ANNUITY

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LESSON OBJECTIVES

At the end of the lesson, you are expected to:

a. distinguishes between simple and general annuities;

b. finds the future value and present value of both simple annuities and general annuities; and

c. calculates the present value and period of deferral of a deferred annuity.

DEFINITION OF TERMS

ANNUITY

- a series of equal payments made at equal intervals of time

PAYMENT INTERVAL

- the period of time between successive payments (e.g. monthly, quarterly)

TERM OF ANNUITY

- the length of time between the beginning of the first payment period and the end of the last payment period.

FUTURE VALUE

- sum of future values of all the payments to be made during the entire term of the annuity.

PRESENT VALUE

- sum of present values of all the payments to be made during the entire term of the annuity

DEFINITION OF TERMS

ORDINARY ANNUITY

- a series of payments where each *periodic payment* is made at the ***end*** of the payment interval.

SIMPLE ANNUITY

- is an annuity whose interest *conversion period* is **equal** to the *payment interval.*

Example:
Monthly payments, and the interest is compounded monthly

GENERAL ANNUITY

- is an annuity whose interest *conversion period* is **not equal** to the *payment interval*.

Example:
Monthly payments, but the interest is compounded semi-annually

Formula for Simple Ordinary Annuity

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

where:

FV = future value of simple ordinary annuity

PV = present value of simple ordinary annuity

R = amount of periodic payment

i = rate of interest per conversion period $\left(\frac{rate\ of\ interest}{conversion\ period}\right)$

n = number of payment $(con.period * term\ of\ annuity)$

ILLUSTRATIVE PROBLEM

REGULAR/PERIODIC PAYMENT (R)

A person made a deposit of ₱2,000 at the end of each six months for 2 years at 5% compounded semi-annually. How much is in his account at the end of 2 years?

RATE OF INTEREST

CONVERSION PERIOD

PAYMENT INTERVAL

TERM OF ANNUITY

Prob 1: A person made a deposit of ₱2,000 at the end of each six months for 2 years at 5% compounded semi-annually. How much is in his account at the end of 2 years?

Given:

$R = 2{,}000$

$r = 0.05$

term of annuity$= 2$

con. period$= 2$

payment int.$= 2$

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.05}{2}$$

$$i = 0.025$$

To find n,

$$n = con.period * term\ of\ annuity$$

$$n = 2 * 2$$

$$n = 4$$

Solution for FV:

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$FV = 2{,}000\left[\frac{(1+0.025)^4 - 1}{0.025}\right]$$

$$FV = 2{,}000\left[\frac{(1.025)^4 - 1}{0.025}\right]$$

$$\boxed{FV = 8{,}305.03}$$

Prob 2: A television (TV) set is for sale at ₱13,499 in cash or on installment terms, ₱2,500 each month for the next 6 months at 9% compounded monthly. If you were the buyer, what would you prefer, cash or installment?

Given:

$R = 2{,}500$

$r = 0.09$

term of annuity$= \frac{6}{12} or \frac{1}{2}$

con. period$= 12$

payment int.$= 12$

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.09}{12}$$

$$i = 0.0075$$

To find n,

$$n = con.period * term\ of\ annuity$$

$$n = 12 * \frac{1}{2}$$

$$n = 6$$

Solution for PV:

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$PV = 2{,}500\left[\frac{1-(1+0.0075)^{-6}}{0.0075}\right]$$

$$PV = 2{,}500\left[\frac{1-(1.0075)^{-6}}{0.0075}\right]$$

$$PV = 14{,}613.99$$

Therefore, it is wiser to buy the television at P13,499 in cash

Prob 3: Mr. Tanjiro paid ₱200,000 as down payment for a car. The remaining amount is to be settled by paying ₱16,200 at the end of each month for 5 years. If interest is 10.5% compounded monthly, what is the cash price of his car? (*Cash price = down payment + present value*)

Given:

$R = 16{,}200$

$r = 0.105$

term of annuity$= 5$

con. period$= 12$

payment int.$= 12$

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.105}{12}$$

$$i = 0.00875$$

To find n,

$$n = con.period * term\ of\ annuity$$

$$n = 12 * 5$$

$$n = 60$$

Solution for PV:

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$PV = 16{,}200\left[\frac{1-(1+0.00875)^{-60}}{0.00875}\right]$$

$$PV = 16{,}200\left[\frac{1-(1.00875)^{-60}}{0.00875}\right]$$

$$PV = 753{,}702.20$$

$$cash\ price = 200{,}000 + 753.702.20$$

$$\boxed{cash\ price = 953{,}702.20}$$

Prob 4: Suppose that you vow to save ₱500 a month for the next three years, with your first deposit one month from today. If your savings can earn 3% converted monthly, determine the total in your account 3 years from now.

Given:

$R = 500$

$r = 0.03$

term of annuity$= 3$

con. period$= 12$

payment int.$= 12$

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.03}{12}$$

$$i = 0.0025$$

To find n,

$$n = con.period * term\ of\ annuity$$

$$n = 12 * 3$$

$$n = 36$$

Solution for FV:

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$FV = 500\left[\frac{(1+0.0025)^{36} - 1}{0.0025}\right]$$

$$FV = 500\left[\frac{(1.0025)^{36} - 1}{0.0025}\right]$$

$$FV = 18{,}810.28$$

Prob 5: A retired employee wished to get ₱15,000 every month for 10 years from her savings deposit. If the money is worth 12% compounded monthly, how much should her money be in the account in order to get the desired amount?

Given:

$R = 15{,}000$

$r = 0.12$

term of annuity$= 10$

con. period$= 12$

payment int.$= 12$

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.12}{12}$$

$$i = 0.01$$

To find n,

$$n = con.period * term\ of\ annuity$$

$$n = 12 * 10$$

$$n = 120$$

Solution for PV:

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$PV = 15{,}000\left[\frac{1-(1+0.01)^{-120}}{0.01}\right]$$

$$PV = 15{,}000\left[\frac{1-(1.01)^{-120}}{0.01}\right]$$

$$PV = 1{,}045{,}507.83$$

GENERAL ANNUITY

STEPS IN SOLVING GENERAL ANNUITY

1. convert the regular/periodic payment in general annuity into its equivalent regular/periodic payment in simple annuity with respect to the interest period;

2. then use the formula for simple annuity.

Formula for General Annuity

$$B = \frac{Ri}{(1+i)^k - 1}$$

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

where:

B = Periodic Payment (in simple interest)

R = Periodic payment in the given general annuity problem

i = rate of interest per conversion period = $\left(\frac{rate\ of\ interest}{conversion\ period}\right)$

k = interest conversion period in a year divided payment interval also in a year

k = $\left(\frac{interest\ conversion\ period}{payment\ interval}\right)$

Prob 6: Nami deposits ₱1,000 at the end of each quarter in her savings account earning interest rate of 3.6% compounded monthly. How much will she have in 5 years?

Given:

$R = 1{,}000$

$r = 0.036$

term of annuity$= 5$

con. period$= 12$

payment int.$= 4$

$$\mathrm{B} = \frac{\mathrm{R}i}{(1+i)^{k}-1}$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.036}{12}$$

$$i = 0.003$$

To find k,

$$k = \frac{conversion\ period}{payment\ interval}$$

$$k = \frac{12}{4}$$

$$k = 3$$

Solution for B:

$$\mathrm{B} = \frac{\mathrm{R}i}{(1+i)^{k}-1}$$

$$\mathrm{B} = \frac{(1{,}000)(0.003)}{(1+0.003)^{3}-1}$$

$$B = R = 332.34$$

To find n,

$$n = con.period. * term\ of\ annuity$$

$$n = 12 * 5$$

$$n = 60$$

Solution for FV:

$$FV = R\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$FV = 332.34\left[\frac{(1+0.003)^{60}-1}{0.003}\right]$$

$$FV = 332.34\left[\frac{(1.003)^{60}-1}{0.003}\right]$$

$$\boxed{FV = 21{,}812.01}$$

Prob 7: Sanji Vinsmoke borrowed an amount of money from Luffy. He agrees to pay the principal plus interest by paying ₱38,973.76 each year for 3 years. How much money did he borrow if interest is 8% compounded quarterly?

Given:

$R = 38{,}973.76$

$r = 0.08$

term of annuity$= 3$

con. period$= 4$

payment int.$= 1$

$$\mathrm{B} = \frac{\mathrm{R}i}{(1+i)^{k}-1}$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.08}{4}$$

$$i = 0.02$$

To find k,

$$k = \frac{conversion\ period}{payment\ interval}$$

$$k = \frac{4}{1}$$

$$k = 4$$

Solution for B:

$$\mathrm{B} = \frac{\mathrm{R}i}{(1+i)^{k}-1}$$

$$\mathrm{B} = \frac{(38{,}973.76)(0.02)}{(1+0.02)^{4}-1}$$

$$B = R = 9{,}455.96$$

To find n,

$$n = con.period. * term\ of\ annuity$$

$$n = 4 * 3$$

$$n = 12$$

Solution for PV:

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$PV = 9{,}455.96\left[\frac{1-(1+0.02)^{-12}}{0.02}\right]$$

$$PV = 9{,}455.96\left[\frac{1-(1.02)^{-12}}{0.02}\right]$$

$$PV = 100{,}000.00$$

Prob 8: Monkey D. Luffy started to deposit ₱300 monthly in a fund that pays 6% compounded quarterly. How much will be in the fund after 15 years?

Given:

$R = 300$

$r = 0.06$

term of annuity$= 15$

con. period$= 4$

payment int.$= 12$

$$\mathrm{B} = \frac{\mathrm{R}i}{(1+i)^{k}-1}$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.06}{4}$$

$$i = 0.015$$

To find k,

$$k = \frac{conversion\ period}{payment\ interval}$$

$$k = \frac{4}{12}$$

$$k = \frac{1}{3}$$

Solution for B:

$$\mathrm{B} = \frac{\mathrm{R}i}{(1+i)^{k}-1}$$

$$\mathrm{B} = \frac{(300)(0.015)}{(1+0.015)^{\frac{1}{3}}-1}$$

$$B = R = 904.49$$

To find n,

$$n = con.period. * term\ of\ annuity$$

$$n = 4 * 15$$

$$n = 60$$

Solution for FV:

$$FV = R\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$FV = 904.49\left[\frac{(1+0.015)^{60}-1}{0.015}\right]$$

$$FV = 904.49\left[\frac{(1.015)^{60}-1}{0.015}\right]$$

$$FV = 87{,}025.19$$

DEFERRED ANNUITY

TERMINOLOGY

Deferred Annuity

- an annuity in which the first periodic payment is made after a certain interval of time, known as the ***deferral period***

Ordinary Deferred Annuity

- when deferral period *ends* **one payment interval** *before the first periodic payment.*

The future value of a deferred annuity

- is the accumulated value of the stream of payments at the end of the annuity period. This is the same procedure as future value of an ordinary annuity (both simple and general annuity).

The present value of a deferred annuity

- is the discounted value of the stream of payments at the beginning of the deferral period.

Ordinary Deferred Annuity

FORMULA

$$PV_{def} = R\left[\frac{1-(1+i)^{-n}}{i(1+i)^{d}}\right]$$

$$FV_{def} = R\left[\frac{(1+i)^{n}-1}{i}\right]$$

where:

PV_{def} = present value of deferred annuity
FV_{def} = future value of deferred annuity
R = amount of periodic payment
i = rate of interest per conversion period
n = number of payment
d = number of deferred period

DETERMINE THE PERIOD OF DEFERRAL

(Assume the annuities are Ordinary)

- Payments of ₱1,000 at the end of each year for ten years with the first payment made three years from now.

  period of deferral = 2

- Payments of ₱5,000 at the end of every 6 months for 15 years with the first payment made 5 years from now

  period of deferral = 9

DETERMINE THE PERIOD OF DEFERRAL

(Assume the annuities are Ordinary)

- A second hand car sells for ₱120,000 down payment and 24 monthly payments of ₱7,000 each, the first payment being due at the end of the 6^{th} month. Find the cash price if the interest rate is 8% compounded monthly.

  period of deferral = 5

- A quarterly payment of ₱8,500 at 6% compounded quarterly, the first payment is due in 1 year and 6 months and the last payment is at the end of 5 years.

  period of deferral = 5

Prob 9: A second hand car sells for ₱120,000 down payment and 24 monthly payments of ₱7,000 each, the first payment being due at the end of the 6th month. Find the cash price if the interest rate is 8% compounded monthly. (*Cash price = down payment + present value*)

Given:

$R = 7{,}000$

$r = 0.08$

term of annuity$= 2$

con. period$= 12$

payment int.$= 12$

$d = 5$

$$\mathrm{PV}_{def} = \mathrm{R}\left[\frac{1-(1+i)^{-n}}{i(1+i)^{d}}\right]$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.08}{12}$$

Retain i, since the answer is repeating

To find n,

$$n = con.period * term\ of\ annuity$$

$$n = 12 * 2$$

$$n = 24$$

Solution for PV:

$$\mathrm{PV}_{def} = \mathrm{R}\left[\frac{1-(1+i)^{-n}}{i(1+i)^{d}}\right]$$

$$\mathrm{PV}_{def} = 7{,}000\left[\frac{1-\left(1+\frac{0.08}{12}\right)^{-24}}{\frac{0.08}{12}\left(1+\frac{0.08}{12}\right)^{5}}\right]$$

$$\mathrm{PV}_{def} = 149{,}716.28$$

$$cash\ price = 120{,}000 + 149{,}716.28$$

$$\boxed{cash\ price = 269{,}716.28}$$

Prob 10: If money is worth 9% compounded semi-annually, find the present value of 6 semi-annual payments of ₱10,000 each, the first payment is due in 4 years.

Given:

$R = 10{,}000$

$r = 0.09$

term of annuity$= 6$

con. period$= 2$

payment int.$= 2$

$d = 7$

$$\mathrm{PV}_{def} = \mathrm{R}\left[\frac{1-(1+i)^{-n}}{i(1+i)^{d}}\right]$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.09}{2}$$

$$i = 0.045$$

To find n,

$$n = con.period * term\ of\ annuity$$

$$n = 2 * 6$$

$$n = 12$$

Solution for PV:

$$\mathrm{PV}_{def} = \mathrm{R}\left[\frac{1-(1+i)^{-n}}{i(1+i)^{d}}\right]$$

$$\mathrm{PV}_{def} = 10{,}000\left[\frac{1-(1+0.045)^{-12}}{0.045(1+0.045)^{7}}\right]$$

$$\mathrm{PV}_{def} = 10{,}000\left[\frac{1-(1.045)^{-12}}{0.045(1.045)^{7}}\right]$$

$$\mathrm{PV}_{def} = 67{,}005.93$$

Prob 11: Find the present value of 24 annual payments of ₱20,000 each, the first payment is due after 3 years and the interest rate is 9% compounded annually.

Given:

$R = 20{,}000$

$r = 0.09$

term of annuity$= 24$

con. period$= 1$

payment int.$= 1$

$d = 2$

$$\mathrm{PV}_{def} = \mathrm{R}\left[\frac{1-(1+i)^{-n}}{i(1+i)^{d}}\right]$$

To find i,

$$i = \frac{rate\ of\ interest}{conversion\ period}$$

$$i = \frac{0.09}{1}$$

$$i = 0.09$$

To find n,

$$n = con.period * term\ of\ annuity$$

$$n = 1 * 24$$

$$n = 24$$

Solution for PV:

$$\mathrm{PV}_{def} = \mathrm{R}\left[\frac{1-(1+i)^{-n}}{i(1+i)^{d}}\right]$$

$$\mathrm{PV}_{def} = 20{,}000\left[\frac{1-(1+0.09)^{-24}}{0.09(1+0.09)^{2}}\right]$$

$$\mathrm{PV}_{def} = 20{,}000\left[\frac{1-(1.09)^{-24}}{0.09(1.09)^{2}}\right]$$

$$\mathrm{PV}_{def} = 163{,}397.22$$