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7 Anticipation Guide

Polynomials

Step 1 *Before you begin Chapter 7*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. When multiplying two powers that have the same base, multiply the exponents.	D
	2. $(k^3)^4$ is equivalent to k^{12}.	A
	3. To divide two powers that have the same base, subtract the exponents.	A
	4. $\left(\frac{2}{5}\right)^3$ is the same as $\frac{2^3}{5}$.	D
	5. A polynomial may contain one or more monomials.	A
	6. The degree of the polynomial $3x^2y^3 - 5y^2 + 8x^3$ is 3 because the highest exponent is 3.	D
	7. The sum of the two polynomials $(3x^2y - 4xy^2 + 2y^3)$ and $(6xy^2 + 2x^2y - 7)$ in simplest form is $5x^2y + 2xy^2 + 2y^3 - 7$.	A
	8. $(4m^2 + 2m - 3) - (m^2 - m + 3)$ is equal to $3m^2 + m$.	D
	9. Because there are different exponents in each factor, the distributive property cannot be used to multiply $3n^3$ by $(2n^2 + 4n - 12)$.	D
	10. The FOIL method of multiplying two binomials stands for *First, Outer, Inner, Last*.	A
	11. The square of $r + t$, $(r + t)^2$, will always equal $r^2 + t^2$.	D
	12. The product of $(x + y)$ and $(x - y)$ will always equal $x^2 - y^2$.	A

Step 2 *After you complete Chapter 7*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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7-1 Study Guide and Intervention

Multiplying Monomials

Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 Simplify $(3x^6)(5x^2)$.

$(3x^6)(5x^2) = (3)(5)(x^6 \cdot x^2)$ Group the coefficients and the variables

$= (3 \cdot 5)(x^{6+2})$ Product of Powers

$= 15x^8$ Simplify.

The product is $15x^8$.

Example 2 Simplify $(-4a^3b)(3a^2b^5)$.

$(-4a^3b)(3a^2b^5) = (-4)(3)(a^3 \cdot a^2)(b \cdot b^5)$

$= -12(a^{3+2})(b^{1+5})$

$= -12a^5b^6$

The product is $-12a^5b^6$.

Exercises

Simplify each expression.

1. $y(y^5)$ **y^6**

2. $n^2 \cdot n^7$ **n^9**

3. $(-7x^2)(x^4)$ **$-7x^6$**

4. $x(x^2)(x^4)$ **x^7**

5. $m \cdot m^5$ **m^6**

6. $(-x^3)(-x^4)$ **x^7**

7. $(2a^2)(8a)$ **$16a^3$**

8. $(rs)(rn^3)(n^2)$ **r^2n^6**

9. $(x^2y)(4xy^3)$ **$4x^3y^4$**

10. $\frac{1}{3}(2a^3b)(6b^3)$ **$4a^3b^4$**

11. $(-4x^3)(-5x^7)$ **$20x^{10}$**

12. $(-3j^2k^4)(2jk^6)$ **$-6j^3k^{10}$**

13. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$ **$a^3b^2c^7$**

14. $(-5xy)(4x^2)(y^4)$ **$-20x^3y^5$**

15. $(10x^3yz^2)(-2xy^5z)$ **$-20x^4y^6z^3$**

Answers

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7-1 Study Guide and Intervention *(continued)*

Multiplying Monomials

Simplify Expressions An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and all integers m and n, $(a^m)^n = a^{mn}$.
Power of a Product	For any number a and all integers m and n, $(ab)^n = a^n b^n$.

We can combine and use these properties to simplify expressions involving monomials.

Example **Simplify $(-2ab^2)^3(a^2)^4$.**

$(-2ab^2)^3(a^2)^4 = (-2ab^2)^3(a^8)$ Power of a Power

$= (-2)^3(a^3)(b^2)^3(a^8)$ Power of a Product

$= (-2)^3(a^3)(a^8)(b^2)^3$ Group the coefficients and the variables

$= (-2)^3(a^{11})(b^2)^3$ Product of Powers

$= -8a^{11}b^6$ Power of a Power

The product is $-8a^{11}b^6$.

Exercises

Simplify each expression.

1. $(y^5)^2$ — y^{10}
2. $(n^7)^4$ — n^{28}
3. $(x^2)^5(x^3)$ — x^{13}
4. $-3(ab^4)^3$ — $-3a^3b^{12}$
5. $(-3ab^4)^3$ — $-27a^3b^{12}$
6. $(4x^2b)^3$ — $64x^6b^3$
7. $(4a^2)^2(b^3)$ — $16a^4b^3$
8. $(4x)^2(b^3)$ — $16x^2b^3$
9. $(x^2y^4)^5$ — $x^{10}y^{20}$
10. $(2a^3b^2)(b^3)^2$ — $2a^3b^8$
11. $(-4xy)^3(-2x^2)^3$ — $512x^9y^3$
12. $(-3j^2k^3)^2(2j^2k)^3$ — $72j^{10}k^9$
13. $(25a^2b)^3\left(\frac{1}{5}abf\right)^2$ — $625a^8b^5f^2$
14. $(2xy)^2(-3x^2)(4y^4)$ — $-48x^4y^6$
15. $(2x^3y^2z^2)^3(x^2z)^4$ — $8x^{17}y^6z^{10}$
16. $(-2n^6y^5)(-6n^3y^2)(ny)^3$ — $12n^{12}y^{10}$
17. $(-3a^3n^4)(-3a^3n)^4$ — $-243a^{15}n^8$
18. $-3(2x)^4(4x^5y)^2$ — $-768x^{14}y^2$

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7-1 Skills Practice

Multiplying Monomials

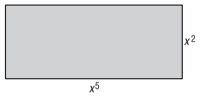
Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. 11 **Yes; 11 is a real number and an example of a constant.**
2. $a - b$ **No; this is the difference, not the product, of two variables.**
3. $\frac{p^2}{r^2}$ **No; this is the quotient, not the product, of two variables.**
4. y **Yes; single variables are monomials.**
5. j^3k **Yes; this is the product of two variables.**
6. $2a + 3b$ **No; this is the sum of two monomials.**

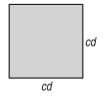
Simplify.

7. $a^2(a^3)(a^6)$ — a^{11}
8. $x(x^2)(x^7)$ — x^{10}
9. $(y^2z)(yz^2)$ — y^3z^3
10. $(\ell^2k^2)(\ell^3k)$ — ℓ^5k^3
11. $(a^2b^4)(a^2b^2)$ — a^4b^6
12. $(cd^2)(c^3d^2)$ — c^4d^4
13. $(2x^2)(3x^5)$ — $6x^7$
14. $(5a^7)(4a^2)$ — $20a^9$
15. $(4xy^3)(3x^3y^5)$ — $12x^4y^8$
16. $(7a^5b^2)(a^2b^3)$ — $7a^7b^5$
17. $(-5m^3)(3m^8)$ — $-15m^{11}$
18. $(-2c^4d)(-4cd)$ — $8c^5d^2$
19. $(10^2)^3$ — 10^6 **or** 1,000,000
20. $(p^3)^{12}$ — p^{36}
21. $(-6p)^2$ — $36p^2$
22. $(-3y)^3$ — $-27y^3$
23. $(3pr^2)^2$ — $9p^2r^4$
24. $(2b^3c^4)^2$ — $4b^6c^8$

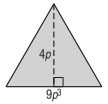
GEOMETRY Express the area of each figure as a monomial.

25. 

x^7

26. 

c^2d^2

27. 

$18p^4$

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7-1 Practice

Multiplying Monomials

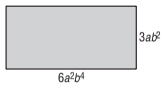
Determine whether each expression is a monomial. Write *yes* or *no*. Explain your reasoning.

1. $\frac{21a^2}{7b}$ **No; this involves the quotient, not the product, of variables.**
2. $\frac{b^3c^2}{2}$ **Yes; this is the product of a number, $\frac{1}{2}$, and two variables.**

Simplify each expression.

3. $(-5x^2y)(3x^4)$ $-15x^6y$
4. $(2ab^2f^2)(4a^3b^2f^2)$ $8a^4b^4f^4$
5. $(3ad^4)(-2a^2)$ $-6a^3d^4$
6. $(4g^3h)(-2g^5)$ $-8g^8h$
7. $(-15xy^4)\left(-\frac{1}{3}xy^3\right)$ $5x^2y^7$
8. $(-xy)^3(xz)$ $-x^4y^3z$
9. $(-18m^2n)^2\left(-\frac{1}{6}mn^2\right)$ $-54m^5n^4$
10. $(0.2a^2b^3)^2$ $0.04a^4b^6$
11. $\left(\frac{2}{3}p\right)^2$ $\frac{4}{9}p^2$
12. $\left(\frac{1}{4}ad^3\right)^2$ $\frac{1}{16}a^2d^6$
13. $(0.4k^3)^3$ $0.064k^9$
14. $[(4^2)^2]^2$ 4^8 **or** $65{,}536$

GEOMETRY Express the area of each figure as a monomial.

15. 

$18a^3b^6$

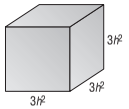
16. 

$(25x^6)\pi$

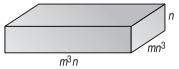
17. 

$12a^3b^4$

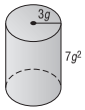
GEOMETRY Express the volume of each solid as a monomial.

18. 

$27h^6$

19. 

m^4n^5

20. 

$(63g^4)\pi$

21. **COUNTING** A panel of four light switches can be set in 2^4 ways. A panel of five light switches can set in twice this many ways. In how many ways can five light switches be set? 2^5 **or** 32

22. **HOBBIES** Tawa wants to increase her rock collection by a power of three this year and then increase it again by a power of two next year. If she has 2 rocks now, how many rocks will she have after the second year? 2^6 **or** 64

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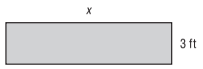
7-1 Word Problem Practice

Multiplying Monomials

1. **GRAVITY** An egg that has been falling for x seconds has dropped at an average speed of $16x$ feet per second. If the egg is dropped from the top of a building, its total distance traveled is the product of the average rate times the time. Write a simplified expression to show the distance the egg has traveled after x seconds. $16x^2$

2. **CIVIL ENGINEERING** A developer is planning a sidewalk for a new development. The sidewalk can be installed in rectangular sections that have a fixed width of 3 feet and a length that can vary. Assuming that each section is the same length, express the area of a 4-section sidewalk as a monomial. $12x$



3. **PROBABILITY** If you flip a coin 3 times in a row, there are 2^3 outcomes that can occur.

Outcomes	
HHH	TTT
HTT	THH
HTH	TTH
HHT	THT

If you then flip the coin two more times, there are $2^3 \times 2^2$ outcomes that can occur. How many outcomes can occur if you flip the quarter as mentioned above plus four more times? Write your answer in the form 2^x. 2^9

4. **SPORTS** The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. Find the volume of air in three different basketballs. Use $\pi = 3.14$. Round your answers to the nearest whole number.

Ball	Radius (in.)	Volume (in³)
Child's	4	268
Women's	4.5	382
HTH	4.8	463

Source: WikiAnswers

5. **ELECTRICITY** An electrician uses the formula $W = I^2R$, where W is the power in watts, I is the current in amperes, and R is the resistance in ohms.

   a. Find the power in a household circuit that has 20 amperes of current and 5 ohms of resistance. **2000 watts**

   b. If the current is reduced by one half, what happens to the power?

   The power is one-fourth the previous amount.

Answers

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7-1 Enrichment

An Wang

An Wang (1920–1990) was an Asian-American who became one of the pioneers of the computer industry in the United States. He grew up in Shanghai, China, but came to the United States to further his studies in science. In 1948, he invented a magnetic pulse controlling device that vastly increased the storage capacity of computers. He later founded his own company, Wang Laboratories, and became a leader in the development of desktop calculators and word processing systems. In 1988, Wang was elected to the National Inventors Hall of Fame.

Digital computers store information as numbers. Because the electronic circuits of a computer can exist in only one of two states, open or closed, the numbers that are stored can consist of only two digits, 0 or 1. Numbers written using only these two digits are called **binary numbers**. To find the decimal value of a binary number, you use the digits to write a *polynomial in 2*. For instance, this is how to find the decimal value of the number 1001101_2. (The subscript 2 indicates that this is a binary number.)

$$\begin{aligned} 1001101_2 &= 1 \times \underline{2}^6 + 0 \times \underline{2}^5 + 0 \times \underline{2}^4 + 1 \times \underline{2}^3 + 1 \times \underline{2}^2 + 0 \times \underline{2}^1 + 1 \times \underline{2}^0 \\ &= 1 \times \underline{64} + 0 \times \underline{32} + 0 \times \underline{16} + 1 \times \underline{8} + 1 \times \underline{4} + 0 \times \underline{2} + 1 \times \underline{1} \\ &= 64 + 0 + 0 + 8 + 4 + 0 + 1 \\ &= 77 \end{aligned}$$

Find the decimal value of each binary number.

1. 1111_2 **15**
2. 10000_2 **16**
3. 11000011_2 **195**
4. 10111001_2 **185**

Write each decimal number as a binary number.

5. 8 **1000_2**
6. 11 **1011_2**
7. 29 **11101_2**
8. 117 **1110101_2**

9. The chart at the right shows a set of decimal code numbers that is used widely in storing letters of the alphabet in a computer's memory. Find the code numbers for the letters of your name. Then write the code for your name using binary numbers. **Answers will vary.**

The American Standard Guide for Information Interchange (ASCII)							
A	65	N	78	a	97	n	110
B	66	O	79	b	98	o	111
C	67	P	80	c	99	p	112
D	68	Q	81	d	100	q	113
E	69	R	82	e	101	r	114
F	70	S	83	f	102	s	115
G	71	T	84	g	103	t	116
H	72	U	85	h	104	u	117
I	73	V	86	i	105	v	118
J	74	W	87	j	106	w	119
K	75	X	88	k	107	x	120
L	76	Y	89	l	108	y	121
M	77	Z	90	m	109	z	122

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7-2 Study Guide and Intervention

Dividing Monomials

Quotients of Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 **Simplify $\frac{a^4b^7}{ab^2}$. Assume that no denominator equals zero.**

$$\begin{aligned} \frac{a^4b^7}{ab^2} &= \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right) && \text{Group powers with the same base.} \\ &= (a^{4-1})(b^{7-2}) && \text{Quotient of Powers} \\ &= a^3b^5 && \text{Simplify.} \end{aligned}$$

The quotient is a^3b^5.

Example 2 **Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that no denominator equals zero.**

$$\begin{aligned} \left(\frac{2a^3b^5}{3b^2}\right)^3 &= \frac{(2a^3b^5)^3}{(3b^2)^3} && \text{Power of a Quotient} \\ &= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3} && \text{Power of a Product} \\ &= \frac{8a^9b^{15}}{27b^6} && \text{Power of a Power} \\ &= \frac{8a^9b^9}{27} && \text{Quotient of Powers} \end{aligned}$$

The quotient is $\frac{8a^9b^9}{27}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{5^5}{5^2}$ **5^3 or 125**
2. $\frac{m^6}{m^4}$ **m^2**
3. $\frac{p^5n^4}{p^2n}$ **p^3n^3**
4. $\frac{a^2}{a}$ **a**
5. $\frac{x^5y^3}{x^5y^2}$ **y**
6. $\frac{-2y^7}{14y^5}$ **$-\frac{1}{7}y^2$**
7. $\frac{xy^6}{y^4x}$ **y^2**
8. $\left(\frac{2a^2b}{a}\right)^3$ **$8a^3b^3$**
9. $\left(\frac{4p^4r^4}{3p^2r^2}\right)^3$ **$\frac{64}{27}p^6r^6$**
10. $\left(\frac{2r^5w^3}{r^4w^3}\right)^4$ **$16r^4$**
11. $\left(\frac{3r^6n^3}{2r^5n}\right)^4$ **$\frac{81}{16}r^4n^8$**
12. $\frac{r^7n^7t^2}{n^3r^3t^2}$ **r^4n^4**

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7-2 Study Guide and Intervention *(continued)*

Dividing Monomials

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example **Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that no denominator equals zero.**

$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$ Group powers with the same base.

$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$ Quotient of Powers and Negative Exponent Properties

$= \frac{1}{4}a^{-5}b^0c^5$ Simplify.

$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$ Negative Exponent and Zero Exponent Properties

$= \frac{c^5}{4a^5}$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{2^2}{2^{-3}}$ **2^5 or 32**
2. $\frac{m}{m^{-4}}$ **m^5**
3. $\frac{p^{-8}}{p^3}$ **$\frac{1}{p^{11}}$**
4. $\frac{b^{-4}}{b^{-5}}$ **b**
5. $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$ **$\frac{w}{4y^2}$**
6. $\frac{(a^2b^3)^2}{(ab)^{-2}}$ **a^6b^8**
7. $\frac{x^4y^0}{x^{-2}}$ **x^6**
8. $\frac{(6a^{-1}b)^2}{(b^2)^4}$ **$\frac{36}{a^2b^6}$**
9. $\frac{(3rt)^2u^{-4}}{r^{-1}t^2u^7}$ **$\frac{9r^3}{u^{11}}$**
10. $\frac{m^{-3}t^{-5}}{(m^2t^3)^{-1}}$ **$\frac{1}{mt^2}$**
11. $\left(\frac{4m^2n^2}{8m^{-1}\ell}\right)^0$ **1**
12. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$ **$-\frac{m^3}{32n^{10}}$**

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7-2 Skills Practice

Dividing Monomials

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{6^5}{6^4}$ **6^1 or 6**
2. $\frac{9^{12}}{9^8}$ **9^4 or 6561**
3. $\frac{x^4}{x^2}$ **x^2**
4. $\frac{r^3t^2}{r^3t^4}$ **$\frac{1}{t^2}$**
5. $\frac{m}{m^3}$ **$\frac{1}{m^2}$**
6. $\frac{9d^7}{3d^6}$ **$3d$**
7. $\frac{12n^5}{36n}$ **$\frac{n^4}{3}$**
8. $\frac{w^4x^3}{w^4x}$ **x^2**
9. $\frac{a^3b^5}{ab^2}$ **a^2b^3**
10. $\frac{m^7p^2}{m^3p^2}$ **m^4**
11. $\frac{-21w^5x^2}{7w^4x^5}$ **$-\frac{3w}{x^3}$**
12. $\frac{32x^3y^2z^5}{-8xyz^2}$ **$-4x^2yz^3$**
13. $\left(\frac{4p^7}{7r^2}\right)^2$ **$\frac{16p^{14}}{49r^4}$**
14. 4^{-4} **$\frac{1}{4^4}$ or $\frac{1}{256}$**
15. 8^{-2} **$\frac{1}{8^2}$ or $\frac{1}{64}$**
16. $\left(\frac{5}{3}\right)^{-2}$ **$\frac{9}{25}$**
17. $\left(\frac{9}{11}\right)^{-1}$ **$\frac{11}{9}$**
18. $\frac{h^3}{h^{-6}}$ **h^9**
19. $k^0(k^4)(k^{-6})$ **$\frac{1}{k^2}$**
20. $k^{-1}(\ell^{-6})(m^3)$ **$\frac{m^3}{k\ell^6}$**
21. $\frac{f^{-7}}{f^4}$ **$\frac{1}{f^{11}}$**
22. $\left(\frac{16p^5w^2}{2p^8w^3}\right)^0$ **1**
23. $\frac{f^{-5}g^4}{h^{-2}}$ **$\frac{g^4h^2}{f^5}$**
24. $\frac{15x^6y^{-9}}{5xy^{-11}}$ **$3x^5y^2$**
25. $\frac{-15t^0u^{-1}}{5u^3}$ **$-\frac{3}{u^4}$**
26. $\frac{48x^6y^7z^5}{-6xy^5z^6}$ **$-\frac{8x^5y^2}{z}$**

Answers

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7-2 Practice

Dividing Monomials

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{8^8}{8^4}$ **8^4 or 4096**
2. $\frac{a^4b^6}{ab^3}$ **a^3b^3**
3. $\frac{xy^2}{xy}$ **y**
4. $\frac{m^5np}{m^4p}$ **mn**
5. $\frac{5c^2d^3}{-4c^2d}$ **$-\frac{5d^2}{4}$**
6. $\frac{8y^7z^6}{4y^6z^5}$ **$2yz$**
7. $\left(\frac{4f^3g}{3h^6}\right)^3$ **$\frac{64f^9g^3}{27h^{18}}$**
8. $\left(\frac{6w^5}{7p^6r^3}\right)^2$ **$\frac{36w^{10}}{49p^{12}r^6}$**
9. $\frac{-4x^2}{24x^5}$ **$-\frac{1}{6x^3}$**
10. $x^3(y^{-5})(x^{-8})$ **$\frac{1}{x^5y^5}$**
11. $p(q^{-2})(r^{-3})$ **$\frac{p}{q^2r^3}$**
12. 12^{-2} **$\frac{1}{144}$**
13. $\left(\frac{3}{7}\right)^{-2}$ **$\frac{49}{9}$**
14. $\left(\frac{4}{3}\right)^{-4}$ **$\frac{81}{256}$**
15. $\frac{22r^3s^2}{11r^2s^{-3}}$ **$2rs^5$**
16. $\frac{-15w^0u^{-1}}{5u^3}$ **$-\frac{3}{u^4}$**
17. $\frac{8c^3d^2f^4}{4c^{-1}d^2f^{-3}}$ **$2c^4f^7$**
18. $\left(\frac{x^{-3}y^5}{4^{-3}}\right)^0$ **1**
19. $\frac{6f^{-2}g^3h^5}{54f^{-5}g^{-5}h^3}$ **$\frac{g^8h^2}{9}$**
20. $\frac{-12t^{-1}u^5x^{-4}}{2t^{-3}ux^5}$ **$-\frac{6t^2u^4}{x^9}$**
21. $\frac{r^4}{(3r)^3}$ **$\frac{r}{27}$**
22. $\frac{m^{-2}n^{-5}}{(m^4n^3)^{-1}}$ **$\frac{m^2}{n^2}$**
23. $\frac{(j^{-1}k^3)^{-4}}{j^3k^3}$ **$\frac{j}{k^{15}}$**
24. $\frac{(2a^{-2}b)^{-3}}{5a^2b^4}$ **$\frac{a^4}{40b^7}$**
25. $\left(\frac{q^{-1}r^3}{qr^{-2}}\right)^{-5}$ **$\frac{q^{10}}{r^{25}}$**
26. $\left(\frac{7c^{-3}d^3}{c^5dh^{-4}}\right)^{-1}$ **$\frac{c^8}{7d^2h^4}$**
27. $\left(\frac{2x^3y^2z}{3x^4yz^{-2}}\right)^{-2}$ **$\frac{9x^2}{4y^2z^6}$**

28. **BIOLOGY** A lab technician draws a sample of blood. A cubic millimeter of the blood contains 22^3 white blood cells and 22^5 red blood cells. What is the ratio of white blood cells to red blood cells? **$\frac{1}{484}$**

29. **COUNTING** The number of three-letter "words" that can be formed with the English alphabet is 26^3. The number of five-letter "words" that can be formed is 26^5. How many times more five-letter "words" can be formed than three-letter "words"? **676**

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7-2 Word Problem Practice

Dividing Monomials

1. **CHEMISTRY** The nucleus of a certain atom is 10^{-13} centimeters across. If the nucleus of a different atom is 10^{-11} centimeters across, how many times as large is it as the first atom? **100**

2. **SPACE** The Moon is approximately 25^4 kilometers away from Earth on average. The Olympus Mons volcano on Mars stands 25 kilometers high. How many Olympus Mons volcanoes, stacked on top of one another, would fit between the surface of the Earth and the Moon?
$25^3 = 15{,}625$

3. **E-MAIL** Spam (also known as junk e-mail) consists of identical messages sent to thousands of e-mail users. People often obtain anti-spam software to filter out the junk e-mail messages they receive. Suppose Yvonne's anti-spam software filtered out 10^2 e-mails, and she received 10^4 e-mails last year. What fraction of her e-mails were filtered out? Write your answer as a monomial.
10^{-2}

4. **METRIC MEASUREMENT** Consider a dust mite that measures 10^{-3} millimeters in length and a caterpillar that measures 10 centimeters long. How many times as long as the mite is the caterpillar?
$10^5 = 100{,}000$

5. **COMPUTERS** In 1995, standard capacity for a personal computer hard drive was 40 megabytes (MB). In 2010, a standard hard drive capacity was 500 gigabytes (GB or Gig). Refer to the table below.

Memory Capacity Approximate Conversions
8 bits = 1 byte
10^3 bytes = 1 kilobyte
10^3 kilobytes = 1 megabyte (meg)
10^3 megabytes = 1 gigabyte (gig)
10^3 gigabytes = 1 terabyte
10^3 terabytes = 1 petabyte

a. The newer hard drives have about how many times the capacity of the 1995 drives? **12,500**

b. Predict the hard drive capacity in the year 2025 if this rate of growth continues. **6.25 petabytes**

c. One kilobyte of memory is what fraction of one terabyte? **$\frac{1}{10^9} = 10^{-9}$**

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7-2 Enrichment

Patterns with Powers

Use your calculator, if necessary, to complete each pattern.

a.		b.		c.	
2^{10} =	1024	5^{10} =	9,765,625	4^{10} =	1,048,576
2^9 =	512	5^9 =	1,953,125	4^9 =	262,144
2^8 =	256	5^8 =	390,625	4^8 =	65,536
2^7 =	128	5^7 =	78,125	4^7 =	16,384
2^6 =	64	5^6 =	15,625	4^6 =	4096
2^5 =	32	5^5 =	3125	4^5 =	1024
2^4 =	16	5^4 =	625	4^4 =	256
2^3 =	8	5^3 =	125	4^3 =	64
2^2 =	4	5^2 =	25	4^2 =	16
2^1 =	2	5^1 =	5	4^1 =	4

Study the patterns for a, b, and c above. Then answer the questions.

1. Describe the pattern of the exponents from the top of each column to the bottom.
   The exponents decrease by one from each row to the one below.

2. Describe the pattern of the powers from the top of the column to the bottom. **To get each power, divide the power on the row above by the base (2, 5, or 4).**

3. What would you expect the following powers to be?
   2^0 **1** 5^0 **1** 4^0 **1**

4. Refer to Exercise 3. Write a rule. Test it on patterns that you obtain using 22, 25, and 24 as bases. **Any nonzero number to the zero power equals one.**

Study the pattern below. Then answer the questions.

$0^3 = 0$ $0^2 = 0$ $0^1 = 0$ $0^0 =$? 0^{-1} does not exist. 0^{-2} does not exist. 0^{-3} does not exist.

5. Why do 0^{-1}, 0^{-2}, and 0^{-3} not exist?
   Negative exponents are not defined unless the base is nonzero.

6. Based upon the pattern, can you determine whether 0^0 exists?
   No, since the pattern $0^n = 0$ breaks down for $n \subset 1$.

7. The symbol 0^0 is called an **indeterminate**, which means that it has no unique value. Thus it does not exist as a unique real number. Why do you think that 0^0 cannot equal 1?
   Answers will vary. One answer is that if $0^0 = 1$, then $1 = \frac{1}{1} = \frac{1^0}{0^0} = \left(\frac{1}{0}\right)^0$, which is a false result, since division by zero is not allowed. Thus, 0^0 cannot equal 1.

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7-3 Study Guide and Intervention

Scientific Notation

Scientific Notation Very large and very small numbers are often best represented using a method known as **scientific notation**. Numbers written in scientific notation take the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer. Any number can be written in scientific notation.

Example 1 **Express 34,020,000,000 in scientific notation.**

Step 1 Move the decimal point until it is to the right of the first nonzero digit. The result is a real number a. Here, $a = 3.402$.

Step 2 Note the number of places n and the direction that you moved the decimal point. The decimal point moved 10 places to the left, so $n = 10$.

Step 3 Because the decimal moved to the left, write the number as $a \times 10^n$.

$34{,}020{,}000{,}000 = 3.4020000000 \times 10^{10}$

Step 4 Remove the extra zeros. 3.402×10^{10}

Example 2 **Express 4.11×10^{-6} in standard notation.**

Step 1 The exponent is –6, so $n = -6$.

Step 2 Because $n < 0$, move the decimal point 6 places to the left.

$4.11 \times 10^{-6} \Rightarrow .00000411$

Step 3 $4.11 \times 10^{-6} \Rightarrow 0.00000411$

Rewrite; insert a 0 before the decimal point.

Exercises

Express each number in scientific notation.

1. 5,100,000 **5.1×10^6**	2. 80,300,000,000 **8.03×10^{10}**	3. 14,250,000 **1.425×10^7**
4. 68,070,000,000,000 **6.807×10^{13}**	5. 14,000 **1.4×10^4**	6. 901,050,000,000 **9.0105×10^{11}**
7. 0.0049 **4.9×10^{-3}**	8. 0.000301 **3.01×10^{-4}**	9. 0.0000000519 **5.19×10^{-8}**
10. 0.000000185 **1.85×10^{-7}**	11. 0.002002 **2.002×10^{-3}**	12. 0.00000771 **7.71×10^{-6}**

Express each number in standard form.

13. 4.91×10^4 **49,100**	14. 3.2×10^{-5} **0.000032**	15. 6.03×10^8 **603,000,000**
16. 2.001×10^{-6} **0.000002001**	15. 1.00024×10^{10} **10,002,400,000**	18. 5×10^5 **500,000**
19. 9.09×10^{-5} **0.0000909**	20. 3.5×10^{-2} **0.035**	21. 1.7087×10^7 **17,087,000**

Answers

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7-3 Study Guide and Intervention *(continued)*

Scientific Notation

Products and Quotients in Scientific Notation You can use scientific notation to simplify multiplying and dividing very large and very small numbers.

Example 1 **Evaluate $(9.2 \times 10^{-3})(4 \times 10^{8})$. Express the result in both scientific notation and standard form.**

$(9.2 \times 10^{-3})(4 \times 10^{8})$	Original Expression
$= (9.2 \times 4)(10^{-3} \times 10^{8})$	Commutative and Associative Properties
$= 36.8 \times 10^{5}$	Product of Powers
$= (3.68 \times 10^{1}) \times 10^{5}$	$36.8 = 3.68 \times 10$
$= 3.68 \times 10^{6}$	Product of Powers
$= 3{,}680{,}000$	Standard Form

Example 2 **Evaluate $\frac{(6.9 \times 10^{7})}{(3 \times 10^{5})}$. Express the result in both scientific notation and standard form.**

$\frac{(2.76 \times 10^{7})}{(6.9 \times 10^{5})} = \left(\frac{2.76}{6.9}\right)\left(\frac{10^{7}}{10^{5}}\right)$	Product rule for fractions
$= 0.4 \times 10^{2}$	Quotient of Powers
$= 4.0 \times 10^{-1} \times 10^{2}$	$0.4 = 4.0 \times 10^{-1}$
$= 4.0 \times 10^{1}$	Product of Powers
$= 40$	Standard form

Exercises

Evaluate each product. Express the results in both scientific notation and standard form.

1. $(3.4 \times 10^{3})(5 \times 10^{4})$
   1.7×10^{8}; 170,000,000
2. $(2.8 \times 10^{-4})(1.9 \times 10^{7})$
   5.32×10^{3}; 5320
3. $(6.7 \times 10^{-7})(3 \times 10^{3})$
   2.01×10^{-3}; 0.00201
4. $(8.1 \times 10^{5})(2.3 \times 10^{-3})$
   1.863×10^{3}; 1863
5. $(1.2 \times 10^{-11})(6 \times 10^{6})$
   7.2×10^{-5}; 0.000072
6. $(5.9 \times 10^{4})(7 \times 10^{-8})$
   4.13×10^{-3}; 0.00413

Evaluate each quotient. Express the results in both scientific notation and standard form.

7. $\frac{(4.9 \times 10^{-3})}{(2.5 \times 10^{-4})}$
   1.96×10^{1}; 19.6
8. $\frac{5.8 \times 10^{4}}{5 \times 10^{-2}}$
   1.16×10^{6}; 1,160,000
9. $\frac{(1.6 \times 10^{5})}{(4 \times 10^{-4})}$
   4.0×10^{8}; 400,000,000
10. $\frac{8.6 \times 10^{6}}{1.6 \times 10^{-3}}$
    5.375×10^{9}; 5,375,000,000
11. $\frac{(4.2 \times 10^{-2})}{(6 \times 10^{-7})}$
    7×10^{4}; 70,000
12. $\frac{8.1 \times 10^{5}}{2.7 \times 10^{4}}$
    3×10^{1}; 30

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7-3 Skills Practice

Scientific Notation

Express each number in scientific notation.

1. 3,400,000,000
2. 0.000000312
3. 2,091,000
4. 980,200,000,000,000
5. 0.00000000008
6. 0.00142

Express each number in standard form.

7. 2.1×10^{5}
8. 8.023×10^{-7}
9. 3.63×10^{-6}
10. 7.15×10^{8}
11. 1.86×10^{-4}
12. 4.9×10^{5}

Evaluate each product. Express the results in both scientific notation and standard form.

13. $(6.1 \times 10^{5})(2 \times 10^{5})$
14. $(4.4 \times 10^{6})(1.6 \times 10^{-9})$
15. $(8.8 \times 10^{8})(3.5 \times 10^{-13})$
16. $(1.35 \times 10^{8})(7.2 \times 10^{-4})$
17. $(2.2 \times 10^{-12})(8 \times 10^{6})$
18. $(3.4 \times 10^{-5})(5.4 \times 10^{-4})$

Evaluate each quotient. Express the results in both scientific notation and standard form.

19. $\frac{(9.2 \times 10^{-8})}{(2 \times 10^{-6})}$
20. $\frac{(4.8 \times 10^{4})}{(3 \times 10^{-5})}$
21. $\frac{(1.161 \times 10^{-9})}{(4.3 \times 10^{-6})}$
22. $\frac{(4.625 \times 10^{10})}{(1.25 \times 10^{4})}$
23. $\frac{(2.376 \times 10^{-4})}{(7.2 \times 10^{-8})}$
24. $\frac{(8.74 \times 10^{-3})}{(1.9 \times 10^{5})}$

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7-3 Practice

Scientific Notation

Express each number in scientific notation.

1. 1,900,000
   1.9×10^6
2. 0.000704
   7.04×10^{-4}
3. 50,040,000,000
   5.004×10^{10}
4. 0.0000000661
   6.61×10^{-8}

Express each number in standard form.

5. 5.3×10^7
   53,000,000
6. 1.09×10^{-4}
   0.000109
7. 9.13×10^3
   9130
8. 7.902×10^{-6}
   0.000007902

Evaluate each product. Express the results in both scientific notation and standard form.

9. $(4.8 \times 10^4)(6 \times 10^6)$
   2.88×10^{11}; 288,000,000,000
10. $(7.5 \times 10^{-5})(3.2 \times 10^7)$
    2.4×10^3; 2400
11. $(2.06 \times 10^4)(5.5 \times 10^{-9})$
    1.133×10^{-4}; 0.0001133
12. $(8.1 \times 10^{-6})(1.96 \times 10^{11})$
    1.5876×10^6; 1,587,600
13. $(5.29 \times 10^8)(9.7 \times 10^4)$
    5.1313×10^{13}; 51,313,000,000,000
14. $(1.45 \times 10^{-6})(7.2 \times 10^{-5})$
    1.044×10^{-10}; 0.0000000001044

Evaluate each quotient. Express the results in both scientific notation and standard form.

15. $\frac{(4.2 \times 10^5)}{(3 \times 10^{-3})}$
    1.4×10^8; 140,000,000
16. $\frac{(1.76 \times 10^{-11})}{(2.2 \times 10^{-5})}$
    8×10^{-7}; 0.0000008
17. $\frac{(7.05 \times 10^{12})}{(9.4 \times 10^7)}$
    7.5×10^4; 75,000
18. $\frac{(2.04 \times 10^{-4})}{(3.4 \times 10^5)}$
    6×10^{-10}; 0.0000000006

19. **GRAVITATION** Issac Newton's theory of universal gravitation states that the equation $F = G\frac{m_1 m_2}{r^2}$ can be used to calculate the amount of gravitational force in newtons between two point masses m_1 and m_2 separated by a distance r. G is a constant equal to 6.67×10^{-11} N m^2 kg^{-2}. The mass of the earth m_1 is equal to 5.97×10^{24} kg, the mass of the moon m_2 is equal to 7.36×10^{22} kg, and the distance r between the two is 384,000,000 m.

   a. Express the distance r in scientific notation. **3.84×10^8 m**

   b. Compute the amount of gravitational force between the earth and the moon. Express your answer in scientific notation. **1.99×10^{20} newtons**

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7-3 Word Problem Practice

Scientific Notation

1. **PLANETS** Neptune's mean distance from the sun is 4,500,000,000 kilometers. Uranus' mean distance from the sun is 2,870,000,000 kilometers. Express these distances in scientific notation.
   Neptune: 4.5×10^9 km; Uranus: 2.87×10^9 km

2. **PATHOLOGY** The common cold is caused by the rhinovirus, which commonly measures 2×10^{-8} m in diameter. The E. coli bacterium, which causes food poisoning, commonly measures 3×10^{-6} m in length. Express these measurements in standard form.
   Rhinovirus: 0.00000002 m; E. coli: 0.000003 m

3. **COMMERCIALS** A 30-second commercial aired during the 2007 Super Bowl cost $2,600,000. A 30-second commercial aired during the 1967 Super Bowl cost $40,000. Express these values in scientific notation. How many times more expensive was it to air an advertisement during the 2007 Super Bowl than the 1967 Super Bowl?
   2007 Super Bowl: $ 2.6×10^6; 1967 Super Bowl: $ 4.0×10^4; 65×10^1 or 65 times more expensive

4. **AVOGADRO'S NUMBER** Avogadro's number is an important concept in chemistry. It states that the number 6.022×10^{23} is approximately equal to the number of molecules in 12 grams of carbon 12. Use Avogadro's number to determine the number of molecules in 5×10^{-7} grams of carbon 12.
   2.509×10^{16} molecules

5. **COAL RESERVES** The table below shows the number of kilograms of coal select countries had in proven reserve at the end of 2006.

Coal Reserves, 2006	
Country	Coal (kg)
United States	2.46×10^{14}
Russia	1.57×10^{14}
India	9.24×10^{13}
Romania	4.94×10^{11}

Source: British Petroleum

   a. Express each country's coal reserves in standard form. **United States: 246,000,000,000,000 kg; Russia: 157,000,000,000,000 kg; India: 92,400,000,000,000 kg; Romania: 494,000,000,000 kg**

   b. How many times more coal does the United States have than Romania?
   4.98×10^2 or 498 times

   c. One kilogram of coal has an energy density of 2.4×10^7 joules. What is the total energy density of the United States' coal reserve? Express your answer in scientific notation.
   5.90×10^{21} joules

Answers

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7-3 Enrichment

Engineering Notation

Engineering notation is a variation on scientific notation where numbers are expressed as powers of 1000 rather than as powers of 10. Engineering notation takes the familiar form of $a \times 10^n$, but n is restricted to multiples of three and $1 \leq |a| < 1000$.

One advantage to engineering notation is that numbers can be neatly expressed using SI prefixes. These prefixes are typically used for scientific measurements.

1000^n	10^n	SI Prefix	Symbol
1000^5	10^{15}	peta	P
1000^4	10^{12}	tera	T
1000^3	10^9	giga	G
1000^2	10^6	mega	M
1000^1	10^3	kilo	k
1000^{-1}	10^{-3}	milli	m
1000^{-2}	10^{-6}	micro	μ
1000^{-3}	10^{-9}	nano	n
1000^{-4}	10^{-12}	pico	p
1000^{-5}	10^{-15}	femto	f

Example **NUCLEAR POWER** **The output of a nuclear power plant is measured to be 620,000,000 watts. Express this number in engineering notation and using SI prefixes.**

To express a number in engineering notation, first convert the number to scientific notation.

Step 1 $620{,}000{,}000 \Rightarrow 6.20000000$ $a = 6.20000000$

Step 2 The decimal point moved 8 places to the left, so $n = 8$.

Step 3 $620{,}000{,}000 = 6.20000000 \times 10^8 = 6.2 \times 10^8$

Because 8 is not a multiple of 3, we need to round down n to the next multiple of 3.

Step 4 $6.2 \times 10^8 = (6.2 \times 10^{-2}) \times 10^8$ $6.2 = 620 \times 10^{-2}$

Step 5 $= 620 \times 10^6$ Product of Powers

The output of the power plant is 620×10^6 watts. Using the chart above, the prefix for 10^6 is found to be mega, or M. The output of the power plant is 620 megawatts, or 620MW.

Exercises

Express each number in engineering notation.

1. 40,000,000,000 **40×10^9**
2. 180,000,000,000,000 **180×10^{12}**
3. 0.00006 **60×10^{-6}**
4. 0.000000000000039 **390×10^{-15}**

Express each measurement using SI prefixes.

5. 0.00000000014 gram
   140 picograms (pg)
6. 40,000,000,000 watts
   40 gigawatts (GW)
7. 63,100,000,000,000 bytes
   63.1 terabytes (TB)
8. 0.0000002 meter
   200 nanometers (nm)

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7-4 Study Guide and Intervention

Polynomials

Degree of a Polynomial A **polynomial** is a monomial or a sum of monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. Polynomials with more than three terms have no special name. The **degree** of a monomial is the sum of the exponents of all its variables. The **degree of the polynomial** is the same as the degree of the monomial term with the highest degree.

Example **Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*. Then find the degree of the polynomial.**

Expression	Polynomial?	Monomial, Binomial, or Trinomial?	Degree of the Polynomial
$3x - 7xyz$	Yes. $3x - 7xyz = 3x + (-7xyz)$, which is the sum of two monomials	binomial	3
-25	Yes. -25 is a real number.	monomial	0
$7n^3 + 3n^{-4}$	No. $3n^{-4} = \frac{3}{n^4}$, which is not a monomial	none of these	—
$9x^3 + 4x + x + 4 + 2x$	Yes. The expression simplifies to $9x^3 + 7x + 4$, which is the sum of three monomials	trinomial	3

Exercises

Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*.

1. 36 **yes; monomial**
2. $\frac{3}{q^2} + 5$ **no**
3. $7x - x + 5$ **yes; binomial**
4. $8g^2h - 7gh + 2$ **yes; trinomial**
5. $\frac{1}{4y^2} + 5y - 8$ **no**
6. $6x + x^2$ **yes; binomial**

Find the degree of each polynomial.

7. $4x^2y^3z$ **6**
8. $-2abc$ **3**
9. $15m$ **1**
10. $r + 5t$ **1**
11. 22 **0**
12. $18x^2 + 4yz - 10y$ **2**
13. $x^4 - 6x^2 - 2x^3 - 10$ **4**
14. $2x^3y^2 - 4xy^3$ **5**
15. $-2r^8x^4 + 7r^2x - 4r^7x^6$ **13**
16. $9x^2 + yz^8$ **9**
17. $8b + bc^5$ **6**
18. $4x^4y - 8zx^2 + 2x^5$ **5**
19. $4x^2 - 1$ **2**
20. $9abc + bc - n^5$ **5**
21. $h^3m + 6h^4m^2 - 7$ **6**

NAME _____ DATE _____ PERIOD _____

7-4 Study Guide and Intervention *(continued)*

Polynomials

Write Polynomials in Standard Form The terms of a polynomial are usually arranged so that the terms are in order from greatest degree to least degree. This is called the **standard form of a polynomial.**

Example **Write $-4x^2 + 9x^4 - 2x$ in standard form. Identify the leading coefficient.**

Step 1: Find the degree of each term.

Polynomial: $-4x^2 + 9x^4 - 2x$

Degree: $\downarrow\ 2 \quad \downarrow\ 4 \quad \downarrow\ 1$

Step 2: Write the terms in descending order: $9x^4 - 4x^2 - 2x$.

The leading coefficient is 9.

Exercises

Write each polynomial in standard form. Identify the leading coefficient.

1. $5x + x^2 + 6$
   $x^2 + 5x + 6$; 1
2. $6x + 9 - 4x^2$
   $-4x^2 + 6x + 9$; −4
3. $x^4 + x^3 + x^2$
   $x^4 + x^3 + x^2$; 1
4. $2x^3 - x + 3x^7$
   $3x^7 + 2x^3 - 7$; 3
5. $2x + x^2 - 5$
   $x^2 + 2x - 5$; 1
6. $20x - 10x^2 + 5x^3$
   $5x^3 - 10x^2 + 20x$; 5
7. $x^3 + x^5 - x^2$
   $x^5 + x^3 - x^2$; 1
8. $x^4 + 4x^3 - 7x^5 + 1$
   $-7x^5 + x^4 + 4x^3 + 1$; −7
9. $-3x^6 - x^5 + 2x^8$
   $2x^8 - 3x^6 - x^5$; 2
10. $2x^7 - x^8$
    $-x^8 + 2x^7$; −1
11. $3x + 5x^4 - 2 - x^2$
    $5x^4 - x^2 + 3x - 2$; 5
12. $-2x^4 + x - 4x^5 + 3$
    $-4x^5 - 2x^4 + x + 3$; −4
13. $2 - x^{12}$
    $-x^{12} + 2$; −1
14. $5x^4 - 12x - 3x^6$
    $-3x^6 + 5x^5 - 12x$; −3
15. $9x^9 - 9 + 3x^3 - 6x^6$
    $9x^9 - 6x^6 + 3x^3 - 9$; 9

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7-4 Skills Practice

Polynomials

Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*.

1. $5mt + t^2$
   yes; binomial
2. $4by + 2b - by$
   yes; binomial
3. -32
   yes; monomial
4. $\frac{3x}{7}$
   yes; monomial
5. $5x^2 - 3x^{-4}$
   no
6. $2c^2 + 8c + 9 - 3$
   yes; trinomial

Find the degree of each polynomial.

7. 12 **0**
8. $3r^4$ **4**
9. $b + 6$ **1**
10. $4a^3 - 2a$ **3**
11. $5abc - 2b^2 + 1$ **3**
12. $8x^5y^4 - 2x^8$ **9**

Write each polynomial in standard form. Identify the leading coefficient.

13. $3x + 1 + 2x^2$
    $2x^2 + 3x + 1$; 3
14. $5x - 6 + 3x^2$
    $3x^2 + 5x - 6$; 3
15. $9x^2 + 2 + x^3 + x$
    $x^3 + 9x^2 + x + 2$; 1
16. $-3 + 3x^3 - x^2 + 4x$
    $3x^3 - x^2 + 4x - 3$; 3
17. $x^2 + 3x^3 + 27 - x$
    $3x^3 + x^2 - x + 27$; 3
18. $25 - x^3 + x$
    $-x^3 + x + 25$; −1
19. $x - 3x^2 + 4 + 5x^3$
    $5x^3 - 3x^2 + x + 4$; 5
20. $x^2 + 64 - x + 7x^3$
    $7x^3 + x^2 - x + 64$; 7
21. $6x^3 - 7x^5 + x - 2x^2 + 1$
    $-7x^5 + 6x^3 - 2x^2 + x + 1$; −7
22. $4 - x + 3x^3 - 2x^2$
    $3x^3 - 2x^2 - x + 4$; 3
23. $13 - 4x^9 + x^3$
    $-4x^9 + x^3 + 13$; −4
24. $17x^5 - 5x^{17} + 2$
    $-5x^{17} + 17x^5 + 2$; −5

Answers

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7-4 Practice

Polynomials

Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*.

1. $7a^2b + 3b^2 - a^2b$
   yes; binomial
2. $\frac{1}{5}y^3 + y^2 - 9$
   yes; trinomial
3. $6g^2h^3k$
   yes; monomial

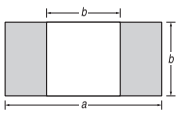
Find the degree of each polynomial.

4. $x + 3x^4 - 21x^2 + x^3$ **4**
5. $3g^2h^3 + g^3h$ **5**
6. $-2x^2y + 3xy^3 + x^2$ **4**
7. $5n^3m - 2m^3 + n^2m^4 + n^2$ **6**
8. $a^3b^2c + 2a^5c + b^3c^2$ **6**
9. $10r^2t^2 + 4rt^2 - 5r^3t^2$ **5**

Write each polynomial in standard form. Identify the leading coefficient.

10. $8x^2 - 15 + 5x^5$
    $5x^5 + 8x^2 - 15$; 5
11. $10x - 7 + x^4 + 4x^3$
    $x^4 + 4x^3 + 10x - 7$; 1
12. $13x^2 - 5 + 6x^3 - x$
    $6x^3 + 13x^2 - x - 5$; 6
13. $4x + 2x^5 - 6x^3 + 2$
    $2x^5 - 6x^3 + 4x + 2$; 2

GEOMETRY Write a polynomial to respect the area of each shaded region.

14. 

$ab - b^2$

15. 

$d^2 - \frac{1}{4}\pi d^2$

16. **MONEY** Write a polynomial to represent the value of t ten-dollar bills, f fifty-dollar bills, and h one-hundred-dollar bills. **$10t + 50f + 100h$**

17. **GRAVITY** The height above the ground of a ball thrown up with a velocity of 96 feet per second from a height of 6 feet is $6 + 96t - 16t^2$ feet, where t is the time in seconds. According to this model, how high is the ball after 7 seconds? Explain.
    -106 ft; The height is negative because the model does not account for the ball hitting the ground when the height is 0 feet.

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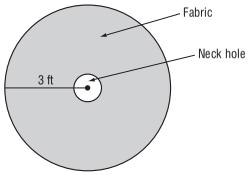
7-4 Word Problem Practice

Polynomials

1. **PRIMES** Mei is trying to list as many prime numbers as she can for a challenge problem for her math class. She finds that the polynomial expression $n^2 - n + 41$ can be used to generate some, but not all, prime numbers. What is the degree of Mei's polynomial? **2**

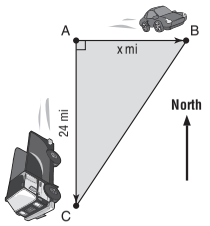
2. **PHONE CALLS** A long-distance telephone company charges a $19.95 standard monthly service fee plus $0.05 per minute of long-distance use. Write a polynomial to express the monthly cost of the phone plan if x minutes of long-distance time are used per month. What is the degree of the polynomial?
   $0.05x + $19.95; 1

3. **COSTUMES** Jack's mother is sewing the cape of his costume for a charity masked ball. The pattern for the cape (lying flat) is shown below. The radius of the neck hole is 6 inches. What is the area, in square feet, of the finished cape?
   27.5 ft²



4. **ARCHITECTURE** Graphing the polynomial function $y = -x^2 + 3$ produces an accurate drawing of the shape of an archway inside a historical library, where x is the horizontal distance in meters from the base of the arch and y is the height of the arch. At $x = 0$, what is the height of the arch? **3 m**

5. **DRIVING** A truck and a car leave an intersection. The truck travels south, and the car travels east. When the truck had gone 24 miles, the distance between the car and truck was four miles more than three times the distance traveled by the car heading east.



   a. Suppose the truck stops at point C and the car stops at point B. Write a polynomial in standard form to express the sum of the distances traveled by the car and the truck.
      $x + 24$

   b. Write a simplified polynomial to express the perimeter of triangle ABC.
      $4x + 28$ mi

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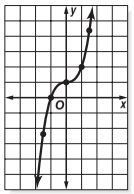
7-4 Enrichment

Polynomial Functions

Suppose a linear equation such as $-3x + y = 4$ is solved for y. Then an equivalent equation, $y = 3x + 4$, is found. Expressed in this way, y is a function of x, or $f(x) = 3x + 4$. Notice that the right side of the equation is a binomial of degree 1.

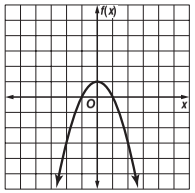
Higher-degree polynomials in x may also form functions. An example is $f(x) = x^3 + 1$, which is a polynomial function of degree 3. You can graph this function using a table of ordered pairs, as shown at the right.

x	y
$-1\frac{1}{2}$	$-2\frac{3}{8}$
-1	0
0	1
1	2
$1\frac{1}{2}$	$4\frac{3}{8}$

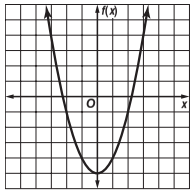


For each of the following polynomial functions, make a table of values for x and $y = f(x)$. Then draw the graph on the grid.

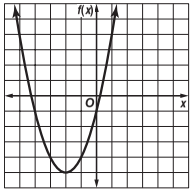
1. $f(x) = 1 - x^2$



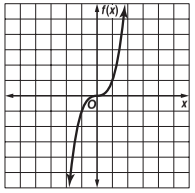
2. $f(x) = x^2 - 5$



3. $f(x) = x^2 + 4x - 1$



4. $f(x) = x^3$



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7-4 Graphing Calculator Activity

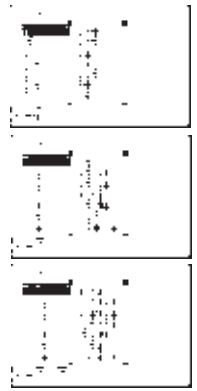
Second Degree Polynomial Functions

Many real world problems can be modeled using polynomial functions. The **TABLE** function can be used to evaluate a function for multiple values.

Example **An object is dropped from the top of a 179-foot cliff to the water below. The height of the object above the water can be modeled by $h(t) = -16t^2 + 179$ where t is time in seconds.**

a. Determine the height of the object after 0.5 second, 1 second, 1.5 seconds, and 2 seconds.

Enter the function into **Y1**. Use **TBLSET** to set up the calculator to display values of t in 0.5 second intervals. Display the table and record the results.



Examine the table. When $x = 0.5$, $y = 175$. This means that $h(0.5) = 175$, or that after 0.5 second, the object is 175 feet above the water. Thus, $h(1) = 163$ feet, $h(1.5) = 143$ feet, and $h(2) = 115$ feet.

b. After how many seconds does the object hit the water? Round to the nearest hundredth.

Scroll through the table. Notice that the y-values change from positive to negative between $x = 3$ and $x = 3.5$. Examine this interval more closely by resetting the table using **TblStart=** 3 and **ΔTbl=** 0.1. Look for the change in sign.

Further examine the interval from $x = 3.3$ to $x = 3.4$ using **TblStart=** 3.3 and **ΔTbl=** 0.01.

The y-value closest to zero occurs when $x = 3.34$. Thus, the object hits the water after about 3.34 seconds.

Exercises

1. An object is dropped from the top of a building that is 412 feet high. The distance, in feet, above the ground at x seconds is given by $P(x) = -16x^2 + 412$.

a. After how many seconds will the object be 100 feet above the ground? **≈ 4.4 s**

b. How many seconds will it take the object to reach the ground? **≈ 5.1 s**

2. A bungee jumper free falls from the Royal Gorge suspension bridge over the Arkansas River, 1053 feet above the river. The height h of the bungee jumper above the river, in feet, after t seconds can be represented by $h = -16t^2 + 1053$. Two seconds after the first bungee jumper falls, another person jumps down with an initial velocity of 80 feet per second. The position of the second jumper can be represented by the equation $h = -16(t - 2)^2 - 80(t - 2) + 1053$.

a. If the bungee cords are designed to stretch just enough so that the jumpers touch the water before springing back up, which jumper will touch the water first? How long does it take each jumper to touch the water? **the second jumper; 8.1 s, 6.0 s**

b. Does the second jumper catch up to the first jumper? If so, how far are they above the river at this point and how long does it take each jumper to reach this point? **yes; 477 ft; 6 s, 4 s**

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7-5 Study Guide and Intervention

Adding and Subtracting Polynomials

Add Polynomials To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms vertically. **Like terms** are monomial terms that are either identical or differ only in their coefficients, such as $3p$ and $-5p$ or $2x^2y$ and $8x^2y$.

Example 1 **Find $(2x^2 + x - 8) + (3x - 4x^2 + 2)$.**

Horizontal Method

Group like terms.

$(2x^2 + x - 8) + (3x - 4x^2 + 2)$

$= [(2x^2 + (-4x^2)] + (x + 3x) + [(-8) + 2)]$

$= -2x^2 + 4x - 6$.

The sum is $-2x^2 + 4x - 6$.

Example 2 **Find $(3x^2 + 5xy) + (xy + 2x^2)$.**

Vertical Method

Align like terms in columns and add.

$3x^2 + 5xy$

$(+)\ 2x^2 + xy$ Put the terms in descending order.

$5x^2 + 6xy$

The sum is $5x^2 + 6xy$.

Exercises

Find each sum.

1. $(4a - 5) + (3a + 6)$ — **$7a + 1$**
2. $(6x + 9) + (4x^2 - 7)$ — **$4x^2 + 6x + 2$**
3. $(6xy + 2y + 6x) + (4xy - x)$ — **$10xy + 5x + 2y$**
4. $(x^2 + y^2) + (-x^2 + y^2)$ — **$2y^2$**
5. $(3p^2 - 2p + 3) + (p^2 - 7p + 7)$ — **$4p^2 - 9p + 10$**
6. $(2x^2 + 5xy + 4y^2) + (-xy - 6x^2 + 2y^2)$ — **$-4x^2 + 4xy + 6y^2$**
7. $(5p + 2q) + (2p^2 - 8q + 1)$ — **$2p^2 + 5p - 6q + 1$**
8. $(4x^2 - x + 4) + (5x + 2x^2 + 2)$ — **$6x^2 + 4x + 6$**
9. $(6x^2 + 3x) + (x^2 - 4x - 3)$ — **$7x^2 - x - 3$**
10. $(x^2 + 2xy + y^2) + (x^2 - xy - 2y^2)$ — **$2x^2 + xy - y^2$**
11. $(2a - 4b - c) + (-2a - b - 4c)$ — **$-5b - 5c$**
12. $(6xy^2 + 4xy) + (2xy - 10xy^2 + y^2)$ — **$-4xy^2 + 6xy + y^2$**
13. $(2p - 5r) + (3p + 6r) + (p - r)$ — **$6p$**
14. $(2x^2 - 6) + (5x^2 + 2) + (-x^2 - 7)$ — **$6x^2 - 11$**
15. $(3z^2 + 5z) + (z^2 + 2z) + (z - 4)$ — **$4z^2 + 8z - 4$**
16. $(8x^2 + 4x + 3y^2 + y) + (6x^2 - x + 4y)$ — **$14x^2 + 3x + 3y^2 + 5y$**

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7-5 Study Guide and Intervention *(continued)*

Adding and Subtracting Polynomials

Subtract Polynomials You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

Example **Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.**

Horizontal Method

Use additive inverses to rewrite as addition. Then group like terms.

$(3x^2 + 2x - 6) - (2x + x^2 + 3)$

$= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]$

$= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]$

$= 2x^2 + (-9)$

$= 2x^2 - 9$

The difference is $2x^2 - 9$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$3x^2 + 2x - 6$

$(-)\ x^2 + 2x + 3$

$3x^2 + 2x - 6$

$(+) -x^2 - 2x - 3$

$2x^2 \quad - 9$

The difference is $2x^2 - 9$.

Exercises

Find each difference.

1. $(3a - 5) - (5a + 1)$ — **$-2a - 6$**
2. $(9x + 2) - (-3x^2 - 5)$ — **$3x^2 + 9x + 7$**
3. $(9xy + y - 2x) - (6xy - 2x)$ — **$3xy + y$**
4. $(x^2 + y^2) - (-x^2 + y^2)$ — **$2x^2$**
5. $(6p^2 + 4p + 5) - (2p^2 - 5p + 1)$ — **$4p^2 + 9p + 4$**
6. $(6x^2 + 5xy - 2y^2) - (-xy - 2x^2 - 4y^2)$ — **$8x^2 + 6xy + 2y^2$**
7. $(8p - 5r) - (-6p^2 + 6r - 3)$ — **$6p^2 + 8p - 11r + 3$**
8. $(8x^2 - 4x - 3) - (-2x - x^2 + 5)$ — **$9x^2 - 2x - 8$**
9. $(3x^2 - 2x) - (3x^2 + 5x - 1)$ — **$-7x + 1$**
10. $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$ — **$5x^2 + 4xy + 7y^2$**
11. $(2h - 6j - 2k) - (-7h - 5j - 4k)$ — **$9h - j + 2k$**
12. $(9xy^2 + 5xy) - (-2xy - 8xy^2)$ — **$17xy^2 + 7xy$**
13. $(2a - 8b) - (-3a + 5b)$ — **$5a - 13b$**
14. $(2x^2 - 8) - (-2x^2 - 6)$ — **$4x^2 - 2$**
15. $(6z^2 + 4z + 2) - (4z^2 + z)$ — **$2z^2 + 3z + 2$**
16. $(6x^2 - 5x + 1) - (-7x^2 - 2x + 4)$ — **$13x^2 - 3x - 3$**

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7-5 Skills Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(2x + 3y) + (4x + 9y)$ **$6x + 12y$**
2. $(6s + 5t) + (4t + 8s)$ **$14s + 9t$**
3. $(5a + 9b) - (2a + 4b)$ **$3a + 5b$**
4. $(11m - 7n) - (2m + 6n)$ **$9m - 13n$**
5. $(m^2 - m) + (2m + m^2)$ **$2m^2 + m$**
6. $(x^2 - 3x) - (2x^2 + 5x)$ **$-x^2 - 8x$**
7. $(d^2 - d + 5) - (2d + 5)$ **$d^2 - 3d$**
8. $(2h^2 - 5h) + (7h - 3h^2)$ **$-h^2 + 2h$**
9. $(5f + g - 2) + (-2f + 3)$ **$3f + g + 1$**
10. $(6k^2 + 2k + 9) + (4k^2 - 5k)$ **$10k^2 - 3k + 9$**
11. $(x^3 - x + 1) - (3x - 1)$ **$x^3 - 4x + 2$**
12. $(b^2 + ab - 2) - (2b^2 + 2ab)$ **$-b^2 - ab - 2$**
13. $(7z^2 + 4 - z) - (-5 + 3z^2)$ **$4z^2 - z + 9$**
14. $(5 + 4n + 2t) + (-6t - 8)$ **$-3 + 4n - 4t$**
15. $(4t^2 + 2) + (-4 + 2t)$ **$4t^2 + 2t - 2$**
16. $(3g^3 + 7g) - (4g + 8g^3)$ **$-5g^3 + 3g$**
17. $(2a^2 + 8a + 4) - (a^2 - 3)$ **$a^2 + 8a + 7$**
18. $(3x^2 - 7x + 5) - (-x^2 + 4x)$ **$4x^2 - 11x + 5$**
19. $(7y^2 + y + 1) - (-4y + 3y^2 - 3)$ **$4y^2 + 5y + 4$**
20. $(2c^2 + 7c + 4) + (c^2 + 1 - 9c)$ **$3c^2 - 2c + 5$**
21. $(n^2 + 3n + 2) - (2n^2 - 6n - 2)$ **$-n^2 + 9n + 4$**
22. $(a^2 + ab - 3b^2) + (b^2 + 4a^2 - ab)$ **$5a^2 - 2b^2$**
23. $(\ell^2 - 5\ell - 6) + (2\ell^2 + 5 + \ell)$ **$3\ell^2 - 4\ell - 1$**
24. $(2m^2 + 5m + 1) - (4m^2 - 3m - 3)$ **$-2m^2 + 8m + 4$**
25. $(x^2 - 6x + 2) - (-5x^2 + 7x - 4)$ **$6x^2 - 13x + 6$**
26. $(5b^2 - 9b - 5) + (b^2 - 6 + 2b)$ **$6b^2 - 7b - 11$**
27. $(2x^2 - 6x - 2) + (x^2 + 4x) + (3x^2 + x + 5)$ **$6x^2 - x + 3$**

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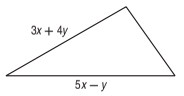
7-5 Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(4y + 5) + (-7y - 1)$ **$-3y + 4$**
2. $(-x^2 + 3x) - (5x + 2x^2)$ **$-3x^2 - 2x$**
3. $(4k^2 + 8k + 2) - (2k + 3)$ **$4k^2 + 6k - 1$**
4. $(2m^2 + 6m) + (m^2 - 5m + 7)$ **$3m^2 + m + 7$**
5. $(5a^2 + 6a + 2) - (7a^2 - 7a + 5)$ **$-2a^2 + 13a - 3$**
6. $(-4p^2 - p + 9) + (p^2 + 3p - 1)$ **$-3p^2 + 2p + 8$**
7. $(x^3 - 3x + 1) - (x^3 + 7 - 12x)$ **$9x - 6$**
8. $(6x^2 - x + 1) - (-4 + 2x^2 + 8x)$ **$4x^2 - 9x + 5$**
9. $(4y^2 + 2y - 8) - (7y^2 + 4 - y)$ **$-3y^2 + 3y - 12$**
10. $(w^2 - 4w - 1) + (-5 + 5w^2 - 3w)$ **$6w^2 - 7w - 6$**
11. $(4u^2 - 2u - 3) + (3u^2 - u + 4)$ **$7u^2 - 3u + 1$**
12. $(5b^2 - 8 + 2b) - (b + 9b^2 + 5)$ **$-4b^2 + b - 13$**
13. $(4d^2 + 2d + 2) + (5d^2 - 2 - d)$ **$9d^2 + d$**
14. $(8x^2 + x - 6) - (-x^2 + 2x - 3)$ **$9x^2 - x - 3$**
15. $(3h^2 + 7h - 1) - (4h + 8h^2 + 1)$ **$-5h^2 + 3h - 2$**
16. $(4m^2 - 3m + 10) + (m^2 + m - 2)$ **$5m^2 - 2m + 8$**
17. $(x^2 + y^2 - 6) - (5x^2 - y^2 - 5)$ **$-4x^2 + 2y^2 - 1$**
18. $(7t^2 + 2 - t) + (t^2 - 7 - 2t)$ **$8t^2 - 3t - 5$**
19. $(k^3 - 2k^2 + 4k + 6) - (-4k + k^2 - 3)$ **$k^3 - 3k^2 + 8k + 9$**
20. $(9j^2 + j + jk) + (-3j^2 - jk - 4j)$ **$6j^2 - 3j$**
21. $(2x + 6y - 3z) + (4x + 6z - 8y) + (x - 3y + z)$ **$7x - 5y + 4z$**
22. $(6f^2 - 7f - 3) - (5f^2 - 1 + 2f) - (2f^2 - 3 + f)$ **$-f^2 - 10f + 1$**
23. **BUSINESS** The polynomial $s^3 - 70s^2 + 1500s - 10{,}800$ models the profit a company makes on selling an item at a price s. A second item sold at the same price brings in a profit of $s^3 - 30s^2 + 450s - 5000$. Write a polynomial that expresses the total profit from the sale of both items. **$2s^3 - 100s^2 + 1950s - 15{,}800$**
24. **GEOMETRY** The measures of two sides of a triangle are given. If P is the perimeter, and $P = 10x + 5y$, find the measure of the third side. **$2x + 2y$**



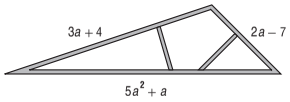
Answers

NAME _____ DATE _____ PERIOD _____

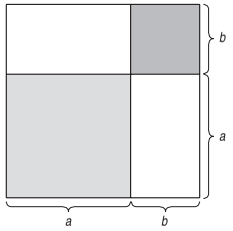
7-5 Word Problem Practice

Adding and Subtracting Polynomials

1. **BUILDING** Find the simplest expression for the perimeter of the triangular roof truss. $5a^2 + 6a - 3$



2. **GEOMETRY** Write a polynomial to show the area of the large square below. $a^2 + 2ab + b^2$

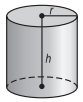


3. **FIREWORKS** Two bottle rockets are launched straight up into the air. The height, in feet, of each rocket at t seconds after launch is given by the polynomial equations below. Write an equation to show how much higher Rocket A traveled.
Rocket A: $H_1 = -16t^2 + 122t$
Rocket B: $H_1 = -16t^2 + 84t$
$D = 38t$

4. **ENVELOPES** An office supply company produces yellow document envelopes. The envelopes come in a variety of sizes, but the length is always 4 centimeters more than double the width. Write a polynomial expression to give the perimeter of any of the envelopes.
$6x + 8$

5. **INDUSTRY** Two identical right cylindrical steel drums containing oil need to be covered with a fire-resistant sealant. In order to determine how much sealant to purchase, George must find the surface area of the two drums. The surface area (including the top and bottom bases) is given by the following formula.

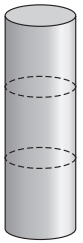
$$S = 2\pi rh + 2\pi r^2$$



a. Write a polynomial to represent the total surface area of the two drums.
$4\pi rh + 4\pi r^2$

b. Find the total surface area if the height of each drum is 2 meters and the radius of each is 0.5 meters. Let $\pi = 3.14$. $15.7\ m^2$

c. The fire resistant sealant must be applied while they are stacked vertically in groups of three. If h is the height of each drum and r is the radius, write a polynomial to represent the exposed surface area.
$6\pi rh + \pi r^2$

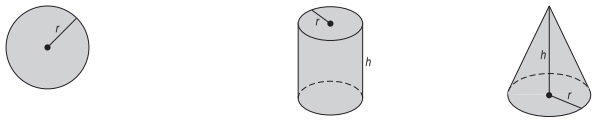


NAME _____ DATE _____ PERIOD _____

7-5 Enrichment

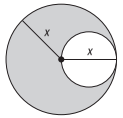
Circular Areas and Volumes

Area of Circle	Volume of Cylinder	Volume of Cone
$A = \pi r^2$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$



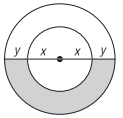
Write an algebraic expression for each shaded area. (Recall that the diameter of a circle is twice its radius.)

1.



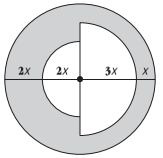
$\pi x^2 - \pi\left(\frac{x}{2}\right)^2 = \pi x^2$

2.



$\frac{\pi}{2}(y^2 + 2xy)$

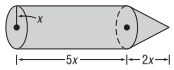
3.



$\frac{19}{2}\pi x^2$

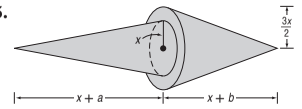
Write an algebraic expression for the total volume of each figure.

4.



$5\frac{2}{3}\pi x^3$

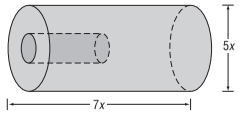
5.



$\frac{\pi}{12}[13x^3 + (4a + 9b)x^2]$

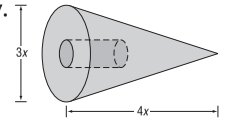
Each figure has a cylindrical hole with a radius of 2 inches and a height of 5 inches. Find each volume.

6.



$\frac{175\pi}{4}x^3 - 20\pi\ in^3$

7.



$3\pi x^3 - 20\pi\ in^3$

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7-6 Study Guide and Intervention

Multiplying a Polynomial by a Monomial

Polynomial Multiplied by Monomial The Distributive Property can be used to multiply a polynomial by a monomial. You can multiply horizontally or vertically. Sometimes multiplying results in like terms. The products can be simplified by combining like terms.

Example 1 **Find $-3x^2(4x^2 + 6x - 8)$.**

Horizontal Method

$-3x^2(4x^2 + 6x - 8)$
$= -3x^2(4x^2) + (-3x^2)(6x) - (-3x^2)(8)$
$= -12x^4 + (-18x^3) - (-24x^2)$
$= -12x^4 - 18x^3 + 24x^2$

Vertical Method

$4x^2 + 6x - 8$
$(\times) \quad -3x^2$
$-12x^4 - 18x^3 + 24x^2$

The product is $-12x^4 - 18x^3 + 24x^2$.

Example 2 **Simplify $-2(4x^2 + 5x) - x(x^2 + 6x)$.**

$-2(4x^2 + 5x) - x(x^2 + 6x)$
$= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x)$
$= -8x^2 + (-10x) + (-x^3) + (-6x^2)$
$= (-x^3) + [-8x^2 + (-6x^2)] + (-10x)$
$= -x^3 - 14x^2 - 10x$

Exercises

Find each product.

1. $x(5x + x^2)$ — $5x^2 + x^3$
2. $x(4x^2 + 3x + 2)$ — $4x^3 + 3x^2 + 2x$
3. $-2xy(2y + 4x^2)$ — $-4xy^2 - 8x^3y$
4. $-2g(g^2 - 2g + 2)$ — $-2g^3 + 4g^2 - 4g$
5. $3x(x^4 + x^3 + x^2)$ — $3x^5 + 3x^4 + 3x^3$
6. $-4x(2x^3 - 2x + 3)$ — $-8x^4 + 8x^2 - 12x$
7. $-4ax(10 + 3x)$ — $-40ax - 12ax^2$
8. $3y(-4x - 6x^3 - 2y)$ — $-12xy - 18x^3y - 6y^2$
9. $2x^2y^2(3xy + 2y + 5x)$ — $6x^3y^3 + 4x^2y^3 + 10x^3y^2$

Simplify each expression.

10. $x(3x - 4) - 5x$ — $3x^2 - 9x$
11. $-x(2x^2 - 4x) - 6x^2$ — $-2x^3 - 2x^2$
12. $6a(2a - b) + 2a(-4a + 5b)$ — $4a^2 + 4ab$
13. $4r(2r^2 - 3r + 5) + 6r(4r^2 + 2r + 8)$ — $32r^3 + 68r$
14. $4n(3n^2 + n - 4) - n(3 - n)$ — $12n^3 + 5n^2 - 19n$
15. $2b(b^2 + 4b + 8) - 3b(3b^2 + 9b - 18)$ — $-7b^3 - 19b^2 + 70b$
16. $-2z(4z^2 - 3z + 1) - z(3z^2 + 2z - 1)$ — $-11z^3 + 4z^2 - z$
17. $2(4x^2 - 2x) - 3(-6x^2 + 4) + 2x(x - 1)$ — $28x^2 - 6x - 12$

NAME _____ DATE _____ PERIOD _____

7-6 Study Guide and Intervention *(continued)*

Multiplying a Polynomial by a Monomial

Solve Equations with Polynomial Expressions Many equations contain polynomials that must be added, subtracted, or multiplied before the equation can be solved.

Example **Solve $4(n - 2) + 5n = 6(3 - n) + 19$.**

$4(n - 2) + 5n = 6(3 - n) + 19$	Original equation
$4n - 8 + 5n = 18 - 6n + 19$	Distributive Property
$9n - 8 = 37 - 6n$	Combine like terms.
$15n - 8 = 37$	Add $6n$ to both sides.
$15n = 45$	Add 8 to both sides.
$n = 3$	Divide each side by 15.

The solution is 3.

Exercises

Solve each equation.

1. $2(a - 3) = 3(-2a + 6)$ — **3**
2. $3(x + 5) - 6 = 18$ — **3**
3. $3x(x - 5) - 3x^2 = -30$ — **2**
4. $6(x^2 + 2x) = 2(3x^2 + 12)$ — **2**
5. $4(2p + 1) - 12p = 2(8p + 12)$ — **-1**
6. $2(6x + 4) + 2 = 4(x - 4)$ — **$-3\frac{1}{4}$**
7. $-2(4y - 3) - 8y + 6 = 4(y - 2)$ — **1**
8. $x(x + 2) - x(x - 6) = 10x - 12$ — **6**
9. $3(x^2 - 2x) = 3x^2 + 5x - 11$ — **1**
10. $2(4x + 3) + 2 = -4(x + 1)$ — **-1**
11. $3(2h - 6) - (2h + 1) = 9$ — **7**
12. $3(y + 5) - (4y - 8) = -2y + 10$ — **-13**
13. $3(2a - 6) - (-3a - 1) = 4a - 2$ — **3**
14. $5(2x^2 - 1) - (10x^2 - 6) = -(x + 2)$ — **-3**
15. $3(x + 2) + 2(x + 1) = -5(x - 3)$ — **$\frac{7}{10}$**
16. $4(3p^2 + 2p) - 12p^2 = 2(8p + 6)$ — **$-\frac{3}{2}$**

Answers

NAME _____ DATE _____ PERIOD _____

7-6 Skills Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $a(4a + 3)$
   $4a^2 + 3a$
2. $-c(11c + 4)$
   $-11c^2 - 4c$
3. $x(2x - 5)$
   $2x^2 - 5x$
4. $2y(y - 4)$
   $2y^2 - 8y$
5. $-3n(n^2 + 2n)$
   $-3n^3 - 6n^2$
6. $4h(3h - 5)$
   $12h^2 - 20h$
7. $3x(5x^2 - x + 4)$
   $15x^3 - 3x^2 + 12x$
8. $7c(5 - 2c^2 + c^3)$
   $35c - 14c^3 + 7c^4$
9. $-4b(1 - 9b - 2b^2)$
   $-4b + 36b^2 + 8b^3$
10. $6y(-5 - y + 4y^2)$
    $-30y - 6y^2 + 24y^3$
11. $2m^2(2m^2 + 3m - 5)$
    $4m^4 + 6m^3 - 10m^2$
12. $-3n^2(-2n^2 + 3n + 4)$
    $6n^4 - 9n^3 - 12n^2$

Simplify each expression.

13. $w(3w + 2) + 5w$
    $3w^2 + 7w$
14. $f(5f - 3) - 2f$
    $5f^2 - 5f$
15. $-p(2p - 8) - 5p$
    $-2p^2 + 3p$
16. $y^2(-4y + 5) - 6y^2$
    $-4y^3 - y^2$
17. $2x(3x^2 + 4) - 3x^3$
    $3x^3 + 8x$
18. $4a(5a^2 - 4) + 9a$
    $20a^3 - 7a$
19. $4b(-5b - 3) - 2(b^2 - 7b - 4)$
    $-22b^2 + 2b + 8$
20. $3m(3m + 6) - 3(m^2 + 4m + 1)$
    $6m^2 + 6m - 3$

Solve each equation.

21. $3(a + 2) + 5 = 2a + 4$ **-7**
22. $2(4x + 2) - 8 = 4(x + 3)$ **4**
23. $5(y + 1) + 2 = 4(y + 2) - 6$ **-5**
24. $4(b + 6) = 2(b + 5) + 2$ **-6**
25. $6(m - 2) + 14 = 3(m + 2) - 10$ **-2**
26. $3(c + 5) - 2 = 2(c + 6) + 2$ **1**

NAME _____ DATE _____ PERIOD _____

7-6 Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $2h(-7h^2 - 4h)$
   $-14h^3 - 8h^2$
2. $6pq(3p^2 + 4q)$
   $18p^3q + 24pq^2$
3. $5jk(3jk + 2k)$
   $15j^2k^2 + 10jk^2$
4. $-3rt(-2t^2 + 3r)$
   $6rt^3 - 9r^2t$
5. $-\frac{1}{4}m(8m^2 + m - 7)$
   $-2m^3 - \frac{1}{4}m^2 + \frac{7}{4}m$
6. $-\frac{2}{3}n^2(-9n^2 + 3n + 6)$
   $6n^4 - 2n^3 - 4n^2$

Simplify each expression.

7. $-2\ell(3\ell - 4) + 7\ell$
   $-6\ell^2 + 15\ell$
8. $5w(-7w + 3) + 2w(-2w^2 + 19w + 2)$
   $-4w^3 + 3w^2 + 19w$
9. $6t(2t - 3) - 5(2t^2 + 9t - 3)$
   $2t^2 - 63t + 15$
10. $-2(3m^3 + 5m + 6) + 3m(2m^2 + 3m + 1)$
    $9m^2 - 7m - 12$
11. $-3g(7g - 2) + 3(g^2 + 2g + 1) - 3g(-5g + 3)$ **$-3g^2 + 3g + 3$**

Solve each equation.

12. $5(2t - 1) + 3 = 3(3t + 2)$ **8**
13. $3(3u + 2) + 5 = 2(2u - 2)$ **-3**
14. $4(8n + 3) - 5 = 2(6n + 8) + 1$ **$\frac{1}{2}$**
15. $8(3b + 1) = 4(b + 3) - 9$ **$-\frac{1}{4}$**
16. $t(t + 4) - 1 = t(t + 2) + 2$ **$\frac{3}{2}$**
17. $u(u - 5) + 8u = u(u + 2) - 4$ **-4**

18. **NUMBER THEORY** Let x be an integer. What is the product of twice the integer added to three times the next consecutive integer? **$5x + 3$**

19. **INVESTMENTS** Kent invested \$5000 in a retirement plan. He allocated x dollars of the money to a bond account that earns 4% interest per year and the rest to a traditional account that earns 5% interest per year.

    a. Write an expression that represents the amount of money invested in the traditional account. **$5000 - x$**

    b. Write a polynomial model in simplest form for the total amount of money T Kent has invested after one year. (*Hint:* Each account has $A + IA$ dollars, where A is the original amount in the account and I is its interest rate.) **$T = 5250 - 0.01x$**

    c. If Kent put \$500 in the bond account, how much money does he have in his retirement plan after one year? **\$5245**

NAME _____ DATE _____ PERIOD _____

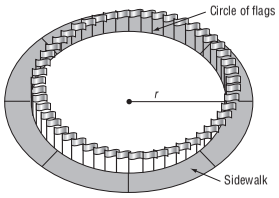
7-6 Word Problem Practice

Multiplying a Polynomial by a Monomial

1. **NUMBER THEORY** The sum of the first n whole numbers is given by the expression $\frac{1}{2}(n^2 + n)$. Expand the equation by multiplying, then find the sum of the first 12 whole numbers.
   $\frac{n^2}{2} + \frac{n}{2}$; 78

2. **COLLEGE** Troy's boss gave him \$700 to start his college savings account. Troy's boss also gives him \$40 each month to add to the account. Troy's mother gives him \$50 each month, but has been doing so for 4 fewer months than Troy's boss. Write a simplified expression for the amount of money Troy has received from his boss and mother after m months.
   \$90m + \$500

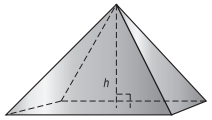
3. **LANDMARKS** A circle of 50 flags surrounds the Washington Monument. Suppose a new sidewalk 12 feet wide is installed just around the outside of the circle of flags. The outside circumference of the sidewalk is 1.10 times the circumference of the circle of flags.



   Write an equation that equates the outside circumference of the sidewalk to 1.10 times the circumference of the circle of flags. Solve the equation for the radius of the circle of flags. Recall that circumference of a circle is $2\pi r$.
   $1.10(2\pi r) = 2\pi(r + 12)$; $r = 120$ *ft*

4. **MARKET** Sophia went to the farmers' market to purchase some vegetables. She bought peppers and potatoes. The peppers were \$0.39 each and the potatoes were \$0.29 each. She spent \$3.88 on vegetables, and bought 4 more potatoes than peppers. If x = the number of peppers, write and solve an equation to find out how many of each vegetable Sophia bought.
   $3.88 = x(0.39) + (x + 4)(0.29)$; 4 peppers and 8 potatoes

5. **GEOMETRY** Some monuments are constructed as rectangular pyramids. The volume of a pyramid can be found by multiplying the area of its base B by one third of its height. The area of the rectangular base of a monument in a local park is given by the polynomial equation $B = x^2 - 4x - 12$.



   a. Write a polynomial equation to represent V, the volume of a rectangular pyramid if its height is 10 centimeters.
   $V = \frac{10}{3}x^2 - \frac{40}{3}x - 40$

   b. Find the volume of the pyramid if $x = 12$.
   280 cm^3

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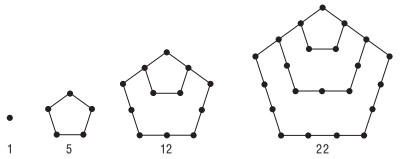
7-6 Enrichment

Figurate Numbers

The numbers below are called **pentagonal numbers**. They are the numbers of dots or disks that can be arranged as pentagons.



1. Find the product $\frac{1}{2}n(3n - 1)$. **$\frac{3n^2}{2} - \frac{n}{2}$**

2. Evaluate the product in Exercise 1 for values of n from 1 through 4. **1, 5, 12, 22**

3. What do you notice? **They are the first four pentagonal numbers.**

4. Find the next six pentagonal numbers. **35, 51, 70, 92, 117, 145**

5. Find the product $\frac{1}{2}n(n + 1)$. **$\frac{n^2}{2} + \frac{n}{2}$**

6. Evaluate the product in Exercise 5 for values of n from 1 through 5. On another sheet of paper, make drawings to show why these numbers are called the triangular numbers.
   1, 3, 6, 10, 15

7. Find the product $n(2n - 1)$. **$2n^2 - n$**

8. Evaluate the product in Exercise 7 for values of n from 1 through 5. Draw these hexagonal numbers. **1, 6, 15, 28, 45**

9. Find the first 5 square numbers. Also, write the general expression for any square number. **1, 4, 9, 16, 25; n^2**

The numbers you have explored above are called the plane figurate numbers because they can be arranged to make geometric figures. You can also create solid figurate numbers.

10. If you pile 10 oranges into a pyramid with a triangle as a base, you get one of the tetrahedral numbers. How many layers are there in the pyramid? How many oranges are there in the bottom layers? **3 layers; 6**

11. Evaluate the expression $\frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$ for values of n from 1 through 5 to find the first five tetrahedral numbers. **1, 4, 10, 20, 35**

Answers

NAME _____ DATE _____ PERIOD _____

7-7 Study Guide and Intervention

Multiplying Polynomials

Multiply Binomials To multiply two binomials, you can apply the Distributive Property twice. A useful way to keep track of terms in the product is to use the FOIL method as illustrated in Example 2.

Example 1 **Find $(x + 3)(x - 4)$.**

Horizontal Method

$(x + 3)(x - 4)$
$= x(x - 4) + 3(x - 4)$
$= (x)(x) + x(-4) + 3(x) + 3(-4)$
$= x^2 - 4x + 3x - 12$
$= x^2 - x - 12$

Vertical Method

$$\begin{array}{r} x + 3 \\ (\times)\quad x - 4 \\ \hline -4x - 12 \\ x^2 + 3x \\ \hline x^2 - x - 12 \end{array}$$

The product is $x^2 - x - 12$.

Example 2 **Find $(x - 2)(x + 5)$ using the FOIL method.**

$(x - 2)(x + 5)$

First Outer Inner Last

$= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)$
$= x^2 + 5x + (-2x) - 10$
$= x^2 + 3x - 10$

The product is $x^2 + 3x - 10$.

Exercises

Find each product.

1. $(x + 2)(x + 3)$ — $x^2 + 5x + 6$
2. $(x - 4)(x + 1)$ — $x^2 - 3x - 4$
3. $(x - 6)(x - 2)$ — $x^2 - 8x + 12$
4. $(p - 4)(p + 2)$ — $p^2 - 2p - 8$
5. $(y + 5)(y + 2)$ — $y^2 + 7y + 10$
6. $(2x - 1)(x + 5)$ — $2x^2 + 9x - 5$
7. $(3n - 4)(3n - 4)$ — $9n^2 - 24n + 16$
8. $(8m - 2)(8m + 2)$ — $64m^2 - 4$
9. $(k + 4)(5k - 1)$ — $5k^2 + 19k - 4$
10. $(3x + 1)(4x + 3)$ — $12x^2 + 13x + 3$
11. $(x - 8)(-3x + 1)$ — $-3x^2 + 25x - 8$
12. $(5t + 4)(2t - 6)$ — $10t^2 - 22t - 24$
13. $(5m - 3n)(4m - 2n)$ — $20m^2 - 22mn + 6n^2$
14. $(a - 3b)(2a - 5b)$ — $2a^2 - 11ab + 15b^2$
15. $(8x - 5)(8x + 5)$ — $64x^2 - 25$
16. $(2n - 4)(2n + 5)$ — $4n^2 + 2n - 20$
17. $(4m - 3)(5m - 5)$ — $20m^2 - 35m + 15$
18. $(7g - 4)(7g + 4)$ — $49g^2 - 16$

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7-7 Study Guide and Intervention *(continued)*

Multiplying Polynomials

Multiply Polynomials The Distributive Property can be used to multiply any two polynomials.

Example **Find $(3x + 2)(2x^2 - 4x + 5)$.**

$(3x + 2)(2x^2 - 4x + 5)$
$= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5)$ Distributive Property
$= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10$ Distributive Property
$= 6x^3 - 8x^2 + 7x + 10$ Combine like terms.

The product is $6x^3 - 8x^2 + 7x + 10$.

Exercises

Find each product.

1. $(x + 2)(x^2 - 2x + 1)$ — $x^3 - 3x + 2$
2. $(x + 3)(2x^2 + x - 3)$ — $2x^3 + 7x^2 - 9$
3. $(2x - 1)(x^2 - x + 2)$ — $2x^3 - 3x^2 + 5x - 2$
4. $(p - 3)(p^2 - 4p + 2)$ — $p^3 - 7p^2 + 14p - 6$
5. $(3k + 2)(k^2 + k - 4)$ — $3k^3 + 5k^2 - 10k - 8$
6. $(2t + 1)(10t^2 - 2t - 4)$ — $20t^3 + 6t^2 - 10t - 4$
7. $(3n - 4)(n^2 + 5n - 4)$ — $3n^3 + 11n^2 - 32n + 16$
8. $(8x - 2)(3x^2 + 2x - 1)$ — $24x^3 + 10x^2 - 12x + 2$
9. $(2a + 4)(2a^2 - 8a + 3)$ — $4a^3 - 8a^2 - 26a + 12$
10. $(3x - 4)(2x^2 + 3x + 3)$ — $6x^3 + x^2 - 3x - 12$
11. $(n^2 + 2n - 1)(n^2 + n + 2)$ — $n^4 + 3n^3 + 3n^2 + 3n - 2$
12. $(t^2 + 4t - 1)(2t^2 - t - 3)$ — $2t^4 + 7t^3 - 9t^2 - 11t + 3$
13. $(y^2 - 5y + 3)(2y^2 + 7y - 4)$ — $2y^4 - 3y^3 - 33y^2 + 41y - 12$
14. $(3b^2 - 2b + 1)(2b^2 - 3b - 4)$ — $6b^4 - 13b^3 - 4b^2 + 5b - 4$

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7-7 Skills Practice

Multiplying Polynomials

Find each product.

1. $(m + 4)(m + 1)$
   $m^2 + 5m + 4$
2. $(x + 2)(x + 2)$
   $x^2 + 4x + 4$
3. $(b + 3)(b + 4)$
   $b^2 + 7b + 12$
4. $(t + 4)(t - 3)$
   $t^2 + t - 12$
5. $(r + 1)(r - 2)$
   $r^2 - r - 2$
6. $(n - 5)(n + 1)$
   $n^2 - 4n - 5$
7. $(3c + 1)(c - 2)$
   $3c^2 - 5c - 2$
8. $(2x - 6)(x + 3)$
   $2x^2 - 18$
9. $(d - 1)(5d - 4)$
   $5d^2 - 9d + 4$
10. $(2\ell + 5)(\ell - 4)$
    $2\ell^2 - 3\ell - 20$
11. $(3n - 7)(n + 3)$
    $3n^2 + 2n - 21$
12. $(q + 5)(5q - 1)$
    $5q^2 + 24q - 5$
13. $(3b + 3)(3b - 2)$
    $9b^2 + 3b - 6$
14. $(2m + 2)(3m - 3)$
    $6m^2 - 6$
15. $(4c + 1)(2c + 1)$
    $8c^2 + 6c + 1$
16. $(5a - 2)(2a - 3)$
    $10a^2 - 19a + 6$
17. $(4h - 2)(4h - 1)$
    $16h^2 - 12h + 2$
18. $(x - y)(2x - y)$
    $2x^2 - 3xy + y^2$
19. $(w + 4)(w^2 + 3w - 6)$
    $w^3 + 7w^2 + 6w - 24$
20. $(t + 1)(t^2 + 2t + 4)$
    $t^3 + 3t^2 + 6t + 4$
21. $(k + 4)(k^2 + 3k - 6)$
    $k^3 + 7k^2 + 6k - 24$
22. $(m + 3)(m^2 + 3m + 5)$
    $m^3 + 6m^2 + 14m + 15$

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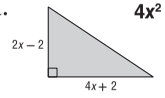
7-7 Practice

Multiplying Polynomials

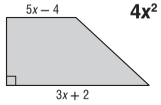
Find each product.

1. $(q + 6)(q + 5)$
   $q^2 + 11q + 30$
2. $(x + 7)(x + 4)$
   $x^2 + 11x + 28$
3. $(n - 4)(n - 6)$
   $n^2 - 10n + 24$
4. $(a + 5)(a - 6)$
   $a^2 - a - 30$
5. $(4b + 6)(b - 4)$
   $4b^2 - 10b - 24$
6. $(2x - 9)(2x + 4)$
   $4x^2 - 10x - 36$
7. $(6a - 3)(7a - 4)$
   $42a^2 - 45a + 12$
8. $(2x - 2)(5x - 4)$
   $10x^2 - 18x + 8$
9. $(3a - b)(2a - b)$
   $6a^2 - 5ab + b^2$
10. $(4g + 3h)(2g + 3h)$
    $8g^2 + 18gh + 9h^2$
11. $(m + 5)(m^2 + 4m - 8)$
    $m^3 + 9m^2 + 12m - 40$
12. $(t + 3)(t^2 + 4t + 7)$
    $t^3 + 7t^2 + 19t + 21$
13. $(2h + 3)(2h^2 + 3h + 4)$
    $4h^3 + 12h^2 + 17h + 12$
14. $(3d + 3)(2d^2 + 5d - 2)$
    $6d^3 + 21d^2 + 9d - 6$
15. $(3q + 2)(9q^2 - 12q + 4)$
    $27q^3 - 18q^2 - 12q + 8$
16. $(3r + 2)(9r^2 + 6r + 4)$
    $27r^3 + 36r^2 + 24r + 8$
17. $(3n^2 + 2n - 1)(2n^2 + n + 9)$
    $6n^4 + 7n^3 + 27n^2 + 17n - 9$
18. $(2t^2 + t + 3)(4t^2 + 2t - 2)$
    $8t^4 + 8t^3 + 10t^2 + 4t - 6$
19. $(2x^2 - 2x - 3)(2x^2 - 4x + 3)$
    $4x^4 - 12x^3 + 8x^2 + 6x - 9$
20. $(3y^2 + 2y + 2)(3y^2 - 4y - 5)$
    $9y^4 - 6y^3 - 17y^2 - 18y - 10$

GEOMETRY Write an expression to represent the area of each figure.

21. 

$4x^2 - 2x - 2$ units^2

22. 

$4x^2 + 3x - 1$ units^2

23. **NUMBER THEORY** Let x be an even integer. What is the product of the next two consecutive even integers? $x^2 + 6x + 8$

24. **GEOMETRY** The volume of a rectangular pyramid is one third the product of the area of its base and its height. Find an expression for the volume of a rectangular pyramid whose base has an area of $3x^2 + 12x + 9$ square feet and whose height is $x + 3$ feet.
    $x^3 + 7x^2 + 15x + 9$ ft^3

Answers

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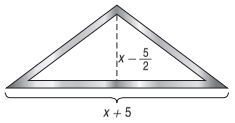
7-7 Word Problem Practice

Multiplying Polynomials

1. **THEATER** The Loft Theater has a center seating section with $3c + 8$ rows and $4c - 1$ seats in each row. Write an expression for the total number of seats in the center section. $12c^2 + 29c - 8$

2. **CRAFTS** Suppose a quilt made up of squares has a length-to-width ratio of 5 to 4. The length of the quilt is $5x$ inches. The quilt can be made slightly larger by adding a border of 1-inch squares all the way around the perimeter of the quilt. Write a polynomial expression for the area of the larger quilt.
$20x^2 + 18x + 4$

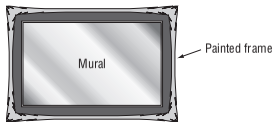
3. **SERVICE** A folded United States flag is sometimes presented to individuals in recognition of outstanding service to the country. The flag is presented folded in a triangle. Often the recipient purchases a case designed to display the folded flag to protect it from wear. One such display case has dimensions (in inches) shown below. Write a polynomial expression that represents the area of wall space covered by the display case.
$\frac{x^2}{2} + \frac{5}{4}x - \frac{25}{4}$



Source: American Flag Store

4. **MATH FUN** Think of a whole number. Subtract 2. Write down this number. Take the original number and add 2. Write down this number. Find the product of the numbers you wrote down. Subtract the square of the original number. The result is always –4. Use polynomials to show how this number trick works.
$(x - 2)(x + 2) - x^2 = -4$
$x^2 - 2x + 2x - 4 - x^2 = -4$
$-4 = -4$

5. **ART** The museum where Julia works plans to have a large wall mural replica of Vincent van Gogh's *The Starry Night* painted in its lobby. First, Julia wants to paint a large frame around where the mural will be. The mural's length will be 5 feet longer than its width, and the frame will be 2 feet wide on all sides. Julia has only enough paint to cover 100 square feet of wall surface. How large can the mural be?



a. Write an expression for the area of the mural. $w(w + 5)$

b. Write an expression for the area of the frame.
$(w + 9)(w + 4) - (w + 5)w$

c. Write and solve an equation to find how large the mural can be.
$(w + 9)(w + 4) - (w + 5)w = 100$
8 ft by 13 ft

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7-7 Enrichment

Pascal's Triangle

This arrangement of numbers is called Pascal's Triangle. It was first published in 1665, but was known hundreds of years earlier.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

1. Each number in the triangle is found by adding two numbers. What two numbers were added to get the 6 in the 5th row?
3 and 3

2. Describe how to create the 6th row of Pascal's Triangle.
The first and last numbers are 1. Evaluate 1 + 4, 4 + 6, 6 + 4, and 4 + 1 to find the other numbers.

3. Write the numbers for rows 6 through 10 of the triangle.

Row 6:	1	5	10	10	5	1				
Row 7:	1	6	15	20	15	6	1			
Row 8:	1	7	21	35	35	21	7	1		
Row 9:	1	8	28	56	70	56	28	8	1	
Row 10:	1	9	36	84	126	126	84	36	9	1

Multiply to find the expanded form of each product.

4. $(a + b)^2$ $a^2 + 2ab + b^2$

5. $(a + b)^3$ $a^3 + 3a^2b + 3ab^2 + b^3$

6. $(a + b)^4$ $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Now compare the coefficients of the three products in Exercises 4–6 with Pascal's Triangle.

7. Describe the relationship between the expanded form of $(a + b)^n$ and Pascal's Triangle.
The coefficients of the expanded form are found in row $n + 1$ of Pascal's Triangle.

8. Use Pascal's Triangle to write the expanded form of $(a + b)^6$.
$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

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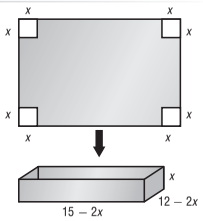
7-7 Spreadsheet Activity

Multiplying Polynomials

Example **A box is made by cutting a square with sides x inches long from each corner of a piece of cardboard and folding up the sides. If the piece of cardboard is 15 inches long and 12 inches wide, what integer value of x allows you to make the box with the greatest volume? What is the volume?**



Step 1 The finished box will be x inches high, $12 - 2x$ inches wide, and $15 - 2x$ inches long. The volume of the box is $x(12 - 2x)(15 - 2x)$ cubic inches.

Step 2 Use Column A of the spreadsheet for the value of x. Enter the formulas for the width, length, and volume in Columns C, D, and E.

	A	B	C	D
1	Height	Width	Length	Volume
2	1	10	13	130
3	2	8	11	176
4	3	6	9	162
5	4	4	7	112
6	5	2	5	50
7				

Sheet 1 Sheet 2 Sheet 3

Notice that x cannot be greater than 5 because $12 - 2x$ must be positive. In this case, the box with the greatest volume is 176 cubic inches when $x = 2$.

Exercises

Use the spreadsheet to find the value of x that allows the box with the greatest volume for each piece of cardboard. State the volume of the box.

1. 16 inches long and 10 inches wide
   2; 144 in³
2. 24 inches long and 18 inches wide
   3; 648 in³
3. 28 inches long and 16 inches wide
   3; 660 in³
4. 36 inches long and 24 inches wide
   5; 1820 in³
5. 48 inches long and 48 inches wide
   8; 8192 in³
6. 108 inches long and 44 inches wide
   10; 21,120 in³
7. Study the spreadsheet you created for Exercise 5. Suppose y is the volume of the box with a height of x inches. If you were to graph the ordered pairs (x, y) and connect them with a smooth curve, what would you expect the graph to look like? Use the graphing tool in the spreadsheet to verify your conjecture. **Sample answer: The graph would rise from (0, 0) to the point where x gives the greatest volume and then fall back down toward the x-axis.**

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7-8 Study Guide and Intervention

Special Products

Squares of Sums and Differences Some pairs of binomials have products that follow specific patterns. One such pattern is called the *square of a sum*. Another is called the *square of a difference*.

Square of a sum	$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
Square of a difference	$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Example 1 **Find $(3a + 4)(3a + 4)$.**

Use the square of a sum pattern, with $a = 3a$ and $b = 4$.

$(3a + 4)(3a + 4) = (3a)^2 + 2(3a)(4) + (4)^2$
$= 9a^2 + 24a + 16$

The product is $9a^2 + 24a + 16$.

Example 2 **Find $(2z - 9)(2z - 9)$.**

Use the square of a difference pattern with $a = 2z$ and $b = 9$.

$(2z - 9)(2z - 9) = (2z)^2 - 2(2z)(9) + (9)(9)$
$= 4z^2 - 36z + 81$

The product is $4z^2 - 36z + 81$.

Exercises

Find each product.

1. $(x - 6)^2$
   $x^2 - 12x + 36$
2. $(3p + 4)^2$
   $9p^2 + 24p + 16$
3. $(4x - 5)^2$
   $16x^2 - 40x + 25$
4. $(2x - 1)^2$
   $4x^2 - 4x + 1$
5. $(2h + 3)^2$
   $4h^2 + 12h + 9$
6. $(m + 5)^2$
   $m^2 + 10m + 25$
7. $(a + 3)^2$
   $a^2 + 6a + 9$
8. $(3 - p)^2$
   $9 - 6p + p^2$
9. $(x - 5y)^2$
   $x^2 - 10xy + 25y^2$
10. $(8y + 4)^2$
    $64y^2 + 64y + 16$
11. $(8 + x)^2$
    $64 + 16x + x^2$
12. $(3a - 2b)^2$
    $9a^2 - 12ab + 4b^2$
13. $(2x - 8)^2$
    $4x^2 - 32x + 64$
14. $(x^2 + 1)^2$
    $x^4 + 2x^2 + 1$
15. $(m^2 - 2)^2$
    $m^4 - 4m^2 + 4$
16. $(x^3 - 1)^2$
    $x^6 - 2x^3 + 1$
17. $(2h^2 - k^2)^2$
    $4h^4 - 4h^2k^2 + k^4$
18. $\left(\frac{1}{4}x + 3\right)^2$
    $\frac{1}{16}x^2 + \frac{3}{2}x + 9$
19. $(x - 4y^2)^2$
    $x^2 - 8xy^2 + 16y^4$
20. $(2p + 4r)^2$
    $4p^2 + 16pr + 16r^2$
21. $\left(\frac{2}{3}x - 2\right)^2$
    $\frac{4}{9}x^2 - \frac{8}{3}x + 4$

Answers

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7-8 Study Guide and Intervention *(continued)*

Special Products

Product of a Sum and a Difference There is also a pattern for the product of a sum and a difference of the same two terms, $(a + b)(a - b)$. The product is called the **difference of squares**.

Product of a Sum and a Difference	$(a + b)(a - b) = a^2 - b^2$

Example **Find $(5x + 3y)(5x - 3y)$.**

$(a + b)(a - b) = a^2 - b^2$ Product of a Sum and a Difference

$(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2$ $a = 5x$ and $b = 3y$

$= 25x^2 - 9y^2$ Simplify.

The product is $25x^2 - 9y^2$.

Exercises

Find each product.

1. $(x - 4)(x + 4)$ — $x^2 - 16$
2. $(p + 2)(p - 2)$ — $p^2 - 4$
3. $(4x - 5)(4x + 5)$ — $16x^2 - 25$
4. $(2x - 1)(2x + 1)$ — $4x^2 - 1$
5. $(h + 7)(h - 7)$ — $h^2 - 49$
6. $(m - 5)(m + 5)$ — $m^2 - 25$
7. $(2d - 3)(2d + 3)$ — $4d^2 - 9$
8. $(3 - 5q)(3 + 5q)$ — $9 - 25q^2$
9. $(x - y)(x + y)$ — $x^2 - y^2$
10. $(y - 4x)(y + 4x)$ — $y^2 - 16x^2$
11. $(8 + 4x)(8 - 4x)$ — $64 - 16x^2$
12. $(3a - 2b)(3a + 2b)$ — $9a^2 - 4b^2$
13. $(3y - 8)(3y + 8)$ — $9y^2 - 64$
14. $(x^2 - 1)(x^2 + 1)$ — $x^4 - 1$
15. $(m^2 - 5)(m^2 + 5)$ — $m^4 - 25$
16. $(x^3 - 2)(x^3 + 2)$ — $x^6 - 4$
17. $(h^2 - k^2)(h^2 + k^2)$ — $h^4 - k^4$
18. $\left(\frac{1}{4}x + 2\right)\left(\frac{1}{4}x - 2\right)$ — $\frac{1}{16}x^2 - 4$
19. $(3x - 2y^2)(3x + 2y^2)$ — $9x^2 - 4y^4$
20. $(2p - 5r)(2p + 5r)$ — $4p^2 - 25r^2$
21. $\left(\frac{4}{3}x - 2y\right)\left(\frac{4}{3}x + 2y\right)$ — $\frac{16}{9}x^2 - 4y^2$

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7-8 Skills Practice

Special Products

Find each product.

1. $(n + 3)^2$ — $n^2 + 6n + 9$
2. $(x + 4)(x + 4)$ — $x^2 + 8x + 16$
3. $(y - 7)^2$ — $y^2 - 14y + 49$
4. $(t - 3)(t - 3)$ — $t^2 - 6t + 9$
5. $(b + 1)(b - 1)$ — $b^2 - 1$
6. $(a - 5)(a + 5)$ — $a^2 - 25$
7. $(p - 4)^2$ — $p^2 - 8p + 16$
8. $(z + 3)(z - 3)$ — $z^2 - 9$
9. $(\ell + 2)(\ell + 2)$ — $\ell^2 + 4\ell + 4$
10. $(r - 1)(r - 1)$ — $r^2 - 2r + 1$
11. $(3g + 2)(3g - 2)$ — $9g^2 - 4$
12. $(2m - 3)(2m + 3)$ — $4m^2 - 9$
13. $(6 + u)^2$ — $36 + 12u + u^2$
14. $(r + t)^2$ — $r^2 + 2rt + t^2$
15. $(3q + 1)(3q - 1)$ — $9q^2 - 1$
16. $(c - d)^2$ — $c^2 - 2cd + d^2$
17. $(2k - 2)^2$ — $4k^2 - 8k + 4$
18. $(w + 3h)^2$ — $w^2 + 6wh + 9h^2$
19. $(3p - 4)(3p + 4)$ — $9p^2 - 16$
20. $(t + 2u)^2$ — $t^2 + 4tu + 4u^2$
21. $(x - 4y)^2$ — $x^2 - 8xy + 16y^2$
22. $(3b + 7)(3b - 7)$ — $9b^2 - 49$
23. $(3y - 3g)(3y + 3g)$ — $9y^2 - 9g^2$
24. $(n^2 + r^2)^2$ — $n^4 + 2n^2r^2 + r^4$
25. $(2k + m^2)^2$ — $4k^2 + 4km^2 + m^4$
26. $(3t^2 - n)^2$ — $9t^4 - 6t^2n + n^2$

27. **GEOMETRY** The length of a rectangle is the sum of two whole numbers. The width of the rectangle is the difference of the same two whole numbers. Using these facts, write a verbal expression for the area of the rectangle. **The area is the square of the larger number minus the square of the smaller number.**

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7-8 Practice

Special Products

Find each product.

1. $(n + 9)^2$
   $n^2 + 18n + 81$
2. $(q + 8)^2$
   $q^2 + 16q + 64$
3. $(x - 10)^2$
   $x^2 - 20x + 100$
4. $(r - 11)^2$
   $r^2 - 22r + 121$
5. $(p + 7)^2$
   $p^2 + 14p + 49$
6. $(b + 6)(b - 6)$
   $b^2 - 36$
7. $(z + 13)(z - 13)$
   $z^2 - 169$
8. $(4j + 2)^2$
   $16j^2 + 16j + 4$
9. $(5w - 4)^2$
   $25w^2 - 40w + 16$
10. $(6h - 1)^2$
    $36h^2 - 12h + 1$
11. $(3m + 4)^2$
    $9m^2 + 24m + 16$
12. $(7v - 2)^2$
    $49v^2 - 28v + 4$
13. $(7k + 3)(7k - 3)$
    $49k^2 - 9$
14. $(4d - 7)(4d + 7)$
    $16d^2 - 49$
15. $(3g + 9h)(3g - 9h)$
    $9g^2 - 81h^2$
16. $(4q + 5t)(4q - 5t)$
    $16q^2 - 25t^2$
17. $(a + 6u)^2$
    $a^2 + 12au + 36u^2$
18. $(5r + s)^2$
    $25r^2 + 10rs + s^2$
19. $(6h - m)^2$
    $36h^2 - 12hm + m^2$
20. $(k - 6y)^2$
    $k^2 - 12ky + 36y^2$
21. $(u - 7p)^2$
    $u^2 - 14up + 49p^2$
22. $(4b - 7v)^2$
    $16b^2 - 56bv + 49v^2$
23. $(6n + 4p)^2$
    $36n^2 + 48np + 16p^2$
24. $(5q + 6t)^2$
    $25q^2 + 60qt + 36t^2$
25. $(6a - 7b)(6a + 7b)$
    $36a^2 - 49b^2$
26. $(8h + 3d)(8h - 3d)$
    $64h^2 - 9d^2$
27. $(9x + 2y^2)^2$
    $81x^2 + 36xy^2 + 4y^4$
28. $(3p^3 + 2m)^2$
    $9p^6 + 12p^3m + 4m^2$
29. $(5a^2 - 2b)^2$
    $25a^4 - 20a^2b + 4b^2$
30. $(4m^3 - 2t)^2$
    $16m^6 - 16m^3t + 4t^2$
31. $(6b^3 - g)^2$
    $36b^6 - 12b^3g + g^2$
32. $(2b^2 - g)(2b^2 + g)$
    $4b^4 - g^2$
33. $(2v^2 + 3x^2)(2v^2 + 3x^2)$
    $4v^4 + 12v^2x^2 + 9x^4$

34. **GEOMETRY** Janelle wants to enlarge a square graph that she has made so that a side of the new graph will be 1 inch more than twice the original side g. What trinomial represents the area of the enlarged graph? $4g^2 + 4g + 1$

35. **GENETICS** In a guinea pig, pure black hair coloring B is dominant over pure white coloring b. Suppose two hybrid Bb guinea pigs, with black hair coloring, are bred.

    a. Find an expression for the genetic make-up of the guinea pig offspring.
    $0.25BB + 0.50Bb + 0.25bb$

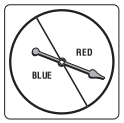
    b. What is the probability that two hybrid guinea pigs with black hair coloring will produce a guinea pig with white hair coloring? 25%

NAME _____ DATE _____ PERIOD _____

7-8 Word Problem Practice

Special Products

1. **PROBABILITY** The spinner below is divided into 2 equal sections. If you spin the spinner 2 times in a row, the possible outcomes are shown in the table below.

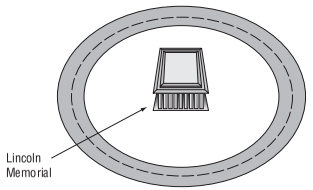


red red	blue red
red blue	blue blue

What is the probability of spinning a blue and a red in two spins? 50%

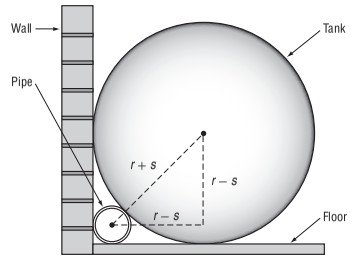
2. **GRAVITY** The height of a penny t seconds after being dropped down a well is given by the product of $(10 - 4t)$ and $(10 + 4t)$. Find the product and simplify. What type of special product does this represent?
   $100 - 16t^2$; product of a sum and difference

3. **TRAFFIC PLANNING** The Lincoln Memorial in Washington, D.C., is surrounded by a circular drive called Lincoln Circle. Suppose the National Park Service wants to change the layout of Lincoln Circle so that there are two concentric circular roads. Write a polynomial equation for the area A of the space between the roads if the radius of the smaller road is 10 meters less than the radius of the larger road.
   $A = 20\pi r - 100\pi$



4. **BUSINESS** The Combo Lock Company finds that its profit data from 2005 to the present can be modeled by the function $y = 4n^2 + 44n + 121$, where y is the profit n years since 2005. Which special product does this polynomial demonstrate? Explain.
   square of a sum; it can also be written as $(2n + 11)^2$

5. **STORAGE** A cylindrical tank is placed along a wall. A cylindrical PVC pipe will be hidden in the corner behind the tank. See the side view diagram below. The radius of the tank is r inches and the radius of the PVC pipe is s inches.



   a. Use the Pythagorean Theorem to write an equation for the relationship between the two radii. Simplify your equation so that there is a zero on one side of the equals sign.
   $0 = r^2 - 6rs + s^2$

   b. Write a polynomial equation you could solve to find the radius s of the PVC pipe if the radius of the tank is 20 inches. $0 = s^2 - 120s + 400$

Answers

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