Answers to Algebra 1 Unit 5 Practice

1. a.

Length (ft)	Width (ft)	Perimeter (ft)	Area (ft²)
5	30	70	150
10	25	70	250
15	20	70	300
20	15	70	300
25	10	70	250
30	5	70	150
l	$35 - l$	70	$l(35 - l)$

b.
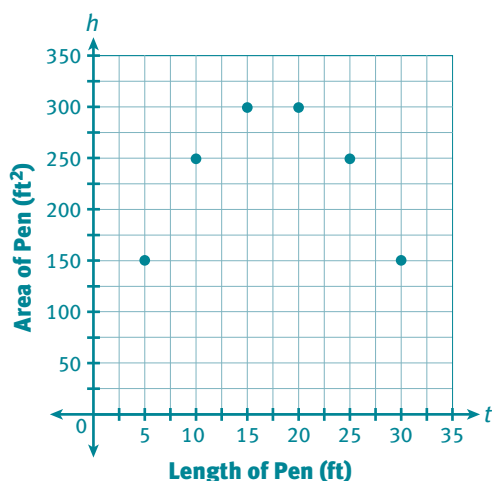
c. $A(x) = x(35 - x)$; $A(x) = -x^2 + 35x$; $a = -1, b = 35, c = 0$

d. The points on Simon's graph will be above the points on Kathleen's graph. For example, at $x = 25$ feet in length, Simon's area will be greater than 250 ft² because his width will be greater than 10 feet.

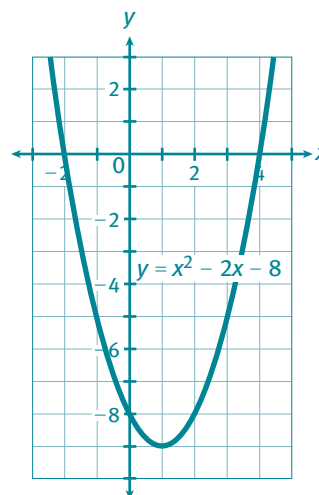**2.** C

3. $f(x) = -x^2 - 2x + 3$; $a = -1, b = -2, c = 3$

4. a. There is a term with an exponent of 3 and there is no x^2-term.

b. The x^2-term is in a denominator.

c. There is no x^2-term.

5. D

6. a. Students' tables will vary.

b. The minimum value is -9.

c. To find the x-intercepts using the graph, identify the points where the graph crosses the x-axis. To find the x-intercepts using the equation, find the values of x for which $f(x) = 0$. In this case $(x + 2)(x - 4) = 0$ means that $x = -2$ or $x = 4$; these are the x-coordinates of the x-intercepts. The x-intercepts are $(-2, 0)$ and $(4, 0)$.

7. The range gives the possible y-values of a function. The minimum is the least y-value, so the range is all values greater than or equal to the minimum. For $f(x)$, the minimum is 3, so the range is $y \geq 3$. The maximum is the greatest y-value, so the range is all values less than or equal to the maximum. For $g(x)$, the maximum is 7, so the range is $y \leq 7$.

8. a. $f(x)$ is increasing when $x > 1$; $g(x)$ is increasing when $x < 3$

b. $f(x)$ is decreasing when $x < 1$; $g(x)$ is decreasing when $x > 3$

c. The maximum or minimum value is the y-coordinate of the vertex. The x-coordinate of the vertex is the value of x for which the function changes from increasing to decreasing, or vice versa.

A1

9. vertical translation 4 units down

10. C

11. $x = -2$; the axis of symmetry is a vertical line through the vertex, so $x =$ the x-coordinate of the vertex is the equation of the axis of symmetry.

12. Answers may vary; $y = x^2 + 3$

13. a. $y = x^2 + 2$

b. $y = (x + 3)^2$

14. A

15. vertical stretch by a factor of 2

16. The graphs of $g(x)$ and $h(x)$ are a shrink and a stretch, respectively, of the graph of $f(x)$. They are not translations. The graphs do not move horizontally or vertically, so the vertex does not change.

17. a. Answers will vary; $q(x) = \dfrac{1}{8}x^2$

b. Infinitely many; any function in the form $q(x) = ax^2$ where $0 < a < \dfrac{1}{3}$ will have a graph that is wider than the graph of $g(x)$.

18. No; although $\dfrac{5}{2}$ is a fraction, its value is greater than 1, so the graph will be a vertical stretch of the graph of $f(x)$.

19. B

20. a. True; the equations indicate that the vertex of $g(x)$ is $(2, 4)$, the vertex of $h(x)$ is $(-1, -2)$, and the vertex of $k(x)$ is $(-3, 4)$. Because the vertex of $k(x)$ has the least x-coordinate, its vertex is farthest to the left.

b. False; because $\dfrac{1}{3}$ is positive, the graph of $g(x)$ opens upward and the function has a minimum value of 4. Because -2 is negative, the graph of $k(x)$ opens downward and the function has a maximum value of 4. Therefore, the range of $g(x)$ is $y \geq 4$ and the range of $k(x)$ is $y \leq 4$.

c. False; the vertex of the graph of $h(x)$ is $(-1, -2)$, so the minimum value of $h(x)$ is -2.

d. True; $k(x)$ is the only function for which the value of a is negative, so it is the only function with a maximum value.

21. From the equation of $g(x)$, the vertex is $(2, 4)$ and the graph opens upward. This means that the minimum value of $g(x)$ is 4; there is no value of x for which $g(x) = 0$ because $0 < 4$. Therefore there are no x-intercepts.

22. $y = 2x^2 - 8x + 6$

23. a. $x = \dfrac{3}{4}$

b. $x = -\dfrac{1}{3}$ or $x = -2$

24. D

25. a. Answers may vary; $x^2 - 9x - 36 = 0$

b. Yes; justifications may vary. Students may point out that multiplying both sides of their equation in part a by any constant will result in an equation with the same roots. Students may also point out that there is more than one parabola with x-intercepts $(-3, 0)$ and $(12, 0)$, so there must be more than one quadratic equation with roots -3 and 12.

26. The equation must be equal to zero in order to use the Zero Product Property. Carmine should have subtracted 12 from each side before factoring. The correct solution is

$$x^2 + 3x - 28 = 12$$
$$x^2 + 3x - 40 = 0$$
$$(x + 8)(x - 5) = 0$$
$$x + 8 = 0 \ \text{or} \ \ x - 5 = 0$$
$$x = -8 \ \ \text{or} \ \ x = 5$$

27. D

28. a. $(-2, 9)$

b. $\left(-\dfrac{3}{2}, -\dfrac{21}{4}\right)$

29. a. 50 feet

b. c must be a negative number greater than -50. The vertex of the graph corresponds to the maximum height of the water. Because the maximum height of the water in this fountain is less than 50 feet, its vertex must be below the vertex of the graph of $f(x)$. Adding a negative number to a function will translate the vertex of its graph down. However, the height of the water must be greater than 0, so the graph cannot be translated down 50 units or more.

30. -2

31. x-intercepts: $(-2, 0)$ and $(5, 0)$; the axis of symmetry is halfway between the x-intercepts, so the axis of symmetry is $x = \dfrac{3}{2}$.
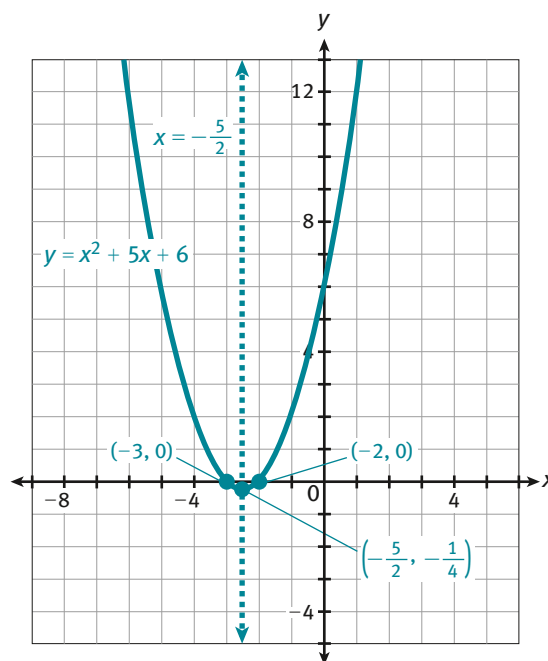
32. Answers may vary. The coordinates of the vertex are not integers. Because the vertex lies on the axis of symmetry, the x-coordinate of the vertex is $\dfrac{3}{2}$. If the equation of the function were given, you could substitute $x = \dfrac{3}{2}$ into the equation to find the y-coordinate of the vertex.
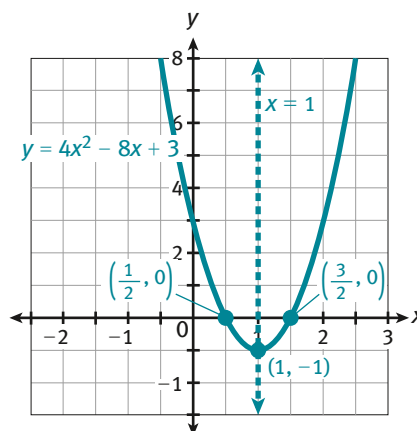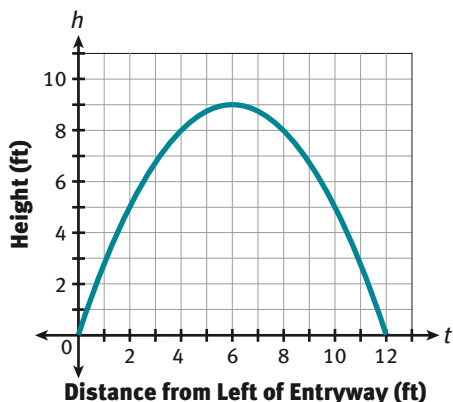
33. C

34. a.

b.

c.

A3
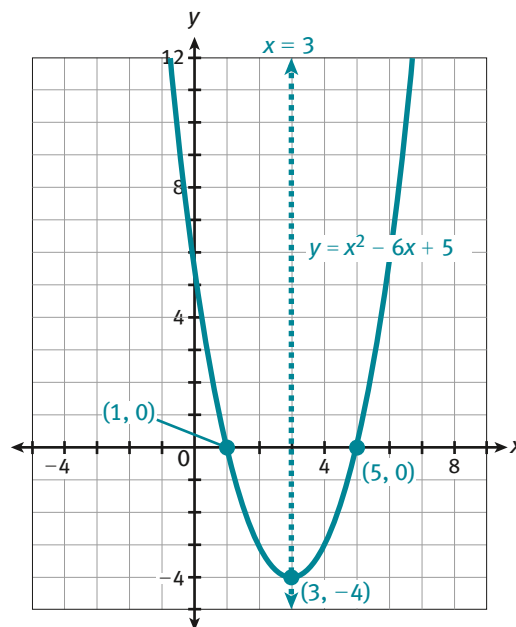
35. a.

b. The distance between the x-intercepts is the width of the entryway. The x-intercepts are $(0, 0)$ and $(12, 0)$ so the width of the entryway is 12 feet. The vertex represents the greatest height of the entryway. The vertex is $(6, 9)$ so the greatest height is 9 feet.

36. B

37. a. $x = \pm \dfrac{5}{2}$

b. $x = \pm \sqrt{6}$

c. $x = 4 \pm \sqrt{2}$

d. $x = 3 \pm \sqrt{5}$

38. a. 11 feet by 11 feet

b. Answers will vary. Any description in which the walkway width and the side length of the garden add to 15 feet is correct. For example, to increase the walkway width to 5 feet, the new dimensions of the garden are 10 feet by 10 feet. To decrease the walkway width to 3 feet, the new dimensions of the garden are 12 feet by 12 feet.

39. Rita is correct because any value of $c < 0$ will result in an equation with no real solutions. Phil is incorrect because only $c = 0$ will result in an equation with exactly one solution.

40. C

41. a. 16; 4; there is only one possible answer because the second number must be $\dfrac{8}{2} = 4$ and the first number must be $4^2 = 16$.

b. 36; -6; there is only one possible answer because the second number must be $\dfrac{-12}{2} = -6$ and the first number must be $(-6)^2 = 36$.

c. Answers will vary; any numbers $2c$, c^2, and c for an integer value of c are correct.

42. a. $x = 2 \pm \sqrt{10}$

b. $x = 13$ or $x = 1$

c. $x = -7$ or $x = 1$

d. $x = -8$ or $x = -4$

43. a. $g(x) = \left(x - \dfrac{5}{2} \right)^2 - \dfrac{9}{4}$

b. The graph of $g(x)$ has been translated $\dfrac{5}{2}$ units to the right and $\dfrac{9}{4}$ units down from the graph of $f(x)$.

c. The answer to part a indicates that the vertex of the graph of $g(x)$ is $\left(\dfrac{5}{2}, -\dfrac{9}{4} \right)$. Because the graph of $g(x)$ is a translation (not a reflection) of the graph of $f(x)$, the graph of $g(x)$ opens in the same direction as $f(x)$, upward. The vertex indicates that the axis of symmetry is $x = \dfrac{5}{2}$; this is also indicated by the fact that the graph of $g(x)$ is translated $\dfrac{5}{2}$ units to the right from the graph of $f(x)$, so the axis of symmetry $x = 0$ is translated 5 units to the right to $x = \dfrac{5}{2}$. The graph of $f(x)$ intersects the x-axis at one point, $(0, 0)$; translating the graph down $\dfrac{9}{4}$ units means that $g(x)$ will intersect the x-axis in two points, so $g(x)$ has two x-intercepts.

d. Yes, Claudia's method is correct because the y-intercept is the point whose x-coordinate is 0. However, it is probably easier to substitute 0 into the original equation $g(x) = x^2 - 5x + 4$; the y-coordinate 4 is apparent upon inspection.

44. $A(w) = -(w - 40)^2 + 1600$

The vertex is (40, 1600). The greatest area is 1600 ft^2 and occurs when the length and width both equal 40 feet.

45. No; Tranh did not write the equation in standard form before identifying a, b, and c. The equation in standard form is $4x^2 + 3x - 2 = 0$, which gives $a = 4$, $b = 3$, and $c = -2$.

46. a. $x = \dfrac{15}{2}$ or $x = -4$

  b. $x = -3 \pm \sqrt{21}$

  c. $x = 5 \pm \sqrt{2}$

47. B

48. a. $-16t^2 + 60 = 30$

  b. $t = \dfrac{\sqrt{30}}{4} \approx 1.37$ seconds; the equation has two solutions, $t = \pm\dfrac{\sqrt{30}}{4}$, but only the positive solution makes sense in the context of the problem.

49. Answers and explanations will vary.

  a. Factoring; the equation factors to $(x - 1)(2x + 5) = 0$.

  b. Square roots; when the equation is in the form $ax^2 = c$, it can be solved by taking the square root of both sides.

  c. Factoring; the equation has constant term 0 so it can be readily factored by finding the GCF of the terms.

  d. Quadratic formula or completing the square; the trinomial does not factor.

  e. Factoring; the equation factors to $(x + 7)(x - 1) = 0$.

  f. Completing the square; $x^2 + 3x - 5$ does not factor and the equation is in the form $ax^2 + bx = c$, so it is ready for completing the square.

50. D

51. a. 1

  b. 2

  c. 0

52. If the solutions are radical, then the discriminant is a perfect square, because the square root of a perfect square is rational. If the discriminant is not a perfect square, its square root will be irrational, so the solutions will be irrational.

**53.** D

54. a. $x = \pm 11i$

  b. $x = -2 \pm i$

55. Answers may vary. You can solve the equation to find that the two solutions are $5 \pm 2i$. You can find that the discriminant, $(-10)^2 - 4(1)(29) = -16$, is negative. You can graph the related function $y = x^2 - 10x + 29$ and observe that the graph does not cross the x-axis.

56. a. $i = \sqrt{-1}, i^2 = -1, i^3 = -\sqrt{-1}, i^4 = 1,$
  $i^5 = \sqrt{-1}, i^6 = -1, i^7 = -\sqrt{-1}, i^8 = 1,$
  $i^9 = \sqrt{-1}, i^{10} = -1, i^{11} = -\sqrt{-1}, i^{12} = 1$

  b. $i^{57} = \sqrt{-1}$; explanations may vary. The pattern repeats in groups of 4. Any time i has a power that is a multiple of 4, it is equal to 1. Because $i^{57} = i^{56} \cdot i$, and $i^{56} = 1, i^{57} = 1 \cdot i, = i$.

57. a. No; explanations may vary. The data points do not lie on a line.

Weekly Profit from Pancakes per Restaurant
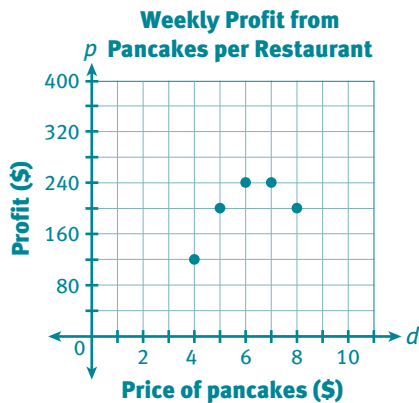
b. As the price of pancakes increases, the profit increases to a maximum and then decreases. This makes sense because, up to a point, a greater price for pancakes will result in more profit. However, there will be a point where pancakes are too expensive and customers will not buy them, resulting in less profit.

58. $p(d) = -20d^2 + 260d - 600$

59. B

60. $245; $6.50

61. $3 < d < 10$, where d is the price of pancakes in dollars; explanations may vary. Solving the equation $0 = -20d^2 + 260d - 600$ shows that the profit is $0 when $d = 3$ or $d = 10$. So, the value of the profit will be positive when $3 < d < 10$.

62. Yes; explanations may vary. Evaluating the function $h(t)$ for $t = 0$ shows that the height of the ball when it is launched is 4 meters.

63. C

64. $3 = -4.9t^2 + 20t + 4$, or equivalent; $t \approx -0.05$ or $t \approx 4.13$. The tennis ball will be at a height of 3 meters approximately 4.13 seconds after it is launched.

65. The equation has two solutions but only the positive solution makes sense in the problem. It is important to keep the context in mind because not all solutions will be meaningful in the situation. In this problem, the negative solution can be rejected because t represents time in seconds and a negative value for the time does not make sense.

66. $1.09 < t < 2.99$, where t is the time in seconds; explanations may vary. I used the quadratic formula to solve the equation $20 = -4.9t^2 + 20t + 4$. The solutions of this equation are $t \approx 1.09$ and $t \approx 2.99$, so the tennis ball will have a height greater than 20 meters between these two times.

67. Cat video: quadratic; explanations may vary. The numbers of people who viewed the cat video increased and then decreased, which suggests a quadratic function.

Dog video: linear; explanations may vary. The numbers of people who viewed the dog video increased by about 15 people each month. A rate of change that is nearly constant suggests a linear function.

68. Cat video: $y = -1.786x^2 + 11.386x + 52.4$, where y is the number of viewers and x is the week number; Dog video: $y = 14.257x + 21.933$, where y is the number of viewers and x is the week number

69.

Viewers of Cat Video
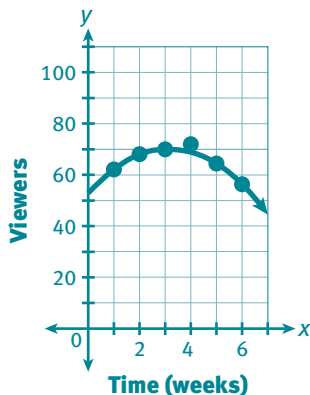
Viewers of Dog Video
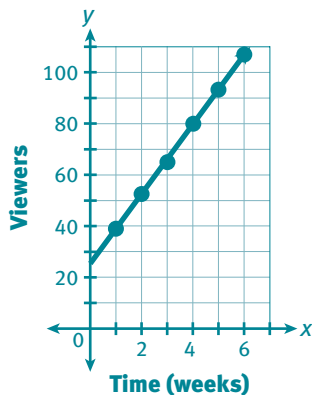
70. Answers may vary. The reasonable domains of the functions are the same: $x > 0$, where x is a whole number. The cat function has a maximum of about 71, while the dog function has no maximum value. The cat function does not have a constant rate of change, while the dog function has a constant rate of change of about 14 viewers per week.

71. B

72. C

73. $100; evaluating both functions for $t = 0$ shows that $f(0) = 1200$ and $m(0) = 1100$. So, Fatima's account initially had $1200, and Mason's account initially had $1100.

74. Yes; justifications may vary. The function that models Fatima's account is an increasing linear function, and the function that models Mason's account is an increasing exponential function. The value of any increasing exponential function will eventually exceed the value of any increasing linear function.

75. Liza's; explanations may vary. Trevon's population does not appear to be modeled by an exponential function because the graph shows that his population first decreases and then increases. Liza's population does appear to be modeled by an exponential function because the rate of change in her population increases over time. The graph shows that the population almost doubles every 10 days.

76. No; the range for Liza's population is $y \geq 20$ because the population starts at 20 and increases from there. The range for Trevon's population is approximately $y \geq 6$ because the population starts at 20, decreases to about 6, and increases from there.

77. A

78. Answers may vary. The water level starts at a depth of 84 inches and remains constant until day 3. On day 3, the water level begins to drop at a constant rate of $\frac{4}{3}$ inches per day until day 6, when it reaches a depth of 80 inches. On day 6, the depth of the water begins to increase at a constant rate of 2 inches per day.

79. No; the depth of the water is increasing at a rate of 2 inches per day, not by a factor of 2 each day.

80. Answers may vary. The ball starts at a maximum height of 8 feet. Between 0 seconds and 0.5 second, the height of the ball decreases from 8 feet to 4 feet. In this part of the graph, the rate of change in the height is not constant; the ball falls faster and faster over time. Between 0.5 second and 0.9 second, the height of the ball decreases from 4 feet to 2 feet, but the rate of change in this part of the graph is constant. From 0.9 second on, the ball remains at its minimum height of 2 feet.

81. Answers may vary. The heights of both balls can be modeled by piecewise functions. Team B's ball starts at a lower height (6 feet) than Team A's ball (8 feet), but both balls end at the same height (2 feet). Team B's ball reaches its minimum height faster than Team A's ball (0.5 second versus 0.9 second). As Team B's ball moves from its maximum height to its minimum height, the rate of change in the ball's height is never constant. By contrast, the rate of change in the height Team A's ball is constant for part of the time that the ball moves from its maximum height to its minimum height.

82. $(2, 4)$; check by substituting $x = 2$ into each equation and verifying that the result is 4: $-3(2) + 10 = -6 + 10 = 4$ and $2^2 = 4$.
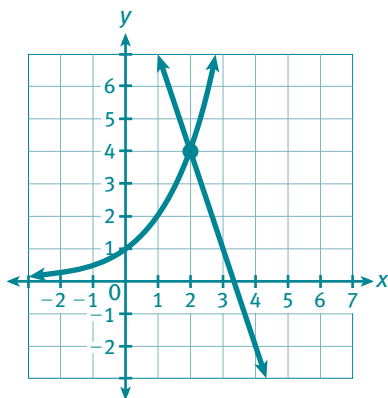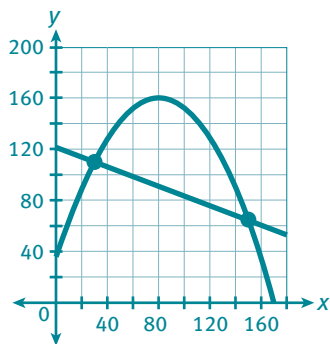

83. Answers will vary.
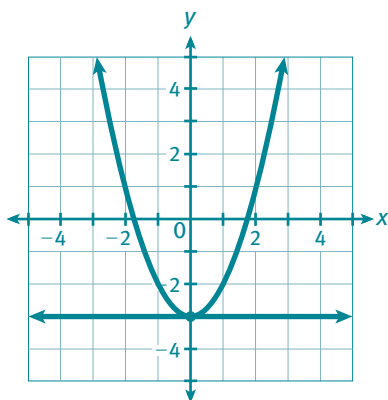
84. B

85. $(30, 110)$ and $(150, 62)$

86. August; explanations may vary. The equation $y = 2^x$ models the percent of the pond's surface covered by Plant A, and the equation $y = 10 + 2x$ models the percent of the pond's surface covered by Plant B. In both equations, x is the number of months since the beginning of April. A graph of this system on a graphing calculator shows that the solutions are approximately $(-4.984, 0.032)$ and $(4.202, 18.404)$. The solution $(-4.984, 0.032)$ can be ignored because a negative value of x does not make sense in this situation. The solution $(4.202, 18.404)$ shows that the percent covered by Plant A will be equal to the percent covered by Plant B in a bit more than 4 months from the beginning of April. So, the percents will be equal sometime in August.

87. $(3, 3)$ and $(7, -5)$

88. $w(t) = 9 + 3t$

89. 1 second; explanations may vary. Write the functions for the heights of the box and the worker's hands as a system of equations. Solving the system using the quadratic formula shows that the solutions are $\left(-\dfrac{19}{16}, \dfrac{87}{16}\right)$ and $(1, 12)$. The first solution can be rejected because a negative value for the time does not make sense. The second solution shows that the box and the worker's hands are at the same height after 1 second.

90. B

91. Answers may vary. Solve each equation in the system for y: $y = 5 - x$ and $y = -x^2 + 3$. Then set the two expressions for y equal to each other: $5 - x = -x^2 + 3$. Write the resulting quadratic equation in standard form: $x^2 - x + 2 = 0$. Next, calculate the discriminant of the quadratic equation, $b^2 - 4ac$: $(-1)^2 - 4(1)(2) = -7$. Because the discriminant is negative, there are no real solutions.