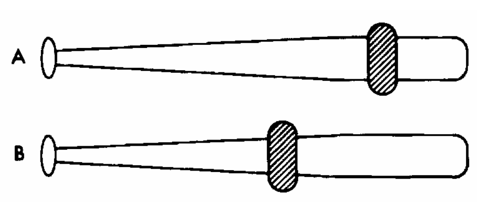
Angular Kinetics

- similar comparison between linear and angular kinematics

Linear	Angular
•Mass →	•Moment of inertia
•Force →	•Torque
•Momentum →	•Angular momentum
• Newton's Laws →	•Newton's Laws (angular analogs)

resistance to angular motion (like linear motion) dependent on mass



however, the more **closely** mass is distributed to the axis of rotation, the easier it is to **rotate**

therefore: resistance to angular motion dependent on both the ***quantity*** and ***distribution of mass***

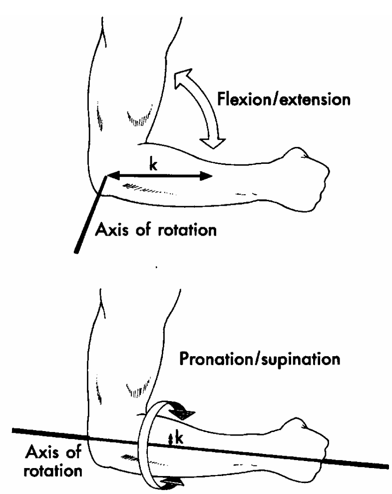
Defined as: **Moment of Inertia**

Moment of Inertia

- ANGULAR FORM OF INERTIA (I)
  - resistance to changes in the state of **angular** motion
- $I = mr^2$
  - for a single particle
  - proportional to **mass** and distance **squared**
- SI unit = $kg{\cdot}m^2$

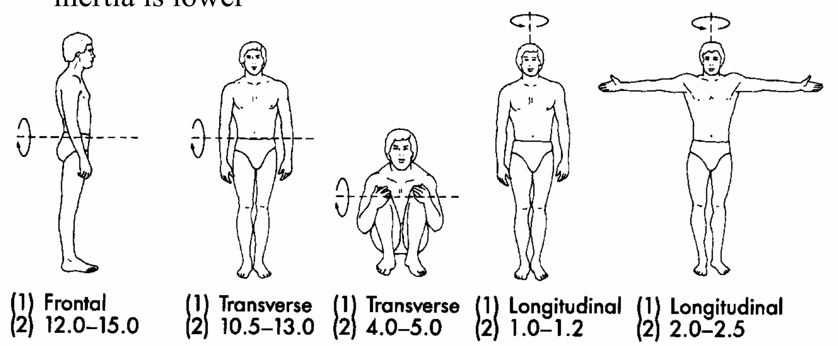
Different Axes

- recognize that rotation can occur about different axes
  - each axis has its own moment of inertia associated with it



Whole Body I

- consider human movement to occur about 3 principal axes
- each principal axis has a principal moment of inertia associated with it
- when mass is distributed closer to axis the moment of inertia is lower

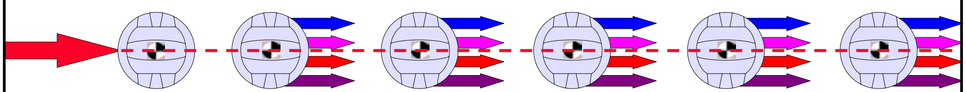


Torque (a.k.a. moment of force)

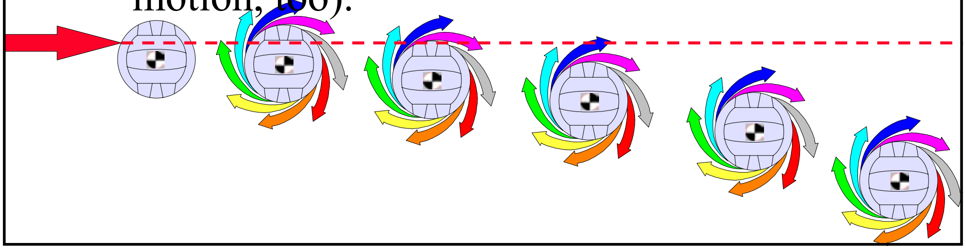
- The turning or rotational effect of an **eccentric force**.
- Equal to the product of **perpendicular** components of **force** and **distance** (from the force's line of action).
  - Any eccentric force will cause a torque
  - **"Moment arm"** is a special name given to the distance from force's line of action and the axis of rotation.

Centric and Eccentric Forces

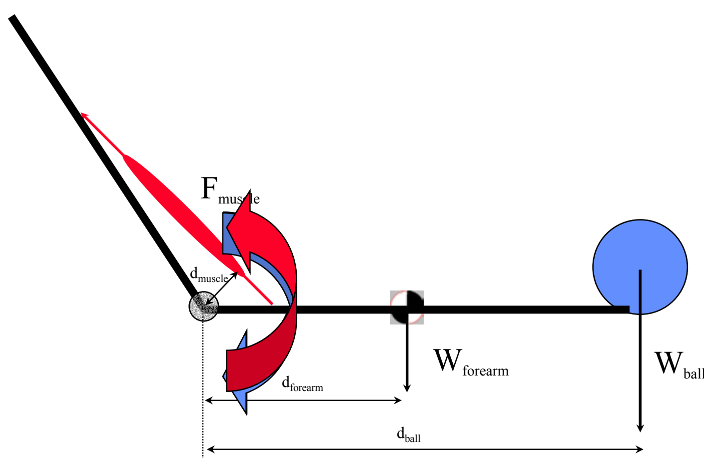
- Centric forces result in **linear** motion *only*.



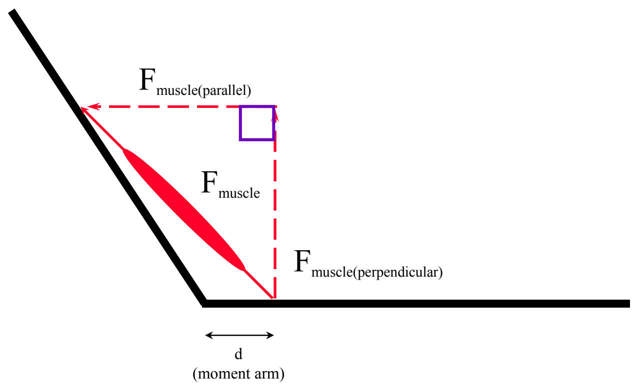
- Eccentric (off-center) forces *always* result in **rotational** motion (sometimes linear motion, too).



Example

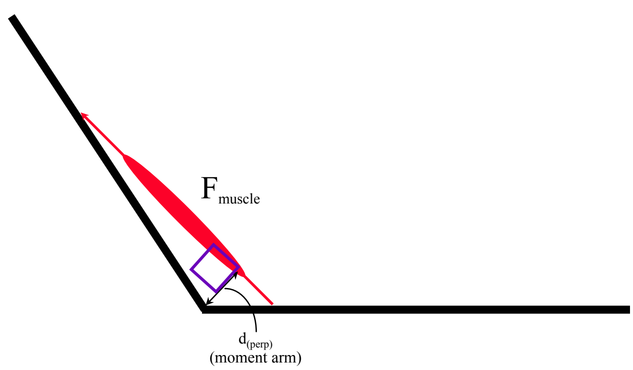


Example



Moment caused by muscle force = $F_{muscle(perp)}$ x d

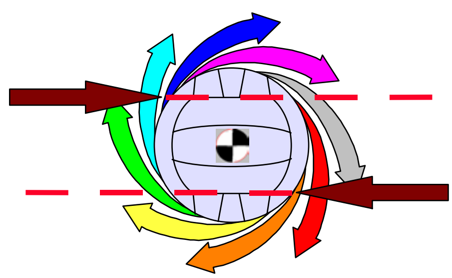
Example



Moment caused by muscle force = F_{muscle} x $d_{(perp)}$

Eccentric Forces: Couple

- A couple is a pair of forces which are **equal** in magnitude but **opposite** in direction, are **equidistant** from the axis of rotation, and act to produce *pure* **rotation**.



Angular Analog
Newton's Laws

1) a rotating body will continue to turn about its axis of rotation with constant angular momentum, unless an external couple or eccentric force is exerted upon it

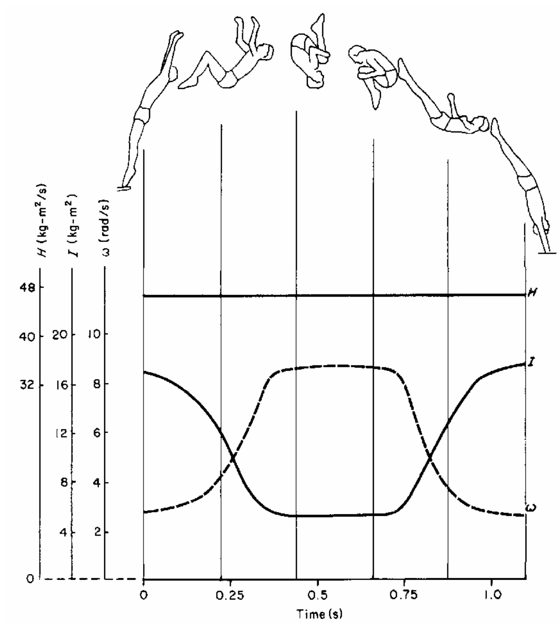
- linear momentum

  $M = m \cdot v$

- angular momentum

  $H = I \cdot \omega$

AKA - The principle of conservation of angular momentum



Angular Analog
Newton's Laws

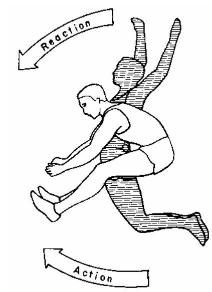
2) the rate of change of angular momentum of a body is proportional to the torque causing it and the change takes place in the direction in which the torque acts

$$\Sigma T = I \frac{\omega_f - \omega_i}{t}$$

$$\Sigma T = I\alpha$$

Angular Analog
Newton's Laws

3) for every torque that is exerted by one body on another there is an equal and opposite torque exerted by the second body on the first



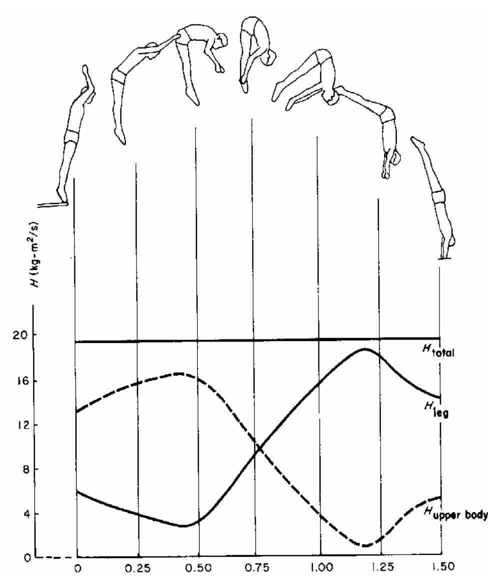
TRANSFER OF ANGULAR MOMENTUM

enter pike - $H_{legs} \downarrow$ because legs slow down

$H_{trunk+arms} \uparrow$ to maintain a constant H_{total}

the opposite occurs at entry - $H_{trunk + arms} \downarrow$ to give a clean entry

$H_{legs} \uparrow$ to maintain H_{total}



Angular Momentum in Long Jump

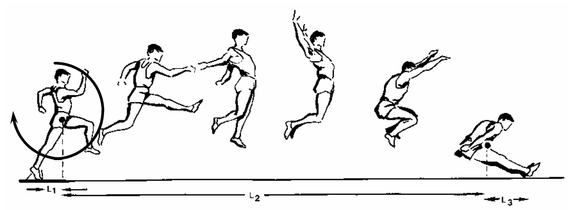


Figure 16-1. Contributions to the length of a hang-style long jump.

$H_{total} = H_{trunk+head} + H_{arms} + H_{legs}$ = constant CW

to prevent trunk+head from rotating forward (CW) rotate arms and legs CW to account for H_{total}

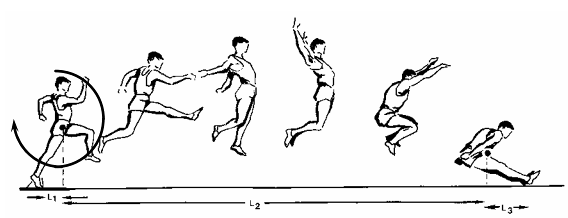


Figure 16-1. Contributions to the length of a hang-style long jump.

I_{arms} and I_{legs} are smaller than I_{total} so

ω_{arms} and ω_{legs} must be larger to produce

H's (respectively) large enough to accommodate H_{total}

Sources of Angular Momentum

$$H = \sum_{s=1}^{s=N} H_s$$

$$H = \sum_{s=1}^{s=N} \left(I_s \omega_{s/G_s} + m_s r^2 \omega_{G_s/G} \right)$$

- Whole body H = sum of all segmental H's
- Each segmental H has 2 sources
  - $I_s \omega_{s/Gs}$ (H caused by **rotation of segment** about its own **CG**)
  - $m_s r^2 \omega_{Gs/G}$ (H caused by **rotation of segment's CG** about the whole body **CG**). ***This is the most important source!***