NAME ______________ DATE ________ PERIOD ________

10 Anticipation Guide

Radical Expressions and Triangles

Chapter Resources

Step 1 ***Before you begin Chapter 10***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. An expression that contains a square root is called a radical expression.	A
	2. It is always true that $\sqrt{xy}$ will equal $\sqrt{x} \cdot \sqrt{y}$.	A
	3. $\frac{1}{\sqrt{3}}$ is in simplest form because $\sqrt{3}$ is not a whole number.	D
	4. The sum of $3\sqrt{3}$ and $2\sqrt{3}$ will equal $5\sqrt{3}$.	A
	5. Before multiplying two radical expressions with different radicands the square roots must be evaluated.	D
	6. When solving radical equations by squaring each side of the equation, it is possible to obtain solutions that are not solutions to the original equation.	A
	7. The longest side of any triangle is called the hypotenuse.	D
	8. Because $5^2 = 4^2 + 3^2$, a triangle whose sides have lengths 3, 4, and 5 will be a right triangle.	A
	9. On a coordinate plane, the distance between any two points can be found using the Pythagorean Theorem.	A
	10. The Distance Formula cannot be used to find the distance between two points on the same vertical line.	D
	11. Two triangles are similar only if their corresponding angles are congruent and the measures of their corresponding sides are in proportion.	A
	12. All right triangles are similar.	D

Step 2 ***After you complete Chapter 10***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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10-1 Study Guide and Intervention

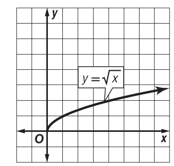
Square Root Functions

Lesson 10-1

Dilations of Radical Functions A **square root function** contains the square root of a variable. Square root functions are a type of **radical function**.

In order for a square root to be a real number, the **radicand**, or the expression under the radical sign, cannot be negative. Values that make the radicant negative are not included in the domain.

Square Root Function	Parent function: $f(x) = \sqrt{x}$ Type of graph: curve Domain: $\{x \mid x \geq 0\}$ Range: $\{y \mid y \geq 0\}$

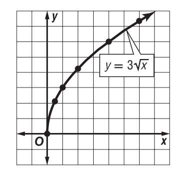


Example **Graph $y = 3\sqrt{x}$. State the domain and range.**

Step 1 Make a table. Choose nonnegative values for x.

x	y
0	0
0.5	≈2.12
1	3
2	≈4.24
4	6
6	≈7.35

Step 2 Plot points and draw a smooth curve.

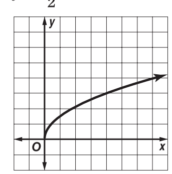


The domain is $\{x \mid x \geq 0\}$ and the range is $\{y \mid y \geq 0\}$.

Exercises

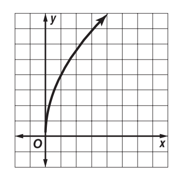
Graph each function, and compare to the parent graph. State the domain and range.

1. $y = \frac{3}{2}\sqrt{x}$



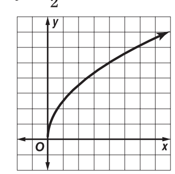
Dilation of $y = \sqrt{x}$; D = $\{x \mid x \geq 0\}$; R = $\{y \mid y \geq 0\}$

2. $y = 4\sqrt{x}$



Dilation of $y = \sqrt{x}$; D = $\{x \mid x \geq 0\}$; R = $\{y \mid y \geq 0\}$

3. $y = \frac{5}{2}\sqrt{x}$



Dilation of $y = \sqrt{x}$; D = $\{x \mid x \geq 0\}$; R = $\{y \mid y \geq 0\}$

Answers

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10-1 Study Guide and Intervention *(continued)*

Square Root Functions

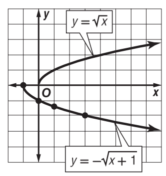
Reflections and Translations of Radical Functions Radical functions, like quadratic functions, can be translated horizontally and vertically, as well as reflected across the x-axis. To draw the graph of $y = a\sqrt{x+h}$, follow these steps.

Graphs of Square Root Functions	**Step 1**	Draw the graph of $y = +c\sqrt{x}$. The graph starts at the origin and passes through the point at $(1, a)$. If $a > 0$, the graph is in the 1st quadrant. If $a < 0$, the graph is reflected across the x-axis and is in the 4th quadrant.
	Step 2	Translate the graph $\lvert c \rvert$ units up if c is positive and down if c is negative.
	Step 3	Translate the graph $\lvert h \rvert$ units left if h is positive and right if h is negative.

Example **Graph $y = -\sqrt{x+1}$ and compare to the parent graph. State the domain and range.**

Step 1 Make a table of values.

x	−1	0	1	3	8
y	0	−1	−1.41	−2	−3

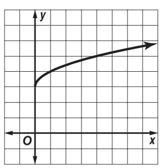


Step 2 This is a horizontal translation 1 unit to the left of the parent function and reflected across the x-axis. The domain is $\{x \mid x \geq 0\}$ and the range is $\{y \mid y \leq 0\}$.

Exercises

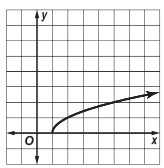
Graph each function, and compare to the parent graph. State the domain and range.

1. $y = \sqrt{x} + 3$



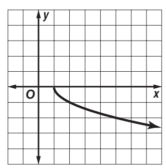
translation of $y = \sqrt{x}$ up 3 units; $D = \{x \mid x \geq 0\}$; $R = \{y \mid y \geq 3\}$

2. $y = \sqrt{x-1}$



translation of $y = \sqrt{x}$ right 1 unit; $D = \{x \mid x \geq 1\}$; $R = \{y \mid y \geq 0\}$

3. $y = -\sqrt{x-1}$



translation of $y = \sqrt{x}$ right 1 unit and reflected across the x-axis; $D = \{x \mid x \geq 1\}$ $R = \{y \mid y \leq 0\}$

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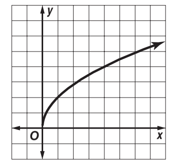
10-1 Skills Practice

Square Root Functions

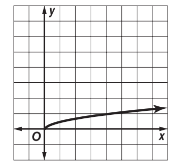
Graph each function, and compare to the parent graph. State the domain and range.

1. $y = 2\sqrt{x}$



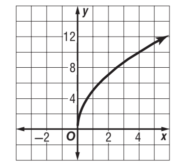
dilation of $y = \sqrt{x}$; D = {x | x ≥ 0}, R = {y | y ≥ 0}

2. $y = \frac{1}{2}\sqrt{x}$



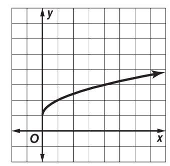
dilation of $y = \sqrt{x}$; D = {x | x ≥ 0}, R = {y | y ≥ 0}

3. $y = 5\sqrt{x}$



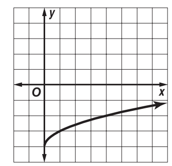
dilation of $y = \sqrt{x}$; D = {x | x ≥ 0}, R = {y | y ≥ 0}

4. $y = \sqrt{x} + 1$



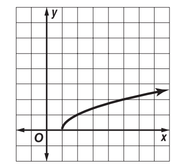
translation of $y = \sqrt{x}$ up 1 unit; D = {x | x ≥ 0}, R = {y | y ≥ 1}

5. $y = \sqrt{x} - 4$



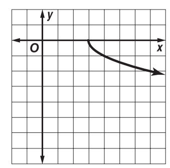
translation of $y = \sqrt{x}$ down 4 units; D = {x | x ≥ 0}, R = {y | y ≥ −4}

6. $y = \sqrt{x-1}$



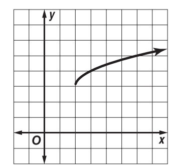
translation of $y = \sqrt{x}$ right 1 unit; D = {x | x ≥ 1}, R = {y | y ≥ 0}

7. $y = -\sqrt{x-3}$



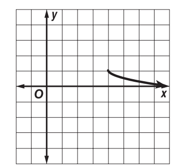
translation of $y = \sqrt{x}$; right 3 units reflected across the x-axis; D = {x | x ≥ 3}, R = {y | y ≤ 0}

8. $y = \sqrt{x-2} + 3$



translation of $y = \sqrt{x}$ right 2 units and up 3 units; D = {x | x ≥ 2}, R = {y | y ≥ 3}

9. $y = -\frac{1}{2}\sqrt{x-4} + 1$



dialation of $y = \sqrt{x}$ reflected across the X-axis translated right 4 units up 1 units; D = {x | x ≥ 4}, R = {y | y ≤ 1}

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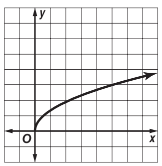
10-1 Practice

Square Root Functions

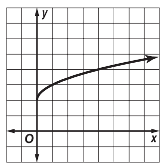
Graph each function, and compare to the parent graph. State the domain and range.

1. $y = \frac{4}{3}\sqrt{x}$



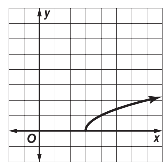
dilation of $y = \sqrt{x}$; D = {x | x ≥ 0}, R = {y | y ≥ 0}

2. $y = \sqrt{x} + 2$



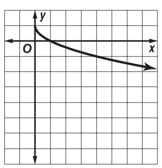
translation of $y = \sqrt{x}$ up 2 unit; D = {x | x ≥ 0}, R = {y | y ≥ 2}

3. $y = \sqrt{x - 3}$



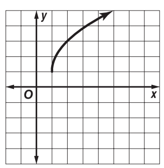
translation of $y = \sqrt{x}$ left 3 units; D = {x | x ≥ –3}, R = {y | y ≥ 0}

4. $y = -\sqrt{x} + 1$



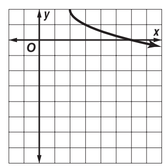
translation of $y = \sqrt{x}$ up 1 unit reflected in the x-axis; D = {x | x ≥ 0}, R = {y | y ≤ 1}

5. $y = 2\sqrt{x - 1} + 1$



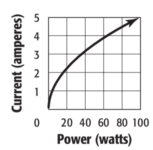
dilation of $y = \sqrt{x}$ translated up 1 unit and right 1 unit; D = {x | x ≥ 1}, R = {y | y ≥ 1}

6. $y = -\sqrt{x - 2} + 2$



translation of $y = \sqrt{x}$; up 2 units and right 2 units, reflected in the x-axis; D = {x | x ≥ 2}, R = {y | y ≤ 2}

7. OHM'S LAW In electrical engineering, the resistance of a circuit can be found by the equation $I = \sqrt{\frac{P}{R}}$, where I is the current in amperes, P is the power in watts, and R is the resistance of the circuit in ohms. Graph this function for a circuit with a resistance of 4 ohms.



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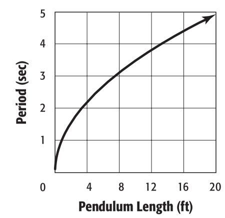
10-1 Word Problem Practice

Square Root Functions

1. PENDULUM MOTION The period T of a pendulum in seconds, which is the time for the pendulum to return to the point of release, is given by the equation $T = 1.11\sqrt{L}$. The length of the pendulum in feet is given by L. Graph this function.



2. EMPIRE STATE BUILDING The roof of the Empire State Building is 1250 feet above the ground. The velocity of an object dropped from a height of h meters is given by the function $V = \sqrt{2gh}$, where g is the gravitational constant, 32.2 feet per second squared. If an object is dropped from the roof of the building, how fast is it traveling when it hits the street below?

approximately 284 ft/s

3. ERROR ANALYSIS Gregory is drawing the graph of $y = -5\sqrt{x + 1}$. He describes the range and domain as $\{x \mid x \geq -1\}$, $\{y \mid y \geq 0\}$. Explain and correct the mistake that Gregory made.

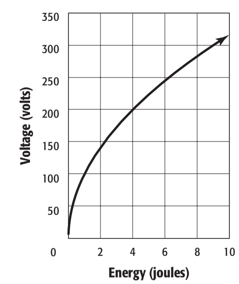
The domain is actually {y | y ≤ 0} because the graph has been reflected across the x-axis.

4. CAPACITORS A capacitor is a set of plates that can store energy in an electric field. The voltage V required to store E joules of energy in a capacitor with a capacitance of C farads is given by $V = \sqrt{\frac{2E}{C}}$.

a. Rewrite and simplify the equation for the case of a 0.0002 farad capacitor.

$V = 100\sqrt{E}$

b. Graph the equation you found in part a.



c. How would the graph differ if you wished to store $E + 1$ joules of energy in the capacitor instead?

translation of $V = 100\sqrt{E}$ one unit to the left

d. How would the graph differ if you applied a voltage of $V + 1$ volts instead?

translation of $V = 100\sqrt{E}$ one unit down

Lesson 10-1

Answers

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10-1 Enrichment

Cubic Root Functions

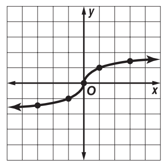
A **cubic root function** contains the cubic root of a variable. The **cubic root** of a number x are the numbers y that satisfy the equation $y \cdot y \cdot y = x$ (or, alternatively, $y = \sqrt[3]{x}$). Unlike square root functions, cubic root functions return real numbers when the radicand is negative.

Example **Graph $y = \sqrt[3]{x}$.**

Step 1 Make a table.

x	y
–5	–1.71
–3	–1.44
–1	–1
0	0
1	1
3	1.44
5	1.71

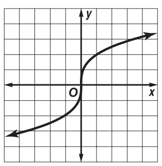
Step 2 Plot points and draw a smooth curve.



Exercises

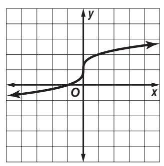
Graph each function, and compare to the parent graph.

1. $y = 2\sqrt[3]{x}$



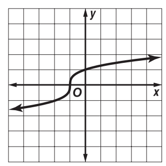
dilation of $y = \sqrt[3]{x}$

2. $y = \sqrt[3]{x} + 1$



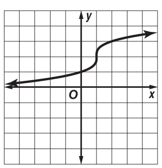
translation of $y = \sqrt[3]{x}$ up 1 unit

3. $y = \sqrt[3]{x + 1}$



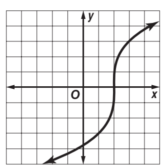
translation of $y = \sqrt[3]{x}$ left 1 unit

4. $y = \sqrt[3]{x - 1} + 2$



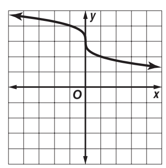
translation of $y = \sqrt[3]{x}$ up 2 units and right 1 unit

5. $y = 3\sqrt[3]{x - 2}$



dilation of $y = \sqrt[3]{x}$ translated right 2 units

6. $y = -\sqrt[3]{x} + 3$



reflection of $y = \sqrt[3]{x}$ across the x-axis translated up 3 units

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10-2 Study Guide and Intervention

Simplifying Radical Expressions

Product Property of Square Roots The **Product Property of Square Roots** and prime factorization can be used to simplify expressions involving irrational square roots. When you simplify radical expressions with variables, use absolute value to ensure nonnegative results.

Product Property of Square Roots	For any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example 1 **Simplify $\sqrt{180}$.**

$\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}$ Prime factorization of 180

$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{5}$ Product Property of Square Roots

$= 2 \cdot 3 \cdot \sqrt{5}$ Simplify.

$= 6\sqrt{5}$ Simplify.

Example 2 **Simplify $\sqrt{120a^2 \cdot b^5 \cdot c^4}$.**

$\sqrt{120a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^3 \cdot 3 \cdot 5 \cdot a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{a^2} \cdot \sqrt{b^4 \cdot b} \cdot \sqrt{c^4}$

$= 2 \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot |a| \cdot b^2 \cdot \sqrt{b} \cdot c^2$

$= 2|a|b^2c^2\sqrt{30b}$

Exercises

Simplify each expression.

1. $\sqrt{28}$ **$2\sqrt{7}$**

2. $\sqrt{68}$ **$2\sqrt{17}$**

3. $\sqrt{60}$ **$2\sqrt{15}$**

4. $\sqrt{75}$ **$5\sqrt{3}$**

5. $\sqrt{162}$ **$9\sqrt{2}$**

6. $\sqrt{3} \cdot \sqrt{6}$ **$3\sqrt{2}$**

7. $\sqrt{2} \cdot \sqrt{5}$ **$\sqrt{10}$**

8. $\sqrt{5} \cdot \sqrt{10}$ **$5\sqrt{2}$**

9. $\sqrt{4a^2}$ **$2|a|$**

10. $\sqrt{9x^4}$ **$3x^2$**

11. $\sqrt{300a^4}$ **$10a^2\sqrt{3}$**

12. $\sqrt{128c^6}$ **$8|c^3|\sqrt{2}$**

13. $4\sqrt{10} \cdot 3\sqrt{6}$ **$24\sqrt{15}$**

14. $\sqrt{3x^2} \cdot 3\sqrt{3x^4}$ **$9|x^3|$**

15. $\sqrt{20a^2b^4}$ **$2|a|b^2\sqrt{5}$**

16. $\sqrt{100x^3y}$ **$10|x|\sqrt{xy}$**

17. $\sqrt{24a^4b^2}$ **$2a^2|b|\sqrt{6}$**

18. $\sqrt{81x^4y^2}$ **$9x^2|y|$**

19. $\sqrt{150a^2b^2c}$ **$5|ab|\sqrt{6c}$**

20. $\sqrt{72a^6b^3c^2}$ **$6|a^3bc|\sqrt{2b}$**

21. $\sqrt{45x^2y^5z^8}$ **$3|x|y^2z^4\sqrt{5y}$**

22. $\sqrt{98x^4y^6z^2}$ **$7x^2|y^3z|\sqrt{2}$**

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10-2 Study Guide and Intervention *(continued)*

Simplifying Radical Expressions

Quotient Property of Square Roots A fraction containing radicals is in simplest form if no radicals are left in the denominator. The **Quotient Property of Square Roots** and **rationalizing the denominator** can be used to simplify radical expressions that involve division. When you rationalize the denominator, you multiply the numerator and denominator by a radical expression that gives a rational number in the denominator.

Quotient Property of Square Roots	For any numbers a and b, where $a \ge 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Example Simplify $\sqrt{\frac{56}{45}}$.

$\sqrt{\frac{56}{45}} = \sqrt{\frac{4 \cdot 14}{9 \cdot 5}}$

$= \frac{2 \cdot \sqrt{14}}{3 \cdot \sqrt{15}}$ — Simplify the numerator and denominator.

$= \frac{2\sqrt{14}}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ — Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$ to rationalize the denominator.

$= \frac{2\sqrt{70}}{15}$ — Product Property of Square Roots

Exercises

Simplify each expression.

1. $\frac{\sqrt{9}}{\sqrt{18}}$ $\mathbf{\frac{\sqrt{2}}{2}}$
2. $\frac{\sqrt{8}}{\sqrt{24}}$ $\mathbf{\frac{\sqrt{3}}{3}}$
3. $\frac{\sqrt{100}}{\sqrt{121}}$ $\mathbf{\frac{10}{11}}$
4. $\frac{\sqrt{75}}{\sqrt{3}}$ **5**
5. $\frac{8\sqrt{2}}{2\sqrt{8}}$ **2**
6. $\sqrt{\frac{2}{5}} \cdot \sqrt{\frac{6}{5}}$ $\mathbf{\frac{2\sqrt{3}}{5}}$
7. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{5}{2}}$ $\mathbf{\frac{\sqrt{30}}{4}}$
8. $\sqrt{\frac{5}{7}} \cdot \sqrt{\frac{2}{5}}$ $\mathbf{\frac{\sqrt{14}}{7}}$
9. $\sqrt{\frac{3a^2}{10b^6}}$ $\mathbf{\frac{|a|\sqrt{30}}{10|b^3|}}$
10. $\sqrt{\frac{x^6}{y^4}}$ $\mathbf{\frac{x^3}{y^2}}$
11. $\sqrt{\frac{100a^4}{144b^8}}$ $\mathbf{\frac{5a^2}{6b^4}}$
12. $\sqrt{\frac{75b^3c^6}{a^2}}$ $\mathbf{\frac{5|bc^3|\sqrt{3b}}{|a|}}$
13. $\frac{\sqrt{4}}{3-\sqrt{5}}$ $\mathbf{\frac{3+\sqrt{5}}{2}}$
14. $\frac{\sqrt{8}}{2+\sqrt{3}}$ $\mathbf{4\sqrt{2} - 2\sqrt{6}}$
15. $\frac{\sqrt{5}}{5+\sqrt{5}}$ $\mathbf{5 - 2\sqrt{5}}$
16. $\frac{\sqrt{8}}{2\sqrt{7}+4\sqrt{10}}$ $\mathbf{\frac{4\sqrt{5}-\sqrt{14}}{33}}$

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10-2 Skills Practice

Simplifying Radical Expressions

Simplify each expression.

1. $\sqrt{28}$ $\mathbf{2\sqrt{7}}$
2. $\sqrt{40}$ $\mathbf{2\sqrt{10}}$
3. $\sqrt{72}$ $\mathbf{6\sqrt{2}}$
4. $\sqrt{99}$ $\mathbf{3\sqrt{11}}$
5. $\sqrt{2} \cdot \sqrt{10}$ $\mathbf{2\sqrt{5}}$
6. $\sqrt{5} \cdot \sqrt{60}$ $\mathbf{10\sqrt{3}}$
7. $3\sqrt{5} \cdot \sqrt{5}$ **15**
8. $\sqrt{6} \cdot 4\sqrt{24}$ **48**
9. $2\sqrt{3} \cdot 3\sqrt{15}$ $\mathbf{18\sqrt{5}}$
10. $\sqrt{16b^4}$ $\mathbf{4b^2}$
11. $\sqrt{81a^2d^4}$ $\mathbf{9|a|d^2}$
12. $\sqrt{40x^4y^6}$ $\mathbf{2x^2|y^3|\sqrt{10}}$
13. $\sqrt{75m^5P^2}$ $\mathbf{5m^2|P|\sqrt{3m}}$
14. $\sqrt{\frac{5}{3}}$ $\mathbf{\frac{\sqrt{15}}{3}}$
15. $\sqrt{\frac{1}{6}}$ $\mathbf{\frac{\sqrt{6}}{6}}$
16. $\sqrt{\frac{6}{7}} \cdot \sqrt{\frac{1}{3}}$ $\mathbf{\frac{\sqrt{14}}{7}}$
17. $\sqrt{\frac{q}{12}}$ $\mathbf{\frac{\sqrt{3q}}{6}}$
18. $\sqrt{\frac{4h}{5}}$ $\mathbf{\frac{2\sqrt{5h}}{5}}$
19. $\sqrt{\frac{12}{b^2}}$ $\mathbf{\frac{2\sqrt{3}}{|b|}}$
20. $\sqrt{\frac{45}{4m^4}}$ $\mathbf{\frac{3\sqrt{5}}{2m^2}}$
21. $\frac{2}{4+\sqrt{5}}$ $\mathbf{\frac{8-2\sqrt{5}}{11}}$
22. $\frac{3}{2-\sqrt{3}}$ $\mathbf{6 + 3\sqrt{3}}$
23. $\frac{5}{7+\sqrt{7}}$ $\mathbf{\frac{35-5\sqrt{7}}{42}}$
24. $\frac{4}{3-\sqrt{2}}$ $\mathbf{\frac{12+4\sqrt{2}}{7}}$

Lesson 10-2

Answers

Answers (Lesson 10-2)

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10-2 Practice

Simplifying Radical Expressions

Simplify.

1. $\sqrt{24}$ **$2\sqrt{6}$**
2. $\sqrt{60}$ **$2\sqrt{15}$**
3. $\sqrt{108}$ **$6\sqrt{3}$**
4. $\sqrt{8} \cdot \sqrt{6}$ **$4\sqrt{3}$**
5. $\sqrt{7} \cdot \sqrt{14}$ **$7\sqrt{2}$**
6. $3\sqrt{12} \cdot 5\sqrt{6}$ **$90\sqrt{2}$**
7. $4\sqrt{3} \cdot 3\sqrt{18}$ **$36\sqrt{6}$**
8. $\sqrt{27tu^3}$ **$3|u|\sqrt{3tu}$**
9. $\sqrt{50p^5}$ **$5p^2\sqrt{2p}$**
10. $\sqrt{108x^6y^4z^5}$ **$6|x^3|y^2z^2\sqrt{3z}$**
11. $\sqrt{56m^2n^4p^5}$ **$2|m|n^2p^2\sqrt{14p}$**
12. $\frac{\sqrt{8}}{\sqrt{6}}$ **$\frac{2\sqrt{3}}{3}$**
13. $\sqrt{\frac{2}{10}}$ **$\frac{\sqrt{5}}{5}$**
14. $\sqrt{\frac{5}{32}}$ **$\frac{\sqrt{10}}{8}$**
15. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{4}{5}}$ **$\frac{\sqrt{15}}{5}$**
16. $\sqrt{\frac{1}{7}} \cdot \sqrt{\frac{7}{11}}$ **$\frac{\sqrt{11}}{11}$**
17. $\frac{\sqrt{3k}}{\sqrt{8}}$ **$\frac{\sqrt{6k}}{4}$**
18. $\sqrt{\frac{18}{x^3}}$ **$\frac{3\sqrt{2x}}{x^2}$**
19. $\sqrt{\frac{4y}{3y^2}}$ **$\frac{2\sqrt{3y}}{3|y|}$**
20. $\sqrt{\frac{9ab}{4ab^4}}$ **$\frac{3\sqrt{b}}{2b^2}$**
21. $\frac{3}{5 - \sqrt{2}}$ **$\frac{15 + 3\sqrt{2}}{23}$**
22. $\frac{8}{3 + \sqrt{3}}$ **$\frac{12 - 4\sqrt{3}}{3}$**
23. $\frac{5}{\sqrt{7} + \sqrt{3}}$ **$\frac{5\sqrt{7} - 5\sqrt{3}}{4}$**
24. $\frac{3\sqrt{7}}{-1 - \sqrt{27}}$ **$\frac{3\sqrt{7} - 9\sqrt{21}}{26}$**

25. **SKY DIVING** When a skydiver jumps from an airplane, the time t it takes to free fall a given distance can be estimated by the formula $t = \sqrt{\frac{2s}{9.8}}$, where t is in seconds and s is in meters. If Julie jumps from an airplane, how long will it take her to free fall 750 meters? **about 12.4 s**

26. **METEOROLOGY** To estimate how long a thunderstorm will last, meteorologists can use the formula $t = \sqrt{\frac{d^3}{216}}$, where t is the time in hours and d is the diameter of the storm in miles.

   a. A thunderstorm is 8 miles in diameter. Estimate how long the storm will last. Give your answer in simplified form and as a decimal. **$\frac{8\sqrt{3}}{9}$ h ≈ 1.5 h**

   b. Will a thunderstorm twice this diameter last twice as long? Explain. **No; it will last about 4.4 h, or nearly 3 times as long.**

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10-2 Word Problem Practice

Simplifying Radical Expressions

1. **SPORTS** Jasmine calculated the height of her team's soccer goal to be $\frac{15}{\sqrt{3}}$ feet. Simplify the expression.

   $5\sqrt{3}$

2. **NATURE** In 2004, an earthquake below the ocean floor initiated a devastating tsunami in the Indian Ocean. Scientists can approximate the velocity (in feet per second) of a tsunami in water of depth d (in feet) with the formula $V = \sqrt{16d}$. Determine the velocity of a tsunami in 300 feet of water. Write your answer in simplified radical form.

   $40\sqrt{3}$ ft/s

3. **AUTOMOBILES** The following formula can be used to find the "zero to sixty" time for a car, or the time it takes for a car to accelerate from a stop to sixty miles per hour.

$$V = \sqrt{\frac{2PT}{M}}$$

   V is the velocity (in meters per second).
   P is the car's average power (in watts).
   M is the mass of the car (in kilograms).
   T is the time (in seconds).

   Find the time it takes for a 900-kilogram car with an average 60,000 watts of power to accelerate from stop to 26.82 meters per second (60 miles per hour). Round your answer to the nearest tenth.

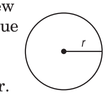
   about 5.4 s

4. **PHYSICAL SCIENCE** When a substance such as water vapor is in its gaseous state, the volume and the velocity of its molecules increase as temperature increases. The average velocity V of a molecule with mass m at temperature T is given by the formula $V = \sqrt{\frac{3kT}{m}}$.

   Solve the equation for k.

   $k = \frac{mV^2}{3T}$

5. **GEOMETRY** Suppose Emeryville Hospital wants to build a new helipad on which medic rescue helicopters can land. The helipad will be circular and made of fire resistant rubber.



   a. If the area of the helipad is A, write an equation for the radius r.

   $r = \sqrt{\frac{A}{\pi}}$

   b. Write an expression in simplified radical form for the radius of a helipad with an area of 288 square meters.

   $r = \frac{12\sqrt{2\pi}}{\pi}$

   c. Using your calculator, find a decimal approximation for the radius. Round your answer to the nearest hundredth.

   9.57 m

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10-2 Enrichment

Squares and Square Roots From a Graph

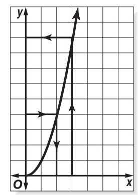
The graph of $y = x^2$ can be used to find the squares and square roots of numbers.

To find the square of 3, locate 3 on the x-axis. Then find its corresponding value on the y-axis.

The arrows show that $3^2 = 9$.

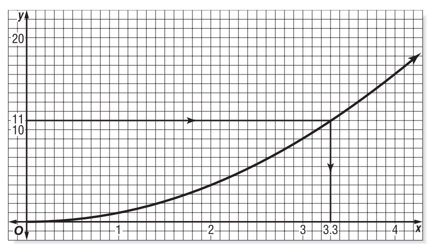
To find the square root of 4, first locate 4 on the y-axis. Then find its corresponding value on the x-axis. Following the arrows on the graph, you can see that $\sqrt{4} = 2$.

A small part of the graph at $y = x^2$ is shown below. A 1:10 ratio for unit length on the y-axis to unit length on the x-axis is used.



Example **Find $\sqrt{11}$.**

The arrows show that $\sqrt{11} = 3.3$ to the nearest tenth.



Exercises

Use the graph above to find each of the following to the nearest whole number.

1. 1.5^2 **2**

2. 2.7^2 **7**

3. 0.9^2 **1**

4. 3.6^2 **13**

5. 4.2^2 **18**

6. 3.9^2 **15**

Use the graph above to find each of the following to the nearest tenth.

7. $\sqrt{15}$ **3.9**

8. $\sqrt{8}$ **2.8**

9. $\sqrt{3}$ **1.7**

10. $\sqrt{5}$ **2.2**

11. $\sqrt{14}$ **3.7**

12. $\sqrt{17}$ **4.1**

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10-3 Study Guide and Intervention

Operations with Radical Expressions

Add or Subtract Radical Expressions When adding or subtracting radical expressions, use the Associative and Distributive Properties to simplify the expressions. If radical expressions are not in simplest form, simplify them.

Example 1 **Simplify $10\sqrt{6} - 5\sqrt{3} + 6\sqrt{3} - 4\sqrt{6}$.**

$10\sqrt{6} - 5\sqrt{3} + 6\sqrt{3} - 4\sqrt{6} = (10 - 4)\sqrt{6} + (-5 + 6)\sqrt{3}$ — Associative and Distributive Properties

$= 6\sqrt{6} + \sqrt{3}$ — Simplify.

Example 2 **Simplify $3\sqrt{12} + 5\sqrt{75}$.**

$3\sqrt{12} + 5\sqrt{75} = 3\sqrt{2^2 \cdot 3} + 5\sqrt{5^2 \cdot 3}$ — Simplify.

$= 3 \cdot 2\sqrt{3} + 5 \cdot 5\sqrt{3}$ — Simplify.

$= 6\sqrt{3} + 25\sqrt{3}$ — Simplify.

$= 31\sqrt{3}$ — Distributive Property

Exercises

Simplify each expression.

1. $2\sqrt{5} + 4\sqrt{5}$ $\mathbf{6\sqrt{5}}$

2. $\sqrt{6} - 4\sqrt{6}$ $\mathbf{-3\sqrt{6}}$

3. $\sqrt{8} - \sqrt{2}$ $\mathbf{\sqrt{2}}$

4. $3\sqrt{75} + 2\sqrt{5}$ $\mathbf{15\sqrt{3} + 2\sqrt{5}}$

5. $\sqrt{20} + 2\sqrt{5} - 3\sqrt{5}$ $\mathbf{\sqrt{5}}$

6. $2\sqrt{3} + \sqrt{6} - 5\sqrt{3}$ $\mathbf{-3\sqrt{3} + \sqrt{6}}$

7. $\sqrt{12} + 2\sqrt{3} - 5\sqrt{3}$ $\mathbf{-\sqrt{3}}$

8. $3\sqrt{6} + 3\sqrt{2} - \sqrt{50} + \sqrt{24}$ $\mathbf{5\sqrt{6} - 2\sqrt{2}}$

9. $\sqrt{8a} - \sqrt{2a} + 5\sqrt{2a}$ $\mathbf{6\sqrt{2a}}$

10. $\sqrt{54} + \sqrt{24}$ $\mathbf{5\sqrt{6}}$

11. $\sqrt{3} + \sqrt{\frac{1}{3}}$ $\mathbf{\frac{4\sqrt{3}}{3}}$

12. $\sqrt{12} + \sqrt{\frac{1}{3}}$ $\mathbf{\frac{7\sqrt{3}}{3}}$

13. $\sqrt{54} - \sqrt{\frac{1}{6}}$ $\mathbf{\frac{17\sqrt{6}}{6}}$

14. $\sqrt{80} - \sqrt{20} + \sqrt{180}$ $\mathbf{8\sqrt{5}}$

15. $\sqrt{50} + \sqrt{18} - \sqrt{75} + \sqrt{27}$ $\mathbf{8\sqrt{2} - 2\sqrt{3}}$

16. $2\sqrt{3} - 4\sqrt{45} + 2\sqrt{\frac{1}{3}}$ $\mathbf{\frac{8\sqrt{3}}{3} - 12\sqrt{5}}$

17. $\sqrt{125} - 2\sqrt{\frac{1}{5}} + \sqrt{\frac{1}{3}}$ $\mathbf{\frac{23\sqrt{5}}{5} + \frac{\sqrt{3}}{3}}$

18. $\sqrt{\frac{2}{3}} + 3\sqrt{3} - 4\sqrt{\frac{1}{12}}$ $\mathbf{\frac{\sqrt{6} + 7\sqrt{3}}{3}}$

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10-3 Study Guide and Intervention *(continued)*

Operations with Radical Expressions

Multiply Radical Expressions Multiplying two radical expressions with different radicands is similar to multiplying binomials.

Example **Multiply $(3\sqrt{2} - 2\sqrt{5})(4\sqrt{20} + \sqrt{8})$.**

Use the FOIL method.

$(3\sqrt{2} - 2\sqrt{5})(4\sqrt{20} + \sqrt{8}) = (3\sqrt{2})(4\sqrt{20}) + (3\sqrt{2})(\sqrt{8}) + (-2\sqrt{5})(4\sqrt{20}) + (-2\sqrt{5})(\sqrt{8})$

$= 12\sqrt{40} + 3\sqrt{16} - 8\sqrt{100} - 2\sqrt{40}$ Multiply.

$= 12\sqrt{2^2 \cdot 10} + 3 \cdot 4 - 8 \cdot 10 - 2\sqrt{2^2 \cdot 10}$ Simplify.

$= 24\sqrt{10} + 12 - 80 - 4\sqrt{10}$ Simplify.

$= 20\sqrt{10} - 68$ Combine like terms.

Exercises

Simplify each expression.

1. $2(\sqrt{3} + 4\sqrt{5})$ **$2\sqrt{3} + 8\sqrt{5}$**
2. $\sqrt{6}(\sqrt{3} - 2\sqrt{6})$ **$3\sqrt{2} - 12$**
3. $\sqrt{5}(\sqrt{5} - \sqrt{2})$ **$5 - \sqrt{10}$**
4. $\sqrt{2}(3\sqrt{7} + 2\sqrt{5})$ **$3\sqrt{14} + 2\sqrt{10}$**
5. $(2 - 4\sqrt{2})(2 + 4\sqrt{2})$ **-28**
6. $(3 + \sqrt{6})^2$ **$15 + 6\sqrt{6}$**
7. $(2 - 2\sqrt{5})^2$ **$24 - 8\sqrt{5}$**
8. $3\sqrt{2}(\sqrt{8} + \sqrt{24})$ **$12 + 12\sqrt{3}$**
9. $\sqrt{8}(\sqrt{2} + 5\sqrt{8})$ **44**
10. $(\sqrt{5} - 3\sqrt{2})(\sqrt{5} + 3\sqrt{2})$ **-13**
11. $(\sqrt{3} + \sqrt{6})^2$ **$9 + 6\sqrt{2}$**
12. $(\sqrt{2} - 2\sqrt{3})^2$ **$14 - 4\sqrt{6}$**
13. $(\sqrt{5} - \sqrt{2})(\sqrt{2} + \sqrt{6})$ **$\sqrt{10} - 2 + \sqrt{30} - 2\sqrt{3}$**
14. $(\sqrt{8} - \sqrt{2})(\sqrt{3} + \sqrt{6})$ **$\sqrt{6} + 2\sqrt{3}$**
15. $(\sqrt{5} - \sqrt{18})(7\sqrt{5} + \sqrt{3})$ **$35 + \sqrt{15} - 21\sqrt{10} - 3\sqrt{6}$**
16. $(2\sqrt{3} - \sqrt{45})(\sqrt{12} + 2\sqrt{6})$ **$12 - 6\sqrt{15} + 12\sqrt{2} - 6\sqrt{30}$**
17. $(2\sqrt{5} - 2\sqrt{3})(\sqrt{10} + \sqrt{6})$ **$4\sqrt{2}$**
18. $(\sqrt{2} + 3\sqrt{3})(\sqrt{12} - 4\sqrt{8})$ **$2 - 22\sqrt{6}$**

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10-3 Skills Practice

Operations with Radical Expressions

Simplify each expression.

1. $7\sqrt{7} - 2\sqrt{7}$ **$5\sqrt{7}$**
2. $3\sqrt{13} + 7\sqrt{13}$ **$10\sqrt{13}$**
3. $6\sqrt{5} - 2\sqrt{5} + 8\sqrt{5}$ **$12\sqrt{5}$**
4. $\sqrt{15} + 8\sqrt{15} - 12\sqrt{15}$ **$-3\sqrt{15}$**
5. $12\sqrt{r} - 9\sqrt{r}$ **$3\sqrt{r}$**
6. $9\sqrt{6a} - 11\sqrt{6a} + 4\sqrt{6a}$ **$2\sqrt{6a}$**
7. $\sqrt{44} - \sqrt{11}$ **$\sqrt{11}$**
8. $\sqrt{28} + \sqrt{63}$ **$5\sqrt{7}$**
9. $4\sqrt{3} + 2\sqrt{12}$ **$8\sqrt{3}$**
10. $8\sqrt{54} - 4\sqrt{6}$ **$20\sqrt{6}$**
11. $\sqrt{27} + \sqrt{48} + \sqrt{12}$ **$9\sqrt{3}$**
12. $\sqrt{72} + \sqrt{50} - \sqrt{8}$ **$9\sqrt{2}$**
13. $\sqrt{180} - 5\sqrt{5} + \sqrt{20}$ **$3\sqrt{5}$**
14. $2\sqrt{24} + 4\sqrt{54} + 5\sqrt{96}$ **$36\sqrt{6}$**
15. $5\sqrt{8} + 2\sqrt{20} - \sqrt{8}$ **$8\sqrt{2} + 4\sqrt{5}$**
16. $2\sqrt{13} + 4\sqrt{2} - 5\sqrt{13} + \sqrt{2}$ **$-3\sqrt{13} + 5\sqrt{2}$**
17. $\sqrt{2}(\sqrt{8} + \sqrt{6})$ **$4 + 2\sqrt{3}$**
18. $\sqrt{5}(\sqrt{10} - \sqrt{3})$ **$5\sqrt{2} - \sqrt{15}$**
19. $\sqrt{6}(3\sqrt{2} - 2\sqrt{3})$ **$6\sqrt{3} - 6\sqrt{2}$**
20. $3\sqrt{3}(2\sqrt{6} + 4\sqrt{10})$ **$18\sqrt{2} + 12\sqrt{30}$**
21. $(4 + \sqrt{3})(4 - \sqrt{3})$ **13**
22. $(2 - \sqrt{6})^2$ **$10 - 4\sqrt{6}$**
23. $(\sqrt{8} + \sqrt{2})(\sqrt{5} + \sqrt{3})$ **$3\sqrt{10} + 3\sqrt{6}$**
24. $(\sqrt{6} + 4\sqrt{5})(4\sqrt{3} - \sqrt{10})$ **$-8\sqrt{2} + 14\sqrt{15}$**

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10-3 Practice

Operations with Radical Expressions

Simplify each expression.

1. $8\sqrt{30} - 4\sqrt{30}$ **$4\sqrt{30}$**
2. $2\sqrt{5} - 7\sqrt{5} - 5\sqrt{5}$ **$-10\sqrt{5}$**
3. $7\sqrt{13x} - 14\sqrt{13x} + 2\sqrt{13x}$ **$-5\sqrt{13x}$**
4. $2\sqrt{45} + 4\sqrt{20}$ **$14\sqrt{5}$**
5. $\sqrt{40} - \sqrt{10} + \sqrt{90}$ **$4\sqrt{10}$**
6. $2\sqrt{32} + 3\sqrt{50} - 3\sqrt{18}$ **$14\sqrt{2}$**
7. $\sqrt{27} + \sqrt{18} + \sqrt{300}$ **$3\sqrt{2} + 13\sqrt{3}$**
8. $5\sqrt{8} + 3\sqrt{20} - \sqrt{32}$ **$6\sqrt{2} + 6\sqrt{5}$**
9. $\sqrt{14} - \sqrt{\frac{2}{7}}$ **$\frac{6\sqrt{14}}{7}$**
10. $\sqrt{50} + \sqrt{32} - \sqrt{\frac{1}{2}}$ **$\frac{17\sqrt{2}}{2}$**
11. $5\sqrt{19} + 4\sqrt{28} - 8\sqrt{19} + \sqrt{63}$ **$-3\sqrt{19} + 11\sqrt{7}$**
12. $3\sqrt{10} + \sqrt{75} - 2\sqrt{40} - 4\sqrt{12}$ **$-\sqrt{10} - 3\sqrt{3}$**
13. $\sqrt{6}(\sqrt{10} + \sqrt{15})$ **$2\sqrt{15} + 3\sqrt{10}$**
14. $\sqrt{5}(5\sqrt{2} - 4\sqrt{8})$ **$-3\sqrt{10}$**
15. $2\sqrt{7}(3\sqrt{12} + 5\sqrt{8})$ **$12\sqrt{21} + 20\sqrt{14}$**
16. $(5 - \sqrt{15})^2$ **$40 - 10\sqrt{15}$**
17. $(\sqrt{10} + \sqrt{6})(\sqrt{30} - \sqrt{18})$ **$4\sqrt{3}$**
18. $(\sqrt{8} + \sqrt{12})(\sqrt{48} + \sqrt{18})$ **$36 + 14\sqrt{6}$**
19. $(\sqrt{2} + 2\sqrt{8})(3\sqrt{6} - \sqrt{5})$ **$30\sqrt{3} - 5\sqrt{10}$**
20. $(4\sqrt{3} - 2\sqrt{5})(3\sqrt{10} + 5\sqrt{6})$ **$2\sqrt{30} + 30\sqrt{2}$**

21. **SOUND** The speed of sound V in meters per second near Earth's surface is given by $V = 20\sqrt{t + 273}$, where t is the surface temperature in degrees Celsius.

a. What is the speed of sound near Earth's surface at 15°C and at 2°C in simplest form? **$240\sqrt{2}$ m/s, $100\sqrt{11}$ m/s**

b. How much faster is the speed of sound at 15°C than at 2°C? **$240\sqrt{2} - 100\sqrt{11} \approx 7.75$ m/s**

22. **GEOMETRY** A rectangle is $5\sqrt{7} + 2\sqrt{3}$ meters long and $6\sqrt{7} - 3\sqrt{3}$ meters wide.

a. Find the perimeter of the rectangle in simplest form. **$22\sqrt{7} - 2\sqrt{3}$ m**

b. Find the area of the rectangle in simplest form. **$190 - 3\sqrt{21}$ m²**

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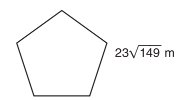
10-3 Word Problem Practice

Operations with Radical Expressions

1. **ARCHITECTURE** The Pentagon is the building that houses the U.S. Department of Defense. Find the approximate perimeter of the building, which is a regular pentagon. Leave your answer as a radical expression.



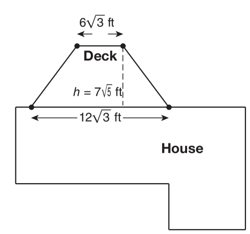
$115\sqrt{149}$ m

2. **EARTH** The surface area of a sphere with radius r is given by the formula $4\pi r^2$. Assuming that Earth is close to spherical in shape and has a surface area of about 5.1×10^8 square kilometers, what is the radius of Earth to the nearest ten kilometers?

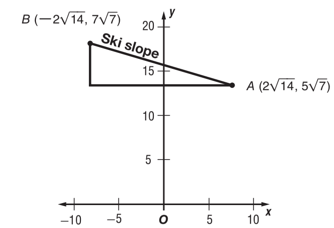
6370 km

3. **GEOMETRY** The area of a trapezoid is found by multiplying its height by the average length of its bases. Find the area of deck attached to Mr. Wilson's house. Give your answer as a simplified radical expression.

$63\sqrt{15}$ ft²



4. **RECREATION** Carmen surveyed a ski slope using a digital device connected to a computer. The computer model assigned coordinates to the top and bottom points of the hill as shown in the diagram. Write a simplified radical expression that represents the slope of the hill.



5. **FREE FALL** Suppose a ball is dropped from a building window 800 feet in the air. Another ball is dropped from a lower window 288 feet high. Both balls are released at the same time. Assume air resistance is not a factor and use the following formula to find how many seconds t it will take a ball to fall h feet.

$$t = \frac{1}{4}\sqrt{h}$$

a. How much time will pass between when the first ball hits the ground and when the second ball hits the ground? Give your answer as a simplified radical expression. **$2\sqrt{2}$ s**

b. Which ball lands first? **The ball dropped from 288 feet lands first.**

c. Find a decimal approximation of the answer for part **a**. Round your answer to the nearest tenth. **about 2.8 s**

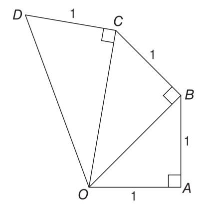
Lesson 10-3

Answers

NAME ______________ DATE ________ PERIOD ________

10-3 Enrichment

The Wheel of Theodorus



The Greek mathematicians were intrigued by problems of representing different numbers and expressions using geometric constructions.

Theodorus, a Greek philosopher who lived about 425 B.C., is said to have discovered a way to construct the sequence $\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \ldots$.

The beginning of his construction is shown. You start with an isosceles right triangle with sides 1 unit long.

Use the figure above. Write each length as a radical expression in simplest form.

1. line segment *AO* $\sqrt{1}$

2. line segment *BO* $\sqrt{2}$

3. line segment *CO* $\sqrt{3}$

4. line segment *DO* $\sqrt{4}$

5. Describe how each new triangle is added to the figure. **Draw a new side of length 1 at right angles to the last hypotenuse. Then draw the new hypotenuse.**

6. The length of the hypotenuse of the first triangle is $\sqrt{2}$. For the second triangle, the length is $\sqrt{3}$. Write an expression for the length of the hypotenuse of the *n*th triangle.

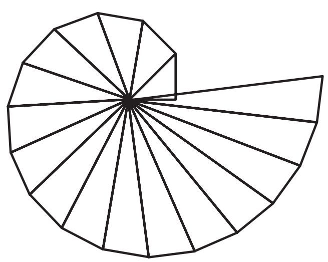
$\sqrt{n+1}$

7. Show that the method of construction will always produce the next number in the sequence. (*Hint:* Find an expression for the hypotenuse of the $(n + 1)$th triangle.)

$\sqrt{(\sqrt{n})^2 + (1)^2}$ **or** $\sqrt{n+1}$

8. In the space below, construct a Wheel of Theodorus. Start with a line segment 1 centimeter long. When does the Wheel start to overlap?

after length $\sqrt{18}$



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10-4 Study Guide and Intervention

Radical Equations

Radical Equations Equations containing radicals with variables in the radicand are called **radical equations**. These can be solved by first using the following steps.

Step 1	Isolate the radical on one side of the equation.
Step 2	Square each side of the equation to eliminate the radical.

Example 1 **Solve $16 = \frac{\sqrt{x}}{2}$ for *x*.**

$16 = \frac{\sqrt{x}}{2}$	Original equation
$2(16) = 2\left(\frac{\sqrt{x}}{2}\right)$	Multiply each side by 2.
$32 = \sqrt{x}$	Simplify.
$(32)^2 = (\sqrt{x})^2$	Square each side.
$1024 = x$	Simplify.

The solution is 1024, which checks in the original equation.

Example 2 **Solve $\sqrt{4x - 7} + 2 = 7$.**

$\sqrt{4x - 7} + 2 = 7$	Original equation
$\sqrt{4x - 7} + 2 - 2 = 7 - 2$	Subtract 2 from each side.
$\sqrt{4x - 7} = 5$	Simplify.
$\left(\sqrt{4x - 7}\right)^2 = 5^2$	Square each side.
$4x - 7 = 25$	Simplify.
$4x - 7 + 7 = 25 + 7$	Add 7 to each side.
$4x = 32$	Simplify.
$x = 8$	Divide each side by 4.

The solution is 8, which checks in the original equation.

Exercises

Solve each equation. Check your solution.

1. $\sqrt{a} = 8$ **64**

2. $\sqrt{a} + 6 = 32$ **676**

3. $2\sqrt{x} = 8$ **16**

4. $7 = \sqrt{26 - n}$ **−23**

5. $\sqrt{-a} = 6$ **−36**

6. $\sqrt{3r^2} = 3$ $\pm\sqrt{3}$

7. $2\sqrt{3} = \sqrt{y}$ **12**

8. $2\sqrt{3a} - 2 = 7$ $6\frac{3}{4}$

9. $\sqrt{x - 4} = 6$ **40**

10. $\sqrt{2m + 3} = 5$ **11**

11. $\sqrt{3b - 2} + 19 = 24$ **9**

12. $\sqrt{4x - 1} = 3$ $\frac{5}{2}$

13. $\sqrt{3r + 2} = 2\sqrt{3}$ $\frac{10}{3}$

14. $\sqrt{\frac{x}{2}} = \frac{1}{2}$ $\frac{1}{2}$

15. $\sqrt{\frac{x}{8}} = 4$ **128**

16. $\sqrt{6x^2 + 5x} = 2$ $\frac{1}{2}, -\frac{4}{3}$

17. $\sqrt{\frac{x}{3}} + 6 = 8$ **12**

18. $2\sqrt{\frac{3x}{5}} + 3 = 11$ $26\frac{2}{3}$

Lesson 10-4

NAME ____________ DATE ________ PERIOD ________

10-4 Study Guide and Intervention *(continued)*

Radical Equations

Extraneous Solutions To solve a radical equation with a variable on both sides, you need to square each side of the equation. Squaring each side of an equation sometimes produces **extraneous solutions**, or solutions that are not solutions of the original equation. Therefore, it is very important that you check each solution.

Example 1 **Solve $\sqrt{x+3} = x - 3$.**

$\sqrt{x+3} = x - 3$	Original equation
$\left(\sqrt{x+3}\right)^2 = (x-3)^2$	Square each side.
$x + 3 = x^2 - 6x + 9$	Simplify.
$0 = x^2 - 7x + 6$	Subtract x and 3 from each side.
$0 = (x-1)(x-6)$	Factor.
$x - 1 = 0$ or $x - 6 = 0$	Zero Product Property
$x = 1$ $\quad$ $x = 6$	Solve.

CHECK

$\sqrt{x+3} = x - 3$	$\sqrt{x+3} = x - 3$
$\sqrt{1+3} \stackrel{?}{=} 1 - 3$	$\sqrt{6+3} \stackrel{?}{=} 6 - 3$
$\sqrt{4} \stackrel{?}{=} -2$	$\sqrt{9} \stackrel{?}{=} 3$
$2 \neq -2$	$3 = 3$ ✓

Since $x = 1$ does not satisfy the original equation, $x = 6$ is the only solution.

Exercises

Solve each equation. Check your solution.

1. $\sqrt{a} = a$ **0, 1**
2. $\sqrt{a+6} = a$ **3**
3. $2\sqrt{x} = x$ **0, 4**
4. $n = \sqrt{2-n}$ **1**
5. $\sqrt{-a} = a$ **0**
6. $\sqrt{10-6k} + 3 = k$ **∅**
7. $\sqrt{y-1} = y - 1$ **1, 2**
8. $\sqrt{3a-2} = a$ **1, 2**
9. $\sqrt{x+2} = x$ **2**
10. $\sqrt{2b+5} = b - 5$ **10**
11. $\sqrt{3b+6} = b + 2$ **1**
12. $\sqrt{4x-4} = x$ **2**
13. $r + \sqrt{2-r} = 2$ **1, 2**
14. $\sqrt{x^2+10x} = x + 4$ **8**
15. $-2\sqrt{\frac{x}{8}} = 15$ **∅**
16. $\sqrt{6x^2-4x} = x + 2$ **$-\frac{2}{5}$, 2**
17. $\sqrt{2y^2-64} = y$ **8**
18. $\sqrt{3x^2+12x+1} = x + 5$ **−4, 3**

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10-4 Skills Practice

Radical Equations

Solve each equation. Check your solution.

1. $\sqrt{f} = 7$ **49**
2. $\sqrt{-x} = 5$ **−25**
3. $\sqrt{5p} = 10$ **20**
4. $\sqrt{4y} = 6$ **9**
5. $2\sqrt{2} = \sqrt{u}$ **8**
6. $3\sqrt{5} = \sqrt{-n}$ **−45**
7. $\sqrt{g} - 6 = 3$ **81**
8. $\sqrt{5a} + 2 = 0$ **ø**
9. $\sqrt{2t-1} = 5$ **13**
10. $\sqrt{3k-2} = 4$ **6**
11. $\sqrt{x+4} - 2 = 1$ **5**
12. $\sqrt{4x-4} - 4 = 0$ **5**
13. $\frac{\sqrt{d}}{3} = 4$ **144**
14. $\sqrt{\frac{m}{3}} = 3$ **27**
15. $x = \sqrt{x+2}$ **2**
16. $d = \sqrt{12-d}$ **3**
17. $\sqrt{6x-9} = x$ **3**
18. $\sqrt{6p-8} = p$ **2, 4**
19. $\sqrt{x+5} = x - 1$ **4**
20. $\sqrt{8-d} = d - 8$ **8**
21. $\sqrt{r-3} + 5 = r$ **7**
22. $\sqrt{y-1} + 3 = y$ **5**
23. $\sqrt{5n+4} = n + 2$ **1, 0**
24. $\sqrt{3z-6} = z - 2$ **5, 2**

Lesson 10-4

Answers

NAME ______________ DATE __________ PERIOD __________

10-4 Practice
Radical Equations

Solve each equation. Check your solution.

1. $\sqrt{-b} = 8$ **−64**
2. $4\sqrt{3} = \sqrt{x}$ **48**
3. $2\sqrt{4r} + 3 = 11$ **4**
4. $6 - \sqrt{2y} = -2$ **32**
5. $\sqrt{k+2} - 3 = 7$ **98**
6. $\sqrt{m-5} = 4\sqrt{3}$ **53**
7. $\sqrt{6t+12} = 8\sqrt{6}$ **62**
8. $\sqrt{3j-11} + 2 = 9$ **20**
9. $\sqrt{2x+15} + 5 = 18$ **77**
10. $\sqrt{\frac{3d}{5}} - 4 = 2$ **60**
11. $6\sqrt{\frac{3x}{3}} - 3 = 0$ $\mathbf{\frac{1}{4}}$
12. $6 + \sqrt{\frac{5r}{6}} = -2$ **∅**
13. $y = \sqrt{y+6}$ **3**
14. $\sqrt{15-2x} = x$ **3**
15. $\sqrt{w+4} = w + 4$ **−4, −3**
16. $\sqrt{17-k} = k - 5$ **8**
17. $\sqrt{5m-16} = m - 2$ **4, 5**
18. $\sqrt{24+8q} = q + 3$ **−3, 5**
19. $\sqrt{4t+17} - t - 3 = 0$ **2**
20. $4 - \sqrt{3m+28} = m$ **−1**
21. $\sqrt{10p+61} - 7 = p$ **−6, 2**
22. $\sqrt{2x^2-9} = x$ **3**

23. **ELECTRICITY** The voltage V in a circuit is given by $V = \sqrt{PR}$, where P is the power in watts and R is the resistance in ohms.
   - a. If the voltage in a circuit is 120 volts and the circuit produces 1500 watts of power, what is the resistance in the circuit? **9.6 ohms**
   - b. Suppose an electrician designs a circuit with 110 volts and a resistance of 10 ohms. How much power will the circuit produce? **1210 watts**

24. **FREE FALL** Assuming no air resistance, the time t in seconds that it takes an object to fall h feet can be determined by the equation $t = \frac{\sqrt{h}}{4}$.
   - a. If a skydiver jumps from an airplane and free falls for 10 seconds before opening the parachute, how many feet does the skydiver fall? **1600 ft**
   - b. Suppose a second skydiver jumps and free falls for 6 seconds. How many feet does the second skydiver fall? **576 ft**

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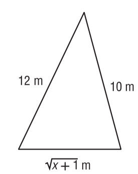
10-4 Word Problem Practice
Radical Equations

1. **SUBMARINES** The distance in miles that the lookout of a submarine can see is approximately $d = 1.22\sqrt{h}$, where h is the height in feet above the surface of the water. How far would a submarine periscope have to be above the water to locate a ship 6 miles away? Round your answer to the nearest tenth. **24.2 ft**

2. **PETS** Find the value of x if the perimeter of a triangular dog pen is 25 meters.
x = 8



3. **LOGGING** Doyle's log rule estimates the amount of usable lumber (in board feet) that can be milled from a shipment of logs. It is represented by the equation $B = L\left(\frac{d-4}{4}\right)^2$, where d is the log diameter (in inches) and L is the log length (in feet). Suppose the truck carries 20 logs, each 25 feet long, and that the shipment yields a total of 6000 board feet of lumber. Estimate the diameter of the logs to the nearest inch. Assume that all the logs have uniform length and diameter.
18 in.

4. **FIREFIGHTING** Fire fighters calculate the flow rate of water out of a particular hydrant by using the following formula.

$$F = 26.9d^2\sqrt{p}$$

F is the flow rate (in gallons per minute), p is the nozzle pressure (in pounds per square inch), and d is the diameter of the hose (in inches). In order to effectively fight a fire, the combined flow rate of two hoses needs to be about 2430 gallons per minute. The diameter of each of the hoses is 3 inches, but the nozzle pressure of one hose is 4 times that of the second hose. What are the nozzle pressures for each hose? Round your answers to the nearest tenth.

11.2 psi and 44.8 psi

5. **GEOMETRY** The lateral surface area s of a right circular cone, not including the base, is represented by the equation $s = \pi r\sqrt{r^2 + h^2}$, where r is the radius of the circular base and h is the height of the cone.
   - a. If the lateral surface area of a funnel is 127.54 square centimeters and its radius is 3.5 centimeters, find its height to the nearest tenth of a centimeter.
   11.1 cm
   - b. What is the area of the opening (i.e., the base) of the funnel?
   38.5 cm²

Lesson 10-4

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10-4 Enrichment

More Than One Square Root

You have learned that to remove the square root in an equation, you first need to isolate the square root, then square both sides of the equation, and finally, solve the resulting equation. However, there are equations that contain more that one square root and simply squaring once is not enough to remove all of the radicals.

Example **Solve $\sqrt{x+7} = \sqrt{x} + 1$.**

$\sqrt{x+7} = \sqrt{x} + 1$	One of the square roots is already isolated.
$\left(\sqrt{x+7}\right)^2 = \left(\sqrt{x}+1\right)^2$	Square both sides to remove the square root.
$x + 7 = x + 2\sqrt{x} + 1$	Simplify. Use the FOIL method to square right side.
$x + 7 - x - 1 = 2\sqrt{x}$	Simplify.
$6 = 2\sqrt{x}$	Simplify. Isolate the square root term again.
$3 = \sqrt{x}$	Divide both sides by 2.
$9 = x$	Square both sides to remove the square root.

Check: Substitute into the *original* equation to make sure your solution is valid.

$\sqrt{9+7} = \sqrt{9} + 1$	Replace *x* with 9.
$\sqrt{16} = 3 + 1$	Simplify.
$4 = 4$ ✓	The equation is true, so $x = 9$ is the solution.

Exercises

Solve each equation.

1. $\sqrt{x+13} - 2 = \sqrt{x+1}$ **3**

2. $\sqrt{x+11} = \sqrt{x+3} + 2$ **−2**

3. $\sqrt{x+9} - 3 = \sqrt{x-6}$ **7**

4. $\sqrt{x+21} = \sqrt{x} + 3$ **4**

5. $\sqrt{x+9} + 3 = \sqrt{x+20} + 2$ **16**

6. $\sqrt{x-6} + 6 = \sqrt{x+1} + 5$ **15**

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10-4 Graphing Calculator Activity

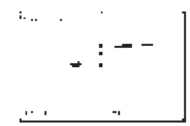
Radical Inequalities

The graphs of radical equations can be used to determine the solutions of radical inequalities through the **CALC** menu.

Example **Solve each inequality.**

a. $\sqrt{x+4} \le 3$

Enter $\sqrt{x+4}$ in **Y1** and 3 in **Y2** and graph. Examine the graphs. Use TRACE to find the endpoint of the graph of the radical equation. Use **CALC** to determine the intersection of the graphs. This interval, −4 to 5, where the graph of $y = \sqrt{x+4}$ is below the graph of $y = 3$, represents the solution to the inequality. Thus, the solution is $-4 \le x \le 5$.



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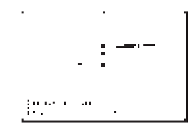

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b. $\sqrt{2x-5} > x - 4$

Graph each side of the inequality. Find the intersection and trace to the endpoint of the radical graph.

The graph of $y = \sqrt{2x-5}$ is above the graph of $y = x - 4$ from 2.5 up to 7. Thus, the solution is $2.5 < x < 7$.

[−10, 10] scl:1 by [−10, 10] scl:1

[−10, 10] scl:1 by [−10, 10] scl:1

Exercises

Solve each inequality.

1. $6 - \sqrt{2x+1} < 3$ **$x > 4$**

2. $\sqrt{4x-5} \le 7$ **$\frac{5}{4} \le x \le \frac{27}{2}$**

3. $\sqrt{5x-4} \ge 4$ **$x \ge 4$**

4. $-4 > \sqrt{3x-2}$ **no solution**

5. $\sqrt{3x-6} + 5 \ge -3$ **$x \ge 2$**

6. $\sqrt{6-3x} < x + 16$ **$-10 < x < 2$**

Lesson 10-4

Answers

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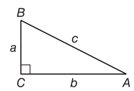
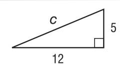
10-5 Study Guide and Intervention

The Pythagorean Theorem

The Pythagorean Theorem The side opposite the right angle in a right triangle is called the **hypotenuse**. This side is always the longest side of a right triangle. The other two sides are called the **legs** of the triangle. To find the length of any side of a right triangle, given the lengths of the other two sides, you can use the **Pythagorean Theorem**.

Pythagorean Theorem	If a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then $c^2 = a^2 + b^2$. 

Example **Find the length of the missing side.**



$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 5^2 + 12^2$ $a = 5$ and $b = 12$

$c^2 = 169$ Simplify.

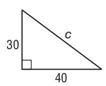
$c = \sqrt{169}$ Take the square root of each side.

$c = 13$

The length of the hypotenuse is 13.

Exercises

Find the length of each missing side. If necessary, round to the nearest hundredth.

1.


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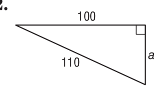
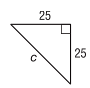
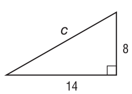
2.


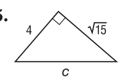
45.83

3.


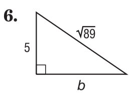
35.36

4.


16.12

5.


5.57

6.


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10-5 Study Guide and Intervention *(continued)*

The Pythagorean Theorem

Right Triangles If a and b are the measures of the shorter sides of a triangle, c is the measure of the longest side, and $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Example **Determine whether the following side measures form right triangles.**

a. 10, 12, 14

Since the measure of the longest side is 14, let $c = 14$, $a = 10$, and $b = 12$.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$14^2 \stackrel{?}{=} 10^2 + 12^2$ $a = 10, b = 12, c = 14$

$196 \stackrel{?}{=} 100 + 144$ Multiply.

$196 \neq 244$ Add.

Since $c^2 \neq a^2 + b^2$, the triangle is not a right triangle.

b. 7, 24, 25

Since the measure of the longest side is 25, let $c = 25$, $a = 7$, and $b = 24$.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$25^2 \stackrel{?}{=} 7^2 + 24^2$ $a = 7, b = 24, c = 25$

$625 \stackrel{?}{=} 49 + 576$ Multiply.

$625 = 625$ Add.

Since $c^2 = a^2 + b^2$, the triangle is a right triangle.

Exercises

Determine whether each set of measures can be sides of a right triangle. Then determine whether they form a Pythagorean triple.

1. 14, 48, 50 **yes; yes**

2. 6, 8, 10 **yes; yes**

3. 8, 8, 10 **no; no**

4. 90, 120, 150 **yes; yes**

5. 15, 20, 25 **yes; yes**

6. 4, 8, $4\sqrt{5}$ **yes; no**

7. 2, 2, $\sqrt{8}$ **yes; no**

8. 4, 4, $\sqrt{20}$ **no; no**

9. 25, 30, 35 **no; no**

10. 24, 36, 48 **no; no**

11. 18, 80, 82 **yes; yes**

12. 150, 200, 250 **yes; yes**

13. 100, 200, 300 **no; no**

14. 500, 1200, 1300 **yes; yes**

15. 700, 1000, 1300 **no; no**

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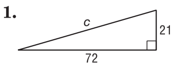
10-5 Skills Practice

The Pythagorean Theorem

Find the length of each missing side. If necessary, round to the nearest hundredth.

1.


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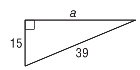
2.


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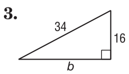
3.


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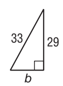
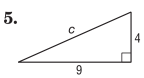
4.


15.75

5.


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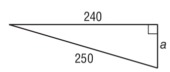
6.


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Determine whether each set of measures can be sides of a right triangle. Then determine whether they form a Pythagorean triple.

7. 7, 24, 25 **yes; yes**

8. 15, 30, 34 **no; no**

9. 16, 28, 32 **no; no**

10. 18, 24, 30 **yes; yes**

11. 15, 36, 39 **yes; yes**

12. 5, 7, $\sqrt{74}$ **yes; no**

13. 4, 5, 6 **no; no**

14. 10, 11, $\sqrt{221}$ **yes; no**

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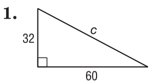
NAME ______________________ DATE ________ PERIOD ________

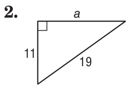
10-5 Practice

The Pythagorean Theorem

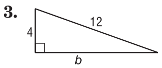
Find the length of each missing side. If necessary, round to the nearest hundredth.

1.


68

2.


15.49

3.


11.31

Determine whether each set of measures can be sides of right triangle. Then determine whether they form a Pythagorean triple.

4. 11, 18, 21
no; no

5. 21, 72, 75
yes; yes

6. 7, 8, 11
no; yes

7. 9, 10, $\sqrt{161}$
no; no

8. 9, $2\sqrt{10}$, 11
yes; no

9. $\sqrt{7}$, $2\sqrt{2}$, $\sqrt{15}$
yes; no

10. STORAGE The shed in Stephan's back yard has a door that measures 6 feet high and 3 feet wide. Stephan would like to store a square theater prop that is 7 feet on a side. Will it fit through the door diagonally? Explain. **No; the greatest length that will fit through the door is $\sqrt{45} \approx 6.71$ ft.**

11. SCREEN SIZES The size of a television is measured by the length of the screen's diagonal.

a. If a television screen measures 24 inches high and 18 inches wide, what size television is it? **30-in. television**

b. Darla told Tri that she has a 35-inch television. The height of the screen is 21 inches. What is its width? **28 in.**

c. Tri told Darla that he has a 5-inch handheld television and that the screen measures 2 inches by 3 inches. Is this a reasonable measure for the screen size? Explain. **No; if the screen measures 2 in. by 3 in., then its diagonal is only about 3.61 in.**

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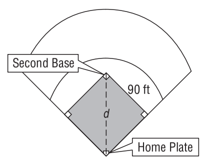
10-5 Word Problem Practice

Pythagorean Theorem

1. **BASEBALL** A baseball diamond is a square. Each base path is 90 feet long. After a pitch, the catcher quickly throws the ball from home plate to a teammate standing by second base. Find the distance the ball travels. Round your answer to the nearest tenth.

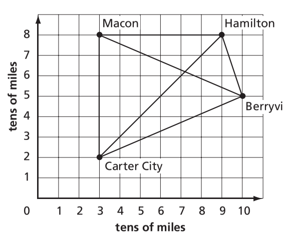
127.3 ft



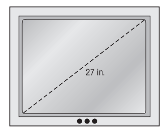
2. **TRIANGLES** Each student in Mrs. Kelly's geometry class constructed a unique right triangle from drinking straws. Mrs. Kelly made a chart with the dimensions of each triangle. However, Mrs. Kelly made a mistake when recording their results. Which result was recorded incorrectly? **Fran's**

Side Lengths							
Student	a	b	c	Student	a	b	c
Amy	3	4	5	Fran	8	14	16
Belinda	7	24	25	Gus	5	12	13
Emory	9	12	15				

3. **MAPS** Find the distance between Macon and Berryville. Round your answer to the nearest tenth. **76.2 mi**



4. **TELEVISION** Televisions are identified by the diagonal measurement of the viewing screen. For example, a 27-inch television has a diagonal screen measurement of 27 inches.

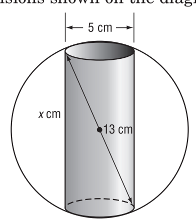


Complete the chart to find the screen height of each television given its size and screen width. Round your answers to the nearest whole number.

TV size	width (in.)	height (in.)
19-inch	15	**12**
25-inch	21	**14**
32-inch	25	**20**
50-inch	40	**30**

Source: Best Buy

5. **MANUFACTURING** Karl works for a company that manufactures car parts. His job is to drill a hole in spherical steel balls. The balls and the holes have the dimensions shown on the diagram.



a. How deep is the hole? **12 cm**

b. What would be the radius of a ball with a similar hole 7 centimeters wide and 24 centimeters deep? **12.5 cm**

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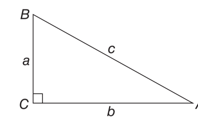
10-5 Enrichment

Pythagorean Triples

Recall the Pythagorean Theorem:
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$a^2 + b^2 = c^2$

Note that c is the length of the hypotenuse.

The integers 3, 4, and 5 satisfy the Pythagorean Theorem and can be the lengths of the sides of a right triangle.

$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$

Furthermore, for any positive integer n, the numbers $3n$, $4n$, and $5n$ satisfy the Pythagorean Theorem.

For $n = 2$: $6^2 + 8^2 = 10^2$
$36 + 64 = 100$
$100 = 100$

If three numbers satisfy the Pythagorean Theorem, they are called a **Pythagorean triple**. Here is an easy way to find other Pythagorean triples.

The numbers a, b, and c are a Pythagorean triple if $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$, where m and n are relatively prime positive integers and $m > n$.

Example **Choose $m = 5$ and $n = 2$.**

$a = m^2 - n^2 = 5^2 - 2^2 = 25 - 4 = 21$

$b = 2mn = 2(5)(2) = 20$

$c = m^2 + n^2 = 5^2 + 2^2 = 25 + 4 = 29$

Check $20^2 + 21^2 \stackrel{?}{=} 29^2$
$400 + 441 \stackrel{?}{=} 841$
$841 = 841$

Exercises

Use the following values of m and n to find Pythagorean triples.

1. $m = 3$ and $n = 2$ **5, 12, 13**
2. $m = 4$ and $n = 1$ **8, 15, 17**
3. $m = 5$ and $n = 3$ **16, 30, 34**
4. $m = 6$ and $n = 5$ **11, 60, 61**
5. $m = 10$ and $n = 7$ **51, 140, 149**
6. $m = 8$ and $n = 5$ **39, 80, 89**

Lesson 10-5

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10-5 Spreadsheet Activity

Pythagorean Triples

A **Pythagorean triple** is a set of three whole numbers that satisfies the equation $a^2 + b^2 = c^2$, where c is the greatest number. You can use a spreadsheet to investigate the patterns in Pythagorean triples. A **primitive Pythagorean triple** is a Pythagorean triple in which the numbers have no common factors other than 1. A **family of Pythagorean triples** is a primitive Pythagorean triple and its whole number multiples.

The spreadsheet at the right produces a family of Pythagorean triples.

Step 1 Enter a Pythagorean triple into cells A1, A2, and A3.

Step 2 Use rows 2 through 10 to find 9 additional Pythagorean triples that are multiples of the primitive triple. Format the rows so that row 2 multiplies the numbers in row 1 by 2, row 3 multiplies the numbers in row 1 by 3, and so on.

Triples.xls

◇	A	B	C
1	3	4	5
2	6	8	10
3	9	12	15
4	12	16	20
5	15	20	25
6	18	24	30
7	21	28	35
8	24	32	40
9	27	36	45
10	30	40	50

Sheet 1 / Sheet 2 / Sl

The formula in cell A10 is A1 * 10.

Exercises

Use the spreadsheet of families of Pythagorean triples.

1. Choose one of the triples other than (3, 4, 5) from the spreadsheet. Verify that it is a Pythagorean triple. **Sample answer: For (6, 8, 10), $6^2 + 8^2 = 36 + 64$ or $100 = 10^2$.**

2. Two polygons are **similar** if they are the same shape, but not necessarily the same size. For triangles, if two triangles have angles with the same measures then they are similar. Use a centimeter ruler to draw triangles with measures from the spreadsheet. Do the triangles appear to be similar? **See students' work.; yes**

Each of the following is a primitive Pythagorean triple. Use the spreadsheet to find two Pythagorean triples in their families.

3. (5, 12, 13) **Sample answer: (10, 24, 26), (15, 36, 39)**

4. (9, 40, 41) **Sample answer: (18, 80, 82), (27, 120, 123)**

5. (20, 21, 29) **Sample answer: (40, 42, 58), (60, 63, 87)**

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10-6 Study Guide and Intervention

The Distance and Midpoint Formulas

Distance Formula The Pythagorean Theorem can be used to derive the **Distance Formula** shown below. The Distance Formula can then be used to find the distance between any two points in the coordinate plane.

Distance Formula	The distance between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1 **Find the distance between the points at (−5, 2) and (4, 5).**

$d = \sqrt{(x_2 - x_1)^2 + (y^2 - y_1)^2}$ Distance Formula

$= \sqrt{(4 - (-5))^2 + (5 - 2)^2}$ $(x_1, y_1) = (-5, 2), (x_2, y_2) = (4, 5)$

$= \sqrt{9^2 + 3^2}$ Simplify.

$= \sqrt{81 + 9}$ Evaluate squares and simplify.

$= \sqrt{90}$

The distance is $\sqrt{90}$, or about 9.49 units.

Example 2 **Jill draws a line segment from point (1, 4) on her computer screen to point (98, 49). How long is the segment?**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(98 - 1)^2 + (49 - 4)^2}$

$= \sqrt{97^2 + 45^2}$

$= \sqrt{9409 + 2025}$

$= \sqrt{11{,}434}$

The segment is about 106.93 units long.

Exercises

Find the distance between the points with the given coordinates.

1. (1, 5), (3, 1) **$2\sqrt{5}$; 4.47**
2. (0, 0), (6, 8) **10**
3. (−2, −8), (7, −3) **$\sqrt{106}$; 10.30**
4. (6, −7), (−2, 8) **17**
5. (1, 5), (−8, 4) **$\sqrt{82}$; 9.06**
6. (3, −4), (−4, −4) **7**
7. (−1, 4), (3, 2) **$2\sqrt{5}$; 4.47**
8. (0, 0), (−3, 5) **$\sqrt{34}$; 5.83**
9. (2, −6), (−7, 1) **$\sqrt{130}$; 11.40**
10. (−2, −5), (0, 8) **$\sqrt{173}$; 13.15**
11. (3, 4), (0, 0) **5**
12. (3, −4), (−4, −16) **$\sqrt{193}$; 13.89**

Find the possible values of *a* if the points with the given coordinates are the indicated distance apart.

13. (1, a), (3, −2); $d = \sqrt{5}$ **−1 or −3**
14. (0, 0), (a, 4); $d = 5$ **3 or −3**
15. (2, −1), (a, 3); $d = 5$ **−1 or 5**
16. (1, −3), (a, 21); $d = 25$ **−6 or 8**
17. (1, a), (−2, 4); $d = 3$ **4**
18. (3, −4), (−4, a); $d = \sqrt{65}$ **−8 or 0**

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10-6 Study Guide and Intervention *(continued)*

The Distance and Midpoint Formulas

Midpoint Formula The point that is equidistance from both of the endpoints is called the **midpoint**. You can find the coordinates of the midpoint by using the Midpoint Formula.

Midpoint Formula	The midpoint M of a line segment with endpoints at (x_1, y_1) and (x_2, y_2) is given by $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Example 1 **Find the coordinates of the midpoint of the segment with endpoints at (–2, 5) and (4, 9).**

$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ Midpoint Formula

$= M\left(\frac{-2+4}{2}, \frac{5+9}{2}\right)$ $(x_1, y_1) = (-2, 5)$ and $(x_2, y_2) = (4, 9)$

$= M\left(\frac{2}{2}, \frac{14}{2}\right)$ Simplify the numerators.

$= M(1, 3)$ Simplify.

Exercises

Find the coordinates of the midpoint of the segment with the given endpoints.

1. (1, 6), (3, 10) **(2, 8)**
2. (4, −2), (0, 6) **(2, 2)**
3. (7, 2), (13, −4) **(10, −1)**
4. (−1, 2), (1, 0) **(1, 0)**
5. (−3, −3), (5, −11) **(1, −7)**
6. (0, 8), (−6, 0) **(−3, 4)**
7. (4, −3), (−2, 3) **(1, 0)**
8. (9, −1), (3, −7) **(6, −4)**
9. (2, −1), (8, 7) **(5, 3)**
10. (1, 4), (−3, 12) **(−1, 8)**
11. (4, 0), (−2, 6) **(1, 3)**
12. (1, 9), (7, 1) **(4, 5)**
13. (12, 0), (2, −6) **(7, −3)**
14. (1, 1), (9, −9) **(5, −4)**
15. (4, 5), (−2, −1) **(1, 2)**
16. (1, −14), (−5, 0) **(−2, −7)**
17. (2, 2), (6, 8) **(4, 5)**
18. (−7, 3), (5, −3) **(−1, 0)**

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10-6 Skills Practice

The Distance and Midpoint Formulas

Find the distance between the points with the given coordinates.

1. (9, 7), (1, 1) **10**
2. (5, 2), (8, −2) **5**
3. (1, −3), (1, 4) **7**
4. (7, 2), (−5, 7) **13**
5. (−6, 3), (10, 3) **16**
6. (3, 3), (−2, 3) **5**
7. (−1, −4), (−6, 0) $\sqrt{41} \approx 6.40$
8. (−2, 4), (5, 8) $\sqrt{65} \approx 8.06$

Find the possible values of *a* if the points with the given coordinates are the indicated distance apart.

9. (−2, −5), (*a*, 7); $d = 13$ **a = −7 or 3**
10. (8, −2), (5, *a*); $d = 3$ **a = −2**
11. (4, *a*), (1, 6); $d = 5$ **a = 2 or 10**
12. (*a*, 3), (5, −1); $d = 5$ **a = 2 or 8**
13. (1, 1), (*a*, 1); $d = 4$ **a = −3 or 5**
14. (2, *a*), (2, 3); $d = 10$ **a = −7 or 13**
15. (*a*, 2), (−3, 3); $d = \sqrt{2}$ **a = −4 or −2**
16. (−5, 3), (−3, *a*); $d = \sqrt{5}$ **a = 2 or 4**

Find the coordinates of the midpoint of the segment with the given endpoints.

17. (−3, 4), (−2, 8) **(−2.5, 6)**
18. (5, −6), (7, −9) **(6, −7.5)**
19. (4, 2), (8, 6) **(6, 4)**
20. (5, 2), (3, 10) **(4, 6)**
21. (12, −1), (4, −11) **(8, −6)**
22. (−3, −1), (−11, 3) **(−7, 1)**
23. (9, 3), (6, −6) **(7.5, −1.5)**
24. (0, −4), (8, 4) **(4, 0)**

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10-6 Practice

The Distance and Midpoint Formulas

Find the distance between the points with the given coordinates.

1. (4, 7), (1, 3) **5**

2. (0, 9), (−7, −2) **$\sqrt{170} \approx 13.04$**

3. (6, 2), + $\left(4, \frac{1}{2}\right)$ **$\frac{5}{2}$ or 2.50**

4. (−1, 7), + $\left(\frac{1}{3}, 6\right)$ **$\frac{5}{3} \approx 1.67$**

5. $(\sqrt{3}, 3)$, $(2\sqrt{3}, 5)$ **$\sqrt{7} \approx 2.65$**

6. $(2\sqrt{2}, -1)$, $(3\sqrt{2}, 3)$ **$3\sqrt{2} \approx 4.24$**

Find the possible values of *a* if the points with the given coordinates are the indicated distance apart.

7. (4, −1), (*a*, 5); *d* = 10 **a = −4 or 12**

8. (2, −5), (*a*, 7); *d* = 15 **a = −7 or 11**

9. (6, −7), (*a*, −4); $d = \sqrt{18}$ **a = 3 or 9**

10. (−4, 1), (*a*, 8); $d = \sqrt{50}$ **a = −5 or −3**

11. (8, −5), (*a*, 4); $d = \sqrt{85}$ **a = 6 or 10**

12. (−9, 7), (*a*, 5); $d = \sqrt{29}$ **a = −14 or −4**

Find the coordinates of the midpoint of the segment with the given endpoints.

13. (4, −6), (3, −9) **(3.5, − 7.5)**

14. (−3, −8), (−7, 2) **(−5, −3)**

15. (0, −4), (3, 2) **(1.5, − 1)**

16. (−13, −9), (−1, −5) **(−7, −7)**

17. $\left(2, -\frac{1}{2}\right)$, $\left(1, \frac{1}{2}\right)$ **$\left(1\frac{1}{2}, 0\right)$**

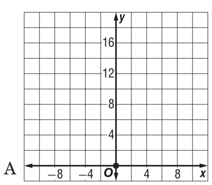
18. $\left(\frac{2}{3}, -1\right)$, $\left(2, \frac{1}{3}\right)$ **$\left(1\frac{1}{3}, -\frac{1}{3}\right)$**

19. **BASEBALL** Three players are warming up for a baseball game. Player B stands 9 feet to the right and 18 feet in front of Player A. Player C stands 8 feet to the left and 13 feet in front of Player A.

   a. Draw a model of the situation on the coordinate grid. Assume that Player A is located at (0, 0).

   b. To the nearest tenth, what is the distance between Players A and B and between Players A and C? **20.1 ft; 15.3 ft**

   c. What is the distance between Players B and C? **17.7 ft**



20. **MAPS** Maria and Jackson live in adjacent neighborhoods. If they superimpose a coordinate grid on the map of their neighborhoods, Maria lives at (−9, 1) and Jackson lives at (5, −4). **about 1.96 mi**

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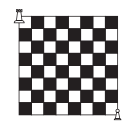
10-6 Word Problem Practice

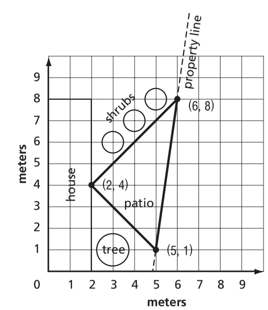
The Distance and Midpoint Formulas

1. **CHESS** Margaret's last two remaining chess pieces are located at the centers of the squares at opposite corners of the board. If the chessboard is a square with 8-inch sides, about how far apart are the pieces? Round your answer to the nearest tenth. **9.9 in.**



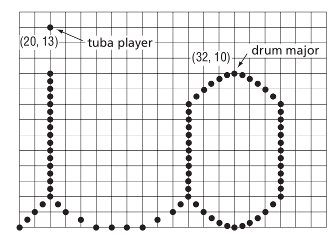
2. **ENGINEERING** Todd has drawn a cul-de-sac for a residential development plan. He used a compass to draw the cul-de-sac so that it would be circular. On his blueprint, the center of the cul-de-sac has coordinates (−1, −1) and a point on the circle is (2, 3). What is the radius of the cul-de-sac? **5 units**

3. **LANDSCAPING** Randy plotted a triangular patio on a landscape plan for a client. What is the length of fencing he will need along the patio edge that borders the property line? Round your answer to the nearest tenth. **7.1 m**



4. **UTILITIES** The electric company is running some wires across an open field. The wire connects a utility pole at (2, 14) and a second utility pole at (7, −8). If the electric company wishes to place a third pole at the midpoint of the two poles, at what coordinates should the pole be placed? **(4.5, 3)**

5. **MARCHING BAND** The Ohio State University marching band performs a famous on-field spelling of O-H-I-O called "Script Ohio". Sometimes they must adjust the usual dimensions of the word to fit it into the limited guest band performance area. The diagram below shows part of the adjusted drill chart. Each point represents one band member, and the coordinates are in yards.



   a. How far is the drum major from the tuba player who dots the "i"? **12.4 yd**

   b. Carol is the band member at the top left of the first O in Ohio. She is located at (0, 26). How far away is Carol from the tuba player? Round your answer to the nearest tenth. **23.9 yd**

Lesson 10-6

Answers

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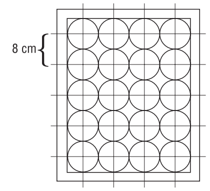
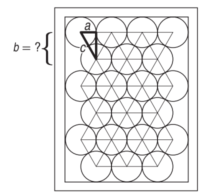
10-6 Enrichment

A Space-Saving Method

Two arrangements for cookies on a 32 cm by 40 cm cookie sheet are shown at the right. The cookies have 8-cm diameters after they are baked. The centers of the cookies are on the vertices of squares in the top arrangement. In the other, the centers are on the vertices of equilateral triangles. Which arrangement is more economical? The triangle arrangement is more economical, because it contains one more cookie.



20 cookies



21 cookies

In the square arrangement, rows are placed every 8 cm. At what intervals are rows placed in the triangle arrangement?

Look at the right triangle labeled a, b, and c. A leg a of the triangle is the radius of a cookie, or 4 cm. The hypotenuse c is the sum of two radii, or 8 cm. Use the Pythagorean theorem to find b, the interval of the rows.

$$c^2 = a^2 + b^2$$
$$8^2 = 4^2 + b^2$$
$$64 - 16 = b^2$$
$$\sqrt{48} = b$$
$$4\sqrt{3} = b$$
$$b = 4\sqrt{3} \approx 6.93$$

The rows are placed approximately every 6.93 cm.

Solve each problem.

1. Suppose cookies with 10-cm diameters are arranged in the triangular pattern shown above. What is the interval b of the rows? **8.66 cm**

2. Find the diameter of a cookie if the rows are placed in the triangular pattern every $3\sqrt{3}$ cm. **6 cm**

3. Describe other practical applications in which this kind of triangular pattern can be used to economize on space. **Sample answer: packaging cans**

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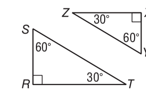
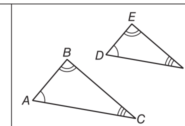
10-7 Study Guide and Intervention

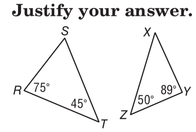
Similar Triangles

Similar Triangles $\triangle RST$ is **similar** to $\triangle XYZ$. The angles of the two triangles have equal measure. They are called **corresponding angles**. The sides opposite the corresponding angles are called **corresponding sides**.



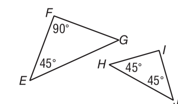
Similar Triangles	If two triangles are similar, then the measures of their corresponding sides are proportional and the measures of their corresponding angles are equal.	$\triangle ABC \sim \triangle DEF$ $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	

Example 1 **Determine whether the pair of triangles is similar. Justify your answer.**



Since corresponding angles do not have the equal measures, the triangles are not similar.

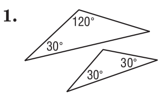
Example 2 **Determine whether the pair of triangles is similar. Justify your answer.**



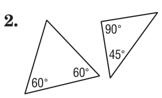
The measure of $\angle G = 180° - (90° + 45°) = 45°$.
The measure of $\angle I = 180° - (45° + 45°) = 90°$.
Since corresponding angles have equal measures, $\triangle EFG \sim \triangle HIJ$.

Exercises

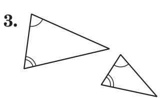
Determine whether each pair of triangles is similar. Justify your answer.

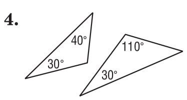
1. 

Yes; corresponding angles have equal measures.

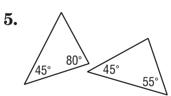
2. 

No; corresponding angles do not have equal measures.

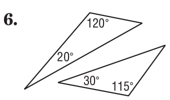
3. 
Yes; corresponding angles have equal measures.

4. 

Yes; corresponding angles have equal measures.

5. 

Yes; corresponding angles have equal measures.

6. 

No; corresponding angles do not have equal measures.

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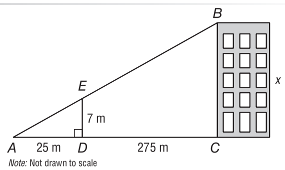
10-7 Study Guide and Intervention *(continued)*

Similar Triangles

Find Unknown Measures If some of the measurements are known, proportions can be used to find the measures of the other sides of similar triangles.

Example **INDIRECT MEASUREMENT**
$\triangle ABC \sim \triangle AED$ in the figure at the right. Find the height of the apartment building.



Let $BC = x$.

$\frac{ED}{BC} = \frac{AD}{AC}$

$\frac{7}{x} = \frac{25}{300}$ ED = 7, AD = 25, AC = 300

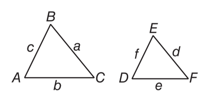
$25x = 2100$ Find the cross products.

$x = 84$

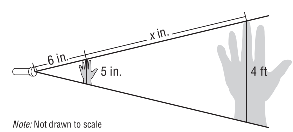
The apartment building is 84 meters high.

Exercises

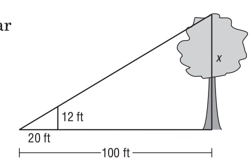
Find the missing measures for the pair of similar triangles if $\triangle ABC \sim \triangle DEF$.



1. $c = 15, d = 8, e = 6, f = 10$ **a = 12; b = 9**
2. $c = 20, a = 12, b = 8, f = 15$ **d = 9; e = 6**
3. $a = 8, d = 8, e = 6, f = 7$ **b = 6; c = 7**
4. $a = 20, d = 10, e = 8, f = 10$ **b = 16; c = 20**
5. $c = 5, d = 10, e = 8, f = 8$ **$a = \frac{25}{4}$; b = 5**
6. $a = 25, b = 20, c = 15, f = 12$ **d = 20; e = 16**
7. $b = 8, d = 8, e = 4, f = 10$ **a = 16; c = 20**
8. **INDIRECT MEASUREMENT** Bruce likes to amuse his brother by shining a flashlight on his hand and making a shadow on the wall. How far is it from the flashlight to the wall? **51.6 in. or 4.3 ft**



9. **INDIRECT MEASUREMENT** A forest ranger uses similar triangles to find the height of a tree. Find the height of the tree. **60 ft**



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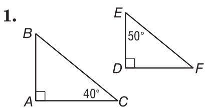
10-7 Skills Practice

Similar Triangles

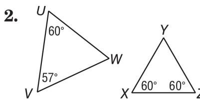
Determine whether each pair of triangles is similar. Justify your answer.

1.



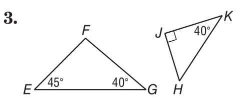
Yes; ∠A = ∠D = 90°; ∠B = 180° − (90° + 40°) = 50° = ∠E; ∠F = 180° − (90° + 50°) = 40° = ∠C. Since the corresponding angles have equal measures, △ABC ~ △DEF.

2.



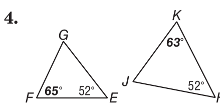
No; ∠Y = 180° − (60° + 60°) = 60°. Since △UVW has a 57° angle, but △XYZ does not, corresponding angles do not all have equal measures, and the triangles are not similar.

3.



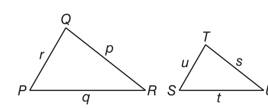
No; ∠F = 180° − (45° + 40°) = 95°. Since △HJK has a 90° angle, but △EFG does not, corresponding angles do not all have equal measures, and the triangles are not similar.

4.



Yes; ∠G = 180° − (65° + 52°) = 63° = ∠K; ∠J = 180° − (63° + 52°) = 65° = ∠F; ∠E = ∠H = 52°. Since the corresponding angles have equal measures, △EFG ~ △HJK.

Find the missing measures for the pair of similar triangles if $\triangle PQR \sim \triangle STU$.



5. $r = 4, s = 6, t = 3, u = 2$ **p = 12, q = 6**
6. $t = 8, p = 21, q = 14, r = 7$ **u = 4, s = 12**
7. $p = 15, q = 10, r = 5, s = 6$ **t = 4, u = 2**
8. $p = 48, s = 16, t = 8, u = 4$ **r = 12, q = 24**
9. $q = 6, s = 2, t = \frac{3}{2}, u = \frac{1}{2}$ **r = 2, p = 8**
10. $p = 3, q = 2, r = 1, u = \frac{1}{3}$ **s = 1, $t = \frac{2}{3}$**
11. $p = 14, q = 7, u = 2.5, t = 5$ **r = 3.5, s = 10**
12. $r = 6, s = 3, t = \frac{21}{8}, u = \frac{9}{4}$ **p = 8, q = 7**

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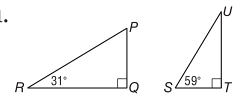
10-7 Practice

Similar Triangles

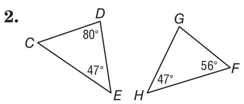
Determine whether each pair of triangles is similar. Justify your answer.

1.



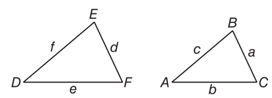
Yes; ∠Q = ∠T = 90°; ∠P = 180° − (90° + 31°) = 59° = ∠S; ∠U = 180° − (90° + 59°) = 31° = ∠R. Since the corresponding angles have equal measures, △PQR, △STU.

2.



No; ∠C = 180° − (47° + 80°) = 53°. Since △FGH has a 56° angle, but △CDE does not, corresponding angles do not all have equal measures, and the triangles are not similar.

Find the missing measures for the pair of similar triangles if △ABC ~ △DEF.



3. $c = 4, d = 12, e = 16, f = 8$ **a = 6, b = 8**

4. $e = 20, a = 24, b = 30, c = 15$ **d = 16, f = 10**

5. $a = 10, b = 12, c = 6, d = 4$ **e = 4.8, f = 2.4**

6. $a = 4, d = 6, e = 4, f = 3$ **$c = 2, b = \frac{8}{3}$**

7. $b = 15, d = 16, e = 20, f = 10$ **$a = 12, c = \frac{15}{2}$**

8. $a = 16, b = 22, c = 12, f = 8$ **$d = \frac{32}{3}, e = \frac{44}{3}$**

9. $a = \frac{5}{2}, b = 3, f = \frac{11}{2}, e = 7$ **$c = \frac{33}{14}, d = \frac{35}{6}$**

10. $c = 4, d = 6, e = 5.625, f = 12$ **a = 2, b = 1.875**

11. SHADOWS Suppose you are standing near a building and you want to know its height. The building casts a 66-foot shadow. You cast a 3-foot shadow. If you are 5 feet 6 inches tall, how tall is the building? **121 ft**

12. MODELS Truss bridges use triangles in their support beams. Molly made a model of a truss bridge in the scale of 1 inch = 8 feet. If the height of the triangles on the model is 4.5 inches, what is the height of the triangles on the actual bridge? **36 ft**

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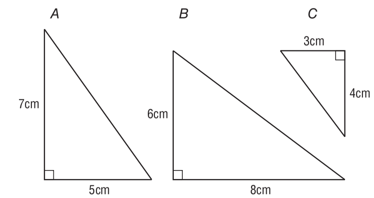
10-7 Word Problem Practice

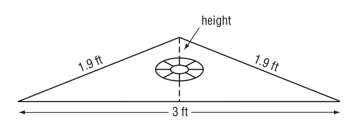
Similar Triangles

1. CRAFTS Layla is wants to buy a set of similar magnets for her refrigerator door. Layla finds the magnets below for sale at a local shop. Which two are similar?
B and C



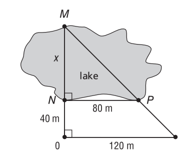
2. EXHIBITIONS The world's largest candle was displayed at the 1897 Stockholm Exhibition. Suppose Lars measured the length of the shadow it cast at 11:00 A.M. and found that it was 12 feet. Suppose that immediately after this, he measured to find that a nearby 25-foot tent pole cast a shadow 5 feet long. How tall was the world's largest candle? **60 ft**

3. LANDMARKS The Toy and Miniature Museum of Kansas City displays a miniature replica of George Washington's Mount Vernon mansion. The miniature house is 10 feet long, 6 feet wide, 8 feet tall, and has 22 rooms. The scale of the model to the original is one inch to one foot. If the roof gable of the miniature has dimensions as shown on the diagram below, what is the height of the roof gable on the original Mount Vernon mansion? **14 ft**

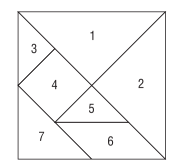


Source: Mount Vernon

4. SURVEYING Surveyors use properties of triangles including similarity and the Pythagorean Theorem to find unknown distances. Use the dimensions on the diagram to find the unknown distance x across the lake. **80 m**



5. PUZZLES The figure below shows an ancient Chinese movable puzzle called a tangram. It has 7 pieces that can be reconfigured to produce an endless number of designs and pictures.



Assume that the side length of this tangram square is $\sqrt{2}$ cm. Leave your answers as simplified radical expressions.

a. What are the side lengths of triangles 1 and 2? **1 cm, 1 cm, $\sqrt{2}$ cm**

b. What are the side lengths of triangle 7? **$\frac{\sqrt{2}}{2}$ cm, $\frac{\sqrt{2}}{2}$ cm, 1 cm**

c. What are the side lengths of triangles 3 and 5? **$\frac{1}{2}$ cm, $\frac{1}{2}$ cm, $\frac{\sqrt{2}}{2}$ cm**

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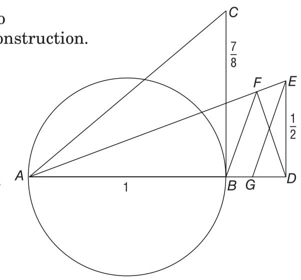
10-7 Enrichment

A Curious Construction

Many mathematicians have been interested in ways to construct the number π. Here is one such geometric construction.

In the drawing, triangles ABC and ADE are right triangles. The length of $\overline{AD}$ equals the length of $\overline{AC}$ and $\overline{FB}$ is parallel to $\overline{EG}$.

The length of $\overline{BG}$ gives a decimal approximation of the fractional part of π to six decimal places.



Follow the steps to find the length of $\overline{BG}$. Round to seven decimal places.

1. Use the length of $\overline{BC}$ and the Pythagorean Theorem to find the length of $\overline{AC}$.

   $AC = \sqrt{1^2 + \left(\frac{7}{8}\right)^2} = 1.3287682$

2. Find the length of $\overline{AD}$.

   $AD = AC = 1.3287682$

3. Use the length of $\overline{AD}$ and the Pythagorean Theorem to find the length of $\overline{AE}$.

   $AE = \sqrt{(AD)^2 + \left(\frac{1}{2}\right)^2} = 1.4197271$

4. The sides of the similar triangles FED and DEA are in proportion. So, $\frac{FE}{0.5} = \frac{0.5}{AE}$. Find the length of $\overline{FE}$.

   $FE = \frac{1}{4(AE)} = 0.1760902$

5. Find the length of $\overline{AF}$.

   $AF = AE - FE = 1.2436369$

6. The sides of the similar triangles AFB and AEG are in proportion. So, $\frac{AF}{AE} = \frac{AB}{AG}$. Find the length of $\overline{AG}$.

   $AG = \frac{AB \cdot AE}{AF} = \frac{AE}{AF} = 1.1415929$

7. Now, find the length of $\overline{BG}$.

   $BG = AG - AB = AG - 1 = 0.1415929$

8. The value of π to seven decimal places is 3.1415927. Compare the fractional part of pi with the length of $\overline{BG}$.

   0.1415929 − 0.1415927 = 0.0000002, an error of less than 1 part in a million

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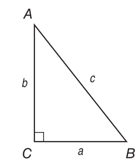
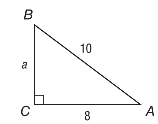
10-8 Study Guide and Intervention

Trigonometric Ratios

Trigonometric Ratios Trigonometry is the study of relationships of the angles and the sides of a right triangle. The three most common trigonometric ratios are the **sine**, **cosine**, and **tangent**.

sine of $\angle A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$ sine of $\angle B = \frac{\text{leg opposite } \angle B}{\text{hypotenuse}}$	$\sin A = \frac{a}{c}$ $\sin B = \frac{b}{c}$	
cosine of $\angle A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$ cosine of $\angle B = \frac{\text{leg adjacent to } \angle B}{\text{hypotenuse}}$	$\cos A = \frac{b}{c}$ $\cos B = \frac{a}{c}$	
tangent of $\angle A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$ tangent of $\angle B = \frac{\text{leg opposite } \angle B}{\text{leg adjacent to } \angle B}$	$\tan A = \frac{a}{b}$ $\tan B = \frac{b}{a}$	

Example **Find the values of the three trigonometric ratios for angle *A*.**



Step 1 Use the Pythagorean Theorem to find BC.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$a^2 + 8^2 = 10^2$ $b = 8$ and $c = 10$

$a^2 + 64 = 100$ Simplify.

$a^2 = 36$ Subtract 64 from both sides.

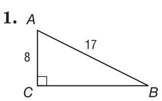
$a = 6$ Take the square root of each side.

Step 2 Use the side lengths to write the trigonometric ratios.

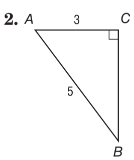
$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5}$ $\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5}$ $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = \frac{3}{4}$

Exercises

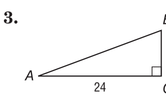
Find the values of the three trigonometric ratios for angle *A*.

1. 

   $\sin A = \frac{15}{17}$, $\cos A = \frac{8}{17}$, $\tan A = \frac{15}{8}$

2. 

   $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$, $\tan A = \frac{4}{3}$

3. 

   $\sin A = \frac{7}{25}$, $\cos A = \frac{24}{25}$, $\tan A = \frac{7}{24}$

Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

4. sin 40° **0.6428**

5. cos 25° **0.9063**

6. tan 85° **11.4301**

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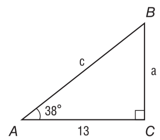
10-8 Study Guide and Intervention *(continued)*

Trigonometric Ratios

Use Trigonometric Ratios When you find all of the unknown measures of the sides and angles of a right triangle, you are **solving the triangle**. You can find the missing measures of a right triangle if you know the measure of two sides of the triangle, or the measure of one side and the measure of one acute angle.

Example **Solve the triangle. Round each side length to the nearest tenth.**



Step 1 Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180.
$180° - (90° + 38°) = 52°$
The measure of $\angle B$ is 52°.

Step 2 Find the measure of $\overline{AB}$. Because you are given the measure of the side adjacent to $\angle A$ and are finding the measure of the hypotenuse, use the cosine ratio.

$\cos 38° = \frac{13}{c}$ Definition of cosine

$c \cos 38° = 13$ Multiply each side by c.

$c = \frac{13}{\cos 38°}$ Divide each side by sin 41°.

So the measure of $\overline{AB}$ is about 16.5.

Step 3 Find the measure of $\overline{BC}$. Because you are given the measure of the side adjacent to $\angle A$ and are finding the measure of the side opposite $\angle A$, use the tangent ratio.

$\tan 38° = \frac{a}{13}$ Definition of tangent

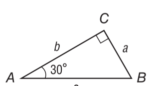
$13 \tan 38° = a$ Multiply each side by 13.

$10.2 \approx a$ Use a calculator.

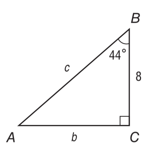
So the measure of $\overline{BC}$ is about 10.2.

Exercises

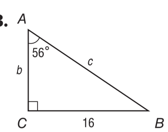
Solve each right triangle. Round each side length to the nearest tenth.

1. 

$\angle B = 60°$, $AC \approx 7.8$, $BC = 4.5$

2. 

$\angle A = 60°$, $AC \approx 7.7$, $AB \approx 11.1$

3. 

$\angle B = 34°$, $AC \approx 19.3$, $AB \approx 10.8$

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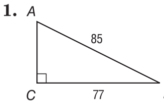
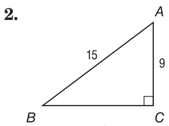
10-8 Skills Practice

Trigonometric Ratios

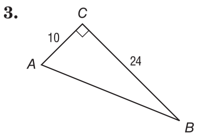
Find the values of the three trigonometric ratios for angle *A*.

1. 

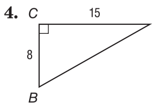
$\sin A = \frac{77}{85}$, $\cos A = \frac{36}{85}$, $\tan A = \frac{77}{36}$

2. 

$\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$, $\tan A = \frac{4}{3}$

3. 

$\sin A = \frac{12}{13}$, $\cos A = \frac{5}{13}$, $\tan A = \frac{12}{5}$

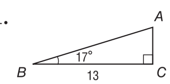
4. 

$\sin A = \frac{8}{17}$, $\cos A = \frac{15}{17}$, $\tan A = \frac{8}{15}$

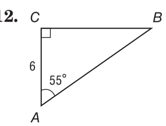
Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

5. sin 18° **0.3090**
6. cos 68° **0.3746**
7. tan 27° **0.5095**
8. cos 60° **0.5**
9. tan 75° **3.7321**
10. sin 9° **0.1564**

Solve each right triangle. Round each side length to the nearest tenth.

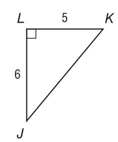
11. 

$\angle A = 73°$, $AB = 13.6$, $AC = 4.0$

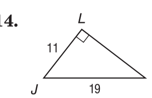
12. 

$\angle B = 35°$, $AB = 10.5$, $BC = 8.6$

Find $m\angle J$ for each right triangle to the nearest degree.

13. 

40°

14. 

55°

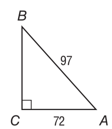
NAME ______________________ DATE ____________ PERIOD ____________

10-8 Practice

Trigonometric Ratios

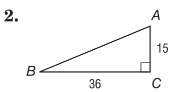
Find the values of the three trigonometric ratios for angle *A*.

1.



$\sin A = \frac{65}{97}, \cos A = \frac{72}{97}, \tan A = \frac{65}{72}$

2.



$\sin A = \frac{36}{39}, \cos A = \frac{15}{39}, \tan A = \frac{36}{15}$

Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

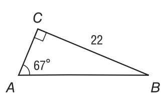
3. tan 26° **0.4877**

4. sin 53° **0.7986**

5. cos 81° **0.1564**

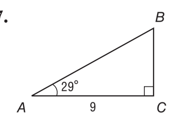
Solve each right triangle. Round each side length to the nearest tenth.

6.



$\angle B = 23°, AB = 23.9, AC = 9.3$

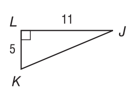
7.



$\angle A = 61°, AB = 10.3, BC = 5.0$

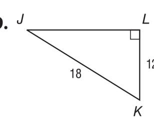
Find $m\angle J$ for each right triangle to the nearest degree.

8.



24°

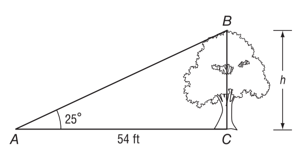
9.



42°

10. **SURVEYING** If point *A* is 54 feet from the tree, and the angle between the ground at point *A* and the top of the tree is 25°, find the height *h* of the tree.

25.2 ft

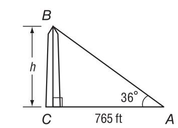


NAME ______________________ DATE ____________ PERIOD ____________

10-8 Word Problem Practice

Trigonometric Ratios

1. **WASHINGTON MONUMENT** Jeannie is trying to determine the height of the Washington Monument. If point *A* is 765 feet from the monument, and the angle between the ground and the top of the monument at point *A* is 36°, find the height *h* of the monument to the nearest foot. **556 ft**

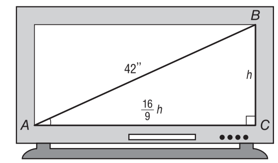


2. **AIRPLANES** A pilot takes off from a runway at an angle of 20° and maintains that angle until it is at its cruising altitude of 2500 feet. What horizontal distance has the plane traveled when it reaches its cruising altitude? **6869 ft**

3. **TRUCK RAMPS** A moving company uses an 11-foot-long ramp to unload furniture from a truck. If the bed of the truck is 3 feet above the ground, what is the angle of incline of the ramp to the nearest degree? **16°**

4. **SPECIAL TRIANGLES** While investigating right triangle *KLM*, Mercedes finds that cos *M* = sin *M*. What is the measure of angle *M*? **45°**

5. **TELEVISIONS** Televisions are commonly sized by measuring their diagonal. A common size for widescreen plasma TVs is 42 inches.



a. A widescreen television has a 16:9 aspect ratio, that is, the screen width is $\frac{16}{9}$ times the screen height. Use the Pythagorean Theorem to write an equation and solve for the height *h* of the television in inches.

$\left(\frac{16}{9}h\right)^2 + h^2 = 42^2$; $h = 20.6$ in.

b. Use the information from part **a** to solve the right triangle.

width = 36.6 in., $\angle A = 29°$,

$\angle B = 61°$

c. What would the measure of angle *A* be on a standard television with a 4:3 aspect ratio? **37°**

Lesson 10-8

Answers

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