ANGLES 2.1.1 – 2.1.5

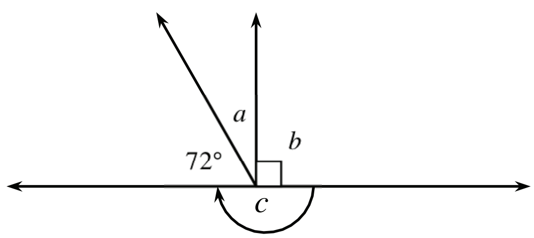
Applications of geometry in "everyday" settings often involve the measures of angles. In this chapter we begin our study of angle measurement. After describing angles and recognizing their characteristics, students complete an Angle Relationships Toolkit (Lesson 2.1.3 Resource Page). The toolkit lists some special angles and then students record important information about them. The list includes vertical angles (which are always equal in measure), straight angles (which measure 180°), corresponding angles, alternate interior angles, and same-side interior angles.

See the Math Notes boxes in Lessons 2.1.1 and 2.1.4 for more information about angle relationships.

Example 1

In each figure below, find the measures of angles a, b, and/or c. Justify your answers.

a.



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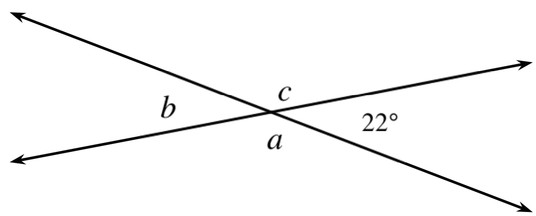

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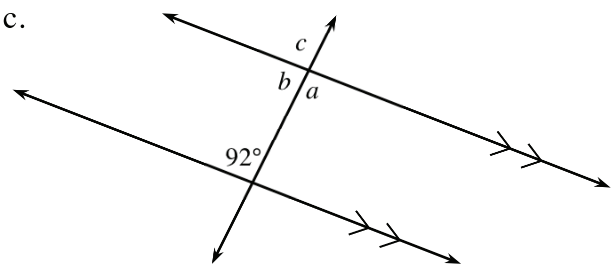

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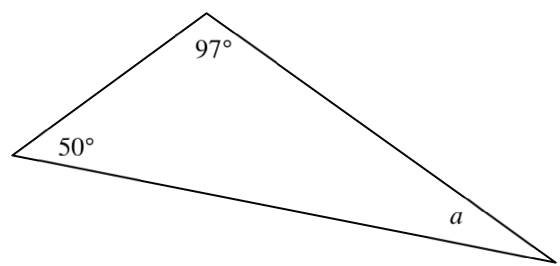

Each figure gives us information that enables us to find the measures of the other angles. In part (a), the little box at angle b tells us that angle b is a right angle, so $m\angle b = 90°$. The angle labeled c is a straight angle (it is opened wide enough to form a straight line) so $m\angle c = 180°$. To calculate $m\angle a$ we need to realize that $\angle a$ and the 72° angle are complementary which means together they sum to 90°. Therefore, $m\angle a + 72° = 90°$ which tells us that $m\angle a = 18°$.

In part (b) we will use two pieces of information, one about supplementary angles and one about vertical angles. First, $m\angle a$ and the 22° angle are supplementary because they form a straight angle (line), so the sum of their measures is 180°. Subtracting from 180° we find that $m\angle a = 158°$. Vertical angles are formed when two lines intersect. They are the two pairs of angles that are opposite (across from) each other where the lines cross. Their angle measures are always equal. Since the 22° angle and $\angle b$ are a pair of vertical angles, $m\angle b = 22°$. Similarly, $\angle a$ and $\angle c$ are vertical angles, and therefore equal, so $m\angle a = m\angle c = 158°$.

The figure in part (c) shows two parallel lines that are intersected by a transversal. When this happens we have several pairs of angles with equal measures. $\angle a$ and the 92° angle are called alternate interior angles, and since the lines are parallel (as indicated by the double arrows on the lines), these angles have equal measures. Therefore, $m\angle a = 92°$. There are several ways to calculate the remaining angles. One way is to realize that $\angle a$ and $\angle b$ are supplementary. Another uses the fact that $\angle b$ and the 92° angle are same-side interior angles, which makes them supplementary because the lines are parallel. Either way gives the same result: $m\angle b = 180° - 92° = 88°$. There is also more than one way to calculate $m\angle c$. We know that $\angle c$ and $\angle b$ are supplementary. Alternately, $\angle c$ and the 92° angle are corresponding angles, which are equal because the lines are parallel. A third way is to see that $\angle a$ and $\angle c$ are vertical angles. With any of these approaches, $m\angle c = 92°$.

Part (d) is a triangle. In class, students investigated the measures of the angles in a triangle. They found that the sum of the measures of the three angles always equals 180°. Knowing this, we can calculate $m\angle a$: $m\angle a + 50° + 97° = 180°$. Therefore, $m\angle a = 33°$.

Problems

Use the geometric properties and theorems you have learned to solve for x in each diagram and write the property or theorem you use in each case.

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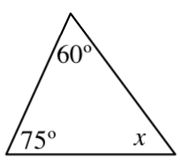

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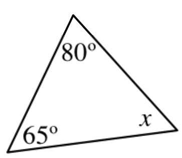

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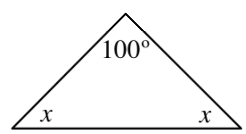

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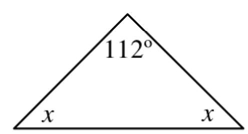

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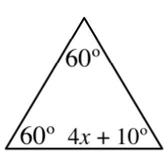

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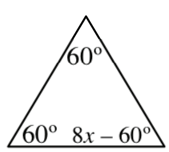

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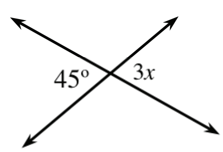

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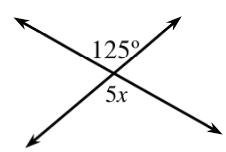

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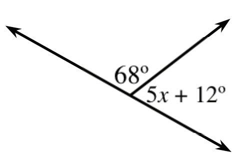

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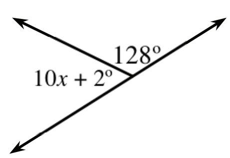

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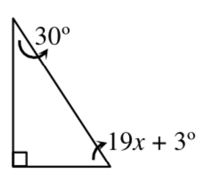

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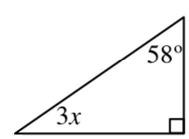

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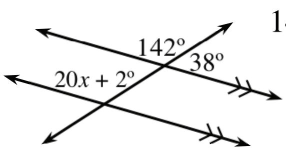

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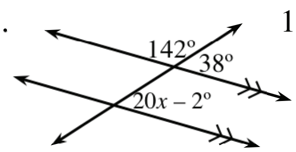

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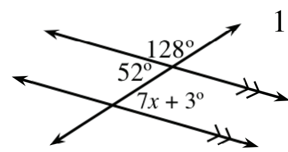

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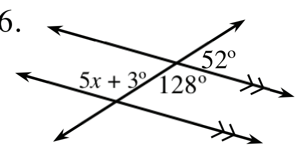

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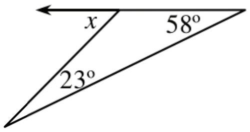

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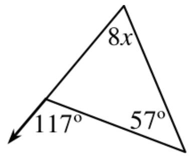

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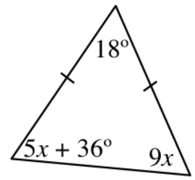

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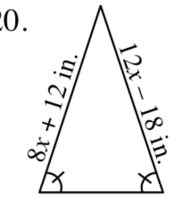

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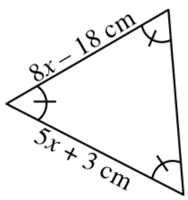

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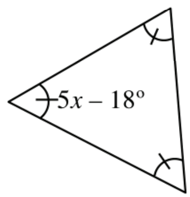

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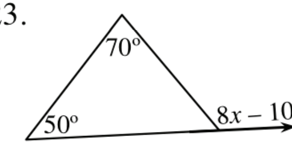

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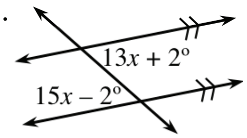

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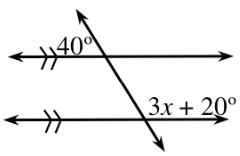

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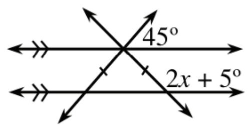

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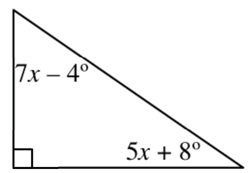

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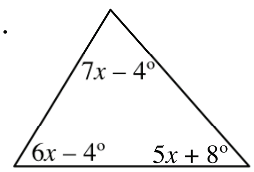
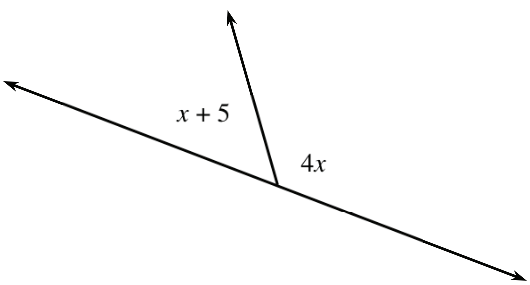


Use what you know about angle measures to find x, y, or z.

29.



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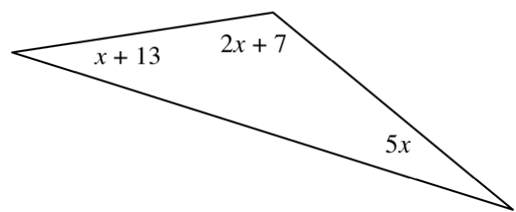

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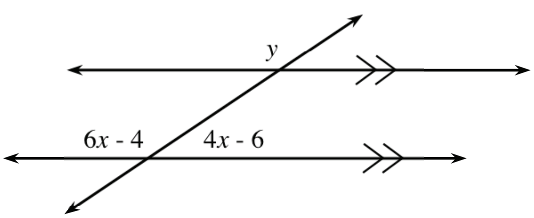

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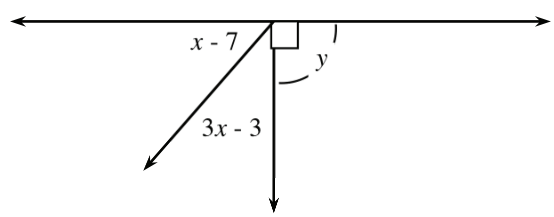

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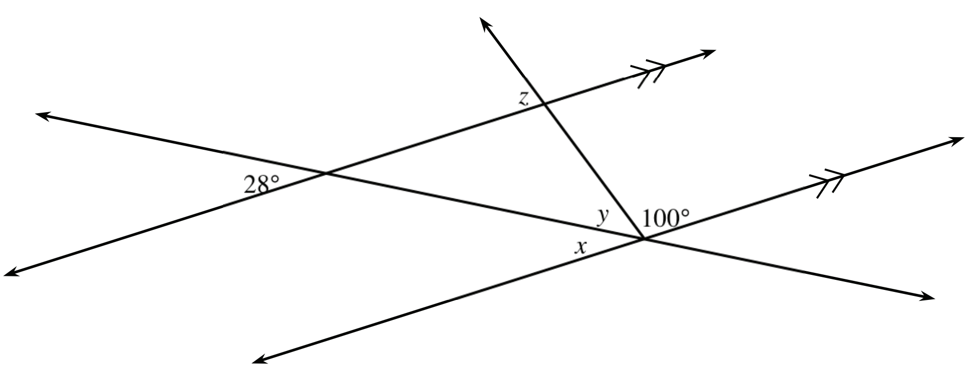
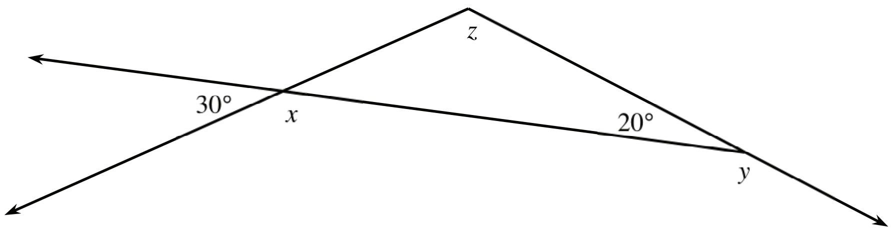


34. 

In Lesson 2.1.5 we used what we have learned about angle measures to create proofs by contradiction. (See the Math Notes box in Lesson 2.1.5.) Use this method of proof to justify each of your conclusions to problems 35 and 36 below.

35. Nik scored 40 points lower than Tess on their last math test. The scores could range from 0 to 100 points. Could Tess have scored a 30 on this test? Justify using a proof by contradiction.

36. Can a triangle have two right angles? Justify your answer with a proof by contradiction.

Answers

1.	$x = 45^\circ$	2.	$x = 35^\circ$	3.	$x = 40^\circ$	4.	$x = 34^\circ$
5.	$x = 12.5^\circ$	6.	$x = 15^\circ$	7.	$x = 15^\circ$	8.	$x = 25^\circ$
9.	$x = 20^\circ$	10.	$x = 5^\circ$	11.	$x = 3^\circ$	12.	$x = 10\frac{2}{3}^\circ$
13.	$x = 7^\circ$	14.	$x = 2^\circ$	15.	$x = 7^\circ$	16.	$x = 25^\circ$
17.	$x = 81^\circ$	18.	$x = 7.5^\circ$	19.	$x = 9^\circ$	20.	$x = 7.5^\circ$
21.	$x = 7^\circ$	22.	$x = 15.6^\circ$	23.	$x = 26^\circ$	24.	$x = 2^\circ$
25.	$x = 40^\circ$	26.	$x = 65^\circ$	27.	$x = 7\frac{1}{6}^\circ$	28.	$x = 10^\circ$

29. $(x + 5) + 4x = 180$, $x = 35^\circ$

30. $(x + 13) + (2x + 7) + 5x = 180$, $x = 20^\circ$

31. $(6x - 4) + (4x - 6) = 180$, $x = 19^\circ$, $y = 110^\circ$

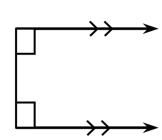
32. $(x - 7) + (3x - 3) = 90$, $x = 25^\circ$, $y = 90^\circ$

33. $x = 28^\circ$, $y = 52^\circ$, $z = 80^\circ$

34. $x = 150^\circ$, $y = 160^\circ$, $z = 130^\circ$

35. If Tess scored 30 points, then Nik's score would be -10, which is impossible. So Tess cannot have a score of 30 points.

36. If a triangle has two right angles, then the measure of the third angle must be zero. However, this is impossible, so a triangle cannot have two right angles. OR: If a triangle has two 90° angles, the two sides that intersect with the side between them would be parallel and never meet to complete the triangle, as shown in the figure.



AREA 2.2.1 – 2.2.4

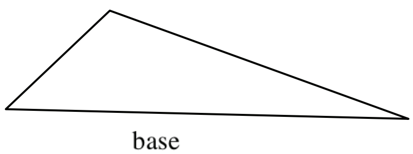
After measuring various angles, students look at measurement in more familiar situations, those of length and area on a flat surface. Students develop methods and formulas for calculating the areas of triangles, parallelograms, and trapezoids. They also find the areas of more complicated shapes by partitioning them into shapes for which they can use the basic area formulas. Students also learn how to determine the height of a figure with respect to a particular base.

See the Math Notes box in Lesson 2.2.4 for more information about area.

Example 1

In each figure, one side is labeled as the "base." For this "base," draw in a corresponding height.

a.



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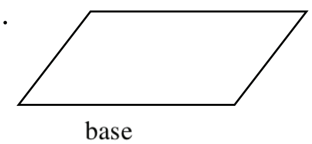

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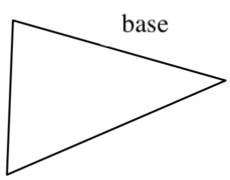

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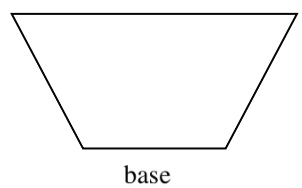
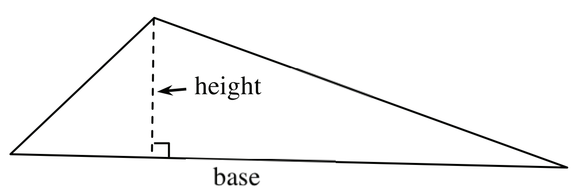
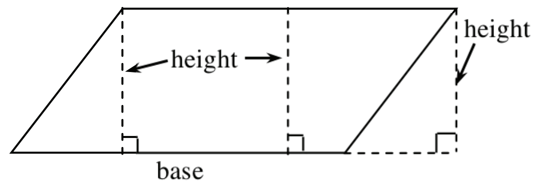


To find how tall a person is, we have them stand straight up and measure the distance from the highest point on their head straight down to the floor. We measure the height of figures in a similar way. One way to calculate the height is to visualize that the shape, with its base horizontal, needs to slide into a tunnel. How tall must the tunnel be so that the shape will slide into it? How tall the tunnel is equals the height of the shape. The height is perpendicular to the base (or a line that contains the base) from any of the shape's "highest" point(s). In class, students also used a 3×5 card to help them draw in the height.

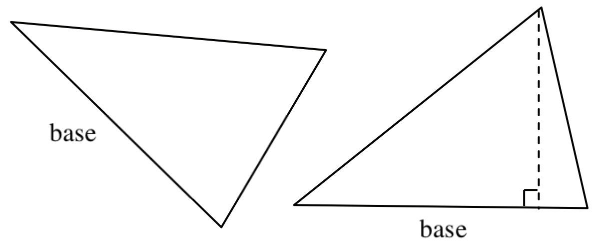
a. It is often easier to draw in the height of a figure when the base is horizontal, or the "bottom" of the figure. The height of the triangle at right is drawn from the highest point down to the base and forms a right angle with the base.



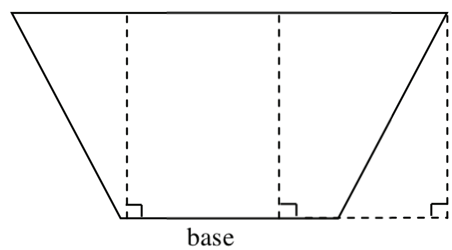
b. Even though the shape at right is not a triangle, it still has a height. In fact, the height can be drawn in any number of places from the side opposite the base. Three heights, all of equal length, are shown.



c. The base of the first triangle at right is different from the one in part (a) in that no side is horizontal or at the bottom. Rotate the shape, then draw the height as we did in part (a).



d. Shapes like the trapezoid at right or the parallelogram in part (b) have at least one pair of parallel sides. Because the base is always one of the parallel sides, we can draw several heights. The height at far right shows a situation where the height is drawn to a segment that contains the base segment.



Example 2

Find the area of each shape or its shaded region below. Be sure to include the appropriate units of measurement.

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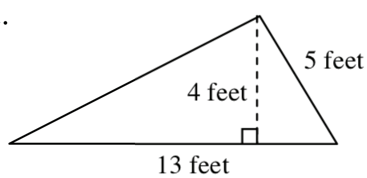

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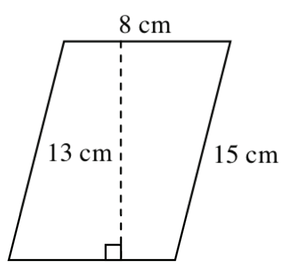

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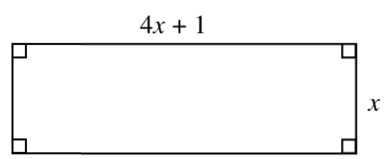

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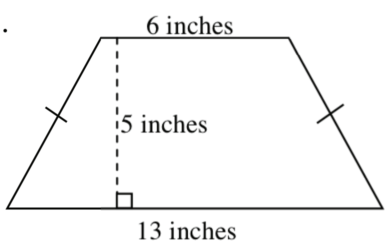

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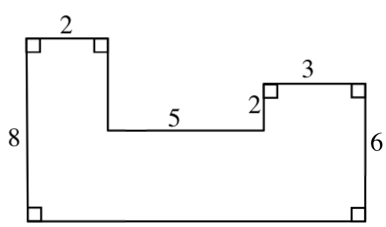

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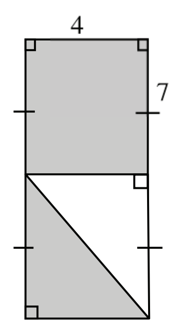
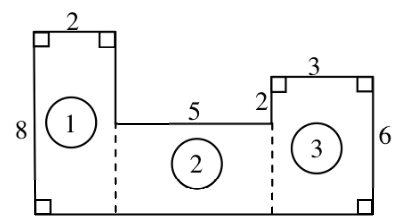


Students have the formulas for the areas of different shapes in their Area Toolkit (Lesson 2.2.4B Resource Page). For part (a), the area of a triangle is $A = \frac{1}{2}bh$, where b and h are perpendicular to each other. In this case, the base is 13 feet and the height is 4 feet. The side which is 5 feet is not a height because it does not meet the base at a right angle. Therefore, $A = \frac{1}{2}(13 \text{ feet})(4 \text{ feet}) = 26 \text{ feet}^2$. Area is measured in square units, while length (such as a perimeter) is measured in linear units, such as feet.

The figure in part (b) is a parallelogram and the area of a parallelogram is $A = bh$ where b and h are perpendicular. Therefore A = (13 cm)(8 cm) = 104 square cm.

The figure in part (c) is a rectangle so the area is also $A = bh$, but in this case, we have variable expressions representing the lengths of the base and height. We still calculate the area in the same way. $A = (4x+1)(x) = 4x^2 + x$ square units. Since we do not know in what units the lengths are measured, we say the area is just "square units."

Part (d) shows a trapezoid; the students found several different ways to calculate its area. The most common way is: $A = \frac{1}{2}(b_1 + b_2)h$ where b_1 is the upper base and b_2 is the lower base. As always, b and h must be perpendicular. The area is $A = \frac{1}{2}(6 \text{ in.} + 13 \text{ in.})5 \text{ in.} = 47.5$ square inches.

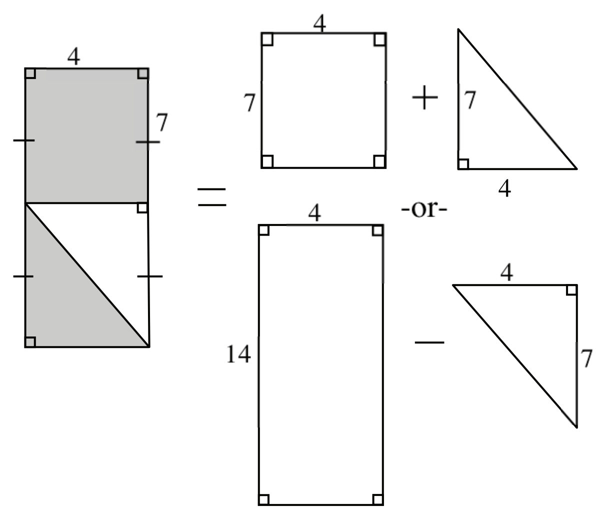


The figures shown in parts (e) and (f) are more complicated and one formula alone will not give us the area. In part (e), there are several ways to divide the figure into rectangles. One way is shown at right. The areas of the rectangles on either end are easy to find since the dimensions are labeled on the figure. The area of rectangle (1) is A = (2)(8) = 16 square units. The area of rectangle (3) is A = (3)(6) = 18 square units. To find the area of rectangle (2), we know the length is 5 but we have to determine its height. The height is 2 shorter than 6, so the height is 4. Therefore, the area of rectangle (2) is A = (5)(4) = 20 square units. Now that we know the area of each rectangle, we can add them together to find the area of the entire figure: A(entire figure) = 16 + 18 + 20 = 54 square units.

In part (f), we are finding the area of the shaded region, and again, there are several ways to do this. One way is to see it as the sum of a rectangle and a triangle. Another way is to see the shaded figure as a tall rectangle with a triangle cut out of it. Either way will give the same answer.

Using the top method,
$A = 4(7) + \frac{1}{2}(4)(7) = 42$ square units.

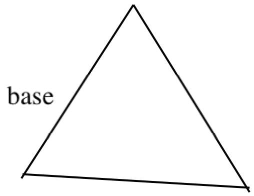
The bottom method gives the same answer:
$A = 4(14) - \frac{1}{2}(4)(7) = 42$ square units.



Problems

For each figure below, draw in a corresponding height for the labeled base.

1.



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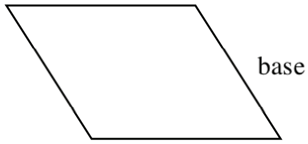

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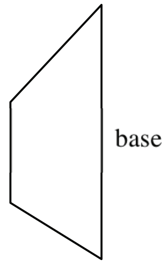

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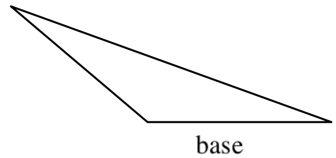
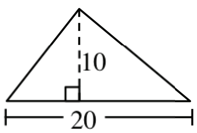


Find the area of the following triangles, parallelograms and trapezoids. Pictures are not drawn to scale. Round answers to the nearest tenth.

5.



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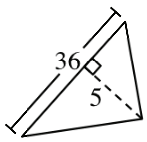

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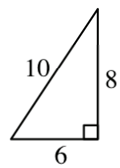

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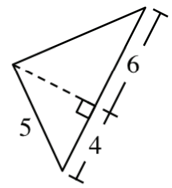

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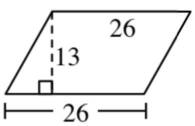

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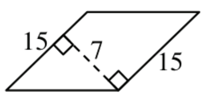

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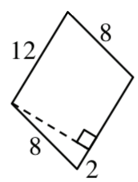

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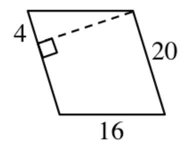

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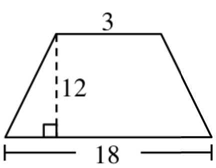

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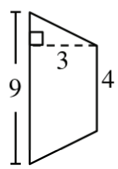

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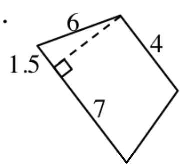

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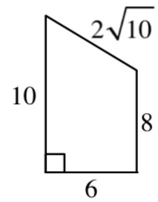

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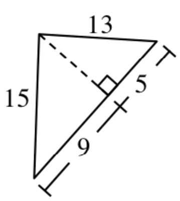

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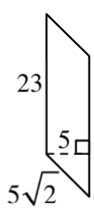

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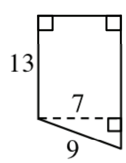

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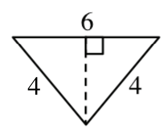
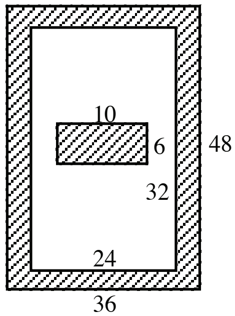


Find the area of the shaded regions.

21.



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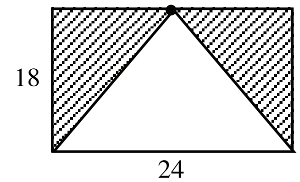

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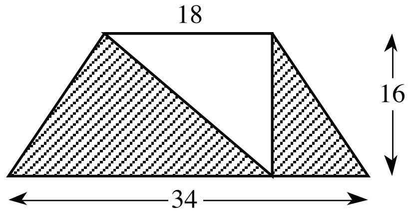

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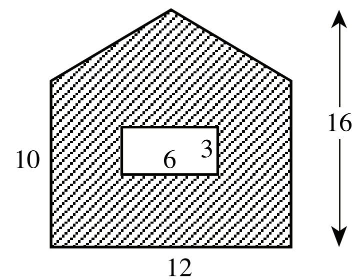
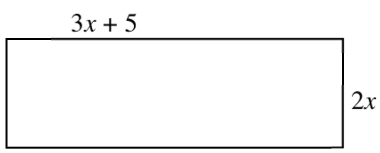


Find the area of each shape and/or shaded region. Be sure to include the appropriate units.

25.



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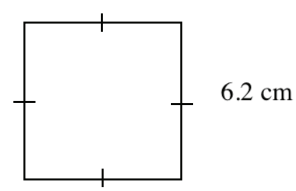

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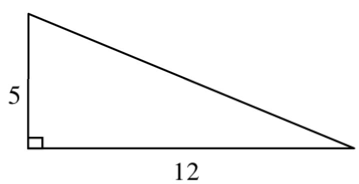

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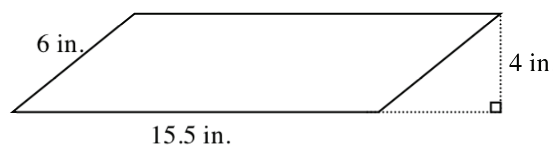

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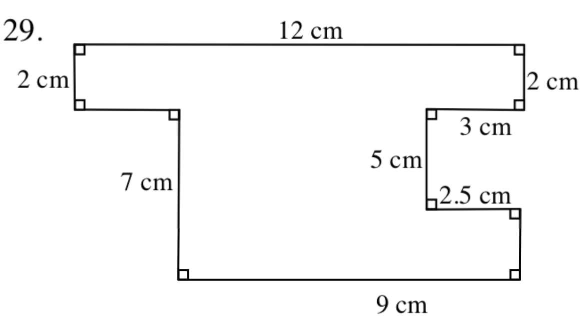

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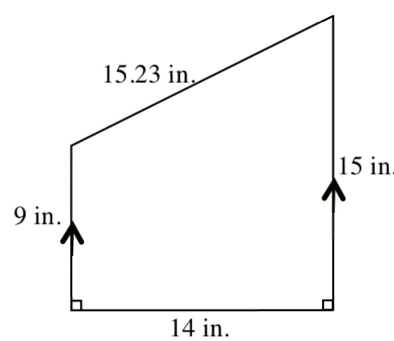

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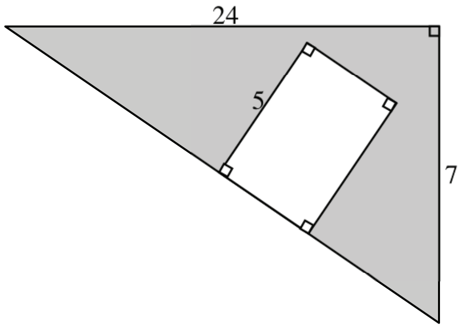

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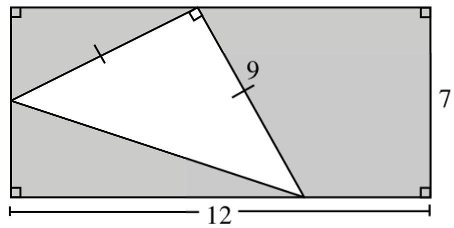
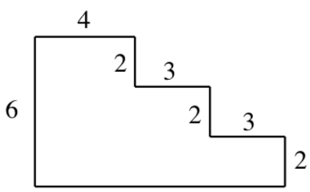


Find the area of each of the following figures. Assume that anything that looks like a right angle is a right angle.

33.



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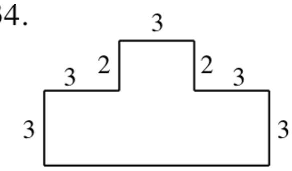

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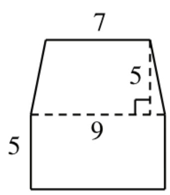

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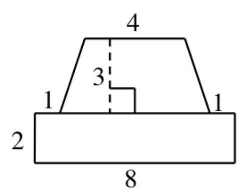

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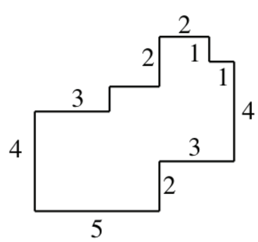

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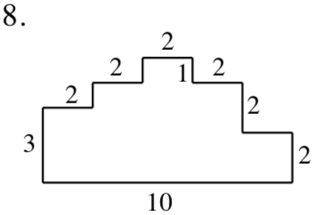

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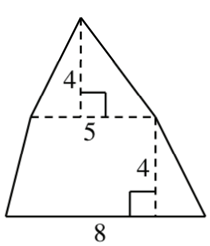

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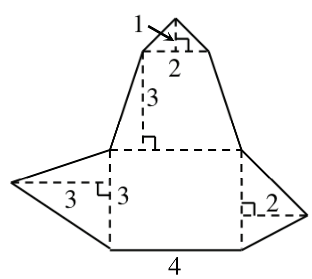

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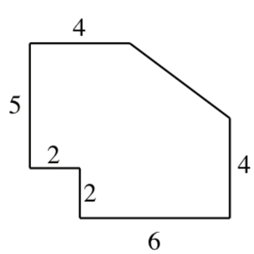
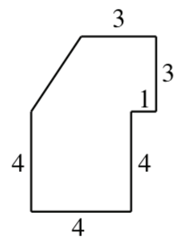
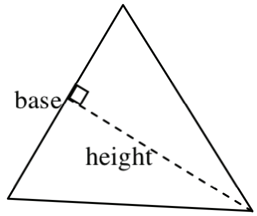


42.



Answers

1.



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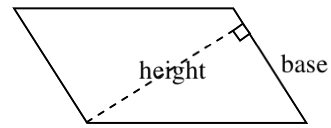

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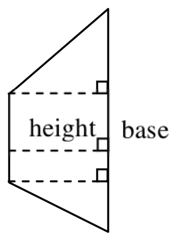

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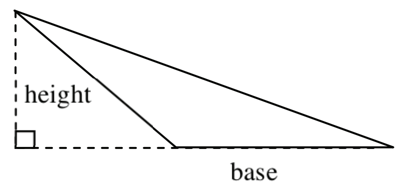


5.	100 sq. units	6.	90 sq. units	7.	24 sq. units	8.	15 sq. units
9.	338 sq. units	10.	105 sq. units	11.	93 sq. units	12.	309.8 sq. units
13.	126 sq. units	14.	19.5 sq. units	15.	36.3 sq. units	16.	54 sq. units
17.	84 sq. units	18.	115 sq. units	19.	110.8 sq. units	20.	7.9 sq. units
21.	1020 sq. units	22.	216 sq. units	23.	272 sq. units	24.	138 sq. units

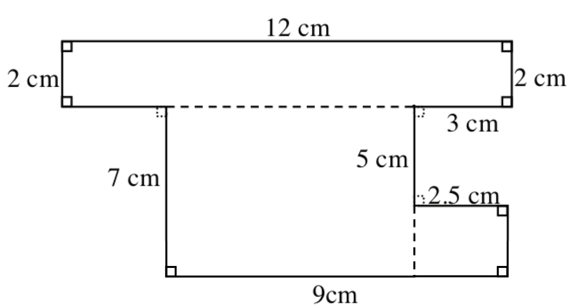
25. $2x(3x+5)=6x^2+10x$ square units

26. $(6.2)^2=38.44$ square cm

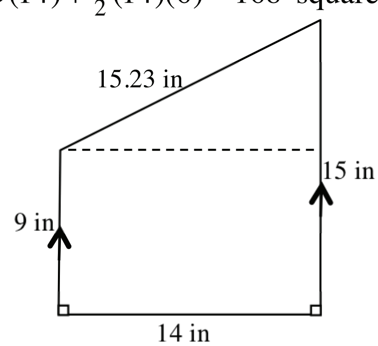
27. $\frac{1}{2}(12)5=30$ square units

28. $(15.5)(4)=62$ square inches

29. $2(12)+7(6.5)+2(2.5)=74.5$ sq. cm



30. $9(14)+\frac{1}{2}(14)(6)=168$ square inches



31. $\frac{1}{2}(7)(24)-(3)(5)=84-15=69$ sq. units

32. $(12)(7)-\frac{1}{2}(9)(9)=84-40.5=43.5$ sq. units

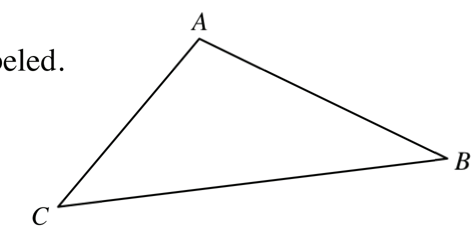
33.	42 units^2	34.	33 units^2	35.	85 units^2	36.	31 units^2	37.	36 units^2
38.	36 units^2	39.	36 units^2	40.	29.5 units^2	41.	46 units^2	42.	28 units^2

SIDES LENGTHS OF TRIANGLES 2.3.1 and 2.3.2

Using technology, students explore the Triangle Inequality, which determines the restrictions on the possible lengths of the third side of a triangle given the lengths of its other two sides. Using technology, students explore different ways to determine lengths of sides of triangles through calculation rather than measurement. The Triangle Inequality determines the restrictions on the possible lengths of the third side of a triangle given the lengths of its other two sides. Students use a method that reinforces the understanding of "square root" then use the method to apply the Pythagorean Theorem in right triangles.

See the Math Notes boxes in Lessons 2.3.1 and 2.3.2 for more information about right triangle vocabulary and the Pythagorean Theorem.

Example 1



The triangle at right does not have the lengths of its sides labeled. Can the sides have lengths of:

a. 3, 4, 5? b. 8, 2, 12?

At first, students might think that the lengths of the sides of a triangle can be any three lengths, but that is not so. The Triangle Inequality says that the length of any side must be less than the sum of the lengths of the other two sides. For the triangle in part (a) to exist, all of these statements must be true:

$$5 \overset{?}{<} 3+4,\ 3 \overset{?}{<} 4+5, \text{ and } 4 \overset{?}{<} 5+3.$$

Since each of them is true, we could draw a triangle with sides of lengths 3, 4, and 5.

In part (b) we need to check if:

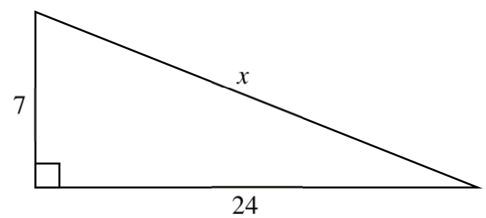
$$12 \overset{?}{<} 8+2,\ 8 \overset{?}{<} 2+12, \text{ and } 2 \overset{?}{<} 12+8.$$

In this case, only two of the three conditions are true, namely, the last two. The first inequality is not true so we cannot draw a triangle with side lengths of 8, 2, and 12. One way to make a convincing argument about this is to cut linguine or coffee stirrers to these lengths and see if you can put the pieces together at their endpoints to form a triangle.

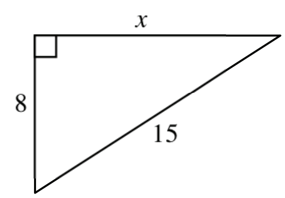
Example 2

Use the Pythagorean Theorem to determine the value of x.

a.



b.



The two sides of a right triangle that form the right angle are called the **legs**, while the third side, the longest side of the triangle, is called the **hypotenuse**. The relationship between the lengths of the legs and the hypotenuse is shown at right.

The Pythagorean Theorem

$$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$$

In part (a), this gives us:
$$\begin{aligned} 7^2 + 24^2 &= x^2 \\ 49 + 576 &= x^2 \\ 625 &= x^2 \end{aligned}$$

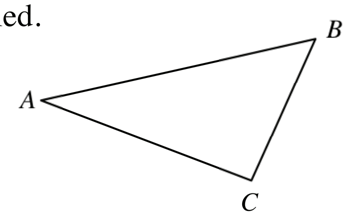
To determine the value of x, use a calculator to find the square root of 625: $x = \sqrt{625}$, so $x = 25$.

Part (b) is a bit different in that the variable is not the hypotenuse. The solution is shown at right.

$$\begin{aligned} 8^2 + x^2 &= 15^2 \\ 64 + x^2 &= 225 \\ x^2 &= 225 - 64 \\ x^2 &= 161 \\ x &= \sqrt{161} \\ x &\approx 12.69 \end{aligned}$$

Problems

The triangle at right does not have any of the lengths of the sides labeled. Can the triangle have side lengths of:



1. 1, 2, 3?
2. 7, 8, 9?
3. 4.5, 2.5, 6?
4. 9.5, 1.25, 11.75?

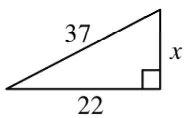
5. A square has an area of 144 square feet. What is the length of one of its sides?

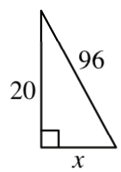
6. A square has an area of 484 square inches. What is the length of one of its sides?

7. A square has an area of 200 square cm. What is the length of one of its sides?

8. A square has an area of 169 square units. What is the perimeter of the square?

Use the Pythagorean Theorem to determine the value of x. Round answers to the nearest tenth.

9.


10.


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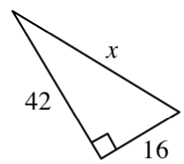

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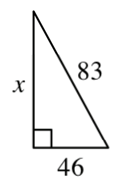

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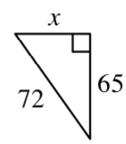

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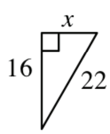

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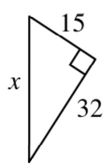

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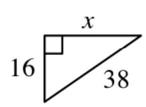

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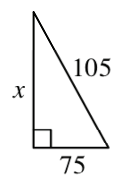

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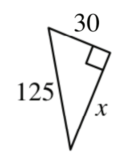


Solve the following problems.

19. A 12 foot ladder is six feet from a wall. How high on the wall does the ladder touch?

20. A 15 foot ladder is five feet from a wall. How high on the wall does the ladder touch?

21. A 9 foot ladder is three feet from a wall. How high on the wall does the ladder touch?

22. A 12 foot ladder is three and a half feet from a wall. How high on the wall does the ladder touch?

23. A 6 foot ladder is one and a half feet from a wall. How high on the wall does the ladder touch?

24. Could 2, 3, and 6 represent the lengths of sides of a right angle triangle? Justify your answer.

25. Could 8, 12, and 13 represent the lengths of sides of a right triangle? Justify your answer.

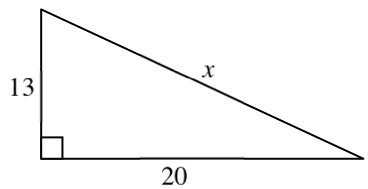
26. Could 5, 12, and 13 represent the lengths of sides of a right triangle? Justify your answer.

27. Could 9, 12, and 15 represent the lengths of sides of a right triangle? Justify your answer.

28. Could 10, 15, and 20 represent the lengths of sides of a right triangle? Justify your answer.

Use the Pythagorean Theorem to find the value of x. When necessary, round your answer to the nearest hundredth.

29.



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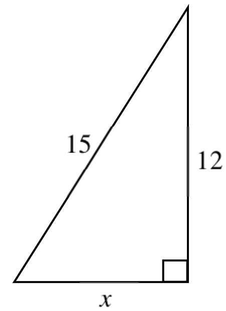

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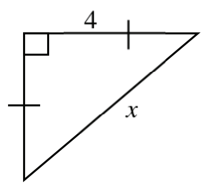

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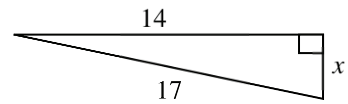

Answers

1. no	2. yes	3. yes
4. no	5. 12 feet	6. 22 inches
7. ≈ 14.14 cm	8. 52 units	9. $x = 27.7$ units
10. $x = 93.9$ units	11. $x = 44.9$ units	12. $x = 69.1$ units
13. $x = 31.0$ units	14. $x = 15.1$ units	15. $x = 35.3$ units
16. $x = 34.5$ units	17. $x = 73.5$ units	18. $x = 121.3$ units
19. 10.4 ft	20. 14.1 ft	21. 8.5 ft
22. 11.5 ft	23. 5.8 ft	24. no
25. no	26. yes	27. yes
28. no	29. $x \approx 23.85$ units	30. $x = 9$ units
31. $x \approx 5.66$ units	32. $x \approx 9.64$ units	