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12 Anticipation Guide

Probability and Statistics

Step 1 ***Before you begin Chapter 12***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. A sample space is a partial list of possible outcomes of an experiment.	D
	2. Two events are called independent if choosing one does not affect choosing the other.	A
	3. According to the Fundamental Counting Principle, if one event can occur in 6 ways and another event can occur in 3 ways, then the events together can occur in 6 + 3 or 9 ways.	D
	4. Since order is not important in a combination, an outcome *ab* is the same as an outcome *ba*.	A
	5. The odds of an event occurring can be expressed as a ratio of the number of successes to the total number of outcomes.	D
	6. If two events are dependent, then the probability of both events occurring is the product of the probabilities of each event.	D
	7. Two events are *mutually exclusive* if they cannot occur at the same time.	A
	8. If a set of data contains outliers, the median would be a good choice to represent the set.	A
	9. Measures of variation are the differences between consecutive values in the set.	D
	10. The curve representing a normal distribution is symmetric.	A
	11. The Binomial Theorem can be used to find probabilities only when there are two possible outcomes.	A
	12. Asking people in a music store how many hours they spend listening to music to determine how many hours people in the city listen to music is an example of an unbiased survey.	D

Step 2 ***After you complete Chapter 12***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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12-1 Lesson Reading Guide

The Counting Principle

Get Ready for the Lesson

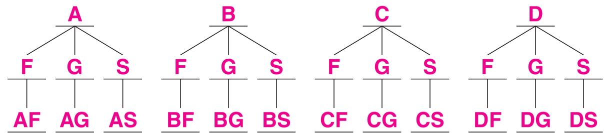
Read the introduction to Lesson 12-1 in your textbook.

Assume that all Florida license plates have three letters followed by three digits, and that there are no rules against using the same letter or number more than once. How many choices are there for each letter? for each digit? **26; 10**

Read the Lesson

1. Shamim is signing up for her classes. Most of her classes are required, but she has two electives. For her arts class, she can chose between Art, Band, Chorus, or Drama. For her language class, she can choose between French, German, and Spanish.

a. To organize her choices, Shamim decides to make a tree diagram. Let A, B, C, and D represent Art, Band, Chorus, and Drama, and F, G, and S represent French, German, and Spanish. Complete the following diagram.



b. How could Shamim have found the number of possible combinations without making a tree diagram? **Sample answer: Multiply the number of choices for her arts class by the number of choices for her language class: $4 \times 3 = 12$.**

2. A jar contains 6 red marbles, 4 blue marbles, and 3 yellow marbles. Indicate whether the events described are *dependent* or *independent*.

a. A marble is drawn out of the jar and is not replaced. A second marble is drawn. **dependent**

b. A marble is drawn out of the jar and is put back in. The jar is shaken. A second marble is drawn. **independent**

Remember What You Learned

3. One definition of *independent* is "not determined or influenced by someone or something else." How can this definition help you remember the difference between *independent* and *dependent* events? **Sample answer: If the outcome of one event does *not* affect or influence the outcome of another, the events are independent. If the outcome of one event *does* affect or influence the outcome of another, the events are dependent.**

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12-1 Study Guide and Intervention

The Counting Principle

Independent Events If the outcome of one event does not affect the outcome of another event and vice versa, the events are called **independent events**.

Fundamental Counting Principle	If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by the event N can occur in $m \cdot n$ ways.

Example FOOD **For the Breakfast Special at the Country Pantry, customers can choose their eggs scrambled, fried, or poached, whole wheat or white toast, and either orange, apple, tomato, or grapefruit juice. How many different Breakfast Specials can a customer order?**

A customer's choice of eggs does not affect his or her choice of toast or juice, so the events are independent. There are 3 ways to choose eggs, 2 ways to choose toast, and 4 ways to choose juice. By the Fundamental Counting Principle, there are $3 \cdot 2 \cdot 4$ or 24 ways to choose the Breakfast Special.

Exercises

Solve each problem.

1. The Palace of Pizza offers small, medium, or large pizzas with 14 different toppings available. How many different one-topping pizzas do they serve? **42**
2. The letters A, B, C, and D are used to form four-letter passwords for entering a computer file. How many passwords are possible if letters can be repeated? **256**
3. A restaurant serves 5 main dishes, 3 salads, and 4 desserts. How many different meals could be ordered if each has a main dish, a salad, and a dessert? **60**
4. Marissa brought 8 T-shirts and 6 pairs of shorts to summer camp. How many different outfits consisting of a T-shirt and a pair of shorts does she have? **48**
5. There are 6 different packages available for school pictures. The studio offers 5 different backgrounds and 2 different finishes. How many different options are available? **60**
6. How many 5-digit even numbers can be formed using the digits 4, 6, 7, 2, 8 if digits can be repeated? **2500**
7. How many license plate numbers consisting of three letters followed by three numbers are possible when repetition is allowed? **17,576,000**
8. How many 4-digit positive even integers are there? **4500**

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12-1 Study Guide and Intervention *(continued)*

The Counting Principle

Dependent Events If the outcome of an event *does* affect the outcome of another event, the two events are said to be **dependent**. The Fundamental Counting Principle still applies.

Example ENTERTAINMENT **The guests at a sleepover brought 8 videos. They decided they would only watch 3 videos. How many orders of 3 different videos are possible?**

After the group chooses to watch a video, they will not choose to watch it again, so the choices of videos are dependent events.

There are 8 choices for the first video. That leaves 7 choices for the second. After they choose the first 2 videos, there are 6 remaining choices. Thus, by the Fundamental Counting Principle, there are $8 \cdot 7 \cdot 6$ or 336 orders of 3 different videos.

Exercises

Solve each problem.

1. Three students are scheduled to give oral reports on Monday. In how many ways can their presentations be ordered? **6**
2. In how many ways can the first five letters of the alphabet be arranged if each letter is used only once? **120**
3. In how many different ways can 4 different books be arranged on the shelf? **24**
4. How many license plates consisting of three letters followed by three numbers are possible when no repetition is allowed? **11,232,000**
5. Sixteen teams are competing in a soccer match. Gold, silver, and bronze medals will be awarded to the top three finishers. In how many ways can the medals be awarded? **3360**
6. In a word-building game each player picks 7 letter tiles. If Julio's letters are all different, how many 3-letter combinations can he make out of his 7 letters? **210**
7. The editor has accepted 6 articles for the newsletter. In how many ways can the 6 articles be ordered? **720**
8. There are 10 one-hour workshops scheduled for the open house at the greenhouse. There is only one conference room available. In how many ways can the workshops be ordered? **3,628,800**
9. The top 5 runners at the cross-country meet will receive trophies. If there are 22 runners in the race, in how many ways can the trophies be awarded? **3,160,080**

Lesson 12-1

NAME ______________________ DATE __________ PERIOD _____

12-1 Skills Practice

The Counting Principle

State whether the events are *independent* or *dependent*.

1. finishing in first, second, or third place in a ten-person race **dependent**

2. choosing a pizza size and a topping for the pizza **independent**

3. Seventy-five raffle tickets are placed in a jar. Three tickets are then selected, one after the other, without replacing a ticket after it is chosen. **dependent**

4. The 232 members of the freshman class all vote by secret ballot for the class representative to the Student Senate. **independent**

Solve each problem.

5. A surveying firm plans to buy a color printer for printing its maps. It has narrowed its choice to one of three models. Each of the models is available with either 32 megabytes of random access memory (RAM), 64 megabytes of RAM, or 128 megabytes of RAM. From how many combinations of models and RAM does the firm have to choose? **9**

6. How many arrangements of three letters can be formed from the letters of the word *MATH* if any letter will not be used more than once? **24**

7. Allan is playing the role of Oliver in his school's production of *Oliver Twist*. The wardrobe crew has presented Allan with 5 pairs of pants and 4 shirts that he can wear. From how many possible costumes consisting of a pair of pants and a shirt does Allan have to choose? **20**

8. The 10-member steering committee that is preparing a study of the public transportation needs of its town will select a chairperson, vice-chairperson, and secretary from the committee. No person can serve in more than one position. In how many ways can the three positions be filled? **720**

9. Jeanine has decided to buy a pickup truck. Her choices include either a V-6 engine or a V-8 engine, a standard cab or an extended cab, and 2-wheel drive or 4-wheel drive. How many possible models does she have to choose from? **8**

10. A mail-order company that sells gardening tools offers rakes in two different lengths. Customers can also choose either a wooden, plastic, or fiberglass handle for the rake. How many different kinds of rakes can a customer buy? **6**

11. A Mexican restaurant offers chicken, beef, or vegetarian fajitas wrapped with either corn or flour tortillas and topped with either mild, medium, or hot salsa. How many different choices of fajitas does a customer have? **18**

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12-1 Practice

The Counting Principle

State whether the events are *independent* or *dependent*.

1. choosing an ice cream flavor and choosing a topping for the ice cream **independent**

2. choosing an offensive player of the game and a defensive player of the game in a professional football game **independent**

3. From 15 entries in an art contest, a camp counselor chooses first, second, and third place winners. **dependent**

4. Jillian is selecting two more courses for her block schedule next semester. She must select one of three morning history classes and one of two afternoon math classes. **independent**

Solve each problem.

5. A briefcase lock has 3 rotating cylinders, each containing 10 digits. How many numerical codes are possible? **1000**

6. A golf club manufacturer makes irons with 7 different shaft lengths, 3 different grips, 5 different lies, and 2 different club head materials. How many different combinations are offered? **210**

7. There are five different routes that a commuter can take from her home to the office. In how many ways can she make a round trip if she uses a different route coming than going? **20**

8. In how many ways can the four call letters of a radio station be arranged if the first letter must be W or K and no letters repeat? **27,600**

9. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and any digit can be repeated? **8,000,000**

10. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and any digit can be repeated? **9,000,000**

11. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and no digit can be repeated? **483,840**

12. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and no digit can be repeated? **544,320**

13. How many 6-character passwords can be formed if the first character is a digit and the remaining 5 characters are letters that can be repeated? **118,813,760**

14. How many 6-character passwords can be formed if the first and last characters are digits and the remaining characters are letters? Assume that any character can be repeated. **45,697,600**

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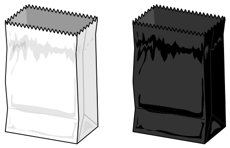
12-1 Word Problem Practice

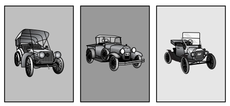
The Counting Principle

1. **CANDY** Amy, Bruce, and Carol can choose one piece of candy from either a white or black bag. The white bag contains various chocolates. The black bag contains small bags of jelly beans. Amy picks from the white bag, and Bruce and Carol both pick from the black bag. Describe whether each of the picks is related as dependent or independent events.



Amy's pick is independent of each of Bruce and Carol's picks; Bruce and Carol's picks are examples of dependent events.

2. **PHOTOS** Morgan has three pictures that she would like to display side by side.



In how many different ways can the pictures be displayed?
6

3. **COMBINATION LOCKS** Eric uses a combination lock for his locker. The lock uses a three number secret code. Each number ranges from 1 to 35, inclusive. How many different combinations are possible with Eric's lock?
42,875

4. **TRUE OR FALSE** Faith is preparing a true or false quiz for her biology class. How many different answer keys can there be for a 10 question true or false quiz?
1024

WEBSITES For Exercises 5-8, use the following information.

Greg is registering to use a website. The website requires him to choose an 8-character alphanumeric password that is not case-sensitive. In other words, for each character, he can choose one of the 26 letters A through Z or one of the 10 digits 0 through 9.

5. How many different passwords are possible?
2,821,109,907,456

6. Greg decides to use a password that does not contain any repeated characters. How many different passwords are possible with this constraint?
1,220,096,908,800

7. Suppose Greg chooses to use only letters with possible repeats. How many different passwords would be possible?
208,827,064,576

8. If Greg's password begins with his first name and ends with his birth month and date, how many possibilities would need to be checked to find his password?
372

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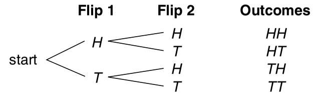
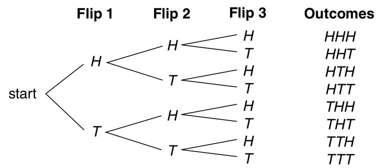
12-1 Enrichment

Tree Diagrams and the Power Rule

If you flip a coin once, there are two possible outcomes: heads showing (H) or tails showing (T). The tree diagram to the right shows the four (2^2) possible outcomes if you flip a coin twice.

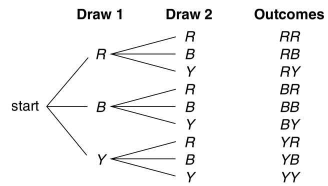


Example 1 **Draw a tree diagram to show all the possible outcomes for flipping a coin three times. List the outcomes.**



There are eight (2^3) possible outcomes. With each extra flip, the number of outcomes doubles. With 4 flips, there would be sixteen (2^4) outcomes.

Example 2 **In a cup there are a red, a blue, and a yellow marble. How many possible outcomes are there if you draw one marble at random, replace it, and then draw another?**



There are nine (3^2) possible outcomes.

The Power Rule for the number of outcomes states that if an experiment is repeated n times, and if there are b possible outcomes each time, there are b^n total possible outcomes.

Find the total number of possible outcomes for each experiment. Use tree diagrams to help you.

1. flipping a coin 5 times **2^5**
2. doing the marble experiment 6 times **3^6**
3. flipping a coin 8 times **2^8**
4. rolling a 6-sided die 2 times **6^2**
5. rolling a 6-sided die 3 times **6^3**
6. rolling a 4-sided die 2 times **4^2**
7. rolling a 4-sided die 3 times **4^3**
8. rolling a 12-sided die 2 times **12^2**

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12-2 Lesson Reading Guide

Permutations and Combinations

Get Ready for the Lesson

Read the introduction to Lesson 12-2 in your textbook.

Suppose that 20 students enter a math contest. In how many ways can first, second, and third places be awarded? (Write your answer as a product. Do not calculate the product.)
$20 \cdot 19 \cdot 18$

Read the Lesson

1. Indicate whether each situation involves a *permutation* or a *combination*.

a. choosing five students from a class to work on a special project **combination**

b. arranging five pictures in a row on a wall **permutation**

c. drawing a hand of 13 cards from a 52-card deck **combination**

d. arranging the letters of the word *algebra* **permutation**

2. Write an expression that can be used to calculate each of the following.

a. number of combinations of n distinct objects taken r at a time $\frac{n!}{(n-r)!r!}$

b. number of permutations of n objects of which p are alike and q are alike $\frac{n!}{p!q!}$

c. number of permutations of n distinct objects taken r at a time $\frac{n!}{(n-r)!}$

3. Five cards are drawn from a standard deck of cards. Suppose you are asked to determine how many possible hands consist of one heart, two diamonds, and two spades.

a. Which of the following would you use to solve this problem: *Fundamental Counting Principle*, *permutations*, or *combinations*? (More than one of these may apply.)

Fundamental Counting Principle, combinations

b. Write an expression that involves the notation $P(n, r)$ and/or $C(n, r)$ that you would use to solve this problem. (Do not do any calculations.)

$C(13, 1) \cdot C(13, 2) \cdot C(13, 2)$

Remember What You Learned

4. Many students have trouble knowing when to use permutations and when to use combinations to solve counting problems. How can the idea of *order* help you to remember the difference between permutations and combinations?

Sample answer: A permutation is an arrangement of objects in which order is important. A combination is a selection of objects in which order is *not* important.

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12-2 Study Guide and Intervention

Permutations and Combinations

Permutations When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**.

Permutations	The number of permutations of n distinct objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$.
Permutations with Repetitions	The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

The rule for permutations with repetitions can be extended to any number of objects that are repeated.

Example **From a list of 20 books, each student must choose 4 books for book reports. The first report is a traditional book report, the second a poster, the third a newspaper interview with one of the characters, and the fourth a timeline of the plot. How many different orderings of books can be chosen?**

Since each book report has a different format, order is important. You must find the number of permutations of 20 objects taken 4 at a time.

$$P(n, r) = \frac{n!}{(n-r)!} \quad \text{Permutation formula}$$
$$P(20, 4) = \frac{20!}{(20-4)!} \quad n = 20, r = 4$$
$$= \frac{20!}{16!} \quad \text{Simplify.}$$
$$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \cancel{16} \cdot \cancel{15} \cdot \ldots \cdot \cancel{1}}{\cancel{16} \cdot \cancel{15} \cdot \ldots \cdot \cancel{1}} \quad \text{Divide by common factors.}$$
$$= 116{,}280$$

Books for the book reports can be chosen 116,280 ways.

Exercises

Evaluate each expression.

1. $P(6, 3)$ **120** **2.** $P(8, 5)$ **6720** **3.** $P(9, 4)$ **3024** **4.** $P(11, 6)$ **332,640**

How many different ways can the letters of each word be arranged?

5. MOM **3** **6.** MONDAY **720** **7.** STEREO **360**

8. SCHOOL The high school chorus has been practicing 12 songs, but there is time for only 5 of them at the spring concert. How may different orderings of 5 songs are possible?
95,040

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12-2 Study Guide and Intervention *(continued)*

Permutations and Combinations

Combinations An arrangement or selection of objects in which order is *not* important is called a combination.

Combinations	The number of combinations of n distinct objects taken r at a time is given by $C(n, r) = \frac{n!}{(n - r)!r!}$.

Example 1 **SCHOOL How many groups of 4 students can be selected from a class of 20?**

Since the order of choosing the students is not important, you must find the number of combinations of 20 students taken 4 at a time.

$C(n, r) = \frac{n!}{(n - r)!r!}$ Combination formula

$C(20, 4) = \frac{20!}{(20 - 4)!4!}$ $n = 20, r = 4$

$= \frac{20!}{16!4!}$ or 4845

There are 4845 possible ways to choose 4 students.

Example 2 **In how many ways can you choose 1 vowel and 2 consonants from a set of 26 letter tiles? (Assume there are 5 vowels and 21 consonants.)**

By the Fundamental Counting Principle, you can multiply the number of ways to select one vowel and the number of ways to select 2 consonants. Only the letters chosen matter, not the order in which they were chosen, so use combinations.

$C(5, 1)$ One of 5 vowels are drawn.
$C(21, 2)$ Two of 21 consonants are drawn.

$C(5, 1) \cdot C(21, 2) = \frac{5!}{(5 - 1)!1!} \cdot \frac{21!}{(21 - 2)!2!}$ Combination formula

$= \frac{5!}{4!} \cdot \frac{21!}{19!2!}$ Subtract.

$= 5 \cdot 210$ or 1050 Simplify.

There are 1050 combinations of 1 vowel and 2 consonants.

Exercises

Evaluate each expression.

1. $C(5, 3)$ **10** **2.** $C(7, 4)$ **35** **3.** $C(15, 7)$ **6435** **4.** $C(10, 5)$ **252**

5. PLAYING CARDS From a standard deck of 52 cards, in how many ways can 5 cards be drawn? **2,598,960**

6. HOCKEY How many hockey teams of 6 players can be formed from 14 players without regard to position played? **3003**

7. COMMITTEES From a group of 10 men and 12 women, how many committees of 5 men and 6 women can be formed? **232,848**

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12-2 Skills Practice

Permutations and Combinations

Evaluate each expression.

1. $P(6, 3)$ **120** **2.** $P(8, 2)$ **56** **3.** $P(2, 1)$ **2**

4. $P(3, 2)$ **6** **5.** $P(10, 4)$ **5040** **6.** $P(5, 5)$ **120**

7. $C(2, 2)$ **1** **8.** $C(5, 3)$ **10** **9.** $C(4, 1)$ **4**

10. $C(8, 7)$ **8** **11.** $C(3, 2)$ **3** **12.** $C(7, 4)$ **35**

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. seating 8 students in 8 seats in the front row of the school auditorium
permutation; 40,320

14. introducing the 5 starting players on the Woodsville High School basketball team at the beginning of the next basketball game
permutation; 120

15. checking out 3 library books from a list of 8 books for a research paper
combination; 56

16. choosing 2 movies to rent from 5 movies
combination; 10

17. the first-, second-, and third-place finishers in a race with 10 contestants
permutation; 720

18. electing 4 candidates to a municipal planning board from a field of 7 candidates
combination; 35

19. choosing 2 vegetables from a menu that offers 6 vegetable choices
combination; 15

20. an arrangement of the letters in the word *rhombus*
permutation; 5040

21. selecting 2 of 8 choices of orange juice at a store
combination; 28

22. placing a red rose bush, a yellow rose bush, a white rose bush, and a pink rose bush in a row in a planter **permutation; 24**

23. selecting 2 of 9 kittens at an animal rescue shelter
combination; 36

24. an arrangement of the letters in the word *isosceles*
permutation; 30,240

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12-2 Practice

Permutations and Combinations

Evaluate each expression.

1. $P(8, 6)$ **20,160**	**2.** $P(9, 7)$ **181,440**	**3.** $P(3, 3)$ **6**
4. $P(4, 3)$ **24**	**5.** $P(4, 1)$ **4**	**6.** $P(7, 2)$ **42**
7. $C(8, 2)$ **28**	**8.** $C(11, 3)$ **165**	**9.** $C(20, 18)$ **190**
10. $C(9, 9)$ **1**	**11.** $C(3, 1)$ **3**	**12.** $C(9, 3) \cdot C(6, 2)$ **1260**

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. selecting a 4-person bobsled team from a group of 9 athletes
combination; 126

14. an arrangement of the letters in the word *Canada*
permutation; 120

15. arranging 4 charms on a bracelet that has a clasp, a front, and a back
permutation; 24

16. selecting 3 desserts from 10 desserts that are displayed on a dessert cart in a restaurant
combination; 120

17. an arrangement of the letters in the word *annually*
permutation; 5040

18. forming a 2-person sales team from a group of 12 salespeople
combination; 66

19. making 5-sided polygons by choosing any 5 of 11 points located on a circle to be the vertices
combination; 462

20. seating 5 men and 5 women alternately in a row, beginning with a woman
permutation; 14,400

21. STUDENT GROUPS Farmington High is planning its academic festival. All math classes will send 2 representatives to compete in the math bowl. How many different groups of students can be chosen from a class of 16 students? **120**

22. PHOTOGRAPHY A photographer is taking pictures of a bride and groom and their 6 attendants. If she takes photographs of 3 people in a group, how many different groups can she photograph? **56**

23. AIRLINES An airline is hiring 5 flight attendants. If 8 people apply for the job, how many different groups of 5 attendants can the airline hire? **56**

24. SUBSCRIPTIONS A school librarian would like to buy subscriptions to 7 new magazines. Her budget, however, will allow her to buy only 4 new subscriptions. How many different groups of 4 magazines can she choose from the 7 magazines? **35**

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12-2 Word Problem Practice

Permutations and Combinations

1. WAITING IN LINE When the 12 students in Mr. Jaybird's class go to lunch, they form a single file line. Does forming a line involve a permutation or a combination of the students?
A permutation

2. ART Isabel needs to select three different colors of construction paper to make a flag for a school project. She can choose from a selection of 15 different colors. In how many ways can she pick her colors?
455

3. SUDOKU A popular game called "Sudoku" involves square arrays of numbers. In a game of Sudoku, every entry is an integer between 1 and 9, inclusive. No number appears twice in any row or column.

7	1	8	6	9	4	2	3	5
9	2	5	7	3	1	6	4	8
4	6	3	8	5	2	7	9	1
5	9	2	1	7	3	4	8	6
8	3	1	4	6	5	9	2	7
9	7	4	2	8	9	5	1	3
3	4	9	5	1	7	8	6	2
2	8	7	3	4	6	1	5	9
1	5	6	9	2	8	3	7	4

For a game of Sudoku, how many different possibilities are there for the first row of numbers?
362,880

4. NAMES Hannah is curious to know how many different 6 letter sequences she can make using each of the letters of her name exactly once. For example, "HANNAH," "AAHHNN," and "NAHNAH" are all possible sequences. How many total sequences are possible?
90

METEORITES For Exercises 5 and 6, use the following information.

Over the course of several years, Kendra managed to collect 7 meteorites. Each one is unique.

5. For a school science fair, Kendra displays her meteorites in a row. How many ways are there to order the meteorites?
5040

6. She decides to trade three of her meteorites for a telescope after the fair. How many ways can she pick out 3 meteorites from her collection?
35

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12-2 Enrichment

Combinations and Pascal's Triangle

Pascal's triangle is a special array of numbers invented by Blaise Pascal (1623–1662). The values in Pascal's triangle can be found using the combinations shown below.

1. Evaluate the expression in each cell of the triangle.

C(1,0) **1**	C(1,1) **1**				
C(2,0) **1**	C(2,1) **2**	C(2,2) **1**			
C(3,0) **1**	C(3,1) **3**	C(3,2) **3**	C(3,3) **1**		
C(4,0) **1**	C(4,1) **4**	C(4,2) **6**	C(4,3) **4**	C(4,4) **1**	
C(5,0) **1**	C(5,1) **5**	C(5,2) **10**	C(5,3) **10**	C(5,4) **5**	C(5,5) **1**

2. The pattern shows the relationship between $C(n, r)$ and Pascal's triangle. In general, it is true that $C(n, r) + C(n, r + 1) = C(n + 1, r + 1)$. Complete the proof of this property. In each step, the denominator has been given.

$$C(n, r) + C(n, r + 1) = \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!}$$
$$= \frac{n!(r+1)}{r!(n-r)!(r+1)} + \frac{n!(n-r)}{(r+1)!(n-r-1)!(n-r)}$$
$$= \frac{n!(r+1)}{(r+1)!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)!}$$
$$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$$
$$= \frac{n!(n+1)}{(r+1)!(n-r)!}$$
$$= \frac{(n+1)!}{(r+1)!(n-r)!}$$
$$= \frac{(n+1)!}{(r+1)![(n+1)-(r+1)]!}$$
$$= C(n + 1, r + 1)$$

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12-2 Spreadsheet Activity

Permutations and Combinations

You have learned the formulas for the number of permutations of n objects taken r at a time, $P(n, r)$, and the number of combinations of n objects taken r at a time, $C(n, r)$. You are going to set up a spreadsheet like the one shown below to perform analyses of these functions.

◇	A	B	C	D	E	F	G
1	*n*	5	5	5	5	5	5
2	*r*	0	1	2	3	4	5
3	*P (n, r)*	1	5	20	60	120	120
4	*C (n, r)*	1	5	10	10	5	1

Sheet 1 | Sheet 2 | Sheet 3

In the spreadsheet, the values in row 1 represent n, the values in row 2 represent r, and the formulas for $P(n, r)$ and $C(n, r)$ are in rows 3 and 4, respectively.

The formula to calculate $P(n, r)$ is =FACT(B1)/FACT(B1-B2).

FACT is a special function from the function list and should not be entered from the letters on the keyboard. Enter the formula in B3. Then drag the cursor across the row to copy the formula into cells C3 through G3.

The formula for $C(n, r)$ is =FACT(B1)/(FACT(B1−B2)*FACT(B2)) and should be entered in cell B4. Copy the formula into cells C4 through G4.

Exercises

1. Compare the values of $P(n, r)$ and $C(n, r)$ for $n = 5$ and $r = 0$ through 5, as well as for two other choices of n and r. **Most of the values of $P(n, r)$ are much larger than the corresponding values of $C(n, r)$. The values of $P(n, r)$ tend to increase, while the values of $C(n, r)$ tend to increase and then decrease.**

2. Several identities hold for $P(n, r)$ and $C(n, r)$. Use the spreadsheet to verify the following identities by finding three examples of each. **2a-2c. See students' work.**

a. $P(n, n) = P(n, n - 1)$

b. $C(n + 1, r) = C(n, r - 1) + C(n, r)$

c. $C(n, 0) + C(n, 1) + C(n, 2) + \ldots + C(n, n) = 2^n$

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12-3 Lesson Reading Guide

Probability

Get Ready for the Lesson

Read the introduction to Lesson 12-3 in your textbook.

What is the probability that a person will *not* be struck by lightning in a given year?
$\frac{749{,}999}{750{,}000}$

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. If an event can never occur, its probability is a negative number. **false**

b. If an event is certain to happen, its probability is 1. **true**

c. If an event can succeed in s ways and fail in f ways, then the probability of success is $\frac{s}{f}$. **false**

d. If an event can succeed in s ways and fail in f ways, then the odds against the event are $s:f$. **false**

e. A probability distribution is a function in which the domain is the sample space of an experiment. **true**

2. A weather forecast says that the chance of rain tomorrow is 40%.

a. Write the probability that it will rain tomorrow as a fraction in lowest terms. $\frac{2}{5}$

b. Write the probability that it will not rain tomorrow as a fraction in lowest terms. $\frac{3}{5}$

c. What are the odds in favor of rain? **2:3**

d. What are the odds against rain? **3:2**

3. Refer to the table in Example 4 on page 646 in your textbook.

a. What other sum has the same probability as a sum of 11? **3**

b. What are the odds of rolling a sum of 8? **5:31**

c. What are the odds against rolling a sum of 9? **8:1**

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose that your friend Roberto is having trouble remembering the difference between probability and odds. What would you tell him to help him remember this easily?

Sample answer: Probability gives the ratio of successes to the total number of outcomes, while odds gives the ratio of successes to failures.

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12-3 Study Guide and Intervention

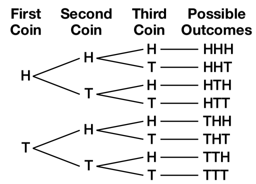
Probability

Probability and Odds In probability, a desired outcome is called a **success**; any other outcome is called a **failure**.

Probability of Success and Failure	If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows. $P(S) = \frac{s}{s+f}$ and $P(F) = \frac{f}{s+f}$.
Definition of Odds	If an event can succeed in s ways and fail in f ways, then the odds of success and of failure are as follows. Odds of success = $s:f$ Odds of failure = $f:s$

Example 1 **When 3 coins are tossed, what is the probability that at least 2 are heads?**

You can use a tree diagram to find the sample space.



Of the 8 possible outcomes, 4 have at least 2 heads. So the probability of tossing at least 2 heads is $\frac{4}{8}$ or $\frac{1}{2}$.

Example 2 **What is the probability of picking 4 fiction books and 2 biographies from a best-seller list that consists of 12 fiction books and 6 biographies?**

By the Fundamental Counting Principle, the number of successes is $C(12, 4) \cdot C(6, 2)$.
The total number of selections, $s + f$, of 6 books is $C(18, 6)$.

$P(\text{4 fiction, 2 biography}) = \frac{C(12, 4) \cdot C(6, 2)}{C(18, 6)}$ or about 0.40

The probability of selecting 4 fiction books and 2 biographies is about 40%.

Exercises

Find the odds of an event occurring, given the probability of the event.

1. $\frac{3}{7}$ **3:4** **2.** $\frac{4}{5}$ **4:1** **3.** $\frac{2}{13}$ **2:11** **4.** $\frac{1}{15}$ **1:14**

Find the probability of an event occurring, given the odds of the event.

5. 10:1 $\frac{10}{11}$ **6.** 2:5 $\frac{2}{7}$ **7.** 4:9 $\frac{4}{13}$ **8.** 8:3 $\frac{8}{11}$

One bag of candy contains 15 red candies, 10 yellow candies, and 6 green candies. Find the probability of each selection.

9. picking a red candy $\frac{15}{31}$

10. not picking a yellow candy $\frac{21}{31}$

11. picking a green candy $\frac{6}{31}$

12. not picking a red candy $\frac{16}{31}$

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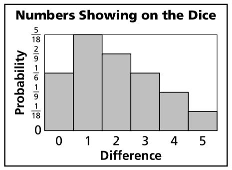
12-3 Study Guide and Intervention *(continued)*

Probability

Probability Distributions A **random variable** is a variable whose value is the numerical outcome of a random event. A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space.

Example **Suppose two dice are rolled. The table and the relative-frequency histogram show the distribution of the absolute value of the difference of the numbers rolled. Use the graph to determine which outcome is the most likely. What is its probability?**

Difference	0	1	2	3	4	5
Probability	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$



The greatest probability in the graph is $\frac{5}{18}$.
The most likely outcome is a difference of 1 and its probability is $\frac{5}{18}$.

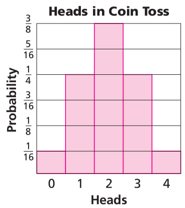
Exercises

Four coins are tossed.

1. Complete the table below to show the probability distribution of the number of heads.

Number of Heads	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

2. Make relative-frequency distribution of the data.



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12-3 Skills Practice

Probability

Ahmed is posting 2 photographs on his website. He has narrowed his choices to 4 landscape photographs and 3 portraits. If he chooses the two photographs at random, find the probability of each selection.

1. P(2 portrait) $\frac{1}{7}$ **2.** P(2 landscape) $\frac{2}{7}$ **3.** P(1 of each) $\frac{4}{7}$

The Carubas have a collection of 28 video movies, including 12 westerns and 16 science fiction. Elise selects 3 of the movies at random to bring to a sleep-over at her friend's house. Find the probability of each selection.

4. P(3 westerns) $\frac{55}{819}$

5. P(3 science fiction) $\frac{20}{117}$

6. P(1 western and 2 science fiction) $\frac{40}{91}$

7. P(2 westerns and 1 science fiction) $\frac{88}{273}$

8. P(3 comedy) 0

9. P(2 science fiction and 2 westerns) 0

For Exercises 10–13, use the chart that shows the class and gender statistics for the students taking an Algebra 1 or Algebra 2 class at La Mesa High School. If a student taking Algebra 1 or Algebra 2 is selected at random, find each probability. Express as decimals rounded to the nearest thousandth.

Class/Gender	Number
Freshman/Male	95
Freshman/Female	101
Sophomore/Male	154
Sophomore/Female	145
Junior/Male	100
Junior/Female	102

10. P(sophomore/female) 0.208

11. P(junior/male) 0.143

12. P(freshman/male) 0.136

13. P(freshman/female) 0.145

Find the odds of an event occurring, given the probability of the event.

14. $\frac{5}{8}$ 5:3 **15.** $\frac{2}{7}$ 2:5 **16.** $\frac{3}{5}$ 3:2

17. $\frac{1}{10}$ 1:9 **18.** $\frac{5}{6}$ 5:1 **19.** $\frac{5}{12}$ 5:7

Find the probability of an event occurring, given the odds of the event.

20. 2:1 $\frac{2}{3}$ **21.** 8:9 $\frac{8}{17}$ **22.** 4:1 $\frac{4}{5}$

23. 1:9 $\frac{1}{10}$ **24.** 2:7 $\frac{2}{9}$ **25.** 5:9 $\frac{5}{14}$

Lesson 12-3

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12-3 Practice

Probability

A bag contains 1 green, 4 red, and 5 yellow balls. Two balls are selected at random. Find the probability of each selection.

1. P(2 red) $\frac{2}{15}$ **2.** P(1 red and 1 yellow) $\frac{4}{9}$ **3.** P(1 green and 1 yellow) $\frac{1}{9}$

4. P(2 green) 0 **5.** P(2 red and 1 yellow) 0 **6.** P(1 red and 1 green) $\frac{4}{45}$

A bank contains 3 pennies, 8 nickels, 4 dimes, and 10 quarters. Two coins are selected at random. Find the probability of each selection.

7. P(2 pennies) $\frac{1}{100}$ **8.** P(2 dimes) $\frac{1}{50}$ **9.** P(1 nickel and 1 dime) $\frac{8}{75}$

10. P(1 quarter and 1 penny) $\frac{1}{10}$ **11.** P(1 quarter and 1 nickel) $\frac{4}{15}$ **12.** P(2 dimes and 1 quarter) 0

Henrico visits a home decorating store to choose wallpapers for his new house. The store has 28 books of wallpaper samples, including 10 books of WallPride samples and 18 books of Deluxe Wall Coverings samples. The store will allow Henrico to bring 4 books home for a few days so he can decide which wallpapers he wants to buy. If Henrico randomly chooses 4 books to bring home, find the probability of each selection.

13. P(4 WallPride) $\frac{2}{195}$ **14.** P(2 WallPride and 2 Deluxe) $\frac{153}{455}$

15. P(1 WallPride and 3 Deluxe) $\frac{544}{1365}$ **16.** P(3 WallPride and 1 Deluxe) $\frac{48}{455}$

For Exercises 17–20, use the table that shows the range of verbal SAT scores for freshmen at a small liberal arts college. If a freshman student is chosen at random, find each probability. Express as decimals rounded to the nearest thousandth.

Range	400–449	450–499	500–549	550–559	600–649	650+
Number of Students	129	275	438	602	620	412

17. P(400–449) 0.052 **18.** P(550–559) 0.243 **19.** P(at least 650) 0.166

Find the odds of an event occurring, given the probability of the event.

20. $\frac{4}{11}$ 4:7 **21.** $\frac{12}{13}$ 12:1 **22.** $\frac{5}{99}$ 5:94 **23.** $\frac{1}{1000}$ 1:999

24. $\frac{5}{16}$ 5:11 **25.** $\frac{3}{95}$ 3:92 **26.** $\frac{9}{70}$ 9:61 **27.** $\frac{8}{15}$ 8:7

Find the probability of an event occurring, given the odds of the event.

28. 2:23 $\frac{2}{25}$ **29.** 2:5 $\frac{2}{7}$ **30.** 15:1 $\frac{15}{16}$ **31.** 9:7 $\frac{9}{16}$

32. 11:14 $\frac{11}{25}$ **33.** 1000:1 $\frac{1000}{1001}$ **34.** 12:17 $\frac{12}{29}$ **35.** 8:13 $\frac{8}{21}$

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12-3 Word Problem Practice

Probability

1. ART The letters “A”, “R”, and “T” are written on three different pieces of paper. The pieces of paper are then put in a bag and mixed up. Logan picks each letter without looking and places them side by side. What is the probability that the letters spell “ART”?

$\frac{1}{6}$

2. AGE There are 24 students in Miss Mason’s third grade class, all born on different days. Eleven students are boys. In the morning, the classroom is empty. One student arrives followed by another. What is the probability that when the first two students arrive, one is a boy and the other a girl?

52%

3. DICE Jamal rolls two six-sided dice, one after the other. What is the probability that the second die shows a number larger than the first die?

$\frac{5}{12}$

4. LANGUAGES Noah cannot decide whether to learn French, German, Italian, Russian, or Chinese. He assigns each language a different number from 0 to 4. He then takes four fair coins and flips them. He decided to take the language corresponding to the number of coins that come up heads. Does Noah’s method for choosing a language give each language the same chance of being chosen? Explain.

No. For example, the probability of getting 0 is 1 in 16 but the probability of getting 1 is 1 in 4.

ICE CREAM For Exercises 5-7, use the following information.

A survey of the students in Mr. Orr’s fifth grade class asked each student to name their favorite flavor of ice cream. The results are shown in the table below.

Flavor	Number of Students
Vanilla	10
Chocolate	9
Butternut	5
Strawberry	4
Banana	1
Coffee	1

5. A student from Mr. Orr’s class is selected at random. What is the probability that the student’s favorite flavor of ice cream is chocolate?

$\frac{3}{10}$

6. A student from Mr. Orr’s class is selected at random. What is the probability that the student’s favorite flavor of ice cream is banana?

$\frac{1}{30}$

7. A student from Mr. Orr’s class is selected at random. Is it more likely that the student prefers either butternut or strawberry or that the student prefers either chocolate or banana?

Chocolate or banana is more likely.

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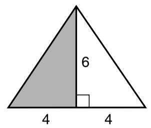
12-3 Enrichment

Geometric Probability

If a dart, thrown at random, hits the triangular board shown at the right, what is the chance that it will hit the shaded region? This chance, also called a probability, can be determined by comparing the area of the shaded region to the area of the board. This ratio indicates what fraction of the tosses should hit in the shaded region.



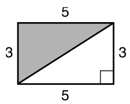
$$\frac{\text{area of shaded region}}{\text{area of triangular board}} = \frac{\frac{1}{2}(4)(6)}{\frac{1}{2}(8)(6)}$$

$$= \frac{12}{24} \text{ or } \frac{1}{2}$$

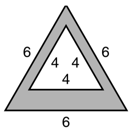
In general, if S is a subregion of some region R, then the probability, $P(S)$, that a point, chosen at random, belongs to subregion S is given by the following.

$$P(S) = \frac{\text{area of subregion } S}{\text{are of region } R}$$

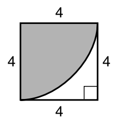
Find the probability that a point, chosen at random, belongs to the shaded subregions of the following regions.

1. 

$\frac{1}{2}$

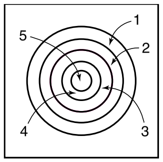
2. 

$\frac{5}{9}$

3. 

$\frac{\pi}{4}$

The dart board shown at the right has 5 concentric circles whose centers are also the center of the square board. Each side of the board is 38 cm, and the radii of the circles are 2 cm, 5 cm, 8 cm, 11 cm, and 14 cm. A dart hitting within one of the circular regions scores the number of points indicated on the board, while a hit anywhere else scores 0 points. If a dart, thrown at random, hits the board, find the probability of scoring the indicated number of points.



4. 0 points $\frac{361 - 49\pi}{361}$

5. 1 point $\frac{75\pi}{1444}$

6. 2 points $\frac{57\pi}{1444}$

7. 3 points $\frac{39\pi}{1444}$

8. 4 points $\frac{21\pi}{1444}$

9. 5 points $\frac{\pi}{361}$

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12-4 Lesson Reading Guide

Multiplying Probabilities

Get Ready for the Lesson

Read the introduction to Lesson 12-4 in your textbook.

Write the probability that Yao Ming made a field goal shot during the 2004–05 season as a fraction in lowest terms. (Your answer should not include a decimal.) $\frac{276}{5}$

Read the Lesson

1. A bag contains 4 yellow balls, 5 red balls, 1 white ball, and 2 black balls. A ball is drawn from the bag and is not replaced. A second ball is drawn.

a. Let Y be the event "first ball is yellow" and B be the event "second ball is black." Are these events *independent* or *dependent*? **dependent**

b. Tell which formula you would use to find the probability that the first ball is yellow and the second ball is black. **C**

A. $P(Y \text{ and } B) = \frac{P(Y)}{P(Y) + P(B)}$

B. $P(Y \text{ and } B) = P(Y) \cdot P(B)$

C. $P(Y \text{ and } B) = P(Y) \cdot P(B \text{ following } Y)$

c. Which equation shows the correct calculation of this probability? **B**

A. $\frac{1}{3} + \frac{2}{11} = \frac{17}{33}$ **B.** $\frac{1}{3} \cdot \frac{2}{11} = \frac{2}{33}$

C. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ **D.** $\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

d. Which equation shows the correct calculation of the probability that if three balls are drawn in succession without replacement, all three will be red? **B**

A. $\frac{5}{12} \cdot \frac{5}{12} \cdot \frac{5}{12} = \frac{125}{1728}$ **B.** $\frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} = \frac{1}{22}$

C. $\frac{5}{12} + \frac{4}{11} + \frac{3}{10} = \frac{713}{660}$

Remember What You Learned

2. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both independent and dependent events? Explain your reasoning. **Sample answer: Just remember the formula for dependent events: $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$. When the events are independent, $P(B \text{ following } A) = P(B)$, so the formula for dependent events simplifies to $P(A \text{ and } B) = P(A) \cdot P(B)$, which is the correct formula for independent events.**

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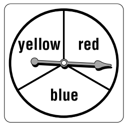
12-4 Study Guide and Intervention

Multiplying Probabilities

Probability of Independent Events

Probability of Two Independent Events	If two events, A and B, are independent, then the probability of both occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

Example **In a board game, each player has 3 different-colored markers. To move around the board, the player first spins a spinner to determine which piece can be moved. He or she then rolls a die to determine how many spaces that colored piece should move. On a given turn, what is the probability that a player will be able to move the yellow piece more than 2 spaces?**



Let A be the event that the spinner lands on yellow, and let B be the event that the die shows a number greater than 2. The probability of A is $\frac{1}{3}$, and the probability of B is $\frac{2}{3}$.

$P(A \text{ and } B) = P(A) \cdot P(B)$ Probability of independent events

$= \frac{1}{3} \cdot \frac{2}{3}$ or $\frac{2}{9}$ Substitute and multiply.

The probability that the player can move the yellow piece more than 2 spaces is $\frac{2}{9}$.

Exercises

A die is rolled 3 times. Find the probability of each event.

1. a 1 is rolled, then a 2, then a 3 $\frac{1}{216}$

2. a 1 or a 2 is rolled, then a 3, then a 5 or a 6 $\frac{1}{54}$

3. 2 odd numbers are rolled, then a 6 $\frac{1}{24}$

4. a number less than 3 is rolled, then a 3, then a number greater than 3 $\frac{1}{36}$

5. A box contains 5 triangles, 6 circles, and 4 squares. If a figure is removed, replaced, and a second figure is picked, what is the probability that a triangle and then a circle will be picked? $\frac{2}{15}$ **or about 0.13**

6. A bag contains 5 red marbles and 4 white marbles. A marble is selected from the bag, then replaced, and a second selection is made. What is the probability of selecting 2 red marbles? $\frac{25}{81}$ **or about 0.31**

7. A jar contains 7 lemon jawbreakers, 3 cherry jawbreakers, and 8 rainbow jawbreakers. What is the probability of selecting 2 lemon jawbreakers in succession providing the jawbreaker drawn first is then replaced before the second is drawn? $\frac{49}{324}$ **or about 0.15**

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12-4 Study Guide and Intervention *(continued)*

Multiplying Probabilities

Probability of Dependent Events

Probability of Two Dependent Events	If two events, A and B, are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

Example 1 **There are 7 dimes and 9 pennies in a wallet. Suppose two coins are to be selected at random, without replacing the first one. Find the probability of picking a penny and then a dime.**

Because the coin is not replaced, the events are dependent.

Thus, $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

$P(\text{penny, then dime}) = P(\text{penny}) \cdot P(\text{dime following penny})$

$\frac{9}{16} \cdot \frac{7}{15} = \frac{21}{80}$

The probability is $\frac{21}{80}$ or about 0.26

Example 2 **What is the probability of drawing, without replacement, 3 hearts, then a spade from a standard deck of cards?**

Since the cards are not replaced, the events are dependent. Let H represent a heart and S represent a spade.

$P(\text{H, H, H, S}) = P(\text{H}) \cdot P(\text{H following H}) \cdot P(\text{H following 2 Hs}) \cdot P(\text{S following 3 Hs})$

$= \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{13}{49}$ or about 0.003

The probability is about 0.003 of drawing 3 hearts, then a spade.

Exercises

Find each probability.

1. The cup on Sophie's desk holds 4 red pens and 7 black pens. What is the probability of her selecting first a black pen, then a red one? $\frac{14}{55}$ **or about 0.25**

2. What is the probability of drawing two cards showing odd numbers from a set of cards that show the first 20 counting numbers if the first card is not replaced before the second is chosen? $\frac{9}{38}$ **or about 0.24**

3. There are 3 quarters, 4 dimes, and 7 nickels in a change purse. Suppose 3 coins are selected without replacement. What is the probability of selecting a quarter, then a dime, and then a nickel? $\frac{1}{26}$ **or about 0.04**

4. A basket contains 4 plums, 6 peaches, and 5 oranges. What is the probability of picking 2 oranges, then a peach if 3 pieces of fruit are selected at random? $\frac{4}{91}$ **or about 0.04**

5. A photographer has taken 8 black and white photographs and 10 color photographs for a brochure. If 4 photographs are selected at random, what is the probability of picking first 2 black and white photographs, then 2 color photographs? $\frac{7}{102}$ **or about 0.07**

Lesson 12-4

Answers

NAME ______________ DATE ______ PERIOD ____

12-4 Skills Practice
Multiplying Probabilities

A die is rolled twice. Find each probability.

1. P(5, then 6) $\frac{1}{36}$ **2.** P(no 2s) $\frac{25}{36}$ **3.** P(two 1s) $\frac{1}{36}$

4. P(any number, then not 5) $\frac{5}{6}$ **5.** P(4, then not 6) $\frac{5}{36}$ **6.** P(not 1, then not 2) $\frac{25}{36}$

A board game uses a set of 6 different cards. Each card displays one of the following figures: a star, a square, a circle, a diamond, a rectangle, or a pentagon. The cards are placed face down, and a player chooses two cards. Find each probability.

7. P(circle, then star), if no replacement occurs $\frac{1}{30}$

8. P(diamond, then square), if replacement occurs $\frac{1}{36}$

9. P(2 polygons), if replacement occurs $\frac{25}{36}$

10. P(2 polygons), if no replacement occurs $\frac{2}{3}$

11. P(circle, then hexagon), if no replacement occurs **0**

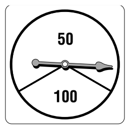
Determine whether the events are *independent* or *dependent*. Then find each probability.

12. A mixed box of herbal teabags contains 2 lemon teabags, 3 orange-mango teabags, 3 chamomile teabags, and 1 apricot-ginger teabag. Kevin chooses 2 teabags at random to bring to work with him. What is the probability that he first chooses a lemon teabag and then a chamomile teabag? **dependent;** $\frac{1}{12}$

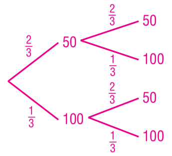
13. The chart shows the selection of olive oils that Hasha finds in a specialty foods catalog. If she randomly selects one type of oil, then randomly selects another, different oil, what is the probability that both selections are domestic, first cold pressed oils? **dependent;** $\frac{21}{820}$

Type of Oil	Domestic	Imported
Pure	2	5
Cold Pressed	4	8
First Cold Pressed	7	15

For Exercises 14 and 15, two thirds of the area of the spinner earns you 50 points. Suppose you spin the spinner twice.



14. Sketch a tree diagram showing all of the possibilities. Use it to find the probability of spinning 50 points, then 100 points. $\frac{2}{9}$



15. What is the probability that you get 100 points on each spin? $\frac{1}{9}$

NAME ______________ DATE ______ PERIOD ____

12-4 Practice
Multiplying Probabilities

A die is rolled three times. Find each probability.

1. P(three 4s) $\frac{1}{216}$ **2.** P(no 4s) $\frac{125}{216}$

3. P(2, then 3, then 1) $\frac{1}{216}$ **4.** P(three different even numbers) $\frac{1}{36}$

5. P(any number, then 5, then 5) $\frac{1}{36}$ **6.** P(even number, then odd number, then 1) $\frac{1}{24}$

There are 3 nickels, 2 dimes, and 5 quarters in a purse. Three coins are selected in succession at random. Find the probability.

7. P(nickel, then dime, then quarter), if no replacement occurs $\frac{1}{24}$

8. P(nickel, then dime, then quarter), if replacement occurs $\frac{3}{100}$

9. P(2 nickels, then 1 quarter), if no replacement occurs $\frac{1}{24}$

10. P(3 dimes), if replacement occurs $\frac{1}{125}$

11. P(3 dimes), if no replacement occurs **0**

For Exercises 12 and 13, determine whether the events are *independent* or *dependent*. Then find each probability.

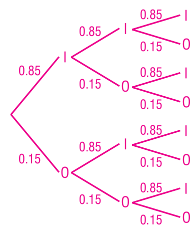
12. Serena is creating a painting. She wants to use 2 more colors. She chooses randomly from 6 shades of red, 10 shades of green, 4 shades of yellow, 4 shades of purple, and 6 shades of blue. What is the probability that she chooses 2 shades of green? **dependent;** $\frac{3}{29}$

13. Kershel's mother is shopping at a bakery. The owner offers Kershel a cookie from a jar containing 22 chocolate chip cookies, 18 sugar cookies, and 15 oatmeal cookies. Without looking, Kershel selects one, drops it back in, and then randomly selects another. What is the probability that neither selection was a chocolate chip cookie? **independent;** $\frac{9}{25}$

14. METEOROLOGY The Fadeeva's are planning a 3-day vacation to the mountains. A long-range forecast reports that the probability of rain each day is 10%. Assuming that the daily probabilities of rain are independent, what is the probability that there is no rain on the first two days, but that it rains on the third day? $\frac{81}{1000}$

RANDOM NUMBERS For Exercises 15 and 16, use the following information.

Anita has a list of 20 jobs around the house to do, and plans to do 3 of them today. She assigns each job a number from 1 to 20, and sets her calculator to generate random numbers from 1 to 20, which can reoccur. Of the jobs, 3 are outside, and the rest are inside.



15. Sketch a tree diagram showing all of the possibilities that the first three numbers generated correspond to inside jobs or outside jobs. Use it to find the probability that the first two numbers correspond to inside jobs, and the third to an outside job. **0.108375**

16. What is the probability that the number generated corresponds to an outside job three times in a row? **0.003375**

Lesson 12-4

NAME ______________________ DATE __________ PERIOD _____

12-4 Word Problem Practice

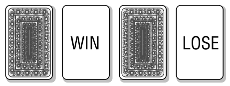
Multiplying Probabilities

1. **BUSSING** Portia and Quinton use the same bus stop when they go to work. They arrive at the bus stop independently of each other. The probability that Portia catches the 7:45 A.M. bus is $\frac{3}{5}$. The probability that Quinton catches the 7:45 A.M. bus is $\frac{1}{2}$. What is the probability that they both catch the 7:45 A.M. bus on the same day? $\frac{3}{10}$

2. **GOODY BAGS** Ryan and Sophia are given goody bags with identical contents. The probability of reaching into either of these goody bags and pulling out a stick of chewing gum is $\frac{1}{10}$. Ryan and Sophia each reach into their own goody bag and randomly pull out something. What is the probability that they both pulled out a stick of chewing gum? $\frac{1}{100}$

3. **PENCILS** A box of pencils contains 11 type 2 pencils and 5 type 3 pencils. Tara picks out a pencil from the box without looking and keeps it. Then, Upton picks out a pencil from the box without looking. What is the probability that Tara picks a type 2 pencil and Upton picks a type 3 pencil? $\frac{11}{48}$

4. **GUESSING GAMES** Valerie is playing a guessing game. Four cards are placed face down before her. The hidden side of each card shows either the word "LOSE" or "WIN". Only one card is labeled "WIN". Valerie is given two chances to find the card labeled "WIN".



What is the probability that she does not pick the "win" card on her first try but does find it with her second? $\frac{1}{4}$

WALLETS For Exercises 5 and 6, use the following information.

Wayne has 1 ten-dollar bill, 2 five-dollar bills, and 5 one-dollar bills in his wallet.

5. Wayne randomly chooses a bill from his wallet, puts it back, then picks another bill, and puts that one back, too. What is the probability that both were five-dollar bills? $\frac{1}{16}$

6. Wayne randomly pulls out a bill from his wallet, and then, without putting it back, randomly pulls a second bill from his wallet. He then puts both bills back into the wallet. What is the probability that both of the bills pulled out were five-dollar bills? $\frac{1}{28}$

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12-4 Enrichment

Conditional Probability

Suppose a pair of dice is thrown. It is known that the sum is greater than seven. Find the probability that the dice match.

The probability of an event given the occurrence of another event is called *conditional probability.* The conditional probability of event A, the dice match, given event B, their sum is greater than seven, is denoted $P(A/B)$.

There are 15 sums greater than seven and there are 36 possible pairs altogether.

$P(B) = \frac{15}{36}$

There are three matching pairs greater than seven.

$P(A \text{ and } B) = \frac{3}{36}$

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(A/B) = \frac{\frac{3}{36}}{\frac{15}{36}} \text{ or } \frac{1}{5}$$

The conditional probability is $\frac{1}{5}$.

A card is drawn from a standard deck of 52 and is found to be red. Given that event, find each of the following probabilities.

1. $P(\text{heart})$ $\frac{1}{2}$
2. $P(\text{ace})$ $\frac{1}{13}$
3. $P(\text{face card})$ $\frac{3}{13}$
4. $P(\text{jack or ten})$ $\frac{2}{13}$
5. $P(\text{six of spades})$ 0
6. $P(\text{six of hearts})$ $\frac{1}{26}$

A sports survey taken at Stirers High School shows that 48% of the respondents liked soccer, 66% liked basketball, and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey and 28% liked soccer and hockey. Finally, 12% liked all three sports. Find each of the following probabilities.

7. The probability Meg likes soccer if she likes basketball. $\frac{30}{66}$ or $\frac{5}{11}$
8. The probability Biff likes basketball if he likes soccer. $\frac{30}{48}$ or $\frac{5}{8}$
9. The probability Muffy likes hockey if she likes basketball. $\frac{22}{66}$ or $\frac{1}{3}$
10. The probability Greg likes hockey and basketball if he likes soccer. $\frac{12}{48}$ or $\frac{1}{4}$

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12-5 Lesson Reading Guide

Adding Probabilities

Get Ready for the Lesson

Read the introduction to Lesson 12-5 in your textbook.

Why do the percentages shown on the bar graph add up to more than 100%? **Sample answer: Many teens do one or more of the listed online activities.**

Read the Lesson

1. Indicate whether the events in each pair are *inclusive* or *mutually exclusive*.

a. Q: drawing a queen from a standard deck of cards
D: drawing a diamond from a standard deck of cards **inclusive**

b. J: drawing a jack from a standard deck of cards
K: drawing a king from a standard deck of cards **mutually exclusive**

2. Marla took a quiz on this lesson that contained the following problem.

Each of the integers from 1 through 25 is written on a slip of paper and placed in an envelope. If one slip is drawn at random, what is the probability that it is odd or a multiple of 5?

Here is Marla's work.

$P(\text{odd}) = \frac{13}{25}$ $\quad P(\text{multiple of } 5) = \frac{5}{25} \text{ or } \frac{1}{5}$

$P(\text{odd or multiple of } 5) = P(\text{odd}) + P(\text{multiple of } 5)$

$= \frac{13}{25} + \frac{5}{25} = \frac{18}{25}$

a. Why is Marla's work incorrect? **Sample answer: Marla used the formula for mutually exclusive events, but the events are inclusive. She should use the formula for inclusive events so that the odd multiples of 5 will not be counted twice.**

b. Show the corrected work.

$P(\text{odd or multiple of } 5) = P(\text{odd}) + P(\text{multiple of } 5) - P(\text{odd multiple of } 5)$

$= \frac{13}{25} + \frac{5}{25} - \frac{3}{25} = \frac{15}{25} = \frac{3}{5}$

Remember What You Learned

3. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both mutually exclusive and inclusive events? Explain your reasoning. **Sample answer: Just remember the formula for inclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. When the events are mutually exclusive, $P(A \text{ and } B) = 0$, so the formula for inclusive events simplifies to $P(A \text{ and } B) = P(A) + P(B)$, which is the correct formula for mutually exclusive events.**

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12-5 Study Guide and Intervention

Adding Probabilities

Mutually Exclusive Events Events that cannot occur at the same time are called mutually exclusive events.

Probability of Mutually Exclusive Events	If two events, A and B, are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

This formula can be extended to any number of mutually exclusive events.

Example 1 **To choose an afternoon activity, summer campers pull slips of paper out of a hat. Today there are 25 slips for a nature walk, 35 slips for swimming, and 30 slips for arts and crafts. What is the probability that a camper will pull a slip for a nature walk or for swimming?**

These are mutually exclusive events. Note that there is a total of 90 slips.

$P(\text{nature walk or swimming}) = P(\text{nature walk}) + P(\text{swimming})$

$= \frac{25}{90} + \frac{35}{90} \text{ or } \frac{2}{3}$

The probability of a camper's pulling out a slip for a nature walk or for swimming is $\frac{2}{3}$.

Example 2 **By the time one tent of 6 campers gets to the front of the line, there are only 10 nature walk slips and 15 swimming slips left. What is the probability that more than 4 of the 6 campers will choose a swimming slip?**

$P(\text{more than 4 swimmers}) = P(5 \text{ swimmers}) + P(6 \text{ swimmers})$

$= \frac{C(10, 1) \cdot C(15, 5)}{C(25, 6)} + \frac{C(10, 0) \cdot C(15, 6)}{C(25, 6)}$

≈ 0.2

The probability of more than 4 of the campers swimming is about 0.2.

Exercises

Find each probability.

1. A bag contains 45 dyed eggs: 15 yellow, 12 green, and 18 red. What is the probability of selecting a green or a red egg? $\frac{2}{3}$

2. The letters from the words LOVE and LIVE are placed on cards and put in a box. What is the probability of selecting an L or an O from the box? $\frac{3}{8}$

3. A pair of dice is rolled, and the two numbers are added. What is the probability that the sum is either a 5 or a 7? $\frac{5}{18}$ **or about 0.28**

4. A bowl has 10 whole wheat crackers, 16 sesame crackers, and 14 rye crisps. If a person picks a cracker at random, what is the probability of picking either a sesame cracker or a rye crisp? $\frac{3}{4}$

5. An art box contains 12 colored pencils and 20 pastels. If 5 drawing implements are chosen at random, what is the probability that at least 4 of them are pastels? **about 0.37**

NAME ______________________ DATE __________ PERIOD _____

12-5 Study Guide and Intervention *(continued)*

Adding Probabilities

Inclusive Events

Probability of Inclusive Events	If two events, A and B, are inclusive, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Example **What is the probability of drawing a face card or a black card from a standard deck of cards?**

The two events are inclusive, since a card can be both a face card and a black card.

$P(\text{face card or black card}) = P(\text{face card}) + P(\text{black card}) - P(\text{black face card})$

$= \frac{3}{13} + \frac{1}{2} - \frac{3}{26}$

$= \frac{8}{13}$ or about 0.62

The probability of drawing either a face card or a black card is about 0.62

Exercises

Find each probability.

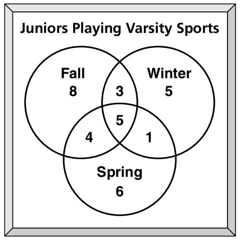
1. What is the probability of drawing a red card or an ace from a standard deck of cards? $\frac{7}{13}$ **or about 0.54**

2. Three cards are selected from a standard deck of 52 cards. What is the probability of selecting a king, a queen, or a red card? $\frac{15}{26}$ **or about 0.58**

3. The letters of the alphabet are placed in a bag. What is the probability of selecting a vowel or one of the letters from the word QUIZ? $\frac{7}{26}$ **or about 0.27**

4. A pair of dice is rolled. What is the probability that the sum is odd or a multiple of 3? $\frac{2}{3}$ **or about 0.67**

5. The Venn diagram at the right shows the number of juniors on varsity sports teams at Elmwood High School. Some athletes are on varsity teams for one season only, some athletes for two seasons, and some for all three seasons. If a varsity athlete is chosen at random from the junior class, what is the probability that he or she plays a fall or winter sport? $\frac{13}{16}$



NAME ______________________ DATE __________ PERIOD _____

12-5 Skills Practice

Adding Probabilities

Eli has 10 baseball cards of 10 different players in his pocket. Three players are pitchers, 5 are outfielders, and 2 are catchers. If Eli randomly selects a card to trade, find each probability.

1. P(pitcher or outfielder) $\frac{4}{5}$ **2.** P(pitcher or catcher) $\frac{1}{2}$ **3.** P(outfielder or catcher) $\frac{7}{10}$

A die is rolled. Find each probability.

4. P(5 or 6) $\frac{1}{3}$ **5.** P(at least a 3) $\frac{2}{3}$ **6.** P(less than 4) $\frac{1}{2}$

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

7. A die is rolled. What is the probability of rolling a 3 or a 4? **mutually exclusive;** $\frac{1}{3}$

8. A die is rolled. What is the probability of rolling an even number or a 4? **inclusive;** $\frac{1}{2}$

9. A card is drawn from a standard deck of cards. What is the probability of drawing a king or a queen? **mutually exclusive;** $\frac{2}{13}$

10. A card is drawn from a standard deck of cards. What is the probability of drawing a jack or a heart? **inclusive;** $\frac{4}{13}$

11. The sophomore class is selling Mother's Day plants to raise money. Susan's prize for being the top seller of plants is a choice of a book, a CD, or a video. She can choose from 6 books, 3 CDs, and 5 videos. What is the probability that Susan selects a book or a CD? **mutually exclusive;** $\frac{9}{14}$

A spinner numbered 1–10 is spun. Find each probability.

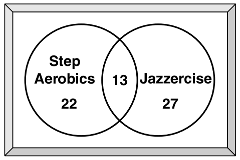
12. P(less than 5 or even) $\frac{7}{10}$ **13.** P(even or odd) **1** **14.** P(prime or even) $\frac{4}{5}$

Two cards are drawn from a standard deck of cards. Find each probability.

15. P(both red or both black) $\frac{25}{51}$ **16.** P(both aces or both red) $\frac{55}{221}$

17. P(both 2s or both less than 5) $\frac{11}{221}$ **18.** P(both black or both less than 5) $\frac{188}{663}$

For Exercises 19 and 20, use the Venn diagram that shows the number of participants in two different kinds of aerobic exercise classes that are offered at a health club. Determine each probability if a person is selected at random from the participants.



19. P(step aerobics or jazzercise, but not both) $\frac{49}{62}$

20. P(step aerobics and jazzercise) $\frac{13}{62}$

Lesson 12-5

Answers

NAME ______________ DATE ________ PERIOD _____

12-5 Practice

Adding Probabilities

An urn contains 7 white marbles and 5 blue marbles. Four marbles are selected without replacement. Find each probability.

1. P(4 white or 4 blue) $\frac{8}{99}$

2. P(exactly 3 white) $\frac{35}{99}$

3. P(at least 3 white) $\frac{14}{33}$

4. P(fewer than 3 white) $\frac{19}{33}$

5. P(3 white or 3 blue) $\frac{49}{99}$

6. P(no white or no blue) $\frac{8}{99}$

Jason and Maria are playing a board game in which three dice are tossed to determine a player's move. Find each probability.

7. P(two 5s) $\frac{5}{72}$

8. P(three 5s) $\frac{1}{216}$

9. P(at least two 5s) $\frac{2}{27}$

10. P(no 5s) $\frac{125}{216}$

11. P(one 5) $\frac{25}{72}$

12. P(one 5 or two 5s) $\frac{5}{12}$

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

13. A clerk chooses 4 CD players at random for floor displays from a shipment of 24 CD players. If 15 of the players have a blue case and the rest have a red case, what is the probability of choosing 4 players with a blue case or 4 players with a red case? **mutual. exclus.; $\frac{71}{506}$**

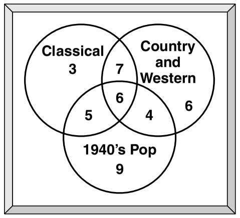
14. A department store employs 28 high school students, all juniors and seniors. Six of the 12 seniors are females and 12 of the juniors are males. One student employee is chosen at random. What is the probability of selecting a senior or a female? **inclusive; $\frac{4}{7}$**

15. A restaurant has 5 pieces of apple pie, 4 pieces of chocolate cream pie, and 3 pieces of blueberry pie. If Janine selects a piece of pie at random for dessert, what is the probability that she selects either apple or chocolate cream? **mutually exclusive; $\frac{3}{4}$**

16. At a statewide meeting, there are 20 school superintendents, 13 principals, and 6 assistant principals. If one of these people is chosen at random, what is the probability that he or she is either a principal or an assistant principal? **mutually exclusive; $\frac{19}{39}$**

17. An airline has one bank of 13 telephones at a reservations office. Of the 13 operators who work there, 8 take reservations for domestic flights and 5 take reservations for international flights. Seven of the operators taking domestic reservations and 3 of the operators taking international reservations are female. If an operator is chosen at random, what is the probability that the person chosen takes domestic reservations or is a male? **inclusive; $\frac{10}{13}$**

18. MUSIC Forty senior citizens were surveyed about their music preferences. The results are displayed in the Venn diagram. If a senior citizen from the survey group is selected at random, what is the probability that he or she likes only country and western music? What is the probability that he or she likes classical and/or country, but not 1940's pop? **$\frac{3}{20}$; $\frac{2}{5}$**



NAME ______________ DATE ________ PERIOD _____

12-5 Word Problem Practice

Adding Probabilities

1. PICK-UP When Tina's parents pick her up from school, there is a $\frac{1}{5}$ chance that she will be in the library, a $\frac{1}{2}$ chance that she will be on the playground, and a $\frac{3}{10}$ chance that she will be in her classroom. What is the probability that when Tina's parents pick her up, she is found in her classroom or on the playground? **$\frac{4}{5}$**

2. TRAVEL John is randomly selected to be given a chance to win a new car. He must choose a red or yellow marble from a bag containing 1 red, 2 yellow, 10 green, and 12 blue marbles. What is the probability he will win the car? **$\frac{3}{25}$**

3. DICE Alexis rolls two identical dice. What is the probability that the sum of the numbers rolled is odd? What is the probability that the sum of the numbers rolled is greater than 7? What is the probability that the sum of the numbers rolled is odd or greater than 7?

$P(\text{odd}) = \frac{1}{2}$; $P(>7) = \frac{5}{12}$;

$P(\text{odd or } >7) = \frac{3}{4}$

4. CLASSES At Jackson High School, 56 of the eleventh graders take physics and 70 of them take biology. There are 400 eleventh graders in total at the school. An eleventh grader is chosen at random from among all the eleventh graders at the high school. The probability that the selected student takes physics and biology is $\frac{11}{40}$. How many students at the high school take physics or biology? **16**

PASSENGERS For Exercises 5 and 6, use the following information.

On an airplane flight, some passengers travel with carry-on luggage while others travel with a suitcase. Some passengers travel with carry-on luggage and a suitcase. Everyone travels with some form of luggage.

5. On one flight, there was no passenger with both carry-on luggage and a suitcase. On this flight are the events of picking a passenger with carry-on luggage and picking a passenger with a suitcase mutually exclusive? **Yes**

6. On another flight, there are 120 passengers. Of those 120 passengers, 80 have carry-on luggage and 70 have a suitcase. What is the probability that a passenger has both carry-on luggage and a suitcase? **$\frac{1}{4}$**

Lesson 12-5

NAME ______________________ DATE ________ PERIOD _____

12-5 Enrichment

Probability and Tic-Tac-Toe

What would be the chances of winning at tic-tac-toe if it were turned into a game of pure chance? To find out, the nine cells of the tic-tac-toe board are numbered from 1 to 9 and nine chips (also numbered from 1 to 9) are put into a bag. Player A draws a chip at random and enters an *X* in the corresponding cell. Player B does the same and enters an *O*.

To solve the problem, assume that both players draw all their chips without looking and all *X* and *O* entries are made at the same time. There are four possible outcomes: a draw, A wins, B wins, and either A or B can win.

There are 16 arrangements that result in a draw. Reflections and rotations must be counted as shown below.

o x o x o x 4 x o x	x o x o o x 4 x x o	o o x x x o 8 o x x

There are 36 arrangements in which either player may win because both players have winning triples.

x x x o o o 4 x o x	x x x x o x 4 o o o	x o x x x x 4 o o o	x x x x x o 8 o o o	x x x o o o 8 x x o	x x o x x x 8 o o o

In these 36 cases, A's chances of winning are $\frac{13}{40}$.

1. Find the 12 arrangements in which B wins and A cannot.

o o x **x o x 8** **x x o**	**o x o** **x o x 4** **x x o**

2. Below are 12 of the arrangements in which A wins and B cannot. Write the numbers to show the reflections and rotations for each arrangement. What is the total number? **62**

o x o x x x **1** o x o	x o x o x o **1** x o x	x x x x o o **4** x o o	x x x o x o **4** o x o	x o o x x x **4** o o x	x o o x x o **4** o o x
x x o o x x **4** o o x	x x x o x o **8** o o x	x x x x o o **8** o x o	x x x x o o **8** o o x	x o o x x x **8** o x o	x x o o x o **8** x o x

3. There are $\frac{9!}{(5!4!)}$ different and equally probable distributions. Complete the chart to find the probability for a draw or for A or B to win.

Draw:	$\frac{16}{126}$	$= \mathbf{\frac{8}{63}}$
A wins:	$\mathbf{\frac{62}{126}} + \frac{13}{40}\left(\frac{36}{126}\right)$	$= \mathbf{\frac{737}{1260}}$
B wins:	$\mathbf{\frac{12}{126}} + \mathbf{\frac{27}{40}\left(\frac{36}{126}\right)}$	$= \mathbf{\frac{121}{420}}$

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12-5 Graphing Calculator Activity

Probabilities

A graphing calculator can be used to perform calculations involving permutations, combinations, and probability.

Example 1 **There are 5 girls and 3 boys on a class committee. A subcommittee of 3 people is being chosen at random. What is the probability that the subcommittee will have at least 2 girls?**

P(at least 2 girls) = *P*(2 girls) + *P*(3 girls). Each probability is the product of the combinations of girls and boys divided by the combinations of all the students taken 3 at a time.

```
(5 nCr 2*3 nCr 1
+5 nCr 3*3 nCr 0
)/8 nCr 3▸Frac
              5/7
```

Keystrokes: (5 MATH ◀ 3 2 × 3 MATH ◀ 3 1 + 5 MATH ◀ 3 3 × 3 MATH ◀ 3 0) ÷ 8 MATH ◀ 3 3 MATH ENTER ENTER .

The probability that the subcommittee has at least 2 girls is $\frac{5}{7}$.

Example 2 **Two cards are randomly selected from a standard deck of cards. Find the probability that both cards are kings or that both cards are red.**

Since these events are mutually inclusive find the combinations of 4 kings taken 2 at a time plus 26 red cards taken 2 at a time minus 2 red kings taken 2 at a time divided by the combinations of 52 cards taken 2 at a time.

```
(4 nCr 2+26 nCr
2-2 nCr 2)/52 nC
r 2▸Frac
           55/221
```

Keystrokes: (4 MATH ◀ 3 2 + 26 MATH ◀ 3 2 − 2 MATH ◀ 3 2) ÷ 52 MATH ◀ 3 2 MATH ENTER ENTER .

The probability of choosing 2 kings or two red cards is $\frac{55}{221}$.

Exercises

Find each probability.

1. There are 5 girls and 4 boys on the school publications committee. A group of 5 members is being chosen at random to attend a workshop on school newspapers. Find each probability.

a. at least 3 girls $\mathbf{\frac{10}{21}}$ **b.** 4 girls or 4 boys $\mathbf{\frac{25}{126}}$ **c.** at least 2 boys $\mathbf{\frac{10}{21}}$

2. Two cards are drawn from a standard deck of cards. Find each probability.

a. both queens or both black $\mathbf{\frac{55}{221}}$ **b.** both kings or both aces $\mathbf{\frac{2}{221}}$ **c.** both face cards or both black $\mathbf{\frac{188}{663}}$

3. Find the probability that a committee of 6 U.S. Representatives selected at random from 7 Democrats and 7 Republicans will have at least 3 Republicans on the committee. $\mathbf{\frac{302}{409}}$

4. Three CDs are randomly selected from a collection of 6 rock and 5 rap CDs. Find the probability that at least 2 are rock. $\mathbf{\frac{19}{33}}$

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12-6 Lesson Reading Guide

Statistical Measures

Get Ready for the Lesson

Read the introduction to Lesson 12-6 in your textbook.

There is more than one way to give an "average" score for this test. Three measures of central tendency for these scores are 94, 76.5 and 73.9. Can you tell which of these is the mean, the median, and the mode without doing any calculations? Explain your answer.

Sample answer: Yes. The mode must be one of the scores, so it must be an integer. The median must be either one of the scores or halfway between two of the scores, so it must be an integer or a decimal ending with .5. Therefore, 94 is the mode, 76.5 is the median, and 73.9 is the mean.

Read the Lesson

1. Match each measure with one of the six descriptions of how to find measures of central tendency and variation.

a. median **vi** **b.** mode **i** **c.** range **iv**

d. variance **iii** **e.** mean **ii** **f.** standard deviation **v**

i. Find the most commonly occurring values or values in a set of data.

ii. Add the data and divide by the number of items.

iii. Find the mean of the squares of the differences between each value in the set of data and the mean.

iv. Find the difference between the largest and smallest values in the set of data.

v. Take the positive square root of the variance.

vi. If there is an odd number of items in a set of data, take the middle one. If there is an even number of items, add the two middle items and divide by 2.

Remember What You Learned

2. It is usually easier to remember a complicated procedure if you break it down into steps. Write the procedure for finding the standard deviation for a set of data in a series of brief, numbered steps.

Sample answer:
1. Find the mean.
2. Find the difference between each value and the mean.
3. Square each difference.
4. Find the mean of the squares.
5. Take the positive square root.

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12-6 Study Guide and Intervention

Statistical Measures

Measures of Central Tendency

Measures of Central Tendency	Use	When
	mean	the data are spread out and you want an average of values
	median	the data contain outliers
	mode	the data are tightly clustered around one or two values

Example **Find the mean, median, and mode of the following set of data: {42, 39, 35, 40, 38, 35, 45}.**

To find the mean, add the values and divide by the number of values.

$$\text{mean} = \frac{42 + 39 + 35 + 40 + 38 + 35 + 45}{7} \approx 39.14.$$

To find the median, arrange the values in ascending or descending order and choose the middle value. (If there is an even number of values, find the mean of the two middle values.) In this case, the median is 39.

To find the mode, take the most common value. In this case, the mode is 35.

Exercises

Find the mean, median, and mode of each set of data. Round to the nearest hundredth, if necessary.

1. {238, 261, 245, 249, 255, 262, 241, 245} **249.5; 247; 245**

2. {9, 13, 8, 10, 11, 9, 12, 16, 10, 9} **10.7; 10; 9**

3. {120, 108, 145, 129, 102, 132, 134, 118, 108, 142} **123.8; 124.5; 108**

4. {68, 54, 73, 58, 63, 72, 65, 70, 61} **64.89; 65; no mode**

5. {34, 49, 42, 38, 40, 45, 34, 28, 43, 30} **38.3; 39; 34**

6. The table at the right shows the populations of the six New England capitals. Which would be the most appropriate measure of central tendency to represent the data? Explain why and find that value. **There is no mode. The population of Boston is an outlier and would raise the mean too high. The median, 79,500, would be the best choice.**

City	Population (rounded to the nearest 1000)
Augusta, ME	19,000
Boston, MA	589,000
Concord, NH	37,000
Hartford, CT	122,000
Montpelier, VT	8,000
Providence, RI	174,000

Source: www.factfinder.census.gov

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12-6 Study Guide and Intervention *(continued)*

Statistical Measures

Measures of Variation The *range* and the **standard deviation** measure how scattered a set of data is.

Standard Deviation	If a set of data consists of the n values $x_1, x_2, \ldots, x_n$ and has mean $\bar{x}$, then the standard deviation is given by $\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}}$.

The square of the standard deviation is called the **variance**.

Example **Find the variance and standard deviation of the data set {10, 9, 6, 9, 18, 4, 8, 20}.**

Step 1 Find the mean.

$$\bar{x} = \frac{10 + 9 + 6 + 9 + 18 + 4 + 8 + 20}{8} = 10.5$$

Step 2 Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n} \quad \text{Standard variance formula}$$

$$= \frac{(10 - 10.5)^2 + (9 - 10.5)^2 + \ldots + (20 - 10.5)^2}{8}$$

$$= \frac{220}{8} \text{ or } 27.5$$

Step 3 Find the standard deviation.

$$\sigma = \sqrt{27.5}$$

$$\approx 5.2$$

The variance is 27.5 and the standard deviation is about 5.2.

Exercises

Find the variance and standard deviation of each set of data. Round to the nearest tenth.

1. {100, 89, 112, 104, 96, 108, 93}
58.5; 7.6

2. {62, 54, 49, 62, 48, 53, 50}
29.4; 5.4

3. {8, 9, 8, 8, 9, 7, 8, 9, 6}
0.9; 0.9

4. {4.2, 5.0, 4.7, 4.5, 5.2, 4.8, 4.6, 5.1}
0.1; 0.3

5. The table at the right lists the prices of ten brands of breakfast cereal. What is the standard deviation of the values to the nearest penny? **$0.33**

Price of 10 Brands of Breakfast Cereal	
$2.29	$3.19
$3.39	$2.79
$2.99	$3.09
$3.19	$2.59
$2.79	$3.29

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12-6 Skills Practice

Statistical Measures

Find the variance and standard deviation of each set of data to the nearest tenth.

1. {32, 41, 35, 35, 46, 42} **23.6, 4.9**

2. {13, 62, 77, 24, 38, 19, 88} **763.8, 27.6**

3. {89, 99, 42, 16, 42, 71, 16} **959.1, 31.0**

4. {450, 400, 625, 225, 300, 750, 650, 625} **30,537.1; 174.7**

5. {17, 23, 65, 94, 33, 33, 33, 8, 57, 75, 44, 12, 11, 68, 39} **630.7, 25.1**

6. {7.2, 3.1, 3.8, 9.5, 8.3, 8.4} **5.8, 2.4**

7. {1.5, 2.5, 3.5, 4.5, 4.5, 5.5, 6.5, 7.5} **3.5, 1.9**

For Exercises 8 and 9, use the table that shows the profit in billions of dollars reported by U.S. manufacturers for the first quarter of the years from 1997 through 2001.

Year	1997	1998	1999	2000	2001
Seasonally-Adjusted Profit ($ billions)	$61.4	$75.6	$60.9	$78.5	$45.3

Source: U. S. Census Bureau

8. Find the mean and median of the data to the nearest tenth. **$64.3 billion, $61.4 billion**

9. Which measure of central tendency best represents the data? Explain.
The median is more representative because the value 45.3 is not close to the other data points, and it lowers the mean.

For Exercises 10 and 11, use the table that shows the percent of fourth grade students reading at or above the proficiency level in a nationally-administered reading assessment.

Year	1992	1994	1998	2000
Percent at or above proficiency level	29%	30%	31%	32%

Source: National Center for Education Statistics

10. Find the mean, median, and standard deviation of the data to the nearest tenth.
30.5%, 30.5%, 1.1%

11. What do the statistics from Exercise 11 tell you about the data?
Sample answer: Since the median and mean are equal and the standard deviation is small, the percent of students reading at or above the proficiency level has not varied much from 1992 to 2000.

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12-6 Practice

Statistical Measures

Find the variance and standard deviation of each set of data to the nearest tenth.

1. {47, 61, 93, 22, 82, 22, 37}
673.1, 25.9

2. {10, 10, 54, 39, 96, 91, 91, 18}
1228.6, 35.1

3. {1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5}
1.6, 1.2

4. {1100, 725, 850, 335, 700, 800, 950}
49,150.0; 221.7

5. {3.4, 7.1, 8.5, 5.1, 4.7, 6.3, 9.9, 8.4, 3.6}
4.7, 2.2

6. {2.8, 0.5, 1.9, 0.8, 1.9, 1.5, 3.3, 2.6, 0.7, 2.5}
0.8, 0.9

7. HEALTH CARE Eight physicians with 15 patients on a hospital floor see these patients an average of 18 minutes a day. The 22 nurses on the same floor see the patients an average of 3 hours a day. As a hospital administrator, would you quote the mean, median, or mode as an indicator of the amount of daily medical attention the patients on this floor receive? Explain. **Either the median or the mode; they are equal and higher than the mean, which is lowered by the smaller amount of time the physicians spend with the patients.**

For Exercises 8–10, use the frequency table that shows the percent of public school teachers in the U. S. in 1999 who used computers or the Internet at school for various administrative and teaching activities.

Activity	Percent Using Computer or Internet
Create instructional materials	39
Administrative record keeping	34
Communicate with colleagues	23
Gather information for planning lessons	16
Multimedia classroom presentations	8
Access research and best practices for teaching	8
Communicate with parents or students	8
Access model lesson plans	6

Source: National Assessment of Educational Progress

8. Find the mean, median, and mode of the data. **17.75%, 12%, 8%**

9. Suppose you believe teachers use computers or the Internet too infrequently. Which measure would you quote as the "average?" Explain. **Mode; it is lowest.**

10. Suppose you believe teachers use computers or the Internet too often. Which measure would you quote as the "average?" Explain. **Mean; it is highest.**

For Exercises 11 and 12, use the frequency table that shows the number of games played by 24 American League baseball players between opening day, 2001 and September 8, 2001.

No. of Games	Frequency
141	4
140	3
139	4
138	5
137	2
136	3
135	3

Source: Major League Baseball

11. Find the mean, median, mode, and standard deviation of the number of games played to the nearest tenth.
138.2, 138; 138, 2.0

12. For how many players is the number of games within one standard deviation of the mean? **14**

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12-6 Word Problem Practice

Statistical Measures

1. SPORTS The table below shows the number of times some teams in the National Football League have won the Super Bowl.

NFL Team	Number of Super Bowl Victories
New England	3
Baltimore	2
Kansas City	1
St. Louis	1
Denver	2
Green Bay	1
Dallas	5
San Francisco	5
Oakland	2
Pittsburgh	5
Miami	2
Washington	3
NY Giants	2
NY Jets	1
Chicago	1

Source: www.pubquizhelp.34sp.com

Which statistical measure represents the team(s) with the least Super Bowl victories? **the mode**

2. SALARIES The median salary in a small company is \$10.20 per hour. What percentage of the employees at the company earns more than \$10.20 per hour? **50%**

3. RANDOM GENERATORS Samuel has written a computer program to generate a random selection of the following two-digit numbers.

25, 67, 54, 99, 41, 87, 90, 18, 32

Find the mean, median, and mode of this data. **57; 54; none**

4. HEIGHTS The following table lists the heights of some of the great NBA players.

Player	Height (in inches)
Kareem Abdul-Jabbar	86
Larry Bird	81
Shaquille O'Neal	85
Wilt Chamberlain	85
Michael Jordan	78

Source: www.sidwell.edu

Find the mean and standard deviation of the data in the table. Round your answer to the nearest hundredth.
83; 3.0

METEORS For Exercises 5-8, use the following information.

Arlene stayed up late one night to watch the Perseid meteor shower. She recorded the number of meteors she saw every ten minutes starting at 1 A.M. and going until 4 A.M. Her data are shown below.

8, 7, 8, 12, 17, 15, 22, 28, 29, 31, 28, 23, 29, 28, 25, 23, 15, 12

5. What is the mean of this data set?
20

6. What is the median of this data set?
22.5

7. What is the mode of this data set?
28

8. What is the standard deviation of this data set? Round your answer to the nearest hundredth.
8.05

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12-6 Enrichment

Standard Deviation of Sample Data

A *population* is the set of all measurements of interest to an investigator. A *sample* is a subset of measurements selected from the population of interest. A *statistic* is any quantity whose value can be calculated from sample data. A common mistake is to use the terms *probability* and *statistics* interchangeably. Probabilities are used to make statements from a population to a sample, but statistics are calculated from a sample and are to make inferences about a population.

The *range* is a statistic calculated by taking the difference between the largest observation and the smallest observation. Range = $x_{max} - x_{min}$.

The *sample variance* is calculated using the formula: $s^2 = \frac{\sum_{i=1}^{n}(\bar{x} - x_i)^2}{n-1}$ where $\bar{x}$ is the sample mean. Therefore, the *sample standard deviation* is the square root of the sample variance, $s = \sqrt{s^2}$.

To calculate the sample variance:

1. Calculate the sample mean. For example, suppose a sample contains the numbers {2, 5, 6, 9, 11}. The sample mean is $\bar{x} = \frac{2+5+6+9+11}{5} = 6.6$.

2. Next use the formula above to calculate the sample variance, in this case:

$$s^2 = \frac{(6.6-2)^2 + (6.6-5)^2 + (6.6-6)^2 + (6.6-9)^2 + (6.6-11)^2}{4} = 12.3.$$

3. Finally, the sample standard deviation is equal to 3.507 by taking the square root of 12.3.

Exercises

1. What are some differences in the formula for the sample variance compared to the formula for the population variance? **It uses the sample mean instead of the population mean, and since the sample mean is an estimator for the population mean, the denominator is *n* − 1 instead of *n*.**

2. Given the random sample {5, 7, 1, 2, 4}, find the sample variance. **5.70**

3. Calculate the sample standard deviation. **2.387**

4. Calculate the range of the sample data {5, 7, 1, 2, 4}. **6**

5. An approximation for the sample standard deviation is given by: $s \approx \frac{Range}{4}$. Compare this answer to your answer from 3. **6**

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12-7 Reading to Learn Mathematics

The Normal Distribution

Get Ready for the Lesson

Read the introduction to Lesson 12-7 in your textbook.

There were 66 players on the team and the mean height was approximately 74.1. About what fraction of the players' heights are between 72 and 75, inclusive?

Sample answer: about $\frac{1}{2}$

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. In a continuous probability distribution, there is a finite number of possible outcomes. **false**

b. Every normal distribution can be represented by a bell curve. **true**

c. A distribution that is represented by a curve that is high at the left and has a tail to the right is negatively skewed. **false**

d. A normal distribution is an example of a skewed distribution. **false**

2. Ms. Rose gave the same quiz to her two geometry classes. She recorded the following scores.

First-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	0	1	0	3	4	5	7	4	3	2

Fifth-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	0	0	0	3	4	9	7	6	1	0

In each class, 30 students took the quiz. The mean score for each class was 6.4. Which set of scores has the greater standard deviation? (Answer this question without doing any calculations.) Explain your answer.

First period class; sample answer: The scores are more spread out from the mean than for the fifth period class.

Remember What You Learned

3. Many students have trouble remembering how to determine if a curve represents a distribution that is *positively skewed* or *negatively skewed*. What is an easy way to remember this?

Sample answer: Follow the tail! If the tail is on the right (positive direction), the distribution is positively skewed. If the tail is on the left (negative direction), the distribution is negatively skewed.

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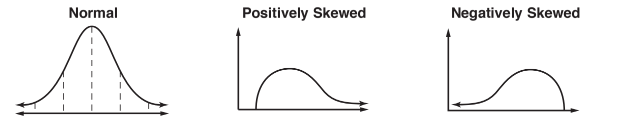
12-7 Study Guide and Intervention

The Normal Distribution

Normal and Skewed Distributions A **continuous probability** distribution is represented by a curve.

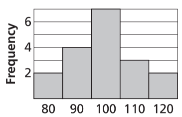
Types of Continuous Distributions: Normal, Positively Skewed, Negatively Skewed



Example **Determine whether the data below appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.**
{100, 120, 110, 100, 110, 80, 100, 90, 100, 120, 100, 90, 110, 100, 90, 80, 100, 90}

Make a frequency table for the data.

Value	80	90	100	110	120
Frequency	2	4	7	3	2

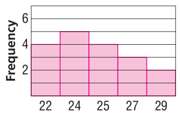


Then use the data to make a histogram.
Since the histogram is roughly symmetric, the data appear to be normally distributed.

Exercises

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. Make a histogram of the data.

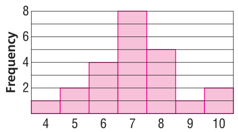
1. {27, 24, 29, 25, 27, 22, 24, 25, 29, 24, 25, 22, 27, 24, 22, 25, 24, 22}
positively skewed



2.

Shoe Size	4	5	6	7	8	9	10
No. of Students	1	2	4	8	5	1	2

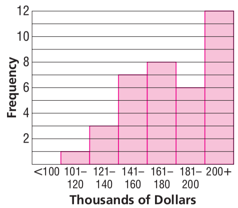
normally distributed



3.

Housing Price	No. of Houses Sold
less than $100,000	0
$100,00–$120,000	1
$121,00–$140,000	3
$141,00–$160,000	7
$161,00–$180,000	8
$181,00–$200,000	6
over $200,000	12

negatively skewed



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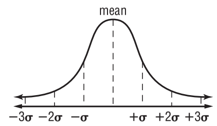
12-7 Study Guide and Intervention *(continued)*

The Normal Distribution

Use Normal Distributions

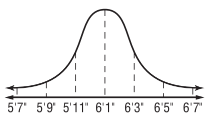


Normal distributions have these properties.
The graph is maximized at the mean.
The mean, median, and mode are about equal.
About 68% of the values are within one standard deviation of the mean.
About 95% of the values are within two standard deviations of the mean.
About 99% of the values are within three standard deviations of the mean.

Example **The heights of players in a basketball league are normally distributed with a mean of 6 feet 1 inch and a standard deviation of 2 inches.**

a. What is the probability that a player selected at random will be shorter than 5 feet 9 inches?

Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.



The value of 5 feet 9 inches is 2 standard deviations below the mean, so approximately 2.5% of the players will be shorter than 5 feet 9 inches.

b. If there are 240 players in the league, about how many players are taller than 6 feet 3 inches?

The value of 6 feet 3 inches is one standard deviation above the mean. Approximately 16% of the players will be taller than this height.

$240 \times 0.16 \approx 38$

About 38 of the players are taller than 6 feet 3 inches.

Exercises

EGG PRODUCTION The number of eggs laid per year by a particular breed of chicken is normally distributed with a mean of 225 and a standard deviation of 10 eggs.

1. About what percent of the chickens will lay between 215 and 235 eggs per year? 68%

2. In a flock of 400 chickens, about how many would you expect to lay more than 245 eggs per year? 10 chickens

MANUFACTURING The diameter of bolts produced by a manufacturing plant is normally distributed with a mean of 18 mm and a standard deviation of 0.2 mm.

3. What percent of bolts coming off of the assembly line have a diameter greater than 18.4 mm? 2.5%

4. What percent have a diameter between 17.8 and 18.2 mm? 68%

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12-7 Skills Practice

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

1.

Miles Run	Track Team Members
0–4	3
5–9	4
10–14	7
15–19	5
20–23	2

normally distributed

2.

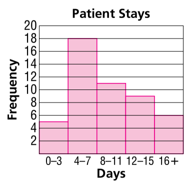
Speeches Given	Political Candidates
0–5	1
6–11	2
12–17	3
18–23	8
24–29	8

negatively skewed

For Exercises 3 and 4, use the frequency table that shows the average number of days patients spent on the surgical ward of a hospital last year.

Days	Number of Patients
0–3	5
4–7	18
8–11	11
12–15	9
16+	6

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.
Positively skewed; the histogram is high at the left and has a tail to the right.

DELIVERY For Exercises 5–7, use the following information.

The time it takes a bicycle courier to deliver a parcel to his farthest customer is normally distributed with a mean of 40 minutes and a standard deviation of 4 minutes.

5. About what percent of the courier's trips to this customer take between 36 and 44 minutes? **68%**

6. About what percent of the courier's trips to this customer take between 40 and 48 minutes? **47.5%**

7. About what percent of the courier's trips to this customer take less than 32 minutes? **2.5%**

TESTING For Exercises 8–10, use the following information.

The average time it takes sophomores to complete a math test is normally distributed with a mean of 63.3 minutes and a standard deviation of 12.3 minutes.

8. About what percent of the sophomores take more than 75.6 minutes to complete the test? **16%**

9. About what percent of the sophomores take between 51 and 63.3 minutes? **34%**

10. About what percent of the sophomores take less than 63.3 minutes to complete the test? **50%**

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12-7 Practice

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

1.

Time Spent at a Museum Exhibit	
Minutes	Frequency
0–25	27
26–50	46
51–75	89
75–100	57
100+	24

normally distributed

2.

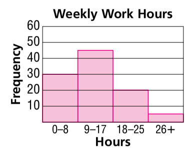
Average Age of High School Principals	
Age in Years	Number
31–35	3
36–40	8
41–45	15
46–50	32
51–55	40
56–60	38
60+	4

negatively skewed

For Exercises 3 and 4, use the frequency table that shows the number of hours worked per week by 100 high school seniors.

Hours	Number of Students
0–8	30
9–17	45
18–25	20
26+	5

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.
Positively skewed; the histogram is high at the left and has a tail to the right.

TESTING For Exercises 5–10, use the following information.

The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 15.

5. About what percent of the scores are between 70 and 130? **95%**

6. About what percent of the scores are between 85 and 130? **81.5%**

7. About what percent of the scores are over 115? **16%**

8. About what percent of the scores are lower than 85 or higher than 115? **32%**

9. If 80 people take the test, how many would you expect to score higher than 130? **2**

10. If 75 people take the test, how many would you expect to score lower than 85? **12**

11. TEMPERATURE The daily July surface temperature of a lake at a resort has a mean of 82° and a standard deviation of 4.2°. If you prefer to swim when the temperature is at least 77.8°, about what percent of the days does the temperature meet your preference? **84%**

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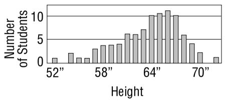
12-7 Word Problem Practice

The Normal Distribution

1. **PARKING** Over several years, Bertram conducted a study of how far into parking spaces people tend to park by measuring the distance from the end of a parking space to the front fender of a car parked in the space. He discovered that the distribution of the data closely approximated a normal distribution with mean 8.5 inches. He found that about 5% of cars parked more than 11.5 inches away from the end of the parking space. What percentage of cars would you expect parked less than 5.5 inches away from the end of the parking space?
5%

2. **HEIGHT** Chandra's graph of the number of tenth grade students of different heights is shown below.



Is the data positively skewed, negatively skewed, or normally distributed?
Negatively skewed

3. **OVENS** An oven manufacturer tries to make the temperature setting on its ovens as accurate as possible. However, if one measures the actual temperatures in the ovens when the temperature setting is 350°F, they will differ slightly from 350°F. The set of actual temperatures for all the ovens is normally distributed around 350°F with a standard deviation of 0.5°F. About what percentage of ovens will be between 350°F and 351°F when their temperature setting is 350°F?
47.5%

4. **LIGHT BULBS** The time that a certain brand of light bulb will last before burning out is normally distributed. About 2.5% of the bulbs last longer than 6800 hours and about 16% of the bulbs last longer than 6500 hours. How long does the average bulb last?
6200 hours

DOGS For Exercises 5-8, use the following information.

The weights of adult greyhound dogs are normally distributed. The mean weight is about 69 pounds and the standard deviation is about 10 pounds.

5. Approximately what percentage of adult greyhound dogs would you expect weigh between 59 and 79 pounds?
68%

6. Approximately what percentage of adult greyhound dogs would you expect weigh more than 99 pounds?
0.5%

7. Approximately what percentage of adult greyhound dogs would you expect weigh less than 49 pounds?
2.5%

8. What would you expect an adult greyhound dog to weigh if it weighed less than 0.5% of an average adult greyhound?
39 lbs or less.

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12-7 Enrichment

Calculating Z-Scores

The normal distribution is the most important probability distribution. Many physical measurements have distributions approximately normal. Examples include height, weight, and measures of intelligence. More importantly, even if the individual variables are not normally distributed, sums and averages tend to still be normally distributed. Unfortunately, normal probability distribution functions are difficult to calculate. Fortunately, statisticians have compiled a table for a normal distribution with mean of zero and standard deviation of one. This is called the Standard Normal Distribution and is typically denoted by N(0, 1), where the N indicates a normal distribution which has mean, μ (mu) = 0, and standard deviation, σ (sigma) = 1.

Suppose the variable x is normally distributed with mean μ and standard deviation σ. In order to calculate probabilities of this normal distribution, we must standardize the variable x by an appropriate transformation. The letter Z denotes the transformed variable and is called the Z-score, which is a measure of relative standing. The following steps are needed to complete the transformation.

- If the mean and standard deviation are not given, then calculate the mean and standard deviation of the given (population) data.
- Define $Z = \frac{x - \mu}{\sigma}$.

Example Find the standard normal variable Z given and $\mu = 15$ and $\sigma = 3$.

Apply the transform to the variable X using the definition above, that is: $Z = \frac{X - 15}{3}$.

1. Suppose that the time, X, to complete an exam is normally distributed. The time, in minutes, of a class of 12 to complete the exam is given in the table. Transform X to a Z-score.

Student	1	2	3	4	5	6	7	8	9	10	11	12
Time	35	42	48	33	32	39	40	52	48	34	36	44

Mean = 40.25, Standard Deviation = 6.34
$Z = (X - 40.25) / 6.34$

2. Suppose that a random variable X is normally distributed with $\mu = 20$ and $\sigma = 5$. Convert the following probability statements to the equivalent statements by standardizing X.

Example $P(X < 25) = P\left(\frac{X - 20}{5} < 25\right) = P(Z < 25)$

a. $P(X > 18)$ **$P\left(Z > \frac{18 - 20}{5}\right) = P\left(Z > -\frac{2}{5}\right)$**

b. $P(17 < X < 23)$ **$P\left(\frac{17 - 20}{5} < Z < \frac{23 - 20}{5}\right) = P\left(-\frac{3}{5} < Z < \frac{3}{5}\right)$**

c. $P(X < 19)$ **$P\left(Z < \frac{19 - 20}{5}\right) = P\left(Z < -\frac{1}{5}\right)$**

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12-8 Lesson Reading Guide

Exponential and Binomial Distribution

Get Ready for the Lesson

Read the introduction to Lesson 12-8 in your textbook.

Is a randomly chosen student likely to be one that talks on the phone for a very short period of time or for a very long period of time? **No, most people will talk an average amount of time.**

Read the Lesson

1. Indicate whether each situation can be represented using an exponential distribution or a binomial distribution.

a. You are trying to predict how many times a coin will land with the tails side up if you flip it 50 times. **binomial distribution**

b. You would like to find the probability that there will be more than 5 pink gumballs in a bag of assorted color gumballs. **binomial distribution**

c. You are trying to predict how long your refrigerator will last. **exponential distribution**

d. You are calculating the probability that a person in your class will be taller than 5 feet, 5 inches **binomial distribution**

e. You would like to determine the percentage of cellular phones that will last longer than 7 years and the percentage that will last longer than 5 years. **exponential distribution**

f. You want to predict the probability that a person in your neighborhood is older than you are. **binomial distribution**

2. Write an equation that can be used to calculate each of the following:

a. The expected number of successes in a binomial distribution that has a 30% rate of success when there are 50 trials. $E = (50)(0.3)$

b. The probability that a randomly selected number from an exponential distribution will be greater than 4 if the mean is 1.5. $P = e^{-\left(\frac{2}{3}\right)(4)}$

Remember What You Learned

3. In binomial distributions, the only possible outcomes are success and failure, but sometimes binomial experiments include events that can have several results. Explain how this is possible. **For example, in 1b, there are assorted colors of gumballs, but you are concerned about how many pink ones there will be. For evaluation as a binomial distribution, a "success" is pink. Any other color is considered a "failure".**

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12-8 Study Guide and Intervention

Exponential and Binomial Distribution

Exponential Distribution

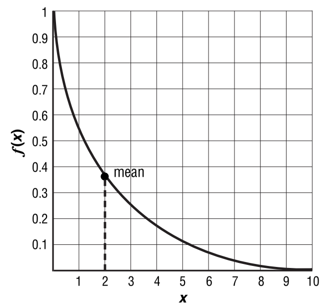
Exponential Distributions are used to predict the probabilities of events based on time.

Probability that a randomly chosen domain value for the exponential function $f(x)$ will be greater than the given value of x. The value m is the multiplicative inverse of the mean.	$f(x) = e^{-mx}$
Probability that a randomly chosen domain value for the exponential function $f(x)$ will be less than the given value of x. The value m is the multiplicative inverse of the mean.	$f(x) = 1 - e^{-mx}$

Example **An exponential distribution function has a mean of 2. Graph the distribution function and label the mean. What is the probability that a randomly chosen value of x will be less than 3?**

The equation for the function will be $f(x) = e^{-mx}$, where m is the multiplicative inverse of the mean. Since the mean is 2, the value of m will be $\frac{1}{2}$ or 0.5. Substituting the value of m into the equation for an exponential distribution, $f(x) = e^{-0.5x}$.

Since the function is applicable when x is greater than zero, the graph only includes the first quadrant x and y values. The y-axis represents the probability, which ranges from 0 to 1.



The probability that a randomly chosen value will be less than 3 can be found using the graph or the equation for the distribution. From the graph, the value of $f(x)$ is about 0.22 when x is 3, which means that the probability that x is greater than 3 is 0.22. Remember to subtract the value from one when you want to know the probability that a randomly chosen value will be less than the given value. The probability that x will be less than 3 is $1 - 0.22$, or 0.88.

To use the equation developed for the function to find the probability, substitute 3 for the value of x into the equation $f(x) = 1 - e^{-0.5x}$ and solve. The probability that x will be greater than 3 is 0.88.

Exercises

1. Write the equation for an exponential distribution that has a mean of 0.5. $f(x) = e^{-2x}$

An exponential distribution has a mean of 6. Find each probability.

2. $x > 7$ **0.31** **3.** $x > 10$ **0.19** **4.** $x > 2$ **0.72** **5.** $x < 9$ **0.78** **6.** $x < 7$ **0.69**