

Analyzing the Parameters of Prey-Predator Models for Simulation Games

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Abstract—We describe and analyze emergent behavior and its effect for a class of prey-predators' simulation models. The simulation uses rule-based agent behavior and follows a prey-predator structure modulated by a number of user-assigned parameters. As part of our analysis, we present key parameter estimations for mapping the prey-predators' simulation parameters to a functional relationship with the LV(Lotka-Volterra) model, and how the parameters interact and drive the evolution of the simulation.

Keywords: *prey-predator systems; Lotka-Volterra model; agent-based simulation; emergent behavior; simulation games; parameter estimation.*

1 Introduction

Simulation games have become a widely accepted and needed tool for training in government agencies and the military. For example, commercial simulation games have had a dramatic effect on the military's education and training programs [1]. In such games, *non-player characters* (NPCs) are an important tool to create complexity, increase realism, and articulate individual and collective behavior. Particularly, in computer games, whether entertainment or educational games, NPCs are agents whose behavior is scripted and automated. When designing such games, one seeks a global emergent behavior but has to implement it from the behavior scripts for individual agents. One difficulty is to relate the agent behavior to global system evolution. Parameter settings, such as life span, speed of motion, etc, affect the system, but it is difficult to predict how changes alter the system evolution. This is especially true for geometric-based behavior, such as cognitive ability and methods of pursuit and evasion. As the expressiveness of agent behavior grows, so does the difficulty understanding its impact on the system.

In this paper, we develop and test a framework for estimating the impact of specific behavior traits on the overall evolution of game worlds. We restrict to the

important subclass of prey-predator simulations that have applications in ecology, biology, economics and sociology, and for which mathematical concepts can be formulated that summarize system properties. For instance, [2] reports a discrete simulation tool to simulate the behavior of predator programs and computer viruses in small scale (1600 node) computer networks. The simulation shows that it is possible to design those predators such that worms and viruses can be eliminated without clogging the network. Thus, the concept of agent and NPC can be understood in very general terms, and applied to a wide variety of situations. Multi-behavioral games between predators and prey have been studied that integrated pre-encounter and post-encounter behaviors [6]. The study included landscape-scale movement by predators and prey, and analyzed to the game using a computer-based evolutionary algorithm. Mitchell and Lima [8] investigated the shell game using an individual-based model that allows predators to update information about prey location. Huige [9] presented results to predict that very little overlap will occur between the players' waiting distributions and that the predator will rarely outwait the prey. We have applied game theory differently, with prey searching for grazing and predators hunting prey in packs. We show that food availability and predators' densities influence patterns of prey distribution. We approach the problem by relating the (extended) Lotka-Volterra (LV) equations [3],[7] to expose essential characteristics of prey-predators simulations. Illustrating our concepts, we relate quantitative parameters, such as birth rate, life span, time to starvation and so on, to the mathematical model, and so try to clarify how changing the driving parameters impacts the simulation.

1.1 Overview

Prey-predator simulations have been studied in biology, to model and understand animal behavior, as well as to consider environmental impact. In many game settings, prey-predator behavior is adversarial or predatory. When designing such simulations, many parameters are introduced, such as life span and reproduction rate, cognition and planning, speed and agility, search and hunt for food resources, and so on. Once implemented, the simulation becomes a complex device and the influence of individual parameter settings and behavioral algorithms on the overall system behavior is not easy to understand. Parameter proliferation is a typical way to better capture real-world behavior, but only further complicates how to value parameters in order to achieve a desired global behavior. So, we test a framework that allows the designer of the simulation to better understand the impact of changing parameter values. We develop our framework relating the behavior parameters to the coefficients of the mathematical model provided by the LV equations [3], so elucidating how the parameters impact the system evolution. This functional relationship is derived by a set of experiments.

2 Relationship to the Lotka-Volterra Model

2.1 Simulation Specifications

Our basic simulation program has been described in [3]. It uses the *OpenSteer* library [4] to implement basic motion algorithms. The parameter set is as in [3]. For

example, using subscript 0 to indicate that the parameter applies to prey, and subscript 1 to indicate that it applies to predators we have:

- *Birth, aging and death:* The L_k is the maximum life span; no agent lives longer than that. Moreover, f_k is the adulthood fraction, that is, the fraction of L_k spent as an adult. The concept here is that juvenile agents have fewer capabilities, qualitatively or quantitatively, than adults. In particular, juveniles do not have offspring as yet. The A_k is the current age, normalized to be between 0 and 1 as the fraction *current age / max lifetime*. The h_k is the hunger rate, measuring the speed with which an agent becomes hungry again, and H is the hunger value. The agent dies if $A_k \geq 1$ or if $A_k < 1$ and $r \in [0, A_k^{18}]$. Here r is a random number between 0 and 1, with uniform distribution. Death from hunger depends on H . Initially, if $H = 0$ then hunger is incremented by

$$\Delta H = h_k * \Delta t * \min \left(1, \frac{A_k}{1-f_k} \right) . \quad (1)$$

A death due to hunger occurs if $H > 0$ and $r < H^{10}$ where r is a random number in $[0,1]$. Feeding diminishes the hunger.

Other parameters include *maximum speed*, *food delay*, the time for an area to be arid until regrowth can take place, and so on.

2.2 The Lotka-Volterra Model

The LV differential equations express the global behavior of a prey-predator system under the assumption of unlimited food supply for prey. Excluding the trivial steady state in which one or both population types are extinct, the steady state is a dynamic balance of the two populations and is given to a cyclic boom-bust cycle. The equations are

$$X' = \alpha X - \beta XY . \quad (2)$$

$$Y' = -\gamma Y + \delta XY . \quad (3)$$

where X and Y are the populations of prey and predators, respectively. The factor α represents the growth rate of the prey population absent predation and so accounts for the birth rate as well as for the rate of death from old age; β is the impact of predation, accounting for death of prey due to being killed by predators. The assumption of unlimited food supply is manifest in that α only assumes death from natural causes. The factor γ quantifies the rate of predators' death from natural causes; δ measures the growth rate of the predator population due to the realized food supply, i.e., due to hunting. Again, change rates are assumed to depend linearly on the population sizes. Assuming nonzero population sizes, steady state is achieved by a zero change rate; that is, when

$$\alpha = \beta Y . \quad (4)$$

$$\gamma = \delta X . \quad (5)$$

See also [5]. The factors α and γ measure intrinsic characteristics of prey and predators: prey life span and fertility determine α , and predator life span only determines γ . The presence of X in the term δXY implies that predators' fertility depends on the food supply X , an assumption justified in many situations. The simulation architecture assumes a limited food supply for prey, so the equations have to be modified accordingly. We assume a stable prey population size X_c , absent predation, that is due to the carrying capacity of the environment. Once reached, birth and natural death rates of prey should become equal, without predation. Assuming linearity, we modify the LV equations to incorporate these considerations:

$$X' = \alpha \left(1 - \frac{X}{X_c}\right) X - \beta XY. \quad (6)$$

$$Y' = -\gamma Y + \delta X. \quad (7)$$

If we assume no predators, then the first equation becomes the logistics equation:

$$X' = \alpha \left(1 - \frac{X}{X_c}\right) X. \quad (8)$$

Assuming the continuous case, the solution is:

$$X(t) = X_c X(0) / (X(0) + (X_c - X(0))e^{-\alpha t}). \quad (9)$$

Here $X(t)$ is the population at time t .

2.3 Carrying Capacity Determination and Base Configuration

We seek to estimate the parameters of the differential equations from the simulation. Since we execute physics-based search and hunt algorithms, it is not quite evident how the coefficients are affected when changing life span, agility, and search strategies. However, we can approach the problem as follows.

Beginning with the logistic equation, we note that the ratio of population size to carrying capacity, for $\alpha > 0$, is 1 in the limit. If $\alpha < 0$, then the ratio goes to zero. The assumption $\alpha > 0$ is the case where the population persists and reaches the carrying capacity of the environment, while for negative α the prey becomes extinct. In the following, we assume that $\alpha > 0$. Note that the magnitude of α only affects how fast the system reaches the carrying capacity. In the stochastic case, we expect that the population at first grows past the carrying capacity and subsequently collapses due to environmental exhaustion. Assuming that the environment can regenerate, the residual population begins to grow again, after some time, and recovers.

The recovery may overshoot the carrying capacity again, in which case the population again reduces in size and the cycle repeats. The carrying capacity X_c is affected by two environmental parameters: the time that the grazing area lies fallow before beginning to regenerate, and the speed with which vegetation grows back, after

that period. In addition, the amount of food needed to sustain a prey and the prey life span also affect the carrying capacity. Finally, the competence finding food, that is, the cognitive ability and the search strategy employed by prey, enter into the carrying capacity as well.

We determine X_c by running the simulation with an initial parameter set approximating the model scenario we have in mind, but without a predator population. We call this the *base configuration*. Then we vary the parameters of the environment and of the prey to determine how these variations impact the carrying capacity. We can do this for each significant parameter individually, so exploring the configuration space locally, near the base configuration.

2.4 Determination of α , β , γ , δ

To determine α , we can run the simulation in the base configuration without any predators. Then the parameter α tracks the average difference of the number of births and deaths at each time step. That is, the intrinsic rate of prey population increase can be estimated as $\alpha(t) = (b(t) - d(t))/X(t)$, where $b(t)$ is the number of births and $d(t)$ is the number of deaths (due to hunger or old age) of the prey, at time t . We can use a moving average to compute this estimate. Since the environment has limited food supply, however, the estimate needs to take into account the logistical equation. The formula is then

$$\alpha(t) = \frac{(b(t)-d(t)) \cdot X_c}{(X_c-X(t)) \cdot X(t)}. \quad (10)$$

When $X(t)$ approaches X_c the denominator vanishes and the coefficient $\alpha(t)$ cannot be determined. Thus, we have to estimate α by measuring population growth with infinite food supply. To accomplish this, we set the food growth parameters to instantaneous regeneration and set the hunger value to zero. The result is an exponential increase in population size where we determine the value of α by fitting the exponential $X(t) = X(0) \cdot (1 + \alpha)^t$. This should be done for several initial population sizes, arriving at an average value for α .

The factor β measures the impact of predation on the prey population. The equations assume linearity in both predators' and prey densities. We determine the factor using the procedure of Sharov [7]: we run the simulation with one predator for a number of time steps during which no new predators are born.¹ The k -values [7] of β for a single time step are averaged over the simulation interval $[t_0, t_1]$. With N the number of steps we obtain

$$-\ln(1 - \beta(t)) = \sum_t \left(-\ln \left(1 - \frac{k(t)}{X(t)} \right) \right) / N. \quad (11)$$

At start-up, the simulation requires a number of steps to reach a semi-stable state. Therefore, we have programmed the possibility of running the simulation for a pre-determined number of steps before introducing the predators at a random location. The parameters γ and δ measure the attrition of the predator population by natural

¹ We assume asexual reproduction for simplicity of the model.

causes, that is, death from old age and hunger, and the effect on the population when prey is available. The parameters are estimated as follows:

- 1) For fixed population size X_0 run the simulation for some time, holding the prey population size constant.
- 2) For X_0 , estimate the aggregate predator growth rate r from $X_1(t) = X_1(0)(1 + r)^t$.

Here, $X_1(t)$ is the predator population size at time t . This is done for a number of different population sizes X_0 . Having determined r , for different prey population sizes, the parameters for the predator population dynamics is found by fitting a line to the data provided by the (r, X_0) data:

$$r = \delta X(t) - \gamma . \quad (12)$$

Holding the prey population constant in this procedure requires a measure of fairness. That is, if the prey population has a particular age distribution, resurrecting killed prey should respect that distribution. This can be accomplished by disabling birth, aging and death from the prey population and reinstating killed prey keeping its age fixed. Moreover, if there are geometric characteristics, such as prey seeking to aggregate in herds, re-instantiation would be in or near a herd of prey.

2.5 Fitness Metrics

Intuitively, a growth in β indicates improving hunting skills of the predators, while a decrease of β indicates that prey is better able to evade predators and defend themselves against predation. We can therefore use β as a competitive fitness metric by which to quantify the survivability of the two agent populations. Similarly, the other parameters can be considered as indirectly contributing to the competitiveness: Larger α implies greater resilience under predation as well as greater impact on the environment; an increase in $|\gamma|$ implies greater mortality of predators, and larger δ indicates a greater dependence on prey.

3 The Environment

When an agent gives birth to a baby, the baby's initial position in the world is random but within a maximum distance from the mother (specified as *baby spawn radius*). When prey or predators are born, the agent spends its initial life period as a juvenile doing nothing but eating. The duration of this period is determined by the 'adult time fraction' which indicates the percentage of the lifetime spent as an adult. Hunger rates are the increase in hunger per second. This appropriate value is added to the hunger value during each update in a time step. The higher the hunger rate is, the faster an agent starves. The birth gap is the minimum gap (in seconds) between births. This is not necessarily the actual gap during the run. When an agent is initialized, a counter is set that indicates the time until giving birth which is this 'birth gap' value. This counter constantly decreases during each time step by a function of the agent's hunger. The more starved an agent is, the more slowly does the counter decrease

which in turn means that healthier agents give birth more frequently compared to starved ones. Food growth is the rate at which the food value of a patch increases per second when it is growing back. We implement a proximity perception model for predator agents; an agent can see prey and another predator agent only when they are within the limited range of visibility. The maximum speed of the prey is higher than the maximum speed of the predators. In order to allow predators to stalk and collectively approach the prey, the range of visibility of predators is greater than that of prey.

Table1. Parameter Values for the Simulation game model

Parameter	Value	Range	Description
Field Size	25.0	0.0-INF	Size of the environment field
Baby Spawn Radius	1.2	0.0- Field Size	Distance from the mother that a new baby gets spawned
Max Prey	4000	Num Prey-INF	Maximum number of prey allowed
Max Predators	4000	Num Predators-INF	Maximum number of predators allowed
Num Prey	40	1- Max Prey	Initial number of prey
Num Predator	10	1- Max Predators	Initial number of predators
Prey Base Speed	4.5	1.0- Prey Max Speed	The initial speed of a prey when its born
Prey Max Speed	6.5	Prey Base Speed-INF	The final speed of a prey when an adult
Predator Base Speed	6.5	1.0- Predator Max Speed	The initial speed of a predator when a newborn
Predator Max Speed	8.8	Predator Base Speed-INF	The final speed of a predator when an adult
Prey Life Time	45.0	1.0-INF	Lifetime of prey in seconds
Predator Life Time	100.0	1.0-INF	Lifetime of predator in seconds
Prey Adult Time	0.9	0.0-1.0	Fraction of the prey's lifetime for which its an adult
Predator Adult Time	0.9	0.0-1.0	Fraction of the predator's lifetime for which its an adult
Predator Hunger Rate	0.07	0.0-INF	Rate per second at which hunger of predator increases
Prey Hunger Rate	0.015	0.0-INF	Rate per second at which hunger of prey increases
Prey Birth Gap	3.26	1.0-INF	Duration in seconds between births for a prey
Predator Birth Gap	4.09	1.0-INF	Duration in seconds between birth for a predator
Prey ROV	8.0	1.0-INF	Range of view of the prey
Predator ROV	12.0	1.0-INF	Range of view of the predator
Prey FOV	155.0	1.0-360.0	Field of view of the prey
Predator FOV	175.0	1.0-360.0	Field of view of the predator
Food growth delay	1.0	0.0-INF	Delay before a food patch grows back after being grazed on
Food growth rate	10.8	0.0-INF	Rate at which a food patch grows back

4 Experimental Results

4.1 Parameter Determination

We determine how the many simulation parameters impact the prey-predator system by estimating the coefficients of the mathematical model of Section 2. This provides guidance when seeking to fine-tune a simulation scenario. The environmental parameter values are given in the Table1 along with their meaning. Recall that each food square in the environment, upon depletion, lies fallow for a time determined by the *food-growth-delay* and then vegetation grows back at a speed set by the *food-growth-rate* parameter. These parameters govern the amount of consumable food per time unit, and directly impact the carrying capacity. Fig. 1a shows the correlation fixing all other parameters. The base configuration values are indicated in red.

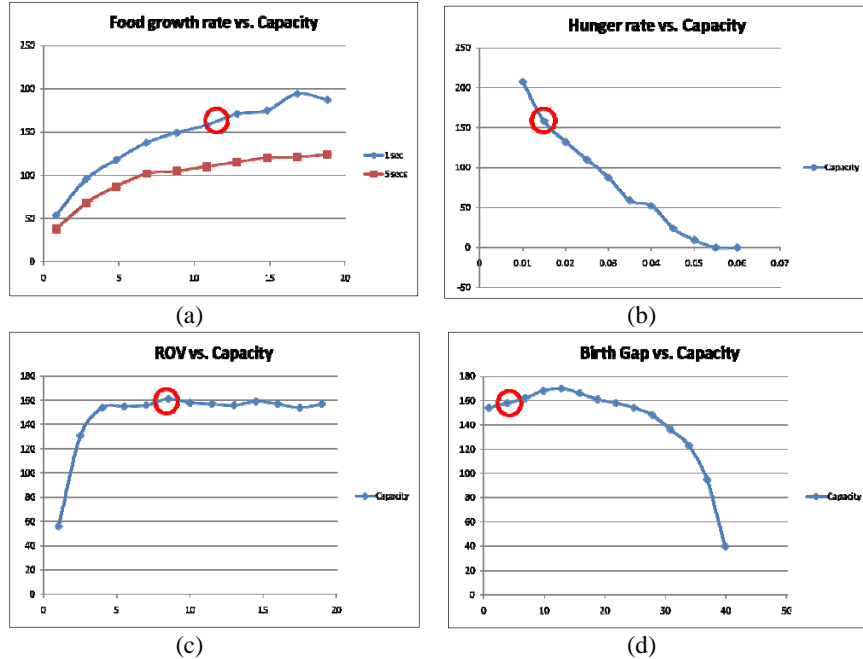


Fig.1. Effect of environmental settings on the carrying capacity (a). Prey characteristics and their effect (b-d). The red circle marks the base configuration values.

The graphs are relatively straightforward. For instance, the carrying capacity first goes up almost linearly with the food growth rate. This is expected, since for the higher growth rate more food is available and the individual prey has a constant expected food intake over their life time. Also as expected, the longer the delay before growth commences, the lower the carrying capacity. However, as the food-growth rate increases, the carrying capacity does not continue to increase linearly and instead approaches a limit value induced by the food-growth-delay. The reason for this is as follows: Beyond a certain rate, re-growth is essentially instantaneous. But grazing takes time and then the food patch has to be fallow for the time specified by the food-growth-delay parameter. This means that the total amount of consumable food cannot exceed a maximum imposed by the rate at which prey grazes and the delay before the food values is fully restored. Fig. 1(a) and (b) show the dependence of carrying capacity on the re-growth speed and regeneration delay. For the base configuration the carrying capacity is about 158. Now consider the impact of the prey parameters on the carrying capacity. The *hunger-rate* parameter governs the amount of food the individual must consume, and the simulation shows in Fig. 1b that the carrying capacity is inversely proportional to it. Beyond a certain rate, the environment cannot produce food fast enough and the population dies out. The ROV (*range-of-vision*) parameter determines how far prey can see. Thus, as the ROV increases, prey can better find the food. This increase in carrying capacity is primarily the absence of grazing time lost to searching for food. When prey can see far enough, no search is

needed to find hidden food patches, so the impact of ROV beyond that point becomes nil. This is supported by the empirical evidence shown in Fig.1c. Finally, the relationship of *birth delay* to carrying capacity is explored in Fig.1d and requires some explanation: The birth consumes some of the parent's energy that must be made up by additional grazing, meaning that more frequent births should depress the carrying capacity.

In summary, the carrying capacity depends directly on the growth rate and inversely on the hunger rate. The dependence on range of vision is direct and linear until a threshold distance is exceeded, around 5 squares distance, where it ceases to matter. The birth gap does not, at first, impact the carrying capacity very much, meaning that in the base configuration the extra food intake of the parent is compensated by the reduced intake of the offspring. However, as the delay becomes large in relation to the life span, the gap impacts the ability to reproduce in sufficient numbers and the carrying capacity decreases owing to the relative infertility of prey. For the base configuration, the carrying capacity is about 155-160 on average. Our sample runs show some variance owing to a small sample size. A more precise measurement would average numerous runs of the base configuration. Fig. 2 shows the various measured quantities throughout the simulation of the base configuration.

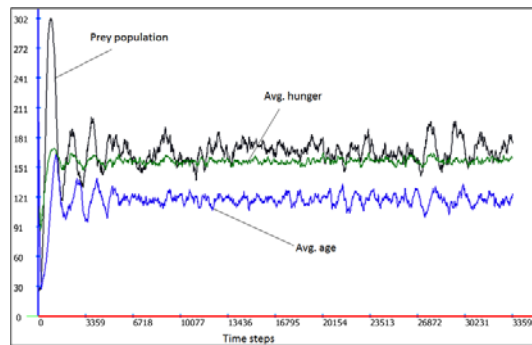


Fig.2. The prey population (black curve) and average hunger value (green) of prey. The average age (blue) shows large variation initially accounting for significant fluctuations in

Fig. 3 shows that the simulation of prey-predators' problem implements the model described above. We initialized each simulation with 40 prey and 10 predators and ran the simulation until both the prey and predators' populations became extinct. The yellow square represents the world used in our experiments. The topology is toroidal, so that an agent leaving on the left border re-enters at the corresponding point on the right border. The grey patches represent food content for the prey. The brightness of a patch indicates its food value. As prey consume a patch, the food value slowly fades out and only grows back after a certain period of time. The agents are represented by the triangular shapes with the red ones prey and the turquoise and pink ones predators. When a prey dies it remains on the screen for some time, turning yellow and slowly fading out. This does not happen for the predators.

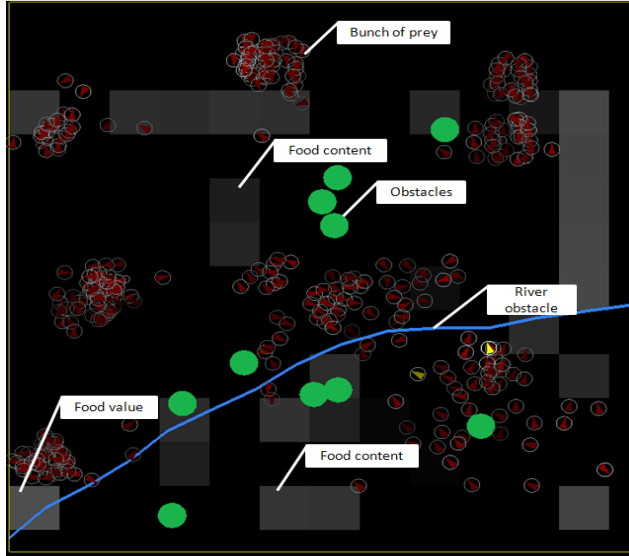


Fig.3. The models of simulation game; Red triangles represent live prey while yellow triangles indicate dead prey with simulation interfaces on the controls.

4.1 Estimation of the Lotka-Volterra Parameters

In Section 2.1 we determined the carrying capacity in the base configuration from a number of simulation runs, without predators, obtaining $X_c \approx 158$. Following the method described in Sharov[7] the four key parameters, in the base configuration, are about $\alpha = 0.075$, $\beta = 0.00543$, $\gamma = 0.0689$, and $\delta = 0.0031$. We integrated the modified LV equations, for a more intense world, with $\alpha = 0.21$, $\beta = 0.024$, $\gamma = 0.23$, and $\delta = 0.002$. These values are the result of more concentrated food growth and hungrier predators. The predicted system evolution is shown in Fig.4:

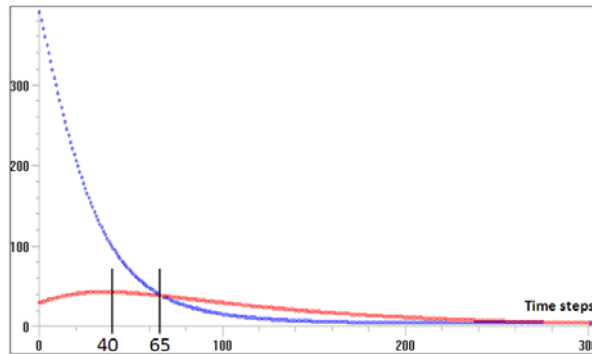


Fig.4. Predicted evolution of the simulation at the measured parameters. The first peak of predator population (red curve) is 42 after about 40 time steps; the crossover of the predator and prey populations (black curve) is 40 after an x-value of about 65 time steps. Carrying capacity is 703.

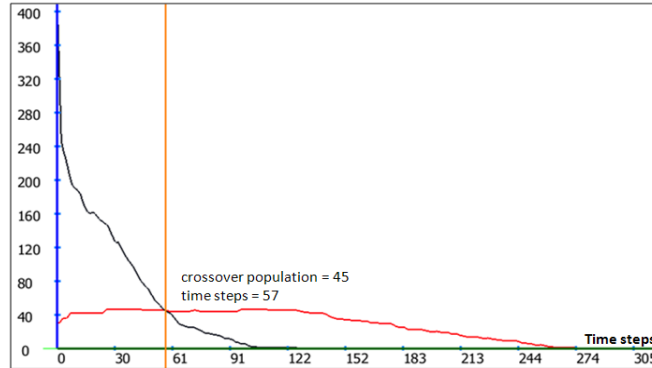


Fig.5. Observed evolution of the configuration. Predator population peaks at 46, holding steady between time steps 25 and 120; crossover is at time step 57 with 45 individuals.

The predictions are in reasonable agreement with the observation, accounting for stochasticity of the simulation. The observed times of extinction are reasonably close considering that, unlike in the equation model, we deal with integer values in the simulation. We have run additional observations and predictions. In a number of cases the times when populations peak or when they cross over are not well matched to the observations, but the number of individuals typically are. The time discrepancy requires additional investigation.

5 Discussion and Conclusions

In this paper, we introduced a simulation model which implements and analyzes NPC roles with a prey-predator relationship. We have investigated and estimated experimentally four key parameters from experiments with the prey-predator simulations that allow predicting the fluctuations of the populations using the augmented LV equation. The augmented equations account for the limited carrying capacity of the environment. This characterization allows a concise and compact description of the fitness of the two populations' skills. The complex relationship of the numerous parameters governing the behavior of prey and predators to the resulting fitness parameters were explored in Section 4 by experiment. The configuration space expressing the system evolution as function of the parameter assignments is a high-dimensional manifold, so local exploration around a particular configuration is an expedient way to assess the survival value resulting from the various parameter settings when making adjustments.

The experiments we have done so far have small population sizes, so that the stochasticity of behavior has a substantial influence on the overall system dynamics. Accordingly, some divergence between the continuous model and the stochastic simulation should be expected. In particular, when the number of prey is small and only a few predators are hunting, it is not unlikely that individual events, success or failure of hunting or finding grazing, can have a large effect on the system evolution. Those effects should diminish for larger population sizes and also over numerous

simulation games. More experimentation is needed to understand better when the sample size is sufficiently large.

Our simulation differs from many other studies by basing prey behavior on Reynolds's flocking algorithms [3]. It is known that the parameter settings of the three basic rules of flocking impact the emergent behavior of a flock [10]. This can influence herd size and cohesion and, with them, the effectiveness of grazing: a large herd could exhaust a sector of the environment so quickly, that not all herd members can have adequate grazing. So, as the herd moves to a new square, the followers in the herd could conceivably starve. Thus, it might be better for prey to roam the environment in herds that are smaller and split and explore in different directions searching for food. Such geometric considerations can be analyzed using our approach.

Acknowledgements

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