

11 Anticipation Guide

Rational Expressions and Equations

Step 1 Before you begin Chapter 11

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. Since a direct variation can be written as $y = kx$, an inverse variation can be written as $y = \frac{x}{k}$.	D
	2. A rational expression is an algebraic fraction that contains a radical.	D
	3. To multiply two rational expressions, such as $\frac{2xy^2}{3c}$ and $\frac{3c^2}{5y}$, multiply the numerators and the denominators.	A
	4. When solving problems involving units of measure, dimensional analysis is the process of determining the units of the final answer so that the units can be ignored while performing calculations.	D
	5. To divide $(4x^2 + 12x)$ by $2x$, divide $4x^2$ by $2x$ and $12x$ by $2x$.	A
	6. To find the sum of $\frac{2a}{(3a - 4)}$ and $\frac{5}{(3a - 4)}$, first add the numerators and then the denominators.	D
	7. The least common denominator of two rational expressions will be the least common multiple of the denominators.	A
	8. A complex fraction contains a fraction in its numerator or denominator.	A
	9. The fraction $\frac{\frac{a}{b}}{\frac{c}{d}}$ can be rewritten as $\frac{ac}{bd}$.	D
	10. Extraneous solutions are solutions that can be eliminated because they are extremely high or low.	D

Chapter Resources

NAME _____ DATE _____ PERIOD _____

Answers (Anticipation Guide and Lesson 11-1)

Lesson 11-1

NAME _____ DATE _____ PERIOD _____

11-1 Study Guide and Intervention

Inverse Variation

Identify and Use Inverse Variations An inverse variation is an equation in the form of $y = \frac{k}{x}$ or $xy = k$. If two points (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1 \cdot y_1 = k$ and $x_2 \cdot y_2 = k$.

Product Rule for Inverse Variation

From the product rule, you can form the proportion $\frac{x_1}{x_2} = \frac{y_1}{y_2}$.

Example If y varies inversely as x and $y = 12$ when $x = 4$, find x when $y = 18$.

Method 1 Use the product rule.

$$\begin{array}{l|l} x_1 \cdot y_1 = x_2 \cdot y_2 & \text{Product rule for inverse variation} \\ 4 \cdot 12 = x_2 \cdot 18 & x_1 = 4, y_1 = 12, y_2 = 18 \\ \frac{48}{18} = x_2 & \text{Divide each side by 18.} \\ \frac{8}{3} = x_2 & \text{Simplify.} \end{array}$$

Both methods show that $x_2 = \frac{8}{3}$ when $y = 18$.

Exercises

Determine whether each table or equation represents an *inverse* or a *direct* variation. Explain.

1.	x	y	direct variation; of the form $y = kx$
3	5	10	
8	16		
12	24		

Assume that y varies inversely as x . Write an inverse variation equation that relates x and y . Then solve.

- If $y = 8$ when $x = 5$, find y when $x = 2$. **xy = 50; 25**
- If $y = -16$ when $x = 4$, find y when $x = 8$. **xy = -64; -2**
- If $y = 100$ when $x = 120$, find x when $y = 20$. **xy = 12,000; 600**
- If $y = -7.5$ when $x = 25$, find y when $x = 5$. **xy = -187.5; -37.5**

Step 2 After you complete Chapter 11

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

11-1 Study Guide and Intervention**Inverse Variation**

(continued)

Graph Inverse Variations Situations in which the values of y decrease as the values of x increase are examples of **inverse variation**. We say that y varies inversely as x , or y is inversely proportional to x .

Inverse Variation Equation | an equation of the form $xy = k$, where $k \neq 0$

Example 1 Suppose you drive 200 miles without stopping. The time it takes to travel a distance varies inversely as the rate at which you travel. Let x = speed in miles per hour and y = time in hours. Graph the variation.

The equation $xy = 200$ can be used to represent the situation. Use various speeds to make a table.

x	y
10	20
20	10
30	6.7
40	5
50	4
60	3.3

Inverse variation equation
 $x = 12$ and $y = 3$

Solve for k .

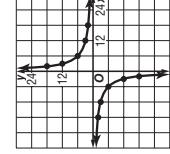
$$12(3) = k$$

$$36 = k$$

Simplify.

$$xy = 36$$

Choose values for x and y , which have a product of 36.



Example 2 Graph an inverse variation in which y varies inversely as x and $y = 3$ when $x = 12$.

Solve for k .

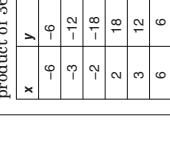
$$12(3) = k$$

$$36 = k$$

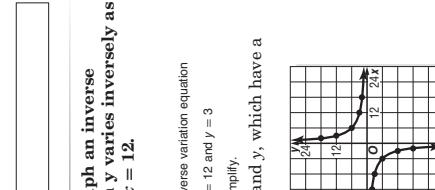
Inverse variation equation
 $x = 12$ and $y = 3$

$$xy = 36$$

Choose values for x and y , which have a product of 36.



- Exercises**
Graph each variation if y varies inversely as x .
1. $y = 9$ when $x = -3$
2. $y = 12$ when $x = 4$
3. $y = -25$ when $x = 5$
4. $y = 4$ when $x = 5$
5. $y = -18$ when $x = -9$
6. $y = 4.8$ when $x = 5.4$
7. $y = -6$ when $x = -6$
8. $y = 4$ when $x = 8$
9. $y = -7$ when $x = 3$
10. $y = -6$ when $x = -2$
11. $y = -24$ when $x = 4$
12. $y = 15$ when $x = 1$
13. $y = 48$ when $x = -4$
14. $y = -4$ when $x = \frac{1}{2}$



1.

x	y
0.5	8
1	4
2	2
4	1

2. $xy = \frac{2}{3}$
inverse, $xy = \frac{2}{3}$

direct, $y = 2x$

3. $-2x + y = 0$

4. $y = -6$ when $x = -6$

5. $y = 10$ when $x = 5$

6. $y = -4$ when $x = -12$

7. $y = 15$ when $x = 3$

8. If $y = 4$ when $x = 8$, find y when $x = 2$. $xy = 32; 16$

9. If $y = -7$ when $x = 3$, find y when $x = -3$. $xy = -21; 7$

10. If $y = -6$ when $x = -2$, find y when $x = 4$. $xy = 12; 3$

11. If $y = -24$ when $x = 4$, find x when $y = -6$. $xy = 72; -12$

12. If $y = 15$ when $x = 1$, find x when $y = -3$. $xy = 15; -5$

13. If $y = 48$ when $x = -4$, find y when $x = 6$. $xy = -192; -32$

14. If $y = -4$ when $x = \frac{1}{2}$, find x when $y = 2$. $xy = -2; -1$

11-1 Skills Practice**Inverse Variation**

Determine whether each table or equation represents an *inverse* or a *direct* variation. Explain.

1.

x	y
0.5	8
1	4
2	2
4	1

2. $xy = \frac{2}{3}$

inverse, $xy = \frac{2}{3}$

3. $-2x + y = 0$

4. $y = -6$ when $x = -6$

5. $y = 10$ when $x = 5$

6. $y = -4$ when $x = -12$

7. $y = 15$ when $x = 3$

8. If $y = 4$ when $x = 8$, find y when $x = 2$. $xy = 32; 16$

9. If $y = -7$ when $x = 3$, find y when $x = -3$. $xy = -21; 7$

10. If $y = -6$ when $x = -2$, find y when $x = 4$. $xy = 12; 3$

11. If $y = -24$ when $x = 4$, find x when $y = -6$. $xy = 72; -12$

12. If $y = 15$ when $x = 1$, find x when $y = -3$. $xy = 15; -5$

13. If $y = 48$ when $x = -4$, find y when $x = 6$. $xy = -192; -32$

14. If $y = -4$ when $x = \frac{1}{2}$, find x when $y = 2$. $xy = -2; -1$

Answers (Lesson 11-1)

11-1 Practice

Inverse Variation

Determine whether each table or equation represents an *inverse* or a *direct* variation. Explain.

x	y
0.25	40
0.5	20
2	5
8	1.25

$$\text{inverse; } xy = k$$

x	y
-2	8
0	0
2	-8
4	-16

$$\text{direct; } y = kx$$

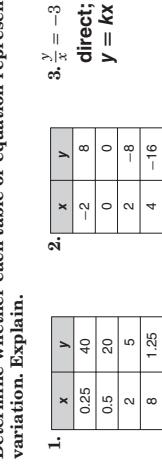
x	y
3	-3
8	8
0	0
2	-8

$$\text{direct; } y = \frac{7}{x}$$

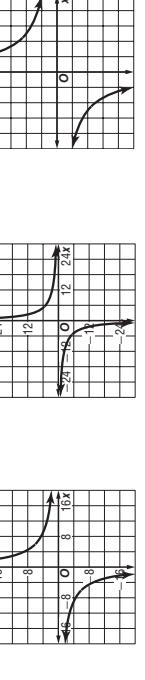
Inverse Variation

11-1 Word Problem Practice

Assume that y varies inversely as x. Write an inverse variation equation that relates x and y. Then graph the equation.



$$xy = 24$$



$$xy = 30$$

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve.

$$8. \text{ If } y = 124 \text{ when } x = 12, \text{ find } y \text{ when } x = -24. \quad \mathbf{xy = 1488; -62}$$

$$9. \text{ If } y = -8.5 \text{ when } x = 6, \text{ find } y \text{ when } x = -2.5. \quad \mathbf{xy = -51; 20.4}$$

$$10. \text{ If } y = 3.2 \text{ when } x = -5.5, \text{ find } y \text{ when } x = 6.4. \quad \mathbf{xy = -17.6; -2.75}$$

$$11. \text{ If } y = 0.6 \text{ when } x = 7.5, \text{ find } y \text{ when } x = -1.25. \quad \mathbf{xy = 4.5; -3.6}$$

12. **EMPLOYMENT** The manager of a lumber store schedules 6 employees to take inventory in an 8-hour work period. The manager assumes all employees work at the same rate.
- Suppose 2 employees call in sick. How many hours will 4 employees need to take inventory? **12 h**
 - If the district supervisor calls in and says she needs the inventory finished in 6 hours, how many employees should the manager assign to take inventory? **8**

13. **TRAVEL** Jesse and Joaquin can drive to their grandparents' home in 3 hours if they average 50 miles per hour. Since the road between the homes is winding and mountainous, their parents prefer they average between 40 and 45 miles per hour. How long will it take to drive to the grandparents' home at the reduced speed? **between 3 h 20 min and 3 h 45 min**

Answers (Lesson 11-1)

Lesson 11-1

1. **PHYSICAL SCIENCE** The illumination I produced by a light source varies inversely as the square of the distance d from the source. The illumination produced 5 feet from the light source is 80 foot-candles. **$Id^2 = k$**
- $$80(5)^2 = k$$
- $$2000 = k$$
- Find the illumination produced 8 feet from the same source. **31.25 foot-candles**
2. **MONEY** A formula called the Rule of 72 approximates how fast money will double in a savings account. It is based on the relation that the number of years it takes for money to double varies inversely as the annual interest rate. Use the information in the table to write the Rule of 72 formula. **$yr = 72$**
- | Years to Double Money | Annual Interest Rate (percent) |
|-----------------------|--------------------------------|
| 18 | 4 |
| 14.4 | 5 |
| 12 | 6 |
| 10.29 | 7 |
3. **ELECTRICITY** The resistance, in ohms, of a certain length of electric wire varies inversely as the square of the diameter of the wire. If a wire 0.04 centimeter in diameter has a resistance of 0.60 ohm, what is the resistance of a wire of the same length and material that is 0.08 centimeters in diameter? **0.15 ohm**

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NAME _____

DATE _____ PERIOD _____

11-1 Enrichment**Direct or Indirect Variation**

Fill in each table below. Then write *inversely*, or *directly* to complete each conclusion.

1. ℓ	2	4	8	16	32
W	4	4	4	4	4
A	8	16	32	64	128

For a set of rectangles with a width of 4, the area varies directly as the length.

3. Oat Bran	$\frac{1}{3}$ cup	$\frac{2}{3}$ cup	1 cup
Water	1 cup	2 cup	3 cup
Servings	1	2	3

The number of servings of oat bran varies directly as the number of cups of oat bran.

5. Miles	100	100	100	100
Rate	20	25	50	100
Hours	5	4	2	1

For a 100-mile car trip, the time the trip takes varies inversely as the average rate of speed the car travels.

Use the table at the right.

7. x varies directly as y .
 8. z varies inversely as y .
 9. x varies inversely as z .

Identify Excluded Values The function $y = \frac{10}{x}$ is an example of a rational function. Because division by zero is undefined, any value of a variable that results in a denominator of zero must be excluded from the domain of that variable. These are called **excluded values** of the rational function.

Lesson 11-2**11-2 Study Guide and Intervention****Rational Functions**

Explain State the excluded value for each function.

a. $y = \frac{3}{x}$
 The denominator cannot equal zero.
 The excluded value is $x = 0$.
 Add 5 to each side.
 $x = 5$
 The excluded value is $x = 5$.

b. $y = \frac{4}{x - 5}$
 Set the denominator equal to 0.
 $x - 5 = 0$
 Add 5 to each side.
 $x = 5$
 The excluded value is $x = 5$.

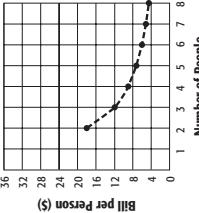
Exercises

State the excluded value for each function.

1. $y = \frac{2}{x} x = 0$
 2. $y = \frac{1}{x - 4} x = 4$
 4. $y = \frac{4}{x - 2} x = 2$
 7. $y = \frac{3x - 2}{x + 3} x = -3$
 10. $y = \frac{x - 7}{2x + 8} x = -4$
 13. $y = \frac{7}{3x + 21} x = -7$

3. $y = \frac{x - 3}{x + 1} x = -1$
 5. $y = \frac{x}{2x - 4} x = 2$
 8. $y = \frac{x - 1}{5x + 10} x = -2$
 11. $y = \frac{x - 5}{6x} x = 0$
 14. $y = \frac{3x - 4}{x + 4} x = -4$
 16. $y = \frac{x}{7x - 35} x = 5$

16. DINING Mya and her friends are eating at a restaurant. The total bill of \$36 is split among x friends. The amount each person pays y is given by $y = \frac{36}{x}$, where x is the number of people. Graph the function.



NAME _____ DATE _____ PERIOD _____

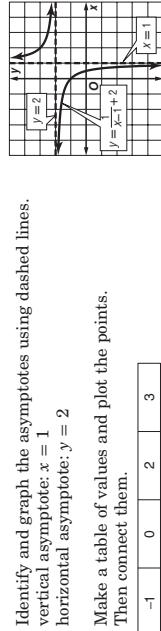
11-2 Study Guide and Intervention

(continued)

Rational Functions

Identify and Use Asymptotes Because excluded values are undefined, they affect the graph of the function. An **asymptote** is a line that the graph of a function approaches. A rational function in the form $y = \frac{a}{x-b} + c$ has a vertical asymptote at the x -value that makes the denominator equal zero, $x = b$. It has a horizontal asymptote at $y = c$.

Example Identify the asymptotes of $y = \frac{1}{x-1} + 2$. Then graph the function.



Step 1 Identify and graph the asymptotes using dashed lines.
vertical asymptote: $x = 1$
horizontal asymptote: $y = 2$

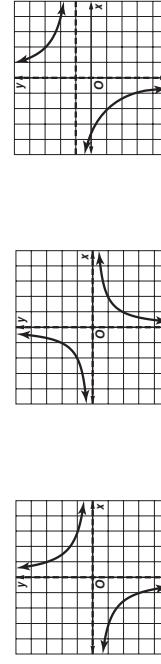
Step 2 Make a table of values and plot the points.
Then connect them.

x	-1	0	2	3	2.5
y	1.5	0	2	3	2.5

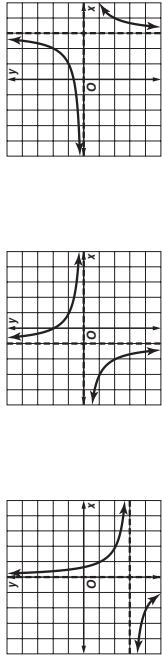
Exercises

Identify the asymptotes of each function. Then graph the function.

1. $y = \frac{3}{x}$ $x = 0$; $y = 0$ 2. $y = \frac{-2}{x}$ $x = 0$; $y = 0$ 3. $y = \frac{4}{x} + 1$ $x = 0$; $y = 1$



4. $y = \frac{2}{x} - 3$ $x = 0$; $y = 3$ 5. $y = \frac{2}{x+1}$ $x = -1$; $y = 0$ 6. $y = \frac{-2}{x-3}$ $x = 3$; $y = 0$



NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

11-2 Skills Practice**Rational Functions**

State the excluded value for each function.

1. $y = \frac{6}{x}$ $x = 0$ 2. $y = \frac{2}{x-2}$ $x = 2$

3. $y = \frac{x}{x+6}$ $x = -6$

4. $y = \frac{x-3}{x+4}$ $x = -4$

5. $y = \frac{3x-5}{x+8}$ $x = -8$

6. $y = \frac{-5}{2x-14}$ $x = 7$

7. $y = \frac{x}{3x+21}$ $x = -7$

8. $y = \frac{x-1}{9x-36}$ $x = 4$

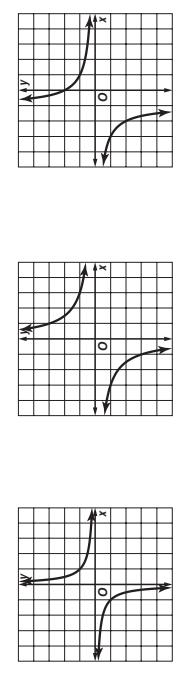
9. $y = \frac{9}{5x+40}$ $x = -8$

Identify the asymptotes of each function. Then graph the function.

10. $y = \frac{1}{x}$ $x = 0$, $y = 0$

11. $y = \frac{3}{x}$ $x = 0$, $y = 0$

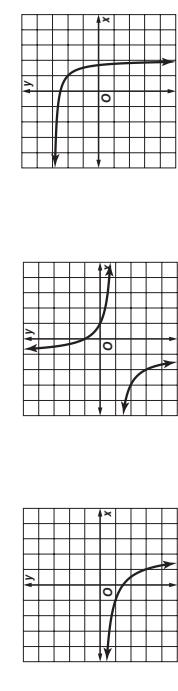
12. $y = \frac{2}{x+1}$ $x = -1$, $y = 0$



13. $y = \frac{-3}{x-2}$ $x = 2$, $y = 0$

14. $y = \frac{2}{x+1} - 1$ $x = -1$, $y = -1$

15. $y = \frac{1}{x-2}$ $x = 2$, $y = 3$



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11-2 Practice**Rational Functions**

State the excluded value for each function.

1. $y = \frac{-1}{x}$ $x = 0$ 2. $y = \frac{3}{x+5}$ $x = -5$

3. $y = \frac{2x}{x-5}$ $x = 5$ 4. $y = \frac{x-1}{12x+36}$ $x = -3$

5. $y = \frac{x+1}{2x+3}$ $x = -\frac{3}{2}$ 6. $y = \frac{1}{5x-2}$ $x = \frac{2}{5}$

7. $y = \frac{1}{x}$ $x = 0, y = 0$ 8. $y = \frac{3}{x}$ $x = 0, y = 0$

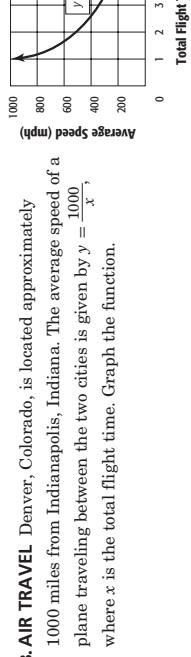
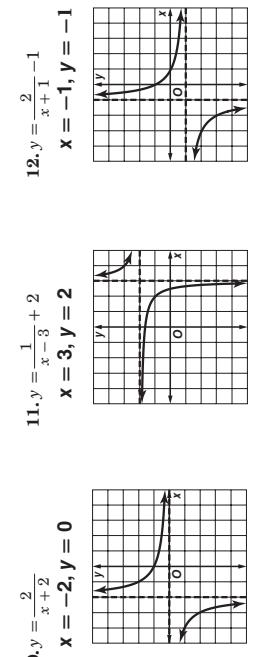
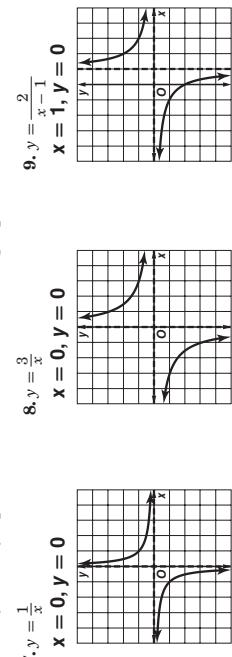
9. $y = \frac{2}{x-1}$ $x = 1, y = 0$

10. $y = \frac{2}{x+2}$ $x = -2, y = 0$

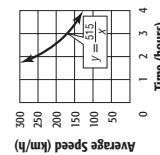
11. $y = \frac{1}{x-3} + 2$ $x = 3, y = 2$

12. $y = \frac{2}{x+1} - 1$ $x = -1, y = -1$

13. $y = \frac{1000}{x}$ $x = 0, y = \infty$

**11-2 Word Problem Practice****Rational Functions**

- 1. BULLET TRAINS** The Shinkansen, or Japanese bullet train network, provides high-speed ground transportation throughout Japan. Trains regularly operate at speeds in excess of 200 kilometers per hour. The average speed of a bullet train traveling between Tokyo and Kyoto is given by $y = \frac{515}{x}$, where x is the total travel time in hours. Graph the function.



- 4. USED CARS** While researching cars to purchase online, Ms. Jacobs found that the value of a used car is inversely proportional to the age of the car. The average price of a used car is given by $y = \frac{17,900}{x+1.2} + 100$, where x is the age of the car. What are the asymptotes of the function? Explain why $x = 0$ cannot be an asymptote.

- 5. FAMILY REUNION** The Gaudet family is holding their annual reunion at Watkins Park. It costs \$50 to get a permit to hold the reunion at the park, and the family is spending \$8 per person on food. The Gaudets have agreed to split the cost of the event evenly among all those attending.

- a.** Write an equation showing the cost per person y if x people attend the reunion. $y = 8 + \frac{50}{x}$
- b.** What are the asymptotes of the equation? $x = 0, y = 8$

- c.** Now assume that the family wants to let a long-lost cousin attend for free. Rewrite the equation to find the new cost per paying person y .

$$y = 8 + \frac{58}{x-1}$$

- d.** What are the asymptotes for the new equation? $x = 1, y = 8$

- 3. ERROR ANALYSIS** Nicolas is graphing the equation $y = \frac{20}{x+3} - 6$ and draws a graph with asymptotes at $y = 3$ and $x = -6$. Explain the error that Nicolas made in his graph. **The asymptotes should be $x = 3, y = -6$.**

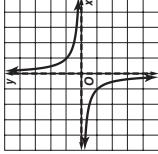
NAME _____

DATE _____ PERIOD _____

11-2 Enrichment**Inequalities involving Rational Functions**

Inequalities involving rational functions can be graphed much like those involving linear functions.

Example Graph $y \geq \frac{1}{x^2}$.



Step 1 Plot the asymptotes, $x = 0$ and $y = 0$, as dashed lines. Inequality involves a greater than or equal to sign, solutions that satisfy $y = \frac{1}{x^2}$ will be a part of the graph.

Step 2 Plot the asymptotes, $x = 0$ and $y = 0$, as dashed lines. Begin testing values. A value must be tested between each set of lines, including asymptotes.

Region 1 Test $(-1, -1)$. This returns a true value for the inequality.

Region 2 Test $(-1, -0.5)$. This returns a true value for the inequality.

Region 3 Test $(-1, -2)$. This returns a false value for the inequality.

Region 4 Test $(1, 2)$. This returns a true value for the inequality.

Region 5 Test $(1, 0.5)$. This returns a false value for the inequality.

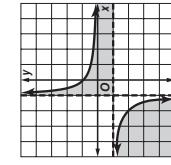
Region 6 Test $(1, -1)$. This returns a true value for the inequality.

Step 4 Shade the regions where the inequality is true.

Exercises

Graph each inequality.

$$1. y \leq \frac{1}{x+1}$$



$$3. y \leq \frac{1}{x+1} - 1$$

$$5. \frac{2n-12}{n^2-4} \quad -2, 2$$

$$7. \frac{x^2-4}{x^2+4x+4} \quad -2$$

$$9. \frac{k^2-2k+1}{k^2+4k+3} \quad -3, -1$$

$$11. \frac{25-n^2}{n^2-4n-5} \quad -1, 5$$

$$13. \frac{n^2-2n-3}{n^2+4n-5} \quad -5, 1$$

$$15. \frac{k^2+2k-3}{k^2-20k+64} \quad 4, 16$$

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

Answers (Lesson 11-2 and Lesson 11-3)

Lesson 11-3

11-3 Study Guide and Intervention**Simplifying Rational Expressions****Identify Excluded Values**

Rational Expression	an algebraic fraction with numerator and denominator that are polynomials
	Example: $\frac{x^2 + 1}{x^2 - 9}$

Because a rational expression involves division, the denominator cannot equal zero. Any value of the denominator that results in division by zero is called an **excluded value** of the denominator.

Example 1	State the excluded value of $\frac{4m-8}{m+2}$.
	Exclude the values for which $m + 2 = 0$. $m + 2 = 0$ The denominator cannot equal 0. $m + 2 - 2 = 0 - 2$ Subtract 2 from each side. $m = -2$ Simplify. Therefore, m cannot equal -2 .

Example 2	State the excluded values of $\frac{x^2+1}{x^2-9}$.
	Exclude the values for which $x^2 - 9 = 0$. $x^2 - 9 = 0$ The denominator cannot equal 0. $(x + 3)(x - 3) = 0$ Factor. $x + 3 = 0$ or $x - 3 = 0$ Zero Product Property $= -3$ Therefore, x cannot equal -3 or 3 .

Exercises

State the excluded values for each rational expression.

1. $\frac{2b}{b^2-8} \quad \sqrt{8}, -\sqrt{8}$
2. $\frac{12-a}{32+a} \quad -32$
3. $\frac{x^2-2}{x^2+4} \quad 2, -2$
4. $\frac{m^2-4}{2m^2-8} \quad 2, -2$
6. $\frac{2x+18}{x^2-16} \quad -4, 4$
7. $\frac{x^2-4}{x^2+4x+4} \quad -2$
8. $\frac{a-1}{a^2+5a+6} \quad -3, -2$
10. $\frac{m^2-1}{2m^2-m-1} \quad -\frac{1}{2}, 1$
12. $\frac{2x^2+5x+1}{x^2-10x+16} \quad 2, 8$
14. $\frac{y^2-y-2}{3y^2-12} \quad -2, 2$
16. $\frac{x^2+4x+4}{4x^2+11x-3} \quad -3, \frac{1}{4}$

Answers (Lesson 11-3)

Lesson 11-3

NAME _____ DATE _____ PERIOD _____

NAME _____

DATE _____ PERIOD _____

11-3 Study Guide and Intervention (continued)

Simplifying Rational Expressions

Simplify Expressions Factoring polynomials is a useful tool for simplifying rational expressions. To simplify a rational expression, first factor the numerator and denominator. Then divide each by the greatest common factor.

Example 1 Simplify $\frac{54x^3}{24yz}$.

$$\begin{aligned} \frac{54x^3}{24yz} &= \frac{(6z)(9x^2)}{(6z)(4y)} \quad \text{The GCF of the numerator and the denominator is } 6z. \\ &= \frac{1(6z)(9x^2)}{1(6z)(4y)} \quad \text{Divide the numerator and denominator by } 6z. \\ &= \frac{9x^2}{4y} \quad \text{Simplify.} \end{aligned}$$

Example 2 Simplify $\frac{3x - 9}{x^2 - 5x + 6}$. State the excluded values of x .

$$\begin{aligned} \frac{3x - 9}{x^2 - 5x + 6} &= \frac{3(x - 3)}{(x - 2)(x - 3)} \quad \text{Factor.} \\ &= \frac{3(x - 2)(x - 3)}{3(x - 2)(x - 3)} \quad \text{Divide by the GCF, } x - 3. \\ &= \frac{3}{x - 2} \quad \text{Simplify.} \end{aligned}$$

Exclude the values for which $x^2 - 5x + 6 = 0$.

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (x - 2)(x - 3) &= 0 \\ x = 2 &\quad \text{or} \quad x = 3 \end{aligned}$$

Therefore, $x \neq 2$ and $x \neq 3$.

Exercises

Simplify each expression. State the excluded values of the variables.

1. $\frac{12ab}{a^2b^2}; a \neq 0; b \neq 0$ 2. $\frac{7n^3}{2\ln^8} \cdot \frac{1}{3n^5}; n \neq 0$
3. $\frac{x + 2}{x^2 - 4} \cdot \frac{1}{x - 2}; x \neq -2 \text{ or } 2$ 4. $\frac{-m^2 - 4}{m^2 + 6m + 8} \cdot \frac{m - 2}{m + 4}; m \neq -4 \text{ or } -2$
5. $\frac{2n - 8}{n^2 - 16} \cdot \frac{2}{n + 4}; n \neq -4 \text{ or } 4$ 6. $\frac{x^2 + 2x + 1}{x^2 - 1} \cdot \frac{x + 1}{x - 1}; x \neq -1 \text{ or } 1$
7. $\frac{x^2 - 4}{x^2 + 4x + 4} \cdot \frac{x - 2}{x + 2}; x \neq -2$ 8. $\frac{a^2 + 3a + 2}{a^2 + 5a + 6} \cdot \frac{a + 1}{a + 3}; a \neq -3 \text{ or } -2$
9. $\frac{k^2 - 1}{k^2 + 4k + 3} \cdot \frac{k - 1}{k + 3}; k \neq -3 \text{ or } -1$ 10. $\frac{m^2 - 2m + 1}{2m^2 - m - 1} \cdot \frac{m - 1}{2m + 1}; m \neq -\frac{1}{2} \text{ or } 1$
11. $\frac{n^2 - 4n - 5}{n^2 + 2n + 12} \cdot \frac{n + 5}{n + 3}; n \neq -1 \text{ or } 5$ 12. $\frac{x^2 + x - 6}{x^2 - 2x - 24} \cdot \frac{x + 3}{2x + 4}; x \neq -2 \text{ or } 2$
13. $\frac{n^2 + 2n - 8}{n^2 + 2n - 8} \cdot \frac{n + 3}{n - 2}; n \neq -4 \text{ or } 2$ 14. $\frac{y^2 - y - 2}{y^2 - 10y + 16} \cdot \frac{y + 1}{y - 8}; y \neq 2 \text{ or } 8$

Skills Practice

Simplifying Rational Expressions

State the excluded values for each rational expression.

$$1. \frac{2p}{p - 7} \quad 7$$

$$2. \frac{4n + 1}{n + 4} \quad -4$$

$$3. \frac{k + 2}{k^2 - 4} \quad -2, 2$$

$$4. \frac{3x + 15}{x^3 - 25} \quad -5, 5$$

$$5. \frac{y^2 - 9}{y^2 + 3y - 18} \quad -6, 3$$

$$6. \frac{b^2 - 2b - 8}{b^2 + 7b + 10} \quad -5, -2$$

Simplify each expression. State the excluded values of the variables.

$$7. \frac{21bc}{28bc^2} \cdot \frac{3}{4c}; 0, 0$$

$$8. \frac{12m^2r}{24mr^3} \cdot \frac{m}{2r^2}; 0, 0$$

$$9. \frac{16x^3y^2}{36x^3y^3} \cdot \frac{4}{9xy}; 0, 0$$

$$10. \frac{8a^2b^3}{40a^3b} \cdot \frac{b^2}{5a}; 0, 0$$

$$11. \frac{n + 6}{3n + 18} \cdot \frac{1}{3}; -6$$

$$12. \frac{4x - 4}{4x + 4} \cdot \frac{x - 1}{x + 1}; -1$$

$$13. \frac{y^2 - 64}{y + 8} \cdot y - 8; -8$$

$$14. \frac{y^2 - 7y - 18}{y - 9} \cdot y + 2; 9$$

$$15. \frac{z + 1}{z^2 - 1} \cdot \frac{1}{z - 1}; -1, 1$$

$$16. \frac{x + 6}{x^2 + 2x - 24} \cdot \frac{1}{x - 4}; -6, 4$$

$$17. \frac{2d + 10}{d^2 - 2d - 35} \cdot \frac{2}{d - 7}; -5, 7$$

$$18. \frac{3h - 9}{h^2 - 7h + 12} \cdot \frac{3}{h - 4}; 3, 4$$

$$19. \frac{t^2 + 5t + 6}{t^2 + 6t + 8} \cdot \frac{t + 3}{t + 4}; -4, -2$$

$$20. \frac{a^2 + 3a - 4}{a^2 + 2a - 8} \cdot \frac{a - 1}{a - 2}; -4, 2$$

$$21. \frac{x^2 + 10x + 24}{x^2 - 2x - 24} \cdot \frac{x + 6}{x - 6}; -4, 6$$

$$22. \frac{b^2 - 6b + 9}{b^2 - 9b + 18} \cdot \frac{b - 3}{b - 6}; 3, 6$$

NAME _____ DATE _____ PERIOD _____

NAME _____

DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

11-3 Practice**Simplifying Rational Expressions**

State the excluded values for each rational expression.

1. $\frac{4n-28}{n^2-49} -7, 7$

2. $\frac{p^2-16}{p^2-13p+36} 4, 9$

3. $\frac{a^2-2a-15}{a^2+8a+15} -5, -3$

Simplify each expression. State the excluded values of the variables.

4. $\frac{12a}{48a^3} \frac{1}{4a^2}; 0$

5. $\frac{6xyz^3}{3x^2y^2z^2} \frac{2z^2}{xy}; 0, 0, 0$

6. $\frac{36k^3np^2}{20k^2np^5} \frac{9k}{5p}; 0, 0, 0$

7. $\frac{5cd^4}{40cd^2+5c'd^2} \frac{c^2d^2}{8+c^3}; c: 0, -2, d: 0$

8. $\frac{p^2-8p+12}{p-2} p-6; 2$

9. $\frac{m^2-4m-12}{m-6} m+2; 6$

10. $\frac{m+3}{m^2-9} \frac{1}{m-3}; -3, 3$

11. $\frac{2b-14}{b^2-9b+14} \frac{2}{b-2}; 2, 7$

12. $\frac{x^3-7x+10}{x^2-2x-15} \frac{x-2}{x+3}; -3, 5$

13. $\frac{y^2+6y-16}{y^2-4y+4} \frac{y+8}{y-2}; 2$

14. $\frac{r^2-7r+6}{r^2+6r-7} \frac{r-6}{r+7}; -7, 1$

15. $\frac{t^2-81}{t^2-12t+27} \frac{t+9}{t-3}; 3, 9$

16. $\frac{r^2+r-6}{r^2+4r-12} \frac{r+3}{r+6}; -6, 2$

17. $\frac{2x^2+18x+36}{3x^2-3x-36} \frac{2(x+6)}{3(x-4)}; -3, 4$

18. $\frac{2y^2+9y+4}{4y^2-4y-3} \frac{y+4}{2y-3}; -\frac{1}{2}, \frac{3}{2}$

19. ENTERTAINMENT Fairfield High spent d dollars for refreshments, decorations, and advertising for a dance. In addition, they hired a band for \$550.

- a. Write an expression that represents the cost of the band as a $\frac{550}{d+550}$ fraction of the total amount spent for the school dance.

- b. If d is \$1650, what percent of the budget did the band account for? 25%

20. PHYSICAL SCIENCE Mr. Kaminski plans to dislodge a tree stump in his yard by using a 6-foot bar as a lever.

He places the bar so that 0.5 foot extends from the fulcrum to the end of the bar under the tree stump. In the diagram, b represents the total length of the bar and r represents the portion of the bar beyond the fulcrum.

- a. Write an equation that can be used to calculate the mechanical advantage. $MA = \frac{b-r}{r}$

- b. What is the mechanical advantage? 11

- c. If a force of 200 pounds is applied to the end of the lever, what is the force placed on the tree stump? **2200 lb**

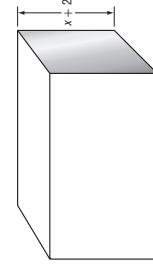
NAME _____ DATE _____ PERIOD _____

11-3 Word Problem Practice**Simplifying Rational Expressions**

1. **PHYSICAL SCIENCE** Pressure is equal to the magnitude of a force divided by the area over which the force acts.

$$P = \frac{F}{A}$$

- Gabe and Shelby each push open a door with one hand. In order to open, the door requires 20 pounds of force. The surface area of Gabe's hand is 10 square inches, and the surface area of Shelby's hand is 8 square inches. Whose hand feels the greater pressure?
- Shelby's:** $\frac{2.5 \text{ lb}}{\text{in}^2}$ (**vs Gabe's** $\frac{2 \text{ lb}}{\text{in}^2}$)



2. **GRAPHING** Recall that the slope of a line is a ratio of the vertical change to the horizontal change in coordinates for two given points. Write a rational expression that represents the slope of the line containing the points at (p, r) and $(7, -3)$.

$$\frac{r+3}{p-7} \text{ or } \frac{-3-r}{7-p}$$

3. **AUTOMOBILES** The force needed to keep a car from skidding out of a turn on a particular road is given by the formula below. What force is required to keep a 2000-pound car traveling at 50 miles per hour on a curve with radius of 750 feet on the road? What value of r is excluded?

$$f = \frac{0.0672ws^2}{r}$$

f = force in pounds
 w = weight in pounds
 s = speed in mph
 r = radius in feet

- a.** How many students attended private school? **6,400,000**
- b.** How many students attended public schools by 42,240,000.

- a.** Write a rational expression to express the ratio of public school students to private school students.

$$\frac{x+42,240,000}{x}$$

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	NAME _____	DATE _____	PERIOD _____

11-3 Enrichment

Shannon's Juggling Theorem

Mathematicians look at various mathematical ways to represent juggling. One way they have found to represent juggling is Shannon's Juggling Theorem. Shannon's Juggling Theorem uses the rational equation

$$\frac{f+d}{v+d} = \frac{b}{h}$$

where f is the flight time, or how long a ball is in the air, d is the dwell time, or how long a ball is in a hand, v is the vacant time, or how long a hand is empty, b is the number of balls, and h is the number of hands (either 1 or 2 for a real-life situation, possibly more for a computer simulation).

So, given the values for f , d , v , and h , it is possible to determine the number of balls being juggled. If the flight time is 9 seconds, the dwell time is 3 seconds, the vacant time is 1 second, and the number of hands is 2, how many balls are being juggled?

$$\begin{aligned} \frac{f+d}{v+d} &= \frac{b}{h} && \text{Original equation} \\ \frac{9+3}{1+3} &= \frac{b}{2} && \text{Replace with the values given.} \\ \frac{12}{4} &= \frac{b}{2} && \text{Simplify.} \\ 24 &= 4b && \text{Cross multiply.} \\ 6 &= b && \text{Divide.} \end{aligned}$$

So, the number of balls being juggled is 6.

Given the following information, determine the number of balls being juggled.

1. flight time = 6 seconds, vacant time = 1 second,
dwell time = 1 second, number of hands = 2

7

2. flight time = 13 seconds, vacant time = 1 second,
dwell time = 5 seconds, number of hands = 1

3

3. flight time = 4 seconds, vacant time = 1 second,
dwell time = 1 second, number of hands = 2

5

4. flight time = 16 seconds, vacant time = 1 second,
dwell time = 2 seconds, number of hands = 2

12

5. flight time = 18 seconds, vacant time = 3 seconds,
dwell time = 2 seconds, number of hands = 1

4

11-4 Study Guide and Intervention

Multiplying and Dividing Rational Expressions

Multiply Rational Expressions To multiply rational expressions, you multiply the numerators and multiply the denominators. Then simplify.

Example 1 Find $\frac{2ac^2f}{5ab^2} \cdot \frac{a^2b}{3cf}$.

$$\begin{aligned} \frac{2ac^2f}{5ab^2} \cdot \frac{a^2b}{3cf} &= \frac{2a^3bc^2f}{15abd^2cf} && \text{Multiply.} \\ &= \frac{4abc^2f(2ac)}{15abcdf(15b)} && \text{Simplify.} \\ &= \frac{2ac}{15b} && \text{Simplify.} \end{aligned}$$

Example 2 Find $\frac{x^3 - 16}{2x + 8} \cdot \frac{x + 4}{x^2 + 8x + 16}$.

$$\begin{aligned} \frac{x^3 - 16}{2x + 8} \cdot \frac{x + 4}{x^2 + 8x + 16} &= \frac{(x - 4)(x + 4)}{2(x + 4)} \cdot \frac{x + 4}{(x + 4)(x + 4)} && \text{Factor.} \\ &= \frac{(x - 4)(x + 4)}{2(x + 4)} \cdot \frac{x + 4}{(x + 4)(x + 4)} && \text{Simplify.} \\ &= \frac{x - 4}{2x + 8} && \text{Multiply.} \end{aligned}$$

Exercises

Find each product.

1. $\frac{6abc}{a^2b^2} \cdot \frac{a^2}{b^3}$

$$2. \frac{mp^2}{3} \cdot \frac{4}{mp} \cdot \frac{4p}{3}$$

3. $\frac{x + 2}{x - 4} \cdot \frac{x - 4}{x - \frac{1}{2}} \cdot \frac{x + 2}{x - 1}$

$$4. \frac{m - 5}{8} \cdot \frac{16}{m - 5} \cdot 2$$

5. $\frac{2n - 8}{n + 2} \cdot \frac{2n + 4}{n - 4} \cdot 4$

$$6. \frac{x^2 - 64}{2x + 16} \cdot \frac{x + 8}{x^2 + 16x + 64} \cdot \frac{x - 8}{2x + 16}$$

7. $\frac{-8x + 8}{-8x - 2x + 1} \cdot \frac{x - 1}{2x + 2} \cdot \frac{4}{x - 1}$

$$8. \frac{a^2 - 25}{a + 2} \cdot \frac{a^2 - 4}{a - 5} \cdot (a + 5)(a - 2)$$

9. $\frac{x^3 + 6x + 8}{2x^2 + 9x + 4} \cdot \frac{2x^3 - x - 1}{x^2 - 3x + 2} \cdot \frac{x + 2}{x - 2}$

$$10. \frac{m^3 - 1}{2m^2 - m - 1} \cdot \frac{2m + 1}{m^2 - 2m + 1} \cdot \frac{m + 1}{(m - 1)^2}$$

11. $\frac{n^2 - 7n + 10}{n^2 - 7n + 1} \cdot \frac{n^2 - 25}{n^2 + 6n + 5} \cdot \frac{n - 1}{n - 2}$

$$12. \frac{3p - 3r}{10pr} \cdot \frac{20p^2r^2}{p^2 - r^2} \cdot \frac{6pr}{p + r}$$

13. $\frac{c^2 + 7a + 12}{a^2 + 2a - 8} \cdot \frac{a^2 + 3a - 10}{(a + 3)(a + 5)}$

$$14. \frac{v^2 - 4v - 21}{3v^2 + 6v} \cdot \frac{v^2 + 8v}{v^2 + 11v + 24} \cdot \frac{v - 7}{3v + 6}$$

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

11-4 Study Guide and Intervention (continued)**Multiplying and Dividing Rational Expressions**

Divide Rational Expressions To divide rational expressions, multiply by the reciprocal of the divisor. Then simplify.

Example 1 Find $\frac{12x^2f}{5a^3b^2} \div \frac{c^3f}{10ab}$.

$$\begin{aligned}\frac{12x^2f}{5a^3b^2} \div \frac{c^3f}{10ab} &= \frac{12x^2f}{5a^3b^2} \times \frac{10ab}{c^3f} \\ &= \frac{12x^2f}{5a^3b^2} \times \frac{10^4ab^2}{c^3f^2} \\ &= \frac{24}{abf}\end{aligned}$$

Example 2 Find $\frac{x^2 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x^2 + x - 2}$.

$$\begin{aligned}\frac{x^2 + 6x - 27}{x^2 + 11x + 18} \div \frac{x - 3}{x^2 + x - 2} &= \frac{x^2 + 6x - 27}{x^2 + 11x + 18} \times \frac{x^2 + x - 2}{x - 3} \\ &= \frac{(x + 9)(x + 3)}{(x + 9)(x + 2)} \times \frac{(x + 2)(x - 1)}{x - 3} \\ &= \frac{(x + 3)(x - 1)}{(x + 2)(x - 1)} \times \frac{(x + 2)(x - 1)}{x - 3} \\ &= x - 1\end{aligned}$$

Exercises
Find each quotient.

$$1. \frac{12ab}{a^3b^2} \div \frac{12}{b^2} \cdot \frac{P}{4}$$

$$2. \frac{n - 5}{8} \div \frac{m - 5}{16}$$

$$3. \frac{3xy^2}{8} \div 6xy \cdot \frac{y}{16}$$

$$4. \frac{2n - 4}{2m} \div \frac{n^2 - 4}{n} \cdot \frac{1}{n + 2}$$

$$5. \frac{x^2 - 5x + 6}{6} \div \frac{x - 3}{15} \cdot 3(x - 2)$$

$$6. \frac{y^2 - 36}{y^2 - 49} \div \frac{y + 6}{y + 7} \cdot \frac{y - 6}{y - 7}$$

$$7. \frac{x^2 - 5x + 6}{5} \div \frac{x - 3}{15} \cdot 3(x - 2)$$

$$8. \frac{a^2b^3c}{3r^4t} \div \frac{6a^3bc}{8rt^4u} \cdot \frac{9r}{4b^2tu}$$

$$9. \frac{x^2 + 6x + 8}{x^2 + 4x + 4} \div \frac{x + 4}{x + 2} \cdot \frac{1}{1}$$

$$10. \frac{m^2 - 49}{m} \div \frac{m^2 - 13m + 42}{3m^2} \cdot \frac{3m(m + 7)}{m - 6}$$

Answers

NAME _____ DATE _____ PERIOD _____

11-4 Skills Practice
Multiplying and Dividing Rational Expressions

Find each product.

$$1. \frac{14}{c^2} \cdot \frac{c^5}{2c} \cdot 7c^2$$

$$3. \frac{2ax^2b}{b^2c} \cdot \frac{b}{a} \cdot \frac{2a}{c}$$

$$4. \frac{2x^2y}{3x^2y} \cdot \frac{3xy}{4y} \cdot \frac{x}{2}$$

$$5. \frac{3(4m - 6)}{18r} \cdot \frac{9r^2}{24m - 6} \cdot \frac{3r}{4}$$

$$6. \frac{4(n + 2)}{n(n - 2)} \cdot \frac{n - 2}{n + 2} \cdot \frac{4}{n}$$

$$7. \frac{(y - 3)(y + 3)}{4} \cdot \frac{8}{y + 3} \cdot \frac{2y - 6}{2}$$

$$9. \frac{(a - 7)(a + 7)}{a(a + 5)} \cdot \frac{a + 5}{a + 7} \cdot \frac{a - 7}{a}$$

$$10. \frac{4(b + 4)}{(b - 4)(b - 3)} \cdot \frac{b - 3}{b + 4} \cdot \frac{4}{b - 4}$$

Find each quotient.

$$11. \frac{c^3}{a^3} \div \frac{d^3}{c^3} \cdot \frac{c^6}{d^6}$$

$$12. \frac{x^3}{y^3} \div \frac{x^3}{y} \cdot \frac{1}{y}$$

$$13. \frac{6a^3}{4f^2} \div \frac{2a^2}{12f^2} \cdot \frac{9a}{8}$$

$$15. \frac{3b + 3}{b + 2} \div (b + 1) \cdot \frac{3}{b + 2}$$

$$17. \frac{x^2 - x - 12}{6} \div \frac{x + 3}{x - 4} \cdot \frac{(x - 4)^2}{6}$$

$$19. \frac{m^2 + 2m + 1}{10m - 10} \div \frac{m + 1}{20} \cdot \frac{2(m + 1)}{m - 1}$$

$$20. \frac{y^2 + 10y + 25}{3y - 9} \div \frac{y + 5}{y - 3} \cdot \frac{y + 5}{3}$$

$$21. \frac{b + 4}{b^2 - 8b + 16} \div \frac{2b + 8}{b - 8} \cdot \frac{b - 8}{2(b - 4)^2}$$

$$22. \frac{6x + 6}{x - 1} \div \frac{x^2 + 3x + 2}{2x - 2} \cdot \frac{12}{x + 2}$$

Answers (Lesson 11-4)

Lesson 11-4

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NAME _____ DATE _____ PERIOD _____

11-4 Practice**Multiplying and Dividing Rational Expressions**

Find each product.

$$1. \frac{18x^2}{10y^2} \cdot \frac{1.5y^3}{24x} \cdot \frac{9xy}{8}$$

$$3. \frac{(x+2)(x+2)}{8} \cdot \frac{72}{(x+2)(x-2)} \cdot \frac{9(x+2)}{x-2}$$

$$5. \frac{a-4}{a^2-a-12} \cdot \frac{a+3}{a-6} \cdot \frac{1}{a-6}$$

$$7. \frac{n^2+10n+16}{5n-10} \cdot \frac{n-2}{n^2+9n+8} \cdot \frac{n+2}{5(n+1)}$$

$$9. \frac{b^2+5b+4}{b^2-36} \cdot \frac{b^2+5b-6}{b^2+2b-8} \cdot \frac{(b+1)(b-1)}{(b-6)(b-2)}$$

$$11. \frac{28x^2}{7b^2} \div \frac{21x^3}{35b} \cdot \frac{20}{3ab}$$

$$13. \frac{2a}{a-1} \div (a+1) \cdot \frac{2a}{(a+1)(a-1)}$$

$$15. \frac{4y+20}{y-3} \div \frac{y+5}{2y-6} \cdot 8$$

$$17. \frac{b^2+2b-8}{b^2-11b+18} \div \frac{2b-8}{2b-18} \cdot \frac{b+4}{b-4}$$

$$19. \frac{a^2+8a+12}{a^2-7a+10} \div \frac{a^2-4a-12}{a^2+3a-10} \cdot \frac{(a+6)(a+5)}{(a-6)(a-5)}$$

$$21. \frac{y^2+6y-7}{y+2}$$

$$23. \frac{y^2+9y+14}{y^2+4y-9} \div \frac{6x-6}{x^2-5x+6} \cdot \frac{x-2}{2(x-3)}$$

$$25. \frac{18x^2-3x-3}{x^2-6x+9} \div \frac{6x}{x^2-5x+6} \cdot \frac{x-2}{2(x-3)}$$

$$27. \frac{20x^2+6y-7}{y+2}$$

21. **BIOLOGY** The heart of an average person pumps about 9000 liters of blood per day. How many quarts of blood does the heart pump per hour? (*Hint:* One quart is equal to 0.946 liter.) Round to the nearest whole number. **396 qt/h**

22. **TRAFFIC** On Saturday, it took Ms. Torres 24 minutes to drive 20 miles from her home to her office. During Friday's rush hour, it took 75 minutes to drive the same distance.

- What was Ms. Torres's average speed in miles per hour on Saturday? **50 mph**
- What was her average speed in miles per hour on Friday? **16 mph**

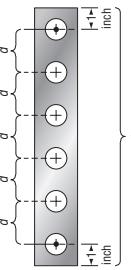
NAME _____

DATE _____ PERIOD _____

11-4 Word Problem Practice**Multiplying and Dividing Rational Expressions**

1. JOBS Rosa earned \$26.25 for babysitting for $3\frac{1}{2}$ hours. At this rate, how much will she earn babysitting for 5 hours? **\$37.50**

5. MANUFACTURING India works in a metal shop and needs to drill equally spaced holes along a strip of metal. The centers of the holes on the ends of the strip must be exactly 1 inch from each end. The remaining holes will be equally spaced.



a. If there are x equally spaced holes, write an expression for the number of equal spaces are there between holes. **$x - 1$**

b. Write an expression for the distance between the end screws if the length is ℓ . **$\ell - 2$**

c. Write a rational equation that represents the distance between the holes on a piece of metal that is ℓ inches long and must have x equally spaced holes. **$d = \frac{\ell - 2}{x - 1}$**

d. How many holes will be drilled in a metal strip that is 6 feet long with a distance of 7 inches between the centers of each screw? **11**

Lesson 11-4

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11-4 Enrichment

Geometric Series

A geometric series is a sum of the terms in a *geometric sequence*. Each term of a geometric sequence is formed by multiplying the previous term by a constant term called the *common ratio*.

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \leftarrow \text{geometric sequence where the common ratio is } \frac{1}{2}$$

The sum of a geometric series can be represented by the rational expression

$$x_0 \frac{(r)^n - 1}{r - 1}, \text{ where } x_0 \text{ is the first term of the series, } r \text{ is the common ratio, and } n \text{ is the number of terms.}$$

$$\text{In the example above, } 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1 + \frac{\left(\frac{1}{2}\right)^4 - 1}{\frac{1}{2} - 1} \text{ or } \frac{15}{8}.$$

You can check this by entering $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ a calculator. The result is the same.

Rewrite each sum as a rational expression and simplify.

$$1. 9 + 3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}$$

$$2. 500 + 250 + 125 + 62 \frac{1}{2} + \frac{1875}{2}$$

$$3. 6 + 1 + \frac{1}{6} + \frac{1}{36} + \frac{259}{36}$$

$$4. 100 + 20 + 4 + \frac{4}{5} + \frac{624}{5}$$

$$5. 1000 + 100 + 10 + 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1,111,111}{1000}$$

$$6. 55 + 5 + \frac{5}{11} + \frac{5}{121} + \frac{7320}{121}$$

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

11-4 Spreadsheet Activity

Revolutions per Minute

One of the characteristics that makes a spreadsheet powerful is the ability to recalculate values in formulas automatically. You can use this ability to investigate real-world situations.

Example 1 Use a spreadsheet to investigate the effect of doubling the diameter of a tire on the number of revolutions the tire makes at a given speed.

Use dimensional analysis to find the formula for the revolutions per minute of a tire with diameter of x inches traveling at y miles per hour.

$$\frac{1 \text{ revolution}}{\pi \cdot x} \times \frac{y}{1} \times \frac{1}{60 \text{ minutes}} \times \frac{63,360}{1} = \frac{1056y}{\pi \cdot x} \text{ revolutions}$$

Step 1 Use Column A of the spreadsheet for diameter of the tire in inches. Use Column B for the speed in miles per hour.

Step 2 Column C contains the formula for the number of rotations per minute. Notice that in Excel, π is entered as $\text{PI}()$.

◊	A	B	Speed (mph)	RPM	C
1	Diameter (in)			=((1056*32)/(A2*B1))	
2				=((1056*33)/(A3*B1))	
3				=((1056*34)/(A4*B1))	
4				=((1056*35)/(A5*B1))	
5				=((1056*36)/(A6*B1))	

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Use the spreadsheet of revolutions per minute.

- How is the number of revolutions affected if the speed of a wheel of a given diameter is doubled? **RPM is cut in half.**
- Name two ways that you can double the RPM of a bicycle wheel. **Double the speed or halve the diameter of the wheel, keeping the same speed.**

Exercises

Answers (Lesson 11-4)

Answers

NAME _____	DATE _____	PERIOD _____
NAME _____	DATE _____	PERIOD _____

11-5 Study Guide and Intervention

Dividing Polynomials

Divide Polynomials by Monomials To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

$$\begin{aligned} \text{Example 1} \quad & \text{Find } (4r^2 - 12r) \div (2r). \\ & (4r^2 - 12r) \div 2r = \frac{4r^2 - 12r}{2r} \\ & \quad = \frac{4r^2}{2r} - \frac{12r}{2r} \quad \text{Divide each term.} \\ & \quad = \frac{2r(2r)}{2r} - \frac{2r(6)}{2r} \\ & \quad = 2r - 6 \quad \text{Simplify.} \end{aligned}$$

Exercises

Find each quotient.

1. $(x^3 + 2x^2 - x) \div x$ $x^2 + 2x - 1$
2. $(2x^3 + 12x^2 - 8x) \div (2x)$ $x^2 + 6x - 4$
3. $(x^2 + 3x - 4) \div x$ $x^2 + 3 - \frac{4}{x}$
4. $(4m^2 + 6m - 8) \div (2m^2)$ $2 + \frac{3}{m} - \frac{4}{m^2}$
5. $(3x^3 + 15x^2 - 21x) \div (3x)$ $x^2 + 5x - 7$
6. $(8m^2p^2 + 4mp - 8p) \div p$ $8m^2p + 4m - 8$
7. $(8y^4 + 16y^2 - 4) \div (4y^2)$ $2y^2 + 4 - \frac{1}{y^2}$
8. $(16x^4y^2 + 24xy + 5) \div (xy)$ $16x^3y + 24 + \frac{5}{xy}$
9. $\frac{15x^4 - 25x + 30}{5}$ $3x^2 - 5x + 6$
10. $\frac{10x^2b + 12ab - 8b}{2a}$ $5ab + 6b - \frac{4b}{a}$
11. $\frac{6x^3 + 9x^2 + 9}{3x}$ $2x^2 + 3x + \frac{3}{x}$
12. $\frac{m^2 - 12m + 42}{3m^2}$ $\frac{1}{3} - \frac{4}{m} + \frac{14}{m^2}$
13. $\frac{m^2p^2 - 5mp + 6}{m^2p^2}$ $1 - \frac{5}{mp} + \frac{6}{m^2p^2}$
14. $\frac{p^2 - 4pr + 6r^2}{pr}$ $\frac{p}{r} - 4 + \frac{6r}{p}$
15. $\frac{6a^2b^2 - 8ab + 12}{2a^2}$ $3b^2 - \frac{4b}{a} + \frac{6}{a^2}$
16. $\frac{2x^2y^3 - 4x^3y^2 - 8xy}{2a^2b^2}$ $xy^2 - 2xy - 4$
17. $\frac{9x^2y^2z - 2xyz + 12x}{xy}$ $\frac{2x^3b^3 + 8a^2b^2 - 10ab + 12}{2a^2b^2}$
18. $\frac{9xyz - 2z + 12}{y}$ $ab + 4 - \frac{5}{ab} + \frac{6}{a^2b^2}$

Answers (Lesson 11-5)

11-5 Study Guide and Intervention (continued)

Dividing Polynomials

Divide Polynomials by Binomials To divide a polynomial by a binomial, factor the dividend if possible and divide both dividend and divisor by the GCF. If the polynomial cannot be factored, use long division.

Example 2 Find $(3x^2 - 8x + 4) \div (4x)$.

$$\begin{aligned} (3x^2 - 8x + 4) \div 4x &= \frac{3x^2 - 8x + 4}{4x} \\ &= \frac{3x^2}{4x} - \frac{8x}{4x} + \frac{4}{4x} \\ &\approx \frac{3x^2}{4x} - \frac{8x}{4x} + \frac{4}{4x} \\ &= \frac{3x^2}{4x} - \frac{8x}{4x} + \frac{4}{4x} \\ &= \frac{3x}{4} - 2 + \frac{1}{x} \end{aligned}$$

Example Find $(x^2 + 7x + 10) \div (x + 3)$.

$$\begin{aligned} x^2 + 7x + 10 &\quad \text{Divide the first term of the dividend, } x^2 \text{ by the first term of the divisor, } x. \\ (-)x^2 + 3x &\quad \text{Multiply } x \text{ and } x + 3. \\ \hline 4x &\quad \text{Subtract.} \end{aligned}$$

Step 2 Bring down the next term, 10. Divide the first term of $4x + 10$ by x .

$$\begin{array}{r} x+4 \\ x+3 \overline{)x^2+7x+10} \\ \underline{x^2+3x} \\ 4x+10 \\ (-)4x+12 \\ \hline -2 \end{array}$$

Multiply 4 and $x + 3$.
Subtract.

The quotient is $x + 4$ with remainder -2 . The quotient can be written as $x + 4 + \frac{-2}{x+3}$.

Exercises

Find each quotient.

1. $(b^2 - 5b + 6) \div (b - 2)$ **b - 3**

2. $(x^2 - x - 6) \div (x - 3)$ **x + 2**

3. $(x^2 + 3x - 4) \div (x - 1)$ **x + 4**

4. $(m^2 + 2m - 8) \div (m + 4)$ **m - 2**

5. $(x^2 + 5x + 6) \div (x + 2)$ **x + 3**

6. $(m^2 + 4m + 4) \div (m + 2)$ **m + 2**

7. $(2y^2 + 5y + 2) \div (y + 2)$ **2y + 1**

8. $(8y^2 - 15y - 2) \div (y - 2)$ **8y + 1**

9. $\frac{8x^2 - 6x - 9}{4x + 3}$ **2x - 3**

10. $\frac{m^2 - 5m - 6}{m - 6}$ **m + 1**

11. $\frac{x^3 + 1}{x - 2}$ **x^2 + 2x + 4 + \frac{9}{x-2}**

12. $\frac{6m^2 + 11m^2 + 4m + 35}{2m + 5}$ **3m^2 - 2m + 7**

13. $\frac{6a^2 + 7a + 5}{2a + 5}$ **3a - 4 + \frac{25}{2a + 5}**

14. $\frac{8p^3 + 27}{2p + 3}$ **4p^2 - 6p + 9**

11-5 Skills Practice

Dividing Polynomials

Find each quotient.

1. $(20x^2 + 12x) \div 4x$ **5x + 3**

2. $(18n^2 + 6n) \div 3n$ **6n + 2**

3. $(b^2 - 12b + 5) \div 2b$ **$\frac{b}{2} - 6 + \frac{5}{2b}$**

4. $(8t^2 + 5t - 20) \div 4t$ **$2t + \frac{5}{4} - \frac{5}{t}$**

5. $\frac{12p^3r^2 + 18p^2r - 6pr}{6p^2r} \quad 2pr + 3 - \frac{1}{p}$

6. $\frac{15k^2u - 10ku + 25u^2}{5ku} \quad 3k - 2 + \frac{5u}{k}$

7. $(x^2 - 5x - 6) \div (x - 6)$ **x + 1**

8. $(a^2 - 10a + 16) \div (a - 2)$ **a - 8**

9. $(n^2 - n - 20) \div (n + 4)$ **n - 5**

10. $(y^2 + 4y - 21) \div (y - 3)$ **y + 7**

11. $(h^2 - 6h + 9) \div (h - 2)$ **h - 4 + $\frac{1}{h-2}$**

12. $(b^2 + 5b - 2) \div (b + 6)$ **b - 1 + $\frac{4}{b+6}$**

13. $(y^2 + 6y + 1) \div (y + 2)$ **y + 4 - $\frac{7}{y+2}$**

14. $(m^2 - 2m - 5) \div (m - 3)$ **m + 1 - $\frac{2}{m-3}$**

15. $\frac{2c^2 - 5c - 3}{2c + 1} \quad \mathbf{c - 3}$

16. $\frac{2r^2 + 6r - 20}{2r - 4} \quad \mathbf{r + 5}$

17. $\frac{x^3 - 3x^2 - 6x - 20}{x - 5} \quad \mathbf{x^2 + 2x + 4}$

18. $\frac{p^3 - 4p^2 + p + 6}{p - 2} \quad \mathbf{p^2 - 2p - 3}$

19. $\frac{n^3 - 6n^2 - 20}{n + 1} \quad \mathbf{n^2 - n - 5 + \frac{3}{n+1}}$

20. $\frac{y^3 - y^2 - 40}{y - 4} \quad \mathbf{y^2 + 3y + 12 + \frac{8}{y-4}}$

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

11-5 Practice

Dividing Polynomials

Find each quotient.

1. $(6q^2 - 18q - 9) \div 9q$

2. $(y^2 + 6y + 2) \div 3y$

3. $\frac{2q}{3} - 2 - \frac{1}{q}$

4. $\frac{2mn^3p^2 + 56mp - 4m^2p^3}{8mp} \quad \mathbf{p + \frac{7}{m^2} - \frac{p^2}{2m}}$

5. $\frac{(x^2 - 3x - 40) \div (x + 5)}{5(x^2 - 3x - 2) \div (x + 7)}$

6. $(3m^2 - 20m + 12) \div (m - 6)$

7. $\frac{a + 8 + \frac{44}{a-3}}{a + 5a + 20) \div (a - 3)}$

8. $(x^2 - 3x - 2) \div (x + 3)$

9. $\frac{x - 10 + \frac{68}{x+7}}{(n^2 - 9n + 25) \div (n - 4)}$

10. $\frac{11. \frac{6t^2 - 5t - 56}{3t + 8}}{(n^2 - 9n + 25) \div (n - 4)}$

11. $\frac{t + 6 + \frac{10}{t+3}}{n - 5 + \frac{5}{n-4}}$

12. $\frac{4w + 3}{2r - 7}$

13. $\frac{14. \frac{t^3 - 11t - 6}{t+3}}{(x^3 + 2x^2 - 16) \div (x - 2)}$

14. $(t^3 - 11t - 6) \div (t + 3)$

15. $x^2 + 4x + 8$

16. $\frac{16. \frac{6a^3 + d^2 - 2d + 17}{2d + 3}}{x^3 + 6x^2 + 3x + 1} \quad \mathbf{x - 2}$

17. $x^2 + 8x + 19 + \frac{39}{x-2}$

18. $\frac{3g^2 - 4d + 5 + \frac{2}{2d+3}}{\frac{2k^3 + 7k^2 - 7}{2k - 3}} \quad \mathbf{3y^2 - 2y + 1 - \frac{3}{3y+2}}$

19. $k^2 + 2k - 3 + \frac{2}{2k+3}$

20. $\frac{12 \text{ ft by } 3.5 \text{ ft}}{12 \text{ ft by } 6 \text{ ft}} \quad \mathbf{6 \text{ ft}}$

21. $\frac{1. \text{ If } x = 3 \text{ feet, what will be the dimensions of the eactus bed in each of the designs?}}{\text{The total area can be modeled by the expression } 2x^2 + 7x + 3, \text{ where } x \text{ is in feet.}}$

22. $\frac{\text{a. Suppose in one design the length of the cactus bed is } 4x, \text{ and in another, the length is } 2x + 1. \text{ What are the widths of the two designs?}}{\text{b. If } x = 3 \text{ feet, what will be the dimensions of the eactus bed in each of the designs?}}$

23. $\frac{\text{20. FURNITURE Teri is upholstering the seats of four chairs and a bench. She needs } \frac{1}{4} \text{ square yard of fabric for each chair, and } \frac{1}{2} \text{ square yard for the bench. If the fabric at the store is } 45 \text{ inches wide, how many yards of fabric will Teri need to cover the chairs and the bench if there is no waste? } \frac{1}{5} \text{ yd}}{\text{Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.}}$

11-5 Word Problem Practice

Dividing Polynomials

- 1. TECHNOLOGY** The surface area (in square millimeters) of a rectangular computer microchip is represented by the expression $x^3 - 12x + 35$, where x is the number of circuits. If the width of the chip is $x - 5$ millimeters, write a polynomial that represents the length, $x - 7$ mm

$G = \text{green time in seconds}$
 $n = \text{average number of vehicles traveling in each lane per light cycle}$
 $\text{Write a simplified expression to represent the average green light time per vehicle. } 2.1 + \frac{3.7}{n}$

- 2. HOMEWORK** Your classmate Ava writes her answer to a homework problem on the chalkboard. She has simplified $\frac{6x^2 - 12x}{6} \text{ as } x^2 - 12x$. Is this correct? If not, what is the correct simplification? **This is not correct.**
She forgot to factor 6 from the $-12x$ term. The correct answer should be } $\frac{6(x^2 - 2x)}{6} = x^2 - 2x$.



- 3. CIVIL ENGINEERING** Suppose 5400 tons of concrete costs $(500 + d)$ dollars. Write a formula that gives the cost C of t tons of concrete. $\frac{500t + dt}{5400}$

- 4. SHIPPING** The Overseas Shipping Company loads cargo into a container to be shipped around the world. The volume of their shipping containers is determined by the following equation.
 $x^3 + 21x^2 + 99x + 135$

- The container's height is $x + 3$. Write an expression that represents the area of the base of the shipping container.
 $x^2 + 18x + 45$

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

11-5 Enrichment

Synthetic Division

You can divide a polynomial such as $3x^3 - 4x^2 - 3x - 2$ by a binomial such as $x - 3$ by a process called **synthetic division**. Compare the process with long division in the following explanation.

Example Divide $(3x^3 - 4x^2 - 3x - 2)$ by $(x - 3)$ using synthetic division.

- Show the coefficients of the terms in descending order.
- The divisor is $x - 3$. Since 3 is to be subtracted, write 3 in the corner.
- Bring down the first coefficient, 3.
- Multiply.
- Add.
- Multiply.
- Add.
- Multiply.
- Add.

Check Use long division.

$$\begin{array}{r} 3x^2 + 5x + 12 \\ 3x^3 - 4x^2 - 3x - 2 \\ \hline 3x^3 - 9x^2 \\ \hline -4x^2 - 3x - 2 \\ \hline -4x^2 - 12x \\ \hline 9x - 2 \\ \hline 9x - 27 \\ \hline 25 \\ \hline \end{array}$$

The result is $3x^2 + 5x + 12 + \frac{34}{x - 3}$.

Divide by using synthetic division. Check your result using long division.

- $(x^3 + 6x^2 + 3x + 1) \div (x - 2)$
 $x^2 + 8x + 19 + \frac{39}{x - 2}$
- $(2x^3 - 5x + 1) \div (x + 1)$
 $2x^2 - 2x - 3 + \frac{4}{x + 1}$
- $(x^3 + 2x^2 - x + 4) \div (x + 3)$
 $x^2 - x + 2 - \frac{2}{x + 3}$

Glencoe Algebra 1

Chapter 11

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35

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

Answers (Lesson 11-5)

Lesson 11-5

Glencoe Algebra 1

NAME _____ DATE _____ PERIOD _____

11-6 Study Guide and Intervention**Adding and Subtracting Rational Expressions**

Add and Subtract Rational Expressions with Like Denominators To add rational expressions with like denominators, add the numerators and then write the sum over the common denominator. To subtract fractions with like denominators, subtract the numerators. If possible, simplify the resulting rational expression.

Example 1 Find $\frac{5n}{15} + \frac{7n}{15}$.

$$\begin{aligned} \frac{5n}{15} + \frac{7n}{15} &= \frac{5n + 7n}{15} \quad \text{Add the numerators.} \\ &= \frac{12n}{15} \quad \text{Simplify.} \\ &= \frac{4n}{5} \end{aligned}$$

Example 2 Find $\frac{3x+2}{x-2} - \frac{4x}{x-2}$.

$$\begin{aligned} \frac{3x+2}{x-2} - \frac{4x}{x-2} &= \frac{3x+2-4x}{x-2} = \frac{-x-2}{x-2} \quad \text{The common denominator is } x-2. \\ &= \frac{2-x}{x-2} \quad \text{Subtract.} \\ &= \frac{-1(x-2)}{-1(x-2)} = \frac{1}{1} \quad 2-x=-1(x-2) \\ &= -\frac{1}{1} \quad \text{Simplify.} \\ &= -1 \end{aligned}$$

Adding and Subtracting Rational Expressions with Unlike Denominators

Adding and Subtracting Rational Expressions with Unlike Denominators Adding or subtracting rational expressions with unlike denominators is similar to adding and subtracting fractions with unlike denominators.

Lesson 11-6

11-6 Study Guide and Intervention (continued)**Adding and Subtracting Rational Expressions**

To add or subtract rational expressions with unlike denominators, change each expression into an equivalent expression with the LCD as the denominator. Add or subtract as with expressions with like denominators. Simplify, if necessary.

Example 1 Find $\frac{n+3}{n} + \frac{8n-4}{4n}$.

$$\begin{aligned} &\text{Factor each denominator.} \\ n &= n \\ 4n &= 4 \cdot n \\ \text{LCD} &= 4n \end{aligned}$$

Since the denominator of $\frac{8n-4}{4n}$ is already $4n$, only $\frac{n+3}{n}$ needs to be renamed.

$$\begin{aligned} \frac{n+3}{n} + \frac{8n-4}{4n} &= \frac{4(n+3)}{4n} + \frac{8n-4}{4n} \\ &= \frac{4n+12}{4n} + \frac{8n-4}{4n} \\ &= \frac{12n+8}{4n} \\ &= \frac{3n+2}{n} \end{aligned}$$

Exercises

Find each sum or difference.

1. $\frac{3}{a} + \frac{4}{a} \frac{7}{a}$

3. $\frac{5x}{9} - \frac{x}{9} \frac{4x}{9}$

5. $\frac{2a-4}{a-4} + \frac{-a}{a-4} \frac{1}{1}$

7. $\frac{y+7}{y+6} - \frac{1}{y+6} \frac{1}{1}$

9. $\frac{x+1}{x-2} + \frac{x-5}{x-2} \frac{2}{2}$

11. $\frac{x^2+x}{x} - \frac{x^2+5x}{x} \frac{-4}{-4}$

13. $\frac{3x+2}{x+2} + \frac{x+6}{x+2} \frac{4}{4}$

14. $\frac{a-4}{a+1} + \frac{a+6}{a+1} \frac{2}{2}$

15. $\frac{y+2}{y+6} + \frac{2-y}{y-6} \frac{0}{0}$

16. $\frac{q}{q^2+5q+4} + \frac{q+1}{q^2+5q+4} \frac{2q-4}{(q-4)(q+4)}$

Example 2 Find $\frac{3x}{x^2-4x} - \frac{1}{x-4}$.

$$\begin{aligned} &\text{Factor the denominator.} \\ x^2-4x &= x(x-4) \quad x-4 \text{ is the denominator.} \\ &= \frac{3x}{x(x-4)} - \frac{1}{x-4} \cdot \frac{x}{x} \quad \text{The LCD is } x(x-4). \\ &= \frac{3x}{x(x-4)} - \frac{x}{x(x-4)} \quad \text{Subtract numerators.} \\ &= \frac{2x}{x(x-4)} \quad \text{Simplify.} \end{aligned}$$

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$$\begin{aligned} 2. \frac{1}{a} + \frac{7}{3a} \frac{10}{10} &2. \frac{1}{6x} + \frac{3}{8} \frac{4+10}{24x} \\ 3. \frac{5}{9x} - \frac{1}{x^2} \frac{5x-9}{9x^2} &4. \frac{6}{x^2} - \frac{3}{x^3} \frac{6x-3}{x^3} \\ 5. \frac{8}{4a^2} + \frac{6}{3a} \frac{2+2a}{a^2} &6. \frac{4}{h+1} + \frac{2}{h+2} \frac{6h+10}{(h+1)(h+2)} \\ 7. \frac{y}{y-3} - \frac{3}{y+3} \frac{y^2+9}{(y-3)(y+3)} &8. \frac{y}{y-7} - \frac{3}{y^2-4y-21} \frac{y-1}{y-7} \\ 9. \frac{a}{a+4} + \frac{4}{a-4} \frac{(a+4)(a-4)}{a^2+16} &10. \frac{6}{3(m+1)} + \frac{2}{3(m-1)} \frac{8m-4}{3(m+1)(m-1)} \\ 11. \frac{4}{x-2y} - \frac{2}{x+2y} \frac{2x+12y}{(x+2y)(x-2y)} &12. \frac{a-6b}{2x^2-5ab+2b^2} - \frac{7}{a-2b} \frac{-13a+b}{(2a-b)(a-2b)} \\ 13. \frac{y+2}{y^2+5y+6} + \frac{2-y}{y^2+y-6} \frac{0}{0} &14. \frac{q}{q^2-16} + \frac{q+1}{q^2+5q+4} \frac{2q-4}{(q-4)(q+4)} \end{aligned}$$

Answers (Lesson 11-6)

Lesson 11-6

<p>NAME _____ DATE _____ PERIOD _____</p> <p>11-6 Skills Practice</p> <p style="text-align: center;">Adding and Subtracting Rational Expressions</p>	<p>NAME _____ DATE _____ PERIOD _____</p>
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Find each sum or difference.

$$1. \frac{2y}{5} + \frac{y}{5} = \frac{3y}{5}$$

$$2. \frac{4r}{9} + \frac{5r}{9} = r$$

$$3. \frac{t+3}{7} - \frac{t-3}{7} = \frac{6}{7}$$

$$4. \frac{c+8}{4} - \frac{c+6}{4} = \frac{1}{2}$$

$$5. \frac{x+2}{3} + \frac{x+5}{3} = \frac{2x+7}{3}$$

$$6. \frac{g+2}{4} + \frac{g-8}{4} = \frac{g-6}{2}$$

$$7. \frac{x-5}{x+2} + \frac{-2}{x+2} = \frac{x-7}{x+2}$$

$$8. \frac{3r}{r+3} - \frac{r}{r+3} = \frac{2r}{r+3}$$

Find the LCM of each pair of polynomials.

$$10. 3ax^2b^3, 18ab^3$$

$$11. w - 4, w + 2$$

$$12. 5d - 20, d - 4$$

$$13. 6p + 1, p - 1$$

$$14. x^2 + 5x + 4, (x + 1)^2$$

$$15. m^2 + 3m - 10, m^2 - 4$$

$$(m + 5)(m - 2)(m + 2)$$

$$16. t + 4, 4t + 16$$

$$17. m + 4, m - 3$$

$$18. \frac{6p}{5x^3} - \frac{2p}{3x} = \frac{18p - 10xp}{15x^2}$$

$$19. \frac{p+1}{p^2 + 3p - 4} + \frac{p}{p+4} = \frac{p^2 + 1}{(p+4)(p-1)}$$

$$20. \frac{t+3}{t^2 - 3t - 10} - \frac{4t - 8}{t^2 - 10t + 25}$$

$$21. \frac{-3t^2 - 2t + 1}{(t+2)(t-5)^2}$$

$$22. \frac{4y}{y^2 - y - 6} - \frac{3y + 3}{y^2 - y - 4}$$

$$23. \frac{(y+3)(y-4)}{(y+4)^2(y-4)}$$

$$24. \frac{p+1}{p^2 + 3p - 4} + \frac{p}{p+4} = \frac{p^2 + 1}{(p+4)(p-1)}$$

$$25. \frac{w+9}{9} + \frac{w+4}{9} = \frac{2w+13}{9}$$

$$26. \frac{6}{c-1} - \frac{-2}{c-1} = \frac{8}{c-1}$$

$$27. \frac{n+14}{5} - \frac{n-14}{5} = \frac{28}{5}$$

$$28. \frac{r+5}{r-5} + \frac{2r-1}{r-5} = \frac{3r+4}{r-5}$$

$$29. \frac{4p+14}{p+4} + \frac{2p+10}{p+4} = 6$$

$$30. \frac{x+2}{x-25} + \frac{x+5}{x+5} = \frac{x^2 - 2x + 15}{(x+5)(x-5)}$$

$$31. \frac{x^2 - 4x + 4}{x^2 - x - 2} + \frac{x+2}{x-2} = \frac{x^2 + x - 7}{(x-2)^2}$$

$$32. \frac{5a+4b}{5a+b} = \frac{3a+2b}{2a+b}$$

$$33. \frac{m^2 + 3m - 10}{m + 5}(m - 2)(m + 2)$$

$$34. \frac{5d - 20}{d - 4}$$

$$35. \frac{w+9}{9} + \frac{w+4}{9} = \frac{2w+13}{9}$$

$$36. \frac{6}{c-1} - \frac{-2}{c-1} = \frac{8}{c-1}$$

$$37. \frac{n+14}{5} - \frac{n-14}{5} = \frac{28}{5}$$

$$38. \frac{r+5}{r-5} + \frac{2r-1}{r-5} = \frac{3r+4}{r-5}$$

$$39. \frac{4p+14}{p+4} + \frac{2p+10}{p+4} = 6$$

NAME _____ DATE _____ PERIOD _____

11-6 Word Problem Practice**Rational Expressions with Unlike Denominators**

- 1. TEXAS** Of the 254 counties in Texas, 4 are larger than 6000 square miles. Another 21 counties are smaller than 300 square miles. What fraction of the counties are 300 to 6000 square miles in size? $\frac{229}{254}$

- 2. SWIMMING** Power Pools installs swimming pools. To determine the appropriate size of pool for a yard, they measure the length of the yard in meters and call that value x . The length and width of the pool are calculated with the diagram below. Write an expression in simplest form for the perimeter of a rectangular pool for the given variable dimensions. $\frac{13x}{10} \text{ m} \quad \frac{\frac{3}{5}m}{\frac{1}{4}m}$

- 4. INSURANCE** For a hospital stay, Paul's health insurance plan requires him to pay $\frac{2}{5}$ the cost of the first day in the hospital and $\frac{1}{5}$ the cost of the second and third days. If Paul's hospital stay is 3 days and cost him \$420, what was the full daily cost? $\$525$

- 5. PACKAGE DELIVERY** Fredricksburg Parcel Express delivered a total of 498 packages on Monday, Tuesday, and Wednesday. On Tuesday, they delivered 71 less than 2 times the number of packages delivered on Monday. On Wednesday, they delivered the average number delivered on Monday and Tuesday.

- a.** Write a rational equation that represents the sum of the numbers of packages delivered on Monday, Tuesday, and Wednesday. **Sample answer:** $x + 2x - 7 + \frac{3x - 7}{2}$ or 498
- b.** How many packages were delivered on Monday? **113**

- 3. EGYPTIAN FRACTIONS** Ancient Egyptians used only unit fractions, which are fractions in the form $\frac{1}{n}$. Their mathematical notation only allowed for a numerator of 1. When they needed to express a fraction with a numerator other than 1, they wrote it as a sum of unit fractions. An example is shown below.

$$\frac{5}{6} = \frac{1}{3} + \frac{1}{2}$$

Simplify the following expression so it is a sum of unit fractions.

$$\frac{5x + 6}{10x^2 + 12x + 8x^2} + \frac{2x}{2x + 1}$$

NAME _____ DATE _____ PERIOD _____

11-6 Enrichment**Sum and Difference of Any Two Like Powers**

The sum of any two like powers can be written $a^n + b^n$, where n is a positive integer. The difference of like powers is $a^n - b^n$. Under what conditions are these expressions exactly divisible by $(a + b)$ or $(a - b)$? The answer depends on whether n is an odd or even number.

Use long division to find the following quotients. (**Hint:** Write $a^3 + b^3$ as $a^3 + 0a^2 + 0a + b^3$.) Is the numerator exactly divisible by the denominator? Write yes or no.

1. $\frac{a^3 + b^3}{a + b}$ yes	2. $\frac{a^3 + b^3}{a - b}$ no	3. $\frac{a^3 - b^3}{a + b}$ yes	4. $\frac{a^3 - b^3}{a - b}$ yes
5. $\frac{a^4 + b^4}{a + b}$ no	6. $\frac{a^4 + b^4}{a - b}$ yes	7. $\frac{a^4 - b^4}{a + b}$ yes	8. $\frac{a^4 - b^4}{a - b}$ yes
9. $\frac{a^5 + b^5}{a + b}$ yes	10. $\frac{a^5 + b^5}{a - b}$ no	11. $\frac{a^5 - b^5}{a + b}$ no	12. $\frac{a^5 - b^5}{a - b}$ yes

13. Use the words *odd* and *even* to complete these two statements.

a. $a^n + b^n$ is divisible by $a + b$ if n is **odd**, and by neither $a + b$ nor $a - b$ if n is **even**.

b. $a^n - b^n$ is divisible by $a - b$ if n is **odd**, and by both $a + b$ and $a - b$ if n is **even**.

14. Describe the signs of the terms of the quotients when the divisor is $a - b$.
The terms are all positive.

15. Describe the signs of the terms of the quotient when the divisor is $a + b$.
The terms are alternately positive and negative.

NAME _____ DATE _____ PERIOD _____

11-7 Study Guide and Intervention

Mixed Expressions and Complex Fractions

Simplify Mixed Expressions Algebraic expressions such as $a + \frac{b}{c}$ and $5 + \frac{x+y}{x+3}$ are called **mixed expressions**. Changing mixed expressions to rational expressions is similar to changing mixed numbers to improper fractions.

Example 1 Simplify $5 + \frac{2}{n}$.

$$\begin{aligned} 5 + \frac{2}{n} &= \frac{5 \cdot n + 2}{n} \quad \text{LCD is } n. \\ &= \frac{5n + 2}{n} \quad \text{Add the numerators.} \\ &\text{Therefore, } 5 + \frac{2}{n} = \frac{5n + 2}{n}. \end{aligned}$$

$$\begin{aligned} \text{Therefore, } 2 + \frac{n+3}{n+3} &= \frac{2n+9}{n+3}. \\ \text{Therefore, } 2 + \frac{3}{n+3} &= \frac{2n+9}{n+3}. \end{aligned}$$

Exercises

Write each mixed expression as a rational expression.

$$1. 4 + \frac{6}{a} \frac{4a + 6}{a}$$

$$2. \frac{1}{9x} - 3 \frac{1 - 27x}{9x}$$

$$3. 3x - \frac{1}{x^2} \frac{3x^3 - 1}{x^2}$$

$$5. 10 + \frac{60}{x+5} \frac{10x + 110}{x+5}$$

$$7. \frac{y}{y-2} + y^2 \frac{y^2 - 2y^2 + y}{y-2}$$

$$9. 1 + \frac{1}{x} \frac{x+1}{x}$$

$$10. \frac{4}{m-2} - 2m \frac{4 - 2m^2 + 4m}{m-2}$$

$$11. x^2 + \frac{x+2}{x-3} \frac{x^3 - 3x^2 + x + 2}{x-3}$$

$$13. 4m + \frac{3p}{2t} \frac{8mt + 3p}{2t}$$

$$15. \frac{2}{y^2 - 1} - 4y^2 \frac{2 - 4y^2 + 4y^2}{y^2 - 1}$$

11-7 Study Guide and Intervention (continued)

Mixed Expressions and Complex Fractions

Simplify Complex Fractions If a fraction has one or more fractions in the numerator or denominator, it is called a **complex fraction**.

Simplifying a Complex Fraction	Any complex fraction $\frac{\frac{a}{b}}{\frac{c}{d}}$, where $b \neq 0, c \neq 0$, and $d \neq 0$, can be expressed as $\frac{ad}{bc}$.
--------------------------------	--

Example Simplify $\frac{2 + \frac{4}{a}}{a + 2}$.	Example Simplify $\frac{2 + \frac{4}{a}}{a + 2}$.
$\begin{aligned} \frac{2 + \frac{4}{a}}{a + 2} &= \frac{\frac{2a + 4}{a}}{a + 2} \\ &= \frac{2a + 4}{a(a + 2)} \\ &= \frac{2a + 4}{a^2 + 2a} \\ &= \frac{2a + 4}{a(a + 2)} \\ &= \frac{2a + 4}{a^2 + 2a} \\ &= \frac{2a + 4}{a(a + 2)} \\ &= \frac{2a + 4}{a^2 + 2a} \\ &= \frac{2a + 4}{a(a + 2)} \\ &= \frac{2a + 4}{a^2 + 2a} \end{aligned}$	$\begin{aligned} \frac{2 + \frac{4}{a}}{a + 2} &= \frac{\frac{2a + 4}{a}}{a + 2} \\ &= \frac{2(a + 2)}{a(a + 2)} \\ &= \frac{2}{a} \end{aligned}$

Find the LCD for the numerator and rewrite as like fractions.
Simplify the numerator.
Rewrite as the product of the numerator and the reciprocal of the denominator.

Exercises

Simplify each expression.

$$1. \frac{2\frac{2}{5}}{3\frac{3}{4}} \frac{16}{25}$$

$$2. \frac{\frac{3}{x}}{\frac{4}{y}} \frac{3y}{4x}$$

$$4. \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}} \frac{x - 1}{x + 1}$$

$$5. \frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}} \frac{x}{x + 1}$$

$$6. \frac{\frac{1}{x - 3}}{\frac{2}{x^2 - 9}} \frac{x + 3}{x - 2}$$

$$7. \frac{\frac{x^2 - 25}{y}}{x^3 - 5x^2} \frac{x + 5}{xy}$$

$$8. \frac{\frac{x - 12}{x - 8}}{\frac{3}{x - 2}} \frac{(x + 3)(x - 2)}{(x + 1)(x + 2)}$$

$$9. \frac{\frac{3}{y + 2} - \frac{2}{y - 2}}{\frac{1}{y + 2} - \frac{2}{y - 2}} \frac{y - 10}{-y - 6}$$

Answers (Lesson 11-7)

Lesson 11-7

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

11-7 Skills Practice**Mixed Expressions and Complex Fractions**

Write each mixed expression as a rational expression.

$$1. 6 + \frac{4}{h} \frac{6h+4}{h}$$

$$2. 7 + \frac{6}{p} \frac{7p+6}{p}$$

$$3. 4b + \frac{b}{c} \frac{4bc+b}{c}$$

$$4. 8q - \frac{2q}{r} \frac{8qr-2q}{r}$$

$$1. 14 - \frac{9}{u} \frac{14u-9}{u}$$

$$2. 7d + \frac{4d}{c} \frac{7dc+4d}{c}$$

$$5. 2 + \frac{4}{d-5} \frac{2d-6}{d-5}$$

$$6. 5 - \frac{6}{f+2} \frac{5f+4}{f+2}$$

$$7. b^2 + \frac{12}{b+3} \frac{b^3+3b^2+12}{b+3}$$

$$8. m - \frac{6}{m-7} \frac{m^2-7m-6}{m-7}$$

$$9. 2a + \frac{a-2}{a} \frac{2a^2+a-2}{a}$$

$$10. 4r - \frac{r+9}{2r} \frac{8r^2-r-9}{2r}$$

$$1. 14 - \frac{9}{u} \frac{14u-9}{u}$$

$$2. 7d + \frac{4d}{c} \frac{7dc+4d}{c}$$

$$11. \frac{\frac{32}{3}}{\frac{55}{81}} \frac{55}{81}$$

$$12. \frac{\frac{32}{3}}{\frac{55}{81}} \frac{55}{81}$$

$$3. \frac{\frac{3^2}{n^2}}{\frac{1}{rn}} \frac{1}{rn}$$

$$14. \frac{\frac{a^2}{b^2}}{\frac{a}{b^2}} \frac{a}{b^2}$$

$$15. \frac{\frac{x^2y}{xy^2}}{\frac{xc}{c^2}} \frac{xc}{y^2}$$

$$16. \frac{\frac{r-2}{r+3}}{\frac{r-2}{3}} \frac{3}{r+3}$$

$$17. \frac{\frac{w+4}{w^2-16}}{\frac{1}{w}} \frac{1}{w}$$

$$18. \frac{\frac{3^2-1}{x}}{\frac{x(x+1)}{x^2}} \frac{x(x+1)}{x^2}$$

$$19. \frac{\frac{b^2-4}{b-2}}{\frac{1}{b-2}} \frac{1}{b+5}$$

$$20. \frac{\frac{k^2+5k+6}{k^2-9}}{\frac{1}{k+2}} \frac{1}{k-3}$$

$$21. \frac{\frac{g+\frac{12}{8}}{g+6}}{\frac{g+2}{g+8}} \frac{g+2}{g+8}$$

$$22. \frac{\frac{p+\frac{9}{p-6}}{p-3}}{\frac{p-3}{p-6}} \frac{p-3}{p-6}$$

Simplify each expression.

$$1. 14 - \frac{9}{u} \frac{14u-9}{u}$$

$$2. 7d + \frac{4d}{c} \frac{7dc+4d}{c}$$

$$3. \frac{\frac{3^2}{n^2}}{\frac{1}{rn}} \frac{1}{rn}$$

$$4. \frac{\frac{r-10}{q^2-16}}{\frac{q-3}{q+4}} \frac{1}{q+4}$$

$$5. \frac{\frac{b^2+b-12}{b^2+3b-4}}{\frac{b-3}{b^2-b}} \frac{b}{b}$$

$$6. \frac{\frac{g-10}{g-9}}{\frac{g-5}{g+4}} \frac{(g+10)(g+4)}{(g+9)(g+5)}$$

Simplify each expression.

$$7. \frac{\frac{3^2}{n^2}}{\frac{1}{rn}} \frac{1}{rn}$$

$$8. \frac{\frac{b^2-7q+12}{b^2+4k-16}}{\frac{b-8}{b^2-8k+8}} \frac{k(k+6)}{k+5}$$

19. TRAVEL Ray and Jan are on a $12\frac{1}{2}$ -hour drive from Springfield, Missouri, to Chicago, Illinois. They stop for a break every $3\frac{1}{4}$ hours.

$$a. \text{ Write an expression to model this situation. } \frac{12\frac{1}{2}}{3\frac{1}{4}}$$

b. How many stops will Ray and Jan make before arriving in Chicago? **3**20. CARPENTRY Tai needs several $2\frac{1}{4}$ -inch wooden rods to reinforce the frame on a futon. She can cut the rods from a $24\frac{1}{2}$ -inch dowel purchased from a hardware store. How many wooden rods can she cut from the dowel? **10**

NAME _____	DATE _____	PERIOD _____	NAME _____	DATE _____	PERIOD _____
11-7 Word Problem Practice					
Mixed Expressions and Complex Fractions					
<p>1. CYCLING Natalie rode in a bicycle event for charity on Saturday. It took her $\frac{2}{3}$ of an hour to complete the 18-mile race. What was her average speed in miles per hour? 27 mph</p> <p>2. QUILTING Mrs. Tantora sews and sells Amish baby quilts. She bought $4\frac{3}{4}$ yards of backing fabric, and $2\frac{1}{4}$ yards are needed for each quilt she sews. How many quilts can she make with the backing fabric she bought? 19</p> <p>3. TRAVEL The Franz family traveled from Galveston to Waco for a family reunion. Driving their van, they averaged 30 miles per hour on the way to Waco and 45 miles per hour on the return trip home to Galveston. What is their average rate for the entire trip? (<i>Hint:</i> Remember that average rate equals total distance divided by total time and that time can be represented as a ratio of distance x to rate.) 36 mph</p>	<p>5. SAFETY The Occupational Safety and Health Administration provides safety standards in the workplace to keep workers free from dangerous working conditions. OSHA recommends that for general construction there be 5 foot-candles of illumination in which to work. A foreman using a light meter finds that the illumination of a construction light on a surface 8 feet from the source is 11 foot-candles. The illumination produced by a light source varies inversely as the square of the distance from the source.</p> <p><i>I</i> is illumination (in foot-candles). $I = \frac{k}{d^2}$ d is the distance from the source (in feet).</p> <p><i>k</i> is a constant.</p> <p>a. Find the illumination of the same light at a distance of $15\frac{3}{4}$ feet. Round your answer to the nearest hundredth. 2.84 foot-candles</p> <p>b. Is there enough illumination at this distance to meet OSHA requirements for lighting? no</p> <p>c. In order to comply with OSHA, what is the maximum allowable working distance from this light source? Round your decimal answer to nearest tenth. 11.9 ft</p> <p>4. PHYSICAL SCIENCE The volume of a gas varies directly as the Kelvin temperature T and inversely as the pressure P, where k is the constant of variation.</p> $V = k \frac{T}{P}$ <p>If $k = \frac{13}{157}$, find the volume in liters of helium gas at 273 degrees Kelvin and $\frac{13}{3}$ atmospheres of pressure. Round your answer to the nearest hundredth. 5.22 L</p>	<p>5. SAFETY The Occupational Safety and Health Administration provides safety standards in the workplace to keep workers free from dangerous working conditions. OSHA recommends that for general construction there be 5 foot-candles of illumination in which to work. A foreman using a light meter finds that the illumination of a construction light on a surface 8 feet from the source is 11 foot-candles. The illumination produced by a light source varies inversely as the square of the distance from the source.</p> <p><i>I</i> is illumination (in foot-candles). $I = \frac{k}{d^2}$ d is the distance from the source (in feet).</p> <p><i>k</i> is a constant.</p> <p>a. Find the illumination of the same light at a distance of $15\frac{3}{4}$ feet. Round your answer to the nearest hundredth. 2.84 foot-candles</p> <p>b. Is there enough illumination at this distance to meet OSHA requirements for lighting? no</p> <p>c. In order to comply with OSHA, what is the maximum allowable working distance from this light source? Round your decimal answer to nearest tenth. 11.9 ft</p> <p>4. PHYSICAL SCIENCE The volume of a gas varies directly as the Kelvin temperature T and inversely as the pressure P, where k is the constant of variation.</p> $V = k \frac{T}{P}$ <p>If $k = \frac{13}{157}$, find the volume in liters of helium gas at 273 degrees Kelvin and $\frac{13}{3}$ atmospheres of pressure. Round your answer to the nearest hundredth. 5.22 L</p>	<p>Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.</p>	<p>Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.</p>	<p>Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.</p>
11-7 Enrichment					
Continued Fractions					
<p>Continued fractions are a special type of complex fraction. Each fraction in a continued fraction has a numerator of 1.</p> <p>Example 1 Evaluate the continued fraction above. Start at the bottom and work your way up.</p> <p>Step 1 $3 + \frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{13}{4}$</p> <p>Step 2 $\frac{1}{\frac{13}{4}} = \frac{4}{13}$</p> <p>Step 3 $2 + \frac{4}{13} = \frac{26}{13} + \frac{4}{13} = \frac{30}{13}$</p> <p>Step 4 $\frac{1}{\frac{30}{13}} = \frac{13}{30}$</p> <p>Step 5 $1 + \frac{13}{30} = \frac{43}{30}$</p> <p>Evaluate each continued fraction.</p> <p>1. $0 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}}$ $\frac{13}{19}$</p> <p>2. $4.1 + \frac{1}{3 + \frac{1}{5 + \frac{1}{7}}}$ $\frac{204}{457}$</p> <p>3. $3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$ $\frac{5}{8}$</p> <p>Change each fraction into a continued fraction.</p> <p>5. $\frac{71}{26} \quad 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2}}}}}$ $\frac{36}{115}$</p>					

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

11-8 Study Guide and Intervention**Rational Functions and Equations**

Solve Rational Equations Rational equations are equations that contain rational expressions. To solve equations containing rational expressions, multiply each side of the equation by the least common denominator. Rational equations can be used to solve **work problems** and **rate problems**.

Example 1 Solve $\frac{x-3}{3} + \frac{x}{2} = 4$.

$$\begin{aligned} \frac{x-3}{3} + \frac{x}{2} &= 4 \\ 6\left(\frac{x-3}{3} + \frac{x}{2}\right) &= 6(4) \quad \text{The LCD is 6.} \\ 2(x-3) + 3x &= 24 \quad \text{Distributive Property} \\ 2x - 6 + 3x &= 24 \\ 5x &= 30 \\ x &= 6 \end{aligned}$$

The solution is 6.

The solution is 6.

$$\begin{aligned} \text{Example 2} \quad \text{Solve } \frac{15}{x^2-1} = \frac{5}{2(x-1)}. \quad \text{State any extraneous solutions.} \\ \frac{15}{x^2-1} &= \frac{5}{2(x-1)} \quad \text{Original equation} \\ 30(x-1) &= 5(x^2-1) \quad \text{Cross multiply.} \\ 30x - 30 &= 5x^2 - 5 \quad \text{Distributive Property} \\ 0 &= 5x^2 - 30x + 30 - 5 \quad \text{Add } -30 \text{ to each side.} \\ 0 &= 5x^2 - 30x + 25 \quad \text{Simplify.} \\ 0 &= 5(x^2 - 6x + 5) \quad \text{Factor.} \\ 0 &= 5(x-1)(x-5) \end{aligned}$$

$x = 1$ or $x = 5$

The number 1 is an extraneous solution, since 1 is an excluded value for x . So, 5 is the solution of the equation.

Exercises

Solve each equation. State any extraneous solutions.

1. $\frac{x-5}{5} + \frac{x}{4} = 8$ **20**

2. $\frac{3}{x} = \frac{6}{x+1}$ **1**

3. $\frac{x-1}{5} = \frac{2x-2}{15}$ **1**

4. $t - \frac{4}{t+3} = t+3$ **-4** **3**

5. $\frac{m+4}{m} + \frac{m}{3} = \frac{m}{3} - 4$ **0** **is extraneous**

6. $\frac{5-2x}{q-1} + \frac{4x+3}{q+1} = 2 - \frac{3}{2}$ **2**

7. $\frac{m+1}{m-1} - \frac{m}{1-m} = 1 - 2$ **-3 or 2**

8. $\frac{5-2x}{6} - \frac{4x+3}{6} = \frac{7x+2}{6} - \frac{10}{17}$ **17**

9. $\frac{x^2-9}{x-3} + x^2 = 9$ **-3 or 2**

10. $\frac{x^2-9}{x-3} + x^2 = 9$ **-3 or 2**

11. $\frac{2}{x^2-36} - \frac{1}{x-6} = 0$ **-4; 6 is extraneous**

12. $\frac{4x}{x^2-4} + \frac{3}{x-4} = 4$ **-6, 2**

13. $\frac{4}{4-p} - \frac{p^2}{p-4} = 4$ **-6, 2**

Answers

NAME _____ DATE _____ PERIOD _____

11-8 Study Guide and Intervention (continued)**Functions and Rational Equations**

Use Rational Equations to Solve Problems Rational equation can be used to solve work problems and rate problems.

Example WORK PROBLEM Marla can paint Percy's kitchen in 3 hours. Percy can paint it in 2 hours. Working together, how long will it take Marla and Percy to paint the kitchen?

In t hours, Marla completes $t \cdot \frac{1}{3}$ of the job and Percy completes $t \cdot \frac{1}{2}$ of the job. So an equation for completing the whole job is $\frac{t}{3} + \frac{t}{2} = 1$.

$$\begin{aligned} \frac{t}{3} + \frac{t}{2} &= 1 \\ 2t + 3t &= 6 \\ 5t &= 6 \\ t &= \frac{6}{5} \end{aligned}$$

Multiply each term by 6.
Add like terms.
Solve.

So it will take Marla and Percy $1\frac{1}{5}$ hours to paint the room if they work together.

Exercises

1. **GREETING CARDS** It takes Kenesha 45 minutes to prepare 20 greeting cards. It takes Paula 30 minutes to prepare the same number of cards. Working together at this rate, how long will it take them to prepare the cards? **18 min**

2. **BOATING** A motorboat went upstream at 15 miles per hour and returned downstream at 20 miles per hour. How far did the boat travel one way if the round trip took 3.5 hours? **30 mi**

3. **FLOORING** Maya and Reginald are installing hardwood flooring. Maya can install flooring in a room in 4 hours. Reginald can install flooring in a room in 3 hours. How long would it take them if they worked together? **$\frac{12}{7}$ h or about 1.71 hours**

4. **BICYCLING** Stefan is bicycling on a bike trail at an average of 10 miles per hour. Erik starts bicycling on the same trail 30 minutes later. If Erik averages 16 miles per hour, how long will it take him to pass Stefan? **50 min**

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11-8 Skills Practice

Rational Functions and Equations

Solve each equation. State any extraneous solutions.

$$1. \frac{5}{c} = \frac{2}{c+3} - 5 \quad 2. \frac{3}{q} = \frac{5}{q+4} \quad \mathbf{6}$$

$$3. \frac{7}{m+1} = \frac{12}{m+2} \quad \mathbf{5}$$

$$5. \frac{y}{y-2} = \frac{y+1}{y-5} \quad \mathbf{2}$$

$$7. \frac{3m}{2} - \frac{1}{4} = \frac{10m}{8} \quad \mathbf{1}$$

$$9. \frac{2a+5}{6} - \frac{2a}{3} = -\frac{1}{2} \quad \mathbf{4}$$

$$10. \frac{n-3}{10} + \frac{n-5}{5} = \frac{1}{2} \quad \mathbf{6}$$

$$12. \frac{3b-4}{6} - \frac{b-7}{b} = 1 \quad \mathbf{-3}$$

$$13. \frac{m-4}{m} - \frac{m-11}{m+4} = \frac{1}{m} \quad \mathbf{2}$$

$$15. \frac{r+3}{r-1} - \frac{r}{r-3} = 0 \quad \mathbf{9}$$

$$16. \frac{u+1}{u-2} - \frac{u}{u+1} = 0 \quad \mathbf{-\frac{1}{4}}$$

$$18. \frac{5}{m-4} - \frac{m}{2m-8} = 1 \quad \mathbf{6}$$

$$17. \frac{-2}{x+1} + \frac{2}{x} = 1 \quad \mathbf{-2, 1}$$

- 19. ACTIVISM** Maury and Tyra are making phone calls to state representatives' offices to lobby for an issue. Maury can call all 120 state representatives in 10 hours. Tyra can call all 120 state representatives in 8 hours. How long would it take them to call all 120 state representatives together?
4 **9 hr**

NAME _____ DATE _____ PERIOD _____

11-8 Practice

Rational Functions and Equations

Solve each equation. State any extraneous solutions.

$$1. \frac{5}{n+2} = \frac{7}{n+6} \quad \mathbf{8}$$

$$4. \frac{2h}{h-1} = \frac{2h+1}{h+2} \quad \mathbf{-1}$$

$$7. \frac{2y-1}{6} - \frac{q}{3} = \frac{q+4}{18} \quad \mathbf{-7}$$

$$10. \frac{4x}{2x+1} - \frac{2x}{2x+3} = 1 \quad \mathbf{3}$$

$$13. \frac{2}{m+2} - \frac{m+2}{m-2} = \frac{7}{3} \quad \mathbf{-1, \frac{2}{5}}$$

$$16. \frac{2p}{p-2} + \frac{p+2}{p^2-4} = 1 \quad \mathbf{-3; extraneous: -2}$$

$$18. \frac{x+7}{x^2-9} - \frac{x}{x+3} = 1 \quad \mathbf{-2, 4}$$

- 19. PUBLISHING** Tracey and Alan publish a 10-page independent newspaper once a month.

At production, Alan usually spends 6 hours on the layout of the paper. When Tracey helps, layout takes 3 hours and 20 minutes.

- a. Write an equation that could be used to determine how long it would take Tracey to do the layout by herself. **Sample answer:** $\frac{1}{6}(\frac{10}{3}) + \frac{1}{6}(\frac{10}{3}) = 1$
- b. How long would it take Tracey to do the job alone? **7 h 30 min**

- 20. TRAVEL** Emilio made arrangements to have Lynda pick him up from an auto repair shop after he dropped his car off. He called Lynda to tell her he would start walking and to look for him on the way. Emilio and Lynda live 10 miles from the auto shop. It takes Emilio $\frac{1}{4}$ hours to walk the distance and Lynda 15 minutes to drive the distance.

- a. If Emilio and Lynda leave at the same time, when should Lynda expect to spot Emilio on the road? **in $13\frac{1}{2}$ min**
- b. How far will Emilio have walked when Lynda picks him up? **1 mi**

11-8 Word Problem Practice

Rational Functions and Equations

- 1. ELECTRICITY** The current in a simple electric circuit varies inversely as the resistance. If the current is 20 amps when the resistance is 5 ohms, find the current when the resistance is 8 ohms. **12.5 amps**

- 4. NAUTICAL** A ferry captain keeps track of the progress of his ship in the ship's log. One day, he records the following entry.

With the recent spring snow melt, the six-mile trip downstream to Whyte's landing was very quick. However, we only covered two miles in the same amount of time when we headed back upstream.

- Write a rational equation using b for the speed of the boat and c for the speed of the stream and solve for b in terms of c .

$$\frac{6}{b+c} = \frac{2}{b-c} \cdot b = 2c$$

- 5. NUMBERS** The formula to find the sum of the first n whole numbers is $\sum = \frac{n^2 + n}{2}$. In order to encourage students to show up early to a school dance, the dance committee decides to charge less for those who come to the dance early. Their plan is to charge the first student to arrive 1 penny. The second student through the door is charged 2 pennies; the third student through the door is charged 3 pennies, and so on. How much money, in total, would paid by the first 150 students? **11,325 pennies or \$113.25**

- 6. HEALTH CARE** The total number of Americans waiting for kidney and heart transplants is approximately 66,500. The ratio of those awaiting a kidney transplant to those awaiting a heart transplant is about 20 to 1.

- a. How many people are on each of the waiting lists? Round your answers to the nearest hundred.
Kidney: 63,300; heart: 3200

- b. These two groups make up about $\frac{3}{4}$ of the transplant candidates for all organs. About how many organ transplant candidates are there altogether? Round your answer to the nearest thousand.
89,000

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

NAME _____ DATE _____ PERIOD _____

11-8 Enrichment

Winning Distances

- In 1999, Hicham El Guerrouj set a world record for the mile run with a time of 3:43.13 (3 min 43.13 s). In 1954, Roger Bannister ran the first mile under 4 minutes at 3:59.4. Had they run those times in the same race, how far in front of Bannister would El Guerrouj have been at the finish?

- Use $\frac{d}{t} = r$. Since 3 min 43.13 s = 223.13 s, and 3 min 59.4 s = 239.4 s, El Guerrouj's rate was $\frac{5280 \text{ ft}}{223.13 \text{ s}}$ and Bannister's rate was $\frac{5280 \text{ ft}}{239.4 \text{ s}}$.

	r	t	d
El Guerrouj	$\frac{5280}{223.13}$	223.13	5280 feet
Bannister	$\frac{5280}{239.4}$	223.13	$\frac{5280}{239.4} \cdot 223.13$ or 4921.2 feet

- Therefore, when El Guerrouj hit the tape, he would be 5280 - 4921.2, or 358.8 feet, ahead of Bannister. Let's see whether we can develop a formula for this type of problem.

Let D = the distance raced.

W = the winner's time,
and L = the loser's time.

Following the same pattern, you obtain the results shown in the table at the right.
The winning distance will be $D - \frac{DW}{L}$.

1. Show that the expression for the winning distance is equivalent to $\frac{DL - W}{L}$.

$$D - \frac{DW}{L} = \frac{DL}{L} - \frac{DW}{L} = \frac{DL - DW}{L}$$

Use the formula winning distance = $\frac{DL - W}{L}$ to find the winning distance to the nearest tenth for each of the following Olympic races.

2. women's 400 meter relay: Canada 48.4 s (1928);
East Germany 41.6 s (1980) **56.2 meters**

3. men's 200 meter freestyle swimming: Yevgeny Sadoyvi 1 min 46.70 s (1992);
Michael Gross 1 min 47.44 s (1984) **1.4 meters**

4. men's 50,000 meter walk: Vyacheslav Ivanenko 3 h 38 min 29 s (1988);
Hartwig Gauthier 3 h 49 min 24 s (1980) **2379.4 meters**

5. women's 400 meter freestyle relay: United States 3 min 39.29 s (1996);
East Germany 3 min 42.71 s (1980) **6.1 meters**

Lesson 11-8

	r	t	d
Winner	$\frac{D}{W}$	w	$\frac{D}{W} \cdot w = D$
Loser	$\frac{D}{L}$	w	$\frac{D}{L} \cdot w = DW$

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