-4)^2 = 8$ **OR** Let R = new radius of circle $R^2 = (2r)^2 = 4(2) = 8$ $(x-6+4)^2 + (y-4)^2 = 8$ $\therefore (x-2)^2 + (y-4)^2 = 8$	✓ $M'(2;4)$ ✓ $r = 2\sqrt{2}$ ✓ equation (3) ✓ $(x-2)^2$ ✓ $(y-4)^2$ ✓ 8 or $(2\sqrt{2})^2$ (3) **[17]**

QUESTION 4

In the diagram below Δ PQR with vertices P$(-1 ; 2)$, Q$(-2 ; -2)$ and R$(3 ; 0)$ is given.



4.1 Calculate the angle that PQ makes with the positive *x*-axis. (3)

4.2 Determine the coordinates of M, the midpoint of PR. (2)

4.3 Determine the perimeter of Δ PQR to the nearest whole number. (5)

4.4 Determine an equation of the line parallel to PQ that passes through M. (3)
[13]

QUESTION 5

5.1 The equation of a circle is $x^2 + y^2 - 8x + 6y = 15$.

5.1.1 Prove that the point (2 ; – 9) is on the circumference of the circle. (2)

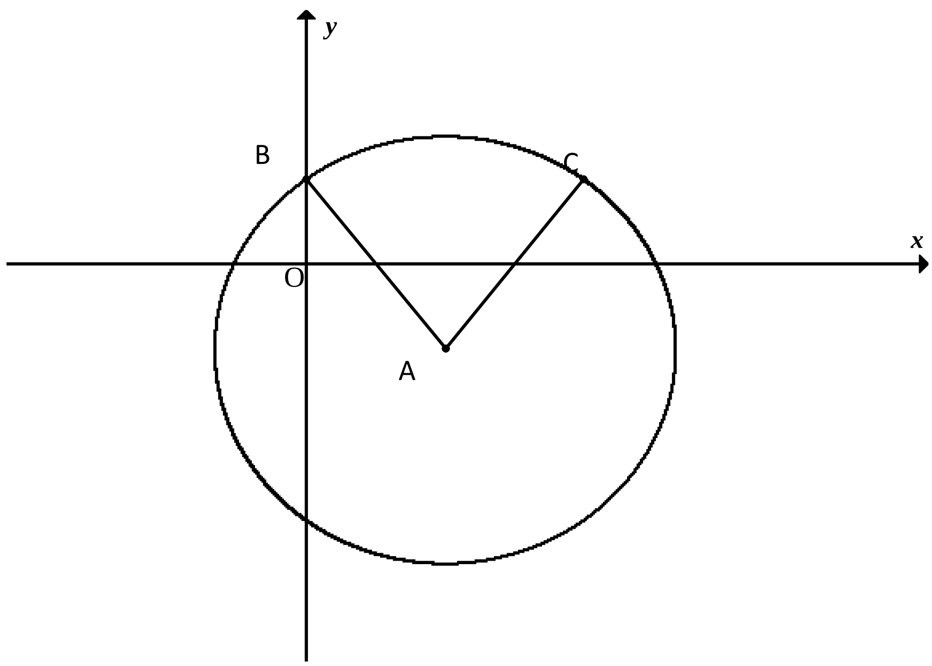
5.1.2 Determine an equation of the tangent to the circle at the point (2 ; – 9). (7)

5.2 Calculate the length of the tangent AB drawn from the point A(6 ; 4) to the circle with equation $(x-3)^2 + (y+1)^2 = 10$. (5)

[14]

QUESTION 6

The circle, with centre A and equation $(x-3)^2 + (y+2)^2 = 25$ is given in the following diagram. B is a *y*-intercept of the circle.



6.1 Determine the coordinates of B. (4)

6.2 Write down the coordinates of C, if C is the reflection of B in the line $x = 3$. (2)

6.3 The circle is enlarged through the origin by a factor of $\frac{3}{2}$.

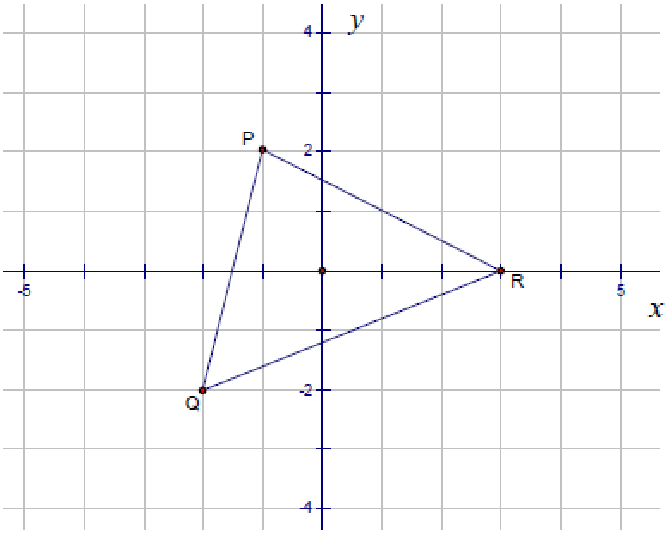
Write down the equation of the new circle, centre A$^{/}$, in the form $(x-a)^2 + (y-b)^2 = r^2$. (2)

6.4 In addition to the circle with centre A and equation $(x-3)^2+(y+2)^2=25$, you are given the circle $(x-12)^2+(y-10)^2=100$ with centre B.

6.4.1 Calculate the distance between the centres A and B. (2)

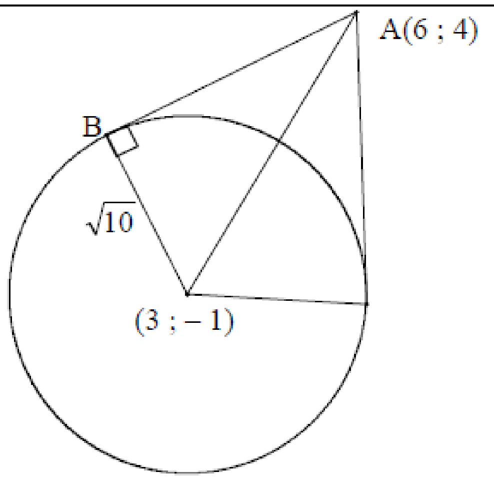
6.4.2 In how many points do these two circles intersect? Justify your answer. (2)
[12]

QUESTION 4



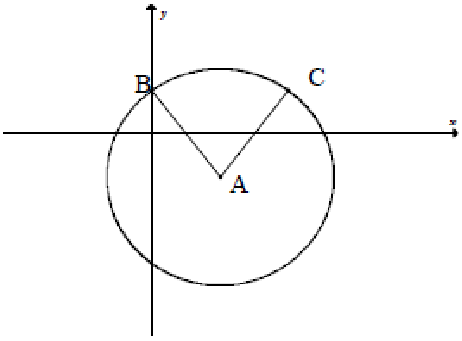
4.1	Let β be the angle of inclination of PQ. $\tan\beta = m_{PQ}$ $\tan\beta = \dfrac{2-(-2)}{-1-(-2)}$ $\tan\beta = 4$ $\beta = 75{,}96°$	✓ $\tan\beta = m_{PQ}$ ✓ $\tan\beta = 4$ ✓ answer (3)
4.2	$M\left(\dfrac{-1+3}{2};\dfrac{2+0}{2}\right)$ M (1 ; 1)	✓ x-value ✓ y-value (2)
4.3	$PQ = \sqrt{(-1+2)^2+(2+2)^2}$ $= \sqrt{17}$ $PR = \sqrt{(-1-3)^2+(2-0)^2}$ $= \sqrt{20}$ $QR = \sqrt{(0-(-2))^2+(3-(-2))^2}$ $= \sqrt{29}$ Perimeter = $\sqrt{29}+\sqrt{20}+\sqrt{17}$ = 13,98 units = 14 to the nearest whole number	✓ substitution into correct formula ✓ answer ✓ answer ✓ sum ✓ answer (5)
4.4	$y-1=4(x-1)$ $y=4x-3$	✓ $m=4$ ✓ substitution of (1 ; 1) ✓ answer (3) **[13]**

QUESTION 5

5.1.1	$x^2+y^2-8x+6y$ $=(2)^2+(-9)^2-8(2)+6(-9)$ $=4+81-16-54$ $=15$ Hence, the point lies on the circumference of the circle. **OR** $x^2+y^2-8x+6y=15$ $(x-4)^2+(y+3)^2=15+16+9$ $(x-4)^2+(y+3)^2=40$ $(x-4)^2+(y+3)^2$ $=(2-4)^2+(-9+3)^2$ $=2^2+6^2$ $=40$ $\therefore$ The point lies on the circumference of the circle.	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2)
5.1.2	$x^2+y^2-8x+6y=15$ $(x-4)^2+(y+3)^2=15+16+9$ $(x-4)^2+(y+3)^2=40$ Circle centre $(4\,;-3)$ $m_{rad}=\frac{-3-(-9)}{4-2}$ $m_{rad}=3$ $m_{\tan}=-\frac{1}{3}$ $y+9=-\frac{1}{3}(x-2)$ $y=-\frac{1}{3}x-\frac{25}{3}$	✓✓ $(x-4)^2+(y+3)^2=40$ ✓ centre ✓ gradient of radius ✓ gradient of tangent ✓substitution ✓ answer (7)
5.2		✓ radius = $\sqrt{10}$

	Radius AB $=\sqrt{10}$ Distance from A to centre of circle is $=\sqrt{(6-3)^2+(4+1)^2}$ $=\sqrt{9+25}$ $=\sqrt{34}$ $AB^2=34-10$ $AB^2=24$ $AB=\sqrt{24}$ $AB=2\sqrt{6}$ $AB=4{,}90$	✓ subs into distance formula ✓ $\sqrt{34}$ ✓ $AB^2=34-10$ ✓ answer (5)
	OR $r^2=10$ $r=\sqrt{10}$ Radius ⊥ tangent By Pythagoras $AB^2=(6-3)^2+(4+1)^2-10$ $=24$ $AB=4{,}90$	✓ $r=\sqrt{10}$ ✓✓ $AB^2=(6-3)^2+(4+1)^2-10$ ✓ AB = 4,90 (5) **[14]**

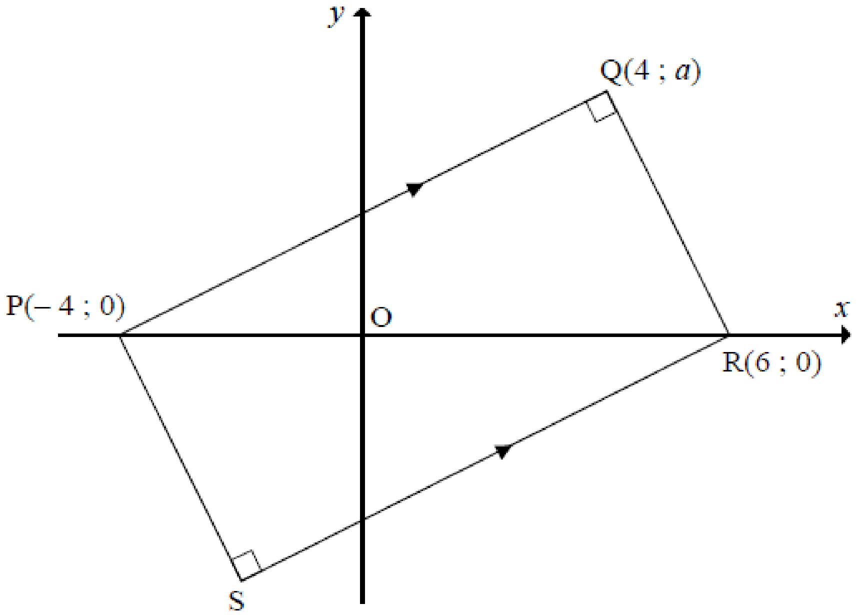
QUESTION 6



6.1	$9+(y+2)^2=25$ $(y+2)^2=16$ $y+2=\pm 4$ $y=2$ or $y=-6$ B(0 ; 2) **OR** $x=0$ $(0)^2-6(0)+y^2+4y=12$ $y^2+4y-12=0$ $(y+6)(y-2)=0$ $y=-6$ or $y=2$ B(0 ; 2)	✓ $x=0$ ✓ factors ✓ answers ✓ answer for B (4) ✓ $x=0$ ✓ factors ✓ answers ✓ answer for B (4)
6.2	C(6 ; 2)	✓✓ answer (2)
6.3	$\left(x-3\times\frac{3}{2}\right)^2+\left(y+2\times\frac{3}{2}\right)^2=\left(5\times\frac{3}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=\left(\frac{15}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=56{,}25$	✓ each part $\times\frac{3}{2}$ ✓ answer (2)
6.4.1	$AB=\sqrt{(12-3)^2+(10-(-2))^2}$ $=\sqrt{9^2+12^2}$ $=15$	✓ substitution ✓ answer (2)
6.4.2	The radii are 5 and 10. $r_A+r_B=5+10$ $=15$ $=AB$ The circles will only intersect at one point.	✓ addition of radii ✓ answer (2) **[12]**

QUESTION 5

In the diagram below, PQRS is a rectangle with vertices P(– 4 ; 0), Q(4 ; a), R(6 ; 0) and S. Q lies in the first quadrant.



5.1 Show that $a = 4$. (4)

5.2 Determine the equation of the straight line passing through the points S and R in the form $y = mx + c$. (4)

5.3 Calculate the coordinates of S. (4)

5.4 Calculate the length of PR. (2)

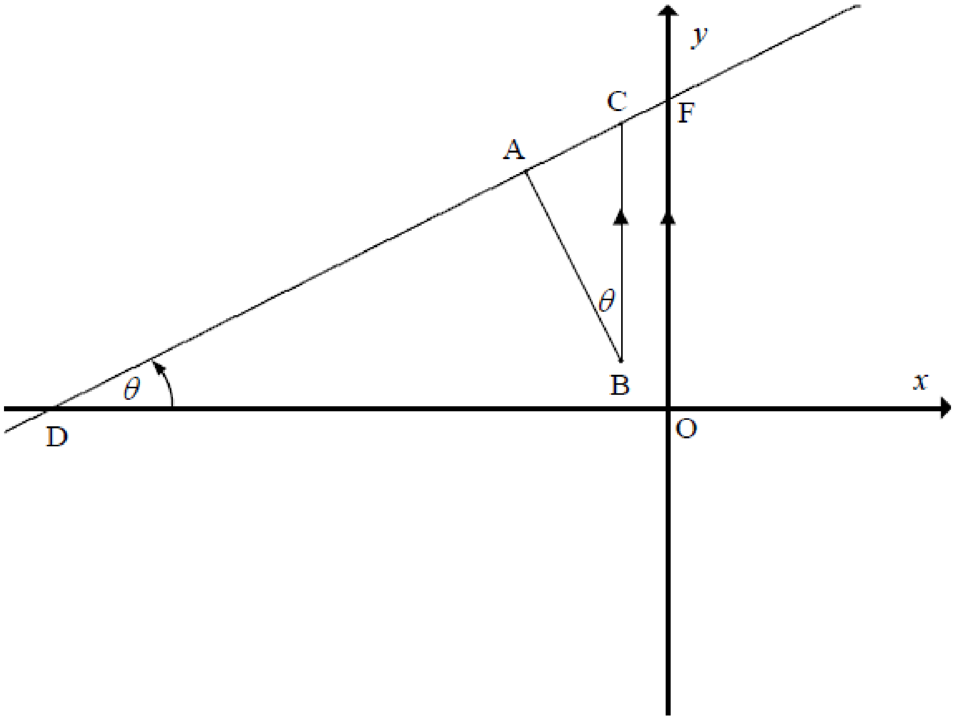
5.5 Determine the equation of the circle that has diameter PR. Give the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

5.6 Show that Q is a point on the circle in QUESTION 5.5. (2)

5.7 Rectangle PQRS undergoes the transformation $(x ; y) \rightarrow (x+k ; y+l)$ where k and l are numbers. What is the minimum value of $k + l$ so that the image of PQRS lies in the first quadrant (that is, $x \geq 0$ and $y \geq 0$)? (3)
[22]

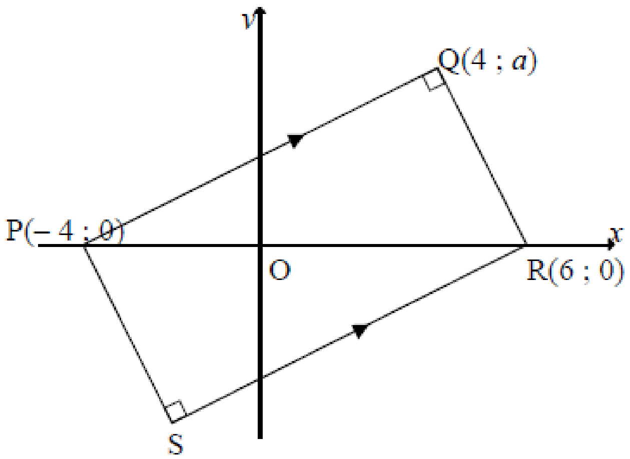
QUESTION 6

The circle with centre B(–1 ; 1) and radius $\sqrt{20}$ is shown. BC is parallel to the *y*-axis and CB = 5. The tangent to the circle at A passes through C.
$A\hat{B}C = A\hat{D}O = \theta$



6.1	Determine the coordinates of C.	(2)
6.2	Calculate the length of CA.	(3)
6.3	Write down the value of tan θ.	(1)
6.4	Show that the gradient of AB is – 2.	(2)
6.5	Determine the coordinates of A.	(6)
6.6	Calculate the ratio of the area of ΔABC to the area of ΔODF. Simplify your answer.	(5)
		[19]

QUESTION 5



5.1	$m_{PQ} \times m_{QR} = -1$ $\left(\frac{a-0}{4+4}\right)\left(\frac{a-0}{4-6}\right) = -1$ $\left(\frac{a}{8}\right)\left(\frac{a}{-2}\right) = -1$ $\frac{a^2}{-16} = -1$ $a^2 = 16$ $a = \pm 4$ $a = 4;$ since $a > 0$	✓ $\frac{a-0}{4+4}$ or $\frac{a}{8}$ ✓ $\frac{a-0}{4-6}$ or $\frac{a}{-2}$ ✓ using gradient of perpendicular lines ✓ $a^2 = 16$ (4)
	OR	
	$PQ^2 + QR^2 = PR^2$ $(8^2 + a^2) + (a^2 + 2^2) = 10^2$ $\therefore\ 2a^2 = 32$ $\therefore\ a^2 = 16$ $\therefore\ a = 4$	✓using Pythagoras ✓$(8^2 + a^2)$ $+ (a^2 + 2^2)$ ✓ 10^2 ✓ $a^2 = 16$ (4)
	OR	
	Let A be the midpoint of diagonal PR. Then $A(\frac{-4+6}{2};\frac{0+0}{2}) = A(1;0)$. AQ = AR (diagonals equal and bisect each other) $AQ^2 = AR^2$ $(1-4)^2 + (0-a)^2 = 5^2$ $9 + a^2 = 25$ $a^2 = 16$ $a = 4$ **Note:** If candidate uses $a = 4$ at the beginning, then zero marks.	✓ (1 ; 0) is centre ✓ AQ = AR ✓ $3^2 + a^2 = 5^2$ ✓ $a^2 = 16$ (4)

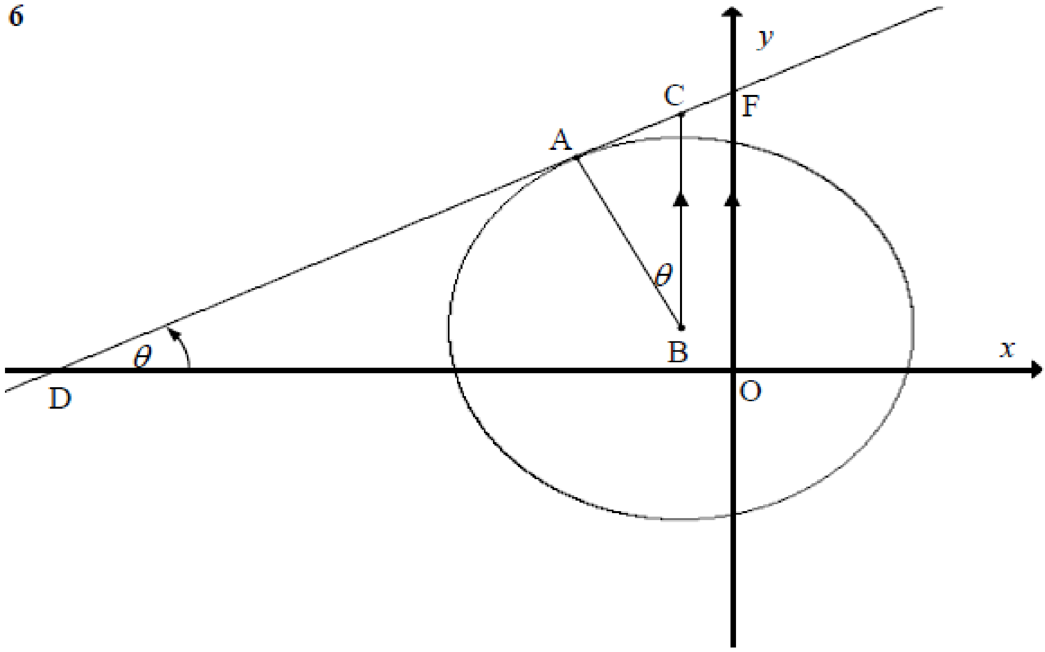
5.2	Equation of line SR: $m_{PQ}=\frac{4-0}{4-(-4)}=\frac{1}{2}$ $m_{SR}=m_{PQ}=\frac{1}{2}$ PQ || SR $y-y_1=m(x-x_1)$ $y-0=\frac{1}{2}(x-6)$ $y=\frac{1}{2}x-3$ **OR**	✓ $m_{PQ}=\frac{1}{2}$ ✓ $m_{SR}=\frac{1}{2}$ ✓ substitution of m and (6 ; 0) ✓ standard form (4)

	$m_{PQ}=\frac{1}{2}$ $m_{PQ}=m_{SR}=\frac{1}{2}$ PQ || SR $y=\frac{1}{2}x+c$ $0=\left(\frac{1}{2}\right)\left(\frac{6}{1}\right)+c$ $-3=c$ $y=\frac{1}{2}x-3$ **OR** S(– 2 ; – 4) (translation) $m_{RS}=\frac{0+4}{6+2}=\frac{1}{2}$ $\therefore y+4=\frac{1}{2}(x+2)$ $\therefore y=\frac{1}{2}x-3$	✓ $m_{PQ}=\frac{1}{2}$ ✓ $m_{SR}=\frac{1}{2}$ ✓ substitution of m and (6 ; 0) ✓ standard form ✓ S(– 2 ; – 4) ✓ $m_{SR}=\frac{1}{2}$ ✓ substitution of m and (– 2 ; – 4) ✓ standard form (4)
5.3	Eq. of RS: $y=\frac{1}{2}x-3$ Eq. of SP: $y-0=-2(x+4)$ $\therefore \frac{1}{2}x-3=-2(x+4)$ $\therefore x=-2$ $y=-4$ Answer only: FULL MARKS **OR**	✓ m = – 2 ✓ eq. of SP ✓ value of x ✓ value of y (4)

	Midpoint PR $= M\left(\frac{-4+6}{2};\frac{0+0}{2}\right) = (1\ ;\ 0)$ Let S$(x;y)$. Then since M(1 ; 0) is this, the midpoint of QS is: $\frac{x_1+x_2}{2}=1$ $\frac{y_1+y_2}{2}=0$ $\therefore \frac{x+4}{2}=1$ and $\frac{y+4}{2}=0$ $x+4=2$ $y+4=0$ $x=-2$ $y=-4$	✓ $\frac{x+4}{2}=1$ ✓ $\frac{y+4}{2}=0$ ✓ value of x ✓ value of y (4)
	OR	
	The translation that sends Q(4 ; 4) to R(6 ; 0) also sends P(– 4 ; 0) to S. (6 ; 0) = (4 + 2 ; 4 – 4) $\therefore$ S = (– 4 + 2 ; 0 – 4) = (– 2 ; – 4)	✓method ✓ 2 or $x + 2$ ✓ – 4 or $y – 4$ ✓ answer (4)
	OR	
	The translation that sends Q(4 ; 4) to P(– 4 ; 0) also sends R(6 ; 0) to S. (– 4 ; 0) = (4 – 8 ; 4 – 4) $\therefore$ S = (6 – 8 ; 0 – 4) = (– 2 ; – 4)	✓method ✓ – 8 or $x – 8$ ✓ – 4 or $y – 4$ ✓ answer (4)
	OR	
	$m_{PQ}=m_{SR}$ $\frac{1}{2}=\frac{y}{x-6}$ $2y=x-6$ (1) $m_{PS}=m_{SR}$ $\frac{y}{x+4}=\frac{4}{-2}$ $-2y=4x+16$ (2) $(1)+(2): 0=5x+10$ $x=-2$ $Substitute: 2y=-2-6=-8$ $y=-4$	✓equations using the gradient ✓adding the equations ✓ value of x ✓ value of y (4)
5.4	$PR=6-(-4)$ $=10$ **OR** [Answer only: FULL MARKS] $PR^2=(6+4)^2+(0-0)^2$ $PR=10$	✓ $6-(-4)$ ✓ 10 (2) ✓ substitution in correct formula ✓ 10 (2)

5.5	midpoint PR= $\left(\frac{6+(-4)}{2} ; \frac{0+0}{2}\right) = (1; 0)$ radius of circle = $\frac{1}{2}$PR = 5 units $\therefore (x-1)^2 + (y-0)^2 = 5^2$ $(x-1)^2 + y^2 = 25$ Answer only: FULL MARKS	✓ midpoint ✓ radius ✓ eq. of circle in correct form (3)
5.6	$(x-1)^2 + y^2 = 25$ substitute Q(4 ; 4): LHS $=(4-1)^2 + 4^2$ $= 25$ $=$ RHS $\therefore$ Q is a point on the circle **Note**: If substitute point into equation resulting in 25 = 25: 1 mark No conclusion: 1 mark **OR** Distance from centre (1 ; 0) to Q(4 ; 4) $\therefore$ Q is a point on circle, r = 5 **OR** PR is the diameter of circle PQR therefore Q lies on circle ($P\hat{Q}R = 90°$) **OR** $(4-1)^2 + y^2 = 25$ $y^2 = 16$ $\therefore y = 4$ $\therefore$ Q is a point on the circle **OR** $(x-1)^2 + 4^2 = 25$ $(x-1)^2 = 9$ $x - 1 = 3$ $x = 4$ $\therefore$ Q is a point on the circle	✓ substitute Q(4;4) ✓ LHS = RHS (2) ✓ = 5 ✓ conclusion (2) ✓ diameter ✓ $P\hat{Q}R = 90°$) (2) ✓ substitute $x = 4$ ✓ conclusion (2) ✓ substitute $y = 4$ ✓ conclusion (2)
5.7	P needs to shift at least 4 units to the right and S needs to shift at least 4 units up for the image of PQRS in first quadrant. $\therefore$ minimum value of k is 4 and minimum value of l is 4 $\therefore$ minimum value of $k + l$ is 8 Answer only: FULL MARKS **Note**: No CA mark applies in 5.7 if k and l are not minimums.	✓ $k = 4$ ✓ $l = 4$ ✓ $k + l = 8$ (3) **[22]**

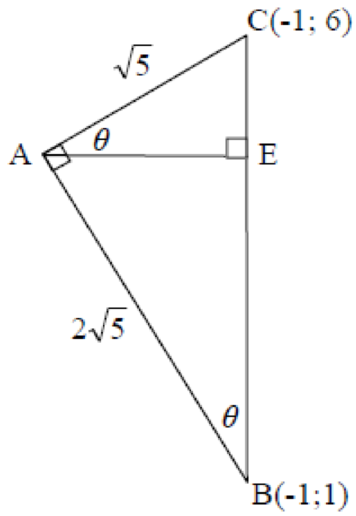
QUESTION 6



6.1	$x_C = x_B = -1$ $y_C = y_B + 5 = 6$ $\therefore C(-1\,;\,6)$	✓ value of x ✓ value of y (2)
6.2	$BA \perp CA$ (tangent ⊥ radius) $\therefore\ CA^2 = BC^2 - AB^2$ (Pythagoras) $= (5)^2 - (\sqrt{20})^2 = 5$ $\therefore\ CA = \sqrt{5}$ or 2,24 units	✓ BA⊥CA or $B\hat{A}C = 90°$ ✓ substitution into Pythagoras ✓ answer (3)
6.3	$\tan\theta = \frac{\sqrt{5}}{\sqrt{20}} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$	✓ tan ratio (in any form) (1)
6.4	$m_{DC} \times m_{AB} = -1$ $m_{DC} = \tan\theta = \frac{1}{2}$ $m_{DC} = \frac{1}{2}$ $m_{AB} = -2$	✓ $m_{DC} \times m_{AB} = -1$ ✓ $m_{DC} = \tan\theta = \frac{1}{2}$ (2)

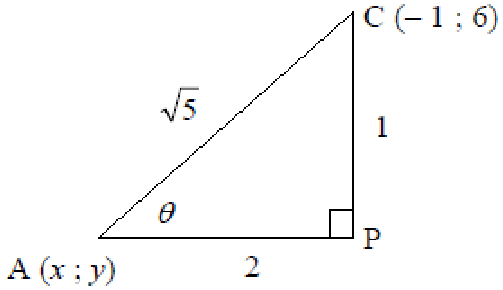
6.5	Eq. of DC: $y - 6 = \frac{1}{2}(x+1)$ $y = \frac{1}{2}x + \frac{13}{2}$ Eq. of AB: $y - 1 = -2(x+1)$ $y = -2x - 1$ $-2x - 1 = \frac{1}{2}x + \frac{13}{2}$ $-\frac{5}{2}x = \frac{15}{2}$ $x = -3$ $y = -2(-3) - 1$ $y = 5$ $\therefore$ A $(-3\ ;\ 5)$ Answer only: $(-3\ ;\ 5)$: 1 mark **OR** Eq. of DC: $y - 6 = \frac{1}{2}(x+1)$ $y = \frac{1}{2}x + \frac{13}{2}$ Eq. of AB: $y - 1 = -2(x+1)$ $y = -2x - 1$ <u>At A:</u> $x - 2(-2x - 1) + 13 = 0$ $x + 4x + 2 + 13 = 0$ $5x = -15$ $x = -3$ and $y = -2(-3) - 1 = 5$ $\therefore$ A$(-3\ ;\ 5)$ **OR**	✓ DC: subst m and $(-1\ ;\ 6)$ ✓ eq. of DC ✓ eq. of AB ✓ equating equations ✓ value of x ✓ value of y (6) ✓ DC: subst m and $(-1\ ;\ 6)$ ✓ eq. of DC ✓subt m and $(-1;1)$ ✓ eq. of AB ✓ value of x ✓ value of y (6)

	Eq. of DC: $y - 6 = \frac{1}{2}(x+1)$ $y = \frac{1}{2}x + \frac{13}{2}$ Eq. of circle: $(x+1)^2 + (y-1)^2 = 20$ <u>At A:</u> $(x+1)^2 + (\frac{1}{2}x + \frac{13}{2} - 1)^2 = 20$ $(x+1)^2 + (\frac{1}{2}x + \frac{11}{2})^2 = 20$ $1\frac{1}{4}x^2 + \frac{15}{2}x + 11\frac{1}{4} = 0$ $\therefore x^2 + 6x + 9 = 0$ $(x+3)^2 = 0$ $\therefore\ x = -3$ and $y = \frac{1}{2}(-3) + \frac{13}{2} = 5$ $\therefore$ A$(-3\ ;\ 5)$	✓ DC: subst m and $(-1\ ;\ 6)$ ✓ eq. of DC ✓ substitution ✓ $x^2 + 6x + 9 = 0$ ✓ value of x ✓ value of y (6)

	OR Draw AE ⊥ BC $\cos\theta = \frac{2\sqrt{5}}{5} = \frac{AE}{\sqrt{5}} = \frac{BE}{2\sqrt{5}}$ $\therefore AE = \frac{2\times5}{5} = 2$ $BE = \frac{4\times5}{5} = 4$ $x_A = -1 - AE = -1 - 2 = -3$ $\therefore y_A = 1 + BE = 4 + 1 = 5$ $\therefore$ A(− 3 ; 5)  	✓ $\frac{2\sqrt{5}}{5} = \frac{AE}{\sqrt{5}}$ ✓AE = 2 ✓ $\frac{2\sqrt{5}}{5} = \frac{BE}{2\sqrt{5}}$ ✓ BE = 1 ✓ − 3 ✓ 5 (6)
	OR $(x+1)^2 + (y-1)^2 = 20 \quad (1)$ $y = -2x - 1 \quad (2)$ $(x+1)^2 + (-2x-2)^2 = 20$ $x^2 + 2x + 1 + 4x^2 + 8x + 4 - 20 = 0$ $5x^2 + 10x - 15 = 0$ $x^2 + 10x - 15 = 0$ $(x+3)(x-1) = 0$ $x = -3$ or $x \neq 1$ subst (1) in (2) $\therefore y = 5$	✓ subst m and (− 1;1) ✓eq of AB ✓eq of circle ✓ substation ✓value of x ✓ value of y (6)

OR

Equation AC : $y = \frac{1}{2}x + 6\frac{1}{2}$



$\tan\theta = \frac{1}{2}$

$\theta = 26,57°$

$AP = \sqrt{5}\cos 26,57°$

$AP = 2$

$CP = \sqrt{5}\sin 26,57°$

$CP = 1$

$\therefore x = -1 - 2 = -3$

$y = 6 - 1 = 5$

$\therefore A(-3 ; 5)$

✓ $\theta = 26,57°$
✓ $AP = \sqrt{5}\cos 26,57°$
✓ $AP = 2$
✓ $CP = 1$
✓ value of x
✓ value of y
(6)

6.6

Area $\Delta ABC = \frac{1}{2}(\sqrt{5})(\sqrt{20}) = 5$

Eqn. of DC is $y = \frac{1}{2}x + \frac{13}{2}$

Therefore $OF = \frac{13}{2}$ and $OD = 13$.

Area $\Delta ODF = \frac{1}{2}\left(\frac{13}{2}\right)(13) = \frac{169}{4}$

Area ΔABC: Area $\Delta ODF = 5 : \frac{169}{4} = 20 : 169$

✓ $\frac{1}{2}(\sqrt{5})(\sqrt{20})$
✓ $OF = \frac{13}{2}$
✓ $OD = 13$
✓ $\frac{1}{2}\left(\frac{13}{2}\right)(13)$
✓ answer
(5)

OR

$DF^2 = 13^2 + (\frac{13}{2})^2 = \frac{845}{4}$

$DF = \frac{13.\sqrt{5}}{2}$

$$\frac{\Delta ABC}{\Delta ODF} = \frac{\frac{1}{2}(5)(\sqrt{20})\sin\theta}{\frac{1}{2}(13)(\frac{13.\sqrt{5}}{2})\sin\theta}$$

$$= \frac{20}{169}$$

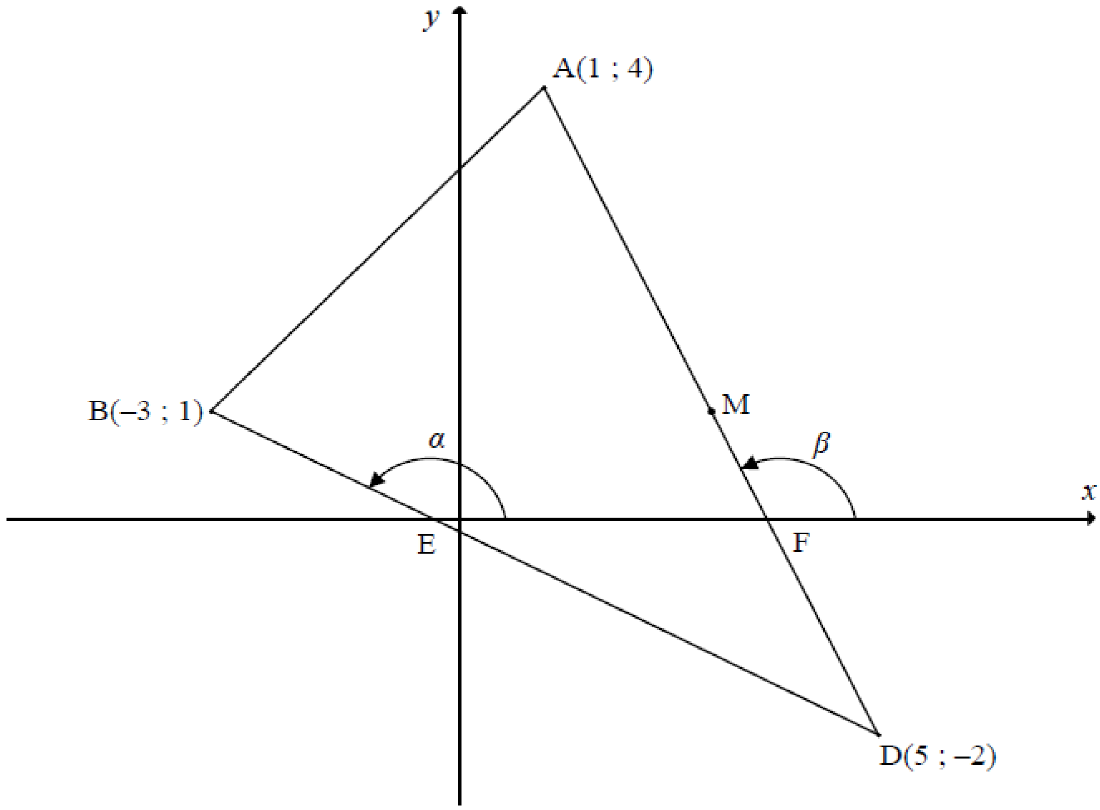
✓ $= 13^2$
✓ $+(\frac{13}{2})^2 = \frac{845}{4}$
✓ $DF = \frac{13.\sqrt{5}}{2}$
✓ $\frac{1}{2}(5)(\sqrt{20})\sin\theta$
✓ $\frac{1}{2}(13)(\frac{13.\sqrt{5}}{2})\sin\theta$
✓ answer (5)

	OR ΔODF is an enlargement of ΔABC $\therefore$ area ΔABC : area ΔODF = $AB^2 : OD^2 = 20 : OD^2$ Equation of DC is $y = \frac{1}{2}x + \frac{13}{2}$ $x_D = -13$ $OD = 13$ $\therefore$ area ΔABC : area ΔODF = $AB^2 : OD^2 = 20 : 169$	✓ enlargement ✓✓ $AB^2 : OD^2 = 20 : OD^2$ ✓ – 13 ✓answer (5) **[19]**

QUESTION 5

In the figure below, A(1 ; 4), B(–3 ; 1) and D(5 ; –2) are the coordinates of the vertices of ΔABD.

- BD and AD intersect the x-axis at E and F respectively.
- The angle of inclination of BD with the x-axis at E is α.
- The angle of inclination of AD with the x-axis at F is β.

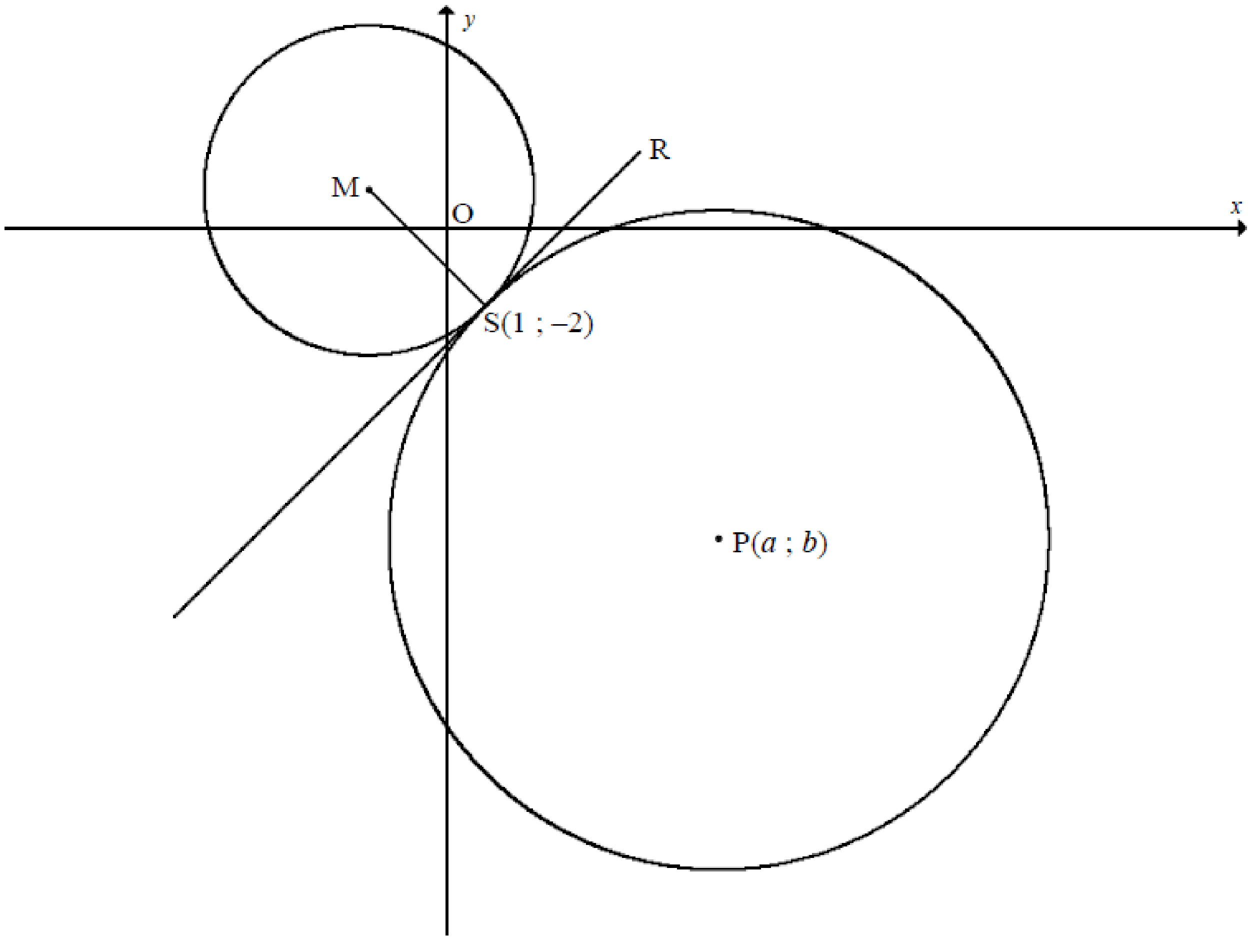


5.1	Calculate the gradient of AD.	(2)
5.2	Determine the length of the line segment AD. (Leave your answer in surd form, if necessary.)	(2)
5.3	Determine the coordinates of M, the midpoint of AD.	(2)
5.4	C is a point such that line BC is parallel to AD. Determine the equation of line BC in the form $ax + by + c = 0$.	(3)
5.5	5.5.1 Calculate the size of β.	(2)
	5.5.2 Calculate ALL the angles of ΔDEF.	(5)
5.6	Determine the equation of a circle, with centre M, which passes through the points A and D. Give your answer in the form: $(x - a)^2 + (y - b)^2 = r^2$.	(2)
5.7	Does the point B lie inside, outside or on the circle in QUESTION 5.6? Show ALL calculations to justify your answer.	(2)

[20]

QUESTION 6

In the figure below, a circle with centre M is drawn. The equation of the circle is $(x+2)^2 + (y-1)^2 = r^2$.
S(1 ; –2) is a point on the circle.
SR is a tangent to the circle.



6.1 Write down the coordinates of M and the radius of the circle centre M. (4)

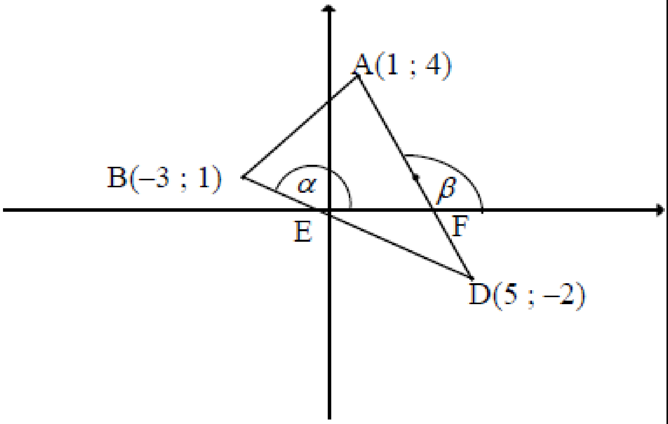
6.2 Determine the equation of the tangent RS in the form $y = mx + c$. (4)

6.3 The circles having centres P and M touch externally at point S. SR is a tangent to both these circles. If MS : MP = 1 : 3, determine the coordinates $(a ; b)$ of point P. (8)

[16]

QUESTION 5

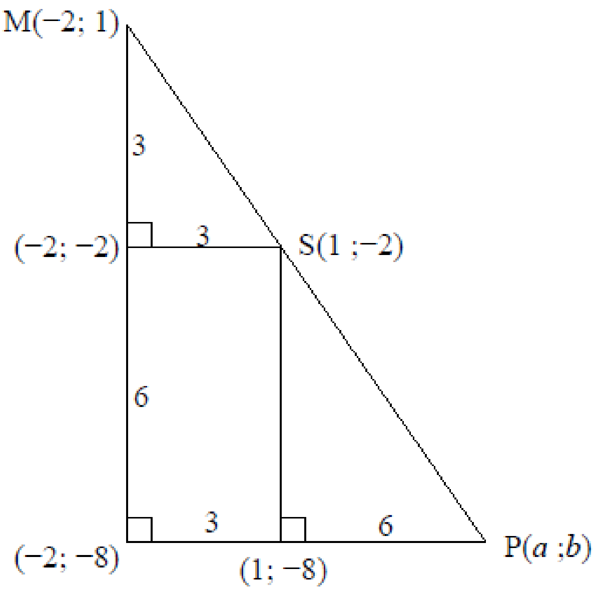
5.1	$m_{AD} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{-2-4}{5-1}$ $= -\dfrac{6}{4} \quad = -\dfrac{3}{2}$	✓ for substitution ✓ for answer (2)
5.2	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5-1)^2 + (-2-4)^2}$ $= \sqrt{16+36}$ $= \sqrt{52}$	✓ for substitution ✓ $\sqrt{52}$ (2)
5.3	$M = \left(\dfrac{x_1 + x_2}{2} ; \dfrac{y_1 + y_2}{2}\right)$ $M = \left(\dfrac{1+5}{2} ; \dfrac{4-2}{2}\right)$ $M = (3\ ;\ 1)$	✓ x-value ✓ y-value (2)
5.4	$m_{BC} = m_{AD}$ Lines are parallel $= -\dfrac{3}{2}$ $y - y_1 = m\,(x - x_1)$ $y - 1 = -\dfrac{3}{2}(x + 3)$ $2y - 2 = -3x - 9$ $3x + 2y + 7 = 0$ **OR** $y = -\dfrac{3}{2}x + c$ $1 = -\dfrac{3}{2}(-3) + c$ $c = -\dfrac{7}{2}$ $y = -\dfrac{3}{2}x - \dfrac{7}{2}$ $3x + 2y + 7 = 0$	✓ value m_{BC} ✓ subst $(-3\ ;\ 1)$ ✓ equation (3) ✓ value m_{BC} ✓ subst $(-3\ ;\ 1)$ ✓ equation (3)

5.5.1	$m_{AD} = -\frac{3}{2}$ $\tan\beta = -\frac{3}{2}$ $\beta = 180° - 56{,}31°$ $\beta = 123{,}69$ 	✓ $\tan\beta = m_{AD}$ ✓ 123,69° (2)
5.5.2	$m_{BD} = \frac{-2-1}{5-(-3)} = \frac{-3}{8}$ $\tan\alpha = -\frac{3}{8}$ $\alpha = 180° - 20{,}56°$ $\alpha = 159{,}44°$ $F\hat{E}D = 180° - 159{,}44° = 20{,}56°$ $E\hat{F}D = 123{,}69°$ $F\hat{D}E = 180° - (20{,}56° + 123{,}69°) = 35{,}75°$	✓ $m_{BD} = \frac{-3}{8}$ ✓159,44° ✓20,56° ✓123,69° ✓35,75° (5)
5.6	Co-ordinates of centre M (3 ; 1) Radius of circle: $\frac{1}{2}$ of AD = $\frac{1}{2}(2\sqrt{13}) = \sqrt{13} = \frac{1}{2}\sqrt{52}$ Equation of the circle is: $(x-3)^2 + (y-1)^2 = 13$ **OR** $r^2 = (3-1)^2 + (1-4)^2 = 13$ Equation of the circle is: $(x-3)^2 + (y-1)^2 = 13$	✓ value of radius ✓ substitution into equation of circle centre form (2) ✓ value of r^2 ✓ substitution into equation of circle centre form (2)
5.7	M(3 ; 1) B(– 3 ; 1) $MB = \sqrt{(3+3)^2 + (1-1)^2}$ MB = 6 Point B lies outside the circle because MB > radius **OR** M(3 ; 1) B(– 3 ; 1) MB = 3 + 3 = 6 Radius of the circle = $\sqrt{13} < 6$ Point B lies outside the circle because MB > radius	✓substitution ✓outside (2) ✓substitution ✓outside (2) **[20]**

QUESTION 6

6.1	Coordinates of centre M $(-2\ ;\ 1)$ $(1+2)^2+(-2-1)^2=18=r^2$ Radius = $\sqrt{18}$ or $3\sqrt{2}$	✓✓ coordinates of centre ✓ calculation ✓ value (4)
6.2	$m_{MS}=\frac{-3}{3}=-1$ $m_{MS} x m_{RS}=-1$ OR tangent $\perp$ radius $m_{RS}=1$ $y-y_1=m(x-x_1)$ $y+2=1(x-1)$ $y=x-3$ **OR** $m_{MS}=\frac{-3}{3}=-1$ $m_{MS} x m_{RS}=-1$ $m_{RS}=1$ $y=x+c$ $-2=1+c$ $c=-3$ $y=x-3$	✓ gradient MS ✓ gradient RS ✓ subst $(1\ ;-2)$ ✓ equation (4) ✓ gradient MS ✓ gradient RS ✓ subst $(1\ ;-2)$ ✓ equation (4)
6.3	$\frac{MS}{MP}=\frac{1}{3}$ $\therefore MP=3MS$ $MP^2=9MS^2$ $(a+2)^2+(b-1)^2=9(3^2+3^2)=162$ (1) $MS \perp SR$ *and* $PS \perp SR$ $\therefore m_{PS}=m_{MS}$ $\frac{b+2}{a-1}=\frac{3}{-3}=-1$ $b+2=-a+1$ $b=-a-1$ (2) Subst (2) into(1)	✓ MP = 3MS ✓ equation ✓ equal gradients ✓ gradient ✓ $b=-a-1$

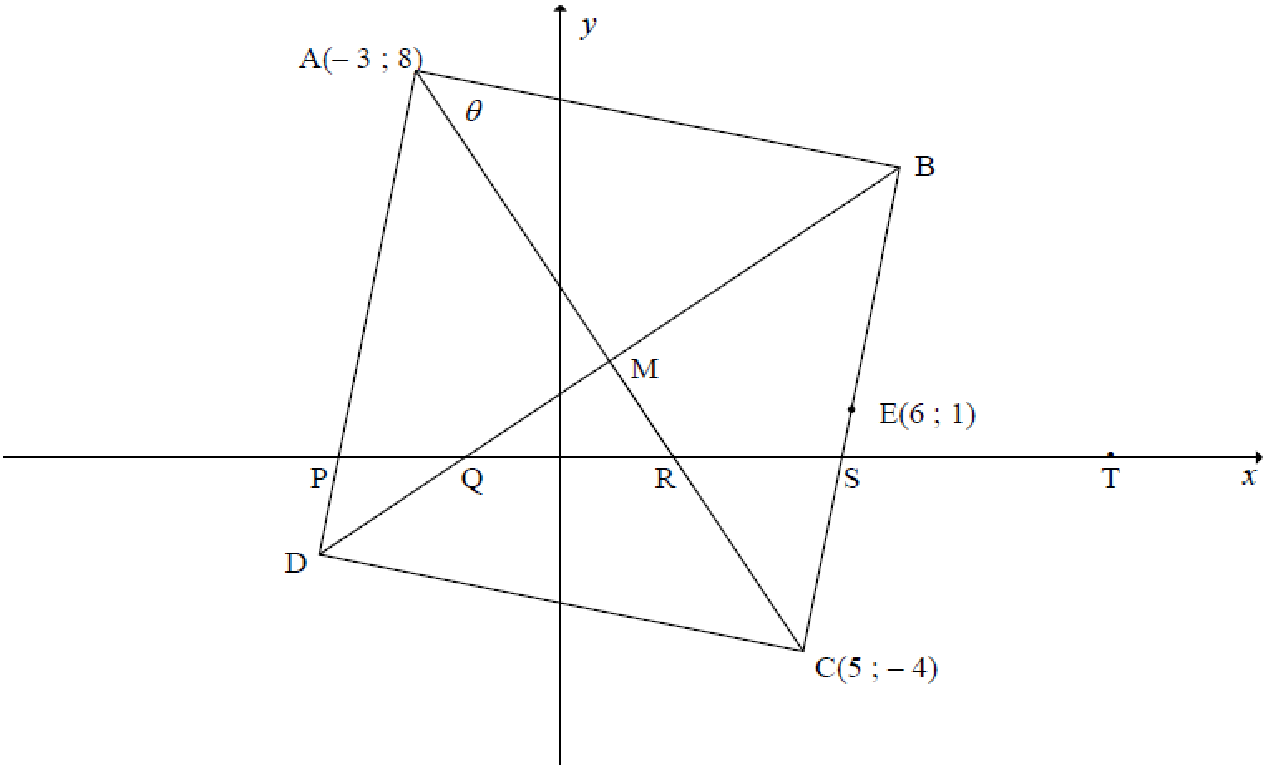
	$(a+2)^2+(-a-1-1)^2=162$ $(a+2)^2+(a+2)^2=162$ $2(a+2)^2=162$ $(a+2)^2=81$ $a+2=9 \text{ or } -9$ $a=7 \text{ or } -11$ $b=-a-1=-8$ $P(7\,;-8)$	✓substitution ✓ $a = 7$ ✓ $b = -8$ (8)
	OR $\frac{MS}{MP}=\frac{1}{3}$ $\therefore MP=3MS$ $MP^2=9MS^2$ $(a+2)^2+(b-1)^2=9(3^2+3^2)=162$ (1) $MS\perp SR$ and $PS\perp SR$ $\therefore m_{PS}=m_{MS}$ $\frac{b+2}{a-1}=\frac{3}{-3}=-1$ $b+2=-a+1$ $b=-a-1$ (2) Subst (2) into(1) $a^2+4a+4+a^2+4a+4=162$ $2a^2+8a-154=0$ $a^2+4a-77=0$ $(a+11)(a-7)=0$ $a=7 \text{ or } -11$ But $a>0$ $\therefore a=7$ $b=-a-1=-8$ $P(7\,;-8)$	✓ MP = 3MS ✓ equation ✓ equal gradients ✓ gradient ✓ $b = -a - 1$ ✓substitution ✓ $a = 7$ ✓ $b = -8$ (8)
	OR	

	P(a ; b) MSP is a straight line (MS ⊥ SR) $m_{PM} = -1$ $\frac{b-1}{a+2} = -1$ $b - 1 = -a - 2$ $b = -a - 1$.......(1) $PS = 2MS = 2\sqrt{9+9} = 2\sqrt{18}$ $PS^2 = 4(18) = 72$ $(a-1)^2 + (b+2)^2 = 72$.......(2) $(a-1)^2 + (-a-1+2)^2 = 72$ $2a^2 - 4a - 70 = 0$ $a^2 - 2a - 35 = 0$ $(a-7)(a+5) = 0$ $a = 7$ or $a \neq -5$ $b = -7 - 1 = -8$ $P(7 ; -8)$ **OR** $2(a-1)^2 = 72$ $(a-1)^2 = 36$ $a - 1 = 6$ or -6 $a = 7$ or -5 $a = 7$ $b = -8$ $P(7 ; -8)$	✓ MSP a straight line ✓ $m_{PM} = -1$ ✓ $\frac{b-1}{a+2}$ ✓ equation 1 ✓ equation 2 ✓ substitution of equation 1 into equation 2 ✓✓ coordinates (8)
	OR 	✓✓ diagram ✓✓ (−2; −8) ✓ (−2; −2) ✓ (1; −8) ✓✓ P(7 ; −8) (8) ✓✓ division of line segment into

	P(a ; b)	given ratio
	$\frac{x_S - x_M}{x_P - x_M} = \frac{y_S - y_M}{y_P - y_M} = \frac{1}{3}$	✓✓substitution ✓ equation
	$\frac{-3}{b-1} = \frac{3}{a+2} = \frac{1}{3}$	
	$-9 = b - 1$	✓equation
	$b = -8$	✓coordinates
	$9 = a + 2$	(8)
	$a = 7$	**[16]**
	$P(7\,;-8)$	

QUESTION 5

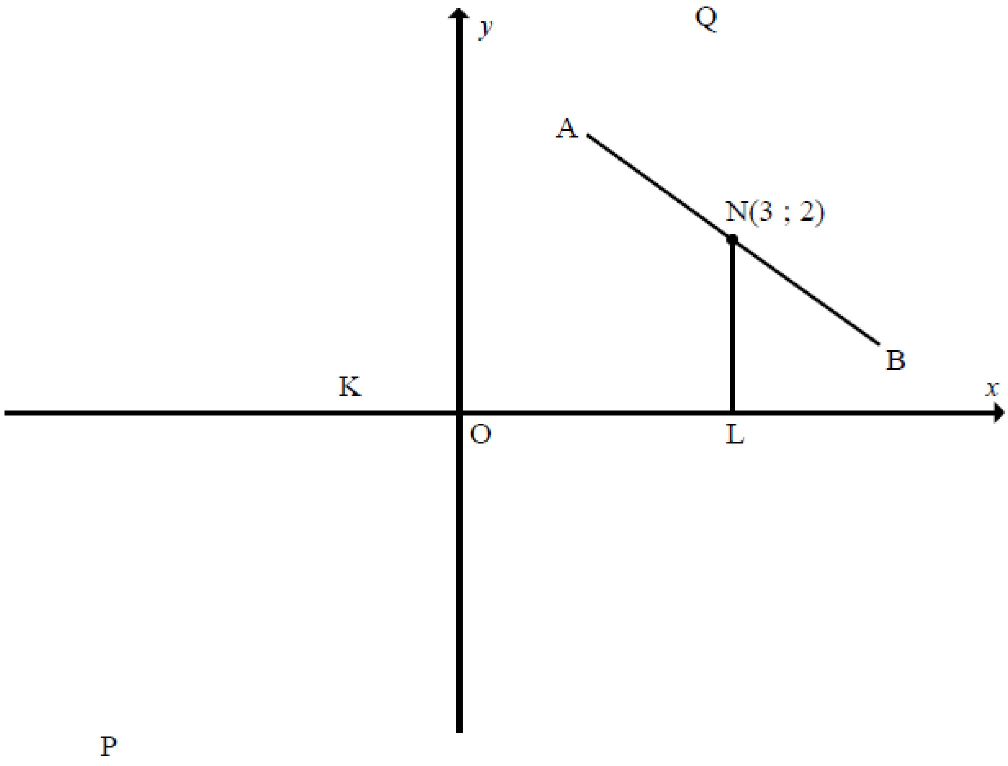
ABCD is a rhombus with A(– 3 ; 8) and C(5 ; – 4). The diagonals of ABCD bisect each other at M. The point E(6 ; 1) lies on BC.



5.1	Calculate the coordinates of M.	(2)
5.2	Calculate the gradient of BC.	(2)
5.3	Determine the equation of the line AD in the form $y = mx + c$.	(3)
5.4	Determine the size of θ, that is $B\hat{A}C$. Show ALL calculations.	(6)
		[13]

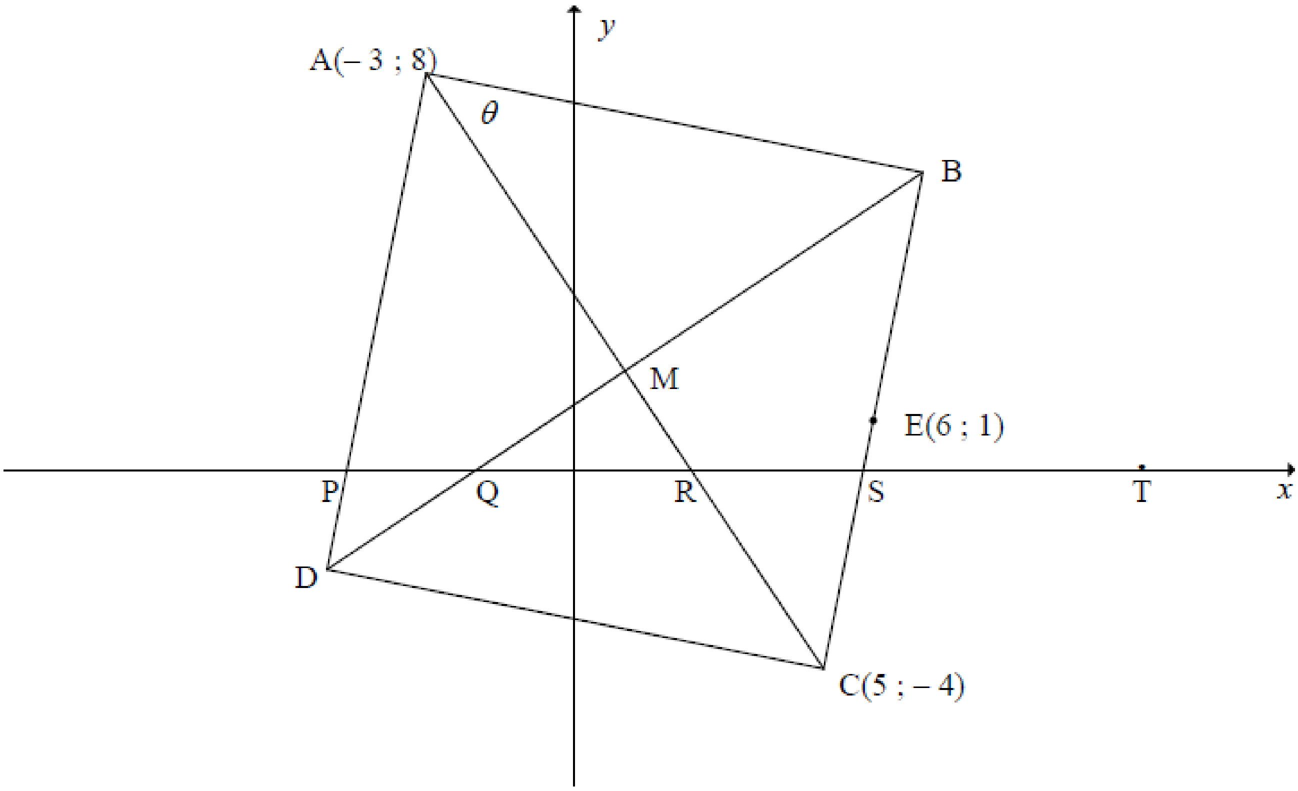
QUESTION 6

A circle centred at N(3 ; 2) touches the x-axis at point L. The line PQ, defined by the equation $y = \frac{4}{3}x + \frac{4}{3}$, is a tangent to the same circle at point A.



6.1	Why is NL perpendicular to OL?	(1)
6.2	Determine the coordinates of L.	(1)
6.3	Determine the equation of the circle with centre N in the form $(x-a)^2 + (y-b)^2 = r^2$	(3)
6.4	Calculate the length of KL.	(3)
6.5	Determine the equation of the diameter AB in the form $y = mx + c$.	(4)
6.6	Show that the coordinates of A are $\left(\frac{7}{5} ; \frac{16}{5}\right)$.	(3)
6.7	Calculate the length of KA.	(3)
6.8	Why is KLNA a kite?	(2)
6.9	Show that $\hat{ABK} = 45°$.	(3)
6.10	If the given circle is reflected about the x-axis, give the coordinates of the centre of the new circle.	(1)
		[24]

QUESTION 5



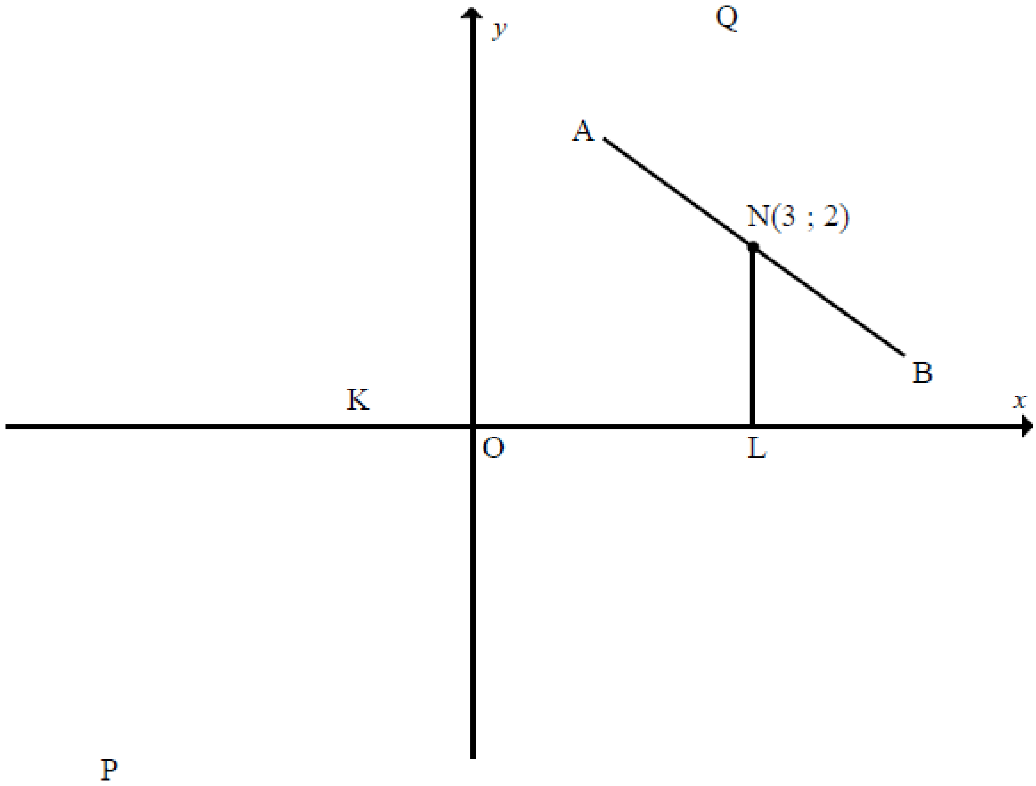
5.1	Diagonals bisect each other at M: $x_M = \frac{-3+5}{2} = 1$; $y_M = \frac{8+(-4)}{2} = 2$ M(1 ; 2)	✓ $x_M = 1$ ✓ $y_M = 2$ (2)
5.2	$m_{BC} = \frac{1+4}{6-5}$ $m_{BC} = 5$ **OR** $m_{BC} = \frac{-4-1}{5-6}$ $m_{BC} = 5$	✓ substitution into gradient formula ✓ 5 (2) ✓ $m_{BC} = \frac{-4-1}{5-6}$ ✓ 5 (2)
5.3	$y - y_1 = m(x - x_1)$ $y - 8 = m(x+3)$ $m_{AD} = m_{BC} = 5$ Lines parallel $y - 8 = 5(x+3)$ $y = 5x + 23$ **OR**	✓ substitute (–3 ; 8) ✓ gradients equal ✓ equation (3)

	$m_{AD} = m_{BC}$ $m_{AD} = 5$ Lines parallel $y = 5x + c$ $8 = 5(-3) + c$ $c = 23$ $y = 5x + 23$	✓ gradients equal ✓ substitute (–3 ; 8) ✓ equation (3)
5.4	ABCD is a rhombus, therefore AB = BC $\theta = B\hat{C}A = A\hat{R}S - R\hat{S}C$ $= A\hat{R}S - B\hat{S}T$ $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5}$ $\tan A\hat{R}S = -\frac{3}{2}$ $A\hat{R}S = 180° - 56{,}3099...$ $A\hat{R}S = 123{,}69°$ $\tan B\hat{S}T = m_{BC} = 5$ $B\hat{S}T = 78{,}69°$ $\theta = B\hat{C}A = 123{,}69° - 78{,}69°$ $\theta = 45°$ **OR** $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$ $A\hat{R}S = 123{,}69°$ $\tan A\hat{P}R = m_{AD} = 5$ $A\hat{P}R = 78{,}69°$ $P\hat{A}R = A\hat{R}S - A\hat{P}R$ Exterior angle of a triangle $= 123{,}69° - 78{,}69°$ $= 45°$ $\theta = P\hat{A}R$ Diagonals of the rhombus bisect opposite angles $= 45°$	✓ $\theta = B\hat{C}A$ ✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ 123,69° ✓ $\tan B\hat{S}T = m_{BC} = 5$ ✓ 78,69° ✓ $\theta = 45°$ (6) ✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ 123,69° ✓ $\tan A\hat{P}R = m_{AD} = 5$ ✓ 78,69° ✓ $P\hat{A}R = 45°$ ✓ $\theta = 45°$ (6)

	OR $\tan A\hat{R}S = m_{AC} = \dfrac{8+4}{-3-5} = -\dfrac{3}{2}$ $A\hat{R}S = 123{,}69°$ $\tan A\hat{P}R = 5$ $A\hat{P}R = 78{,}69°$ $\theta = P\hat{A}R$ Diagonals of the rhombus bisect opposite angles $\theta = A\hat{R}S - A\hat{P}R$ $\theta = 123{,}69° - 78{,}69°$ Exterior angle of a triangle $\theta = 45°$	✓ $\tan A\hat{R}S = -\dfrac{3}{2}$ ✓ $123{,}69°$ ✓ $\tan A\hat{P}R = m_{AD} = 5$ ✓ $78{,}69°$ ✓ $\theta = P\hat{A}R$ ✓ $\theta = 45°$ (6)
	OR $\tan A\hat{R}S = m_{AC} = \dfrac{8+4}{-3-5} = -\dfrac{3}{2}$ $A\hat{R}S = 123{,}69°$ $\tan B\hat{S}T = 5$ $B\hat{S}T = 78{,}69°$ $\theta = R\hat{C}S$ BA=BC $R\hat{C}S + B\hat{S}T = R\hat{C}S + R\hat{S}C$ $= A\hat{R}S$ $\theta = A\hat{R}S - B\hat{S}T$ $= 123{,}69° - 78{,}69°$ $= 45°$	✓ $\tan A\hat{R}S = -\dfrac{3}{2}$ ✓ $123{,}69°$ ✓ $\tan B\hat{S}T = 5$ ✓ $78{,}69°$ ✓ $\theta = R\hat{C}S$ ✓ $\theta = 45°$ (6)
	OR ABCD is a rhombus, therefore AB = BC $\therefore A\hat{C}B = B\hat{A}C$ $\tan\theta = \tan A\hat{C}B$ $= \tan(A\hat{R}S - B\hat{S}T)$ $= \dfrac{\tan A\hat{R}S - \tan B\hat{S}T}{1 + \tan A\hat{R}S . \tan B\hat{S}T}$ $= \dfrac{\left(\frac{12}{-8}\right) - \left(\frac{-5}{-1}\right)}{1 + \left(\frac{12}{8}\right)\left(\frac{5}{1}\right)}$ $= 1$ $\theta = 45°$	✓ $A\hat{C}B = B\hat{A}C$ ✓ $\tan\theta = \tan A\hat{C}B$ ✓ formula ✓ substitution ✓ $\tan\theta = 1$ ✓ $\theta = 45°$ (6)

	OR	
	From 5.1, M has coordinates (1 ; 2)	
	Join ME	
	$m_{ME} = \frac{2-1}{1-6} = -\frac{1}{5}$	✓gradient of ME
	From 5.2,	
	$m_{BC} = 5$	✓gradient of BC
	$\therefore m_{ME} \times .m_{BC} = -1$	
	$\therefore M\hat{E}C = 90°$	✓ $M\hat{E}C = 90°$
	$ME = \sqrt{(1-6)^2 + (2-1)^2} = \sqrt{26}$	✓ $ME = \sqrt{26}$
	$EC = \sqrt{(5-6)^2 + (-4-1)^2} = \sqrt{26}$	✓ $EC = \sqrt{26}$
	$\therefore$MEC is a right-angled triangle.	
	$E\hat{C}M = 45°$	✓ $E\hat{C}M = 45°$
	ABCD is a rhombus, therefore	
	AB = BC	(6)
	$\therefore \theta = B\hat{C}M = 45°$	
	OR	
	$AM = \sqrt{(-3-1)^2 + (8-2)^2} = 2\sqrt{13}$	✓ $AM = 2\sqrt{13}$
	Now to calculate the coordinates of B:	
	$m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$	
	$m_{BD} \times m_{AC} = -1$ diagonals bisect at right angles	
	$m_{BD} = \frac{2}{3}$	
	Equation of BD is $y = \frac{2}{3}x + \frac{4}{3}$	✓ $y = \frac{2}{3}x + \frac{4}{3}$
	Equation of BC is $y = 5x - 29$	✓ $y = 5x - 29$
	BD and BC intersect at B.	✓ B(7 ; 6)
	Solve equations simultaneously to get B(7 ; 6).	
	$BM = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{52} = 2\sqrt{13}$	✓ $BM = 2\sqrt{13}$
	$\therefore BM = AM$	
	Since $A\hat{M}B = 90°$	
	$\tan\theta = \frac{BM}{AM}$	✓ 45°
	$\therefore \tan\theta = 1$	(6)
	$\theta = 45°$	**[13]**

QUESTION 6



6.1	The radius (NL) of a circle is perpendicular to the tangent (OL) at the point of contact.	✓ radius ⊥ tangent (1)
6.2	L(3 ; 0)	✓ (3 ; 0) (1)
6.3	Centre N (3 ; 2) and r = NL = 2 Equation of the circle N: $(x-a)^2+(y-b)^2=r^2$ $(x-3)^2+(y-2)^2=4$	✓$r = 2$ ✓ $(x-3)^2+(y-2)^2$ ✓ 4 (3)
6.4	Coordinates of K. K is the x-intercept of the tangent. $y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ KL = 3 – (–1) **OR** KL = 3 + 1 KL = 4	✓ substitute $y = 0$ into equation of tangent ✓ $x = -1$ ✓ $KL = 4$ (3)

	OR $y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ $KL=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $KL=\sqrt{(3+1)^2+(0-0)^2}$ $KL=\sqrt{16}$ $KL=4$	✓ substitute $y=0$ into equation of tangent ✓ $x=-1$ ✓ $KL=4$ (3)
	OR For AK, $m=\frac{4}{3}$, $c=\frac{4}{3}$ $\frac{\frac{4}{3}}{OK}=\tan A\hat{K}O=\frac{4}{3}$ $OK=1$ $\therefore KL=4$	✓ $\frac{\frac{4}{3}}{OK}=\frac{4}{3}$ ✓ OK = 1 ✓ $KL=4$ (3)
	OR $y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ $KN^2=NL^2+KL^2$ Theorem of Pythagoras $(-1-3)^2+(0-2)^2=4+KL^2$ $20=4+KL^2$ $16=KL^2$ $KL=4$	✓ $x=-1$ ✓ $KN^2=NL^2+KL^2$ ✓ $KL=4$ (3)

6.5	$m_{AB} \times m_{AK} = -1$ tangent ⊥ radius $m_{AK} = \frac{4}{3}$ $\therefore m_{AB} = -\frac{3}{4}$ $y - y_1 = m(x - x_1)$ $y - 2 = -\frac{3}{4}(x - 3)$ $y = -\frac{3}{4}x + \frac{9}{4} + \frac{8}{4}$ $y = -\frac{3}{4}x + \frac{17}{4}$ **OR** $m_{AB} \times m_{AK} = -1$ tangent ⊥ radius $m_{AK} = \frac{4}{3}$ $\therefore m_{AB} = -\frac{3}{4}$ $y = -\frac{3}{4}x + c$ $2 = \left(-\frac{3}{4}\right)(3) + c$ $c = \frac{8}{4} + \frac{9}{4}$ $c = \frac{17}{4}$ $y = -\frac{3}{4}x + \frac{17}{4}$	✓ $m_{AK} = \frac{4}{3}$ ✓ $m_{AB} = -\frac{3}{4}$ ✓ substitution of point (3;2) into equation ✓ equation (4) ✓ $m_{AK} = \frac{4}{3}$ ✓ $m_{AB} = -\frac{3}{4}$ ✓ substitution of point (3;2) into equation ✓ equation (4)

6.6	Point A lies on PQ and AB. Therefore	
	$\frac{4}{3}x+\frac{4}{3}=-\frac{3}{4}x+\frac{17}{4}$	✓ equation
	$16x+16=-9x+51$	
	$25x = 35$	✓ $25x=35$
	$x = \frac{7}{5}$	
	$y = -\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$	✓ substitution of x
	$y = \frac{16}{5}$	
	$A\left(\frac{7}{5};\frac{16}{5}\right)$	(3)
	OR	
	Point A lies on PQ and the circle. Therefore	
	$(x-3)^2+(\frac{4}{3}x+\frac{4}{3}-2)^2=4$	✓ equation
	$(x-3)^2+(\frac{4}{3}x-\frac{2}{3})^2=4$	
	$25x^2-70x+49=0$	
	$(5x-7)^2=0$	✓ $(5x-7)^2=0$
	$x=\frac{7}{5}$	
	$y = -\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$	✓ substitution of x
	$y = \frac{16}{5}$	(3)
	OR	

Point A lies on the circle and line AB

$(x-3)^2+(y-2)^2=4 \quad ----------(1)$

$y=-\frac{3}{4}x+\frac{17}{4} \quad ----------(2)$

✓ equation

Subs (2) in (1): $x^2-6x+9+(-\frac{3}{4}x+\frac{17}{4}-2)^2=4$

$x^2-6x+9+(-\frac{3}{4}x+\frac{9}{4})^2=4$

$25x^2-150x+161=0$

$(5x-23)(5x-7)=0$

✓ $(5x-23)(5x-7)=0$

$x=\frac{7}{5}$

$y=-\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$

✓ substitution of x

$y=\frac{16}{5}$

(3)

OR

Using rotation:

Let $\theta = A\hat{K}N = L\hat{K}N$

Move diagram 1 unit to the right. Then $A^{/}$ is $L^{/}$ rotated through 2θ .

$\tan\theta=\frac{AN}{KA}=\frac{2}{4}=\frac{1}{2}$

$\therefore \sin 2\theta = 2\sin\theta\cos\theta = 2(\frac{1}{\sqrt{5}})(\frac{2}{\sqrt{5}})=\frac{4}{5}$

$\cos 2\theta=\cos^2\theta-\sin^2\theta=(\frac{2}{\sqrt{5}})^2-(\frac{1}{\sqrt{5}})^2=\frac{3}{5}$

✓ values of $\sin 2\theta$ and $\cos 2\theta$

$\therefore x_{A'}=x_{L'}\cos 2\theta - y_{L'}\sin 2\theta = 4(\frac{3}{5})-(0)(\frac{4}{5})=\frac{12}{5}$

$y_{A'}=x_{L'}\sin 2\theta + y_{L'}\cos 2\theta = 4(\frac{4}{5})-(0)(\frac{3}{5})=\frac{16}{5}$

$A'(\frac{12}{5}\ ;\ \frac{16}{5})$

✓ substitution into rotation formulae

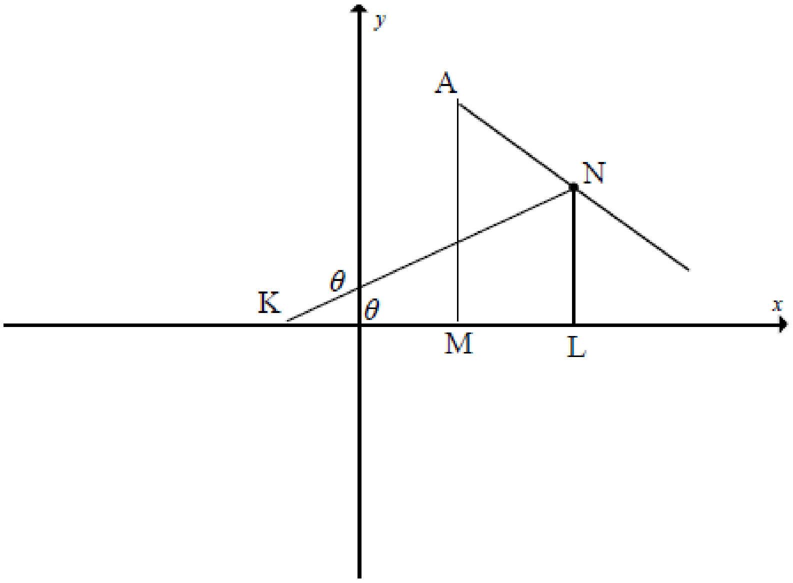
✓ $A'(\frac{12}{5}\ ;\ \frac{16}{5})$

Now to get back to A, move back 1 unit to the left.

$\therefore A(\frac{7}{5}\ ;\ \frac{16}{5})$

(3)

OR



Let $N\hat{K}L = \theta$. So, $\tan\theta = \frac{NL}{KN} = \frac{2}{4} = \frac{1}{2}$.

✓ $\tan\ \theta = \frac{1}{2}$

Hence $\sin\theta = \frac{1}{\sqrt{5}}$ and $\cos\theta = \frac{2}{\sqrt{5}}$

Let $AM \perp x$ – axis with M on x - axis

$\Delta NAK \equiv \Delta NLK$

$A\hat{K}N = N\hat{K}L = \theta$

$\therefore A\hat{K}L = 2\theta$

$y_A = AM = AK\sin 2\theta = KL\sin 2\theta = 4\sin 2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$

✓ $\sin 2\theta = \frac{4}{5}$

$y_A = 4\left(\frac{4}{5}\right) = \frac{16}{5}$

✓ solve for x and y

$x_A = OL - NA\sin M\hat{A}N$

$= 3 - 2\sin\left(90° - M\hat{A}K\right)$

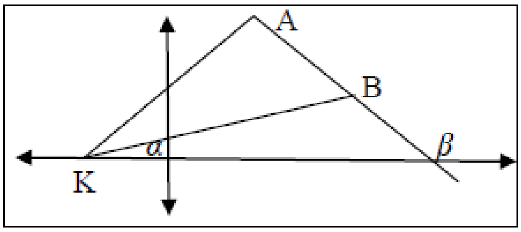
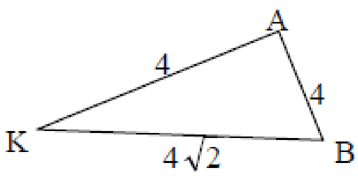
$= 3 - 2\sin 2\theta$

$= 3 - \frac{8}{5}$

$= \frac{7}{5}$

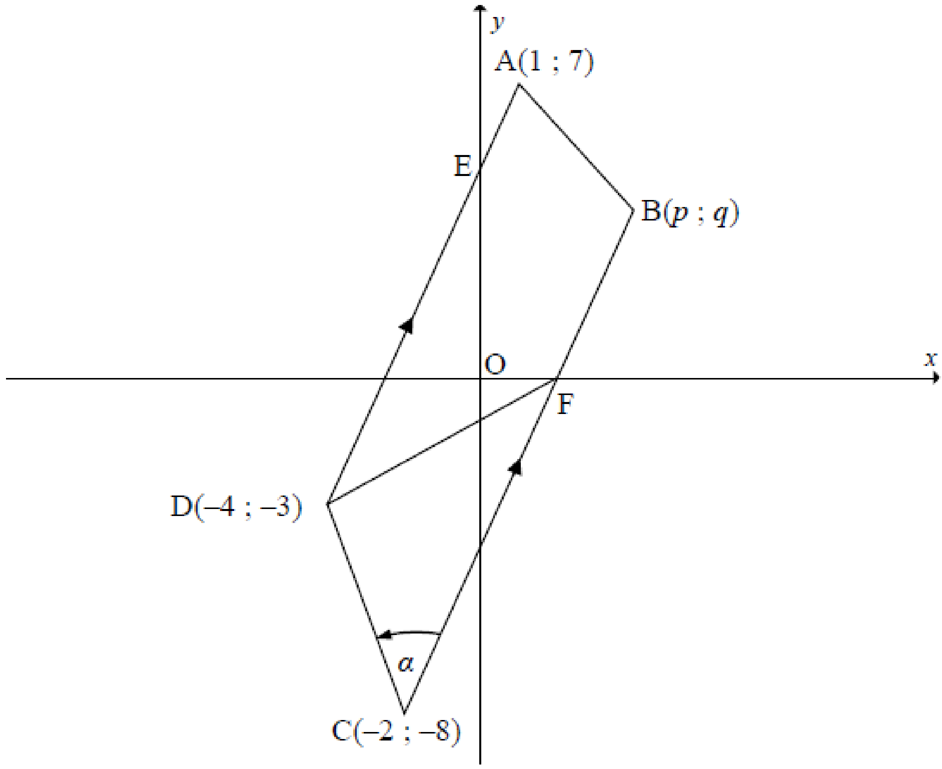
(3)

6.7	$KA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{\left(\frac{7}{5}+1\right)^2+\left(\frac{16}{5}-0\right)^2}$ $=4$ **OR** $KN=\sqrt{4^2+2^2}=\sqrt{20}$ $KA^2=KN^2-AN^2$ $=20-4$ $=16$ $KA=4$ **OR** KA = KL Tangents from a common point are equal KA = 4	✓ distance formula ✓ substitution ✓ 4 (3) ✓ $KN=\sqrt{20}$ ✓ $KA^2=KN^2-AN^2$ ✓ 4 (3) ✓ KA=KL ✓ reason ✓ 4 (3)
6.8	AN = NL Radii are equal KA = KL ∴KLNA is a kite two pairs of adjacent sides are equal.	✓ AN = NL ✓ KA = KL (2)
6.9	AB = AN + NB = 2 + 2 = 4 AK = 4= AB $K\hat{A}B=90°$ tangent ⊥ radius ∴ ΔAKB is a right – angled isosceles triangle $A\hat{K}B+A\hat{B}K=90°$ $2A\hat{B}K=90°$ $\therefore A\hat{B}K=45°$ **OR**	✓ AB = 4 ✓ AK = AB ✓ $K\hat{A}B=90°$ (3)

	N is midpoint of AB Let B be $(x_B;y_B)$ $\frac{x_B+\frac{7}{5}}{2}=3$ $\quad$ $\frac{y_B+\frac{16}{5}}{2}=2$ $\therefore x_B=\frac{23}{5}$ $\quad$ $\therefore y_B=\frac{4}{5}$ $\therefore B\left(\frac{23}{5};\frac{4}{5}\right)$  $\tan\beta=m_{AB}=-\frac{3}{4}$ $\beta=180°-36{,}87°$ $\beta=143{,}13°$ $\tan\alpha=m_{KB}=\frac{\frac{4}{5}-0}{\frac{23}{5}+1}=\frac{1}{7}$ $\alpha=8{,}13°$ $A\hat{B}K=\alpha+(180°-\beta)$ $=8{,}13°+36{,}87°$ $=45°$ **OR** N is midpoint of AB Let B be $(x_B;y_B)$ $\frac{x_B+\frac{7}{5}}{2}=3$ $\quad$ $\frac{y_B+\frac{16}{5}}{2}=2$ $\therefore x_B=\frac{23}{5}$ $\quad$ $\therefore y_B=\frac{4}{5}$ $\therefore B\left(\frac{23}{5};\frac{4}{5}\right)$  $KB=\sqrt{(\frac{23}{5}+1)^2+(\frac{4}{5})^2}=4\sqrt{2}$ $4^2=4^2+(\sqrt{32})^2-2(4)(\sqrt{32})\cos\theta$ $\cos\theta=\frac{\sqrt{2}}{2}$ $\therefore\theta=45°$	✓ $143{,}13°$ ✓ $8{,}13°$ ✓ $A\hat{B}K=\alpha+(180°-\beta)$ (3) ✓ $4\sqrt{2}$ ✓ substitution into cosine formula ✓ $\cos\theta=\frac{\sqrt{2}}{2}$ (3)
6.10	$N'(3;-2)$	✓ $N'(3;-2)$ (1) **[24]**

QUESTION 4

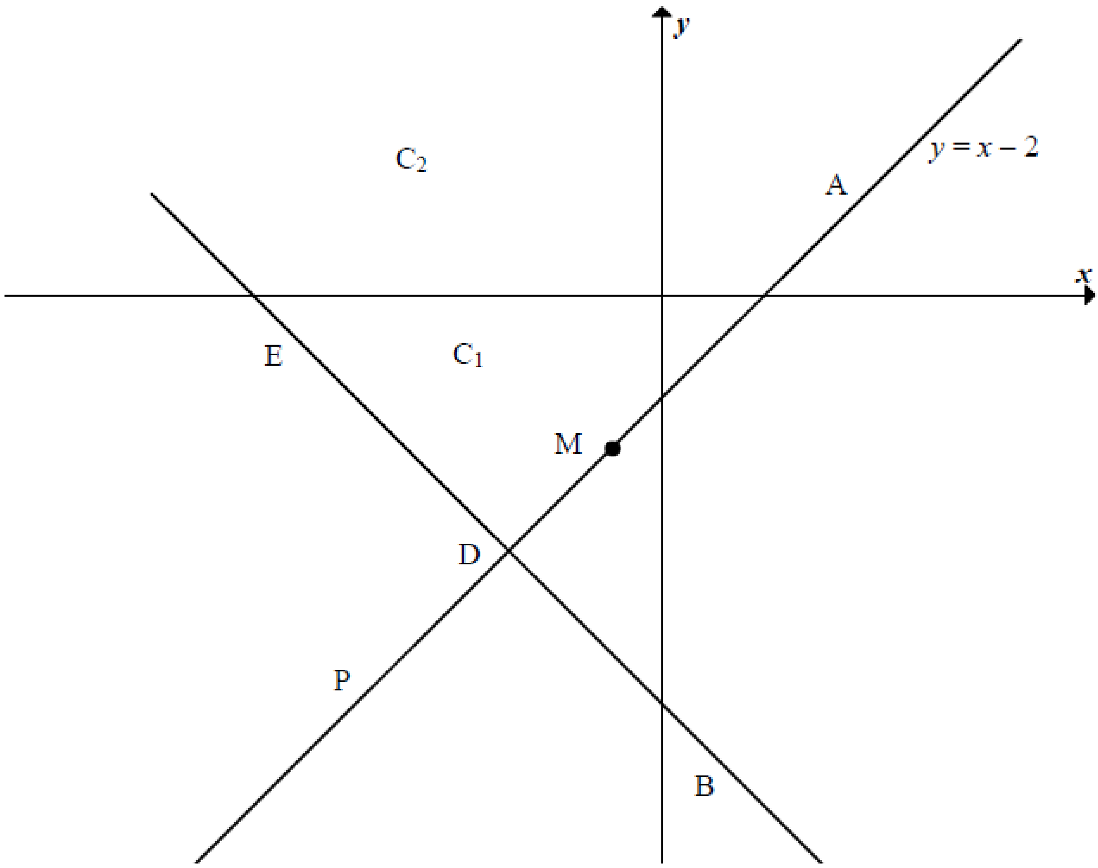
In the diagram below, trapezium ABCD with AD // BC is drawn. The coordinates of the vertices are A(1 ; 7); B(p ; q); C(–2 ; –8) and D(–4 ; –3). BC intersects the x-axis at F. $D\hat{C}B = \alpha$.



4.1	Calculate the gradient of AD.	(2)
4.2	Determine the equation of BC in the form $y = mx + c$.	(3)
4.3	Determine the coordinates of point F.	(2)
4.4	AB$^{/}$CD is a parallelogram with B$^{/}$ on BC. Determine the coordinates of B$^{/}$, using a transformation $(x\,;\,y) \rightarrow (x + a\,;\,y + b)$ that sends A to B$^{/}$.	(2)
4.5	Show that $\alpha = 48{,}37°$.	(4)
4.6	Calculate the area of ΔDCF.	(6) **[19]**

QUESTION 5

Circles C_1 and C_2 in the figure below have the same centre M. P is a point on C_2. PM intersects C_1 at D. The tangent DB to C_1 intersects C_2 at B. The equation of circle C_1 is given by $x^2 + 2x + y^2 + 6y + 2 = 0$ and the equation of line PM is $y = x - 2$.



5.1 Determine the following:

5.1.1 The coordinates of centre M (3)

5.1.2 The radius of circle C_1 (1)

5.2 Determine the coordinates of D, the point where line PM and circle C_1 intersect. (5)

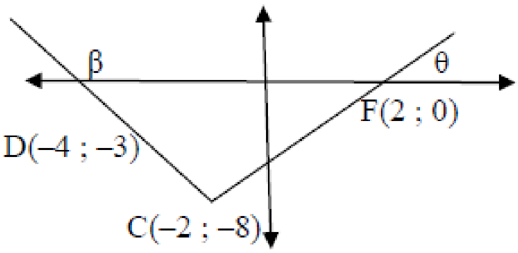
5.3 If it is given that $DB = 4\sqrt{2}$, determine MB, the radius of circle C_2. (3)

5.4 Write down the equation of C_2 in the form $(x-a)^2 + (y-b)^2 = r^2$. (2)

5.5 Is the point $F\left(2\sqrt{5}\ ;\ 0\right)$ inside circle C_2? Support your answer with calculations. (4)

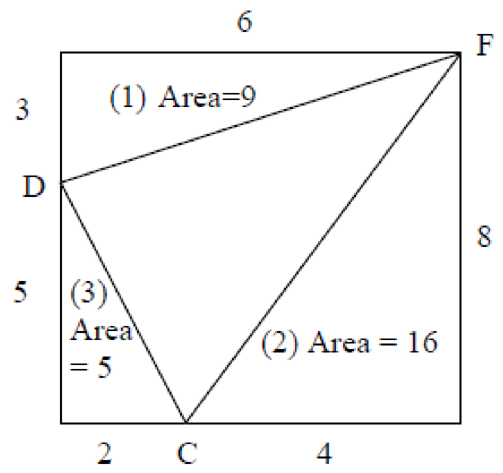
[18]

QUESTION 4

4.1	$m_{AD} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{7-(-3)}{1-(-4)}$ $= 2$	✓ substitution ✓ 2 (2)
4.2	AD//BC $m_{AD} = m_{BC} = 2$ $y - y_1 = m(x - x_1)$ $y-(-8) = 2(x-(-2))$ $\therefore y = 2x - 4$	✓ $m_{AD} = 2$ ✓ substitute into formula ✓ $y = 2x - 4$ (3)
4.3	At F: $y = 0$ $0 = 2x - 4$ $x = 2$ F(2 ; 0)	✓ $y = 0$ ✓ $x = 2$ (2)
4.4	D is translated C according to the rule: $D(x;y) \rightarrow C(x+2\,;y-5)$ A must also be translated according to this rule to B$'$. $\therefore$A(1 ; 7) $\rightarrow$ B$'$ (3 ; 2) **OR** $x_{B'} = -2+(1+4) = 3$ $y_{B'} = -8+(7+3) = 5$	✓ $x = 3$ ✓ $y = 2$ (2) ✓ $x = 3$ ✓ $y = 2$ (2)
4.5	$m_{BC} = 2$ $\tan\theta = 2$ $\theta = 63{,}43°$ $m_{DC} = \dfrac{-8-(-3)}{-2-(-4)} = -\dfrac{5}{2}$ $\tan\beta = -\dfrac{5}{2}$ $\beta = 180° - 68{,}20° = 111{,}80°$ $\alpha = 111{,}80° - 63.43° = 48{,}37°$  **OR**	✓ 63,43° ✓ $\tan\beta = -\dfrac{5}{2}$ ✓ 111,8° ✓ 48,37° (4)

	$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ $CF = \sqrt{(-2-2)^2 + (-8-0)^2}$ $= \sqrt{80}$ $DF = \sqrt{(2+4)^2 + (0+3)^2}$ $= \sqrt{45}$ $\cos\alpha = \dfrac{29+80-45}{2(\sqrt{29})(\sqrt{80})}$ $= 0{,}6643...$ $\alpha = 48{,}37°$ **OR** $DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ $DB = \sqrt{(3+4)^2 + (2+3)^2}$ $= \sqrt{74}$ $BC = \sqrt{(3+2)^2 + (2+8)^2}$ $= \sqrt{125}$ $\cos\alpha = \dfrac{29+125-74}{2(\sqrt{29})(\sqrt{125})}$ $= 0{,}6643...$ $\alpha = 48{,}37°$	✓Subst in cos-formula ✓cos α subject ✓0,6643... ✓48,37° (4) ✓Subst in cos-formula ✓cos α subject ✓0,6643... ✓48,37° (4)
4.6	$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ $CF = \sqrt{(-2-2)^2 + (-8-0)^2}$ $= \sqrt{80}$ $\text{Area } \Delta DCF = \frac{1}{2}.DC.CF.\sin\alpha$ $= \frac{1}{2}(\sqrt{29})(\sqrt{80})\sin 48{,}37°$ $= 18 \text{ units}^2$	✓ substitution into formula ✓ $\sqrt{29}$ ✓ substitution into formula ✓ $\sqrt{80}$ ✓ substitution into the area rule ✓ 18 (6)

OR



Area ΔDCF = Area of rectangle – (1) – (2) – (3)

= 48 – 9 – 5 – 16

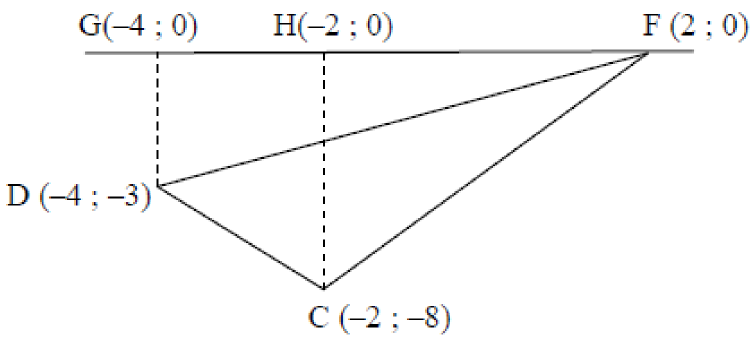
= 18 sq units

✓ establishing rectangle and area

✓ relationship of areas
✓ (1) = 9
✓(2) = 16
✓(3) = 5
✓18 units^2

(6)

OR



Area CDF = Area CHF + Area CDGH - Area DGF

$$= \frac{1}{2} \times 4 \times 8 + 2 \times \frac{1}{2}(3 \times 8) - \frac{1}{2} \times 6 \times 3$$

$$= 16 + 11 - 9$$

$$= 18$$

✓ drawing perpendiculars

✓ relationship of areas
✓ 16
✓ 11
✓ 9
✓18 units^2

(6)

[19]

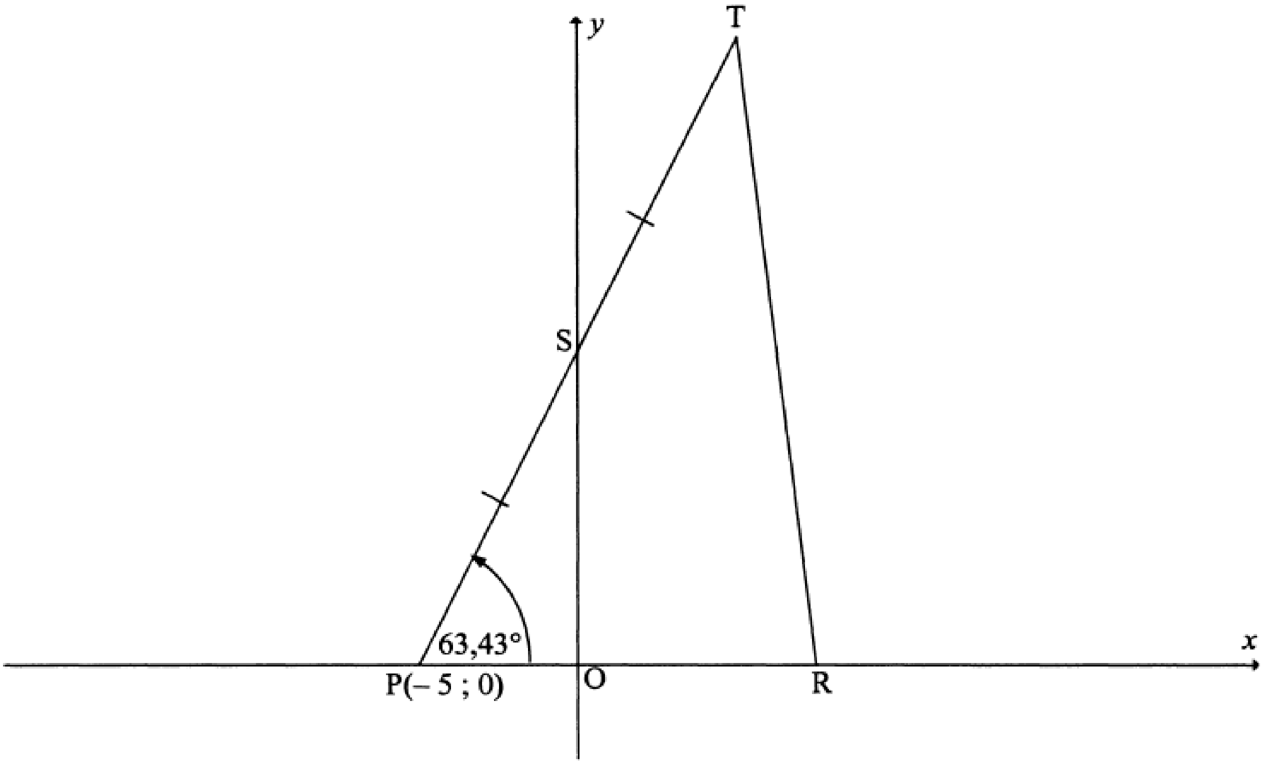
QUESTION 5

5.1.1	$x^2 + y^2 + 2x + 6y + 2 = 0$ $x^2 + 2x + 1 + y^2 + 6y + 9 = -2 + 10$ $(x+1)^2 + (y+3)^2 = 8$ $M(-1\,;\,-3)$	✓ $(x+1)^2 + (y+3)^2 = 8$ ✓ -1 ✓ -3 (3)
5.1.2	radius of circle $C_1 = \sqrt{8}$	✓ $\sqrt{8}$ (1)
5.2	$x^2 + (x-2)^2 + 2x + 6(x-2) + 2 = 0$ $x^2 + x^2 - 4x + 4 + 2x + 6x - 12 + 2 = 0$ $2x^2 + 4x - 6 = 0$ $x^2 + 2x - 3 = 0$ $(x+3)(x-1) = 0$ $x = -3 \text{ or } x \neq 1$ $y = -3 - 2 = -5$ $\therefore D(-3;-5)$ **OR** $(x+1)^2 + (y+3)^2 = 8$ *subst.* $y = x - 2$ $(x+1)^2 + (x-2+3)^2 = 8$ $(x+1)^2 + (x+1)^2 = 8$ $x^2 + 2x - 3 = 0$ $(x+3)(x-1) = 0$ $x = -3 \text{ or } x \neq 1$ $y = -3 - 2 = -5$ **OR** $(x+1)^2 + (y+3)^2 = 8$ *subst.* $y = x - 2$ $(x+1)^2 + (x-2+3)^2 = 8$ $(x+1)^2 + (x+1)^2 = 8$ $(x+1)^2 = 4$ $x + 1 = \pm 2$ $x = -3 \text{ or } x \neq 1$ $y = -3 - 2 = -5$ **OR**	✓ substitution ✓ standard form ✓ factors ✓ value of x ✓ value of y (5) ✓ substitution ✓ standard form ✓ factors ✓ value of x ✓ value of y (5) ✓ substitution ✓ simplification ✓ square root of both sides ✓ value of x ✓ value of y

	PM makes 45° with the x-axis. $\sqrt{8}=\sqrt{2^2+2^2}$ Therefore: $x_D=x_M-2=-1-2=-3$ $y_D=-3-2=-5$	✓✓ $\sqrt{8}=\sqrt{2^2+2^2}$ ✓ value of x ✓ value of y (5)
5.3	MD ⊥ DB (tangent ⊥ radius) $MB^2=MD^2+DB^2$ (Pythagoras) $=(\sqrt{8})^2+(4\sqrt{2})^2$ $=40$ MB is the radius of C_2 $MB=\sqrt{40}$	✓ tangent ⊥ radius ✓ substitution into Pythagoras ✓ $\sqrt{40}$ (3)
5.4	$(x+1)^2+(y+3)^2=40$	✓ LHS ✓ RHS (2)
5.5	Distance from $(2\sqrt{5}\,;0)$ to centre $=\sqrt{(2\sqrt{5}+1)^2+(0+3)^2}$ $=6{,}24$ $6{,}24<6{,}32\ (\sqrt{40})$ Distance from $(2\sqrt{5}\,;0)$ to centre < radius of circle. $(2\sqrt{5}\,;0)$ lies inside the circle.	✓substitution into distance formula ✓ 6,24 ✓6,24 < 6,32 ✓ conclusion (4) **[18]**

QUESTION 5

In the diagram below, P is a point (– 5 ; 0). The inclination of line PT is 63,43°. S is the midpoint and the y-intercept of PT. R is a point on the x-axis such that PO : OR = 2 : 3.



5.1 Determine:

5.1.1 The gradient of PT, correct to the nearest integer value (2)

5.1.2 The equation of PT in the form $y = mx + c$ (2)

5.1.3 The distance PS in surd form (3)

5.1.4 The coordinates of T (2)

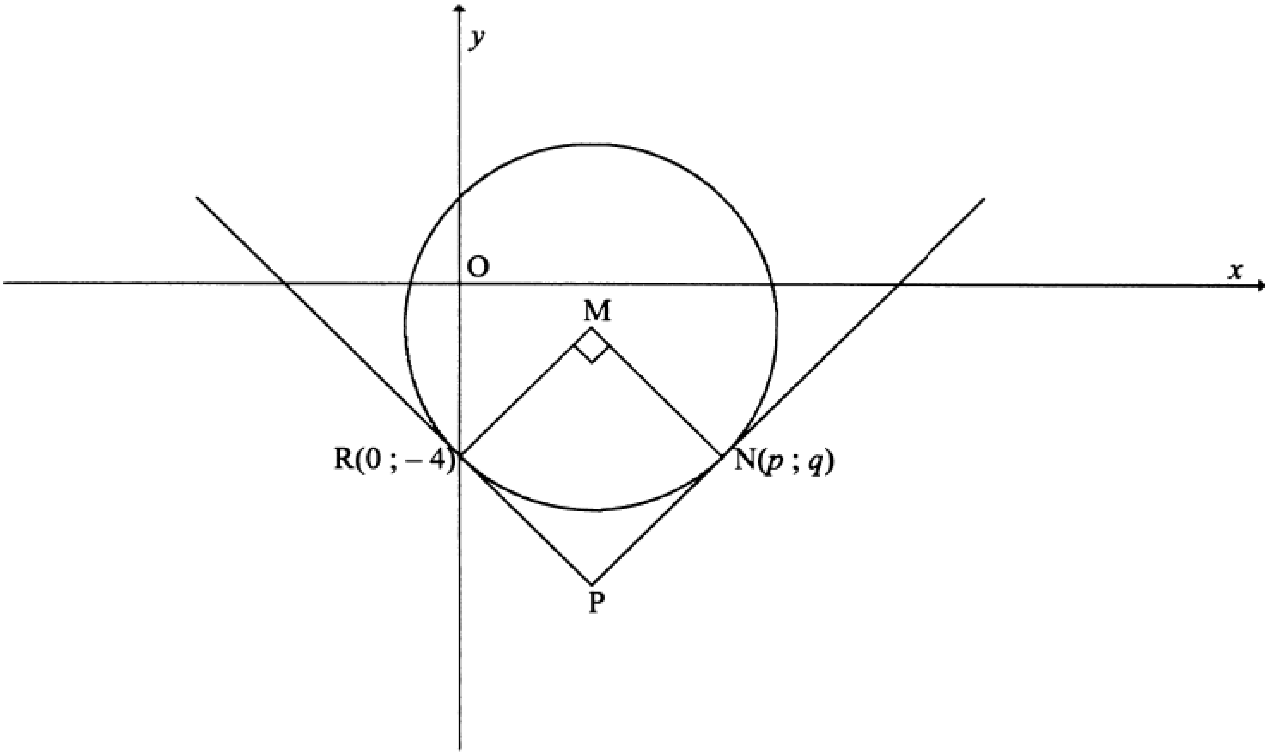
5.2 Determine the coordinates of R. (2)

5.3 Calculate the area of ΔPTR. (4)

[15]

QUESTION 6

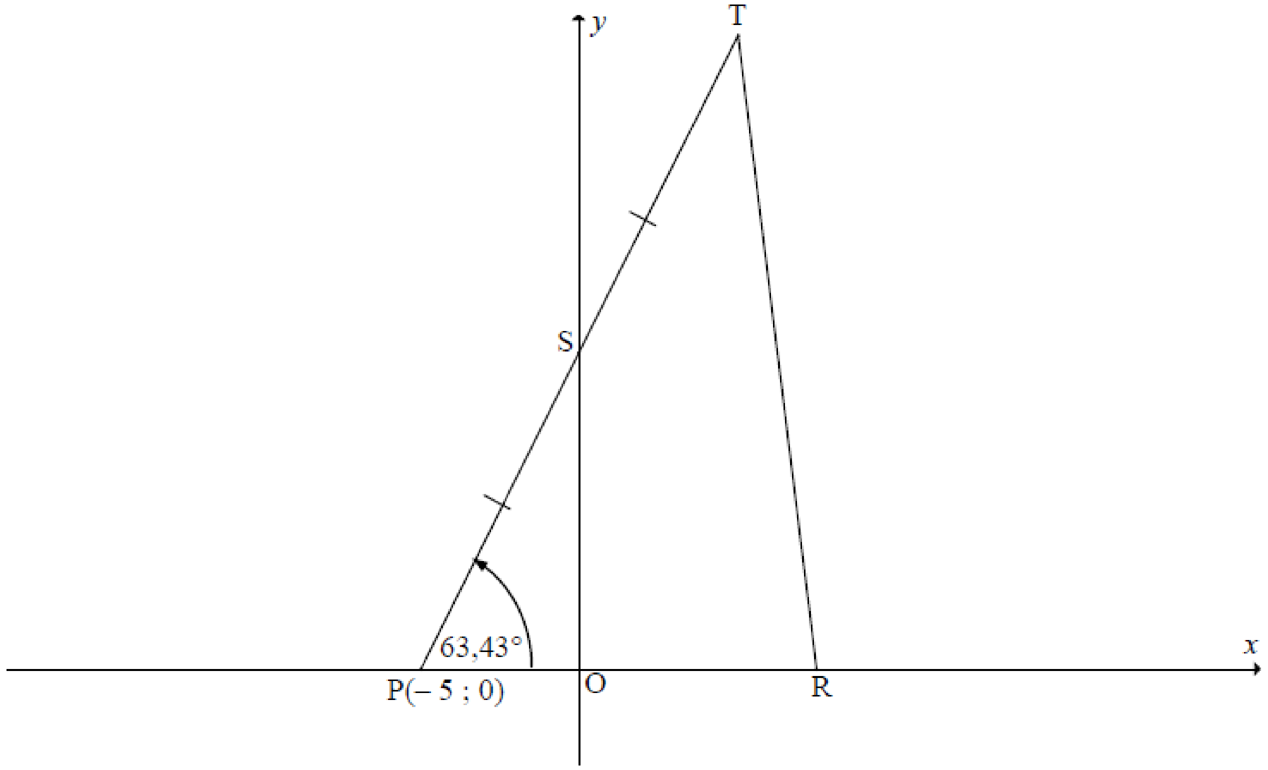
In the diagram below, M is the centre of the circle having the equation $x^2 + y^2 - 6x + 2y - 8 = 0$. The circle passes through R(0 ; – 4) and N(p ; q). $\hat{RMN} = 90°$. The tangents drawn to the circle at R and N meet at P.



6.1	Show that M is the point (3 ; – 1).	(4)
6.2	Determine the equation of MR in the form $y = mx + c$.	(3)
6.3	Show that $q = 2 - p$.	(4)
6.4	Determine the values of p and q.	(5)
6.5	Determine the equation of the circle having centre O and passing through N.	(2)
6.6	Calculate the area of the circle centred at M.	(2)
6.7	Calculate the ratio in its simplest form: $\frac{\text{NP}}{\text{MP}}$	(4)

[24]

QUESTION/*VRAAG* 5

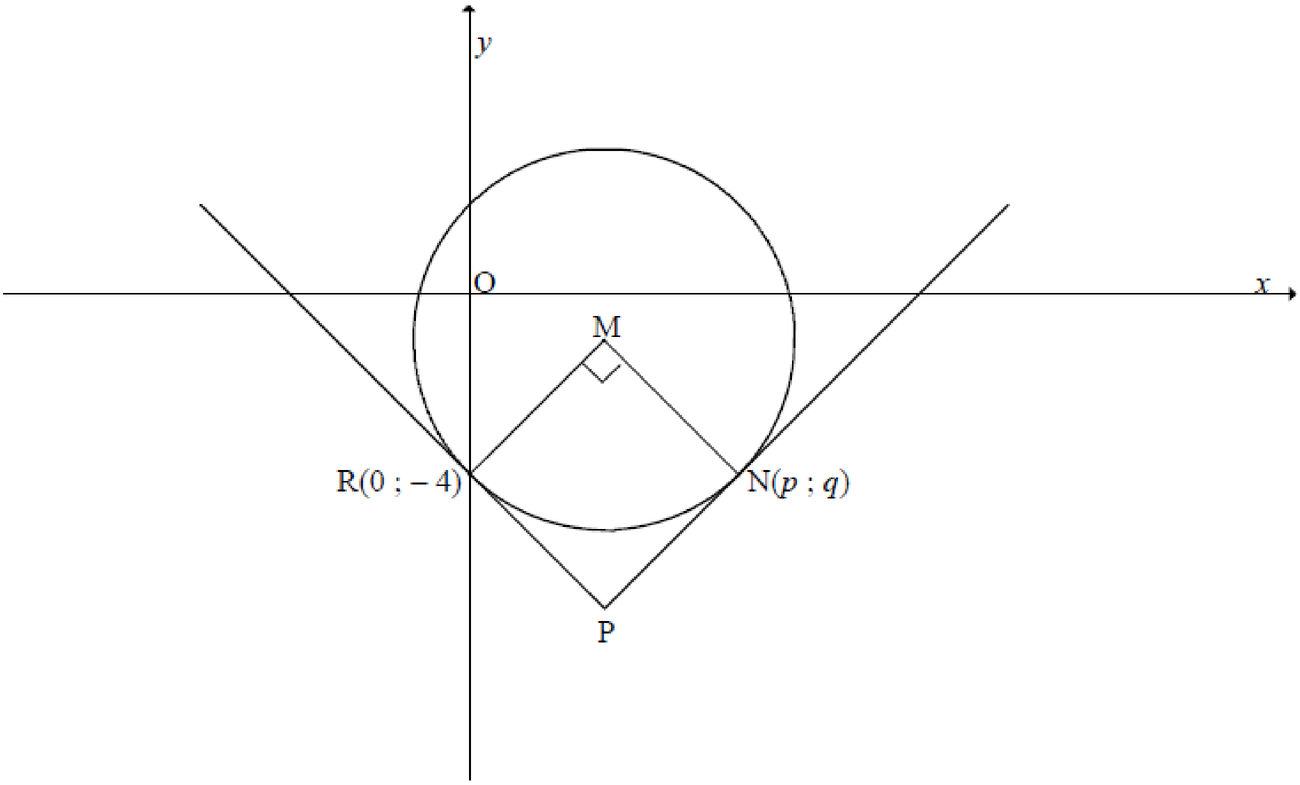


5.1.1	$m_{PT} = \tan\ 63{,}43°$ $= 2$	✓ tan 63,43° ✓ 2 (2) Answer only: full marks
5.1.2	Coordinates of P(– 5 ; 0) $y - y_1 = m(x - x_1)$ $y - 0 = 2(x + 5)$ $y = 2x + 10$ **OR** $y = mx + c$ $0 = (2)(-5) + c$ $c = 10$ $y = 2x + 10$ **OR** $m_{PT} = 2 = \tan 63{,}43°$ $\tan 63{,}43° = \frac{OS}{OP} = \frac{OS}{5} = 2$ $\therefore OS = 10$ $y = 2x + 10$	✓substitution of P(–5 ; 0) and m = 2 into equation ✓ equation (2) ✓substitution of P(–5 ; 0) and m = 2 into equation ✓ equation (2) ✓ $\frac{OS}{5} = 2$ ✓ equation (2)

5.1.3	$OS = 10$ units $PS^2 = (5)^2 + (10)^2$ $= 125$ $PS = \sqrt{125} = 5\sqrt{5}$ [Accept PS = 11,18] **OR** P(–5 ; 0) ; OS = 10 units $PS^2 = (-5-0)^2 + (0-10)^2$ $= 25 + 100$ $= 125$ $PS = \sqrt{125} = 5\sqrt{5}$ [Accept PS = 11,18] **OR** $\frac{PS}{5} = \frac{1}{\cos 63{,}43°}$ $\therefore PS = \frac{5}{\cos 63{,}43°}$ $PS = 11{,}18$ **OR** $\frac{PS}{10} = \frac{1}{\sin 63{,}43°}$ $\therefore PS = \frac{10}{\sin 63{,}43°}$ $PS = 11{,}18$	✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3) ✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3) ✓ ratio ✓ $PS = \frac{5}{\cos 63{,}43°}$ ✓ 11,18 (3) ✓ ratio ✓ $PS = \frac{10}{\sin 63{,}43°}$ ✓ 11,18 (3)
5.1.4	Let T be $(x ; y)$. Then $\frac{-5+x}{2} = 0$ and $\frac{0+y}{2} = 10$ $x = 5$ $y = 20$ T(5 ; 20) **OR** by inspection: T(5 ; 20)	✓ 5 ✓ 20 (2) ✓ 5 ✓ 20 (2)
5.2	$OR = \left(\frac{3}{2}\right)(5) = \frac{15}{2} = 7{,}5$ $R\left(\frac{15}{2} ; 0\right)$ [If only x-coordinate : 2 marks]	✓ $x = 7{,}5$ / $\frac{15}{2}$ ✓ $y = 0$ (2)

5.3	Area $\Delta PTR = \frac{1}{2}(\text{base PR}) \times (\text{height})$ $= \frac{1}{2}(5 + \frac{15}{2}) \times 20$ $= 125$ square units	✓ area formula ✓ $5 + \frac{15}{2} = 12,5$ ✓ 20 ✓125 (4)
	OR	
	Area $\Delta PTR = \frac{1}{2} PT.PR.\sin T\hat{P}R$ $= \frac{1}{2}\left(10\sqrt{5}\right)\left(\frac{25}{2}\right)\sin 63,43°$ $= 124,99$ square units	✓ area formula ✓ $10\sqrt{5}$ ✓ $\frac{25}{2}$ ✓124,99 (4) **[15]**

QUESTION/*VRAAG* 6



6.1	$x^2 + y^2 - 6x + 2y - 8 = 0$ $x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$ $(x-3)^2 + (y+1)^2 = 18$ $\therefore$ M(3 ; – 1) If only $(x-3)^2 + (y+1)^2 = r^2$ $(r^2 \neq 18)$, then 2 marks **OR** $x_M = -\frac{1}{2}(\text{coefficient of } x)$ $x_M = -\frac{1}{2}(-6)$ $x_M = 3$ $y_M = -\frac{1}{2}(\text{coefficient of } y)$ $y_M = -\frac{1}{2}(2)$ $y_M = -1$ $\therefore$ M(3 ; –1)	✓ $x^2 - 6x + 9$ ✓ $y^2 + 2y + 1$ ✓ $(x-3)^2$ ✓ $(y+1)^2$ (4) ✓ $x_M = -\frac{1}{2}(-6)$ ✓ $x_M = 3$ ✓ $y_M = -\frac{1}{2}(2)$ ✓ $y_M = -1$ (4)

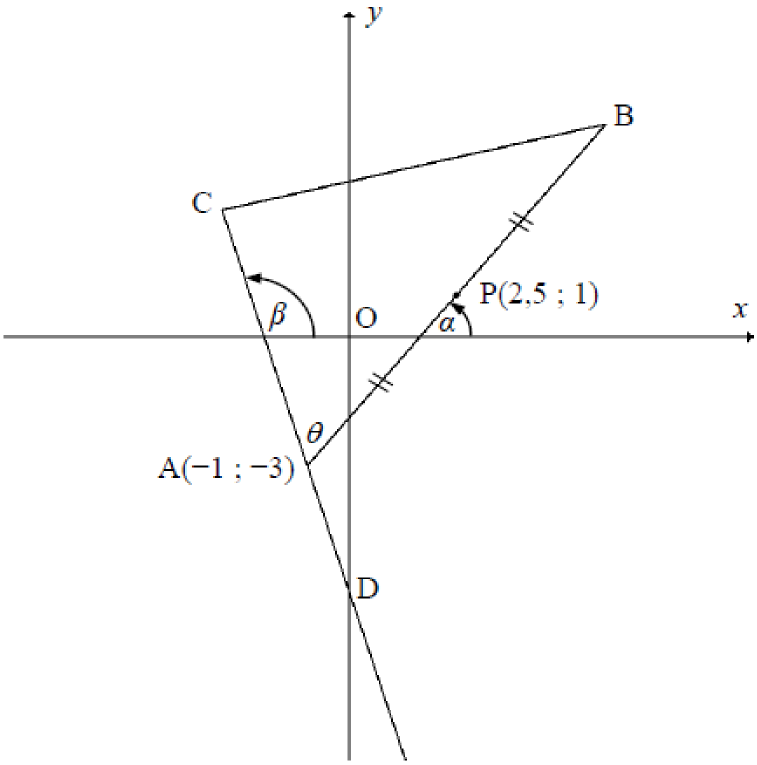
6.2	$m_{RM} = \dfrac{-1-(-4)}{3-0}$ $=1$ y-intercept is -4 $y = x - 4$	✓ substitution into gradient formula ✓ $m_{RM} = 1$ ✓ equation (3)
6.3	MR ⊥ RP (radius ⊥ tangent/*raaklyn*) $m_{MN} = m_{PR} = -1$ $\dfrac{q-(-1)}{p-3} = -1$ $-p+3 = q+1$ $q = 2-p$ **OR** MR ⊥ RP (radius ⊥ tangent/*raaklyn*) $m_{MN} = m_{PR} = -1$ $y-(-1) = -1(x-3)$ $y+1 = -x+3$ $y = -x+2$ $q = 2-p$	✓✓ $m_{MN} = -1$ ✓ substitution into gradient formula ✓ $-p+3 = q+1$ (4) ✓✓ $m_{MN} = -1$ ✓ $y = -x+2$ ✓ substitution into equation of line (4)
6.4	$(x-3)^2 + (y+1)^2 = 18$ $(p-3)^2 + (q+1)^2 = 18$ $(2-q-3)^2 + (q+1)^2 = 18$ $q^2 + 2q + 1 + q^2 + 2q + 1 - 18 = 0$ $2q^2 + 4q - 16 = 0$ $q^2 + 2q - 8 = 0$ $(q+4)(q-2) = 0$ $q = -4 \text{ or } q \neq 2$ $p = 6$ **OR** MRPN is a square/*vierkant* (rectangle with/*reghoek met* MN = MR) $\therefore\ M\hat{P}N = 45°$ But MR has a slope/gradient of 1, so RN | | x-axis $\therefore\ q = -4$ and $p = 2-(-4) = 6$ **OR**	✓ method ✓✓ $q = -4$ ✓✓ $p = 6$ (5) ✓ method ✓✓ $q = -4$ ✓✓ $p = 6$ (5)

	$q = 2 - p$ $(p-3)^2 + (2-p+1)^2 = 18$ $(p-3)^2 = 9$ $\therefore \quad p - 3 = 3 \quad (p > 0)$ $p = 6$ $\therefore q = -4$ **OR** Using symmetry: $q = -4$ (since $y_M = y_R$) $-4 = 2 - p$ $p = 6$ OR $p = 2 \times 3$ (since $x_M = 2x_N$)	✓ method ✓✓ $p = 6$ ✓✓ $q = -4$ (5) ✓method ✓✓ q ✓✓ p (5)
6.5	$r^2 = (6)^2 + (-4)^2$ $= 36+16 = 52$ $x^2 + y^2 = 52$ **OR** $p^2 + q^2 = (6)^2 + (-4)^2$ $= 36+16 = 52$ $x^2 + y^2 = p^2 + q^2$ $= 52$	✓ substitution ✓ equation (2) ✓ substitution ✓ equation (2)
6.6	area of circle M $= \pi r^2$ $= \pi(\sqrt{18})^2$ $= 18\pi$ square *units* $= 56{,}55$*square units*	✓ $r = \sqrt{18}$ ✓ area of circle (2)
6.7	MRPN is a square (all angles equals 90°, adj sides equal) $N\hat{M}P = 45°$ (diagonals of a square bisect the angles/*hoeklyne van vierkant halveer hoeke)* $\frac{NP}{MP} = \sin N\hat{M}P$ $= \sin 45°$ $= \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$ **OR** MRPN is a square (all angles equals 90°, adj sides equal) $MP^2 = 18 + 18$ $= 36$ $MP = 6$ $\frac{NP}{MP} = \frac{\sqrt{18}}{6} = \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓ $N\hat{M}P = 45°$ ✓ ✓ $\frac{NP}{MP} = \sin N\hat{M}P$ ✓ $\frac{1}{\sqrt{2}}$ (4) ✓ $MN^2 = 18$ ✓$MP^2 = 36$ ✓6 ✓ $\frac{1}{\sqrt{2}}$ (4)

	OR By inspection: P(3 ; – 7) $\frac{NP}{MP} = \frac{\sqrt{(6-3)^2+(4-7)^2}}{\sqrt{(3-3)^2+(-7+1)^2}}$ $= \frac{\sqrt{18}}{6} = \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓ P(3 ; – 7) ✓$NP^2 = 18$ ✓MP = 6 ✓ $\frac{1}{\sqrt{2}}$ (4) **[24]**

QUESTION 4

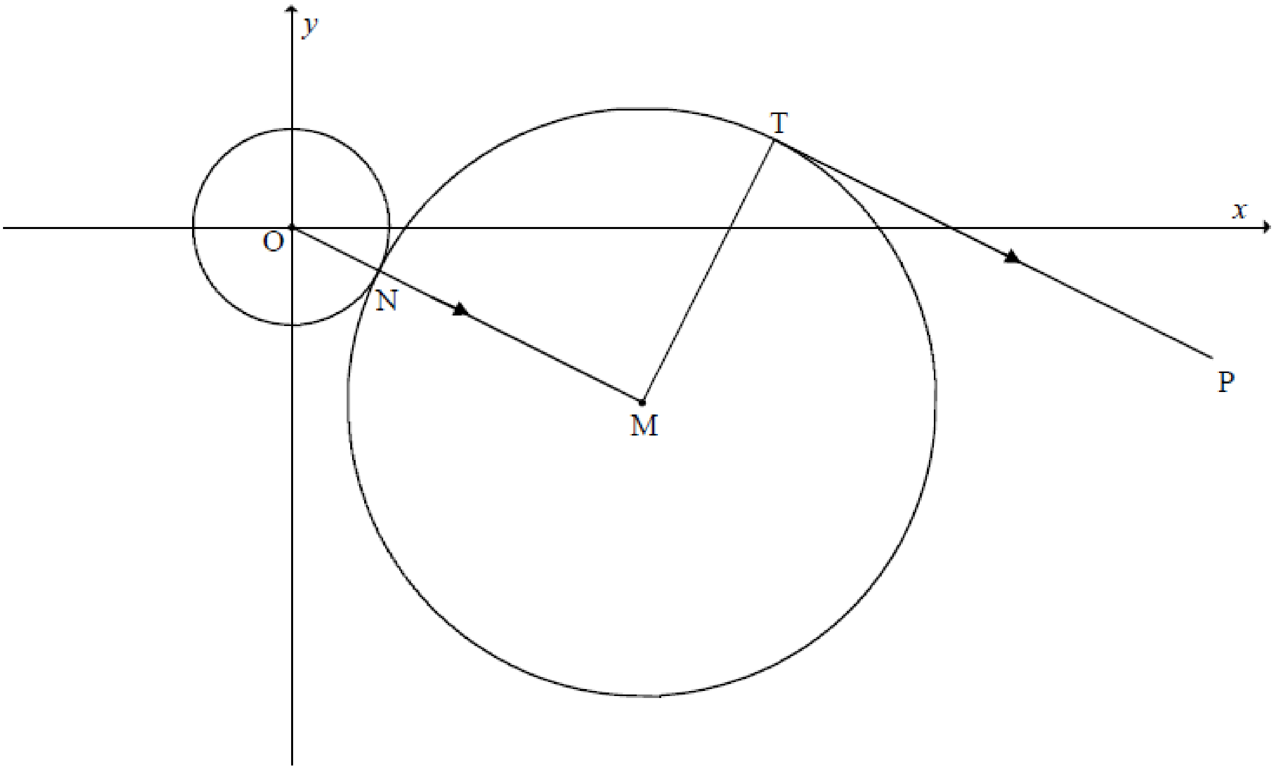
In the diagram below, A(−1 ; −3), B and C are the vertices of a triangle.
P(2,5 ; 1) is the midpoint of AB. CA extended cuts the y-axis at D.
The equation of CD is $y = -3x + k$. $C\hat{A}B = \theta$.
α and β are the angles that AB and AC respectively make with the x-axis.



4.1	Determine the value of k.	(2)
4.2	Determine the coordinates of B.	(2)
4.3	Determine the gradient of AB.	(2)
4.4	Calculate the size of θ.	(5)
4.5	Calculate the length of AD. Leave your answer in surd form.	(2)
4.6	If AC = 2AD and $AB = \sqrt{113}$, calculate the length of CB.	(5)
		[18]

QUESTION 5

In the diagram below, the equation of the circle with centre M is $(x - 8)^2 + (y + 4)^2 = 45$. PT is a tangent to this circle at T and PT is parallel to OM. Another circle, having centre O, touches the circle having centre M at N.



5.1	Write down the coordinates of M.	(1)
5.2	Calculate the length of OM. Leave your answer in simplest surd form.	(2)
5.3	Calculate the length of ON. Leave your answer in simplest surd form.	(3)
5.4	Calculate the size of $O\hat{M}T$.	(2)
5.5	Determine the equation of MT in the form $y = mx + c$.	(5)
5.6	Calculate the coordinates of T.	(6)
		[19]

QUESTION/*VRAAG* 4

4.1	$y = -3x + k$ $-3 = (-3)(-1) + k$ **OR** By inspection, using the gradient: $k = -6$ $k = -6$	✓ substitution of $(-1\ ;\ -3)$ ✓ $k = -6$ (2)
4.2	$\frac{x_A + x_B}{2} = x_P$ $\frac{y_A + y_B}{2} = y_P$ $\frac{-1 + x_B}{2} = \frac{5}{2}$ **and** $\frac{-3 + y_B}{2} = 1$ **OR** By using translation: B(6 ; 5) $x_B = 6$ $y_B = 5$ $\therefore$ B (6 ; 5)	✓ 6 ✓ 5 (2)
4.3	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - (-3)}{6 - (-1)}$ $= \frac{8}{7}$ **OR** $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1 - (-3)}{2{,}5 - (-1)}$ $= \frac{8}{7}$	✓ substitution ✓ gradient (2)

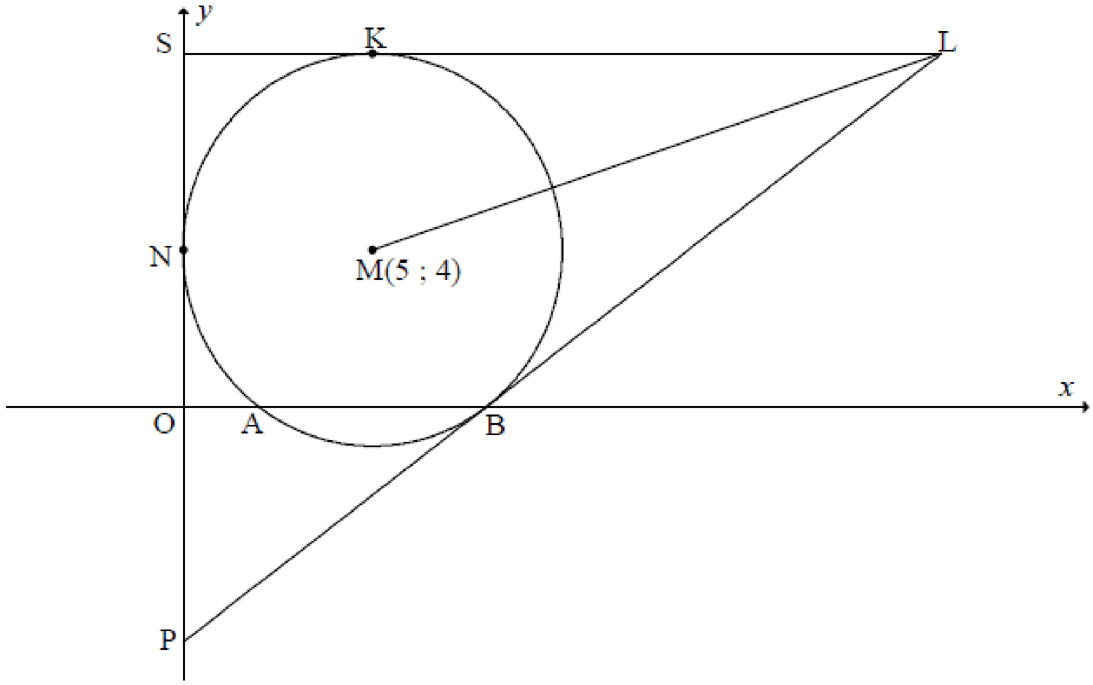
4.4	$\tan\beta = m_{AD} = -3$ $\beta = 108{,}43°$ $\tan\alpha = m_{AB} = \frac{8}{7}$ $\alpha = 48{,}81°$ $\theta = 108{,}43° - 48{,}81°$ $\theta = 59{,}62°$ **OR** $\tan\beta = m_{AD} = -3$ $\beta = 108{,}43°$ $C\hat{D}O = 18{,}43°$ $\tan\alpha = m_{AB} = \frac{8}{7}$ $\alpha = 48{,}81°$ $\theta = 18{,}43° + (90° - 48{,}81°)$ $\theta = 59{,}62°$	✓ $\tan\beta = -3$ ✓ $\beta = 108{,}43°$ ✓ $\tan\alpha = \frac{8}{7}$ ✓ $\alpha = 48{,}81°$ ✓ $\theta = 59{,}62°$ (5) ✓ $\tan\beta = -3$ ✓ $\beta = 108{,}43°$ ✓ $\tan\alpha = \frac{8}{7}$ ✓ $\alpha = 48{,}81°$ ✓ $\theta = 59{,}62°$ (5)
4.5	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(0+1)^2 + (-6+3)^2}$ $= \sqrt{10}$	✓ substitution into distance formula ✓ $\sqrt{10}$ (2)
4.6	AC = 2 AD $= 2\sqrt{10}$ $CB^2 = AC^2 + AB^2 - 2AC.AB.\cos\theta$ $= (2\sqrt{10})^2 + (\sqrt{113})^2 - 2(2\sqrt{10})(\sqrt{113})\cos 59{,}62°$ $= 84{,}998...$ CB = 9,22 units. **OR** D(0 ; –6), A(–1 ; –3), AC = 2AD So $x_c - x_A = 2(x_A - x_D)$ $x_C + 1 = 2(-1 - 0)$, $x_C = -3$ $y_c - y_A = 2(y_A - y_D)$ $y_C + 3 = 2(-3 + 6)$, $y_C = 3$ The coordinates of C are (–3 ; 3). $CB = \sqrt{(6-(-3))^2 + (5-3)^2}$ = 9,22 units	✓ $AC = 2\sqrt{10}$ ✓ using cosine rule ✓ substitution ✓ 84,998... ✓ 9,22 (5) ✓✓✓ C(–3 ; 3) ✓ substitution into distance formula ✓ 9,22 (5) **[18]**

QUESTION/*VRAAG* 5

5.1	M(8 ; – 4)	✓ coordinates (1)
5.2	$OM = \sqrt{(8-0)^2 + (-4-0)^2}$ $= \sqrt{80}$ or $4\sqrt{5}$ units	✓ substitution into distance formula ✓ $\sqrt{80}$ *or* $4\sqrt{5}$ (2)
5.3	$ON = OM - NM$ $= \sqrt{80} - \sqrt{45}$ $= 4\sqrt{5} - 3\sqrt{5}$ $= \sqrt{5}$ *units*	✓ $ON = OM - NM$ ✓ length of NM ✓ answer (3)
5.4	$M\hat{T}P = 90°$ (tangent/*raaklyn* ⊥ radius) $\therefore O\hat{M}T = 90°$ (alternate ∠'s /*verwissellende ∠'e* ; TP | | OM)	✓ Statement + reason ✓ answer (2)
5.5	$m_{MT} \cdot m_{OM} = -1$ $m_{OM} = \frac{-4-0}{8-0} = -\frac{1}{2}$ $m_{MT} = 2$ $y + 4 = 2(x-8)$ $y = 2x - 20$ **OR** $y = 2x + c$ $-4 = 2(8) + c$ $c = -20$ $y = 2x - 20$	✓ ✓ m_{OM} ✓ m_{MT} ✓ substitution of m and (8 ; –4) ✓ equation MT (5)
5.6	$(x-8)^2 + (y+4)^2 = 45$ $(x-8)^2 + (2x-20+4)^2 = 45$ $(x-8)^2 + (2x-16)^2 = 45$ $x^2 - 16x + 64 + 4x^2 - 64x + 256 - 45 = 0$ $5x^2 - 80x + 275 = 0$ $x^2 - 16x + 55 = 0$ $(x-11)(x-5) = 0$ $x = 11$ $y = 2(11) - 20$ $y = 2$ $\therefore$ T(11 ; 2)	✓ substitution ✓ expansion ✓ standard form ✓ factors ✓ $x = 11$ ✓ substitution (6) **[19]**

QUESTION 3

In the diagram below, a circle with centre M(5 ; 4) touches the y-axis at N and intersects the x-axis at A and B. PBL and SKL are tangents to the circle where SKL is parallel to the x-axis and P and S are points on the y-axis. LM is drawn.



3.1 Write down the length of the radius of the circle having centre M. (1)

3.2 Write down the equation of the circle having centre M, in the form $(x-a)^2+(y-b)^2=r^2$. (1)

3.3 Calculate the coordinates of A. (3)

3.4 If the coordinates of B are (8 ; 0), calculate:

3.4.1 The gradient of MB (2)

3.4.2 The equation of the tangent PB in the form $y=mx+c$ (3)

3.5 Write down the equation of tangent SKL. (2)

3.6 Show that L is the point (20 ; 9). (2)

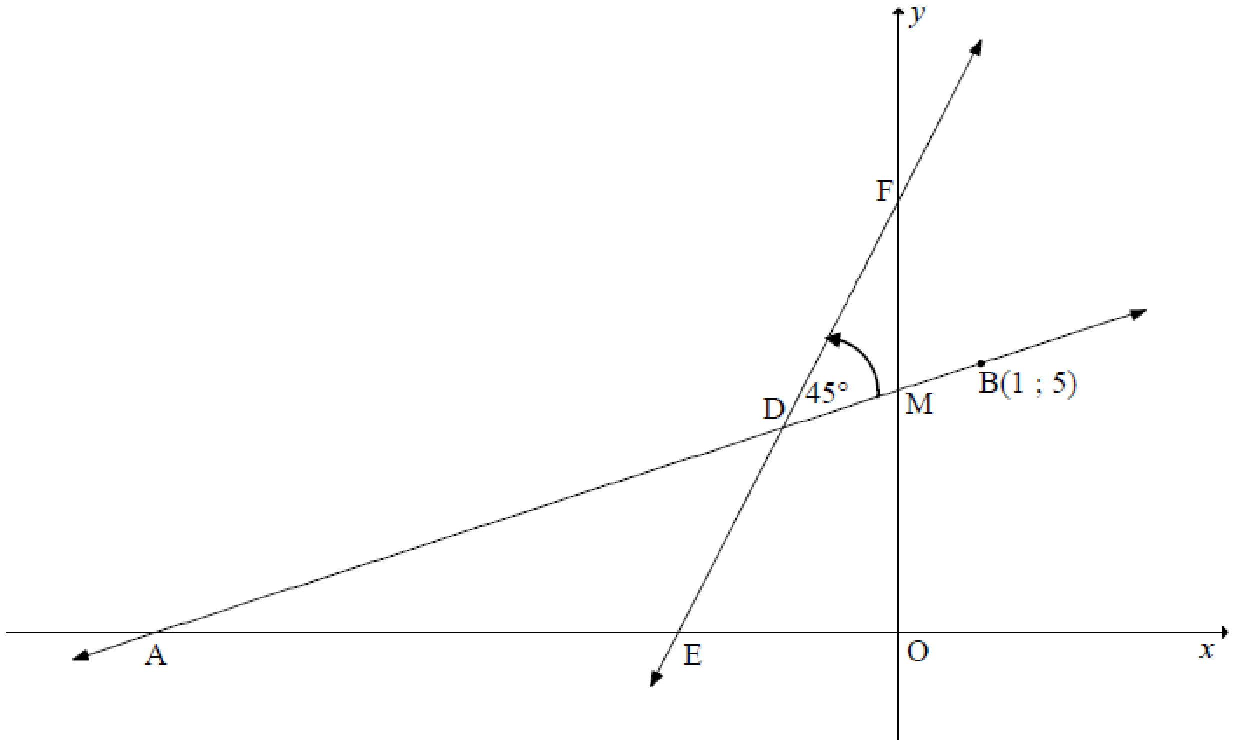
3.7 Calculate the length of ML in surd form. (2)

3.8 Determine the equation of the circle passing through points K, L and M in the form $(x-p)^2+(y-q)^2=c^2$ (5)

[21]

QUESTION 4

In the diagram below, E and F respectively are the x- and y-intercepts of the line having equation $y = 3x + 8$. The line through B(1 ; 5) making an angle of 45° with EF, as shown below, has x- and y-intercepts A and M respectively.



4.1 Determine the coordinates of E. (2)

4.2 Calculate the size of $D\hat{A}E$. (3)

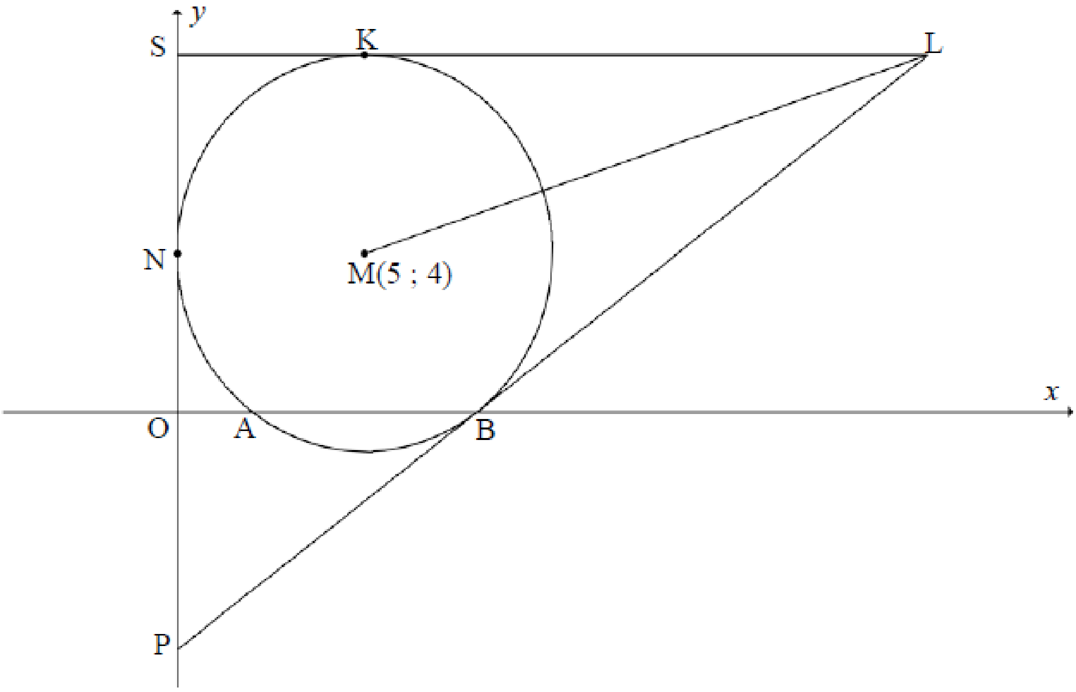
4.3 Determine the equation of AB in the form $y = mx + c$. (4)

4.4 If AB has equation $x - 2y + 9 = 0$, determine the coordinates of D. (4)

4.5 Calculate the area of quadrilateral DMOE. (6)

[19]

QUESTION/*VRAAG* 3

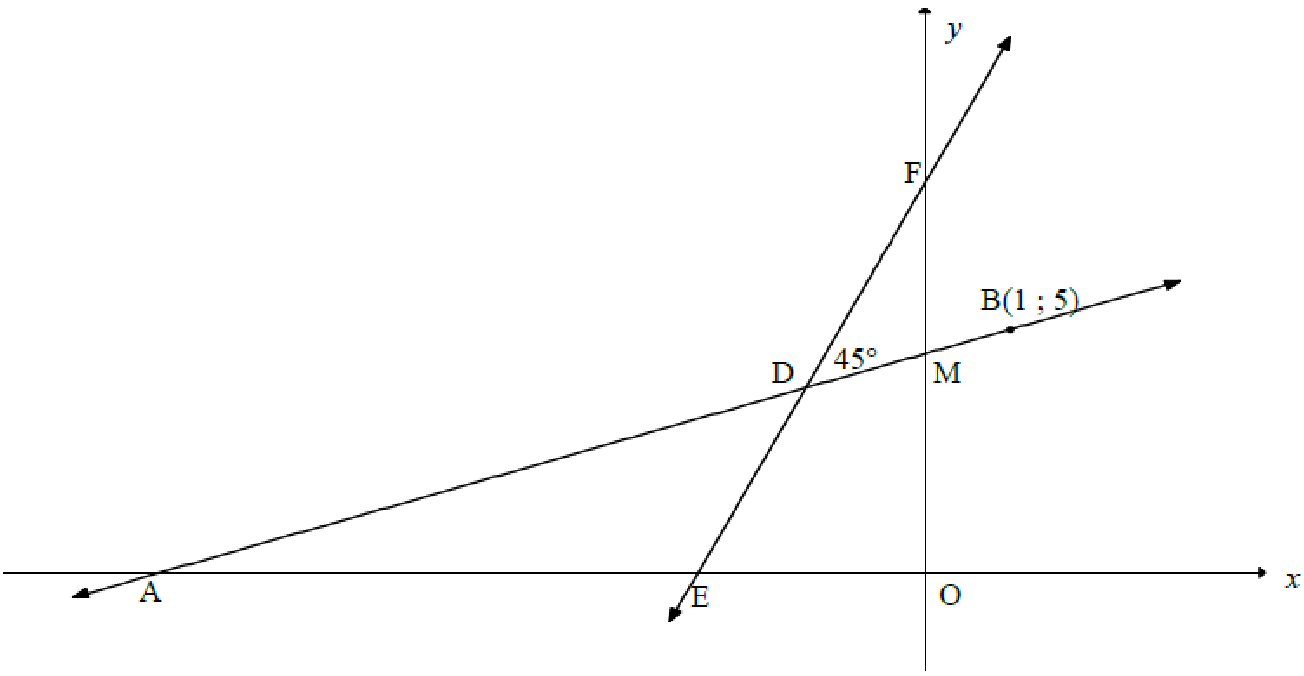


3.1	$r = MN = 5$	✓ answer/*antw* (1)
3.2	$(x-5)^2 + (y-4)^2 = 25$	✓equation/*vgl* (1)
3.3	A(x ; 0) $(x-5)^2 + (0-4)^2 = 25$ $x^2 - 10x + 25 + 16 = 25$ $x^2 - 10x + 16 = 0$ $(x-8)(x-2) = 0$ $\therefore x = 8$ or/*of* $x = 2$ $\therefore$ A(2 ; 0) **OR/*OF*** $(x-5)^2 + (0-4)^2 = 25$ $(x-5)^2 + 16 = 25$ $(x-5)^2 = 9$ $(x-5) = \pm 3$ $\therefore x = 8$ or/*of* $x = 2$ $\therefore$ A(2 ; 0)	✓ substitute into eq/ *vervang in vgl* $y = 0$ ✓ standard form/ *standaardvorm* or perfect square form/*kwadr vorm* ✓ answer/*antw* (3)
3.4.1	$m_{MB} = \frac{4-0}{5-8}$ $= -\frac{4}{3}$	✓ subst M and B into form/*vervang M and B in form* ✓ $m_{MB} = -\frac{4}{3}$ (2)

3.4.2	$m_{MB} \times m_{PB} = -1$ (tangent $\perp$ radius/ *rkl* $\perp$ *radius*) $m_{PB} = \frac{3}{4}$ $y = \frac{3}{4}x + c$ **OR/*OF*** $y - y_1 = \frac{3}{4}(x - x_1)$ $0 = \frac{3}{4}(8) + c$ $y - 0 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 6$ $y = \frac{3}{4}x - 6$	✓ $m_{MB} \times m_{PB} = -1$ ✓ $m_{PB} = \frac{3}{4}$ ✓ equation/*vgl* (3)
3.5	$y_K = y_M + r = 4 + 5$ $y = 9$	✓ 9 ✓ equation/*vgl* (2)
3.6	At/*By* L: $\frac{3}{4}x - 6 = 9$ $3x - 24 = 36$ $3x = 60$ $x = 20$ $\therefore$ L(20 ; 9)	✓ equating simultaneously ✓ simplification (2)
3.7	L(20 ; 9) $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **OR/OF** $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(20-5)^2 + (9-4)^2}$ $= \sqrt{(15)^2 + (5)^2}$ $= \sqrt{225 + 25}$ $= \sqrt{(5)^2(9+1)}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$	✓ correct subst into distance formula/ *korrekte subst in afstand-formule* ✓ answer in surd form/*antw in wortelvorm* (2)
3.8	**MK $\perp$ KL OR/OF $M\hat{K}L = 90°$** (radius $\perp$ tangent/*radius $\perp$ rkl*) $\therefore$ ML is a diameter as it subtends a right angle/*ML is middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore (x - 12{,}5)^2 + (y - 6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR/*OF***	✓ S ✓ value of/*waarde van r* ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ answer in correct form/ *antw in korrekte vorm* (5)

	MK ⊥ KL OR/OF $\hat{MKL} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*)	✓ S
	∴ML is a diameter as it subtends a right angle/*ML is middellyn*	
	Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML*	
	$x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$	✓ $x = 12{,}5$ ✓ $y = 6{,}5$
	Centre/*midpt*: (12,5 ; 6,5)	
	Equation of the circle KLM /*Vgl van sirkel KLM*:	
	$(x-12{,}5)^2 + (y-6{,}5)^2 = r^2$	
	subst (5 ; 4): $(5-12{,}5)^2 + (4-6{,}5)^2 = r^2$ $62{,}5 = r^2$	✓ value of/*waarde van* r^2
	$\therefore \quad (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$	✓ answer in correct form/*antw in korrekte vorm*
	OR/*OF*	(5)
	By symmetry about LM/*deur simmetrie om LM*:	
	MK ⊥ KL OR/OF $\hat{MKL} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*)	✓ S
	∴ML is a diameter as it subtends a right angle/*ML is middellyn*	
	ML is a diameter /*ML is 'n middellyn*	
	$r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or /of 7,91	✓ value of/*waarde van r*
	Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML*	
	$x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$	✓ $x = 12{,}5$ ✓ $y = 6{,}5$
	Centre/*midpt*: (12,5 ; 6,5)	
	Equation of the circle KLM /*Vgl van sirkel KLM*:	
	$\therefore \quad (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$	✓ answer in correct form/*antw in korrekte vorm*
		(5) **[21]**

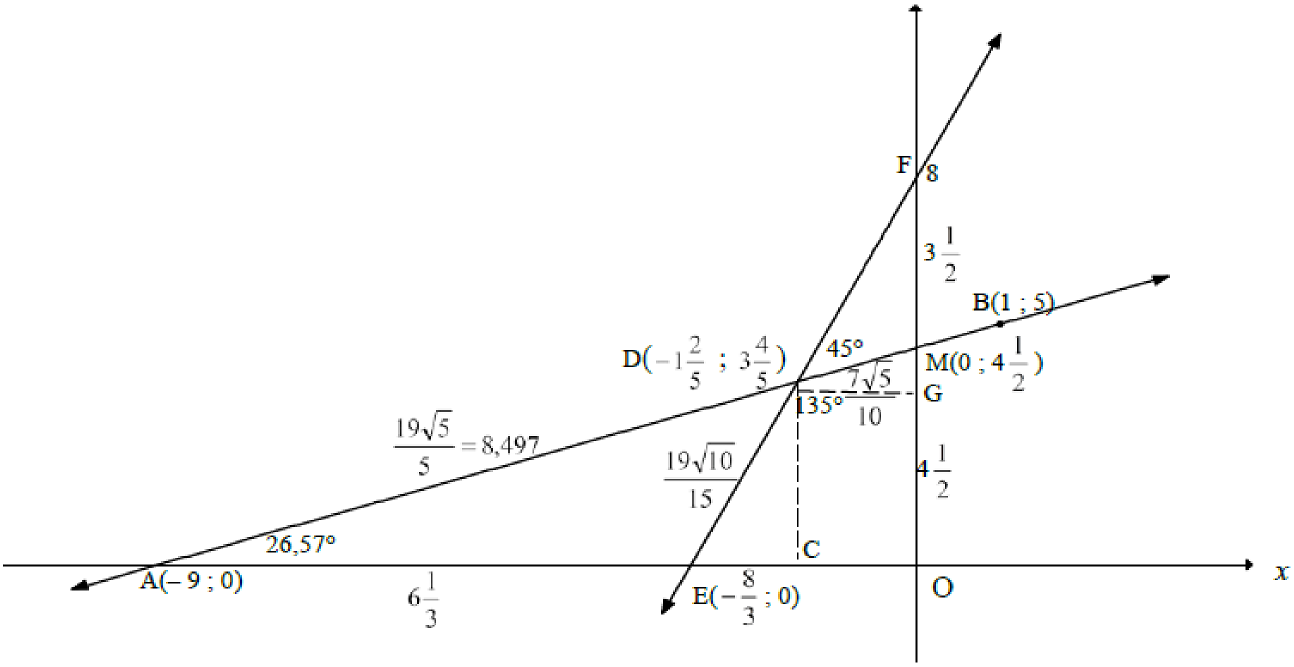
QUESTION/VRAAG 4



4.1	$y = 0$: $3x + 8 = 0$ $x = -\frac{8}{3}$ $\therefore E\left(-2\frac{2}{3};0\right)$ **OR/*OF*** $E\left(-\frac{8}{3};0\right)$	✓ *y*-value/*waarde* ✓ *x*-value/*waarde* (2)
4.2	$\tan D\hat{E}O = m_{DE} = 3$ $\therefore D\hat{E}O = 71{,}565... = 71{,}57°$ $D\hat{A}E = 71{,}565...° - 45°$ $= 26{,}57°$	✓ $\tan D\hat{E}O = 3$ ✓ 71,565…° ✓ 26,57° (3)
4.3	$m_{AB} = \tan 26{,}57°$ $= \frac{1}{2}$ $y = \frac{1}{2}x + c$ **OR/*OF*** $y - y_1 = \frac{1}{2}(x - x_1)$ $5 = \frac{1}{2}(1) + c$ $y - 5 = \frac{1}{2}(x - 1)$ $y = \frac{1}{2}x + 4\frac{1}{2}$ $y = \frac{1}{2}x + \frac{9}{2}$	✓ $m_{AB} = \tan 26{,}57°$ ✓ $m_{AB} = \frac{1}{2}$ ✓ subst of *m* and (1 ; 5)into formula/ *subst m en (1 ; 5) in formule* ✓ equation/*vgl* (4)

4.4	Solve $x - 2y + 9 = 0$ and $y = 3x + 8$ simultaneously:	
	$x - 2(3x+8) + 9 = 0$	✓ subst/*vervang*
	$x - 6x - 16 + 9 = 0$	
	$-5x = 7$	
	$x = -1\frac{2}{5}$	✓ x-value/*waarde*
	$\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $-1\frac{2}{5} - 2y + 9 = 0$	✓ subst/*vervang*
	$y = 3\frac{4}{5}$ $\quad$ $y = 3\frac{4}{5}$	✓ y-value/*waarde*
		(4)
	$\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	
	OR/OF	
	$x = 2y - 9$	
	$y = 3(2y - 9) + 8$	✓ subst/*vervang*
	$y = 6y - 27 + 8$	
	$\therefore y = 3\frac{4}{5}$	✓ y value/*waarde*
	$x = 2(3\frac{4}{5}) - 9$ **OR/OF** $3\frac{4}{5} = 3x + 8$	✓ subst/*vervang*
	$x = -1\frac{2}{5}$ $\quad$ $x = -1\frac{2}{5}$	✓ x-value/*waarde*
	$\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	
	OR/*OF*	(4)
	$3x + 8 = \frac{1}{2}x + 4\frac{1}{2}$	✓ equating/*gelyk stel*
	$6x + 16 = x + 9$	
	$5x = -7$	
	$\therefore \quad x = -1\frac{2}{5}$	✓ x value/*waarde*
	$\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $y = \frac{1}{2}(-1\frac{2}{5}) + 4\frac{1}{2}$	✓ subst/*vervang*
	$y = 3\frac{4}{5}$ $\quad$ $y = 3\frac{4}{5}$	✓ y-value/*waarde*
	$\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	
		(4)
	OR/*OF*	

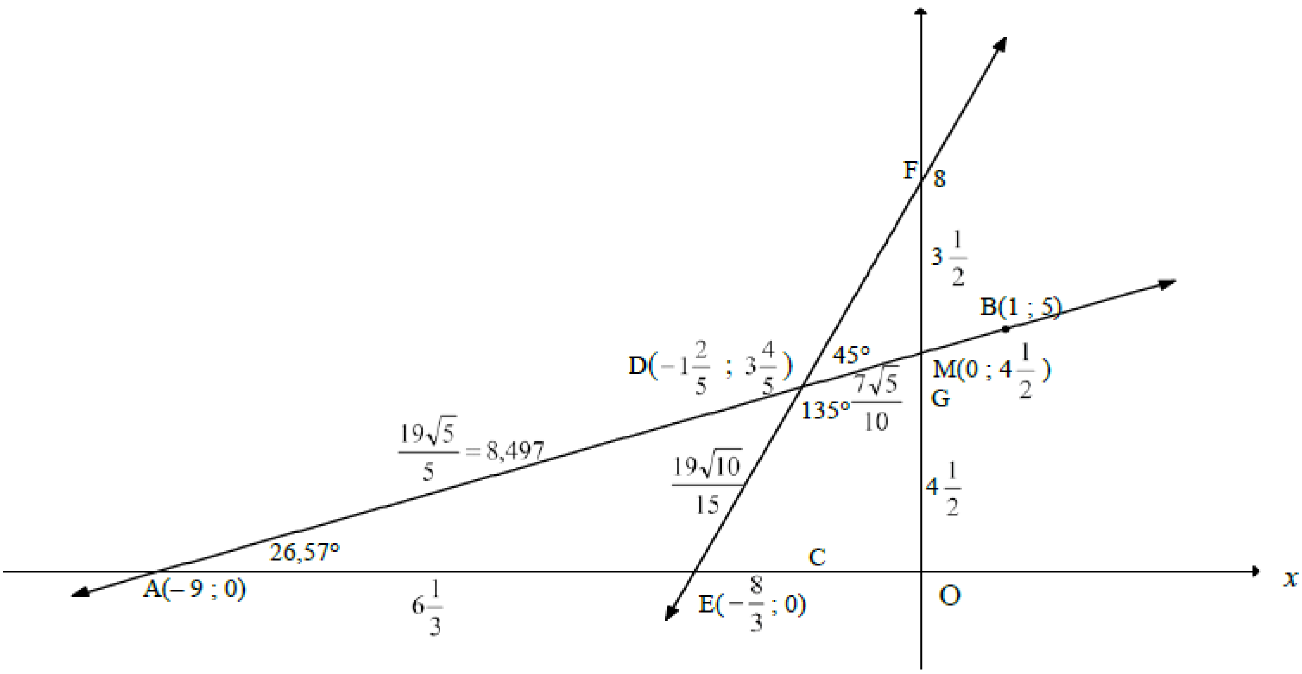
	$x-2y=-9$ ……(1) $-6x+2y=16$ ……(2) (1) + (2): $-5x=7$ $\therefore x=-1\frac{2}{5}$ $\therefore -1\frac{2}{5}-2y=-9$ **OR/*OF*** $y=3(-1\frac{2}{5})+8$ $y=3\frac{4}{5}$ $y=3\frac{4}{5}$ $\therefore \text{D}(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ adding/*optelling* ✓ x-value/*waarde* ✓ subst/*vervang* ✓ y-value/*waarde* (4)
	OR/*OF* $y=3x+8$ ……….(1) $6y=3x+27$ …….(2) (1) – (2): $-5y=-19$ $\therefore y=3\frac{4}{5}$ $3\frac{4}{5}=3x+8$ **OR/*OF*** $x=2(3\frac{4}{5})-9$ $x=-1\frac{2}{5}$ $x=-1\frac{2}{5}$ $\therefore \text{D}(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ subtracting/*aftrekking* ✓ y-value/*waarde* ✓ subst/*vervang* ✓ x-value/*waarde* (4)



4.5	area DMOE = area ΔAMO – area ΔADE $x_A = 2(0) - 9 \quad \therefore A(-9\ ;\ 0)$ area ΔAMO $= \frac{1}{2}.\ AO.\ OM$ $= \frac{1}{2}(9)(4\frac{1}{2})$ $= 20{,}25$ area ΔADE $= \frac{1}{2}.\ AE\ .\ y_D$ $= \frac{1}{2}.\ (AO - EO).\ y_D$ $= \frac{1}{2}\left(9 - 2\frac{2}{3}\right)\left(3\frac{4}{5}\right)$ $= 12{,}03$ **OR/OF** area ΔADE $= \frac{1}{2}AD.AE.\sin\ D\hat{A}E$ $= \frac{1}{2}\ (\frac{19\sqrt{5}}{5}).6\frac{1}{3}.\sin 26{,}57°$ $= 12{,}03$ $\therefore$ area DMOE = 8,22 square units/*vk eenh* **OR/*OF***	✓ correct method/ *korrekte metode* ✓ $x_A = -9$ ✓ $\frac{1}{2}(9)(4\frac{1}{2})$ ✓ $AE = 9 - 2\frac{2}{3} = 6\frac{1}{3}$ ✓ $y_D = 3\frac{4}{5}$ **OR/OF** ✓ $AD = \frac{19\sqrt{5}}{5}$ ✓ $AE = 6\frac{1}{3}$ ✓ answer/*antw* (6)

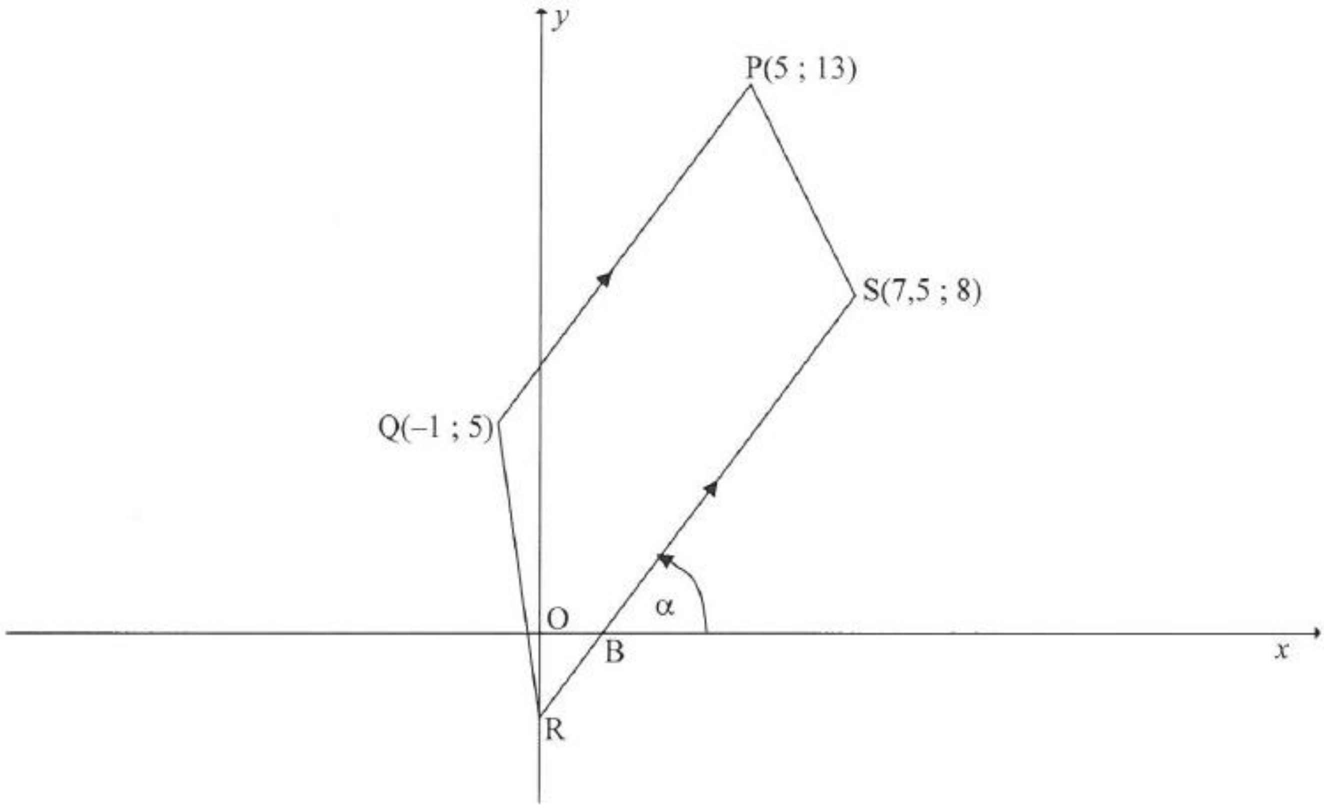
	area DMOE = area rectangle DCOG + area ΔDMG + area ΔDEC $= (1\frac{2}{5} \times 3\frac{4}{5}) + \frac{1}{2}\left(1\frac{2}{5}\right)(\frac{7}{10}) + \frac{1}{2}(3\frac{4}{5})(\frac{19}{15})$ = 8,22 square units/*vk eenh*	✓ correct method/ *korrekte metode* ✓ $3\frac{4}{5}$ ✓ $1\frac{2}{5}$ ✓ 0,7 ✓ $\frac{19}{15}$ ✓ answer (6)
	OR/*OF*	
	area DMOE = area ΔEDO + area ΔODM $= \frac{1}{2}(EO \times y_D) + \frac{1}{2}(OM \times -x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times \frac{19}{5}\right) + \left(\frac{9}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{304+189}{30}\right)$ $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*vk eenh*	✓ correct method/ *korrekte metode* ✓ $y_D = \frac{19}{5}$ or $3\frac{4}{5}$ ✓ $EO = \frac{8}{3}$ ✓ $-x_D = \frac{7}{5}$ ✓ $OM = \frac{9}{2}$ or $4\frac{1}{2}$ ✓ answer/*antw* (6)
	OR/*OF*	
	area DMOE = area ΔEOF – area ΔDMF $= \frac{1}{2}(EO \times OF) - \frac{1}{2}(OF - OM)(-x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times 8\right) + \left(\frac{7}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{640-147}{30}\right)$ $= \frac{493}{60}$ or $8\frac{13}{60}$ or 8,22 square units/*vk eenh*	✓ correct method/ *korrekte metode* ✓ $y_F = 8$ ✓ $EO = \frac{8}{3}$ ✓ $-x_D = \frac{7}{5}$ ✓ $FM = 3\frac{1}{2}$ ✓ answer/*antw* (6)
	OR/*OF*	

	$\text{area } \Delta EOM = \frac{1}{2}(EO \times OM)$ $= \frac{1}{2}\left(\frac{8}{3} \times \frac{9}{2}\right)$ $= 6$ sq units/*vk eenh* $ED = \sqrt{\left(-\frac{7}{5}+\frac{8}{3}\right)^2 + \left(\frac{19}{5}\right)^2}$ and $DM = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{9}{2}-\frac{19}{5}\right)^2}$ $= \frac{19\sqrt{10}}{15}$ or 4,005... $\quad = \frac{7\sqrt{5}}{10}$ or 1,565.. $\text{area } \Delta EDM = \frac{1}{2}(ED \times DM \times \sin E\hat{D}M)$ $= \frac{1}{2}\left(\frac{19\sqrt{10}}{15}\right)\left(\frac{7\sqrt{5}}{10}\right)\sin 135°$ $= \frac{133}{60}$ or 2,216... $\therefore$ area DMOE = area ΔEOM + area ΔEDM $= 6 + 2,216...$ $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*eenh*^{2}	✓ area ΔEOM ✓ $ED = \frac{19\sqrt{10}}{15}$ ✓ $DM = \frac{7\sqrt{5}}{10}$ ✓ area ΔEDM ✓ correct method/ *korrekte metode* ✓ answer/*antw* (6) **[19]**



QUESTION 3

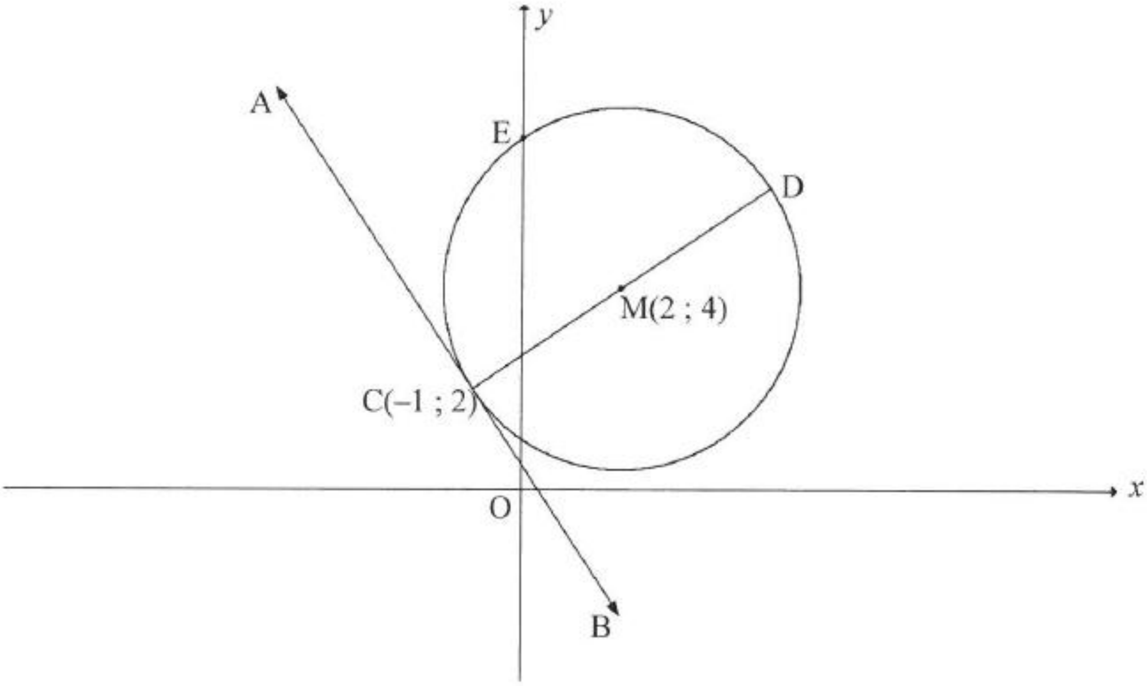
In the diagram below points P(5 ; 13), Q(–1 ; 5) and S(7,5 ; 8) are given. SR | | PQ where R is the y-intercept of SR. The x-intercept of SR is B. QR is joined.



3.1	Calculate the length of PQ.	(3)
3.2	Calculate the gradient of PQ.	(2)
3.3	Determine the equation of line RS in the form $ax + by + c = 0$.	(4)
3.4	Determine the x-coordinate of B.	(2)
3.5	Calculate the size of $O\hat{R}B$.	(3)
3.6	Prove that QBSP is a parallelogram.	(4)
		[18]

QUESTION 4

4.1 In the diagram below, the circle centred at M(2 ; 4) passes through C(–1 ; 2) and cuts the y-axis at E. The diameter CMD is drawn and ACB is a tangent to the circle.



4.1.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.1.2 Write down the coordinates of D. (2)

4.1.3 Determine the equation of AB in the form $y=mx+c$. (5)

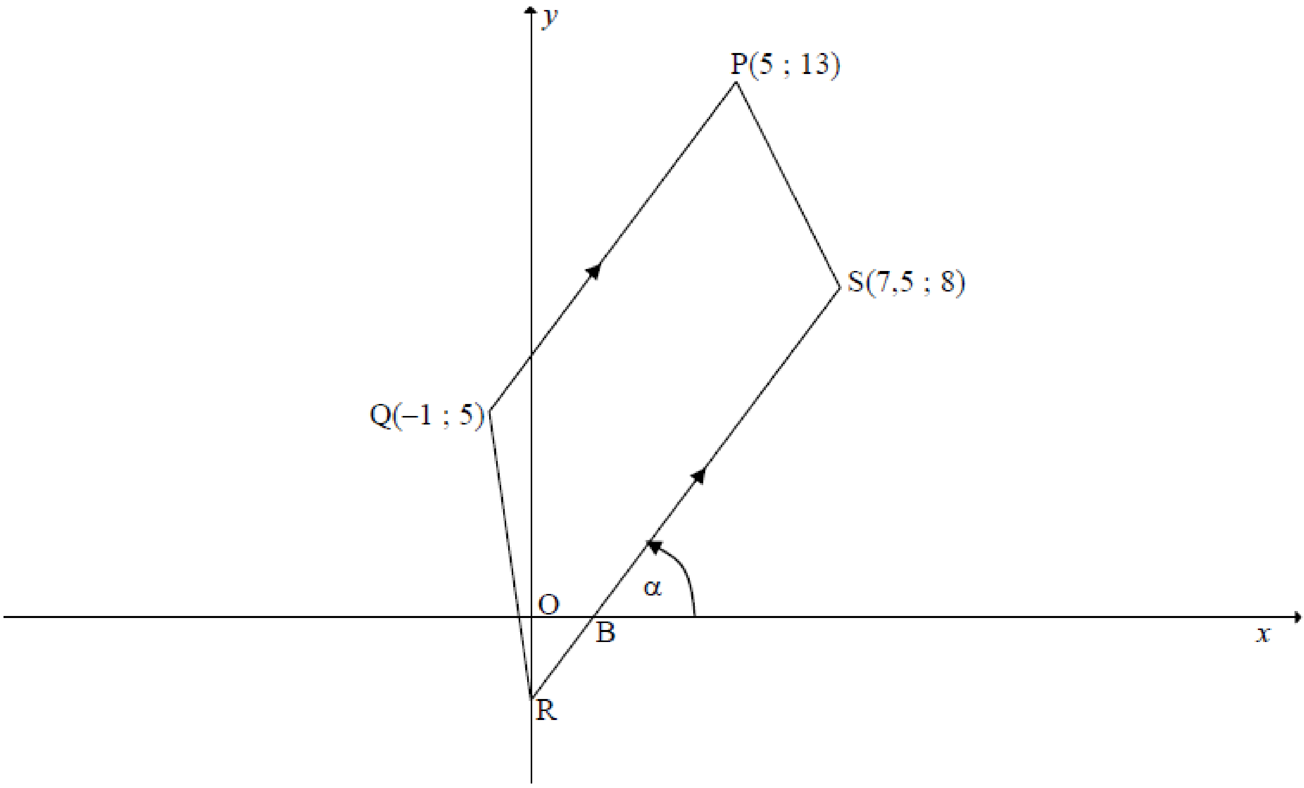
4.1.4 Calculate the coordinates of E. (4)

4.1.5 Show that EM is parallel to AB. (2)

4.2 Determine whether or not the circles having equations $(x+2)^2+(y-4)^2=25$ and $(x-5)^2+(y+1)^2=9$ will intersect. Show ALL calculations. (6)

[22]

QUESTION/VRAAG 3

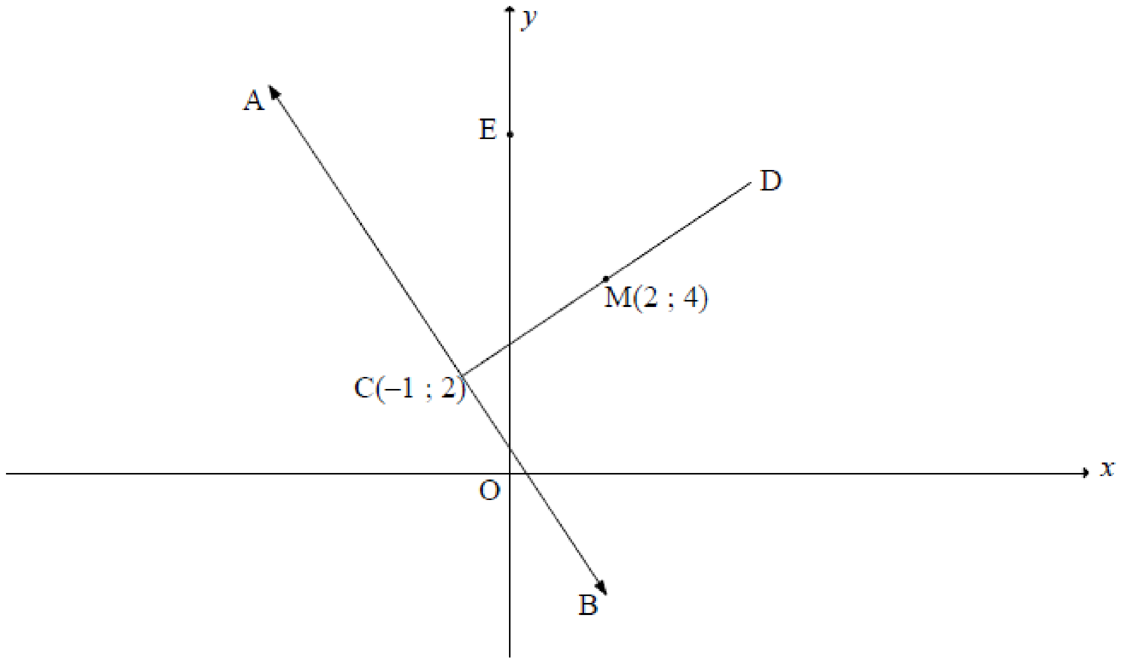


3.1	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5+1)^2 + (13-5)^2}$ $= 10$	✓ use of distance formula/*gebruik afstandformule* ✓ correct subst into form/*korrekte subst in formule* ✓ 10 (3)
3.2	$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{13-5}{5-(-1)}$ $= \frac{8}{6} = \frac{4}{3}$ Answer only: Full marks *slegs antw: volpunte*	✓ correct subst into gradient formula/ *korrekte subst in gradiëntformule* ✓ gradient/*gradiënt* (2)

3.3	Equation of line RS/*Vgl van lyn RS*: $m_{RS} = m_{PQ} = \frac{4}{3}$ (= gradients, || lines/=*gradiënte*, || *lyne*) $y = mx + c$ $8 = \frac{4}{3}\left(\frac{15}{2}\right) + c$ $c = -2$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $y - 8 = \frac{4}{3}\left(x - \frac{15}{2}\right)$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$	✓ $m_{RS} = \frac{4}{3}$ ✓ subst of S(7,5 ; 8) and m into eq /*subst van S(7,5 ; 8) en m in vgl* ✓ value of c /*waarde van c* or/of st form/*st vorm* ✓ equation/*vgl* (4)
3.4	B is the x-intercept of/*is die x-afsnit van* $y = \frac{4}{3}x - 2$ $0 = \frac{4}{3}x - 2$ $4x - 6 = 0$ $x = \frac{3}{2}$ **OR/*OF*** $4x - 3(0) - 6 = 0$ $4x - 6 = 0$ $x = \frac{3}{2}$	✓ $y = 0$ ✓ $x = \frac{3}{2}$ (2)
3.5	$\tan\ \alpha = \frac{4}{3}$ $\alpha = 53{,}13° = O\hat{B}R$ (vert opp ∠s/*regoorst* ∠*e*) $O\hat{R}B = 180° - (90° + 53{,}13°)$ (∠s of Δ/∠*e van Δ*) $= 36{,}87°$	✓ $\tan\ \alpha = \frac{4}{3}$ ✓ 53,13° ✓ 36,87° (3)
3.6	$BS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{15}{2} - \frac{3}{2}\right)^2 + (8 - 0)^2}$ $= 10$ PQ || BS and/*en* PQ = BS PQBS = parallelogram (1 pair opp sides = and ||/*1 pr tos sye =en* ||) **OR/*OF*** midpoint of/*midpt van* QS: $\left(\frac{-1+7.5}{2};\frac{5+8}{2}\right) = \left(\frac{13}{4};\frac{13}{2}\right)$ midpoint of/*midpt van* PB: $\left(\frac{5+1.5}{2};\frac{13+0}{2}\right) = \left(\frac{13}{4};\frac{13}{2}\right)$ PQBS = parallelogram (diags bisect each other/*hoekl halv mekaar)* **OR/*OF***	✓ correct subst into form/*korrekte subst in formule* ✓ BS = 10 ✓ BS = PQ ✓ reason/*rede* (4) ✓ $\left(\frac{-1+7.5}{2};\frac{5+8}{2}\right)$ ✓ $\left(\frac{5+1.5}{2};\frac{13+0}{2}\right)$ ✓ $\left(\frac{13}{4};\frac{13}{2}\right)$ ✓ reason/*rede* (4)

	$m_{QB} = \frac{5-0}{-1-1,5} = \frac{5}{-2,5} = -2$ $m_{PS} = \frac{13-8}{5-7,5} = \frac{5}{-2,5} = -2$ $m_{QB} = m_{PS}$ $\therefore$ QB || PS PQ || BS PQBS = parallelogram (both pairs opp sides ||/*beide pr tos sye* ||)	✓ m_{QB} ✓ m_{PS} ✓ QB || PS ✓ reason/*rede* (4)
	OR/*OF* $BS = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{\left(\frac{15}{2}-\frac{3}{2}\right)^2+(8-0)^2}$ $\therefore$ PQ = BS $= 10$ $QB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(-1-1,5)^2+(5-0)^2} = \sqrt{(2,5)^2+(5)^2} = \frac{5\sqrt{5}}{2}$ or 5,59 $PS = \sqrt{(5-7,5)^2+(13-8)^2} = \sqrt{(2,5)^2+(5)^2} = \frac{\sqrt{125}}{2}$ or 5,59 QB = PS PQBS = parallelogram (both pairs opp sides =/ *beide pr tos sye* =)	✓ correct subst into form/*korrekte subst in formule* ✓ PQ = 10 ✓ QB = PS ✓ reason/*rede* (4) **[18]**

QUESTION/VRAAG 4



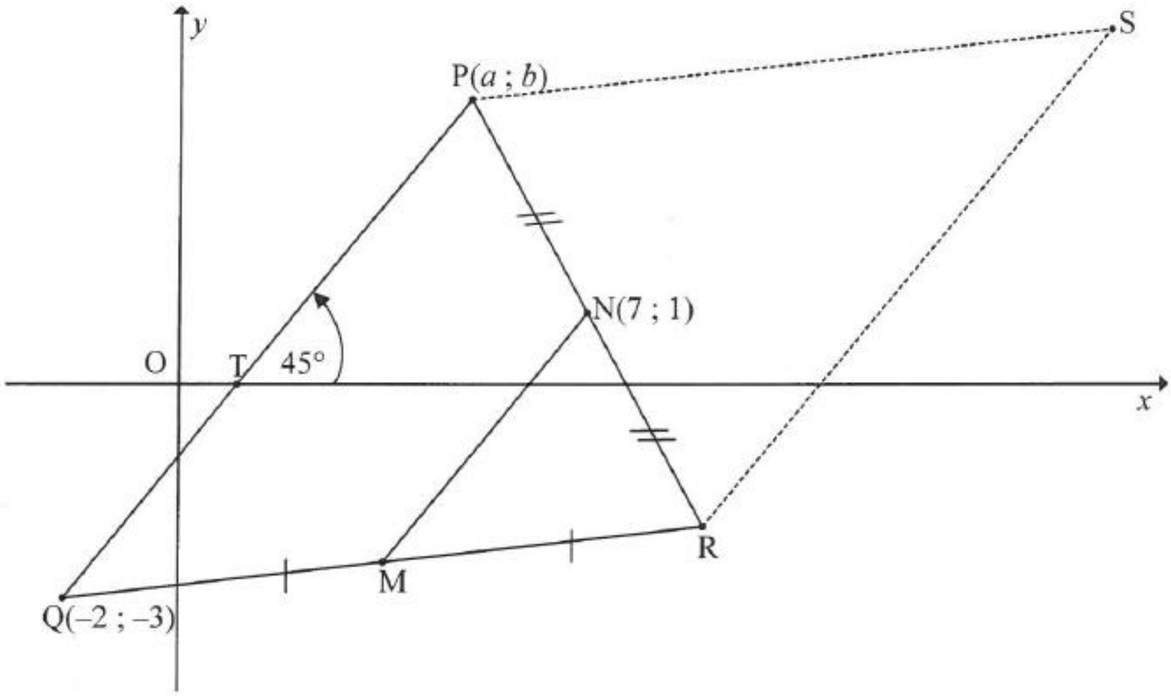
4.1.1	Radius = $\sqrt{(2+1)^2+(4-2)^2}$ $r = \sqrt{13}$ Equation of circle/*vgl van sirkel*: $(x-2)^2+(y-4)^2=13$ **OR/*OF*** $(x-2)^2+(y-4)^2=r^2$ $(-1-2)^2+(2-4)^2=r^2$ $r^2=13$ $\therefore\ (x-2)^2+(y-4)^2=13$	✓ $\sqrt{(2+1)^2+(4-2)^2}$ or/*of* $\sqrt{13}$ ✓ $(x-2)^2+(y-4)^2$ ✓ 13 (3) ✓ $(x-2)^2+(y-4)^2$ ✓ $(-1-2)^2+(2-4)^2$ ✓ 13 (3)
4.1.2	At/*by* D: $\frac{-1+x_D}{2}=2$ $\frac{2+y_D}{2}=4$ $-1+x_D=4$ and/*en* $2+y_D=8$ $x_D=5$ $y_D=6$ D(5 ; 6) **OR/*OF*** By inspection/*deur inspeksie*: D(5 ; 6)	✓ x – value/*waarde* ✓ y - value/*waarde* (2) ✓ x - value/*waarde* ✓ y - value/*waarde* (2)

4.1.3	$m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ $m_{AB} \times m_{MC} = -1$ (Tangent ⊥ radius/*raaklyn ⊥ radius*) $m_{AB} = -\frac{3}{2}$ $y - y_1 = m(x - x_1)$ ***OR/OF*** $y = mx + c$ $y - 2 = -\frac{3}{2}(x+1)$ $2 = -\frac{3}{2}(-1) + c$ $y = -\frac{3}{2}x + \frac{1}{2}$ $y = -\frac{3}{2}x + \frac{1}{2}$	✓ $m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ ✓ $m_{AB} \times m_{MC} = -1$ ✓ $m_{AB} = -\frac{3}{2}$ ✓subst m and $(-1 ; 2)$ into eq /*subst m en (–1 ; 2) in vgl* ✓ eq in standard form/ *vgl in st vorm* (5)
4.1.4	At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $(y-4)^2 = 9$ $y - 4 = \pm 3$ $y = 7$ or $y = 1$ E(0 ; 7) **OR/*OF*** At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $4 + y^2 - 8y + 16 = 13$ $y^2 - 8y + 7 = 0$ $(y-7)(y-1) = 0$ $y = 7$ or $y = 1$ E(0 ; 7)	✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ y - values/*waardes* ✓ E(0 ; 7) (4) ✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ y - values/*waardes* ✓ E(0 ; 7) (4)
4.1.5	$m_{EM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-7}{2-0}$ $= -\frac{3}{2}$ $m_{AB} = -\frac{3}{2}$ $\therefore$ EM || AB ($m_{EM} = m_{AB}$)	✓ $m_{EM} = -\frac{3}{2}$ ✓ reason/*rede* (2)

4.2	The centres of the circles are / *Die middelpunte van die sirkels is* P(–2 ; 4) and / *en* Q(5 ; –1) $QP^2 = (-2-5)^2 + (4-(-1))^2$ $QP = \sqrt{74} \approx 8{,}60$ units $r_M + r_P = 5 + 3$ $= 8$ $\therefore\ r_M + r_P < QP$ ∴ The two circles do not intersect/*Die twee sirkels sny nie*	✓ both centres/*albei Midpte* ✓ QP ✓ correct subst into form/*korrekte subst in formule* ✓ distance between 2 centres/*afstand tussen 2 midpte* ✓✓ $r_M + r_P < QP$ (6) **[22]**

QUESTION 3

In the diagram below, the line joining Q(–2 ; –3) and P(a ; b), a and $b > 0$, makes an angle of 45° with the positive x-axis. QP = $7\sqrt{2}$ units. N(7 ; 1) is the midpoint of PR and M is the midpoint of QR.



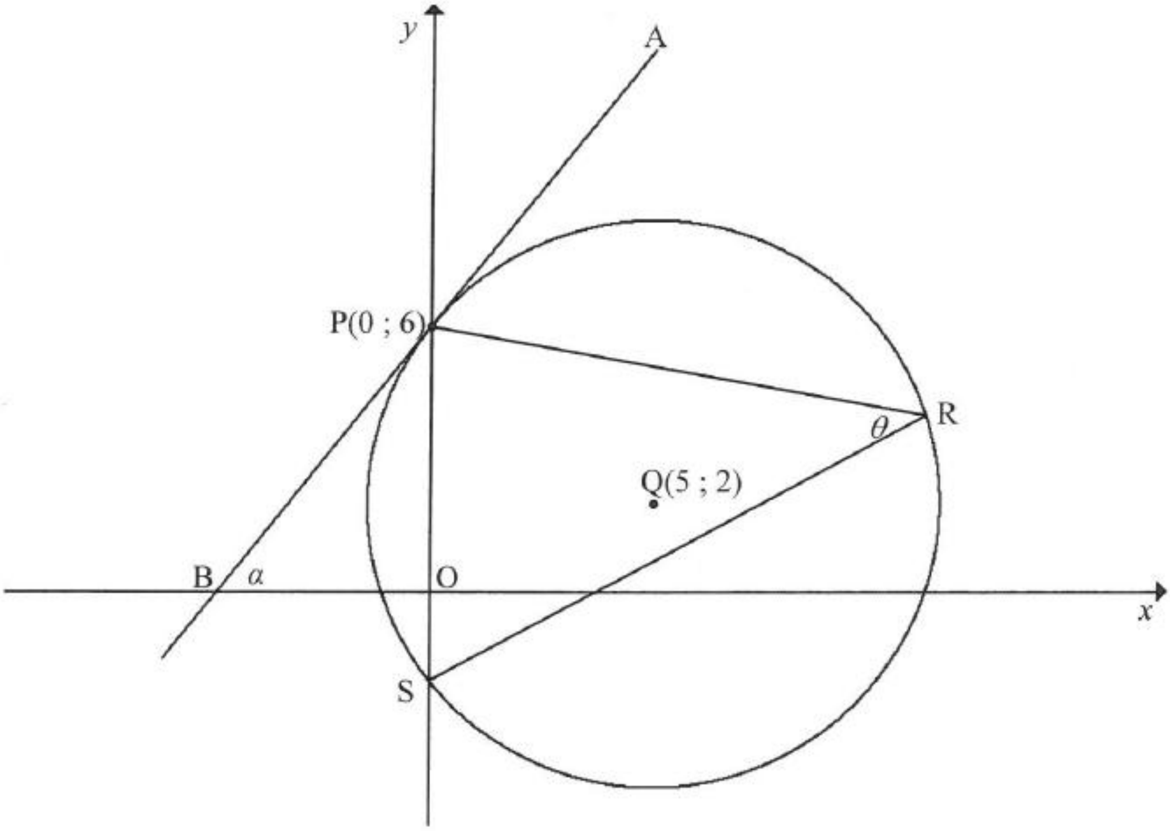
Determine:

3.1	The gradient of PQ	(2)
3.2	The equation of MN in the form $y = mx + c$ and give reasons	(4)
3.3	The length of MN	(2)
3.4	The length of RS	(1)
3.5	The coordinates of S such that PQRS, in this order, is a parallelogram	(3)
3.6	The coordinates of P	(6)

[18]

QUESTION 4

In the diagram below, Q(5 ; 2) is the centre of a circle that intersects the y-axis at P(0 ; 6) and S. The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $P\hat{R}S = \theta$.



4.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.2 Calculate the coordinates of S. (3)

4.3 Determine the equation of the tangent APB in the form $y = mx + c$. (4)

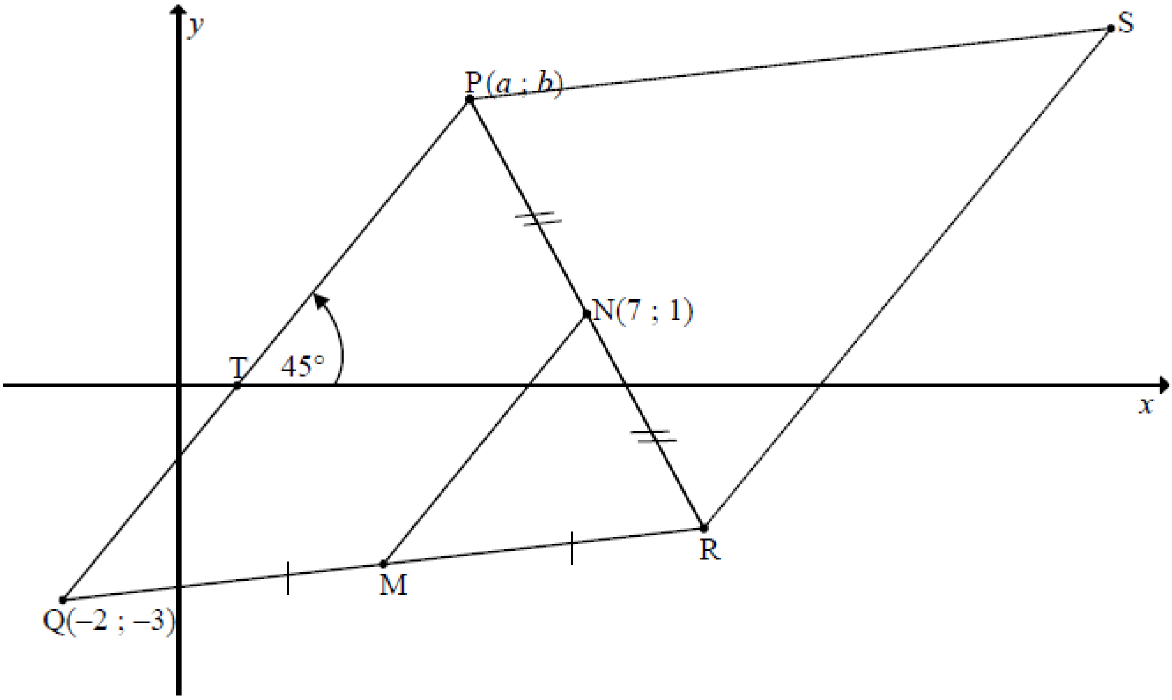
4.4 Calculate the size of α. (2)

4.5 Calculate, with reasons, the size of θ. (4)

4.6 Calculate the area of ΔPQS. (4)

[20]

QUESTION/*VRAAG* 3



3.1	$m_{PQ} = \tan 45°$ $= 1$	✓ $m = \tan 45°$ ✓ answ/*antw* (2)
3.2	MN | | QP [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore \quad y = x - 6$ **OR/*OF*** MN | | PQ [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore \quad y = x - 6$	✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4) ✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4)
3.3	$\text{MN} = \frac{1}{2}\text{PQ}$ [midpoint theorem/midp stelling] $\therefore \text{MN} = \frac{7\sqrt{2}}{2} \approx 4{,}95$	✓ S ✓ answ/antw (2)

<table>
<tr>
<td>3.5</td>
<td>QN = NS [diag of ||m/hoekl van ||m]
$\frac{-2+x_S}{2}=7$ and/en $\frac{-3+y_S}{2}=1$
$\therefore\ x_S = 16$ $\therefore\ y_S = 5$
OR/OF
QN = NS [diag of ||m/hoekl van ||m]
∴ by inspection/deur inspeksie:
S(16 ; 5)</td>
<td>✓ method/metode
✓ x-value/waarde
✓ y-value/waarde
(3)

✓ method/metode
✓ x-value/waarde
✓ y-value/waarde
(3)</td>
</tr>
<tr>
<td>3.6</td>
<td>Equation of/Vgl van PQ: $y = x + c$
$-3 = -2 + c$
$y = x - 1$ $\therefore a = b + 1$ .....(1)
From distance formula/Van afstandsformule:
$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$
$\therefore 98 = (a+2)^2 + (b+3)^2$ .....(2)
Subst (1) into (2):
$98 = (b+1+2)^2 + (b+3)^2$
$98 = b^2 + 6b + 9 + b^2 + 6b + 9$
$0 = 2b^2 + 12b - 80$
$0 = b^2 + 6b - 40$
$\therefore 0 = (b+10)(b-4)$
$\therefore\ b = 4$ (since $b > 0$)
Subst $b = 4$ into (1):
$\therefore a = 4 + 1 = 5$
∴ P(5 ; 4)

OR/OF
Equation of/Vgl van PQ: $y = x + c$
$-3 = -2 + c$
$y = x - 1$ $\therefore a = b + 1$ .....(1)
From distance formula/Van afstandsformule:
$7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$
$\therefore 98 = (a+2)^2 + (b+3)^2$ .....(2)
Subst (1) into (2):
$98 = (b+1+2)^2 + (b+3)^2$
$98 = 2(b+3)^2$
$49 = (b+3)^2$
$\pm 7 = b + 3$
$\pm 7 - 3 = b$
$\therefore\ b = 4$ (since $b > 0$)
Subst $b = 4$ into (1):
$\therefore a = 4 + 1 = 5$
∴ P(5 ; 4)</td>
<td>✓ eq of/vgl van PQ

✓ subst Q & $7\sqrt{2}$ into/in distance formula/afstandsformule

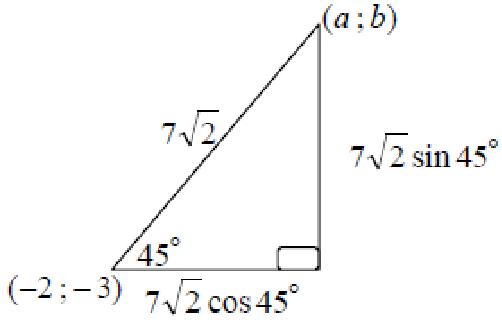
✓ subst eq of/vgl v. PQ
✓ st form/st vorm

✓ value of/waarde van b
✓ value of/waarde van a
(6)

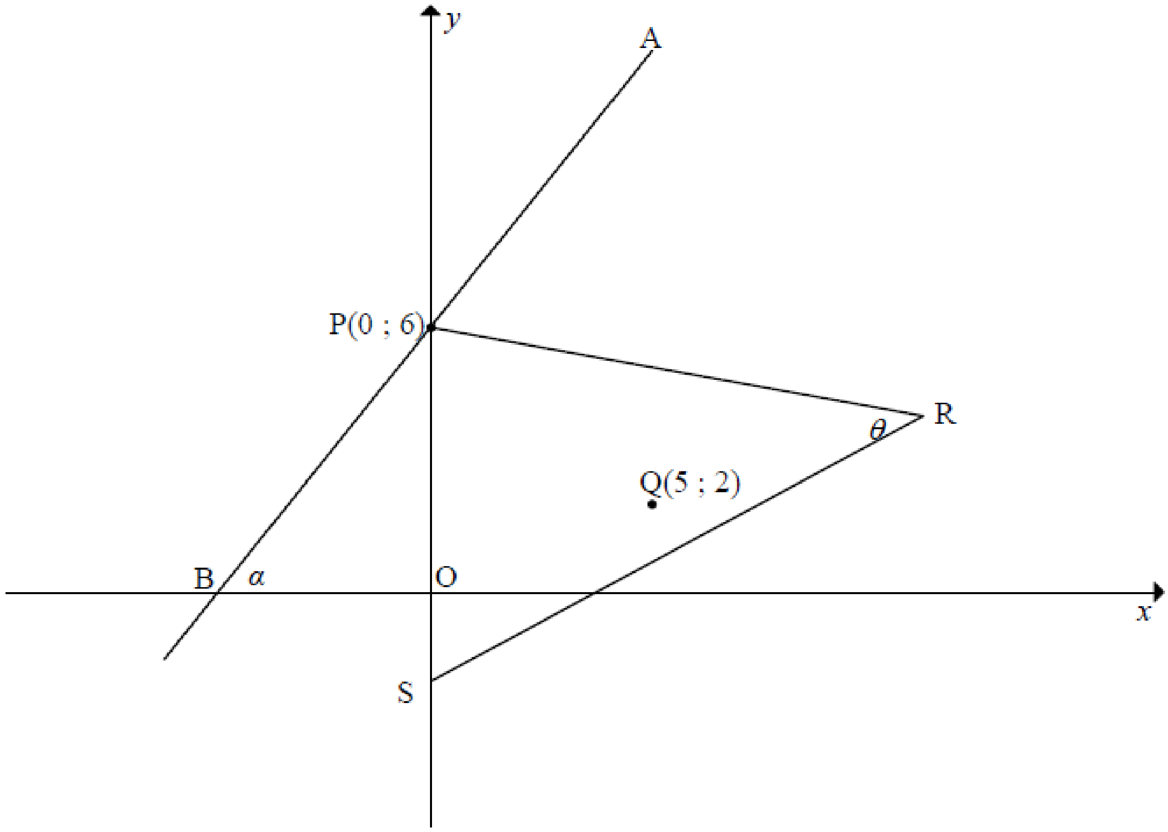
✓ eq of/vgl van PQ

✓ subst Q & $7\sqrt{2}$ into/in distance formula/afstandsformule
✓ subst eq of/vgl v. PQ
✓ simplification/vereenvoudig

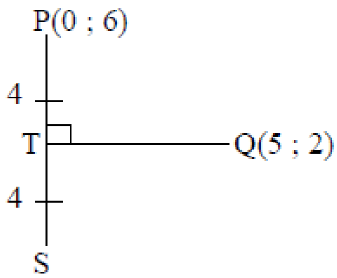
✓ value of/waarde van b
✓ value of/waarde van a
(6)</td>
</tr>
</table>

	OR/*OF*	
	Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\quad\therefore a = b + 1$ .....(1)	✓ eq of/*vgl van* PQ
	From distance formula/*Van afstandsformule*: $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$	✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule*
	$98 = (a+2)^2 + (a-1+3)^2$ $= 2(a+2)^2$	✓ subst eq of/*vgl v.* PQ
	$\therefore a + 2 = 7$ (since/*aangesien* $a > 0$)	✓ simplification/ *vereenvoudig*
	$\therefore a = 5$ Subst $a = 4$ into (1): $\therefore b = 5 - 1 = 4$ $\therefore$ P(5 ; 4)	✓ value of/*waarde van a* ✓ value of/*waarde van b* (6)
	OR/*OF*	
		✓✓✓✓
	$a = -2 + 7\sqrt{2}\cos 45^\circ = 5$	✓
	$b = -3 + 7\sqrt{2}\sin 45^\circ = 4$	✓ (6) **[17]**

QUESTION/*VRAAG* 4



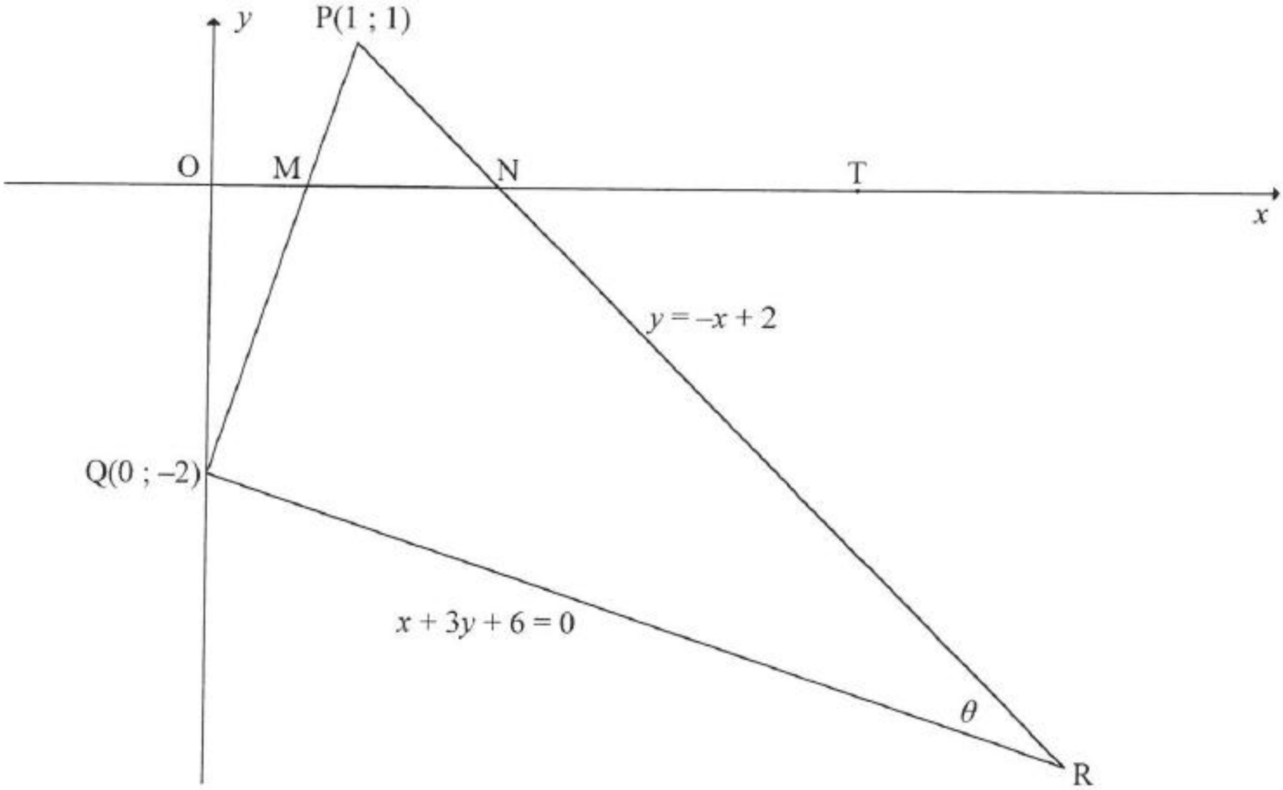
4.1	$(x-5)^2+(y-2)^2=r^2$ $(0-5)^2+(6-2)^2=r^2$ $25+16=r^2$ $41=r^2$ $\therefore\ (x-5)^2+(y-2)^2=41$ **OR/*OF*** $PQ=\sqrt{(0-5)^2+(6-2)^2}$ $=\sqrt{25+16}$ $r=\sqrt{41}$ $\therefore\ (x-5)^2+(y-2)^2=41$	✓ subst (5 ; 2) into circle eq/*in sirkelvgl* ✓ value of/*waarde van* r^2 ✓ equation/*vgl* (3) ✓ subst (5 ; 2) & (0 ; 6) into dist. form/*in afst. form* ✓ value of/*waarde van* r ✓ equation/*vgl* (3)
4.2	$(0-5)^2+(y-2)^2=41$ $25+(y-2)^2=41$ $25+y^2-4y+4=41$ $y^2-4y-12=0$ $(y-6)(y+2)=0$ $y\neq 6\quad or/of\quad y=-2$ $\therefore$ S(0 ; –2) or $y=-2$	✓ $x=0$ ✓ st form/*st. vorm* ✓ answ/*antw* (neg value) (3)

	OR/*OF* $(0-5)^2+(y-2)^2 = 41$ $25+(y-2)^2 = 41$ $(y-2)^2 = 16$ $y-2 = \pm 4$ $y = 2 \pm 4$ $y \neq 6$ or / *of* $y = -2$ $\therefore$ S(0 ; –2)	✓ $x = 0$ ✓ square form/ *kwadraatvorm* ✓ answ/*antw* (neg value) (3)
	OR/*OF* Draw/*Trek* QT ⊥ PS PT = TS [line from centre ⊥ to chord/ *lyn van midpt ⊥ koord*] PT = $y_P - y_Q = 6 - 2 = 4$ $y_Q - y_S = 4$ $y_S = 2 - 4 = -2$ $\therefore$ S(0 ; –2)  	✓ $x = 0$ ✓✓ $y = -2$ (3)
4.3	$m_{PQ} = \frac{6-2}{0-5}$ $= -\frac{4}{5}$ $m_{PQ} \times m_{APB} = -1$ [tan/*raakl* ⊥ radius] $\therefore m_{APB} = \frac{5}{4}$ $\therefore y = \frac{5}{4}x + 6$	✓ subst (0 ; 6) & (5 ; 2) into grad form/*in grad. formule* ✓ m_{PQ} ✓ m_{APB} ✓ equation/*vgl* (4)
4.4	$\tan\alpha = \frac{5}{4}$ $\therefore \alpha = 51{,}34°$	✓ $\tan\alpha = m_{APB}$ ✓ answ/*antw* (2)
	OR/*OF* B(4,8 ; 0) $\therefore \tan\alpha = \frac{6}{4{,}8}$ $\therefore \alpha = 51{,}34°$	✓ $\tan\alpha = \frac{6}{4{,}8}$ ✓ answ/*antw* (2)

4.5	$\theta = B\hat{P}S$ [tan-chord th/*raakl-koordst.*] $= 90° - \alpha$ [∠ sum in Δ/∠ *som van* Δ] $= 90° - 51,34°$ $= 38,66°$ **OR/*OF*** PS = 8 $PQ = SQ = \sqrt{41}$ $PS^2 = PQ^2 + SQ^2 - 2.PQ.SQ.\cos P\hat{Q}S$ $64 = 41 + 41 - 2.41.\cos P\hat{Q}S$ $\cos P\hat{Q}S = \frac{18}{82}$ $P\hat{Q}S = 77,32°$ $\theta = \frac{1}{2} P\hat{Q}S$ [∠ at centre = 2 × ∠ circumf] $= 38,66°$	✓ S ✓ R ✓ 90° − α ✓ answ/*antw* (4) ✓ correct subst into cosine rule ✓ $P\hat{Q}S = 77,32°$ ✓ R ✓ answ/*antw* (4)
4.6	Area $\Delta PQS = \frac{1}{2}$ PS ×height/*hoogte* $= \frac{1}{2}(8)(5)$ = 20 sq units/*vk eenh* **OR/*OF*** $P\hat{Q}S = 2 \times 38,66°$ [∠at centre = 2×∠ at circum/ *midpts∠=2omtreks∠*] $= 77,32°$ Area $\Delta PQS = \frac{1}{2} PQ.QS.\sin P\hat{Q}S$ $= \frac{1}{2}.\sqrt{41}.\sqrt{41}.\sin 77,32°$ = 20 sq units/*vk eenh*	✓ area formula/*e*: ΔPQS ✓ PS = 8 ✓ ⊥*h* = 5 ✓ answ/*antw* (4) ✓ size of/*grootte v* $P\hat{Q}S$ ✓ area rule/*reël*: ΔPQS ✓ subst correctly/ *subst korrek* ✓ answ/*antw* (4) **[20]**

QUESTION 3

In the diagram below, P(1 ; 1), Q(0 ; –2) and R are the vertices of a triangle and $P\hat{R}Q = \theta$. The x-intercepts of PQ and PR are M and N respectively. The equations of the sides PR and QR are $y = -x + 2$ and $x + 3y + 6 = 0$ respectively. T is a point on the x-axis, as shown.

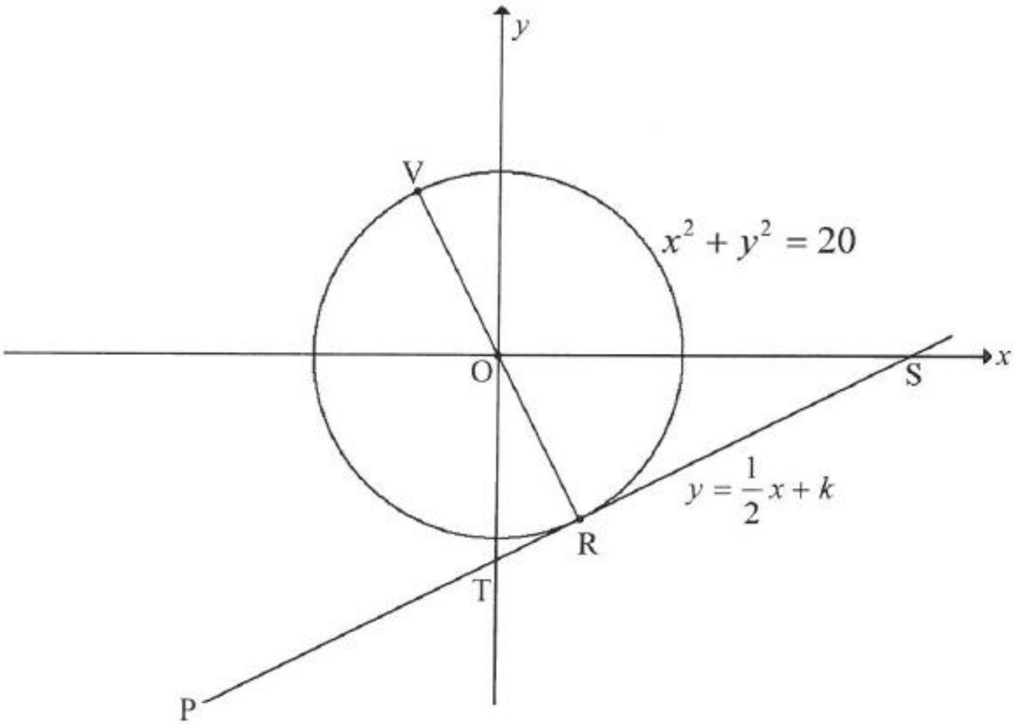


3.1	Determine the gradient of QP.	(2)
3.2	Prove that $P\hat{Q}R = 90°$.	(2)
3.3	Determine the coordinates of R.	(3)
3.4	Calculate the length of PR. Leave your answer in surd form.	(2)
3.5	Determine the equation of a circle passing through P, Q and R in the form $(x-a)^2 + (y-b)^2 = r^2$.	(6)
3.6	Determine the equation of a tangent to the circle passing through P, Q and R at point P in the form $y = mx + c$.	(3)
3.7	Calculate the size of θ.	(5)

[23]

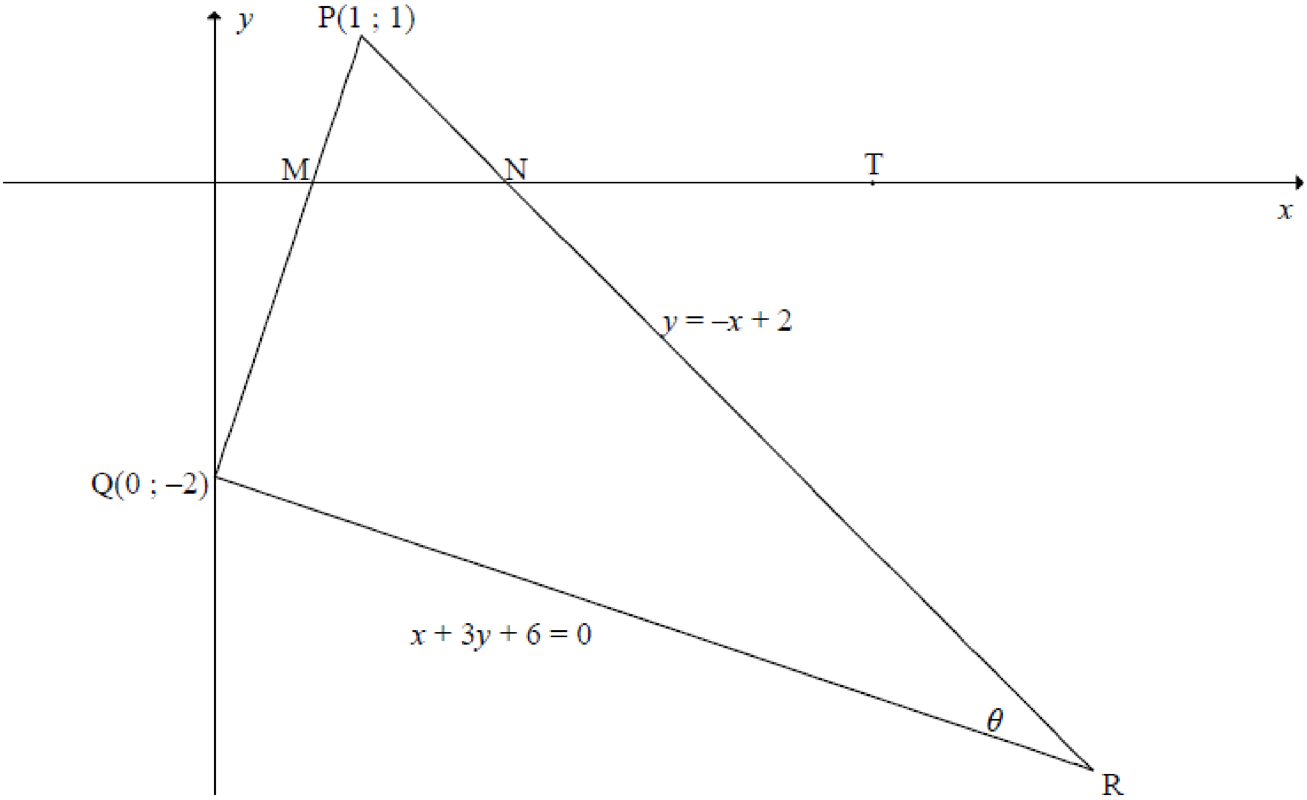
QUESTION 4

In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$. The tangent PRS to the circle at R has the equation $y = \frac{1}{2}x + k$. PRS cuts the y-axis at T and the x-axis at S.



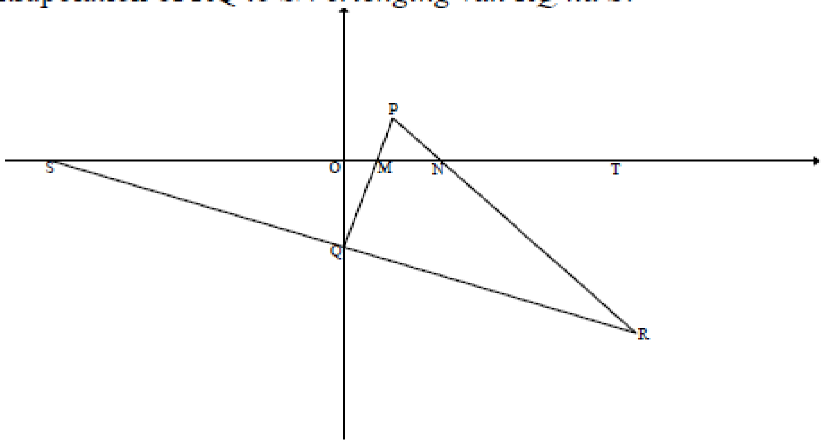
4.1	Determine, giving reasons, the equation of OR in the form $y = mx + c$.	(3)
4.2	Determine the coordinates of R.	(4)
4.3	Determine the area of ΔOTS, given that R(2 ; –4).	(6)
4.4	Calculate the length of VT.	(4)
		[17]

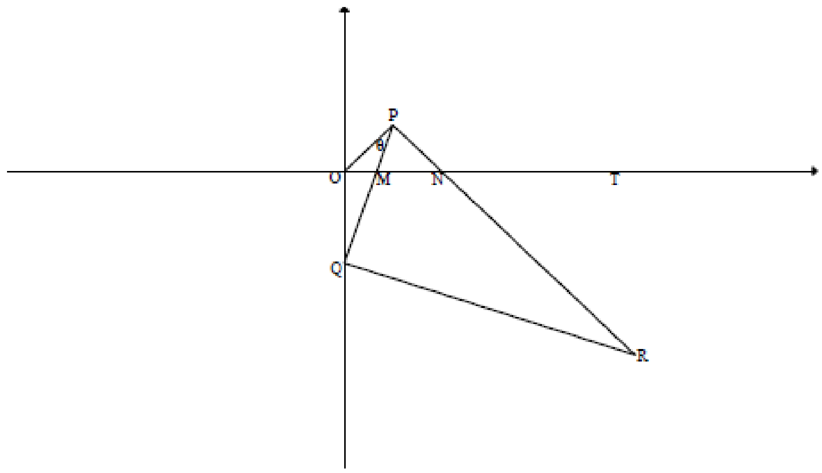
QUESTION/*VRAAG* 3



3.1	$m_{PQ} = \frac{1-(-2)}{1-0}$ $= 3$	✓subst (1 ; 1) & (0 ; –2) ✓answ/*antw* (2)
3.2	QR: $y = -\frac{1}{3}x - 2$ $\therefore m_{QR} = -\frac{1}{3}$ $m_{PQ} \times m_{QR} = 3 \times -\frac{1}{3}$ $= -1$ $\therefore PQ \perp QR \quad \therefore P\hat{Q}R = 90°$	✓ $m_{QR} = -\frac{1}{3}$ ✓ $m_{PQ} \times m_{QR} = -1$ (2)

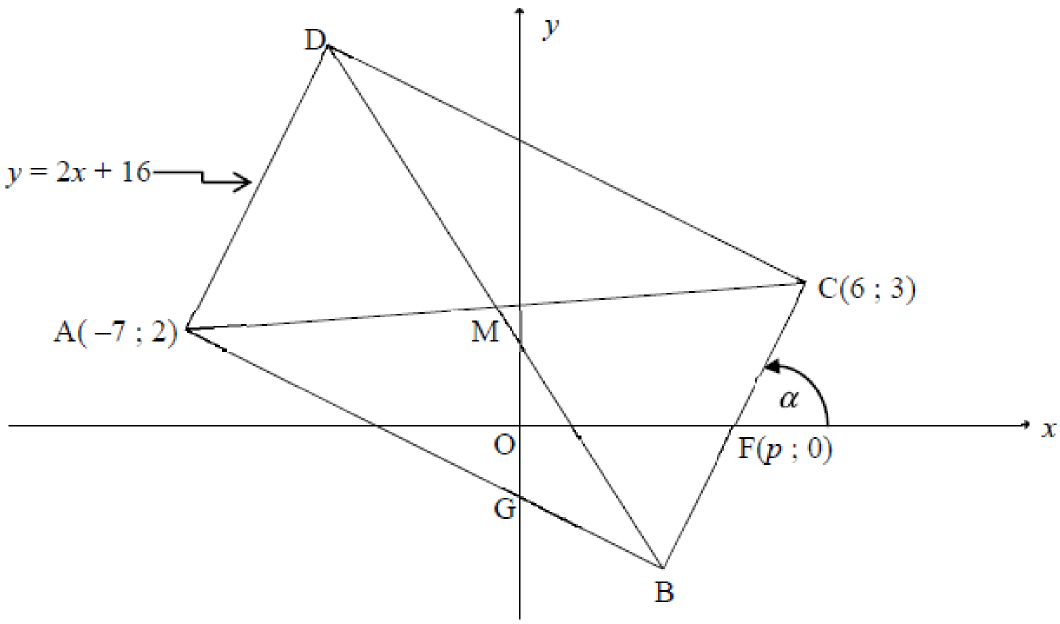
3.3	$-\frac{1}{3}x-2=-x+2$ $\frac{2}{3}x=4$ $x=6$ $y=-4$ $\therefore R(6\,;-4)$	✓equating/*gelyk stel* ✓x-value/*waarde* ✓y-value/*waarde* (3)
3.4	$PR=\sqrt{(1-6)^2+(1-(-4))^2}$ $=\sqrt{50}=5\sqrt{2}$ **OR/*OF*** $PR^2=(1-6)^2+(1-(-4))^2$ $=50$ $\therefore PR=\sqrt{50}=5\sqrt{2}$	✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2) ✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2)
3.5	PR is a diameter/ *'n middellyn* [chord subtends/*kd onderspan* 90°] Centre of circle/*Midpt v sirkel*: $\left(\frac{1+6}{2}\,;\frac{1-4}{2}\right)$ $=\left(3\frac{1}{2}\,;-1\frac{1}{2}\right)$ $r=\frac{\sqrt{50}}{2}$ OR $\frac{5\sqrt{2}}{2}$ OR 3,54 $\therefore\left(x-\frac{7}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{50}{4}$ OR $\frac{25}{2}$ OR 12,5	✓✓S ✓✓ $\left(3\frac{1}{2}\,;-1\frac{1}{2}\right)$ ✓r-value/*waarde* ✓answ/*antw* (6)
3.6	m of/*van* radius $=-1$ $\therefore m$ of/*van* tangent/*raaklyn* $=1$ Equation of tangent/*Vgl van raaklyn*: $y-y_1=(x-x_1)$ $y-1=x-1$ $\therefore y=x$ **OR/*OF*** $y=x+c$ $1=1+c$ $y=x$	✓m of tang/*rkl* ✓subst m & P(1 ; 1) into/*in* eq of line/*vgl v lyn* ✓answ/*antw* (3)
3.7	$\tan P\hat{N}T=m_{PR}=-1$ $\therefore P\hat{N}T=135°$ $\tan P\hat{M}T=m_{PQ}=3$ $\therefore P\hat{M}T=71{,}57°$ $\hat{P}=63{,}43°$ [ext ∠ of Δ/*buite ∠ v Δ*] $\therefore\theta=26{,}57°$ [sum of ∠s in Δ/*som v ∠e in Δ*] **OR/*OF***	✓ $\tan P\hat{N}T=-1$ ✓ $P\hat{N}T=135°$ ✓ $P\hat{M}T=71{,}57°$ ✓ $\hat{P}=63{,}43°$ ✓answ/*antw* (5)

	Extrapolation of RQ to S/*Verlenging van RQ na S*: 	
	$\tan \hat{PNT} = m_{PR} = -1$ $\therefore \hat{SNR} = 135°$ $\tan \hat{NSR} = m_{RS} = -\frac{1}{3}$ $\therefore \hat{NSR} = 18{,}43°$ $\theta = 180° - (135° + 18{,}43°)$ [sum of ∠s in Δ/*som v ∠e in* Δ] $= 26{,}57°$	✓ $\tan \hat{PNT} = -1$ ✓ $\hat{SNR} = 135°$ ✓ $\tan \hat{NSR} = -\frac{1}{3}$ ✓ $\hat{NSR} = 18{,}43°$ ✓answ/*antw* (5)
	OR/*OF*	
	$PQ^2 = 1^2 + 3^2 = 10$ $PQ = \sqrt{10}$ $\therefore \sin\theta = \frac{PQ}{PR} = \frac{\sqrt{10}}{\sqrt{50}} = \frac{1}{\sqrt{5}}$ $\therefore \theta = 26{,}57°$	✓subst into/*in* distance formula/ *afstandsformule* ✓distance/*afst* PQ ✓correct trig ratio/ *korrekte trig vh* ✓correct trig eq/ *korrekte trig vgl* ✓answ/*antw* (5)
	OR/*OF*	
	$QR^2 = 6^2 + 2^2 = 40$ $QR = 2\sqrt{10}$ $\therefore \cos\theta = \frac{2\sqrt{10}}{\sqrt{50}} = \frac{2}{\sqrt{5}}$ $\therefore \theta = 26{,}57°$	✓subst into/*in* distance formula/ *afstandsformule* ✓distance/*afst* PQ ✓correct trig ratio/ *korrekte trig vh* ✓correct trig eq/ *korrekte trig vgl* ✓answ/*antw* (5)
	OR/*OF*	

	$\tan\theta = \dfrac{m_{RQ} - m_{PR}}{1 + m_{RQ}.m_{PR}}$ $= \dfrac{-\frac{1}{3} - (-1)}{1 + (-\frac{1}{3})(-1)}$ $= \dfrac{1}{2}$ $\therefore\ \theta = 26{,}57°$	✓correct formula/ *korrekte formule* ✓ $m_{RQ} = -\frac{1}{3}$ ✓correct subst/ *subst korrek* ✓ $\tan\ \theta = \dfrac{1}{2}$ ✓ $\theta = 26{,}57°$ (5)
	 tangent OP goes through the origin/*raakl OP gaan deur oorsprong* $P\hat{O}M = 45°$ $O\hat{P}M = \theta = \hat{P}$ [tan-chord theorem/*raakl-kdst*] $\tan P\hat{M}T = m_{PQ} = 3$ $\therefore P\hat{M}T = 71{,}57°$ $\therefore\ \theta + 45° = 71{,}57°$ [ext ∠ of Δ/*buite-∠ v* Δ] $\therefore\ \theta = 26{,}57°$	✓ $P\hat{O}M = 45°$ ✓R ✓ $P\hat{M}T = 71{,}57°$ ✓S ✓ $\theta = 26{,}57°$ (5) **[23]**

QUESTION 3

In the diagram, A(–7 ; 2), B, C(6 ; 3) and D are the vertices of rectangle ABCD. The equation of AD is $y = 2x + 16$. Line AB cuts the y-axis at G. The x-intercept of line BC is F(p ; 0) and the angle of inclination of BC with the positive x-axis is α. The diagonals of the rectangle intersect at M.

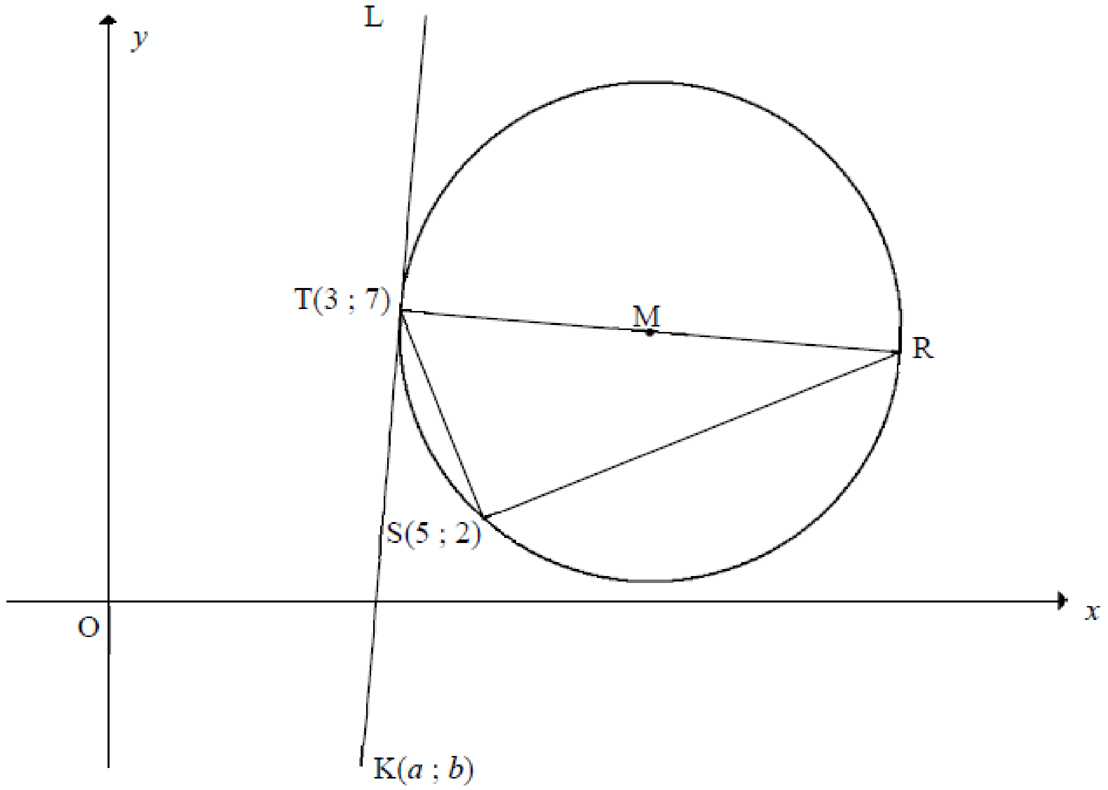


3.1	Calculate the coordinates of M.	(2)
3.2	Write down the gradient of BC in terms of p.	(1)
3.3	Hence, calculate the value of p.	(3)
3.4	Calculate the length of DB.	(3)
3.5	Calculate the size of α.	(2)
3.6	Calculate the size of $O\hat{G}B$.	(3)
3.7	Determine the equation of the circle passing through points D, B and C in the form $(x-a)^2+(y-b)^2=r^2$.	(3)
3.8	If AD is shifted so that ABCD becomes a square, will BC be a tangent to the circle passing through points A, M and B, where M is now the intersection of the diagonals of the square ABCD? Motivate your answer.	(2)

[19]

QUESTION 4

In the diagram, M is the centre of the circle passing through T(3 ; 7), R and S(5 ; 2). RT is a diameter of the circle. K(a ; b) is a point in the 4th quadrant such that KTL is a tangent to the circle at T.



4.1 Give a reason why $T\hat{S}R = 90°$. (1)

4.2 Calculate the gradient of TS. (2)

4.3 Determine the equation of the line SR in the form $y = mx + c$. (3)

4.4 The equation of the circle above is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

4.4.1 Calculate the length of TR in surd form. (2)

4.4.2 Calculate the coordinates of R. (3)

4.4.3 Calculate $\sin R$. (3)

4.4.4 Show that $b = 12a - 29$. (3)

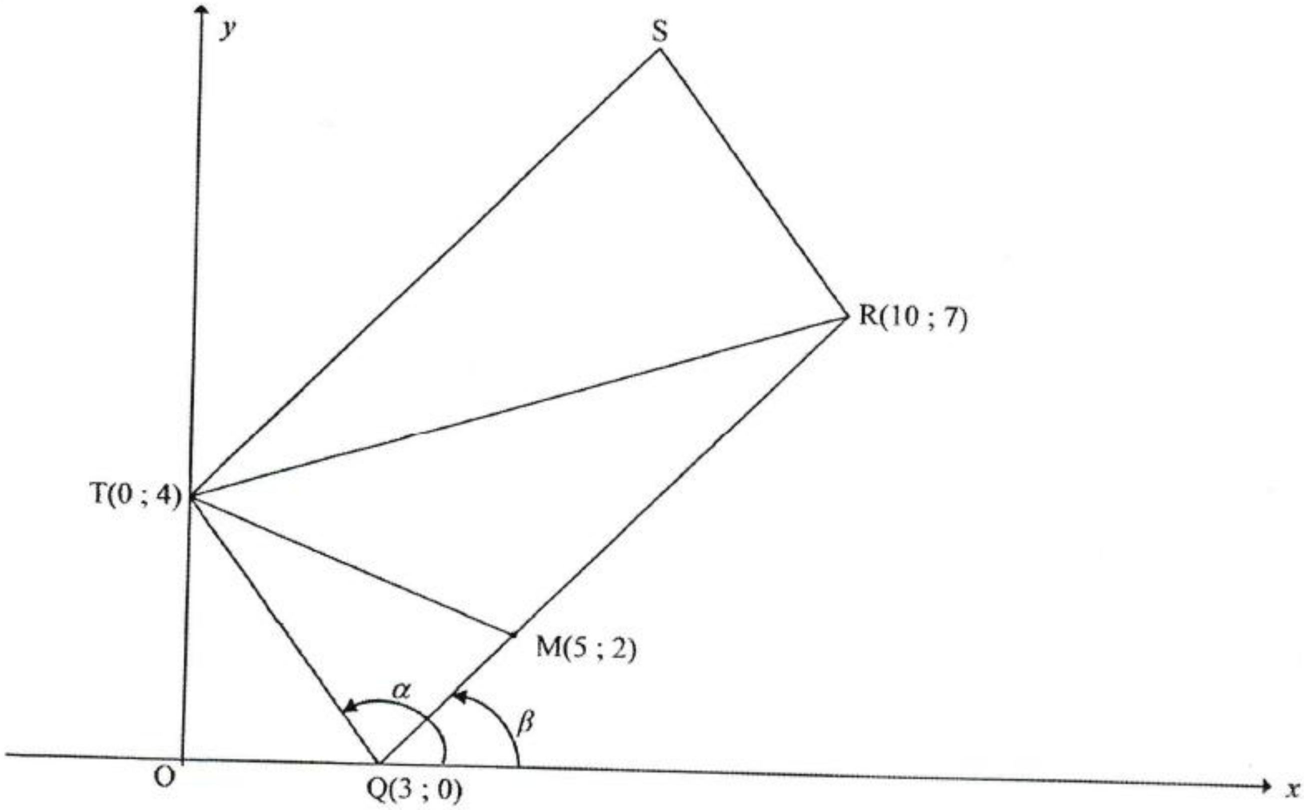
4.4.5 If TK = TR, calculate the coordinates of K. (6)

[23]

***** **SOLUTIONS TO FOLLOW** *****

QUESTION 3

In the diagram, Q(3 ; 0), R(10 ; 7), S and T(0 ; 4) are the vertices of parallelogram QRST. From T a straight line is drawn to meet QR at M(5 ; 2). The angles of inclination of TQ and RQ are α and β respectively.



3.1 Calculate the gradient of TQ. (1)

3.2 Calculate the length of RQ. Leave your answer in surd form. (2)

3.3 F(k ; –8) is a point in the Cartesian plane such that T, Q and F are collinear. Calculate the value of k. (4)

3.4 Calculate the coordinates of S. (4)

3.5 Calculate the size of $T\hat{S}R$. (6)

3.6 Calculate, in the simplest form, the ratio of:

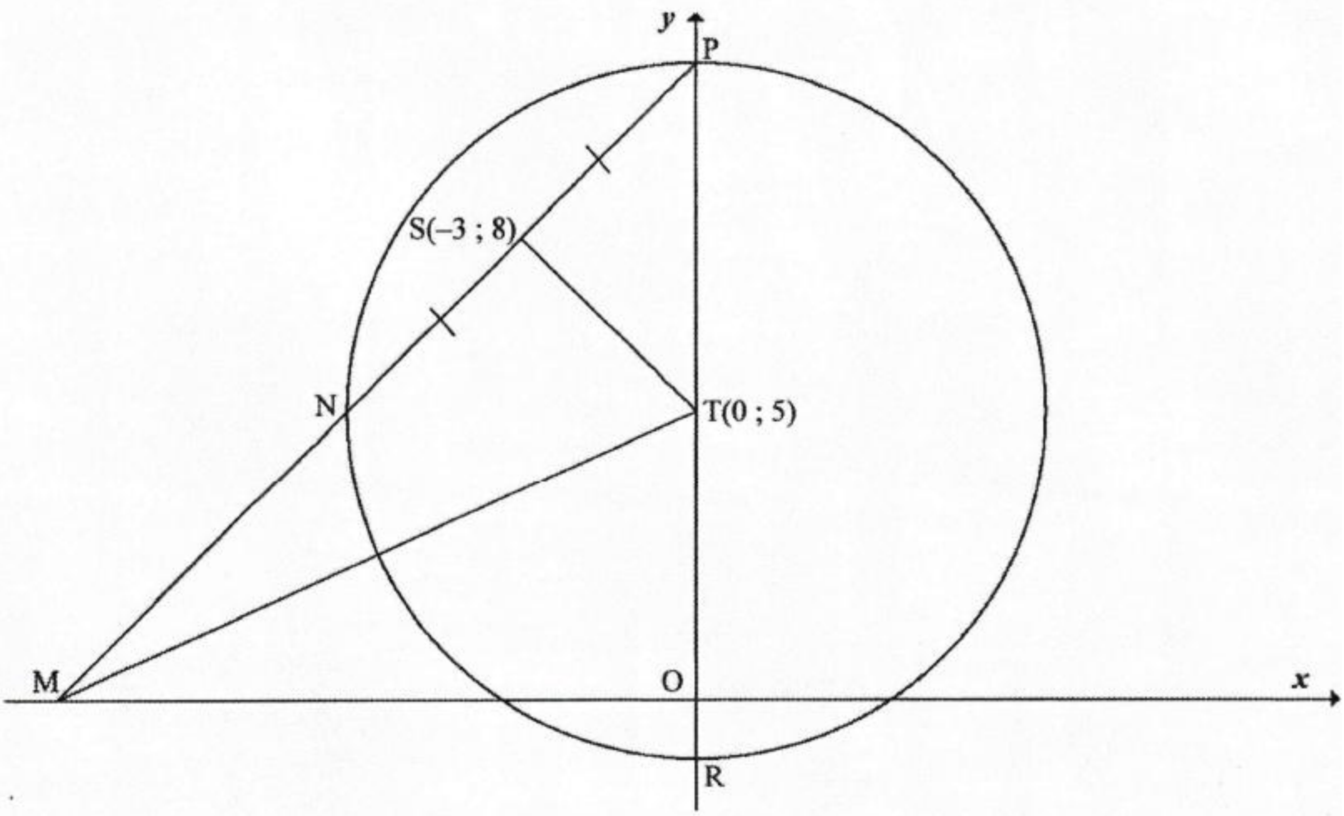
3.6.1 $\frac{MQ}{RQ}$ (3)

3.6.2 $\frac{\text{area of } \Delta TQM}{\text{area of parallelogram RQTS}}$ (3)

[23]

QUESTION 4

In the diagram, the circle, having centre T(0 ; 5), cuts the y-axis at P and R. The line through P and S(–3 ; 8) intersects the circle at N and the x-axis at M. NS = PS. MT is drawn.



4.1	Give a reason why TS ⊥ NP.	(1)
4.2	Determine the equation of the line passing through N and P in the form $y = mx + c$.	(5)
4.3	Determine the equations of the tangents to the circle that are parallel to the x-axis.	(4)
4.4	Determine the length of MT.	(4)
4.5	Another circle is drawn through the points S, T and M. Determine, with reasons, the equation of this circle STM in the form $(x-a)^2 + (y-b)^2 = r^2$.	(5)
		[19]

***** **SOLUTIONS TO FOLLOW** *****