2012 NCTM Regional Conference

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Singapore's Visual Models Enable All Students Develop Algebraic Thinking

ARRIVED FROM SINGAPORE

Top Scoring Nation for 15 years!

Hong Kong SAR	607
Singapore	599
Chinese Taipei	576
Japan	568
Kazakhstan	549
Russian Federation	544
England	541
Latvia	537
Netherlands	535
Lithuania	530
United States	529
Germany	525
Denmark	523
Australia	516
Hungary	510
Italy	507
Austria	505
Sweden	503
Slovenia	502
TIMSS Scale Average	500
Armenia	500
Slovak Republic	496
Scotland	494
New Zealand	492
Czech Republic	486
Norway	473
Ukraine	469
Georgia	438
Iran, Islamic Rep. of	402
Algeria	378
Colombia	355
Morocco	341
El Salvador	330
Tunisia	327
Kuwait	316
Qatar	296
Yemen	224

TRENDS IN INTERNATIONAL MATHEMATICS AND SCIENCE STUDY

TIMSS

HOUGHTON MIFFLIN HARCOURT

Wednesday, October 31, 12

Grade 4	Sweden	Median	USA	Japan	Singapore
Advanced	3	5	10	23	41
High	24	26	40	61	74
Intermediate	69	67	77	89	92
Low	93	90	95	98	98

% of students who reached at least this level
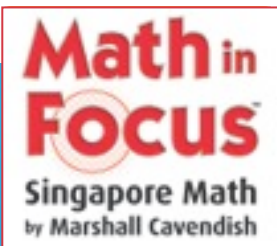
TIMSS – 4th Grade Problem
Trends in International Math and Science Study

Math in Focus
Singapore Math
by Marshall Cavendish

Wednesday, October 31, 12

Example of 8th Grade TIMSS Problem

Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils?

% Received full credit

Singapore: 59%

United States 37%

Math in Focus

Singapore Math by Marshall Cavendish

Example of 8th Grade TIMSS Advanced Problem

Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils?

Pencil   = 3 zeds

Pen   1   = 4   zeds

1 pen and 2 pencils = 4 + 6=10

} 17

1   1

5   = 15
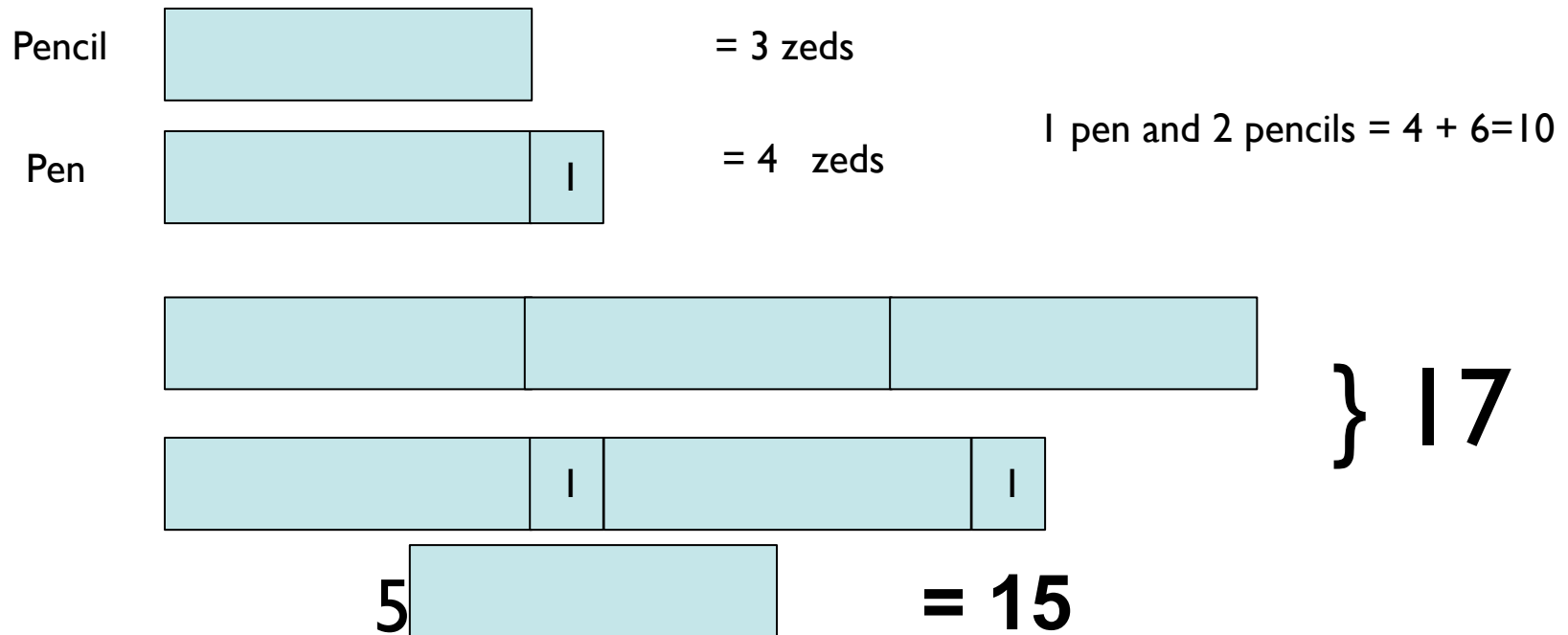
Example of 8th Grade TIMSS Problem

The length of a rectangle is 6 cm, and its perimeter is 16 cm. What is the area of the rectangle in square centimeters?

Answer

Measurement: Singapore, 86%; Korea, 66%; Japan, 65%;
Canada, 41%; France, 36%;
U.S., 22%

Highest performing country for 15 year olds:
Singapore (562)

Highest performing economic zone (or city):
Shanghai, China (600)

US 487 Singapore **A World Leader**

Pedag... d upon 20 years
of... and success

MATH	PISA SCORE
Shanghai, China	600
Singapore	562
Hong Kong, China	555
Korea	546
Taiwan	543
Finland	541
Liechtenstein	536
Switzerland	534
Japan	529
Canada	527
Netherlands	526
Macao, China	525
New Zealand	519
Belgium	515
Australia	514
Germany	513
Estonia	512
Iceland	507
Denmark	503
Slovenia	501
Norway	498
France	497
Slovakia	497
AVERAGE SCORE	497
Austria	496
Poland	495
Sweden	494
Czech Republic	493
Britain	492
Hungary	490
Luxembourg	489
United States	487

PISA 2009 Results:
What Students Know and Can Do

STUDENT PERFORMANCE IN READING, MATHEMATICS AND SCIENCE

VOLUME I

Teach to Mastery

"…mathematics curriculum in the United States must…address the problem of a curriculum that is a "mile wide and an inch deep."

Common Core State Standards

Math in Focus
Singapore Math by Marshall Cavendish

What can we learn?

Singapore: The Model for CCSS

> **Overall, the CCSS are well aligned to Singapore's Mathematics Syllabus. Policymakers can be assured that in adopting the CCSS, they will be setting learning expectations for students that are similar to those set by Singapore in terms of rigor, coherence and focus.**
>
> —*Achieve*[*], (achieve.org/CCSSandSingapore)

Math in FOCUS

Singapore Math by Marshall Cavendish

What can we learn?

Mathematics I Standards for Mathematical Practice

- Able :
  - to make sense of problems and persevere
  - to reason abstractly and quantitatively
  - to construct viable arguments
  - to model with mathematics
  - to use tools appropriately
  - to attend to precision
  - to looks for and make use of structure
  - to look for regularity

COMMON CORE
STATE STANDARDS FOR

Mathematics

Preparing for Common Core

Preparing for SBAC

42960

The point on the number line shows the location of $-3\frac{1}{2}$.

Move each expression into a box to show its correct location on the number line.

7th Grade

$-3\frac{1}{2}$

0

$-3\frac{1}{2} - 3\frac{1}{2}$

$-3\frac{1}{2} - (-5)$

$-3\frac{1}{2} + 3\frac{1}{2}$

$-3\frac{1}{2} + (-5)$
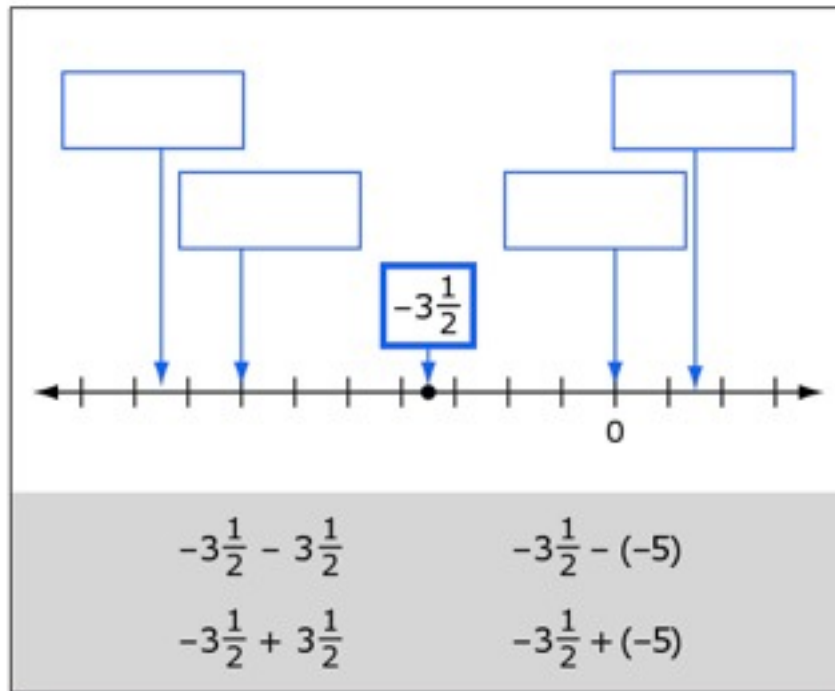
Math in Focus

Singapore Math by Marshall Cavendish

Preparing for Common Core

Write your answer to the following problem in your answer booklet.

San Francisco Giants' stadium: 41,915 seats	Washington Nationals' stadium: 41,888 seats	San Diego Padres' stadium: 42,445 seats

Compare these statements from two students.

Jeff said, "I get the same number when I round all three numbers of seats in these stadiums."

Sara said, "When I round them, I get the same number for two of the stadiums but a *different* number for the other stadium."

Can Jeff and Sara both be correct? Explain how you know.

PARCC Assessment

Grade 7 - Speed

Object A	Object B
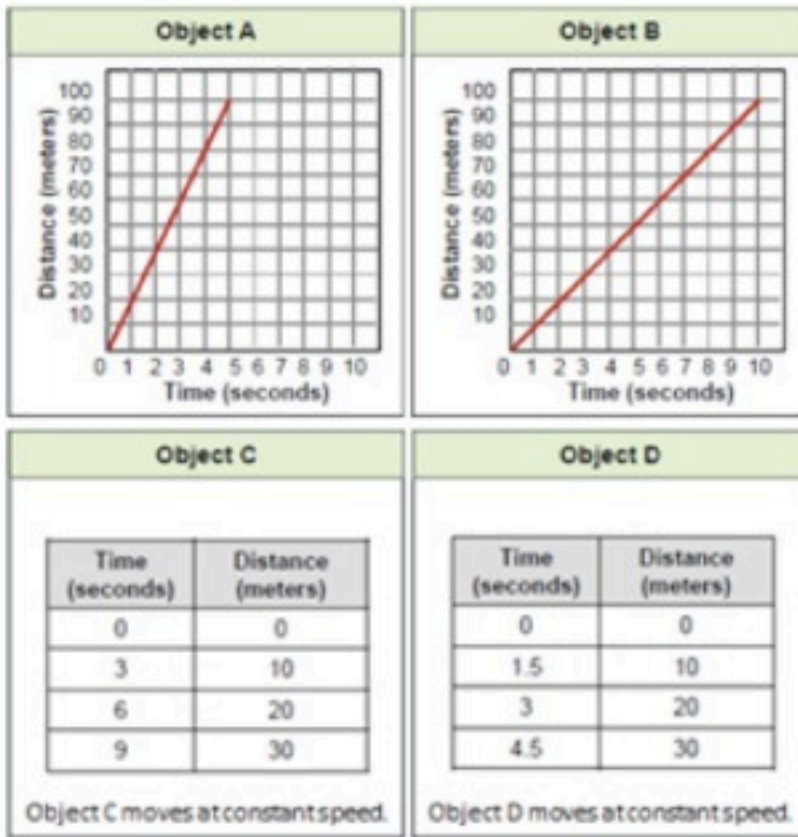
The speed of an object is defined as the change in distance divided by the change in time.

Information about objects A, B, C and D are shown. Objects C and D both have constant speed.

Based on the information given, drag and drop the object names in order from greatest speed to least speed in the table provided.

Object C

Time (seconds)	Distance (meters)
0	0
3	10
6	20
9	30

Object C moves at constant speed.

Object D

Time (seconds)	Distance (meters)
0	0
1.5	10
3	20
4.5	30

Object D moves at constant speed.

Object A

Object B

Object C

Object D

Greatest Speed ⬇ Least Speed

Preparing for Common Core

7
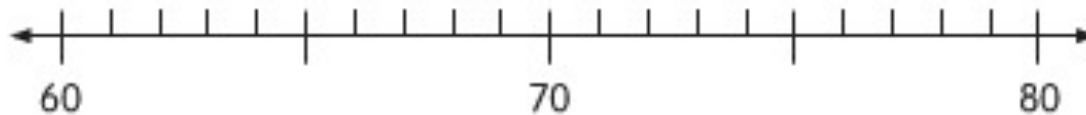
A number when rounded to the nearest 10 is 70.

a Find all the numbers that give 70 when rounded to the nearest ten. Mark each number with a ✗ on the number line.

b Which number is the least?

c Which number is the greatest?

Math in Focus

Singapore Math by Marshall Cavendish

Preparing for CCSS Assessments

The total area of the two squares is 89, what is the length of the sides of the two squares? What is the perimeter of the figure?
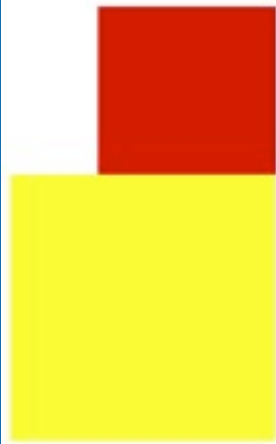
One of the three models shows the sum of $\frac{1}{2}$ and $\frac{1}{7}$. The other two models are incorrect.

$\frac{1}{2}$ $\frac{1}{7}$

Model 1:

?

$\frac{1}{2}$ $\frac{1}{7}$

Model 2:

?

$\frac{1}{2}$ $\frac{1}{7}$

Model 3:

?
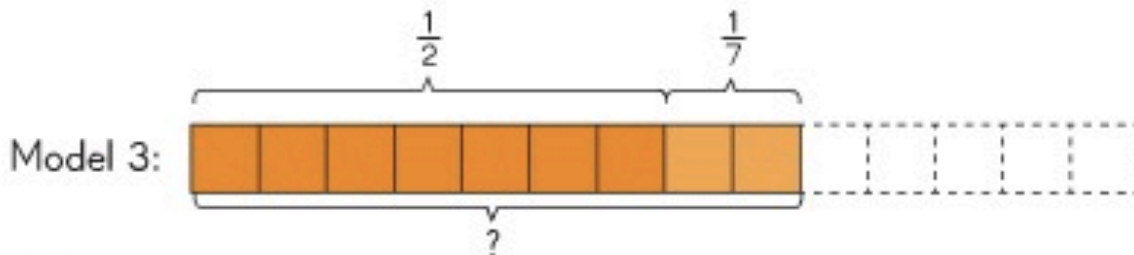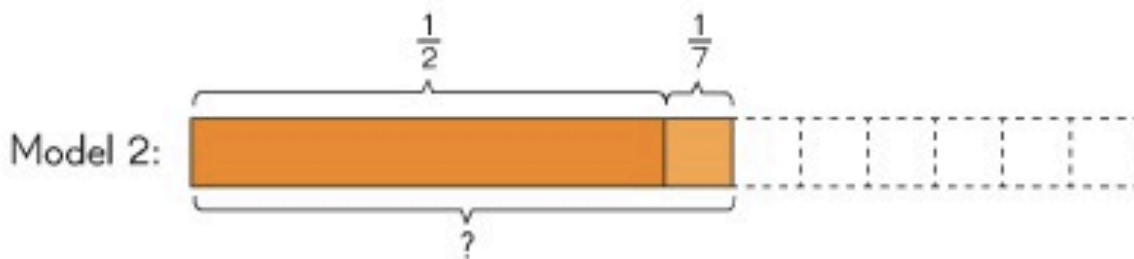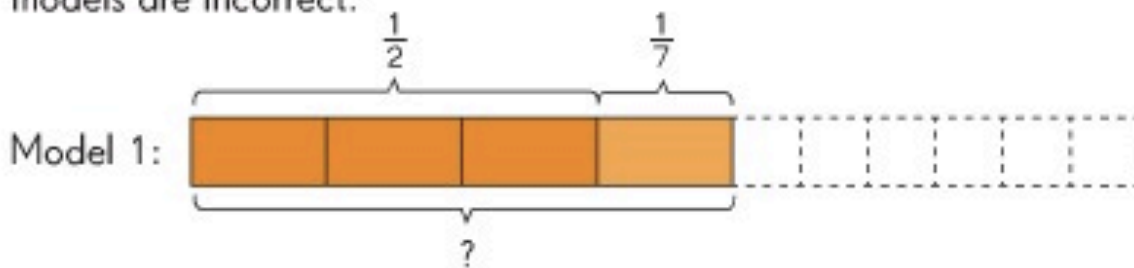
a Identify the correct one of the three.

b Explain why the other two are incorrect.

Wednesday, October 31, 12

Common Core Instructional emphases

- Focus: Prioritize topics and focus on them
- Coherence: Topics taught to mastery so each grade level builds on the previous ones
- Depth of understanding: Concepts are taught so students understand how, why, and when they work
- Balance conceptual understanding and procedural fluency: students understand concepts deeply and become fluent with procedures and facts

What can we learn from Singapore?

- Teach math as a vehicle for teaching thinking

- Use an effective pedagogy – concrete to pictorial to abstract - that develops understanding and fluency

- Recognize the importance of visualization

- Teach fewer topics to mastery and greater proficiency in each grade

Math in Focus

Singapore Math by Marshall Cavendish

- # Teach math as a vehicle for teaching thinking

Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals.

COMMON CORE STATE STANDARDS FOR

Mathematics

Mathematics Framework created by the Singapore Ministry of Education
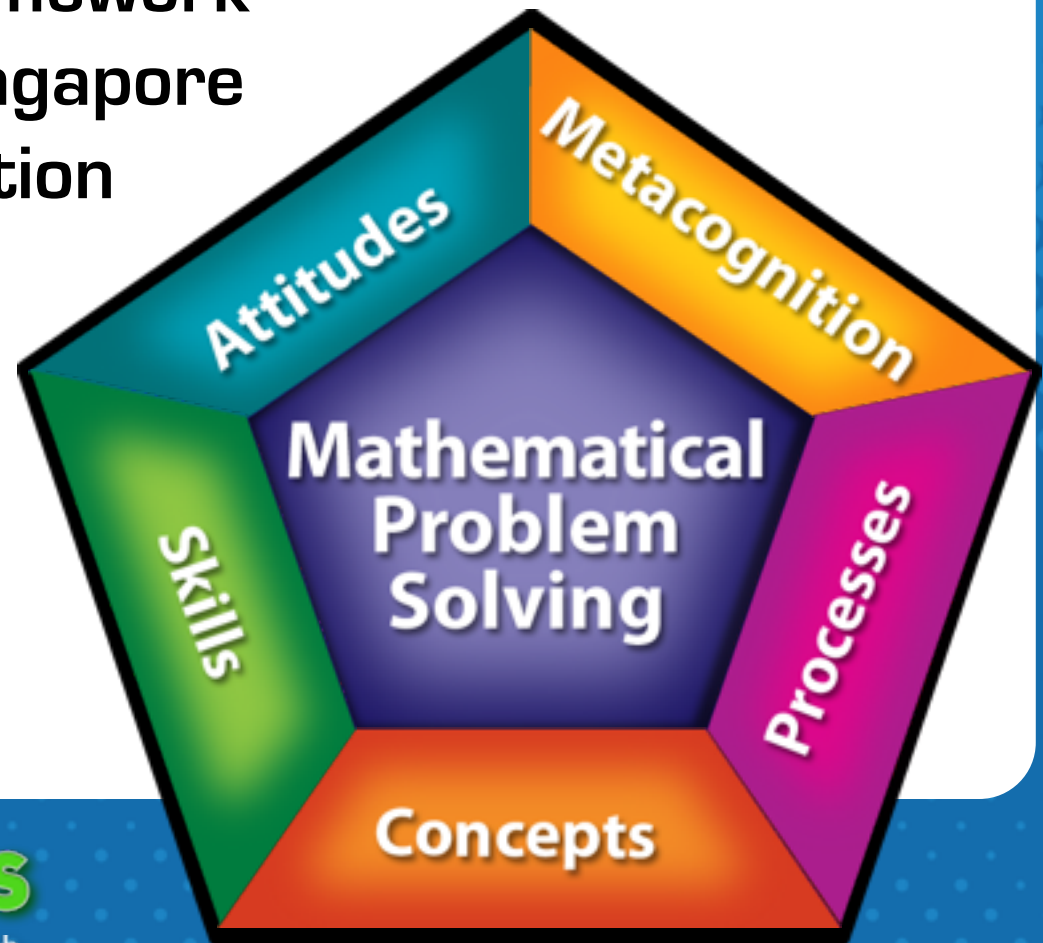
Teach math as a vehicle for teaching thinking: examples

⑬ Math Journal The ratio of the number of beads collected by Jane to the number of beads collected by Jill is 7 : 3. Jane gave some beads to Jill. Is it possible for both Jane and Jill to have the same number of beads after Jane gave some beads to Jill? Explain why you think so.

Make sense of problems and persevere
Reason abstractly and quantitatively
Construct viable arguments

CCSS

Math As Thinking

10 The petal of a paper flower is created by cutting along the outlines of two overlapping quadrants within a square. Use 3.14 as an approximation for π.

a) Find the distance around the shaded part.

b) Find the area of the shaded part.

10 cm
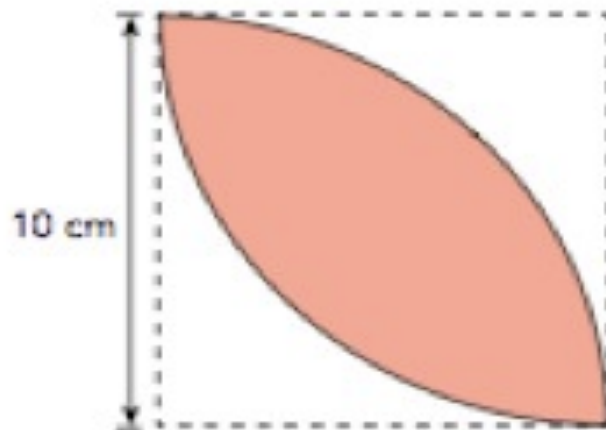
Teach math as a vehicle for teaching thinking

"Mathematics is an excellent vehicle for the development and improvement of a person's intellectual competence in logical reasoning, spatial visualization, analysis and abstract thought."

"Mathematics is also a subject of enjoyment and excitement, which offers students opportunities for creative work and moments of enlightenment and joy."

Singapore MOE Math Syllabus

Use an effective instructional strategy: Concrete - Pictorial - Abstract

Grade 6

Concrete

Pictorial

Abstract

Use an effective instructional strategy:
Concrete - Pictorial - Abstract
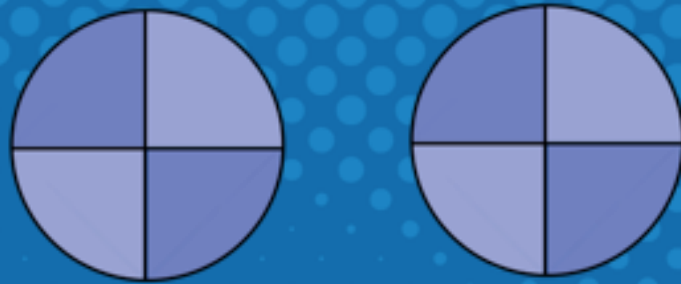
How many fourths in 1 whole? In 2

How many fourths in 6 wholes? In 20?
If you know one whole, how can you find the number
in any number of wholes

#wholes x 4 = number of fourths

Concrete - Pictorial - Abstract

How many 2/3 in 1 whole?

How many 2/3 in 2 wholes?

Now how many 2/3 in 1 whole? Half of 3 or 3/2

So if we know how many in 1, how can we
find how many 2/3 in 5 wholes?

5 x 3/2 = 5 ÷ 2/3
7 x 3/2 = 7 ÷ 2/3

Mine is not to reason why…

$$2 \div \frac{2}{3} =$$

$$2 \div \frac{2}{3} =$$

Han ✋

DIVIDING WHOLE N

Use 5 paper strips o___ ___p represents 1 whole.

Materials:
- 5 paper strips

1 Take 2 paper strips. Divide each of them into thirds using vertical lines and
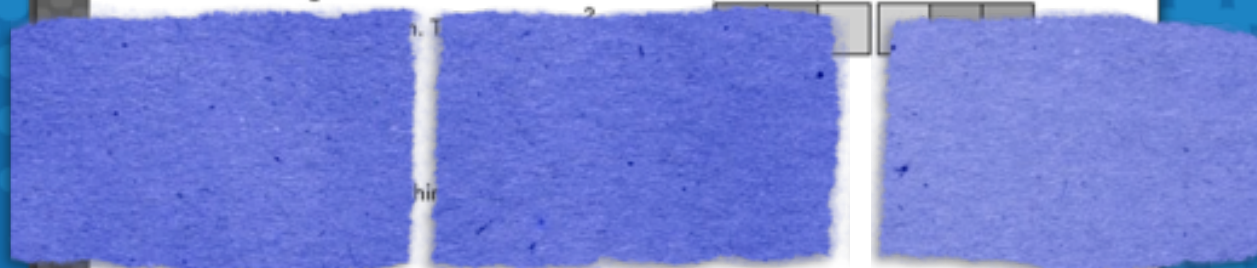
2 Divide each of the other 3 paper strips into fourths using vertical lines and place them as shown. Then find $3 \div \frac{3}{4}$.
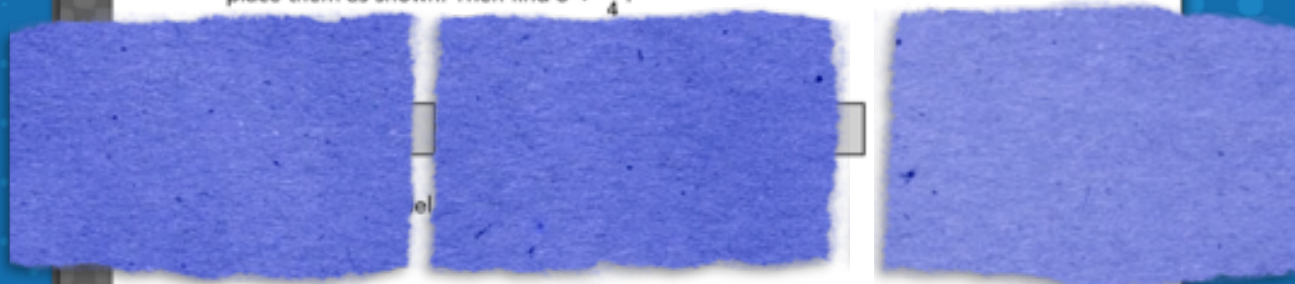
There are __?__ three-fourths in the 3 paper strips.

So, $3 \div \frac{3}{4} = $ __?__.

Middle School Course 1, Book A, Page 67

Concrete - Pictorial - Abstract

A plank is $\frac{4}{5}$ meter in length. A worker cuts it into some pieces, each of which is $\frac{1}{10}$ meter long. Into how many pieces did he cut the plank?

Plank

1/10
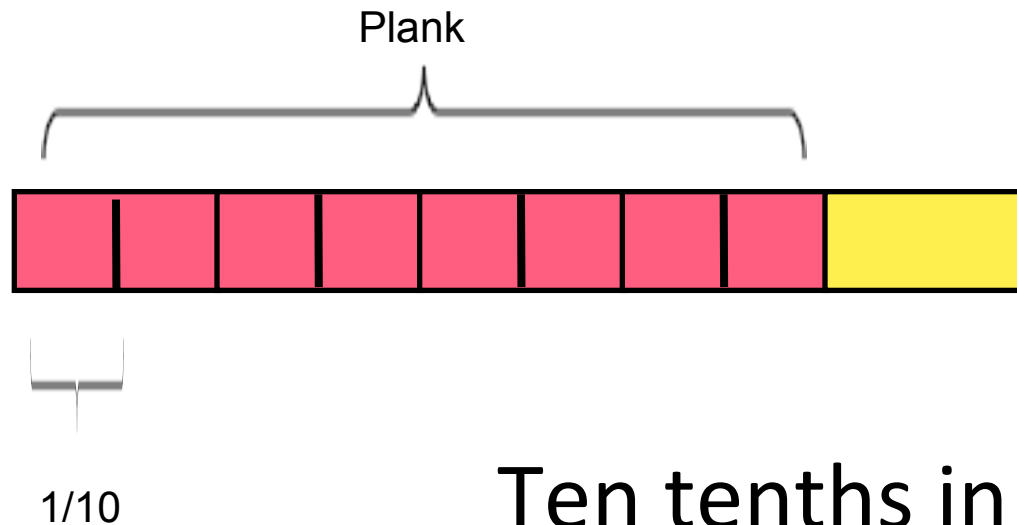
Ten tenths in 5/5, so
4/5 x 10 in 4/5 of a yard

Concrete - Pictorial - Abstract

A pitcher contains $\frac{4}{5}$ quart of juice. If the juice is poured into glasses that hold $\frac{3}{10}$ quart, how many glasses can be filled? How much juice is left in the pitcher?

Concrete - Pictorial - Abstract

$$1/2 \div 1/4$$
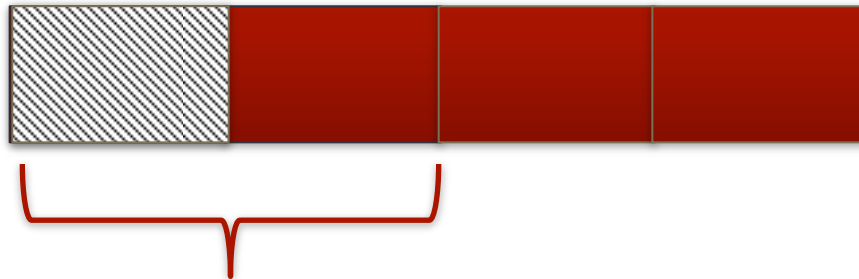

1/2

4 fourths in 1, so 1/2 x 4 in 1/2 of a whole
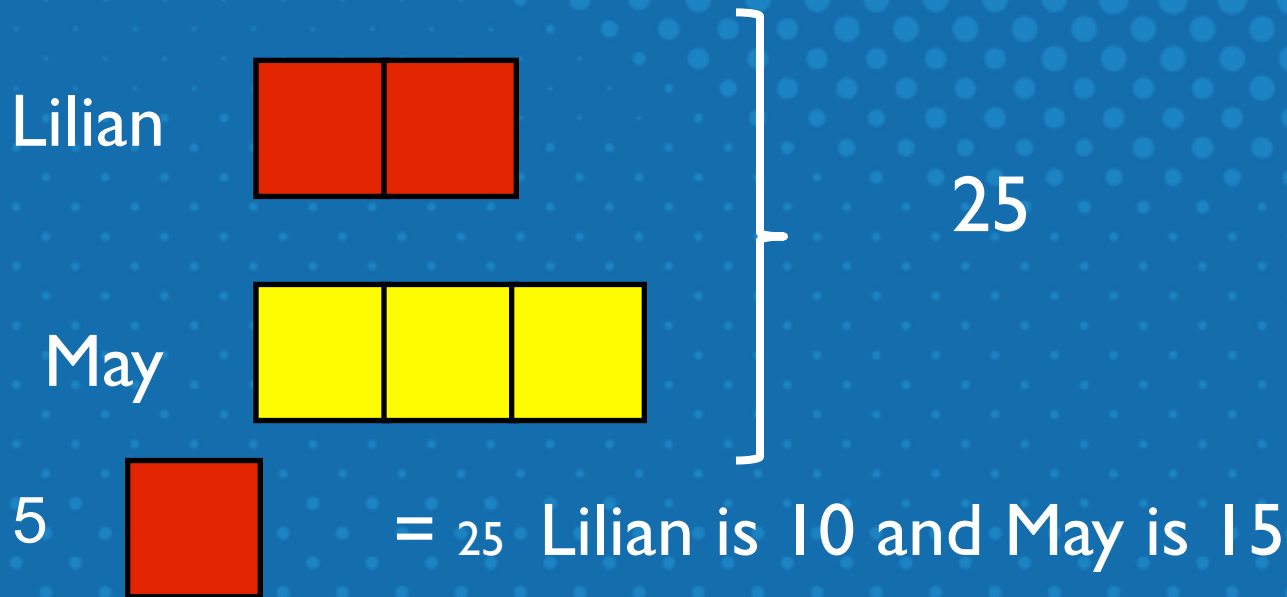
Concrete - Pictorial - Abstract

$$1/4 \div 1/2$$

1/2

Two halves in 1, so 1/4 times 2 = 1/2 in a quarter

Why visualization?

Lilian's present age is 2/3 times May's age.

a) Find the ratio of Mays age to Lilian's age.

b) How many times the total age of the two girls is Lilian's age?

c) Their combined age is 25 years. Find the age of

Lilian

May

25

5 ▪ = 25 Lilian is 10 and May is 15

- Model drawing, often called bar modeling in the U.S., is a systematic method of representing word problems and number relationships that is explicitly taught in Singapore beginning in **second grade** and extending all the way to algebra

Math in Focus
Singapore Math by Marshall Cavendish

Multiplicative comparison

Ahmad sells 32 video games. He sells 4 times as many video games as Justin. How many video games does Justin sell?
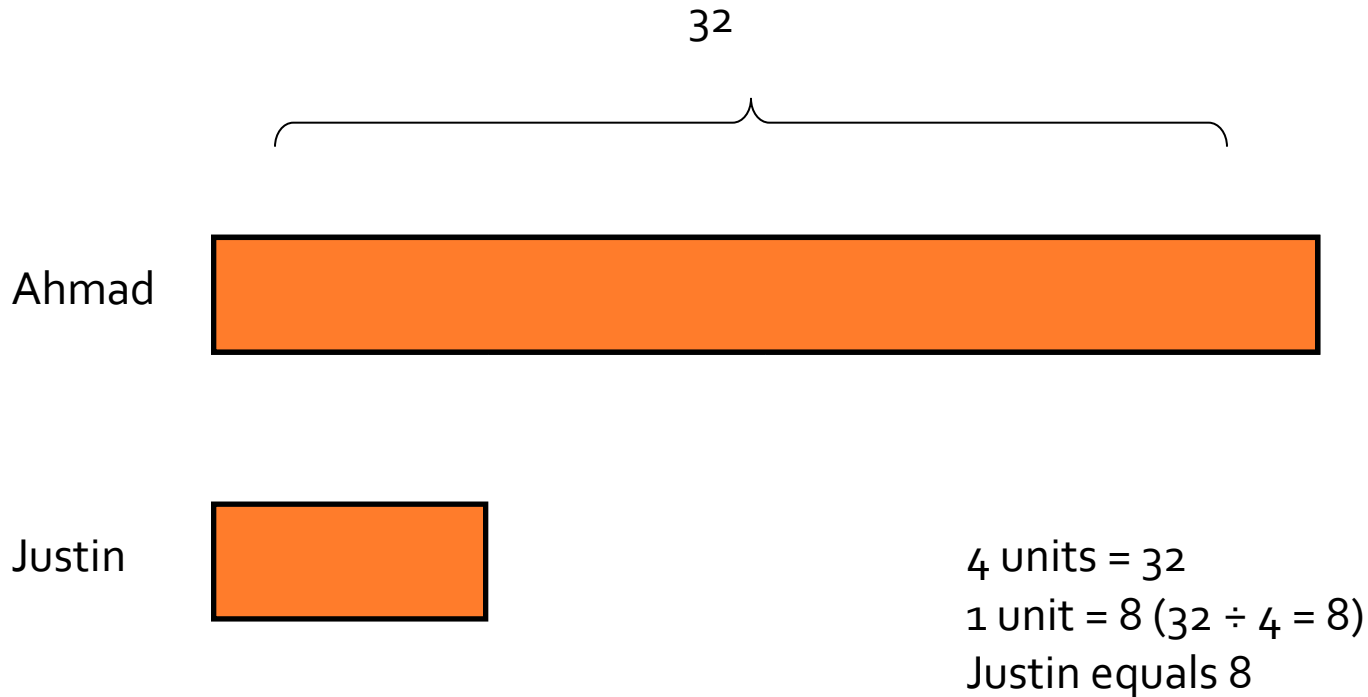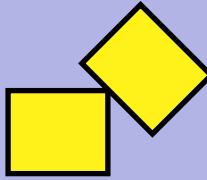
32
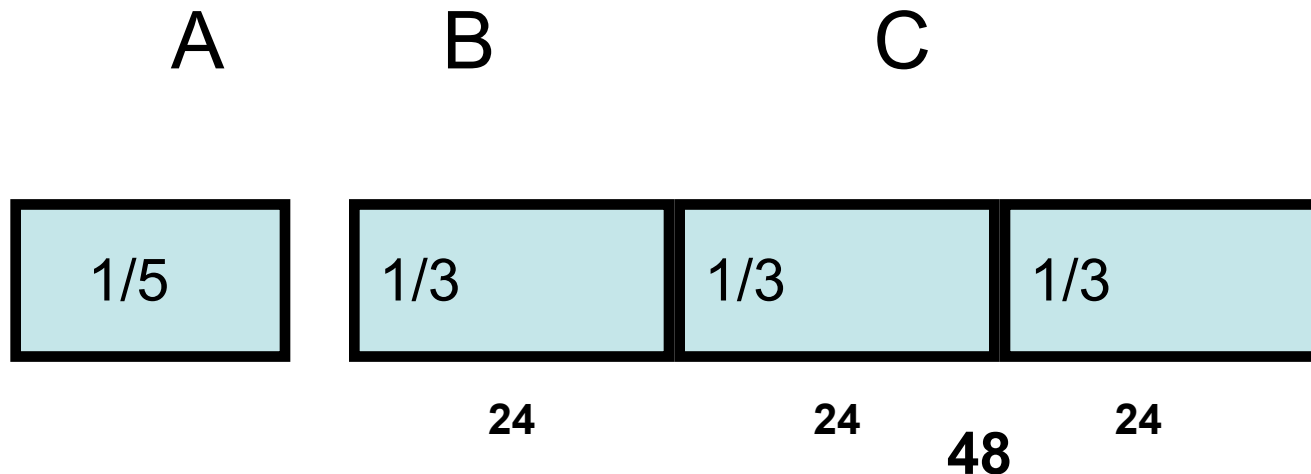
Ahmad

Justin

4 units = 32
1 unit = 8 (32 ÷ 4 = 8)
Justin equals 8

- Ben took a test with 3 sections A,B, and C. Ben spent 1/5 of his time on Section A and 1/3 of the remaining time on Section B. He spent 48 minutes on section C. How much time did Ben take to complete the whole test.

- Ben took a test with 3 sections A,B, and C. Ben spent 1/5 of his time on Section A and 1/3 of the remaining time on Section B. He spent 48 minutes on section C. How much time did Ben take to complete the whole test.

A   B   C

| 1/5 | | 1/3 | 1/3 | 1/3 |

24   24   24

48

72 = 4/5, so 72 ÷ 4 = 1/5
1/5 = 18, 5/5 = 90 He spent 90 minutes on test

GRADE 5: Fifth Grade

- Ben took a test with 3 sections A,B, and C. Ben spent 1/5 of his time on Section A and 1/3 of the remaining time on Section B. He spent 48 minutes on section C. How much time did Ben take to complete the whole test.

A   B

? **what fraction?**

| 1/5 | 1/3 rest | 48 | | | | | | |

1- 1/5 = 4/5 = 12/15
1/3 of 12/15 = 4/15
12/15 - 4/15 = 8/15

8/15 = 48 so
1/15 = 6 minutes
15/15 = 90 minutes

Ricardo spends **8/9** hour reading the newspaper. He spends **1/4** of the time reading the world news and splits the remaining time equally between the sports and the comics.

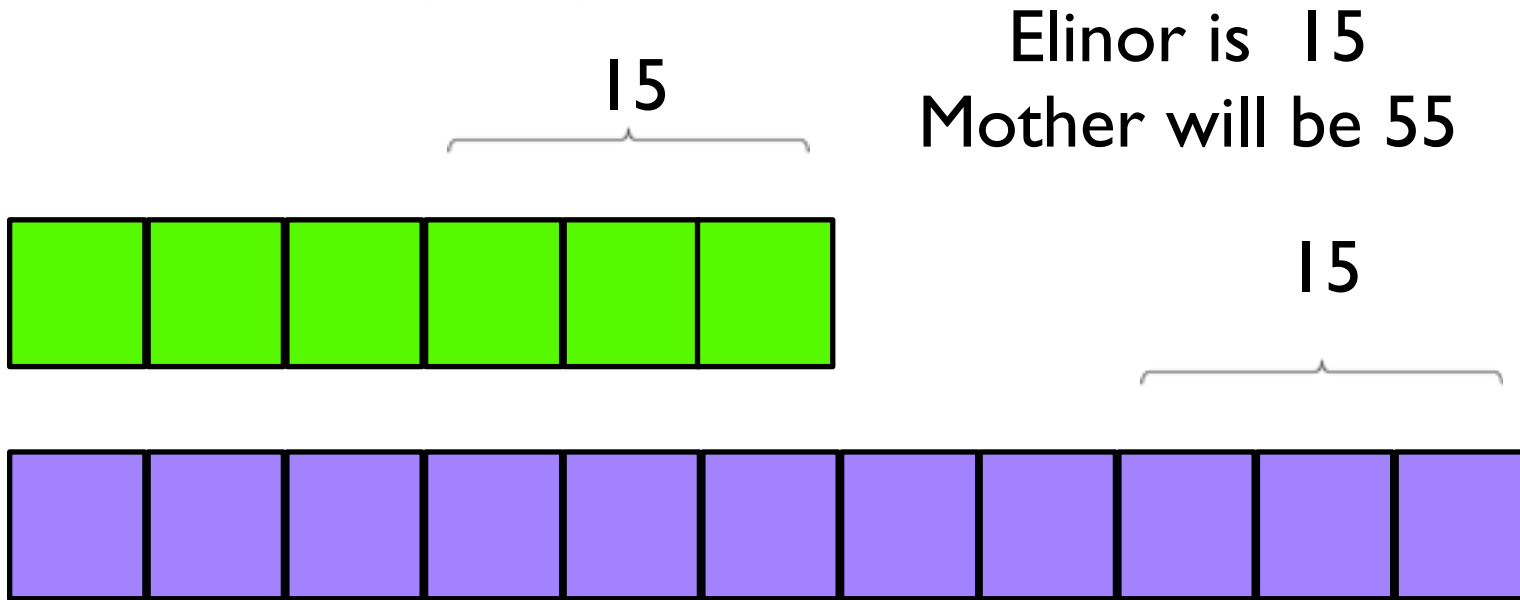How much time does he spend reading the comics?

hour
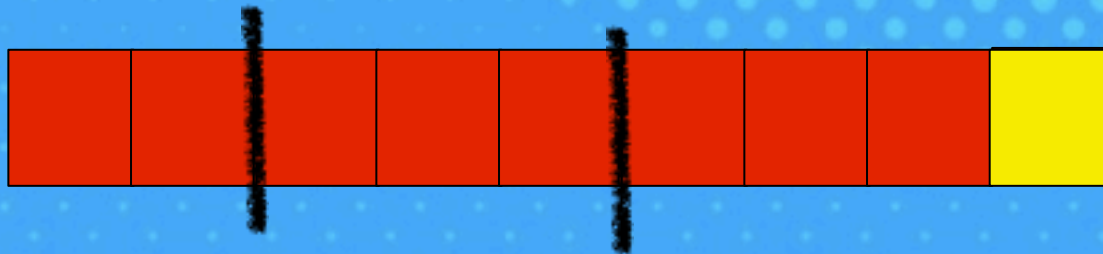
world news

Reads comics for 3/9 OR 1/3 of an hour

**14** Today the ratio of Elinor's age to her mother's age is 3 : 8. After 15 years, the ratio will become 6 : 11.

a) Find Elinor's age today.

b) Find her mother's age after 15 years.

Elinor is 15
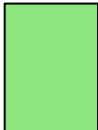Mother will be 55

15

15

Grade 6: ratio problems

- The ratio of the number of beads Karen had to # of beads Patricia had was 2:5. After Patricia bought another 75 beads, the ratio became 4:15 How many beads did each girl have at first?

Karen

Pat

75
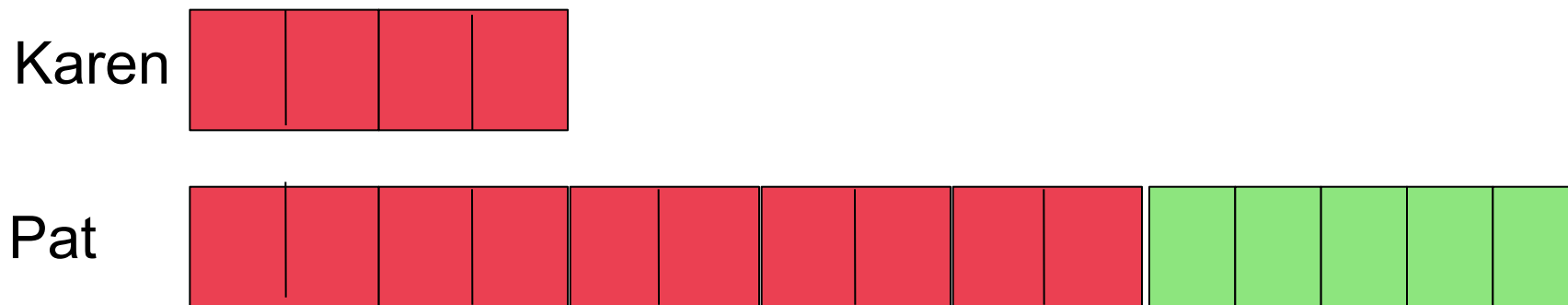
$75 \div 5 = 15$
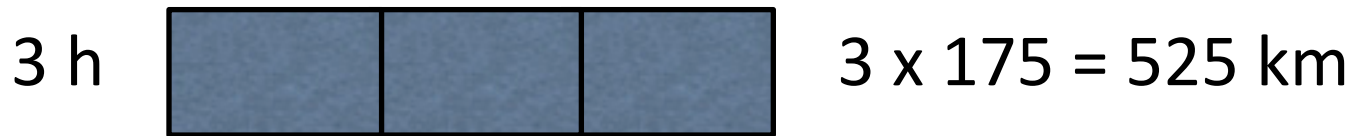
Karen = 60
Patricia had 150

= 15

Grade 6: From ratios to rates

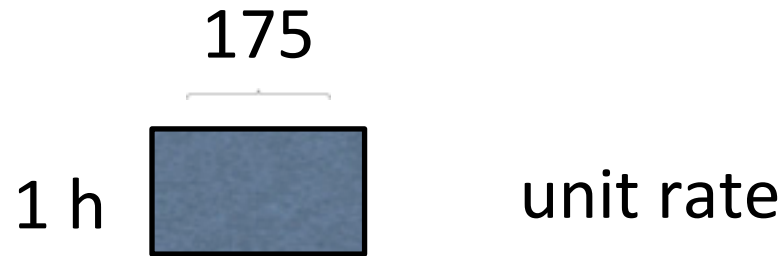"Solve unit rates problems including those involving unit pricing and constant speed."

COMMON CORE
STATE STANDARDS FOR

Mathematics

A racing car can travel at a speed of 175 km per hour. How far can the racing car travel in 3 hours?

175

1 h   unit rate

3 h   3 x 175 = 525 km

or method 2:

Distance = Rate x Time
Distance = 175 km/hour x 3 hours = 525 km

11 Mr. Alan drove for $2\frac{1}{5}$ hours at a speed of 70 kilometers per hour. He then drove another 224 kilometers. He took 5 hours for the whole journey. What was Mr. Alan's average speed for the whole journey?

12 A family took 2 hours to drive from City A to City B at a speed of 55 miles per hour. On the return trip, due to a snowstorm, the family took 3 hours to travel back to City A.

a)   How many miles did the family travel in all?

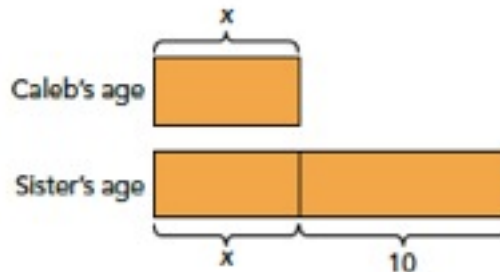b)   What was the average speed for the entire trip?

Students learn to write linear equations

Write a linear equation to represent a given situation.

a) Caleb is x years old. His sister is 10 years older than he is. If his sister is y years old, write an equation that relates their two ages.

From the model, an expression for the sister's age is $x + 10$.

To make an equation using this expression, notice that the problem says that the sister's age is another variable y.

So you can write:

$y = x + 10$

The equation $y = x + 10$ is called a linear equation.

Math in Focus
Singapore Math
by Marshall Cavendish

Grade 7: Variable expressions

$$3(2x + 4) = 3(2x) + 3(4)$$

Use the distributive property.

$$= 6x + 12$$

Multiply.

$$2(5x - 1) = 2(5x) - 2(1)$$

Use the distributive property.

$$= 10x - 2$$

Multiply.

You obtain an expression equivalent to the original expression after expandir

$$3(2x + 4)$$

$$6x + 12$$

Method 1

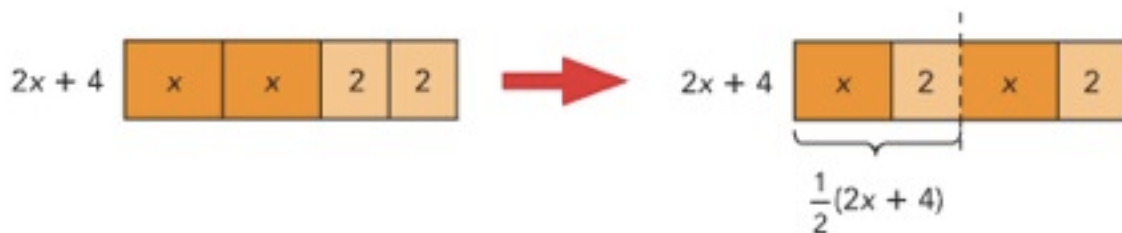You can rearrange the bar model into 2 equal groups.

2x + 4 [x | x | 2 | 2] → 2x + 4 [x | 2 ¦ x | 2]

$\frac{1}{2}(2x + 4)$

From the bar model,

$\frac{1}{2}(2x + 4) = x + 2$

> You can rearrange the bar model into 2 equal groups to find one half of (2x + 4).

Method 2

You can also use the distributive property to expand $\frac{1}{2}(2x + 4)$.

$\frac{1}{2}(2x + 4) = \frac{1}{2}(2x) + \frac{1}{2}(4)$ Use the distributive property.

          $= x + 2$ Multiply.

$\frac{1}{2}(2x + 4)$ and x + 2 are equivalent expressions.

Grade 7: Solving equations

Lisa wrote a riddle: a positive number is 1/3 of another positive number. If their difference is 48, find the two positive numbers

Large #

Small #

48
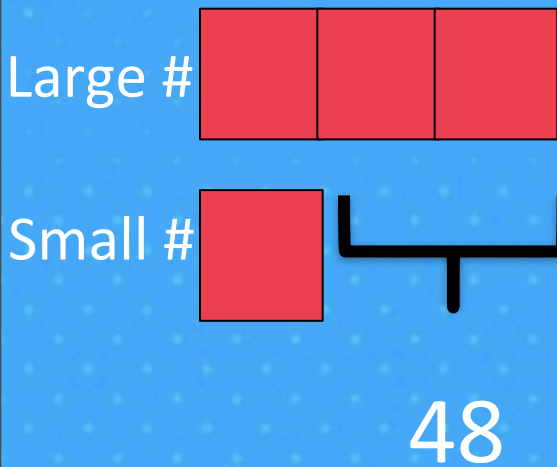
or method 2

x - 1/3x = 48

3/3x - 1/3x = 48

2/3 x = 48

x = 72, other is 24

Two apples and a mango cost $4.
Two apples and three mangos cost $9.
Find the cost of a mango.

$ 4

?

$ 9

$$x + y = 8$$
$$x + 2y = 10$$

8

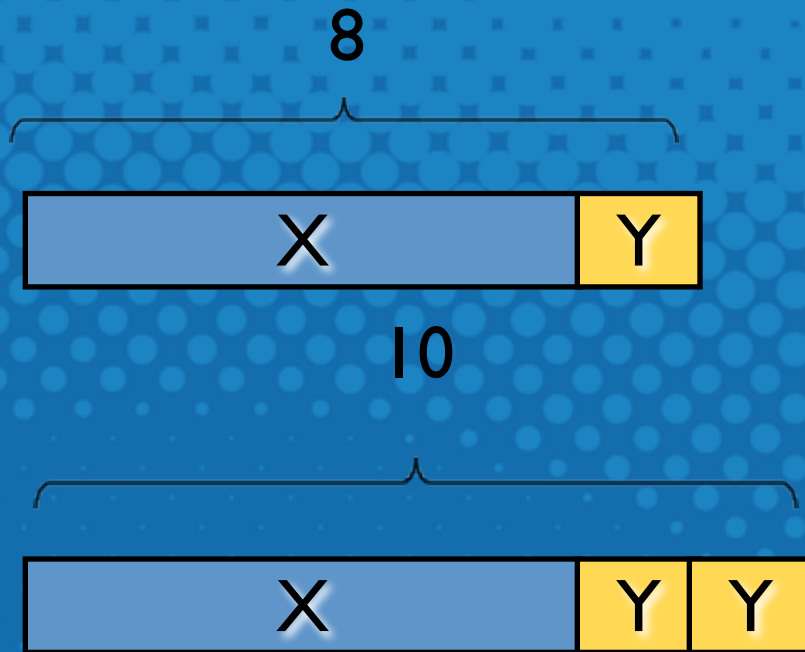X   Y

10

X   Y Y
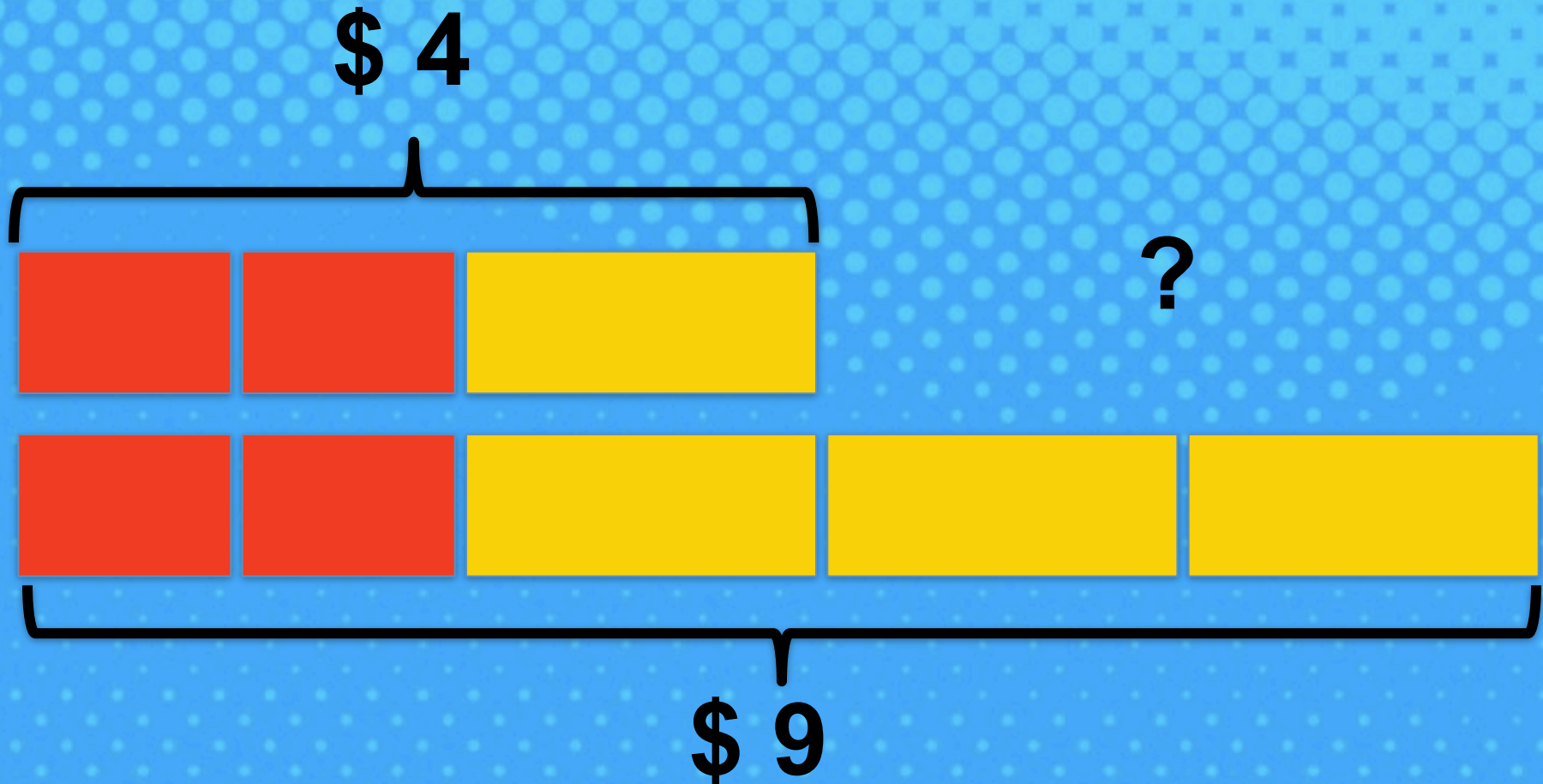
Solve algebraically, first with elimination, then with substitution

$$(x + 2y) - (x + y) = 10 - 8 = 2$$

Trajectory Algebra 8th Grade

7

$$X \quad X \quad Y \quad Y \quad Y$$

8

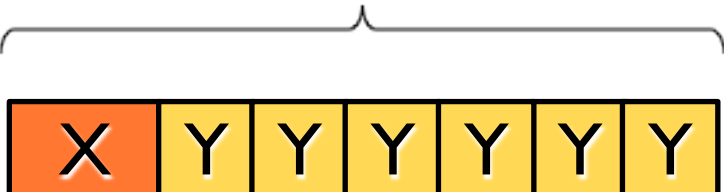$$X \quad Y \quad Y \quad Y \quad Y \quad Y \quad Y$$

$2x + 3y = 7$
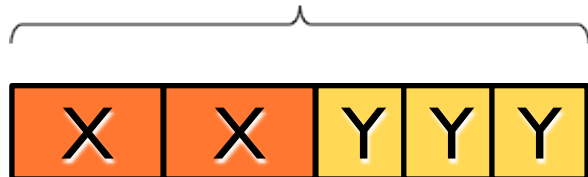
$x + 6y = 8$

If we double the bottom equation, we will have 2 x and 3 y to subtract

? 16 - 7 = 9   y = 1, x = 2

$$X \quad X \quad Y \quad Y \quad Y$$

$$X \quad X \quad Y \quad Y \quad Y \quad Y \quad Y \quad Y \quad Y \quad Y \quad Y \quad Y \quad Y \quad Y$$

16

Grade 8: Systems of equations

Example 9 **Solve real-world systems of linear equations by the graphical method.**

Two cars are traveling along a highway in the same direction. Their motions are described by the linear equations

$$d = 60t$$
$$d = 50t + 20$$

where t is the time (hours) and d is the distance (miles) from point A on the highway.

a) Solve the system of equations graphically.

b) When will the cars meet?

Solution

a) Sketch the graphs of the two equations using the slope and y-intercept values.

Journey of Two Cars

$d = 50t + 20$
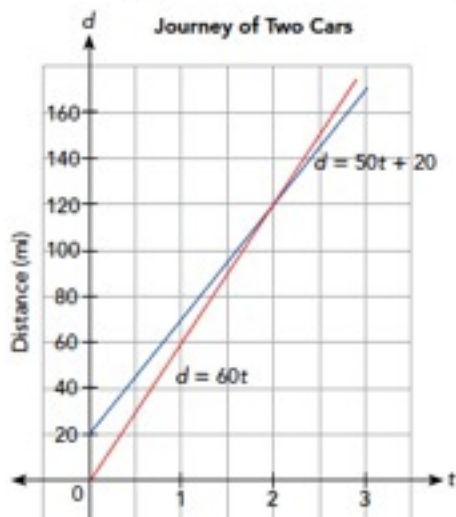
$d = 60t$

Because Distance = Rate × Time, these two equations tell me much about the situation. What is the speed of each car? Do they start at the same time? Do they start at the same place?

Then graphical solutions for problems

What does d = 60t mean?

What does d = 50t + 20 mean?

NAEP 2011 8th Grade

Which of the following is an equation of a line that passes through the point (0, 5) and has a negative slope?

Ⓐ $y = 5x$

Ⓑ $y = 5x - 5$

Ⓒ $y = 5x + 5$

Ⓓ $y = -5x - 5$

Ⓔ $y = -5x + 5$

Percentage of eighth-grade students in each response category: 2011

Choice A	Choice B	Choice C	Choice D	Choice E	Omitted
12	27	9	20	**31**	1

- Teach fewer topics to mastery and greater proficiency in each grade

  - Concepts covered in one grade are covered in later grades, but only at a more advanced level
  - Concepts are covered thoroughly when introduced and each grade builds on the previous grade

Math in **Focus**

Singapore Math by Marshall Cavendish

Teach to mastery and proficiency

Write an expression for the area of the figure. Expand and simplify.

51
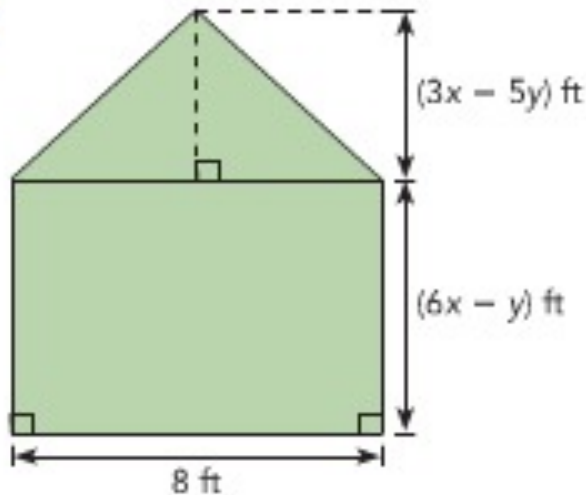
$(3x - 5y)$ ft

$(6x - y)$ ft

8 ft

• Teach to mastery and proficiency

Guided Practice

Solve. Copy and complete.

1. Mark wrote a riddle: A negative number is $\frac{2}{5}$ of another negative number. If the sum of the two negative numbers is -35, find the two negative numbers.

-35

• Teach to mastery and proficiency

Guided Practice

Solve. Copy and complete.

1 Mark wrote a riddle: A negative number is $\frac{2}{5}$ of another negative number. If th sum of the two negative numbers is -35, find the two negative numbers.

x + 2/5x = -35

7/5 x = -35

divide both sides by 7/5

x = -25

Math in Focus

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What can we learn from Singapore?

- Teach math as a vehicle for teaching thinking

- Use an effective pedagogy – concrete to pictorial to abstract - that develops understanding and fluency

- Recognize the importance of visualization

- Teach fewer topics to mastery and greater proficiency in each grade

Math in Focus

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